September, 2018

MULUNGUSHI UNIVERSITY (SSET)

ENGINEERING MATHEMATICS (EGM 211)

APPLICATION OF DIFFERENTIATION SHEET 1

1. An alternating current, i amperes, is given by $i = 10\sin 2\pi f\, t$, where f is the frequency in hertz and t the time in seconds. Determine the rate of change of current when $t = 20ms$, given that $f = 150Hz$.
2. The luminous intensity, I candelas, of a lamp is given by $I = 6 \times 10^{-4} V^2$, where V is the voltage. Find (a) the rate of change of luminous intensity with voltage when $V = 200volts$, and

   (b) the voltage at which the light is increasing at a rate of 0.3 candelas per volt.
3. The voltage across the plates of a capacitor at any time t seconds is given by $v = V\, e^{-\frac{t}{CR}}$, where V, C and R are constants. Given $V = 300volts$, $C = 0.12 \times 10^{-6}F$ and $R = 4 \times 10^6$.

   find

   (a) the initial rate of change of voltage, and

   (b) the rate of change of voltage after 0.5 s.
4. The pressure p of the atmosphere at height h above ground level is given by $p = p_o e^{-\frac{h}{c}}$, where p_o is the pressure at ground level and c is a constant. Determine the rate of change of pressure with height when $p_o = 1.013 \times 10^5$ pascals and $c = 6.05 \times 10^4$ at 1450 metres.
5. A missile fired from ground level rises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find

   (a) the initial velocity of the missile,

(b) the time when the height of the missile is a maximum,

(c) the maximum height reached,

(d) the velocity with which the missile strikes the ground.

6. The distance s metres travelled by a car in t seconds after the brakes are applied is given by $s = 25\,t - 2.5t^2$. Find

   (a) the speed of the car (in km/h) when the brakes are applied,

   (b) the distance the car travels before it stops.

7. At any time t seconds the distance x metres of a particle moving in a straight line from a fixed point is given by $x = 4t + ln(1 - t)$. Determine (a) the initial velocity and acceleration (b) the velocity and acceleration after 1.5s. (c) The time when the velocity is zero.

8. The angular displacement θ of a rotating disc is given by $\theta = 6\sin\frac{t}{4}$, where t is the time in seconds.

   Determine,

   (a) the angular velocity of the disc when t is $1.5s$,

   (b) the angular acceleration when t is $5.5\ s$, and

   (c) the first time when the angular velocity is zero.

9. $x = \frac{20}{3}\,t^3 - \frac{23}{2}t^{\,2} + 6t + 5$ represents the distance, x metres, moved by a body in t seconds.Determine

   (a) the velocity and acceleration at the start,

   (b) the velocity and acceleration when $t = 3\ s$,

   (c) the values of t when the body is at rest,

   (d) the value of t when the acceleration is $37\ m/s^2$ and

   (e) the distance travelled in the third second.

10. Find the turning points and distinguish between them.

    a. $y = x^2 - 6x$ b. $y = 4\,x^3 + 3\,x^2 - 60\,x - 12$ c. $y = 5\,x - 2\,lnx$

    d. $y = 2\,x - e^x$

11. Show that the curve $y = \frac{2}{3}(t\ -1)^3 + 2t\ (t\ -2)$ has a maximum value of $\frac{2}{3}$ and a minimum value of -2.
12. The speed, v, of a car $(in\ m/s)$ is related to time t s by the equation $v = 3 + 12t\ - 3t^2$. Determine the maximum speed of the car in km/h.
13. Determine the maximum area of a rectangular piece of land that can be enclosed by 1200m of fencing.
14. A lidless box with square ends is to be made from a thin sheet of metal. Determine the least area of the metal for which the volume of the box is 3.5 m^3.
15. A closed cylindrical container has a surface area of $400cm^2$. Determine the dimensions for maximum volume.
16. Calculate the height of a cylinder of maximum volume which can be cut from a cone of height 20cm and base radius 80cm.
17. The power developed in a resistor R by a battery of emf E and internal resistance r is given by

    $P = \frac{E^2R}{(R+r)^2}$. Differentiate P with respect to R and show that the power is a maximum when $R = r$.
18. Resistance to motion, F, of a moving vehicle, is given by $F\ = \frac{5}{x} + 100x$. Determine the minimum value of resistance.
19. An electrical voltage E is given by $E\ = (15sin50\pi t + 40cos50\pi t\ )$ volts, where t is the time in seconds. Determine the maximum value of voltage.