

Q5

Find the sequence x satisfying $x * \langle 1, 1, -1 \rangle = \langle 1, 0, -1, 2, -1 \rangle$

Answer :

$$\text{Let } x = \langle A_0, A_1, A_2, A_3, \dots \rangle$$

$$x = A_0 + A_1x + A_2x^2 + A_3x^3 + \dots$$

$$x * \langle 1, 1, -1 \rangle = \langle 1, 0, -1, 2, -1 \rangle$$

$$(A_0 + A_1x + A_2x^2 + A_3x^3 + \dots)(1 + x - x^2) = 1 - x^2 + 2x^3 - x^4$$

$$\begin{aligned} (A_0 + A_1x + A_2x^2 + \dots) + (A_0x + A_1x^2 + A_2x^3 + \dots) - (A_0x^2 + A_1x^3 + A_2x^4 \dots) \\ = 1 - x^2 + 2x^3 - x^4 \end{aligned}$$

Now compare

$$A_0 = 1$$

$$A_1 + A_0 = 0 \Rightarrow A_1 = -1$$

$$A_2 + A_1 - A_0 = -1 \Rightarrow A_2 = 1$$

$$A_3 + A_2 - A_1 = 2 \Rightarrow A_3 = 0$$

$$A_4 + A_3 - A_2 = -1 \Rightarrow A_4 = 0$$

For $i \geq 3$ all $A_i = 0$

$$\Rightarrow x = \langle 1, -1, 1 \rangle$$