Mubarra Khan z5161249 Wednesday, 5 August 2020

Q2

You are given a usual n × n chess board with k white bishops on the board at the given cells (ai , bi), (1 ≤ ai , bi ≤ n, 1 ≤ i ≤ k). You have to determine the largest number of black rooks you can place on the board so that no two rooks are in the same row or in the same column and are not under the attack of any of the k bishops (recall that bishops go diagonally).(20 pts)

Hints: Make a bipartite graph with all the columns as vertices on the left hand side and all the rows as vertices on the right hand side. Each square sij can now be represented by and edge from column j to row i. Think which edges you allow.

Solution: This is a maximum flow problem

- Think of a bipartite graph with all the columns as vertices on the left hand side and all the rows as vertices on the right hand side.
- We connect column to a super source by a directed edge of capacity 1.
- Next we connect each row with a super sink by a directed edge of capacity 1
- There is also an edge between a row and column if they intersect, forming a square. However we do not allow all edges to ensure no rocks are under attack of any of the k Bishops. Therefore on each edge we need to check the validity of the path. We make a judgement on each cell which can be represented as $C\left(a_m, b_m\right),\ (1 \le a_m, b_m \le n,).$
  - For the bishop there is k white bishops at the cells $C\left(a_i, b_i\right), (1 \le a_i, b_i \le n,\ 1 < i < k).$ Therefore we do not add an edge for a square which is in the range of a bishop
    - As the bishop goes diagonally we can not put rocks on the positions $(a_i + 1, b_i + 1), (a_i + 1, b_i - 1), (a_i - 1, b_i + 1), (a_i - 1, b_i - 1).$
    - We check if $C(a_m, b_m)$ equals $C(a_i, b_i)$ or any of the diagonal positions above . If so then the path is not feasible and we need to give it up -> do not connect an edge.
- Since we set **each** edge capacity to 1, no two rocks are in the same row or in the same column
- Now we have constructed a bipartite graph and can turn it into a flow network
- Using the Ford - Fulkerson algorithm we can get the max flow which is the largest number of black rooks you can place on the board.