

COMPLEMENT OPERATION

In mathematics and computing, the **method of complements** is a technique used to subtract one number from another using only addition of positive numbers. This method was commonly used in mechanical calculators and is still used in modern computers in order to simplify the subtraction operation and for the logical manipulations. To subtract a number y (the subtrahend) from another number x (the minuend), the radix complement of y is added to x and the initial '1' of the result is discarded. Discarding the initial '1' is especially convenient on calculators or computers that use a fixed number of digits: there is nowhere for it to go so it is simply lost during the calculation.

For any number there are two types of complements.

Base or Radix complement

1. For decimal number: 10's complement
2. For binary number: 2's complement
3. For octal number: 8's complement
4. For hexadecimal number: 16's complement

One less than the base complement

1. For decimal number: 10's complement
2. For decimal number: 1's complement
3. For decimal number: 70's complement
4. For decimal number: 15's complement

DECIMAL COMPLEMENT

Obtain 9's complement of 187

$$\begin{array}{r} 9 \ 9 \ 9 \\ - 1 \ 8 \ 7 \\ \hline 8 \ 1 \ 2 \end{array}$$

Ans: 812

Obtain 10's complement of 187

$$\begin{array}{r} 9 \ 9 \ 9 \\ - 1 \ 8 \ 7 \\ \hline 8 \ 1 \ 2 \\ + \quad 1 \\ \hline 8 \ 1 \ 3 \end{array}$$

Ans: 813

Subtraction using decimal complement

9's complement method

a) $554 - 475$

9's complement of the subtrahend 475:

$$999 - 475 = 524$$

$$\begin{array}{r} 5 \ 5 \ 4 \\ + 5 \ 2 \ 4 \\ \hline 1 \ 0 \ 7 \ 8 \\ + 1 \\ \hline 7 \ 9 \end{array}$$

$554 - 475 = 79$ Ans.

b) $475 - 554$

9's complement of the subtrahend 554:

$$999 - 554 = 445$$

$$\begin{array}{r} 4 \ 7 \ 5 \\ + 4 \ 4 \ 5 \\ \hline 9 \ 2 \ 0 \end{array}$$

No carry

- 9's complement of 920 = -79 **Ans.**

10's complement method

a) $554 - 475$

10's complement of the subtrahend 475:

$$999 - 475 = 524 + 1 = 525$$

b) $475 - 554$

10's complement of the subtrahend 554:

$$999 - 554 = 445 + 1 = 446$$

$$\begin{array}{r}
 554 \\
 + 525 \\
 \hline
 1079
 \end{array}$$

Ignore carry

Ans: 79

$$\begin{array}{r}
 475 \\
 + 446 \\
 \hline
 921
 \end{array}$$

No carry

$475 - 554 = - (10\text{'s complement of } 921)$

$= - (78 + 1) = - 79 \text{ Ans.}$

BINARY COMPLEMENT

Obtain 1's complement of 1010_2

1's complement of $1010_2 = 0101_2$

Ans: 0101_2

Obtain 2's complement of 1010_2

1's complement of $1010_2 = 0101_2$

Now $0101 + 1 = 0110_2$ is the 2's complement

Ans: 0110_2

Subtraction using binary complement

1's complement method

a) $1101_2 - 1001_2$

1's complement of the subtrahend $1001 = 0110$

$$\begin{array}{r}
 1101 \\
 + 0110 \\
 \hline
 10011
 \end{array}$$

Ignore carry

0100

Therefore, $1101_2 - 1001_2 = 100_2 \text{ Ans.}$

b) $1001_2 - 1101_2$

1's complement of the subtrahend $1101 = 0010$

$$\begin{array}{r}
 1001 \\
 + 0010 \\
 \hline
 1011
 \end{array}$$

No carry

Therefore, $1001_2 - 1101_2$

$= - (1\text{'s complement of } 1011)$

$= -0100_2 = -100_2 \text{ Ans.}$

2's complement method

a) $1101_2 - 1001_2$

2's complement of the subtrahend $1001 = 0110 + 1 = 0111$

$$\begin{array}{r}
 1101 \\
 + 0111 \\
 \hline
 10100
 \end{array}$$

Ignore carry

Therefore, $1101_2 - 1001_2 = 100_2 \text{ Ans.}$

b) $1001_2 - 1101_2$

2's complement of the subtrahend $1101 = 0010 + 1 = 0011$

$$\begin{array}{r}
 1001 \\
 + 0011 \\
 \hline
 1100
 \end{array}$$

No carry

Therefore, $1001_2 - 1101_2$

$= - (2\text{'s complement of } 1100)$

$= -(0011 + 1) = -100_2 \text{ Ans.}$

OCTAL COMPLEMENT

Obtain 7's complement of 467₈

$$\begin{array}{r} 7 \ 7 \ 7 \\ - \ 4 \ 6 \ 7 \\ \hline 3 \ 1 \ 0 \end{array}$$

Ans: 310₈

Obtain 8's complement of 467₈

$$\begin{array}{r} 7 \ 7 \ 7 \\ - \ 4 \ 6 \ 7 \\ \hline 3 \ 1 \ 0 \\ + \quad \quad 1 \\ \hline 3 \ 1 \ 1 \end{array}$$

Ans: 311₈

Subtraction using binary complement

7's complement method

a) 764 – 467

7's complement of the subtrahend 467

$$= (777-467)=310$$

$$\begin{array}{r} 7 \ 6 \ 4 \\ + \quad 3 \ 1 \ 0 \\ \hline \boxed{1} \ 2 \ 7 \ 4 \\ \quad \quad \quad \rightarrow + 1 \\ \hline 2 \ 7 \ 5 \end{array}$$

Therefore, $764_8 - 467_8 = 275_8$ **Ans.**

b) 467 – 764

7's complement of the subtrahend 746

$$= (777-764)=013$$

$$\begin{array}{r} 4 \ 6 \ 7 \\ + \quad 0 \ 1 \ 3 \\ \hline 5 \ 0 \ 2 \end{array}$$

No carry

Therefore, $467_8 - 764_8$

$$= - (7's \text{ complement of } 502)$$

$$= -275_8 \text{ **Ans.**}$$

8's complement method

a) 764 – 467

8's complement of the subtrahend 467

$$= (777-467)=310+1=311$$

$$\begin{array}{r} 7 \ 6 \ 4 \\ + \quad 3 \ 1 \ 1 \\ \hline \boxed{1} \ 2 \ 7 \ 5 \end{array}$$

Ignore carry

Therefore, $764_8 - 467_8 = 275_8$ **Ans.**

b) 467 – 764

8's complement of the subtrahend 746

$$= (777-764)=013+1=14$$

$$\begin{array}{r} 4 \ 6 \ 7 \\ + \quad 0 \ 1 \ 4 \\ \hline 5 \ 0 \ 3 \end{array}$$

No carry

Therefore, $467_8 - 764_8$

$$= - (8's \text{ complement of } 503)$$

$$= - (274+1) = - 275_8 \text{ **Ans.**}$$

HEXADECIMAL COMPLEMENT

Obtain 15's complement of $7CA_{16}$

$$\begin{array}{r} 15 \ 15 \ 15 \\ - \ 7 \ C \ A \\ \hline 8 \ 3 \ 5 \end{array}$$

Ans: 835_{16}

Obtain 16's complement of $7CA_{16}$

$$\begin{array}{r} 15 \ 15 \ 15 \\ - \ 7 \ C \ A \\ \hline 8 \ 3 \ 5 \\ + \quad \quad 1 \\ \hline 8 \ 3 \ 6 \end{array}$$

Ans: 836_8

Subtraction using binary complement

15's complement method

a) $8AB_{16} - 7CA_{16}$

15's complement of the subtrahend $7CA = 835$

$$\begin{array}{r} 8 \ A \ B \\ + \ 8 \ 3 \ 5 \\ \hline \boxed{1} \ 0 \ E \ 0 \\ \quad \quad \quad + \ 1 \\ \hline 0 \ E \ 1 \end{array}$$

Therefore, $8AB_{16} - 7CA_{16} = E1_{16}$ Ans.

b) $7CA_{16} - 8AB_{16}$

15's complement of the subtrahend $8AB =$

$$\begin{array}{r} 15 \ 15 \ 15 \\ - \ 8 \ A \ B \\ \hline 7 \ 5 \ 4 \\ 7 \ C \ A \\ + \ 7 \ 5 \ 4 \\ \hline F \ 1 \ E \end{array}$$

No carry

Therefore, $7CA_{16} - 8AB_{16} =$

$-(15' \text{ complement of } F1E) = -E1_{16}$ Ans.

16's complement method

a) $8AB_{16} - 7CA_{16}$

16's complement of the subtrahend $7CA =$
 $835 + 1 = 836$

$$\begin{array}{r} 8 \ A \ B \\ + \ 8 \ 3 \ 6 \\ \hline \boxed{1} \ 0 \ E \ 1 \\ \quad \quad \quad \text{Ignore carry} \\ \hline 0 \ E \ 1 \end{array}$$

Therefore, $8AB_{16} - 7CA_{16} = E1_{16}$ Ans.

b) $7CA_{16} - 8AB_{16}$

15's complement of the subtrahend $8AB =$
 $754 + 1 = 755$

$$\begin{array}{r} 7 \ C \ A \\ + \ 7 \ 5 \ 5 \\ \hline F \ 1 \ F \end{array}$$

No carry

Therefore, $7CA_{16} - 8AB_{16} =$

$-(16' \text{ complement of } F1F) = -(E0=1)_{16}$
 $= -E1_{16}$ Ans.