$\vec{A}\cdot\vec{B} = |A|\,|B|\cos\theta$

$\cos\theta = \dfrac{1+16}{\sqrt{65}\sqrt{5}} = \dfrac{17}{5\sqrt{13}}$

$\theta = \cos^{-1}\left(\dfrac{17}{5\sqrt{13}}\right)$

Uses of the Dot Product

1. Find the angle between the vectors $\mathbf{A} = \mathbf{i} + 8\mathbf{j}$ and $\mathbf{B} = \mathbf{i} + 2\mathbf{j}$.

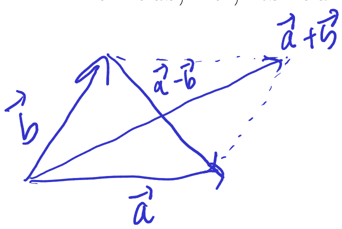
2. Take points $P = (a, 1, -1)$, $Q = (0, 1, 1)$, $R = (a, -1, 3)$. For what value(s) of a is PQR a right angle?

$\vec{PQ} = (-a, 0, 2)$ $\quad \vec{QR} = (a, -2, 2)$ $\quad PQ \perp PR \Rightarrow -a^2 - 2\times 0 + 4 = 0$

$\Rightarrow |a| = 2$

$\Rightarrow a = \pm 2$

3. Show that the diagonals of a parallelogram are perpendicular if and only if it is a rhombus, i.e., its four sides have equal lengths.



$\left(\dfrac{\vec{a}+\vec{b}}{2}\right)\cdot\left(\dfrac{\vec{a}-\vec{b}}{2}\right) = 0$

$|a|^2 - |b|^2 = 0$

$\Rightarrow |a| = |b|$ proved