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On a possible quantum limit for the stabilization of moduli in brane-world scenarios

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ABSTRACT

I consider the implications for brane-world scenarios of the rather robust quantum-gravity expectation that there should be a quantum minimum limit on the uncertainty of all physical length scales. In order to illustrate the possible significance of this issue, I observe that, according to a plausible estimate, the quantum limit on the length scales that characterize the bulk geometry could affect severely the phenomenology of a recently-proposed brane-world scenario.

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An extensive research effort (see, *e.g.*, Refs. [1, 2, 3, 4, 5, 6, 7, 8] and references therein) has been recently devoted to the possibility that the non-gravitational degrees of freedom be confined to one or more p-branes while gravitational degrees of freedom have access to some extra dimensions. With respect to the development of related formalism an important observation is that in certain string theories it is quite natural [2] to obtain this type of different properties for gravitational and non-gravitational degrees of freedom. Concerning phenomenological implications, since it is the gravitational realm which is most affected by these "brane-world scenarios", it is not surprising that significant constraints come from the requirement that classical gravity should behave as observed in the regimes we have already explored experimentally. On the quantum-gravity side some constraints also emerge; in particular, interestingly, while more conventional pictures lead to graviton effects that are negligibly small, one finds [3] that certain portions of the parameter space of a given brane-world scenario turn out to be excluded for predicting graviton effects that are inconsistent with data obtained at existing particle colliders. Larger portions of these parameter spaces will be probed at planned colliders, such as LHC at CERN.

In this brief note I observe that, in addition to graviton contributions to processes studied at particle colliders, there is another class of quantum-gravity effects which could have important implications for brane-world scenarios. These effects are associated with the rather robust quantum-gravity expectation [9, 10, 11, 12, 13, 14] that physical length scales should not be definable with perfect accuracy, there should be a minimum length uncertainty, and there should be quantum fluctuations of lengths. This is conventionally (and somewhat generically) expressed with formulas of the type $\Delta R \geq L_{min}$, intended to be valid for any physical length scale. I shall argue that, if such quantum limitations on the stabilization of length scales apply to the length scales that characterize the bulk geometry, there might be implications also for observables on the brane where the Standard Model fields reside.

In conventional quantum-gravity scenarios [9, 11, 12, 13] L_{min} is expected to coincide with L_{QG}, the length scale that characterizes the strength of gravitational interactions (L_{QG} would be given by the Planck length $L_p \sim 10^{-35}m$ in the conventional picture with only 3+1 space-time dimensions, but in the bulk of a brane-world scenario one can have $L_{QG} \gg L_p$).

In quantum-gravity scenarios based on string theory traditionally there has been the expectation [10] that the measurability bound should be even more stringent: $L_{min} \sim L_s > L_{QG}$, where L_s is the string length ($L_s > L_{QG}$ in the perturbative regime). More recently the analysis of certain stringy scenarios with several length scales [14] has suggested that in presence of appropriate hierarchies of scales it may be possible to have $L_{min} < L_{QG}$. For example, in the scenario considered in Ref. [14] it appears that $L_{min} \sim (M_{D0} L_s)^{-1/12} L_{QG} < L_{QG}$, where M_{D0} is the mass of D-particles.

For brane-world scenarios in which quantum gravity (possibly in the guise of a string theory) behaves in the bulk in such a way that $L_{min} \geq L_{QG}$ one would find that every given length scale R_{bulk} characterizing the bulk geometry (*e.g.*, a curvature radius or an overall length of a finite extra dimension) would be affected by a quantum limitation: $\Delta R_{bulk} \geq L_{min} \geq L_{QG}$. In the ordinary case, in which $L_{QG} \sim L_p$, such quantum limits are very weak for all lengths R that we can access experimentally (extremely small relative uncertainty $\Delta R/R \sim L_{QG}/R$), but in the bulk of a brane-world scenario they can be significant because $L_{QG} \gg L_p$ and some of the length scales R_{bulk} are not much larger than L_{QG}.

In the mentioned stringy scenarios with several length scales and an appropriate hierarchy of scales it might be possible to have $\Delta R_{bulk} \sim L_{min} < L_{QG}$, but values of ΔR_{bulk} that are significantly smaller than L_{QG} may require a strong hierarchy of scales. For example, in the scenario considered in Ref. [14] even just the availability of ΔR_{bulk} of order, say, $L_{QG}/1000$, would already require a very strong hierarchy between the mass of D-particles and the string scale: $M_{D0} \geq 10^{36}/L_s$. The fact that the availability of ΔR_{bulk} significantly smaller than L_{QG} may require such strong hierarchies can be quite significant since most brane-world scenarios intend to solve the ordinary "hierarchy problem" and may therefore loose most of their motivation if requiring for other reasons (see below) that some new hierarchy problems arise.

Let me now discuss the possible implications of this set of ideas in the significant illustrative example provided by the model proposed by Randall and Sundrum in Ref. [4], which in particular assumes that all length scales characterizing the bulk geometry are not too far from the fundamental bulk-gravitational length scale L_{QG} (which in the model [4] is taken to be close to the TeV scale). In this model of Ref. [4] the mass m of an ordinary Standard-Model field and the mass scale $M_p = 1/L_p$ setting the strength (weakness) of gravity on the brane where the Standard Model fields reside can be related through an exponential of the ratio between two of the length scales characterizing the bulk geometry: $m = M_p \cdot exp(-\pi R_{bulk,1}/R_{bulk,2})$ (in the notation of Ref. [4] $R_{bulk,1} = r_c$ and $R_{bulk,2} = 1/k$). Values of $exp(\pi R_{bulk,1}/R_{bulk,2})$ close to 10^{15} are of interest for a solution of the ordinary hierarchy problem [4], but if $R_{bulk,1}$ (and/or $R_{bulk,2}$) is not much bigger than L_{min} one would then predict a rather significant limitation[2] on the accuracy of the ratio m/M_p, while instead we measure with great accuracy both the masses of Standard Model particles and the strength of ordinary gravitational interactions.

If the quantum gravity (or string theory) appropriate for the model of Ref. [4] behaves in the bulk in such a way that $L_{min} \geq L_{QG}$ the length scales $R_{bulk,1}$ and $R_{bulk,2}$ should indeed not be much bigger than L_{min}, since, as mentioned, in the model of Ref. [4] all length scales characterizing the bulk geometry are not too far from L_{QG}. In this case the alarming prediction of significant limitations on the accuracy of the ratio m/M_p appears to be inevitable.

If the model of Ref. [4] could be embedded in a stringy quantum-gravity scenario of the type considered in Ref. [14], with several length scales and a hierarchy of scales appropriate for having $L_{min} \ll L_{QG}$, one might be able to escape the prediction of significant limitations on the accuracy of the ratio m/M_p, but then, as mentioned, one would easily end up having a new hierarchy problem associated with the requirement $L_{min} \ll L_{QG}$.

In summary, at least at the heuristic level of the present discussion, it would seem that the quantum-gravity expectation that there should be a limit $\Delta R_{bulk} \geq L_{min}$ on the measurability of any length scale R_{bulk} might affect non-trivially the analysis of the scenario proposed in Ref. [4]. Of course, a definite statement must await more rigorous and quantitative analyses of the quantum properties of the bulk geometry in models based on the scenario proposed in Ref. [4]. The analyses should tell us whether $L_{min} < L_{QG}$ (actually, even though the evidence for an $L_{min} > 0$ is quite robust [9, 10, 11, 12, 13, 14], one cannot exclude that $L_{min} = 0$ might be found in some quantum-gravity or stringy-quantum-gravity pictures) or $L_{min} \geq L_{QG}$, and if $L_{min} < L_{QG}$ an estimate should be given of the amount of tuning required to eliminate the ordinary hierarchy problem.

[2]For example, the relation $m = M_p \cdot exp(-\pi R_{bulk,1}/R_{bulk,2})$ for $exp(\pi R_{bulk,1}/R_{bulk,2}) = 10^{15}$ and $R_{bulk,1} \sim 100 L_{min} \sim 100 \Delta R_{bulk,1}$ leads to $\Delta(m/M_p)$ of order m/M_p.

The example of the model proposed in Ref. [4] might also indicate that in general any claim of consistency of a brane-world scenario must await the results of at least the level of analysis of the quantum properties of the bulk geometry necessary to address rigorously the issues I considered heuristically here. It is important that such analyses be performed in very physical terms, always resorting to operative definitions of gravitational observables. It is in fact well known [12, 13, 15] that formal estimates of quantum uncertainties in geometric observables can be misleading. For example, one finds [12, 13, 15] that it is not sufficient to identify formally one of the objects in the formalism as a distance observable; it is instead necessary to analyze an operative definition of distance and consider all the possible limitations which might be caused by each of the elements of the measurement procedure. It is perhaps worth emphasizing that the operative definition of gravitational observables, which is already a delicate task in more conventional physical scenarios [13, 15, 16], might be a formidable task in the case of those observables of a brane-world scenario that concern the bulk geometry; in fact, the measurement procedures that have been discussed in the conventional quantum-gravity literature all rely [13, 15, 16] on several non-gravitational elements, while the bulk is not accessible to non-gravitational degrees of freedom. It might be nontrivial even to establish what is genuinely observable [16] in the bulk, and what type of measurement procedures, particularly with respect to the probes to be exchanged, would be appropriate.

Besides the analysis of possible quantum limits for the stabilization of geometric observables in the bulk, it might be also necessary [17] to consider quantum limits for the stabilization of geometric observables of the brane where the Standard Model fields reside; in fact, on some of these observables we start to have significant experimental constraints [17, 18].

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