SCALE DEPENDENT DIMENSIONALITY

B.G. Sidharth*
B.M. Birla Science Centre, Hyderabad 500 063 (India)

Abstract

We argue that dimensionality is not absolute, but that it depends on the scale of resolution, from the Planck to the macro scale.

1 Introduction

Is dimensionality dependent on the scale of resolution, or is it independent of this scale. This question becomes relevant in the light of some recent work (for example Cf.[1]). It has ofcourse been pointed out that the spin half character of a collection of Fermions leads to the usual three dimensionality of our space[2, 3], while the spin half itself is associated with the Compton wavelength as discussed in recent papers (Cf. for example[4]). Further, it was argued that as we approach the Compton scale, we encounter lower dimensionality[5, 6]. In this paper we point out that indeed the dimensionality is scale dependent.

2 Scale Dependence

We first notice that at Planck scale l_P, we have

$$N^{3/4} l_P \sim R \tag{1}$$

where $N \sim 10^{80}$ is the number of elementary particles and $R \sim 10^{28} cm$ the radius of the universe. This is not an empirical relation but rather can be

[0]*E-mail:birlasc@hd1.vsnl.net.in

deduced on the basis of a fluctuational creation of particles scheme recently discussed (Cf. for example[7, 8]). In this scheme, $\sqrt{N}$ particles are fluctuationally created and this happens in the Compton time τ of a typical elemental particle, a pion.
Further this corresponds to a fluctuational creation of $N^{1/4}$ Planck particles as recently argued[9], in the Planck time τ_P. Indeed, we have

$$\dot{N} \sim N^{1/4}/\tau_P \sim \sqrt{N}/\tau \tag{2}$$

Equation (2) leads to (1).
(1) shows that at the Planck length, the fluctuational dimensionality is 4/3. Interestingly this is the dimension of a Koch curve and a coastline [10]. With this dimensionality we should have

$$M \propto R^{4/3},$$

which indeed is true[11].
At the Compton scale of resolution, we have[7], as indeed can be deduced from (2) the well known Eddington formula,

$$R \sim \sqrt{N} l \tag{3}$$

(3) shows the two dimensional character at the Compton length. Indeed as noted in the introduction three dimensionality is at scales much greater than the Compton wavelength - as we approach the Compton wavelength we encounter two dimensionality as can be seen from (3) - indeed this was the key to explain puzzling characteristics of quarks including their fractional charge and handedness[6].
Finally at scales $L \sim 10cm$, we have

$$N^{1/3} L \sim R \tag{4}$$

(4) shows up the usual three dimensionality.
Interestingly, if we take the typical elementary particle the pion, and consider it successively as a 4/3 dimensional object at the Planck scale, a two dimensional object at the Compton scale and three dimensional at our macro scale, and consider successive densities

$$\rho_P \sim m/(l_P)^{4/3}, \rho_\pi \sim m/l^2 \quad \text{and} \quad \rho \sim m/L^3,$$

we have,

$$M \sim \rho_P R^{4/3} \sim \rho_\pi R^2 \sim \rho R^3,$$

as required.

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