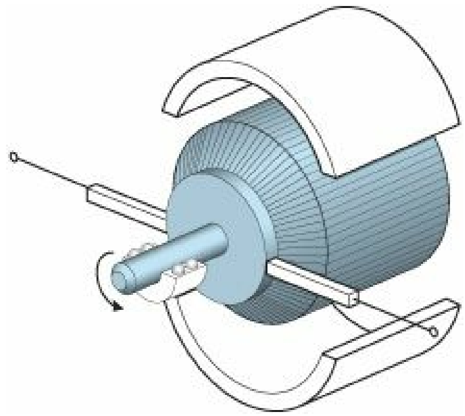
4 A First Analysis of Feedback



A Perspective on the Analysis of Feedback

In the next three chapters we will introduce three techniques for the design of controllers. Before doing so, it is useful to develop the assumptions to be used and to derive the equations that are common to each of the design approaches we describe. As a general observation, the dynamics of systems to which control is applied are nonlinear and very complex. However, in this initial analysis, we assume that the plant to be controlled as well as the controller can be represented as dynamic systems which are linear and time invariant (LTI). We also assume that they have only single inputs and single outputs, for the most part, and may thus be represented by simple scalar transfer functions. As we mentioned in Chapter 1, our basic concerns for control are stability, tracking, regulation, and sensitivity. The goal of the analysis in this chapter is to revisit each of these requirements in a linear dynamic setting and to develop equations that will expose constraints placed on the controller and identify elementary objectives to be suggested for the controllers.

Open-loop and closed-loop control

The two fundamental structures for realizing controls are the open-loop structure as shown in Fig. 4.1, and the closed-loop structure, also known as feedback control, as shown in Fig. 4.2. The definition of open-loop control is that there is no closed signal path whereby the output influences the control effort. In the structure shown in Fig. 4.1, the controller transfer function modifies the reference input signal before it is applied to the plant. This controller might cancel the unwanted dynamics of the plant and replace them with the more desirable dynamics of the controller. In other cases open-loop control actions are taken on the plant as the environment changes, actions that are calibrated to give a good response but are not dependent on measuring the actual response. An example of this would be an aircraft autopilot whose parameters are changed with altitude or speed but not by feedback of the craft's motion. Feedback control, on the other hand, uses a sensor to measure the output and by feedback indirectly modifies the dynamics of the system. Although it is possible that feedback may cause an otherwise stable system to become unstable (a vicious circle), feedback gives the designer more flexibility and a preferable response to each of our objectives when compared to open-loop control.

Chapter Overview

The chapter begins with consideration of the basic equations of a simple open-loop structure and of an elementary feedback structure. In Section 4.1 the equations for the two structures are presented in general form and compared in turn with respect to stability, tracking, regulation, and sensitivity. In Section 4.2 the steady-state errors in response to polynomial inputs are analyzed in more detail. As part of the language of steady-state performance, control systems are assigned a type number according to the maximum degree of the input polynomial for which the steady-state error is a finite constant. For each type an appropriate error constant is defined, which allows the designer to easily compute the size of this error.

Although Maxwell and Routh developed a mathematical basis for assuring stability of a feedback system, design of controllers from the earliest days was largely trial and error based on experience. From this tradition there emerged an almost universal controller, the proportional–integral–derivative (PID) structure considered in Section 4.3. This device has three elements: a Pro-portional term to close the feedback loop, an Integral term to assure zero error to constant reference and disturbance inputs, and a Derivative term to improve (or realize!) stability and good dynamic response. In this section these terms are considered and their respective effects illustrated. As part of the evolution of the PID controller design, a major step was the development of a simple procedure for selecting the three parameters, a process called “tuning the controller.” Ziegler and Nichols developed and published a set of experiments to be run, characteristics to be measured, and tuning values to be recommended as a result. These procedures are discussed in this section. Finally, in optional Section 4.4, a brief introduction to the increasingly common digital implementation of controllers is given. Sensitivity of time response to parameter changes is discussed in Appendix W4 on the web.

4.1 The Basic Equations of Control

We begin by collecting a set of equations and transfer functions that will be used throughout the rest of the text. For the open-loop system of Fig. 4.1, if we take the disturbance to be at the input of the plant, the output is given by

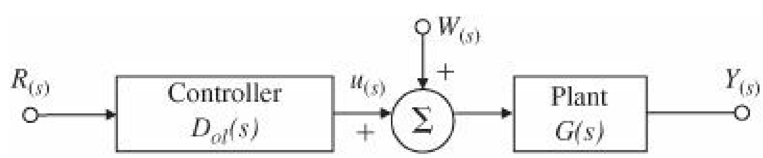


Figure 4.1 Open-loop system showing reference, R, control, U, disturbance, W, and output Y

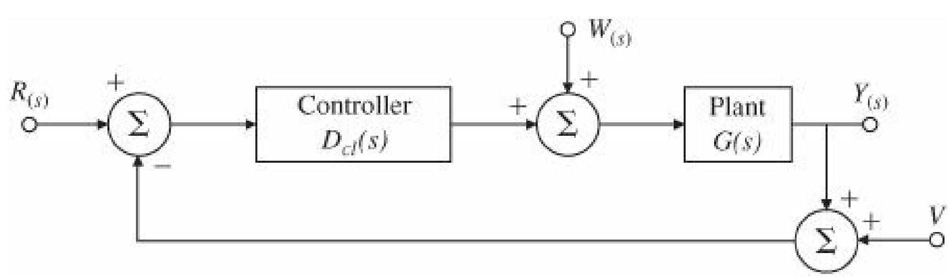


Figure 4.2 Closed-loop system showing the reference, R, control, U, disturbance, W, output, Y, and sensor noise, V

$$Y_{ol} = GD_{ol}R + GW \tag{4.1}$$

and the error, the difference between reference input and system output, is given by

$$\begin{aligned} E_{ol} &= R - Y_{ol} && (4.2) \\ &= R - [GD_{ol}R + GW] && (4.3) \\ &= [1 - GD_{ol}]R - GW. && (4.4) \end{aligned}$$

The open-loop transfer function in this case is $T_{ol}(s) = G(s)D_{ol}(s)$.

For feedback control, Fig. 4.2 gives the basic unity feedback structure of interest. There are three external inputs: the reference, R, which the output is expected to track, the plant disturbance, W, which the control is expected to counteract so it does not disturb the output, and the sensor noise, V, which the controller is supposed to ignore.

For the feedback block diagram of Fig. 4.2, the equations for the output and the control are given by the superposition of the responses to the three inputs individually, as follows:

$$Y_{cl} = \frac{GD_{cl}}{1+GD_{cl}}R + \frac{G}{1+GD_{cl}}W - \frac{GD_{cl}}{1+GD_{cl}}V. \tag{4.5}$$

$$U = \frac{D_{cl}}{1+GD_{cl}}R - \frac{GD_{cl}}{1+GD_{cl}}W - \frac{D_{cl}}{1+GD_{cl}}V. \tag{4.6}$$

Perhaps more important than these is the equation for the error, $E_{cl} = R - Y_{cl}$.

$$E_{cl} = R - \left[\frac{GD_{cl}}{1+GD_{cl}}R + \frac{G}{1+GD_{cl}}W - \frac{GD_{cl}}{1+GD_{cl}}V\right] \tag{4.7}$$

$$= \frac{1}{1+GD_{cl}}R - \frac{G}{1+GD_{cl}}W + \frac{GD_{cl}}{1+GD_{cl}}V. \tag{4.8}$$

In this case, the closed-loop transfer function is $T_{cl} = \dfrac{GD_{cl}}{1+GD_{cl}}$

With these equations we will explore the four basic objectives of stability, tracking, regulation, and sensitivity for both the open-loop and the closed-loop cases.

4.1.1 Stability

As we saw in Chapter 3, the requirement for stability is simply stated: All poles of the transfer function must be in the left half-plane (LHP). In the open-loop case described by Eq. (4.1), these are the poles of GD_{ol}. To see the restrictions this requirement places on the controller, we define the polynomials a(s), b(s), c(s), and d(s) so that $G(s) = \frac{b(s)}{a(s)}$ and $D_{ol}(s) = \frac{c(s)}{d(s)}$. Therefore $GD_{ol} = \frac{bc}{ad}$. With these definitions, the stability requirement is that neither a(s) nor d(s) may have roots in the right half-plane (RHP). A naive engineer might believe that if the plant is unstable with a(s) having a root in the RHP, the system might be made stable by canceling this pole with a zero of c(s). However, the unstable pole remains and the slightest noise or disturbance will cause the output to grow until stopped by saturation or system failure. Likewise, if the plant shows poor response because of a zero of b (s) in the RHP, an attempt to fix this by cancellation using a root of d(s) will similarly result in an unstable system. We conclude that an open-loop structure cannot be used to make an unstable plant to be stable and therefore cannot be used if the plant is already unstable.

For the feedback system, from Eq. (4.8), the system poles are the roots of $1 + GD_{cl} = 0$. Again using the polynomials defined above, the system characteristic equation is

$$1 + GD_{cl} = 0 \tag{4.9}$$

$$1+\frac{b(s)c(s)}{a(s)d(s)}=0 \tag{4.10}$$

$$a(s)d(s)+b(s)c(s)=0. \tag{4.11}$$

From this equation, it is clear that the feedback case grants considerably more freedom to the controller design than does the open-loop case. However, one must still avoid unstable cancellations. For example, if the plant is unstable and therefore a (s) has a root in the RHP, we might cancel this pole by putting a zero of c(s) at the same place. However, Eq. (4.11) shows that as a result, the unstable pole remains a pole of the system and this method will not work. However, unlike the open-loop case, having a pole of a(s) in the RHP does NOT prevent our designing a feedback controller that will make the system stable. For example, in Chapter 2 we derived the transfer function for the inverted pendulum, which, for simple values, might be $G(s)=\frac{1}{s^2-1}$ for which we have b(s) = 1 and a(s) = s^2-1 = (s + 1)(s – 1). Suppose we try $D(s)=\frac{K(s+\gamma)}{s+\delta}$. The characteristic equation that results for the system is

$$(s+1)(s-1)(s+\delta)+K(s+\gamma)=0. \tag{4.12}$$

This is the problem that Maxwell faced in his study of governors, namely under what conditions on the parameters will all the roots of this equation be in the LHP? The problem was solved by Routh. In our case, a simple solution is to take y = 1 and the common (stable) factor cancels. The resulting second-order equation can be easily solved to place the remaining two poles at any point desired.

Exercise. If we wish to force the characteristic equation to be $s^2+2\xi\omega s+\omega^2=0$, solve for K and δ in terms of ξ and ω

4.1.2 Tracking

The tracking problem is to cause the output to follow the reference input as closely as possible. In the open-loop case, if the plant is stable and has neither poles nor zeros in the RHP, then in principle the controller can be selected to cancel the transfer function of the plant and substitute whatever desired transfer function the engineer wishes. This apparent freedom, however, comes with three caveats. First, in order to physically build it, the controller transfer function must be proper, meaning that it cannot be given more zeros than it has poles. Second, the engineer must not get greedy and request an unrealistically fast design. This entire analysis has been based on the assumption that the plant is linear and a demand for a fast response will demand large inputs to the plant, inputs that will be sure to saturate the system if the demand is too great. Again, it is the responsibility of the engineer to know the limits of the plant and to set the desired overall transfer function to a reasonable value with this knowledge. Third and finally, although one can, in principle, stably cancel any pole in the LHP, the next section on sensitivity faces up to the fact that the plant transfer function is subject to change and if one tries to cancel a pole that is barely inside the LHP there is a good chance of disaster as that pole moves a bit and exposes the system response to unacceptable transients.

Exercise. For a plant having the transfer function $\frac{1}{s^2+3s+9}$ it is proposed to use a controller in a unity feedback system and having the transfer function $\frac{c_2s^2+c_1s+c_0}{s(s+d_1)}$. Solve for the parameters of this controller so that the closed-loop will have the characteristic equation $(s+6)(s+3)(s^2+3s+9)=0$[1].

{ans: $c_2 = 34$, $c_1 = 36$, $c_0 = 162$, $d_1 = 11$}

Exercise. Show that if the reference input to the system of the above exercise is a step of amplitude A, the steady-state error will be zero.

4.1.3 Regulation

The problem of regulation is to keep the error small when the reference is at most a constant set point and disturbances are present. A quick look at the open-loop block diagram reveals that the controller has no influence at all on the system response to either of the disturbances, w, or v, so this structure is useless for regulation. We turn to the feedback case. From Eq. (4.8) we find a conflict between w and v in the search for a good controller. For example, the term giving the contribution of the plant disturbance to the system error is $\frac{G}{1+GD_{cl}}W$. To select D_{cl} to make this term small, we should make D_{cl} as large as possible and infinite if that is feasible. On the other hand, the error term for the sensor noise is $\frac{GD_{cl}}{1+GD_{cl}}V$. In this case, unfortunately, if we select D_{cl} to be large, the transfer function tends to unity and the sensor noise is not reduced at all! What are we to do? The resolution of the dilemma is to observe that each of these terms is a function of frequency so one of them can be large for some frequencies and small for others. With this in mind, we also note that the frequency content of most plant disturbances occurs at very low frequencies and in fact, the most common case is a bias, which is all at zero frequency! On the other hand, a good sensor will have no bias and can be constructed to have very little noise over the entire range of low frequencies of interest. Thus, using this information, we design the controller transfer function to be large at the low frequencies, where it will reduce the effect of w, and we make it small at the higher frequencies, where it will reduce the effects of the high frequency sensor noise. The control engineer must determine in each case the best place on the frequency scale to make the cross over from amplifying to attenuation.

Exercise. Show that if w is a constant bias and if D_{cl} has a pole at $s = 0$ then the error due to this bias will be zero. However, show that if G has a pole at zero, it does not help with a disturbance bias.

4.1.4 Sensitivity

Suppose a plant is designed with gain G at a particular frequency but in operation it changes to be $G + \delta G$. This represents a fractional or percent change of gain of $^{\delta G}/G$. For the purposes of this analysis, we set the frequency at zero and take the open-loop controller gain to be fixed at $D_{ol}(0)$. In the open-loop case the nominal overall gain is thus $T_{ol} = GD_{ol}$, and with the perturbed plant gain, the overall gain would be

$$T_{ol} + \delta T_{ol} = D_{ol}(G + \delta G) = D_{ol}G + D_{ol}\delta G = T_{ol} + D_{ol}\delta G.$$

Therefore, the gain change is $\delta T_{ol} = D_{ol}\delta G$. The sensitivity, S_G^T, of a transfer function, T_{ol}, to a plant gain, G, is defined to be the ratio of the fractional change in T_{ol} defined as $\frac{\delta T_{ol}}{T_{ol}}$ to the fractional change in G. In equation form