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Domain Walls and Solitons in Odd Dimensions

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Abstract

We discuss the existence of smooth soliton solutions which interpolate between supersymmetric vacua in odd-dimensional theories. In particular we apply this analysis to a wide class of supergravities to argue against the existence of smooth domain walls interpolating between supersymmetric vacua. We find that if the superpotential changes sign then any Goldstino modes will diverge.

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1 Introduction

Soliton solutions are a central theme in the study of supersymmetric theories. In particular domain wall solitons in five-dimensions have received substantial attention because of their potential to admit chiral fermion zero modes. In this paper we wish to describe some observations about chiral fermion zero modes of domain walls viewed as stable finite energy solutions which interpolate between two vacua of the theory. Indeed we will argue, at least in a wide class of supersymmetric theories in odd dimensions, that no smooth domain walls exist.

The analysis given below was initiated by the question as to whether or not a Randall-Sundrum scenario [1] can be extended to a smooth domain wall in a supergravity theory. This question has several motivations. It was pointed out in [2] that such an embedding would solve the fine-tuning problem associated with matching the domain wall tension and bulk cosmological constant needed in [1]. Indeed without supersymmetry one is led to question the general stability of a domain wall [3]. In addition, with a smooth domain wall solution one can improve upon the thin wall approximation in [1] and provide a complete non-linear analysis of the Randall-Sundrum scenario [3]. Finally there is widespread belief that supersymmetry and supergravity are relevant phenomenologically and in this context it is natural to embed our universe in a higher dimensional theory containing supergravity. Certainly from a theoretical point of view one would like to place such a "brane-world" in the context of supergravity and ultimately string theory. The difficulty in obtaining a smooth Randall-Sundrum domain wall in five-dimensional supergravity has been discussed recently [4, 5, 6] and a no-go theorem can be proven in various cases [5, 6, 7, 8][1]. In this paper we argue against the existence of smooth domain walls interpolating between supersymmetric vacua on rather general grounds in a wide class of odd-dimensional supergravities (although not all, e.g. see [10]).

Supersymmetric domain wall spacetimes have also received interest recently due to their role in the AdS/CFT correspondence. In particular the domain wall central charge has been identified with the c-function of a four-dimensional field theory [10, 11]. From

[1]Recently the original but discontinuous Randall-Sundrum domain wall has been embedded into a supergravity [9].

this perspective the absence of smooth Randall-Sundrum domain walls in a particular supergravity is interpreted as the statement that the (monotonic) c-function [11, 10]

$$C(r) = \frac{C_0}{|W(r)|^{D-2}} \ , \tag{1}$$

is bounded along the renormalisation group flow (i.e. that $W(r)$ does not pass through zero).

The obstruction to finding supergravity domain walls in five dimensions seems to be obtaining solutions where the real superpotential W changes sign [4, 5, 6, 7]. In supergravity theories W appears in the mass terms for the fermions. Domain walls that interpolate between regions in which W changes sign connect regions with positive fermion mass to those with negative fermion mass. In even dimensions the sign of a fermion mass term has no physical significance; it may be reversed by multiplying the fermion field by Γ^{D+1}. By contrast, in odd dimensions, $\Gamma^{D+1} = \pm 1$, and a fermion mass term breaks parity. Furthermore in odd dimensions there are two inequivalent irreducible representations of the Clifford algebra labelled by the sign of Γ^{D+1}. Theories with different signs for the Γ-matrices are rather like different superselection sectors and one would not expect that these two sectors could be realised in single connected spacetime. However a change in sign of all fermion masses may be effected by a change in sign of the Γ-matrices. Thus a domain wall in an odd-dimensional theory in which W changes sign looks as if it connects two distinct superselection sectors and one might doubt that this is physically sensible. Perhaps this is the reason why domain walls with W changing sign have not been found in supergravity. A similar reservation was raised in [5].

This also raises the question of whether domain walls coupled to fermions in which mass terms change sign can exist in flat space theories, supersymmetric or not. As is well-known such domain walls admit localised zero energy fermion modes which are chiral. As long as the worldvolume theory is not anomalous, this would seem to lead to no contradiction. Of course if it were anomalous, there would have to be some inflow to balance the anomaly and such domain walls might be incompatible with being supersymmetric, i.e. BPS.

Thus it seems that the key to understanding the absence of supersymmetric domain wall solutions lies in understanding the Goldstino fermions. Therefore the rest of this paper is organised as follows. In section two we shall discuss non-gravitational domain walls. We find that even if four-dimensional supersymmetric domain walls exist their Goldstinos are non-chiral and hence non-anomalous. Finally in section three we discuss domain walls in supergravity. There we find that the Goldstino fermion modes would diverge if W changed sign.

2 Non-Gravitational Domain Walls

2.1 Fermion Zero-Modes

Let us consider a general theory which includes a fermionic field ψ. Around any vacuum of this theory we may consider the fluctuations of the fermion which we assume satisfy the Dirac equation (we use a "mostly" plus metric in D spacetime dimensions, $m, n = 0, 1, 2, ..., D-1$)

$$\Gamma^m \nabla_m \psi + M\psi = 0 \ . \tag{2}$$

As is well known this equation admits both positive and negative energy solutions $\psi^{(\pm)}$. In particular, particles at rest have one-particle wave functions given by "plane-wave" solutions $\psi^{(\pm)} = e^{\mp i|M|t}\eta_\pm$ where $\eta_\pm$ is a constant spinor and $i\Gamma^0\eta_\pm = \pm sign(M)\eta_\pm$. The resolution of this "energy crisis" in the quantum theory is to simply assert that in a given vacuum all the negative energy states $\psi^{(-)}$ are filled.

Now imagine that there is a domain solution associated with a scalar field $\phi(r)$ which interpolates between two vacua, where r is the coordinate transverse to the domain wall. As a consequence the fermion mass $M(r)$ becomes dependent upon r. We must now look for solutions of the form

$$\psi = e^{ip_\mu x^\mu}\chi(r) \ , \tag{3}$$

where $\mu = 0, 1, 2, ..., D-2$. There are two cases to consider. In the first case $\Gamma^\mu p_\mu \chi = 0$ so that $p_\mu p^\mu = 0$. The solution then takes the form

$$\chi(r) = \chi_\pm e^{ip_\mu x^\mu} exp\left(\mp \int_0^r M(r')dr'\right) \ , \tag{4}$$

where $\Gamma^r\chi_\pm = \pm\chi_\pm$. If the spacetime dimension is *odd*, $\Gamma^0\Gamma^1 \ldots \Gamma^r = \pm 1$ and hence Γ^r determines the chirality of the fermions with respect to the wall. If $M(r)$ changes sign then one chiral mode in (4) will be normalisable and the other non-normalisable. This implies that a massless chiral fermion is bound to the domain wall. If $M(r)$ does not change sign then neither mode is normalisable. We will not be interested in this situation in this paper.

In the case that $\Gamma^\mu p_\mu \chi \neq 0$ we may, without loss of generality, take $p_\mu = (-E, 0, \dots, 0)$. Using the Γ-matrix algebra we have $i\Gamma^0 \chi_\pm = \chi_\mp$ leading to the following system of equations

$$\begin{aligned} (\partial_r + M)\chi_+ &= E\chi_- \ , \\ (\partial_r - M)\chi_- &= -E\chi_+ \ . \end{aligned} \tag{5}$$

Elimination leads to a second order differential equation in which $\chi_\pm$ decouple

$$\left(-\partial_r^2 + V_\pm(r)\right)\chi_\pm = E^2\chi_\pm \ , \quad V_\pm(r) = M^2(r) \mp \partial_r M(r) \ . \tag{6}$$

For an explicit example we let $M(r) = m\tanh(mr)$. We can solve exactly these equations since $V_+ = m^2$ and hence

$$\chi_+ = Ae^{i\sqrt{E^2-m^2}\ r} + Be^{-i\sqrt{E^2-m^2}\ r} \ , \tag{7}$$

where A and B are arbitrary constants. These represent plane wave solutions for $E^2 \geq m^2$ but are non-normalisable if $E^2 < m^2$. We may then obtain

$$\chi_- = E^{-1}(\partial_r + m\tanh(mr))\chi_+ \ . \tag{8}$$

This completes the spectrum of fermion modes in the domain wall background. In summary, if $M(r)$ changes sign then there is a single chiral fermion with zero energy localised on the wall. In addition there are modes with non-vanishing energy. Note that we may perform a boost to transform these modes into their rest frame where $E = m$ so that the fermion wave function is

$$\psi = e^{-imt}(\chi_+^0 + \tanh(mr)\chi_-^0) \ , \tag{9}$$

which smoothly interpolates between the constant eigenstates $\eta_\pm^0 = \chi_+^0 \pm \chi_-^0$ of $i\Gamma^0$ that appear in the Dirac sea in each vacuum.

This example shows that one can have domain walls in odd dimensions which interpolate between two vacua with opposite signs for the fermion mass. Thus if the domain wall has a $\mathbf{Z}_2$ symmetry then globally parity is a good symmetry, even though it is broken in each vacua. We also see that fermions with positive energy can travel freely between the two vacua. In particular there is no mixing of positive and negative frequencies which would indicate a quantum instability.

2.2 Chiral Goldstinos

Let us now consider supersymmetric domain walls in five dimensions where the fermion zero modes arise as Goldstinos. First we note that the mass term in (2) comes from a term of the form $M(\phi)\bar{\psi}\psi$ in the Lagrangian. For a supersymmetric theory in five dimensions, the scalar ϕ belongs in a vector multiplet, tensor multiplet or a hyper multiplet. For a vector multiplet a coupling of the form $\phi\bar{\psi}\psi$ might seem natural, since this comes from the dimensional reduction of a covariant derivative term for ψ in six dimensions. However in this case there can be no potential for ϕ by gauge invariance and hence no domain wall. In fact in general, if a five-dimensional multiplet with eight supercharges comes via compactification from six dimensions, then it must come from a chiral multiplet. Therefore there are no fermion mass terms possible (unless we consider theories with sixteen supercharges in which case there are no scalar potentials possible).

In fact regardless of which kind of multiplet ϕ belongs in five dimensions, when solving the domain wall equations we effectively compactify the system to the two dimensions t and r. The action will then have $(4,4)$ supersymmetry and this constrains the potential to take the form $V = g_{ij}k^i k^j$ where $g_{ij}(\phi)$ is a hyper-Kähler metric appearing in the scalar kinetic term and $k^i(\phi)$ is a tri-holomorphic Killing vector. The Yukawa term is $\nabla_i k_j(\phi)\bar{\psi}^i\Gamma^{0r}\psi^j$ [12]. Let us again write $\Gamma^r\psi^i_\pm = \pm\psi^i_\pm$, $i\Gamma^0\psi^i_\pm = \psi^i_\mp$ and $\psi^i = e^{-iEt}\chi^i$. In this case the equations of motion for the fermions become

$$\begin{aligned} \nabla_r\chi^i_+ - M^i{}_j\chi^j_- &= E\chi^i_- \ , \\ \nabla_r\chi^i_- - M^i{}_j\chi^j_+ &= -E\chi^i_+ \ , \end{aligned} \qquad (10)$$

where $M^i{}_j = \nabla^i k_j$, $\nabla_r \psi^i_\pm = \partial_r \psi^i_\pm + \Gamma^i_{jk}\partial_r \phi^j \psi^k_\pm$ and we have ignored a term cubic in the fermions involving the curvature of the metric g_{ij}.

The system (10) is quite different to (5) because the left hand side contains both χ^i_- and χ^i_+. In particular if $E = 0$, χ^i_+ and χ^i_- do not decouple. Moreover if $E = 0$, χ^i_+ and χ^i_- satisfy the same equation and hence the fermion zero modes come in pairs containing both chiralities. Note that this result no longer holds in lower dimensions since the resulting two-dimensional transverse theories need only have $(1,1)$ or $(2,2)$ supersymmetry and equations of motion of the form (5) are possible [12]. We also observe that this form of the Yukawa term does not break parity.

3 Domain Walls in Supergravity

We now wish to discuss domain wall solitons in supergravity. First let us review some basic features of supergravity domain walls. We assume that the bosonic action takes the form

$$S = \int d^D x \sqrt{-g} \left(R - \gamma_{AB}(\phi)\partial_m \phi^A \partial^m \phi^B - V(\phi) \right) \ , \tag{11}$$

where ϕ^A, $A = 1, 2, 3, ..., N$ are scalar modes and we assume that metric γ_{AB} appearing in their kinetic term is positive definite. We further assume that (11) is the consistent truncation of a supergravity theory which is invariant under supersymmetry transformations of the form

$$\begin{aligned} \delta\psi_m &= (\nabla_m \epsilon + W\Gamma_m \epsilon + \partial_m W_2 \epsilon) \ , \\ \delta\lambda_A &= (-\frac{1}{2}\gamma_{AB}\Gamma^m \partial_m \phi^B + W_{3A})\epsilon \ . \end{aligned} \tag{12}$$

Here W, W_2 and W_{3A} are functions of the scalars ϕ^A which we will avoid specifying in order to keep our argument as general as possible. In fact we can remove the term in (12) involving W_2 by performing the field redefinitions $\epsilon \to e^{-W_2}\epsilon$, $\psi_m \to e^{-W_2}\psi_m$ and $\lambda_A \to e^{-W_2}\lambda_A$. Therefore, without loss of generality, we set $W_2 = 0$. We have also assumed that any internal indices on the spinors ϵ may be ignored. This form for the supersymmetry transformation is quite general for $N = 2$ supergravity in five dimensions but does not include all extended supergravities (e.g. see [10]). We will also ignore any higher order fermion terms since it is clear that their inclusion would not affect our discussion.

Let us now look for a supersymmetric domain wall. Without loss of generality we may choose the spacetime to have the metric

$$ds^2 = dr^2 + e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu \ , \tag{13}$$

where $\mu, \nu = 0, 1, 2, ..., D-2$ and the scalars depend only on r. The requirement that some supersymmetry is preserved gives rise to the Bogomoln'yi equations

$$\begin{aligned} A' &= \mp 2W \ , \\ \phi^{A\prime} &= \pm 2\gamma^{AB}W_{3B} \ , \end{aligned} \tag{14}$$

where a prime denotes differentiation with respect to r. The preserved supersymmetries (i.e. Killing Spinors) for these domain walls are

$$\epsilon = e^{\frac{1}{2}A}\epsilon_\pm \ , \tag{15}$$

where $\Gamma_{\underline{r}}\epsilon_\pm = \pm\epsilon_\pm$ and an underlined index refers to the tangent frame.

It is instructive to consider supersymmetric vacua of this theory. Here we set all the scalars to constants $\phi^A = \phi^A_0$. Clearly this can only occur at the "critical" points where $W_{3A}(\phi^A_0) = \partial V/\partial\phi^A = 0$. The spacetime (13) is now just pure AdS space with $A = \mp 2W(\phi^A_0)r$. In this case there are additional Killing Spinors given by

$$\epsilon = \left(e^{-\frac{1}{2}A} - 2W(\phi^A_0)e^{\frac{1}{2}A}x^\nu\Gamma_{\underline{\nu}}\right)\epsilon_\mp \ . \tag{16}$$

There may also be non-supersymmetric vacua where $\partial V/\partial\phi^A = 0$ but $W_{3A} \neq 0$. However we will have little so say about these cases.

In a Randall-Sundrum domain wall $A(r) \sim -|r|$ as $r \to \pm\infty$ [1]. Thus asymptotically $g_{00} = e^{2A}$ falls off exponentially and gravity is is localised to the domain wall. This will be the case for a domain wall of the theory (11) if W changes sign between the two vacua. For example in the original proposal [1] there are no scalars ϕ^A or fermions λ_A

and $V \sim -W^2$ is constant. The domain wall is obtained by simply choosing the sign of W to be positive on one side and negative on the other, i.e. $W(r)$ is discontinuous. Note that from the point of view of the supergravity equations this domain wall is equivalent to keeping W fixed everywhere but choosing one representation for the Γ-matrices on one side and the opposite representation (obtained by $\Gamma^m \to -\Gamma^m$) on the other. Given the comments in the introduction it is natural to be concerned that this is unphysical.

In supergravity theories there is a standard argument for the stability of BPS backgrounds. We will briefly review it here and note that it is insensitive to a change in sign of W. We construct a "Nester" tensor [13]

$$N^{mn} = \bar{\epsilon}\Gamma^{mnp}\delta\psi_p, \tag{17}$$

with $\delta\psi_m$ given in (12). Such a tensor has the property that, on shell,

$$\nabla_m N^{mn} = \delta\bar{\psi}_m \Gamma^{mnp}\delta\psi_p + \gamma^{AB}\delta\bar{\lambda}_A\Gamma^n\delta\lambda_B \ . \tag{18}$$

So in particular $\nabla_m N^{m0}$ is negative definite (provided that we impose the Witten condition $\Gamma^m\delta\psi_m = 0$ [14]) and vanishes if and only if some supersymmetry is preserved. In our case this case the requirement that N^{mn} satisfies (18) implies [15]

$$\begin{aligned} W_{3A} &= (D-2)\frac{\partial W}{\partial \phi^A} \ , \\ V &= 4(D-2)^2\left[\gamma^{AB}\frac{\partial W}{\partial \phi^A}\frac{\partial W}{\partial \phi^B} - \left(\frac{D-1}{D-2}\right)W^2\right] \ . \end{aligned} \tag{19}$$

The Nester tensor can be used to provide a bound on the tension of an arbitrary domain wall in terms of a central charge of the supersymmetry algebra which in turn provides a non-perturbative proof of the stability of the solution. Following [16, 2] we integrate N^{mn} over a spacelike boundary which encloses the domain wall

$$\frac{1}{2}\int d\Sigma_{mn}N^{mn} = \int d\Sigma_{0r}N^{0r} = -\int d\Sigma_0 \nabla_m N^{m0} \geq 0 \ . \tag{20}$$

On the other hand we can directly evaluate the surface integral

$$\int d\Sigma_{0r}N^{0r} = \sigma - |W(r=\infty) - W(r=-\infty)| \ , \tag{21}$$

where σ is the tension of the domain wall and we have assumed that the domain wall interpolates smoothly between two AdS vacua. Combining these two equations we learn that $\sigma \geq |W(r=\infty) - W(r=-\infty)|$ for all domain walls with equality if and only if some supersymmetry is preserved.

Note that this proof does not actually require that the action (11) admit a supersymmetric completion. The proof of stability merely requires that the identities (19) hold and that there are solutions to the supersymmetry Killing spinor equations (12). In particular it places no restriction on the function W and hence any domain wall satisfying (14) will be stable in the purely bosonic theory [17]. On the other hand we will shortly see that some choices of the function $W(\phi)$ can never appear in a consistent supergravity because one could not consistently couple the theory to fermions.

To begin our discussion of the fermions we first obtain their equations of motion by constructing the most general form and then imposing the condition that their variation under supersymmetry vanishes when the scalars are on-shell. After a lengthy calculation we find

$$\begin{aligned} \Gamma^m \nabla_m \lambda_A \;&+\; M_A{}^B \lambda_B - (D-2)\gamma^{CD} \frac{\partial \gamma_{BD}}{\partial \phi^A} \frac{\partial W}{\partial \phi^C} \lambda^B - \frac{1}{2} \frac{\partial \gamma_{BD}}{\partial \phi^A} \Gamma^m \nabla_m \phi^B \lambda^D \\ &+\; \frac{1}{2} \gamma_{AB} \Gamma^m \Gamma^n \nabla_n \phi^B \psi_m - (D-2) \frac{\partial W}{\partial \phi^A} \Gamma^m \psi_m = 0 \ , \end{aligned} \tag{22}$$

$$\begin{aligned} \Gamma^{mnp} \nabla_n \psi_p \;&-\; (D-2) W \Gamma^{mn} \psi_n + (D-2) \frac{\partial W}{\partial \phi^A} \Gamma^m \lambda^A \\ &+\; \frac{1}{2} (g^{mn} - \Gamma^{mn}) \nabla_n \phi^A \lambda_A = 0 \ , \end{aligned} \tag{23}$$

where

$$M_A{}^B = 2(D-2) \frac{\partial W}{\partial \phi^A \partial \phi^C} \gamma^{BC} - (D-2) W \delta_A^B \ . \tag{24}$$

Therefore, in a supersymmetric AdS vacuum, we may set $\psi_m = 0$ and obtain the equation of motion

$$\Gamma^m \nabla_m \lambda_A + M_A{}^B \lambda_B = 0 \ . \tag{25}$$

Consider now a stable domain wall, i.e. one that satisfies (14). In particular since half of the supersymmetries are broken, one expects that a finite tension domain wall has massless Goldstino λ_A modes bound to it. Therefore we should look for a solutions to the fermion equations in a background given by (14) which are invariant under the Poincare symmetry of the domain wall, i.e. with $\partial_\mu = \psi_\mu = 0$. It is important to note that the two fermion equations (22) and (23) do not decouple in this case and we must have $\psi_r \neq 0$. Specifically we find from the $m = r$ component of the ψ_m equation (23) that $\Gamma^r \lambda_A = \mp\lambda_A$. For $m \neq r$ the ψ_m equation implies that

$$\psi_r = \mp\frac{1}{W}\frac{\partial W}{\partial\phi^A}\lambda^A \ . \tag{26}$$

Thus the fermion zero modes are chiral, as expected from the chiral form of the broken supersymmetries. Substituting (26) into the λ_A equation (22) and using (14) yields the equation

$$\pm\partial_r\lambda_A = W\lambda_A + 2(D-2)\left(\frac{\partial^2 W}{\partial\phi^A\partial\phi^B} - \frac{1}{W}\frac{\partial W}{\partial\phi^A}\frac{\partial W}{\partial\phi^B}\right)\lambda^B \ . \tag{27}$$

Thus we obtain the wavefunctions for the chiral Goldstino modes

$$\begin{aligned} \lambda_A &= \frac{1}{W}\frac{\partial W}{\partial\phi^A}e^{-\frac{1}{2}A}\epsilon_\mp \ , \\ \psi_r &= \mp\frac{1}{W^2}\gamma^{AB}\frac{\partial W}{\partial\phi^A}\frac{\partial W}{\partial\phi^B}e^{-\frac{1}{2}A}\epsilon_\mp \ , \end{aligned} \tag{28}$$

where $\epsilon_\mp$ is a constant spinor satisfying $\Gamma^r\epsilon_\mp = \mp\epsilon_\mp$. From (28) it is clear that if $W(r)$ passes through zero (e.g. if W changes sign) the Goldstino modes will diverge on the domain wall (or more precisely where $W = 0$) and will not be normalisable. From supersymmetry we expect that a smooth finite tension domain wall should have Goldstinos. We therefore conclude that W can not change sign (by passing through zero) in a supergravity theory. In particular there are no smooth domain walls of the Randall-Sundrum type.

To be more explicit consider a single scalar and suppose that near the point where W changes sign we may write $W \sim (\phi-\phi_0)^\gamma$ with $\gamma \leq 1$ so that ϕ_0 is not a critical point. We then find that

$$\phi - \phi_0 \sim (r-r_0)^{\frac{1}{2-\gamma}} \ , \quad A \sim (r-r_0)^{\frac{2}{2-\gamma}} \ , \quad \lambda \sim (r-r_0)^{-\frac{1}{2-\gamma}} \ . \tag{29}$$

Thus the Goldstinos diverge where W changes sign. Note that the metric and Killing spinors are bounded near $r = r_0$ but they will have a cusp singularity for $\gamma < 0$ (i.e. if W diverges). In addition the norm $\int dr \sqrt{-g}\bar{\lambda}\lambda$ will be convergent at $r = 0$ only for $\gamma < 0$.

The argument just given depends crucially on the form of the supersymmetry transformation rules (12). In four dimensions, for example, other possibilities arise and our results on the divergence of the Goldstino modes will not necessarily apply. Indeed four-dimensional supersymmetric supergravity domain walls do exist [16].

To illustrate the above points we may consider a case with just one scalar and a superpotential of the form

$$W(\phi) = \alpha \left(\phi - \frac{1}{3}\beta^2 \phi^3 \right) , \tag{30}$$

where α and β are constants and $\gamma_{AB} = \delta_{AB}$. The critical points occur at $\phi_0 = \pm\beta^{-1}$ where $W = \pm 2\alpha/3$ and indeed one can find smooth supersymmetric domain walls [18, 19, 5]. The stability of these domain walls in the bosonic theory follows from the equations (20) and (21). However we see that this superpotential can never be consistently embedded in a supergravity because W changes sign between the two critical points.

4 Conclusion

In this paper we have discussed the existence of domain walls in supersymmetric odd-dimensional theories. In the case of global supersymmetry we argued that no supersymmetric domain walls exist with purely chiral Goldstino modes. In the case of supergravities in odd dimensions we argued that the superpotential W cannot change sign because if it did the Goldstino modes would diverge.

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References

[1] L. Randall and R. Sundrum, *An Alternative to Compactification*, hep-th/9906064.

[2] K. Behrndt and M. Cvetic, *Supersymmetric Domain-Wall World from D=5 Simple Gauged Supergravity* , hep-th/9909058.

[3] A. Chamblin and G.W. Gibbons, *Supergravity on the Brane*, hep-th/9909130.

[4] M. Cvetic, H. Lü and C.N. Pope, *Domain Walls and Massive Gauged Supergravity Potentials*, hep-th/0001002.

[5] R. Kallosh and A. Linde, *Supersymmetry and the Brane World*, hep-th/0001071.

[6] K. Behrndt and M. Cvetic, *Anti-de-Sitter Vacua of Gauged Supergravities with 8 Supercharges*, hep-th/0001159.

[7] R. Kallosh, A. Linde and M. Shmakova, *Supersymmetric Multiple Basin Attractors*, hep-th/9910021.

[8] G.W. Gibbons, unpublished.

[9] R. Altendorfer, J. Bagger and D. Nemeschansky, *Supersymmetric Randall-Sundrum Scenario*, hep-th/0003117.

[10] D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, *Renormalization Group Flows from Holography–Supersymmetry and a c-Theorem*, hep-th/9904017.

[11] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel Local CFT and Exact Results on Perturbations of N=4 Super Yang Mills from AdS Dynamics*, JHEP **9812** (1998) 022, hep-th/9810126.

[12] L. Alvarez-Gaume and D.Z. Freedman, *Potentials for the Supersymmetric Non-Linear Sigma Model*, Comm. Math. Phys. **91** (1983) 87.

[13] J.M. Nester, *A New Gravitational Energy Expression with a Simple Positivity Proof*, Phys. Lett. **A83** (1981) 241.

[14] E. Witten, *A Simple Proof of the Positive Energy Theorem*, Commm. Math. Phys. **80** (1981) 241.

[15] P.K. Townsend, *Positive Energy And The Scalar Potential In Higher Dimensional (Super)Gravity*, Phys. Lett. **B148** (1984) 55.

[16] M. Cvetic, S. Griffies and S-J. Rey, *Static Domain Walls in N=1 Supergravity*, hep-th/9201007.

[17] W. Boucher, *Positive Energy Without Supersymmetry*, Nucl. Phys. **B242** (1984) 282.

[18] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, *Modeling the Fifth Dimension with Scalars and Gravity*, hep-th/9909134.

[19] M. Gremm, *Four-Dimensional Gravity on a Thick Domain Wall*, hep-th/9912060.