

# The Hebrew University of Jerusalem Introduction to Artificial Intelligence Problem Set 2- Adversarial search & logic

## Adversarial search

[25 points]

1. Using the definitions of MIN and MAX functions from the minimax algorithm learned in class Prove the following assertion:

For every game tree, the utility obtained by MAX using minimax decisions against any other min strategy will never be lower than the utility obtained playing against MIN.

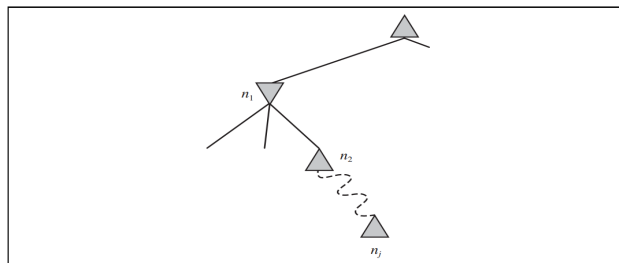
Find a game tree and a min function in which max player can do better using a different strategy (than using MAX).

(Draw the game tree and explain the strategies of min and max)

[30 points]

2. This problem exercises the basic concepts of game playing, using tic-tac-toe (noughts and crosses) as an example. We define  $X_n$  as the number of rows, columns, or diagonals with exactly  $n$  X's and no O's. Similarly,  $O_n$  is the number of rows, columns, or diagonals with just  $n$  O's. The utility function assigns  $+1$  to any position with  $X_3 = 1$  and  $-1$  to any position with  $O_3 = 1$ . All other terminal positions have utility 0. For nonterminal positions, we use a linear evaluation function defined as  $Eval(s) = 3X_2(s) + X_1(s) - 3O_2(s) - O_1(s)$

- Approximately how many possible games of tic-tac-toe are there?
- Show the whole game tree starting from an empty board down to depth 2 (i.e., one X and one O on the board), taking symmetry into account.
- Mark on your tree the evaluations of all the positions at depth 2.
- Using the minimax algorithm, mark on your tree the backed-up values for the positions at depths 1 and 0, and use those values to choose the best starting move.
- Circle the nodes at depth 2 that would not be evaluated if alpha-beta pruning were applied, assuming the nodes are generated in the optimal order for alpha-beta pruning.



[20 Points]

Adversarial Search Problem 3 :

Find the max-min binary game tree with 4 terminal states with utilities {1, 2, 3, 4}-values in the nodes that achieves the most effective alpha-beta pruning. (The tree does not have to be balanced).

## Logic

[25 points]

1. Convert the following formulae into Conjunctive Normal Form (CNF).

- a.  $Q \rightarrow P$
- b.  $(P \rightarrow \neg Q) \rightarrow R$
- c.  $\neg(P \wedge \neg Q) \rightarrow (\neg R \vee \neg Q)$
- d.  $\neg(P \rightarrow \neg Q) \rightarrow R$
- e.  $\neg(P \rightarrow (\neg R \vee \neg Q)) \rightarrow \neg R$