Chapter 2: Algorithm Discovery and Design

Invitation to Computer Science,

C++ Version, Fourth Edition

Objectives

In this chapter, you will learn about

- Representing algorithms
- Examples of algorithmic problem solving

Introduction

- This chapter uses four problems to discuss algorithms and algorithmic problem solving
  - Multiplying two numbers
  - Searching lists
  - Finding maxima and minima
  - Matching patterns

Representing Algorithms

- Natural language
  - Language spoken and written in everyday life
  - Examples: English, Spanish, Arabic, and so on
  - Problems with using natural language for algorithms
    - Verbose
    - Imprecise
    - Relies on context and experiences to give precise meaning to a word or phrase

Initially, set the value of the variable *carry* to 0 and the value of the variable *i* to 0. When these initializations have been completed, begin looping as long as the value of the variable *i* is less than or equal to $(m - 1)$. First, add together the values of the two digits a_i and b_i and the current value of the carry digit to get the result called c_i. Now check the value of c_i to see whether it is greater than or equal to 10. If c_i is greater than or equal to 10, then reset the value of *carry* to 1 and reduce the value of c_i by 10; otherwise, set the value of *carry* to zero. When you are done with that operation, add 1 to *i* and begin the loop all over again. When the loop has completed execution, set the leftmost digit of the result c_m to the value of *carry* and print out the final result, which consists of the digits $c_m\ c_m{-}1 \ldots c_0$. After printing the result, the algorithm is finished, and it terminates.

Figure 2.1

The Addition Algorithm of Figure 1.2 Expressed in Natural Language

Representing Algorithms (continued)

- High-level programming language
  - Examples: C++, Java
  - Problem with using a high-level programming language for algorithms
    - During the initial phases of design, we are forced to deal with detailed language issues

```
{
int i, m, Carry;
int[] a = new int[100];
int[] b = new int[100];
int[] c = new int[100];
m = Console.readInt();
for (int j = 0; j < = m-1; j++)    {
            a[j] = Console.readInt();
            b[j] = Console.readInt();
}
Carry = 0;
i = 0;
while (i < m)    {
            c[i] = a[i] + b[i] + Carry;
            if (c[i] > = 10)
            .
            .
            .
```

Figure 2.2

The Beginning of the Addition Algorithm of Figure 1.2 Expressed in a High-Level Programming Language

Pseudocode

- English language constructs modeled to look like statements available in most programming languages
- Steps presented in a structured manner (numbered, indented, and so on)
- No fixed syntax for most operations is required

Pseudocode (continued)

- Less ambiguous and more readable than natural language
- Emphasis is on process, not notation
- Well-understood forms allow logical reasoning about algorithm behavior
- Can be easily translated into a programming language

Sequential Operations

- Types of algorithmic operations
  - Sequential
  - Conditional
  - Iterative

Sequential Operations (continued)

- Computation operations
  - Example
    - Set the value of “variable” to “arithmetic expression”
  - Variable
    - Named storage location that can hold a data value

Sequential Operations (continued)

- Input operations
  - To receive data values from the outside world
  - Example
    - Get a value for *r*, the radius of the circle
- Output operations
  - To send results to the outside world for display
  - Example
    - Print the value of *Area*

Average Miles per Gallon Algorithm (Version 1)

Step	Operation
1	Get values for *gallons used, starting mileage, ending mileage*
2	Set value of *distance driven* to (*ending mileage* – *starting mileage*)
3	Set value of *average miles per gallon* to (*distance driven* ÷ *gallons used*)
4	Print the value of *average miles per gallon*
5	Stop

Figure 2.3
Algorithm for Computing Average Miles per Gallon

Conditional and Iterative Operations

- Sequential algorithm
  - Also called straight-line algorithm
  - Executes its instructions in a straight line from top to bottom and then stops
- Control operations
  - Conditional operations
  - Iterative operations

Conditional and Iterative Operations (continued)

- Conditional operations
  - Ask questions and choose alternative actions based on the answers
  - Example
    - if *x* is greater than 25 then

      print *x*

      else

      print *x* times 100

Conditional and Iterative Operations (continued)

- Iterative operations
  - Perform “looping” behavior, repeating actions until a continuation condition becomes false
  - Loop
    - The repetition of a block of instructions

Conditional and Iterative Operations (continued)

- Examples
  - while $j > 0$ do

    set s to $s + a_j$

    set j to $j - 1$

  - repeat

    print a_k

    set k to $k + 1$

    until $k > n$

Average Miles per Gallon Algorithm (Version 2)

Step	Operation
1	Get values for *gallons used*, *starting mileage*, *ending mileage*
2	Set value of *distance driven* to (*ending mileage* – *starting mileage*)
3	Set value of *average miles per gallon* to (*distance driven* ÷ *gallons used*)
4	Print the value of *average miles per gallon*
5	If *average miles per gallon* is greater than 25.0 then
6	Print the message 'You are getting good gas mileage'
	Else
7	Print the message 'You are NOT getting good gas mileage'
8	Stop

Figure 2.5
Second Version of the Average Miles per Gallon Algorithm

Conditional and Iterative Operations (continued)

- Components of a loop
  - Continuation condition
  - Loop body
- Infinite loop
  - The continuation condition never becomes false
  - An error

Average Miles per Gallon Algorithm (Version 3)

Step	Operation
1	*response* = Yes
2	While (*response* = Yes) do steps 3 through 11
3	Get values for *gallons used*, *starting mileage*, *ending mileage*
4	Set value of *distance driven* to (*ending mileage* – *starting mileage*)
5	Set value of *average miles per gallon* to (*distance driven* ÷ *gallons used*)
6	Print the value of *average miles per gallon*
7	If average miles per gallon > 25.0 then
8	Print the message 'You are getting good gas mileage'
	Else
9	Print the message 'You are NOT getting good gas mileage'
10	Print the message 'Do you want to do this again? Enter Yes or No'
11	Get a new value for *response* from the user
12	Stop

Figure 2.7

Third Version of the Average Miles per Gallon Algorithm

Conditional and Iterative Operations (continued)

- Pretest loop
  - Continuation condition tested at the beginning of each pass through the loop
  - It is possible for the loop body to never be executed
  - While loop

Conditional and Iterative Operations (continued)

- Posttest loop
  - Continuation condition tested at the end of loop body
  - Loop body must be executed at least once
  - Do/While loop

COMPUTATION:

Set the value of "variable" to "arithmetic expression"

INPUT/OUTPUT:

Get a value for "variable", "variable"...

Print the value of "variable", "variable", ...

Print the message 'message'

CONDITIONAL:

```
If "a true/false condition" is true then
    first set of algorithmic operations
Else
    second set of algorithmic operations
```

ITERATIVE:

```
While ("a true/false condition") do step i through step j
        Step i: operation
        .
        .
        .
        Step j: operation

While ("a true/false condition") do
        operation
        .
        .
        .
        operation
End of the loop

Do
        operation
        operation
        .
        .
        .
While ("a true/false condition")
```

Figure 2.9

Summary of Pseudocode Language Instructions

Examples of Algorithmic Problem Solving

- Go Forth and Multiply: Multiply two numbers using repeated addition
- Sequential search: Find a particular value in an unordered collection
- Find maximum: Find the largest value in a collection of data
- Pattern matching: Determine if and where a particular pattern occurs in a piece of text

Example 1: Go Forth and Multiply

- Task
  - Implement an algorithm to multiply two numbers, *a* and *b*, using repeated addition
- Algorithm outline
  - Create a loop that executes exactly *b* times, with each execution of the loop adding the value of *a* to a running total

Multiplication via Repeated Addition

```
Get values for a and b
If (either a = 0 or b = 0) then
    Set the value of product to 0
Else
    Set the value of count to 0
    Set the value of product to 0
    While (count < b) do
        Set the value of product to (product + a)
        Set the value of count to (count+1)
    End of loop
Print the value of product
Stop
```

Figure 2.10
Algorithm for Multiplication via Repeated Addition

Example 2: Looking, Looking, Looking

- Task
  - Find a particular person’s name from an unordered list of telephone subscribers
- Algorithm outline
  - Start with the first entry and check its name, then repeat the process for all entries

Example 2: Looking, Looking, Looking (continued)

- Algorithm discovery
  - Finding a solution to a given problem
- Naïve sequential search algorithm
  - For each entry, write a separate section of the algorithm that checks for a match
  - Problems
    - Only works for collections of exactly one size
    - Duplicates the same operations over and over

Example 2: Looking, Looking, Looking (continued)

- Correct sequential search algorithm
  - Uses iteration to simplify the task
  - Refers to a value in the list using an index (or pointer)
  - Handles special cases (such as a name not found in the collection)
  - Uses the variable *Found* to exit the iteration as soon as a match is found

Sequential Search Algorithm

Step	Operation
1	Get values for *NAME*, $N_1, \ldots, N_{10,000}$, and $T_1, \ldots, T_{10,000}$
2	Set the value of i to 1 and set the value of *Found* to NO
3	While both (*Found* = NO) and ($i \le 10,000$) do steps 4 through 7
4	If *NAME* is equal to the ith name on the list N_i then
5	Print the telephone number of that person, T_i
6	Set the value of *Found* to YES
	Else (*NAME* is not equal to N_i)
7	Add 1 to the value of i
8	If (*Found* = NO) then
9	Print the message 'Sorry, this name is not in the directory'
10	Stop

Figure 2.13
The Sequential Search Algorithm

Example 2: Looking, Looking, Looking (continued)

- The selection of an algorithm to solve a problem is greatly influenced by the way the data for that problem is organized

Example 3: Big, Bigger, Biggest

- Task
  - Find the largest value from a list of values
- Algorithm outline
  - Keep track of the largest value seen so far (initialized to be the first in the list)
  - Compare each value to the largest seen so far, and keep the larger as the new largest

Example 3: Big, Bigger, Biggest (continued)

- Once an algorithm has been developed, it may itself be used in the construction of other, more complex algorithms
- Library
  - A collection of useful algorithms
  - An important tool in algorithm design and development

Example 3: Big, Bigger, Biggest (continued)

- Find Largest algorithm
  - Uses iteration and indices as in previous example
  - Updates *location* and *largest so far* when needed in the loop

Find Largest Algorithm

Get a value for n, the size of the list
Get values for $A_1, A_2, \ldots, A_n$, the list to be searched
Set the value of *largest so far* to A_1
Set the value of *location* to 1
Set the value of i to 2
While ($i \le n$) do
    If $A_i >$ *largest so far* then
        Set *largest so far* to A_i
        Set *location* to i
    Add 1 to the value of i
End of the loop
Print out the values of *largest so far* and *location*
Stop

Figure 2.14
Algorithm to Find the Largest Value in a List

Example 4: Meeting Your Match

- Task
  - Find if and where a pattern string occurs within a longer piece of text
- Algorithm outline
  - Try each possible location of pattern string in turn
  - At each location, compare pattern characters against string characters

Example 4: Meeting Your Match (continued)

- Abstraction
  - Separating high-level view from low-level details
  - Key concept in computer science
  - Makes difficult problems intellectually manageable
  - Allows piece-by-piece development of algorithms

Example 4: Meeting Your Match (continued)

- Top-down design
  - When solving a complex problem
    - Create high-level operations in the first draft of an algorithm
    - After drafting the outline of the algorithm, return to the high-level operations and elaborate each one
    - Repeat until all operations are primitives

Example 4: Meeting Your Match (continued)

- Pattern-matching algorithm
  - Contains a loop within a loop
    - External loop iterates through possible locations of matches to pattern
    - Internal loop iterates through corresponding characters of pattern and string to evaluate match

Pattern-Matching Algorithm

Get values for n and m, the size of the text and the pattern, respectively
Get values for both the text $T_1\ T_2 \ldots T_n$ and the pattern $P_1\ P_2 \ldots P_m$
Set k, the starting location for the attempted match, to 1
While ($k \le (n - m + 1)$) do
    Set the value of i to 1
    Set the value of *Mismatch* to NO
    While both ($i \le m$) and (*Mismatch* = NO) do
        If $P_i \ne T_{k+(i-1)}$ then
            Set *Mismatch* to YES
        Else
            Increment i by 1 (to move to the next character)
    End of the loop
    If *Mismatch* = NO then
        Print the message 'There is a match at position'
        Print the value of k
    Increment k by 1
End of the loop
Stop, we are finished

Figure 2.16
Final Draft of the Pattern-Matching Algorithm

Summary

- Algorithm design is a first step in developing an algorithm
- Algorithm design must
  - Ensure the algorithm is correct
  - Ensure the algorithm is sufficiently efficient
- Pseudocode is used to design and represent algorithms

Summary

- Pseudocode is readable, unambiguous, and able to be analyzed
- Algorithm design is a creative process; uses multiple drafts and top-down design to develop the best solution
- Abstraction is a key tool for good design