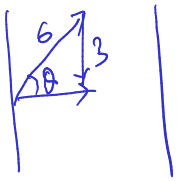
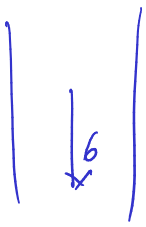


$\theta = \sin^{-1}(3/6) = 30^\circ = \pi/6$

Vector problems



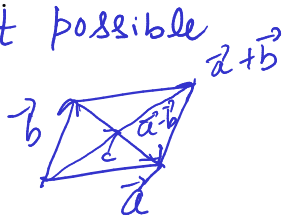
1. a) A river flows at 3 mph and a rower rows at 6 mph. What heading should the rower take to go straight across a river?

b) Answer the same question if the river flows at 6 mph and the rower rows at 3 mph.

not possible

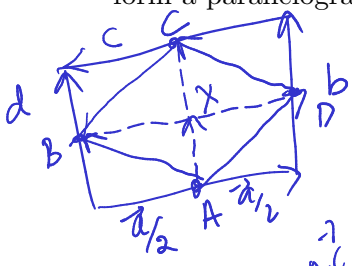
2. Find a unit vector in the direction of $\langle 2, 3\rangle$.

$$\frac{\langle 2,3\rangle}{|\langle 2,3\rangle|} = \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$$



3. Use vectors to prove that the diagonals of a parallelogram bisect each other.

4. Prove using vector methods that the midpoints of the sides of a space quadrilateral form a parallelogram



$\vec{AC} = \frac{\vec{a}}{2} + \vec{b} + \frac{\vec{c}}{2}$

$\vec{AX} + \vec{XD} = \frac{\vec{a}+\vec{b}}{2}$

$\vec{BX} + \vec{XC} = \vec{BC} = \frac{\vec{d}-\vec{c}}{2}$

$\vec{AD} - \vec{BC} = \frac{\vec{a}+\vec{b}+\vec{c}-\vec{d}}{2} = 0$

$\vec{AD} = \vec{BC}$

thus $\vec{AD} \parallel \vec{BC}$

Similarly, it can be proved that $\vec{AB} \parallel \vec{DC}$

$\vec{c} = k(\vec{a}+\vec{b})$

$\vec{c} - \vec{b} = l(\vec{a}-\vec{b})$

$k\vec{a} + k\vec{b} - \vec{b} = l\vec{a} - l\vec{b}$

$k\vec{a} + (k-1)\vec{b} = l\vec{a} - l\vec{b}$

On comparing coefficients on both sides,

$k = l$

$k - 1 = -l$

$k - 1 = -k$

$k = 1/2 = l$

18.02SC Multivariable Calculus
Fall 2010