0.1 Homework 2

0.1.1 Angular momentum for 4-dimensional space

Consider a 4-dimensional space with coordinates (x, y, z, w)**.**

1. **Show that the operators** $L_i = \epsilon_{ijk} x_j p_k$ **and** $K_i = w p_i - x_i p_w$ **generate rotations in this space by showing that the transformations generated by these operators leave the four dimensional radius, defined by** $R^2 = x^2 + y^2 + z^2 + w^2$**, invariant.**

2. **Compute the commutators** $[L_i, K_j]$ **and** $[K_i, K_j]$**.**

0.1.2 Harmonic oscillator

1. **Find the energy eigenvalues E_n and the corresponding wave functions $\psi_n(x)$ for a one-dimensional quantum harmonic oscillator system.**

2. **Calculate $\langle m|x|n\rangle$, $\langle m|p|n\rangle$, $\langle m|x^2|n\rangle$, and $\langle m|p^2|n\rangle$.**

3. **Assume the quantum harmonic oscillator is in a thermal bath at temperature T; find the partition function Z and the average energy $\langle E \rangle$ of the system.**

4. **Prove that the inner product of coherent states is given by:**

$$\langle \alpha | \beta \rangle = e^{-\frac{1}{2}(|\alpha|^2 + |\beta|^2) + \alpha^* \beta}$$