MTH 202

1. Ordinary Differential Equation (ODE): an equation involving 1 dependent and 1 independent variable. Eg $\frac{d^2y}{dx^2} + 3y = 4$ where y is the dependent variable & x is the independent variable.

2. Partial differential Equation (PDE): a differential eqn involving one or more dependent variable and 2 or more independent variables. Eg

$$\frac{\partial^2 u}{\partial x^2} + \frac{3\partial u}{\partial y} = 0$$

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x^2} = e^t$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial z^2} = 0$$

3. Total differential Equation (TDE): an equation that expresses the total change of a function with respect to changes in all of its independent variable often used for approximating changes in multi-variable function.

Eg $u = u(x, y, z)$

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{du}{\partial z}dz$$

$$Pdx + Qdy + Rdz$$

$$P = \frac{\partial u}{\partial x}, \quad Q = \frac{\partial u}{\partial y}, \quad R = \frac{\partial u}{\partial z}$$

Classification of differential Equation

1. Homogenous
2. Non-homogenous

Examples: i. $3\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = \sin x$ ⟹ Non homogenous

ii) $\frac{d^3y}{dx^2} - 5\left(\frac{dy}{dx}\right)^3 = 4y$

$\frac{d^3y}{dx^3} - 5\left(\frac{dy}{dx}\right)^3 - 4y = 0$ ⟶ Homogenous

$$\frac{dy}{dx} + y = 0 \Rightarrow \text{homogenous}$$

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + e^{-x} = 4$$

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 4 - e^{-x}$$

Order of a differential equation

The order of a differential equation is the order of the highest derivative present in the differential equation.

Example 1.

i. $\frac{d^2y}{dx^2} + y = x^2$ is of second order or order 2

ii $(x+y)\left(\frac{dy}{dx}\right)^2 - 1 = 0$ is of 1st order or order 1

iii) $\left(\frac{d^3y}{dx^3}\right)^2 + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + x^2\left(\frac{dy}{dx}\right)^3 = 0$ is order 3 or 3rd order

Degree of a differential equation

This is the highest exponent of the highest order derivative appearing in it after the equation has been expressed in the form free from radicals and any fractional power of the derivatives or negative power.

Example 1

i. $y - x\frac{dy}{dx} = r\sqrt{1+\left(\frac{dy}{dx}\right)^3}$

$\cancel{y^2} - x^2\cancel{\left(\frac{dy}{dx}\right)^2}$ Square both sides

$$\left(y - \frac{xdy}{dx}\right)^2 = \left(r\sqrt{1+\left(\frac{dy}{dx}\right)^3}\right)^2$$

Independent → [illegible]
Dependent → [illegible]

$y^2 - x^2\frac{dy}{dx}$ $\quad y^2 - x^2\left(\frac{dy}{dx}\right)^2 = r^2 + r\left(\frac{dy}{dx}\right)^3$

The degree of the derivative is of degree 3

iii) $y = x\frac{dy}{dx} + \frac{a}{dy/dx}$

multiply both sides by dy/dx

$y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + a$

The degree is 2 or is of degree 2

iii) $e^x\left(\frac{d^2y}{dx^2}\right)^7 + 4\left(\frac{dy}{dx}\right)^4 + \frac{d^3y}{dx^3} = 0$

iv $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$

is of degree 2

$$\left.\begin{aligned} \frac{\partial^2 z}{\partial x^2} &= \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\ \frac{\partial^2 z}{\partial y^2} &= \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \\ \frac{\partial^2 z}{\partial x \partial y} &= \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \end{aligned}\right\} \text{explain each expression}$$

Linear and Non-Linear differential Equation

Types of function

1. Algebraic function → Linear function or Non-linear funct
2. Logarithm Function
3. Exponential Function
4. Trigonometric Function

(2–4) } Non-linear function

A linear function is when the highest power of unknown is 1.

Conditions of a linear differential Equation

1. Any DE involving product of derivatives is also non-linear. For Example, DEs involving terms like $(dy/dx)^2$, $\left(\frac{d^2y}{dx^2}\right)^5$ are non-linear.

2. For functions, non linearity affects only functions of dependent variables (y) and its derivatives but NOT independent variables (x). For example, functions like e^x, $\sin x$, $\cos x$ do not make a DE non-linear. But function like e^y, $\sin y$, $\cos y$ make a DE nonlinear.

$\frac{dy}{dx}$ of $\cos x = -\sin x$

$\sin x \ 2 \equiv \cos x$

Solution of a Differential Equation.

A solution to a differential equation is a function (of the independent variables) that, when substituted into the equation, ~~radaa~~ produces a true statement. For instance, if ~~your~~ $y(t) = e^{2t}$, then $\frac{dy}{dt} = 2e^{2t}$

$$y(t) = e^{2t}$$

$$\frac{dy}{dt} = 2e^{2t} = 2y$$

$$\therefore \frac{dy}{dt} - 2y = 0$$

$$\frac{d}{dt}(y) - 2y = \frac{d}{dt}(e^{2t}) - 2e^{2t}$$

$$= 2e^{2t} - 2e^{2t}$$

$$= 0$$

If $y(x) = e^x \cos x$, we can verify that y is a solution to the D.E

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

$$\frac{dy}{dx} = \cos x e^x + e^x(-\sin x) = e^x \cos x - e^x \sin x = e^x(\cos x - \sin x)$$

$$\frac{d^2y}{dx^2} = e^x(-\sin x - \cos x) + e^x(\cos x - \sin x)$$

$$= -e^x \sin x - e^x \sin x$$

$$= -2e^x \sin x$$

On substitution:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = -2e^x \sin x - 2[e^x \cos x - e^x \sin x) + 2e^x \cos x$$

$$= -2e^x \cancel{\sin x} - 2e^x \cancel{\cos x} + 2e^x \cancel{\sin x} + 2e^x \cos x$$

~~Types of Differential Solution~~

~~Each initial condition~~

no of initial condition = no of order

Find the general & particular solution of the DE — Initial condition ⇒ $y(1) = 2$

1. $\frac{dy}{dx} = 3x^2$ that is satisfied at point $(1, 2)$

ii. $\frac{d^2y}{dx^2} = 6x + 2$ at points $(1, 0)$ and $(0, 1)$

i) Sol

$\frac{dy}{dx} = 3x^2$ (Integrate b-s)

$\int \frac{dy}{dx}\, dx = \int 3x^2\, dx$

$y = x^3 + C$ ⟹ General Solution

For the particular solution; $x = 1$ and $y = 2$

$2 = 1^3 + C$

$2 - 1 = C$

$C = 1$

∴ The particular solution is $y = x^3 + 1$

ii $\frac{d^2y}{dx^2} = 6x + 2$

NOTE: The no of order of the equation is the no of constants expected

$\iint \frac{d^2y}{dx^2} = \iint 6x + 2$

$\int \frac{dy}{dx} = \int 3x^2 + 2x + A$

$y = x^3 + x^2 + Ax + B$ ⟹ general solutions

For PS: $(1, 0)$ $0 = 1 + 1 + A + B$

$A + B = -2$ — (i)

$(0, 1)$ $1 = 0 + 0 + 0 + B$

$B = 1$ — (ii)

$A + 1 = -2$

$A = -3$, $B = 1$

Hence, $y = x^3 + x^2 - 3x + 1$ ⟹ P.S

Constant coeffice $3\frac{d^3y}{dy^3}$

Variable coeffict $4x\frac{dy}{dx}$

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
Fronmed

Example 3: Show that for real constats a and b, the functions $y_1(x) = a\cos 2x$ and $y_2(x) = b\sin 2x$ are solutions of the D.E $\frac{d^2y}{dx^2} + 4y = 0$

For $y_1(x) = a\cos 2x$

$$\frac{dy_1}{dx} = a(-2\sin 2x) = -2a\sin 2x$$

$$\frac{d^2y_1}{dx^2} = -2a(2\cos 2x)$$

$$\frac{d^2y_1}{dx^2} = -4a\cos 2x$$

$$\frac{d^2y}{dx^2} + 4y = -4a\cos 2x + 4a\cos 2x = 0 \quad \therefore y_1(x) \text{ is a solution}$$

For $y_2(x) = b\sin 2x$

$$\frac{dy_2}{dx} = 2b\cos 2x$$

$$\frac{d^2y}{dx^2} = -4b\cos 2x$$

$$\frac{d^2y}{dx^2} + 4y = -4b\cos 2x + 4(b\cos 2x)$$

$$= -4b\cos 2x - 4b\cos 2x = 0$$

Variable Separable

If a DE is of the form $f(x)\,dx + g(y)\,dy = 0$ or can be changed into such form then it is said to belong to the family of DE called variable separable. To obtain a solution of a DE that is of variable separable, we must first reduce it to the form above and integrate directly.

Find the general solution of each of the D.E below

1. $y^2 + x^2\frac{dy}{dx} = 0$
2. $s\,ds - \frac{1(1+s^2)}{1+t^2}\,dt = 0$
3. $1(-x^2)^{1/2}\,dy/dx + 1 + y^2 = 0$
4. $xy(1+x^2)\,dy/dx - (1+y^2) = 0$

Integrate both sides

$$\frac{-1}{y} = \frac{1}{x} + K$$

4. $xy(1+x^2)\frac{dy}{dx} - (1+y^2) = 0$

$$xy(1+x^2)\frac{dy}{dx} = 1+y^2$$

divide b-s by y

$$x(1+x^2)\frac{dy}{dx} = \frac{1+y^2}{y}$$

$$y\left[x(1+x^2)\frac{dy}{dy}\right] = dx(1+y^2)$$

$$\int \frac{y}{1+y^2}\,dy = \int \frac{dx}{x(1+x^2)}$$

2. $S\,ds - \frac{1(1+s^2)}{1+t^2}\,dt = 0$

$$S\,ds = \frac{1(1+s^2)}{1+t^2}\,dt$$

$$\frac{S\,ds}{1} = \frac{1+s^2}{1+t^2}\,dt$$

$$\int \frac{S\,ds}{1+s^2} = \int \frac{dt}{1+t^2}$$

let $u = 1+s^2$ $\frac{du}{ds} = 2s$

$\therefore$ $S\,ds = \frac{du}{2}$

$$\int \frac{s}{1+s^2}\,ds = \int \frac{1}{u}\cdot\frac{du}{2}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln u + c$$

$$= \frac{1}{2}\ln|1+s^2| + C$$

$$\int \frac{1}{1+t^2}\,dt = \tan^{-1}(t) + C$$

$$\therefore \int \frac{s}{1+s^2}\,ds = \int \frac{1}{1+t^2}\,dt$$

$$= \frac{1}{2}\ln(1+s^2) = \tan^{-1}(t) + c$$

3. $(1-x^2)^{1/2}\frac{dy}{dx} + 1 + y^2 = 0$

$$(1-x^2)^{1/2}\frac{dy}{dx} = -1(1+y^2)$$

$\frac{a}{b} = \frac{c}{d}$ $\quad ad = cb \quad \frac{ad}{c} = \frac{b}{1} \quad \frac{ad}{c} = \frac{b}{d} \quad \frac{a}{c} = \frac{b}{d}$

$\frac{a}{b} = \frac{c}{d}$

$$(1-x^2)^{1/2} dy = (-1-y^2) dx$$

$$\frac{1}{(-1-y^2)} dy = \frac{1}{(1-x^2)^{1/2}} dx$$

$$\frac{-1}{(1+y^2)} dy = \frac{dx}{(1-x^2)^{1/2}}$$

$$\int \frac{-dy}{1+y^2} = \int \frac{dx}{(1-x^2)^{1/2}}$$

$$\frac{dy}{1+y^2} = -\frac{dx}{(1-x^2)^{1/2}}$$

$$\int \frac{dy}{1+y^2} = \int -\frac{dx}{\sqrt{1-x^2}}$$

$=$

For $\int \frac{1}{1+y^2} dy = \tan^{-1} y + C$

For $\int \frac{-dx}{\sqrt{1+x^2}}$

$$\int \frac{dx}{\sqrt{1+x^2}} = \sin^{-1}(x)$$

$$\therefore \int \frac{-dx}{\sqrt{1+x^2}} = -\int \frac{dx}{\sqrt{1+x^2}} = -\sin^{-1}(x)$$

$$\therefore \int \frac{1}{1+y^2} dy = \int \frac{-dx}{\sqrt{1+x^2}}$$

$$= \tan^{-1} y = -\sin^{-1}(x) + C.$$

differentiation / Equation Table

Derivatives		Integrals	
$\sin x$	$\cos x$	dx	$x + C$
$-\cos x$	$\sin x$	$\cos x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\sin x$	$-\cos x + C$
$-\cot x$	$\csc^2 x$	$\sec^2 x$	$\tan x + C$
$\sec x$	$\sec x \tan x$	$\csc^2 x$	$\cot x$
$-\csc x$	$\csc x \cot x$	$\csc x \cot x$	$-\csc x + C$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{dx}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$	$-\frac{dx}{\sqrt{1-x^2}}$	$\cos^{-1} x + C$
$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\frac{dx}{1+x^2}$	$\tan^{-1} x + C$
$\cot^{-1} x$	$-\frac{1}{1+x^2}$	$-\frac{dx}{1+x^2}$	$\cot^{-1} x + C$
$\sec^{-1} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{dx}{\lvert x\rvert\sqrt{x^2-1}}$	$\sec^{-1} x + C$
$\csc^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$-\frac{dx}{\lvert x\rvert\sqrt{x^2-1}}$	$\csc^{-1} x + C$
e^x	e^x	$e^x dx$	$e^x + C + C$
$\log\lvert x\rvert$	$\frac{1}{x}$	$\frac{dx}{x}$	$\log\lvert x\rvert + C$

Homogenous D.E.

A D.E $M[x, y]\,dx + N[x, y]\,dy = 0$ is homogenous if $M(x, y)$ and $N(x, y)$ are homogenous expression of the same degree.

N.B: Anytime we have a homogenous D.E of [x and y] and a substitution of the form $Z = y/x$ or $Z = x/y$ reduces the homogenous D.E to a variable separable form.

$$\boxed{M \text{ is } dx \text{ \& } N \text{ is } dy}$$

Show if the following differential equations are homogenous, hence; solve the equation

1. $(x^2 + y^2)\,dx + 2xy\,dy = 0$
2. $xy^2 \dfrac{dy}{dx} = x^3 + y^3$

1. $M(x, y) = x^2 + y^2$

$N(x, y) = 2xy$

$M(Sx, Sy) = (Sx)^2 + (Sy)^2 = S^2x^2 + S^2y^2$

$= S^2(x^2 + y^2)$

$= \cancel{S^2 M(x} = S^2 M(x, y)$

$M(Sx, Sy) = 2(Sx)(Sy) = 2S^2xy$

$= S^2(2xy)$

$= S^2 M(x, y)$

Since the degree are equal (power of S are equal), hence the D.E is homogenous.

To solve the D.E, let $Z = \dfrac{x}{y} \Rightarrow x = yz$, $y = \dfrac{x}{z}$

~~Substitute~~ $x = yz$ and $y = \cancel{z/x}$ in the D.E

Substitute $x = yz$ and $y = \dfrac{x}{z}$ in the D.E

$\therefore (x^2 + y^2)\,dx + 2xy\,dy = 0$

will be $\left(y^2z^2 + \dfrac{x^2}{z^2}\right)dx + 2(yz)\left(\dfrac{x}{z}\right)dy = 0$

substitute one

so you try to completely eliminate ~~of~~ of the variables by using the newly introduced variable z. In this case, we are ~~eliminating~~ substituting for y

$\therefore$ we have

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$dy = \frac{v\,du - u\,dv}{v^2}$$

$y = \frac{x}{z}$ $\frac{u}{v}$

differentiating $= \frac{z\,dx - u\,dz}{z^2}$

$$(x^2+y^2)\,dx + 2xy\,dy$$

$$= \left(x^2 + \frac{x^2}{z^2}\right)dx + (2x)\left(\frac{x}{z}\right)\left(\frac{z\,dx - x\,dz}{z^2}\right) = 0$$

$$\frac{x^2z^2+x^2}{z^2}\,dx + \frac{2x^2}{z}\left(\frac{1}{z}dx - \frac{x}{z^2}dz\right) = 0$$

$$\frac{x^2z^2+x^2}{z^2}\,dx + \frac{2x^2}{z^2}\,dx - \frac{2x^3}{z^3}\,dz = 0$$

$$\frac{x^2z^2+3x^2}{z^2}\,dx - \frac{2x}{z^3}\,dz = 0$$

$$\frac{x^2(z^2+3)}{z^2}\,dx = \frac{2x^3}{z^3}\,dz$$

$$(z^2+3)\,dx = \frac{2x}{z}\,dz$$

If you differentiate the denominator & it gives the numerator then the

$$\frac{2z}{3(z^2+3)}$$

Since $\int z^2+3 = 2z$

$$\therefore \int \frac{2z}{3(z^2+3)}$$

$$= \int \frac{1}{3}\cdot\frac{2z}{z^2+3}$$

$$= \frac{1}{3}\ln z^2+3$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$$

$$\frac{dx}{x} = \frac{2}{z(z^2+3)}\,dz$$

Integrate both sides

$$\int \frac{1}{x}\,dx = \int \frac{2}{z(z^2+3)}\,dz$$

Using partial fraction

$$\frac{2}{z(z^2+3)} = \frac{A}{z} + \frac{Bz+C}{z^2+3}$$

$$\frac{2}{z(z^2+3)} = \frac{A(z^2+3) + z(Bz+c)}{z(z^2+3)}$$

$$2 = Az^2 + Bz^2 + Cz + 3A$$

$A + B = 0 \Rightarrow B = -2/3$

$C = 0$

$3A = 2 \Rightarrow A = 2/3$ $\quad\therefore \frac{2}{3z} + -\frac{2z}{3(z^2+3)}$

$$\therefore \frac{dx}{x} = \left(\frac{2}{3z} - \frac{2z}{3(z^2+3)}\right)dz$$

$$\ln x = \frac{2}{3}\ln z - \frac{1}{3}\ln(z^2+3) + C$$

$$\ln x = \frac{2}{3}\ln z - \frac{1}{3}\ln(z^2+3) + \ln K$$

$\ln x = \frac{1}{3}\left(\frac{y}{x}\right)^2 + C$ OR

$kx = e^{y^3/3x^3}$

$x^3 + 3xy^2 = k$

$\boxed{\text{If } y = e^x \text{ then } \ln(y) = x}$

$\ln x = \frac{2}{3}\ln z - \frac{1}{3}\ln(z^2+3) + \ln k$

$\cancel{\ln x - \frac{2}{3}\ln z}$ $\ln x - \ln z^{2/3} + \ln(z^2+3)^{1/3} = \ln k$

[+ = product, − = division] → Rule of logarithm

$\cancel{\ln}\left[\frac{x(z^2+3)^{1/3}}{z^{2/3}}\right] = \cancel{\ln} k$

$$\boxed{\begin{array}{l}\text{If } y = e^x \text{ then}\\ \ln(y) = x\\ e^{\ln(x)} = x\\ \ln(e^x) = x\end{array}}$$

~~But~~ Recall, $z = x/y$ on substitute

$\therefore \frac{x(z^2+3)^{1/3}}{z^{2/3}} = k$

$x\left[\frac{((x/y)^2+3)^{1/3}}{(x/y)^{2/3}}\right] = k$

$x\left[(x/y)^2+3\right]^{1/3} = k\left(\frac{x}{y}\right)^{2/3}$

~~By~~ cube both sides

$\therefore x^3\left((x/y)^2+3\right) = k\left(\frac{x}{y}\right)^2$

$x^3\left(\frac{x^2}{y^2}+3\right) = k\left(\frac{x}{y}\right)^2$

$x^3\left(\frac{x^2}{y^2}+3\right) = k\left(\frac{x^2}{y^2}\right)$

$x\left(\frac{x^2}{y^2}+3\right) = k\left(\frac{1}{y^2}\right)$

$x\left(\frac{x^2+3y^2}{y^2}\right) = k\left(\frac{1}{y^2}\right)$

$x(x^2+3y^2) = k$

$k = x^3 + 3y^2$

Pard because each is a func of 2 variables

A D.E can be expressed in the form $A(x)\,dx + B(y)\,dy = 0$, A set of solution can be determined by integrating the function whose derivative is $A(x)\,dx + B(y)\,dy = 0$. The concept can be extended to some equations of the form

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad \text{(i)}$$

$$dF = M\,dx + N\,dy \quad \text{(ii)}$$

$$F(x,y) = 0 \quad \text{(iii)}$$

From equation 2, it follows that $dF = 0$ implies that $M\,dx + N\,dy = 0$. We want to find out under what condition on M and N does F exist such that its total differential is $M\,dx + N\,dy$. We call equation 1 an exact equation if there exists the function F such that $df = M\,dx + N\,dy$

The necessary and sufficient condition that makes equation 1 to be exact is that ∂/∂o (∂ is partial) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Example

Test for the Exactness of the D.E below and solve

1. $3x(xy-2)\,dx + (x^3+2y)\,dy = 0$

ii $(2x^3 - xy^2 - 2y + 3)\,dx - (x^2y + 2x)\,dy = 0$

Sol

$M = 3x(xy-2) = 3x^2y - 6x$

$N = x^3 + 2y$

$\frac{\partial M}{\partial y} = 3x^2, \quad \frac{\partial N}{\partial x} = 3x^2$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the D.E is exact

To solve the D.E, we find a differential equation such that $F(x,y) = C$ —(1)

M is $\partial f/dx$ and N is $\partial f/\partial y$ ∴

$$\partial f/\partial x = M(x,y) = 3x^2y - 6x \quad \text{—(2)}$$

$$\partial f/\partial y = N(x,y) = x^3 + 2y \quad \text{—(3)}$$

$\frac{\partial f}{\partial y}$ with respect to y

~~In~~ If you integrate a partial derivative, the constant of the equation will be the function of the other variable x.

~~differentiating~~ integrating $\partial f/\partial x$, we have

$$F = \int (3x^2y - 6x)\,dx + T(y)$$

$$= x^3y - 3x^2 + T(y)$$

$$\therefore \frac{\partial f}{\partial y} = x^3 + T'(y)$$ → Partial derivative of the integral of the $\partial f/\partial x$.

$$\frac{\partial f}{\partial y} = x^3 + T'(y) \quad \text{——} (4)$$

equations 3 and 4

$$x^3 + 2y = x^3 + T'(y)$$

$$T'(y) = 2y$$

$$T(y) = y^2$$

$$\therefore F(x,y) = x^3y - 3x^2 + T(y) = x^3y - 3x^2 + y^2$$

$\therefore$ $x^3y - 3x^2 + y^2 = C$ is the solution to the D.E

check

$$\boxed{\begin{gathered} x^3\frac{dy}{dx} + 3x^2y - 6x + 2y\frac{dy}{dx} = 0 \quad \text{→ Product rule of } x^3y \\ \frac{dy}{dx}(x^3+2y) - 6x + 3x^2y \\ (x^3+2y)\,dy + (3x^2y - 6x)\,dx = 0 \end{gathered}}$$

Assignment

1. Show if this d.e is homogenous hence solve the equation $xy^2\, dy/dx = x^3 + y^3$
2. Test for the Exactness of this D.E & solve
$$(2x^3 - xy^2 - 2y + 3)\,dx - (x^2y + 2x)\,dy = 0$$

Che

$$\frac{V\frac{du}{dx}-U\frac{dv}{dx}}{V^2}$$

$xy^2 \, dy/dx = x^3+y^3$

$\cancel{xy^2}\, dy = (x^3+y^3)dx \qquad xy^2dy = (x^3+y^3)dx$

$M(x,y) = x^3+y^3$

$N(x,y) \quad xy^2$

$M(Sx, Sy) = (Sx)^3+(Sy)^3 \qquad = S^3x^3 + S^3y^3$

$N(Sx,Sy) = (Sx)(Sy)^2 \qquad = Sx S^2y^2$

$M(Sx, Sy) = S^3(x^3+y^3) \qquad = S^3(M(x,y))$

$N(Sx, Sy) = S^3(xy^2) \qquad = S^3N(x,y)$

Since $M(Sx, Sy)$ & $N(Sx, Sy)$ have the same power, the equation is homogenous

$xy^2 \, dy/dx = x^3+y^3$

$(x^3+y^3)dx = (xy^2)dy$

let $Z = \frac{x}{y}$ $\therefore$ $y = x/Z$ in the D.E

$$\left(x^3+\left(\frac{x}{z}\right)^3\right)dx = \left(x\left(\frac{x}{z}\right)^2\right)\left(\frac{Zdx-xdv}{Z^2}\right)$$

$$\left(x^3+\frac{x^3}{Z^3}\right)dx = \frac{x^3}{Z^2}\left(\frac{Zdx-xdZ}{Z^2}\right)$$

$$\left(\frac{Z^3x^3+x^3}{Z^3}\right)dx = \frac{x^3}{Z^4}(Zdx-xdZ)$$

$$x^3\left(\frac{Z^3+1}{Z^3}\right)dx = \frac{x^3}{Z^4}(Zdx-xdz)$$

$$(Z^3+1)dx = \frac{Zdx-xdZ}{Z}$$

$$Z^4dx+\cancel{Z}dx = \cancel{Z}dx-xdZ$$

$$Z^4dx = -xdZ$$

$$\frac{Z^4}{dz} = \frac{-x}{dx} \qquad \int \frac{dZ}{Z^4} = \frac{-dx}{x} \qquad = \frac{-dz}{Z^4} = \frac{dx}{x}$$

recall $\cancel{y = x/}$ $Z = x/y$ $\therefore$ $dZ = \frac{y}{x}$

$\cancel{y = x/}$ $Z = x/y$

Z^{-4} $\quad \frac{x^2}{2x}$ $\quad 2x^1$ $\quad -\frac{Z^{-3}}{3}$ $\quad -x^{-1}$ $\quad -x$

$-\frac{1}{Z^4}dZ = +\frac{1}{x}dx$

$+\frac{1}{3Z^3} = \ln x +$

$\ln x = \frac{1}{3Z^3} + C$

$\ln x = \frac{1}{3}\frac{y^3}{x^3} + C$

2. $(2x^3 - xy^2 - 2y + 3)\,dx - (x^2y + 2x)\,dy = 0$

$\cancel{2x^3 - xy^2}$

$M = 2x^3 - xy^2 - 2y + 3$

$N = -(x^2y + 2x) = -x^2y - 2x$

$\cancel{\frac{\partial M}{\partial y} = -2yx - 2}$ $\quad$ $\cancel{\frac{\partial N}{\partial x} = -2xy}$

$\frac{\partial M}{\partial y} = -2xy - 2$ $\quad$ $\frac{\partial N}{\partial x} = -2xy - 2$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\therefore$ It is exact

~~Ans~~

$\partial f / \partial x$ $M(x,y) = 2x^3 - xy^2 - 2y + 3$ — (i) $\rightarrow \partial f / \partial x$

$N(x,y) = -x^2y - 2x$ — (ii) $\rightarrow \partial f / \partial y$

integrating ~~differentiate~~ $\partial f / \partial x$ $\quad$ $F(x,y) = c$ — (iii)

$F = \int (2x^3 - xy^2 - 2y + 3)\,dx + T(y)$

$= \frac{x^4}{2} - \frac{x^2y^2}{2} - 2xy + 3x + T(y)$

$\frac{\partial F}{\partial y} = -x^2y - 2x + T(y)$ — equating 2 & 3

$-\cancel{x^2y} - \cancel{2x} + T(y) = -\cancel{x^2y} - \cancel{2x}$

$T(y) = 0$

$F(x,y) = \frac{x^4}{2} - \frac{x^2y^2}{2} - 2xy + 3x + 0$