Module 2: MODELLING AND SIMULATION CONCEPTS

Module Introduction

This module is divided into four (4) units

Unit 1: Simulation and Modelling
Unit 2: Modelling Methods
Unit 3: Physics-Based Finite Element Model
Unit 4: Statistics for Modelling and Simulation

Unit 1: Simulation and Modelling

Contents

1.0 Introduction

Real world phenomenon are very dynamic thus difficult to exactly predict. To make decisions in such circumstances, we need a tool or verifiable procedures that guide decision makers to an informed and provable decision and action. In this unit we will look at one such tool; simulation which has become cornerstone to many probabilistic projects.

2.0 Intended Learning Outcomes (ILOs)

At the end of this unit you should be able to:

- Say what simulation is about
- State why we need simulation
- Describe how simulations are done
- Describe various types of Simulations

- Give examples of Simulation
- Show areas of applications of Simulation

3.0 Main Content

3.1 What is Simulation?

The term **simulation** is used in different ways by different people. As used here, simulation is defined as the process of creating a *model* (i.e., an abstract representation or exact copy) of an existing or proposed *system* (e.g., a project, a business, a mine, a forest, the organs in your body, etc.) in order to identify and understand those factors which control the system and/or to predict (forecast) the future behaviour of the system. Almost any system which can be quantitatively described using equations and/or rules can be simulated.

The underlying purpose of simulation is to shed light on the underlying mechanisms that control the behaviour of a system. More practically, simulation can be used to predict (forecast) the future behaviour of a system, and determine what you can do to influence that future behaviour. That is, simulation can be used to predict the way in which the system will evolve and respond to its surroundings, so that you can identify any necessary changes that will help make the system perform the way that you want it to.

For example, a fisheries biologist could dynamically simulate the salmon population in a river in order to predict changes to the population, and quantitatively understand the impacts on the salmon of possible actions (e.g., fishing, loss of habitat) to ensure that they do not go extinct at some point in the future.
Also flight simulator on PC is also a computer model of some aspect of the flight; it shows on the screen the controls and what the pilot is supposed to see from the “cockpit” (his armchair).

Simulation therefore, is a technique (not a method) for representing a dynamic real world system by a model and experimenting with the model in order to gain information about the system and hence take appropriate decision. Simulation can be done by hand or by a computer.

Simulation is a powerful and important tool because it provides a way in which alternative designs, plans and/or policies can be evaluated without having to experiment on a real system, which may be prohibitively costly, time-consuming, or simply impractical to do. That is, it allows you to ask *"What if?" questions* about a system without having to experiment on the actual system itself (and hence incur the costs of field tests, prototypes, etc.).

3.2 When to Use simulation

Simulation is used in systems that change with time, such as a gas station, where cars come and go (called dynamic systems) and involve randomness. In such a system nobody can guess at exactly which time the next car should arrive at the station. Modelling complex dynamic systems theoretically need too many simplifications and the emerging models may not therefore be valid. Simulation does not require many simplifying assumptions, making it the only tool even in absence of randomness.

Simulation is used to observe the dynamic behaviour of a model of real or imaginary system. Indeed, by simulating a complex system we are able to understand the behaviour at low cost. Otherwise we would have to carry out a complicated theoretical research or to build a device (an electric heater, a building or a plane), and observe how it changes to get hints for improvements in the design.

If you run a shop, an hospital or a bank, then computer simulation may show you bottlenecks, service time, flows, and queues of clients and provide important information on how to improve your business.

Note that often we describe a real world system by:

1. A physical model
2. A mathematical or analytic model
3. An analogue model.

What happens when a system is not amenable to treatment using the above model? Constructing a real physical system could be very expensive and what more testing it with live human beings and observing what happens could be fatal. Training a new pilot using an airplane is suicidal. This is why simulation is designed and utilized.

Thus simulation is the answer to our question. Many operations Research analysts consider simulation to be a method of last resort. This is because it may be useful when other approaches cannot be used, for example when a real world situation is complex. Note that nothing prevents you from using simulation approach to analytic problem. Results can at least be compared!

Thus, before designing and implementing a real life system, it is necessary to find out via simulation studies whether the system will work otherwise the whole exercise will be a wild goose chase. Inevitably huge sums of money might have been wasted.

Unlike the situation in mathematical programming, so far there are no clear cut underlying principle guiding the formulation of simulation models. Each application is ad-hoc to a large extent. In general there are three basic objectives of simulation studies:

1. To Describe a Current System – Suppose that a manufacturing company has suddenly observed a marked deterioration in meeting due-dates of customers order. It may be necessary to build a simulation model to see how the current procedures for estimating due dates, scheduling production and ordering raw materials are giving rise to observed delays.
2. To explore a Hypothetical System – such as installing a new system, which will cost a lot of money, it might be better to build a hypothetical model of the system and learn from its behaviour.
3. To Design an Improved System – for example consider a supermarket that has one payment counter. Due to increase in patronage, it is considering to increase the

number of pay points. A simulation experiment may identify if one, two or more additional points are needed or not needed.

3.3 Types of Simulations

Computer models can be classified according to several criteria including:

- Stochastic or deterministic (and as a special case of deterministic, chaotic)
- Steady-state or dynamic
- Continuous or discrete (and as an important special case of discrete, discrete event or DE models)
- Local or distributed.

For example:

- Steady-state models use equations defining the relationships between elements of the modelled system and attempt to find a state in which the system is in equilibrium. Such models are often used in simulating physical systems, as a simpler modelling case before dynamic simulation is attempted.
- Dynamic simulations model changes in a system in response to (usually changing) input signals.
- *Stochastic* models use *random number generators* to model the chance or random events; they are also called Monte Carlo simulations.

There are two basic types of simulation for which models are built, and the process of choosing the subset of characteristics or features is different for each. The distinction between the two types is based on how time is represented; either as a **continuous** variable or as a **discrete** variable.

3.3.1 Continuous simulation

Continuous simulations treat time as continuous and express changes in terms of a set of differential equations that reflect the relationships among the set of characteristics. Thus the characteristics or features chosen to model the system must be those whose behaviour is understood mathematically.

Continuous simulation is used in systems where the state changes all the time, not just at the time of some discrete events. For example, the water level in a reservoir due to in and outflow changes all the time. In such cases ***continuous simulation*** is more appropriate, although discrete events simulation can serve as an approximation.

A meteorological modelling falls is another example in this category. The characteristics of weather models are wind components, temperature, water vapour, cloud formation, precipitation, and so on. The interaction of these components over time can be modelled by a set of partial differential equations, which measure the rate of change of each component over some three-dimensional region.

A *continuous dynamic simulation* performs numerical solution of differential-algebraic equations or differential equations (either partial or ordinary). Periodically, the simulation program solves all the equations, and uses the numbers to change the state and output of the

simulation. Applications include flight simulators, racing-car games, chemical process modelling, and simulations of electrical circuits. Originally, these kinds of simulations were actually implemented on analog computers, where the differential equations could be represented directly by various electrical components such as operational amplifiers. By the late 1980s, however, most "analogue" simulations were run on conventional digital computers that emulate the behaviour of an analog computer.
A typical Continuous (stochastic) system has a large number of control parameters that can have a significant impact on the performance of the system. To establish a basic knowledge of the behaviour of a system under variation of input parameters, sensitivity analysis is usually performed, which applies small changes from one state to the nominal values of input parameters. For such simulation, variations of the input parameter cannot be made infinitely small. The sensitivity of the performance measure with respect to an input parameter is therefore defined as (partial) derivative.
Sensitivity analysis is concerned with evaluating sensitivities (gradient) of performance measures with respect to parameter of interest. It provides guidance for design and operational decisions and plays a pivotal role in identifying the most significant system parameters, as well as bottleneck of subsystems.
In designing, analysing and operating such complex systems, one is interested not only in performance evaluation but also in sensitivity analysis and optimisation.

3.3.2 Discrete-event simulation,

Discrete event models are made up of *entitie*s, *attributes*, and *events*. An entity represents some object in the real system that must be explicitly defined. That is, the characteristic or feature of the system or an object. For example, if we were modelling a manufacturing plant, the different machines, and the product being created, would be entities. An attribute is some characteristic of a particular entity. The identification number, the purchase date, and the maintenance history would be attributes of a particular machine. An event is an interaction between entities. For example, the sending of the output from one machine as input to the next machine would be an event.

Suppose we are interested in a gas station. We may describe the behaviour of this system graphically by plotting the number of cars in the station; the state of the system. Every time a car arrives the graph increases by one unit while a departing car causes the graph to drop by one unit. This graph (called sample path), could be obtained from observation of real station, but could also be artificially constructed. Such *artificial construction and analysis of the resulting sample path (or more sample paths in more complex cases) consists of the simulation*.

The path consists of only horizontal and vertical lines, as cars arrivals and departures occurred, at distinct points in time, what we refer to as events. Between two consecutive events, nothing happens – the graph is horizontal. When the number of events are finite, we call the simulation ***discrete event***.

Discrete event systems (DES) are dynamic systems, which evolve in time by the occurrence of events at possible irregular time intervals. DES abound in real-world applications. Examples include traffic systems, flexible manufacturing systems, computer communication systems, production lines, flow networks etc. Most of these systems can be modelled in terms of discrete events whose occurrence causes the system to change from one state to another. Simulations may be performed manually. Most often, however, the system model is written either as a computer program or as some kind of input into simulator software.

A *discrete event simulation* (DE) manages events in time. Most computer, logic-test and fault-tree simulations are of this type. In this type of simulation, the simulator maintains a queue of events sorted by the simulated time they should occur. The simulator reads the queue and triggers new events as each event is processed. It is not important to execute the simulation in real time. It's often more important to be able to access the data produced by the simulation, to discover logic defects in the design, or the sequence of events.

A special type of discrete simulation which does not rely on a model with an underlying equation, but can nonetheless be represented formally, is *agent-based simulation*. In agent-based simulation, the individual entities (such as molecules, cells, trees or consumers) in the model are represented directly (rather than by their density or concentration) and possess an internal *state* and set of behaviours or *rules* which determine how the agent's state is updated from one time-step to the next.

3.4 Steps In Constructing A Simulation Model.

1. Formulate the model (see modelling)
2. Design the Experiment – Workout details of experimental procedures before running the model subsystems, parameters, relationships, data structures, etc.
3. Develop the Computer Programs – Each historical evolution of the model, including generation of random events and generation of objects, will take place within the computer. if a model has a simple structure, you can use **BASIC, FORTRAN, PASCAL or C** and so on to develop the computerized version. However, it is better to use a simulation language such as **SIMULATIONSCRIPT, GPSS, SIMULATIONULA (SIMULA), SIMULATIONNET (SIMNET) II, QMS**, etc.

3.4.1 To Extract the terms in Simulation

Let us consider building a simulation of gas station with a single pump served by a single service man. Assumptions: arrival of cars as well as their times are random.
At first identify the:
State: number of cars waiting for service and number of cars served at any moment.
Event: arrival of cars, start of service, end of service.
Entities: these are the cars.
Queue: the queue of cars in front of the pump waiting for service.
Random realization: interval times, service times.
Distribution: we shall assume exponential distributions for the interval time and service time.

Next, specify what to do at each event. The above example would look like this:

At event of entity arrival: Create next arrival. If the server is free, send entity for start of service. Otherwise it joins the queue. At event of service start: Server becomes occupied. Schedule end of service for this entity. At event of service end: Server becomes free. If any entity is waiting in the queue: remove the first entity from the queue; send it for start of service.

Some initiation is still required, for example, the creation of the first arrival. Lastly, the above is translated into code. This is easy with appropriate library function, which has subroutine for creation, scheduling, proper timing of events, queue manipulations, random variate generation and statistics collection.

3.4.2 Simulation terminologies:

State – A variable characterizing an attribute in the system such as level of stock in inventory or number of jobs in waiting for processing.

Event: - An occurrence at a point in time which may change the state of the system, such as arrival of a customer or start of work on a job.

Entity: An object that passes through the system, such as cars in an intersection or orders in a factory. Often an event (e.g., arrival) is associated with an entity (e.g., customer).

Queue: A queue is not only a physical queue of people, or cars, etc it can also be a task list, a buffer of finished goods waiting for transportation or any place where entities are waiting for something to happen for any reason.

Creating: Is causing an arrival of new entity into the system at some point in time.

Scheduling: is the act of assigning a new future event to an existing entity. **Random variable**: is a quantity that is uncertain, such as interval time between two incoming flights or number of defectives parts in a shipment.

Random Variate: is an artificially generated random variable.

Distribution: is the mathematical law, which governs the probabilistic features of a random variable.

3.5 Applications of Computer Simulation

Computer simulation has become a useful part of modelling many natural systems in physics, chemistry and biology, and human systems in economics and social science (the computational sociology) as well as in engineering to gain insight into the operation of those systems. A good example of the usefulness of using computers to simulate can be found in the field of network traffic simulation. In such simulations the model behaviour will change each simulation according to the set of initial parameters assumed for the environment. Computer simulations are often considered to be *human out of the loop* simulations.

Computer graphics can be used to display the results of a computer simulation. Animations can be used to experience a simulation in real-time e.g. in training simulations. In some cases animations may also be useful in faster than real-time or even slower than real-time modes. For example, faster than real-time animations can be useful in visualizing the build up of queues in the simulation of humans evacuating a building.

There are many different types of computer simulation; the common feature they all share is the attempt to generate a sample of representative scenarios for a model in which a complete enumeration of all possible states of the model would be prohibitive or impossible. Several software packages also exist for running computer-based simulation modelling that makes the modelling almost effortless and simple.

a. Simulation in computer science

In computer science, simulation has an even more specialized meaning: Alan Turing uses the term "simulation" to refer to what happens when a digital computer runs a state transition table (runs a program) that describes the state transitions, inputs and outputs of a subject discrete-state machine. The computer simulates the subject machine.

In computer programming, a simulator is often used to execute a program that has to run on some inconvenient type of computer, or in a tightly controlled testing environment For example, simulators are usually used to debug a micro program or sometimes commercial application programs. Since the operation of the computer is simulated, all of the information about the computer's operation is directly available to the programmer, and the speed and execution of the simulation can be varied at will.

Simulators may also be used to interpret fault trees, or test very large scale integration (VLSI) logic designs before they are constructed. In theoretical computer science the term *simulation* represents a relation between state transition systems.

b. Simulation in training

Simulation is often used in the training of civilian and military personnel. This usually occurs when it is prohibitively expensive or simply too dangerous to allow trainees to use the real equipment in the real world. In such situations they will spend time learning valuable lessons in a "safe" virtual environment. Often the convenience is to permit mistakes during training for a safety-critical system.

Training simulations typically come in one of three categories:

- **live** simulation (where real people use simulated (or "dummy") equipment in the real world);
- **virtual** simulation (where real people use simulated equipment in a simulated world (or "virtual environment")), or
- **constructive** simulation (where simulated people use simulated equipment in a simulated environment). Constructive simulation is often referred to as "wargaming" since it bears some resemblance to table-top war games in which players command armies of soldiers and equipment which move around a board.

c. Simulation in Education

Simulations in education are somewhat like training simulations. They focus on specific tasks. In the past, video has been used for teachers and students to observe, problem solve and role play; however, a more recent use of simulation in education include animated narrative vignettes (ANV). ANVs are cartoon-like video narratives of hypothetical and reality-based stories involving classroom teaching and learning. ANVs have been used to assess knowledge, problem solving skills and dispositions of children, and pre-service and in-service teachers.

Another form of simulation has been finding favour in business education in recent years. Business simulations that incorporate a dynamic model enable experimentation with business strategies in a risk free environment and provide a useful extension to case study discussions.

d. Medical Simulators

Medical simulators are increasingly being developed and deployed to teach therapeutic and diagnostic procedures as well as medical concepts and decision making to personnel in the health professions. Simulators have been developed for training procedures ranging from the basics such as blood draw, to laparoscopic surgery and trauma care.

Many medical simulators involve a computer connected to a plastic simulation of the relevant anatomy. Sophisticated simulators of this type employ a life size mannequin which responds to injected drugs and can be programmed to create simulations of life- threatening emergencies. In others simulations, visual components of the procedure are reproduced by computer graphics techniques, while touch-based components are reproduced by haptic feedback devices combined with physical simulation routines computed in response to the user's actions. Medical simulations of this sort will often use 3D CT or MRI scans of patient data to enhance realism.

Another important medical application of a simulator -- although, perhaps, denoting a slightly different meaning of *simulator* -- is the use of a *placebo* drug, a formulation which simulates the active drug in trials of drug efficacy.

e. City Simulators / Urban Simulation

A City Simulator is a tool used by urban planners to understand how cities are likely to evolve in response to various policy decisions. UrbanSim. The City Simulator developed at the University of Washington and ILUTE developed at the University of Toronto are examples of modern, large-scale urban simulators designed for use by urban planners. City simulators are generally agent-based simulations with explicit representations for land use and transportation.

f. Flight simulators

A flight simulator is used to train pilots on the ground. It permits a pilot to crash his simulated "aircraft" without being hurt. Flight simulators are often used to train pilots to operate aircraft in extremely hazardous situations, such as landings with no engines, or complete electrical or hydraulic failures. The most advanced simulators have high-fidelity visual systems and hydraulic motion systems. The simulator is normally cheaper to operate than a real trainer aircraft.

g. Marine simulators

This bears resemblance to flight simulators. The marine simulators are used to train ship personnel. Simulators like these are mostly used to simulate large or complex vessels, such as cruise ships and dredging ships. They often consist of a replication of a ships' bridge, with operating desk(s), and a number of screens on which the virtual surroundings are projected.

Simulation in Engineering (Technology) Process

Simulation is an important feature in engineering systems or any system that involves many processes. For example in electrical engineering, delay lines may be used to simulate propagation delay and phase shift caused by an actual transmission line. Similarly, dummy loads may be used to simulate impedance without simulating propagation, and is used in situations where propagation is unwanted. A simulator may imitate only a few of the operations and functions of the unit it simulates. *Contrast with*: emulate.

Most engineering simulations entail mathematical modelling and computer assisted investigation. There are many cases, however, where mathematical modelling is not reliable. Simulation of fluid dynamics problems often requires both mathematical and physical simulations. In these cases the physical models require dynamic similitude. Physical and chemical simulations have also direct realistic uses, rather than research uses; in chemical engineering, for example, process simulations are used to give the process parameters immediately used for operating chemical plants, such as oil refineries.

Discrete Event Simulation is often used in industrial engineering, operations management and operational research to model many systems (commerce, health, defence, manufacturing, logistics, etc.); for example, the value-adding transformation processes in businesses, to optimize business performance. Imagine a business, where each person could do 30 tasks, where thousands of products or services involved dozens of tasks in a sequence, where customer demand varied seasonally and forecasting was inaccurate- this is the domain where such simulation helps with business decisions across all functions.

h. Simulation and games

Strategy games - both traditional and modern - may be viewed as simulations of abstracted decision-making for the purpose of training military and political leaders. In a narrower sense, many video games are also simulators, implemented inexpensively. These are sometimes called "sim games". Such games can simulate various aspects of reality, from economics to piloting vehicles, such as flight simulators (described above).

i. The "classroom of the future"

The "classroom of the future" will probably contain several kinds of simulators, in addition to textual and visual learning tools. This will allow students to enter school better prepared, and with a higher skill level. The advanced student or postgraduate will have a more concise and comprehensive method of retraining -- or of incorporating new academic contents into their skill set -- and regulatory bodies and institution managers will find it easier to assess the proficiency and competence of individuals.

In classrooms of the future, the simulator will be more than a "living" textbook; it will become an integral a part of the practice of Education and training. The simulator environment will also provide a standard platform for curriculum development in educational institutions.

3.6 Model Evaluation

An important part of the modelling process is the evaluation of an acquired model. *How do we know if a mathematical model describes the system well?* This is not an easy question to answer. Usually the engineer has a set of measurements from the system which are used in creating the model. Then, if the model was built well, the model will adequately show the relations between system variables for the measurements at hand. The question then becomes: How do we know that the measurement data are a representative set of possible values? Does the model describe well the properties of the system between the measurement data (interpolation)? Does the model describe well events outside the measurement data (extrapolation)?

A common approach is to split the measured data into two parts; training data and verification data. The training data are used to *train* the model, that is, to estimate the model parameters (see above). The verification data are used to evaluate model performance. Assuming that the training data and verification data are not the same, we can assume that if the model describes the verification data well, then the model describes the real system well.

However, this still leaves the *extrapolation question* open. How well does this model describe events outside the measured data? Consider again Newtonian classical mechanics-model. Newton made his measurements without advanced equipment, so he could not measure properties of particles travelling at speeds close to the speed of light. Likewise, he did not measure the movements of molecules and other small particles, but macro particles only. It is then not surprising that his model does not extrapolate well into these domains, even though his model is quite sufficient for ordinary life physics.

The reliability and the trust people put in computer simulations depends on the validity of the simulation model, therefore verification and validation are of crucial importance in the development of computer simulations. Another important aspect of computer simulations is that of reproducibility of the results, meaning that a simulation model should not provide a different answer for each execution. Although this might seem obvious, this is a special point of attention in stochastic simulations, where random numbers should actually be semi-random numbers. An exception to reproducibility are human in the loop simulations such as flight simulations and computer games. Here a human is part of the simulation and thus influences the outcome in a way that is hard if not impossible to reproduce exactly.

Case Studies

Operations study to add a new plane arrival at La Guardia southwest terminal

LaGuardia airport planned to add a new flight to the schedule of the southwest terminal. The airport administration wanted to understand how the introduction of a new flight would influence terminal capacity.

Problem

In order to understand the scale of the problem, the developers conducted a preliminary static pedestrian flow analysis based on data of how long before the flight passengers arrived at the airport. In the picture, the solid line represented the number of seats in the waiting area, the red stacks represented the number of passengers in the terminal before introducing the new flight, and additional passengers from the new flight were represented by purple areas. The graph showed that if the new plane took off in the afternoon at 5:00 pm, the already crowded waiting area would have to bear an additional burden that could lead to a significant problem.

The developers used the AnyLogic Pedestrian Library to create a crowd simulation model of the terminal in order to examine the use of seats under different scenarios. The basic model displayed the operation of all terminal areas before the introduction of the new flight, and then various assumptions could be checked against this model. The best situation was when people were waiting for departure at their gates, but the consultants wanted to check how far they would have to move away from their gates to wait for their departure.

To set up the crowd simulation model, the developers used tables of passenger preference for waiting areas.

The model showed how far from their gate people would have to wait. The results of modeling the base scenario, without the new flight, showed that some of the peaks were reduced compared with the static analysis. This was due to passengers lining up 30 minutes before their flights. The model also showed where the people would actually wait. From this, it could be verified that there was no overflow and that the situation was stable.

In the afternoon, the waiting area was a lot more heavily utilized. There were a lot of passengers mixing in different areas and waiting for different gates. With the new flight at this peak time, some of these areas would get extremely overloaded. This pedestrian simulation was very useful in showing the operations of this terminal and how adding the new flight would affect the passengers in this area, including how far they would have to move to wait for their flights.

Solution

Designing large transport facilities requires careful consideration and agreement on every detail. That means that such projects must go through a great deal of decision making. The initial task of engineers usually produces alternatives and functional designs. These consider physical requirements and standards, but whether business or operating objectives will be met can be hard to determine accurately. It is here that AnyLogic based modeling helps by enabling faster decision-making and significantly improving insight into the various tasks that engineers face when planning large transport facilities.

4.0 Self-Assessment Exercise(s)

Answer the following questions:

- What are the objectives of simulation?
- In one sentence for each distinguish between different types of simulation

- Briefly describe simulation in five application areas.

5.0 Conclusion

Simulation is used to shed light on the underlying mechanisms that control the behaviour of a system. It can be used to predict (forecast) the future behaviour of a system, and determine what you can do to influence that future behaviour. We simulation when we require information to solve bottlenecks, service time, flows, and queues of clients and provide important information on how to improve your business.

We simulate when a system is not amenable to treatment using any of the physical model, mathematical or analogue models. Other reasons to resort to simulation include when it is very expensive to construct a real physical system and what more testing it with live human beings and observing what happens could be fatal. Training a new pilot using an airplane is suicidal. These are where and when simulations are designed and utilized.

6.0 Summary

In this unit we:

- defined simulation as the process of creating a *model* (i.e., an abstract representation or exact copy) of an existing or proposed *system* (e.g., a project, a business, a mine, a forest, the organs in your body, etc.) in order to identify and understand those factors which control the system and/or to predict (forecast) the future behaviour of the system.
- Stated that simulation is required when a system is not amenable to treatment using any existing model or when it is very expensive to construct a real physical system or when testing it with live human beings could be fatal.
- classified simulation into:
  - Stochastic or deterministic (and as a special case of deterministic, chaotic)
  - Steady-state or dynamic
  - Continuous or discrete (and as an important special case of discrete, discrete event or DE models)
  - Local or distributed
- Stated that simulations are done by: Formulating the model, Design the Experiment and Developing the Computer Programs.
- Listed areas of applications of Simulation to include: Computer science, Medicine, Education, City/Urban planning, Training etc.

7.0 Further Readings

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