Three-particle GHZ correlations without nonlocality

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Abstract

The formalism employing local complex amplitudes that resolved the Einstein-Podolsky-Rosen puzzle (C. S. Unnikrishnan, quant-ph/0001112) is applied to the three-particle GHZ correlations. We show that the GHZ quantum correlations can be reproduced without nonlocality.

We have recently shown that quantum correlations can be reproduced starting from local probability amplitudes, by calculating the correlations from amplitudes directly rather than by multiplying the outcomes and integrating over some hidden variable values [1]. In the local hidden variable theories the correlations are calculated from eigenvalues and this procedure *does not preserve the phase information*. The situation has some analogy to the description of interference in quantum mechanics. Any attempt to reproduce the interference pattern using locality and the information on 'which-path' will fail since the phase information is lost or modified in such an attempt. In the local amplitude formalism, measurement on one particle does not cause the companion particle to acquire, or to collapse into, a definite state. (The present interpretation of quantum teleportation and entanglement swapping will change in this local picture, without affecting the actually measured correlations).

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The statement of locality is at the level of the probability amplitudes and can be written as

$$C_{1\pm} = C_{1\pm}(\mathbf{a}, \phi_1), \quad C_{2\pm} = C_{2\pm}(\mathbf{b}, \phi_2) \tag{1}$$

where $\mathbf{a}$ and $\mathbf{b}$ are local settings for analyzers, and ϕ_1 and ϕ_2 are the internal variables ('hidden variables') associated with the individual particles and appear in the amplitudes as a phase [1]. A definite value for these variables does not imply a define state for the particles before the measurement.

The locality assumption also implies the locality for observables A and B,

$$A(\mathbf{a},\phi_1) = \pm 1, \quad B(\mathbf{b},\phi_2) = \pm 1 \tag{2}$$

This is the same locality assumption as in local realistic theories. But, this has a meaning different from its meaning in standard local realistic theories. Here, this means that the outcomes, *when measured*, depend only on the local setting and the local internal variable. There is no objective reality to A and B before a measurement. There is objective reality to ϕ_1 and ϕ_2, but there is no way to observe these absolute phases.

In this framework the correlation function is not $P(\mathbf{a}, \mathbf{b}) = \frac{1}{N}\sum(A_i B_i)$ or $\int d\phi \rho(\phi) A(\mathbf{a},\phi_1) B(\mathbf{b},\phi_2)$. The correct correlations are of the form,

$$U(\mathbf{a}, \mathbf{b}) = \ Real(N C_i C_j^*) \tag{3}$$

where N is a normalization factor. It is the square of this correlation function that would give a joint probability. The correlation of the eigenvalues $P(\mathbf{a}, \mathbf{b}) = \frac{1}{N}\sum(A_i B_i)$ also can be derived from the absolute square of $U(\mathbf{a}, \mathbf{b})$ [1]. *The crucial difference from local realistic theories is that the correlation is calculated from quantities which preserve the relative phases.*

We now apply this formalism for the description of correlations of the three particle G-H-Z state [2] defined as

$$|\Psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|1, 1, 1\rangle - |-1, -1, -1\rangle) \tag{4}$$

where the eigenvalues in the kets are with respect to the z-axis basis.

The conflict between a local realistic theory and quantum mechanics is the following statement [2]:

The prediction from quantum mechanics for the measurement represented by the operator $\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$ is given by

$$\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3 |\Psi_{GHZ}\rangle = - |\Psi_{GHZ}\rangle \tag{5}$$

Local realistic theories predict that the product of the outcomes in the x direction for the three particles should be $+1$. This contradicts Eq. 5.

We now show that the quantum prediction can be reproduced using local amplitudes. The general idea is that the three particle correlation, analogous to our scheme for two-particle states [1], is the real part of a complex number Z obtained as a suitable product of three complex amplitudes. We choose the different phases such that the correlation represented by $\text{Real}(Z---)$ is ± 1 (i.e. $(Z---)$ is pure real) to satisfy the condition that the joint probability for the outcome $(-,-,-)$ is unity according to Eq. 5. The rest of the correlations follow without any additional input since flipping the sign once (for example $\text{Real}(Z+--)$) amounts to rotating Z through the phase $\pi/2$. This is because the amplitudes for $+$ and $-$ are orthogonal. The joint probability itself is the square of the correlation function and clearly these joint probabilities are unity for the outcomes containing an odd number of $(-)$.

We define the local amplitudes for the outcomes $+$ and $-$ at the analyzer (with respect to the x basis) for the first particle as $C_{1+} = \frac{1}{\sqrt{2}} \exp(i\theta_1)$, and $C_{1-} = \frac{1}{\sqrt{2}} \exp(i(\theta_1 + \pi/2))$. The amplitude C_{1-} contains the added angle $\pi/2$ because this amplitude is orthogonal to C_{1+}. Similarly, we have $C_{2+} = \frac{1}{\sqrt{2}} \exp(i\theta_2)$, and $C_{2-} = \frac{1}{\sqrt{2}} \exp(i(\theta_2 + \pi/2))$ for the second particle and $C_{3+} = \frac{1}{\sqrt{2}} \exp(i\theta_3)$, and $C_{3-} = \frac{1}{\sqrt{2}} \exp(i(\theta_3 + \pi/2))$ for the third particle. Our aim is to choose the various phases such that the following is true:

$$
\begin{aligned}
P(+,+,+) &= 0 \\
P(-,-,-) &= 1 \\
P(+,+,-) &= 1 \\
P(+,-,+) &= 1 \\
P(-,+,+) &= 1 \\
P(-,-,+) &= 0 \\
P(+,-,-) &= 0 \\
P(-,+,-) &= 0
\end{aligned}
\tag{6}
$$

These are the quantum mechanical predictions for the joint probabilities for getting the outcomes indicated.

We choose the following definition for the correlation function whose square is the relevant joint probability. (The final results are independent of the particular

definition we use. Once a definition is chosen the phases can be solved for the outcomes).

Correlation function is obtained from the definition $N\text{Real}(C_1 C_2^* C_3^*)$, where N is a normalization constant. Since we want $N\text{Real}(C_{1-} C_{2-}^* C_{3-}^*) = \pm 1$, we choose $C_{1-} C_{2-}^* C_{3-}^*$ to be pure real. This gives

$$\frac{N}{2\sqrt{2}} Real(\exp i(\theta_1 - \theta_2 - \theta_3 - \pi/2)) = \pm 1$$

$$\theta_1 - \theta_2 - \theta_3 - \pi/2 = 0 \text{ or } \pm \pi$$

We can choose the relevant relative phases to satisfy this condition. Then we get

$$P(-, -, -) = 1$$

Rest of the joint probabilities given in Eq. 6 automatically follow, since flipping sign once rotates the complex number $C_{1-} C_{2-}^* C_{3-}^*$ through $\pi/2$. The square of $N\text{Real}(C_1 C_2^* C_3^*)$ is then 1 for an odd number of $(-)$ outcomes and 0 for even number of $(-)$ outcomes.

This completes the construction of local amplitudes for the three particle maximally entangled state. Similar construction also applies to four- particle maximally entangled state [3] and general multiparticle maximally entangled states.

References

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