Mathematics | Rules of Inference

Prerequisite: Predicates and Quantifiers Set 2, Propositional Equivalences Every Theorem in Mathematics, or any subject for that matter, is supported by underlying proofs. These proofs are nothing but a set of arguments that are conclusive evidence of the validity of the theory. The arguments are chained together using Rules of Inferences to deduce new statements and ultimately prove that the theorem is valid.

Important Definitions :

1. Argument – A sequence of statements, **premises**, that end with a conclusion.

2. Validity – A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false.

3. Fallacy – An incorrect reasoning or mistake which leads to invalid arguments.

Structure of an Argument : As defined, an argument is a sequence of statements called premises which end with a conclusion.

Premises - $p_1, \ p_2, \ p_3, \ldots, \ p_n$Conclusion - q

$if(p_1 \wedge p_2 \wedge p_3 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology, then the argument is termed valid otherwise termed as invalid. The argument is written as –

$$\begin{array}{l}\text{First Premise}\\ \text{Second Premise}\\ \text{Third Premise}\\ .\\ .\\ \text{Nth Premise}\\ \hline \therefore Conclusion\end{array}$$

Rules of Inference : Simple arguments can be used as building blocks to construct more complicated valid arguments. Certain simple arguments that have been established as valid are very important in terms of their usage. These arguments are called Rules of Inference. The most commonly used Rules of Inference are tabulated below –

Rule of Inference	Tautology	Name
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus Ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus Tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$\neg p$ $p \lor q$ $\therefore q$	$(\neg p \land (p \lor q)) \to q$	Disjunctive Syllogism
p $\therefore (p \lor q)$	$p \to (p \lor q)$	Addition
$(p \land q) \to r$ $\therefore p \to (q \to r)$	$((p \land q) \to r) \to (p \to (q \to r))$	Exportation
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to q \lor r$	Resolution

Similarly, we have Rules of Inference for quantified statements –

Rule of Inference	Name
$\dfrac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\dfrac{\text{P(c) for an arbitrary c}}{\therefore \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore P(c)\ for\ some\ c}$	Existential instantiation
$\dfrac{\text{P(c) for some c}}{\therefore \exists x P(x)}$	Existential generalization

Let's see how Rules of Inference can be used to deduce conclusions from given arguments or check the validity of a given argument. **Example :** Show that the hypotheses "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip", and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". The first step is to identify propositions and use propositional variables to represent them. $p-$"It is sunny this afternoon" $q-$ "It is colder than yesterday" $r-$"We will go swimming" $s-$"We will take a canoe trip" $t-$"We will be home by sunset" The hypotheses are – $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$, and $s \rightarrow t$. The conclusion is – tTo deduce the conclusion we must use Rules of Inference to construct a proof using the given hypotheses.

Step	Reason
1. $\neg p \wedge q$	Hypothesis
2. $\neg p$	Simplification
3. r $\rightarrow p$	Hypothesis
4. $\neg r$	Modus Tollens using (2) and (3)
5. $\neg r \rightarrow s$	Hypothesis
6. s	Modus Ponens using (4) and (5)
7. s$\rightarrow t$	Hypothesis
8. t	Modus Ponens Using (6) and (7)

Resolution Principle : To understand the Resolution principle, first we need to know certain definitions.

1. **Literal** – A variable or negation of a variable. Eg- $p, \neg q$
2. **Sum –** Disjunction of literals. Eg- $p \vee \neg q$

3. **Product** – Conjunction of literals. Eg- $p \wedge \neg q$
4. **Clause** – A disjunction of literals i.e. it is a sum.
5. **Resolvent** – For any two clauses $C_1$and C_2, if there is a literal L_1in C_1that is complementary to a literal L_2in C_2, then removing both and joining the remaining clauses through a disjunction produces another clause C. $C$is called the resolvent of $C_1$and C_2

For example,

$C_1 = p \vee q \vee r$[Tex]C_{2} = \neg p\vee \neg s \vee t[/Tex]

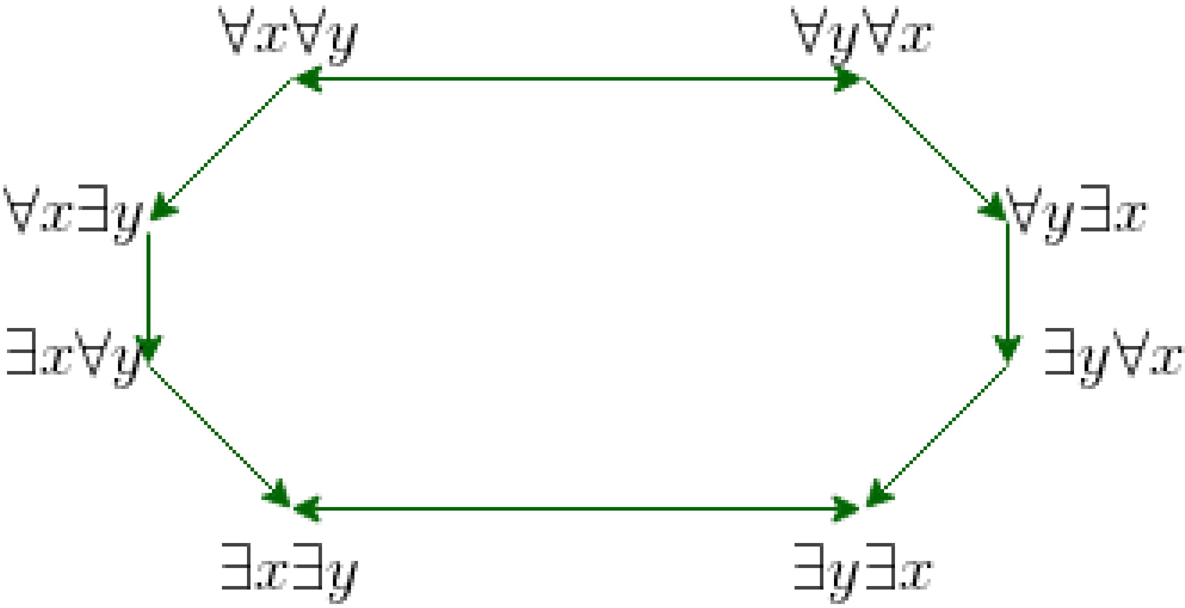
Here, $\neg p$and $p$are complementary to each other. Removing them and joining the remaining clauses with a disjunction gives us- $q \vee r \vee \neg s \vee t$We could skip the removal part and simply join the clauses to get the same resolvent. $p \vee \neg p \equiv T \; and, \; T \vee q \equiv q$ This is also the Rule of Inference known as Resolution. **Theorem** – If $C$is the resolvent of $C_1$and C_2, then $C$is also the logical consequence of $C_1$and C_2. **The Resolution Principle** – Given a set $S$of clauses, a (resolution) deduction of $C$from $S$is a finite sequence $C_1, C_2, ..., C_k$of clauses such that each $C_i$is either a clause in Sor a resolvent of clauses preceding $C$and $C_k = C$. We can use the resolution principle to check the validity of arguments or deduce conclusions from them. Other Rules of Inference have the same purpose, but Resolution is unique. It is complete by it's own. You would need no other Rule of Inference to deduce the conclusion from the given argument. To do so, we first need to convert all the premises to clausal form. The next step is to apply the resolution Rule of Inference to them step by step until it cannot be applied any further. For example, consider that we have the following premises –

$p \rightarrow (q \vee r)$[Tex]s\rightarrow \neg r[/Tex]$p \wedge s$

The first step is to convert them to clausal form –

$C_1 : \; \neg p \vee q \vee r \;\; C_2 : \; \neg s \vee \neg r$[Tex]C_{3}: \:p[/Tex]$C_4 : \; s$From the resolution of C_1 and C_2, $C_5 : \; \neg p \vee q \vee \neg s$From the resolution of C_5 and C_3, $C_6 : q \vee \neg s$From the resolution of C_6 and C_4, $C_7 : q$Therefore, the conclusion is q.

Note:Implications can also be visualised on octagon as,



It shows how implication changes on changing order of their exists and for all symbols. **GATE CS Corner Questions** Practicing the following questions will help you test your knowledge. All questions have been asked in GATE in previous years or in GATE Mock Tests. It is highly recommended that you practice them. 1. GATE CS 2004, Question 70 2. GATE CS 2015 Set-2, Question 13 **References-** Rules of Inference – Simon Fraser University Rules of Inference – Wikipedia Fallacy – Wikipedia Book – Discrete Mathematics and Its Applications by Kenneth Rosen This article is contributed by **Chirag Manwani**. If you like GeeksforGeeks and would like to contribute, you can also write an article using write.geeksforgeeks.org or mail your article to review-team@geeksforgeeks.org. See your article appearing on the GeeksforGeeks main page and help other Geeks. Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Last Updated : 25 Aug, 2022

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