OCHA-PP-152
KEK-TH-686
May (2000)

Chamseddine, Fr$\ddot{o}$hlich, Grandjean Metric and Localized Higgs Coupling on Noncommutative Geometry

R.Kuriki and A.Sugamoto[♯]

Laboratory for Particle and Nuclear Physics,

High Energy Accelerator Research Organization (KEK),

Tsukuba, Ibaraki 305-0801, Japan

♯ *Department of Physics, Ochanomizu University*

1-1 Otsuka 2, Bunkyo-Ku Tokyo 112-0012, Japan

Keywords:

Noncommutative geometry, vielbein, Connes-Lott Higgs field, localized Higgs coupling.

Abstract

We propose a geometrized Higgs mechanism based on the gravitational sector in the Connes-Lott formulation of the standard model, which has been constructed by Chamseddine, Fröhlic and Grandjean. The point of our idea is that Higgs-like couplings depend on the local coordinates of the four-dimensional continuum, M_4. The localized couplings can be calculated by the Wilson loops of the $U(1)_{EM}$ gauge field and the connection, which is defined on $Z_2 \times M_4$.

1 Introduction

There are some proposals as constructive definitions of string theory. One of them is called "M-theory" which is limited to eleven-dimensional supergravity at low energy, and that corresponds to the strong coupling limit of the type-IIA superstring [1]. More concretely, M-theory is described by the IIA matrix model, which governs the dynamics of D0-branes. One of the others is the IIB matrix model, which has been proposed by Ishibashi, Kawai, Kitazawa and Tsuchiya [2]. Recently, the authors of [3] have shown that the twisted reduced model can be interpreted as noncommutative Yang-Mills theory. They have obtained noncommutative Yang-Mills theory with D-brane backgrounds in the IIB matrix model. Furthermore, some of the same authors have calculated the Wilson loop in noncommutative Yang-Mills, and have found an open string-like object [4], whose length is $|C^{\mu\nu}k_\nu|$. $C^{\mu\nu}$ delineates the noncommutative scale. In [5], a non-local basis, called bi-local basis, is introduced to provide a simple description for high-momentum $|k^\mu| > \lambda$, where λ is the spacing quanta and k^μ is the eigenvalue of adjoint $\hat{P}^\mu$. As can be recognized above, some new mathematical methods are required to more exactly investigate superstring theory. One of new mathematics is noncommutative geometry, which was introduced to physicists by a mathematician, Alain Connes [6].

There are many papers in which authors apply noncommutative geometry to rebuilding ordinary physics. The initiative is as follows. A trial for constructing the standard model, $U(1) \times SU(2) \times SU(3)$, has been partially successed in the framework of noncommutative geometry [7]. In [7] the base manifold is $E_4(M) \times Z_2$ and the algebra is $\mathcal{A}$=$C^\infty(M) \otimes \{M_2(C) \oplus M_1(C)\}$. The acting Hilbert space is given by that of spinors of $\begin{pmatrix} l\{\text{doublet}\} \\ e\{\text{singlet}\} \end{pmatrix}$. To introduce an $SU(3)$ gauge group for the quarks color, the authors have taken notice of the bimodule structure relating two algebras $(\mathcal{A}, \mathcal{B})$, where $\mathcal{A}$=algebra of quaternions$\oplus C$ and $\mathcal{B}$=$C \oplus M_3(C)$. Finally, the Higgs field appears as a gauge field between two points represented by Z_2. From the unification of the standard model and gravitational theory, this work is very meaningful, because it gives a geometrical interpretation of the standard model.

However, the Connes-Lott formulation [8] could not be beyond the standard model, as they have already described in the literature. For example, the generation matrix is given by

C^{N_G}; namely, the authors could not introduce the inside structure of the generation. In [6] A.Connes represents ultimate noncommutative space including the color symmetry $SU(3)$ as the 4-dimensional continuum times the finite space 'F'. What is the quantum group of the finite space 'F' ? This is one of problems which he discusses on the last page of [6], which one should solve when trying to make up a beyond standard model on noncommutative geometry. We have already recognized that we could not excel A.Connes in mathematical abilities. Hence, we attack the open problem from the opposite side to mathematicians.

We expect that noncommutative geometry may better extract the potential abilities of the existing physical models, for instance the standard model, though there are still a few papers which have suggested that noncommutative geometry can expose a new thing in ordinary things. In the future we would like to determine the masses of quarks based on geometric quantities on some noncommutative algebraic structure. Our purpose is to propose a gravitational model coupled with an $SU(2)$ doublet complex Higgs-like field. In this paper we artificially elevate the ordinary Higgs couplings to some components of one-form basis for our noncommutative space. We then impose the unitarity condition and the torsion-less condition for the basis of our noncommutative differential geometry as the authors of [8][9] have performed. Finally, in virtue of imposing these conditions, we will find that our Higgs-like couplings are represented by the Wilson loops of the spin connections of the noncommutative space and $U(1)_{EM}$ gauge fields. These gauge fields are expressed by the linear combinations of the $U(1)_Y$ and $SU(2)_W$ gauge fields. In section two we briefly review [8]. In section three we will introduce our vielbein while comparing it to Chamseddine, Frölich, Grandjean's metric. Our discussion is given in the last section. The unitarity conditions and the components of torsion are expressed in the appendices.

2 Review of Chamseddine, Fröhlich, Grandjean metric

In this section we introduce [8] in order to describe the underlying geometric structure of our proposal. The authors have studied the gravitational action of the noncommutative geometry underlying the above Connes-Lott construction of the standard model [8]. Ultimately, they have found the Einstein-Hilbert action in the leptonic sector and one in the quark sector. In these actions they have shown that the distance function between the two points is determined by a real scalar field, σ, whose vacuum expectation value sets the weak scale.

They set up the noncomutative differential geometry on two copies of the four-dimensional continuum, M_4, defined by

$$M_4 \times Z_2. \tag{1}$$

They choose the algebra $\mathcal{A}$ describing the above noncommutative space as

$$\mathcal{A} = (\mathcal{A}_1 \oplus \mathcal{A}_2) \otimes C^\infty(M_4), \tag{2}$$

where $\mathcal{A}_1 = M_2(C)$ and $\mathcal{A}_2 = C$. They firstly consider the leptonic part and the Higgs sector of the standard model.

$a \in \mathcal{A}$ is represented by

$$a = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \tag{3}$$

where a_1 (for $\mathcal{A}_1$),a_2 (for $\mathcal{A}_2$) are the C^∞ function on M_4. Notice that a_1 is a two times two matrix.

They define the representation space for $\mathcal{A}$ and the Hilbert space as follows:

$$\mathcal{H} = L^2(S_1, dv_1) \oplus L^2(S_2, dv_2), \tag{4}$$

where $S_i = S_0 \otimes V_i$. S_0 is the bundle of Dirac spinors on M_4 and V_i is a representation space for $\mathcal{A}_i$. They give the representation space as $V_1 = C^2, V_2 = C$. dv_i is the volume form corresponding to a Riemannian metric on M_4.

They express the Dirac operator on the noncommutative space as

$$D = \begin{pmatrix} \nabla_1 \otimes \mathbf{1}_2 \otimes \mathbf{1}_3 & \gamma^5 \otimes \mathbf{M}_{12} \otimes \mathbf{k} \\ \gamma^5 \otimes \mathbf{M}_{12}^* \otimes \mathbf{k}^* & \nabla_2 \otimes \mathbf{1}_3 \end{pmatrix}, \tag{5}$$

where ∇_i is defined by

$$\nabla_i = e^\mu_{ia} \gamma^a (\partial_\mu + i\omega_{i\mu}). \tag{6}$$

e^{μ}_{ia} is a vielbein of M_4. The index $i = 1, 2$ distinguishes $\mathcal{A}_i$ and μ represents the vector index on M_4; μ=0,1,2,3; a is the coordinate of the tangent bundle M_4; a=1,2,3,4. $\omega_{i\mu}$ is the spin connection in (1). $\{\gamma^a\}^4_{a=1}$ are the anti-hermitian Euclidean Dirac matrices, with $\{\ \gamma^a\ ,\ \gamma^b\ \} = \gamma^a\gamma^b + \gamma^b\gamma^a = -2\delta^{ab}$, $\gamma^5 = \gamma^1\gamma^2\gamma^3\gamma^4$, $\gamma^5 = (\gamma^5)^*$, and $-\gamma^a = (\gamma^a)^*$. γ^* means the hermitian conjugate. We follow their notation in the section three. The components of the spin connection are chosen based on the Cartan structure equation of (1) as the Riemannian geometry. k is a 3×3 family mixing matrix. In the leptonic sector the detailed structure inside of k is not necessary. M_{12}, called a doublet, is written as $M_{12} = \begin{pmatrix} \alpha(x) \\ \beta(x) \end{pmatrix}$. Functions $\alpha(x)$ and $\beta(x)$ are restricted by the requirements the consistency of $\Omega^2_D(\mathcal{A})$, which is isomorphism to $\pi(\Omega^2(\mathcal{A}))/\mathrm{Aux}^2$, where

$$\mathrm{Aux}^2 = \{\sum_i [D, a_i]\,[D, b_i] = 0 : \sum_i a_i\,[D, b_i] = 0\}. \tag{7}$$

In other words, we should pull the following elements a_i, b_i from $\mathcal{A}$:

$$\begin{aligned} \text{If} \quad \rho \quad &= \quad \sum_i a_i db_i \ \in \ \ker\pi \qquad \Longleftrightarrow \pi(\rho) = 0, \\ \text{then} \quad \pi(d\rho) \quad &= \quad \sum_i [D, a_i]\,[D, b_i] = 0. \end{aligned} \tag{8}$$

The authors find three possibilities which can satisfy Eq.(8), and in detail compute one of these in [8],

$$M_{12} = \exp(-\sigma) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{9}$$

where $\sigma(x)$ is a real scaler field.

They introduce a system of generators of $\tilde{\Omega}^1_D(\mathcal{A})$, $\{E^A\}$, which are suitable for the Hilbert space (4) and the representation space defined by themselves:

$$E^a \quad = \quad \gamma^a \begin{pmatrix} \mathbf{1}_2 & 0 \\ 0 & 1 \end{pmatrix}, \quad a = 1, 2, 3, 4, \tag{10}$$

$$E^r \quad = \quad \gamma^5 \begin{pmatrix} \mathbf{0}_2 & k e_r \\ -k^* e_r^\top & 0 \end{pmatrix}, \quad r = 5, 6, \tag{11}$$

where $e_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $e_6 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and $\top$ means the transposed matrix. [1]

[1] The authors of [8] define $\tilde{\Omega}(\mathcal{A}) := \tilde{\pi}(\Omega^{\cdot}(\mathcal{A}))$, $\tilde{\Omega}^n_D(\mathcal{A}) := \tilde{\pi}(\Omega^n(\mathcal{A}))/\tilde{\pi}(dJ_{n-1})$. Here J_n is the intersection of the kernel which is given by Eq.(8). $\tilde{\pi}$ is a *-representation of $\Omega^*(\mathcal{A})$.

They list up the conditions of vanishing torsion T^A, $A = 1, \ldots, 6$, where $T^A = dE^A + \Omega^A{}_B E^B$ and the unitarity conditions on (10) and (11). The components $\Omega^A{}_B$ which are defined by

$$\nabla E^A = -\Omega^A{}_B \otimes E^B, \tag{12}$$

are the connection coefficients on $\tilde{\Omega}^1_D(\mathcal{A})$ [8]. After imposing these conditions, they declare that the field σ becomes non-dynamical. They then weaken the condition of vanishing torsion as

$$Tr_k T^A = 0 \tag{13}$$

where Tr_k is the trace over the family mixing matrix. As the result of Eq.(13), the σ field behaves as a dynamical field in the Hilbert-Einstein action of the leptonic sector.

They also construct the gravitational action of the quark sector. It is the same form of action of the leptonic sector, but with different coefficients and with dependence on the generation-mixing matrices of the quarks masses. The action is given by

$$\int d^4x \left[-\frac{1}{2}(3c_l + 4c_q)R + \alpha(\nabla_a \sigma)^2 + \beta e^{-2\sigma} \right], \tag{14}$$

where c_l, c_q are arbitrary constants and α,β depends on the structure of the generation matrices of the quark masses and c_l, c_q [8]. However, as the authors of [8] described, since the gravitational action is non-renormalizable and nobody understands the quantum noncommutative geometry, these coefficients do not have any physical significance. R is the scalar curvature in $M_4 \times Z_2$.

In string theory we have known a theory whose action is almost the same form as (14). It is the Liouville field theory, where the action contains an exponential tachyon field, $\exp(\alpha_- x^1)$. Here, x^1 is one-direction of the target space, and the effective string coupling is diverging at large x^1. α_- is given by $(\frac{26-D}{6\alpha'})^{1/2} - (\frac{2-D}{6\alpha'})^{1/2}$. For $D > 2$ this is complex, hence this term oscillates, but for $D \leq 2$, $\alpha_- > 0$, we have a real exponential. [2] The difference between (14) and the Liouville action is that the latter contains the term $R \times x^1$. The difference between the sign of the exponential interactions can be eliminated by redefining of the field σ to $-\sigma$. What we want to stress here is that in string theory we may express the degree of freedom, x^1,

[2] In the discussion of the Liouville field theory, we have referred to the explanation on pp.323-327 of [10].

as a differential operator for some discrete space, since the authors of [8] can gain (14), which is similar to the Liouville action, from the gravitational theory on $M_4 \times Z_2$.

3 Our proposal- geometrized Higgs mechanism

An outline of our idea is that we would like to finally generalize an ordinary Higgs mechanism of the standard model to a gravitational theory on a noncommutative geometry. We attempt to raise the Higgs coupling from a constant to one of components of vielbein on noncommutative geometry, $M_4 \times Z_2$. Furthermore, we make our Higgs couplings [3] depend on the local coordinates of M_4. In the second subsection we impose the torsion-less condition and the unitarity condition on the vielbein and the spin connection of $M_4 \times Z_2$, following [8]. As a result, we find that the Higgs-like couplings can be calculated by the Wilson loops of the spin connection and the $U(1)_{EM}$ gauge.

3.1 Dynamical vielbein and localized Higgs couplings

We choose the algebra $\mathcal{A}$, defining the noncommutative space underlying our model as

$$\mathcal{A} = (\mathcal{A}_1 \oplus \mathcal{A}_2 \oplus \mathcal{A}_3) \otimes C^\infty(M_4), \tag{15}$$

where M_4 is a smooth, compact, four-dimensional Riemannian spin manifold; $\mathcal{A}_1$ is the algebra of complex 2×2 matrices. $\mathcal{A}_2$ and $\mathcal{A}_3$ are C. Elements, a, of $\mathcal{A}$ are written as

$$a = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}, \tag{16}$$

where a_i is a C^∞-function on M_4 with values in $\mathcal{A}_i$, $i = 1, 2, 3$.

The Hilbert space is defined as the spinors of the form $L = \begin{pmatrix} u_L \\ d_L \\ u_R \\ d_R \end{pmatrix}$, where R, L respectively mean the two kinds of chiralities which are defined by the four-dimensional γ^5. We want to regard u_R, d_R as an $SU(2)$ singlet and u_L, d_L as an $SU(2)$ doublet. Here, we image the Hilbert space of one-family quarks.

[3] We call our Higgs couplings Higgs-like couplings.

We introduce the following basis as

$$
\begin{aligned}
\mathcal{E}^a &= \gamma^a \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, && (17) \\
\mathcal{E}^{\bar{r}} &= \gamma^5 \begin{pmatrix} \mathbf{0} & \bar{\mathcal{A}} e_{\bar{r}} \\ -e^*_{\bar{r}} \bar{\mathcal{A}} & \mathbf{0} \end{pmatrix}, && (18)
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{\mathcal{A}} &= \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \\ \frac{v}{\sqrt{2}} & 0 \end{pmatrix}, && (19) \\
e_u = e_{\bar{u}} &= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad e_d = e_{\bar{d}} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, && (20)
\end{aligned}
$$

$$
e^*_u = e^*_{\bar{u}} = e_d = e_{\bar{d}}, \quad e^*_d = e^*_{\bar{d}} = e_u = e_{\bar{u}}. \tag{21}
$$

$a = 1, \ldots, 4$ and $\bar{r} = \bar{u}, \bar{d}$. We call this basis flat basis. (3.25) and (3.26) in [8] inspire us (10) and (18). v is a constant. We would like to first treat (17) and (18) as a system of generators of the one-form $\tilde{\Omega}^1(\mathcal{A})$ though (17) and (18) contain auxiliary parts as we will explain in the section 4.

Let us introduce a local basis $e^a_\mu(x)$, $\mu = 1, \ldots, 4$ of orthonormal tangent vectors to M_4 for each a. a is the local Lorentz index on the flat tangent plane. In curved M_4 we introduce gamma matrices by $\gamma^\mu \equiv e^\mu_a \gamma^a$ and define a curved base which we would like to regard as one-form of $\tilde{\Omega}^1(\mathcal{A})$,

$$
\mathcal{E}^\mu = \gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{22}
$$

Moreover, let us consider remaining components for the curved base. These contain Higgs-like couplings $f(x)$, $\tilde{f}(x)$, which depend on the local coordinates of M_4 as

$$
\mathcal{E}^r = \gamma^5 \begin{pmatrix} \mathbf{0} & \mathcal{A}^* e_r \\ -e^*_r \mathcal{A} & 0 \end{pmatrix}, \tag{23}
$$

where

$$
\mathcal{A} = \begin{pmatrix} 0 & f(x)\frac{v}{\sqrt{2}} \\ \tilde{f}(x)\frac{v}{\sqrt{2}} & 0 \end{pmatrix}, \quad \mathcal{A}^* = \begin{pmatrix} 0 & \tilde{f}^*(x)\frac{v}{\sqrt{2}} \\ f^*(x)\frac{v}{\sqrt{2}} & 0 \end{pmatrix}. \tag{24}
$$

We express the dual basis, following (3.29) of [8],

$$\omega = \left(\begin{array}{cccc} \gamma^\mu \omega_{1\mu,ij}(x) & & {\omega'}_2(x) & 0 \\ & & 0 & \omega_2(x) \\ {\omega'}_1(x) & 0 & \gamma^\mu \omega_{2\mu,ij}(x), & \\ 0 & \omega_1(x) & & \end{array} \right), \tag{25}$$

where $\omega_{1\mu,ij}(x)$ and $\omega_{2\mu,ij}(x)$ are two-times-two matrices, $i, j = 1, 2$.

Following (3.28) and (3.29) in [8], we write down the following connection coefficients of (22) and (23), which have been defined in (12), as

$$\Omega^A{}_B = \left(\begin{array}{cc} \gamma^\mu {\omega_{1\mu}}^A{}_{B,\,ij} & \mathcal{A}\gamma^5 e^{-\sigma} \left(\begin{array}{cc} {\omega'_2}^A{}_B & 0 \\ 0 & {\omega_2}^A{}_B \end{array} \right) \\ \left(\begin{array}{cc} {\omega'_1}^A{}_B & 0 \\ 0 & {\omega_1}^A{}_B \end{array} \right) \gamma^5 e^{-\sigma} \mathcal{A}^* & \gamma^\mu {\omega_{2\mu}}^A{}_{B,\,ij} \end{array} \right), \tag{26}$$

$$\Omega^A{}_B{}^* = \left(\begin{array}{cc} -\gamma^\mu {\omega_{1\mu}}^A{}_B{}^\top & \mathcal{A}\gamma^5 e^{-\sigma} \left(\begin{array}{cc} {\omega'_1}^A{}_B & 0 \\ 0 & {\omega_1}^A{}_B \end{array} \right) \\ \left(\begin{array}{cc} {\omega'_2}^A{}_B & 0 \\ 0 & {\omega_2}^A{}_B \end{array} \right) \gamma^5 e^{-\sigma} \mathcal{A}^* & -\gamma^\mu {\omega_{2\mu}}^A{}_B{}^\top \end{array} \right), \tag{27}$$

where $A, B, C = 1, 2, 3, 4, 5, 6$ and $5, 6$ respectively, refers to $\bar{u}, \bar{d}$.

We define the Dirac operator on the curved space, which is described by (22) and (23), as

$$\hat{D} = \left(\begin{array}{ccc} i\nabla_R & 0 & \gamma^5 \mathbf{M}^* \\ 0 & i\nabla_R & \\ & & \\ \gamma^5 \mathbf{M} & & i\nabla_L \end{array} \right), \tag{28}$$

$$\mathbf{M}^* = \left(\begin{array}{c} f^*(x)(\ \phi^{0*}\ ,\ \phi^+\ ) \\ \tilde{f}^*(x)(\ -\phi^-\ ,\ \phi^0\ ) \end{array} \right), \quad \mathbf{M} = \left(\begin{array}{cc} f(x) \left(\begin{array}{c} \phi^0 \\ \phi^- \end{array} \right) & \tilde{f}(x) \left(\begin{array}{c} -\phi^+ \\ \phi^{0*} \end{array} \right) \end{array} \right), \tag{29}$$

where ϕ^0 is the neutral Higgs-like field and $\phi^{+*} = \phi^-$.

$$i\nabla_R = \gamma^\mu (i\partial_\mu - \frac{g'}{2} Y B_\mu), \tag{30}$$

$$i\nabla_L = \gamma^\mu (i\partial_\mu - \frac{g}{2}\tau_a A_{a\mu} - \frac{g'}{2} Y B_\mu), \tag{31}$$

where $A_{a\mu}(x)$ is the $SU(2)_W$ gauge field and $B_\mu(x)$ is the $U(1)_Y$ gauge field. τ_a are the Pauli matrices. [4]

---

[4] $\tau_1 = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right)$, $\tau_2 = \left(\begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right)$, $\tau_3 = \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right)$.

3.2 On-shell Higgs couplings

Following [8] we impose the unitarity condition and the torsion-less condition on these gravitational components given in the previous subsection 3.1. First, we would like to impose the unitarity condition for the components. There are four possibilities from α to δ: α. (10) and (18), β. (10) and (23), γ. (22) and (18), δ. (22) and (23). We here treat the second case, β. In the curved quark space, u, d, the generalized Higgs couplings depend on the local coordinate, $f = f(x), \quad \tilde{f} = \tilde{f}(x)$.

In order to discuss the suitable Dirac operator, which we will calculate the unitarity condition, let us remember here the spontaneous symmetry breaking down in the standard model. In $SU(2)_W \times U(1)_Y$ non-abelian gauge theory, if one gives the following vacuum expectation value of the Higgs doublet field:

$$\langle 0|\Phi(x)|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{32}$$

then the mass terms of the gauge fields appear as $M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$ in the Lagrangian. Here, $W_\mu = \frac{1}{\sqrt{2}}(A_\mu^1 - iA_\mu^2)$ corresponds to the W boson and $Z_\mu = \cos\theta_W A_\mu^3 - \sin\theta_W B_\mu$ is the Z boson. From the action one can easily recognize that a linear combination of $SU(2)_W \times U(1)_Y$ gauge fields, $A_\mu = \sin\theta_W A_\mu^3 + \cos\theta_W B_\mu$, is still a mass-less gauge field.

Next, let us recall the leptonic sector after spontaneous symmetry breaking. The left-handed sector is $SU(2)_W$ doublet and the right-handed sector is $SU(2)_W$ singlet, because the charged current weak interaction of leptonic fields is the V-A interaction. When the vacuum expectation value of the Higgs doublet is (32), the masses of leptonic fields, m_j, are in proportion to the Higgs coupling as $f_j = \sqrt{2}m_j/v$. The masses of quarks are in proportion to the Higgs couplings, similar to the leptonic sector. The only different point between the leptonic sector and the quarks sector is that the lower quarks of the left handed ($SU(2)_W$ doublet) are not mass eigenstates.

We apply the following Dirac operator on our Hilbert space and we will impose the unitarity condition and the torsion-less condition on our bases,

$$\hat{D}_{EM} = \left(\begin{array}{cc} \begin{array}{cc} i\partial_{EM} & 0 \\ 0 & i\partial_{EM} \end{array} & \frac{v}{\sqrt{2}}\gamma^5 \left(\begin{array}{cc} f^*(x) & 0 \\ 0 & \tilde{f}^*(x) \end{array} \right) \\ \frac{v}{\sqrt{2}}\gamma^5 \left(\begin{array}{cc} f(x) & 0 \\ 0 & \tilde{f}(x) \end{array} \right) & \begin{array}{cc} i\partial_{EM} & 0 \\ 0 & i\partial_{EM} \end{array} \end{array} \right), \tag{33}$$

where

$$\begin{aligned} i\partial_{EM} &= \gamma^\mu(i\partial_\mu - LA_\mu^{EM}), \\ A_\mu^{EM} &= \sin\theta_W A_\mu^3 + \cos\theta_W B_\mu. \end{aligned} \tag{34}$$

L is the $U(1)_{EM}$ charge and θ_W is the Weinberg angle.

We summarize the unitarity condition and the components of torsion in appendices. From (68) and (69) in appendix A, we ultimately acquire

$$|\ \tilde{f}\ |^2(x) = C_1 \exp i \int^x dy^\nu (2\omega_{1\nu}{}^u{}_{u,\,11} - LA_\nu^{EM}) \quad (y), \tag{35}$$

$$|\ \tilde{f}\ |^2(x) = C_2 \exp i \int^x dy^\nu (2\omega_{2\nu}{}^u{}_{u,\,11} - LA_\nu^{EM})(y). \tag{36}$$

From (74) and (75) in the appendix, we obtain

$$|\ f\ |^2(x) = C_3 \exp i \int^x dy^\nu (2\omega_{1\nu}{}^d{}_{d,\,22} - LA_\nu^{EM}) \quad (y), \tag{37}$$

$$|\ f\ |^2(x) = C_4 \exp i \int^x dy^\nu (2\omega_{2\nu}{}^d{}_{d,\,22} - LA_\nu^{EM})(y). \tag{38}$$

In the left-hand side, $|\ f\ |^2$ and $|\ \tilde{f}\ |^2$, are real, we can actually observe these. On the other hand, we can calculate the right-hand sides of (76), (77), (78) and (79) as the Wilson loops of the spin connections, which is a kind of gauge field and the $U(1)_{EM}$ gauge field. In ordinary gauge theories and string theory it is well-known that the Wilson loops are one of the observables. Therefore, we may expect that these equations are geometrically meaningful for our noncommutative geometry, whose algebra is $\{M_2(C) \oplus C \oplus C\} \otimes C^\infty(M_4)$.

In the list of the components of the torsion given by the appendix B we find some solvable differential equations:

$$T_{42}^d = -\frac{v}{\sqrt{2}}(i\gamma^\mu\gamma^5\partial_\mu f \cdot - \left[\gamma^5, \gamma^\mu\right]_- f i\partial_{EM\mu}) - \frac{v}{\sqrt{2}} f\gamma^\mu\gamma^5\omega_{2\mu}{}^d{}_{d,22} = 0, \tag{39}$$

$$T_{24}^d = \gamma^\mu\gamma^5\frac{v}{\sqrt{2}} f^*\omega_{1\mu}{}^d{}_{d,22} + \frac{v}{\sqrt{2}}(i\gamma^\mu\gamma^5\partial_\mu f^* \cdot - \left[\gamma^5, \gamma^\mu\right]_- f^* i\partial_{EM\mu}) = 0, \tag{40}$$

$$T^u_{13} = \gamma^\mu\gamma^5 \frac{v}{\sqrt{2}}\omega_{1\mu}{}^u{}_{u,11}\tilde{f}^* + i\frac{v}{\sqrt{2}}\gamma^\mu\gamma^5\partial_\mu\tilde{f}^* \cdot +i\frac{v}{\sqrt{2}}\tilde{f}^*\left[\gamma^\mu,\gamma^5\right]_- \partial_{EM\mu} = 0, \tag{41}$$

$$T^u_{31} = -i\frac{v}{\sqrt{2}}\gamma^\mu\gamma^5\partial_\mu\tilde{f}\cdot - \frac{v}{\sqrt{2}}\tilde{f}\left[\gamma^\mu,\gamma^5\right]_- i\partial_{EM\mu} - \gamma^\mu\gamma^5\tilde{f}\frac{v}{\sqrt{2}}\omega_{2\mu}{}^u{}_{u,11} = 0, \tag{42}$$

where '$\cdot$' means that the differential operator, ∂_μ, only acts on the next f.

If at a quantum level we imposed the torsion-less condition on the following ground state

$$\partial_\mu|0>=0, \tag{43}$$

Solutions of Eq.(39)-Eq.(42) would be given by:

$$f^*(x) = Q_1 : \exp i\int^x dy^\nu(\hat{\omega}_{1\nu}{}^d{}_{d,22} - 2L\hat{A}^{EM}_\nu)(y) :, \tag{44}$$

$$f(x) = Q_2 : \exp i\int^x dy^\nu(\hat{\omega}_{2\nu}{}^d{}_{d,22} - 2L\hat{A}^{EM}_\nu)(y) :, \tag{45}$$

$$\tilde{f}^*(x) = Q_3 : \exp i\int^x dy^\nu(\hat{\omega}_{1\nu}{}^u{}_{u,11} - 2L\hat{A}^{EM}_\nu)(y) :, \tag{46}$$

$$\tilde{f}(x) = Q_4 : \exp i\int^x dy^\nu(\hat{\omega}_{2\nu}{}^u{}_{u,11} - 2L\hat{A}^{EM}_\nu)(y) :, \tag{47}$$

where Q_i, $i = 1,\ldots,4$ are arbitrary constants. $\hat{\omega}$, $\hat{A}^{EM}_\nu$ denote operators at the quantum level. : expresses a normal-ordering. This normal ordering should be defined which can be consistent with (76), (77), (78) and (79).

4 Conclusion and discussion

We have proposed a gravitational model coupled with $SU(2)$ doublet complex Higgs-like fields whose couplings depend on the local coordinates of M_4. As the results of some on-shell conditions (the unitarity condition and the torsion-less condition)in the noncommutative space, these Higgs couplings have been represented by the Wilson loops of the connections and the gauge fields, (76), (77), (78) and (79). Moreover if we use these equations, we can *geometrically* present the conditions that the Higgs couplings become zero. The choices of the topological properties for the four dimensional continuum M_4 are not free, but are restricted by the other conditions,

$$|\tilde{f}|^2 = -\frac{2}{v^2}\frac{\omega_{1\mu}{}^{ua}{}_{,11}}{\omega_{1\mu}{}^a{}_{u,11}} = -\frac{2}{v^2}\frac{\omega_{1\mu}{}^{ua}{}_{,12}}{\omega_{1\mu}{}^a{}_{u,21}} = \ldots$$

Notice that these restrictions contain the connections which the spread over the noncommutative space $M_4 \times Z_2$. Therefore, we wish to stress in this paper that if we construct the standard model on the non-commutative space and we elevate the Higgs couplings to some geometrical objects on the noncommutative space, we would gain a system beyond the standard model, which could represent the masses of leptons and quarks as geometrical things of noncommutative space.

Lastly, we confess what we should improve in our construction. We have not decomposed the components of our bases containing the local Higgs couplings as the authors have performed in [8]. They have described the decomposition of their one-form as $E^r = e^\sigma [D, m^r]$, where $m^5 = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $m^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. In other words, we have not exactly presented the one-form of $\tilde{\Omega}^1_D(\mathcal{A})$ for our algebra (15). The one-form $\tilde{\Omega}^1_D(\mathcal{A})$ should be represented by a zero-form times a commutator between a zero-form and a Dirac operator. After excluding the auxiliary part of our base given in this paper, a final description of these conditions with respect to the Higgs-like couplings would be changed. However if we add a one-form of some algebra, we may revive the expressions of the above Wilson loops.

We hope that our attempt will be one of the advantages to found a formulation beyond the standard model.

Acknowledgments

We would like to thank Professor Watamura for giving his seminar at our university and for stimulating discussion. One of the authors (R.K) would also like to thank him for discussing about [8] and [9]. After submitting version 2 of this paper to arXiv, R.K. had a chance to talk about [7]and [8] to some members of theoretical physics group in KEK. They commented on the renormalizability of noncommutative field theories. R.K. would like to thank them for the valuable discussions and comments. R.K. is supported by the Research Fellowships of the Japan Society for the Promotion of Science.

A Unitarity conditions

We calculate the unitarity condition by using

$$d_{EM}\langle\mathcal{E}^A,\mathcal{E}^B\rangle_D = -\Omega^A{}_C\langle\mathcal{E}^C,\mathcal{E}^B\rangle_D + \langle\mathcal{E}^A,\mathcal{E}^C\rangle_D(\Omega^B{}_C)^*. \tag{48}$$

In order to calculate the unitarity condition of the bases and the spin connection, we need to estimate the generalized hermitian inner product between the above one-form bases as (3.30) and (3.31) in [8],

$$\langle\mathcal{E}^A,\mathcal{E}^B\rangle_D = -\frac{1}{2}\{\gamma(A),\gamma(B)\}_+\hat{\mathcal{E}}^A\hat{\mathcal{E}}^B, \tag{49}$$

where $\{\gamma(A),\gamma(B)\}_+$ means the anti-commutator between two gamma matrixes which are respectively containing $\mathcal{E}^A$ and $\mathcal{E}^B$. $\hat{\mathcal{E}}^B$ means $\mathcal{E}^B$ removing the gamma matrix, γ^B. The inner products between each (10), (18) (22) and (23) are respectively expressed by

$$\begin{aligned}
\langle\mathcal{E}^a,\mathcal{E}^b\rangle_D &= \delta^{ab},\\
\langle\mathcal{E}^{\bar{u}},\mathcal{E}^{\bar{u}}\rangle_D &= \frac{v^2}{2}\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix},\\
\langle\mathcal{E}^{\bar{d}},\mathcal{E}^{\bar{d}}\rangle_D &= \frac{v^2}{2}\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix},\\
\langle\mathcal{E}^{\bar{u}},\mathcal{E}^{\bar{d}}\rangle_D &= \langle\mathcal{E}^{\bar{d}},\mathcal{E}^{\bar{u}}\rangle_D = 0.
\end{aligned} \tag{50}$$

For the bases of the curved space:

$$\langle\mathcal{E}^\mu,\mathcal{E}^\nu\rangle_D = g^{\mu\nu}(x), \quad g^{\mu\nu} = e^\mu_a e^\nu_b\delta^{ab}, \tag{51}$$

$$\langle\mathcal{E}^u,\mathcal{E}^u\rangle_D = |\,\tilde{f}(x)\,|^2\frac{v^2}{2}\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix}, \tag{52}$$

$$\langle\mathcal{E}^d,\mathcal{E}^d\rangle_D = |\,f(x)\,|^2\frac{v^2}{2}\begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix}, \tag{53}$$

$$\langle\mathcal{E}^u,\mathcal{E}^d\rangle_D = \langle\mathcal{E}^d,\mathcal{E}^u\rangle_D = 0. \tag{54}$$

$(\mathbf{A}, \mathbf{B}) = (a, b)$

$$\begin{aligned} {\omega_{1\mu}}^{ab},_{11} + {\omega_{1\mu}}^{ba},_{11} &= 0, \\ {\omega_{1\mu}}^{ab},_{12} + {\omega_{1\mu}}^{ba},_{21} &= 0, \\ {\omega_2}^{ab} - {\omega_1}^{ba} = 0, \end{aligned} \tag{55}$$

$$\begin{aligned} {\omega_{1\mu}}^{ab},_{21} + {\omega_{1\mu}}^{ba},_{12} &= 0, \\ {\omega_{1\mu}}^{ab},_{22} + {\omega_{1\mu}}^{ba},_{22} &= 0, \\ {\omega'_2}^{ab} - {\omega'_1}^{ab} = 0, \end{aligned} \tag{56}$$

$$\begin{aligned} {\omega'_1}^{ab} - {\omega'_2}^{ba} &= 0, \\ {\omega_{2\mu}}^{ab},_{11} + {\omega_{2\mu}}^{ba},_{11} &= 0, \\ {\omega_{2\mu}}^{ab},_{12} + {\omega_{2\mu}}^{ba},_{21} = 0, \end{aligned} \tag{57}$$

$$\begin{aligned} {\omega_1}^{ab} - {\omega_2}^{ba} &= 0, \\ {\omega_{2\mu}}^{ab},_{21} + {\omega_{2\mu}}^{ba},_{12} &= 0, \\ {\omega_{2\mu}}^{ab},_{22} + {\omega_{2\mu}}^{ba},_{22} = 0. \end{aligned} \tag{58}$$

$(\mathbf{A}, \mathbf{B}) = (a, u)$

$$\begin{aligned} \mid \tilde{f} \mid^2 \frac{v^2}{2} {\omega_{1\mu}}^{a}{}_{u},_{11} + {\omega_{1\mu}}^{ua},_{11} &= 0, \\ {\omega_{1\mu}}^{ua},_{21} = 0, \quad {\omega_1}^{ua} = 0, \\ \mid \tilde{f} \mid^2 \frac{v^2}{2} {\omega_{1\mu}}^{a}{}_{u},_{21} + {\omega_{1\mu}}^{ua},_{12} &= 0, \\ -\mid \tilde{f} \mid^2 \frac{v^2}{2} {\omega'_2}^{a}{}_{u} + {\omega'_1}^{ua} &= 0, \\ {\omega'_2}^{ua} &= 0, \\ \mid \tilde{f} \mid^2 \frac{v^2}{2} {\omega_{2\mu}}^{a}{}_{u},_{11} + {\omega_{2\mu}}^{ua},_{11} &= 0, \\ {\omega_{2\mu}}^{ua},_{21} &= 0, \\ -\mid \tilde{f} \mid^2 \frac{v^2}{2} {\omega_1}^{a}{}_{u} + {\omega_2}^{ua} &= 0, \\ \mid \tilde{f} \mid^2 \frac{v^2}{2} {\omega_{2\mu}}^{a}{}_{u},_{21} + {\omega_{2\mu}}^{ua},_{12} &= 0, \\ {\omega_{2\mu}}^{ua},_{22} = 0. \end{aligned} \tag{59}$$

$(\mathbf{A},\mathbf{B})=(a,d)$

$$\begin{aligned}
{\omega_{1\mu}}^{da},_{11} &= 0, \\
\mid f\mid^2\frac{v^2}{2}{\omega_{1\mu}}^a{}_d,_{12}+{\omega_{1\mu}}^{6a},_{21} &= 0, \\
-\mid f\mid^2\frac{v^2}{2}{\omega_2}^a{}_d+{\omega_1}^{da}=0,
\end{aligned} \tag{60}$$

$$\begin{aligned}
{\omega_{1\mu}}^{da},_{12} &= 0, \\
\mid f\mid^2\frac{v^2}{2}{\omega_{1\mu}}^a{}_d,_{22}+{\omega_{1\mu}}^{da},_{22} &= 0, \\
{\omega'_1}^{da}=0,
\end{aligned} \tag{61}$$

$$\begin{aligned}
-\mid f\mid^2\frac{v^2}{2}{\omega'_1}^a{}_d+{\omega'_2}^{da} &= 0, \\
{\omega_{2\mu}}^{da},_{11} &= 0, \\
\mid f\mid^2\frac{v^2}{2}{\omega_{2\mu}}^a{}_d,_{12}+{\omega_{2\mu}}^{da},_{21}=0,
\end{aligned} \tag{62}$$

$$\begin{aligned}
{\omega_2}^{da}=0,\quad {\omega_{2\mu}}^{da},_{12}=0, \\
\mid f\mid^2\frac{v^2}{2}{\omega_{2\mu}}^a{}_d,_{22}+{\omega_{2\mu}}^{da},_{22}=0.
\end{aligned} \tag{63}$$

$(\mathbf{A},\mathbf{B})=(u,b)$

$$\begin{aligned}
{\omega_{1\mu}}^{ub},_{11}+\mid \tilde{f}\mid^2\frac{v^2}{2}{\omega_{1\mu}}^b{}_u,_{11} &= 0, \\
{\omega_{1\mu}}^{ub},_{12}+\mid \tilde{f}\mid^2\frac{v^2}{2}{\omega_{1\mu}}^b{}_u,_{21} &= 0, \\
-{\omega_2}^{ub}+\mid \tilde{f}\mid^2\frac{v^2}{2}{\omega_1}^b{}_u=0,
\end{aligned} \tag{64}$$

$$\begin{aligned}
{\omega_{1\mu}}^{ub},_{21} &= 0, \\
{\omega_{1\mu}}^{ub},_{22} &= 0, \\
{\omega'_2}^{ub}=0,
\end{aligned} \tag{65}$$

$$\begin{aligned}
-{\omega'_1}^{ub}+\mid \tilde{f}\mid^2\frac{v^2}{2}{\omega'_2}^b{}_u &= 0, \\
{\omega_{2\mu}}^{ub},_{11}-\mid \tilde{f}\mid^2\frac{v^2}{2}{\omega_{2\mu}}^b{}_u,_{11} &= 0, \\
{\omega_{2\mu}}^{ub},_{12}+\mid \tilde{f}\mid^2\frac{v^2}{2}{\omega_{2\mu}}^b{}_u,_{21}=0,
\end{aligned} \tag{66}$$

$$
\begin{aligned}
{\omega_1}^{ub} &= 0, \\
{\omega_{2\mu}}^{ub}{}_{,\,21} &= 0, \\
{\omega_{2\mu}}^{ub}{}_{,\,22} = 0. && (67)
\end{aligned}
$$

$(\mathbf{A}, \mathbf{B}) = (u, u)$

$$
\begin{aligned}
i\partial_\mu^{EM} |\ \tilde{f}\ |^2 \frac{v^2}{2} \cdot + v^2 |\ \tilde{f}\ |^2 {\omega_{1\mu}}^{u}{}_{u,\,11} &= 0, && (68) \\
{\omega_{1\mu}}^{u}{}_{u,\,21} &= 0, \\
{\omega_1}^{u}{}_{u} &= 0, \\
{\omega'_2}^{u}{}_{u} &= 0, \\
i\partial_\mu^{EM} |\ \tilde{f}\ |^2 \frac{v^2}{2} \cdot + v^2 |\ \tilde{f}\ |^2 {\omega_{2\mu}}^{u}{}_{u,\,11} &= 0, && (69) \\
{\omega_{2\mu}}^{u}{}_{u,\,21} = 0. && (70)
\end{aligned}
$$

$(\mathbf{A}, \mathbf{B}) = (u, d)$

$$
\begin{aligned}
{\omega_{1\mu}}^{d}{}_{u,\,11} &= 0, \\
|\ f\ |^2 {\omega_{1\mu}}^{u}{}_{d,\,12} + |\ \tilde{f}\ |^2 {\omega_{1\mu}}^{d}{}_{u,\,21} &= 0, \\
-|\ f\ |^2 {\omega_2}^{u}{}_{d} + |\ \tilde{f}\ |^2 {\omega_1}^{d}{}_{u} &= 0, \\
{\omega_{1\mu}}^{u}{}_{d,\,22} &= 0, \\
-|\ f\ |^2 {\omega'_1}^{u}{}_{d} + |\ \tilde{f}\ |^2 {\omega'_2}^{d}{}_{u} &= 0, \\
{\omega_{2\mu}}^{d}{}_{u,\,11} &= 0, \\
|\ f\ |^2 {\omega_{2\mu}}^{u}{}_{d,\,12} + |\ \tilde{f}\ |^2 {\omega_{2\mu}}^{d}{}_{u,\,21} &= 0, \\
{\omega_{2\mu}}^{u}{}_{d,\,22} = 0. && (71)
\end{aligned}
$$

$(\mathbf{A}, \mathbf{B}) = (d, b)$

$$
\begin{aligned}
{\omega_{1\mu}}^{db}{}_{,\,11} = {\omega_{1\mu}}^{db}{}_{,\,12} &= {\omega_2}^{db} = 0, \\
{\omega_{1\mu}}^{db}{}_{,\,21} + |\ f\ |^2 \frac{v^2}{2} {\omega_{1\mu}}^{b}{}_{d,\,12} &= 0, \\
{\omega_{1\mu}}^{db}{}_{,\,22} + |\ f\ |^2 \frac{v^2}{2} {\omega_{1\mu}}^{b}{}_{d,\,22} &= 0, \\
-{\acute{\omega}_2}^{db} + |\ f\ |^2 \frac{v^2}{2} {\omega'_1}^{b}{}_{d} &= 0,
\end{aligned}
$$

$$\begin{aligned} {\omega'_1}^{db} = \omega^{db}_{2\mu},{}_{11} = {\omega_{2\mu}}^{db},{}_{12} &= 0, \\ -{\omega_1}^{db} + \mid f\mid^2 \frac{v^2}{2} {\omega_2}^b{}_d &= 0, \\ {\omega_{2\mu}}^{db},{}_{21} + \mid f\mid^2 \frac{v^2}{2} {\omega_{2\mu}}^b{}_d,{}_{12} &= 0, \\ {\omega_{2\mu}}^{db},{}_{22} + \mid f\mid^2 \frac{v^2}{2} {\omega_{2\mu}}^b{}_d,{}_{22} = 0. \end{aligned} \tag{72}$$

$(\mathbf{A}, \mathbf{B}) = (d, u)$

$$\begin{aligned} {\omega_{1\mu}}^d{}_u,{}_{11} &= 0, \\ \mid \tilde{f}\mid^2 {\omega_{1\mu}}^d{}_u,{}_{21} + \mid f\mid^2 {\omega_{1\mu}}^u{}_d,{}_{12} &= 0, \\ {\omega_{1\mu}}^u{}_d,{}_{22} &= 0, \\ -\mid \tilde{f}\mid^2 {\omega'_2}^d{}_u + \mid f\mid^2 {\omega'_1}^u{}_d &= 0, \\ {\omega_{2\mu}}^d{}_u,{}_{11} &= 0, \\ -\mid \tilde{f}\mid {\omega_1}^d{}_u + \mid f\mid^2 {\omega_2}^u{}_d &= 0, \\ \mid \tilde{f}\mid^2 {\omega_{2\mu}}^d{}_u,{}_{21} + \mid f\mid^2 {\omega_{2\mu}}^u{}_d,{}_{12} &= 0, \\ {\omega_{2\mu}}^u{}_d,{}_{22} = 0. \end{aligned} \tag{73}$$

$(\mathbf{A}, \mathbf{B}) = (d, d)$

$$\begin{aligned} {\omega_{1\mu}}^d{}_d,{}_{12} = {\omega_2}^d{}_d &= 0, \\ i\partial^{EM}_\mu \mid f\mid^2 \frac{v^2}{2} \cdot + \mid f\mid^2 v^2 {\omega_{1\mu}}^d{}_d,{}_{22} &= 0, \end{aligned} \tag{74}$$

$$\begin{aligned} {\omega'_1}^d{}_d &= 0, \\ {\omega_{2\mu}}^d{}_d,{}_{12} &= 0, \\ i\partial^{EM}_\mu \mid f\mid^2 \frac{v^2}{2} \cdot + \mid f\mid^2 v^2 {\omega_{2\mu}}^d{}_d,{}_{22} = 0. \end{aligned} \tag{75}$$

From (68) and (69), we obtain

$$\mid \tilde{f}\mid^2(x) = C_1 \exp i \int^x dy^\nu (2{\omega_{1\nu}}^u{}_u,{}_{11} - LA^{EM}_\nu) \quad (y), \tag{76}$$

$$\mid \tilde{f}\mid^2(x) = C_2 \exp i \int^x dy^\nu (2{\omega_{2\nu}}^u{}_u,{}_{11} - LA^{EM}_\nu)(y). \tag{77}$$

From (74) and (75), we obtain

$$\mid f\mid^2(x) = C_3 \exp i \int^x dy^\nu (2{\omega_{1\nu}}^d{}_d,{}_{22} - LA^{EM}_\nu) \quad (y), \tag{78}$$

$$| \ f \ |^2(x) = C_4 \exp i \int^x dy^\nu (2\omega_{2\nu}{}^d{}_{d,\,22} - LA_\nu^{EM})(y). \tag{79}$$

B Components of torsion

We have calculated the torsion $T^A = dE^A + \Omega^A{}_B E^B$ where $dE^A = \left[\hat{D}_{EM} \ , \ E^A\right]$ and (23) [5]. The each component of T^A is given by

$$\begin{aligned}
T^a_{11} &= \gamma^\mu \gamma^b \omega_{1\mu}{}^a{}_{b,11} + i\gamma^\mu \partial_\mu \gamma^a \cdot + [\gamma^\mu, \gamma^a]_- \, i\partial_\mu^{EM} \\
T^a_{12} &= \gamma^\mu \gamma^b \omega_{1\mu}{}^a{}_{b,12} - \mathbf{1}_{4\times4} \exp(-\sigma) f^2 \frac{v^2}{2} \omega_2{}^a{}_d \\
T^a_{13} &= \gamma^\mu \gamma^5 \frac{v}{\sqrt{2}} \omega_{1\mu}{}^a{}_{u,11} \tilde{f}^* + \frac{v}{\sqrt{2}} \left[\gamma^5, \gamma^a\right]_- f^* \\
T^a_{14} &= \gamma^\mu \gamma^5 \frac{v}{\sqrt{2}} f^* \omega_{1\mu}{}^a{}_{d,12} + \frac{v}{\sqrt{2}} f \gamma^5 \gamma^b \exp(-\sigma) \omega_2{}^a{}_b \\
T^a_{21} &= \gamma^\mu \gamma^b \omega_{1\mu}{}^a{}_{b,21} - \mathbf{1}_{4\times4} \exp(-\sigma) \frac{v^2}{2} \tilde{f}^2 \omega'_2{}^a{}_u \\
T^a_{22} &= \gamma^\mu \gamma^b \omega_{1\mu}{}^a{}_{b,22} + \gamma^\mu i\partial_\mu \gamma^a \cdot + [\gamma^\mu, \gamma^a]_- \, i\partial_\mu^{EM} \\
T^a_{23} &= \gamma^5 \gamma^b \exp(-\sigma) \tilde{f} \frac{v}{\sqrt{2}} \omega'_2{}^a{}_b + \gamma^\mu \gamma^5 \omega_{1\mu}{}^a{}_{u,21} \frac{v}{\sqrt{2}} \tilde{f}^* \\
T^a_{24} &= \gamma^\mu \gamma^5 \frac{v}{\sqrt{2}} f^* \omega_{1\mu}{}^a{}_{d,22} + \frac{v}{\sqrt{2}} \tilde{f}^* \left[\gamma^5, \gamma^a\right]_- \\
T^a_{31} &= -\gamma^\mu \gamma^5 \tilde{f} \frac{v}{\sqrt{2}} \omega_{2\mu}{}^a{}_{u,11} + \frac{v}{\sqrt{2}} \left[\gamma^5, \gamma^a\right]_- f \\
T^a_{32} &= \omega'_1{}^a{}_b \tilde{f}^* \frac{v}{\sqrt{2}} \gamma^5 \gamma^b \exp(-\sigma) - \frac{v}{\sqrt{2}} f \gamma^\mu \gamma^5 \omega_{2\mu}{}^a{}_{d,12} \\
T^a_{33} &= i\gamma^\mu \partial_\mu \gamma^a \cdot + [\gamma^\mu, \gamma^a]_- \, i\partial_\mu^{EM} + \gamma^\mu \gamma^b \omega_{2\mu}{}^a{}_{b,11} \\
T^a_{34} &= \gamma^\mu \gamma^b \omega_{2\mu}{}^a{}_{b,12} + \mathbf{1}_{4\times4} \exp(-\sigma) \frac{v^2}{2} f^* \tilde{f}^* \omega'_1{}^a{}_d \\
T^a_{41} &= \omega_1{}^a{}_b f^* \frac{v}{\sqrt{2}} \gamma^5 \gamma^b \exp(-\sigma) - \frac{v}{\sqrt{2}} \gamma^\mu \gamma^5 \tilde{f} \omega_{2\mu}{}^a{}_{u,21} \\
T^a_{42} &= \frac{v}{\sqrt{2}} \left[\gamma^5, \gamma^a\right]_- \tilde{f} - \frac{v}{\sqrt{2}} f \gamma^\mu \gamma^5 \omega_{2\mu}{}^a{}_{d,22} \\
T^a_{43} &= \gamma^\mu \gamma^b \omega_{2\mu}{}^a{}_{b,21} + \mathbf{1}_{4\times4} \exp(-\sigma) \tilde{f}^* f^* \frac{v^2}{2} \omega_1{}^a{}_u \\
T^a_{44} &= \gamma^\mu \gamma^b \omega_{2\mu}{}^a{}_{b,22} + \gamma^\mu i\partial_\mu \gamma^a \cdot + [\gamma^\mu, \gamma^a]_- \, i\partial_\mu^{EM}
\end{aligned} \tag{80}$$

$$T^d_{11} = \gamma^\mu \gamma^b \omega_{1\mu}{}^d{}_{b,11}$$

[5] As we described in the discussion, we did not eliminate the auxiliary part which has been defined by (7) and (8) from (22)

$$T^d_{12} = \gamma^\mu\gamma^b\omega_{1\mu}{}^d{}_{b,12} - \mathbf{1}_{4\times 4}\exp(-\sigma)f^2\frac{v^2}{2}\omega_2{}^d{}_d$$
$$T^d_{13} = \gamma^\mu\gamma^5\frac{v}{\sqrt{2}}\omega_{1\mu}{}^d{}_{u,11}\tilde{f}^*$$
$$T^d_{14} = \gamma^\mu\gamma^5\frac{v}{\sqrt{2}}f^*\omega_{1\mu}{}^d{}_{d,12}$$
$$T^d_{21} = \gamma^\mu\gamma^b\omega_{1\mu}{}^d{}_{b,21} - \mathbf{1}_{4\times 4}\exp(-\sigma)\frac{v^2}{2}\omega'_2{}^d{}_u$$
$$T^d_{22} = -\frac{v^2}{2}(\tilde{f}^*f + f^*\tilde{f})\mathbf{1}_{4\times 4}$$
$$T^d_{23} = \gamma^5\gamma^b\exp(-\sigma)\tilde{f}\frac{v}{\sqrt{2}}\omega'_2{}^d{}_b + \gamma^\mu\gamma^5\omega_{1\mu}{}^d{}_{u,21}\frac{v}{\sqrt{2}}\tilde{f}^*$$
$$T^d_{24} = \gamma^\mu\gamma^5\frac{v}{\sqrt{2}}f^*\omega_{1\mu}{}^d{}_{d,22} + \frac{v}{\sqrt{2}}(i\gamma^\mu\gamma^5\partial_\mu f^* \cdot - \left[\gamma^5,\gamma^\mu\right]_- f^* i\partial_{EM\mu})$$
$$T^d_{31} = -\gamma^\mu\gamma^5\tilde{f}\frac{v}{\sqrt{2}}\omega_{2\mu}{}^d{}_{u,11}$$
$$T^d_{32} = \omega'_1{}^d{}_b\tilde{f}^*\frac{v}{\sqrt{2}}\gamma^5\gamma^b\exp(-\sigma) - \frac{v}{\sqrt{2}}f\gamma^\mu\gamma^5\omega_{2\mu}{}^d{}_{d,12}$$
$$T^d_{33} = \gamma^\mu\gamma^b\omega_{2\mu}{}^d{}_{b,11}$$
$$T^d_{34} = \gamma^\mu\gamma^b\omega_{2\mu}{}^d{}_{b,12} + \mathbf{1}_{4\times 4}\exp(-\sigma)\frac{v^2}{2}f^*\tilde{f}^*\acute{\omega}_1{}^d{}_d$$
$$T^d_{41} = \omega_1{}^d{}_b f^*\frac{v}{\sqrt{2}}\gamma^5\gamma^b\exp(-\sigma) - \frac{v}{\sqrt{2}}\gamma^\mu\gamma^5\tilde{f}\omega_{2\mu}{}^d{}_{u,21}$$
$$T^d_{42} = -\frac{v}{\sqrt{2}}(i\gamma^\mu\gamma^5\partial_\mu f \cdot - \left[\gamma^5,\gamma^\mu\right]_- f i\partial_{EM\mu}) - \frac{v}{\sqrt{2}}f\gamma^\mu\gamma^5\omega_{2\mu}{}^d{}_{d,22}$$
$$T^d_{43} = \gamma^\mu\gamma^b\omega_{2\mu}{}^d{}_{b,21} + \mathbf{1}_{4\times 4}\exp(-\sigma)\tilde{f}^*f^*\frac{v^2}{2}\omega_1{}^d{}_u$$
$$T^d_{44} = \gamma^\mu\gamma^b\omega_{2\mu}{}^d{}_{b,22} + \frac{v^2}{2}(\tilde{f}f^* + f\tilde{f}^*)\mathbf{1}_{4\times 4} \tag{81}$$

$$T^u_{11} = -\frac{v^2}{2}\mathbf{1}_{4\times 4}(f^*\tilde{f} + \tilde{f}^*f) + \gamma^\mu\gamma^b\omega_{1\mu}{}^u{}_{b,11}$$
$$T^u_{12} = \gamma^\mu\gamma^b\omega_{1\mu}{}^u{}_{b,12} - \mathbf{1}_{4\times 4}\exp(-\sigma)f^2\frac{v^2}{2}\omega_2{}^u{}_d$$
$$T^u_{13} = \gamma^\mu\gamma^5\frac{v}{\sqrt{2}}\omega_{1\mu}{}^u{}_{u,11}\tilde{f}^* + i\frac{v}{\sqrt{2}}\gamma^\mu\gamma^5\partial_\mu\tilde{f}^* \cdot + i\frac{v}{\sqrt{2}}\tilde{f}^*\left[\gamma^\mu,\gamma^5\right]_-\partial_{EM\mu}$$
$$T^u_{14} = \gamma^\mu\gamma^5\frac{v}{\sqrt{2}}f^*\omega_{1\mu}{}^u{}_{d,12} + f\frac{v}{\sqrt{2}}\gamma^5\gamma^b\exp(-\sigma)\omega_2{}^u{}_b$$
$$T^u_{21} = \gamma^\mu\gamma^b\omega_{1\mu}{}^u{}_{b,21} - \mathbf{1}_{4\times 4}\exp(-\sigma)\frac{v^2}{2}\tilde{f}^2\omega'_2{}^u{}_u$$
$$T^u_{22} = \gamma^\mu\gamma^b\omega_{1\mu}{}^u{}_{b,22}$$
$$T^u_{23} = \gamma^5\gamma^b\exp(-\sigma)\tilde{f}\frac{v}{\sqrt{2}}\omega'_2{}^u{}_b + \gamma^\mu\gamma^5\omega_{1\mu}{}^u{}_{u,21}\frac{v}{\sqrt{2}}\tilde{f}^*$$

$$
\begin{aligned}
T^u_{24} &= \gamma^\mu\gamma^5\frac{v}{\sqrt{2}}f^*\omega_{1\mu}{}^u{}_{d,22} \\
T^u_{31} &= -i\frac{v}{\sqrt{2}}\gamma^\mu\gamma^5\partial_\mu\tilde{f}\cdot-\frac{v}{\sqrt{2}}\tilde{f}\left[\gamma^\mu,\gamma^5\right]_- i\partial_{EM\mu}-\gamma^\mu\gamma^5\tilde{f}\frac{v}{\sqrt{2}}\omega_{2\mu}{}^u{}_{u,11} \\
T^u_{32} &= \omega'_1{}^u{}_b\tilde{f}^*\frac{v}{\sqrt{2}}\gamma^5\gamma^b\exp{(-\sigma)}-\frac{v}{\sqrt{2}}f\gamma^\mu\gamma^5\omega_{2\mu}{}^u{}_{d,12} \\
T^u_{33} &= \gamma^\mu\gamma^b\omega_{2\mu}{}^u{}_{b,11}+\frac{v^2}{2}\mathbf{1}_{4\times4}(f\tilde{f}^*+\tilde{f}f^*) \\
T^u_{34} &= \gamma^\mu\gamma^b\omega_{2\mu}{}^u{}_{b,12}+\mathbf{1}_{4\times4}\exp{(-\sigma)}\frac{v^2}{2}f^*\tilde{f}^*\omega'_1{}^u{}_d \\
T^u_{41} &= \omega_1{}^u{}_bf^*\frac{v}{\sqrt{2}}\gamma^5\gamma^b\exp{(-\sigma)}-\frac{v}{\sqrt{2}}\gamma^\mu\gamma^5\tilde{f}\omega_{2\mu}{}^u{}_{u,21} \\
T^u_{42} &= -\frac{v}{\sqrt{2}}f\gamma^\mu\gamma^5\omega_{2\mu}{}^u{}_{d,22} \\
T^u_{43} &= \gamma^\mu\gamma^b\omega_{2\mu}{}^u{}_{b,21}+\mathbf{1}_{4\times4}\exp{(-\sigma)}\tilde{f}^*f^*\frac{v^2}{2}\omega_2{}^u{}_u \\
T^u_{44} &= \gamma^\mu\gamma^b\omega_{2\mu}{}^u{}_{b,22}
\end{aligned}
\tag{82}
$$

In section three we have imposed the torsion-less condition for a part of these components.

References

[1] T.Banks, W.Fischler, S.H.Shenker, L.Susskind, *Phys.Rev.*__D55__,5112(1997), *M Theory As A Matrix Model: A Conjecture.*

[2] N.Ishibashi, H.Kawai, Y.Kitazawa, A.Tsuchiya, *Nucl.Phys.*__B498__, 467(1997), hep-th/9612115, *A Large-N Reduced Model as Superstring.*

[3] H.Aoki, N.Ishibashi, S.Iso, H.Kawai, Y.Kitazawa and T.Tada, *Nucl.Phys.*__B565__, 176(2000), hep-th/990841, *Noncommutative Yang-Mills in IIB matrix model.*

[4] N.Ishibashi, S.Iso, H.Kawai and Y.Kitazawa, hep-th/9910004 v2, *Wilson Loops in Noncommutative Yang Mills.*

[5] S.Iso, H.Kawai and Y.Kitazawa, arXiv:hep-th/0001027, *Bilocal Fields in Noncommutative Field Theory.*

[6] A.Connes, *Noncommutative Geometry,*__Academic Press__, 1-661(1994).

[7] A.Connes, J.Lott,*Nucl.Phys.***B(Proc.Suppl.)18**, 29-47(1990), *PARTICLE MODELS AND NONCOMMUTATIVE GEOMETRY*.

[8] A.H.Chamseddine, J.Fröhlich, O.Grandjean, *J.Math.Phys.***36**, 6255-6275(1995), *The Gravitational Sector in the Connes-Lott Formulation of the Standard Model*

[9] A.H.Chamseddine, G.Felder and J.Fröhlich, *Commun.Math.Phys.***155**, 205-218(1993), *Gravity in Non-Commutative Geometry.*

[10] J.Polchinski, STRING THEORY,volume 1 an introduction to the bosonic string, Cambridge university press