

1.

Determine whether each of the following is a tautology, a contradiction or neither:

1. $p \rightarrow (p \vee q)$

2. $(p \rightarrow q) \wedge (\bar{p} \vee q)$

3. $(p \vee q) \leftrightarrow (q \vee p)$

4. $(p \wedge q) \rightarrow p$

5. $(p \wedge q) \wedge (\overline{p \vee q})$

2. Prove the following logical implications:

(i) $(p \wedge q) \vdash q$

(ii) $(p \wedge q) \vdash p$

(iii) $[(p \rightarrow q) \wedge p] \vdash q$

3. Prove each of the following logical equivalences using replacement laws

(i) $(p \wedge p) \vee (\bar{p} \vee \bar{p}) \equiv t.$

(ii) $(p \wedge q) \wedge q \equiv p \wedge q.$

(iii) $p \rightarrow q \equiv \overline{p \wedge \bar{q}}.$

(iv) $(p \wedge q) \rightarrow r \equiv (\bar{p} \vee \bar{q}) \vee r.$

4. Test the validity of the following arguments:

- i. If you gamble you're stupid. You're not stupid therefore you don't gamble.
- ii. If I leave college then I'll get a job in a bank. I'm not leaving college so I won't get a job in a bank.
- iii. James is either a Programmer or a footballer. If he's a programmer then he is intelligent. James isn't intelligent so he's a footballer.
- iv. I shall be a lecturer or a banker (but not both). If I become a lecturer then I shall never be rich. Therefore I shall be rich only if I become a banker.