Visual Analysis of Algorithms

Version 0.0.4

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Algorithms and data structures notes.

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2.1 Common Complexity Classes

Name	Notation
Constant	$O(n)$
Logarithmic	$O(log\ n)$
Linear	$O(n)$
Linearithmic	$O(n\ log\ n)$
Quadratic	$O(n^2)$
Cubic	$O(n^3)$
Exponential	$O(2^n)$
Factorial	$O(n!)$

2.2 Binary Logarithm

Logarithm is the inverse function to exponentiation which means that the logarithm of a number n to the base b is the power to which b must be raised to produce n. The binary logarithm is the power to which the number 2 must be raised to obtain the value n.

$x = \log_2 n \quad \iff \quad 2^x = n$

- The number of bits in the binary representation of a positive integer n is the integral part of $\log_2(n) + 1$.

- The binary logarithm also frequently appears in the analysis of algorithms because binary logarithms occur in the analysis of algorithms based on two-way branching. If a problem initially has n choices for its solution, and each iteration of the algorithm reduces the number of choices by a factor of two, then the number of iterations needed to select a single choice is again the integral part of $\log_2(n)$.

— https://en.wikipedia.org/wiki/Binary_logarithm

2.3 Amortized Analysis

The motivation for amortized analysis is that looking at the worst-case run time can be too pessimistic. Instead, amortized analysis averages the running times of operations in a sequence over that sequence. Amortized analysis is a useful tool that complements other techniques such as worst-case and average-case analysis.

Imagine a dynamic list backed by a fixed size array that doubles once capacity is reached and require n inserts into the new fixed size array.

n	# of inserts
1	1
2	1
3	1
...	1
n	n
...	n + 1

$\frac{\text{\# of inserts}}{n} \rightarrow \frac{n+n+1}{n} \rightarrow \frac{2n+1}{n} \rightarrow \frac{2n}{n} \rightarrow 2$

— https://en.wikipedia.org/wiki/Amortized_analysis

Chapter 3

Data Structures

3.1 Heaps

3.1.1 Applications

- Heapsort
- Priority queue
- K-way merge

Chapter 4

Problem Solving Methods

4.1 Greedy Algorithms

4.1.1 Overview

A greedy algorithm is any algorithm that follows the problem-solving heuristic of making the locally optimal choice at each stage. In many problems, a greedy strategy does not produce an optimal solution, but a greedy heuristic can yield locally optimal solutions that approximate a globally optimal solution in a reasonable amount of time.

— https://en.wikipedia.org/wiki/Greedy_algorithm

4.1.2 Dijkstra's Shortest Path Algorithm

Dijkstra's shortest path algorithm is a search algorithm that finds the shortest path between a vertex and other vertices in a weighted graph.

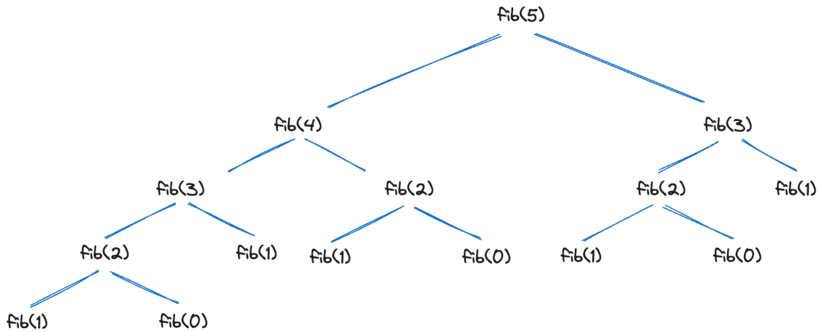
4.2 Recursion

The power of recursion evidently lies in the possibility of defining an infinite set of objects by a finite statement. In the same manner, an infinite number of computations can be described by a finite recursive program, even if this program contains no explicit repetitions.

— Niklaus Wirth, Algorithms + Data Structures = Programs, 1976[1]

4.2.1 Fibonacci Sequence

$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2} \qquad for\ n > 1$



$F_n = F_{n-1} + F_{n-2}$
$F_5 = F_4 + F_3$
$F_5 = (F_3 + F_2) + (F_2 + F_1)$
$F_5 = ((F_2 + F_1) + (F_1 + F_0)) + ((F_1 + F_0) + F_1)$
$F_5 = (((F_1 + F_0) + F_1) + (F_1 + F_0)) + ((F_1 + F_0) + F_1)$
$F_5 = (((1 + 0) + 1) + (1 + 0)) + ((1 + 0) + 1)$
$F_5 = 5$

```
export function fib(n: number): number {
    if (n == 0 || n == 1) {
        return n
    }
    return fib(n - 1) + fib(n - 2)
}
```

4.3 Dynamic Programming

To apply DP, must have the following:

[1] https://archive.org/details/algorithmsdatast00wirt/page/126]

1. Optimal substructure - Problem can be broken into sub-problems and solved recursively.
2. Overlapping sub-problems: Recursive algorithm solves the same sub-problems over and over.

If okay, then we can approach in 2 ways.

1. Top-down - Memoize with cache table.
2. Bottom-up - Solve sub-problems first and use their solutions to solve bigger sub-problems.

Chapter 5

Appendix

5.1 Resources

- LeetCode
- Project Euler
- The Algorithm Design Manual
- Elements of Programming Interviews