# Further Effects of Varying $G$

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**Abstract**

The correct perihelion precession was recently deduced within the frame work of a time varying Gravitational constant G. Here, we deduce also the observed gravitational bending of light and flattening of galactic rotational curves.

## 1 Introduction

In a recent communication[1] we saw that it is possible to account for the precession of the perihelion of Mercury, for example, only in terms of the time varying universal constant of gravitation G. It may be mentioned that Dirac had argued[2] that a time varying G could be reconciled with General Relativity and the perihelion precession by considering a suitable redefinition of units. We will now show that it is also possible to account for the bending of light on the one hand and on the other, the flat galactic rotation curves without invoking dark matter, with the same time variation of G.

## 2 Bending of Light

It may also be mentioned that some varying G cosmologies have been reviewed by Narlikar and Barrow, while a fluctuational cosmology with the

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above G variation has been considered by the author [3, 4, 5, 6] and [7].
We start by observing that, as is well known, the bending of light can be deduced in Newtonian theory also, though the amount of bending is half of that predicted by General Relativity[8, 9, 10, 11]. In this case the equations for the orbit of a particle of mass $m$ are used in the limit $m \to 0$ with due justification. A quick way of obtaining the result is to observe that we have the well known orbital equations[1, 12].

$$\frac{1}{r} = \frac{GM}{L^2}(1 + ecos\Theta) \tag{1}$$

where $M$ is the mass of the central object, $L$ is the angular momentum per unit mass, which in our case is $bc$, $b$ being the impact parameter or minimum approach distance of light to the object, and $e$ the eccentricity of the trajectory is given by

$$e^2 = 1 + \frac{c^2 L^2}{G^2 M^2} \tag{2}$$

For the bending of light, if we substitute in (1), $r = \pm\infty$, and then use (2) we get

$$\alpha = \frac{2GM}{bc^2} \tag{3}$$

$\alpha$ being the deflection or bending of the light. This is half the General Relativistic value.
We also note that the effect of time variation is given by (cf.ref.[1])

$$G = G_0(1 - \frac{t}{t_0}), r = r_0(1 - \frac{t}{t_0}) \tag{4}$$

where $t_0$ is the present age of the universe and $t$ is the time elapsed from the present epoch.
Using (4) the well known equation for the trajectory is given by (Cf.[13],[12],[14])

$$u" + u = \frac{GM}{L^2} + u\frac{t}{t_0} + 0\left(\frac{t}{t_0}\right)^2 \tag{5}$$

where $u = \frac{1}{r}$ and primes denote differenciation with respect to $\Theta$.
The first term on the right hand side represents the Newtonian contribution

while the remaining terms are the contributions due to (4). The solution of (5) is given by

$$u = \frac{GM}{L^2}\left[1 + ecos\left\{\left(1 - \frac{t}{2t_0}\right)\Theta + \omega\right\}\right] \tag{6}$$

where $\omega$ is a constant of integration. Corresponding to $-\infty < r < \infty$ in the Newtonian case we have in the present case, $-t_0 < t < t_0$, where $t_0$ is large and infinite for practical purposes. Accordingly the analogue of the reception of light for the observer, viz., $r = +\infty$ in the Newtonian case is obtained by taking $t = t_0$ in (6) which gives

$$u = \frac{GM}{L^2} + ecos\left(\frac{\Theta}{2} + \omega\right) \tag{7}$$

Comparison of (7) with the Newtonian solution obtained by neglecting terms $\sim t/t_0$ in equations (4),(5) and (6) shows that the Newtonian $\Theta$ is replaced by $\frac{\Theta}{2}$, whence the deflection obtained by equating the left side of (6) or (7) to zero, is

$$cos\Theta\left(1 - \frac{t}{2t_0}\right) = -\frac{1}{e} \tag{8}$$

where $e$ is given by (2). The value of the deflection from (8) is twice the Newtonian deflection given by (3). That is the deflection $\alpha$ is now given not by (3) but by

$$\alpha = \frac{4GM}{bc^2},$$

which is the correct General Relativistic Formula.

# 3 Galactic Rotation

The problem of galactic rotational curves is well known (cf.ref.[8]). We would expect, on the basis of straightforward dynamics that the rotational velocities at the edges of galaxies would fall off according to

$$v^2 \approx \frac{GM}{r} \tag{9}$$

whereas it is found that the velocities tend to a constant value,

$$v \sim 300km/sec \tag{10}$$

This has lead to the hypothesis of as yet undetected dark matter, that is that the galaxies are more massive than their visible material content indicates. We observe that from (4) it can be easily deduced that

$$a \equiv (\ddot{r}_o - \ddot{r}) \approx \frac{1}{t_o}(t\ddot{r}_o + 2\dot{r}_o) \approx -2\frac{r_o}{t_o^2} \tag{11}$$

as we are considering infinitesimal intervals $t$ and nearly circular orbits. Equation (11) shows (Cf.ref[1] also) that there is an anomalous inward acceleration, as if there is an extra attractive force, or an additional central mass. So,

$$\frac{GMm}{r^2} + \frac{2mr}{t_o^2} \approx \frac{mv^2}{r} \tag{12}$$

From (12) it follows that

$$v \approx \left(\frac{2r^2}{t_o^2} + \frac{GM}{r}\right)^{1/2} \tag{13}$$

From (13) it is easily seen that at distances within the edge of a typical galaxy, that is $r < 10^{23} cms$ the equation (9) holds but as we reach the edge and beyond, that is for $r \geq 10^{24} cms$ we have $v \sim 10^7 cms$ per second, in agreement with (10).

Thus the time variation of G given in equation (4) explains observation without taking recourse to dark matter.

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