

Index

1-forms, 98
 2-adic metric, 23

Abelian, 43
 affine automorphisms, 221
 angle, 87
 angle deficit, 209
 antipodal points, 69
 area preserving maps, 219
 atlas, 136

Banach–Tarski Theorem, 17, 275
 basepoint, 46, 48
 basic reflection, 105
 basic rotation, 105
 bijection, 21
 billiards, 213
 binary icosahedral group, 63
 Bolyai–Gerwein Theorem, 96
 Bolzano–Weierstrass Theorem, 72
 boundary, 23

Cauchy Integral Formula, 163, 165, 167
 Cauchy Rigidity Theorem, 17, 295
 Cauchy sequence, 154
 Cauchy’s Arm Lemma, 302
 Cauchy’s Theorem, 165
 Cauchy–Riemann equations, 165
 Cauchy–Schwarz inequality, 87
 Cayley’s Theorem, 44

chain, 32
 chain rule, 164
 circle, 56, 116
 circular polygon, 166
 classification of isometries, 130
 classification of surfaces, 36
 Clean Dissections, 292
 closed set, 23
 closure, 23
 combinatorial Gauss–Bonnet Theorem, 209
 compact, 26, 36
 compact Riemann surfaces, 203
 complex analysis, 11
 complex analytic function, 163
 composing loops, 46
 composition, 24
 cone surface, 13
 connect sum, 36
 continued fractions, 241, 245
 continuity, 24
 convex geodesic polygon, 145
 convexity, 110
 coordinate chart, 136
 covering group, 66
 covering space, 8, 65
 curve, 133
 cylinder, 34
 cylinder decomposition, 234

deck group, 66, 71
 deck transformation, 66
 Dehn twist, 235
 Dehn’s Dissection Theorem, 17, 98, 287
 determinant, 30
 diffeomorphism, 135
 differential representation, 223
 disk model, 125
 disk rigidity, 177
 disk-like sets, 143, 144
 dodecahedron, 63
 dot product, 22, 87
 double octagon, 232

edges, 5
 elliptic isometry, 130
 equivalence class, 32
 equivalence relation, 32
 essential curves, 5
 Euclidean cone, 207
 Euclidean cone surface, 208
 Euclidean plane, 1
 Euclidean space, 1, 87
 Euler characteristic, 37
 exponential function, 175

faces, 5
 Farey graph, 242, 245
 Farey related, 242
 figure 8, 69
 flat cone surface, 207
 flat torus, 16, 67, 195, 252
 functoriality, 49
 fundamental group, 43, 46, 71
 Fundamental Theorem of Algebra, 57, 180

Gauss map, 239
 Gauss–Bonnet Theorem, 107, 127
 Gauss–Bonnet Theorem, 151
 generalized circle, 116
 geodesic, 145
 geodesic half-plane, 150
 geodesic hyperbolic triangle, 228
 geodesic polygon, 127, 145
 geodesic triangle, 107
 geodesic triangulation, 150
 geodesics, 9, 105, 120, 123

Giraud’s Theorem, 107
 gluing, 2, 3, 31, 145
 gluing recipe, 147
 good metric space, 80
 Green’s Theorem, 100, 166
 group, 43

Hadamard’s Theorem, 154
 harmonic function, 179
 holomorphic function, 163
 homeomorphism, 25
 homomorphism, 44
 homotopy, 45
 homotopy equivalence, 45
 Hopf–Rinow Theorem, 154
 hyperbolic geometry, 9
 hyperbolic group actions, 224
 hyperbolic isometry, 121, 130, 143
 hyperbolic plane, 9, 224
 hyperbolic reflections, 229
 hyperbolic surface, 143, 147, 195

ideal triangle, 127, 242
 ideal vertices, 127
 Implicit Function Theorem, 28
 infimum, 31
 infinite differentiability, 169
 inner product, 121, 134
 interior, 23
 Invariance of Domain, 189
 Inverse Function Theorem, 135, 185
 irrational numbers, 247
 isometry, 135
 isomorphism, 44
 isomorphism theorem, 71, 74, 79

Jordan curve, 199
 Jordan domain, 199

Klein bottle, 35

latitude, 105
 lens space, 59
 Lie group, 30
 lifting property, 73
 line integrals, 98
 linear fractional transformation, 11, 115, 130
 linear functionals, 98

linear reflections, 229
 Liouville's Theorem, 179
 longitude, 105
 loop, 46

 manifold, 28, 59
 map, 24
 mapping class group, 260, 263
 marked loop, 190
 marked parallelograms, 251
 matrix group, 30
 maximal atlas, 136
 Maximum Principle, 170
 metric, 22
 metric completeness, 154
 metric space, 22
 Möbius band, 34
 Möbius geometry, 11
 Möbius transformation, 11, 115
 modular group, 14, 242, 244, 254
 moduli space, 16, 256, 263

 negative genus, 258
 nonorientable, 35
 nonorientable surfaces, 41
 norm, 87

 octagon, 4
 octagon surface, 9, 34
 one-to-one, 21
 onto, 21
 open set, 23
 orientable, 35
 overlap function, 136

 pairs of pants, 263
 pants decomposition, 265
 parabolic isometry, 130
 parallelogram tiling, 68
 path connectivity, 48
 path homotopic, 70
 pathification, 32
 Peano curve, 51
 periodic billiard paths, 219
 permutation group, 43
 Pick's Theorem, 92
 Poincaré homology sphere, 62
 Poincaré Recurrence Theorem, 219

Poincaré Uniformization Theorem, 201
 Polygon Dissection Theorem, 96
 polygonal billiards, 213
 power series, 172
 product space, 51
 projective plane, 34, 59, 69
 projective space, 59
 properly discontinuous action, 224
 Pythagorean Theorem, 90

 quadratic irrationals, 248
 quaternions, 62

 real linear transformation, 121
 removable singularity, 171
 Riemann Mapping Theorem, 188, 199
 Riemann surface, 195
 Riemann surface covering, 196
 Riemannian cover, 152
 Riemannian geometries, 9
 Riemannian isometry, 135
 Riemannian length, 139
 Riemannian manifold, 152
 Riemannian metric, 134, 139
 Riemannian surface, 139
 right-angled hexagon, 150

 saddle connection, 227
 Schwarz–Christoffel Transformation, 188
 scissors congruent, 287
 sectors, 207
 set, 21
 simple connectivity, 70
 Sinai Robins, 92
 Small Picard Theorem, 202
 smooth curve, 133, 137
 smooth surface, 136
 sphere, 1, 58
 spherical isometries, 103
 spherical metrics, 103
 spherical triangle, 107
 square torus, 1
 stereographic projection, 111, 181
 surface, 21, 27
 tangent plane, 103, 137

Taylor series, 163, 174
 Teichmüller space, 16, 254, 258, 263
 The Riemann sphere, 196
 Thomas Harriot, 107
 topological space, 25
 torus, 1, 58
 translation surface, 13, 211, 221
 tree, 69
 triangle group, 228
 triangle inequality, 22, 32, 87

 unit convergence condition (UCC), 172
 universal cover, 79, 156
 universal covering map, 57
 universal covering theorem, 80
 upper half-plane, 118

 Veech group, 14, 224, 228
 vertices, 5
 volume, 87

 William Wallace, 96
 winding number, 53

 X Theorem, 91