CSE-345: Artificial Intelligence

The search techniques we have seen so far...

- Breadth first search
- Uniform cost search
- Depth first search
- Depth limited search
- Iterative Deepening
- Bi-directional Search



...are all too slow for most real world problems

8-Puzzle

Solution in Node 46

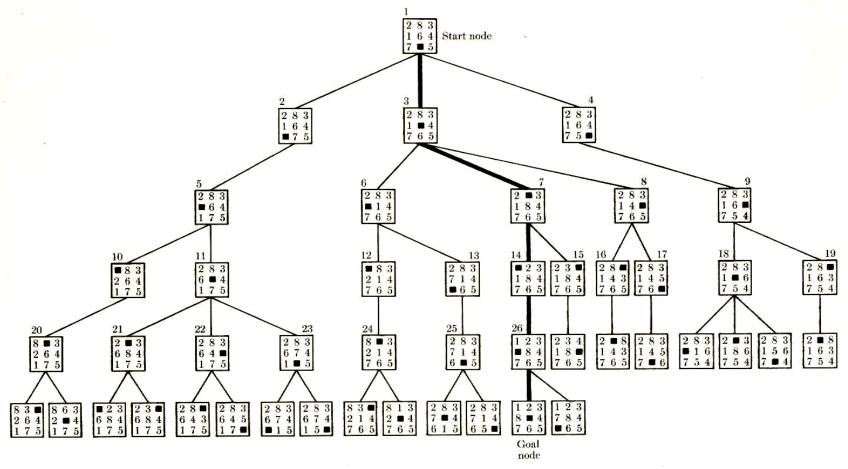


FIG. 3-2 The tree produced by a breadth-first search.

Solution in Node 31

8-Puzzle

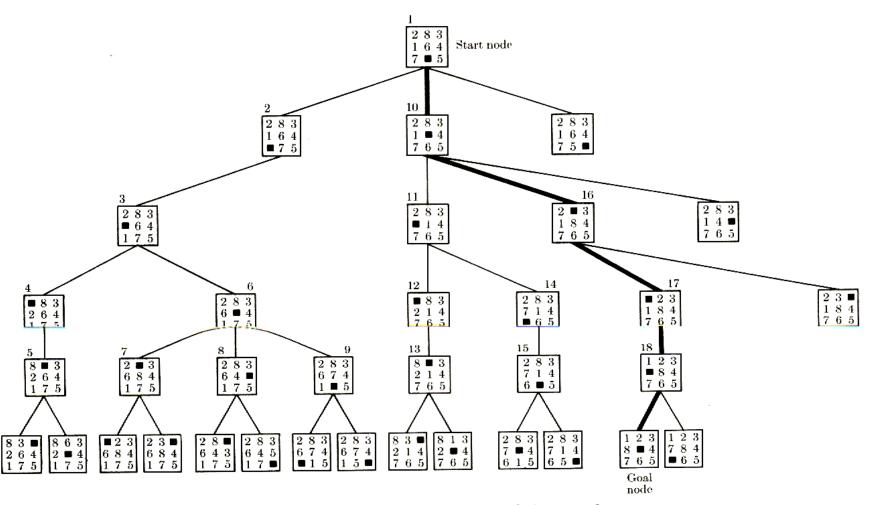
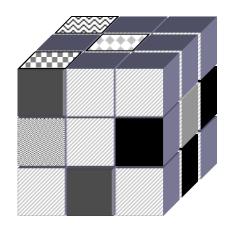


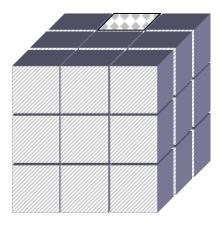
FIG. 3-5 The tree produced by a depth-first search.

Sometimes we can tell that some states appear better than others...

7	8	4
3	5	1
6	2	

1	2	3
4	5	6
7		8





...we can use this knowledge of the relative merit of states to guide search

Heuristic Search (informed search)

A **Heuristic** is a function that, when applied to a state, returns a number that is an estimate of the merit of the state, with respect to the goal.

In other words, the heuristic tells us approximately how far the state is from the goal state*.

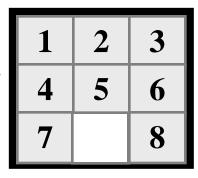
Note we said "approximately". Heuristics might underestimate or overestimate the merit of a state. But for reasons which we will see, heuristics that *only* underestimate are very desirable, and are called admissible.

*I.e Smaller numbers are better

Heuristics for 8-puzzle I

•The number of misplaced tiles (not including the blank)

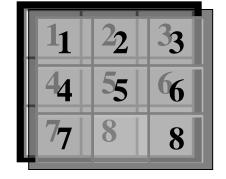
Current State





Goal State

1	2	3
4	5	6
7	8	



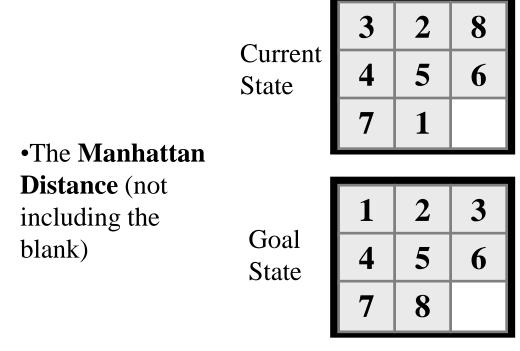
In this case, only "8" is misplaced, so the heuristic function evaluates to 1.

In other words, the heuristic is *telling* us, that it *thinks* a solution might be available in just 1 more move.

N	N	N
N	N	N
N	Y	

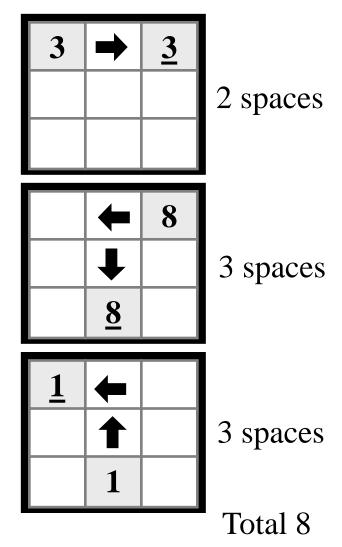
Notation: h(n) h(current state) = 1

Heuristics for 8-puzzle II



In this case, only the "3", "8" and "1" tiles are misplaced, by 2, 3, and 3 squares respectively, so the heuristic function evaluates to 8.

In other words, the heuristic is *telling* us, that it *thinks* a solution is available in just 8 more moves.

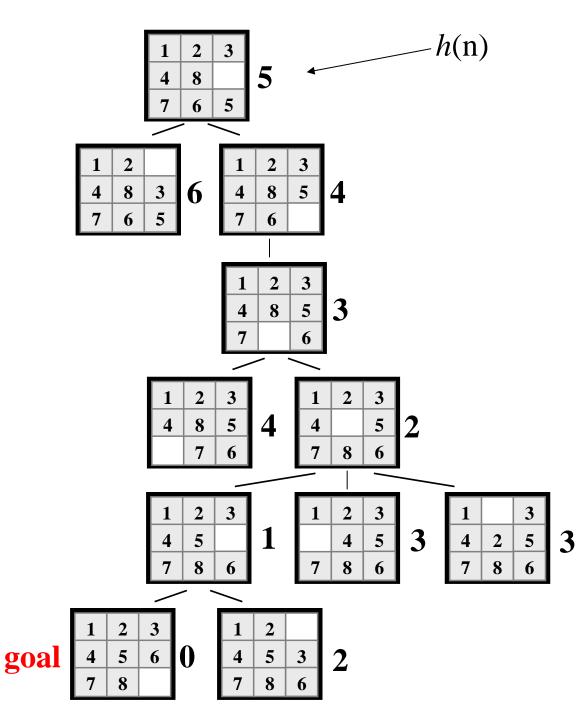


Notation: h(n)

h(current state) = 8

We can use heuristics to guide search.

In this example, the Manhattan Distance heuristic helps us quickly find a solution to the 8-puzzle.



- **☐** Heuristic
 - **heuriskein (Greek word)**-to discover
 - > original: eureka
 - -heurika (I have found): Archimedes uttering when determining the purity of gold
- **☐** Heuristic Function
 - > a way to inform the search about the direction to a goal.
 - > provides an informed way to guess which neighbor of a node will lead to a goal.
- □ Pros
 - ➤ Like a tour guide
 - > Good to the extent that they point in generally interesting direction
 - > Improve the quality of the paths that are explored
 - ➤ Using good heuristic, we can hope to get good solutions to hard problem
 - ➤ It is possible to construct special-purpose heuristics that exploit domain-specific knowledge to solve particular problems

- A *heuristic* is a method that
 - -might not always find the best solution
 - *-but* is guaranteed to find a good solution in reasonable time.
- ✓ By sacrificing completeness it increases efficiency.
- ✓ Useful in solving tough problems which
 - could not be solved any other way.
 - solutions take an infinite time or very long time to compute.
- The *classic* example of heuristic search methods is the travelling salesman problem.

☐ Traveling Salesman Problem

- 1. Arbitrarily select a starting city
- 2. To select the next city, look at all cities not yet visited,
 - -Select the one closest to the current city.
 - -Go to it next
- 3. Repeat step 2 until all cities have been visited

Heuristic Function

- A *heuristic function* is a function that maps from problem state descriptions to measure of desirability, usually represented as numbers
 - -Which aspects of the problem state are considered?
 - -How those aspects are evaluated?
 - -The weights given to individual aspects are chosen in such a way that the value of the heuristic function at a given node in the search process gives as good an estimate as possible of whether that node is on the desired path to a solution
- Well-designed heuristic functions can play an important part in efficiently guiding a search process toward a solution
- The purpose of a heuristic function is to guide the search process in the most profitable direction by suggesting which path to follow first when more than one is available
- The more accurately the heuristic function estimates the true merits of each node in the search tree/graph, the more direct the solution process

- Generate –and- test
- Hill Climbing
 - Simple Hill Climbing
 - Steepest-Ascent Hill Climbing
 - Simulated Annealing
- Best-First Search
 - o OR Graphs
 - o A* Algorithm
 - Greedy Best-First Search
- Problem Reduction
 - o AND-OR Graphs
 - o AO* Algorithm
- Constraint Satisfaction
- Means-ends Analysis

Generate-and-Test

Algorithm

- 1. Generate a possible solution.
- 2. Test to see if this is actually a solution.
- 3. Quit if a solution has been found. Otherwise, return to step 1.
- ✓ Acceptable for simple problems.
- *Inefficient for problems with large space.

Hill Climbing

- A variant of generate-and-test in which feedback from the test procedure is used to help the generator decide which direction to move in the search space.
- Searching for a goal state = Climbing to the top of a hill
- Generate-and-test + direction to move.
- Heuristic function to estimate how close a given state is to a goal state.

Simple Hill Climbing

Algorithm

- 1. Evaluate the initial state.
- 2. Loop until a solution is found or there are no new operators left to be applied:
 - Select and apply a new operator
 - Evaluate the new state:

```
goal \rightarrow quit
```

better than current state \rightarrow new current state

Steepest-Ascent Hill Climbing (Gradient Search)

- Considers all the moves from the current state.
- Selects the best one as the next state.

Steepest-Ascent Hill Climbing (Gradient Search)

Algorithm

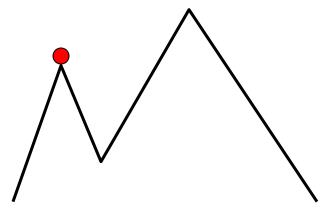
- 1. Evaluate the initial state.
- 2. Loop until a solution is found or a complete iteration produces no change to current state:
 - SUCC = a state such that any possible successor of the current state will be better than SUCC (the worst state).
 - For each operator that applies to the current state, evaluate the new state:

```
goal \rightarrow quit
better than SUCC \rightarrow set SUCC to this state
```

- SUCC is better than the current state \rightarrow set the current state to SUCC.

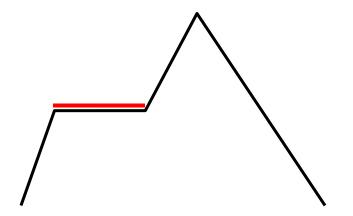
Local maximum

A state that is better than all of its neighbours, but not better than some other states far away.



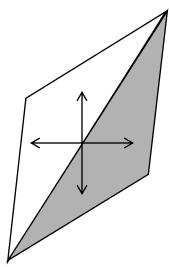
Plateau

A flat area of the search space in which all neighbouring states have the same value.



Ridge

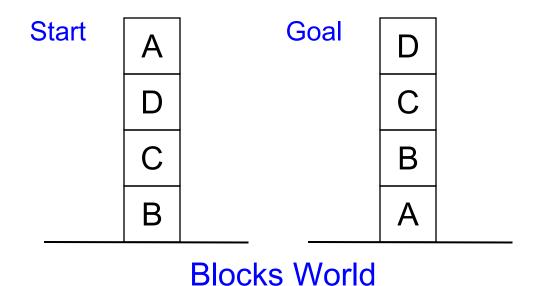
The orientation of the high region, compared to the set of available moves, makes it impossible to climb up. However, two moves executed serially may increase the height.

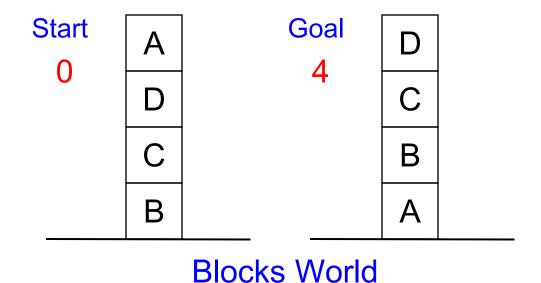


Ways Out

- Backtrack to some earlier node and try going in a different direction.
- Make a big jump to try to get in a new section.
- Moving in several directions at once.

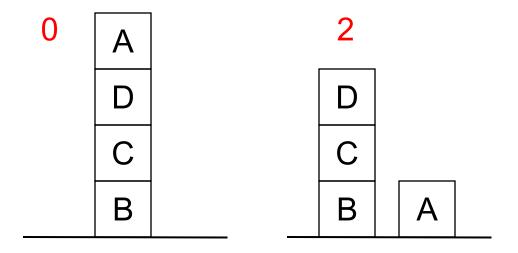
- Hill climbing is a local method:
 Decides what to do next by looking only at the "immediate" consequences of its choices.
- Global information might be encoded in heuristic functions.

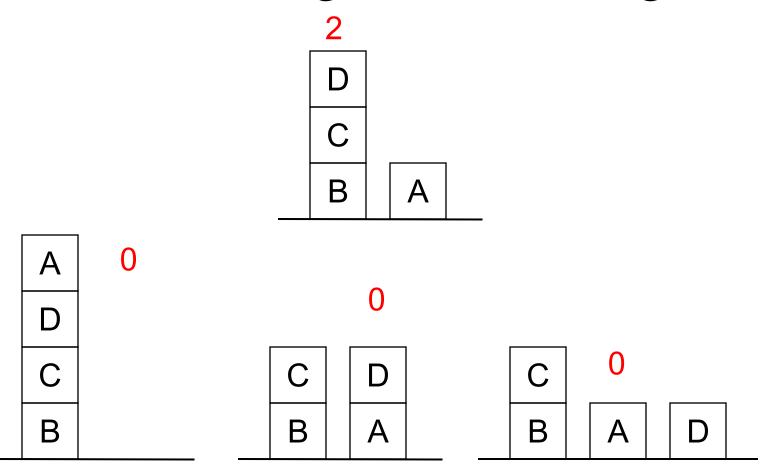


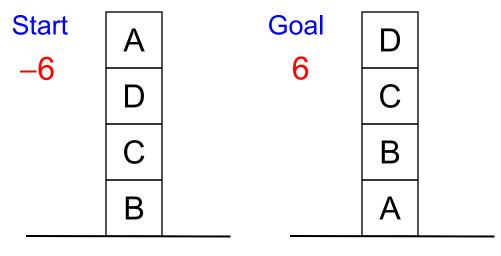


Local heuristic:

- +1 for each block that is resting on the thing it is supposed to be resting on.
- -1 for each block that is resting on a wrong thing.





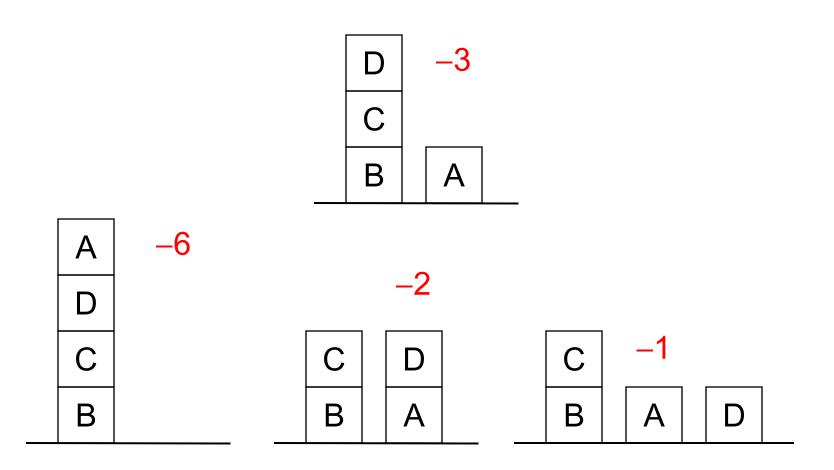


Blocks World

Global heuristic:

For each block that has the correct support structure: +1 to every block in the support structure.

For each block that has a wrong support structure: –1 to every block in the support structure.



Hill Climbing: Conclusion

- Can be very inefficient in a large, rough problem space.
- Global heuristic may have to pay for computational complexity.
- ➤ Often useful when combined with other methods.

Simulated Annealing

- A variation of hill climbing in which, at the beginning of the process, some downhill moves may be made.
- To do enough exploration of the whole space early on, so that the final solution is relatively insensitive to the starting state.
- Lowering the chances of getting caught at a local maximum, or plateau, or a ridge.

Simulated Annealing

Physical Annealing

- Physical substances are melted and then gradually cooled until some solid state is reached.
- The goal is to produce a minimal-energy state.
- Annealing schedule: if the temperature is lowered sufficiently slowly, then the goal will be attained.
- Nevertheless, there is some probability for a transition to a higher energy state: $p=e^{-\Delta E/kT}$. $\Delta E: positive change in the energy level T: temperature$

k: Boltzmann's Constant

Simulated Annealing

Algorithm

- 1. Evaluate the initial state.
- 2. Loop until a solution is found or there are no new operators left to be applied:
 - Set T according to an annealing schedule
 - Selects and applies a new operator
 - Evaluate the new state:

```
goal \rightarrow quit

\Delta E = Val(current state) - Val(new state)

\Delta E < 0 \rightarrow new current state

else \rightarrow new current state with probability e^{-\Delta E/kT}.
```

Difference from Simple Hill Climbing

- 1. Annealing schedule must be maintained
- 2. Moves to worse states may be accepted
- ➤ It is a good idea to maintain, current state, the best state found so far.
- Then, if the final state is worse than that earlier state, the earlier state is still available

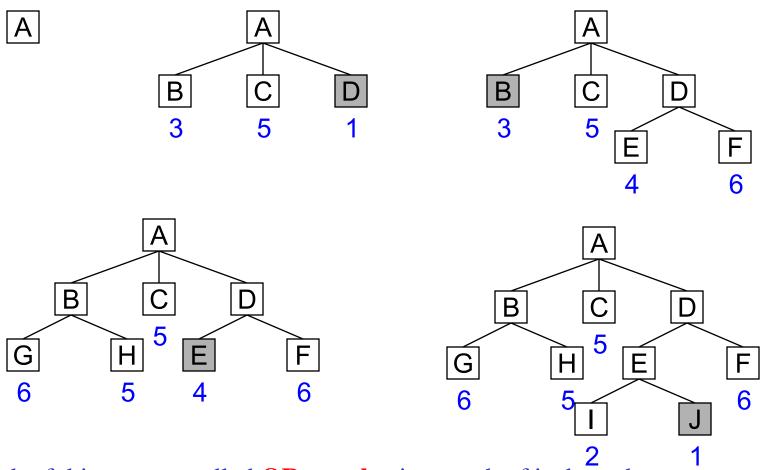
Best-First Search

➤ A way of Combining the advantage of both DFS and BFS into a single method.

Best-First Search: OR Graphs

- Depth-first search: not all competing branches having to be expanded.
- Breadth-first search: not getting trapped on deadend end paths.
 - ⇒ Combining the two is to follow a single path at a time, but switch paths whenever some competing path look more promising than the current one.

Best-First Search: OR Graphs



A graph of this sort are called **OR graph**, since each of its branches represent an alternative problem-solving path.

Best-First Search: OR Graphs

• OPEN: nodes that have been generated, but have not examined.

This is organized as a priority queue.

• CLOSED: nodes that have already been examined.

Whenever a new node is generated, check whether it has been generated before.

Best-First Search

Algorithm

- 1. $OPEN = \{initial state\}.$
- 2. Loop until a goal is found or there are no nodes left in OPEN:
 - Pick the best node in OPEN
 - Generate its successors
 - For each successor:

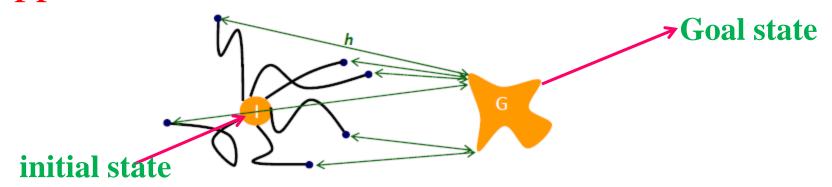
new \rightarrow evaluate it, add it to OPEN, record its parent generated before \rightarrow change parent, update successors

Best-First Search

- Some authors have used "best-first search" to refer specifically to a search with a heuristic that attempts to predict how close the end of a path is to a solution, so that paths which are judged to be closer to a solution are extended first.
- This specific type of search is called greedy best-first search
- Special cases:
 - greedy best-first search
 - A* search

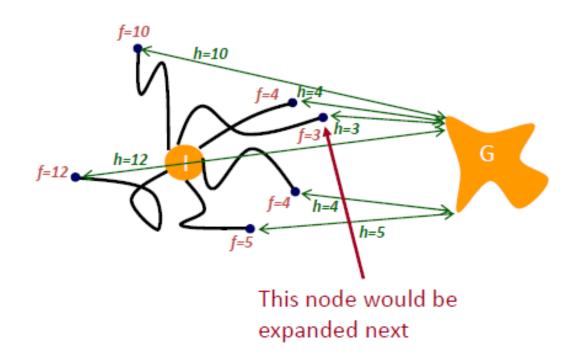
Greedy Best-First Search

- Evaluation function f(n) = h(n) (heuristic) = estimate of cost from n to goal
- e.g., $h_{SLD}(n) = \text{straight-line distance from } n$ to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal



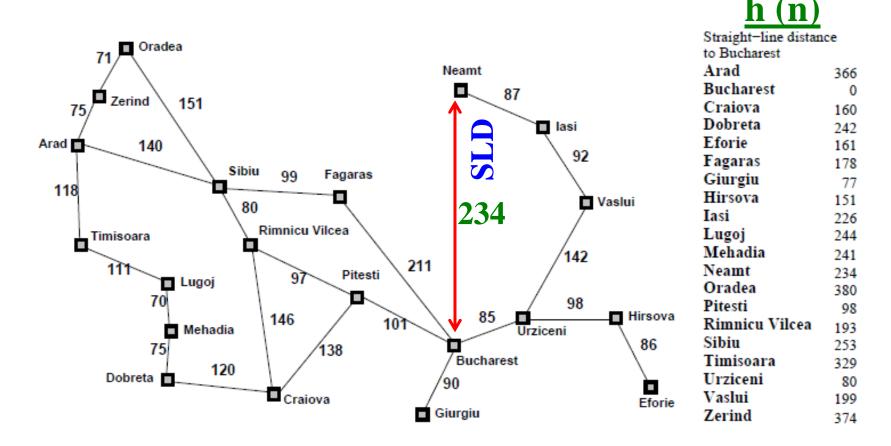
Greedy Best-First Search

• f(n) = h(n): Expand node that appears to be closest to the goal

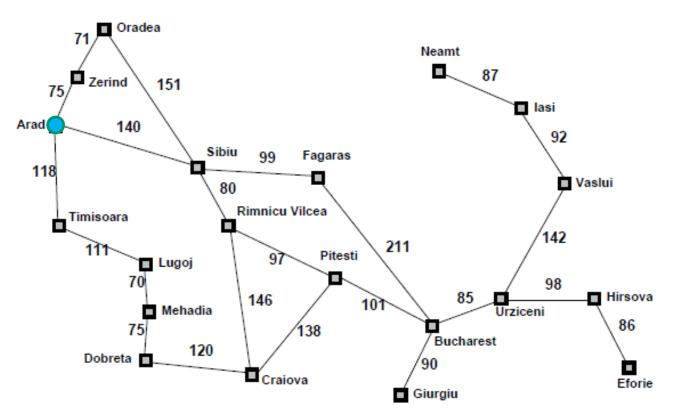


Go from Arad to Bucharest

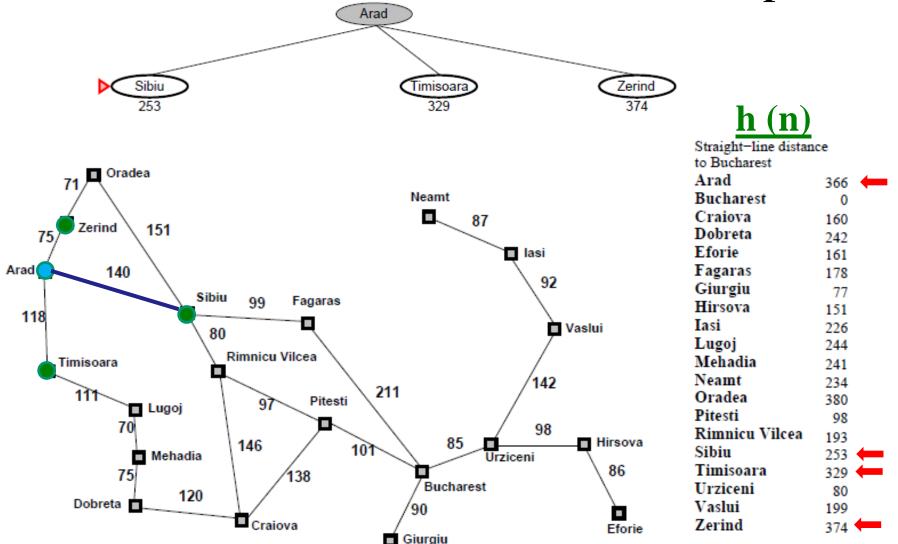
• $f(n) = h_{SLD}(n) = \text{straight-line distance from } n \text{ } STATE \text{ to } Bucharest$

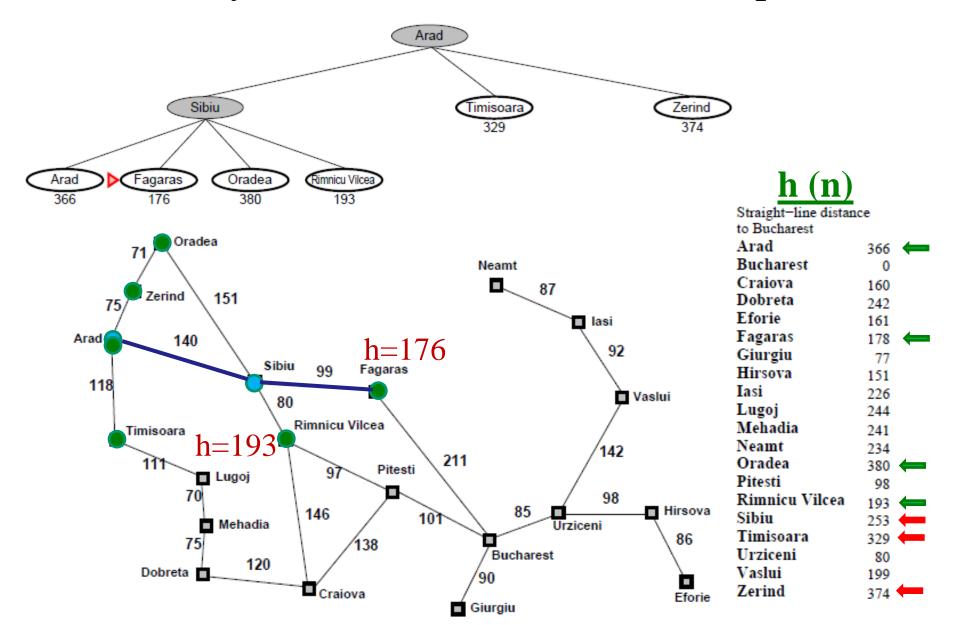


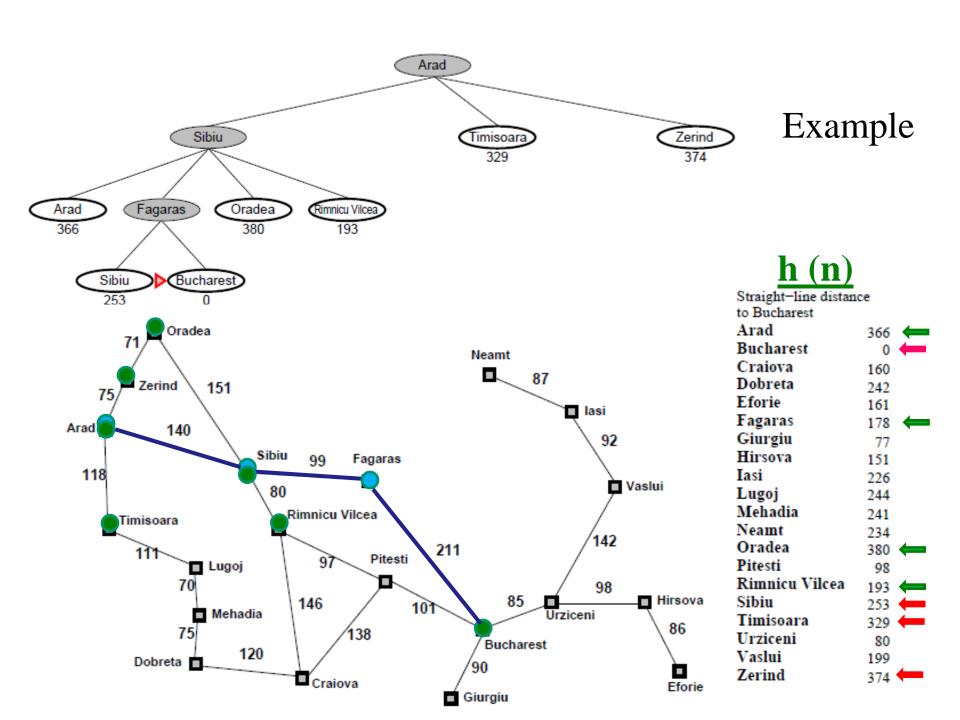


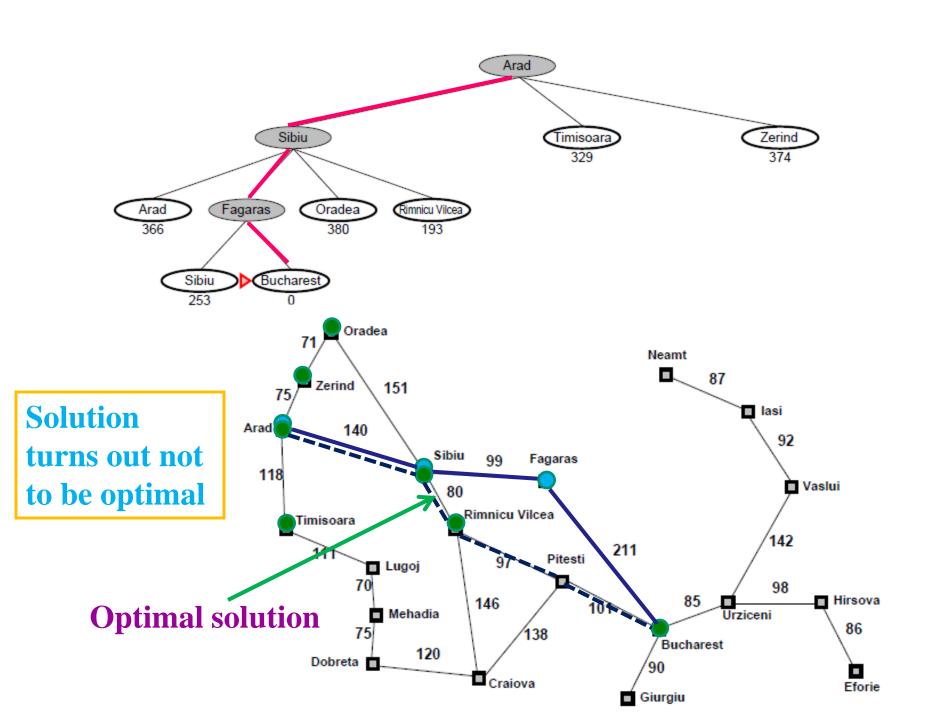


h (**n**) Straight-line distance to Bucharest Arad 366 Bucharest 0 Craiova 160 Dobreta 242 Eforie 161 Fagaras 178 Giurgiu 77 Hirsova 151 Iasi 226 Lugoj 244 Mehadia 241 Neamt 234 Oradea 380 Pitesti 98 Rimnicu Vilcea 193 Sibiu 253 Timisoara 329 Urziceni 80 Vaslui 199 Zerind 374









Properties of greedy best-first search

- Complete? No can get stuck in loops, e.g.,
 Iasi → Neamt → Iasi → Neamt →
- Complete in finite space with repeatedstate checking
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

Greedy Best-First Search

• Greedy search:

h(n) = estimated cost of the cheapest path from node n to a goal state.

Neither optimal nor complete

Uniform-cost search:
 g(n) = cost of the cheapest path from the initial state to node n.

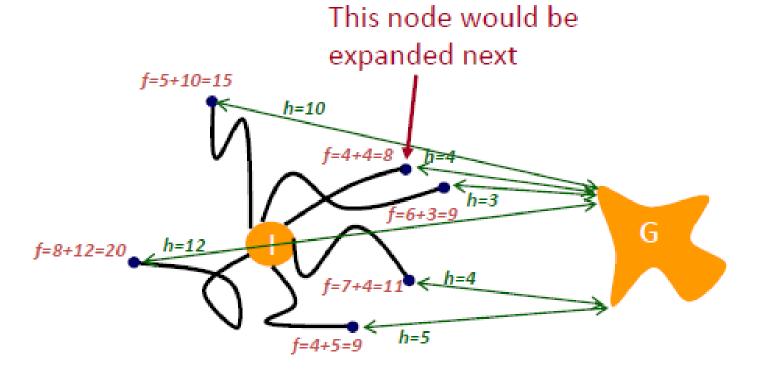
Optimal and complete, but very inefficient

A* search

- Idea: avoid expanding paths that are already expensive
- include cost of reaching node
- $-g(n) = cost \ of \ reaching \ n$
- -h(n) = estimated cost of reaching goal from state of n
- -f(n) = estimated cost of the cheapest path to a goal state that goes through path of <math>n

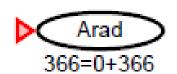
A* Search

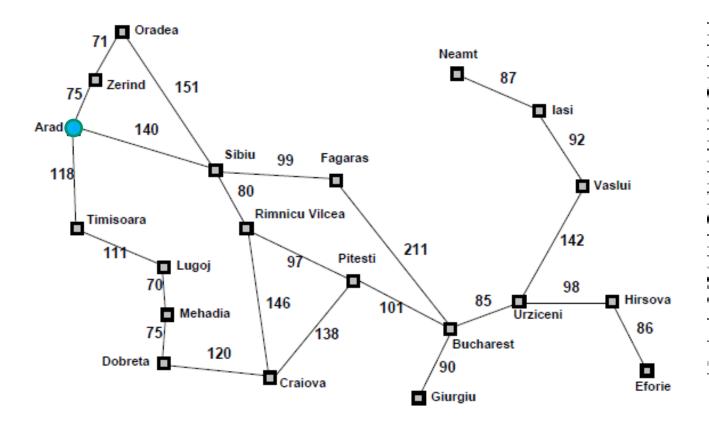
• f(n) = g(n) + h(n): Expand node that appears to be on cheapest paths to the goal



Go from Arad to Bucharest

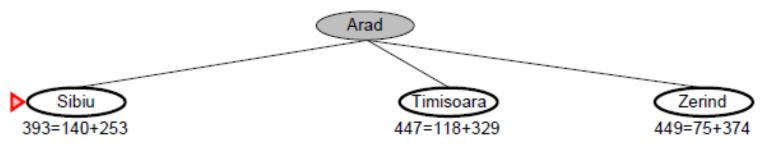
• $f(n) = g(n) + h_{SLD}(n)$

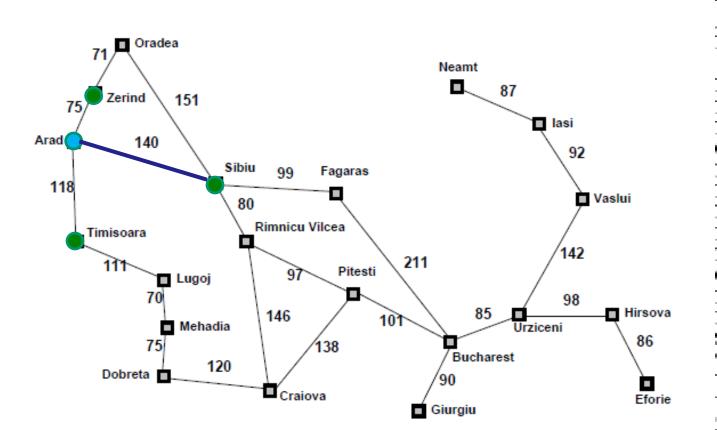




<u>h(n)</u>

Straight-line distance to Bucharest Arad 366 Bucharest 0 Craiova 160 Dobreta 242 Eforie 161 **Fagaras** 176 Giorgio 77 Hirsova 151 Lasi 226 Lugoj 244 Mehadia 241 Neamt 234 Oradea 380 Pitesti 10 Rimnicu Vilcea 193 Sibiu 253 Timiscara 329 Urziceni 80 Vashui 199 Zerind 374

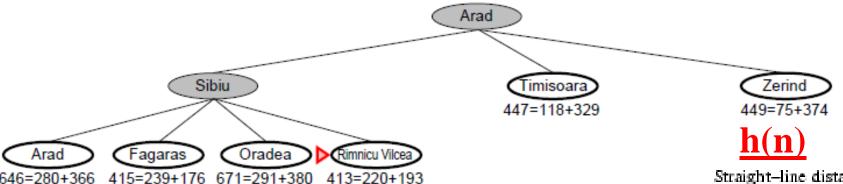


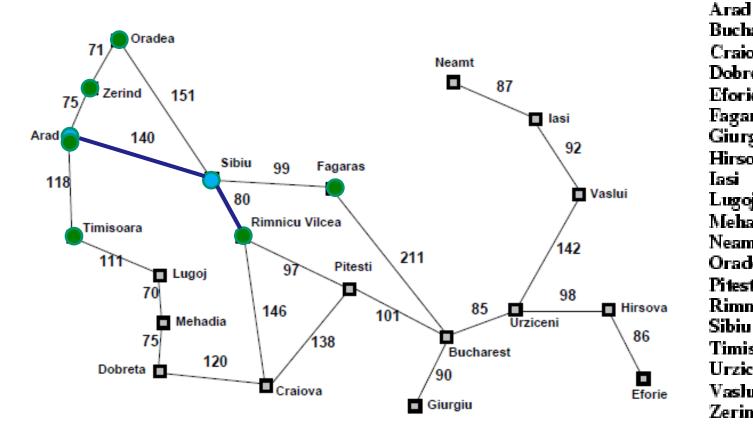


<u>h(n)</u>

Straight-line distance

o Bucharest	
\rad	366
Bucharest	0
Craiova	160
Oobreta	242
Eforie	161
agaras	176
3iurgiu	77
Tirsova	151
asi	226
ugoj	244
(fehadia	241
Veam t	234
Oradea	380
² itesti	10
Rimnicu V ilcea	193
Sibiu	253
limisoara	329
Jrziceni	80
/ashui	199
Zerind	374

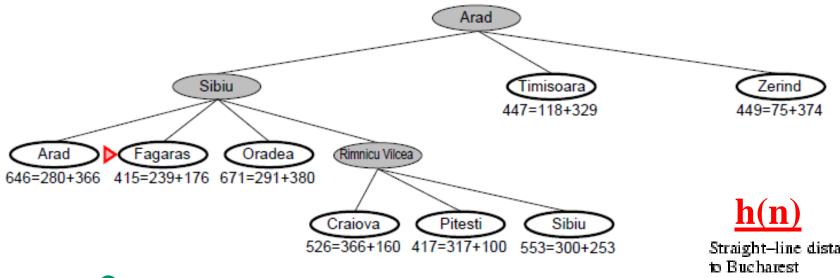


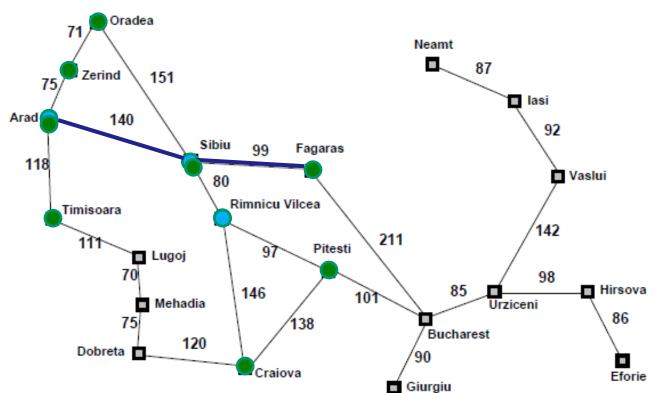


Straight-line distance to Bucharest

COLUMN TO A SECURITY

ATao	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
[asi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu V ilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374



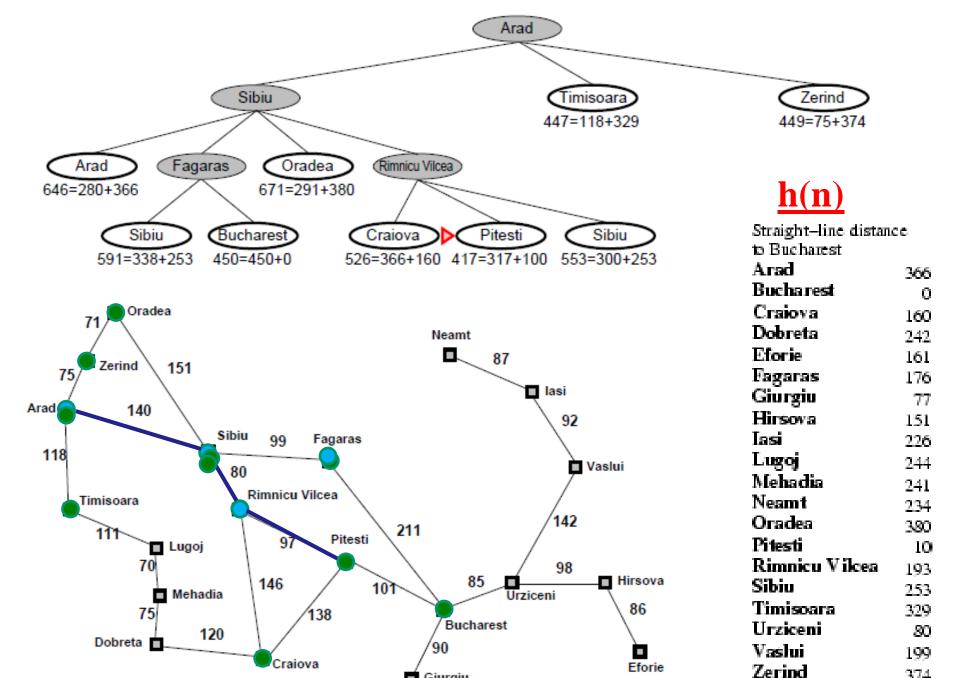


Straight-line distance

366
0
160
242
161
176
77
151
226
244
241
234
380
10
193
253
329
80
199

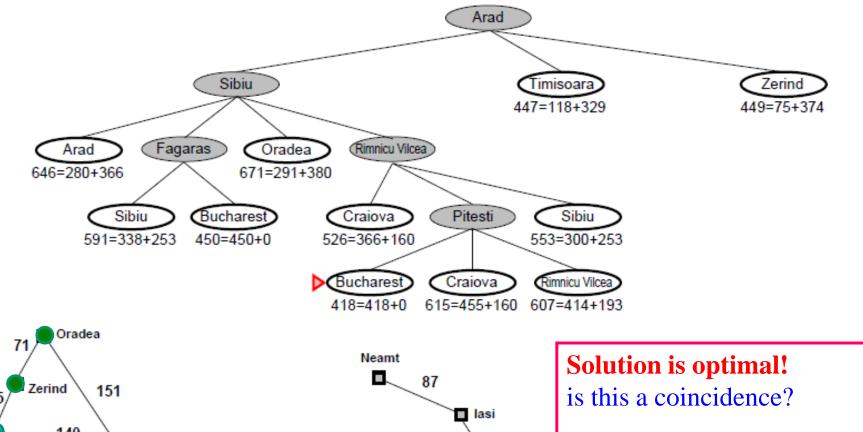
374

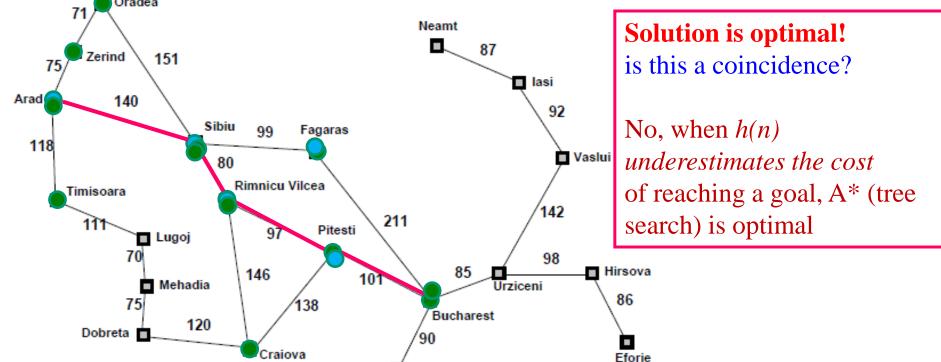
Zerind



Giurgiu

374





Giurgiu

Admissible heuristic

- Let h*(N) be the **true** cost of the optimal path from N to a goal node
- Heuristic h(N) is admissible if:

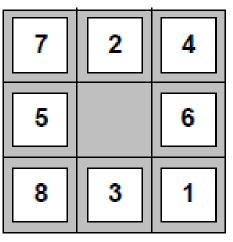
$$0 \le h(N) \le h^*(N)$$

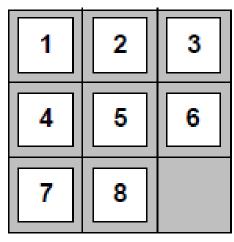
An admissible heuristic is always optimistic

Admissible heuristics

- For the 8-puzzle: If we want to find the shortest solutions, we need a heuristic function that never overestimates the number of steps to the goal. Here are two candidates:
- $h_1(n)$ = number of misplaced tiles
- $h_2(n) = total Manhattan distance (no. of squares from desired)$

location of each tile)





Goal State

$$h1(S) = ??$$

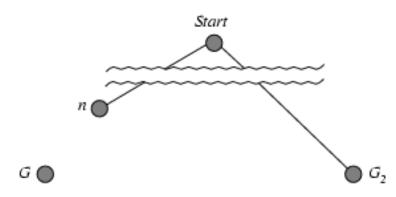
$$h2(S) = ??$$

Start State
$$h1(S) = ?? 6$$

$$h2(S) = ?? 4+0+3+3+1+0+2+1 = 14$$

Optimality of A* (proof)

Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



$$f(G_2) = g(G_2)$$

$$g(G_2) > g(G)$$

$$f(G) = g(G)$$

$$f(G_2) > f(G)$$

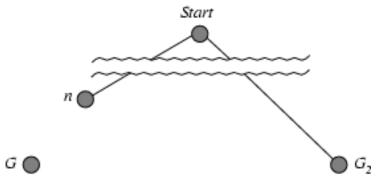
since
$$h(G_2) = 0$$

since
$$h(G) = 0$$

from above

Optimality of A* (proof)

Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



$$\begin{array}{ll} f(G_2) & > f(G) & \text{from above} \\ h(n) & \leq h^*(n) & \text{since h is admissible} \\ g(n) + h(n) & \leq g(n) + h^*(n) \\ f(n) & \leq f(G) \end{array}$$

Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion

Properties of A

- Complete?? Yes, unless there are infinitely many nodes with $f \le f(G)$
- Time?? Exponential in [relative error in h x length of soln.]
- Space?? Keeps all nodes in memory
- Optimal?? Yes-cannot expand f_{i+1} until f_i is finished
- \triangleright A expands all nodes with $f(n) < C^*$
- \triangleright A expands some nodes with $f(n) = C^*$
- \triangleright A expands no nodes with f(n) > C*

Proof of lemma: Consistency

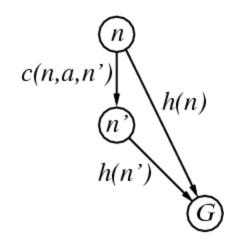
• A heuristic is consistent if

$$h(n) \le c(n, a, n') + h(n')$$

• If h is consistent, we have

$$f(n') = g(n') + h(n')$$

= $g(n) + c(n,a,n') + h(n')$
 $\geq g(n) + h(n)$
 $\geq f(n)$



i.e., f(n) is always greater at a successor node, so once a node has been visited, it cannot be visited again at a smaller cost and thus will never be reopened

Theorem: If h(n) is consistent, A* using GRAPH-SEARCH is optimal

Dominance

• If $h_2(n) \ge h_1(n)$ for all n (both admissible)

then h_2 dominates h_1 and is better for search

• For 8-puzzle heuristics h1 and h2, typical search costs (average number of nodes expanded for solution depth d):

IDS = 3,644,035 nodes

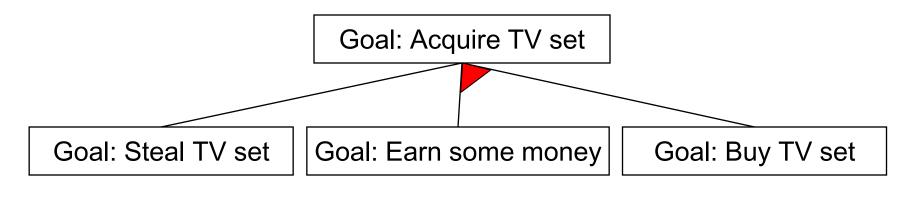
$$A^*(h_1) = 227 \text{ nodes}$$

 $A^*(h_2) = 73 \text{ nodes}$
 $d=24$ IDS = 54,000,000,000 nodes
 $A^*(h_1) = 39,135 \text{ nodes}$
 $A^*(h_2) = 1,641 \text{ nodes}$

Relaxed Problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem
- Example:
- ➤ If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then **h**₁(**n**) gives the shortest solution
- For the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution
- **Key point**: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Problem Reduction

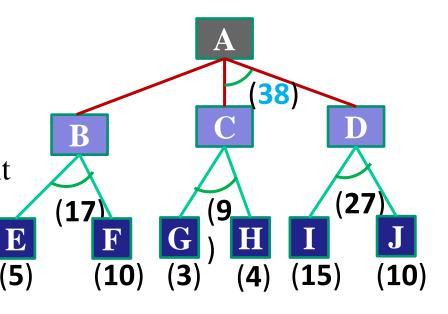


AND-OR Graphs

Algorithm AO* (Martelli & Montanari 1973, Nilsson 1980)

AND-OR Graph

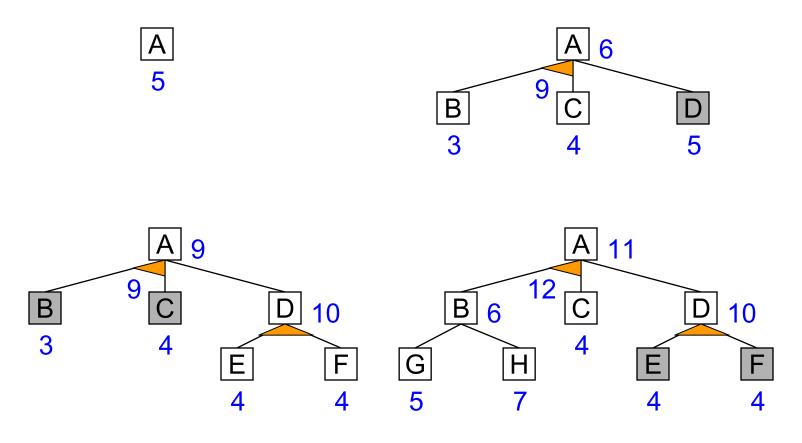
- Most promising single node: G
 (f = 3)
- Arc: **G-H** (7+2=9)
- But that arc is not part of the current best path since to use it, we must also use the arc: I-J (27)
- Path A-B-E-F is better (5+10+2+1=18)
- Should not expand **G** next; rather should examine either **E/F**



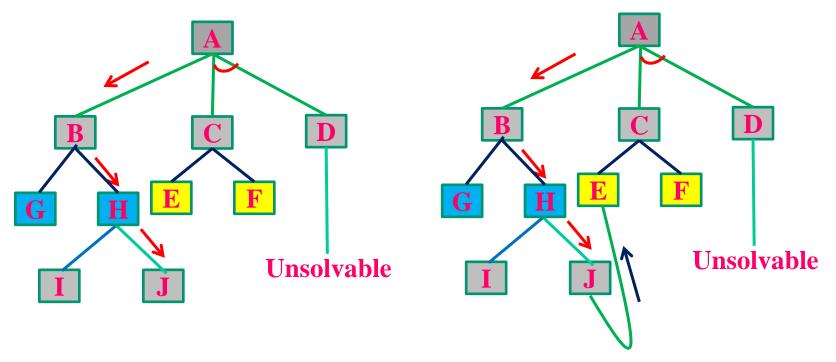
Problem Reduction

- 1. Initialize the graph to the starting node
- 2. Loop until the starting node is labeled **SOLVED** or until its cost goes above **FUTILITY**
 - (a) Traverse the graph, starting at the initial node & following the current best path, & accumulate the set of nodes that are on that path & have not yet been expanded/labeled as solved
 - (b) Pick one of these unexpanded nodes & expand it. if there are no successors, assign FUTILITY as the value of this node. Otherwise, add its successors to the graph & for each of them compute f` (use only h` & ignore g). If f` of any node is 0, mark that node as SOLVED
- (c) Change the f estimate of the newly expanded node to reflect the new info provided by its successors
- -propagate this change backward through the graph
- -if any node contains a successor are whose descendants are all solved, label the node itself as SOLVED
- -at each node that is visited while going up the graph, decide which of its successor arcs is the most promising & mark it as part of the current best path
- This may cause the current best path to change
- This propagation of revised cost estimates back up the tree was not necessary in the BFS algo because only unexpanded nodes were examined
- But now expanded nodes must be reexamined so that the best current path can be selected
- Thus it is important that their f value estimates available

Problem Reduction(Example)



Problem Reduction: Limitation



- -suppose J is expanded at the next step
- -that one of its successors in node E
- ☐Long path may be better

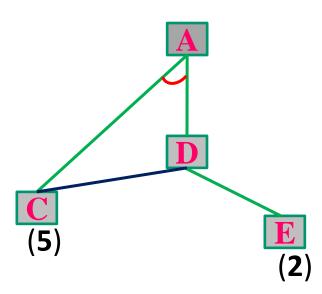
-This new path to E is longer than the previous path to E going through C

-but since the path through **C** will only lead to a solution if there is also a solution to **D**, which we Know there is not, the path through

J is better

Problem Reduction: Limitation

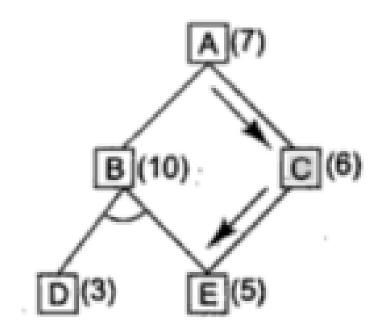
- Fails to take into account any interaction between sub goals
- Both node C/E ultimately lead to a solution, algorithm will report a complete solution that includes both of them
- AND-OR graph states that for A to be solved, both C/D must be solved
- The algo considers the solution of D as a completely separate process from the solution of C
- Looking just at the alternatives from D, E is the best path
- But it turns out that C is necessary anyway, so it would be better also to use it to satisfy D
- But the algo does not consider such interactions, it will find a non-optimal path.



AO* Algorithm

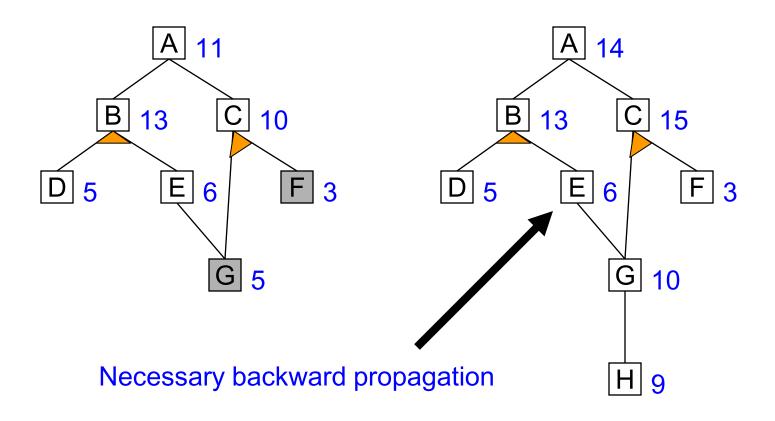
- Algorithm for searching AND-OR graph is called AO*
- Here single structure G, representing the part of the search graph that has been explicitly generated so far is used rather than two lists, OPEN and CLOSED in previous algorithms.
- Each node in the graph will point both down to its immediate successors and up to its immediate predecessor.
- Each node in a graph will also have associated with it an h value (an estimate of the cost of a path from current node to a set of solution nodes).
- We will not store g (cost from start to current node) as it is not possible to compute a single such value since there may be many paths to the same state.
- Also such value is not necessary because of the topdown traversing of the best-known path which guarantees that only nodes that are on the best path will ever be considered for expansion. So h will be good estimate for AND/OR graph search.

AO* Algorithm (Example)



Unnecessary backward propagation

AO* Algorithm (Example)



Assignment

• Why not best-first search algorithm is not adequate for searching AND-OR graphs?