

Orientifolds, M–Theory, and the $ABCD$ ’s of the Enhançon

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Abstract

Supergravity solutions related to large N $SU(N)$ pure gauge theories with eight supercharges have recently been shown to give rise to an “enhançon”, a new type of hypersurface made of D–branes. We show that enhançons also arise in similar situations pertaining to $SO(2N+1)$, $USp(2N)$ and $SO(2N)$ gauge theories, using orientifolds. Enhançons therefore appear to come in types A , B , C , and D . The latter three differ globally from type A by having an extra \mathbf{Z}_2 identification, and are distinguished locally by their subleading behaviour in large N . We focus mainly on 2+1 dimensional gauge theory, where a relation to M–theory and the Atiyah–Hitchin and Taub–NUT manifolds enables the construction of the smooth supergravity solution and the study of some of the $1/N$ corrections. The role of the enhançon in eleven dimensional supergravity is also uncovered. There is a close relation to certain multi–monopole moduli space problems.

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1 Opening Remarks

In studying brane configurations related to large N $SU(N)$ pure gauge theories with eight supercharges, the authors of ref.[1] considered the BPS supergravity solutions which ought to result in taking gN large, where g is the string coupling, and N is the number of constituent branes. Such supergravity solutions are afflicted by a naked singularity known as a “repulson”[2] which is unphysical, and incompatible with the physics of the gauge theory.

Upon closer examination (by investigating how such a geometry could have arisen by constructing it out of a large number of its constituent BPS parts) it was argued[1] that the repulson is not present. The supergravity solution may only be taken as physical down to a radius of closest approach. At that locus of points (in the case of a $(p + 1)$ -dimensional gauge theory, it is a $(4 - p)$ -sphere, S^{4-p}), there is an enhanced gauge symmetry in the parent string theory and new physics, consistent with the related $SU(N)$ gauge theory, takes over.

That locus of points —called the “*enhançon*”— is new type of hypersurface essentially made of D-branes. The entire curved geometry is produced by a large number of identical BPS objects. An individual unit, when separated from all the others, is an object which has a simple description in terms of (roughly) a pair of D-branes, one of which is partially wrapped on a $K3$ surface, and the other which is induced by the wrapping (see later). It is therefore a sharply localised and heavy object. Upon approaching the geometry produced by a large number of its counterparts, the unit becomes lighter and less sharply defined, ultimately going to zero mass while spreading out completely at the *enhançon* locus.

Ref. [1] also went on to display a variety of familiar dual situations in string theory (with related gauge theory physics) in which the *enhançon* phenomena described above play a crucial role. For this reason, and also because it is a genuinely new mechanism by which string theory avoids an

important class of spacetime singularity[3], the enhançon deserves to be better understood and characterised.

We show in this paper that enhançons also arise naturally in similar situations pertaining to large N $SO(2N+1)$, $USp(2N)$ and $SO(2N)$ $(p+1)$ -dimensional gauge theories, and construct these new classes using orientifolds. It is therefore clear that enhançons may be broadly classified into types A , B , C , and D . (There is no natural E -type which has a smooth geometrical interpretation, since the rank of those groups cannot be made arbitrarily large in order to make contact with a supergravity discussion.) The latter three types differ globally from type A by having an extra \mathbf{Z}_2 identification, making them into $\mathbb{RP}^{4-p} \equiv S^{4-p}/\mathbf{Z}_2$ instead of S^{4-p} . All types can be distinguished locally by examining their subleading behaviour in N .

As a concrete example, we shall focus in particular on 2+1 dimensional gauge theory for N large. The reason that we focus on this case is that we can explicitly write the relevant part of the supergravity solution, using the fact that one of the relevant orientifold 6-planes has a smooth M-theory realization as the Atiyah–Hitchin manifold[4, 5], while D6-branes are related to Taub–NUT[6]. While much of the structure of the final result can be deduced on general grounds (the overall global \mathbf{Z}_2 is the main feature), we show that a whole family of $1/N$ corrections can be completely characterised using the construction that we present.

An overview is as follows: In the next section, we orient the reader and set up our notation by reviewing the salient features of ref.[1]. In section 3, we discuss generally how orientifolds yield the other types of enhançon. Crucially, we use a familiar gauge theory fact to help us make a general statement about the result of wrapping branes and orientifolds on $K3$.

In section 4, in preparation for the device of using the Atiyah–Hitchin manifold to construct the eleven dimensional solution for the $2+1$ dimensional B, C, D gauge theory cases, we lift the A -type case to M-theory and

reconsider it in M-theory terms. In particular, we observe that while in ten dimensions there is a discussion of the geometry in terms of its constituents being dynamical objects (D-branes), there is no analogous discussion involving dynamical objects in M-theory. The geometry can be discussed only in terms of the non-dynamical $K3$ and multi-Taub-NUT. Taking a probe to be an M2-brane fails to show the enhançon, since the geometry cannot be constructed out of them. Fortunately, long before one reaches the “repulson” singularity in the geometry, there is a sensible dual heterotic string description (one of the duals in ref.[1] (see also refs.[7, 8])), where the geometry again has natural brane probes revealing the enhançon. So we see again that the enhançon mechanism resolves the physics of a supergravity situation, this time by driving eleven dimensional supergravity back to string theory.

In section 5, we show how to modify the discussion to make it pertinent to the orientifolded enhançon, using the Atiyah-Hitchin manifold combined with multi-Taub-NUT. On returning to string theory, we study the supergravity solution, and extract the expression for the enhançon radius and the leading $1/N$ correction which follows from the orientifold’s presence. We point out that an entire class of $1/N$ corrections can be concisely summarised in terms of the exponentially small differences between the smooth Atiyah-Hitchin manifold and the (negative mass) Taub-NUT solution. Unfortunately, we do not have such control over all of the corrections present in the geometry.

We also present the result of a probe computation analogous to that performed in ref.[1] which yields the one-loop result for the metric on a subspace of the Coulomb branch of moduli space for the B, C, D gauge theories. Again, they differ from the $SU(N)$ case by a global \mathbf{Z}_2 action, and the leading behaviour for the $1/N$ corrections we computed. In all cases, there is a related monopole moduli space problem, in the spirit of refs.[9, 10, 11].

2 The A -Type Enhancement

Consider wrapping N coincident D6-branes on a $K3$ surface of volume V . This results in an effective 2+1 dimensional object, with $-N$ units of D2-brane charge, due to the interaction[12]

$$-\frac{\mu_6}{48} \int C_{(3)} \wedge p_1(\mathcal{R}) \quad (2.1)$$

on the D6-brane world-volume. The precise value $-N$ comes about since $\mu_6 = (2\pi)^{-6} \alpha'^{-7/2}$, $\mu_2 = (2\pi)^{-2} \alpha'^{-3/2}$, $\mathcal{R} = 4\pi^2 \alpha' R$, and because for $K3$

$$p_1(R) \equiv \frac{1}{8\pi^2} R \wedge R, \quad (2.2)$$

integrates to 48. We will call this wrong-sign D2-brane a “D2*-brane”. It preserves the same supersymmetries as a correct sign D2-brane with the same orientation, and therefore is *not* an anti-D2-brane. It is useful to think of it as a brane which is bound inside the D6-brane worldvolume, resulting from the curvature of the $K3$. It is quite analogous to the (correct sign) D2-brane which would be bound inside the worldvolume of a D6-brane if there was a field theory instanton configuration, due to the term[13]

$$\frac{\mu_6}{2} \int C_{(3)} \wedge \mathcal{F} \wedge \mathcal{F}, \quad (2.3)$$

where $\mathcal{F} = 2\pi\alpha' F$. An instanton in the 6+1 dimensional gauge theory has $(8\pi^2)^{-1} \int F \wedge F = 1$, and consequently has the charge of a single D2-brane. In the limit where the instanton shrinks to zero size, there is a good description[14] in the full string theory corresponding to a fully localised pointlike D2-brane.

Similarly, one would recover pointlike D2*-branes from wrapping the D6-branes if $K3$'s curvature was located at a finite number of points, such as in an orbifold limit. This situation (no doubt) has a good string theory description, and is worth investigating. In the present case, the curvature of the $K3$ is distributed everywhere, and correspondingly the D2*-branes are delocalised everywhere on it.

Imagine that the $K3$ surface lies in the x^6, x^7, x^8, x^9 directions, and that the remaining (unwrapped) part of the D6-brane lies in the x^0, x^1, x^2 directions. There is an $SU(N)$ gauge theory on the 2+1 dimensional worldvolume, with eight supercharges. The gauge supermultiplet consists of a gauge field A_μ and three scalars ϕ_i , where $i = 3, 4, 5$. The scalars parameterise the positions of the D6–D2* system in the transverse directions, x^3, x^4, x^5 . This vector supermultiplet transforms in the adjoint representation of $SU(N)$. The gauge theory has a scalar potential of the form $\text{Tr}[\phi_i, \phi_j]^2$. Supersymmetric solutions of the theory, giving a moduli space of vacua, may be found by choosing vacuum expectation values (“vevs”) of the scalars such that they are in the Cartan subalgebra of $SU(N)$. This breaks $SU(N) \rightarrow U(1)^{N-1}$, giving the “Coulomb branch” of moduli space.

Classically, the moduli space is

$$\mathcal{M}_{\text{cl}}^N = \frac{(\mathbb{R}^3 \times S^1)^{N-1}}{S_{N-1}}, \quad (2.4)$$

where the S^1 factors represent the periodic scalars resulting from dualising the gauge fields (recall that we are in $2 + 1$ dimensions). The S_{N-1} is the Weyl group of $SU(N)$, which acts as permutations of the $N - 1$ eigenvalues of the ϕ ’s, which are now in the Cartan subalgebra. $U(1)^{N-1}$ is the gauge symmetry on N separated, but wrapped D-branes, where the extra $U(1)$ we would naively expect corresponds to the overall centre of mass of the system.

We will focus on the situation where all of the branes are coincident, which is to say that the vev’s of all of the fields are given the same value, except for a complete set of four making a multiplet giving the location of a probe brane in the background of all the others. In the gauge theory, this is equivalent to focusing on a particular subspace of the relative moduli space. In another, equivalent description[9, 10, 11], it is the four dimensional $(1, N-1)$ subspace representing relative moduli space of the full moduli space of N $SU(2)$ monopoles; $N - 1$ of them are coincident, and one is separated.

The classical moduli space is then

$$\mathcal{M}_{\text{cl}}^{(1,N-1)} = \mathbb{R}^3 \times S^1 . \quad (2.5)$$

One of the results of ref.[1] (see below) is the computation of the one-loop result for the metric on this moduli space. Here, we shall compute a closely related version, representing a similar subspace of the Coulomb branch of the $SO(2N+1)$, $USp(2N)$, or $SO(2N)$ gauge theory. These will also have an interpretation as multimonopole moduli spaces, where the monopoles are of an $SU(2)$ gauge theory with a \mathbf{Z}_2 identification.

For gN large, (g is the closed string coupling) we have a chance of obtaining a good description of the geometry of the system in terms of a ten dimensional type IIA supergravity solution, which is

$$\begin{aligned} ds^2 &= Z_2^{-\frac{1}{2}} Z_6^{-\frac{1}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + Z_2^{\frac{1}{2}} Z_6^{\frac{1}{2}} dx^i dx^i + V^{\frac{1}{2}} Z_2^{\frac{1}{2}} Z_6^{-\frac{1}{2}} ds_{K3}^2 , \\ e^{2\Phi} &= g^2 Z_2^{\frac{1}{2}} Z_6^{-\frac{3}{2}} , \\ C_{(3)} &= g^{-1} (Z_2^{-1} - 1) dx^0 \wedge dx^1 \wedge dx^2 , \\ C_{(7)} &= g^{-1} (Z_6^{-1} - 1) dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^6 \wedge dx^7 \wedge dx^8 \wedge dx^9 . \end{aligned} \quad (2.6)$$

Here, $\mu, \nu = 0, 1, 2$; $i = 3, 4, 5$ and the x^6, x^7, x^8, x^9 directions contain ds_{K3}^2 , the (unknown) metric of a unit volume $K3$. The 345-harmonic functions representing the D2*- and D6-branes respectively are:

$$Z_2 = 1 + \frac{r_2}{r} \quad \text{and} \quad Z_6 = 1 + \frac{r_6}{r} , \quad (2.7)$$

(recall that the D2*'s are delocalised in $K3$), with

$$r = |\vec{r}|, \quad \vec{r} \in \mathbb{R}_{3,4,5}^3, \quad r_2 = -\frac{(2\pi)^4 g N \alpha'^{5/2}}{2V} \quad \text{and} \quad r_6 = \frac{g N \alpha'^{1/2}}{2} . \quad (2.8)$$

The latter are written so as to give the masses of the BPS object which is formed when we wrap a D6-brane to make the D6-D2* object:

$$\tau = \frac{N}{g} (\mu_6 V - \mu_2) = \frac{N}{g} \mu_6 (V - V_*) = \frac{N}{g} \mu_2 \left(\frac{V}{V_*} - 1 \right) , \quad (2.9)$$

where $V_* = (2\pi\sqrt{\alpha'})^4$.

There are a number of things to note about this supergravity solution. First, note that g appears as the asymptotic value of the string coupling far away from the core of the solution ($r \rightarrow \infty$). The actual string coupling in the interior of the solution is given by the value of e^Φ , as usual, and varies with r . Similarly, the volume of $K3$ is a function of r : $V(r) = V Z_2(r)/Z_6(r)$, which approaches V asymptotically, and decreases, becoming zero at the singularity $r = |r_2|$.

One of the key points noticed in ref.[1] is that while a naive examination of the supergravity solution shows an unsettling naked singularity (the “repulson”[2]) at $r = |r_2|$, this part of the geometry is actually non-physical. The geometry should only be taken at face value down to radius

$$r_e = \frac{2V}{V - V_*} |r_2| . \quad (2.10)$$

This is the radius at which a number of special things happen:

- The volume of $K3$ is equal to the special value $V_* = (2\pi\sqrt{\alpha'})^4$.
- The 5+1 dimensional $K3$ -compactified string theory has an R–R sector $U(1)$ which becomes enhanced to an $SU(2)$ gauge symmetry.
- A D6–D2* probe is a monopole of this $U(1)$, and becomes massless at the enhanced symmetry point. It also ceases to be pointlike, and dissolves into the “enharon” locus at r_e , which is an S^2 .

The interpretation of these and other facts uncovered in ref.[1] is that there are no brane sources for $r < r_e$, and therefore the supergravity solution inside that radius is simply the trivial flat solution with no R–R fields switched on. The smooth interpolating region between the two solutions in the neighbourhood of the enharon radius is described by the relatively innocuous (but nonetheless interacting) monopole physics.

On the one hand, this situation represents another remarkable method by which string theory rids itself of potentially troublesome singularities, while on the other hand, it potentially teaches us something about gauge theories. For example, in seeking for a limit in which the gauge theory decouples from the rest of bulk physics, we take $\alpha' \rightarrow 0$ while holding fixed the 2+1 dimensional gauge coupling given by:

$$g_{\text{YM}}^2 = (2\pi)^4 g \alpha'^{3/2} V^{-4} \quad (2.11)$$

and hold fixed $U = r/\alpha'$. In this limit, it was found that the metric on the moduli space, as seen by the D6–D2* probe, can be read off from the effective Lagrangian for the monopole probe moving in the transverse space with coordinates $(U, \theta, \phi, \sigma)$:

$$\mathcal{L} = f(U) \left(\dot{U}^2 + U^2 \dot{\Omega}_2^2 \right) + f(U)^{-1} \left(\dot{\sigma} - \frac{(N-1)}{8\pi^2} A_\phi \dot{\phi} \right)^2, \quad (2.12)$$

where¹

$$f(U) = \frac{1}{8\pi^2 g_{\text{YM}}^2} \left(1 - \frac{g_{\text{YM}}^2 (N-1)}{U} \right), \quad \dot{\Omega}_2^2 = \dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2, \quad (2.13)$$

with $0 \leq \theta < \pi$, $0 \leq \phi < 2\pi$ and $U_e < U < \infty$. Here, $U_e = g_{\text{YM}}^2 (N-1)$ and $A_\phi = \pm 1 - \cos \theta$ is a $U(1)$ monopole potential. The metric in (2.12) is the Euclidean Taub–NUT metric, with a negative mass. It is a hyperKähler manifold, because $\nabla f = \nabla \times A$, where $A = ((N-1)/8\pi^2) A_\phi d\phi$. The coordinate σ is periodic with period 4π , and is the dual of the $U(1)$ centre-of-mass gauge field on the 2+1 dimensional worldvolume of the monopole probe.

This result is completely in accord with the expectation from gauge theory, being the one-loop result for the metric on moduli space, in the special

¹Note that we have inserted $N-1$ instead of the N which appears in the supergravity solution (2.6) and also in the probe result exhibited in ref.[1]. Strictly speaking, there are $N-1$ D6–D2* units being probed by one separated unit, giving N in total. The difference is a $1/N$ effect, not considered in ref.[1], but should be included here since we will later be discussing a family of corrections at that order.

case where the $N - 1$ coordinates parameterising the Cartan subalgebra are chosen to be equal, corresponding to making all of the branes coincident. The enhançon is at $U = U_e$, and corresponds to the Landau pole, representing in gauge theory the place where the one-loop correction makes the gauge coupling diverge.

In the equivalent monopole language, this is (an approximation to) the metric on a subspace $\mathcal{M}^{(1,N-1)}$ (described above eqn. (2.5)) of the full moduli space of N $SU(2)$ monopoles. This $SU(2)$ is the enhanced gauge symmetry from whence comes the name “enhançon”.

There are exponential corrections to this metric which will remove the singular behaviour and complete it into a smooth hyperKähler manifold, $\mathcal{M}^{(1,N-1)}$. This space generalises the Atiyah–Hitchin manifold, which is the metric on the relative two-monopole moduli space $\mathcal{M}^{(1,1)}$ which governs the case of $SU(2)$ gauge theory[9].

A natural question arises about the nature of the story for the case where one studies gauge groups other than $SU(N)$. It is straightforward to construct gauge groups $SO(2N)$, $SO(2N + 1)$, and $USp(2N)$ in perturbative string theory by combining D-branes with orientifolds. Studying the wrapping of such a system on $K3$ should therefore be our first step in answering the question. Let us do that.

3 Including Orientifolds

On general grounds, one expects a similar story to that which was constructed above, as all of the constituent features which are present to make the physics work as it should are still present after we insert an orientifold six-plane (O6-plane) parallel with the D6-branes. Of course, the details of precisely where the enhançon is located (corresponding to where in \mathbb{R}_{345}^3 the $K3$ volume reaches the value V_*) will be modified, but only at subleading order in N . Globally, the orientifold will also place a \mathbf{Z}_2 identification on \mathbb{R}_{345}^3 (\mathbf{Z}_2 acts

by multiplying each of x^3, x^4, x^5 by -1), turning the S^2 of the enhançon into $\mathbb{RP}^2 \equiv S^2/\mathbf{Z}_2$. The basic problem is to understand the nature of the supergravity solution in the presence of the orientifolds, which we will do below in a particular case.

First, let us understand the physics of the perturbative string theory description, containing the weakly coupled gauge theory. For small gN , we have N D6-branes, and an O6-plane parallel to them. This gives a gauge group $SO(2N+1)$, $USp(2N)$ or $SO(2N)$. In the latter case, the O6-plane has negative charge, equal to $-2\mu_6$ and is often denoted $O6^-$. We can obtain the former case by trapping a half D6-brane on the $O6^-$ -plane: this combination is often referred to as an $\widetilde{O6}$ -plane, with charge $-3\mu_6/2$. In the middle case, the O6-plane has charge $+2\mu_6$ and is written $O6^+$. To be concise, we will use the symbol α to denote these O6-charges, measured in D2-brane units. It takes the values $\alpha = -3/2, +2, -2$, respectively.

We now wrap the whole system on $K3$. This results in the induced D2*-branes as described above, but with an additional contribution. This is due to a curvature coupling, this time on the world-volume of the O6-plane[16], similar to eqn.(2.1). The couplings are different in each case $\widetilde{O6}$, $O6^+$, $O6^-$:

$$-\frac{\mu_6}{32} \int C_3 \wedge p_1(\mathcal{R}) , \quad -\frac{5\mu_6}{48} \int C_3 \wedge p_1(\mathcal{R}) , \quad -\frac{\mu_6}{48} \int C_3 \wedge p_1(\mathcal{R}) , \quad (3.14)$$

which, after wrapping on $K3$ will induce some $C_{(3)}$ charge, β , which in D2-brane units, is respectively $\beta = -3/2, -5, -1$. This will modify the contribution to the effective amount of D2*-brane present².

Note that we can introduce extra (correct sign) D2-branes parallel to the D2*-branes into the story while preserving the eight supercharges. Open

²The temptation to interpret these extra charges as induced wrong sign O2-planes should, we believe, be firmly resisted. First of all, the resulting charges are hard to interpret, given the existing types of O2-brane already known. Secondly, one would have to insert a \mathbf{Z}_2 identification on the $K3$ part of the spacetime, which is hard to justify as the result of a smooth wrapping process. The most economical interpretation is the one presented here.

strings stretching between these new branes and the wrapped system play the role of extra hypermultiplets in the 2+1 dimensional gauge theory. For M D2-branes, we have M species of such hypermultiplets. For consistency, in the presence of the orientifold, these D2-branes will have the *opposite* orientifold projection acting on them from that acting on the D6's, as follows from T-duality to the situations studied in refs.[14, 15].

So, for example, while there is an $SO(2N)$ (or $USp(2N)$) gauge symmetry on the D6-branes, the M -flavour sector has a $USp(2M)$ (or $SO(2M)$) symmetry. This is indeed correct from the perspective of gauge theory[17], and this fact has featured in the physics of orientifolds before. In ref.[18], it was shown to correspond to the phenomenon that the orientifold must change its sign when it passes through an NS5-brane. Actually, one of the dual realizations of the enhançon story involves NS5-branes[1]. We display it with the inclusion of the orientifold in figure 1, where our case here is $p = 2$. The D3-branes in the interior are dual to the D6-D2* units, while the those on the exterior (supplying matter multiplets) are dual to the ordinary D2-branes. The orientifold runs through the whole system, having a minus sign on the interior (giving $SO(2N)$) and a plus sign on the exterior, giving $USp(2M)$.

So we see that the sign flip of the O3-plane on either side of the NS5-brane is dual to the fact that the O6-plane has the opposite projection on the D2's from that on the D6's. Of course, this discussion clearly generalises to all Dp -branes and Dp^* -branes with orientifolds, and the dual situation involving $D(p+1)$ -branes stretched between NS5-branes with an $O(p+1)$ -plane passing through.

In this way, we see that we can consistently construct wrapped D-brane and orientifold systems giving gauge groups $SO(2N+1)$, $USp(2N)$, and $SO(2N)$. The amount of D2*-branes induced from the wrapping is modified from $-N$ (for the $SU(N)$ case) to $-N-3/2$, $-N-5$ and $-N-1$, respectively. The changes are to be thought of as $1/N$ corrections to the original case,

Figure 1: D($p+1$)–branes ending on NS5–branes, in the presence of an $O(p+1)^-$ –plane (central dotted line). (a) The N branes and their images in the interior give an $SO(2N)$ gauge group. On the exterior, M branes and their images supply matter hypermultiplets, and carry a gauge group $USp(2M)$, as the orientifold plane changes sign when it goes through an NS5–brane. (b) At large gN , the fivebranes will be bent, touching at an $\mathbb{RP}^{4-p}=S^{4-p}/\mathbf{Z}_2$, (whose double cover is a circle in the figure), the D –type enhançon, where the NS5–branes carry an enhanced gauge symmetry. (In case (b), for clarity, no matter branes are shown.)

and are different for each type of orientifold. We can now consider taking gN large, and expect that the phenomena which occurred for $SU(N)$ will happen again, giving an enhançon for each case. We shall name the types of enhançon which can occur in each situation the A –type (for $SU(N)$), B –type (for $SO(2N+1)$), C –type (for $USp(2N)$) and D –type ($SO(2N)$).

Of course, there is no natural definition of an E –type, for (at least) two reasons: There is no known perturbative way to make $E_{6,7,8}$ gauge symmetry with D–branes, and furthermore, the enhançon as a smooth geometric object is a large N phenomenon, which is incompatible with the fact that the maximum rank of the exceptional groups is eight.

The next step in our story will be to write down the geometry corresponding to the large gN physics of the wrapped system of D–branes and orientifolds. The observation that we shall use to achieve this is the fact that

for the $O6^-$ case (giving 2+1 dimensional $SO(2N)$), the supergravity geometry of the system can be written down accurately enough for us to make progress. Along the way, we will see that we can study cases $SO(2N+1)$ and $USp(2N)$ accurately enough for our purposes using similar techniques.

4 Uplifting the Enhançon

Before we proceed to the new types, let us pause for a moment to consider the A -type enhançon story in eleven dimensional terms. Recall that the metric of the Taub–NUT space, made into an eleven dimensional supergravity solution (by adding $\mathbb{R}^{6,1}$ for the world-volume directions) is (defining an eleventh direction $\psi = x^{\sharp}/16m$):

$$ds_{11}^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2 + F(r)(dr^2 + r^2 d\Omega_2^2) + F^{-1}(r)(d\psi + C_\phi d\phi)^2, \quad (4.15)$$

where, with $0 \leq \psi < 4\pi$,

$$d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad F = 1 + \frac{4mN}{r}, \quad C_\phi = 4mN \cos \theta. \quad (4.16)$$

Reducing along the ψ -circle, the relation between eleven dimensional metrics and ten dimensional type IIA fields is[19]:

$$ds_{11}^2 = e^{-\frac{2}{3}\phi} ds_{10}^2 + e^{\frac{4}{3}\phi} (d\psi + C_{(1)})^2, \quad (4.17)$$

and so we recover the now standard fact[6] that Taub–NUT corresponds to a familiar ten-dimensional solution:

$$ds_{10}^2 = Z_6^{-\frac{1}{2}} (-dt^2 + dx_1^2 + dx_2^2 + dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2) + Z_6^{\frac{1}{2}} (dr^2 + r^2 d\Omega_2^2) \\ Z_6 = F(r); \quad e^\phi = Z_6^{-\frac{3}{4}}(r), \quad C_\phi = 4mN \cos \theta, \quad (4.18)$$

which is precisely the D6-brane solution, if we identify $4mN = r_6$ (and set the asymptotic value of the dilaton to $\log g$.) The one-form potential

$C_{(1)} = C_\phi d\phi$ can be Hodge-dualised in ten dimensions to give an electric source for $C_{(7)}$ of precisely the form given in eqn. (2.6).

Turning to the enhançon, by using the prescription of eqn. (4.17), supplemented with a direct uplift of the three-form potential $C_{(3)}$ to give the components of the eleven dimensional three-form $A_{(3)}$, it is easy to write an eleven dimensional solution for the uplifted D6–D2* system:

$$ds_{11}^2 = \tilde{Z}_2^{-\frac{2}{3}}(-dt^2 + dx_1^2 + dx_2^2) + \tilde{Z}_2^{\frac{1}{3}} \tilde{V}^{\frac{1}{2}} ds_{K3}^2 \\ + \tilde{Z}_2^{\frac{1}{3}} \left[\tilde{Z}_6 (dr^2 + r^2 d\Omega_2^2) + \tilde{Z}_6^{-1} (d\psi + C_\phi d\phi)^2 \right] ,$$

with $A_{(3)} = (\tilde{Z}_2^{-1} - 1) dx^0 \wedge dx^1 \wedge dx^2 ,$ (4.19)

$$\text{where } \tilde{V} = g^2 V , \quad \tilde{Z}_2 = g Z_2 , \quad \tilde{Z}_6 = g^{-1} Z_6 . \quad (4.20)$$

It is interesting to contrast the interpretation of this solution with the ten dimensional discussion. Recall that from the point of view of ten dimensions, there is the geometry of $K3$, accompanied by D6-branes wrapped on it. The wrapping induced some D2*-branes, completely delocalised in the $K3$. We were able to probe the geometry of the supergravity solution (4.19) with one of its basic constituents, a single D6–D2* BPS object.

From the point of view of the eleven dimensional supergravity solution, everything is geometry: there are no branes here. The Taub–NUT part lies in the 345 directions, while $K3$ lies in the 6789 directions. Together, they act as a source for the three-form potential $A_{(3)}$, due to the supergravity term[21]:

$$\int A_{(3)} \wedge X_8 , \quad \text{with } X_8 = \frac{1}{24} \left(p_2 - \frac{1}{4} p_1 \wedge p_1 \right) . \quad (4.21)$$

Given that for Taub–NUT of charge N , $p_1 = 2N$ and, as stated before, for $K3$ we know that $p_1 = 48$, we get $-N$ units of $A_{(3)}$ charge, as the solution shows. This fits, as (4.21) is the M-theory ancestor of the brane world-volume term (2.1).

Sadly, there is no natural extended dynamical object which we can use as a candidate for the basic constituent of the geometry. Thus, we cannot

perform a world-volume probe computation to deduce the true geometry. It is tempting to read the \tilde{Z}_2 part of the geometry as representing a “wrong sign” M2-brane which is otherwise dynamical, (perhaps restoring a $1/r^3$ behaviour to make it also localised in the x^4 direction.) Unfortunately, this cannot work. The putative M2*-brane necessarily would have negative tension at all locations in \mathbb{R}_{345}^3 , and since there is no larger wrapped brane with positive tension to combine it with to make a positive tension object, we cannot write a sensible worldvolume action. Of course, a probe computation with a correct sign M2-brane (which preserves the same amount of supersymmetry) gives a sensible result: simply the pure (with mass parameter of $+4mN = r_6$) Taub-NUT metric, as it should, with no sign of either enhançon or repulson. This is in accord with our expectation that the repulson (still present at $r = |r_2|$) is an artifact, while the enhançon should be invisible to an M2-brane because its world-volume theory does *not* relate to the $SU(N)$ gauge theory.

To get at the enhançon, there is a pertinent supergravity question to be asked all the same: Can we envision a supergravity mechanism by which the troublesome repulson singularity at $r = |r_2|$ is avoided? In the string theory situation, we saw that the $K3$ reached the natural value $V_* = (2\pi\sqrt{\alpha'})^4$, before it reached its singular value (zero), and the physics of the enhanced gauge symmetry took over. Is there a special value for the volume of $K3$ in this case? Here, the natural length scale is of course set by the Planck length, $\ell_{11} = g^{1/3}\ell_s$, which the system again reaches before the singular value of zero. Once $K3$ has shrunk to that size, we should search for a better description than eleven dimensional supergravity. The alternative to using the full (unknown) M-theory is to search for a dual description. Happily, eleven dimensional supergravity on such a small $K3$ is well described[19] by the heterotic string on T^3 . The distinguished ψ -circle which is fibred to make the Taub-NUT joins the rest to become a T^4 , and the Taub-NUT structure becomes a “warped”[20] (not “wrapped”) NS-fivebrane/Kaluza-Klein monopole structure giving rise to a monopole membrane whose mass

goes to zero at an $SU(2)$ enhanced point of the torus[1, 7] (see also ref.[8]). So we see that again, stringy physics (now heterotic) takes over before we get to the repulson radius, and repairs the geometry with the same $SU(2)$ physics that we saw in type IIA string theory.

5 The Orientifolded Enhancement

Just as the D6-brane has an interpretation as the Taub–NUT spacetime upon going to low energy M–theory (eleven dimensional supergravity), in a similar fashion, the $O6^-$ –plane becomes[5] the Atiyah–Hitchin manifold[4], described by a metric:

$$ds_{11}^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2 + f(\rho)dr^2 + 8m^2 \left(a^2(\rho)\sigma_1^2 + b^2(\rho)\sigma_2^2 + c^2(\rho)\sigma_3^2 \right) , \quad (5.22)$$

where

$$\begin{aligned} \sigma_1 &= -\sin\psi d\theta + \cos\psi \sin\theta d\phi , & \sigma_2 &= \cos\psi d\theta + \sin\psi \sin\theta d\phi \\ \sigma_3 &= d\psi + \cos\theta d\phi , & \rho &= \frac{r}{8m} , & \psi &= \frac{x^{\natural}}{16m} , \end{aligned} \quad (5.23)$$

and the functions f, a, b, c, d are given in terms of elliptic integrals in ref.[22]. There is also an identification by:

$$(r, \theta, \phi, \psi) \rightarrow (r, \pi - \theta, \pi + \phi, -\psi) , \quad (5.24)$$

which, in terms of the coordinates (x^3, x^4, x^5) and the M–direction x^{\natural} , is simply a multiplication by a minus sign on all directions. The displayed metric (5.22) has a conical singularity at $r = 8\pi m$. The space made by imposing the \mathbf{Z}_2 identification (5.24) is the Atiyah–Hitchin space, and it is free of conical singularities.

While a closed form for the metric cannot be written, for large r the metric becomes[22]

$$ds_{11}^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2 +$$

$$+G(r)(dr^2 + r^2 d\Omega_2^2) + G^{-1}(r)(d\psi + C_\phi d\phi)^2, \quad (5.25)$$

where

$$G = 1 - \frac{16m}{r}, \quad C_\phi = 16m \cos \theta, \quad (5.26)$$

which we recognise as the metric for Taub–NUT, but with a negative mass. Clearly, it can be reduced to ten dimensions in the same way as before, and we see [5] that it has -2 units of D6–brane charge, which is in accord with our knowledge of the charge of an $O6^-$ –plane from perturbative string theory. (The actual appearance of $-16m$ in the metric instead of $-8m$ follows from the fact that the displayed metric is the double cover of the actual solution: recall that we must divide by the \mathbf{Z}_2 action.)

Now we are in a position to construct the geometry which gives rise to the D –type enhançon. We simply combine the geometry of the Atiyah–Hitchin manifold with that of N coincident-centred Taub–NUT. The exact smooth metric certainly exists (the $N = 1$ case is known, and is Dancer’s manifold[23]), but we need not be able to write it exactly to get at the physics we require. The radius at which the enhançon appears can be tuned to be arbitrarily large by making N as large as we like, so we can rest assured that if we take the approximate expression for the Atiyah–Hitchin manifold, we can capture the essential physics for large N .

Once we relax the condition of exactness, and focus on the large r part of the solution, we can include the cases of the B – and C –type enhançons. While a precise relation to a cousin of the smooth Atiyah–Hitchin+Taub–NUT geometry is not known, at large r , the difference is immaterial, as only the leading behaviour is needed to characterise the enhançon at large enough N . We can simply use the same supergravity solution as before, but with different numbers inserted into the $1/N$ corrections to the harmonic functions.

It is clear therefore, that for all cases our solution can be written (for large enough r) in the covering space in the precise form of eqn. (4.19), but

with the replacement of \tilde{Z}_2 and \tilde{Z}_6 by (respectively):

$$\tilde{Z}'_2 = g \left(1 - \frac{2|r_2|(1 - \beta/N)}{r} \right) \quad \text{and} \quad \tilde{Z}'_6 = \frac{1}{g} \left(1 + \frac{2r_6(1 + \alpha/N)}{r} \right). \quad (5.27)$$

Here³ $\beta = -3/2, -5, -1$, and $\alpha = -3/2, +2, -2$ for types B, C, D , respectively. We have deduced \tilde{Z}'_2 's asymptotic form⁴ from the fact that it must give the correct induced D2*-brane mass and charge at large r in the string theory limit.

This should be taken to mean the metric on the covering space of our solution, and we must divide by the \mathbf{Z}_2 action in order to reconstruct the correct solution, as before. This also accounts for the factors of two we have inserted into the harmonic functions. Notice that the contribution to the harmonic functions of (what will become) the orientifolds is simply a $1/N$ correction to the geometry. This will turn into part of the family of $1/N$ corrections to the location and shape of the enhançon locus, once we return to string theory.

The final step is clear. We return to type IIA string theory by reducing on the ψ -circle, recovering a supergravity solution representing the large gN geometry of system of wrapped D6-brane and $O6^-$ -plane, as promised in section 2. The solution is simply the geometry (2.6) with Z_2 and Z_6 replaced by their $1/N$ corrected counterparts in (5.27) with the factors of g and $1/g$ removed. Crucially, there is a \mathbf{Z}_2 identification on the (x^3, x^4, x^5) directions, making it globally distinct from the A -type case, in addition to the different structure of the subleading behaviour in N .

³It is amusing to note that the sum $\alpha + \beta$ is the same in each case. We do not know if this has any physical significance. Later, in eqn. (5.31), we shall see that it is $\alpha - \beta$ which controls the leading $1/N$ correction to the enhançon in each case. Were it the sum which appeared, we would have had a remarkably universal result.

⁴While we know (in the D -type case) precisely how the harmonic function of \tilde{Z}'_6 gets corrected into the smooth Atiyah-Hitchin+Taub-NUT solution, we do not know how \tilde{Z}'_2 , which owes its presence to the $K3$ part of the eleven dimensional geometry, gets corrected. In its current form, it must be there in order to measure the correct mass and charge at large r , but the small r details are unknown to us.

Again, in string theory, the natural object to construct this geometry out of is the D6–D2* at large gN , now in the presence of an orientifold, and we may examine the nature of the geometry as seen by the probe by a computation precisely along the lines of ref.[1]. The structure of the computation is almost identical to that carried out there, and we refer the reader to that work for the details. A crucial difference is that we are working on the *covering space* of the actual geometry, and so we should insert a mirror image of the probe at the image position obtained by reflecting through the orientifold fixed point. The result is structurally identical:

$$\mathcal{L} = F'(r) \left(\dot{r}^2 + r^2 \dot{\Omega}_2^2 \right) + F'(r)^{-1} \left(\dot{s}/2 - \mu_2 C_\phi \dot{\phi}/2 \right)^2, \quad (5.28)$$

where now

$$F'(r) = \frac{1}{2g} (\mu_6 V Z'_2 - \mu_2 Z'_6), \quad (5.29)$$

with

$$\begin{aligned} Z'_2 &= \left(1 - \frac{2|r_2|(1 - (\beta + 1)/N)}{r} \right) \quad \text{and} \\ Z'_6 &= \left(1 + \frac{2r_6(1 + (\alpha - 1)/N)}{r} \right), \end{aligned} \quad (5.30)$$

where we have shifted N to $N - 1$ to represent separating off the probe (see footnote 1). Here, s is the fourth modulus obtained by dualising the world-volume centre of mass gauge field. The location r'_e of the D -type enhançon can be read off as the place in r where the mass of the probe becomes zero (equivalent to $V(r'_e) = V_*$):

$$r'_e = \frac{2V}{V - V_*} |r_2| \left(1 - \frac{\gamma}{N} \right), \quad (5.31)$$

with

$$\gamma = - \left(\frac{\alpha - \beta - 2}{2} \right) = 1, -\frac{5}{2}, \frac{3}{2} \quad (5.32)$$

in each case B, C, D . (The analogous expression for the A case —with the $1/N$ correction from separating off the probe (*c.f.* eqn. (2.10))— has $\gamma = 1$.)

Note that for case B the effect of the O6–plane is precisely cancelled by the effect of the D2*–brane contribution which is produced from wrapping, giving the same leading $1/N$ contribution as for type A .)

Correspondingly, when we take the limit where we decouple the gauge theory with $\alpha' \rightarrow 0$ holding g_{YM}^2 fixed, we recover the prediction for the metric on the moduli space of the gauge theory at large N (in the coincident limit):

$$\mathcal{L} = f'(U) \left(\dot{U}^2 + U^2 \dot{\Omega}_2^2 \right) + f'(U)^{-1} \left(\dot{\phi} - \frac{N(1 - \gamma/N)}{8\pi^2} A_\phi \dot{\phi} \right)^2, \quad (5.33)$$

where

$$f'(U) = \frac{1}{8\pi^2 g_{\text{YM}}^2} \left(1 - \frac{g_{\text{YM}}^2 N}{U} \left(1 - \frac{\gamma}{N} \right) \right). \quad (5.34)$$

This is the one-loop expression for the metric on moduli space for the $SO(2N+1)$, $USp(2N)$ or $SO(2N)$ 2+1 dimensional gauge theory. On general grounds, the classical moduli space has the geometry

$$\mathcal{M}_{\text{cl}} = \frac{(\mathbb{R}^3 \times S^1)^N}{S_N \times \mathbf{Z}_2} \quad (5.35)$$

where there is a natural \mathbf{Z}_2 action reflecting the N eigenvalues into (minus) themselves. In the subspace where we set all the vev's (but four) to be equal, we are reduced to

$$\mathcal{M}_{\text{cl}} = \frac{\mathbb{R}^3 \times S^1}{\mathbf{Z}_2} \quad (5.36)$$

for the classical moduli space. Our metric above, with the \mathbf{Z}_2 action (imposed, recall, for smoothness of the Atiyah–Hitchin manifold representing the O6[−]–plane), is the one-loop expression for the metric on the full moduli space.

Finally, we point out that once again, these results have a dual interpretation as an approximate result for the metric on moduli space of N monopoles[9, 10, 11]. This time, they are monopoles of a spontaneously broken $SU(2)$ theory which has an identification by \mathbf{Z}_2 , which can be understood

as follows⁵: The relation between the moduli space of $2+1$ dimensional gauge theories and that of monopoles is readily seen in the string realization of such theories by D3-branes stretched between NS5-branes[11]. The spontaneously broken $SU(2)$ lives on the world-volume of the NS5-branes. The ends of the D3-branes in the NS5-brane worldvolumes are the monopoles. We need only look at the orientifolded version of that picture, drawn in figure 1, to see the origin of the \mathbf{Z}_2 action on the $SU(2)$ theory. By passing through the world-volume of the NS5-branes, the O3-plane places a spacetime \mathbf{Z}_2 identification on the $SU(2)$ gauge theory.

6 Closing Remarks

The enhançon locus which appears in the study of spacetime geometry associated to $SU(N)$ $(p+1)$ -dimensional gauge theory (at large N) with eight supercharges has three natural counterparts: Those pertaining to $SO(2N+1)$, $USp(2N)$ and $SO(2N)$ gauge theory. The four classes deserve to be called types A , B , C , and D . (There is no natural E -type which has a smooth geometrical interpretation, since the rank of those groups cannot be made arbitrarily large.)

We presented the general scenario for the case of $(p+1)$ -dimensions and exhibited and studied the orientifolded enhançon for the case of $2+1$ dimensional gauge theory. Guided by the case of the D -type, where the fact that the $O6^-$ -plane has a known eleven dimensional supergravity description in terms of the Atiyah-Hitchin manifold, we were able to study aspects of all three new types: While the Atiyah-Hitchin manifold cannot be written explicitly, it reduces to (negative mass) Taub-NUT at large r (up to exponentially small corrections in r) which was enough for us to study explicitly the relevant features of the supergravity solution which results from placing many D6-branes and an O6-plane on $K3$. This multi-Taub-NUT solution

⁵See ref.[24] for comments on such theories in a closely related stringy context.

can also be reliably modified to capture the local asymptotic behaviour of the B - and C -type cases. The Atiyah–Hitchin structure imposes a global \mathbf{Z}_2 identification on the entire geometry. Correspondingly, we found that there is a global \mathbf{Z}_2 identification inherited by the enhançon locus, making the enhançon a natural $\mathbb{RP}^2 \equiv S^2/\mathbf{Z}_2$ geometry, in contrast to the S^2 geometry of the A -type. (We should also note that we observed that in all cases A, B, C , or D , the apparent repulson singularity in eleven dimensional supergravity is naturally removed; not by M-branes, but by being forced back to ten dimensional heterotic string theory because the $K3$ becomes small. The heterotic string phenomena dual to the enhançon[1] then take over the description.)

We displayed some leading $1/N$ corrections to the location of the all three types of orientifolded enhançon, as compared to the location of the A -type, and hence also the $1/N$ corrections to the one-loop metric on moduli space. Note that the A -type enhançon already has a series of exponential corrections of the form $\exp(-1/g_{\text{YM}}^2)$. On general grounds, our new types have a similar class of corrections, which can be phrased in terms of field theory instanton corrections[9, 25], and equivalently, in terms of corrections from D1-brane world-sheets[11, 26].

It is amusing to note that the $1/N$ corrections we studied here, which are of a different type, can all be written in terms of exponential corrections too. This follows from the fact that the part of the geometry of the Atiyah–Hitchin (–like) manifold that we neglected in writing the explicit supergravity solution is a series of exponential corrections in r . These corrections should also have an interpretation in terms of an instanton problem. Perhaps one can always organise the exponential corrections to these geometries in terms of structures reminiscent of the geometry of the Atiyah–Hitchin manifold, regardless of whether they are non-perturbative in g_{YM}^2 or $1/N$.

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