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Vortex configurations in the large N limit.

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ABSTRACT

We study the properties of vortex-like configurations which are solutions of the SU(N) Yang-Mills classical equations of motion. We show that these solutions are concentrated along a two-dimensional wall with size growing with the number of colors.

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1 Introduction.

The vortex condensation theory is one of the most promising ideas to explain the property of confinement. This idea was proposed by many authors at the end of 70's [1] and has received a renewed interest in a set of recent works [2–14]. A gauge dependent approach is used in [2–13] to study the relation between the confinement property and the vortex contain of the vacuum. By first going to the maximal center gauge and then center projecting, it is found evidence of a center vortex origin for the confinement property. In references [14] is presented another approach to the same problem, in this case relating the vortex contain of the vacuum and confinement in a gauge invariant way.

One of the problems of this idea is that it can not explain the behaviour of the string tension in the adjoint representation. This quantity shows casimir scaling for small distances and vanishes for large distances. Casimir scaling is a property of the ratio between the string tension in the adjoint representation, σ_A, and the string tension in the fundamental one, σ_F. This quantity σ_A/σ_F is equal to the ratio between the casimirs of the two representations in the small distance region [15]. In the large distance region the adjoint string breaks because quarks in the adjoint representation and gluons can mix to built a bound state. Nevertheless, in the limit of large number of colors it is expected that the property of confinement also holds. This fact is expected because of the factorization property for gauge invariant operators, which holds in the large N limit. The expected value of a Wilson loop along a path C calculated in the adjoint representation $\langle W_A(C)\rangle$ can be written as $\langle W_A(C)\rangle = \langle W_F(C)W_F(C)^\dagger\rangle$, being $W_F(C)$ the loop in the fundamental representation, and, using the factorization property, we obtain that $\langle W_A(C)\rangle = \langle W_F(C)\rangle^2$. Then, in the large N limit it is expected that the adjoint string tension is twice the fundamental one. These two things, casimir scaling of the string tension and confinement of adjoint quarks in the large N limit, could be in contradiction with the model of confinement based in physical objects carrying flux quantized in elements of the center of the group, because the adjoint representation is blind to these elements. The proposal presented in reference [3] to solve this problem is that the approximate casimir scaling is an effect due to the finite thickness of the vortex configuration, and also that the non vanishing adjoint string tension in the large N limit can be explained if this thickness grows with the number of colors. Following this idea, we study the behaviour with the number of colors of the vortex solution presented in [17,18] for the SU(2) and SU(3) groups, and we found that for $N > 3$ the solution has the same properties and his size grows with the number of colors, N.

The paper is structured as follows. In section 2 we show the properties of the solution we have found and in section 3 we present our conclusions.

2 The solutions.

Following the same methods used in [16–18] we isolate solutions of the SU(N) Yang-Mills classical equations of motion on a four torus with sizes ${l_L}^2 \times {l_S}^2$, being $l_L \gg l_S$ (two large, t and x, and two small, y and z, directions). These solutions satisfy twisted boundary conditions given by the twist vectors $\vec{k} = \vec{m} = (1, 0, 0)$, have topological charge $|Q| = 1/N$ and action $S = 8\pi^2/N = 8\pi^2|Q|$ (a review of the properties of Yang-Mills fields on the torus can be found in [19]). The choice of the twist vectors is based in the results presented in reference [17]. In this article it is shown that the necessary condition to obtain a SU(2) solution which is localized in the two large directions, flat in the other two and having the properties of a vortex is that the twist in the small plane must be non trivial. The same property also holds for SU(3) [18] and we expect the same result for SU(N) groups, with $N > 3$. For this reason we choose $m_1 = 1$, because this is the component of the twist linked with the small plane, the yz plane.

We have isolated lattice configurations for values of the number of colors $N = 4, 5, 6, 7, 8, 9, 10$. As in [16–18] these configurations are obtained using standard cooling algorithms (our procedure is a local minimization of the Wilson action using the Cabbibo-Marinari-Okawa update [20]). Our criteria to stop the cooling procedure is that the Wilson action is stable up to the eight significant digit and close to the value $S = 8\pi^2/N$. In table 1 we give the set of studied solutions, specifying the number of colors, lattice size $(N_L)^2 \times (N_S)^2$ and the values of the action S (in $8\pi^2/N$ units), topological charge Q multiplied by N and the electric and magnetic parts of the action, S_e and S_b respectively (also in $8\pi^2/N$ units). These quantities are calculated from the field strength $\mathbf{F}_{\mu\nu}$ obtained from the clover average of plaquettes 1×1 and 2×2, combined in such a way that the discretization errors are $O(a^4)$. The continuum expected values for these quantities are: $SN/8\pi^2 = 1$, $QN = \pm 1$, $S_e/8\pi^2 = 0.5$ and $S_b/8\pi^2 = 0.5$. We observe that our lattice results are very near to the continuum values and also that the lattice corrections are smaller for larger values of N.

The choice of the sizes of the lattice is based in references [17, 18]. From the work [17] we learn that for the SU(2) case the obtained results show a very nice scaling towards continuum functions for lattice sizes $(4N_S)^2 \times (N_S)^2$ with $N_S = 4, 5, 6, 7$. The SU(3) case is studied in [18] and it is shown in this reference that for lattice sizes $(6N_S)^2 \times (N_S)^2$ with $N_S = 4, 5, 6$ it is also found a very nice scaling towards continuum functions. Following these results we use lattices with sizes $(2NN_S)^2 \times (N_S)^2$. For number of colors $N = 4$ we isolate solutions with $N_S = 2, 3, 4$ and study their scaling properties. We observe that with only two points in the small directions the obtained results show a very nice continuum behaviour (to set the scale we fix the length of the small direction, l_S, equal to 1, being therefore the lattice spacing $a = 1/N_S$). We illustrate this in figure 1 in which we show the energy profile $\epsilon(t)$, defined as,

$$\epsilon(t) = \int S(t, x, y, z) dx dy dz, \tag{1}$$

where $S(t, x, y, z)$ is the action density, for these three SU(4) configurations. This is a property which also holds for bigger values of N and allows us to use lattices with sizes $N_S = 2$ and still obtain good continuum results. In fact, we can use this lattice size because these solutions are almost flat in the small directions and then very few points are needed to describe it.

Now, we describe the shape and the N behaviour of the action density and the integrated quantity defined before, the energy profile. We start with the N behaviour of the energy profile. The result we get is that this function has a typical width which grows with the numbers of colors as the square root of this number. We illustrate this property in figure 2 in which we plot $\epsilon(t)$ multiplied by $N^{3/2}$ as a function of $\sqrt{N}$. From this figure we can clearly see that the size of the solution grows with the square root of the number of colors, and also the scaling of the energy profile with $N^{3/2}$.

The properties of the action density $S(t, x, y, z)$ for our solutions are the following. The function S is almost independent of the y and z coordinates, and has a instanton-like dependence in the t and x coordinates. The function in these coordinates, t and x, has only one maximum, that we put at point $t = x = 0$, and decreases with $|t|$ and $|x|$ up to the value $S = 0$, showing an exponential behaviour at the tails. The size of the solution in these two directions, t and x, is approximately the square root of the number of colors (remember that the length scale is set by the size of the small direction $l_S = 1$).

The main result of this work is that these solutions have vortex properties for all the values of N studied and that the size of the object grows with the number of colors. To illustrate these properties we calculate the Wilson loop $W_C(r)$ around this object, being the path C a $r \times r$ square loop in the x,t plane centered at the maximum of the action density. We parametrize $W_C(r)$ by the functions $L(r)$ (its module) and $\phi(r)$ (its phase). Now we describe the behaviour of these two functions with the size of the loop r. The module L takes the value $L = 1$ at point $r = 0$, decreases with r up to reach the minimum of the function, and then increases up to the value $L \sim 1$ at a point r between $\sqrt{N}$ and N. The phase ϕ takes the value $\phi = 2\pi$ at $r = 0$ and then decreases up to the value $\phi = 2\pi(1 - \frac{1}{N})$ at a point r between $\sqrt{N}$ and N. Then, for a enough big value of r the Wilson Loop $W_C(r)$ takes the value of an element of the center of the group, $W_C \to exp(-i2\pi/N)$. We illustrate this property in figures 3 and 4. In figure 3 we show the phase ϕ for the three SU(4) solutions as a function of the size of the loop r. First, we can see the nice scaling towards a continuum function of the points coming from different lattice sizes, $N_s = 2, 3, 4$. And second, how the phase ϕ rotate from the value $\phi = 2\pi$ at point $r = 0$ to the value $\phi = 3\pi/2$ at point $r \sim N = 4$. Finally, we study the N behaviour of the phase ϕ. In figure 4 we show the quantity $\Phi = (2\pi - \phi)N$ as a function of $r/\sqrt{N}$ for the solutions with $N = 4, 7, 10$. If the Wilson loop $W_C(r)$ takes the value $W_C \to exp(-i2\pi/N)$ for a enough big value of r then this phase Φ must take the value $\Phi = 2\pi$ for the same value of r. From this figure we can clearly see that this phase Φ reach the value $\Phi = 2\pi$, and that the value of r in which this is happening, grows with the number of colors with a tendency a bit faster than the square root of the number of colors.

3 Conclusions.

In this work we have shown that there are solutions of the SU(N) Yang-Mills equations of motion having vortex properties and a size growing with the number of colors. These solutions live on a four torus T^4 with sizes ${l_L}^2 \times {l_S}^2$, being $l_L \gg l_S$, and satisfy twisted boundary conditions. Looking at the action density we have seen that are flat in the small directions and are localized in the two large directions, having a size which grows as the square root of the number of colors N. We calculate a square Wilson loop in the plane formed by the two large directions centered at the maximum of the solution, and observe that for a enough big size of the loop the value obtained is an element of the center of the group, $exp(-i2\pi/N)$.

Note that, if we consider the limit $l_L \to \infty$, we can say that the solutions live on $R^2 \times T^2$. And further on, if we repeat the solutions in the small directions , we can say that we have a solution of the SU(N) Yang-Mills equations of motion in R^4. In this case the solution will have infinite action and infinite topological charge, will be flat in two of the coordinates and localized in the other two, in a region of size approximately equal to the square root of the number of colors.

The relevant point of these solutions is that they are presenting the expected behaviour described in [3] for a confining configuration. They are bidimensional objects carrying a flux quantized in elements of the center of the group, and have a thickness which grows with the number of colors. Remember that, as we mention in the introduction, this finite thickness may be the explanation for the casimir scaling of the string tension, and also, that the growing of this thickness with the number of colors may take into account the confinement property of quarks in the adjoint representation which holds in the limit of large number of colors.

Another interesting point is the one presented in reference [18]. After going to the maximal center gauge and center projecting the configuration, the vortex solution for the SU(2) and SU(3) group appears as a projected vortex, the objects to which is attributed the confinement property. We expect that the same property holds for group SU(N) with $N > 3$, and then, these solutions will be identified as projected vortices after going to maximal center gauge and center projecting the configuration.

Finally, we want to say that our claim in this paper is not that our solution is the basic ingredient to explain the confinement property. Nevertheless, we think that this kind of configurations may play a role in the confinement mechanism, and, in any model based in a bunch of classical solutions as, for example, the instanton liquid model, these solutions must be included. This kind of model has been presented in [21] and some favourable results have been reported in [22, 23]. In this model the object which was included is also a fractional charge solution, in this case having point-like properties.

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Table 1: Set of studied solutions. We specify the number of colors N, the lattice size $(N_L)^2 \times (N_S)^2$ and the values of the action S (in $8\pi^2/N$ units), topological charge multiplied by N, electric part of the action S_e and magnetic part of the action S_b (both in $8\pi^2/N$ units).

N	$(N_L)^2 \times (N_S)^2$	$\frac{S\mathrm{N}}{8\pi^2}$	Q N	$\frac{S_e\mathrm{N}}{8\pi^2}$	$\frac{S_b\mathrm{N}}{8\pi^2}$
4	$16^2 \times 2^2$	0.96074	-0.95659	0.49696	0.46378
4	$24^2 \times 3^2$	0.99643	-0.99603	0.50412	0.49231
4	$32^2 \times 4^2$	0.99934	-0.99921	0.50313	0.49621
5	$20^2 \times 2^2$	0.98237	-0.98053	0.50211	0.48026
5	$30^2 \times 3^2$	0.99851	-0.99826	0.50392	0.49459
6	$24^2 \times 2^2$	0.99061	-0.98957	0.50380	0.48680
6	$36^2 \times 3^2$	0.99930	-0.99912	0.50351	0.49579
7	$28^2 \times 2^2$	0.99450	-0.99382	0.50439	0.49011
8	$32^2 \times 2^2$	0.99653	-0.99605	0.50445	0.49209
9	$36^2 \times 2^2$	0.99770	-0.99733	0.50439	0.49331
10	$40^2 \times 2^2$	0.99843	-0.99813	0.50420	0.49422

Figure 1: The energy profiles $\epsilon(t)$ for the three SU(4) solutions, each one obtained on a lattice of size $(8N_s)^2 \times (N_s)^2$, are shown as a function of t.

Figure 2: The energy profiles $\epsilon(t)$ multiplied by $N^{3/2}$ for the solutions with N=4,7,10 are shown as a function of $t/N^{1/2}$.

Figure 3: The phase $\phi(r)$ of the Wilson loop $W_C(r)$ is shown as a function of r for the three SU(4) solutions, each one obtained on a lattice with size $(8N_s)^2 \times (N_s)^2$.

Figure 4: The phase $\Phi = (2\pi - \phi(r)) \times N$ of the Wilson loop $W_C(r)$ is shown as a function of $r/N^{1/2}$ for the solutions with $N = 4, 7, 10$.