11.2 Introduction to Difference Equations II

1. Difference Equation II
2. Interest
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Difference Equation II

The difference equation $y_n = ay_{n-1} + b$ (y_0 given) has solution

$$\begin{cases} y_n = \dfrac{b}{1-a} + \left(y_0 - \dfrac{b}{1-a} \right) a^n, \text{if } a \neq 1 \\ y_n = y_0 + bn, \text{if } a = 1. \end{cases}$$

ple Difference Equation II

a) Solve the difference equation $y_n = y_{n-1} + 2$, $y_0 = 3$.

b) Find y_{100}.

a) Here $a = 1$, $b = 2$ and $y_0 = 3$.

Therefore, $y_n = 3 + 2n$.

b) $y_{100} = 3 + 2(100) = 203$.

Interest

- Let an amount of money be deposited in a savings account. If interest is paid only on the initial deposit (and not on accumulated interest), then the interest is called *simple*.
- If interest is paid on the current amount in the account (initial deposit and accumulated interest), then the interest is called *compound*.

ample Simple Interest

Find the amount y_n at the end of n years if \$40 is deposited into a savings account earning 6% simple interest.

[new balance] = [previous balance] + [interest].

Simple interest is computed on the original balance.

So $y_n = y_{n-1} + .06y_0 = y_{n-1} + .06(40)$

$= y_{n-1} + 2.40.$

Since $a = 1$, $y_n = 40 + 2.4n$.

nple Compound Interest

Find the amount y_n at the end of n years if \$40 is deposited into a savings account earning 6% interest compounded annually.

[new balance] = [previous balance] + [interest].

Compound interest is computed on the previous balance, y_{n-1}.

So $y_n = y_{n-1} + .06y_{n-1} = 1.06y_{n-1}$.

Since $a \neq 1$, $y_n = 40(1.06)^n$.

Amount with Interest

If y_0 dollars is deposited at interest rate i per period, then the amount after n periods is

Simple interest: $y_n = y_0 + (iy_0)n$

Compound interest: $y_n = y_0(1 + i)^n$.

The initial value y_0 is called the *principal*.

If the annual interest rate is r compounded k times per year, then $i = r/k$.

Consumer Loan

- A *consumer loan* is one in which an item is bought and paid for with a series of equal payments until the original cost plus interest is paid off. Each time period, part of the payment goes toward paying off the interest and part goes toward reducing the balance of the loan.
- A consumer loan used to purchase a house is called a *mortgage*.

mple Consumer Loan

- A consumer loan of \$2400 carries an interest rate of 12% compounded annually and a yearly payment of \$1000.
- *a*) Find the difference equation for the balance, y_n, owed after n years.
- *b*) Compute the balances after 1, 2 and 3 years.

ple Consumer Loan (*a*)

- *a*) At the end of each year
- [new balance] =
-     [previous balance] + [interest] - [payment]
- Since interest is compound, it is computed on the previous balance, y_{n-1}.
  - $y_n = y_{n-1} + .12\, y_{n-1} - 1000 = 1.12\, y_{n-1} - 1000$

ple Consumer Loan (*b*)

- *b*) $y_n = 1.12\, y_{n-1} - 1000$
- $y_1 = 1.12(2400) - 1000 = \$1688$
- $y_2 = 1.12(1688) - 1000 = \$890.56$
- $y_3 = 1.12(890.56) - 1000 \approx \$0$

Summary Section 11.2 - Part 1

- The value of the n^{th} term of a difference equation $y_n = y_{n-1} + b$ (y_0 given) can be obtained directly (that is, without generating the preceding terms) with the formula $y_n = y_0 + bn$.
- Money deposited in a savings account is called the *principal*.

Summary Section 11.2 - Part 2

- Suppose y_0 dollars is deposited at a yearly simple interest rate r. The balance after n years, y_n, satisfies the difference equation
$$y_n = y_{n-1} + ry_0.$$
- Suppose y_0 dollars is deposited at a yearly interest rate r compounded k times a year. The period interest rate is $i = r/k$. The balance after n interest periods, y_n, satisfies the difference equation $y_n = (1 + i)y_{n-1}$.