Q3

There are N computers in a network, labelled {1, 2, 3, . . . , N}. There are M one-directional links which connect pairs of computers. Computer 1 is trying to send a virus to computer N. This can happen as long as there is a path of links from computer 1 to computer N. To prevent this, you've decided to remove some of the links from the network so that the two computers are no longer connected. For each link, you've calculated the cost of removing it. What is the minimum total cost to disconnect the computers as required, and which edges should be removed to achieve this minimum cost? (20 pts)

Solution:

- We first construct a corresponding flow network with the computers as vertices and with computer 1 as the source and computer N as the sink.
- If there are k one- directional links between computer a and b, you combine them into a one directed edge of capacity k in the flow network. This capacity will also represent the "cost of removing" that particular edge.
  - Note: you only combine the links which are going in the same direction. As it is possible to have two or more links going in the opposite direction between computers.
    - In this case you treat links of the opposite direction as a separate bundle.
- The main idea now is to use the Ford-Fulkerson algorithm to find the the maximum flow and then corresponding minimum cut.
- We do this by first creating a **residual graph** from our original graph.
  - Each edge in the residual graph will "undo" the flow sent and will have strictly positive residual capacity.
  - Using the residual graph we will find the augmenting path using breadth first search:
    - Such an augmenting path:
      - Increases the flow along forward edges
      - Decreases the flow along backward edges.
  - While there exists an augmenting path in the residual graph
    - we will find the augmenting path *P*
    - Compute the bottleneck capacity of *P*

    - Augment each edge and the total flow

  - This will lead to he maximum flow and as the Max-flow min-cut theorem states The value of the max flow is equal to the capacity of the min cut.
  - Thus the value of the maximum flow tells us what link is causing the bottleneck. limiting the cost of creating a flow to the sink.

- Once identifying the bottleneck and where to cut, we need to sever all connections that contributed to that edges capacity (this will be the minimum cost).
    - Note: we only remove the edges in the forward direction if we want the minimum cost