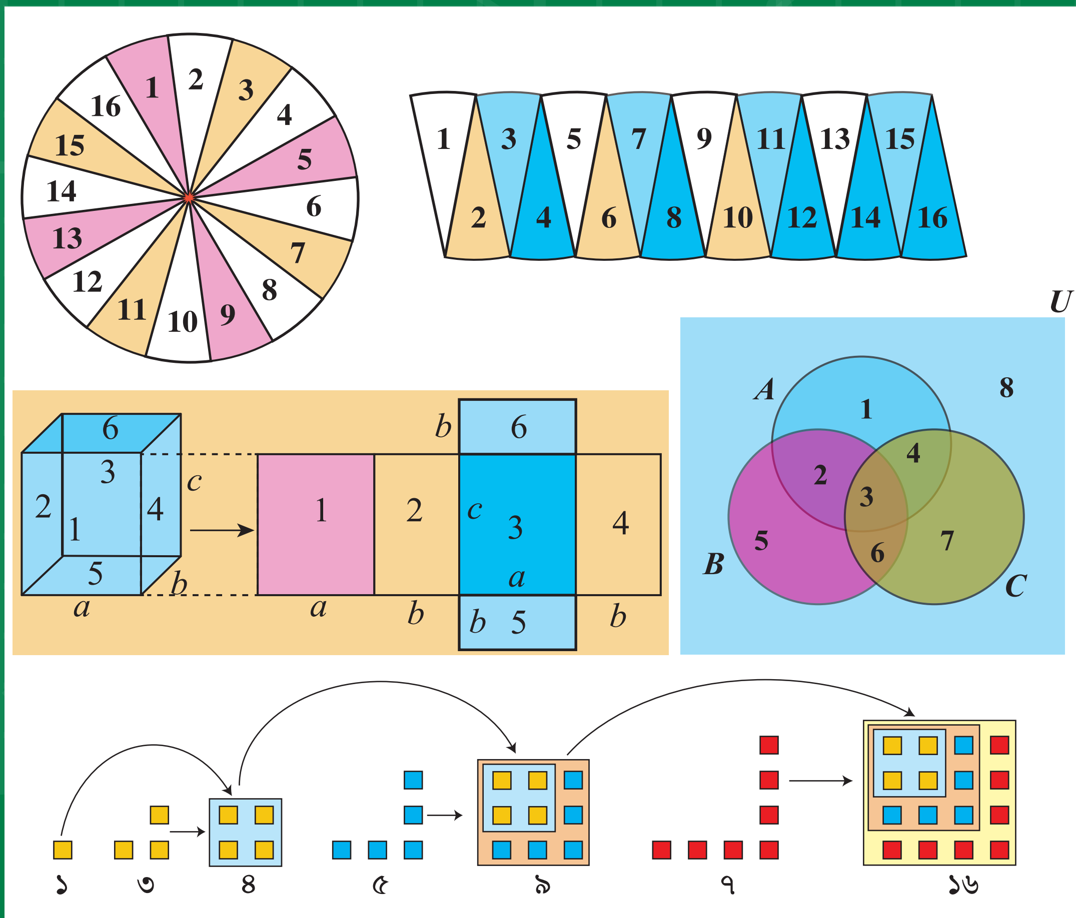
Mathematics

Class Eight



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Mathematics

Class Eight

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Preface

The aim of secondary education is to make the learners fit for entry into higher education by flourishing their latent talents and prospects with a view to building the nation with the spirit of the Language Movement and the Liberation War. To make the learners skilled and competent citizens of the country based on the economic, social, cultural and environmental settings is also an important issue of secondary education.

The textbooks of secondary level have been written and compiled according to the revised curriculum 2012 in accordance with the aims and objectives of National Education Policy-2010.Contents and presentations of the textbooks have been bselected according to the moral and humanistic values of Bengali tradition and culture and the spirit of Liberation War 1971 ensuring equal dignity for all irrespective of caste and creed of different religions and sex.

The present government is committed to ensure the successful implementation of Vision 2021. Honorable Prime Minister, Government of the People's Republic of Bangladesh, Sheikh Hasina expressed her firm determination to make the country free from illiteracy and instructed the concerned authority to give free textbooks to every student of the country. National Curriculum and Textbook Board started to distribute textbooks free of cost since 2010 according to her instruction.

Mathematics plays an important role in developing scientific knowledge at this time of the 21st century. Not only that, the application of Mathematics has increased in family and social life including personal life. With all these things under consideration Mathematics has been presented easily and nicely at the Secondary level to make it useful and delightful to the learners, and a good number of new topics have been included in the textbook.

I thank sincerely all for their intellectual labor who were involved in the process of revision, writing, editing, art and design of the textbook.

Prof. Narayan Chandra Saha
Chairman
National Curriculum and Textbook Board, Bangladesh

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Chapter One
Patterns

The diverse nature is full of various patterns. We experience this diversity through numbers and patterns. Patterns are involved with our life in various ways. When a child separates red and blue blocks by putting red ones on one side and blues on the other, it is a pattern. He learns to count numbers which is also a pattern. The multiples of 5 end with either 0 or 5, this is also a pattern. To recognize a number-pattern is an important part to gain efficiency in solving mathematical problems. Again, we see different designs in our dresses, artistic designs on different constructions; these are geometric patterns. In this chapter, we shall discuss numerical patterns as well as geometrical patterns.

At the end of this chapter, the students will be able to -

- Explain what patterns are.
- Write and explain linear patterns.
- Write and explain different geometrical patterns.
- Write and explain simple linear patterns set by certain conditions.
- Express the linear patterns as algebraic expressions by using variables.
- Find the particular term of the linear pattern.

1.1 Patterns

Let us have a look at the tiles of figure- 1 below. These are arranged in a pattern. The alternate tiles are arranged vertically and horizontally. This rule of arrangement creates a pattern.

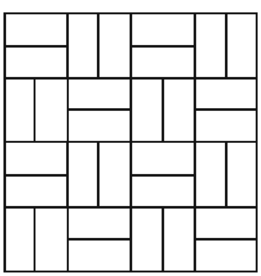


Figure-1

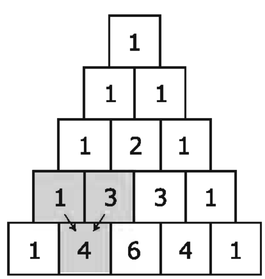


Figure-2

In the figure-2, some numbers are arranged in a triangular form. The numbers are chosen according to a certain rule. The rule is: Put 1 at the beginning and at the end of each row and the other numbers in a row is the sum of two consecutive numbers just above it. This rule of arrangement of sum creates another pattern.

Again, the numbers 1,4,7,10,13, ... exhibit a pattern. If we closely look at the numbers, we will find a rule. The rule is, begin with 1 and add 3 to get the next number. Another example: 2, 4, 8, 16, 32 where each number is double the previous number.

1.2 Patterns of Natural Numbers

Determining Prime Numbers

We know that the numbers those are greater than 1 and having no factor other than 1 and itself are prime numbers. With the help of sieve of Eratosthenes we can easily check whether a number is prime or not. Let us write down the numbers 1 to 100 in a table. Pick out the lowest prime number 2 and cross out all the multiples of 2. Then cross out the multiples of 3, 5 and 7 etc. successively. The uncrossed numbers in the list are the prime numbers.

1	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Determining the Particular number from a list of numbers.

Example 1. Find the next two consecutive numbers from the following list of numbers:

3, 10, 17, 24, 31, ...

Solution: Given numbers in the list : 3, 10, 17, 24, 31, ...

difference 7 7 7 7

Note that each time the difference is 7. Hence the next two numbers are 31+7=38 and 38+7=45.

Example 2. Find the next number from the following list of numbers:

1, 4, 9, 16, 25, ...

Solution: Given numbers in the list : 1, 4, 9, 16, 25, ...

difference 3 5 7 9

Note that each time the difference increases by 2.

Hence, the next number is $25 + 11 = 36$.

Example 3. Find the next number from the following list of numbers:

1, 5, 6, 11, 17, 28, ...

Solution : Given numbers in the list 1, 5, 6, 11, 17, 28, ...

Sum of consecutive two numbers 6 11 17 28 45 ......

The numbers in the list are written in a pattern. The sum of two consecutive numbers is the next number. Again, the difference between two consecutive numbers of the sum produces the original list except the first one. So, the next number of the list will be 17+28=45.

Activity:

1. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, are Fibonacci numbers. Do you find any pattern in the list?

Hints: The sum of any two consecutive numbers is the next number; for example, 2= 1 + 1, 3= 1 + 2, 21 =8+ 13 and so on. Find the next 10 Fibonacci numbers.

Determining the Sum of Consecutive Natural Numbers

There is a fine formula to find out the sum of consecutive natural numbers. We can find out the formula easily:

Let S be the sum of first ten consecutive natural numbers.

that is, $S = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

Note that the sum of the first and last number is 1 + 1 O= 11. The sum of the numbers second from left and second from right is also 2+9= 11. By following this pattern we will get five pairs of numbers having the same sum. So, the sum of the numbers will be $11 \times 5 = 55$. Thus we have got a technique for finding the sum of consecutive natural numbers.

The technique is :

write down the given numbers in reverse order and add :

$$\begin{array}{rcl} S & = & 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 \\ S & = & 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 \\ \hline 2\,S & = & (1+10) + (2+9) + \ldots \ldots + (9+2) + (10+1) \\ 2\,S & = & (1+10) \times 10 = 11 \times 10 \\ S & = & \dfrac{(1+10) \times 10}{2} = \dfrac{(11 \times 10)}{2} = 55 \end{array}$$

That is, $\text{Sum} = \dfrac{(\text{first number} + \text{last number}) \times \text{number of terms}}{2}$

Activity : Finding the sum of natural numbers from 1 to 15, establish the formula.

Determining the sum of the first ten odd numbers

What is the sum of the first ten odd numbers? Using our calculator, we get the sum that is 100.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$$

In this way, it is not easy to find out the sum of the first fifty odd numbers. Rather to determine the sum of this type of numbers let us derive a useful mathematical formula. If the odd numbers from 1 to 19 are noted, we find $1 + 19 = 20$, $3 + 17 = 20$, $5 + 15 = 20$. There are five pairs of such numbers whose sum of each pair is 20. Therefore, the sum of the numbers is $20 \times 5 = 100$.

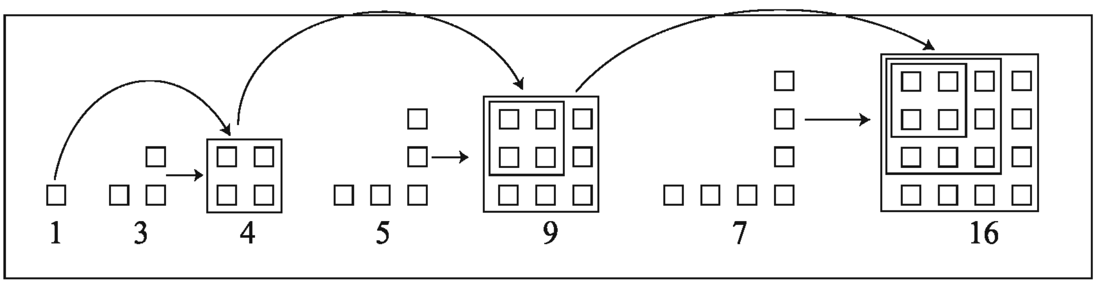
We note,

$1+3 = 4$, a perfect square

$1 + 3 + 5 = 9$, a perfect square

$1 + 3 + 5 + 7 = 16$, a perfect square etc.

Each time the sum is a perfect square number. This can be explained as a geometric pattern. Let us observe the pattern of the sum by the help of small squares.



We see that in the sum of first two consecutive odd numbers we find 2 small squares are placed in each side in the figure. In the sum of 3 consecutive odd numbers we find three small squares are placed in each side the figure. Hence in the sum of ten consecutive odd numbers, there will be 10 small squares in each side i.e. it will need $10 \times 10 = 10^2$ or 100 squares. In general, we can say that the sum of first n consecutive odd numbers will be n^2 .

Activity:

1. Find the sum: $1 + 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31$

1.3 Expression of a number as the sum of two squares

There are some numbers which can be expressed as the sum of two perfect squares. For example,

$$2 = 1^2 + 1^2$$

$$5 = 1^2 + 2^2$$

$$8 = 2^2 + 2^2$$

$$10 = 1^2 + 3^2$$

$$13 = 2^2 + 3^2 \text{ etc.}$$

In this way between 1 and 100 there are 34 such numbers which can be expressed as a sum of two squares.

Again, there are certain numbers which can be expressed as a sum of two squares in two or more than two ways. For example,

$$50 = 1^2 + 7^2 = 5^2 + 5^2$$

$$65 = 1^2 + 8^2 = 4^2 + 7^2$$

Activity:

1. Express 130, 170, 185 as the sum of two squares in two different ways.
2. Express 325 as the sum of two squares in three different ways.

1.4 Formation of Magic Square

(a) Magic square of order 3

If we devide a square in three parts along its length and breadth, we get 9 small squares. In this case, 15 is the magic number. If we arrange the numbers 1 to 9 horizontally, vertically and diagonally and add respectively, the sum will be the same that is 15 for the magic square of order 3. There are various ways to arrange the numbers. In one such arrangement put 5 in the central grid and place the even numbers in the corner grids so that the sum of the numbers of each diogonal will be 15. Fill the vacant grids with the remaining odd numbers so that the sum of the numbers in each of horizontal and vertical grid is 15. We see that the sum of the numbers in each of horizontal, vertical and diagonal grids is 15.

	5	

⟶

2		4
	5	
6		8

⟶

2	9	4
	5	
6	1	8

⟶

2	9	4
7	5	3
6	1	8

(b) Magic square of order 4

If we divide a square in four parts along its length and breadth, we get 16 samll squares. If we arrange the numbers 1 to 16 horizontally, vertically and diagonally and add respectively, the sum will be the same, that is 34. In this case 34 is the magic number for the magic square of order 4. There are various ways to arrange the numbers. In one such arrangement beginning from any corner, place the natural numbers horizontally and then vertically. Cross out the numbers which are placed diagonally. Fill the vacant grids with the crossed out numbers starting from the opposite corner. We see that the sum of the numbers in each horizontal, vertical and diagonal grid is 34.

⟶

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

	2	3	
5			8
	14	15	

⟶

16			13
	11	10	
	7	6	
4			1

⟶

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Activity:

1. Construct a magic square of order 4 by a different technique.
2. Try to construct a magic square of order 5 as a group work.

1.5 Playing with numbers

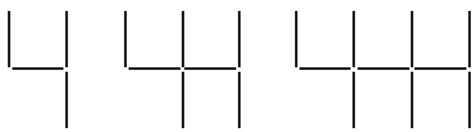
1. Take any two-digit number. Interchange the digits of the number and add to the original number. Now divide the sum by 11. The remainder is 0.

2. Interchange the digits of any two-digit number. Of the two numbers, subtract the smaller one from the larger one and divide the result by 9. The remainder is 0.

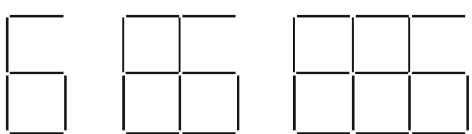
3. Take any three-digit number. Write down the digits in reverse order. Now subtract the smaller number from the larger one and divide the result by 99. The remainder is again 0.

1.6 Geometric pattern

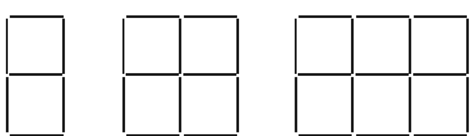
The numbers of the following figures are made of equal line segments. We see some figures of this type of numbers are:



4 7 10 13 $3n+1$



6 11 16 21 $5n+1$



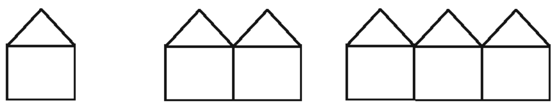
7 12 17 22 $5n+2$

We look at the pattern of the number of line segments required to construct the pictures. The number of line segments required to construct the n such numbers are shown at the end of each pattern by an algebraic expression.

We complete the table of patterns with the help of algebraic expressions:

Serial no.	Expression	Term								
		1st	2nd	3rd	4th	5th		10th		100th
1	$2n+1$	3	5	7	9	11		21		201
2	$3n+1$	4	7	10	13	16		31		301
3	n^2-1	0	3	8	15	24		99		9999
4	$4n+3$	7	11	15	19	23		43		403

4.



Figure

The above geometric figures are formed with sticks of equal length.

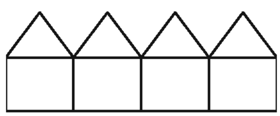
A. Form the 4th pattern and find out the number of lines.

B. Which algebraic expression is followed by the pattern? Present it with logic.

C. Find, how many lines will be required to form the first 50 patterns of the pattern.

Solution:

A. According to the stimulus, the 4th pattern is as follow:



Figure

In the pattern no. of equal lines = 21

B. In the figure 1, number of lines $= 6$
$= 5+1$
$= 5\times1 + 1$

In the figure 2, number of lines $= 11$
$= 10+1$
$= 5\times2 + 1$

In the figure 3, number of lines $= 16$
$= 15+1$
$= 5\times3 + 1$

In the figure 4, number of lines $= 21$
$= 20+1$
$= 5\times4 + 1$

....

In the same way in the figure, A number of lines $= 5\times A+1$
$= 5A+1$

$\therefore$ The patterns can be expressed in Algebraic Expression : $5A+1$

C. From B, we get

Algebraic expression of the pattern $= 5A+1$

In the 50^{th} pattern, number of sticks $= 5\times50+1$
$= 250+1$
$= 251$

Now, the summation of the sticks in the patterns
$= 6+11+16+21+.........+251$

Here, the 1^{st} term $= 6$

the last term $= 251$

number of terms $= 50$

$\therefore$ Summation $= \frac{6+251}{2}\times50$
$= \frac{257}{2}\times50$
$= 257\times25$
$= 6425.$

$\therefore$ To form 50 patterns the number of lines required is 6425.

Exercise 1

1. In the formation of magic square of order 3 –

   i. The magic number will be 15

   ii. At the centre, the number in the small square will be 5

   iii. In the small squares the integers 1–15 are set.Which one of the following is correct?

   A. i and ii B. i and iii C. ii and iii D. i, ii and iii

2. Which one of the following terms will be divisible by 9?

A. 52+25 B. 527+725 C. 412+234 D. 75-57

3. In which algebraic expressions 9999 is the 100^{th} term?

A. 99A+1 B. 99A-1 C. A^2+1 D. A^2-1

4. What is the sum of 'A' numbered series of normal odd numbers?

A. A B. 2A-1 C. A^2 D. 2A+1

5. How many integers from 1 to 100 can be expressed as the sum of two square numbers.

A. 10 B. 20 C. 34 D. 50

According to the stimuli answer to the question no. 6 and 7 :

12	19	14
17	A	13
16	11	18

← A magic square

6. What would be the right number in the square marked 'A'?

A. 45 B. 20 C. 15 D. 3

7. In the magic square, what is the magic number?

A. 15 B. 34 C. 35 D. 45

8. The sum of the 1^{st} three odd integers is-

i. square number

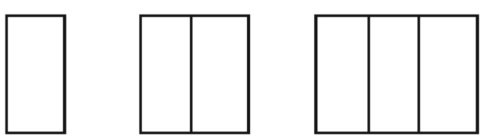
ii. odd number

iii. Prime number

Which one of the following is correct?

A. i and ii B. i and iii C. ii and iii D. i, ii and iii

9. The following geometrical figures are constructed with sticks.

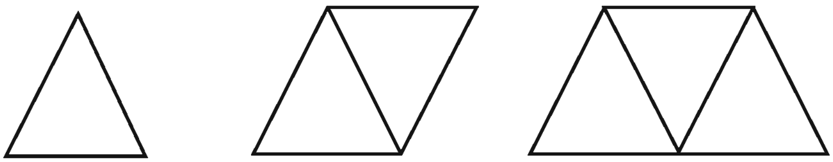


(a) Make a list of the numbers of sticks.

(b) Explain how you can find the next number in the list.

(c) Construct the next figure with sticks and verify your result.

10. The pattern of the triangles is constructed with match sticks.



1st 2nd 3rd

(a) Find the number of match sticks in the fourth patterns.

(b) Explain how you can find the next number in the patterns.

(c) How many match sticks are required to construct the hundredth pattern?

11. 5, 13, 21, 29, 37......

A. Express 29 and 37 as the sum of two squares.

B. Find the next four number in the list.

C. Find the sum of the first 50 numbers in the list.

Chapter Two
Profits

In day to day life all of us are associated with buying-selling and mutual transactions. Someone invests money in industry, produces commodities and sell those commodities to the wholesale traders in the markets. Then the wholesale traders sell those commodities to the retail traders. Finally, the retail traders sell their commodities to the common buyers. In each stage everyone wants to make a profit. But, there may also be loss due to different causes, such as, in the share market, as there is profit, there is also loss due to the fall of market price. Again, we deposit our money in the banks for safety. Bank invests that money in different sectors and makes a profit. Bank also gives profits to the depositors. Therefore, everyone needs to have a clear idea about the investment and the profit. In this chapter, we have discussed profits and losses, especially profits.

At the end of the chapter, the students will be able to -

- Explain what profit is.
- Explain the rate of simple profit and solve the related problems.
- Explain the rate of compound profit and solve the related problems.
- Understand and explain the bank's statements.

2.1 Profit and Loss

A businessman adds the shop-rent, transport cost and other related expenditures with the buying price of commodities and fixes the actual cost price, or briefly the cost price. This actual cost price is called investment and this investment is considered to be the buying price for determining profit or loss. The price of selling the commodities is its selling price. If selling price is more than its cost price, there will be a profit. On the other hand, if the selling price is less than the cost price, there will be a loss. Again, if the cost price and the selling price are equal, there will be no profit and no loss. Profit or loss is determined by its cost price.

We can write, Profit = selling price – cost price

Loss = cost price – selling price

From the above relations the cost price or the selling price can be determined.

For comparison, profit or loss is also expressed in percentage.

Example 1. If a shopkeeper buys eggs at Tk. 25 per quad (hali) and sells at Tk. 56 per 2 quads, how much profit will he make ?

Solution : The cost price of 1 quad of eggs = Tk. 25

∴ The cost price of 2 quads of eggs = Tk. 25×2 = Tk. 50

Again, the selling price of 2 quads of eggs = Tk. 56

Since the selling price is more than the cost price, there will be a profit.

Here, Profit = Tk. $(56 - 50)$ = Tk. 6

In Tk. 50, profit = Tk. 6

In Tk. 1, profit = Tk. $\frac{6}{50}$

In Tk. 100, profit = Tk. $\frac{6\times 1\cancel{00}^{2}}{\cancel{50}_{1}}$ = Tk. 12

∴ The profit is 12%

Example 2. A goat is sold at the loss of Tk. 8%. If it were sold at Tk. 800 more, there would be a profit of Tk. 8%. What was the cost price of the goat ?

Solution : If the cost price of the goat was Tk. 100, its selling price would be Tk. $(100 - 8)$, or Tk. 92 at the loss of Tk. 8%.

Again, if the goat was sold at the profit of Tk. 8%, the selling price would be Tk. $(100 + 8)$, or Tk. 108.

∴ the cost price is more by Tk. $(108 - 92)$, or Tk. 16

If the selling price were Tk. 16 more, the cost price would be Tk. 100

If the selling price „ „ 1 „ „ „ Tk. $\frac{100}{16}$

„ „ „ „ „ 800 „ „ „ Tk. $\frac{100\times \cancel{800}^{50}}{\cancel{16}_{1}}$

= Tk. 5000

∴ the cost price of the goat is Tk. 5000.

Activity : Fill in the blank spaces :			
Cost Price (Tk.)	**Selling price (Tk.)**	**Profit/Loss**	**Percentage of Profit/Loss**
600	660	Profit Tk. 60	Profit 10%
600	552	Loss Tk. 48	Loss 8%
	583	Profit Tk. 33	
856		Loss Tk. 107	
		Profit Tk. 64	Profit 8%

2.2 Profit :

Farida Begum decided to deposit her savings in a bank. She deposited Tk. 10,000 in a bank. After one year she went to the bank to take a bank statement. She noticed that her deposited amount of money had been increased by Tk. 700 and her balance became Tk. 10,700. How was the amount of money of Farida Begum increased by Tk. 700?

When money is deposited in a bank, the bank invests that money as loans in different sectors, such as business, house-building etc. and gets profit from those sectors. From that profit, the bank gives some money to the depositor. This money is the profit of the depositor. The money which was first deposited in the bank was her principal amount. To deposit or give money as a loan and to take money from anyone as a loan is accomplished by a process. This process relates to the principal, the rate of profit and time and profit.

We observe :

Rate of Profit : A profit of Tk. 100 in 1 year is called the rate of profit or percentage of profit per annum.

Period of time : The time for which a profit is calculated, is called the period of time.

Simple profit : The profit which is accounted each year only on the primary or initial principal, is called the simple profit. Usually, profit means the simple profit. In this chapter we shall use the following algebraic symbols :

Principal = P Rate of profit = r (rate of interest) Time = n Profit = I Profit-principal = A (total amount)	Profit-Principal = Principal + Profit i.e. ; $A = P + I$ which gives $P = A - I$, $I = A - p$

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2.3 Problems related to profit

If any three of the four data namely principal, rate of profit, time and profit are known, the remaining one can be found. It is discussed as follows :

(a) Determination of profit :

Example 3. Mr. Ramiz deposited Tk. 5,000 in a bank and decided not to withdraw any amount from the bank till next 6 years.The annual profit given by the bank is 10%. How much profit will he get after 6 years ? What will be the profit-principal ?

Solution : The profit of Tk. 100 in 1 year is Tk. 10

$$\text{,, Tk. 1 ,, 1 ,, ,, Tk. } \frac{10}{100}$$

$$\text{,, Tk. 5,000 ,, 1 ,, ,, Tk. } \frac{10\times 5{,}000}{100}$$

$$\text{,, Tk. 5,000 ,, 6 ,, ,, Tk. } \frac{10\times \cancel{5000}^{50}\times 6}{\cancel{100}_{1}} = \text{Tk. } 3{,}000$$

$\therefore$ Profit-principal = principal + profit

= Tk. (5,000 + 3,000)

= Tk. 8,000.

$\therefore$ Profit is Tk. 3,000 and Profit-principal is Tk. 8,000.

We observe : Profit of Tk. 5,000 in 6 years at the percentage of Tk. 10 per annum = Tk. $\left(5{,}000\times\frac{10}{100}\times 6\right)$.

Formula : Profit = Principal × Rate of Profit × time, $I=Prn$
Profit-Principal = Principal + Profit, $A=P+I=P+Prn=P(1+rn)$

Alternative solution of example 3 :

We know, $I=Prn$, i.e., Profit = Principal × rate of profit × time

$\therefore$ Profit $= \text{Tk. }\left(5{,}000\times\frac{10}{100}\times 6\right)$

= Tk. 3,000

$\therefore$ Profit-Principal = Principal + Profit

= Tk. (5,000 + 3,000) or Tk. 8,000

$\therefore$ Profit is Tk. 3,000 and Profit-Principal is Tk. 8,000.

(b) Determination of Principal :

Example 4. If the rate of profit is $8\frac{1}{2}\%$ per annum, how much will be the profit of Tk. 2,550 in 6 years ?

Solution : Rate of profit $= 8\frac{1}{2}\%$ or $\frac{17}{2}\%$

We know, $I = Prn$ or, $P = \frac{I}{rn}$

$$\therefore \text{Principal} = \frac{\text{profit}}{\text{rate of profit} \times \text{time}}$$

$$= \text{Tk. } \frac{2{,}550}{\frac{17}{2 \times 100} \times 6} = \text{Tk. } \frac{{}^{50}\cancel{{}^{150}}\cancel{2550} \times \cancel{2}^{1} \times 100}{\cancel{17}_{1} \times \cancel{6}\cancel{{}_{3}}_{1}}$$

$= \text{Tk. } (50 \times 100) = \text{Tk. } 5{,}000.$

$\therefore$ Principal is Tk. 5,000.

Where,

P = Principal = Required

I = Profit = Tk. 2,550

r = Rate of Profit

$= 8\frac{1}{2}\%$ or $\frac{17}{2}\%$

n = Time = 6 years

(c) Determination of the rate of profit:

Example 5. What is the rate of profit by which the profit of Tk. 3,000 will be Tk. 1,500 in 5 years ?

Solution : We know, $I = Prn$ or, $r = \frac{I}{Pn}$

$$\therefore \text{rate of profit} = \frac{\text{profit}}{\text{principal} \times \text{time}}$$

$$\frac{{}^{1}\cancel{1500}}{\cancel{3000}_{2} \times 5}$$

$$= \frac{1}{10} = \frac{1 \times 100}{10 \times 100}$$

$= 10\%$

$\therefore$ The rate of profit is 10%

Where,

P = Principal = Tk. 3,000

I = Profit = Tk. 1,500

r = Rate of Profit = Required

n = Time = 5 years

Example 6. The profit-principal of some principal is Tk. 5,500 in 3 years. If the profit is $\frac{3}{8}$ part of the principal, find the principal and the rate of profit ?

Solution : We know, principal + profit = profit-principal

i.e. $P+I=A$

or, $P+\frac{3}{8}P=A$

or, $\left(1+\frac{3}{8}\right)\times \mathrm{P}=5{,}500$

or, $\frac{11}{8}\times \mathrm{P}=5{,}500$

$\therefore\ \mathrm{P}=\text{Tk. }\frac{\overset{500}{\cancel{5500}}\times 8}{\underset{1}{\cancel{11}}}$

$=\text{Tk. }4{,}000.$

Where,
P = Principal

$\therefore$ Profit = profit-principal – principal

= Tk. (5,500 – 4,000) or, Tk. 1,500

Again, we know, $I=Prn$

or, $r=\frac{I}{Pn}$

$=\text{Tk. }\frac{1500}{4000\times 3}$

$=\frac{\overset{25}{\cancel{\overset{500}{\cancel{1500}}}}\times\cancel{100}}{\underset{2}{\cancel{\underset{40}{\cancel{4000}}}}\times 3}\%$ or, $\frac{25}{2}\%$ or, $12\frac{1}{2}\%$

Where,
P = Principal = Tk. 4,000
I = Profit = Tk. 1,500
r = Rate of Profit = Required
n = Time = 3 years

$\therefore$ Principal is Tk. 4,000 and rate of profit is $12\frac{1}{2}\%$

(D) Determination of time :

Example 7. In how many years will the profit of Tk. 10,000 be Tk. 4,800 at the rate of 12% profit?

Solution : We know, $I=Prn$

or, $n=\frac{I}{Pr}$

where profit I = Tk. 4,800, Principal P = Tk. 10,000

rate of profit r =12%, time n = ?

$$\therefore \text{ Time } n = \frac{\text{profit}}{\text{principal} \times \text{rate of profit}}$$

$$\therefore \text{ Time} = \frac{4{,}800}{10{,}000 \times \frac{12}{100}} \text{ years}$$

$$= \frac{\overset{4}{\cancel{48}}\,\cancel{4800} \times \cancel{100}^{1}}{\cancel{10000}_{\cancel{100}_{1}} \times \cancel{12}_{1}} \text{ years}$$

$$= 4 \text{ years}$$

$\therefore$ Time is 4 years.

Exercise 2.1

1. On selling a commodity, the profit of a wholesale seller is 20% and the profit of retail seller is 20%. If the retail selling price of the commodity is Tk. 576, what is the cost price of the wholesale seller ?
2. A shopkeeper sold some amount of pulses for Tk. 2,375 at the loss of Tk. 5%. What would be the selling price of pulses to make a profit of Tk. 6% ?
3. An equal number of bananas is bought at 10 and 15 pieces per Tk. 30 and all the bananas are sold at 12 pieces per Tk. 30. What will be the percentage of profit or loss?
4. What will be the profit for Tk. 2,000 in 5 years if the percentage of profit is Tk. 10.50 per annum?
5. How much less will be the profit of Tk. 3,000 in 3 years if the percentage of profit per annum is decreased from Tk. 10 to Tk. 8 ?
6. What is the percentage of profit per annum by which Tk. 13,000 will be Tk. 18,850 as profit-principal in 5 years ?
7. For what percentage of profit per annum, some principal will be double in profit-principal in 8 years ?
8. How much money will become Tk. 10,200 as profit-principal in 4 years at the same rate of profit at which Tk. 6,500 becomes Tk. 8,840 as the profit-principal in 4 years ?

9. Mr. Riaz deposited some money in a bank and got the profit of Tk. 4,760 after 4 years. If the percentage of profit of the bank is Tk. 8.50 per annum, what amount of money did he deposit in the bank ?

10. What amount of money will become Tk. 2,050 as profit-principal in 4 years, at the same rate of profit at which some principal becomes double as profit-principal in 6 years ?

11. For how much money will the profit at the rate of Tk. 5 per annum in 2 years 6 months be same as that ofTk. 500 at the rate ofTk. 6 per annum in 4 years?

12. Due to increase in the rate of profit from 8% to 10%, the income of Tisha Marma was increased by Tk. 128 in 4 years. How much was her principal?

13. Some principal becomes Tk. 1,578 as profit-principal in 3 years and Tk. 1,830 as profit-principal in 5 years. Find the principal and the rate of profit.

14. Tk. 3,000 at the rate of 10% profit and Tk. 2,000 at the rate of 8% profit are invested. What will be the average percentage of profit on the total sum of the principals?

15. Rodrick Gomage borrowed Tk. 10,000 for 3 years and Tk. 15,000 for 4 years from a bank and paid Tk. 9,900 in total as a profit. In both cases if the rate of profit is same, find the rate of profit.

16. Some principal becomes its double as profit-principal in 6 years at the same percentage of profit. In how many years will it be thrice of it as profit-principal at the same percentage of profit ?

17. The profit-principal for a certain period of time is Tk. 5,600 and the profit is $\frac{2}{5}$ of the principal. If the percentage of profit is Tk. 8, find the time.

18. After having the pension, Mr. Jamil bought pension savings certificates of Tk. 10 lac for five years term on the basis of having the profit in three months' interval. If the percentage of profit is 12% per annum, what amount of profit will he get at the first installment, that is, after first three months ?

19. A fruit seller bought some bananas at the cost of Tk. 36 for 12 pieces from Jessore and Tk. 36 for 18 pieces from Kustia. He bought equal pieces of bananas both from Jessore and Kustia. His salesman sold the bananas at Tk. 36 for 15 pieces.

A. What was the cost price of 100 pieces from Jessore?
B. If the salesman sold all bananas, how much would be profit or loss?
C. If the fruit seller wants to make 25% profit, what would be the selling price for 4 pcs of banana?

20. A principal is turned to an amount in 3 years of Tk. 28,000 and in 5 years of Tk. 30,000 at simple interest.
    A. Using the symbols in details, write the formula for Principal.
    B. Find out the rate of interest or profit.
    C. How much principal should be deposited to get the amount of Tk. 48,000 at the same rate of interest ?

2.4 Compound Profit

In the case of compound profit, the profit of any amount of principal is added to the principal at the end of each year and the total sum is considered as the new principal. If a depositor deposits Tk. 1,000 in a bank and the bank gives him the profit at the rate of 12%, the depositor will get profit on Tk. 1,000 at the end of one year.

$$\begin{aligned} 12\% \text{ of Tk. } 1{,}000 &= \text{Tk. } 1{,}000 \times \frac{12}{100} \\ &= \text{Tk. } 120. \end{aligned}$$

Then, for the 2nd year, his principal will be Tk. (1000 + 120) = Tk. 1,120, which is his compound principal. Again, 12% profit will be given on Tk. 1,120 at the end of 2nd year.

$$\begin{aligned} 12\% \text{ of Tk.} &= \overset{224}{\cancel{1120}} \times \frac{\cancel{12}}{\underset{\underset{5}{\cancel{25}}}{\cancel{100}}} \\ &= \text{Tk. } \frac{672}{5} \\ &= \text{Tk. } 134.40. \end{aligned}$$

So, for the 3rd year the compound principal of the depositor will be Tk. (1120+ 134.40) = Tk. 1254.40

In this way, the principal of the depositor at the end of each year will go on to be increased. This increased principal is called the compound principal and the profit which is given on the increased principal, is called the compound profit. Terms for giving profit may be for three months or six months or may be less than those periods.

Formation of the formulae for compound principal and compound profit :

Let the initial principal be P and the percentage of compound profit be r per annum. Then, at the end of 1st year, compound principal = principal + profit

$$=P+P\times r$$
$$=P(I+r)$$

Again, at the end of 2nd year, Compound principal = compound principal of the 1st year + profit

$$= P(1+r) + P(1+r)\times r$$
$$= P(1+r)(1+r)$$
$$= P(1+r)^2$$

At the end of 3rd year, Compound principal = compound principal of the 2nd year + profit

$$= P(1+r)^2 + P(1+r)^2\times r$$
$$= P(1+r)^2(1+r)$$
$$= P(1+r)^3$$

We observe : In the compound principal

at the end of 1st year, index of $(1+r)$ is 1

at the end of 2nd year, index of $(1+r)$ is 2

at the end of 3rd year, index of $(1+r)$ is 3

$\therefore$ at the end of nth year, index of $(1+r)$ will be n .

$\therefore$ if C be the compound principal at the end of C years, then $P(1+r)^n$.

Again, compound profit = Compound principal - Initial principal= $P(1+r)^n - P$

Formula : Compound principal $C = P(1+r)^n$
Compound profit $= P(1+r)^n - P$

Now, we apply the formula for compound principal in case where the initial principal of Tk. 1,000 and profit of 12% were taken at the beginning of the discussion about compound principal :

At the end of 1st year, compound principal $= P(1+r)$

$$= \text{Tk. } 1{,}000 \times \left(1+\frac{12}{100}\right)$$
$$= \text{Tk. } 1{,}000 \times (1+0.12)$$
$$= \text{Tk. } 1{,}000 \times 1.12$$
$$= \text{Tk. } 1{,}120$$

At the end of 2nd year, compound principal $= P(1+r)^2$

$$= \text{Tk. } 1{,}000 \times \left(1+\frac{12}{100}\right)^2$$
$$= \text{Tk. } 1{,}000 \times (1+0.12)^2$$
$$= \text{Tk. } 1{,}000 \times (1.12)^2$$
$$= \text{Tk. } 1{,}000 \times 1.2544$$
$$= \text{Tk. } 1{,}254.40$$

At the end of 3rd year, compound principal $= P(1+r)^3$

$$= \text{Tk. } 1{,}000 \times \left(1+\frac{12}{100}\right)^3$$
$$= \text{Tk. } 1{,}000 \ (1+0.12)^3$$
$$= \text{Tk. } 1{,}000 \times (1.12)^3$$
$$= \text{Tk. } 1{,}000 \times 1.404928$$
$$= \text{Tk. } 1{,}404.93 \text{ (nearly)}$$

Example 1. Find the compound principal of Tk. 62,500 in 3 years at the profit of Tk. 8 percent per annum.

Solution : We know, compound principal. $C = P\,(1+r)^n$.

Given, initial principal P = Tk. 62,500

percentage of profit r = 8%

and time n = 3 years

$$\therefore C = 62{,}500 \times \text{Tk.}\left(1+\frac{\overset{2}{\cancel{8}}}{\underset{25}{\cancel{100}}}\right)^3 = \text{Tk.}\,62{,}500 \times \left(\frac{27}{25}\right)^3$$

$$= \text{Tk. } 62{,}500 \times (1.08)^3$$

$$= \text{Tk. } 78{,}732$$

$\therefore$ Compound principal is Tk. 78,732

Example 2. Find the compound profit of TK. 5000 at the profit of 10.50% per annum in 2 years.

Solution : To find out the compound profit, at first we find the compound principal.

We know,

compound principal $C = P(1+r)^n$, where principal P = Tk. 5,000

percentage of profit $r = 10.50\% = \dfrac{21}{200}$

and time n = 2 years.

$$\therefore C = P(1+r)^2$$

$$= \text{Tk. } 5{,}000 \times \left(1+\frac{21}{200}\right)^2$$

$$= \text{Tk. } 5{,}000 \times \left(\frac{221}{200}\right)^2$$

$$= \text{Tk. } \overset{\overset{1}{\cancel{25}}}{\cancel{5000}} \times \frac{221}{\underset{1}{\cancel{200}}} \times \frac{221}{\underset{8}{\cancel{200}}}$$

$= \text{Tk. } \frac{48,841}{8}$ or Tk. 6,105.13 (nearly)

$\therefore$ Compound profit $= C - P = P\,(1+r)^2 - P$

$= \text{Tk. } (6,105.13 - 5,000)$

$= \text{Tk. } 1,105.13$ (nearly)

Example 3. A flat owners welfare association deposited the surplus money of Tk. 2,00,000 from its service charges in a bank in the fixed deposit scheme on the basis of compound profit for six months' interval. If the percentage of profit is Tk. 12, how much profit will be credited to the account of the association ? What will be the compound principal after one year ?

Solution : Given, principal $P = \text{Tk. } 2,00,000$

rate of profit $r = 12\%$, time $n = 6$ months $= \frac{1}{2}$ year

$\therefore$ profit $= \text{Tk. } I = Prn$

$$= \text{Tk. } \overset{2,000}{\cancel{2,00,000}} \times \frac{\overset{6}{\cancel{12}}}{\underset{1}{\cancel{100}}} \times \frac{1}{\underset{1}{\cancel{2}}}$$

$= \text{Tk. } 12,000$

$\therefore$ profit after 6 months will be Tk. 12,000

After 6 months compound principal=Tk. (2,00,000+12,000) = Tk. 2,12,000

Again, profit-principal after 6 months $= \text{Tk. } 2,12,000\,(1+\frac{12}{100} \times \frac{1}{2})$

$= \text{Tk. } 2,12,000 \times 1.06$

$= \text{Tk. } 2,24,720$

$\therefore$ Compound principal after 1 year will be Tk. 2,24,720.

Example 4. present population of a city is 80 lac.What will be the population of the city after 3 years if the growth rate of population of that city is 30 per thousand?

Solution : present population of the city is $P = 80{,}00{,}000$

growth rate of population $= \frac{30}{1000} \times 100\% = 3\%$

time $n = 3$ years.

Here, in the case of growth of population, formula for compound principal is applicable.

$\therefore \; C = P\,(1+r)^n$

$= 80{,}00{,}000 \times \left(1 + \frac{3}{100}\right)^3$

$= 80{,}00{,}000 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100}$

$= 8 \times 103 \times 103 \times 103$

$= 87{,}41{,}816$

$\therefore$ After 3 years, population of the city will be 87,41,816.

Example 5. To meet an urgent family need, Monowara Begum takes a loan of taka 'x' at the rate of 6% interest and taka 'y' at 4% interest. She takes in total taka 56,000 as a loan and pays Tk. 2,840 as interest.

A. What will be annual interest if 5% interest is imposed on total loan?

B. Find out the value of x and y.

C. How much interest will be paid by Monowara Begum if 5% compound interest is imposed for 2 years ?

Solution :

A. Total amount of loan P = Tk 56,000

Rate of interest, $r = 5\% = \frac{5}{100}$

Time, n = 1 year

$\therefore$ Interest I $= Pnr$

$= $ Tk. $(56000 \times 1 \times \frac{5}{100})$

$=$ Tk. 2800

B. The annual Interest on Tk. x at 6% $= \text{Tk } (x \times 1 \times \frac{6}{100})$

$= \text{Tk } \frac{6x}{100}$

Again, The annual Interest on Tk 'y' at 4% $= \text{Tk } (y \times 1 \times \frac{4}{100})$

$= \text{Tk } \frac{4y}{100}$

Now, according to the information of the stimulus,

$x + y = 56{,}000$ ......(i)

And $\frac{6x}{100} + \frac{4y}{100} = 2{,}840$

Or, $6x+4y = 2{,}84{,}000$

Or, $3x+2y = 1{,}42{,}000$ ......(ii)

Now multiply equation (i) by 3 and subtract equation (ii)

$$\begin{aligned} 3x+3y &= 168000 \\ \underline{3x+2y} &\underline{= 142000} \\ y &= 26000 \end{aligned}$$

Now, putting the value of 'y' in equation (i)

We get, $x = 30{,}000$

$\therefore$ x = Tk. 30,000 and y = Tk. 26,000

C. Total amount of loan P = Tk. 56,000

Rate of Interest $r = \frac{5}{100}$

Time n = 2 years.

Now, for compound rate of Interest, the amount of loan $= P(1+r)^n$

$\therefore$ After 2 years, the amount of loan $= \text{Tk. } 56{,}000\ (1+\frac{5}{100})^2$

$= \text{Tk. } 56{,}000 \text{ x } (1.05)^2$

$= \text{Tk. } 61{,}740$

$\therefore$ The interest to be paid by Monowara $= \text{Tk. } (61{,}740-5{,}600)$

$= \text{Tk. } 5{,}740$

Exercise 2.2

1. Which one of the following is 8% of Tk. 1050 ?

2018 a. Tk. 80 b. Tk. 82 c. Tk. 84 d. Tk. 86

2. What is the simple profit of Tk. 1,200 in 4 years at the rate of simple profit of 10% per annum?

a. Tk. 120 b. Tk. 240 c. Tk. 360 d. Tk. 480

3. The cost price of something is 5 pieces at Tk. 1 and selling price is 4 pieces at Tk. 1. What will be the percentage of profit or loss?

A. Profit 25% B. Loss 25% C. Profit 20% D. Loss 20%

4. Counting Profit :

i. Profit = profit-principal - principal.

ii. Profit = $\frac{\text{Principal} \times \text{Profit} \times \text{Principal}}{2}$

iii. Compound Profit = Compound principal- principal

According to the above information, which one of the following is correct ?

a. *i* and *ii* b. *i* and *iii* c. *ii* and *iii* d. *i, ii* and *iii*

5. At 10% simple profit for principal Tk. 2000

i. Profit in 1 year is Tk. 200

ii. Amount in 5 years is $1\frac{2}{3}$ times of principal.

iii. In 6 years the profit will be equal to principal.

Which one of the following is correct?

A. i and ii B. i and iii C. ii and iii D. i, ii and iii

6. Mr.Jamil deposited Tk. 2,000 in a bank at the rate of profit 10% per annum:

(1) What will be the profit-principal at the end of 1st year ?

a. Tk. 2,050 b. Tk. 2,100 c. Tk. 1,200 d. Tk. 2,250

(2) In simple profit, what will be the profit-principal at the end of 2nd year?

a. Tk. 2,400 b. Tk. 2,420 c. Tk. 2,440 d. Tk. 2,450

(3) What will be the compound principal at the end of 1st year ?

a. Tk. 2,050 b. Tk. 2,100 c. Tk. 21,500 d. Tk. 2,200

7. If the rate of profit is 10% per annum, find the compound principal ofTk. 8000 in 3 years.

8. What will be the difference of simple profit and compound profit of Tk. 5,000 in 3 years if the rate of profit is Tk. 10 percent per annum?

9. What was the principal if the compound principal of any amount of principal at the end of one year is Tk. 6,500 and at the end of two years is Tk. 6,760 at the same rate of profit ?

10. If the rate of compound profit is Tk. 8.50 percent per annum, find the compound principal and compound profit ofTk. 10000 in 2 years.

11. Present population of a city is 64 lac. What will be the population of the city after 2 years if growth rate of population of the city is 25 per thousand ?

12. A person borrows Tk. 5,000 from a lending organization at the rate of 8% compound profit. At the end of every year he paid off Tk.2,000. How much more money will he have as loan after paying off the 2nd installment ?

13. At the same rate of compound profit a principal will amount to Tk. 19,500 after 1 year and Tk. 20,280 after 2 years.
   A. Write down the formula for profit.
   B. Find out the principal.
   C. Find the difference between simple profit and compound profit after 3 years at the same rate for the principal.

14. Shipra Barua deposited Tk. 3,000 in a bank and got Tk. 3,600 together with the profit after 2 years.
   a. Find the percentage of simple profit.
   b. What will be the profit-principal after another 3 years ?
   c. What would be the compound principal after 2 years if Tk. 3,000 was deposited at the same percentage of compound profit ?

Chapter three
Measurement

The method of measurement of different types of commodities and other materials used in our day to day life depends on their shapes, sizes and types. There are different systems for measuring lengths, weights and volumes of liquid. The system for the measurement of length is used to derive a system for measuring areas and volumes. Again, we also need to know the number of population, animals, fruits, rivers and streams, houses, vehicles etc. These are measured by simple counting.

At the end of this chapter, the students will be able to -

- Explain national, British and international systems of measurement and solve the problems involving determination of length, area, weight, volume of liquid by related systems.

- Measure by the daily used scales in national, British and international systems.

3.1 Concept of Measurement and Units

A unit is required in any counting or measurement. The unit for counting, 1 is the first natural number. For measuring length a definite length is chosen to be 1 unit. Similarly, a definite weight is chosen to be a unit weight which is known as the unit of weight. Again, the unit for measuring the volume of liquid is also determined in such a way. A square with a side of 1 unit length is taken to be the unit of area and is termed as 1 square unit. Similarly, the volume of a cube with sides of 1 unit length is called 1 cube unit. In all cases, the concept of whole measurement is obtained through units. But there are different units in different countries for measurement.

3.2 Measurement in Metric System

The different systems of measurement in different countries cause problems in international trades and transactions. That is why, the system international (SI) or the metric system has been used for measurement in trade and transaction. The characteristic of this system is that it is a system of multiples of ten. In this system the measurement of fractions can easily be expressed by the decimal fraction. In the eighteenth century it was first introduced in France.

The unit of measurement of length is 1 metre. One part of 1 crore parts of the distance from the North Pole of the earth to the equator along the longitude over Paris is considered to be one metre. Later, the length of a platinum rod kept in Paris Museum has been recognised as one metre. Linear measurements are made considering this length as unit. For measurement of small length, centimetre is used while larger length is expressed in kilometres. The term **metric system** is derived from this unit of length- **metre**.

The metric unit of measurement of weight is gram. For measurement of small weights, gram is used while larger weights are expressed in kilograms (kg).

The unit of measurement of volume of liquid is litre. It is also a unit of metric system. For measurement of small volumes of liquids, litre is used and to measure larger quantity of liquids, kilolitre is used.

In metric system, to convert from larger unit into smaller unit and vice versa, the digits are written side by side and decimal point is moved right or left as required.

For example, 5 km. 4 hm.7 deca.m 6m. 9 deci.m 2 cm 3 mm

$$= (50{,}00{,}000 + 4{,}00{,}000 + 70{,}000 + 6{,}000 + 900 + 20 + 3) \text{ mm}$$

$$= 54{,}76{,}923 \text{ mm} = 5{,}47{,}692.3 \text{ cm} = 54{,}769.23 \text{ deci.m} = 5{,}476.923 \text{ m}$$

$$= 547.6923 \text{ deca.m} = 54.76923 \text{ h.m} = 5.4768213 \text{ km}$$

We know that in any decimal numbers the place value of any digit is ten times of the place value of the digit just to the right of it and the place value of that digit is one tenth of the place value of the number just to the left. In metric system, there exists such relation among the units of measurement of length, weight and volume. Hence, in metric system, the measured length, weight or volume can be easily expressed in any other unit. The list of place value taken from Greek and Latin is as follows :

From Greek			Unit	From Latin		
thousand	hundred	ten		One tenth	One hundredth	One thousandth
1000 kilo	100 hecto	10 deca	1 metre gram liter	$\frac{1}{10} = .1$ deci	$\frac{1}{100} = .01$ centi	$\frac{1}{1000} = .001$ milli

The multiples from Greek and parts from Latin have been added as prefixes to the units.

Deca means 10 times, hecto means 100 times and kilo means 1000 times in Greek language and in Latin deci means one tenth, centi means one hundredth and milli means one thousandth.

3.3 The units of measuring length

Metric System			British System		
10 millimetres (mm)	=	1 centimetre (cm)	12 inches	=	1 foot
10 centimetres (cm)	=	1 decimetre (deci m)	3 feet	=	1 yard
10 decimetres (deci m)	=	1 metre (m)	1760 yards	=	1 mile
10 metres (m)	=	1 decametre (deca m)	6080 feet	=	1 nautical mile
10 decametres (deca m)	=	1 hectometre (hm)	220 yards	=	1 furlong
10 hectometres (h.m)	=	1 kilometre (km)	8 furlongs	=	1 mile

Unit of measurement of length : metre

3.4 Relation between British and Metric System

1 inch	=	2.54 cm (approximate)	1 metre	=	39.37 inches (approximate)
1 yard	=	0.9144 cm (approximate)	1 km	=	0.62 miles
1 mile	=	1.61 km (approximate)			

The relation between the British and the Metric System can not be determined exactly. That is why, this relation is expressed approximately with a few decimal places. The ruler is used to measure short lengths and tapes are used for measuring larger lengths. Usually the length of a tape is about 30 metres or 100 feet.

Activity :

1. Measure the length of your bench in inches and centimetres by a ruler. Determine from this, how many inches equal to 1 metre .
2. Determine from above relation, how many kilometres equal to a mile.

Example 1. A runner ran 24 rounds in a circular track of a length of 400 metres. How much distance did he run?

Solution : 1 round is 400 metres.

$\therefore$ The distance of 24 rounds will be (400×24) metres or 9600 metres or 9 kilometres 600 metres.

Therefore, the runner ran 9 kilometres 600 metres.

3.5 Measurement of Weights

Objects around us have weights. Their weights are measured by using different units in different countries.

Metric Units of Measurement of Weights

10 milligrams (mg)	=	1 centigram (cgm)
10 centigrams (cgm)	=	1 decigram (deci gm)
10 decigrams (deci gm)	=	1 gram (gm)
10 grams (gm)	=	1 decagram (deca gm)
10 decagrams (deca gm)	=	1 hectogram (hgm)
10 hectograms (hgm)	=	1 kilogram (kg)

Unit of weight : gm	1 kilogram or 1 kg = 1000 grams

There are two more units used for measurement in metric system. The units quintal and metric ton are used in order to measure large quantity of goods.

100 kilograms	=	1 quintal
1000 kilograms	=	1 metric ton

Activity :

1. Find the weight of your 5 books by the balance with a pointer.

2. Find your weight by a digital balance.

Example 2. How much rice each of them will get if 1 metric ton rice is distributed among 64 labours ?

Solution : 1 metric ton = 1000 kg

64 labours get 1000 kg rice

$\therefore$ 1 ,, ,, ,, $\frac{1000}{64}$ kg rice

= 15 kg 625 gm rice

$\therefore$ Each labour will get 15 kg 625 gm rice.

3.6 Measurement of Volume of Liquids

The space occupied by any liquid is its volume. A solid body has length, breadth and height. But no liquid material has definite length, breadth and height. Liquid takes the shape of the container it is put in. That is why, liquid is measured by a pot of definite volume. Usually, we use litre pots. These are conically or cylindrically shaped mug made of aluminum or iron sheet, having capacities $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 3, 4 etc. litres. Again, vertical pots made of transparent glass with 25, 50, 100, 200, 300, 500, 1000 millilitre marks are also used. Usually, those pots are used to measure milk, oil etc.

At present for the convenience of buyers and sellers edible oils are sold in bottles. In such cases, also, bottles of 1, 2, 5 and 8 literes are widely used. Different types of soft drinks are sold in 250, 500, 1000, 2000 millilitre bottles.

Metric Units for measurement of Volume of Liquids

10 millilitres (ml)	= 1 centilitre (cl)
10 centilitres	= 1 decilitre (dl)
10 decilitres	= 1 litre (l)
10 litres	= 1 decalitre (decal)
10 decalitres	= 1 hectolitre (hl)
10 hectolitres	= 1 kilolitre (kl)

The unit of measuring volume of liquid : litre

Remarks : The weight of 1 cubic centimetre of pure water at 4° Celsius is 1 gram. Cubic centimetre is abbreviated as cc in English.

Weight of 1 litre of pure water is 1 kilogram

In metric units, if the unit of any measurement is known, the others can easily be derived. If the units of measurement of length are known, the measurement of weight and volume of liquid are found by putting gram or litre in place of metre only.

Activity :

1. Measure the capacity of your water container in c.c and express it in cubic inches.
2. Assume the volume of a pot of an unknown volume given by your teacher. Then find the exact volume and estimate the error.

Example 3. The length of a tank is 3 metres, the breadth is 2 metres and the height is 4 metres. How many litres and kilograms of pure water will it contain?

Solution : The length of the tank = 3 metres, breadth = 2 metre and height = 4 metres

$\therefore$ The volume of the tank = $(3 \times 2 \times 4)$ cubic metres = 24 cubic meters

= 24000000 cubic cm

= 24000 litres [1000 cubic cm = 1 litre]

The weight of 1 litre pure water is 1 kilogram

$\therefore$ The weight of 24000 litres of pure water is 24000 kilogram.

Therefore, the tank contains 24000 litres of water and its weight is 24000 kilograms.

3.7 Measurement of Area

Measurement of area of a rectangle = length × breadth

Measurement of area of a square = $(\text{side})^2$

Measurement of area of a triangle = $\frac{1}{2} \times$ base × height

Unit of measure of area : square metre

Metric Units in Measuring Area

100 square centimetres (sq. cm)	=	1 square decimetre (sq. deci m)
100 square decimetres (sq. deci m)	=	1 square metre (sq. m)
100 square metres (sq. m)	=	1 are (squre decametre)
100 are (squre decametre)	=	1 hector (or 1 square hecto metre)
100 hectors (or 1 square hecto metre)	=	1 square kilometre

Metric units

144 square inches = 1 sq. feet
9 sq. feet = 1 sq. yard
4840 sq. yards = 1 acre
100 decimals = 1 acre

Local Units

1 sq. arm = 1 Ganda
20 Ganda = 1 Chatak
16 Chatak = 1 Katha
20 Katha = 1 Bigha

Relation between Metric and British System in Measuring Area

1 sq. centimetre	=	0.16 sq. inches (approx.)
1 sq. metre	=	10.76 sq. feet (approx.)
1 hector	=	2.47 acres (approx.)
1 sq. inch	=	6.45 acres (approx.)
1 sq. feet	=	929 sq. centimetres (approx.)
1 sq. yard	=	0.84 sq. metres (approx.)
1 sq. mile	=	640 acres

Relation between Metric, British and National Units in Measuring Area

1 sq. arm	=	324 sq. inches
1 sq. yard or 4 ganda	=	9 sq. feet = 0.836 sq. metres (approx.)
1 Katha	=	720 sq. feet = 80 sq. yard = 66.89 sq. metres (approx.)
1 Bigha	=	1600 sq. yards = 1337.8 sq. metres (approx.)
1 Acre	=	3 Bigha 8 chatak = 4046.86 sq. metres (approx.)
1 decimal	=	435.6 sq. feet = 1000 sq. kari (100 kari = 66 feet)
1 mile	=	1936 Bigha
1 sq. metre	=	4.78 ganda (approx.) = 0.239 chatak (approx.)
1 are	=	23.9 chatak (approx.)

Activity :

1. Measure the length and the breadth of a book and table in inches and centimetres by a scale and find their areas in both units. From this find the relation between 1 sq. inch and 1 sq. centimetre.
2. Measure the length and the breadth of a bench, table, door, window etc. in a group in inches and centimetres by scale and find their areas.

Example 4. 1 inch = 2.54 centimetres and 1 acre = 4840 yards. How many square metres are there in 1 acre ?

Solution : 1 inch = 2.54 centimtres

$\therefore$ 36 inches or 1 yard $= 2.54 \times 36$ centimetres

$= 91.44$ centimetres

$= \dfrac{91.44}{100}$ metres $= 0.9144$ metres

$\therefore$ 1 yard × 1 yard $= 0.9144$ metres × 0.9144 metres

or, 1sq. yard $= 0.83612736$ sq. metres

$\therefore$ 4840 sq. yard $= 0.83612736 \times 4840$ sq. metres

$= 4046.85642240$,, ,,

$= 4046.86$ sq. metres (app.)

$\therefore$ 1 acre $= 4046.86$ sq. metres (app.)

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Example 5. The area of Jahangirnagar University is 700 acres. Express it in hectors in the nearest integer.

Solution : 2.47 acres = 1 hector

$$1 \text{ ,, } = \frac{1}{2.47} \text{ ,,}$$

$$700 \text{ ,, } = \frac{1 \times 700 \times 100}{247} \text{ hectors } = 283.4 \text{ hectors}$$

Therefore, required area is 283 hectors (app.)

Example 6. The length of a rectangle is 40 metres and the breadth is 30 metres 20 cm. What is the area of the rectangle?

Solution : Length of the rectangle = 40 metres = (40×100) cm = 4000 cm

and breadth = 30 metres 30 cm

= (30×100) cm + 30 cm

= 3030 cm

∴ Required area = (4000×3030) sq. cm = 12120000 sq. cm

= 1212 metres = 12 ares 12 sq. metres

Therefore, the area of the rectangle is 12 ares 12 sq. metres.

3.8 Volume

Volume is the cubic measurement of solid

Volume of rectangular solid = length × breadth × height

Volume of a solid is determined by expressing length, breadth and height of the solid in the same units. The volume of a solid body of 1 cm length, 1 cm breadth and 1 cm height is 1 cubic centimetre.

Metric Units of Measuring Volume

1000 cubic centimetres (c.cm)	=	1 cubic decimetre (c.dm.) = 1 litre
1000 cubic deci metres	=	1 cubic metre (c.m)
1 cubic metre	=	1 stayor
10 stayor	=	1 deca stayor
1 cubic cm. (cc) = 1 millilitre		1 cubic inch = 16.39 millilitres(app.)

Relation between Metric and British Systems of Volume

1 Stayor	=	35.3 cubic feet (app.)
1 decastayor	=	13.08 cubic yards (app.)
1 cubic feet	=	28.67 litres (app.)

Activity :

1. Measure the length, the breadth and the height of your most volumous book and find its volume.

2. Guess the volume of a box specified by your class teacher. Then find the exact volume and determine the error.

Example 7. The length of a box is 2 metres, the breadth is 1 metre 50 cm and the height is 1 metre. What is the volume of the box?

Solution : length = 2 metres = 200 cm

breadth = 1 metre 50 cm = 150 cm

and height = 1 metre = 100 cm

$\therefore$ Volume of the box = length × breadth × height

= (200 × 150 × 100) cubic centimetres

= 3000000 cubic cm

= 3 cubic metres

Alternative method : length = 2 metres, breadth = 1 metre 50 cm = $1\frac{1}{2}$ metres and height = 1 metre

$\therefore$ Volume of the box = length × breadth × height

$$= \left(2 \times \frac{3}{2} \times 1\right) \text{ cubic metres}$$

= 3 cubic metres

$\therefore$ Required volume is 3 cubic metres.

Example 8. The capacity of containing water of a tank is 8000 litres. The length of the tank is 2.56 metres and breadth is 1.25 metres. What is the depth of the tank ?

Solution : The area of the bottom $= 2.56 \times 1.25$ metres

$= 256$ cm $\times 125$ cm

$= 32000$ sq. cm

The capacity of containing water is 8000 litres or 8000×1000 cc [1 litre=1000 cc]

Therefore, the volume is 8000000 cubic cm

$\therefore$ The depth of the tank $= \dfrac{8000000}{32000}$ cm $= 250$ cm

$= 2.5$ metres

Alternative method

The area of the bottom of the tank $= 2.56$ metres $\times 1.25$ metres

$= 3.2$ sq. metres

The capacity of the tank is 8000 litres or 8000×1000 cubic cm

$\therefore$ Volume of the tank $= \dfrac{8000 \times 1000}{1000000}$ cubic cm $= 8$ cubic metres [1000000 cc = 1 cubic m]

$\therefore$ Depth of the tank $= \dfrac{8}{3.2}$ metres

$= 2.5$ metres

Example 9. The length of a house is 3 times the breadth. To cover the house by carpet an amount of Tk. 1102.50 is spent at the rate of Tk. 7.50 per sq. metre of carpet. Find the length and the breadth of the house.

Solution : Tk. 7.50 is spent for 1 sq. metre

$\therefore$ Tk. 1 ,, ,, ,, $\dfrac{1}{7.50}$ sq. metres

$\therefore$ Tk. 1102.50 ,, ,, ,, $\dfrac{1 \times 1102.50}{7.50}$ sq. metres

$= 147$ sq metres.

i.e., the area of the house is 147 sq. metres.

Let, the breadth $= x$ metres

$\therefore$ the length $= 3x$ metres

$\therefore$ Area = (length × breadth) sq. units

$= (3x \times x)$ sq. units $= 3x^2$ sq. units

According to the condition

$$3x^2 = 147$$

$$\text{or, } x^2 = \frac{147}{3}$$

$$\text{or, } x^2 = 49$$

$$\therefore x = \sqrt{49} = 7$$

Therefore, breadth = 7 metres

and length = (3 × 7) metres or 21 metres.

Example 10. Air is 0.00129 times heavier than water. How many kilograms of air are there in the house whose length, breadth and height are 16 metres, 12 metres and 4 metres respectively?

Solution : Volume of the house = length × breadth × height

= 16 metres × 12 metres × 4 metres

= 768 cubic metres

= 768 × 1000000 cubic cm

= 768000000 cubic cm

Air is 0.00129 times heavier than water.

$\therefore$ The weight of 1 cubic cm of air = 0.00129 grams

So, the quantity of air = 76800000 × 0.00129 gm

$= 990720$ gm

$= 990.72$ kg

$\therefore$ There are 990.72 kg of air in the house.

Example 11. There is a 2-metre wide road around the outside of a garden of the length of 21 metres and the breadth of 15 metres. How much money will be spent to planting grass at Tk. 2.75 per sq. metre?

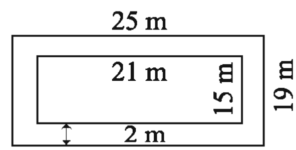
Solution :

The length of the garden along with the road = 21 m + (2+2) m = 25 metres

The breadth of the garden along including the road = 15 m + (2 + 2) m = 19 m

The area of the garden including the road $= (25 \times 19)$ sq. m

$= 475$ sq. m



The area of the garden excluding the road $= (21 \times 15)$ sq. m

$= 315$ sq. metres

$\therefore$ Area of the road $= (475 - 315)$ sq. metres

$= 160$ sq. metres

The total cost to planting grass = Tk. (160×2.75)

= Tk. 440.00

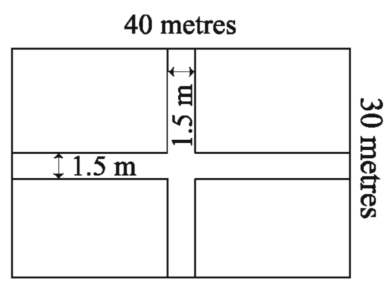
Therefore, the total cost for planting grass is Tk. 440.

Example 12. There are two crosswise roads of breadth 1.5 metres just in the middle of a field of length 40 metres and breadth 30 metres. What is the area of the two roads?

Solution : The area of the road along the length $= 40 \times 1.5$ sq. metres

$= 60$ sq. metres

The area of the road long the breadth



$= (30 - 1.5) \times 1.5$ sq. metres

$= 28.5 \times 1.5$ sq. metres

$= 42.75$ sq. metres

Therefore, the area of the two roads

$$= (60 + 42.75) \text{ sq. metres}$$
$$= 102.75 \text{ sq. metres}$$

$\therefore$ Total area of the two roads is 102.75 sq. metres.

Example 13. An amount of Tk. 7,500 is spent to carpet a room of length 20 metres. If the breadth of that room is reduced by 4 metres, an amount of Tk. 6,000 would be spent. What is the breadth of that room?

Solution : Length of the room is 20 metres. For a decrease of 4 metres in length, the area decreases by (20 metre × 4 metres)= 80 sq. metres

So, for a decrease of 80 sq. metres, the cost reduces by Tk. (7,500 - 6,000) = Tk. 1,500

Tk. 1,500 is spent for 80 sq. metres

$$\therefore 1 \quad ,, \quad ,, \quad ,, \quad = \frac{80}{1,500} \quad ,, \quad ,,$$

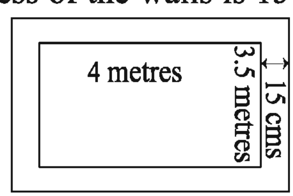
$$\therefore 7,500 \quad ,, \quad ,, \quad ,, \quad = \frac{80 \times 7,500}{1,500} \quad ,, \quad ,, \quad \text{or, } 400 \text{ sq. metres}$$

Therefore, the area of the room is 400 sq. metres.

$$\therefore \text{Breadth of the room} = \frac{\text{Area}}{\text{Length}}$$
$$= \frac{400}{20} \text{ metres}$$
$$= 20 \text{ metres}$$

$\therefore$ The breadth of the room is 20 metres.

Example 14. The length of the floor of a house is 4 metres and the breadth is 3.5 metres. The height of the house is 3 metres and the thickness of the walls is 15 cm. What is the volume of the four walls?



Solution : Thickness of the walls is15 cm = 0.15 metres

According to the figure, the volume of the two walls along the length

$= (4 + 2 \times 0.15) \times 3 \times 0.15 \times 2$ cubic metres = 3.87 cubic metres

And the volume of the two walls along the breadth $= 3.5 \times 3 \times 0.15 \times 2$ cubic metres

$= 3.15$ cubic metres

$\therefore$ Total volume of walls $= (3.87 + 3.15)$ cubic metres

$= 7.02$ cubic metres

$\therefore$ Required volume is 7.02 cubic metres.

Example 15. There are 3 doors and 6 windows in a house. Each of the doors is 2 metres long and 1.25 metres wide and each of the windows is 1 metre long and 1 metre wide. How many planks of 5 metres long and 0.60 metres wide are required to make the doors and windows?

Solution : Areas of 3 doors $= (2 \times 1.25) \times 3$ sq. metres

$= 7.5$ sq. metres

Areas of 6 windows $= (1.25 \times 1) \times 6$ sq. metres $= 7.5$ sq. metres

Area of a plank $= (5 \times 0.6)$ sq. metres $= 3$ sq. metres

Required numbers of planks = Total area of doors and windows ÷ area of a plank

$= (7.5 + 7.5) \div 3$

$= 15 \div 3$

$= 5$

Example 16.

A rectangular iron bar is 8.8 c.m. long, 6.4 c.m. wide and 2.5 c.m. high. The iron bar is kept in a pot measuring 15 c.m. $\times$ 6.25 c.m. $\times$ 4 c.m. and the pot is filled with water. Iron is 7.5 times heavier than water.

A. Find out the volume of the water pot.
B. Find out the weight of the iron bar.
C. The iron bar is taken out of the fully filled water pot. What will be the height of the water in the pot?

Solution :

A. Length of the water pot, $l = 15$ c.m.

Width of the water pot, $w = 6.25$ c.m.

Height of the water pot, $h = 4$ c.m.

$\therefore$ Volume of the water pot $= (15 \times 6.25 \times 4)$ cubic c.m. $[V = l \times w \times h]$

$= 375$ cubic c.m.

B. Length of the iron bar = 8.8 c.m.
Width of the iron bar = 6.4 c.m.
and height = 2.5 c.m.

$\therefore$ Volume of the iron bar $= (8.8\times6.4\times2.5)$ cubic c.m.
$= 140.8$ cubic c.m.

Now, we know that the weight of 1 cubic c.m.water = 1 gm

And given that iron is 7.5 times heavier than water.

$\therefore$ Weight of 1 cubic c.m. iron rod $= (1\times7.5)$ gm

$\therefore$ Weight of 140.8 cubic c.m. iron rod $= (7.5\times140.8)$ gm
$= 1056$ gm
$= 1.056$ kg [$\because$ 1 kg = 1000 gm]

$\therefore$ The weight of the iron bar is 1.056 kg.

C. Volume of the water pot = 375 cubic cm

Volume of the iron bar = 140.8 cubic cm

Hence, the iron rod is taken out of the fully filled pot.

$\therefore$ The volume of the remaining water in the pot $= (375-140.8)$ cubic cm
$= 234.2$ cubic cm

Let the height of the remaining water in the pot be x cm

$$\therefore x \times 15 \times 6.25 = 234.2$$

$$\text{Or, } x = \frac{234.2}{15\times6.25}$$

$$= \frac{234.2}{93.75}$$

$$= 2.50 \text{ (approx)}$$

$\therefore$ Height of the water remaining in the pot is 2.50 cm (approx).

Exercise 3

1. In Greek Deca means-

   A. 10 times B. 100 times C. Tenth D. Hundredth

2. 1 Stayor is—

   i. 13.08 cubic yards ii. 1 cubic metre iii. 35.3 cubic feet

   Which one of the following is correct?

   A. i and ii B. i and iii C. ii and iii D. i, ii and iii

3. What is the area of a cubic of the side of 4 cm?

   A. 16 sq.cm B. 24 sq.cm C. 64 sq.cm D. 96 sq.cm

4. The area of a rectangular field is 10 hectors. What is its value in 'are'-

   A. 2.47 B. 4.049 C. 100 D. 1000

5. A water tank is filled with water. Its length is 3m, breadth is 2m and height is 1m.

   i. The volume of the water tank is 6 cubic meters.
   ii. The weight of the water in the tank is 6 kg.
   iii. The volume of the tank full of water is 6000 cubic metre.

   Which one of the following is correct?

   A. i and ii B. i and iii C. ii and iii D. i, ii and iii

#Answer to the questions 6 and 7 in accordance with the following statement:
Area of a rectangular garden is 400 sq.m and its breadth is 16 m.

6. What is the parameter of the rectangle in metre?

   A. 16 B. 25 C. 41 D. 82

7. What is the diagonal of the garden in metre?

   A. 29.68 B. 29.86 C. 32.68 D. 41

8. The circumference of the wheel of a cart is 5m.How many times will the wheel move to go a distance of 1km 500m?

   A. 200 B.250 C.300 D.350

9. International unit system is----

   i. its characteristic is 10 times
   ii. it is institutionalised first in France in 18^{th} Century.
   iii. it is institutionalised on July 01 in 1982 in Bangladesh.

   Which one of the following is correct?

   A.i and ii B. i and iii C. ii and iii D. i, ii and iii

10. The length of a pond is 60 metres and the breadth is 40 metres. If the breadth of its bank is 3 metres, find the area of the bank.

11. The area of a rectangle is 10 acres and its length is 4 times the breadth. What is the length of the rectangle in metres?

12. The length of a rectangular house is one and a half time its breadth. If the area of the house is 216 sq. metres, what is its perimeter?

13. The base of a triangular region is 24 metres and the height is 15 metres 50 cm. Find its area.

14. The length of a rectangle is 48 metres and its breadth is 32 metres 80 cm. Around outside there is a road of breadth 3 metres. What is the area of the road?

15. The length of one side of a square is 300 metres and around its outside, there is a road of breadth 4 metres. Find the area of the road.

16. The area of a triangular land is 264 sq. metres. Find the height if the base is 22 metres.

17. A reservoir contains 19200 litres of water. Its depth is 2.56 metres and its breadth is 2.5 metres. What is its length ?

18. Gold is 19.3 times heavier than water. The length of a rectangular gold bar is 7.8 cm, the breadth is 6.4 cm and the height is 2.5 cm. What is the weight of the gold bar?

19. The length of a small box is 15 cm 2.4 mm, the breadth is 7 cm 6.2 mm and the height is 5 cm 8 mm. What is the volume of the box in cubic centimetres.

20. The length of a rectangular reservoir is 5.5 metres, the breadth is 4 metres and the height is 2 metres. If the reservoir is full of water, what is the volume of water in litres and its weight in kg?

21. The length of a rectangular field is 1.5 times its breadth. An amount of Tk. 10260 is spent to plant grass at Tk. 1.90 per sq. metres. How much money will be spent at Tk. 2.50 per metre to erect a fence around that field?

22. An amount of Tk. 7,200 is spent to cover the floor of a room by carpet. An amount of Tk. 576 would be saved if the breadth were 3 metres less. What is the breadth of the room?

23. Around inside a rectangular garden of length 80 metres and breadth 60 metres, there is a road of breadth 4 metres. How much money will be spent to construct that road at Tk. 7.25 per square metre?

24. A square open reservoir of depth 2.5 metres contains 28,900 litres of water inside. How much money will be spent to put a lead sheet in the innerside at Tk. 12 50 per sq. metres?

25. The length of the floor of a house is 26 metres and breadth is 20 metres. How many mats of length 4 metres and breadth 2.5 metres will be required to cover the floor completely? How much money will be spent if the price of each mat is Tk. 27.50?

26. The length of a book is 25 cm and the breadth is 18 cm. The number of pages of the book is 200 and the thickness of each page is 0.1 mm. Find the volume of the book.

27. The length of a pond is 32 metres, breadth is 20 metres and the depth of water of the pond is 3 metres. The pond is being made empty by a machine which can remove 0.1 cubic metres of water per second. How much time will be required to make the pond empty?

28. A solid cube of sides 50 cm is kept in an empty reservoir of length 3 metres, breadth 2 metres and height 1 metre. The cube is taken out after filling the reservoir with water. What is the depth of water now?

29. The breadth of a room is $\frac{2}{3}$ times of its length. The length and height of the room are 15 m and 4 m respectively. The floor of the rooms is set with stone of the size 50 sq.cm leaving lm margin in all sides. Air is 0.00129 times heavier than water.

    A. Find out the parameter of the room.
    B. How many pieces of stone will be needed?
    C. How much air is their in the room?

30. The length of a rectangular plot of land is 80 m. Its breadth is 60 m. In the middle of the land a tank with 3 m depth is dug-keeping 4 m wide bank. 0.1 cubic meter water is emptied by a machine.

    A. Express the depth of the tank in inches.
    B. Find out the area of the bank of the pond.
    C. How much time is required to empty the tank?

31. The area of a rectangular school campus is 10 acres. Its length is four times the breadth. The size of the auditorium is 40m × 35m × 10m and the thickness of the wall is 15 cm.

    A. What is the area of the campus in hectre?
    B. Find out the length of the boundary wall in metre.
    C. Find out the volume of the 4 walls of the auditorium.

Chapter Four
Algebraic Formulae and Applications

In day to day life applications and uses of algebra are widely in practice in solving mathematical problems. Any general rule or corollary expressed by Algebraic symbols is known as algebraic formulae or in short formulae. Different types of mathematical problems can be solved by algebraic formulae. The first four formulae and the corollaries related to them have been discussed in detail in class VII. In this chapter those are repeated and some examples are given to show their applications so that the students can acquire sufficient knowledge regarding their applications. In this chapter, finding of the squares and cubes of binomial and trinomial expressions, middle term distribution, factorization by the use of algebraic formulae and by their help how to find H.C.F. and L.C.M. of algebraic expressions have been discussed in detail.

At the end of the chapter, the students will be able to-

- Find the square of binomial and trinomial expressions, simplify and evaluate by applying algebraic formulae.
- Find the cube of binomial and trinomial expressions, simplify and evaluate by applying algebraic formulae.
- Factorize the expressions with the help of middle term distribution.
- Find H.C.F. and L.C.M. of algebraic expressions.

4.1 Algebraic Formulae :

The first four formulae and the corollaries related to them have been elaborately discussed in class VII. Here, those are repeated.

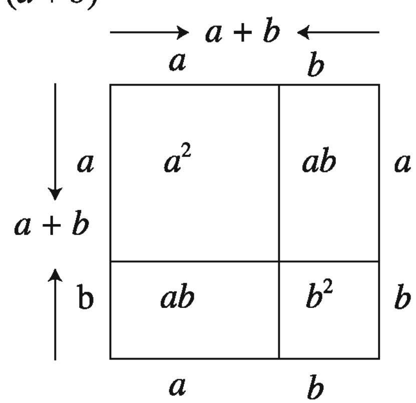
The geometric explanation of $(a+b)^2$ is as follows:

The area of the whole square $= (a+b) \times (a+b) = (a+b)^2$

$\therefore \ (a+b)^2 = a \times (a+b) + b \times (a+b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$



Again, the sum of the areas of the parts of the square

$a \times a + a \times b + b \times a + b \times b$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

Observe that, the area of the whole square = the sum of the areas of the parts of the square. $\therefore$ $(a+b)^2 = a^2 + 2ab + b^2$

In class VII, Formulae and corollary, which we have known, are as follows:

Formula 1. $(a+b)^2 = a^2 + 2ab + b^2$

In words, the square of the sum of two quantities = the square of first quantity + 2 × first quantity × second quantity + the square of second quantity.

Formula 2. $(a-b)^2 = a^2 - 2ab + b^2$

In words, the square of the difference of two quantities = the square of first quantity -2 × first quantity × second quantity + the square of second quantity.

Formula 3. $a^2 - b^2 = (a+b)(a-b)$

In words, the difference of squares of two quantities = the sum of two quantities × the difference of two quantities.

Formula 4. $(x+a)(x+b) = x^2 + (a+b)x + ab$

In words, if the first terms of two binomial expressions are the same, their product will be equal to the sum of square of the first term, product of the first term with the sum of their second terms with their usual signs and product of the second two terms with their usual signs. That is, $(x+a)(x+b) = x^2 +$ (algebraic sum of a and b) x + (product of a and b).

Corollary 1. $a^2 + b^2 = (a+b)^2 - 2ab$

Corollary 2. $a^2 + b^2 = (a-b)^2 + 2ab$

Corollary 3. $(a+b)^2 = (a-b)^2 + 4ab$

Corollary 4. $(a-b)^2 = (a+b)^2 - 4ab$

Corollary 5. $2(a^2 + b^2) = (a+b)^2 + (a-b)^2$

Corollary 6. $4ab = (a+b)^2 - (a-b)^2$

$$\text{or, } ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$$

Example 1. Find the square of $3x + 5y$.

Solution : $(3x+5y)^2 = (3x)^2 + 2 \times 3x \times 5y + (5y)^2$

$= 9x^2 + 30xy + 25y^2$

Example 2. Find the square of 25 by applying the formula of square.

Solution : $(25)^2 = (20+5)^2 = (20)^2 + 2 \times 20 \times 5 + (5)^2$

$= 400 + 200 + 25$

$= 625$

Example 3. Find the square of $4x - 7y$.

Solution : $(4x - 7y)^2 = (4x)^2 - 2 \times 4x \times 7y + (7y)^2$

$= 16x^2 - 56xy + 49y^2$

Example 4. If $a + b = 8$ and $ab = 15$, find the value of $a^2 + b^2$.

Solution : $a^2 + b^2 = (a+b)^2 - 2ab$

$= (8)^2 - 2 \times 15$

$= 64 - 30$

$= 34$

Example 5. If $a - b = 7$ and $ab = 60$, find the value of $a^2 + b^2$.

Solution : $a^2 + b^2 = (a-b)^2 + 2ab$

$= (7)^2 + 2 \times 60$

$= 49 + 120$

$= 169$

Example 6. If $x - y = 3$ and $xy = 10$, find the value of $(x+y)^2$.

Solution : $(x+y)^2 = (x-y)^2 + 4xy$

$= (3)^2 + 4 \times 10$

$= 9 + 40$

$= 49$

Example 7. If $a + b = 7$ and $ab = 10$, find the value of $(a-b)^2$.

Solution : $(a-b)^2 = (a+b)^2 - 4ab$

$= (7)^2 - 4 \times 10$

$= 49 - 40$

$= 9$

Example 8. If $x-\dfrac{1}{x}=5$, find the value of $\left(x+\dfrac{1}{x}\right)^2$

Solution : $\left(x+\dfrac{1}{x}\right)^2=\left(x-\dfrac{1}{x}\right)^2+4\times x\times\dfrac{1}{x}$

$$=(5)^2+4$$

$$=25+4$$

$$=29$$

Activity:

1. Find the square of $2a+5b$.
2. Find the square of $4x-7$.
3. If $a+b=7$ and $ab=9$, find the value of a^2+b^2.
4. If $x-y=5$ and $xy=6$, find the value of $(x+y)^2$.

Example 9. Multiply $3p+4$ by $3p-4$ by an appropriate formula.

Solution : $(3p+4)(3p-4)=(3p)^2-(4)^2$ $[\because (a+b)(a-b)=a^2-b^2]$

$$=9p^2-16$$

Example 10. Multiply $5m+8$ by $5m+9$ by an appropriate of formula.

Solution : We know, $(x+a)(x+b)=x^2+(a+b)x+ab$

$\therefore$ $(5m+8)(5m+9)$

$$=(5m)^2+(8+9)\times 5m+8\times 9$$

$$=25m^2+17\times 5m+72$$

$$=25m^2+85m+72$$

Example 11. Simplify : $(5a-7b)^2+2(5a-7b)(9b-4a)+(9b-4a)^2$.

Solution : Let, $(5a-7b)=x$ and $9b-4a=y$

$\therefore$ Given expression $= x^2 + 2xy + y^2$

$= (x+y)^2$

$= (5a - 7b + 9b - 4a)^2$ [Substituting the value of x and y]

$= (a+2b)^2$

$= a^2 + 4ab + 4b^2$

Example 12. Express $(x + 6)$ $(x + 4)$ as the difference of two squares.

Solution : We know, $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

$$\therefore (x+6)(x+4) = \left(\frac{x+6+x+4}{2}\right)^2 - \left(\frac{x+6-x-4}{2}\right)^2$$

$$= \left(\frac{2x+10}{2}\right)^2 - \left(\frac{2}{2}\right)^2$$

$$= (x+5)^2 - 1^2$$

Example 13. If $x = 4, y = -8$ and $z = 5$, what is the value of

$25(x+y)^2 - 20(x+y)(y+z) + 4(y+z)^2$?

Solution : Let, $x+y = a$ and $y+z = b$

$\therefore$ Given expression $= 25a^2 - 20ab + 4b^2$

$= (5a)^2 - 2 \times 5a \times 2b + (2b)^2$

$= (5a - 2b)^2$

$= \{5(x+y) - 2(y+z)\}^2$ [Putting the value of a and b]

$= (5x + 5y - 2y - 2z)^2$

$= (5x + 3y - 2z)^2$

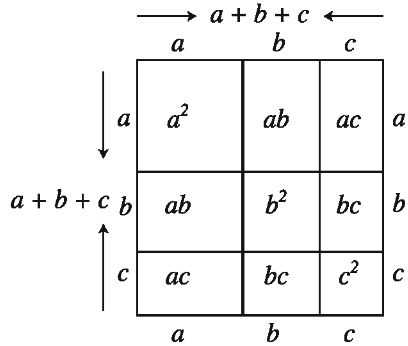
$= (5 \times 4 + 3 \times (-8) - 2 \times 5)^2$ [Putting the value of x, y and z]

$= (20 - 24 - 10)^2$

$= (-14)^2 = 196$

Activity : 1. Find the product of (5x+ 7y) and (5x- 7y) by an appropriate formula.
2. Find the product of (x+ 10) and (x- 14) by an appropriate formula.
3. Express (4x- 3y) (6x+ Sy) as the difference of two squares.

Geometric explanation of $(a+b+c)^2$:



The area of the whole square

$(a+b+c)\times(a+b+c)=(a+b+c)^2$

$\therefore\ (a+b+c)^2$

$=a\times(a+b+c)+b\times(a+b+c)+c\times(a+b+c)$

$=a^2+ab+ac+ab+b^2+bc+ca+bc+c^2$

$=a^2+2ab+2ac+b^2+2bc+c^2$

$\therefore\ (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac$

Again, the sum of the area of the parts of a square

$=a^2+ab+ac+ab+b^2+bc+ac+bc+c^2$

$=a^2+2ab+2ac+b^2+2bc+c^2$

$=a^2+b^2+c^2+2ab+2bc+2ac$

Observe that, the area of the whole square = the sum of the areas of the parts of a square.

$\therefore\ (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac$

Example 14. Find the square of $2x+3y+5z$.

Solution : Let, $2x=a,\ 3y=b$ and $5z=c$

$\therefore$ Given square of expression = $(a+b+c)^2$

$=a^2+b^2+c^2+2ab+2bc+2ac$

$=(2x)^2+(3y)^2+(5z)^2+2\times2x\times3y+2\times3y\times5z+2\times2x\times5z$ [putting the value of *a*, *b* and *c*]

$=4x^2+9y^2+25z^2+12xy+30yz+20xz$

$\therefore\ (2x+3y+5z)^2=4x^2+9y^2+25z^2+12xy+30yz+20xz$

Example 15. Find the square of $5a - 6b - 7c$.

Solution : $(5a - 6b - 7c)^2 = \{5a - (6b + 7c)\}^2$

$$= (5a)^2 - 2 \times 5a \times (6b + 7c) + (6b + 7c)^2$$

$$= 25a^2 - 10a(6b + 7c) + (6b)^2 + 2 \times 6b \times 7c + (7c)^2$$

$$= 25a^2 - 60ab - 70ac + 36b^2 + 84bc + 49c^2$$

$$= 25a^2 + 36b^2 + 49c^2 - 60ab + 84bc - 70ac$$

Alternative Solution :

We know, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$

Here, let, $5a = x$, $-6b = y$ and $-7c = z$

$\therefore (5a - 6b - 7c)^2 = (5a)^2 + (-6b)^2 + (-7c)^2$

$$+ 2 \times (5a) \times (-6b) + 2 \times (-6b) \times (-7c) + 2 \times 5a \times (-7c)$$

$$= 25a^2 + 36b^2 + 49c^2 - 60ab + 84bc - 70ac$$

Activity : Find the square by appropriate formula :
1. $ax + by + c$ 2. $4x + 5y - 7z$

Exercise 4.1

1. Find the square of the following expressions with the help of formulae :

(a) $5a + 7b$ (b) $6x + 3$ (c) $7p - 2q$

(d) $ax - by$ (e) $x^3 + xy$ (f) $11a - 12b$

(g) $6x^2y - 5xy^2$ (h) $-x - y$ (i) $-xyz - abc$

(j) $a^2x^3 - b^2y^4$ (k) 108 (l) 606

(m) 597 (n) $a - b + c$ (o) $ax + b + 2$

(p) $xy + yz - zx$ (q) $3p + 2q - 5r$ (r) $x^2 - y^2 - z^2$

(s) $7a^2 + 8b^2 - 5c^2$

2. Simplify :

(a) $(x+y)^2+2(x+y)(x-y)+(x-y)^2$

(b) $(2a+3b)^2-2(2a+3b)(3b-a)+(3b-a)^2$

(c) $(3x^2+7y^2)^2+2(3x^2+7y^2)(3x^2-7y^2)+(3x^2-7y^2)^2$

(d) $(8x+y)^2-(16x+2y)(5x+y)+(5x+y)^2$

(e) $(5x^2-3x-2)^2+(2+5x^2-3x)^2-2(5x^2-3x+2)(2+5x^2-3x)$

3. Find the product by applying formulae:

(a) $(x+7)(x-7)$

(b) $(5x+13)(5x-13)$

(c) $(xy+yz)(xy-yz)$

(d) $(ax+b)(ax-b)$

(e) $(a+3)(a+4)$

(f) $(ax+3)(ax+4)$

(g) $(6x+17)(6x-13)$

(h) $(a^2+b^2)(a^2-b^2)(a^4+b^4)$

(i) $(ax-by+cz)(ax+by-cz)$

(j) $(3a-10)(3a-5)$

(k) $(5a+2b-3c)(5a+2b+3c)$

(l) $(ax+by+5)(ax+by+3)$

4. If $a=4$, $b=6$ and $c=3$, find the value of $4a^2b^2-16ab^2c+16b^2c^2$.

5. If $x-\frac{1}{x}=3$, find the value of $x^2+\frac{1}{x^2}$.

6. If $a+\frac{1}{a}=4$, what is the value of $a^4+\frac{1}{a^4}$?

7. If $m=6$, $n=7$, find the value of

$16(m^2+n^2)^2+56(m^2+n^2)(3m^2-2n^2)+49(3m^2-2n^2)^2$.

8. If $a-\frac{1}{a}=m$, show that $a^4+\frac{1}{a^4}=m^4+4m^2+2$

9. If $x-\frac{1}{x}=4$, prove that $x^2+\left(\frac{1}{x}\right)^2=18$

10. If $m+\frac{1}{m}=2$, prove that $m^4+\frac{1}{m^4}=2$

11. If $x+y=12$ and $xy=27$, find the value of $(x-y)^2$ and x^2+y^2.

12. If $a+b=13$ and $a-b=3$, find the value of $2a^2+2b^2$ and ab.

13. Express as the difference of the square of two expressions :

(a) $(5p-3q)(p+7q)$ (b $(6a+9b)(7b-8a)$

(c) $(3x+5y)(7x-5y)$ (d) $(5x+13)(5x-13)$

14. The two numbers are a and b. Here a>b.The Sum of two numbers is 12 and the product is 32.

A. Multiply with the help of formulae : (2x+3) (2x-7)

B. Find out the value of $2a^2+2b^2$.

C. Prove that, $(a+2b)^2-5b^2 = 176$

4.2 Formulae of cubes and corollaries

Formula 5. $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$$= a^3+b^3+3ab(a+b)$$

Proof : $(a+b)^3 = (a+b)(a+b)^2$

$= (a+b)(a^2+2ab+b^2)$

$= a(a^2+2ab+b^2)+b(a^2+2ab+b^2)$

$= a^3+2a^2b+ab^2+(a^2b+2ab^2+b^3)$

$= a^3+3a^2b+3ab^2+b^3$

$= a^3+3ab(a+b)+b^3$

$= a^3+b^3+3ab(a+b)$

Corollary 7. $(a^3+b^3) = (a+b)^3-3ab(a+b)$

Formula 6. $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

$$= a^3-b^3-3ab(a-b)$$

Proof : $(a-b)^3 = (a-b)(a-b)^2$

$= (a-b)(a^2-2ab+b^2)$

$= a(a^2-2ab+b^2)-b(a^2-2ab+b^2)$

$= a^3-2a^2b+ab^2-a^2b+2ab^2-b^3$

$= a^3-3a^2b+3ab^2-b^3$

$= a^3-b^3-3ab(a-b)$

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Corollary 8. $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

Example 16. Find the cube of $3x + 2y$.

Solution : $(3x+2y)^3 = (3x)^3 + 3 \times (3x)^2 \times (2y) + 3 \times (3x) \times (2y)^2 + (2y)^3$

$$= 27x^3 + 3 \times 9x^2 \times 2y + 3 \times 3x \times 4y^2 + 8y^3$$

$$= 27x^3 + 54x^2y + 36xy^2 + 8y^3$$

Example 17. Find the cube of $2a + 5b$.

Solution : $(2a+5b)^3 = (2a)^3 + 3 \times (2a)^2 \times (5b) + 3 \times (2a) \times (5b)^2 + (5b)^3$

$$= 8a^3 + 3 \times 4a^2 \times 5b + 3 \times 2a \times 25b^2 + 125b^3$$

$$= 8a^3 + 60a^2b + 150ab^2 + 125b^3$$

Example 18. Find the cube of $m - 2n$.

Solution : $(m-2n)^3 = (m)^3 - 3 \times (m)^2 \times (2n) + 3 \times m \times (2n)^2 - (2n)^3$

$$= m^3 - 3m^2 \times 2n + 3m \times 4n^2 - 8n^3$$

$$= m^3 - 6m^2n + 12mn^2 - 8n^3$$

Example 19. Find the cube of $4x - 5y$.

Solution : $(4x-5y)^3 = (4x)^3 - 3 \times (4x)^2 \times (5y) + 3 \times (4x) \times (5y)^2 - (5y)^3$

$$= 64x^3 - 3 \times 16x^2 \times 5y + 3 \times 4x \times 25y^2 - 125y^3$$

$$= 64x^3 - 240x^2y + 300xy^2 - 125y^3$$

Example 20. Find the cube of $x + y - z$.

Solution : $(x+y-z)^3 = \{(x+y) - z\}^3$

$$= (x+y)^3 - 3(x+y)^2 \times z + 3(x+y) \times z^2 - z^3$$

$$= (x^3 + 3x^2y + 3xy^2 + y^3) - 3(x^2 + 2xy + y^2) \times z + 3(x+y) \times z^2 - z^3$$

$$= x^3 + 3x^2y + 3xy^2 + y^3 - 3x^2z - 6xyz - 3y^2z + 3xz^2 + 3yz^2 - z^3$$

$$= x^3 + y^3 - z^3 + 3x^2y + 3xy^2 - 3x^2z - 3y^2z + 3xz^2 + 3yz^2 - 6xyz$$

Activity : Find the cube with the help of an appropriate formulae :

1. $ab + bc$ 2. $2x - 5y$ 3. $2x - 3y - z$

Example 21. Simplify :

$$(4m + 2n)^3 + 3(4m + 2n)^2(m - 2n) + 3(4m + 2n)(m - 2n)^2 + (m - 2n)^3$$

Solution : Let, $4m + 2n = a$ and $m - 2n = b$

$\therefore$ Given expression $= a^3 + 3a^2b + 3ab^2 + b^3$

$= (a + b)^3$

$= \{(4m + 2n) + (m - 2n)\}^3$

$= (4m + 2n + m - 2n)^3$

$= (5m)^3 = 125m^3$

Example 22. Simplify :

$$(4a - 8b)^3 - (3a - 9b)^3 - 3(a + b)(4a - 8b)(3a - 9b)$$

Solution : Let, $4a - 8b = x$ and $3a - 9b = y$

$\therefore x - y = (4a - 8b) - (3a - 9b) = 4a - 8b - 3a + 9b = a + b$

Now given expression $= x^3 - y^3 - 3(x - y) \times x \times y$

$= x^3 - y^3 - 3xy(x - y)$

$= (x - y)^3$

$= (a + b)^3$

Example 23. If $a + b = 3$ and $ab = 2$, find the value of $a^3 + b^3$.

Solution : $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (3)^3 - 3 \times 2 \times 3$ [putting the value of $(a + b)$ and ab]

$= 27 - 18 = 9$

Alternative Solution : Given that, $a+b=3$ and $ab=2$

Now, $a+b=3$

or, $(a+b)^3=(3)^3$ [cube both the sides]

or, $a^3+b^3+3ab(a+b)=27$

or, $a^3+b^3+3\times 2\times 3=27$

or, $a^3+b^3+18=27$

or, $a^3+b^3=27-18$

$\therefore\ a^3+b^3=9$

Example 24. If $x-y=10$ and $xy=30$, find the value of x^3-y^3.

Solution : $x^3-y^3=(x-y)^3+3xy(x-y)$

$=(10)^3+3\times 30\times 10$

$=1000+900$

$=1900$

Example 25. If $x+y=4$, what is the value of x^3+y^3+12xy ?

Solution : $x^3+y^3+12xy=x^3+y^3+3\times 4\times xy$

$=x^3+y^3+3(x+y)\times xy$

$=x^3+y^3+3xy(x+y)$

$=(x+y)^3$

$=(4)^3$

$=64$

Example 26. If $a+\dfrac{1}{a}=7$, find the value of $a^3+\dfrac{1}{a^3}$.

Solution : $a^3+\dfrac{1}{a^3}=a^3+\left(\dfrac{1}{a}\right)^3$

$$= \left(a+\frac{1}{a}\right)^3 - 3\times a\times\frac{1}{a}\left(a+\frac{1}{a}\right)$$
$$= (7)^3 - 3\times 7$$
$$= 343 - 21$$
$$= 322$$

Example 27. If $m = 2$, find the value of $27m^3 + 54m^2 + 36m + 3$.

Solution : Given expression $= (3m)^3 + 3\times(3m)^2\times 2 + 3\times(3m)\times(2)^2 + (2)^3 - 5$

$$= (3m+2)^3 - 5$$
$$= (3\times 2+2)^3 - 5 \qquad \text{[putting the value of } m\text{]}$$
$$= (6+2)^3 - 5 = 8^3 - 5$$
$$= 512 - 5 = 507$$

Activity :

1. Simplify : $(7x-6)^3 - (5x-6)^3 - 6x(7x-6)(5x-6)$.
2. If $a+b=10$ and $ab=21$, find the value of a^3+b^3.
3. If $a+\frac{1}{a}=3$, show that, $a^3+\frac{1}{a^3}=18$.

4.3 Two more Formulae related to Cubes :

Formula 7. $a^3+b^3 = (a+b)(a^2-ab+b^2)$

Proof : $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

$$= (a+b)\{(a+b)^2 - 3ab\}$$
$$= (a+b)(a^2+2ab+b^2-3ab)$$
$$= (a+b)(a^2-ab+b^2)$$

Conversely, $(a+b)(a^2-ab+b^2)$

$$= a(a^2-ab+b^2) + b(a^2-ab+b^2)$$
$$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$$
$$= a^3 + b^3$$

$\therefore\ (a+b)(a^2-ab+b^2) = a^3+b^3$

Formula 8. $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Proof : $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

$$= (a-b)\{(a-b)^2 + 3ab\}$$

$$= (a-b)(a^2 - 2ab + b^2 + 3ab)$$

$$= (a-b)(a^2 + ab + b^2)$$

Conversely, $(a-b)(a^2+ab+b^2)$

$$= a(a^2+ab+b^2) - b(a^2+ab+b^2)$$

$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$

$$= a^3 - b^3$$

$\therefore\ (a-b)(a^2+ab+b^2) = a^3 - b^3$

Solution : $24x^4 - 81y^3 = 3(8x^3 - 27y^3)$

$$= 3\{(2x)^3 - (3y)^3\}$$

$$= 3(2x-3y)\{(2x)^2 + (2x)\times(3y) + (3y)^2\}$$

$$= 3(2x-3y)(4x^2 + 6xy + 9y^2)$$

Example 28. Find the product of (x^2+2) and $(x^4 - 2x^2 + 4)$ with the help of formula.

Solution : $(x^2+2)(x^4-2x^2+4)$

$$= (x^2+2)\{(x^2)^2 - x^2 \times 2 + 2^2)\}$$

$$= (x^2)^3 + (2)^3$$

$$= x^6 + 8$$

Example 29. Find the product of $(4a-5b)$ and $(16a^2+20ab+25b^2)$ with the help of formula.

Solution : $(4a-5b)(16a^2+20ab+25b^2)$

$$= (4a-5b)\{(4a)^2 + 4a\times 5b + (5b)^2\}$$

$$= (4a)^3 - (5b)^3$$

$$= 64a^3 - 125b^3$$

Activity :

1. Find the product of $(2a+3b)$ and $(4a^2-6ab+9b^2)$ by an appropriate formula.

Exercise 4·2

1. Find the cube of the following expressions with the help of formula :

   (a) $3x+y$ (b) x^2+y (c) $5p+2q$ (d) a^2b+c^2d (e) $6p-7$ (f) $ax-by$

   (g) $2p^2-3r^2$ (h) x^3+2 (i) $2m+3n-5p$ (j) $x^2-y^2+z^2$ (k) $a^2b^2-c^2d^2$

   (l) a^2b-b^3c (m) x^3-2y^3 (n) $11a-12b$ (o) x^3+y^3

2. Simplify :

   (a) $(3x+y)^3+3(3x+y)^2(3x-y)+3(3x+y)(3x-y)^2+(3x-y)^3$

   (b) $(2p+5q)^3+3(2p+5q)^2(5q-2p)+3(2p+5q)(5q-2p)^2+(5q-2p)^3$

   (c) $(x+2y)^3-3(x+2y)^2(x-2y)+3(x+2y)(x-2y)^2-(x-2y)^3$

   (d) $(6m+2)^3-3(6m+2)^2(6m-4)+3(6m+2)(6m-4)^2-(6m-4)^3$

   (e) $(x-y)^3+(x+y)^3+6x(x^2-y^2)$

3. If $a+b=8$ and $ab=15$, what is the value of a^3+b^3 ?

4. If $x+y=2$, show that $x^3+y^3+6xy=8$.

5. If $2x+3y=13$ and $xy=6$, find the value of $8x^3+27y^3$.

6. If $p-q=5,\ pq=3$, find the value of p^3-q^3.

7. If $x-2y=3$, find the value of x^3-8y^3-18xy.

8. If $4x-3=5$, prove that $64x^3-27-180x=125$

9. If $a=-3$ and $b=2$, find the value of $8a^3+36a^2b+54ab^2+27b^3$.

10. If $a=7$, find the value of $a^3+6a^2+12a+1$.

11. If $x=5$, what is the value of $x^3-12x^2+48x-64$?

12. If $a^2+b^2=c^2$, prove that $a^6+b^6+3a^2b^2c^2=c^6$.

13. If $x+\frac{1}{x}=4$, prove that $x^3+\frac{1}{x^3}=52$.

14. If $a-\frac{1}{a}=5$, what is the value of $a^3-\frac{1}{a^3}$?

15. Find the product with the help of formula :

(a) $(a^2+b^2)(a^4-a^2b^2+b^4)$ (b) $(ax-by)(a^2x^2+abxy+b^2y^2)$

(c) $(2ab^2-1)(4a^2b^4+2ab^2+1)$ (d) $(x^2+a)(x^4-ax^2+a^2)$

(e) $(7a+4b)(49a^2-28ab+16b^2)$ (f) $(2a-1)(4a^2+2a+1)(8a^3+1)$

(g) $(x+a)(x^2-ax+a^2)(x-a)(x^2+ax+a^2)$

(h) $(5a+3b)(25a^2-15ab+9b^2)(125a^3-27b^3)$

4·4 Resolving into Factors :

Factor : If an expression is the product of two or more expressions, each of these two or more latter expression is termed as factor of the first expression. For example, $a^2-b^2=(a+b)(a-b)$, here $(a+b)$ and $(a-b)$ are two factors of the expression (a^2-b^2).

Resolving into factors : When any expression is expressed as the product of two or more of expressions, it is said to have been resolved into factors and each of such expressions is called the factor of the first expression.

For example, $x^2+2x=x(x+2)$ [here, x and $(x+2)$ are the factors].

Rules of resolving expressions into factors are stated below :

(a) Arranging conveniently :

$px-qy+qx-py$ is arranged as, $px+qx-py-qy$.

Now, $px+qx-py-qy=x(p+q)-y(p+q)=(p+q)(x-y)$.

Again, $px-qy+qx-py$ is arranged as, $px-py+qx-qy$.

Now, $px-py+qx-qy=p(x-y)+q(x-y)=(x-y)(p+q)$.

(b) Expressing an expression in the form of square :

$$x^2+4xy+4y^2=(x)^2+2\times x\times 2y+(2y)^2$$
$$=(x+2y)^2=(x+2y)(x+2y)$$

(c) Expressing an expression as the difference of two squares and applying the formula a^2-b^2:

$a^2+2ab-2b-1$

$=a^2+2ab+b^2-b^2-2b-1$ [Here, b^2 is added and then subtracted. In this way, there is no change of the value of expression]

$= (a^2 + 2ab + b^2) - (b^2 + 2b + 1)$
$= (a + b)^2 - (b + 1)^2$
$= (a + b + b + 1)(a + b - b - 1)$
$= (a + 2b + 1)(a - 1)$

Alternative rule :

$a^2 + 2ab - 2b - 1$
$= (a^2 - 1) + (2ab - 2b)$
$= (a + 1)(a - 1) + 2b(a - 1)$
$= (a - 1)(a + 1 + 2b)$
$= (a - 1)(a + 2b + 1)$

(d) Applying the formula, $x^2 + (a + b)x + ab = (x + a)(x + b)$:

$x^2 + 7x + 10 = x^2 + (2 + 5)x + 2 \times 5$
$= (x + 2)(x + 5)$

(e) Expressing the expression in the form of cubes :

$8x^3 + 36x^2 + 54x + 27$
$= (2x)^3 + 3 \times (2x)^2 \times 3 + 3 \times 2x \times (3)^2 + (3)^3$
$= (2x + 3)^3$
$= (2x + 3)(2x + 3)(2x + 3)$

(f) Applying two formulae : $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$:

$8x^3 + 125 = (2x)^3 + (5)^3 = (2x + 5)\{(2x)^2 - (2x) \times 5 + (5)^2\}$
$= (2x + 5)(4x^2 - 10x + 25)$
$27x^3 - 8 = (3x)^3 - (2)^3 = (3x - 2)\{(3x)^2 + (3x) \times 2 + (2)^2\}$
$= (3x - 2)(9x^2 + 6x + 4)$

Example-1: Resolve into factors : $27x^4 + 8xy^3$.

Solution : $27x^4 + 8xy^3 = x(27x^3 + 8y^3)$
$= x\{(3x)^3 + (2y)^3\}$
$= x(3x + 2y)\{(3x)^2 - (3x) \times (2y) + (2y)^2\}$
$= x(3x + 2y)(9x^2 - 6xy + 4y^2)$

Example-2: Resolve into factors : $24x^3 - 81y^3$.

Solution : $24x^3 - 81y^3 = 3(8x^3 - 27y^3)$
$= 3[(2x)^3 - (3y)^3]$
$= 3[(2x - 3y)\{(2x)^2 + 3x \cdot 2y + (3y)^2\}]$
$= 3(2x - 3y)(4x^2 + 6xy + 9y^2)$

Activity : Resolve into factors :
1. $4x^2 - y^2$ 2. $6ab^2 - 24a$ 3. $x^2 + 2px + p^2 - 4$ 4. $x^3 + 27y^3$ 5. $27a^3 - 8$

4·5 Factors of the expression of the form $x^2 + px + q$.

We know, $x^2 + (a+b)x + ab = (x+a)(x+b)$. If the expression of the left-hand side of this formula is compared with the expression $x^2 + px + q$, it is found that in both the expressions there are three terms. The first term is x^2 whose coefficient is 1 (one), the second or middle term x whose coefficients are $(a+b)$ and p respectively and the third term is free from x, where there are ab and q respectively.

$\therefore$ $x^2 + (a+b)x + ab$ consists of two factors. Therefore, the expression $x^2 + px + q$ has also two factors.

Let, two factors of $x^2 + px + q$ are $(x+a)$ and $(x+b)$

Hence, $x^2 + px + q = (x+a)(x+b) = x^2 + (a+b)x + ab$

Then, $p = a + b$ and $q = ab$

Now, in order to find the factors of $x^2 + px + q$, q is to be expressed in two such factors that their algebraic sum is equal to p. This method is called Middle term breakup.

If it is required to resolve $x^2 + 7x + 12$ into factors, the number 12 is to be expressed into two such factors whose sum is 7 and the product is 12. The possible pairs of factors of 12 are 1, 12; 2, 6 and 3, 4. Of them, the sum of the pair 3, 4 is $3 + 4 = 7$ and the product is $3 \times 4 = 12$.

$\therefore$ $x^2 + 7x + 12 = (x+3)(x+4)$

Remark : In each case, considering both p and q positive, in order to resolve into factors the expressions $x^2 + px + q$, $x^2 - px + q$, $x^2 + px - q$ and $x^2 - px - q$, both the factors of q will be of the same sign i.e. both the factors will be either positive or negative since q is positive in the first and the second expressions. In this case, if p is positive, both the factors of q are positive and if p is negative, both the factors of q are negative.

In the third and the fourth expression, q is negative i.e. $(-q)$ and hence two factors of q will be of opposite sign and if p is positive, the positive number of two factors will be greater than the absolute value of the negative number and if p is negative, the absolute value of negative number of two factors will be greater than the positive number.

Example 3. Resolve into factors : $x^2 + 5x + 6$.

Solution : We have to find two such positive numbers whose product is 6 and their sum is 5.

The possible pairs of factors of 6 are 1, 6 and 2, 3.

Of them, the sum of numbers of the pair 2, 3 is $2+3=5$ and the product is $2 \times 3 = 6$.

$$\therefore x^2+5x+6 = x^2+2x+3x+6$$
$$= x(x+2)+3(x+2)$$
$$= (x+2)(x+3)$$

Example 4. Resolve into factors : $x^2-15x+54$.

Solution : We have to find two such numbers whose product is 54 and their sum is -15. Here the sum of two numbers is negative but the product is positive. Therefore, both the numbers will be negative.

The possible pairs of factors of 54 are -1,-54; -2, -27; -3,-18; -6,-9. Of them, the sum of numbers of the pair -6, -9 is - 6 - 9 = -15 and the product is (-6) × (-9) = 54.

$$\therefore x^2-15x+54 = x^2-6x-9x+54$$
$$= x(x-6)-9(x-6)$$
$$= (x-6)(x-9)$$

Example 5. Resolve into factors : $x^2+2x-15$.

Solution : We have to find such two numbers whose product is -15 and their sum is 2. Here the sum of two numbers is positive but their product is negative. Hence of two numbers, the number whose absolute value is greater than that of the other is positive and that number is negative whose absolute value is smaller than the other. The possible pairs of factors of (-15) are

$-1, 15;\ -3,\ 5.$

Of them, the sum of the numbers of the pair $(-3, 5)$ is $-3+5=2$.

$$\therefore x^2+2x-15 = x^2+5x-3x-15$$
$$= x(x+5)-3(x+5)$$
$$= (x+5)(x-3)$$

Example 6. Resolve into factors : $x^2-3x-28$.

Solution : We have to find two such numbers whose product is (–28) and their sum is (-3) . Here, the sum of two numbers is negative and their product is negative. Hence, of the two numbers, the number whose absolute value is greater than that of the other is negative and the number whose absolute value is smaller than that of the other is positive. The possible pairs of factors of

(–28) are $-1,\ 28;\ 2,\ -14$ and $(4,\ \ 7)$. Of them, the sum of numbers of the pair $(4,-7)$ is $-7+4=-3$.

$$\therefore\ x^2-3x-28=x^2-7x+4x-28$$
$$=x(x-7)+4(x-7)$$
$$=(x-7)(x+4)$$

Activity : Resolve into factors :

1. $x^2-18x+72$ 2. $x^2-9x-36$ 3. $x^2-23x+132$

4.6 Factors of the expression in the form of ax^2+bx+c :

Let, $ax^2+bx+c=(rx+p)(sx+q)$

$$=rsx^2+(rq+sp)x+pq$$

Then, $a=rs,\ b=rq+sp$ and $c=pq$

Hence, $ac=rspq=rq\times sp$ and $b=rq+sp$

Now, to find the factors of ax^2+bx+c, the product of the coefficient a of x^2 and the constant c is to be expressed in two such factors that their algebraic sum is equal to b, the coefficient of x and the product is equal to a & c. To factorize $2x^2+11x+15$, $(2\times15)=30$ is to be expressed into two such factors whose sum is 11 and their product is 30.

The pairs of factors of 30 are 1, 30; 2, 15; 3, 10 and 5, 6. Of them, the sum of the pair 6, 5 is 5 + 6 = 11 and their product is $5\times6=30$.

$$\therefore 2x^2+11x+15=2x^2+5x+6x+15$$
$$=x(2x+5)+3(2x+5)=(2x+5)(x+3)$$

Remark : To factorize ax^2+bx+c, the rules which are followed for different values of p, q having positive and negative signs of x^2+px+q are also followed for different values of a, b, c having positive and negative signs. Here b for p and $(a\times c)$ for q are to be considered.

Example 7. Resolve into factors : $2x^2+9x+10$

Solution : Here, $2\times 10=20$ [product of coefficient of x^2 and constant term]

Now, $4\times 5=20$ and $4+5=9$

$$\therefore\ 2x^2+9x+10=2x^2+4x+5x+10$$
$$=2x(x+2)+5(x+2)=(x+2)(2x+5)$$

Example 8. Resolve into factors : $3x^2+x-10$.

Solution : Here, $3\times(-10)=-30$

Now, $(-5)\times 6=-30$ and $(-5)+6=1$

$$\therefore\ 3x^2+x-10=3x^2+6x-5x-10$$
$$=3x(x+2)-5(x+2)$$
$$=(x+2)(3x-5)$$

Example 9. Resolve into factors : $4x^2-23x+33$.

Solution : Here, $4\times 33=132$

Now, $(-11)\times(-12)=132$ and $(-11)+(-12)=-23$

$$\therefore\ 4x^2-23x+33=4x^2-11x-12x+33$$
$$=x(4x-11)-3(4x-11)$$
$$=(4x-11)(x-3)$$

Example 10. Resolve into factors : $9x^2-9x-4$.

Solution : Here, $9\times(-4)=-36$

Now, $3\times(-12)=-36$ and $3+(-12)=-9$

$$\therefore\ 9x^2-9x-4=9x^2+3x-12x-4$$
$$=3x(3x+1)-4(3x+1)$$
$$=(3x+1)(3x-4)$$

Activity : Resolve into factors :
1. $8x^2+18x+9$ 2. $27x^2+15x+2$ 3. $2a^2-6a-20$

Exercise 4.3

Resolve into factors :

1. a^3+8 2. $8x^3+343$ 3. $8a^4+27ab^3$
4. $8x^3+1$ 5. $64a^3-125b^3$ 6. $729a^3-64b^3c^6$
7. $27a^3b^3+64b^3c^3$ 8. $56x^3-189y^3$
9. $3x-75x^3$ 10. $4x^2-y^2$ 11. $3ay^2-48a$
12. $a^2-2ab+b^2-p^2$ 13. $16y^2-a^2-6a-9$ 14. $8a+ap^3$
15. $2a^3+16b^3$ 16. $x^2+y^2-2xy-1$ 17. $a^2-2ab+2b-1$
18. x^4-2x^2+1 19. $36-12x+x^2$ 20. x^6-y^6
21. $(x-y)^3+z^3$ 22. $64x^3-8y^3$ 23. $x^2+14x+40$
24. $x^2+7x-120$ 25. $x^2-51x+650$ 26. $a^2+7ab+12b^2$
27. $p^2+2pq-80q^2$ 28. $x^2-3xy-40y^2$ 29. $(x^2-x)^2+3(x^2-x)-40$
30. $(a^2+b^2)^2-18(a^2+b^2)-88$ 31. $(a^2+7a)^2-8(a^2+7a)-180$
32. $x^2+(3a+4b)x+(2a^2+5ab+3b^2)$ 33. $6x^2-x-15$ 34. $x^2-x-(a+1)(a+2)$
35. $3x^2+11x-4$ 36. $3x^2-16x-12$ 37. $2x^2-9x-35$
38. $2x^2-5xy+2y^2$ 39. $x^3-8(x-y)^3$ 40. $10p^2+11pq-6q^2$
41. $2(x+y)^2-3(x+y)-2$ 42. $ax^2+(a^2+1)x+a$ 43. $15x^2-11xy-12y^2$
44. $a^3-3a^2b+3ab^2-2b^3$

4·7 H.C.F. and L.C.M. of Algebraic Expressions :

The clear concept for finding H.C.F. and L.C.M. of not more than three algebraic expressions including numerical coefficients has already been discussed in class VII. A brief discussion is made here again.

Common factor : The expression which is a factor of each of two or more expressions, is called common factor. For example x is the common factor of the expressions $x^2y,\ xy,\ xy^2,\ 5x$.
Again, $(a+b)$ is the common factor of the expressions $(a^2-b^2),\ (a+b)^2,\ (a^3+b^3)$.

4.7.1 Highest Common Factor (H.C.F.)

The product of common factors of two or more expressions is called the Highest Common Factor or in brief H.C.F. of those two or more expressions. For example, the H.C.F. of three expressions $a^3b^2c^3, a^5b^3c^4$ and $a^4b^3c^2$ is $a^3b^2c^2$.
Again, the H.C.F. of three expressions $(x+y)^2, (x+y)^3$ and (x^2-y^2) is $(x+y)$.

Rules of finding H.C.F.

The H.C.F. of the numerical coefficients of expressions of those algebraic expressions should be determined first by applying the rules of Arithmetic. Then the prime factors of those algebraic expressions have to be found. After that, the successive product of the H.C.F. of numerical coefficients and the highest number of algebraic common factors of given expressions is the required H.C.F.

Example 1. Find the H.C.F. of $9a^3b^2c^2,\ 12a^2bc,\ 15ab^3c^3$.

Solution : H.C.F. of $9, 12, 15 = 3$
H.C.F. of $a^3, a^2, a = a$
H.C.F. of $b^2, b, b^3 = \text{b}$
H.C.F. of $c^2, c, c^3 = c$

$\therefore$ the required H.C.F. is $3abc$.

Example 2. Find the H.C.F. of $x^3-2x^2, x^2-4, xy-2y$.

Solution : Here, the first expression $= x^3-2x^2 = x^2(x-2)$
The second expression $= x^2-4 = (x+2)(x-2)$
The third expression $= xy-2y = y(x-2)$

Here the common factor of the expressions is $(x-2)$.
$\therefore$ H.C.F. $= (x-2)$

Example 3. Find the H.C.F. of $x^2y(x^3-y^3), x^2y^2(x^4+x^2y^2+y^4)$ and $x^3y^2+x^2y^3+xy^4$.

Solution : Here, the first expression $= x^2y(x^3-y^3)$

$$= x^2y(x-y)(x^2+xy+y^2)$$

The second expression $= x^2y^2(x^4+x^2y^2+y^4)$

$$= x^2y^2\{(x^2)^2+2x^2y^2+(y^2)^2-x^2y^2\}$$
$$= x^2y^2\{(x^2+y^2)^2-(xy)^2\}$$
$$= x^2y^2\{(x^2+y^2+xy)(x^2+y^2-xy)\}$$
$$= x^2y^2(x^2+xy+y^2)(x^2-xy+y^2)$$

third expression $= x^3y^2+x^2y^3+xy^4 = xy^2(x^2+xy+y^2)$

Here, the common factor of the first, the second and the third expression $xy(x^2+xy+y^2)$

$\therefore$ H.C.F. $= xy(x^2+xy+y^2)$

Activity : Find the H.C.F. of:
1. $15a^3b^2c^4$, $25a^2b^4c^3$ and $20a^4b^3c^2$
2. $(x+2)^2, (x^2+2x)$ and (x^2+5x+6)
3. $6a^2+3ab$, $2a^2+5a-12$ and a^4-8a

Common Multiple :

If any expression is completely divisible by two or more expressions, the dividend is called the common multiple of those two or more divisors. For example, the expression a^2b^2c is divisible by each expression of $a, b, c, ab, bc, ac, a^2b, ab^2, a^2c, b^2c, a^2b^2$. Hence, the expression a^2b^2c is the common multiple of expressions $a, b, c, ab, bc, ac, a^2b, ab^2, a^2c, b^2c, a^2b^2$. Again, the expression $(a+b)^2(a-b)$ is the common multiple of three expressions $(a+b), (a+b)^2$ and (a^2-b^2).

4.7.2 Least Common Multiple (L.C.M.)

Among different multiples of two or more expressions the common multiple which consists of lowest number of factors is called Least Common Multiple or L.C.M. in short.

For example, the expression x^2y^2z is the L.C.M. of three expressions, x^2yz, xy^2 and xyz. Again, the expression $(x+y)^2(x-y)$ is the L.C.M. of three expressions $(x+y), (x+y)^2$ and (x^2-y^2).

Rules of finding L.C.M.

At first, the L.C.M. of the given expressions of the numerical coefficients have to be determined. Then, we have to find the highest power of common factors. After that, the product of the both is the L.C.M. of the given expressions.

Example 4. Find the L.C.M. of $4a^2bc, 8ab^2c, 6a^2b^2c$.
Solution : Here, the L.C.M. of $4, 8$ and $6 = 24$
The common factors with highest common power among the given expressions are a^2, b^2, c respectively.

$\therefore$ L.C.M. $= 24a^2b^2c$.

Example 5. Find the L.C.M. of $x^3+x^2y,\ x^2y+xy^2, x^3+y^3$ and $(x+y)^3$.

Solution : Here, the first expression $= x^3+x^2y = x^2(x+y)$

The second expression $= x^2y+xy^2 = xy(x+y)$

The third expression $= x^3+y^3 = (x+y)(x^2-xy+y^2)$

The forth expression $= (x+y)^3 = (x+y)(x+y)(x+y)$

$\therefore$ L.C.M. $= x^2y(x+y)^3(x^2-xy+y^2) = x^2y(x+y)^2(x^3+y^3)$

Example 6. Find the L.C.M. of $4(x^2+ax)^2, 6(x^3-a^2x)$ and $14x^3(x^3-a^3)$
Solution : Here, the first expression $= 4(x^2+ax)^2 = 2\times2\times x^2(x+a)^2$

The second expression $= 6(x^3-a^2x) = 2\times3\times x(x^2-a^2) = 2\times3\times x(x+a)(x-a)$

The third expression $= 14x^3(x^3-a^3) = 2\times7\times x^3(x-a)(x^2+ax+a^2)$

$\therefore$ L.C.M. $= 2\times2\times3\times7\times x^3(x+a)^2(x-a)(x^2+ax+a^2)$

$= 84x^3(x+a)^2(x^3-a^3)$

Activity : Find the L.C.M. of :
1. $5x^3y,\ 10x^2y,\ 20x^4y^2$
2. $x^2-y^2, 2(x+y), 2x^2y+2xy^2$
3. $a^3-1,\ a^3+1,\ a^4+a^2+1$

Exercise 4.4

1. Which one of the following is the square of $(-5-y)$?

   A. $y^2+10y+25$ B. $y^2-10y+25$ C. $25-10y+y^2$ D. $y^2-10y-25$

2. Which one of the following is the product of $(x-2)$ and $(4x+3)$?

   A. $4x^2-5x+6$ B. $4x^2-11x-6$ C. $4x^2+5x-6$ D. $4x^2-5x-6$

3. What is the H.C.F of x^2-2x-3 and x^2+2x-3?

   A. $x+1$ B. $x-1$ C. 1 D. 0

4. Which one of the following will be right if we express $(3x-5)$ $(5+3x)$ in the form of the difference between two squares ?

   A. $3x^2-25$ B. $9x^2-5$ C. $(3x)^2-5^2$ D. $9x^2-25$

Answer to the questions no. 5-7 in accordance with the information given below:

If $x^2-\sqrt{3}\,x+1=0$

5. Which one of the following is the value of $x+\frac{1}{x}$?

   A. $-\sqrt{3}x$ B. $\sqrt{3}x$ C. $-\sqrt{3}$ D. $\sqrt{3}$

6. Which one of the following is the value of $x^2+\frac{1}{x^2}$?

   A. 1 B. 5 C. 7 D. 11

7. Which one of the following is the value of $x^3+\frac{1}{x^3}$?

   A. 12 B. $6\sqrt{3}$ C. $3\sqrt{3}+3$ D. 0

8. Which one of the following expressions is the factors of x^2-x-30 ?

   A. $(x-5)(x+6)$ B. $(x+5)(x-6)$ C. $(x-5)(x-6)$ D. $(x+5)(x+6)$

9. If $x^2-10x+21$ and x^2+6x-7 are two algebraic expressions-

   i. The H.S.F. of the two expressions is $x-7$

   ii. The L.C.M. of the two expressions is $(x+1)(x-3)(x-7)$

   iii. The Product of the two expressions is x^4-60x^2-147

   Which one of the following is correct?

   A. i and ii B. i and iii C. ii and iii D. i, ii and iii

10. বীজগণিতের সূত্রাবলিতে

(i) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

(ii) $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

(iii) $x^3 + y^3 = (x+y)^3 + 3xy(x+y)$

Which one of the following is correct according to the above information?

(a) i and ii (b) i and iii (c) ii and iii (d) i, ii and iii

11. If $x+y=5$ and $x-y=3$, then

(1) What is the value of x^2+y^2 ?

(a) 15 (b) 16 (c) 17 (d) 18

(2) What is the value of xy ?

(a) 10 (b) 8 (c) 6 (d) 4

(3) What is the value of x^2-y^2 ?

(a) 13 (b) 14 (c) 15 (d) 16

12. If $x+\frac{1}{x}=2$, then

(1) What is the value of $\left(x-\frac{1}{x}\right)^2$?

(a) 0 (b) 1 (c) 2 (d) 4

(2) What is the value of $x^3+\frac{1}{x^3}$?

(a) 1 (b) 2 (c) 3 (d) 4

(3) What is the value of $x^4+\frac{1}{x^4}$?

(a) 8 (b) 6 (c) 4 (d) 2

Find the H.C.F. of the following (13 – 20) :

13. $36a^2b^2c^4d^5$, $54a^5c^2d^4$ and $90a^4b^3c^2$

14. $20x^3y^2a^3b^4$, $15x^4y^3a^4b^3$ and $35x^2y^4a^3b^2$

15. $15x^2y^3z^4a^3$, $12x^3y^2z^3a^4$ and $27x^3y^4z^5a^7$

16. $18a^3b^4c^5$, $42a^4c^3d^4$, $60b^3c^4d^5$ and $78a^2b^4d^3$

17. x^2-3x, x^2-9 and x^2-4x+3

18. $18(x+y)^3$, $24(x+y)^2$ and $32(x^2-y^2)$

19. $a^2b(a^3-b^3)$, $a^2b^2(a^4+a^2b^2+b^4)$ and $a^3b^2+a^2b^3+ab^4$

20. a^3-3a^2-10a, a^3+6a^2+8a and $a^4-5a^3-14a^2$

Find the L.C.M. of the following (21 – 28) :

21. a^5b^2c, ab^3c^2 and $a^7b^4c^3$

22. $5a^2b^3c^2$, $10ab^2c^3$ and $15ab^3c$

23. $3x^3y^2$, $4xy^3z$, $5x^4y^2z^2$ and $12xy^4z^2$

24. $3a^2d^3$, $9d^2b^2$, $12c^3d^2$, $24a^3b^2$ and $36c^3d^2$

25. x^2+3x+2, x^2-1 and x^2+x-2

26. x^2-4, x^2+4x+4 and x^3-8

27. $6x^2 - x - 1$, $3x^2 + 7x + 2$ and $2x^2 + 3x - 2$

28. $a^3 + b^3$, $(a+b)^3, (a^2 - b^2)^2$ and $(a^2 - ab + b^2)^2$

29. If $x^2 + \frac{1}{x^2} = 3$,

(a) Determine the value of $\left(x + \frac{1}{x}\right)^2$.

(b) What is the value of $\frac{x^6 + 1}{x^3}$?

(c) Determine the cube of $x^2 - \frac{1}{x^2}$ and find its value.

30. $3x - 5y + 3z$ and $3x + 5y - z$ are two algebraic expressions.

A. Find out the square of the first expression.

B. Express the product of the two expressions in the form of difference of two squares.

C. If the second expression is '0' (zero), prove that $27x^3 + 125y^3 + 45xy = z^3$

31. $P = 3x^2 - 16x - 12$, $Q = 3x^2 + 5x + 2$, $R = 3x^2 - x - 2$ are three algebraic expressions.

A. What do you mean by factorization?

B. If $Q = 0$, find out the value of $9x^2 + \frac{4}{x^2}$

C. Find out the L.C.M of P, Q and R.

Chapter Five

Algebraic Fractions

In day to day life we use a complete object along with its different parts. Each of these different parts is a fraction. In class VII we have learnt the algebraic fraction, the reduction of algebraic fraction and the common denomination form. We have also learnt addition, subtraction and simplification of fractions in detail. In this chapter, we shall discuss addition and subtraction of fractions as review and multiplication, division and simplification of fractions in detail.

At the end of this chapter, the students will be able to –

- Add, subtract, multiply and divide the algebraic fractions, simplify and solve the problems related to fractions.

5·1 Algebraic Fraction

If m and n are two algebraic expressions, $\frac{m}{n}$ is an algebraic fraction where $n \neq 0$. Here, m is called numerator and n is called denominator of the fraction $\frac{m}{n}$. For example, $\frac{a}{b}, \frac{x+y}{y}, \frac{x^2+a^2}{x+a}$ etc. are algebraic fractions.

5·2 Lowest Form of Fraction

If there are common factors of both numerator and denominator of any algebraic fraction, and if numerator and denominator are divided by H.C.F of the numerator and the denominator of the fraction, a new fraction is formed and the new fraction is called lowest form of the fraction.

For example, $$\frac{a^3b^2 - a^2b^3}{a^3b - ab^3} = \frac{a^2b^2(a-b)}{ab(a^2-b^2)}$$
$$= \frac{a^2b^2(a-b)}{ab((a+b)(a-b)}$$
$$= \frac{ab}{a+b}$$

Here, the smallest form of fraction is formed by dividing the numerator and the denominator by H.C.F $ab(a-b)$ of the numerator and the denominator.

5·3 Fractions in the form of common denominator

If we convert two or more fractions in the form of common denominator, we have to follow the steps below :

1. The L.C.M of the denominators has to be determined.
2. The L.C.M has to be divided by the denominator of the fractions.
3. The numerator and the denominator of the respective fraction has to be multiplied by the obtained quotient.

For example, $\frac{x}{y}, \frac{a}{b}, \frac{m}{n}$ are three fractions, we have to convert them into the fractions with common denominator.
Here, the denominators of the fractions are respectively y, b and n.
L.C.M of them is ybn.
y is the denominator of the first fraction $\frac{x}{y}$. If we divide the L.C.M ybn by y, the quotient will be bn. Now, we have to multiply both the numerator and the denominator of the fraction $\frac{x}{y}$ by bn.

$$\therefore \frac{x}{y} = \frac{x \times bn}{y \times bn} = \frac{xbn}{ybn}$$

Similarly, b is the denominator of the second fraction $\frac{a}{b}$. If we divide the L.C.M ybn by b, the quotient will be yn. Now, we have to multiply both numerator and denominator of the fraction $\frac{a}{b}$ by yn.

$$\therefore \frac{a}{b} = \frac{a \times yn}{b \times yn} = \frac{ayn}{ybn}.$$

n is the denominator of the third fraction $\frac{m}{n}$. If we divide the L.C.M ybn by n, the quotient will be yb. Now we have to multiply both the numerator and the denominator of the fraction $\frac{m}{n}$ by yb.

$$\therefore \frac{m}{n} = \frac{m \times yb}{n \times yb} = \frac{myb}{ybn}.$$

Therefore, $\frac{xbn}{ybn}, \frac{ayn}{ybn}$ and $\frac{myb}{ybn}$ are respectively the fractions with common denominator of the fractions $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$.

Example 1. Express the following two fractions in the lowest form :

(a) $\frac{16a^2b^3c^4y}{8a^3b^2c^5x}$ (b) $\frac{a(a^2+2ab+b^2)(a^3-b^3)}{(a^3+b^3)(a^4b-b^5)}$

Solution : (a) Given fraction $= \frac{16a^2b^3c^4y}{8a^3b^2c^5x}$

Here, H.C.F of 16 and 8 is 8

,, ,, a^2 and a^3 is a^2

,, ,, b^3 and b^2 is b^2

,, ,, c^4 and c^5 is c^4

y and x is 1

H.C.F of $16a^2b^3c^4y$ and $8a^3b^2c^5x$ is $8a^2b^2c^4$

Dividing the numerator and the denominator of the fraction $\frac{16a^2b^3c^4y}{8a^3b^2c^5x}$ by $8a^2b^2c^4$,

we get $\frac{2by}{acx}$

The lowest form of $\frac{16a^2b^3c^4y}{8a^3b^2c^5x}$ is $\frac{2by}{acx}$.

(b) Given fraction $= \frac{a(a^2+2ab+b^2)(a^3-b^3)}{(a^3+b^3)(a^4b-b^5)}$

Here, the numerator $= a(a^2+2ab+b^2)(a^3-b^3)$

$= a(a+b)^2(a-b)(a^2+ab+b^2)$

and the denominator $=(a^3+b^3)(a^4b-b^5)$

$=(a+b)(a^2-ab+b^2)\{b(a^4-b^4)\}$

$=b(a+b)(a^2-ab+b^2)(a^2-b^2)(a^2+b^2)$

$=b(a+b)(a^2-ab+b^2)(a+b)(a-b)(a^2+b^2)$

$=b(a+b)^2(a-b)(a^2+b^2)(a^2-ab+b^2)$

$\therefore$ H.C.F of the numerator and the denominator $= (a+b)^2(a-b)$

Dividing the numerator and the denominator of the given fraction by $(a+b)^2(a-b)$,

we get $\frac{a(a^2+ab+b^2)}{b(a^2+b^2)(a^2-ab+b^2)}$

The lowest form of the fraction is $\frac{a(a^2+ab+b^2)}{b(a^2+b^2)(a^2-ab+b^2)}$.

Example 2. Express the fractions $\frac{x}{x^3y-xy^3}, \frac{a}{xy(a^2-b^2)}, \frac{m}{m^3n-mn^3}$ with a common denominator.

Solution : Here the fractions are $\frac{x}{x^3y-xy^3}, \frac{a}{xy(a^2-b^2)}, \frac{m}{m^3n-mn^3}$

Here, the denominator of 1st fraction $= x^3y - xy^3$
$= xy(x^2 - y^2)$

The denominator of 2nd fraction $= xy(a^2 - b^2)$

The denominator of 3rd fraction $= m^3n - mn^3$
$= mn(m^2 - n^2)$

$\therefore$ L.C.M of the denominators $= xy(x^2 - y^2)(a^2 - b^2)(m^2 - n^2)mn$

Therefore, $\dfrac{x}{x^3y - xy^3} = \dfrac{x(a^2 - b^2)(m^2 - n^2)mn}{xy(x^2 - y^2)(a^2 - b^2)(m^2 - n^2)mn}$

$$\frac{a}{xy(a^2 - b^2)} = \frac{a(x^2 - y^2)(m^2 - n^2)mn}{xy(x^2 - y^2)(a^2 - b^2)(m^2 - n^2)mn}$$

and $\dfrac{m}{m^3n - mn^3} = \dfrac{xym(x^2 - y^2)(a^2 - b^2)}{xy(x^2 - y^2)(a^2 - b^2)(m^2 - n^2)mn}$

$\therefore$ Required fractions are

$\dfrac{x(a^2 - b^2)(m^2 - n^2)mn}{xy(x^2 - y^2)(a^2 - b^2)(m^2 - n^2)mn}$, $\dfrac{a(x^2 - y^2)(m^2 - n^2)mn}{xy(x^2 - y^2)(a^2 - b^2)(m^2 - n^2)mn}$ and

$$\frac{xym(x^2 - y^2)(a^2 - b^2)}{xy(x^2 - y^2)(a^2 - b^2)(m^2 - n^2)mn}$$

Activity : Express the fractions with a common denominator :

1. $\dfrac{x^2 + xy}{x^2y}$ and $\dfrac{x^2 - xy}{xy^2}$ 2. $\dfrac{a - b}{a + 2b}$ and $\dfrac{2a + b}{a^2 - 4b}$

5·4 Addition of Fractions

If we want to add two or more fractions, at first, we have to express all fractions with a common denominator. By adding all the numerators, we get a new fraction of which the numerator is the sum of the numerators of the given fractions and the denominator is the L.C.M of the denominators of the given fractions.

For example,

$$\frac{a}{x} + \frac{b}{y} + \frac{b}{z} = \frac{ayz}{xyz} + \frac{bxz}{xyz} + \frac{bxy}{xyz} = \frac{ayz + bxz + bxy}{xyz}$$

Example 3. Add the following three fractions :

$$\frac{1}{x-y}, \frac{x}{x^2+xy+y^2}, \frac{y^2}{x^3-y^3}$$

Here, the 1st fraction $= \dfrac{1}{x-y}$

The 2nd fraction $= \dfrac{x}{x^2+xy+y^2}$

The 3rd fraction $= \dfrac{y^2}{x^3-y^3} = \dfrac{y^2}{(x-y)(x^2+xy+y^2)}$

$\therefore$ The L.C.M of denominators $= (x-y)(x^2+xy+y^2) = (x^3-y^3)$

Therefore, the sum of $\dfrac{1}{x-y}, \dfrac{x}{x^2+xy+y^2}$ and $\dfrac{y}{x^3-y^3}$

is $\dfrac{1}{x-y} + \dfrac{x}{x^2+xy+y^2} + \dfrac{y^3}{x^3-y^3}$

$$= \frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)} + \frac{x(x-y)}{(x-y)(x^2+xy+y^2)} + \frac{y^2}{x^3-y^3}$$

$$= \frac{x^2+xy+y^2}{x^3-y^3} + \frac{x^2-xy}{x^3-y^3} + \frac{y^2}{x^3-y^3}$$

$$= \frac{x^2+xy+y^2+x^2-xy+y^2}{x^3-y^3}$$

$$= \frac{2(x^2+y^2)}{x^3-y^3}$$

Required summation $= \dfrac{2(x^2+y^2)}{x^3-y^3}$.

Example 4. Find the sum $\dfrac{3a}{a^2+3a-4} + \dfrac{2a}{a^2-1} + \dfrac{a}{a^2+5a+4}$

Solution : The given expression is $\dfrac{3a}{a^2+3a-4} + \dfrac{2a}{a^2-1} + \dfrac{a}{a^2+5a+4}$

$$= \frac{3a}{a^2+4a-a-4} + \frac{2a}{(a+1)(a-1)} + \frac{a}{a^2+a+4a+4}$$

$$= \frac{3a}{(a+4)(a-1)} + \frac{2a}{(a+1)(a-1)} + \frac{a}{(a+1)(a+4)}$$

$$= \frac{3a(a+1)+2a(a+4)+a(a-1)}{(a+4)(a+1)(a-1)}$$

$$= \frac{3a^2+3a+2a^2+8a+a^2-a}{(a+4)(a+1)(a-1)}$$

$$= \frac{6a^2+10a}{(a+4)(a+1)(a-1)}$$

$$= \frac{2a(3a+5)}{(a+4)(a^2-1)}$$

Example 5. Find the sum

(a) $\frac{a-b}{bc} + \frac{b-c}{ca} + \frac{c-a}{ab}$

(b) $\frac{1}{a^2-5a+6} + \frac{1}{a^2-9} + \frac{1}{a^2+4a+3}$

(c) $\frac{1}{a-2} + \frac{a-2}{a^2+2a+4}$

Solution : (a) $\frac{a-b}{bc} + \frac{b-c}{ca} + \frac{c-a}{ab}$

$$= \frac{a^2-ab+b^2-bc+c^2-ca}{abc}$$

$$= \frac{a^2+b^2+c^2-ab-bc-ca}{abc}$$

(b) $\frac{1}{a^2-5a+6} + \frac{1}{a^2-9} + \frac{1}{a^2+4a+3}$

$$= \frac{1}{a^2-2a-3a+6} + \frac{1}{(a+3)(a-3)} + \frac{1}{a^2+3a+a+3}$$

$$= \frac{1}{a(a-2)-3(a-2)} + \frac{1}{(a+3)(a-3)} + \frac{1}{a(a+3)+1(a+3)}$$

$$= \frac{1}{(a-2)(a-3)} + \frac{1}{(a+3)(a-3)} + \frac{1}{(a+3)(a+1)}$$

$$= \frac{(a+1)(a+3)+(a+1)(a-2)+(a-2)(a-3)}{(a+1)(a-2)(a+3)(a-3)}$$

$$= \frac{a^2+4a+3+a^2-a-2+a^2-5a+6}{(a+1)(a-2)(a+3)(a-3)}$$

$$= \frac{3a^2-2a+7}{(a+1)(a-2)(a^2-9)}$$

(c) $\frac{1}{a-2} + \frac{a+2}{a^2+2a+4}$

$$= \frac{a^2+2a+4+(a-2)(a+2)}{(a-2)(a^2+2a+4)}$$

$$= \frac{a^2+2a+4+a^2-4}{a^3-8}$$

$$= \frac{2a^2+2a}{a^3-8}$$

$$= \frac{2a(a+1)}{a^3-8}$$

Activity : Add the expressions :

1. $\frac{2a}{3x^2y}, \frac{3b}{2xy^2}, \frac{a+b}{xy}$ 2. $\frac{2}{x^2y-xy^2}, \frac{3}{xy(x^2-y^2)}, \frac{1}{x^2y^2}$

5·5 Subtraction of fractions

For the subtraction of two fractions, at first, we have to form all fractions having a common denominator. By subtracting one numerator from the other numerator, we get a new fraction of which the numerator is the subtraction of the two numerators of given fractions and the denominator is the L.C.M of the denominators of the given fractions.

For example, $\frac{a}{xy} - \frac{b}{yz} = \frac{az}{xyz} - \frac{bx}{xyz}$

$$= \frac{az - bx}{xyz}$$

Example 6. Find the difference

(a) $\frac{x}{4a^2bc^2} - \frac{y}{9ab^2c^3}$

(b) $\frac{x}{(x-y)^2} - \frac{x+y}{x^2-y^2}$

(c) $\frac{a^2+9y^2}{a^2-9y^2} - \frac{a-3y}{a+3y}$

Solution : (a) $\frac{x}{4a^2bc^2} - \frac{y}{9ab^2c^3}$

Here, the L.C.M of $4a^2bc^2$ and $9ab^2c^3$ is $36a^2b^2c^3$

$$\therefore \frac{x}{4a^2bc^2} - \frac{y}{9ab^2c^3}$$

$$= \frac{9xbc - 4ya}{36a^2b^2c^3}$$

(b) $\frac{x}{(x-y)^2} - \frac{x+y}{x^2-y^2}$

Here, the L.C.M of $(x-y)^2$ and x^2-y^2 is $(x-y)^2(x+y)$

$$\therefore \frac{x}{(x-y)^2} - \frac{x+y}{x^2-y^2}$$

$$= \frac{x(x+y) - (x+y)(x-y)}{(x-y)^2(x+y)}$$

$$= \frac{x^2 + xy - x^2 + y^2}{(x-y)^2(x+y)}$$

$$= \frac{xy + y^2}{(x-y)^2(x+y)}$$

$$= \frac{y(x+y)}{(x-y)^2(x+y)}$$

$$= \frac{y}{(x-y)^2}$$

(c) $\frac{a^2+9y^2}{a^2-9y^2} - \frac{a-3y}{a+3y}$

Here, the L.C.M of a^2-9y^2 and $a+3y$ is a^2-9y^2

$$\therefore \quad \frac{a^2+9y^2}{a^2-9y^2} - \frac{a-3y}{a+3y}$$

$$= \frac{a^2+9y^2-(a-3y)(a-3y)}{a^2-9y^2}$$

$$= \frac{a^2+9y^2-(a^2-6ay+9y^2)}{a^2-9y^2}$$

$$= \frac{a^2+9y^2-a^2+6ay-9y^2}{a^2-9y^2}$$

$$= \frac{6ay}{a^2-9y^2}$$

Activity : Simplify:

1. $\frac{x}{x^2+xy+y^2} - \frac{xy}{x^3-y^3}$ 2. $\frac{1}{1+a+a^2} - \frac{2a}{1+a^2+a^4}$

Observation: While adding and subtracting the algebraic fraction, the given fractions are to be expressed in the lowest form, if necessary.

For example, $\frac{a^2bc}{ab^2c} + \frac{ab^2c}{abc^2} + \frac{abc^2}{a^2bc}$

$$= \frac{a}{b} + \frac{b}{c} + \frac{c}{a}$$

$$= \frac{a \times ca}{b \times ca} + \frac{b \times ab}{c \times ab} + \frac{c \times bc}{a \times bc}$$ [L.C.M. of the denominators *b, c, a* is *abc*]

$$=\frac{ca^2}{abc}+\frac{ab^2}{abc}+\frac{bc^2}{abc}$$

$$=\frac{ca^2+ab^2+bc^2}{abc}.$$

Solution 7. Simplify :

(a) $\frac{x-y}{(y+z)(z+x)}+\frac{y-z}{(x+y)(z+x)}+\frac{z-x}{(x+y)(y+z)}$

(b) $\frac{1}{x-2}-\frac{1}{x+2}-\frac{4}{x^2+4}$

(c) $\frac{1}{1-a+a^2}-\frac{1}{1+a+a^2}-\frac{2a}{1+a^2+a^4}$

Solution : (a) $\frac{x-y}{(y+z)(z+x)}+\frac{y-z}{(x+y)(z+x)}+\frac{z-x}{(x+y)(y+z)}$

Here, the L.C.M of $(y+z)(z+x)$, $(x+y)(z+x)$ and $(x+y)(y+z)$ is $(x+y)(y+z)(z+x)$

$$\therefore \frac{x-y}{(y+z)(z+x)}+\frac{y-z}{(x+y)(z+x)}+\frac{z-x}{(x+y)(y+z)}$$

$$=\frac{(x-y)(x+y)+(y-z)(y+z)+(z-x)(z+x)}{(x+y)(y+z)(z+x)}$$

$$=\frac{x^2-y^2+y^2-z^2+z^2-x^2}{(x+y)(y+z)(z+x)}$$

$$=\frac{0}{(x+y)(y+z)(z+x)}$$

$$=0.$$

(b) $\frac{1}{x-2}-\frac{1}{x+2}-\frac{4}{x^2+4}$

$$=\frac{x+2-x+2}{(x-2)(x+2)}-\frac{4}{x^2+4}$$

$$= \frac{4}{x^2-4} - \frac{4}{x^2+4}$$

$$= 4\left[\frac{1}{x^2-4} - \frac{1}{x^2+4}\right]$$

$$= 4\left[\frac{x^2+4-x^2+4}{(x^2-4)(x^2+4)}\right]$$

$$= \frac{4\times 8}{(x^2-4)(x^2+4)}$$

$$= \frac{32}{x^4-16}$$

(c) $\frac{1}{1-a+a^2} - \frac{1}{1+a+a^2} - \frac{2a}{1+a^2+a^4}$

Here, $1+a^2+a^4 = 1+2a^2+a^4-a^2$

$$= (1+a^2)^2 - a^2$$

$$= (1+a^2+a)(1+a^2-a)$$

$$= (a^2+a+1)(a^2-a+1)$$

The L.C.M of the denominators $1-a+a^2, 1+a+a^2, 1+a^2+a^4$ is

$$(1-a+a^2)(1+a+a^2) = 1+a^2+a^4$$

$$\therefore \frac{1}{1-a+a^2} - \frac{1}{1+a+a^2} - \frac{2a}{1+a^2+a^4}$$

$$= \frac{1+a+a^2-1+a-a^2-2a}{1+a^2+a^4}$$

$$= \frac{0}{1+a^2+a^4}$$

$$= 0$$

Exercise 5.1

1. Express the following fractions in the lowest form.

(a) $\dfrac{4x^2y^3z^5}{9x^5y^2z^3}$ (b) $\dfrac{16(2x)^4(3y)^5}{(3x)^3.(2y)^6}$

(c) $\dfrac{x^3y+xy^3}{x^2y^3+x^3y^2}$ (d) $\dfrac{(a-b)(a+b)}{a^3-b^3}$

(e) $\dfrac{x^2-6x+5}{x^2-25}$ (f) $\dfrac{x^2-7x+12}{x^2-9x+20}$

(g) $\dfrac{(x^3-y^3)(x^2-xy+y^2)}{(x^2-y^2)(x^3+y^3)}$ (h) $\dfrac{a^2-b^2-2bc-c^2}{a^2+2ab+b^2-c^2}$

2. Express the following fractions in the form of a common denominator.

(a) $\dfrac{x^2}{xy},\dfrac{y^2}{yz},\dfrac{z^2}{zx}$ (b) $\dfrac{x-y}{xy},\dfrac{y-z}{yz},\dfrac{z-x}{zx}$

(c) $\dfrac{x}{x-y},\dfrac{y}{x+y},\dfrac{z}{x(x+y)}$ (d) $\dfrac{x+y}{(x-y)^2},\dfrac{x-y}{x^3+y^3},\dfrac{y-z}{x^2-y^2}$

(e) $\dfrac{a}{a^3+b^3},\dfrac{b}{(a^2+ab+b^2)},\dfrac{c}{a^3-b^3}$

(f) $\dfrac{1}{x^2-5x+6},\dfrac{1}{x^2-7x+12},\dfrac{1}{x^2-9x+20}$ (g) $\dfrac{a-b}{a^2b^2},\dfrac{b-c}{b^2c^2},\dfrac{c-a}{c^2a^2}$

(h) $\dfrac{x-y}{x+y},\dfrac{y-z}{y+z},\dfrac{z-x}{z+x}$

3. Find the sum :

(a) $\dfrac{a-b}{a}+\dfrac{a+b}{b}$ (b) $\dfrac{a}{bc}+\dfrac{b}{ca}+\dfrac{c}{ab}$

(c) $\dfrac{x-y}{x}+\dfrac{y-z}{y}+\dfrac{z-x}{z}$ (d) $\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}$

(e) $\dfrac{1}{x^2-3x+2}+\dfrac{1}{x^2-4x+3}+\dfrac{1}{x^2-5x+4}$

(f) $\dfrac{1}{a^2-b^2}+\dfrac{1}{a^2+ab+b^2}+\dfrac{1}{a^2-ab+b^2}$

(g) $\dfrac{1}{x-2}-\dfrac{1}{x+2}+\dfrac{4}{x^2-4}$ (h) $\dfrac{1}{x^2-1}+\dfrac{1}{x^4-1}+\dfrac{4}{x^8-1}$

4. Find the difference

(a) $\dfrac{a}{x-3}-\dfrac{a^2}{x^2-9}$ (b) $\dfrac{1}{y(x-y)}-\dfrac{1}{x(x+y)}$

(c) $\dfrac{x+1}{1+x+x^2}-\dfrac{x-1}{1-x+x^2}$ (d) $\dfrac{a^2+16b^2}{a^2-16b^2}-\dfrac{a-4b}{a+4b}$

(e) $\dfrac{1}{x-y}-\dfrac{x^2-xy+y^2}{x^3+y^3}$

5. Simplify :

(a) $\dfrac{x-y}{xy}+\dfrac{y-z}{yz}+\dfrac{z-x}{zx}$

(b) $\dfrac{x-y}{(x+y)(y+z)}+\dfrac{y-z}{(y+z)(z+x)}+\dfrac{z-x}{(z+x)(x+y)}$

(c) $\dfrac{y}{(x-y)(y-z)}+\dfrac{x}{(z-x)(x-y)}+\dfrac{z}{(y-z)(z-x)}$

(d) $\dfrac{1}{x+3y}+\dfrac{1}{x-3y}-\dfrac{2x}{x^2-9y^2}$ (e) $\dfrac{1}{x-y}-\dfrac{2}{2x+y}+\dfrac{1}{x+y}-\dfrac{2}{2x-y}$

(f) $\dfrac{1}{x-2}-\dfrac{x-2}{x^2+2x+4}+\dfrac{6x}{x^3+8}$ (g) $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}+\dfrac{4}{x^4+1}$

(h) $\dfrac{x-y}{(y-z)(z-x)}+\dfrac{y-z}{(z-x)(x-y)}+\dfrac{z-x}{(x-y)(y-z)}$

(i) $\dfrac{1}{a-b-c}+\dfrac{1}{a-b+c}+\dfrac{a}{a^2+b^2-c^2-2ab}$

(j) $\dfrac{1}{a^2+b^2-c^2+2ab}+\dfrac{1}{b^2+c^2-a^2+2bc}+\dfrac{1}{c^2+a^2-b^2+2ca}$

5·6 Multiplication of fractions

By multiplying two or more fractions, we can also get a fraction. Its numerator is equal to the product of the numerators of two or more fractions and the denominator is equal to the product of their denominators. If we convert this type of fractions into the lowest form, both the numerators and the denominators are changed.

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For example, $\frac{x}{y}$ and $\frac{a}{b}$ are two fractions.

The product of this two fractions is

$$\frac{x}{y} \times \frac{a}{b} = \frac{x \times a}{y \times b}$$

$$= \frac{xa}{yb}$$

Here, xa is the numerator of the fraction which is the product of the numerators of two fractions and yb is the denominator of the fraction which is the product of the denominators of two fractions.

Again, the product of three fractions $\frac{x}{by}, \frac{ya}{z}$ and $\frac{z}{x}$ is

$$\frac{x}{by} \times \frac{ya}{z} \times \frac{z}{x} = \frac{xyza}{xyzb}$$

$$= \frac{a}{b} \quad \text{[By reducing]}$$

Here the numerator and the denominator were changed by reducing the product of the fractions.

Example 8 : Multiply :

(a) $\frac{a^2b^2}{cd}$ by $\frac{ab}{c^2d^2}$

(b) $\frac{x^2y^3}{xy^2}$ by $\frac{x^3b}{ay^3}$

(c) $\frac{10x^5b^4z^3}{3x^2b^2z}$ by $\frac{15y^5b^2z^2}{2y^2a^2x}$

(d) $\frac{x^2-y^2}{x^3+y^3}$ by $\frac{x^2-xy+y^2}{x^3-y^3}$

(e) $\frac{x^2-5x+6}{x^2-9x+20}$ by $\frac{x-5}{x-3}$

Solution :

(a) $\frac{a^2b^2}{cd} \times \frac{ab}{c^2d^2} = \frac{a^2b^2 \times ab}{cd \times c^2d^2}$

$\therefore$ Required product $= \frac{a^3b^3}{c^3d^3}$

(b) $\dfrac{x^2y^3}{xy^2} \times \dfrac{x^3b}{ay^3}$

$= \dfrac{x^2y^3 \times x^3b}{xy^2 \times ay^3}$

$= \dfrac{x^5y^3b}{xy^5a}$

$\therefore$ Required product $= \dfrac{x^4b}{y^2a}$

(c) $\dfrac{10x^5b^4z^3}{3x^2b^2z} \times \dfrac{15y^5b^2z^2}{2y^2a^2x}$

$= \dfrac{10x^5b^4z^3 \times 15y^5b^2z^2}{3x^2b^2z \times 2y^2a^2x}$

$= \dfrac{25x^5y^5z^6}{x^3y^2z^3a^2b^2}$

$\therefore$ Required product $= \dfrac{25b^4x^2y^3z^4}{a^2}$

(d) $\dfrac{x^2-y^2}{x^3+y^3} \times \dfrac{x^2-xy+y^2}{x^3-y^3}$

$= \dfrac{(x+y)(x-y)\times(x^2-xy+y^2)}{(x+y)(x^2+xy-y^2)(x-y)(x^2+xy+y^2)}$

$\therefore$ Required product $= \dfrac{1}{x^2+xy+y^2}$

(e) $\dfrac{x^2-5x+6}{x^2-9x+20} \times \dfrac{x-5}{x-3}$

$= \dfrac{x^2-2x-3x+6}{x^2-4x-5x+20} \times \dfrac{x-5}{x-3}$

$= \dfrac{x(x-2)-3(x-2)}{x(x-4)-5(x-4)} \times \dfrac{x-5}{x-3}$

$= \dfrac{(x-2)(x-3)}{(x-4)(x-5)} \times \dfrac{x-5}{x-3}$

$= \dfrac{(x-2)(x-3)(x-5)}{(x-4)(x-5)(x-3)}$

$\therefore$ Required product $= \dfrac{x-2}{x-4}$.

Activity : Multiply :

1. $\frac{7a^2b}{36a^3b^2}$ by $\frac{24ab^2}{35a^4b^5}$ 2. $\frac{x^2+3x-4}{x^2-7x+12}$ by $\frac{x^2-9}{x^2-16}$.

5·7 Division of fractions

Division of one fraction by another fraction means multiplication of the first fraction by the inverse of the second fraction.

For example, to divide $\frac{x}{y}$ by $\frac{z}{y}$,

then $\frac{x}{y} \div \frac{z}{y}$

$= \frac{x}{y} \times \frac{y}{z}$ [Here $\frac{y}{z}$ is the inverse fraction of $\frac{z}{y}$]

$= \frac{x}{z}$

Example 9. Divide :

(a) $\frac{a^3b^2}{c^2d}$ by $\frac{a^2b^3}{cd^3}$

(b) $\frac{12a^4x^3y^2}{10x^4y^3z^2}$ by $\frac{6a^3b^2c}{5x^2y^2z^2}$

(c) $\frac{a^2-b^2}{a^2+ab+b^2}$ by $\frac{a+b}{a^3-b^3}$

(d) $\frac{x^3-27}{x^2-7x+6}$ by $\frac{x^2-9}{x^2-36}$

(e) $\frac{x^3-y^3}{x^3+y^3}$ by $\frac{x^2-y^2}{(x+y)^2}$

Solution :

(a) The 1st fraction $= \frac{a^3b^2}{c^2d}$

The 2nd fraction $= \frac{a^2b^3}{cd^3}$

The multiplicative inverse of 2nd fraction is $\frac{cd^3}{a^2b^3}$

$$\frac{a^3b^2}{c^2d} \div \frac{a^2b^3}{cd^3}$$

$$= \frac{a^3b^2}{c^2d} \times \frac{cd^3}{a^2b^3}$$

$\therefore$ Required quotient $= \frac{a^3b^2cd^3}{a^2b^3c^2d} = \frac{ad^2}{bc}$

(b) $\frac{12a^4x^3y^2}{10x^4y^3z^2} \div \frac{6a^3b^2c}{5x^2y^2z^2}$

$$= \frac{12a^4x^3y^2}{10x^4y^3z^2} \times \frac{5x^2y^2z^2}{6a^3b^2c}$$

$\therefore$ Required quotient $= \frac{axy}{b^2c}$

(c) $\frac{a^2 - b^2}{a^2 + ab + b^2} \div \frac{a+b}{a^3 - b^3}$

$$= \frac{(a+b)(a-b)}{(a^2+ab+b^2)} \times \frac{(a-b)(a^2+ab+b^2)}{a+b}$$

$$= (a-b)(a-b)$$

$\therefore$ Required quotient $= (a-b)^2$

(d) $\frac{x^3 - 27}{x^2 - 7x + 6} \div \frac{x^2 - 9}{x^2 - 36}$

$$= \frac{x^3 - 3^3}{x^2 - 6x - x + 6} \times \frac{x^2 - 6^2}{x^2 - 3^2}$$

$$= \frac{(x-3)(x^2+3x+3^2)}{(x-6)(x-1)} \times \frac{(x+6)(x-6)}{(x+3)(x-3)}$$

$\therefore$ Required quotient $= \frac{(x^2+3x+9)(x+6)}{(x-1)(x+3)}$

(e) $\frac{x^3 - y^3}{x^3 + y^3} \div \frac{x^2 - y^2}{(x+y)^2}$

$$=\frac{(x-y)(x^2+xy+y^2)}{(x+y)(x^2-xy+y^2)}\times\frac{(x+y)^2}{(x+y)(x-y)}$$

$\therefore$ Required quotient $=\dfrac{x^2+xy+y^2}{x^2-xy+y^2}$.

Activity : Divide :

1. $\dfrac{16a^2b^2}{21z^2}$ by $\dfrac{28ab^4}{35xy}$ 2. $\dfrac{x^4-y^4}{x^2-2xy+y^2}$ by $\dfrac{x^3+y^3}{x-y}$.

Example 10. Simplify :

(a) $\left(1+\dfrac{1}{x}\right)\div\left(1-\dfrac{1}{x^2}\right)$

(b) $\left(\dfrac{x}{x+y}+\dfrac{y}{x-y}\right)\div\left(\dfrac{x}{x-y}-\dfrac{y}{x+y}\right)$

(c) $\dfrac{a^3+b^3}{(a-b)^2+3ab}\div\dfrac{(a+b)^2-3ab}{a^3-b^3}\times\dfrac{a+b}{a-b}$

(d) $\dfrac{x^2+3x-4}{x^2-7x+12}\div\dfrac{x^2-16}{x^2-9}\times\dfrac{(x-4)^2}{(x-1)^2}$

(e) $\dfrac{x^3+y^3+3xy(x+y)}{(x+y)^2-4xy}\div\dfrac{(x-y)^2+4xy}{x^3-y^3-3xy(x-y)}$

Solution : (a) $\left(1+\dfrac{1}{x}\right)\div\left(1-\dfrac{1}{x^2}\right)$

$$=\frac{x+1}{x}\div\frac{x^2-1}{x^2}$$

$$=\frac{x+1}{x}\times\frac{x^2}{(x+1)(x-1)}$$

$$=\frac{x}{x-1}$$

(b) $\left(\dfrac{x}{x+y}+\dfrac{y}{x-y}\right)\div\left(\dfrac{x}{x-y}-\dfrac{y}{x+y}\right)$

$$= \frac{x^2 - xy + xy + y^2}{(x+y)(x-y)} \div \frac{x^2 + xy - xy + y^2}{(x-y)(x+y)}$$
$$= \frac{x^2 + y^2}{x^2 - y^2} \div \frac{x^2 + y^2}{x^2 - y^2}$$
$$= \frac{x^2 + y^2}{x^2 - y^2} \times \frac{x^2 - y^2}{x^2 + y^2}$$
$$= 1$$

(c) $$\frac{a^3 + b^3}{(a-b)^2 + 3ab} \div \frac{(a+b)^2 - 3ab}{a^3 - b^3} \times \frac{a+b}{a-b}$$
$$= \frac{(a+b)(a^2 - ab + b^2)}{a^2 - 2ab + b^2 + 3ab} \div \frac{a^2 + 2ab + b^2 - 3ab}{(a-b)(a^2 + ab + b^2)} \times \frac{a+b}{a-b}$$
$$= \frac{(a+b)(a^2 - ab + b^2)}{(a^2 + ab + b^2)} \times \frac{(a-b)(a^2 + ab + b^2)}{(a^2 - ab + b^2)} \times \frac{a+b}{a-b}$$
$$= (a+b)(a+b)$$
$$= (a+b)^2$$

(d) $$\frac{x^2 + 3x - 4}{x^2 - 7x + 12} \div \frac{x^2 - 16}{x^2 - 9} \times \frac{(x-4)^2}{(x-1)^2}$$
$$= \frac{x^2 + 4x - x - 4}{x^2 - 3x - 4x + 12} \times \frac{x^2 - 3^2}{x^2 - 4^2} \times \frac{(x-4)^2}{(x-1)^2}$$
$$= \frac{(x+4)(x-1)}{(x-3)(x-4)} \times \frac{(x+3)(x-3)}{(x+4)(x-4)} \times \frac{(x-4)^2}{(x-1)^2}$$
$$= \frac{x+3}{x-1}$$

(e) $$\frac{x^3 + y^3 + 3xy(x+y)}{(x+y)^2 - 4xy} \div \frac{(x-y)^2 + 4xy}{x^3 - y^3 - 3xy(x-y)}$$
$$= \frac{(x+y)^3}{(x-y)^2} \div \frac{(x+y)^2}{(x-y)^3}$$
$$= \frac{(x+y)^3}{(x-y)^2} \times \frac{(x-y)^3}{(x+y)^2}$$
$$= (x+y)(x-y)$$
$$= x^2 - y^2$$

Exercise 5.2

1. Which one is correct if $\frac{a}{x}, \frac{b}{y}, \frac{c}{z}, \frac{p}{q}$ are reduced to the common denominator?

(a) $\frac{ayzq}{xyzq}, \frac{bxzq}{xyzq}, \frac{cxyq}{xyzq}, \frac{pxyz}{xyzq}$ (b) $\frac{axy}{xyzq}, \frac{byz}{xyzq}, \frac{czx}{xyzq}, \frac{pxy}{xyzq}$

(c) $\frac{a}{xyzq}, \frac{b}{xyzq}, \frac{c}{xyzq}, \frac{p}{xyzq}$ (d) $\frac{axyzq}{xyzq}, \frac{bxyzq}{xyzq}, \frac{cxyzq}{xyzq}, \frac{pxyzq}{xyzq}$

2. Which one of the following is the product of $\frac{x^2y^2}{ab}$ and $\frac{c^3d^2}{x^5y^3}$?

(a) $\frac{x^2y^2c^3d^2}{abx^3y^2}$ (b) $\frac{c^3d^2}{abx^3y}$ (c) $\frac{x^2y^2c^3}{x^3y}$ (d) $\frac{xyd^2}{ab}$

3. What is the quotient if $\frac{x^2-2x+1}{a^2-2a+1}$ is divided by $\frac{x-1}{a-1}$?

(a) $\frac{x+1}{a-1}$ (b) $\frac{x-1}{a-1}$ (c) $\frac{x-1}{a+1}$ (d) $\frac{a-1}{x-1}$

4. Which one of the following is the simple value of $\frac{a-b}{a} - \frac{a+b}{b}$?

A. $\frac{a^2-2ab-b^2}{ab}$ B. $\frac{a^2-2ab+b^2}{ab}$ C. $\frac{-a^2-b^2}{ab}$ D. $\frac{a^2-b^2}{ab}$

5. Which one of the following is the value of $\frac{P+x}{P-x} \div \frac{(P+x)^2}{P^2-x^2}$?

A. 1 B. $p-x$ C. $p+x$ D. $\frac{P-x}{P+x}$

6. Which one of the following expressions will be if $\frac{x+y}{x-y}$ and $\frac{x-y}{x+y}$ are turned into fractions with common denominator?

A. $\frac{(x+y)^2}{x^2-y^2}, \frac{(x-y)^2}{x^2-y^2}$ B. $\frac{(x+y)^2}{x-y}, \frac{(x-y)^2}{x+y}$ C. $\frac{(x+y)^2}{x^2+y^2}, \frac{(x-y)^2}{x^2+y^2}$ D. $\frac{x-y}{(x+y)^2}, \frac{x+y}{(x-y)^2}$

Answer questions 7-9 in the light of the following information of the stimulus:

$\frac{x^2+4x-21}{x^2+5x-14}$ is an algebraic fraction.

7. Which one is the factorized form of the numerator ?

A. (x+7) (x-3) B. (x-1) (x+21) C. (x-3) (x-7) D. (x+3) (x-7)

8. Which one of the following is the lowest value of the fraction ?

A. $\frac{x-7}{x+7}$ B. $\frac{x-3}{x+2}$ C. $\frac{x+7}{x-2}$ D. $\frac{x-3}{x-2}$

9. What would be added to the lowest value to get the sum $\frac{1}{2-x}$?

A. –1 B. 1 C. x–2 D. x–3

10. The following will be the equivalent fraction of $\frac{x^2+6x+5}{x^2+10x+5}$

i. $\frac{x+1}{x+5}$ ii. $\frac{x^2-2x-3}{x^2+2x-15}$ iii. $\frac{x^2+2x+1}{x^2-3x-10}$

Which one of the following is correct?

A. i and ii B. i and iii C. ii and iii D. i, ii and iii

11. Which one of the following is the quotient of $\frac{x^2+2x-3}{x^2+x-2}$ and $\frac{x^2+x-6}{x^2-4}$

A. $\frac{x+3}{x+2}$ B. $\frac{x-1}{x+3}$ C. 1 D. 0

12. What is the simplified value of $\frac{1}{x-12}-\frac{1}{x+2}-\frac{4}{x^2-4}$

A. $\frac{8}{x^2-4}$ B. $\frac{2x}{x^2-4}$ C. 1 D. 0

13. Multiply :

(a) $\frac{9x^2y^2}{7y^2z^2}, \frac{5b^2c^2}{3z^2x^2}$ and $\frac{7c^2a^2}{x^2y^2}$

(b) $\frac{16a^2b^2}{21z^2}, \frac{28z^4}{9x^3y^4}$ and $\frac{3y^7z}{10x}$

(c) $\frac{yz}{x^2}, \frac{zx}{y^2}$ and $\frac{xy}{z^2}$

(d) $\frac{x-1}{x+1}, \frac{(x-1)^2}{x^2+x}$ and $\frac{x^2}{x^2-4x+5}$

(e) $\frac{x^4-y^4}{x^2-2xy+y^2}, \frac{x-y}{x^3+y^3}$ and $\frac{x+y}{x^3+y^3}$

(f) $\frac{1-b^2}{1+x}, \frac{1-x^2}{b+b^2}$ and $\left(1+\frac{1-x}{x}\right)$

(g) $\frac{x^2-3x+2}{x^2-4x+3}, \frac{x^2-5x+6}{x^2-7x+12}$ and $\frac{x^2-16}{x^2-9}$

(h) $\dfrac{x^3+y^3}{a^2b+ab^2+b^3}$, $\dfrac{a^3-b^3}{x^2-xy+y^2}$ and $\dfrac{ab}{x+y}$

(i) $\dfrac{x^3+y^3+3xy(x+y)}{(a+b)^3}$, $\dfrac{a^3+b^3+3ab(a+b)}{x^2-y^2}$ and $\dfrac{(x-y)^2}{(x+y)^2}$

14. Divide (1st expression by 2nd expression)

(a) $\dfrac{3x^2}{2a}$, $\dfrac{4y^2}{15zx}$

(b) $\dfrac{9a^2b^2}{4c^2}$, $\dfrac{16a^3b}{3c^3}$

(c) $\dfrac{21a^4b^4c^4}{4x^3y^3z^3}$, $\dfrac{7a^2b^2c^2}{12xyz}$

(d) $\dfrac{x}{y}$, $\dfrac{x+y}{y}$

(e) $\dfrac{(a+b)^2}{(a-b)^2}$, $\dfrac{a^2-b^2}{a+b}$

(f) $\dfrac{x^3-y^3}{x+y}$, $\dfrac{x^2+xy+y^2}{x^2-y^2}$

(g) $\dfrac{a^3+b^3}{a-b}$, $\dfrac{a^2-ab+b^2}{a^2-b^2}$

(h) $\dfrac{x^2-7x+12}{x^2-4}$, $\dfrac{x^2-16}{x^2-3x+2}$

(i) $\dfrac{x^2-x-30}{x^2-36}$, $\dfrac{x^2+13x+40}{x^2+x-56}$

15. Simplify

(a) $\left(\dfrac{1}{x}+\dfrac{1}{y}\right)\times\left(\dfrac{1}{y}-\dfrac{1}{x}\right)$

(b) $\left(\dfrac{1}{1+x}+\dfrac{2x}{1-x^2}\right)\left(\dfrac{1}{x}-\dfrac{1}{x^2}\right)$

(c) $\left(1-\dfrac{c}{a+b}\right)\left(\dfrac{a}{a+b+c}-\dfrac{a}{a+b-c}\right)$

(d) $\left(\dfrac{1}{1+a}+\dfrac{a}{1-a}\right)\left(\dfrac{1}{1+a^2}-\dfrac{1}{1+a+a^2}\right)$

(e) $\left(\dfrac{x}{2x-y}+\dfrac{x}{2x+y}\right)\left(4+\dfrac{3y^2}{x^2-y^2}\right)$

(f) $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{y}{x+y}\right)$

(g) $\left(\frac{a}{a+b}+\frac{b}{a-b}\right)\div\left(\frac{a}{a-b}-\frac{b}{a+b}\right)$

(h) $\left(\frac{a^2+b^2}{2ab}-1\right)\div\left(\frac{a^3-b^3}{a-b}-3ab\right)$

(i) $\frac{(x+y)^2-4xy}{(a+b)^2-4ab}\div\frac{x^3-y^3-3xy(x-y)}{a^3-b^3-3ab(a-b)}$

(j) $\left(\frac{a}{b}+\frac{b}{a}+1\right)\div\left(\frac{a^2}{b^2}+\frac{a}{b}+1\right)$

16. Simplify :

(a) $\frac{x^2+2x-15}{x^2+x-12}\div\frac{x^2-25}{x^2-x-20}\times\frac{x-2}{x^2-5x+6}$

(b) $\left(\frac{x}{x-y}-\frac{x}{x+y}\right)\div\left(\frac{y}{x-y}-\frac{y}{x+y}\right)+\left(\frac{x+y}{x-y}+\frac{x-y}{x+y}\right)\div\left(\frac{x+y}{x-y}-\frac{x-y}{x+y}\right)$

(c) $\frac{x^2+2x-3}{x^2+x-2}\div\frac{x^2+x-6}{x^2-4}$

(d) $\frac{a^4-b^4}{a^2+b^2-2ab}\times\frac{(a+b)^2-4ab}{a^3-b^3}\div\frac{a+b}{a^2+ab+b^2}$

17. $\frac{a^4-b^4}{a^2-2ab+b^2}$, $\frac{a-b}{a^3+b^3}$, $\frac{a+b}{a^3+b^3}$ are three algebraic expressions.

A. Express the first expression into the lowest form.

B. Show that, the product of the three expressions is $\frac{a^2+b^2}{(a^2-ab+b^2)}$

C. Divide the first expression by $\frac{a^3+a^2b+ab^2+b^3}{(a+b)^2-4ab}$. Add $\frac{a^2}{a+b}$ to the quotient you get.

18. $A = x^2-5x+6$, $B = x^2-7x+12$, $C = x^2-9x+20$ are three algebraic expressions.

A. Find out the difference between $\frac{x}{y}$ and $\frac{x+y}{y}$.

B. Express $\frac{1}{B}+\frac{1}{C}$ into the lowest form.

C. Turn $\frac{1}{A}$, $\frac{1}{B}$, $\frac{1}{C}$ into the fractions with a common denominator.

19. $A = x-2$, $B = x^2+2x+4$, $C = x^3-8$ are three algebraic expressions.

A. Find out the sum of $\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}+\frac{a-b}{ac}$

B. Simplify : $\frac{1}{A}\times\frac{x-2}{B}+\frac{6x}{C}$

C. Prove that, $\frac{1}{A}\times\frac{x+2}{B}\div\frac{x+2}{C}=1$.

20. $A=\frac{x^2+3x-4}{x^2+7x+12}$, $B=\frac{x^2+2x-3}{x^2+6x-7}$, $C=\frac{x^2+12x+35}{x^2+4x-5}$ are three algebraic expressions.

A. Turn the expression A into the lowest form.

B. Simplify A + B

C. Show that, $B\times C\div\frac{x^2-9}{x-1}=\frac{1}{x-3}$.

Chapter Six
Simple Simultaneous Equations

The role of equations for solving mathematical problems is very important. In class VI and VII, we have learnt how to form and solve simple equations in a single variable, and equations of real problems related to this. In class VII, we have learnt the laws of transposition, cancellation, cross multiplication and symmetric properties of equations. Besides, we have learnt how to solve equations with the help of graph. In this chapter, various methods of solving simple simultaneous equations, both algebraic and graphical, have been discussed in detail.

At the end of this chapter, the students will be able to –

- Explain the method of substitution and elimination.
- Solve simple simultaneous equations with two variables.
- Form and solve simple simultaneous equations of mathematical problems.
- Show the solution of simple simultaneous equation in graph.
- Solve simple simultaneous equations with the help of graph.

6.1 Simple Simultaneous equations

$x+y=5$ is an equation. Here x and y are two unknown expressions or variables. The variables are of single power. This is an example of simple equation. Here, the pair of numbers whose sum is equal to 5, will satisfy the equation. For example, the equation will be satisfied by an infinite number of such pairs of numbers $x=4, y=1$; or, $x=3$, $y=2$; or, $x=2, y=3$; or, $x=1$, $y=4$ etc. Again, if we consider the equation $x-y=3$, we see that the equation is satisfied by the infinite number of the following pairs of numbers $x=4$, $y=1$; or, $x=5, y=2$, or, $x=6$, $y=3$, or, $x=7$, $y=4$, or, $x=8$, $y=5$, or, $x=2$, $y=-1$, or, $x=1$, $y=-2$, $x=0$, $y=-3$ etc.

Here, if we consider the equations $x+y=5$ and $x-y=3$, both equations are simultaneously satisfied by $x=4, y=1$. If two or more equations are connected by the same set of variables, the equations are called simultaneous equations, and if each of the variables is of one dimension, they are known as simple simultaneous equations.

The values of the variables by which equations are satisfied simultaneously, are called the roots or solution of simultaneous equations. Here the equations x + y = 5 and x – y = 3 are simultaneous equations. Their only solution is x = 4, y = 1 which can be expressed by (x, y) = (4, 1).

6.2 Method of solution of simple simultaneous equations of two variables.

Here, the following two methods of the solution of simple simultaneous equations of two variables have been discussed.

(1) Method of substitution

(2) Method of elimination

Method of substitution

Applying this method, we can solve the equations by following the steps below:

(a) From any equation express one variable in terms of the other variable.

(b) Substitute the value of the obtained variable in the other equation and solve the equation in one variable.

(c) Put the obtained value in any of the given equations to find the value of other variable.

Example 1. Solve the equations

$$x + y = 7$$
$$x - y = 3$$

Solution : Given equations are

$$x + y = 7 \ldots\ldots\ldots\ldots(1)$$
$$x - y = 3 \ldots\ldots\ldots\ldots(2)$$

In equation (2) expressing x in terms if y we get

$$x = y + 3 \ldots\ldots\ldots\ldots(3)$$

From (3) putting the value of x in (1) we get , $y + 3 + y = 7$

or, $2y = 7 - 3$

or, $2y = 4$

or, $y = 7 - 3$

$\therefore\ y = 2$

Here, putting $y=2$ in equation (3) we get ,

$$x=2+3$$
$$\therefore\ x=5$$

$\therefore$ Required solution is $(x, y) = (5, 2)$.

[**Verification** : If we put $x=5$ and $y=2$ in both the equations, L.H.S of (1) is $5+2=7$ = R.H.S and L.H.S of (2) $5-2=3$ = R.H.S.]

Example 2. Solve the equations

$$x+2y=9$$
$$2x-y=3$$

Solution : The given equations are

$$x+2y=9\text{...........}(1)$$
$$2x-y=3\text{............}(2)$$

From (2) we get $y = 2x - 3$.........(3)

Putting the value of y in equation (1) we get, $x + 2(2x - 3) = 9$

or, $x + 4x - 6 = 9$

or, $5x = 6 + 9$

or, $5x = 15$

or, $x = \dfrac{15}{5}$

$\therefore\ x = 3$

Now putting the value of x in (3) we get,

$$y=2\times3-3$$
$$=6-3$$
$$=3$$

$\therefore$ Required solution : $(x, y) = (3, 3)$.

Example 3 . Solve the equations

$$2y+5z=16$$
$$y-2z=-1$$

Solution : The given equations are

$$2y+5z=16 \text{.......... } (1)$$
$$y-2z=-1 \text{............} (2)$$

From (2) we get, $y = 2z - 1$ .... (3)

Putting the value of y in equation (1) we get, $2(2z - 1) + 5z = 16$

or, $4z - 2 + 5z = 16$

or, $9z = 16 + 2$

or, $9z = 18$

or $z = \dfrac{18}{9}$

$\therefore\ z = 2$

Here, putting the value of z in (3) we get

$y = 2\times 2-1$

$= 4-1$

$\therefore\ y=3$

$\therefore$ Required solution is $(y, z) = (3, 2)$.

Example 4. Solve the equations

$$\frac{2}{x} + \frac{1}{y} = 1$$
$$\frac{4}{x} - \frac{9}{y} = -1$$

Solution : The given equations are

$$\frac{2}{x} + \frac{1}{y} = 1 \text{............................ } (1)$$
$$\frac{4}{x} - \frac{9}{y} = -1 \text{ .... } (2)$$

Putting $\dfrac{1}{x} = u$ and $\dfrac{1}{y} = v$, we get

from (1) and (2)

$$2u + v = 1 \text{................................ } (3)$$
$$4u - 9v = -1 \text{............................ } (4)$$

from (3), we get

$$v = 1 - 2u \text{................................ } (5)$$

Now, we get from (4) and (5)

$$4u - 9\,(1-2u) = -\,1$$
or, $4u - 9 + 18u = -\,1$
or, $22u = 9 - 1$
$\therefore\ u = \dfrac{8}{22} = \dfrac{4}{11}$
or, $\dfrac{1}{x} = \dfrac{4}{11}\quad \therefore\ x = \dfrac{11}{4}\quad (\therefore\ \dfrac{1}{x} = u)$

Putting the value of u in (5), we get

$$v = 1 - 2 \cdot \frac{4}{11}$$
$$= \frac{11-8}{11}$$
$$v = \frac{3}{11}$$
or, $\dfrac{1}{y} = \dfrac{3}{11}$
$\therefore\ y = \dfrac{11}{3}$
$\because\ [\dfrac{1}{y} = v]$

$\therefore$ Required solution is $(x, y) = (\frac{11}{4}, \frac{11}{3})$

(2) Method of Elimination

Applying by this method, we can solve the equations by following the steps below :

(a) Multiply both the equations by two such numbers separately so that the coefficients of one variable become equal.

(b) If the coefficients of a variable are of the same or opposite sign, subtract or add the equations. The equation after subtraction (or addition) will be reduced to an equation of one variable.

(c) Find the value of a variable by the method of solution of simple equation.

(d) Put the obtained value of the variable in any one of the given equations and find the value of the other variable.

Example 5. Solve the equations

$$5x-4y=6$$
$$x+2y=4$$

Solution : The given equations are

$$5x-4y=6 \text{..............}(1)$$
$$x+2y=4 \text{...............}(2)$$

Here, multiplying equation (1) by 1 and equation (2) by 2, we get

$$5x-4y=6 \text{................}(3)$$
$$2x+4y=8 \text{...............}(4)$$

Adding (3) and (4) we get,

$$7x=14$$
$$\text{or, } x=\frac{14}{7} \text{...............}(4)$$
$$\therefore x=2$$

Putting the value of x in equation (2), we get

$$2+2y=4$$
$$\text{or, } 2y=4-2$$
$$\text{or } y=\frac{2}{2}$$
$$\therefore y=1$$

$\therefore$ Required solution is $(x, y)=(2, 1)$.

Example 6. Solve the equations

$$x+4y=14$$
$$7x-3y=5$$

Solution : The given equations are

$$x+4y=14 \text{...............}(1)$$
$$7x-3y=5 \text{...............}(2)$$

Now, multiplying equation (1) by 3 and equation (1) by 4, we get

$$3x+12y=42 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$
$$28x-12y=20 \ldots\ldots\ldots\ldots\ldots (4)$$

$$31x=62 \text{ (Adding)}$$

or, $x=\frac{62}{31}$

$\therefore x=2$

Now, putting the value of x in equation (1), we get

$$2+4y=14$$

or, $4y = 14 - 2$

or, $4y = 12$

or, $y = \frac{12}{4}$

$\therefore y = 3$.

$\therefore$ Required solution is $(x, y) = (2, 3)$.

Example 7. Solve the following equations :

$$5x-3y=9$$
$$3x-5y=-1$$

Solution : The given equations are

$$5x-3y=9 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$3x-5y=-1 \ldots\ldots\ldots\ldots\ldots (2)$$

Multiplying equation (1) by 5 and equation (2) by 3 we get

$$25x-15y=45 \ldots\ldots\ldots\ldots\ldots\ldots (3)$$
$$9x-15y=-3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$
$$(-) \quad (+) \quad\quad (+)$$

$$16x=48 \quad \text{[by subtracting]}$$

or, $x=\frac{48}{16}$

$\therefore x=3$

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Putting the value of x in equation (1) we get

$$5 \times 3 - 3y = 9$$

or, $15 - 3y = 9$

or, $-3y = 9 - 15$

or, $-3y = -6$

or, $y = \dfrac{-6}{-3}$

$\therefore$ $y = 2.$

$\therefore$ Required solution is $(x, y) = (3, 2)$.

Example 8. Solve the equations

$$\frac{x}{5} + \frac{3}{y} = 3$$

$$\frac{x}{2} - \frac{6}{y} = 2$$

Solution : The given equations are

$$\frac{x}{5} + \frac{3}{y} = 3 \text{.............................} (1)$$

$$\frac{x}{2} - \frac{6}{y} = 2 \text{ ....} (2)$$

Multiplying equation (1) by 2 and then adding with equation (2), we get

$$\frac{2x}{5} + \cancel{\frac{6}{y}} = 6 \text{...........................} (3)$$

$$\frac{x}{2} - \cancel{\frac{6}{y}} = 2 \text{...........................} (4)$$

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or, $\dfrac{2x}{5} + \dfrac{x}{2} = 8$

or, $\dfrac{4x + 5x}{10} = 8$

or, $\dfrac{9x}{10} = 8$

$\therefore$ $x = \dfrac{80}{9}$

Putting the value of $x = \frac{80}{9}$ in equation (1), we get

$$\frac{1}{5}\cdot\frac{80}{9} + \frac{3}{y} = 3$$

or, $\frac{11}{9} + \frac{3}{y} = 3$

$\therefore$ $\frac{3}{y} = 3 - \frac{16}{9}$

or, $\frac{3}{y} = \frac{27-16}{9}$

or, $\frac{3}{y} = \frac{11}{9}$

$\therefore$ $\frac{1}{y} = \frac{11}{27}$

$\therefore$ $y = \frac{27}{11}$

$\therefore$ Required solution is $(x, y) = (\frac{80}{9}, \frac{27}{11})$

Exercise 6.1

(a) Solve the following by using the method of substitution (1-12) :

1. $x + y = 4$
   $x - y = 2$

2. $2x + y = 5$
   $x - y = 1$

3. $3x + 2y = 10$
   $x - y = 0$

4. $\frac{x}{a} + \frac{y}{b} = \frac{1}{a} + \frac{1}{b}$
   $\frac{x}{a} - \frac{y}{b} = \frac{1}{a} - \frac{1}{b}$

5. $3x - 2y = 0$
   $17x - 7y = 13$

6. $x - y = 2a$
   $ax - by = a^2 + b^2$

7. $ax + by = ab$
   $bx + ay = ab$

8. $ax - by = ab$
   $bx - ay = ab$

9. $ax - by = a - b$
   $ax + by = a + b$

10. $\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$
    $\frac{1}{x} - \frac{1}{y} = \frac{1}{6}$

11. $\frac{x}{a} + \frac{y}{b} = \frac{2}{a} + \frac{1}{b}$
    $\frac{x}{b} - \frac{y}{a} = \frac{2}{b} - \frac{1}{a}$

12. $\frac{a}{x} + \frac{b}{y} = \frac{a}{2} + \frac{b}{3}$
    $\frac{a}{x} - \frac{b}{y} = \frac{a}{2} - \frac{b}{3}$

(b) Solve the following by using the method of elimination (13-26):

13. $x - y = 4$
$x + y = 6$

14. $2x + 3y = 7$
$6x - 7y = 5$

15. $4x + 3y = 15$
$5x + 4y = 19$

16. $3x - 2y = 5$
$2x + 3y = 12$

17. $4x - 3y = -1$
$3x - 2y = 0$

18. $3x - 5y = -9$
$5x - 3y = 1$

19. $\frac{x}{2} + \frac{y}{2} = 3$
$\frac{x}{2} - \frac{y}{2} = 1$

20. $x + ay = b$
$ax - by = c$

21. $\frac{x}{2} + \frac{y}{3} = 3$
$x - \frac{y}{3} = 3$

22. $\frac{x}{3} - \frac{2}{y} = 1$
$\frac{x}{4} + \frac{3}{y} = 3$

23. $\frac{x}{a} + \frac{y}{b} = \frac{2}{a} + \frac{1}{b}$
$\frac{x}{b} - \frac{y}{a} = \frac{2}{b} - \frac{1}{a}$

24. $\frac{a}{x} + \frac{b}{y} = \frac{a}{2} + \frac{b}{3}$
$\frac{a}{x} - \frac{b}{y} = \frac{a}{2} - \frac{b}{3}$

25. $\frac{x}{6} + \frac{2}{y} = 2$
$\frac{x}{4} - \frac{1}{y} = 1$

26. $x + y = a - b$
$ax - by = a^2 + b^2$

6·3 Formation and solution to simultaneous equations of real life problems

From the conception of simple simultaneous equations, we can solve many problems of real life. We use more than one variable in many problems. We form the equations using a separate symbol for each variable. In this case, the number of symbols used to form the equations is equal to the number of variables. Then, by solving the equations simultaneously, we can determine the values of variables.

Remark : If the graphs of given simultaneous equations are parallel, there is no solution.

Example 1. If the sum and difference of two numbers are 60 and 20 respectively, find both the numbers.

Solution : Let the two numbers be x and y. Where x>y.

According to the 1st condition, $x+y=60$...............(1)

According to the 2nd condition , $x-y=20$...........(2)

Adding equations (1) and (2), we get

$$2x=80$$

$$\text{or } x=\frac{80}{2}=40$$

Again, subtracting equation (2) from equation (1) , we get

$$2y=40$$

$$\therefore y=\frac{40}{2}=20$$

Required two numbers are 40 and 20.

Example 2. Faiaj and Ayaj had some jujubes (apple kul). If Faiaj gives 10 jujubes to Ayaj, the number of jujubes of Ayaj would be thrice the number of those of Faiaj. And if Ayaj gives 20 jujubes to Faiaj, the number of jujubes of Faiaj would be double the number of those of Ayaj. How many jujubes did they have each ?

Solution : Let, the number of jujube of Faiaj be x and
the number of jujube of Ayaj be y.

According to the 1st condition, $y+10=3(x-10)$

or, $y+10=3x-30$

or, $3x-y=10+30$

or, $3x-y=40$............(1)

According to the 2nd condition, $x+20=2(y-20)$

or, $x+20=2y-40$

or, $x-2y=-40-20$

or, $x-2y=-60$............(2)

Multiplying equation (1) by 2 and then subtracting equation (2) from it, we get

$$5x=140$$

$$\therefore x=\frac{140}{5}=28$$

Putting the value of x in equation (1), we get

$$3\times 28 - y = 40$$
$$\text{or, } -y = 40 - 84$$
$$\text{or, } -y = -44$$
$$\therefore\ y = 44$$

$\therefore$ The number of jujube of Faiaj is 28 and the number of jujube of Ayaj is 44.

Example 3. 10 years ago the ratio of ages of father and son was 4 : 1. After 10 years the ratio of father and son will be 2 : 1. Find the present age of father and son individually.

Solution : Let, at present father's age is x year and son's age is y year.

According to the 1st condition, $(x-10):(y-10) = 4:1$

$$\text{or, } \frac{x-10}{y-10} = \frac{4}{1}$$
$$\text{or, } x - 10 = 4y - 40$$
$$\text{or, } x - 4y = 10 - 40$$
$$\therefore x - 4y = -30 \ldots\ldots\ldots(i)$$

According to the 2nd condition, $(x+10):(y+10) = 2:1$

$$\text{or, } \frac{x+10}{y+10} = \frac{2}{1}$$
$$\text{or, } x + 10 = 2y + 20$$
$$\text{or } x - 2y = 20 - 10$$
$$\therefore x - 2y = 10 \ldots\ldots\ldots(ii)$$

From equation (1) and (2) we get

$$\begin{array}{r} x - 4y = -30 \\ x - 2y = \ \ 10 \\ - \quad + \qquad - \\ \hline -2y = -40 \end{array} \quad \text{[by subtracting]}$$

$$\therefore y = \frac{-40}{-2} = 20$$

Putting the value of y in (2), we get,

$$x - 2\times 20 = 10$$
$$\text{or } x = 10 + 40$$
$$\therefore x = 50$$

$\therefore$ At present father's age is 50 years and son's age is 20 years.

Example 4. If 7 is added with the sum of digits of a number of two digits, the summation will be thrice the digit of ten place. But, if we subtract 18 from the number, the digits change their position. Find the number.

Solution : Let x and y be the digits of ones and tens place of the two digit number respectively.

$\therefore$ The number is $x+10y$.

According to the 1st condition, $x+y+7=3y$

or, $x+y-3y=-7$

or, $x-2y=-7$.........(1)

According to the 2nd condition, $x+10y-18=y+10x$

or, $x+10y-y-10x=18$

or, $9y-9x=18$

or, $9(y-x)=18$

or, $y-x=\frac{18}{9}=2$

$\therefore$ $y-x=2$.........(2)

Adding (1) and (2), we get, $-y=-5$

$\therefore$ $y=5$

Putting the value of y in equation (1), we get

$x-2\times5=-7$

$\therefore$ $x=3$

Required number is $3+10\times5=3+50=53$

Example 5. If 7 is added with the numerator of a fraction, the fraction will be 2 and if we subtract 2 from the denominator, the fraction will be 1. Find the fraction.

Solution : Let the fraction be $\frac{x}{y}$, $y\neq0$.

According to the 1st condition, $\frac{x+7}{y}=2$

$x+7=2y$

$x-2y=-7$..........(1)

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According to the 2nd condition, $\dfrac{x}{y-2}=1$

$$x=y-2$$
$$x-y=-2\text{...........}(2)$$

From the equations (1) and (2) we get,

$$\begin{array}{r} x-2y=-7 \\ x-\ \ y=-2 \\ -\ \ \ +\ \ \ \ \ \ + \\ \hline -y=-5 \end{array} \quad \text{[by subtracting]}$$

$$\therefore y=5$$

Putting $y=5$ in equation (2) we get

$$x-5=-2$$
$$\therefore\ x=5-2=3$$

Required fraction is $\frac{3}{5}$.

6.4 Graphical solution of simple simultaneous equations

There are two equations in simple simultaneous equations with two variables. By drawing the graphs of two simple equations, we get two straight lines. The point of intersection of these lines lies on both the straight lines. The co-ordinates (x, y) of this point of intersection will be the solution of the given simple simultaneous equations. The two equations are satisfied simultaneously by the obtained values of x and y. Therefore, only solution to a pair of simple simultaneous equations is the abscissa and the ordinate of the point of intersection.

Remark : If the graphs of given simultaneous equations are parallel there is no solution.

Example 6 Solve with the help of graphs :

$$x+y=7\text{.................}(i)$$
$$x-y=1\text{..................}(ii)$$

Solution : From the given equation (i) we get,

$$y=7-x\text{...............}(iii)$$

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We construct the table below by finding values of y for different values of x :

x	-2	-1	0	1	2	3	4
y	9	8	7	6	5	4	3

Table - 1

Again, from equation (ii) we get,

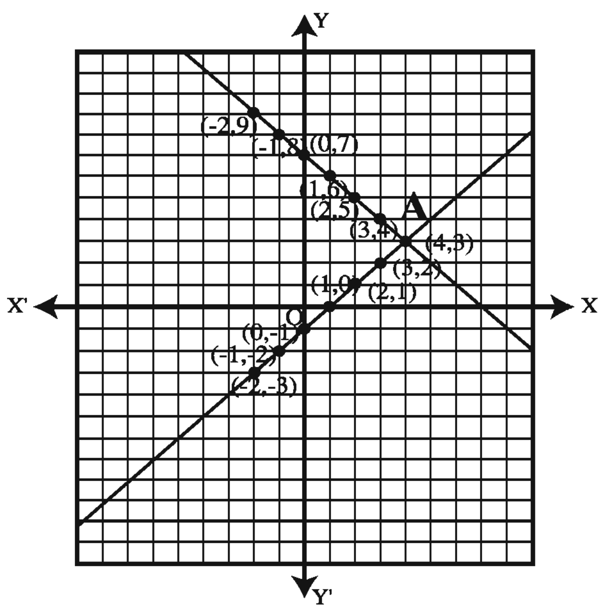
$$y = x - 1 \ldots\ldots\ldots\ldots\ldots(iv)$$

We construct the table below by finding values of y for different values of x :

x	-2	-1	0	1	2	3	4
y	-3	-2	-1	0	1	2	3

Table - 2

Let XOX' and YOY' are x-axis and y-axis respectively and O is the origin. Let the length of a side of the smallest square of both axes be considered a unit. We put the points of table-1 (-2, 9), (-1, 8), (0, 7), (1, 6), (2, 5), (3, 4) and (4, 3) on the graph paper. Joining the points and extending the line in both directions, we get the graph of the straight line represented by the equation (i).



Graph

Again, we put the points table-2 (–2,–3), (–1,–2), (0,–1), (1, 0), (2, 1), (3, 2) and (4, 3), on the graph paper. Joining the points, we get the graph of the straight line which represents the equation (ii). This straight line intersects the previous one at the point A. A is the common point of both the straight lines. So, both equations are satisfied by co-ordinates of A. From graph, we see that the abscissa of A is 4 and the ordinate is 3.

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So, the required solution is $(x, y) = (4, 3)$

Example 7. Solve with the help of graphs :

$$3x + 4y = 10 \ldots\ldots\ldots\ldots(i)$$
$$x - y = 1 \ldots\ldots\ldots\ldots\ldots(ii)$$

From equation (i) we get,

$$4y = 10 - 3x$$
$$y = \frac{10 - 3x}{4}$$

We construct the table below from the values of y for different values of x:

x	-2	0	2	4	6
y	4	$\frac{5}{2}$	1	$\frac{-1}{2}$	-2

Table - 1

From equation (ii) we get

$$y = x - 1$$

We again construct the table below from the values of y for the different values of x:

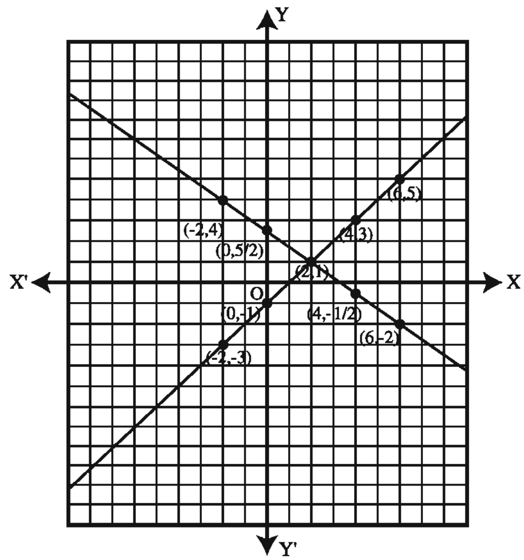
x	-2	0	2	4	6
y	-3	-1	1	3	5

Table - 2

Let XOX' and YOY' be x-axis and y-axis respectively with O as the origin. Let the length of a side of the smallest square of both axes be chosen as a unit. We put the points of table-1 (-2, 4), $(0, \frac{5}{2})$, (2, 1), $(4, -\frac{1}{2})$ and (6, –2) on the graph paper. Adding the points and extending the line in both directions, we get the graph of the equation (i).

Again, we put the points of table-2 (0, –1), (2, 1), (4, 3) and (6, 5) on the graph paper. Joining the points, we get the graph of the straight line which represents the equation (ii).

This straight line intersects the previous one at the point A. A is the common point of both the straight lines. Both the equations are satisfied by coordinates of A. From the graph, we see that the abscissa of A is 2 and the ordinate of A is 1. Therefore, the required solution is $(x, y) = (2, 1)$.



Graph

Exercise 6·2

1. Given that $x+y = 5$, $x-y = 3$

   Which one of the following will be the value of (x, y)?

   A. (4, 1) B. (1, 4) C. (2, 3) D. (3, 2)

2. Which one of the following does not denote the equation for the straight line?

   A. $3x-3y = 0$ B. $x+y = 5$ C. $x = \frac{1}{y}$ D. $4x+5y = 9$

3. What would be the value of x in the system of equations $x-2y = 8$ and $3x-2y = 4$?

   A. -5 B. -2 C. 2 D. 5

4. How many variables are there in the equation- $4x+5y = 9$?

   A. 0 B. 1 C. 2 D. 3

5. Which one is the co-ordinate of the main point?

A. (0, 0) B. (0, 1) C. (1, 0) D. (1, 1)

6. In which quadrant the point $(-3, -5)$ will be?

A. First B. Second C. Third D. Fourth

7. The points on the graphs of the equation- $x+2y = 30$

i. (10, 10)

ii. (0, 15)

iii. (10, 20)

Which one of the following is correct?

A. i and ii B. i and iii C. ii and iii D. i, ii and iii

Answer to the questions 8 and 9 on the basis of the following statement: Half of the difference between the numbers x and y is 4.How many times of smaller number will be added to get the sum 20. Here, $x>y$.

8. Which one is the first condition?

A. $x-y=4$ B. $x-y=8$ C. $y-x=4$ D. $y-x=8$

9. Which one is the value of (x, y)?

A. (3, 11) B. (7, 3) C. (11, 7) D. (11, 3)

10. If the sum and the difference of two numbers are 100 and 20 respectively, find the two numbers.

11. If the sum of two numbers is 160 and one number is thrice the other, find the two numbers.

12. Of two numbers the sum of thrice of the first and double of the second is 59. Again, the difference of the second number from double of the first is 9. Find the two numbers.

13. 5 years ago the ratio of the ages of father and son was 3 : 1 and after 15 years that will be 2 : 1. Find the present age of father and son.

14. If 5 is added to the numerator of a fraction, it will be 2. Again, if 1 is subtracted from the denominator, it will be 1. Find the fraction.

15. If the sum and the difference of the numerator and the denominator of a proper fraction are 14 and 8 respectively, find the fraction.

16. If the sum and the difference of two digits of a two digit number are 10 and 4 respectively, find the number.

17. The length of a rectangle is 25 metre more than the breadth. If the perimeter of the rectangle is 150 metre, find the length and the breadth of the rectangle.

18. A boy bought 15 notebooks and 10 pencils at Tk. 300. Again, another boy bought same type of 10 note-books and 15 pencils at Tk. 250. Find the price of each notebook and each pencil.

19. A person has Tk. 5,000. He divides that amount between two persons in such a way that the first person's share is 4 times than the second one. Again, if the first person gives Tk. 1,500 to the second person, both the amounts become equal. Find the amount of each person.

20. Solve with the help of graphs :

a. $x+y=6$
$x-y=2$

b. $x+4y=11$
$4x-y=10$

c. $3x+2y=21$
$2x-3y=1$

d. $x+2y=1$
$x-y=7$

e. $x-y=0$
$x+2y=-15$

f. $4x+3y=11$
$3x-4y=2$

21. If 11 is added to the numerator of a fraction, the value of the fraction becomes 2. Again, if 2 is subtracted from that fraction, the value of the fraction becomes 1.

    A. Form the system of equation considering the fraction $\frac{x}{y}$.

    B. Find out the value of (x, y) by the method of elimination.

    C. Draw the graph and find out the abscissa and the ordinate of the intersect point.

22. The length of a rectangular garden is 5m more than 2 times of its breadth and the peremeter is 40m.

    A. Form two equations by considering its length x m and breadth y m in the light of the information above.

    B. Solve the equation by elimination method.

    C. Solve the system of equations with the help of the graph.

23. $7x-3y = 31$ and $9x-5y = 41$ are two equations.

    A. Which equation is satisfied by the co-ordinate $(4,-1)$.

    B. Find out (x, y) by elimination method.

    C. Solve the equation with the help of the graph.

Chapter Seven
Set

The word "Set" is familiar to us. For example: tea set, sofa set, dinner set, set of books etc. German mathematician George Cantor (1845-1918) explained the idea about set. His explanation of set is known as Set Theory in mathematics. Starting from the elementary concepts of set through symbols and figures we need to acquire knowledge about set. In this chapter, different types of sets, operations of sets and properties of sets have been discussed.

At the end of the chapter, the students will be able to–

- Explain and form sets.
- Explain finite set, universal set, complementary set, empty set and express the formation of these sets by symbols.
- Explain the formation of union and intersections of sets.
- Verify and prove simple properties of set operations by Venn diagram and examples.
- Solve problems by applying the properties of set.

7·1 Set

A well-defined collection of objects of real or imaginative world is called set. Examples of sets of well-defined objects are: the first five English alphabet, the countries of Asia, natural numbers etc. It is to be determined particularly which object is included in the considered set and which is not. There is no repetition and order of the objects in set.

Each object of a set is called an element of the set. Set is generally denoted by capital letters of English alphabet as A, B, C,.........X, Y, Zand the elements are expressed in small letters as a, b, c,..........x, y, z.

The set is expressed by the symbol { } which includes the elements of the set. For example : the set of a, b, c is { a, b, c}. The set of the Tista, the Meghna, the Jamuna and the Brahmaputra rivers is {Tista, Megna, Jamuna, Brahmaputra}. The set of the first two even natural numbers is { 2, 4} ; the set of the factors of 6 is {1, 2, 3, 6} etc.

Let x be an element of set A. Mathematically, it is expressed by $x \in A$. $x \in A$ is read x is an the element of the set A, (*x belongs to A)*. For example, if $B = \{m, n\}$, then $m \in B$ and $n \in B$.

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Example 1. If the set of the first five odd numbers is A, $A = \{1, 3, 5, 7, 9\}$

Activity :
1. Write a set of SAARC countries.
2. Write a set of prime numbers from 1 to 20.
3. Write a set of any four numbers between 300 and 400 which are divisible by 3.

7·2 Methods of expressing set

Set can be expressed mainly in two methods : (1) Tabular Method (2) Set Builder Method.

(1) Tabular Method : In this method, all the elements of a set are mentioned particularly by enclosing them in second brackets { } and if there is more than one element, the elements are separated by using comma(,). For example :

A= {1, 2, 3}, B = {x, y, z}, C = {100}, D = {Rose, Tube-rose}, E = {Rahim, Sumon, Suvro, Changpai} etc.

(2) Set Builder Method : In this method, conditions are given to determine the elements of sets without mentioning them particularly. For example, if the set of natural even numbers which are smaller than 10 is A, A= {x: x natural even number, x < 10}. Here":" means "such that" or in brief "such". In set builder method, one unknown quantity or variable is placed before ":" sign inside { } and then required conditions are applied to the variable. For example, let us express the set {3, 6, 9, 12} in set builder method. Observe that 3, 6, 9, 12 are natural numbers which are divisible by 3 and which are not greater than 12. In this case, if the element of the set is considered to be the variable y. the conditions that will be applied are : y is a natural number, multiple of 3 and not more than 12 ($y \leq 12$).

Therefore, in set builder method, it will be { y : y is a natural number, multiple of 3 and $y \leq 12$ } .

Example 2. Express the set $P = \{4, 8, 12, 16, 20\}$ in set builder method.

Solution : The elements of the set P are 4, 8, 12, 16, 20.

Here, each element is an even number, multiple of 4 and not greater than 20.

$\therefore$ $P = \{x : x$ is x even number, multiple of 4 and $x \leq 20\}$

Example 3. Express the set $Q = \{x : x$ are all the factors of $42\}$ in Tabular method.

Solution : Q is the set of factors of 42

Here, $42 = 1 \times 42 = 2 \times 21 = 3 \times 14 = 6 \times 7$

$\therefore$ The factors of 42 are $1, 2, 3, 6, 7, 14, 21, 42$.

Required set $Q = \{1, 2, 3, 6, 7, 14, 21, 42\}$

> **Activity :**
> 1. Express the set $A = \{3, 6, 9, 12, 15, 18\}$ in set builder method.
> 2. Express the set $B = \{x : x$ is a factor of $24\}$ in tabular method.

7·3 Classification of Sets

Finite Set :

If the number of elements of a set can be determined by counting, it is called a finite set. For example: $A = \{a, b, c, d\}$, $B = \{5, 10, 15, ..., 100\}$ etc. are finite sets. Here, there are 4 elements in set A and 20 elements in set B.

Infinite Set :

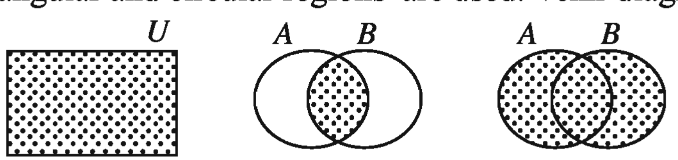
The set whose number of elements can not be determined by counting is called an infinite set. One example of infinite set is, a set of natural numbers, $N = \{1, 2, 3, 4........\}$. Here, the number of elements of set N is innumberable, which can not be determined. In this chapter, only the finite sets will be discussed.

Empty Set :

The set which has no element is called an empty set. An empty set is expressed by the symbol Ø.

7·4 Venn-diagram

John Venn (1834-1883) introduced the method of expressing sets by diagrams. These diagrams are named after Venn and called Venn diagram. Generally, in Venn-diagram, rectangular and circular regions are used. Venn-diagrams are shown below :



By using Venn-diagrams, the properties of sets and operations on sets can be easily determined.

7.5 Subset

Let, $A=\{a,b\}$ is a set. With the elements of the set A, we can form the sets $\{a,b\},\{a\},\{b\}$. The sets $\{a,b\},\{a\},\{b\}$ are the subsets of A.

As many sets are formed from the elements of a set, each of those sets is a subset of the given set.

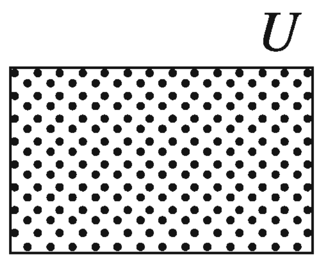
For example : if $P=\{2,3,4,5\}$ and $Q=\{3,5\}$, the set Q is the subset of P. That means, $Q \subseteq P$ becau se the elements 3, 5, of set Q are also included in set P. Subsets are initiated by using '$\subseteq$' symbol.

Example 4 : Write the subsets of the set $A=\{1,2,3\}$.

Solution : The subsets of the set A are shown below :

$\{1,2,3\},\{1,2\},\{1,3\},\{2,3\},\{1\},\{2\},\{3\}$, Ø.

Universal Set :



If all the sets in the discussion are the subsets of a particular set, that particular set is called the universal set. *A* universal set is expressed by the symbol *U*. For example, in a school, the set of all the students is a universal set and the set of the students of class eight is the subset of the universal set.

All sets are subsets of the universal set in a given context.

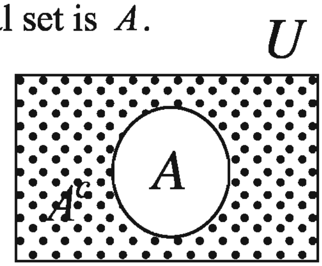
Example 5. If $A=\{1,2,3,4,5,6\}, B=\{1,3,5\}, C=\{3,4,5,6\}$ determine the universal set.

Solution : Given, $A=\{1,2,3,4,5,6\}, B=\{1,3,5\}, C=\{3,4,5,6\}$.

Here, the elements of set B are 1, 3, 5 and the elements of C are 3,4, 5 that are included in set A.

$\therefore$ in respect of these two sets B and C, the universal set is A.

Complement of a Set :



If U is an universal set and set A is the subset of U, then the set of all elements that are excluded from set A, is called the complement set of set A. The complement set of A is denoted by A^c or A'.

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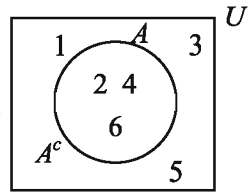
Suppose, in class eight, out of 60 students, 9 students are absent. If the set of all the students of class eight is considered as a universal set, the set of (60 - 9) or 51 present students will be the complement set of the set of those 9 absent students.

Example 6. If $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{2, 4, 6\}$, determine A^c.

Solution : Given, $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{2, 4, 6\}$.



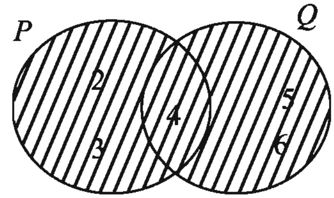
$\therefore$ A^c = The complement set of A
= The set of the elements excluding the elements of A
= $\{1, 3, 5\}$

Required set $A^c = \{1, 3, 5\}$

Activity : If $A = \{a, b, c\}$, find the subsets of A and find the complementary set of any three of them.

7.6 Set operations

Union of sets



$P \cup Q$

Let P=$\{2, 3, 4\}$ and $\{Q = 4, 5, 6\}$ be two sets. Here, all the elements included in sets P and Q are $2, 3, 4, 5, 6$. The set formed by all the elements of sets P and Q is $\{2, 3, 4, 5, 6\}$.

The set formed by all the elements of two or more sets is called a union set. Let, A and B be two sets. The set formed by all the elements of the sets A and B is expressed as $A \cup B$ and read as 'A union B'.
In set builder method, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

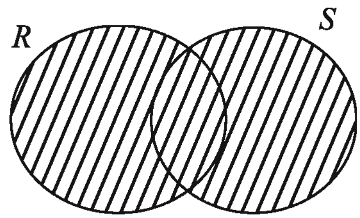
Example 7. C = {Razzaq, Sakib, Alok} and D = {Alok, Mushfiq}, then find $C \cup D$.

Solution : Given that, C = {Razzaq, Sakib, Alok} and D = {Alok, Moshfiq}
$\therefore$ $C \cup D$ = {Razzaq, Sakib, Alok} $\cup$ {Alok, Mushfiq}
= {Razzaq, Sakib, Alok, Mushfiq}

Example 8. $R = \{x : x \text{ is factor of } 6\}$ and $S = \{x : x \text{ is a factor of } 8\}$, then find $R \cup S$.

Solution : Given that, $R = \{x : x \text{ is a factor of } 6\}$
$= \{1, 2, 3, 6\}$
and $S = \{x : x \text{ is a factor of } 8\}$
$= \{1, 2, 4, 8\}$

$\therefore\ R \cup S = \{1, 2, 3, 6\} \cup \{1, 2, 4, 8\}$
$= \{1, 2, 3, 4, 6, 8\}$



$R \cup S$

Intersection of sets

Let, Rina can read and write both Bangla and Arabic and Joya can read and write both Bangla and Hindi. The set of the languages that Rina can read and write is {Bangla, Arabic}, the set of the languages that Joya can read and write is {Bangla, Hindi}. We observe, that the language that both Rina and Joya can read and write is Bangla and the set of it is {Bangla}. Here, the set {Bangla} is the intersection of the two sets.

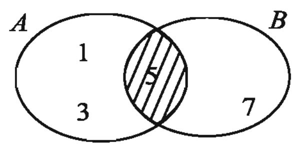
The set formed by the common elements of two or more sets is called intersection of sets.

Let, A and B are two sets. The intersection of the sets A and B is denoted by $A \cap B$ and read 'A intersection B'. In the set builder notation the set is $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Example 9. If $A = \{1, 3, 5\}$ and $B = \{5, 7\}$, find $A \cap B$.

Solution : Given that, $A = \{1, 3, 5\}$ and $B = \{5, 7\}$

$\therefore\ A \cap B = \{1, 3, 5\} \cap \{5, 7\} = \{5\}$



$A \cap B$

Example 10. If $P = \{x : x$ is the multiple of 2 and $x \leq 8\}$ and $Q = \{x : x$ is the multiple of 4 and $x \leq 12\}$, find $P \cap Q$.

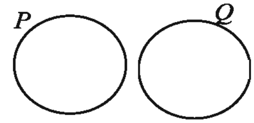
Solution : Given that, $P = \{x : x \text{ is the multiple of 2 and } x \leq 8\}$
$= \{2, 4, 6, 8\}$
and $Q = \{x : x \text{ is the multiple of } 4,\ x \leq 12\}$
$= \{4, 8, 12\}$

$\therefore\ p \cap Q = \{2, 4, 6, 8\} \cap \{4, 8, 12\} = \{4, 8\}$

> **Activity :** $U = \{1, 2, 3, 4\}, A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{1, 3\}$
> show the sets $U \cap A,\ C \cap A$ and $B \cup C$ in Venn diagram.

Disjoint Set

Let, there are two villages side-by-side in Bangladesh. The farmers of one village grow paddy and jute and the farmers of the other village grow potato and vegetables in their fields. If we consider two sets of the cultivated crops, we get {paddy, jute} and {potato, vegetables}.



There is no common crop between the two sets. That means, the farmers of two villages do not grow the same crops. Here, the two sets are disjoint sets to each other.

If there is no common element between the elements of two sets, the sets are called disjoint sets.

Let, , A and B are two sets, A and B will be disjoint sets to each other if $A \cap B = \emptyset$.

If the intersection of two sets is an empty set, they are disjoint to each other.

Example 11. $A = \{x : x$ is odd natural number and $1 < x < 7\}$ and $B = \{x : x$ is a factor of $8\}$, then show that the sets A and B are disjoint.

Solution : Given that, $A = \{x : x$ is odd natural number and $1 < x < 7\}$

$$= \{3, 5\}$$

and $B = \{x, x$ is the factor of $8\}$

$$= \{1, 2, 4, 8\}$$

$$\therefore \; A \cap B = \{3, 5\} \cap \{1, 2, 4, 8\}$$

$$= \emptyset$$

Hence, A and B are disjoint to each other.

Example 12. If $C = \{3, 4, 5\}$ and $D = \{4, 5, 6\}$, find $C \cup D$ and $C \cap D$.

Solution : Given that, $C = \{3, 4, 5\}$ and $D = \{4, 5, 6\}$

$\therefore \; C \cup D = \{3, 4, 5\} \cup \{4, 5, 6\} = \{3, 4, 5, 6\}$

and $C \cap D = \{3, 4, 5\} \cap \{4, 5, 6\} = \{4, 5\}$

Activity :
If $P = \{2, 3, 4, 5, 6, 7\}$ and $Q = \{4, 6, 8\}$
1. Find $P \cup Q$ and $P \cap Q$.
2. Express $P \cup Q$ and $P \cap Q$ in set builder form.

Example 13. Express the set $E = \{x : x \text{ is a prime number and } x < 30\}$ in tabular method.

Solution : The required set will be the set of prime numbers less than 30.
Here, the prime numbers less than 30 are $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$

$\therefore$ Required set is $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$.

Example 14. If A and B are sets of all factors of 42 and 70 respectively, find $A \cap B$.

Solution :

Here $42 = 1 \times 42 = 2 \times 21 = 3 \times 14 = 6 \times 7$

Factors of 42 are $1, 2, 3, 6, 7, 14, 21, 42$

$\therefore \; A = \{1, 2, 3, 6, 7, 14, 21, 42\}$

Again, $70 = 1 \times 70 = 2 \times 35 = 5 \times 14 = 7 \times 10$

Factors of 70 are $1, 2, 5, 7, 10, 14, 35, 70$

$\therefore \; B = \{1, 2, 5, 7, 10, 14, 35, 70\}$

$\therefore \; A \cap B = \{1, 2, 7, 14\}$

Exercise 7

1. How many systems are there to express set?
   A. 1 B. 2 C. 3 D. 4
2. Which one of the following is the subset of any set?
   A. { 0 } B. { ∅ } C. ∅ D. (∅)
3. How many elements are there in the set { 0 }?
   A. 0 B. 1 C. 2 D. 3

4. S = { x:x even number and $1 \leq x \leq 7$ } which one of the following is correct in Tabular set system?

A. {2, 3, 4} B. {2, 4, 6} C. {1, 3, 5} D. {3, 5, 7}

5. Express the following sets in tabular form:

(a) $\{x : x$ is odd number and $3 < x < 15\}$

(b) $\{x : x$ is a prime factor of $48\}$

(c) $\{x : x$ is a multiple of 3 and $x < 36\}$

(d) $\{x : x$ is an integer and $x^2 < 10\}$

6. Express the following sets in set builder form.

(a) $\{3, 4, 5, 6, 7, 8\}$ (b) $\{4, 8, 12, 16, 20, 24\}$ (c) $\{7, 11, 13, 17\}$

7. Find the subsets and the number of subsets of the following two sets :

(a) $C = \{m, n\}$ (b) $D = \{5, 10, 15\}$

8. If A = {2, 3, 4} and B = {5, 7}, which one will be $A \cap B$?

A. φ B. { 0 } C. {5, 7} D. {2, 3, 4, 5, 7}

9. Which one is the tabular form of $A = \{x : x$ is even number and $4 < x < 6\}$:

(a) $\{5\}$ (b) $\{4, 6\}$ (c) $\{4, 5, 6\}$ (d) Ø

10. If $P = \{x, y, z\}$, which one of the following is not subset of P ?

(a) $\{x, y\}$ (b) $\{x, w, z\}$ (c) $\{x, y, z\}$ (d) Ø

11. What is the set of factors of 10 ?

(a) $\{1, 2, 5, 10\}$ (b) $\{1, 10\}$ (c) $\{10\}$ (d) $\{10, 20, 30\}$

12. If $A=\{1,2,3\}, B=\{2,a\}$ and $C=\{a,b\}$, find the following sets :

(a) $A \cup B$ (b) $B \cap C$

(c) $A \cap (B \cup C)$ (d) $(A \cup B) \cup C$

(e) $(A \cap B) \cup (B \cap C)$

13. If $U=\{1,2,3,4,5,6,7\}$, $A=\{1,2,5\}$, $B=\{2,4,7\}$ and $C=\{4,5,6\}$ justify the correctness of the following relations :

(a) $A \cap B = B \cap A$

(b) $(A \cap B)' = A' \cup B'$

(c) $(A \cup C)' = A' \cap C'$

14. If P and Q are the sets of all the factors of 21 and 35 respectively, find $P \cup Q$.

15. If, A = {2, 3, 5}

i. A = {X∈N : 1 <x<6 and x is a prime number}

ii. A = {X∈N : 2 ≤ x <7 and x is a prime number}

iii. A = {X∈N : 2 ≤ x ≤5 and x is a prime number}

Which one of the following is correct?

A. i and ii B. i and iii C. ii and iii D. i, ii and iii

Answer the questions 16 and 17 in light of the information below :

U {2, 3, 5, 7}, A = {2, 5}, B = {3, 5, 7}

16. Which one is A^C?

A. {2, 5} B. {3, 5} C. {3, 7} D. {2, 7}

17. Which one is $A \cap B^C$?

A. { 2 } B. { 5 } C. {2, 5} D. 3, 7}

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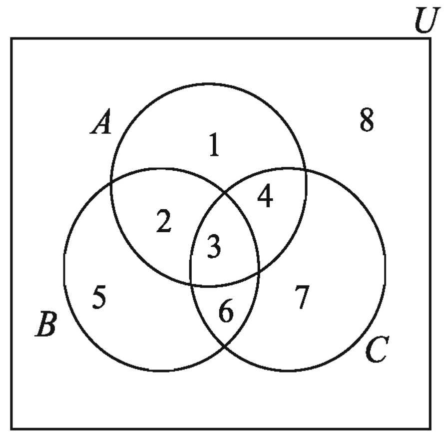
Answer the questions from 18 to 21 in respect of the adjoining Venn diagram.



18. Which one is universal set ?

(a) A (b) B (c) C (d) U

19. Which one is the set B^c ?

(a) $\{5, 6, 7, 8\}$ (b) $\{2, 3, 5, 6\}$ (c) $\{1, 4, 7, 8\}$ (d) $\{3, 6\}$

20. Which one is the set $A \cap B$?

(a) $\{2, 3\}$ (b) $\{2, 3, 5, 6\}$ (c) $\{3, 4, 6, 7\}$ (d) $\{2, 3, 4, 5, 6, 7\}$

21. Which one is the set $A \cup B$?

(a) $\{1, 2, 3, 4, 5, 6\}$ (b) $\{5, 6, 7\}$ (c) $\{8\}$ (d) $\{3\}$

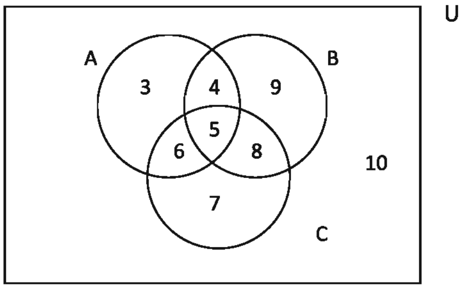
22. In a hostel, 65% of the students like fish, 55% of the students like meat and 40% of the students like both.

(a) Express the stated information by Venn diagram with short explanation.

(b) Find out the number of students who dislike both dishes.

(c) Find out the intersection set of the sets of factors of those students, who like only one dish.

23.



Figure

A. Write Set A in Set Builder Method.

B. Express A, B and C in Tabular method and find $A \cap C$ and $A \cup B$.

C. Prove that $(A \cup B)' = A' \cap B'$.

24. Universal set U = {1, 2, 3, 4, 5, 6, 7} and its three subsets are-

$A = \{X \in N : x<7 \text{ and } x \text{ is an odd number}\}$

$B = \{X \in N : x<7 \text{ and } x \text{ is an even number}\}$

$C = \{X \in N : x \leq 3 \text{ and } x \text{ is a prime number}\}$

A. Express sets A and B in set Builder form.

B. Find out $(A \cup B) \cap (A \cup C)$.

C. Write the subsets of $(B \cup C)'$

25. The sets of the integers by which the numbers 346 and 556 are divided with remainder 31 in each case are A and B.

A. Express set A in Set Builders Form.

B. Find $A \cap B$.

C. Show $A \cap B$ in Venn-diagram and write the subsets of $A \cap B$.

Chapter Eight
Quadrilaterals

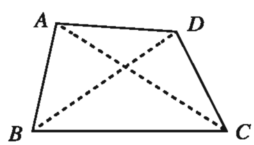
In previous classes, we have discussed triangles and quadrilaterals. In constructing a particular triangle, we have seen that three measures are required. So, naturally the question arises whether four measures are enough to draw a quadrilateral. In this chapter, we shall discuss this matter. Besides, different types of quadrilaterals, such as parallelograms, rectangles, squares and rhombuses have various properties. The properties of these quadrilaterals and their constructions have also been discussed in this chapter.

At the end of the chapter, the students will be able to -

- Verify and logically prove the properties of quadrilaterals.
- Construct quadrilaterals from given data.
- Find the area of quadrilaterals by the formula of triangles.
- Construct rectangular solids.
- Find the area of surfaces of cubic and rectangular solids.

8.1 Quadrilaterals

A quadrilateral is a closed figure bounded by four line segments. The closed region is also known as quadrilateral. The quadrilateral has four sides. The four line segments by which the region is bounded, are the sides of the quadrilateral.



In the figure any three points of the points A, B, C, D are not collinear. The quadrilateral $ABCD$ is constructed by the four line segments AB, BC, CD and DA. The points A, B, C and D are the vertices of the quadrilateral. $\angle ABC$, $\angle BCD$, $\angle CDA$ and $\angle DAB$ are the four angles of the quadrilateral. The vertices A and B are the opposite vertices of C and D respectively. The pairs of sides AB and CD, AD and BC are sides opposite to each other. The two sides that meet at a vertex are the adjacent sides. For example, the sides, AB and BC are the adjacent sides. The line segments AC and BD are the diagonals of $ABCD$. The sum of the lengths of sides is the perimeter of the quadrilateral. The perimeter of the quadrilateral $ABCD$ is equal to the length of $(AB+BC+CD+DA)$. We often denote quadrilaterals by the symbol '□'.

8.2 Types of Quadrilaterals

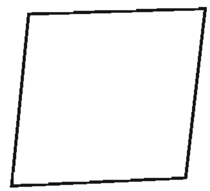
Parallelogram:A parallelogram is a quadrilateral with opposite sides parallel. The region bounded by a parallelogram is also known as parallelogram.

Parallelogram

Rectangle:A rectangle is a parallelogram with a right angle. The region bounded by a rectangle is a rectangular region.

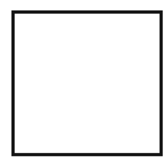
Rectangle

Rhombus:A rhombus is a parallelogram with equal adjacent sides, i.e., the opposite sides of a rhombus are parallel and the lengths of four sides are equal. The region bounded by a rhombus is also called rhombus.



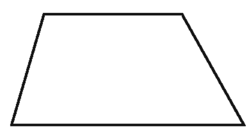
Rhombus

Square: A square is a rectangle with equal adjacent sides, i.e., a square is a parallelogram with all sides equal and all right angles. The area bounded by square is also called a square.



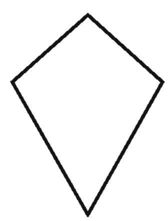
Square

Trapezium: A trapezium is a quadrilateral with a pair of parallel sides.The region bounded by trapezium is also called a trapezium.



Trapezium

Kite: A kite is a quadrilateral with exactly two distinct consecutive pairs of sides of equal lengths.



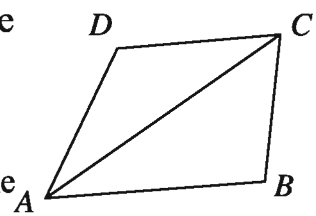
Kite

Activity:
1. Identify parallelograms, rectangles, squares and rhombuses from objects of your day to day life.
2. State whether True or False :
(a) A square is a rhombus and also a rectangle.
(b) A trapezium is a parallelogram.
(c) A parallelogram is a trapezium.
(d) A rectangle or a rhombus is not a square.
3. A square is defined as a rectangle with equal sides. Can you define a square by using a rhombus?

8.3 Theorems related to Quadrilaterals

Different types of quadrilaterals have some common properties. These properties are proved as theorems.

Theorem 1
The sum of four angles of a quadrilateral equals to four right angles.



Proposition: Let $ABCD$ be a quadrilateral and AC be one of its diagonals. It is to be proved that $\angle A + \angle B + \angle C + \angle D = 4$ right angles.

Construction: Join A and C. The diagonal AC divides the quadrilateral into two triangles ΔABC and ΔADC.

Proof:

Steps	Justification
(1) In ΔABC $\angle BAC + \angle ACB + \angle B = 2$ right angles.	[The sum of three angles of a triangle is 2 right angles]
(2) Similarly, in ΔDAC $\angle DAC + \angle ACD + \angle D = 2$ right angles.	[sum of three angles of a triangle is 2 right angles]
(3) Therefore, $\angle DAC + \angle ACD + \angle D + \angle BAC + \angle ACB + \angle B = (2+2)$ right angles.	[From (1) and (2)]
(4) $\angle DAC + \angle BAC = \angle A$ and $\angle ACD + \angle ACB = \angle C$ Therefore, $\angle A + \angle B + \angle C + \angle D = 4$ right angles. (Proved)	[The sum of adjacent angles] [The sum of adjacent angles] [from (3)]

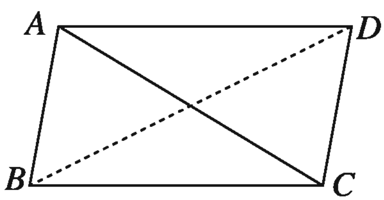
Theorem 2

The opposite sides and angles of a parallelogram are equal.

Proposition: Let $ABCD$ be a parallelogram and AC and BD be its two diagonals. It is required to prove that

(a) $AB = CD$ and $AD = BC$

(b) $\angle BAD = \angle BCD, \angle ABC = \angle ADC$.

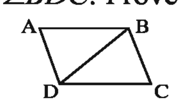


Proof:

Steps	**Justification**
(1) $AB \parallel DC$ and AC is their transversal, therefore, $\angle BAC = \angle ACD$.	[alternate angles are equal]
(2) Again, $BC \parallel AD$ and AC is their transversal, therefore, $\angle ACB = \angle DAC$.	[alternate angles are equal]
(3) Now in ΔABC and ΔADC, $\angle BAC = \angle ACD$, $\angle ACB = \angle DAC$ and AC is common. $\therefore \Delta ABC \cong \Delta ADC$. Therefore, $AB = CD, BC = AD$ and $\angle ABC = \angle ADC$. Similarly it can be proved that $\Delta ABD \cong \Delta BDC$ Therefore, $\angle BAD = \angle BCD$ [Proved].	[ASA theorem]

Activity

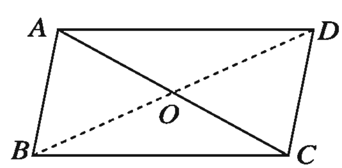
1. Prove that if a pair of opposite sides of a quadrilateral is parallel and equal, it is a parallelogram.
2. Given that in the quadrilateral $ABCD$, $AB = CD$ and $\angle ABD = \angle BDC$. Prove that $ABCD$ is a parallelogram.



Theorem 3

The diagonals of a parallelogram bisect each other.

Proposition: Let the diagonals AC and BD of the parallelogram $ABCD$ intersect at O. It is required to prove that $AO = CO$, $BO = DO$.



Proof:

Steps	**Justification**
(1) The lines AB and DC are parallel and AC is their transversal. Therefore, $\angle BAC$ = alternate $\angle ACD$.	[Alternate angles are equal]
(2) The lines BC and AD are parallel and BD is their transversal Therefore, $\angle BDC$ = alternate $\angle ABD$.	[Alternate angles are equal]
(3) Now, between ΔAOB and ΔCOD $\angle OAB = \angle OCD$, $\angle OBA = \angle ODC$ and $AB = DC$.	$\because$ $\angle$BAC = $\angle$ ACD and $\angle$BDC = $\angle$ ABD
So, $\Delta AOB \cong \Delta COD$.	[ASA theorem]
Therefore, $AO = CO$ and $BO = DO$. (Proved)	

Activity

1. Prove that, if the diagonals of a quadrilateral bisect each other, it is a parallelogram.

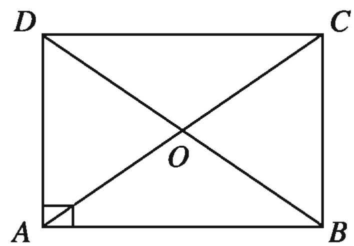
Theorem 4

Two diagonals of a rectangle are equal and bisect each other.

Proposition: : Let the diagonals AC and BD of the rectangle $ABCD$ intersect at O. It is required to prove that,

(i) $AC = BD$

(ii) $AO = CO$, $BO = DO$.



Proof:

Steps	**Justification**
(1) A rectangle is also a parallelogram. Therefore, $AO = CO$, $BO = DO$.	[Diagonals of a parallelogram bisect each other]
(2) Now between ΔABD and ΔACD, $AB = DC$	[Opposite sides of a parallelogram are equal]
and $AD = AD$. $\angle DAB = \angle ADC$	[each angle is a right angle]
Therefore, $\Delta ABD \cong \Delta ACD$.	[ASA theorem]
Therefore, $AC = BD$ (Proved)	

Activity : 1. Prove that every angle of a rectangle is the right angle.

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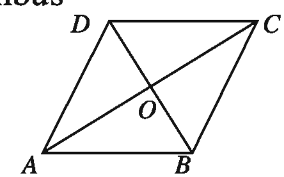
Theorem 5
Two diagonals of a rhombus bisect each other at right angles.

Proposition: Let the diagonals AC and BD of the rhombus $ABCD$ intersect at O. It is required to prove that,
(i) $\angle AOB = \angle BOC = \angle COD = \angle DOA =$ 1 right angle
(ii) $AO = CO,\ BO = DO$.



Proof:

Steps	Justification
(1) A rhombus is a parallelogram. Therefore, $AO = CO,\ BO = DO$.	[Diagonals of a parallelogram bisect each other]
(2) Now, in $\Delta\, AOB$ and ΔBOC, $AB = BC$ $AO = CO$ and $OB = OB$. So $\Delta AOB \simeq \Delta BOC$.	 [sides of a rhombus are equal] [from (1)] [common side] [SSS theorem]

Therefore, $\angle AOB = \angle BOC$.
$\angle AOB + \angle BOC =$ 1 straight angle = 2 right angles.
$\angle AOB = \angle BOC =$1 right angle.
Similarly, it can be proved that, $\angle COD = \angle DOA =$ 1 right angle. (Proved)

Activity
1. Prove that the diagonals of a square are equal and bisect each other.
2. A worker has made a rectangular concrete slab. In how many different ways can he be sure that the slab is really rectangular?

8.4 Area of Quadrilaterals

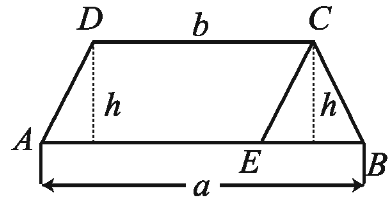
A quadrilateral region is divided into two triangular region by one of its diagonals. So, the area of the quadrilateral region is equal to the sum of area of the triangular regions. In our previous classes, we have learnt how to find areas of square and rectangular regions. We have seen that rectangle and parallelogram with same base and height have equal areas. Here, the methods of finding the area of rhombus and trapezium have been discussed.

(a) The Area of Trapezium Region

$ABCD$ is a trapezium. Where $AB \parallel CD$, $AB = a$, $CD = b$ and perpendicular distance $= h$. Construct $DA \parallel CE$ at C.

$\therefore$ $AECD$ is a parallelogram. From the figure,

area of trapezium = area of parallelogram $AECD$ + area of triangle CEB



$$= b \times h + \frac{1}{2}(a-b) \times h$$

$$= \frac{1}{2}(a+b) \times h$$

Area of trapezium region = average of the sum of two parallel sides × height

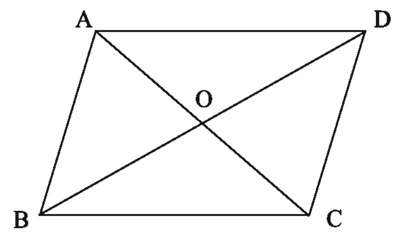
Activity :
1. Find the area of trapezium region by an alternative method.

(b) The Area of Rhombus

The diagonals of a rhombus bisect each other at right angles. If we know the lengths of two diagonals, we can find the area of rhombus.

Let the diagonals AC and BD of a rhombus $ABCD$ intersect each other at O. Denote the lengths of two diagonals by a and b respectively.

Area of rhombus = area of triangle DAC + area of triangle BAC .



$$= \frac{1}{2} \cdot a \times \frac{1}{2} b + \frac{1}{2} a \times \frac{1}{2} b$$

$$= \frac{1}{2} a \times b$$

Area of rhombus = half of the product of two diagonals.

8.5 Solid

Book, Box, Brick, Football etc. are solid bodies. Solid bodies may be the forms of rectangular, square, spherical and also of any other form. A solid body has its length, breadth and height.

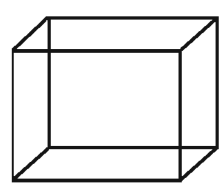


Figure-1

The solid of figure 1 is a rectangular solid. It has six rectangular faces or surfaces each of which is a rectangular region. The two mutual opposite faces are equal and parallel. So, the area of each pair of opposite faces are equal.

The solid of figure 2 is a square solid. It has six mutually equal square faces or surfaces each of which is a square region.

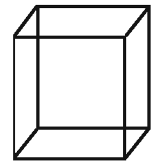
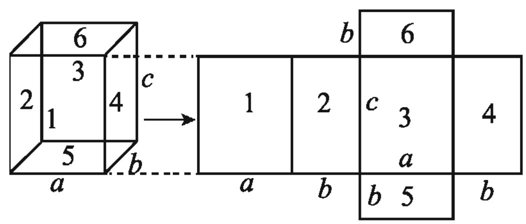


Figure-2

Again, two mutually opposite faces are parallel. The square solid is called cube. The intersecting line segments of each of two faces are called the edges or sides of the cube. All the edges or sides of a cube are equal. So, the area of all faces are equal.

Determination of area of the faces of a solid

(a) Rectangular solid



If the length, breadth and height of a rectangular solid are a unit, b unit and c unit respectively, Then according to the figure, area of the whole face of the solid $= \{(ab + ab) + (bc + bc) + ac + ac)\}$ sq. unit

$$= 2(ab + bc + ac) \text{ sq. unit}$$

(b) Cube

If the side of a cube is a unit, then area of each of the six faces of the cube $= a \times a$ sq. unit $= a^2$ sq. unit.

Therefore, area of the whole faces of the cube $= 6a^2$ sq. unit.

Example : The length, the breadth and the height of a rectangular solid are respectively 7.5 cm, 6 cm and 4 cm. Find the area of the entire faces of the solid.

Solution : We know, if the length, the breadth and the height of a solid are respectively a unit, b unit and c unit, then the area of the entire faces of the solid $= 2(ab + bc + ac)$ sq. unit.

Here, $a = 7.5$ cm, $b = 6$ cm, $c = 4$.

$\therefore$ Area of the entire faces of the given solid.

$= 2\ (7.5 \times 6 \times 6 \times 4 + 7.5 \times 4)$ sq. cm

$= 2(45 + 24 + 30)$ sq. cm

$= 2 \times 99$ sq. cm

$= 198$ sq. cm

Exercise 8.1

1. Which one of the following is true for a parallelogram?

(a) opposite sides are not parallel (b) if one angle is right angle, it is a rectangle

(c) opposite sides are unequal (d) the diagonals are equal

2. Which one of the following is a property of a rhombus?

(a) the diagonals are equal (b) all angles are right angles

(c) opposite angles are unequal (d) all sides are equal

3. *i.* The sum of four angles of a quadrilateral is four right angles.

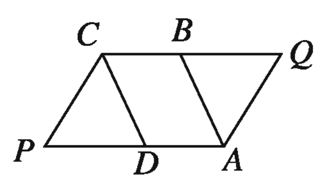
ii If the adjacent sides of a rectangle are equal, it is a square.

iii All rhombuses are parallelograms.

In view of the above information, which one of the following is correct?

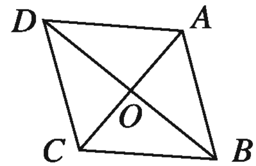
(a) *i* and *ii* (b) *i* and *iii* (c) *ii* and *iii* (d) *i, ii*, and *iii*

4. In the quadrilateral $PAQC$, $PA = CQ$ and $PA \parallel CQ$. If the bisectors of $\angle A$ and $\angle C$ are AB and CD respectively, what is the name of the region $ABCD$?



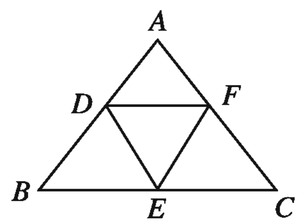
(a) parallelogram (b) rhombus (c) rectangle (d) square

5. The median BO of ΔABC is produced up to D so that $BO = OD$. Prove that, $ABCD$ is a parallelogram.

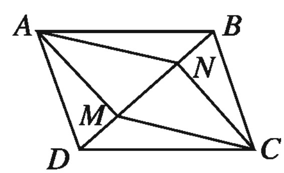


6. Prove that a diagonal of a parallelogram divides it into two congruent triangles.
7. Prove that, if the opposite sides of a quadrilateral are equal and parallel, it is a parallelogram.
8. Prove that, if two diagonals of a parallelogram are equal, it is a rectangle.
9. Prove that, if two diagonals of a quadrilateral are mutually equal and bisect each other at right angles, it is a square.
10. Prove that, the quadrilateral formed by joining the mid-points of adjacent sides of a rectangle is a rhombus.
11. Prove that, the bisectors of any two opposite angles of a parallelogram are parallel to each other.
12. Prove that, the bisectors of any two adjacent angles of a parallelogram are perpendicular to each other.

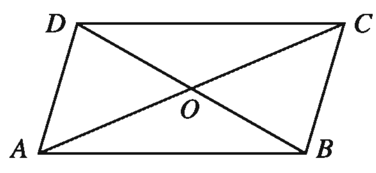
13. In the figure, ABC is an equilateral triangle. D,E and F are the mid-points of AB, BC and AC respectively. a) Prove that, $\angle BDF + \angle DFE + \angle FEB + \angle EBD = 4$ right angles.
(b) Prove that, $DF \parallel BC$ and $DF = \frac{1}{2}BC$.



14. In the parallelogram $ABCD$, AM and CN are both perpendiculars to DB. Prove that $ANCM$ is also a parallelogram.



15. In the figure, $AB=CD$ and $AB \parallel CD$.
   (a) Name the two triangles on base AB·
   (b) Prove that, A and BC are equal and parallel to each other.
   (c) Show that, $OA=OC$ and $OB=OD$.



16. ABCD is a parallelogram. The diagonals AC and BD bisect at O.

A. Find out the value of $\angle ABC$ if $\angle BAD = 70^0$.

B. If AC = BD, Prove that ABCD is a rectangle.

C. If AB = AD, Prove that AC and BD bisect at the point '0' at right angle.

17. The diagonals AC and BD of the quadrilateral BACD are unequal and the sum of any two adjacent angles is two right angles.

A. Define 'Kite' with the figure.

B. Prove that AB = CD and AD = BC.

C. If the perpendiculars BP and PQ are drawn from the points B and D on AC, prove that BPDQ is a parallelogram.

18. The length, the breadth and the height of a rectangular solid are 10 cm, 8 cm and 5 cm respectively. Find the area of the entire faces of the solid.

19. If the edge of a cubic box is 6.5 cm, find the area of the entire faces of the box.

Constructions

8.6 Construction of Quadrilaterals

In previous classes, we have learnt that if three sides are given, a particular triangle can be constructed. But, if the four sides of a quadrilateral are given, it is not possible to construct a particular quadrilateral. For the construction of a quadrilateral more data are required. A quadrilateral has four sides, four angles and two diagonals. Essentially, five unique data are required for constructing a quadrilateral. For example, if the four sides and a particular angle are given, a quadrilateral can be constructed. A particular quadrilateral can be constructed if any one of the following combinations of data is known:

(a) Four sides and an angle

(b) Four sides and a diagonal

(c) Three sides and two diagonals

(d) Three sides and two included angles

(e) Two sides and three angles.

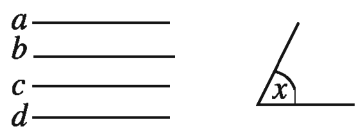
Sometimes special quadrilaterals can be constructed with fewer data. In such cases five data can be retrieved logically.

- A square can be constructed if only one side is given. Here, four sides are equal and an angle is a right angle.
- A rectangle can be constructed if two adjacent sides are given. Here the opposite sides are equal to each other and one angle is a right angle.
- A rhombus can be constructed if a side and an angle are given. Here four sides are equal.
- A parallelogram can be constructed if two adjacent sides and the included angle are given. Here the opposite sides are equal.

Construction 1

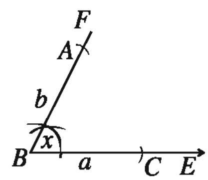
To construct a quadrilateral when four sides and an angle are given.

Let the lengths of four sides of a quadrilateral be a,b,c,d and $\angle x$ be the angle included between a and b. The quadrilateral is to be constructed.

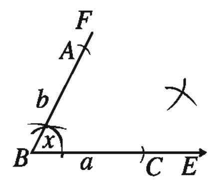


Construction:

(1) From any ray BE, take $BC=a$ and draw $\angle EBF = \angle x$ at B.



(2) From BF, take $BA=b$. With A and C as centre, draw two arcs of radius c and d respectively within the angle $\angle ABC$. The arcs intersect at D.

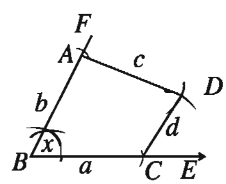


(3) Join A and D, C and D. Then, $ABCD$ is the required quadrilateral.

Proof: According to construction,

$AB = b, BC = a, AD = c, DC = d$ and $\angle ABC = \angle x$.

Therefore, $ABCD$ is the required quadrilateral.

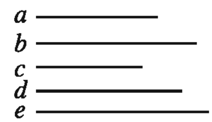


Activity 1. Four sides and an angle are required to construct a quadrilateral. Are you successful in constructing the quadrilateral by five of any measurement? Explain.

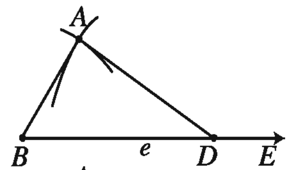
Construction 2

To construct a quadrilateral when four sides and a diagonal are given.

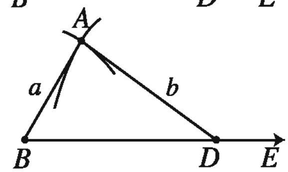
Let the lengths of four sides of a quadrilateral be a,b,c,d and e be the length of a diagonal, where $a+b>e$ and also $c+d>e$. The quadrilateral is to be constructed.



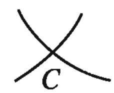
Construction:



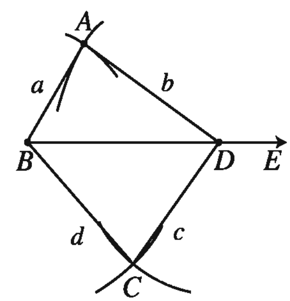
(1) From any ray BE, take $BD=e$. With B and D as centres, draw two arcs of radius a and b respectively on the same side of BD. The arcs intersect at A.



(2) Again, with B and D as centres, draw two arcs of radius d and c respectively on the side of BD opposite to A. The arcs intersect at C.



(3) Join A and B, A and D, B and C, C and D. Then, $ABCD$ is the required quadrilateral.



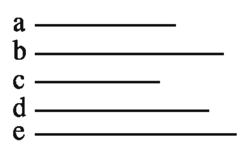
Proof: According to construction,
$AB = a$, $AD = b$, $BC = d$, $CD = e$ and the diagonal $BD = e$..
Therefore, $ABCD$ is the required quadrilateral.

Activity
1. The lengths of four sides and a diagonal are required to construct a quadrilateral. Do you think five of any measurement can construct the quadrilateral? Justify your answer.
2. A student attempted to draw a quadrilateral PLAY where PL = 3 cm, LA = 4 cm, AY = 4.5 cm, PY = 2 cm and LY = 6 cm, but could not draw it. What is the reason?

Construction 3

To construct a quadrilateral when three sides and two diagonals are given.

Let the lengths of three sides of a quadrilateral be a,b,c and d and e be the length of two diagonals respectively, where $a+b>e$. The quadrilateral is to be constructed.



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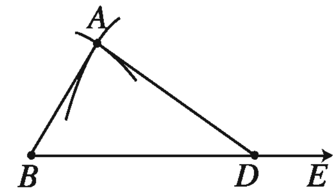
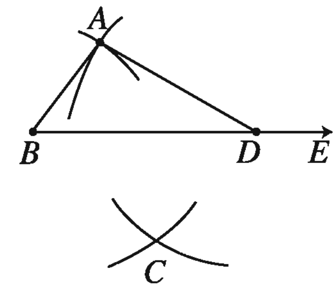
Construction:



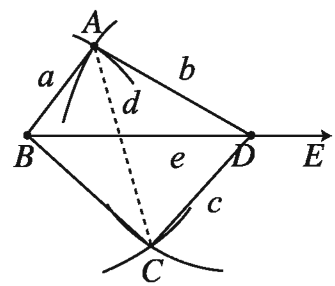
(1) From any ray BE, take $BD=e$. With B and D as centre, draw two arcs of radius a and b respectively on the same side of BD. The arcs intersect at A.



(2) Again, with D and A as centre, draw two arcs of radius c and d respectively on the side of BD opposite to A. The arcs intersect at C.

(3) Join A and B, A and D, B and C, C and D,

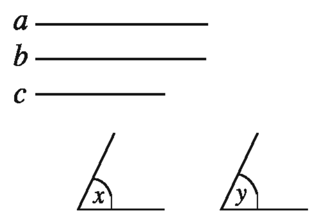
Then, $ABCD$ is the required quadrilateral.



Proof: According to construction,
$AB = a$, $AD = b$, $CD = c$ and the diagonals $BD = e$ and $AC = d$. Therefore, $ABCD$ is the required quadrilateral.

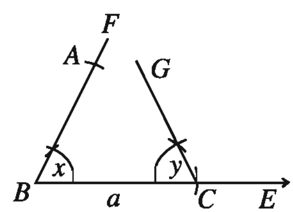
Construction 4

To construct a quadrilateral when three sides and two included angles are given.



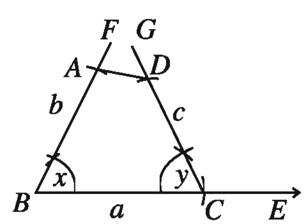
Let the lengths of three sides of a quadrilateral be a,b,c and two included angles of the sides a, b, a, c be $\angle x$ and $\angle y$ respectively. The quadrilateral is to be constructed.

Construction:



From any ray BE, take $BC=a$. Construct two angles $\angle CBF = \angle x$ and $\angle BCG = \angle y$ at B and C respectively. Cut $BA = b$ and $CD = c$ from BF and CG respectively.

Join A and D. Then $ABCD$ is the required quadrilateral.



Proof: According to construction,
$AB = b$, $BC = a$, $CD = c$ and $\angle ABC = \angle x$, $\angle BCD = \angle y$.
Therefore, $ABCD$ is the required quadrilateral.

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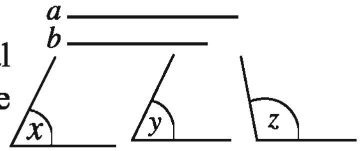
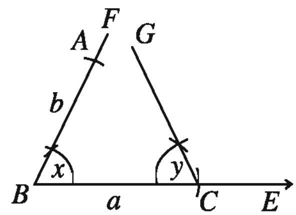
Construction 5

To construct a quadrilateral when two adjacent sides and three angles are given.

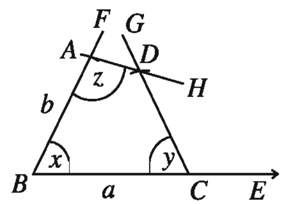


Let a, b be the two adjacent sides of a quadrilateral and three angles be $\angle x$, $\angle y$ and $\angle z$. The quadrilateral is to be constructed.

Construction:
From any ray BE, take $BC=a$. Construct two angles $\angle CBF = \angle x$ and $\angle BCG = \angle y$ at B and C respectively.
Cut $BA=b$ from BF. Construct an angle $\angle BAH = \angle z$ at A. AH and CG intersect at D. Then, $ABCD$ is the required quadrilateral.



Proof: According to construction,
$AB = b$, $BC = a$, $\angle ABC = \angle x$, $\angle DCB = \angle y$ and $\angle BAD = \angle z$.
Therefore, $ABCD$ is the required quadrilateral.

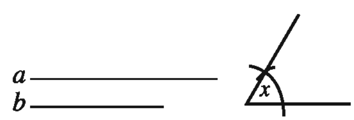


Activity :
1.The lengths of two sides which are not adjacent and three angles are given. Can you construct the quadrilateral?
2. A student attempted to draw a quadrilateral $STOP$ where $ST = 5$ cm, $TO = 4$ cm, $\angle S = 20°$, $\angle T = 30°$, $\angle O = 40°$; but could not draw it. What is the reason?

Construction 6

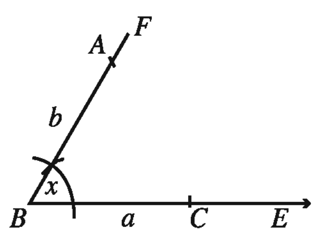
To construct a parallelogram when two adjacent sides and the included angle are given.

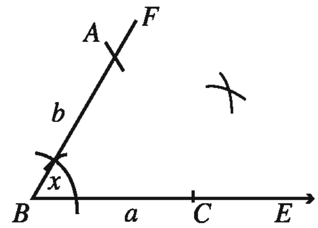
Let a,b be the two adjacent sides of a parallelogram and $\angle x$ be the included angle between them. The parallelogram is to be constructed.



Construction:
From any ray BE, take $BC=a$. Construct an angle $\angle EBF = \angle x$ at B. Take $BA=b$ from BF.

With A and C as centre, draw two arcs of radius a and b respectively within $\angle ABC$. The arcs intersect at D. Join A and D, C and D. Then, $ABCD$ is the required quadrilateral.





Proof: Join A,C. In ΔABC and ΔADC $AB=CD=b$, $AD=BC=a$ and AC is the common side.

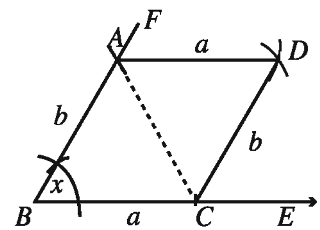
$\therefore \Delta ABC \cong \Delta ADC$.

Therefore, $\angle BAC=\angle DCA$. But, these are alternate angles. $\therefore AB \parallel CD$.

Similarly, it can be proved that, $BC \parallel AD$. Hence, $ABCD$ is a parallelogram.

Again, according to the construction $\angle ABC=\angle x$.

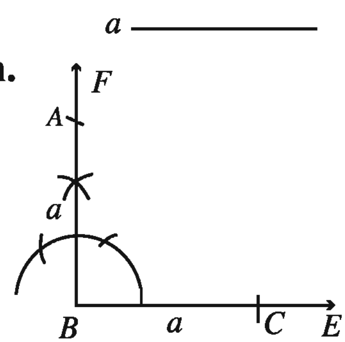
Therefore, $ABCD$ is the required parallelogram.



> **Observe :** A square can be constructed when the length of only one side is given. The sides of a square are equal and the angles are all right angles. So the necessary five conditions can easily be satisfied.

Construction 7

To construct a square when one side is given.

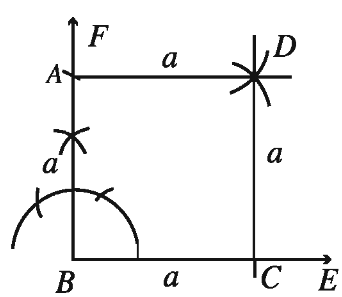


Let a be the length of a side of a square. The square is to be constructed.

Construction:

From any ray BE, take $BC=a$. Construct $BF \perp BC$ at B. Take $BA=a$ from BF.

With A and C as centre, draw two arcs of radius a within the angle $\angle ABC$. The arcs intersect each other at D. Join A and D, C and D. Then, $ABCD$ is the required square.



Proof : In the quadrilateral $AB = BC = CD = DA = a$ and $\angle ABC= 1$ right angle.

So, it is a square.

Therefore, $ABCD$ is the required square.

Exercise 8.2

1. How many independent and unique data are necessary to draw a quadrilateral?
(a) 3 (b) 4 (c) 5 (d) 6

2. In which of the following the diagonals bisect at right angles?
A. Square and Rectangle B. Rhombus and Parallelogram
C. Rectangle and Kite D. Rhombus and Kite.

3. What will be the length of its side if the length of two diagonal of a rhombus are 6c.m. and 4 c.m.?
A. 4.9c.m.(approx.) B. 5 c.m. C. 6.9 c.m(approx.) D. 7 c.m.

4. The peremeter of a Kite is 24 c.m.and the ratio of the unequal sides is 2 : 1. What will be the length of the smallest side in c.m.?
A. 8 B. 6 C. 4 D. 3

5. Distance between the parallel sides of a trapezium is 3 c.m. and its area is 48 sq.c.m.What will be the average length of the two parallel sides in c.m.?
A. 8 B. 16 C. 24 D. 32

6. For all parallelograms
i. opposite sides are equal and parallel.
ii. bisectors of the opposite angles are parallel to each other.
iii. Area = the product of two adjacent sides.
Which one of the following is correct?
A. i and ii B. i and iii C. ii and iii D. i, ii and iii

7. The length of the adjacent sides of a rectangle are 4 c.m. and 3 c.m.
i. Half of the peremeter is 7 c.m.
ii. The length of the diagonal is 5 c.m.
iii. The area is 12 sq.c.m.
Which one of the following is correct?
A. i and ii B. i and iii C. ii and iii D. i, ii and iii

8. i. If two adjacent sides are given, a rectangle can be drawn.
ii. If four angles are given, a quadrilateral can be drawn.
iii. If a side of a square is given, the square can be drawn.
Which one of the following is correct in view of the above information ?
(a) i and ii (b) i and iii (c) ii and iii (d) i, ii and iii

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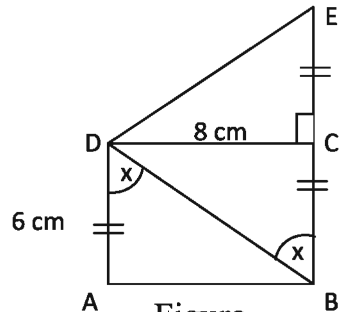
Answer the questions 9, 10, 11 and 12 in the light of the figure below:



Figure

9. What is the length of BD in c.m.?

   A. 7 B. 8 C. 10 D. 12

10. What is the peremeter of the quadrilateral ABED in c.m.?

    A. 24 B. 26 C. 30 D. 36

11. What is the area in sq.c.m. of Δ BDE?

    A. 48 B. 36 C. 28 D. 24

12. How much sq.c.m. will be the area of the quadrilateral ABED?

    A. 48 B. 64 C. 72 D. 96

13. Construct quadrilateral from the following given data :

a) The lengths of four sides are 3 c.m., 3.5 c.m., 2.8 c.m., 3 c.m. and one angle is 45°.

b) The lengths of four sides are 4 c.m., 3 c.m., 3.5 c.m., 4.5 c.m. and one angle is 60°.

c) The lengths of four• sides are 3.2 c.m., 3.5 c.m., 2.5 c.m., 2.8 .cm. and one diagonal is 5c.m.

d) The lengths of four sides are 3.2 c.m., 3 c.m., 3.5 c.m., 2.8c.m. and one diagonal is 5 c.m.

e) The lengths of three sides are 3 c.m., 3.5 c.m., 2.5 c.m. and two angles are 60° and 45°.

f) The lengths of three sides are 3 c.m., 4 c.m., 4.5 c.m. and two diagonals are 5.2 cm and 6 c.m.

14. The length of a side of a square is 4 c.m. Construct the square.

15. The length of a side of a rhombus is 3.5 c.m. and one angle is 75°. Construct the rhombus.

16. The lengths of the adjacent two sides of a rectangle are 3 c.m. and 4 c.m. respectively. Construct the rectangle.

17. Two diagonals AC and BD of a quadrilateral $ABCD$ intersect each other at the point O, in such a way that OA = 4.2 c.m., OB = 5.8 c.m., OC = 3.7 c.m., OD = 4.5 c.m. and $\angle AOB = 100^0$. Construct the quadrilateral $ABCD$.

18. The lengths of two adjacent sides of a rectangle are given. Construct the rectangle.

19. The length of a diagonal and a side are given. Construct the rectangle.

20. The length of one side and two diagonals are given. Construct the parallelogram.

21. The length of one side and a diagonal are given. Construct the rhombus.

22. The lengths of two diagonals are given. Construct the rhombus.

23. Two adjacent sides of a parallelogram are 4 cm and 3cm and their included angle is 60°.

    (a) Express the above information in a figure.

    (b) Draw the parallelogram with the description of drawing.

    (c) Draw a square with a diagonal equal to the larger diagonal of the parallelogram. Give the description of the drawing.

24. Two specific lines are a = 6 cm, b = 4.5 cm and two angles are $\angle x = 75^0$ and $\angle y = 85^0$.

    A. Draw $\angle x$ with pencil and campus.

    B. Consider two lines adjacent sides. Draw a rectangle (Labelling and construction is needed).

    C. Draw a Trapezium considering a and b two parallel sides and two given angles on the side 'a' (Labelling and construction is needed)

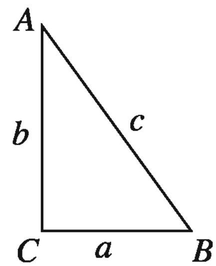
Chapter Nine
Pythagoras Theorem

In 6th century B.C. Greek philosopher Pythagoras discovered a special property of right-angled triangle. This property of right-angled triangle is known as Pythagorean property. It is believed that before the birth of Pythagoras, in Egyptian and Greek era, this special property of right-angled triangle was in use. In this chapter, we shall discuss this property of right-angled triangle. We know that the sides of a right-angled triangle have got special names - the side opposite to right angle as hypotenuse and the sides containing the right angle as base and height. In this chapter, relation among these three sides will be discussed.

At the end of this chapter, the students will be able to –

- Verify and prove Pythagoras theorem.
- Verify whether the triangle is right-angled when the lengths of three sides of a triangle are given.
- Use Pythagoras theorem to solve problems.

9·1 Right angled Triangle



In the figure ABC is a right-angled triangle with $\angle ACB$ as a right angle. Therefore, AB is the hypotenuse of the triangle. In the figure, we denote the sides by a, b, c.

> **Activity :**
> 1. Draw a right angle and locate two points on its two sides at 3 cm and 4 cm apart. Join the two points to draw a right-angled triangle. Measure the length of the hypotenuse. Is the length 5 cm?

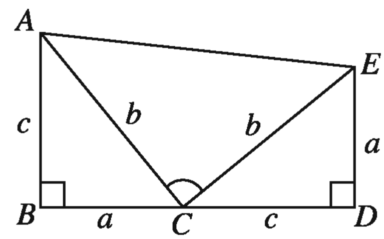
Observe, $3^2 + 4^2 = 5^2$ i.e. the sum of the squares of two sides is equal to the square of the measurement of the hypotenuse. Therefore, for a right-angled triangle with sides a, b and c, $c^2 = a^2 + b^2$. This is the key point of Pythagoras theorem. This theorem has been proved in various methods. A few simple proofs of this theorem are given below.

9·2 Pythagoras Theorem

In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the two other sides.

(Proof with the help of two right angled triangles)

Proposition: Let in the triangle ABC, $\angle B = 90^\circ$, the hypotenuse $AC = b, AB = c$ and $BC = a$. It is required to prove that $AC^2 = AB^2 + BC^2$, i.e. $b^2 = c^2 + a^2$.



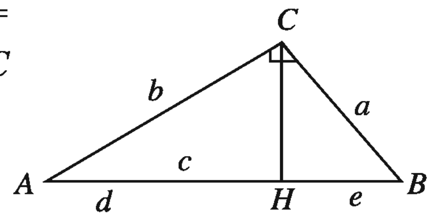
Construction : Produce BC up to D in such a way that $CD = AB = c$. Also, draw perpendicular DE at D on BC produced, so that $DE = BC = a$. Join C, E and A, E.

Proof :

Steps	**Justification**
(1) In ΔABC and ΔCDE , $AB = CD = c, BC = DE = a$ and included $\angle ABC$ = included $\angle CDE$	[each right angle]
Hence, $\Delta ABC \cong \Delta CDE$.	[SAS theorem]
$\therefore$ $AC = CE = b$ and $\angle BAC = \angle ECD$. (2) Again, since $AB \perp BD$ and $ED \perp BD$, $\therefore$ $AB \parallel ED$. Therefore, $ABDE$ is a trapezium.	
(3) Moreover, $\angle ACB + \angle BAC = \angle ACB + \angle ECD = 1$ right angle	$\therefore \angle BAC = \angle ECD$
$\therefore \angle ACE = 1$ right angle and ΔACE is a right-angled triangle	
Now the area of the trapezium $ABDE$ = the area of (Δ region $ABC + \Delta$ region $CDE + \Delta$ region ACE) or, $\frac{1}{2}BD(AB+DE) = \frac{1}{2}ac + \frac{1}{2}ac + \frac{1}{2}b^2$ or, $\frac{1}{2}(BC+CD)(AB+DE) = \frac{1}{2}[2ac+b^2]$ or, $(a+c)(a+c) = 2ac + b^2$ [multiplying by 2] or, $a^2 + 2ac + c^2 = 2ac + b^2$ or, $b^2 = a^2 + c^2$ (Proved)	[Area of trapezium $= \frac{1}{2}$ sum of parallel sides × distance between parallel sides]

Alternative Proof of Pythagoras theorem

(By using similar triangles)



Proposition : Let in the triangle ABC, $\angle C = 90^\circ$ and hypotenuse $AB = c$, $BC = a$ and $AC = b$. It is required to prove that

$AB^2 = AC^2 + BC^2$,

i.e. $c^2=a^2 +b^2$.

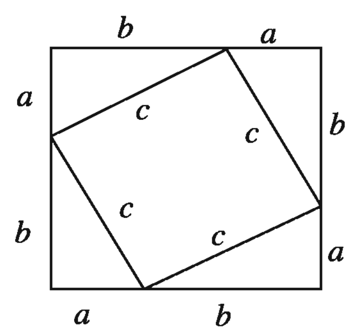
Construction: Draw a perpendicular CH from C on hypotenuse AB. The hypotenuse AB is divided at H into the parts of d and e.

Proof :

Steps	Justification
In ΔBCH and ΔABC, $\angle BHC = \angle ACB$ and $\angle CBH = \angle ABC$ (1) $\therefore$ ΔCBH and ΔABC are similar. $\therefore \frac{BC}{AB} = \frac{BH}{BC}$ $\therefore \frac{a}{c} = \frac{e}{a}$ …… (1)	[(i) Both triangles are right angled (ii) angle $\angle B$ is common]
(2) Similary, ΔACH and ΔABC are similar. $\therefore \frac{b}{c} = \frac{d}{b}$ …… (2)	$\because$ $c = e+d$
(3) From the above two ratios, we get $a^2 = c\times e$, $b^2 = c\times d$ Therefore, $a^2 + b^2 = c\times e + c\times d$ $= c(e+d)$ c × c $= c^2$ $\therefore c^2 = a^2 + b^2$ [Proved]	

Alternative Proof of Pythagoras theorem

(Algebraic proof)



Proposition: Let in the triangle ABC, c is the hypotenuse and a , b be the two other sides. It is required to prove that, $c^2=a^2 +b^2$.

Construction : Draw four triangles congruent to ΔABC as shown in the figure.

Proof :

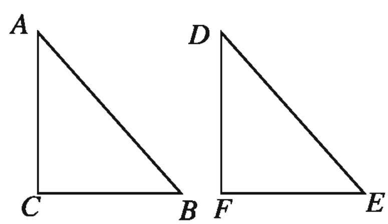
Steps	**Justification**
(1) The larger region is a square with area $(a+b)^2$.	[The length of each of the sides is $a+b$ and the angles are right angles]
(2) The inner smaller quadrilateral is also a square with area c^2.	[The length of each side is c]
(3) From the figure, the area of the larger square is equal to sum of the areas of four triangular regions and the area of smaller square i.e., $(a+b)^2 = 4\times\frac{1}{2}\times a\times b + c^2$ or, $a^2+2ab+b^2 = 2ab+c^2$ or, $c^2 = a^2+b^2$ (Proved)	

Activity :

1. Prove the Pythagoras theorem by using the expansion of $(a-b)^2$.

9·3 Converse of Pythagoras theorem

If the square of a side of any triangle is equal to the sum of the squares of other two sides, the angle between the latter two sides is a right angle.



Proposition: Let in ΔABC, $AB^2 = AC^2 + BC^2$

It is required to prove that $\angle C$ is a right angle.

Construction:

Draw a triangle DEF so that $\angle F = 1$ right angle $EF=BC$ and $DF=AC$.

Proof:

Steps	**Justification**
(1) $DE^2 = EF^2 + DF^2$ $= BC^2 + AC^2 = AB^2$ $\therefore DE = AB$	[Since in $\Delta DEF, \angle F$ is a right angle]
Now, in ΔABC and ΔDEF, $BC = EF$, $AC = DF$ and $AB = DE$.	[supposition]
$\therefore$ $\Delta ABC \cong \Delta DEF$; $\therefore \angle C = \angle F$ $\because \angle F$ =1 right angle. $\therefore \angle C$= 1 right angle [proved]	[SSS theorem]

Exercise 9

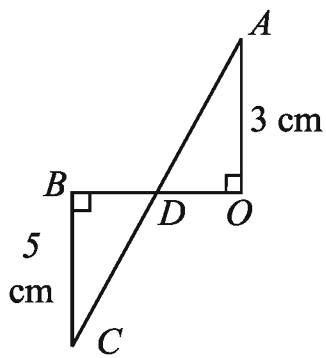
1. ABC is a right angled triangle. AD is the perpendicular to BC. Prove that, $AB^2 + BC^2 + CA^2 = 4AD^2$

2. Two diagonals of the quadrilateral $ABCD$ intersect each other at right-angled. Prove that, $AB^2 + CD^2 = BC^2 + AD^2$

3. In ΔABC, $\angle A$ is a right angle and CD is a median. Prove that, $BC^2 = CD^2 + 3AD^2$

4. In ΔABC, $\angle A$ is a right angle. BP and CQ is are two medians. Prove that, $5BC^2 = 4(BP^2 + CQ^2)$

5. Prove that, the area of square region on the diagonal of square is the double of the area of the square region.

6. 

In figure, if $OB = 4$ cm find the length of BD and AC.

7. Prove that any square region is one half of the square region drawn on its diagonal.

8. In triangle ABC, A= 1 right angle and D is a point on AC. Prove that $BC^2 + AD^2 = BD^2 + AC^2$.

9. In triangle ABC, $\angle A$= 1 right angle. If D and E are respectively the mid points of AB and AC, prove that $DE^2 = CE^2 + BD^2$.

10. In ΔABC, AD is the perpendicular to BC and $AB>AC$. Prove that $AB^2 - AC^2 = BD^2 - CD^2$.

11. In ΔABC, AD is the perpendicular to BC and P is any point on AD and $AB>AC$. Prove that $PB^2 - PC^2 = AB^2 - AC^2$.

12. The ratio of the three sides of a triangle is $1:1:\sqrt{2}$. What is the value of the greatest angle?

A. 80^0 B. 90^0 C. 100^0 D. 120^0

13. For a right angled triangle, the difference between two acute angles is 5^0. What is the value of the smallest one?

A. 40^0 B. 42.5^0 C. 47.5^0 D. 50^0

14. The hypotenuse of a right angled triangle is x unit and one of the other two sides is y unit. What will be the length of the third side?

A. x^2+y^2 B. $\sqrt{x^2+y^2}$ C. $\sqrt{x^2-y^2}$ D. x^2-y^2

15. For which of the following measurements, is it possible to draw a right angled triangle?

A. 4, 4, 5 B. 5, 12, 13 C. 8, 10, 12 D. 2, 3, 4

16. In Δ ABC, $\angle A = 1$ right angle

i. Hypotenuse is BC

ii. Area $= \frac{1}{2}.AB.AC$

iii. $BC^2 = AB^2 + AC^2$

Which one of the following is correct?

A. i & ii B. i & iii C. ii & iii D. i, ii & iii

17. For a right angled triangle-

i. the longest side in hypotenuse.

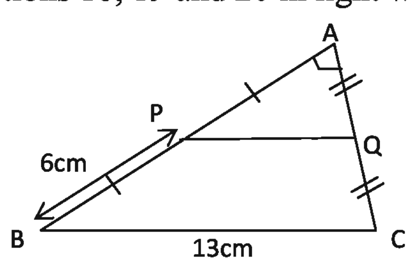
ii. sum of the square of the smaller sides is equal to the square of the longest side.

iii. Acute angles are complementary to each other.

Which one of the following is correct?

A. i and ii B. i and iii C. ii and iii D. i, ii, iii

Answer questions 18, 19 and 20 in light with the following figure :



In the figure, $<A = 90^0$

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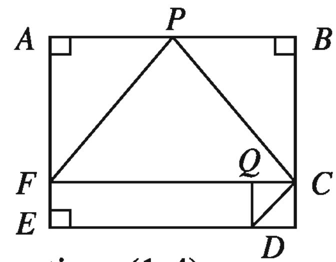
18. What will be the length of PQ in cm?
    A. 6 B. 6.5 C. 7 D. 9.5
19. What will be the area of Δ ABC in sq.cm?
    A. 39 B. 32.5 C. 30 D. 15
20. What will be the peremeter of Δ APQ in cm?
    A. 15 B. 12.5 C. 10 D. 7.5

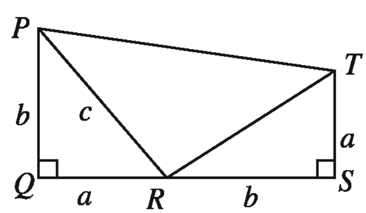
21. In the polygon $ABCDE$, $AE \parallel BC$, $CF \perp AE$ and $DQ \perp CF$. ED= 10 mm, EF= 2 mm. BC= 8 mm, AB=12 mm.



On the basis of the above information, answer the questions (1-4) :

(1) What is the area of the quadrilateral $ABCF$ in square millimetres?
   a. 64 b. 96 c. 100 d. 144
(2) Which one of the following indicates the area of the triangle FPC in sq. m.?
   a. 32 b. 48 c. 72 d. 60
(3) Which one of the following expresses the length of CD in millimetre?
   a. $2\sqrt{2}$ b. 4 c. $4\sqrt{2}$ d. 8
(4) Which one of the following indicates the difference between the areas of ΔFPC and ΔDQC?
   a. 46 sq. unit b. 48 sq. unit c. 50 sq. unit d. 52 sq. unit

22. a. What type of quadrilateral $PQST$ is? Justify your answer.
    b. Show that ΔPRT is a right angled triangle.
    c. Prove that $PR^2 = PQ^2 + QR^2$.



23. For, Δ PQR, $\angle P = 90^0$, Mid-points of PQ and PR are M and N respectively.
    A. Draw the triangle.
    B. Prove from the figure A that $PR^2+PQ^2 = QR^2$.
    C. Prove that, $5RQ^2 = 4\ (RN^2+QM^2)$.

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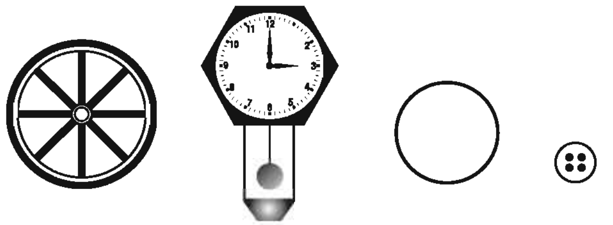
Chapter Ten
Circle

In our day to day life we observe and use some objects which are circular in shape. For example, wheel of a car, bangle, clock, button, plate, coin etc. We notice that the second's hand of watch goes rapidly in a round path. The path traced by the tip of second's hand is a circle. We use circular bodies in a variety of ways.



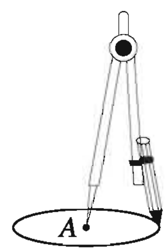
Wheel Clock Bangle Button

At the end of the chapter, the students will be able to–

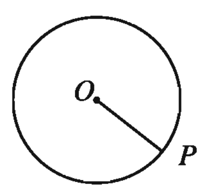
- Develop the concept of circle.
- Explain the concept of Pi (π).
- Find the circumference and the area of a circular region solving related problems.
- Use theorems related to circle to solve problems and using measuring tape measure the circumference and the area of a circular region.
- Measure the area of the outer surfaces of a cylinder with the help of the area a of quadrilateral and a circle.

10.1 Circle

Put a Bangladeshi one taka coin on a piece of paper and press it with the left thumb at the middle. Now, move a pencil around the coin. Remove the coin and notice the closed curved line. The traced line is a circle.



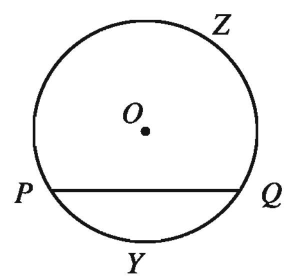
A pencil compass is used to draw a circle precisely. Put its pointed leg on a point on a sheet of paper. Open the other leg to some distance. Keeping the pointed leg fixed, rotate the other leg through one revolution. The closed figure traced by the pencil on paper as shown in the picture is a circle. So, we can draw circle at a fixed distance from a fixed point. The fixed point is called the ***centre*** of the circle and the fixed distance is called the ***radius*** of the circle.



Activity

1. Draw a circle of radius 4 cm with centre at O with the help of pencil compass. Take a few points A, B, C, D on the circle and draw the line segments from O to the points. Measure the lengths of the line segments. What do you see ?

10.2 Chord and Arc of a circle

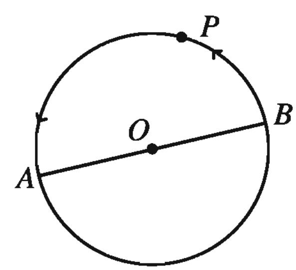


In the adjacent figure, a circle is drawn with the centre at *O*. Taking any two points *P*, *Q* on the circle, draw their joining line segment *PQ*. The line segment *PQ* is called a chord of the circle . The chord divides a circle into two parts.

Taking two points *Y*, *Z* on two sides of the chord and then we get two parts is *PYQ* and *PZQ*. Each part of the circle divided by the chord is called an arc of circle or in brief an arc. In the picture, two arcs, arc *PYQ* and arc *PZQ* are produced by the chord *PQ*.

The joining line segment at any two points of a circle is the chord of the circle. Each chord divides a circle into two arcs.

10.3 Diameter and Circumference



In the adjacent figure, a such chord *AB* of a circle is drawn which passes through the centre at *O*. In that case we call the chord a diameter of the circle. The length of a diameter is also called diameter. The arcs made by the diameter *AB* are equal; they are known as semi-circle. Any chord that passes through the centre is a diameter. The diameter is the largest chord of the circle. Half of the diameter is the radius of the circle. Obviously, diameter is twice the radius.

The complete length of the circle is called its circumference. That means, starting from a point P, the distance covered along the circle until you reach the point *P*, is the circumference.

The circle is not a straight line, so its circumference can not be measured with a ruler. We can apply an easy trick to measure the circumference, Draw a circle in an art paper and cut along the circle. Mark a point on the circumference. Now, draw a line segment on a paper and put the circle in upright position so that the marked point can coincide with the end point of line segment. Now roll the circle along the line segment until the marked point touches the line segment again. Locate the touching point and measure the length from the other end of line segment. This is the length of the circumference. Observe that the diameter of a small circle is small; so is the circumference. On the other hand, the diameter of a larger circle is large, the circumference is also larger.

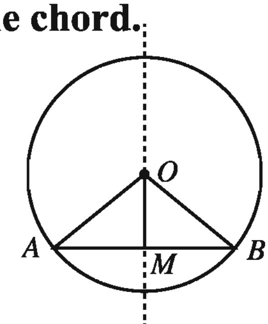
10·4 Theorems Related to Circle

Activity :

1. Draw in a tracing paper, a circle of any radius with the centre at 0. Also draw a chord AB other than diameter. Fold the paper through the point O in such a way that the end points of the chord AB coincide. Now, draw the line segment OM along the crease which meets the chord at M. Then M is the midpoint of the chord. Measure the angles LOMA and LOMB. Is each of them equal to one right angle?

Theorem 1

The line segment joining the centre of a circle to the midpoint of a chord other than diameter, is perpendicular to the chord.



Proposition: Let AB be a chord other than diameter of a circle with centre O and O is joined to the midpoint M of AB . It is to be proved that OM is perpendicular to AB.

Construction: Join O,A and O,B.

Proof:

Steps	Justification
(1) In ΔOAM and ΔOBM,	
$AM = BM$	[M is the midpoint of AB]
$OA = OB$	[radius of same circle]
and $OM = OM$	[common side]
Therefore, $\Delta OAM \cong \Delta OBM$	[SSS theorem]
$\therefore \angle OMA = \angle OMB$	
(2) Since the two angles are equal and make a straight angle	
$\angle OMA = \angle OMB = 1$ right angle.	
Therefore, $OM \perp AB$. (Proved)	

Activity :

1. Prove that the perpendicular from the centre of a circle to a chord bisects the chord.

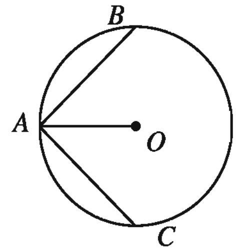
[Hints: Make the use of congruence of right-angled triangles]

Corollary 1: The perpendicular bisector of any chord passes through the centre of the circle.

Corollary 2: A straight line cannot intersect a circle in more than two points.

Exercise 10.1

1. Prove that, if two chords of a circle bisect each other, their point of intersection will be the centre of the circle.
2. Prove that the line joining the midpoints of two parallel chords passes through the centre and is perpendicular to the two chords.
3. Two chords AB and AC of a circle make equal angles with the radius through A. Prove that $AB=AC$.
4. In the figure, O is the centre of the circle and the chord AB = chord AC. Prove that $\angle BAO = \angle CAO$.

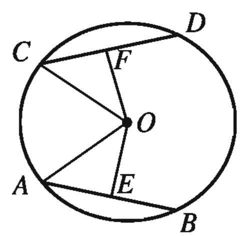


5. A circle passes through the vertices of a right-angled triangle. Show that the centre lies on the midpoint of the hypotenuse.
6. A chord AB of one of two concentric circles intersect the other at C and D. Prove that $AC=BD$.

Theorem 2

Equal chords of a circle are equidistant from the centre.

Proposition: Let AB and CD be two equal chords of a circle with the centre O. It is to be proved that the chords AB and CD are equidistant from the centre.



Construction: Draw from O, the perpendiculars OE and OF to the chords AB and CD respectively. Join O,A and O,C.

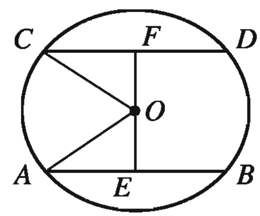
Steps	Justification
(1) $OE \perp AB$ and $OF \perp CD$ Therefore, $AE=BE$ and $CF=DF$. $\therefore AE=\frac{1}{2}AB$ and $CF=\frac{1}{2}CD$	[Perpendicular from the centre bisects the chord]
(2) But $AB = DC$ $\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$ $\therefore AE = CF.$	[supposition] [radius of same circle] [Step 2]
(3) Now between the right-angled ΔOAE and ΔOCF	[RHS theorem]

hypotenuse OA = hypotenuse OC and $AE = CF$. $\therefore \quad \Delta OAE \cong \Delta OCF$ $\therefore \quad OE = OF$. (4) But OE and OF are the distances from O to the chords AB and CD respectively. Therefore, the chords AB and CD are equidistant from the centre of the circle. (Proved)	

Theorem 3

Chords equidistant from the centre of a circle are equal.

Proposition: Let AB and CD be two chords of a circle with centre O. OE and OF are the perpendiculars from O to the chords AB and CD respectively. Then OE and OF represent the distances from centre to the chords AB and CD respectively.



If $OE = OF$, it is to be proved that $AB = CD$.

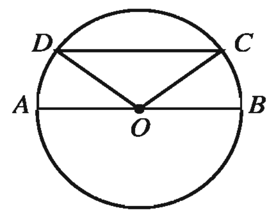
Construction : Join O,A and O,C.

Proof:

Steps	**Justification**
(1) Since $OE \perp AB$ and $OF \perp CD$. Therefore, $\angle OEA = \angle OFC =$ 1 right angle	[right angles]
(2) Now, between the right-angled ΔOAE and ΔOCF hypotenuse OA= hypotenuse OC and $OE = OF$ $\therefore \quad \Delta OAE \cong \Delta OCF$ $\therefore \quad AE = CF$.	[radius of same circle] [supposition] [RHS theorem]
(3) $AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$ (4) Therefore, $\frac{1}{2}AB = \frac{1}{2}CD$ i.e., $AB = CD$	[Perpendicular from the centre bisects the chord]

Example 4. Prove that the diameter is the greatest chord of a circle.

Proposition: Let O be the centre of the circle $ABCD$. Let AB be the diameter and CD be a chord other than diameter of the circle. It is required to prove that $AB > CD$.



Construction: Join O,C and O,D.

Proof: $OA = OB = OC = OD$ [radius of the same circle]

Now, in ΔOCD,

$OC + OD > CD$

or, $OA + OB > CD$ [∵ the sum of two ides is greater than the third side of a triangle.]

Therefore, $AB > CD$.

Exercise 10.2

1. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.
2. Prove that the bisecting points of equal chords lie on a circle.
3. Show that equal chords drawn from the end points on opposite sides of a diameter are parallel.
4. Show that parallel chords drawn from the end points of a diameter are equal.
5. Prove that between the two chords the larger one is nearer to the centre than the smaller one.
6. In a circle with centre 'O', PQ and RS are two equal chords and their mid points are M and N respectively.

   A. Find out the radius of the circle with area 314 sq.cm.

   B. Prove that OM= ON.

   C. If the chords PQ and RS bisect each other, prove that the two parts of one chord are equal to the two parts of the other.

10.5 Ratio of Circumference and Diameter of a Circle (π)

Let us see if there is any relationship between the diameter and the circumference of circles. Work in groups and do the following activity.

Activity:

1. Draw three circles of different radius of your choice and complete the table below by measuring diameter and circumference. Are the ratios of circumference and radius approximately the same?

Circle	Radius	Circumference	Diameter	Circumference/ Diameter
1	3.5 cm	22 cm	7.0 cm	22/7 =3.142

The ratio of the circumference and the diameter of a circle is constant. This ratio is denoted by the Greek letter π (pi). Thus, if the circumference and the diameter are denoted by c and d respectively, we can say that the ratio is $\frac{c}{d} = \pi$ or $c = \pi d$.

We know that diameter (d) of a circle is twice the radius i.e., $d = 2r$; Therefore, $c = 2\pi r$. Since ancient times mathematicians put efforts towards evaluation of π approximately. Indian mathematician Arya Bhatta (476-550 AD)estimated π as $\frac{62832}{20000}$ which is approximately 3.1416. Mathematician Sreenibash Ramanujan (1887-1920) estimated π correct to million places after decimal. Exactly speaking, π is an irrational number. In our day to day life we approximate π by $\frac{22}{7}$.

Example 1. The diameter of a circle is 10 cm. What is the circumference of the circle ? (use $\pi \approx 3 \cdot 14$)

Solution:

Diameter of the circle, $d = 10$ cm

Circumference of the circle = πd

$\approx 3.14 \times 10$ cm $= 31{\cdot}4$ cm

Therefore, the circumference of the circle with radius 10 cm is 31·4 cm.

Example 2. What is the circumference of the circle with a radius of 14 cm ? (use $\pi \approx \frac{22}{7}$)

Solution: Radius of the circle, r =14 cm

Circumference of the circle = $2\pi r$

$\approx 2 \times \frac{22}{7} \times 14$ cm $= 88$ cm.

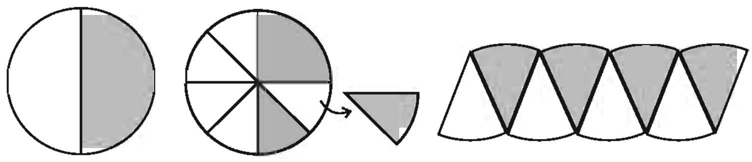
Therefore, circumference of the circle is 88 cm.

10.6 Area of a Circle

The area of the region in a plane bounded by a circle is known as the circular region. Let us do the following activity to find the area of a circular region.

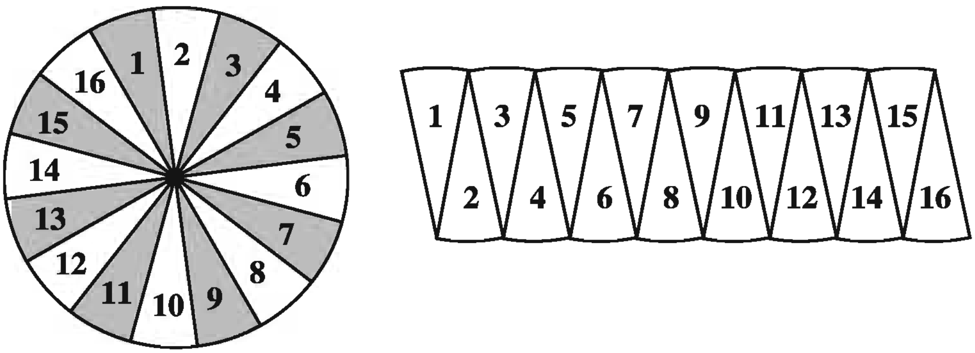
Activity:

1.(a) Draw a circle and colour one half of the circle. Now, fold the circle three times successively along the middle and cut along the folds. The circle is divided into **eight** equal pieces. what do you get arranging the pieces as shown in figure ? Is it not roughly a parallelogram?

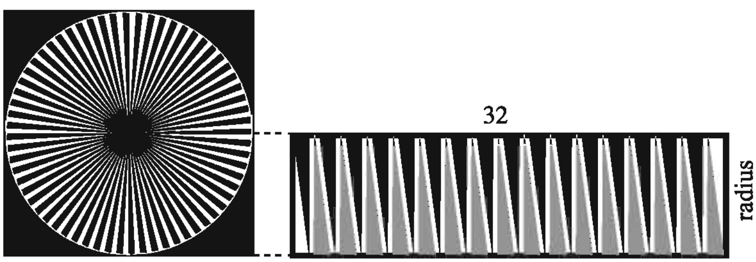


(i) (ii)

(b) Divide the circle into 16 equal parts and arrange in the same way the pieces. What do you get?



(c) Divide the circle into 64 equal sectors as done above and arrange these sectors. What you get ? Is it not roughly a rectangle ?



(d) What is the length and the breadth of this rectangle? What is its area?

Area of the circle = Area of rectangle = length $\times$ breadth

$$= (\text{Half of circumference}) \times \text{radius}$$

$$= \frac{1}{2} \times 2\pi r \times r = \pi r^2$$

So, the area of the circle $= \pi r^2$

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Activity:

1. (a) Draw a circle of radius 5 cm on a graph paper. Count the small squares within the circle region and estimate its area.
   (b) Find the area of the same circle by using the formula. Then find the difference between the evaluated and estimated area of the circle.

Example 3. What is the area of a circular garden of diameter 9.8 m ?

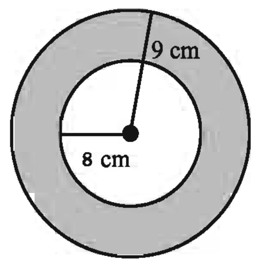
Solution: The diameter of the circular garden, $d = 9{\cdot}8$ m.

The radius of the circular garden $r = \dfrac{9{\cdot}8}{2}$ m$= 4{\cdot}9$ m

The area of the circular garden $= \pi r^2$

$\approx 3{\cdot}14 \times 4{\cdot}9^2$ sq. m $=$ 75.39 square metre.

Example 4. The adjoining figure shows two circles with the same centre. The radius of the larger circle is 9 cm and the radius of the smaller circle is 4 cm. What is the area of the shaded region between the two circles ?



Solution :

The radius of the larger circle $r = 9$ cm

So, the area of the larger circle $= \pi r^2$ sq. cm

$\approx 3.14 \times 9^2$ sq. cm $= 254.34$ sq. cm

The radius of the smaller circle $r = 4$ cm

$\therefore$ The area of the smaller circle $= \pi r^2$ sq. cm

$\approx 3.14 \times 4^2$ sq. cm $= 50.24$ sq. cm

$\therefore$ The area of the shaded region $= (254.34 - 50.24)$ sq. cm (approx.)

$= 204.10$ sq. cm (approx.)

10.7 Cylinder

If a rectangular region (fig-1) or a square region is revived once completely by keeping its one side fixed, than a solid will be produced (fig-2). Such a solid is called a right circular cylinder. The fixed line is called the axis of the cylinder and its opposite side is called its generator; it is the height of the cylinder. The length of the other side is the radius of the cylinder.

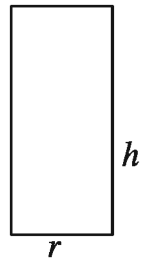


Fig-1

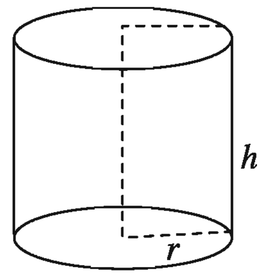


Fig-2

Determination of the area of the faces of a right circular cylinder

Let r be the radius of a right circular cylinder and h be its height.

If the cylinder (as a hollow carket of tin) is cut perpendicularly to the end circular surfaces and is made as plane surface, it will be a rectangular region whose length will be $2\pi r$ (circumference of a circle) and the other side will be the height h of the cylinder.


Circumference = $2\pi r$

So, the area of the whole surface of the right circular cylinder

= area of the two end surfaces (which are circular regions)
+ area of the curved surface (which is a rectangular region)

$$= 2 \times \pi r^2 + 2\pi r \times h$$
$$= 2\pi r^2 + 2\pi rh$$
$$= 2\pi r\,(r + h)$$

Example 5 : The radius of a right circular cylinder is 4.5 cm and its height is 6 cm. Find the area of the curved surface of the cylinder ($\pi = 3.14$)

Solution : The radius of the given right circular cylinder is $r = 4.5$ cm and its height $h = 6$ cm.

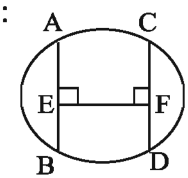
$\therefore$ The area of the curved surface of the cylinder

$$= 2\pi rh = 2 \times 3.14 \times 4.5 \times 6 \text{ sq. cm}$$
$$= 28 \times 27 \text{ sq. cm} = 169.56 \text{ sq. cm}$$

Exercise 10·3

1. On a plane
   i. Innumerable circles can be drawn with two particular points.
   ii. the three points are not on one straight line. Hence, only one circle can be drawn.
   iii. A straight line can intersect at more than two points in a circle.
   Which one of the following is correct.
   A. i & ii B. i & iii C. ii & iii D. i, ii & iii
2. In a circle with radius 2r-
   i. Circumference is 4π r unit.
   ii. Diameter is 4r unit.
   iii. Area is $2\pi^2r^2$ sq. unit.
   Which one of the following is correct?
   A. i & ii B. i & iii C. ii & iii D. i, ii & iii
3. For a circle with radius 3 cm, What will be the length of the chord 6 cm from the centre in cm?
   A. 6 B. 3 C. 2 D. 0
4. What will be the area of a circle with unit radius?
   A. 1 sq. unit B. 2 sq. unit C. π sq. unit D. π^2 sq. unit
5. What will be the length of a radius of a circle with circumference 23 cm?
   A. 2.33 cm (approx) B. 3.66 cm (approx)
   C. 7.32 cm (approx) D. 11.5 cm (approx)
6. What will be the area in between the space of the two uni centered circles with radii 3 cm and 2 cm?
   A. π sq.cm B. 3π sq.cm C. 4π sq.cm D. 5π sq.cm
7. The diametre of a wheel of a vehicle is 38 cm. What will be the distance covered by two complete round?
   A. 59.69 cm B. 76 cm C. 119.38 cm D. 238.76 cm

Answer questions 8, 9 and 10 on the basis of the following figure:



In the figure, 0 is the centre of the circle? BE = 4 cm.

8. If OE = OF, what will be the lengths of CD in cm?
   A. 3 cm B. 4 cm C. 6 cm D. 8 cm

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9. AB = CD, OE = 3 cm. What will be the radius of the circle in cm?
 A. 3 B. 4 C. 5 D. 6

10. If AB>CD, which of the following will be correct?
 A. CF<BE B. OE>OF C. OE<OF D. OE = OF

11. Construct with a pencil compasses a circle with a suitable centre and a radius. Draw a few radius in the circle and measure them to see if they all are of equal length.

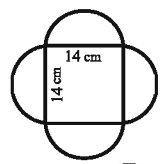
12. Find the circumference of the circles with the following radius:
 (a) 10 cm (b) 14 cm (c) 21 cm

13. Find the area of the circles given below :
 (a) radius = 12 cm (b) diameter = 34 cm (c) radius = 21 cm

14. If the circumference of a circular sheet is 154 cm, find its radius. Also, find the area of the sheet.

15. A gardener wants to fence a circular garden of diameter 21m. Find the length of the rope he needs to purchase if he makes 2 rounds of the fence. Also, find the cost of the rope if it costs Tk. 18 per metre.

16. Find the perimeter of the given shape.



17. From a circular board sheet of radius 14 cm, two circular parts of radius 1.5 cm and a rectangle of length 3 cm and breadth 1cm are removed. Find the area of the remaining board.

18. The height of a right circular cylinder of radius 5.5 cm is 8 cm. Find the area of the whole surfaces of the cylinder ($\pi = 3.14$).

Chapter Eleven
Information and Data

Information and data have an important role in and contribution to the wide expansion and rapid development of knowledge and science. Based on information and data, research is carried out and continuous research results in the unthinkable development of knowledge and science. The use of numbers has expanded largely in the presentation of information and data. The number based information is statistics. So, the fundamental concepts and related contents of statistics are essential to learn. The basic contents of statistics have been presented in the previous class gradually. In continuation of the presentation, the central tendency and its measure namely mean, median and mode have been discussed in detail in this chapter.

At the end of the chapter, the students will be able to–

- Explain the central tendency.
- Determine average, median and mode with the help of mathematical formulae and solve related problems.
- Draw histogram and pie-chart.

11·1 Information and data

We have gained the fundamental concept and learnt it in detail in the previous class. Now, we shall discuss it in a small scale. We know, any information or data based on numbers or events is statistics. And the numbers used for information or events are the data of statistics. Let, out of 50 marks, the marks obtained by 20 participants in a competitive examination are 25, 45, 40, 20, 35, 30, 35, 30, 40, 41, 46, 20, 25,30, 45, 42, 45, 47, 50, 30. Here the numbers used for marks obtained in Mathematics is statistics and the numbers are the data of statistics. The data can be collected directly from the source. The data collected directly from the sourceis more reliable. The data collected directly from the source are primary data. Since the secondary data are collected from indirect sources, it is less reliable. The numbers of the above data are not organized. They are not arranged in any order. The data of this type are disorganized data. The numbers of the data if arranged in any order would be organized data. The numbers arranged in ascending order will be 20, 20, 25, 25, 30, 30, 30, 30, 35, 35, 40, 40, 41, 42, 45, 45, 45, 46, 47, 50, which are organized data. The arrangement of data in this wayis very difficult and there is every possibility of making mistakes. The disorganized data can be made organized easily through classification and can be a presented in a frequency table.

11·2 Frequency Distribution Table

The following steps are used to make a frequency distribution table :
Determination of (1) range (2) number of classes (3) class interval (4) frequency using tally. Range of data to be investigated = (highest number – lowest number) + 1.

Class Interval : After the determination of the range of data under investigation it is required to find the class interval. The data are divided into some class taking convenient intervals. Generally, the classification is made depending upon the number of data. There is no hard and fast rule of classification. But usually, the limit of class interval is maintained between minimum 5 and maximum 15. Hence, there is a highest and a lowest value of each interval. The lowest value of any class is its lower limit and the highest value is its higher limit. The difference between the higher and lower limits of any class is its class interval. For example, let, 10-20 be a class; its minimum value is 10 and maximum value is 20 and (20-10) = 10 and its class interval = 10 + 1 = 11. It is always better to keep the class intervals equal.

Number of class : The range divided by the class interval is number the of classes.

Hence, number of classes = $\dfrac{\text{range}}{\text{class interval}}$ (converted into integer)

Tally Marks : The numerical information of the data must belong to some class. For a numerical value, tally mark is put against the class. If the number of tally in a class is 5, the 5th one is put crosswise.

Frequency : The numerical values of information in the classes are expressed by tally marks and frequency is determined by the numbers of tally marks. The number of frequency of a class will be the number of the tally marks, which is written in frequency column against the tally marks.

Range, class interval and number of classes of the above data under consideration are as follows :
Range = (highest numerical value of the data – lowest numerical value) + 1
= (50 – 20) + 1 = 31.
If the class interval is taken to be 5, the number of classes will be $\dfrac{31}{5} = 6 \cdot 2$
which will be 7 after converting into integer. Hence the number of classes is 7.
In respect of above discussion, the frequency distribution table of the stated data is:

Class interval	Tally marks	Frequency
20-24	//	2
25-29	//	2
30-34	////	4
35-39	//	2
40-44	////	4
45-49	~~////~~	5
50-54	/	1
total	20	20

Activity : Form groups of 20 from your class and put the heights of the members in a frequency table.

11·3 Diagram

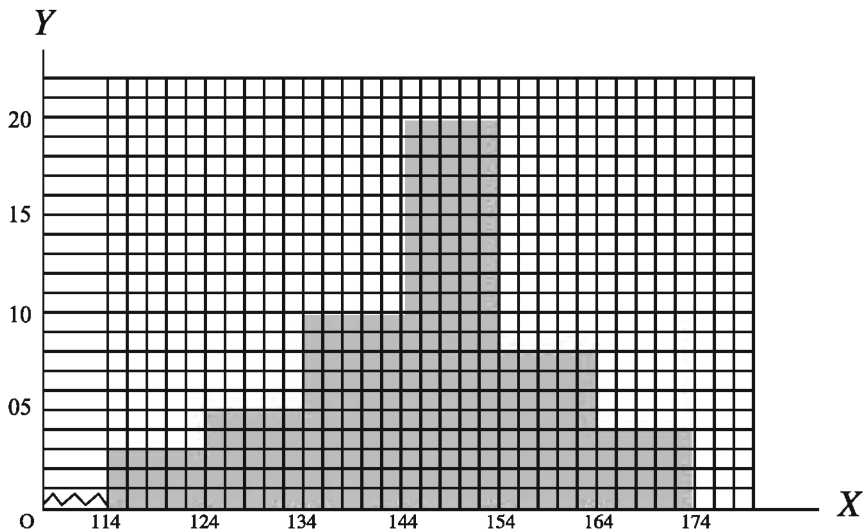
The presentation of information and data by diagram is a widely used method. If the data used in any statistics are presented through diagram, they become easy to comprehend and convenient to draw conclusion. Moreover, the data presented through diagram also become attractive. That is why frequency distribution of data is presented in diagram for easy comprehension and for drawing conclusion. Though there are different types of diagrams in presenting the frequency diagrams, here only Histogram and Pie-chart will be discussed.

Histogram : One of the diagrams of frequency distribution is histogram. For drawing histogram x-axis and y-axis are drawn in a graph paper. The class interval and the frequency are placed along x-axis and y-axis respectively and the histogram is drawn. The base of rectangle is the class interval and height is the frequency.

Example 1. The frequency distribution table of the heights of 50 students is as follows. Draw a histogram.

Class interval of heights (in cm)	114-124	124-134	134-144	144-154	154-164	164-174
Frequency (number of students)	3	5	10	20	8	4

Considering one unit of graph paper to represent 2 of the class interval along the x-axis and one unit of graph paper to denote 1 of the frequency along the y-axis, the histogram of frequency distribution has been drawn. The broken segments from the origin of x-axis to 114 indicate that the previous intervals are omitted.



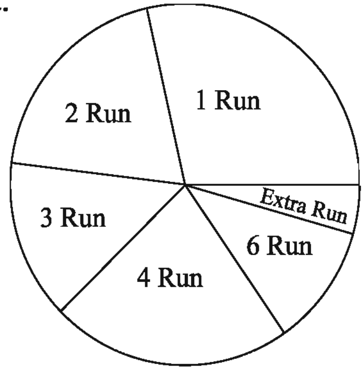
Activity :
1. Form groups of 30. Put the frequency distribution of the marks obtained in Mathematics of the members.
2. Draw histogram of the frequency distribution.

Pie-chart : A pie-chart is also a diagram. Sometimes the collected statistics consists of the sum of the elements or it is divided into some classes. If these classes are expressed by different slices of a circle, the diagram thus obtained is a pie-chart. A pie-chart is also known as a circular diagram. We know that the angle subtended at the centre of a circle is 360°. If statistics is presented as a part of 360°, it will be a pie-chart.

We know that the runs are scored by 1, 2, 3 ,4 and 6 in a cricket game. Extra runs are also scored by no-ball and wide ball. The runs scored by Bangladesh cricket team in a game is placed in the following table.

Run scored	1	2	3	4	6	Extra	Total
Scored Run in different ways	66	50	36	48	30	10	240

If the data of cricket game is shown by a pie-chart, it becomes attractive as well as co easy to understand. When a data is presented through a circle, the diagram is called a pie-chart. Hence a pie-chart is a circular diagram. We know that the angle subtended at the centre is 360°. If the above stated data is presented as a parts of 360°, we get the pie-chart of the data.



For 240 runs, the angle is 360°

$\therefore$ ” 1 ” ” ” $\frac{360^\circ}{240}$

$\therefore$ ” 66 ” ” $\frac{\overset{33}{\cancel{66}}\times\overset{3^\circ}{\cancel{360^\circ}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{240}}}=99^\circ$

Similarly for 50 runs, the angle will be $\frac{50}{240}\times 360^\circ = 75^\circ$

” ” 36 ” ” ” ” ” $\frac{36}{240}\times 360^\circ = 54^\circ$

” ” 48 ” ” ” ” ” $\frac{48}{240}\times 360^\circ = 72^\circ$

” ” 30 ” ” ” ” ” $\frac{30}{240}\times 360^\circ = 45^\circ$

” ” 10 ” ” ” ” ” $\frac{10}{240}\times 360^\circ = 15^\circ$

Here, the angles obtained are drawn as parts of 360°, which is the pie-chart of the data.

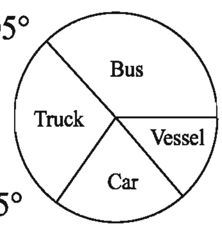
Example 2. The table of death due to accidents in a year is given below. Draw a pie-chart :

Accident	bus	truck	car	vessel	total
Number of deaths	450	350	250	150	1200

Solution :

The angle for death of 450 due to bus accident = $\frac{\overset{45}{\cancel{450}}}{\underset{\cancel{10}}{\cancel{1200}}}\times\overset{3}{\cancel{360}} = 135^\circ$

The angle for death of 350 due to truck accident = $\frac{350}{1200}\times 360^\circ = 105^\circ$

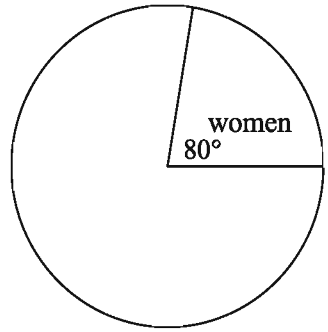


The angle for death of 250 due to car accident = $\frac{250}{1200}\times 360^\circ = 75^\circ$

The angle for death of 150 due to vessel accident = $\frac{150}{1200}\times 360^\circ = 45^\circ$

Here, the angles are drawn as parts of 360° to form the required pie-chart.

Example 3. How many of 450 death due to accident are women, men and children is shown through pie-chart. How many of them are women ? The angle subtended for women is 80°.



Solution : The angle at the centre is 360^0.

Hence 360° represents 450 persons

$\therefore$ " 1° " $\frac{450}{360}$ "

$\therefore$ " 80° " $\frac{450}{360} \times 80 = 100$ "

$\therefore$ Required number of women is 100.

Activity :

1. Form groups of 6 the students in your class. Measure the heights of the members of the groups and show the data through a pie-chart.
2. Draw a pie-chart with the ages of all members of your family. Exchange your notebook with the next student to find the ages of individuals from the fixed angle of the individual.

11·4 Central Tendency

Let the time (in second) taken by 25 girl students to solve a problem be as follows:

22,16,20,30,25,36,35,37,40,43, 40,43,44,43,44,46,45,48,50,64,50,60,55,62,60.

The numbers arranged in ascending order are :

16, 20, 22, 25, 30, 35, 36, 37, 40, 40, 43, 43, 43, 44, 44, 45, 46, 48, 50, 50, 55, 60, 60, 62, 64. The stated data are centred round the middle value of 43 or 44. This tendency is also seen in frequency distribution table. The frequency distribution table of the data is

Interval	16-25	26-35	36-45	46-55	56-65
Frequency	4	2	10	5	4

From this frequency distribution table, it is to be noted that the maximum of the frequency occurs in the class 36−45. Hence, it is clear from the above discussion that the data cluster round the value at centre or middle. The tendency of clustering of the data to the value at middle or centre is called central tendency. The central value of the data is a representative number which measures the central tendency. Generally, measurement of central tendency are (1) Arithmetic Average, (2) Median, (3) Mode.

11·5 Arithmetic Mean

We know that if the sum of the numerical values of data is divided by the number of data, we get Arithmetic mean. Let the number of data be *n* and their numerical values are $x_1, x_2, x_3........x_n$. If the arithmetic mean of the data is $\overline{x}$,

$$\overline{x} = \frac{x_1 + x_2 + x_3 + ...x_n}{n} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Example 4. Out of 50 in an examination, the marks obtained by 20 students are 40,41,45,18,41,20,45,41,45,25,20,40,18,20,45,47,48,48,49,19.Find the arithmetic mean of the marks.

Solution : Here n = 20, $x_1 = 40$, $x_2 = 41$, $x_3 = 45$,......... etc.

If the arithmetic mean is $\overline{x}$, $\overline{x} = \dfrac{\text{sum of numbrs}}{\text{number of numbers}}$

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \sum_{c=1}^{20} \frac{x_i}{20} = \frac{40+41+45+.........+19}{20} = \frac{715}{20} = 35\cdot75$$

∴ Arithmetic Mean is 35.75

Arithmetic Mean of disorganized Data (short-cut method) :

If the numbers of data are large, to find the arithmetic mean by the previous method is difficult and there is every possibility to make mistakes in finding the sum of such large numbers of the data. In this context, it is convenient to use a short-cut method.

In the short-cut method, the possible arithmetic mean is estimated through proper and careful observation of central tendency. Through careful observation of central tendency of the above example, it is clear that the arithmetic mean is a number between 30 and 46. Let the mean be 30. Here the estimated arithmetic mean 30 has to be subtracted from each of the numbers to determine the subtracted value. If the number is larger than 30, the result will be positive and if the number is less than 30, the result will be negative. Then the algebraic sum of the differences has to be determined. The two successive differences are added to find the cumulative sum and the process contiues. The sum of all differences is equal to the final cumulative sum . Here the arithmetic mean of the data used in the above example can be determined by the short-cut method. Let, the data is x_i (i = 1, 2,....., n) and the estimated mean of the data is a, (a = 30).

Data x_i	$x_i - a$	Cumulative Sum	Data x_i	$x_i - a$	Cumulative Sum
40	$40-30=10$	10	20	$20-30=-10$	$61-10=51$
41	$41-30=11$	$10+11=21$	40	$40-30=10$	$51+10=61$
45	$45-30=15$	$21+15=36$	18	$18-30=-12$	$61-12=49$
18	$18-30=-12$	$36-12=24$	20	$20-30=-10$	$49-10=39$
41	$41-30=11$	$24+11=35$	45	$45-30=15$	$39+15=54$
20	$20-30=-10$	35-10 = 25	47	$47-30=17$	$54+17=71$
45	$45-30=15$	$25+15=40$	48	$48-30=18$	$71+18=89$
41	$41-30=11$	$40+11=51$	48	$48-30=18$	$89+18=107$
45	$45-30=15$	$51+15=66$	49	$49-30=19$	$107+19=126$
25	$25-30=-5$	$66-5=61$	19	$19-30=-11$	126-11 = 115

It is evident from the above table that the sum of the differences = 115

$\therefore$ The average of the differences $= \dfrac{115}{20} = 5 \cdot 75$

Hence actual mean = Estimated mean + average of differences

$= 30 + 5.75$

$= 35.75$

Remark : For convenience and for saving time, the subtraction and addition between columns can be calculated mentally and the resultant can be written directly.

Arithmetic Mean of Organized Data

Of the marks obtained in Mathematics by 20 students in example 4, more than one student have obtained the same marks. The frequency distribution table of the marks obtained is placed below :

Marks obtained $x_i\ i=1,....,k$	Frequency $f_i\ i=1,....,k$	f_ix_i
18	2	36
19	1	19
20	3	60
25	1	25
40	2	80
41	3	123
45	4	180
47	1	47
48	2	96
49	1	49
$k=10$	$k=10, n=20$	Total =715

$$\text{Arithmetic Mean} = \frac{\text{Sum of } f_ix_i}{\text{Total Frequency}} = \frac{715}{20}$$

$$= 35.75$$

Formula 1. Arithmetic Mean (Organized Data) : If frequency of k numbers of $x_1, x_2, x_3........, x_k$ of n number of data is $f_1, f_2,.......f_k$, arithmetic mean of the data

$$\bar{x} = \frac{\sum_{i=1}^{k} f_ix_i}{n} = \frac{1}{n}\sum_{i=1}^{k} f_ix_i \text{ where } n \text{ is the number of data}$$

Example 5. The frequency distribution table of the marks obtained in Mathematics by 100 students of a class is as follows. Find the arithmetic mean.

Class Interval	25-34	35-44	45-54	55-64	65-74	75-84	85-94
Frequency	5	10	15	20	30	16	4

Solution : It is not possible to know the individual marks of the students as the class interval is given. In this case, it is necessary to find the class mid-value of the class

$$\text{Class mid-value} = \frac{\text{class higher value} - \text{class lower value}}{2}$$

If the class mid-value is $x_i\ (i = 1,2,.........,k)$, the table containing mid-values will be as follows :

Class interval	Class mid-value (x_i)	Frequency (f_i)	$(f_i x_i)$
25 – 34	29·5	5	147·5
35 – 44	39·5	10	395·0
45 – 54	49·5	15	742·5
55 – 64	59·5	20	1190·0
65 – 74	69·5	30	2085·0
75 – 84	79·5	16	1272·0
85 – 94	89·5	4	348·0
	Total	100	6190·00

$$\text{Required arithmetic mean} = \frac{1}{n}\sum_{i=1}^{k} f_i x_i = \frac{1}{100} \times 6190$$
$$= 61{\cdot}9$$

11·6 Median

We have already learnt about median of the data under consideration in statistics in class VII.

Let 5, 3, 4, 8, 6, 7, 9, 11, 10 be a few numbers. If arranged in ascending order, they will be 3, 4, 5, 6, 7, 8, 9, 10, 11. If the ordered arranged numbers are divided into two equal parts, they will be

$$\boxed{3,\ 4,\ 5,\ 6,}\ 7\ \boxed{8,\ 9,\ 10,\ 11}$$

It is evident that the number 7 divides the numbers in two equal parts and its position is in the middle. Hence, here the mid-term is the 5th term. The value of the 5th term or mid-term is 7. Therefore, the median of the numbers is 7. Here, the number given data is odd. If the number of data is even such as 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 21, 22, what will be the median ? If the numbers are divided into two equal parts, they will be,

$$\boxed{8,\ 9,\ 10,\ 11,\ 12,}\ 13, 15\ \boxed{16,\ 18,\ 19,\ 21,\ 22}$$

It is evident from the above that 13 and 15 divide the numbers into two equal parts and their positions are in the middle. Here mid-terms are 6^{th} and 7^{th} terms. Therefore, the median will be average of the numbers of 6^{th} and 7^{th} terms. The average of the numbers of 6^{th} and 7^{th} terms is $\frac{13+15}{2}$ or 14 i.e. the median is 14. From the above discussion, we can conclude that if there is n number of data and if n is add, the median of the data will be the value of $\frac{n+1}{2}$th term. But if n is even number, the median will be average of the numerical values of $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$th terms.

If the data are arranged either in ascending or descending order, the value which divides the data into two equal parts is the median.

Example 6. Find the median of the following numbers : 23, 11, 25, 15, 21, 12, 17, 18, 22, 27, 29, 30, 16, 19.

Solution : The numbers if arranged in ascending order will be

11, 12, 15, 16, 17, 18, 19, 21, 22, 23, 25, 27, 29, 30

Here the number of data is even, i.e. $n = 14$

$$\therefore \text{ Median} = \frac{\text{Value of sum of } \frac{14}{2}\text{th and } \left(\frac{14}{2}+1\right)\text{th terms}}{2}$$

$$= \frac{\text{value of sum of } 7^{th} \text{ and } 8^{th} \text{ terms}}{2}$$

$$\therefore \text{ Median} = \frac{19+21}{2} = \frac{40}{2} = 20$$

Therefore, median is 20.

Activity : Form 3 groups of 19, 20 and 21 students studying in your class. Each group will find the median of their roll numbers.

Example 7. The frequency distribution table of the marks obtained by 50 students in Mathematics are given below. Find the median.

Marks obtained	45	50	60	65	70	75	80	90	95	100
Frequency	3	2	5	4	10	15	5	3	2	1

Solution : Frequency table for finding median

Marks obtained	Frequency	Cumulative frequency
45	3	3
50	2	5
60	5	10
65	4	14
70	10	24
75	15	39
80	5	44
90	3	47
95	2	49
100	1	50

Here, $n = 50$ which is an even number.

$$\therefore \text{ Median} = \frac{\text{Sum of numerical values of } \frac{50}{2}\text{th and} \left(\frac{50}{2}+1\right)\text{th}}{2}$$

$$= \frac{\text{Sum of numerical values of } 25^{\text{th}} \text{ and } 26^{\text{th}} \text{ terms}}{2}$$

$$= \frac{75+75}{2} \text{ or } 75$$

$\therefore$ Median of marks obtained is 75

Observe : The numerical value of the terms from 25th to 29th is 75.

Activity: Form 2 groups by all the students of your class.
(a) Make a frequency distribution table of time taken to solve a problem by each of the students
(b) From the frequency distribution table, find the median.

11·7 Mode

Let 11, 9, 10, 12, 11, 12, 14, 11, 10, 20, 21, 11, 9 and 18 be a data. If the data are arranged in ascending order, it will be

9, 9, 10, 10, 11, 11, 11, 11, 12, 12, 14, 18, 20, 21.

It is to be noted that in arranged data, 11 appears 4 times which is maximum times of repetition. Since 11 appears maximum times, 11 is the mode of the data.

The number which appears maximum time is the mode of the data.

Example 8. The marks obtained in social science by 30 students in annual examination are as follows. Find the mode of the data. 75,35,40,80,65,80,80,90,95,80,65,60,75,80,40,67,70,72,69,78,80,80,65, 75,75,88,93,80, 75,65.

Solution: The data are arranged in ascending order: 35, 40, 40, 60, 65, 65, 65, 65,67,69, 70, 72, 75, 75, 75, 75, 75, 78,80,80,80,80,80,80,80,80,88,90, 93, 95.

In presentation of the data, 40 repeats 2 times, 65 repeats 4 times, 75 repeats 5 times, 80 repeats 8 times and the rest appears once. Hence the mode is 80.
:. Required mode is 80.

Example 9. Find the mode of the following :
4, 6, 9, 20, 10, 8, 18, 19, 21, 24, 23, 30.

Solution : If the data are arranged in ascending order, they are : 4, 6, 8, 9, 10, 18, 19, 20, 21, 23, 24, 30.

It is to be noted that no number appears more than once. So, the data don't have any mode.

Exercise 11

1. Which one of the following defines a class interval ?
   (a) The difference between first and last data
   (b) The sum of last and first data
   (c) The sum of largest and smallest data
   (d) The difference between highest and lowest numbers of each class.
2. Which one of the following indicates the data included in a class ?
   (a) Frequency of the class (b) Mid-point of the class
   (c) Limit of the class (d) Cumulative frequency
3. What is the arithmetic mean of the numbers 8, 12, 16, 17, 20 ?
   (a) 10·5 (b) 12·5 (c) 13·6 (d) 14·6

4. What is the median of the numbers 10, 12, 14, 18, 19, 25 ?

(a) 11·5 (b) 14·6 (c) 16 (d) 18·6

5. What are the modes of the numbers 6, 12, 7, 12, 11, 12, 11, 7, 11 ?

(a) 11 and 7 (b) 11 and 12 (c) 7 and 12 (d) 6 and 7

The frequency distribution table of the marks obtained in Mathematics by 40 students of your class is as follows :

Class interval	41 – 55	56 – 70	71 – 85	86 – 100
Frequency	6	10	20	4

In the context of the table, answer the questions (6 – 8) :

6. Which one is the class interval ?

(a) 5 (b) 10 (c) 12 (d) 15

7. Which one is mid-value of the 2^{nd} class ?

(a) 45 (b) 63 (c) 78 (d) 93

8. Which is the lower value of the class of mode in the table above ?
(a) 41 (b) 56 (c) 71 (d) 86

9. The marks obtained by 25 students in the annual examination are given below:

72,85, 78,84, 78, 75,69,67,88,80, 74, 77, 79,69, 74, 73,83,65, 75,69, 63, 75, 86, 66, 71.

(a) Find the arithmetic mean of the marks obtained directly.

(b) Make the frequency distribution table with 5 as class interval and find the arithmetic mean from the table.

(c) Show the difference between the arithmetic means found in two different ways.

10. A table is given below. Find the arithmetic mean. Draw the histogram of the data :

Marks obtained	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45
Frequency	5	17	30	38	35	10	7	3

11. Find the arithmetic mean from the following table :

Daily Income (in Tk.)	2210	2215	2220	2225	2230	2235	2240	2245	2250
Frequency	2	3	5	7	6	5	5	4	3

12. Weekly savings (in taka) of 40 house wives are as follows :
155, 173, 166, 143, 168, 160, 156, 146, 162, 158, 159, 148, 150, 147, 132, 156, 140, 155, 145, 135, 151, 141, 169, 140, 125, 122, 140, 137, 175, 145, 150, 164, 142, 156, 152, 146, 148, 157, 167.

Find the arithmetic mean, median and mode of weekly savings.

13. Find the arithmetic mean and draw the histogram of the data :

Age (in years)	5 – 6	7 – 8	9 – 10	11 – 12	13 – 14	15 – 16	17 – 18
Frequency	25	27	28	31	29	28	22

14. The frequency distribution table of monthly wages of 100 labours of an industry is given below. What is the arithmetic mean of monthly wages of the labours ? Draw the histogram of the following data.

Monthly wages (in 100 taka)	51–55	56–60	61–65	66–70	71–75	76–80	81–85	86–90
Frequency	6	20	30	15	11	8	6	4

15. Marks obtained in English by 30 students of class VIII are :

45, 42, 60, 61, 58, 53, 48, 52, 51, 49, 73, 52, 57, 71, 64, 49, 56, 48, 67, 63, 70, 59, 54, 46, 43, 56, 59, 43, 68, 52.

(a) What are the numbers of classes with 5 as class interval ?

(b) Make a frequency distribution table with 5 as class interval.

(c) Find the arithmetic mean from the table.

16. Daily savings of 50 students are given below :

Saving (in taka)	41–50	51–60	61–70	71–80	81–90	91–100
Frequency	6	8	13	10	8	5

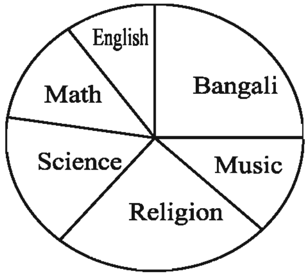
(a) Make a cumulative frequency table.

(b) Find the arithmetic mean from the table.

17. The favourite fruits of 200 students are given in the table. Draw a pie-chart :

Fruit	Mango	Jackfruit	Lichi	Jambolic
Number of students	70	30	80	20

18. The subjects chosen by 720 students are presented in the pie-chart. Express in numbers.



Bengali - 90°
English - 30°
Math - 50°
Science - 60°
Religion - 80°
Music - 50°

360°

19. The frequency distribution table of the marks obtained in Mathematics by 50 students is as follows:

Marks Obtained	60	65	70	75	80	85
Frequency	5	8	11	15	8	3

A. Find out the Median.

B. Find out the Arithmetic mean.

C. Draw the pie-chart of the given data.

20. A table is given below :

Class Interval	20-29	30-39	40-49	50-59	60-69
Frequency	10	6	18	12	8

A. Find the median of the data : 7, 5, 4, 9, 3, 8.

B. Find out the Arithmetic mean from the data (table)

C. Draw the Histogram on the data.

21. Weekly savings (in taka) of 40 house-wives are given below :
155, 173, 166, 143, 168, 160, 156, 146, 162, 158, 159, 148, 150, 147, 132, 136, 154, 140, 155, 145, 135, 151, 141, 169, 140, 125, 122, 140, 137, 175, 145, 150, 164, 142, 156, 152, 146, 148, 157, 167.

A. Arrange the data in ascending order.

B. Find out the Median and Mode.

C. Make a frequency distribution table with 5 as class interval and find out the Arithmetic mean

Answer

Exercise 2·1

1. Tk. 400 2. Tk. 2650 3. There will no loss or profit

4. Tk. 1050 5. Tk. 180 6. 9% 7. 12.5% 8. Tk. 7500

9. Tk. 14000 10. Tk. 1230 11. Tk. 960

12. Tk. 1600 13. Capital Tk. 1200, Profit 10.5% 14. 9.2%

15. 11% 16. 12 years 17. 5 years 18. Tk. 30,000

Exercise 2.2

1. c 2. d 4.a 6. (1) c, (2) a, (3) d 7. Tk. 10648 8. Tk. 155

9. Tk. 6250 10. Tk. 11772.25, Tk. 1772.25 11. 67,24,000 12. Tk. 1672

14. a. 10%, b. Tk. 4500, c. Tk. 3630

Exercise 3

10. 636 sq. metres 11. 402.31 metres (app.) 12. 60 metres (app.) 13. 186 sq. metres 14. 520.8 sq. metres 15. 4864 sq. metres 16. 24 metrse 17. 3 metres 18. 2408.64 grams 19. 673.547 cubic c.m. 20. 44000 litres, 44000 kg 21. Tk. 750 22. 37.5 metres 23. Tk. 7656 24. Tk. 569.50 25. 52; Tk. 1430 26. 450 cubic c.m. 27. 5 hours 20 minutes 28. 97.92 c.m. (app.)

Exercise 4·1

1. (a) $25a^2 + 70ab + 49b^2$ (b) $36x^2 + 36x + 9$ (c) $49p^2 - 28pq + 4q^2$

(d) $a^2x^2 - 2abxy + b^2y^2$ (e) $x^6 + 2x^4y + x^2y^2$ (f) $121a^2 - 264ab + 144b^2$

(g) $36x^4y^2 - 60x^3y^3 + 25x^2y^4$ (h) $x^2 + 2xy + y^2$ (i) $x^2y^2z^2 + 2abcxyz + a^2b^2c^2$

(j) $a^4x^6 - 2a^2b^2x^3y^4 + b^4y^8$ (k) 11664 (l) 367236 (m) 356409

(n) $a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$ (o) $a^2x^2 + b^2 + 2abx + 4b + 4ax + 4$

(p) $x^2y^2 + y^2z^2 + z^2x^2 + 2xy^2z - 2xyz^2 - 2x^2yz$

(q) $9p^2 + 4q^2 + 25r^2 + 12pq - 20qr - 30pr$

(r) $x^4 + y^4 + z^4 - 2x^2y^2 + 2y^2z^2 - 2z^2x^2$

(s) $49a^4 + 64b^4 + 25c^4 + 112a^2b^2 - 80b^2c^2 - 70c^2a^2$

2. (a) $4x^2$ (b) $9a^2$ (c) $36x^4$ (d) $9x^2$ (e) 16

3. (a) $x^2 - 49$ (b) $25x^2 - 169$ (c) $x^2y^2 - y^2z^2$

(d) $a^2x^2 - b^2$ (e) $a^2 + 7a + 12$ (f) $a^2x^2 + 7ax + 12$

(g) $36x^2 + 24x - 221$ (h) $a^8 - b^8$ (i) $a^2x^2 - b^2y^2 - c^2z^2 + 2bcyz$

(j) $9a^2 - 45a + 50$ (k) $25a^2 + 4b^2 - 9c^2 + 20ab$

(l) $a^2x^2 + b^2y^2 + 8ax + 8by + 2abxy + 15$

4. 576 5. 11 6. 194 7. 168100 11. 36, 90 12. 178, 40

13. (a) $(3p + 2q)^2 - (2p - 5q)^2$ (b) $(8b - a)^2 - (b + 7a)^2$

(c) $(5x)^2 - (2x - 5y)^2$ (d) $(5x)^2 - (13)^2$

Exercise 4·2

1. (a) $27x^3+27x^2y+9xy^2+y^3$ (b) $x^6+3x^4y+3x^2y^2+y^3$

(c) $125p^3+150p^2q+60pq^2+8q^3$ (d) $a^6b^3+3a^4b^2c^2d+3a^2bc^4d^2+c^6d^3$

(e) $216p^3-756p^2+882p-343$ (f) $a^3x^3-3a^2x^2by+3axb^2y^2-b^3y^3$

(g) $8p^6-36p^4r^2+54p^2r^4-27r^6$ (h) $x^9+6x^6+12x^3+8$

(i) $8m^3+27n^3+125p^3+36m^2n-60m^2p+54mn^2+150mp^2-135n^2p+225p^2n-180mnp$

(j) $x^6-y^6+z^6-3x^4y^2+3x^2y^4+3x^4z^2+3y^4z^2+3x^2z^4-3y^2z^4-6x^2y^2z^2$

(k) $a^6b^6-3a^4b^4c^2d^2+3a^2b^2c^4d^4-c^6d^6$ (l) $a^6b^3-3a^4b^5c+3a^2b^7c^2-b^9c^3$

(m) $x^9-6x^6y^3+12x^3y^6-8y^9$ (n) $1331a^3-4356a^2b+4752ab^2-1728b^3$

(o) $x^9+3x^6y^3+3x^3y^6+y^9$

2. (a) $216x^3$ (b) $1000q^3$ (c) $64y^3$ (d) 216 (e) $8x^3$ (f) $8x^3$

3. 152 5. 793 6. 170 7. 27 9. 0 10. 722 11. 1

14. 140 15. (a) a^6+b^6 (b) $a^3x^3-b^3y^3$ (c) $8a^3b^6-1$ (d) x^6+a^3

(e) $343a^3+64b^3$ (f) $64a^6-1$ (g) x^6-a^6 (h) $15625a^6-729b^6$

Exercise 4·3

1. $(a+2)(a^2-2a+4)$ 2. $(2x+7)(4x^2-14x+49)$

3. $a(2a+3b)(4a^2-6ab+9b^2)$ 4. $(2x+1)(4x^2-2x+1)$

5. $(4a-5b)(16a^2+20ab+25b^2)$ 6. $(9a-4bc^2)(81a^2+36abc^2+16b^2c^4)$

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7. $b^3(3a+4c)(9a^2-12ac+16c^2)$ 8. $7(2x-3y)(4x^2+6xy+9y^2)$

9. $3x(1+5x)(1-5x)$ 10. $(2x+y)(2x-y)$ 11. $3a(y+4)(y-4)$

12. $(a-b+p)(a-b-p)$ 13. $(4y+a+3)(4y-a-3)$ 14. $a(2+p)(4-2p+p^2)$

15. $2(a+2b)(a^2-2ab+4b^2)$ 16. $(x-y+1)(x-y-1)$ 17. $(a-1)(a-2b+1)$

18. $(x^2+1)(x-1)^2$ 19. $(x-6)^2$

20. $(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

21. $(x-y+z)(x^2+y^2-2xy-xz+yz+z^2)$

22. $8(2x-y)(4x^2+2xy+y^2)$ 23. $(x+4)(x+10)$ 24. $(x+15)(x-8)$

25. $(x-26)(x-25)$ 26. $(a+3b)(a+4b)$ 27. $(p+10q)(p-8q)$

28. $(x-8y)(x+5y)$ 29. $(x^2-x+8)(x^2-x-5)$ 30. $(a^2+b^2+4)(a^2+b^2-22)$

31. $(a+2)(a-2)(a+5)(a+9)$ 32. $(x+a+b)(x+2a+3b)$ 33. $(2x+3)(3x-5)$

34. $(x+a+1)(x-a-2)$ 35. $(x+4)(3x-1)$ 36. $(3x+2)(x-6)$

37. $(x-7)(2x+5)$ 38. $(x-2y)(2x-y)$ 39. $(2y-x)(7x^2-10xy+4y^2)$

40. $(2p+3q)(5p-2q)$ 41. $(x+y-2)(2x+2y+1)$ 42. $(x+a)(ax+1)$

43. $(3x-4y)(5x+3y)$ 44. $(a-2b)(a^2-ab+b^2)$

Exercise 4·4

10. a

11.(1). (c) 11(2). (d) 11(3). (c) 12(1). (a) 12(2). (b) 12(3). (d)

13. $18a^2c^2$ 14. $5x^2y^2a^3b^2$ 15. $3x^2y^2z^3a^3$ 16. 6 17. $(x-3)$ 18. $2(x+y)$

19. $ab(a^2+ab+b^2)$ 20. $a(a+2)$ 21. $a^7b^4c^3$ 22. $30a^2b^3c^3$ 23. $60x^4y^4z^2$

24. $72a^3b^2c^3d^3$ 25. $(x^2-1)(x+2)$ 26. $(x+2)^2(x^3-8)$

27. $(2x-1)(3x+1)(x+2)$

28. $(a-b)^2(a+b)^3(a^2-ab+b^2)^2$ 29. (a) 5 (b) $2\sqrt{5}$ (c) $5\sqrt{5}$

Exercise 5·1

1. (a) $\dfrac{4yz^2}{9x^3}$ (b) $\dfrac{36x}{y}$ (c) $\dfrac{x^2+y^2}{xy(x+y)}$ (d) $\dfrac{a+b}{a^2+ab+b^2}$ (e) $\dfrac{x-1}{x+5}$

(f) $\dfrac{x-3}{x-5}$ (g) $\dfrac{x^2+xy+y^2}{(x+y)^2}$ (h) $\dfrac{a-b-c}{a+b-c}$

2. (a) $\dfrac{x^2z}{xyz}, \dfrac{xy^2}{xyz}, \dfrac{yz^2}{xyz}$ (b) $\dfrac{z(x-y)}{xyz}, \dfrac{x(y-z)}{xyz}, \dfrac{y(z-x)}{xyz}$

(c) $\dfrac{x^2(x+y)}{x(x^2-y^2)}, \dfrac{xy(x-y)}{x(x^2-y^2)}, \dfrac{z(x-y)}{x(x^2-y^2)}$

(d) $\dfrac{(x+y)(x^3+y^3)}{(x-y)^2(x^3+y^{3)}}, \dfrac{(x-y)^3}{(x-y)^2(x^3+y^{3)}}, \dfrac{(y-z)(x-y)(x^2-xy+y^2)}{(x-y)^2(x^3+y^{3)}}$

(e) $\dfrac{a(a^3+b^3)}{(a^3+b^3)(a^3-b^3)}, \dfrac{b((a-b)(a^3+b^3)}{(a^3+b^3)(a^3-b^3)}, \dfrac{c(a^3+b^3)}{(a^3+b^3)(a^3-b^3)}$

(f) $\dfrac{(x-4)(x-5)}{(x-2)(x-3)(x-4)(x-5)}, \dfrac{(x-2)(x-5)}{(x-2)(x-3)(x-4)(x-5)}, \dfrac{(x-2)(x-3)}{(x-2)(x-3)(x-4)(x-5)}$

(g) $\dfrac{c^2(a-b)}{a^2b^2c^2}, \dfrac{a^2(b-c)}{a^2b^2c^2}, \dfrac{b^2(c-a)}{a^2b^2c^2}$

(h) $\dfrac{(x-y)(y+z)(z+x)}{(x+y)(y+z)(z+x)}, \dfrac{(y-z)(x+y)(z+x)}{(x+y)(y+z)(z+x)}, \dfrac{(z-x)(x+y)(y+z)}{(x+y)(y+z)(z+x)}$

3. (a) $\dfrac{a^2+2ab-b^2}{ab}$ (b) $\dfrac{a^2+b^2-c^2}{abc}$ (c) $\dfrac{3xyz-x^2y-y^2z-z^2x}{xyz}$

(d) $\dfrac{2(x^2+y^2)}{x^2-y^2}$ (e) $\dfrac{3x^2-18x+26}{(x-1)(x-2)(x-3)(x-4)}$ (f) $\dfrac{3a^4+a^2b^2-b^4}{(a^3+b^3)(a^3-b^3)}$

(g) $\dfrac{2}{x-2}$ (h) $\dfrac{x^6+2x^4+x^2+6}{x^8-1}$

4. (a) $\dfrac{ax+3a-a^2}{x^2-9}$ (b) $\dfrac{x^2+y^2}{xy(x^2-y^2)}$ (c) $\dfrac{2}{x^4+x^2+1}$ (d) $\dfrac{8ab}{a^2-16b^2}$ (e) $\dfrac{2y}{x^2+y^2}$

5. (a) 0 (b) $\dfrac{x^2+y^2+z^2-xy-yz-zx}{(y+z)(x+y)(z+x)}$ (c) o (d) o

(e) $\dfrac{6xy^2}{(x^2-y^2)(4x^2-y^2)}$ (f) $\dfrac{12x^4}{x^6-64}$ (g) $\dfrac{8x^4}{x^8-1}$

(h) $\dfrac{2(x^2+y^2+z^2-xy-yz-zx)}{(x-y)(y-z)(z-x)}$

(i) $\dfrac{3a-2b}{a^2+b^2-c^2-2ab}$ (j) $\dfrac{2ab+2bc+2ca-a^2-b^2-c^2}{(a+b+c)(a+b-c)(b+c-a)(c+a-b)}$

Exercise 5·2

13. (a) $\dfrac{15a^2b^2c^4}{x^2y^2z^4}$ (b) $\dfrac{32a^2b^2y^3z^3}{45x^4}$ (c) 1 (d) $\dfrac{x(x-1)^3}{(x+1)^2(x^2-4x+5)}$

(e) $\dfrac{x^2+y^2}{(x^2-xy+y^2)^2}$

(f) $\dfrac{(1-b)(1-x)}{bx}$ (g) $\dfrac{(x-2)^2(x+4)}{(x-3)^2(x+3)}$ (h) $a(a-b)$ (i) $(x-y)$

14. (a) $\dfrac{45zx^3}{8ay^2}$ (b) $\dfrac{27bc}{64a}$ (c) $\dfrac{9a^2b^2c^2}{x^2y^2z^2}$ (d) $\dfrac{x}{x+y}$ (e) $\dfrac{(a+b)^2}{(a-b)^3}$ (f) $(x-y)^2$

(g) $(a+b)^2$ (h) $\dfrac{(x-1)(x-3)}{(x+2)(x+4)}$ (i) $\dfrac{(x-7)}{(x+6)}$

15. (a) $\dfrac{x^2-y^2}{x^2y^2}$ (b) $-\dfrac{1}{x^2}$ (c) $\dfrac{-2ca}{(a+b)(a+b+c)}$ (d) $\dfrac{a}{(1-a^2)(1+a+a^2)}$

(e) $\dfrac{4x^2}{x^2-y^2}$ (f) 1 (g) 1 (h) $\dfrac{1}{2ab}$ (i) $\dfrac{a-b}{x-y}$ (j) $\dfrac{b}{a}$

16. (a) $\dfrac{1}{x-3}$ (b) $\dfrac{3x^2+y^2}{2xy}$ (c) 1 (d) (a^2+b^2)

Exercise 6·1

(a) 1. $(3,1)$ 2. (2, 1) 3. (2, 2) 4. (1, 1) 5. (2, 3) 6. $(a+b, b-a)$

7. $\left(\dfrac{ab}{a+b}, \dfrac{ab}{a+b}\right)$ 8. $\left(\dfrac{ab}{a+b}, \dfrac{-ab}{a+b}\right)$ 9. (1, 1) 10. $(2,3)$ 11. (2, 1) 12. (2, 3)

(b) 13. (5, 1) 14. (2, 1) 15. (3, 1) 16. (3, 2) 17. (2, 3) 18. (2, 3)

19. (4, 2) 20. $\left(\dfrac{b^2+ab}{a^2+b}, \dfrac{ab-c}{a^2+b}\right)$ 21. (4, 3) 22. (6, –2) 23. (2, 1)

24. (2, 3) 25. (6, 2) 26. $(a, -b)$

Exercise 6·2

10. 60, 40 11. 120, 40 12. 11, 13 13. Father's age 65 years and Son's age 25 years 14. Fraction $\dfrac{3}{4}$ 15. Proper fraction $\dfrac{3}{11}$ 16. 37 or 73 17. width 25 m and length 50 m 18. Price of Notebook Tk 16 and price pencil Tk. 6 19. Tk. 4000 and Tk. 1000 20. (a) $(4,2)$ (b) $(3,2)$ (c) $(5,3)$ (d) $(5,-2)$ (e) $(-5,-5)$ (f) $(2,1)$

Exercise 7

5. (a) $\{5, 7, 9, 11, 13\}$ (b) $\{2, 3\}$

   (c) $\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33\}$ (d) $\{-3, -2, -1, 0, 1, 2, 3\}$

6. (a) $\{x : x$ is a natural number and $2 < x < 9\}$

   (b) $\{x : x$ is a multiple of 4 and $x < 28\}$

   (c) $\{x : x$ is a prime number and $5 < x < 19\}$

7. (a) $\{m, n), \{m\}, \{n\}, \phi$, 4

   (b) {5, 10,15), {5, 10}, {5, 15}, {10, 15}, {5}, {10}, {15}, ϕ, 8.

12. (a) $\{1, 2, 3, a\}$ (b) $\{a\}$ (c) $\{2\}$ (d) $\{1, 2, 3, a, b\}$ (e) $\{2, a\}$.

14. $\{1, 3, 5, 7, 21, 35\}$ 22. (b) 20 % (c) $\{1, 5\}$

Exercise 8.1

18. 340 sq. cm 19. 253.5 sq. cm

Exercise 10.3

12. (a) 62.8 cm (Approx) (b) 87.92 cm (Approx)

    (c) 131.88 cm (Approx)

13. (a) 452.16 sq. cm (Approx) (b) 907.46 sq. cm (Approx)

    (c) 1384.74 sq. cm (Approx)

14. 24.5 cm; 886.5 cm (Approx) 15. 4752 taka 17. 598.86 sq. cm (Approx)

18. 466.29 sq. cm.

Answer 11

1. (d) 2. (a) 3. (d) 4. (c) 5. (b) 6. (a) 7. (b) 8 (c)

9. (a) 75 (b) 75·02 (c) 0·02 10. 23·31 11. Tk. 2230·33

12. Arithmetic mean Tk. 150·43 Median Tk. 150 Mode Tk. 140 and Tk. 156

13. Arithmetic mean 11·44 years.

14. Arithmetic mean Tk. 66·65.

15. (a) 7 (c) 55.83 Approx. 16. (b) 69.7

18. Bengali 180, English 160, Mathematich 100, Science 120, Religion 160, Music 100.

THE END

2018

Academic Year

8-Math

সমৃদ্ধ বাংলাদেশ গড়ে তোলার জন্য যোগ্যতা অর্জন কর

– মাননীয় প্রধানমন্ত্রী শেখ হাসিনা

বিদ্যা পরম ধন

নারী ও শিশু নির্যাতনের ঘটনা ঘটলে প্রতিকার ও প্রতিরোধের জন্য ন্যাশনাল হেল্পলাইন সেন্টারে ১০৯ নম্বর-এ (টোল ফ্রি, ২৪ ঘণ্টা সার্ভিস) ফোন করুন

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড বাংলাদেশ

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