

Q4

a) Compute the convolution $\underbrace{\langle 1, 0, 0, \dots, 0, 1 \rangle}_k * \underbrace{\langle 1, 0, 0, \dots, 0, 1 \rangle}_k$

The polynomial for $\langle 1, 0, 0, \dots, 0, 1 \rangle$ is $P(x) = 1 + x^{k+1}$

$$\begin{aligned} P(x)^2 &= (1 + x^{k+1})^2 \\ &= 1 + 2x^{k+1} + x^{2k+2} \end{aligned}$$

\therefore The convolution for the above is $\underbrace{\langle 1, 0, 0, \dots, 0, 2, 0, 0, \dots, 0, 1 \rangle}_k$

b) Compute the DFT of the sequence $\underbrace{\langle 1, 0, 0, \dots, 0, 1 \rangle}_k$.

Since $B = \underbrace{\langle 1, 0, 0, \dots, 0, 1 \rangle}_k$, the corresponding polynomial is $P_B(x) = 1 + x^{k+1}$ and

$$\begin{aligned} DFT(B) &= \langle P_B(\omega_{k+2}^0), P_B(\omega_{k+2}^1), \dots, P_B(\omega_{k+2}^{k+1}) \rangle \\ &= \langle 1 + \omega_{k+2}^{0 \cdot (k+1)}, 1 + \omega_{k+2}^{1 \cdot (k+1)}, \dots, 1 + \omega_{k+2}^{(k+1) \cdot (k+1)} \rangle \\ &= \underbrace{\langle 2, 1 + \omega_{k+2}^{(k+1)}, 1 + \omega_{k+2}^{2 \cdot (k+1)}, \dots, 1 + \omega_{k+2}^{(k-1)(k+1)}, 1 + \omega_{k+2}^{k(k+1)}, 1 + \omega_{k+2}^{(k+1)^2} \rangle}_k \\ &= \underbrace{\langle 2, 1 + \omega_{k+2}^{-1}, 1 + \omega_{k+2}^{-2}, \dots, 1 + \omega_{k+2}^{-(k+1)}, 1 + \omega_{k+2}^{-k}, 1 + \omega_{k+2}^{-(k+1)} \rangle}_k \end{aligned}$$