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WAKISSHA JOINT MOCK EXAMINATIONS
MARKING GUIDE
Uganda Advanced Certificate of Education
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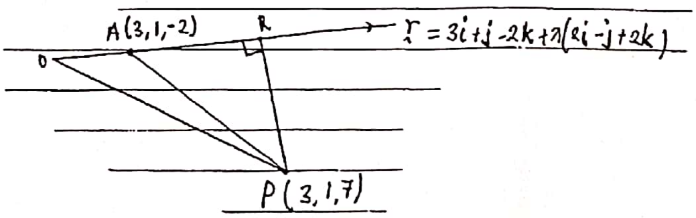
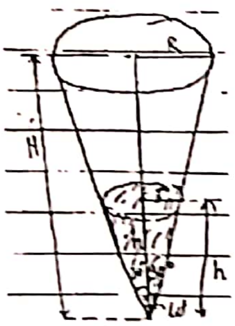
MATHEMATICS P425/1

3, 4, 10, 12

1.	$(a+b)^3 = a^2 + 3a^2b + 3ab^2 + b^3$ $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$ $(a+b)^3 = 6ab(a+b) + 3ab(a+b)$ $(a+b)^3 = 9ab(a+b)$ $(a+b)^2 = 9ab$ $\sqrt{\frac{(a+b)^2}{9}} = \sqrt{ab}$ $\frac{a+b}{3} = a^{\frac{1}{2}}b^{\frac{1}{2}}$	B_1 stating the identity B_1 substituting in identity B_1 simplifying 05
	$\log\frac{a+b}{3} = \log(ab)^{\frac{1}{2}}$ $\log\frac{a+b}{3} = \frac{1}{2}\log(ab)$ $\log\left(\frac{a+b}{3}\right) = \frac{1}{2}(\log a + \log b)$	B_1 Applying logarithm B_1 Drawing conclusion
		05
2.	Let $y = \text{cosec}^{-1}(x)$ Cosec$y = x$ - Cosec$y$ cot$y$ $\frac{dy}{dx} = 1$ $\frac{dy}{dx} = \frac{1}{\text{cosec}\,y \cot y}$ A_1 (making $\frac{dy}{dx}$ the subject) but cot$y$ = $\sqrt{(\text{cosec}^2 y - 1)}$ $\frac{dy}{dx} = \frac{-1}{(\text{cosec}\,y)\left(\sqrt{(\text{cosec}^2 y - 1)}\right)}$ $\frac{dy}{dx} = \frac{-1}{x\left(\sqrt{(x^2-1)}\right)}$ $\therefore \frac{d}{dx}\left(\text{cosec}^{-1}(x)\right) = \frac{-1}{x\sqrt{(x^2-1)}}$	M_1 ~~A_1~~ for differentiating and correct derivatives A_1 for making $\frac{dy}{dx}$ a subject. 05 M_1 for replacing cot$y$ and cosec$y$. A_1 for correct derivative B_1 (conclusion)
		05

3.	$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1+\tan x}\,dx$ Let $u = \tan x$ $du = \sec^2 x\,dx$ $dx = \frac{1}{\sec^2 x}du$ <table><tr><th>x</th><th>u</th></tr><tr><td>0</td><td>0</td></tr><tr><td>$\frac{\pi}{4}$</td><td>1</td></tr></table>	B_1 derivative of tan x B_1 change limits
		05
	$\int_0^1 \frac{\sec^2}{1+u}.\frac{1}{\sec^2 x}du$ $\int_0^1 \frac{1}{1+u}du$ $= [\ln(1+u)]_0^1$ $= \ln 2 - \ln 1$ $= \ln 2$ or 0.693	M_1 substituting u for tan x M_1 integration and substitution of limits. A_1 CAO
		05
4.	$\cos x + \sin 2x = 0$ $\cos x + 2\sin x \cos x = 0$ $\cos x(1+2\sin x) = 0$	B_1 (for expanding sin $2x$) ~~M_1 (for factorizing)~~
	$\cos x = 0$ $x = \cos^{-1(0)}$ $90^0, 270^0$ $= \frac{\pi}{2}$ *and* $\frac{3}{2}\pi$	M_1 (for reading angle) A_1 (for angles in radians) 05
	OR	
	$\sin x = \frac{-1}{2}$ $x = \sin{-1}\left(\frac{-1}{2}\right)$ $= 210^0, 330^0$ $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ $x = \pi/2, \frac{7}{6}\pi, \frac{3}{2}\pi, \frac{11}{6}\pi$.	M_1 A_1 (for angles in radius)
5.	Comparing $x^2+y^2-4x-5=0$ with $x^2+y^2+2gx+2fy+c=0$ ~~g = 0, f = 2 Centre(0,2)~~ c = (2, 0) Radius $r_1 = \sqrt{g^2+f^2-c}$ $= \sqrt{(0^2+(-2)^2 - {}^-5)} = \sqrt{9} = 3$	B_1 Finding the radius and centre

	Comparing $x^2+y^2-8x+12y+1=0$ with $x^2+y^2+2gx+2+y+c=0$ $= g = -4, \quad f = 1 \quad e = 1$ centre (4,-1) Radius $r_2 = \sqrt{(-4)^2+1^2-1} = 4$	B_1 Finding the radius and centre
	$c_1c_2 = \sqrt{(0-4)^2+(2-^-1)^2} = \sqrt{16+9} = 5$ $\overline{c_1c_2}^2 = \sqrt{25} = 5$ $r_1^2+r_2^2 = 3^2+4^2 = 25$ Since $c_1c_2 \neq r_1^2+r_2^2$ the circles are not orthogonal. ~~For two circles to be Orthogonal to each other~~ $\cancel{r_1^2+r_2^2 = \overline{c_1c_2}^2}$ ~~∴ The circles are orthogonal.~~	M_1 finding the $\overline{c_1c_2}^2$ ✓ M_1 Funding sum r_1^2 and r_2^2 ✓ A_1 drawing conclusion. (Not orthogonal)
6.	$\sqrt{\left(\frac{1+3x}{1-3x}\right)} = \sqrt{\frac{(1+3x)(1+3x)}{(1-3x)(1+3x)}} = (1+3x)(1-9x^2)^{\frac{-1}{2}}$	B_1 simplifying
	$(1-9x^2)^{\frac{-1}{2}} = 1+\frac{-1}{2}(-9x^2)+\ldots$ $= 1+\frac{9}{2}x^2+ \ldots$	B_1 for expanding $(1-9x^2)^{\frac{-1}{2}}$
	$(1+3x)\left(1+\frac{9x^2}{2}\right) = 1+3x+\frac{9x^2}{2}+\frac{27}{2}x^3+\ldots$ $\sqrt{\left(\frac{1+3x}{1-3x}\right)} = 1+3x+\frac{9x^2}{2}+\frac{27}{2}x^3+\ldots$	B_1 for correct expansion
	$\left(\frac{1+\frac{3}{7}}{1-\frac{3}{7}}\right)^{\frac{1}{2}} = 1+3\left(\frac{1}{7}\right)+\frac{9}{2}\left(\frac{1}{7}\right)^2+\frac{27}{2}\left(\frac{1}{7}\right)^3+\ldots$	M_1 for substitution
	$\sqrt{10} = 2(1+0.428571428+0.091836734+0.0393586) = 3.12$	A_1 (2dps) — CAO
		05
7.		
	$\underset{\sim}{AP} = \begin{pmatrix}3\\1\\7\end{pmatrix} - \begin{pmatrix}3\\1\\-2\end{pmatrix} = \begin{pmatrix}0\\0\\9\end{pmatrix}$	B_1 (for AP) ✓
	$\underset{\sim}{U} = \begin{pmatrix}2\\^-1\\2\end{pmatrix}$	

	$\underset{\sim}{AP}\wedge\underset{\sim}{U}=\begin{vmatrix} i & j & k \\ 0 & 0 & 9 \\ 2 & -1 & 12 \end{vmatrix}$	M_1 for crossing
	$\underset{\sim}{i}\,(0+9)-\underset{\sim}{j}(0-18)+\underset{\sim}{k}(0-0)$ $=9\underset{\sim}{i}+18\underset{\sim}{j}$	A1 for normal vector obtained
	$\lvert\underset{\sim}{AP}\wedge\underset{\sim}{U}\rvert=\sqrt{9^2+18^2}=\sqrt{405}$ $\lvert\underset{\sim}{U}\rvert=\sqrt{2^2+(-1)^2+2^2}=3$	M1 for magnitudes of both $AP\wedge U$ and U
	$D=\lvert\underset{\sim}{PR}\rvert\frac{\sqrt{405}}{3}=6.7082$ units $\quad D=PR=\frac{\sqrt{405}}{3}=\underline{6.7082}$	A1 for distance from P to the line.
		05
	OR	
	$\underset{\sim}{PR}=\left[\begin{pmatrix}3\\1\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\2\end{pmatrix}\right]-\begin{pmatrix}3\\1\\7\end{pmatrix}=\begin{pmatrix}0 & + & 2\lambda\\0 & - & -\lambda\\-9 & + & 2\lambda\end{pmatrix}$ $\underset{\sim}{PR}\cdot\underset{\sim}{U}=\begin{pmatrix}0 & + & 2\lambda\\0 & - & \lambda\\-9 & + & 2\lambda\end{pmatrix}\begin{pmatrix}2\\-1\\2\end{pmatrix}=0$ $4\lambda+\lambda-18+4\lambda=0$ $\lambda=2$ $\underset{\sim}{PR}=\begin{pmatrix}4\\-2\\-5\end{pmatrix}\Rightarrow D=\lvert\underset{\sim}{PR}\rvert=\sqrt{16+4+25}=6.7082$	B_1 for $\underset{\sim}{PR}$ M1(for dotting) A_1(for $\lambda=2$) M_1 (for both $\underset{\sim}{PR}$ and getting it's magnitude) A_1 = (for distance from P to the line)
8.	 $\frac{R}{H}=\tan 30^0$ $\frac{r}{h}=\frac{R}{H}$ $\frac{r}{h}\tan 30^0$ $r=\frac{h}{3}$	B_1 (Finding r)
	V= $\frac{\pi}{3}\left(\frac{h}{\sqrt{3}}\right)^2 h=\frac{\pi}{9}h^3$ $\frac{dv}{dh}=\pi h^2$ but $\frac{\pi h^3}{9}=9\pi$ h= $\sqrt[3]{81}$ or 4.331 $\frac{dh}{dt}=\frac{dv}{dt}\div\frac{dv}{dh}$	$M_1$for $\frac{dv}{dh}$ (Finding the correct derivative) B_1 for h = $\sqrt[3]{81}$ when volume left is $9\pi cm^3$ $M_1$for substituting for $\frac{dv}{dh}$ and h

$= -9 \times \frac{3}{\pi h^2}$

$= -9 \times \frac{3}{\pi (3)^2} = \underline{0.4591 \text{ cm s}^{-1}}$

~~$= \frac{1}{11}$ of $0.3183cm$ per minute~~

~~Hence it is decreasing at rate of 0.3183 cm per minute.~~

A_1 for rate with correct units.

05

9. (a) Let $Z_1 = 2 - i$, $Z_2 = 3i - 1$ and $Z_3 = (i+3) = -3 - i$

(i) $|Z_1| = \sqrt{2^2 + (-1)} = \sqrt{5} = |Z_1|^2 = \left(\sqrt{5}^2\right) = 5$

$|Z_2| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

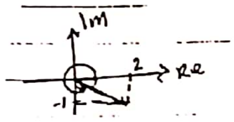
$|Z_3| = \sqrt{1^2 + 3^2} = \sqrt{10} = |z_3|^3 = \cancel{10}\sqrt{10}$

~~$|Z|) = 666.86^0$~~ $|z| = \left|\frac{z_1 z_2}{z_3}\right| = \frac{5 \times \sqrt{10}}{10\sqrt{10}} = \underline{\frac{1}{2}}$

M_1 method of getting modulus. (any method)

A_1 for modulus of Z.

(ii) Arg $Z_1 = \tan^{-1}\left(\frac{1}{2}\right)$



$360^0 \tan^{-1}\left(\frac{1}{2}\right)$

333.43^0 ✓✓ or $-26.$

Arg $\left(Z^2{}_1\right) = 666.86^0$

$Arg\left(Z_2\right) = \tan^{-1}\left(\frac{3}{-1}\right)$

$= 180^0 - \tan^{-1}(3)$

$= 108.43^0$ ✓✓

M_1 for method of getting argument (any method)

$Arg\left(Z_3\right) = \tan^{-1}\left(\frac{+1}{+3}\right) = 18.43$ ~~$180^0 = 198.43$~~ Arg $Z_3 = 3 \times 18.43$ ✓ $= \underline{55.29}$

A_1(argument of Z)

~~$Arg\left(Z_3{}^3\right) = 198.43 \times 3 = 595.29^0$~~

~~Arg Z $= 666.86^0 + 108.43^0 - 595.29^0 = 180^0$~~ Arg $\frac{z_1 z_2}{z_3} = 666.86 + 108.43 - 55.29$

$= \underline{720^0} = \underline{0^0}$.

Z= $\frac{1}{2}\left(\cos 180 + \sin 180^0\right)$ $z = \frac{1}{2}(\cos 0^\circ + \sin 0^\circ)$

For substitution and correct polar form of Z M_1 A_1 (correct polar form)

06

OR

$$Z = \frac{(2-i)^2 (+3i-1)}{\left[-(i+3)^3\right]}$$

$$= \frac{\left[(2-i)(2-i)\right](+3i-1)}{-(i+3)\left[(i+3)(i+3)\right]}$$

M_1

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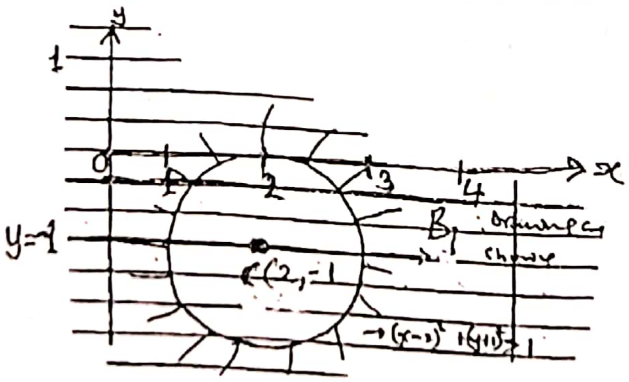
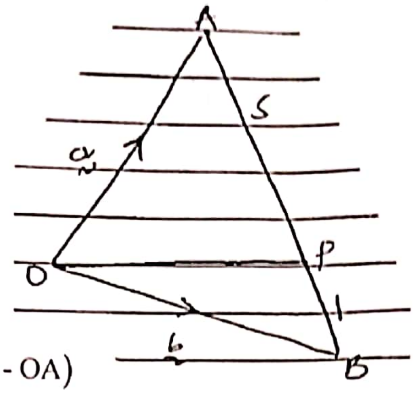
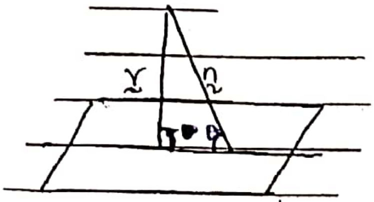
	$= \frac{(4-2i-2i+i^2)(3i-1)}{-(i+3)(i^2+3i+3i+9)}$ $\frac{(4-1-4i)(3i-1)}{-(i+3)(-1+6i+9)}$ $\frac{(3-4i)(3i-1)}{-(i+3)(8+6i)}$ $= \frac{9i-3-12i^2+4i}{-(8i+6i^2+24+18i)} = \frac{9+13i}{(26i+18)} = \frac{+9+13i}{(18+26i)}$ $\frac{(9i+13i)1(8-26i)+4i}{(18+26i18-26i)} = \frac{162-234i+234i+338}{324-468i+468i+676} = \frac{+500}{1000} = \frac{+1}{2}$	A_1
	$\lvert Z\rvert = \sqrt{\left(\frac{-1}{2}\right)^2 + (0)^2} = \frac{1}{2}$ $Arg Z = \tan^{-1}\left(\frac{0}{\frac{-1}{2}}\right) = \cancel{180^\circ} - \tan^{-1}\left(\frac{0}{\frac{1}{2}}\right) = 0^\circ$ $\cancel{180^\circ - 0^\circ}$ $\cancel{= 180^\circ}$ $Z = \frac{1}{2}(\cos 0^\circ + i\sin 0^\circ)$ $\cancel{Z = \frac{1}{2}(\cos 180^\circ + i\sin 180^\circ)}$	M_1 A_1 M_1A_1
9.	(b) Let $Z = x+iy$ $z-2+i = x+iy-2+i$ $\lvert z-2+i\rvert = \sqrt{(x-2)^2+(y+1)^2}$ $\lvert z-2+i\rvert^2 = (x-2)^2+(y+1)^2$ Is the equation of the circle of centre (2, 1) and $\sqrt{(x-2)^2+(y+1)^2} = 1$ means the radius is 1 unit.  Complex number of it's centre is 2-i	B_1 finding the modulus of complex B_1 Squaring both sides M1 writing the coordinates of radius and centre. A_1 CAO (correct equation of the locus) B_1 Drawing and showing and unwanted region. B_1 for centre of wanted region in complex form.
		06

10.	**(a)**	
		
	$\underset{\sim}{OP} = \underset{\sim}{OA} + \underset{\sim}{AP}$	
	$OP = \underset{\sim}{a} + \frac{5}{6}(OB - OA)$	
	$= \underset{\sim}{a} + \frac{5}{6}(\underset{\sim}{b} - \underset{\sim}{a})$	M_1 finding vector OP in terms of a and b
	$= \underset{\sim}{a} + \frac{5\underset{\sim}{b}}{6} - \frac{5}{6}\underset{\sim}{a}$	
	$= \frac{\underset{\sim}{a} + 5\underset{\sim}{b}}{6}$	A_1 C.A.O
	$\underset{\sim}{OP} = \frac{1}{6}\left[i + k + 5(i - j + 3k)\right]$	
	$\underset{\sim}{OP} = \frac{1}{6}[i + k + 5i + 15k]$	M_1 Substituting for a and b
	$\underset{\sim}{OP} = \frac{1}{6}[6i - 5j + 16k]$ $\quad \underset{\sim}{OP} = \underset{\sim}{i} - \frac{5}{6}\underset{\sim}{j} + \frac{8}{3}\underset{\sim}{k}$	A_1 C.A.O (for OP) 04
	Therefore the position vector of P is $\frac{1}{6}(6i - 5j + 16k)$	
	(b)(i)	
	$n = \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ -1 & -3 & 2 \end{vmatrix}$	
	$\underset{\sim}{\hat{n}} = i\begin{vmatrix} -3 & 1 \\ -3 & 2 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} + k\begin{vmatrix} 1 & -3 \\ -1 & -3 \end{vmatrix}$	M_1 (Cross product. Getting the normal vector)
	$\hat{n} = \underset{\sim}{i}(-6+3) - \underset{\sim}{j}(2+1) + \underset{\sim}{k}(-3+3)$	
	$\hat{n} = -3\underset{\sim}{i} - 3\underset{\sim}{j} - 6\underset{\sim}{k}$.	A_1 (Value of ~~Finding~~ the normal vector)
	Hence vector normal to the plane is $n = -\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$.	
	Suppose the plane contains any point Q (x, y, z)	
	Using $\underset{\sim}{r} \cdot \underset{\sim}{n} = \underset{\sim}{n} \cdot \underset{\sim}{a}$ $\quad r \cdot n = n \cdot a$	
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \cancel{\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}} = \cancel{\begin{pmatrix} -3 \\ 5 \\ 6 \end{pmatrix}} \cdot \cancel{\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}}$ $\quad \begin{pmatrix} x \\ y \\ 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$	M_1 Applying the dot product
	$x + y + 2z = 1 - 2 + 6$ $x + y + 2z = 5$	A_1 – Equation of the plane 04

	$-3x+5y+6z=-3-10+18$ $-3x+5y+6z=5$ *OR* $3x-5y-6z+5=0$	A_1 for equation of the plane
		04
	(b) (ii) Let r_1 = vector parallel to the plane $r_1 = 4i + 3j + 2k$ Plane : $3x + 3y + 6Z = 5$	
	 $r.n = \lvert r \rvert . \lvert n \rvert \sin\theta \rvert$ $\theta = \sin^{-1}\left[\dfrac{r.n}{\lvert r\rvert \lvert n\rvert}\right]$	B_1 stating the dot product (cos θ – deducts from 90°)
	$\theta = \sin^{-1}\left[\dfrac{\begin{pmatrix}4\\3\\2\end{pmatrix}.\begin{pmatrix}1\\1\\2\end{pmatrix}}{\sqrt{(4^2+3^2+2^2)((1)^2+1^2+2^2)}}\right]$	M_1 substituting for r and n.
	$\theta = \sin^{-1}\dfrac{15}{\sqrt{29.\times 70}}$ $\theta = 19.44625999$ $\theta = \sin^{-1}\left(\dfrac{11}{\sqrt{29}.\sqrt{6}}\right)$ $\theta = 56.5^\circ$	M_1 Finding the value of θ A_1 CAO. (04)
	∴ The angle the line makes with the plane is 19.4463^0	
		04
11	**(a)** $x=\dfrac{t}{1+t}$, $y=\dfrac{t^2}{1+t}$ $x+xt=t$ $t(1-x)=x$ $t=\dfrac{x}{1-x}$ $y=\dfrac{\left(\dfrac{x}{1-x}\right)^2}{1+\dfrac{x}{1-x}}$	M_1 Substituting for t in y.

$$y=\frac{\frac{x^2}{(1-x)^2}}{\frac{1-x+x}{1-x}}$$

$$y=\frac{x^2}{(1-x)^2}\times\frac{1-x}{1}$$

02

$$y=\frac{x^2}{1-x}$$ ✓

A_1 CAO

(b)

$$y=\frac{x^2}{1-x}$$

$$\frac{dy}{dx}=\frac{(1-x)2x-x^2(-1)}{(1-x)^2}$$

M_1 (Differentiation)

$$\frac{dy}{dx}=\frac{2x-x^2}{(1-x)^2}$$

~~[illegible]~~

At turning point $\frac{dy}{dx}=0$

$$\frac{2x-x^2}{(1-x)^2}=0$$

$x=0$ or $x=2$

A_1 (Getting $x=0$ & $x=2$ when $\frac{dy}{dx}=0$)

When x = 0 y = 0 (0, 0)

When x = 2 y = - 4 (2, -4)

B_1 getting turning points

05

	L	x=0	R	L	x=2	R
$\frac{dy}{dx}$	-ve	0	+ve	+ve	0	-ve
		MIN			MAX	

M_1 (for testing the nature)

= (0, 0) minimum turning point.
= (2, -4) Maximum turning point.

For both minimum turning point A_1 maximum turning point

(c)

Vertical asymptote fix is not defined.

1- x = 0

x = 1

A_1 (vertical asymptote)

Slanting asymptotes

$$1-x\overline{)x^2}\quad -x-1$$

$$\frac{-x^2-x}{x}$$

$$\frac{-x-1}{1}$$

$$y=-x-1+\frac{1}{1-x}$$

$$As\ x\to\pm\propto\ \frac{1}{1-x}\to 0$$

$$y=-x-1$$

Is the stating asymptote ✓

03

B₁ Finding vertical Asymptote

Critical values are 0, 1

	X < 0	0 < X < 1	X > 1
X^2	+	+	+
1-X	+	+	-
y	+	+	-

M₁ for the table

Graph at the back

B2 (Graph).

02

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12 (a)

$y = e^{2x}\sin 3x$

$$y=e^{2x}\sin 3x$$

$$\frac{dy}{dx}=2e^{2x}\sin 3x+3e^{2x}\cos 3x$$

M₁ (for 1st derivative)

$$\frac{dy}{dx}=2y+3e^{2x}\cos 3x$$

$$3e^{2x}\cos 3x=\frac{dy}{dx}-2y$$

B₁ (for making $3e^{2x}\cos 3x$ a subject)

$$\frac{d^2y}{dx^2}=2\frac{dy}{dx}+6e^{2x}\cos 3x-9e^{2x}\sin 3x$$

M₁ (for 2nd derivative)

$$\frac{d^2y}{dx^2}=\frac{2dy}{dx}+2\left(3e^{2x}\cos 3x\right)-9y$$

06

$$\frac{d^2y}{dx^2}=\frac{2dy}{dx}+2\left(\frac{dy-2y}{dx}\right)-9y$$

M₁ (for substituting $3e^{2x}\cos 3x$)

$$\frac{d^2y}{dx^2}=\frac{2dy}{dx}+\frac{2dy}{dx}-4y-9y$$

$$\frac{d^2y}{dx^2}=\frac{4dy}{dx}-13y$$

A₁ (for $\frac{d^2y}{dx^2}$)

$$\frac{d^2y}{dx^2}-\frac{4dy}{dx}+13y=0$$

B₁ (for as required)

	(b) $\int_0^{\frac{3}{2}} \frac{x^3}{\sqrt{1-x^2}}dx$	
	Let $x = \sin\theta$ $dx = \cos\theta d\theta$	B_1 Differentiate x with respect to x.
	$\begin{array}{c\|c\|c} x & \sin\theta & \theta \\ 0 & 0 & 0 \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\pi}{3} \end{array}$	B_1 Changing the limits
	$\int_0^{\frac{\pi}{3}} \frac{\sin^3\theta}{\cos\theta}\cos\theta d\theta$	M_1 substitues for dX and changing limits.
	$\int_0^{\frac{\pi}{3}} \sin^3\theta d\theta$ $\int_0^{\frac{\pi}{3}} \sin\theta\left(1-\cos^2\theta\right)du$ $\int_0^{\frac{\pi}{3}} \left(\sin\theta - \sin\theta\cos^2\theta\right)d\theta$	1.
	$\left\|-\cos\theta + \frac{1}{3}\cos^3\theta\right\|_o^{\frac{\pi}{3}}$	M_1 Finding the integral
	$\left(-\cos\frac{\pi}{3} + \frac{1}{3}\cos^3\frac{\pi}{3}\right) - \left(-\cos + \frac{1}{3}\cos^3 o\right)$	M_1 Substituting the new Limits
	$= \frac{-1}{2} + \frac{1}{3}\left(\frac{1}{2}\right)^3 - \left(-1+\frac{1}{3}\right)$ $== \frac{-1}{2} + \frac{1}{24} + 1 - \frac{1}{3}$ $= 0.208333$ or $5/24$	A_1 CAO
		12
13	**(a)** L.H S Sin $(2\sin^{-1}x + \cos^{-1}x)$ Let $A = \sin^{-1}x$; $\sin A = x$ $B = \cos^{-1}x$; $\cos B = x$	B_1 Expressing both sin A and cos B in terms of x — making x the subject.
	Sin (2A +B) = Sin 2Acos B + sin B cos 2A $= 2\sin A\cos A\cos B + \sin B(1-2\sin^2 A)$	B_1 Expanding the identity

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	But $\sin B = \sqrt{1-\cos^2 B}$ $= \sqrt{1-x^2}$ $\cos A = \sqrt{1-\sin^2 A}$ $= \sqrt{1-x^2}$ $\Rightarrow \sin(2A+B) = 2\sin A\cos A\cos B + \sin B(1-2\sin^2 A)$ $= 2x\left(\sqrt{1-x^2}\right)x + \left(\sqrt{1-x^2}\right)(1-2x^2)$ $= 2x^2\sqrt{1-x^2} + \sqrt{1-x^2}(1-2x^2)$ $= \sqrt{1-x^2}(2x^2+1-2x^2)$ $= \sqrt{1-x^2}$ $\therefore \sin(2\sin^{-1}x + \cos^{-1}x) = \sqrt{1-x^2}$	For both expressing sin B and Cos A interms x B_1 M_1Substituting in the formular 05 A_1 CAO
		05
13	**(b)** $Sin3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$ $= 2\sin\theta\cos^2\theta + \sin\theta(1-2\sin^2\theta)$ $= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$ $= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$ Hence From $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ $1 = 6t - 8t^3$ $1 = 2(3t - 4t^3)$ $t = \sin\theta$ $1 = 2(3\sin\theta - 4\sin^3\theta)$ $1 = 2\sin 3\theta$ $\text{Sin}3\theta = \frac{1}{2}$ $3\theta = \sin^{-1}\left(\frac{1}{2}\right)$. $3\theta = 30^0, 150^0, 390^0, 510^0, 750^0, 870^0$ $\theta = 10^0, 50^0, 130^0, 170^0, 250^0, 290^0$ $t = \sin 10^0, \sin 50^0, \sin 130^0, \sin 170^0, \sin 250^0, \sin 290^0$ $t = 0.1736, 0.7660, 0.1736, -0.9397, -0.9397.$ $\therefore t = 0.1736, 0.7660, -0.9397.$	M_1 Expanding compound angle M_1 change to single angle $A_1 =$ correct expansion B_1 solving for 3θ 07 M_1 for reading angles A_1 correct values of θ B_1 All 3 correct values of t
		07

14	$\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2}=1$ $b^2x^2+a^2m^2mcx+a^2c^2=a^2b^2$ $(b^2+a^2m^2)x^2+(2a^2mc)x+(a^2c^2-a^2b^2)=0$ *For tangency* $B^2-4AC=0$ $=(2a^2cm)^2-4(b^2+a^2m^2)(a^2c^2-a^2b^2)=0$ $=4a^4m^2c^2-4b^2a^2c^2+4b^4a^2-4a^4m^2c+4a^4m^2b^2=0$ $=\frac{4b^2a^2c^2}{4a^4b^2}=\frac{4b^4a^2}{4b^2a^2}+\frac{4a^4m^2b^2}{4b^2a^2}$ $c^2=b^2+a^2m^2$	M_1(for substituting for y) B_1 (for tangency) and substituting M_1 (method) 04 A_1(for required equation) **04**
	(i) Given $a^2=23 and\quad b^2=3$ $=c^2=3+23m^2\rightarrow(1)$ Given $a^2=14$ and $b^2=4$ $=c^2=4+14m^2\rightarrow(2)$ (1) - (2) $C^2 = 3 + 23m^2$ $\underline{C^2 = 4 + 14m^2}$ - $0 = -1 + 9m^2$ $m=\pm\frac{1}{3}$ ✓ $c^2=4+14\left(\frac{1}{9}\right)$ $9c^2=36+14$ $9c^2=50$ $c=\frac{\pm\sqrt{50}}{3}$ $y=\pm\frac{1}{3}x\pm\frac{\sqrt{50}}{3}$ or $3y=\pm x\pm\sqrt{50}$ or $3y=\pm x\pm5\sqrt{2}$ ✓✓	 M_1(for (1) and (2) equation A_1 (for $M = \pm ^1/_3$) 04 B_1 (for $C = \pm\frac{5\sqrt{2}}{3}$) B_1 (Correct equation by substituting them)
		04
14	(ii) Given $a^2=16$ and $b^2 = 9$ but $c^2 = b^2 + a + 2m^2$ $c^2 = 9 + 16m^2$ ……….(1) From $y = mx + c$ $3= -3m + c$ $c = 3 (1+m)$………….(2) $[3(1+m)]^2=9+16m^2$ $9(1+2m+m^2)=9+16m^2$ $9+18m+9m^2=9+16m^2$ $18m=7m^2$	 M_1 (for both (1) and (2) M_1 (for substituting for c in equation (1)from equation (2))

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	$(7m-18)m=0$ $m=0\,,\ \frac{18}{7}$	A_1(for both m = 0 and m = $^{18}/_7$)
	When m = 0 ⇒ c = 3(1) = 3 = y = 3 is equation of tangent at (-3, 3) When m = $\frac{18}{7}$ = c = 3(1 + $\frac{18}{7}$) = $\frac{75}{7}$ = y = $\frac{18}{7}x+\frac{75}{7}$ or $7y=18x+75$ is other equation of tangent at (-3, 3)	B_1(for both equations of tangents at (-3, 3)
		04
15	(a) Let the first term be a the common difference be d n^{th} term of AP = a+ (n-1)d a + 7d = 2(a + 2d) ....(1) sum of n term is given by Sn = $\frac{n}{2}$ (2a + (n - 1)d) $s_8=39=\frac{8}{2}(2a+7d)$.............(2)	
	From equation (1) a + 7d = 2a + 4d a = 3d ....⊘	M_1 for the sum of eight terms
	Substituting equation ⊘ in equation (ii) $\frac{8}{2}(2(3d)+7d)=3a$ $4(6d+7d)=3a$ $52d=3a$ $d=\frac{3}{4}$........⊘⊘	B_1 C.A.O
	Substituting equation ⊘⊘ into ⊘ a = 3 ($\frac{3}{4}$) a = $\frac{9}{4}$	B_1 C.A.O
	The first three term of the progression are given by a + a + d + a + 2d $\frac{9}{4}+3+\frac{15}{4}$ or $1^{st} = 9/4$, $2^{nd} = 3$, $3^{rd} = 15/4$	B_1 C.A.O
	= Sum of n term is given by $Sn=\frac{n}{2}(2a+(n-1)d)$ $Sn=\frac{n}{2}\left(2\left(\frac{9}{4}\right)+(n-1)\frac{3}{4}\right)$	M_1 for substitution in the formula of sum of n^{th} term

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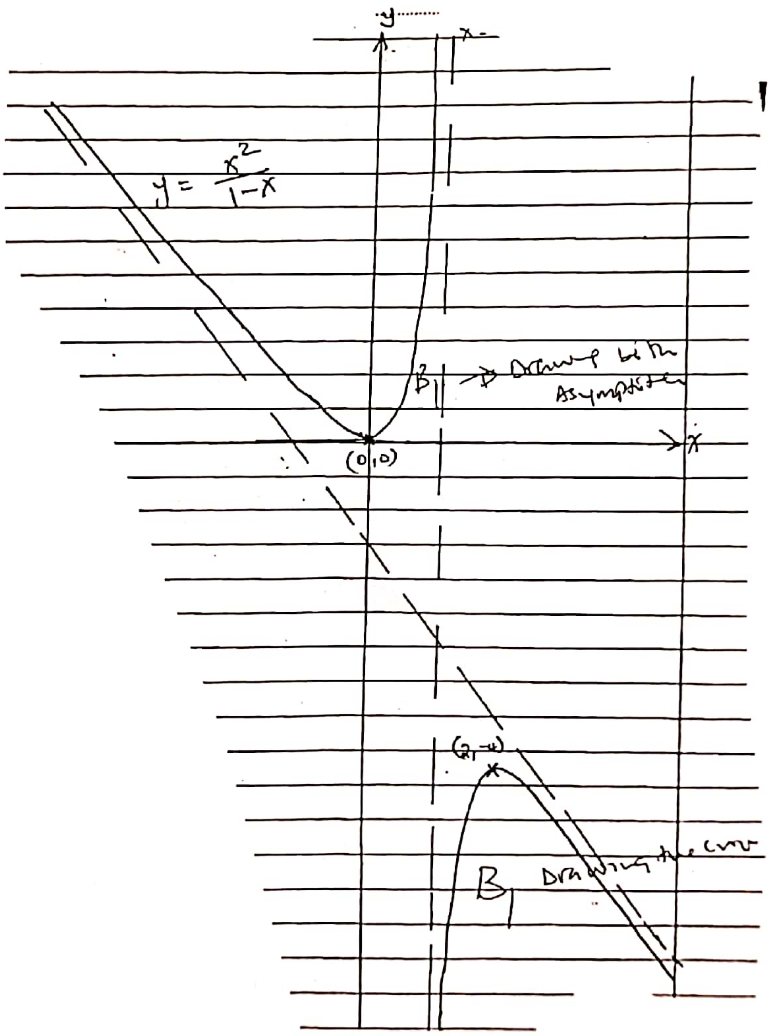
	$S_n=\frac{n}{2}\left(\frac{18}{4}+\frac{3}{4}n-\frac{3}{4}\right)$ $=\frac{n}{2}\left(\frac{15}{4}+\frac{3}{4}n\right)$ $\therefore Sn=\frac{3}{8}n(n+5)$	A_1 C.A.O
		06
15	**(b)** The series $1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+$.......... is a G.P with 1^{st} term a = 1 common ratio $r=\frac{1}{5}$ Sum to infinity $S\infty=\frac{a}{1-r}$ $=\frac{1}{1-\frac{1}{5}}$ $=\frac{1}{\frac{4}{5}}$ $S\infty=\frac{5}{4}$	M_1 for substitution A_1 (Getting the sum to infinity)
	Let the number of term whose sum will differ from sum to infinity by less that 10^{-6} ~~then.~~ be n. $Sn=\frac{a(1-r^n)}{1-r}$ $\quad \|r\|\angle 1$ $a=1 \qquad r=\frac{1}{5}$ $\frac{1\left(1-\left(\frac{1}{5}\right)^n\right)}{1-\frac{1}{5}}$ $=\frac{1-\left(\frac{1}{5}\right)^n}{\frac{4}{5}}$ $=\frac{5}{4}\left(1-\left(\frac{1}{5}\right)^n\right)$	B_1 (getting the sum of n - terms)

		But $S\alpha - Sn \angle 10^{-6}$	
		$\frac{5}{4} - \frac{5}{4}\left(1-\left(\frac{1}{5}\right)^n\right) \angle 10^{-6}$	
		$\frac{5}{4}\left(\frac{1}{5}\right)^n \angle 10^{-6}$	B_1(Difference between $S_\infty - S_n$)
		$5\left(\frac{1}{5}\right)^n \angle 10^{-6} \times 4$	
		$5^{1-n} \angle 4\times 10^{-6}$	
		Introducing $\log_{10}$	
		$\log 5^{1-n} < \log\left(4x10^{-6}\right)$	M_1 for introducing $\log_{10}$ or In.
		$1-n < \frac{\log\left(4\times 10^{-6}\right)}{\log 5}$	
		$1-n < -7.7227$	
		$n > 1+7.7227$	
		$n > 8.7227$	
		$\therefore n = 9 \quad terms$ ✓	A_1 (correct value of n)
			06
16	(a)		
		x = number of antelopes present	
		$\frac{dx}{dt} \propto (x+5)$	M_1 writing differential equation
		$\frac{dx}{dt} = -k(x+5)$	M_1 introducing the scalar k.
		$\int \frac{dx}{(5+x)} = \int -kdt$	M_1(for separating variables)
		$In(5+x) = -kt + c$	M_1(integrating on both sides).
		When t = 0 $\quad$ x = 60	A_1 correct integral.
		ln (5 + 60) = -k(0) + c	M_1 substitute for t and x
		ln (65) = c ✓	A_1 correct expression of c.
		ln (5+ x) = -kt + ln 65	
		ln (5 + x) –ln (65) = -kt	
		$In\left(\frac{5+x}{65}\right) = -kt$	
		$5+x = 65e^{-kt}$	
		$x = \left(65e^{-kt}\right) - 5$	

	When t = 6 months, $x = 40$ $40 = 65e^{-6k} - 5$ $45 = 65e^{-6k}$ $\frac{9}{13} = e^{-6k}$ $k = \frac{-1}{6} ln\left(\frac{9}{13}\right)$ or 0.0613 $x = \frac{1}{6}e^{\frac{t}{6}ln\left(\frac{9}{13}\right)} - 5$	B_1 (for value of k) A_1 for correct solution of D.E
		09
	(b) On 15th November 2019, t = 8.5 $x = 65e^{\frac{8.5}{6}ln\left(\frac{9}{13}\right)} - 5$ $x = 33.6$ antelopes $x = 33$ antelopes.	B_1 for t = 8.5 M_1 substituting t = 8.5 A_1 CAO
		03



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END