Mubarra Khan
Z5161249

Q5 solution

a)
$f(n) = (log_2(n))^2$
$g(n) = log_2(n^{(}log_2 n))^2$

Using $\log_a b = b \log a$ we simplify f(n) and g(n) to get
$f(n) = (log_2(n)) * (log_2(n))$
$g(n) = log_2(n^{log_2 n})^2 = log_2(n^{2log_2 n})$
$= 2log_2(n) \, . \, log_2(n)$

N has to be greater than 0 because of log when n = 1, f(n) = g(n) = 0.

Then $0 \leq f(n) \leq c * g(n)$ when $n \geq 1, c \geq 1$

Also $0 \leq c * g(n) \leq f(n)$ when $n \geq 1, c \leq 1/2$

So the answer is $f(n) = \theta(g(n))$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{log_2(n) * log_2(n)}{2log_2(n) * log_2(n)}$$

$$= \lim_{n\to\infty} \frac{log_2(n) * log_2(n)}{log_2(n) * log_2(n)}$$

$$= \lim_{n\to\infty} \frac{1}{1} = 1$$

Which satisfies $0 <= \lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty$

So the answer is $f(n) = \theta(g(n))$

b) $f(n) = n^{10}$
$g(n) = 2^{10\sqrt{n}}$

Using $log_a a = 1$ we simplify f(n) and g(n)
$f(n) = log_2 n^{10} = 10 \log_2(n)$
$g(n) = log_2 2^{10\sqrt{n}} = 10\sqrt{n}$

Then $0 \leq log_2 n \leq c * n$ when $n \geq 1, c \geq 1$
Then $0 \leq f(n) \leq c * g(n)$ when $n \geq 1, c \geq 1$
So the answer is $f(n) = O(g(n))$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{log_2 n^{10}}{log_2^{10\sqrt{n}}} = \lim_{n\to\infty} \frac{10 \log_2 n}{10\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{log_2 n}{10\sqrt{n}}$$

Using L'Hopital's rule

$$= \lim_{n\to\infty} \frac{1/n}{1/10n^{9/10}}$$

$$= \frac{10n^{9/10}}{n} = \frac{10}{n^{0.1}} -> 0$$

Which satisfies $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$

So the answer is $f(n) = O(g(n))$

c) $f(n) = n^{1+(-1)^n}$
$g(n) = n$

Considering the pattern by which $f(n)$ progresses for:

1, 4, 1, 16, 1, 36, 1, 64 …

Which translates to

$n^0, n^2, n^0, n^2, n^0, n^2, \ldots = 1, n^2, 1, n^2, 1, n^2 \ldots$

Case 1: $f(n) = n^0 = 1$

In this case it is clear that $g(n)$ grows much more quickly than $f(n)$ and hence :

$$f(n) = O(g(n))$$

Case 2 : $f(n) = n^2$

in this case it is clear that $f(n)$ grows much more quickly than $g(n)$ and hence

$$f(n) = \Omega(g(n))$$

From the above it is evident that $f(n)$ oscillates and thus $f(n)$ and $g(n)$ are not comparable. $f(n)$ cannot be related to $g(n)$ via any notation θ, Ω, O.