AI 2021 B EX5

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Problem 1

1

	R	P	S
R	0, 0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0, 0

it is easy to see that there isnt any pure Nash equilibrium for this game since for any result atleast 1 player will get ≤0 and he could always have a move that makes it so he will get 1.

lets solve for mixed strategy $\begin{pmatrix} x_1 \\ y_1 \\ 1-x_1-y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \\ 1-x_2-y_2 \end{pmatrix}$

~~if $x_1 > 0, y_1 > 0, 1-x_1-y_1 > 0$ must be indiffren~~

~~for $\begin{pmatrix} x_1 \\ y_1 \\ 1-x_1-y_1 \end{pmatrix}$~~

lets first observe that $x_1, y_1, 1-x_1-y_1 \neq 0$ since if one of them where then there will be a stratgy (pure one) that will always defeat or tie, and since it is symmetrical 0 sum game that cant be.

lets see that for any player $\begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$ is nash equilibria, let say there is a strategy $\begin{pmatrix} x \\ y \\ 1-x-y \end{pmatrix}$ that those better and lets say that $x > \frac{1}{3}$, and $1-x-y < \frac{1}{3}$ with no regard and $x \geq y > 1-x-y$ to generalety then couter strateg $\begin{pmatrix} 1-x-y \\ x \\ y \end{pmatrix}$ will get a positive net value in a symetric zero sum game, thus $\begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$ nash equilibrium for both.

2.

	R	P	S	L	Sp
R	0, 0	-1, 1	1, -1	1, -1	-1, 1
P	1, -1	0, 0	-1, 1	1, -1	-1, 1
S	-1, 1	1, -1	0, 0	1, -1	-1, 1
L	-1, 1	-1, 1	-1, 1	0, 0	1, -1
Sp	1, -1	1, -1	1, -1	-1, 1	0, 0

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ 1-x_1-x_2-x_3-x_4 \end{pmatrix}$$

they will be the same from symetry, the value will be 0 since it is symetrical 0 sum game

$0 = 0x_1 + 1\cdot x_2 - 1x_3 - 1x_4 + 1(1-x_1-x_2-x_3-x_4) = 1 - x_1 - 2x_3 - 2x_4$

$0 = -1x_1 + 0x_2 + 1x_3 - 1x_4 + 1(1-x_1-x_2-x_3-x_4) = 1 - 2x_1 - x_2 - 2x_4$

$0 = 1\cdot x_1 - 1\cdot x_2 + 0\cdot x_3 - 1\cdot x_4 + 1\cdot(1-x_1-x_2-x_3-x_4) = 1 - 2x_2 - x_3 - 2x_4$

$0 = 1\cdot x_1 + 1\cdot x_2 + 1\cdot x_3 + 0\cdot x_4 - 1(1-x_1-x_2-x_3-x_4) = -1 + 2x_1 + 2x_2 + 2x_3 + x_4$

$0 = -1\cdot x_1 - 1\cdot x_2 - 1\cdot x_3 + 1\cdot x_4 + 0\cdot(1-x_1-x_2-x_3-x_4) = -x_1 - x_2 - x_3 + x_4$

symetry Between R, P, S $x_1 = x_2 = x_3 \overset{\text{mark}}{=} x$
$\Rightarrow$ + equsions 1-3

$1 = 3x + 2x_4$
$1 = 3x + 2x_4$
$1 = 3x + 2x_4$ $\Rightarrow$
$1 = 6x + x_4$
$3x = x_4$

$\Rightarrow 3x_4 = 1 \quad x_4 = \frac{1}{3},\ x = \frac{1}{9} \underset{\text{eq.}}{\overset{\text{nash}}{\Rightarrow}} \begin{pmatrix} 1/9 \\ 1/9 \\ 1/9 \\ 1/3 \\ 1/3 \end{pmatrix} \begin{pmatrix} 1/9 \\ 1/9 \\ 1/9 \\ 1/3 \\ 1/3 \end{pmatrix}$

Problem 2

let a A be ~~dominant~~ dominat strotgy equibrium
spuse that it aint nash eq, than ak is better (strictly) for player 2 or kA is better for player 1 than A or a isnt dominant thus contredict.

look at

	A	B
a	4, 4	0, 0
b	0, 0	1, 1

bB is nash eq but by no means dominant.

Problem 3

1. all pure nash eq are as followed
(A, a), (C, c)

2. for player 1 B is dominated by A
for player 2 ~~even~~ on the base game
there aren't any dominated stratgys
on the surface sinc if P1 plays A
then a is the best, if P1 plays B
b is the best and if P1 plays c
then c is the best however
since P1 wont play B (dominated by A)
then on the updated matrix b wuld
be dominated by c

3.

	a	b	c
A	4, 3	7, 0	2, 2
B	1, 7	5, 6	0, 2
C	0, 2	9, 7	3, 8

A dominate B ⇒

	a	b	c
A	4, 3	7, 0	2, 2
C	0, 2	9, 7	3, 8

c dominates b ⇒

	a	c
A	4, 3	2, 2
C	0, 2	3, 8

Problem [4]: Decision Trees:

PlayTennis

- Four attributes used for classification:
  1. Outlook = {Sunny, Overcast, Rain}
  2. Temperature = {Hot, Mild, Cool}
  3. Humidity = {High, Normal}
  4. Wind = {Weak, Strong}
- One predicted (target) attribute (binary):
  PlayTennis = {Yes,No}
- Given 14 Training examples:
  9 positive, 5 negative.

We will choose the variable to split on such that the corresponding information gain is maximal. We will use the formula from Tirgul (10):

$$IG_{Ex}(Goal;a) = H_{Ex}(Goal) - \sum_{v \in val(a)} \frac{|Ex_{a,v}|}{|Ex|} H_{Ex_{a,v}}(Goal)$$

The entropy of a random variable X is:

$$H(X) = -\sum_{x} p(x) \log_b p(x)$$

So the initial entropy of the training sample:

$$H_{Ex}(Goal) = -\left(\frac{5}{14} log_2 \frac{5}{14} + \frac{9}{14} log_2 \frac{9}{14}\right) = 0.9403$$

The information gains of the attributes are:

$$IG_{Ex}(Goal; \text{Outlook}) = 0.9403 - \left[\frac{5}{14} H\left(\frac{2}{5},\frac{3}{5}\right) + \frac{4}{14} H\left(\frac{4}{4},\frac{0}{4}\right) + \frac{5}{14} H\left(\frac{3}{5},\frac{2}{5}\right)\right] = 0.2468$$

$$IG_{Ex}(Goal; \text{Temperature}) = 0.9403 - \left[\frac{4}{14} H\left(\frac{2}{4},\frac{2}{4}\right) + \frac{6}{14} H\left(\frac{4}{6},\frac{2}{6}\right) + \frac{4}{14} H\left(\frac{3}{4},\frac{1}{4}\right)\right] = 0.0292$$

$$IG_{Ex}(Goal; \text{Humidity}) = 0.9403 - \left[\frac{7}{14} H\left(\frac{4}{7},\frac{3}{7}\right) + \frac{7}{14} H\left(\frac{6}{7},\frac{1}{7}\right)\right] = 0.1518$$

$$IG_{Ex}(Goal; \text{Wind}) = 0.9403 - \left[\frac{8}{14} H\left(\frac{6}{8},\frac{2}{8}\right) + \frac{6}{14} H\left(\frac{3}{6},\frac{3}{6}\right)\right] = 0.0481$$

Outlook is selected as the decision attribute for the root node, and branches are created below the root for each of its possible values (i.e., Sunny, Overcast, and Rainy). Next, we choose an attribute to split on in every leaf of the tree:

First, the ***Sunny*** leaf:

$$H_{Ex_{Outlook,Sunny}}(Goal) = H\left(\frac{2}{5}, \frac{3}{5}\right) = 0.971$$

$$IG_{Ex_{Outlook,Sunny}}(Goal; \text{Temperature}) = 0.971 - \left[\frac{2}{5}H\left(\frac{0}{2}, \frac{2}{2}\right) + \frac{2}{5}H\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{1}{5}H\left(\frac{1}{1}, \frac{0}{1}\right)\right]$$
$$= 0.570$$

$$IG_{Ex_{Outlook,Sunny}}(Goal; \text{H}umidity) = 0.971 - \left[\frac{3}{5}H\left(\frac{0}{3}, \frac{3}{3}\right) + \frac{2}{5}H\left(\frac{2}{2}, \frac{0}{2}\right)\right] = 0.971$$

$$IG_{Ex_{Outlook,Sunny}}(Goal; \text{W}ind) = 0.971 - \left[\frac{3}{5}H\left(\frac{1}{3}, \frac{2}{3}\right) + \frac{2}{5}H\left(\frac{1}{2}, \frac{1}{2}\right)\right] = 0.019$$

Humidity is selected as the leftmost son of the root node, and branches are created below it for each of its possible values (i.e., High and Normal).

Second, the ***Overcast*** leaf. Notice that because the training examples associated with this leaf all have same target attribute value (i.e., True) we stopped expanding this node.

Third, the ***Rainy*** leaf. Calculations are similar to those we've done before:

$$H_{Ex_{Outlook,Rainy}}(Goal) = H\left(\frac{3}{5}, \frac{2}{5}\right) = 0.971$$

$$IG_{Ex_{Outlook,Rainy}}(Goal; \text{Temperature}) = 0.019$$
$$IG_{Ex_{Outlook,Rainy}}(Goal; \text{H}umidity) = 0.019$$
$$IG_{Ex_{Outlook,Rainy}}(Goal; \text{W}ind) = 0.971$$

Wind is selected as the rightmost son of the root node, and branches are created below it for each of its possible values (i.e., Strong and Week).

And because the training examples associated with each one of the current leaves, which are, Humidity attributes and Wind attributes, have the same target attribute value, (High-> False, Normal->True, Strong->True, Weak->False), our algorithm will stop at this point.

The complete decision tree will look like: