Problem Set 3
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Due: April 24th, 2025 at 11.59pm

Your answers should be produced in LaTeX, or Jupyter, and should include all relevant graphs and code. Code should be in the appropriate verbatim environment and properly documented. My python code is in the GitHub folder and should be more than enough if you are stuck.
The data are `rust_data_2020.csv`. The fields are `bus_id`, `period_id`, `y_it` (replacement decision) and `x_it` (mileage state).
Part 0 shouldn't take more than 5 minutes. You can choose to do EITHER Part 1 OR Part 2 - both are challenging.

Part 0: Pre-Estimation

1. Compute $P(x'|x, y = 0) = Pr(\Delta x_{it}|y_{it} = 0)$ in a separate first stage – **you should have 5 of them**.
2. Populate a transition probability matrix with states of $x_{it} \in \{0, \ldots, 200\}$.
   (Adjust your matrix so that rows sum to one!)

Part 1: Estimation via NFXP

Estimate the model using the NFXP approach of Rust. Computers are fast enough - you can probably get away with a finite differences for the gradient, but analytic gradients will be more reliable in addition to faster. If you are feeling ambitious `Jax` can do the gradient for you.

1. Compute $EV(x, \theta)$ for a given guess of the parameters via the fixed point.
2. Construct the CCP given your $EV(x, \theta)$ for every possible mileage state.
3. Construct the likelihood and its gradient contribution with respect to θ for every possible mileage state.
4. Finally, apply your likelihood and gradient contributions to observed data.

Part 2: Estimation via MPEC

Estimate the model using the MPEC method of Su and Judd. If you are feeling ambitious, you can try a modeling language like `pyMC` or `JuMP` or `AMPL` or `GAMS`. (I've solved this in `AMPL` but have no idea if other approaches work).

1. You will probably want to work out the analytic Jacobian where EV_θ is a free parameter.
2. Compare the results in a table and discuss them.
3. Plot the $EV(\cdot)$ you have obtained for both estimators.