



1.5. Acknowledgments. The author is very thankful to O. Ramaré for his help and feedback, especially regarding §5 and Appendix B. He is also much indebted to A. Booker, B. Green, H. Kadiri, D. Platt, T. Tao and M. Watkins for many discussions on Goldbach's problem and related issues. Thanks are also due to B. Bukh, A. Granville and P. Sarnak for their valuable advice.

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The present work would most likely not have been possible without free and publicly available software: PARI, Maxima, Gnuplot, VNODE-LP, PROFIL / BIAS, SAGE, and, of course, L^AT_EX, Emacs, the gcc compiler and GNU/Linux in general. Some exploratory work was done in SAGE and Mathematica. Rigorous calculations used either D. Platt's interval-arithmetic package (based in part on Crlibm) or the PROFIL/BIAS interval arithmetic package underlying VNODE-LP.

2. PRELIMINARIES

2.1. Notation. As is usual, we write μ for the Moebius function, Λ for the von Mangoldt function. We let $\tau(n)$ be the number of divisors of an integer n and $\omega(n)$ the number of prime divisors. For p prime, n a non-zero integer, we define $v_p(n)$ to be the largest non-negative integer α such that $p^\alpha | n$.

We write (a, b) for the greatest common divisor of a and b . If there is any risk of confusion with the pair (a, b) , we write $\gcd(a, b)$. Denote by (a, b^∞) the divisor $\prod_{p|b} p^{v_p(a)}$ of a . (Thus, $a/(a, b^\infty)$ is coprime to b , and is in fact the maximal divisor of a with this property.)

As is customary, we write $e(x)$ for $e^{2\pi i x}$. We write $|f|_r$ for the L_r norm of a function f .

We write $O^*(R)$ to mean a quantity at most R in absolute value.

2.2. Dirichlet characters and L functions. A Dirichlet character $\chi : \mathbb{Z} \rightarrow \mathbb{C}$ of modulus q is a character χ of $(\mathbb{Z}/q\mathbb{Z})^*$ lifted to \mathbb{Z} with the convention that $\chi(n) = 0$ when $(n, q) \neq 1$. Again by convention, there is a Dirichlet character of modulus $q = 1$, namely, the *trivial character* $\chi_T : \mathbb{Z} \rightarrow \mathbb{C}$ defined by $\chi_T(n) = 1$ for every $n \in \mathbb{Z}$.

If χ is a character modulo q and χ' is a character modulo $q'|q$ such that $\chi(n) = \chi'(n)$ for all n coprime to q , we say that χ' *induces* χ . A character is *primitive* if

it is not induced by any character of smaller modulus. Given a character χ , we write χ^* for the (uniquely defined) primitive character inducing χ . If a character $\chi \bmod q$ is induced by the trivial character χ_T , we say that χ is *principal* and write χ_0 for χ (provided the modulus q is clear from the context). In other words, $\chi_0(n) = 1$ when $(n, q) = 1$ and $\chi_0(n) = 0$ when $(n, q) = 0$.

A Dirichlet L -function $L(s, \chi)$ (χ a Dirichlet character) is defined as the analytic continuation of $\sum_n \chi(n)n^{-s}$ to the entire complex plane; there is a pole at $s = 1$ if χ is principal.

A non-trivial zero of $L(s, \chi)$ is any $s \in \mathbb{C}$ such that $L(s, \chi) = 0$ and $0 < \Re(s) < 1$. (In particular, a zero at $s = 0$ is called “trivial”, even though its contribution can be a little tricky to work out. The same would go for the other zeros with $\Re(s) = 0$ occurring for χ non-primitive, though we will avoid this issue by working mainly with χ primitive.) The zeros that occur at (some) negative integers are called *trivial zeros*.

The *critical line* is the line $\Re(s) = 1/2$ in the complex plane. Thus, the generalized Riemann hypothesis for Dirichlet L -functions reads: for every Dirichlet character χ , all non-trivial zeros of $L(s, \chi)$ lie on the critical line. Verifiable finite versions of the generalized Riemann hypothesis generally read: for every Dirichlet character χ of modulus $q \leq Q$, all non-trivial zeros of $L(s, \chi)$ with $|\Im(s)| \leq f(q)$ lie on the critical line (where $f : \mathbb{Z} \rightarrow \mathbb{R}^+$ is some given function).

2.3. Fourier transforms. The Fourier transform on \mathbb{R} is normalized as follows:

$$\widehat{f}(t) = \int_{-\infty}^{\infty} e(-xt)f(x)dx$$

for $f : \mathbb{R} \rightarrow \mathbb{C}$.

The trivial bound is $|\widehat{f}|_{\infty} \leq |f|_1$. Integration by parts gives that, if f is differentiable k times outside finitely many points, then

$$(2.1) \quad \widehat{f}(t) = O^* \left(\frac{|\widehat{f^{(k)}}|_{\infty}}{2\pi t} \right) = O^* \left(\frac{|f^{(k)}|_1}{(2\pi t)^k} \right).$$

It could happen that $|f^{(k)}|_1 = \infty$, in which case (2.1) is trivial (but not false). In practice, we require $f^{(k)} \in L_1$. In a typical situation, f is differentiable k times except at x_1, x_2, \dots, x_k , where it is differentiable only $(k-2)$ times; the contribution of x_i (say) to $|f^{(k)}|_1$ is then $|\lim_{x \rightarrow x_i^+} f^{(k-1)}(x) - \lim_{x \rightarrow x_i^-} f^{(k-1)}(x)|$.

2.4. Mellin transforms. The *Mellin transform* of a function $\phi : (0, \infty) \rightarrow \mathbb{C}$ is

$$(2.2) \quad M\phi(s) := \int_0^{\infty} \phi(x)x^{s-1}dx.$$

In general, $M(f *_M g) = Mf \cdot Mg$ and

$$(2.3) \quad M(f \cdot g)(s) = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} Mf(z)Mg(s-z)dz \quad [\text{GR00, §17.32}]$$

provided that z and $s-z$ are within the strips on which Mf and Mg (respectively) are well-defined.

The Mellin transform is an isometry, in the sense that

$$(2.4) \quad \int_0^{\infty} |f(t)|^2 t^{2\sigma} \frac{dt}{t} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |Mf(\sigma + it)|^2 dt.$$

Table 22. Number and percent distribution of emergency department visits resulting in hospital admission, with corresponding standard errors, by selected characteristics: United States, 2006

Selected characteristic	Number of visits in thousands	Standard error in thousands	Percent distribution	Standard error of percent
All admissions	15,263	896
Age				
Under 15 years	947	99	6.2	0.5
15–24 years	828	73	5.4	0.4
25–44 years	2,698	222	17.7	0.8
45–64 years	4,683	314	30.7	0.9
65–74 years	2,223	191	14.6	0.7
75 years and over	3,884	234	25.4	1.1
Unit to which admitted				
Other bed or unit	10,566	742	69.2	2.2
Critical care unit	2,255	227	14.8	1.3
Operating room or catheterization lab.	479	71	3.1	0.5
Unknown	1,964	323	12.9	2.0
Hospital discharge status				
Alive	12,745	777	83.5	2.2
Died	312	40	2.0	0.3
Unknown	2,205	389	14.4	2.3
Length of stay ¹				
1–2 days	3,236	262	21.2	1.2
3–4 days	4,627	309	30.3	1.2
5–6 days	2,483	180	16.3	0.9
7–8 days	1,247	117	8.2	0.6
9–10 days	652	77	4.3	0.4
More than 10 days	1,139	105	7.5	0.5
Unknown	1,878	381	12.3	2.3
Mode of arrival				
Ambulance	6,155	447	40.3	1.2
Other	9,108	515	59.7	1.2
Immediacy with which patient should be seen				
Immediate or emergent ²	5,238	473	34.3	2.3
Other	10,025	667	65.7	2.3
Patient seen in this ED within the last 72 hours ³				
Yes	545	68	3.6	0.4
No	14,718	865	96.4	0.4
Patient discharged from any hospital within the last 7 days				
Yes	738	106	4.8	0.6
No or unknown	14,525	844	95.2	0.6

... Category not applicable.

¹The mean length of stay was 5.3 days (standard error = 0.1).²Emergent is 1 to 14 minutes.³ED is emergency department.

NOTE: Numbers may not add to totals because of rounding.

Form Approved OMB No. 0920-0278 Exp. Date 05/31/2007 CDC 64.136

FORM NHAMCS-100(ED) (8-1-2005)		<small>U.S. DEPARTMENT OF COMMERCE Economics and Statistics Administration U.S. CENSUS BUREAU ACTING AS DATA COLLECTION AGENT FOR THE U.S. Department of Health and Human Services Centers for Disease Control and Prevention National Center for Health Statistics</small>	
NATIONAL HOSPITAL AMBULATORY MEDICAL CARE SURVEY 2006 EMERGENCY DEPARTMENT PATIENT RECORD			
Assurance of confidentiality - All information which would permit identification of an individual, a practice, or an establishment will be held confidential, will be used only by persons engaged in and for the purpose of the survey and will not be disclosed or released to other persons or used for any other purpose without consent of the individual or the establishment in accordance with section 308(d) of the Public Health Service Act (42 USC 242m).			

NHAMCS-100(ED) (8-1-2005)

1. PATIENT INFORMATION																														
a. Date of visit Month: <input type="text"/> Day: <input type="text"/> Year: <input type="text"/> <div style="text-align: center; font-size: large;">2 0 0 6</div>		b. ZIP code <input type="text"/>																												
c. Date of birth Month: <input type="text"/> Day: <input type="text"/> Year: <input type="text"/>		d. Time of day <input type="checkbox"/> AM <input type="checkbox"/> Military <input type="checkbox"/> PM																												
e. Patient residence 1 <input type="checkbox"/> Private residence 2 <input type="checkbox"/> Nursing home 3 <input type="checkbox"/> Other institution 4 <input type="checkbox"/> Other residence 5 <input type="checkbox"/> Homeless 6 <input type="checkbox"/> Unknown		f. Mode of arrival - Mark (X) one. 1 <input type="checkbox"/> Ambulance (air/ground) 2 <input type="checkbox"/> Walk-in 3 <input type="checkbox"/> Public service (nonambulance, e.g., police, social services) 4 <input type="checkbox"/> Unknown																												
g. Sex 1 <input type="checkbox"/> Female 2 <input type="checkbox"/> Male		(1) Arrival <input type="text"/> : <input type="text"/> : <input type="text"/> (2) Time seen by physician <input type="text"/> : <input type="text"/> : <input type="text"/> <input type="checkbox"/> Not seen by physician (3) ED discharge <input type="text"/> : <input type="text"/> : <input type="text"/> Mark (X) if ED discharge is more than 24 hours from arrival. <input type="checkbox"/>																												
h. Ethnicity 1 <input type="checkbox"/> Hispanic or Latino 2 <input type="checkbox"/> Not Hispanic or Latino		i. Race - Mark (X) one or more. 1 <input type="checkbox"/> White 2 <input type="checkbox"/> Black/ African American 3 <input type="checkbox"/> Asian 4 <input type="checkbox"/> Native Hawaiian/ Other Pacific Islander 5 <input type="checkbox"/> American Indian/ Alaska Native																												
j. Expected source(s) of payment for this visit - Mark (X) all that apply. 1 <input type="checkbox"/> Private insurance 2 <input type="checkbox"/> Medicare 3 <input type="checkbox"/> Medicaid/SCHIP 4 <input type="checkbox"/> Worker's compensation 5 <input type="checkbox"/> Self-pay 6 <input type="checkbox"/> No charge/Charity 7 <input type="checkbox"/> Other 8 <input type="checkbox"/> Unknown																														
2. TRIAGE																														
a. Initial vital signs (1) Temperature <input type="text"/> °C <input type="text"/> °F (2) Pulse <input type="text"/> beats per minute		(3) Blood pressure <input type="text"/> / <input type="text"/> (4) Oriented X 3 1 <input type="checkbox"/> Yes 2 <input type="checkbox"/> No 3 <input type="checkbox"/> Unknown																												
b. Immediacy with which patient should be seen 1 <input type="checkbox"/> Immediate 2 <input type="checkbox"/> 1-14 minutes 3 <input type="checkbox"/> 15-60 minutes 4 <input type="checkbox"/> >1 hour-2 hours 5 <input type="checkbox"/> >2 hours-24 hours 6 <input type="checkbox"/> No triage 7 <input type="checkbox"/> Unknown		c. Presenting level of pain 1 <input type="checkbox"/> None 2 <input type="checkbox"/> Mild 3 <input type="checkbox"/> Moderate 4 <input type="checkbox"/> Severe 5 <input type="checkbox"/> Unknown																												
3. PREVIOUS CARE																														
Has patient been: a. Seen in this ED within the last 72 hours? 1 <input type="checkbox"/> Yes 2 <input type="checkbox"/> No 3 <input type="checkbox"/> Unknown		b. Discharged from any hospital within the last 7 days? 1 <input type="checkbox"/> Yes 2 <input type="checkbox"/> No 3 <input type="checkbox"/> Unknown																												
4. REASON FOR VISIT																														
a. Patient's complaint(s), symptom(s), or other reason(s) for this visit Use patient's own words. (1) Most important: _____ (2) Other: _____ (3) Other: _____		b. Is this visit work related? 1 <input type="checkbox"/> Yes 2 <input type="checkbox"/> No 3 <input type="checkbox"/> Unknown																												
5. INJURY/POISONING/ADVERSE EFFECT																														
a. Is this visit related to an injury, poisoning, or adverse effect of medical treatment? 1 <input type="checkbox"/> Yes 2 <input type="checkbox"/> No - SKIP to item 6.		b. Is this injury/poisoning intentional? 1 <input type="checkbox"/> Yes, self inflicted 2 <input type="checkbox"/> Yes, assault 3 <input type="checkbox"/> No, unintentional 4 <input type="checkbox"/> Unknown																												
c. Cause of injury, poisoning, or adverse effect - Describe the place and events that preceded the injury, poisoning, or adverse effect (e.g., allergy to penicillin, bee sting, pedestrian hit by car driven by drunk driver, spouse beaten with fists by spouse, heroin overdose, infected shunt, etc.). _____ _____ _____																														
6. PHYSICIAN'S DIAGNOSIS FOR THIS VISIT																														
As specifically as possible, list diagnoses related to this visit including chronic conditions.		(1) Primary diagnosis: _____ (2) Other: _____ (3) Other: _____																												
7. DIAGNOSTIC/SCREENING SERVICES																														
Mark (X) all ordered or provided at this visit. 1 <input type="checkbox"/> NONE Blood tests: 2 <input type="checkbox"/> CBC (complete blood count) 3 <input type="checkbox"/> BUN/Creatinine _____ % 4 <input type="checkbox"/> Cardiac enzymes 5 <input type="checkbox"/> Electrolytes 6 <input type="checkbox"/> Glucose 7 <input type="checkbox"/> Liver function tests 8 <input type="checkbox"/> Arterial blood gases 9 <input type="checkbox"/> BAC (blood alcohol) 10 <input type="checkbox"/> HIV serology 11 <input type="checkbox"/> Other blood test Other tests: 12 <input type="checkbox"/> EKG/ECG 13 <input type="checkbox"/> Cardiac monitor 14 <input type="checkbox"/> Pulse oximetry 15 <input type="checkbox"/> Pregnancy test 16 <input type="checkbox"/> Urinalysis (UA) 17 <input type="checkbox"/> Other test/service Imaging: 18 <input type="checkbox"/> X-ray 19 <input type="checkbox"/> Ultrasound 20 <input type="checkbox"/> MRI 21 <input type="checkbox"/> CT scan 22 <input type="checkbox"/> Other imaging																														
8. PROCEDURES																														
Mark (X) all provided at this visit. Exclude medications. 1 <input type="checkbox"/> NONE 2 <input type="checkbox"/> Bladder catheter 3 <input type="checkbox"/> CPR 4 <input type="checkbox"/> Endotracheal intubation 5 <input type="checkbox"/> IV fluids 6 <input type="checkbox"/> Nebulizer therapy 7 <input type="checkbox"/> NG tube/gastric suction 8 <input type="checkbox"/> OB/GYN care 9 <input type="checkbox"/> Orthopedic care 10 <input type="checkbox"/> Thrombolytic therapy 11 <input type="checkbox"/> Wound care 12 <input type="checkbox"/> Other																														
9. MEDICATIONS & IMMUNIZATIONS																														
List up to 8 drugs given at this visit or prescribed at ED discharge. Include Rx and OTC drugs, immunizations, and anesthetics. <input type="checkbox"/> NONE <table border="1" style="width: 100%; border-collapse: collapse;"><thead><tr><th></th><th style="text-align: center;">Given in ED</th><th style="text-align: center;">Rx at discharge</th></tr></thead><tbody><tr><td>(1) _____</td><td style="text-align: center;">1 <input type="checkbox"/></td><td style="text-align: center;">2 <input type="checkbox"/></td></tr><tr><td>(2) _____</td><td style="text-align: center;">1 <input type="checkbox"/></td><td style="text-align: center;">2 <input type="checkbox"/></td></tr><tr><td>(3) _____</td><td style="text-align: center;">1 <input type="checkbox"/></td><td style="text-align: center;">2 <input type="checkbox"/></td></tr><tr><td>(4) _____</td><td style="text-align: center;">1 <input type="checkbox"/></td><td style="text-align: center;">2 <input type="checkbox"/></td></tr><tr><td>(5) _____</td><td style="text-align: center;">1 <input type="checkbox"/></td><td style="text-align: center;">2 <input type="checkbox"/></td></tr><tr><td>(6) _____</td><td style="text-align: center;">1 <input type="checkbox"/></td><td style="text-align: center;">2 <input type="checkbox"/></td></tr><tr><td>(7) _____</td><td style="text-align: center;">1 <input type="checkbox"/></td><td style="text-align: center;">2 <input type="checkbox"/></td></tr><tr><td>(8) _____</td><td style="text-align: center;">1 <input type="checkbox"/></td><td style="text-align: center;">2 <input type="checkbox"/></td></tr></tbody></table>					Given in ED	Rx at discharge	(1) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>	(2) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>	(3) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>	(4) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>	(5) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>	(6) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>	(7) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>	(8) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>
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(1) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>																												
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(6) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>																												
(7) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>																												
(8) _____	1 <input type="checkbox"/>	2 <input type="checkbox"/>																												
10. PROVIDERS																														
Mark (X) all providers seen at this visit. 1 <input type="checkbox"/> ED attending physician 2 <input type="checkbox"/> ED resident/intern 3 <input type="checkbox"/> On call attending physician/fellow 4 <input type="checkbox"/> RN/LPN 5 <input type="checkbox"/> Nurse practitioner 6 <input type="checkbox"/> Physician assistant 7 <input type="checkbox"/> EMT 8 <input type="checkbox"/> Other																														
11. VISIT DISPOSITION																														
Mark (X) all that apply. 1 <input type="checkbox"/> No follow-up planned 2 <input type="checkbox"/> Return if needed, PRN/appointment 3 <input type="checkbox"/> Return/Refer to physician/clinic for FU 4 <input type="checkbox"/> Refer to social services 5 <input type="checkbox"/> Left AMA 6 <input type="checkbox"/> Left without being seen 7 <input type="checkbox"/> DOA/died in ED 8 <input type="checkbox"/> Transfer to different hospital - Reason _____ 9 <input type="checkbox"/> Admit to observation unit 10 <input type="checkbox"/> Admit to hospital 11 <input type="checkbox"/> Other <div style="border: 1px solid black; padding: 5px; margin-top: 10px;">If "Admit to hospital" was marked, then please continue with Item 12 - HOSPITAL ADMISSION on the reverse side.</div>																														

2006 ED

Figure 10. 2006 Emergency Department Patient Record