Exercises: Advanced Derivatives

1–4 ■ Use the quotient rule to compute $f'(x)$. Simplify your answer.

1. $f(x) = \dfrac{x^3}{x^2 + 1}$

2. $f(x) = \dfrac{4 + x^2}{4 - x^2}$

3. $f(x) = \dfrac{e^{3x}}{1 + e^x}$

4. $f(x) = \dfrac{2x}{1 + \sqrt{x}}$

5–6 ■ Differentiate. Look up any derivative formulas that you need.

5. $2^x \cot x$

6. $\sec(5x^2)$

7. $\tan^{-1}(3x^4)$

8. $\sin^{-1}(e^x)$

9. $\arctan(\sqrt{x})$

10. $3^{\csc x}$

11–22 ■ Differentiate.

11. $e^{\sin(4x)}$

12. $\ln(\sin \sqrt[3]{x})$

13. $\sin^2(e^{-x})$

14. $\sin(\cos(\ln x))$

15. $\dfrac{1}{\sqrt{\sin 4x}}$

16. $\dfrac{1}{(1 + e^{2x})^3}$

17. $\sin(x^2 e^x)$

18. $\sin\left(\dfrac{x}{1 + x^2}\right)$

19. $\ln(1 + x^3 e^x)$

20. $e^{x \sin x}$

21. $x e^x \ln x$

22. $x^3 e^{2x} \cos x$

Answers

1. $\dfrac{x^4+3x^2}{\left(x^2+1\right)^2}$ **2.** $\dfrac{16x}{\left(4-x^2\right)^2}$ **3.** $\dfrac{3e^{3x}+2e^{4x}}{\left(1+e^x\right)^2}$ **4.** $\dfrac{2+\sqrt{x}}{\left(1+\sqrt{x}\right)^2}$ **5.** $2^x\ln(2)\cot x \,-\, 2^x\csc^2 x$ **6.** $10x\sec\left(5x^2\right)\tan\left(5x^2\right)$

7. $\dfrac{12x^3}{1+9x^8}$ **8.** $\dfrac{e^x}{\sqrt{1-e^{2x}}}$ **9.** $\dfrac{1}{2\sqrt{x}\,(1+x)}$ **10.** $-3^{\csc x}\ln 3\,\csc x\cot x$ **11.** $4e^{\sin(4x)}\cos(4x)$ **12.** $\dfrac{\cot\sqrt[3]{x}}{3x^{2/3}}$

13. $-2e^{-x}\sin\left(e^{-x}\right)\cos\left(e^{-x}\right)$ **14.** $-\dfrac{\cos\left(\cos(\ln x)\right)\sin(\ln x)}{x}$ **15.** $-\dfrac{2\cos 4x}{\left(\sin 4x\right)^{3/2}}$ **16.** $-\dfrac{6e^{2x}}{\left(1+e^{2x}\right)^4}$

17. $\left(x^2+2x\right)e^x\cos\left(x^2e^x\right)$ **18.** $\dfrac{1-x^2}{\left(1+x^2\right)^2}\cos\left(\dfrac{x}{1+x^2}\right)$ **19.** $\dfrac{\left(x^3+3x^2\right)e^x}{1+x^3e^x}$ **20.** $e^{x\sin x}\left(\sin x+x\cos x\right)$

21. $e^x + e^x\ln x + xe^x\ln x$ **22.** $\left(3x^2+2x^3\right)e^{2x}\cos x \,-\, x^3e^{2x}\sin x$