

Technical Details of Formalization of Rate Distortion

Duolei Wang*

January 10, 2026~

Contents

1	Main Proof	2
2	Convex Function and Optimization	2
3	Probability Theory	2
4	Information Theory	2
5	Rate Distortion Theory	2

*duoleiwang@link.cuhk.edu.cn

1 Main Proof

Theorem 1.1 (Rate Distortion Theorem). Suppose \mathcal{X} and \mathcal{Y} are finite alphabet, the distortion function d is bounded by a constant d_0 . For any probability distribution $p(x)$ defined on \mathcal{X} , we have:

1. If $R > R_I(D)$, the rate distortion pair (R, D) is achievable;
2. If $R < R_I(D)$, the rate distortion pair (R, D) is not achievable.

2 Convex Function and Optimization

Definition 2.1 (Optimization Minimizing Problem). The minimizing optimization problem with:

3 Probability Theory

4 Information Theory

5 Rate Distortion Theory

Definition 5.1 (Achievable). A rate distortion-pair is called achievable if there exists a sequence of (n, M_n) rate distortion codes with increasing length, such that the rates are less than or equal to R , and:

$$\limsup E \left\{ \frac{1}{n} d(\mathbf{X}^n, \hat{\mathbf{X}}) \right\} \leq D$$

Definition 5.2 (Information Rate Distortion Function). For $D \geq 0$, define:

$$R_I :: \text{Real} \rightarrow \text{Optmin}(\text{DistPr}(X, X), \text{Expt-Bounded}, \text{Obj})$$

and

$$R_I(D) := \min_{P(X, \hat{X})} E \left[d(X, \hat{X}) \right] \leq D : I(X; \hat{X})$$

where Optmin is the minimizing optimization problem structure in [2.1](#).