LARGE EXTRA DIMENSIONS AT LINEAR COLLIDERS

K. SRIDHAR *

Department of Theoretical Physics,
Tata Institute of Fundamental Research,
Homi Bhabha Road, Mumbai 400 005, INDIA

In this talk, I first present the motivation for theories wherein the extra spacetime dimensions can be compactified to have large magnitudes. In particular, I discuss the Arkani-Hamed, Dimopoulos, Dvali (ADD) scenario. I present the constraints that have been derived on these models from current experiments and the expectations from future colliders. I concentrate particularly on the possibilities of probing these extra dimensions at future linear colliders.

1. Introduction to the Kaluza-Klein theory

Very soon after the formulation of General Relativity, and its success in giving gravity a geometrical meaning, attempts at unifying electromagnetism with gravity were made. These theories, known as Kaluza-Klein theories, attempted to obtain gravity and electromagnetism from the geometry of an underlying higher-dimensional theory. Non-observation of these extra dimensions implies that these have to be compactified to sizes which are unobservably small. As a simple example, consider a scalar field in 5 dimensions, $\phi(x, y)$ where x is the 4-dimensional space-time co-ordinate and y is the 5th dimension. Assume that the fifth dimension is compactified to a circle with radius R, where R is independent of x. The 5-dimensional field can be expanded in a Fourier series as

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} \phi_n(x) \exp(iny/R), \tag{1}$$

where n is an integer and $\phi_n(x)$ are 4-dimensional fields. Substituting the above expansion in the 5-dimensional Klein-Gordon equation which $\phi(x, y)$ satisfies, one can show that one ends up with an infinite number of equations in 4 dimensions, one for each $\phi_n(x)$ and with a mass $|n|/R$ for every mode n. These modes are called *pyrgons*. The definition of spin in D dimensions depends on the D-dimensional Lorentz symmetry. The light-cone symmetry that leaves the motion of a massless particle unchanged in D dimensions is $SO(D-2)$ and the D-dimensional helicity corresponds to the representations of $SO(D-2)$. A given Kaluza-Klein level in $D = 4 + m$ dimensions has one spin-2 state, $(m-1)$ spin-1 states and $m(m-1)/2$ spin-0 states. A higher dimensional field, therefore, unifies different fields of different masses and spins in 4 dimensions.

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*sridhar@theory.tifr.res.in

The problem, however, is these extra dimensions are not observed. In fact, if the periodicity in the fifth dimension is related to the quantisation of the electric charge then the length of the extra dimension turns out to be of the order of 10^{-30} cm which is only somewhat bigger than the Planck length, making the hypothesis of extra dimensions untestable in any experiment. This is an unattractive feature of these theories, nevertheless the idea that all interactions are the consequence of space-time symmetries is so attractive that there have been vigorous attempts to generalise the attempt of Kaluza and Klein to include other interactions using more complicated compactification schemes.

2. Large Extra Dimensions

Recently, new incarantaions of Kaluza-Klein theories have been discussed in the literature which can be a way of getting around the so-called hierarchy problem. What is the hierarchy problem? The Standard Model (SM) has proved enormously successful in providing a description of particle physics upto energy scales probed by current experiments, which is in the region of several hundred GeV. In the SM, however, one assumes that effects of gravity can be neglected, because the scale where the effects of gravity become large i.e. the Planck scale ($M_P = 1.2 \times 10^{19}$ GeV) is vastly different from the TeV scale. The separation between the TeV scale and the Planck scale is what manifests itself as the hierarchy problem, whose solution has become one of the foci of the search for the correct physics beyond the SM. This problem is exacerbated in traditional unification scenarios: the scale of grand-unification is of the order of 10^{16} GeV and again implies a huge desert. Further, in spite of the unification scale being so close to the Planck scale, traditional unification models make no reference whatsoever to gravity.

Recent advances in the understanding of the strong-coupling regime of string theories has led to a major paradigm shift. The tool that has made it possible to understand the strong-coupling regime is duality. This duality, which is quite similar to the concept of duality in field theories, relates a theory at weak coupling to another theory at strong coupling. In field theories, this relationship also entails an electric/magnetic duality where duality takes a theory of weakly coupled point-like electric charges (and strongly coupled magnetic charges) to one with magnetic charges that are weakly coupled and pointlike. The strongly coupled theory maps on to the weakly coupled theory in which the basic quanta carry magnetic charges. In field theory, therefore, the duality multiplets include the elementary quanta which are pointlike and solitonic modes which are extended configurations. The situtation in string theory is more complicated where, in addition to the elementary strings, the spectrum of particles includes solitonic objects which are called D-branes. These are best thought of as topological defects of varying dimensionality: a D-brane is a dynamical $D + 1$-dimensional surface. The weak coupling string theory is not sensitive to these modes because they are very heavy compared to the stringy modes but, on the other hand, in the dual theory they become lighter and so in the dual

theory it is best to think of the D-branes as the elementary quanta. An interesting feature of the D-branes is that they act as surfaces on which open strings end.

Now let us consider a theory with 3-branes or $3 + 1$-dimensional hypersurfaces which are embedded in a D-dimensional spacetime. This theory would have typically open and closed strings along with the 3-branes. The gauge particles, which correspond to the open strings, will end on the 3-branes while the gravitons, which correspond to the closed strings, are not restricted to lie on the 3-brane. This implies that the gauge particles (i.e. the SM particles) are confined to the 3-brane or the 3+1 dimensional surface and only the gravitons are free to propagate in the full D dimensions. As usual, the extra $D - 4$ dimensions have to be compactified to obtain the $3 + 1$ dimensional theory [1]. But, since these extra dimensions are only 'seen' by gravity, these need not be compactified to length scales which are of the order of M_P^{-1} but it is can be arranged that n of these extra dimensions are compactified to a common scale R which is relatively large, while the remaining dimensions are compactified to much smaller length scales which are of the order of the inverse Planck scale. In this context, the idea of large extra dimensions was first discussed by Arkani-Hamed, Dimopoulos and Dvali [2] and is referred to as the ADD scenario, though earlier attempts at making the extra dimensions large have been made [3]. The relation between the scales in $4 + n$ dimensions and in 4 dimensions is given by [2]

$$M_{\rm P}^2 = M_S^{n+2} R^n \ , \tag{2}$$

where M_S is the low-energy effective string scale. This equation has the interesting consequence that we can choose M_S to be of the order of a TeV and thus get around the hierarchy problem *. For such a value of M_S, it follows that $R = 10^{32/n-19}$ m, and so we find that M_S can be arranged to be a TeV for any value $n > 1$. Effects of non-Newtonian gravity can become apparent at these surprisingly low values of energy. For example, for $n = 2$ the compactified dimensions are of the order of 1 mm, just below the experimentally tested region for the validity of Newton's law of gravitation and within the possible reach of ongoing experiments [5].

3. The View from the Braneless End: The Low-Energy Effective Theory

Below the scale M_S the following effective picture emerges [6], [7], [8]: there are the Kaluza-Klein states, in addition to the usual SM particles. The graviton corresponds to a tower of Kaluza-Klein states which contain spin-2, spin-1 and spin-0 excitations. The spin-1 modes do not couple to the energy-momentum tensor and their couplings to the SM particles in the low-energy effective theory are not important. The scalar modes couple to the trace of the energy-momentum tensor, so they do not couple to massless particles. Other particles related to brane dynamics (for example, the Y modes which are related to the deformation of the brane) have effects which are subleading, compared to those of the graviton. The only

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*A more recent scenario due to Randall and Sundrum [4] is, in fact, a better way of handling the hierarchy problem. The phenomenlogy of this scenario, however, is not very different than the ADD scenario discussed in this paper.

states, then, that contribute are the spin-2 Kaluza-Klein states. These correspond to a massless graviton in the $4 + n$ dimensional theory, but manifest as an infinite tower of massive gravitons in the low-energy effective theory. For graviton momenta smaller than the scale M_S, the effective description reduces to one where the gravitons in the bulk propagate in the flat background and couple to the SM fields which live on the brane via a (four-dimensional) induced metric $g_{\mu\nu}$. Starting from a linearized gravity Lagrangian in n dimensions, the four-dimensional interactions can be derived after a Kaluza-Klein reduction has been performed. The interaction of the SM particles with the graviton, $G_{\mu\nu}$, can be derived from the following Lagrangian:

$$\mathcal{L} = -\frac{1}{\bar{M}_P} G_{\mu\nu}^{(j)} T^{\mu\nu} \ , \tag{3}$$

where j labels the Kaluza-Klein mode and $\bar{M}_P = M_P/\sqrt{8\pi}$, and $T^{\mu\nu}$ is the energy-momentum tensor.

In view of the fact that the effective Lagrangian given in Eq. 3 is suppressed by $1/\bar{M}_P$, it may seem that the effects at colliders will be hopelessly suppressed. However, in the case of real graviton production, the phase space for the Kaluza-Klein modes cancels the dependence on $\bar{M}_P$ and, instead, provides a suppression of the order of M_S. For the case of virtual production, we have to sum over the whole tower of Kaluza-Klein states and this sum when properly evaluated [8], [7] provides the correct order of suppression ($\sim M_S$). The summation of time-like propagators and space-like propagators yield exactly the same form for the leading terms in the expansion of the sum [8] and this shows that the low-energy effective theories for the s and t-channels are equivalent.

4. The Experimental Constraints

There have been several studies exploring the consequences of the above effective Lagrangian for particle phenomenology and astrophysics. Production of gravitons giving rise to characteristic missing energy or missing p_T signatures at e^+e^- or hadron colliders have been studied resulting in bounds on M_S which are around 500 GeV to 1.2 TeV at LEP2 [9], [10] and around 600 GeV to 750 GeV at Tevatron [9]. Production of gravitons at the Large Hadron Collider (LHC) and in high-energy e^+e^- collisions at the Next Linear Collider (NLC) have also been considered. Virtual effects of graviton exchange in dilepton production at Tevatron yields a bound of around 950 GeV [11] to 1100 GeV [12] on M_S, in $t\bar{t}$ production at Tevatron a bound of about 650 GeV is obtained while at the LHC this process can be used to explore a range of M_S values upto 4 TeV [13]. Virtual effects in deep-inelastic scattering at HERA put a bound of 550 GeV on M_S[14], while from jet production at the Tevatron strong bounds of about 1.2 TeV are obtained [15]. Pair production of gauge bosons and fermions in e^+e^- collisions at LEP2 [16], [17], [18] can probe values of M_S upto 0.6 TeV. Other processes studied include associated production of gravitons with gauge bosons and virtual effects in gauge boson pair production at hadron colliders [19], [20]. Higgs production [21], [22] and electroweak precision observables [23] in the light of this

new physics have also been discussed. Astrophysical constraints, like bounds from energy loss for supernovae cores, have also been discussed [24]. In general, the processes which involve real production of gravitons give stronger constraints for $n = 2$ than the processes involving virtual exchange of gravitons but the advantage of the virtual processes is that the bounds obtained from them have a mild n dependence whereas the bounds from real production processes fall rapidly with increasing n.

5. The Linear Collider and Extra Dimensions

The Next Linear Collider (NLC) is an ideal testing ground of the SM and a very effective probe of possible physics that may lie beyond the SM. The collider is planned to be operated in the e^+e^-, e^-e^-, $\gamma\gamma$ and the $e\gamma$ modes. For operation in the latter two modes, the photons are produced in the Compton back-scattering of a highly monochromatic low-energy laser beam off a high energy electron beam [25]. Control over the e^- and laser beam parameters allow for control over the parameters of the $\gamma\gamma$ and $e\gamma$ collisions. The physics potential of the NLC is manifold and the collider is expected to span several steps of ee energy between 500 GeV and 1.5 TeV. The experiments at the NLC also provide a great degree of precision because of the relatively clean initial state, and indeed the degree of precision can be enhanced by using polarised initial beams.

We first discuss the case of e^+e^- collisions. The production of gravitions in e^+e^- collisions at the NLC for a $\sqrt{s}$ of 1 TeV and 100 fb^{-1} luminosity has been studied [9]. Bounds on M_S which are around 3-7 TeV are obtained (for n between 2 and 6). Virtual effects of graviton exchange in fermion pair production at the NLC [11] can also give strong bounds of upto 5 TeV for a $\sqrt{s} = 1$ TeV. These bounds can be enhanced by studying the angular distributions instead of looking at the integrated cross-sections. Virtual effects of graviton exchange in gauge boson pair production at the NLC have also been studied [26], [16] and lead to similarly strong bounds.

In the e^-e^- mode, Möller scattering $e^-e^- \to e^-e^-$ may be used to study the virtual exchange of gravitons and this process is similar to the process $e^+e^- \to e^+e^-$ as far as the gaviton exchange contribution is concerned. The e^-e^- mode, in fact, is advantageous in that it provides a initial state even cleaner than in e^+e^-. The e^-e^- initial state can also be polarised to a greater degree. However, the major advantage that the the e^+e^- mode has over the e^-e^- mode is that there are several $f\bar{f}$ states that are accessible and by summing over all these states the bound can be significantly improved.

If operated in a mode where there laser-back scattering used, both the e^+e^- and the $e - e^-$ colliders can be used to study $\gamma\gamma$ and $e\gamma$ scattering processes. As an example consider the effects of large extra dimensions in top production in photon-photon collisions at the NLC, spanning the energy range between 500 GeV and 1.5 TeV [27].

The basic scattering is described by a $\gamma\gamma$ scattering subprocess, with each γ resulting from the electron-laser back scattering. The energy of the back-scattered photon, E_γ, follows a distribution characteristic of the Compton scattering process

and can be written in terms of the dimensionless ratio $x = E_\gamma/E_e$. The subprocess cross-section is convoluted with the luminosity functions, $f_\gamma^i(x)$, which provide information on the photon flux produced in Compton scattering of the electron and laser beams. The cross-section for the $\gamma\gamma \to t\bar{t}$ process has the usual t- and u-channel SM contributions, but in addition, we also have the s-channel exchange of virtual spin-2 Kaluza-Klein particles. The 2σ limits that we obtain for $\sqrt{s} = 500,\ 1000,\ 1500$ GeV are 1600, 4000 and 5400 GeV, respectively. Tighter cuts on the rapidity can be used to improve the bounds significantly. The use of polarisation enhances the bounds on M_S quite significantly by several 100 GeV in each case. The other processes that have been studied in the case of $\gamma\gamma$ collisions are gauge-boson pair production [28], [16] and dijet production [29].

Finally, the effects of extra dimensions in $e\gamma$ collisions has also been studied. The virtual exchange of gravitons in the $e\gamma \to e\gamma$ Compton scattering yields bounds in the range of 5 TeV [30] and the real production of gravitons *via* $e\gamma \to eG$ also yields bounds in the region of around 5 TeV [31].

6. Conclusions

The possibility that extra dimensions could be compactified to sizes as large as a millimeter, and consequently have effects of quantum gravity in the TeV range, has led to several exciting investigations of these effects at high energy colliders. In this talk, I have essentially summarised the motivation for expecting these extra dimensions to be large, discussed the low-energy effective theory and then reviewed the bounds on the string scale M_S that have been obtained from present experiments and the values of M_S that will be probed in future colliders. In particular, I have discussed these studies made for a future linear collider and conclude that these theories can be probed to very large values of effective string scale M_S.

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