FAST- National University of Computer and Emerging Sciences, Karachi.

FAST School of Computing

Assignment # 3 -- Solution, Fall 2021.

CS1005-Discrete Structures

Instructions: Max. Points: 100

1- This is hand written assignment.
2- Just write the question number instead of writing the whole question.
3- You can only use A4 size paper for solving the assignment.

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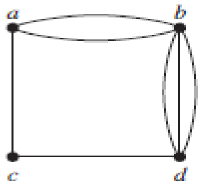
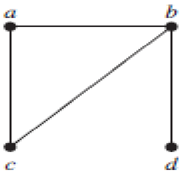
1. Determine whether the graph shown in figure i to iv has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph.

Solution:

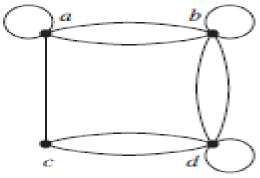
i) It has undirected edges.
It has multiple edges.
It has no loops.
It is undirected Multigraph.



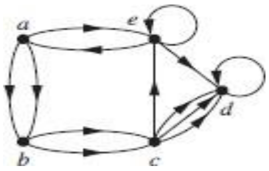
ii) It has undirected edges.
It has no multiple edges.
It has no loops.
It is undirected simple graph.



iii) It has undirected edges.
It has multiple edges.
It has three loops.
It is undirected Pseudo graph.



iv) It has directed edges.
It has multiple edges.
It has two loops.
It is directed Multi graph.



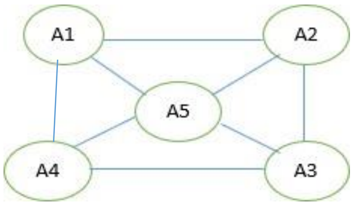
2. The intersection graph of a collection of sets A1, A2,..., An is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

i) A1 = {0, 2, 4, 6, 8}, A2 = {0, 1, 2, 3, 4}, A3 = {1, 3, 5, 7, 9}, A4 = {5, 6, 7, 8, 9}, A5 = {0, 1, 8, 9}

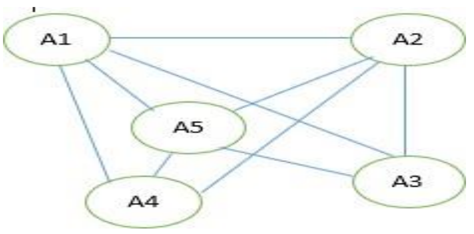
ii) A1 = {...,−4,−3,−2,−1, 0}, A2 = {...,−2,−1, 0, 1, 2,...}, A3 = {...,−6,−4,−2, 0, 2, 4, 6,...}, A4 = {...,−5,−3,−1, 1, 3, 5,...}, A5 = {...,−6,−3, 0, 3, 6,...}

Solution:

i)



ii)



3. **(a) Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Also find the neighborhood vertices of each vertex in given graphs.**

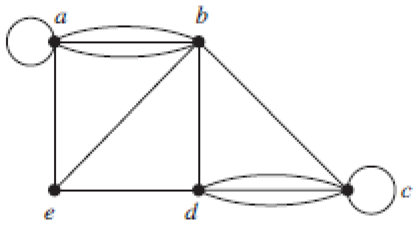
i) Number of Vertices: 5 Number of edges:13

Degree of vertices:

deg(*a*) = deg(*b*) = deg(*c*) = 6, deg(*d*) = 5, deg(*e*) = 3.

Neighborhood Vertices:

***N*(*a*) = {a, *b, e*}, *N*(*b*) = {*a, c, d, e*}, *N*(*c*) = {*b, c, d*}, *N*(*d*) = {b, *c, e*}, *N*(*e*) = a, *b, d*}**



ii) Number of Vertices: 9 Number of edges:12

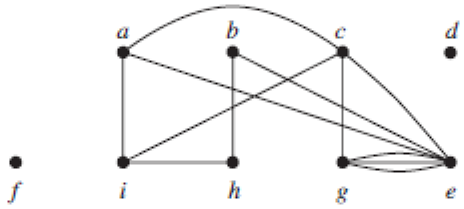
Degree of vertices:

deg(*a*) =3, deg(*b*) =2, deg(*c*) = 4, deg(*d*) = 0, deg(*e*) = 6.

deg(*f*) =0, deg(*g*) =4, deg(*h*) = 2, deg(*i*) = 3.

Neighborhood Vertices:

***N*(*a*) = {*c, e, i*}, *N*(*b*) = {*e, h*}, *N*(*c*) = {*a, e, g, i*}, *N*(*d*) = ∅ , *N*(e) = {a, *b, c, g*}, *N*(*f*) = ∅, *N*(*g*) = {*c, e*}, *N*(*h*) = {*b, i*}, *N*(*i*) = {*a, c, h*}.**



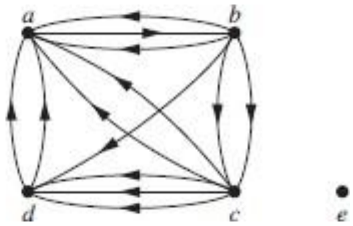
(b) Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

i) In-degree of a vertices

$\deg^-(a) = 6$, $\deg^-(b) = 1$, $\deg^-(c) = 2$, $\deg^-(d) = 4$, $\deg^-(e) = 0$.

Out-degree of a vertices

$\deg^+(a) = 1$, $\deg^+(b) = 5$, $\deg^+(c) = 5$, $\deg^+(d) = 2$, $\deg^+(e) = 0$.

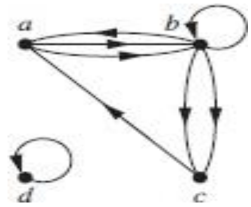


ii) In-degree of a vertices

$\deg^-(a) = 2$, $\deg^-(b) = 3$, $\deg^-(c) = 2$, $\deg^-(d) = 1$.

Out-degree of a vertices

$\deg^+(a) = 2$, $\deg^+(b) = 4$, $\deg^+(c) = 1$, $\deg^+(d) = 1$.

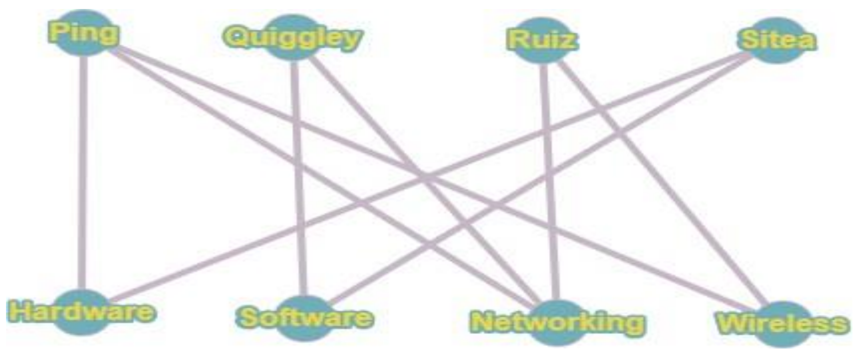


4. **(a) Suppose that a new company has five employees: Zamora, Agraharam, Smith, Chou, and Macintyre. Each employee will assume one of six responsibilities: planning, publicity, sales, marketing, development, and industry relations. Each employee is capable of doing one or more of these jobs: Zamora could do planning, sales, marketing, or industry relations; Agraharam could do planning or development; Smith could do publicity, sales, or industry relations; Chou could do planning, sales, or industry relations; and Macintyre could do planning, publicity, sales, or industry relations. Model the capabilities of these employees using appropriate graph.**

Solution: Bipartite Graph

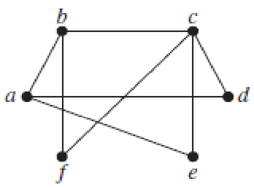
(b) Suppose that there are four employees in the computer support group of the School of Engineering of a large university. Each employee will be assigned to support one of four different areas: hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware, networking, and wireless; Quiggley is qualified to support software and networking; Ruiz is qualified to support networking and wireless, and Sitea is qualified to support hardware and software. Use appropriate graph to model the four employees and their qualifications.

Solution: Bipartite Graph

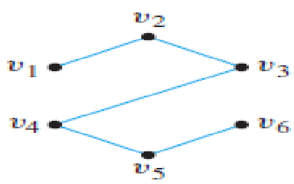


5. Find which of the following graphs are bipartite. Redraw the bipartite graphs so that their bipartite nature is evident. Also write the disjoint set of vertices.

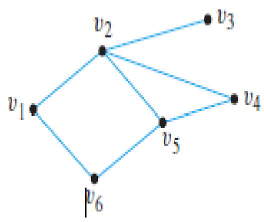
i)



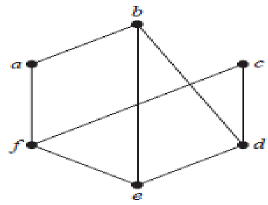
ii)



iii)



iv)



Solution:

(a) Not bipartite (since a is adjacent to b & f vertices)

(b) Bipartite (A (V_1, V_3, V_5) & B (V_2, V_4, V_6))

(c) Not bipartite (since V_4 & V_5 are adjacent vertices)

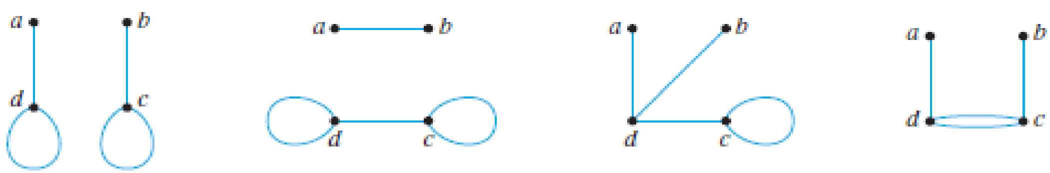
(d) Not Bipartite (since b is adjacent to d & e vertices)

6. Draw a graph with the specified properties or show that no such graph exists.
   a) A graph with four vertices of degrees 1, 1, 2, and 3
   b) A graph with four vertices of degrees 1, 1, 3, and 3
   c) A simple graph with four vertices of degrees 1, 1, 3, and 3

   Solution:

   a) No such graph is possible. By Handshaking theorem, the total degree of a graph is even. But a graph with four vertices of degrees 1, 1, 2, and 3 would have a total degree of 1 + 1 + 2 + 3 = 7, which is odd.

   b) Let G be any of the graphs shown below.



In each case, no matter how the edges are labeled, deg(a) = 1, deg(b) = 1, deg(c) =3, and deg(d) = 3.

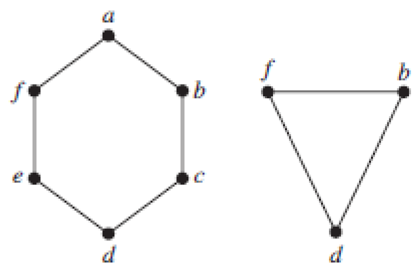
c) There is no simple graph with four vertices of degrees 1, 1, 3, and 3.

7. **a) In a group of 15 people, is it possible for each person to have exactly 3 friends? Explain. (Assume that friendship is a symmetric relationship: If x is a friend of y, then y is a friend of x.)**
Solution:
By using Handshaking theorem.
No! there is no graph possible, such that 15 vertices have degree 3. Since (15 * 3) ≠ 2e.

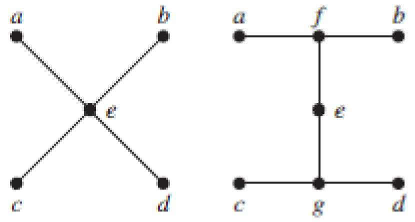
b) In a group of 4 people, is it possible for each person to have exactly 3 friends? Why?
Solution:
By using Handshaking theorem.
Yes! there is graph possible, such that 4 vertices have degree 3. Since (4 * 3) = 2e.

8. **(a) Find the union of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)**

i)

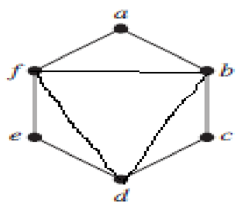


ii)



Solution:

i)



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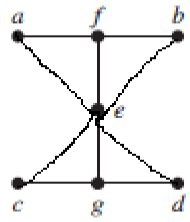


b) How many vertices does a regular graph of degree four with 10 edges have?
Solution:

We want to determine a regular graph of degree four with $m = 10$ edges.

Let the graph contain n vertices $v_1, v_2, ..., v_n$, then each of these n vertices have degree 4.

$$\deg(v_i) = 4$$
$$i = 1, 2, ..., n$$

By the Handshaking theorem, the sum of degrees of all vertices is equal to twice the number of edges:

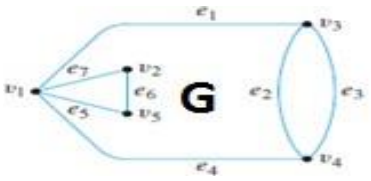
$$20 = 2(10) = 2m = \sum_{v=1}^{n} \deg(v_i) = \sum_{v=1}^{n} 4 = 4n$$

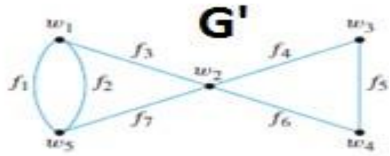
We then obtained the equation $20 = 4n$. Divide each side of the equation by 4:

$$n = \frac{20}{4} = 5$$

9. For given pair (G, G') of graphs. Determine whether they are isomorphic. If they are, give function g: V (G) →V (G') that define the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.

i)

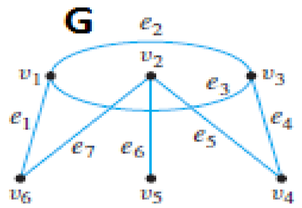


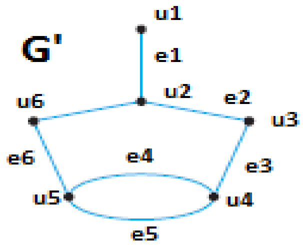


Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: g(V_1) = W_2, g(V_2) = W_3 g(V_3) = W_1 g(V_4) = W_5 g(V_5) = W_4

ii)

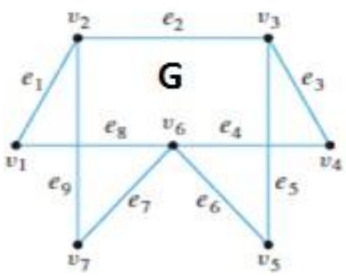


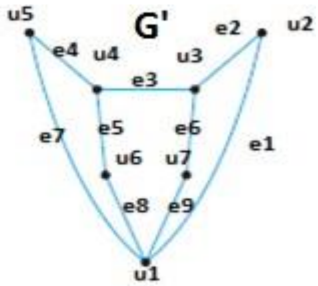


Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: g(V_1) = U_5, g(V_2) = U_2 g(V_3) = U_4 g(V_4) = U_3 g(V_5) = U_1 g(V_6) = U_6

iii)



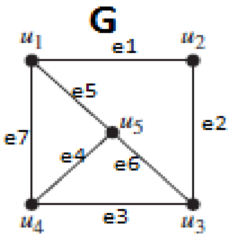


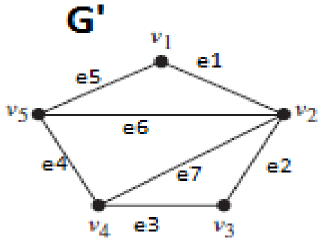
Solution: Both graph G and G' are satisfying all the invariant. Hence, they are isomorphic.

Function: g(V_1) = U_5, g(V_2) = U_4 g(V_3) = U_3 g(V_4) = U_2

g(V_5) = U_7 g(V_6) = U_1 g(V_7) = U_6

iv)

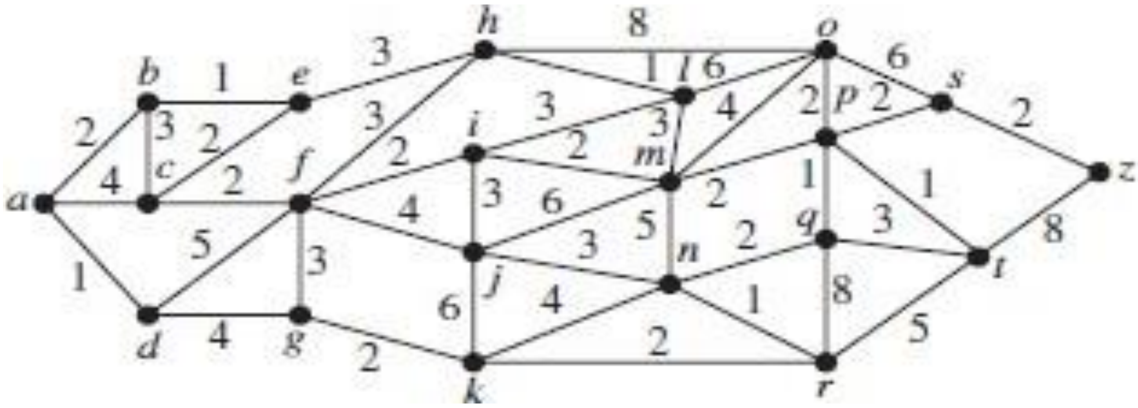




Solution: Graph G has no vertex of degree 4 where G' has vertex V_2 with degree 4. Hence, they are not isomorphic.

10. **Find the length of a shortest path between *a* and *z* in the given weighted graph by using Dijkstra's algorithm.**

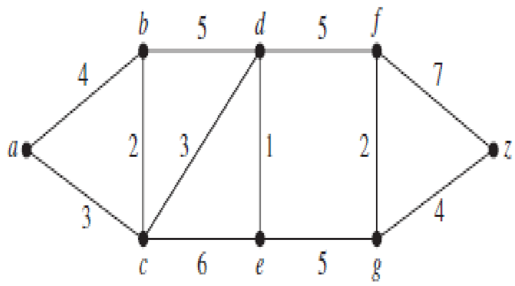
i)



N	D(b)	D (c)	D(d)	D(e)	D(f)	D(g)	D(h)	D(i)	D(j)	D(k)	D(l)	D(m)	D(n)	D(o)	D(p)	D(q)	D(r)	D(s)	D(t)	D(z)
a	2,a	4,a	1,a																	
ad	2,a	4,a			6,d	5,d														
adb		4,a		3,b	6,d	5,d														
adbe		4,a			6,d	5,d	6,e													
adbec					6,c	5,d	6,e													
adbeg					6,c		6,e			7,g										
adbegf							6,e	8,f	10,f	7,g										
adbegfh								8,f	10,f	7,g	7,h			14,h						
adbegfhk								8,f	10,f		7,h		11,k	14,h			9,k			
adbegfhkl								8,f	10,f			10,l	11,k	13,l			9,k			
adbegfhkli									10,f			10,l	11,k	13,l			9,k			
adbegfhklir									10,f			10,l	10,r	13,l		17,r			14,r	
adbegfhklirj												10,l	10,r	13,l		17,r			14,r	
adbegfhklirjm													10,r	13,l	12,m	17,r			14,r	
adbegfhklirjmn														13,l	12,m	12,n			14,r	
adbegfhklirjmnp														13,l		12,n		14,p	13,p	
adbegfhklirjmnpq														13,l				14,p	13,p	
adbegfhklirjmnpqo																		14,p	13,p	
adbegfhklirjmnpqot																		14,p		21,t
adbegfhklirjmnpqots																				16,s
adbegfhklirjmnpqotsz																				

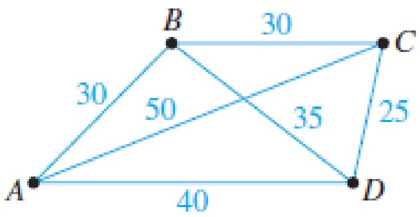
ii)

N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(z)
a	*4,a*	*3,a*	∞	∞	∞	∞	∞
ac			*6,c*	*9,c*	∞	∞	∞
acb			*6,c*	*9,c*	∞	∞	∞
acbd				*7,d*	*11,d*	∞	∞
acbde					*11,d*	*12,e*	∞
acbdef						*12,e*	*18,f*
acbdefg							*16,g*
acbdefgz	*4,a*	*3,a*	*6,c*	*7,d*	*11,d*	*12,e*	*16,g*

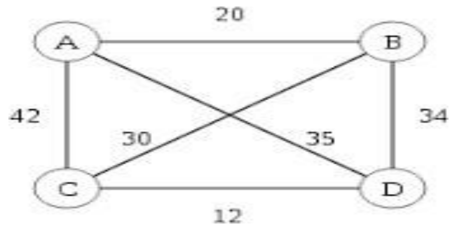


11. Imagine that the drawing below is a map showing four cities and the distances in kilometers between them. Suppose that a salesman must travel to each city exactly once, starting and ending in city A. Which route from city to city will minimize the total distance that must be traveled?

i)



ii)



i) Solution:

Hamiltonian Circuit are: ABCDA = 125; ABDCA = 140; ACBDA = 155.

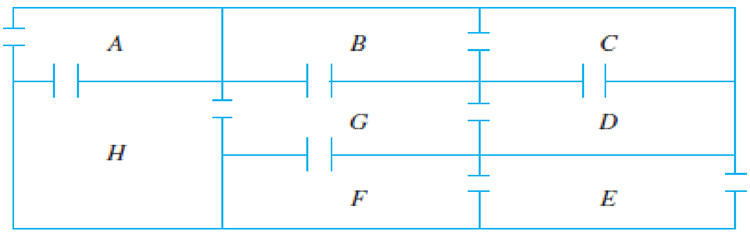
Hence ABCDA = 125 is the minimum distance travelled.

ii) Solution:

Hamiltonian Circuit are: ABCDA = 97; ABDCA = 108; ACBDA = 141.

Hence ABCDA = 97 is the minimum distance travelled.

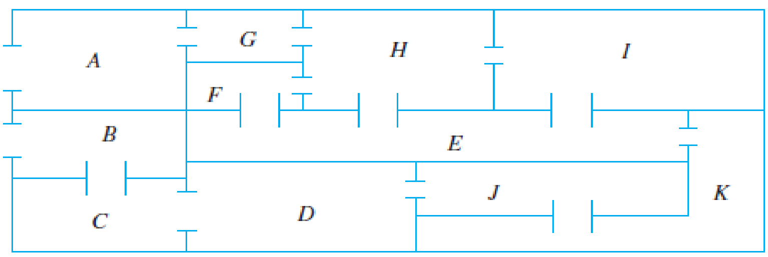
12. (a) The following is a floor plan of a house. Is it possible to enter the house in room A, travel through every interior doorway of the house exactly once, and exit out of room E? If so, how can this be done?



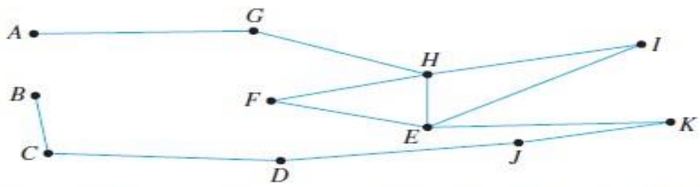
Solution:

Yes! Path: A →H→G→B→C→D→G→F→E

(b) The floor plan shown below is for a house that is open for public viewing. Is it possible to find a trail that starts in room A, ends in room B, and passes through every interior doorway of the house exactly once? If so, find such a trail.



Solution Let the floor plan of the house be represented by the graph below.

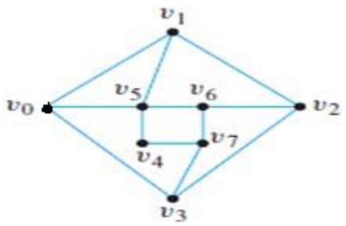


Each vertex of this graph has even degree except for A and B, each of which has degree 1. Hence by Corollary 10.2.5, there is an Euler path from A to B. One such trail is

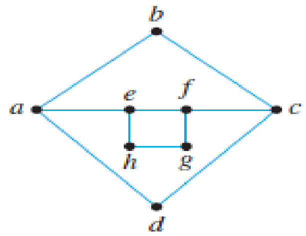
$$AGHFEIHEKJDCB.$$

13. **Find Hamiltonian circuits AND Path for those graphs that have them. Explain why the other graphs do not.**

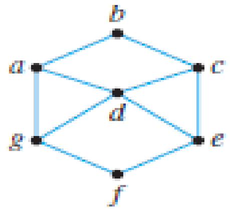
i)



ii)



iii)



i) Solution:

Hamiltonian Circuit: $V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3, V_0$

Hamiltonian Path: $V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3$

ii) Solution:

Hamiltonian Circuit: doesn't exist

Hamiltonian Path: b, c, f, g, h, e, a, d

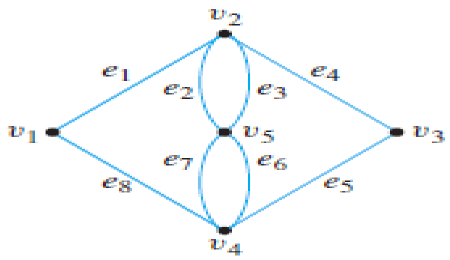
iii) Solution:

Hamiltonian Circuit: d, c, b, a, g, f, e, d

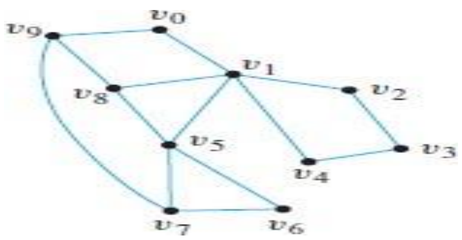
Hamiltonian Path: d, c, b, a, g, f, e

14. **a) Determine which of the graphs have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.**

i)



ii)



i) Solution: All vertices have even degree so circuit exists.

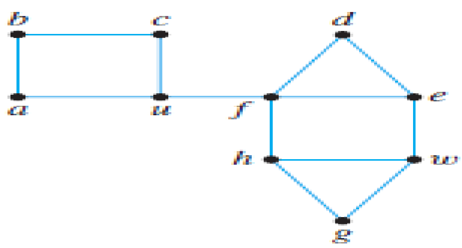
Euler Circuit: $V_1, V_2, V_5, V_4, V_5, V_2, V_3, V_4, V_1$

ii) Solution:

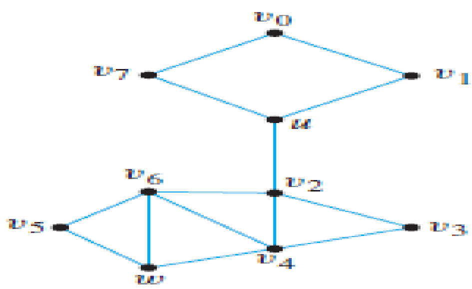
Euler Circuit do not exist because all vertices don't have even degree.

b) Determine whether there is an Euler path from u to w. If the graph does not have an Euler path, explain why not. If it does have an Euler path, describe one.

i)



ii)



i) Solution:

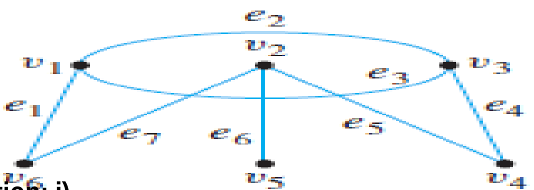
Euler Path doesn't exist because four vertices have odd degree.

ii) Solution: Euler Path exists because exact two vertices have odd degree.

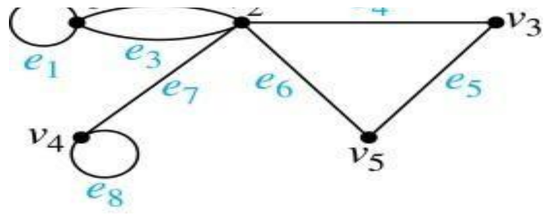
Euler path: $U, V_1, V_0, V_7, U, V_2, V_3, V_4, V_2, V_6, V_5, W, V_6, V_4, W$

15. (a) Use an incidence matrix to represent the graph shown below.

i)



ii)



Solution: i)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

ii)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(b) Draw a graph using below given incidence matrix.

i)

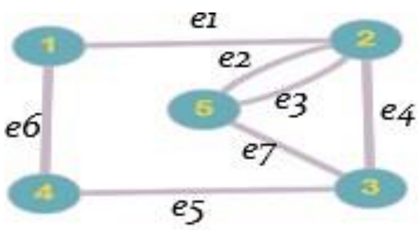
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

ii)

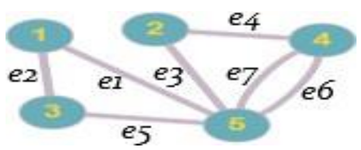
$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Solution:

i)

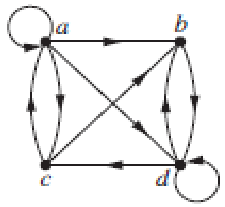


ii)



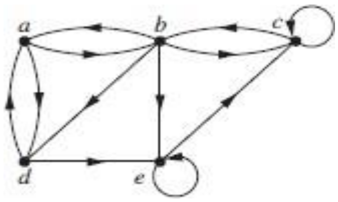
16. Use an adjacency list and adjacency matrix to represent the given graph.

i)



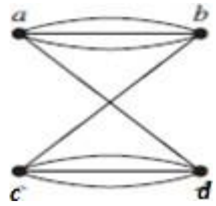
Initial Vertex	Terminal Vertices
a	a, b, c, d
b	d
c	a, b
d	b, c, d

(ii)



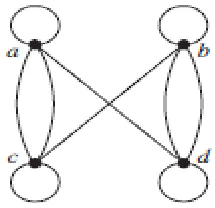
Initial Vertex	Terminal Vertices
a	b, d
b	a, c, d, e
c	b, c,
d	a, e
e	c, e

iii)



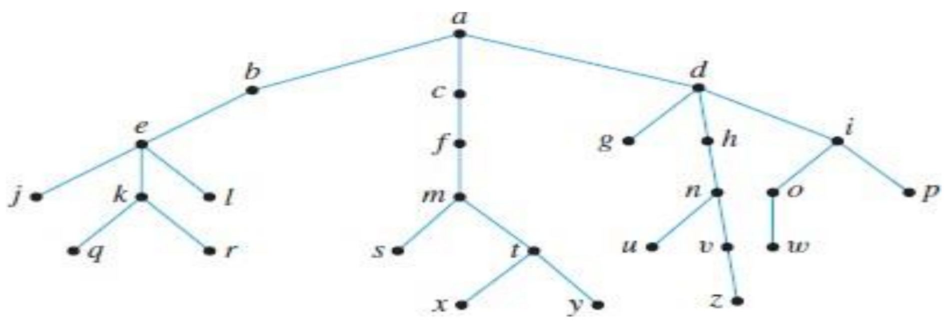
Vertex	Adjacent Vertices
a	b, d
b	a, c
c	b, d
d	a, c

iv)



Vertex	Adjacent Vertices
a	a, c, d
b	b, c, d
c	a, b, c
d	a, c, d

17. Consider the tree shown at right with root a.



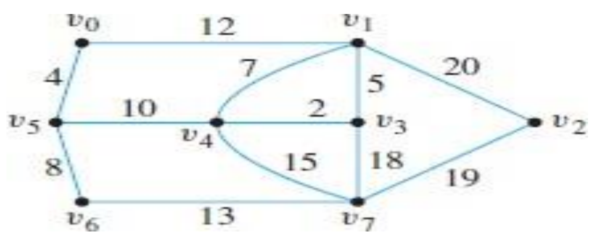
Solution:

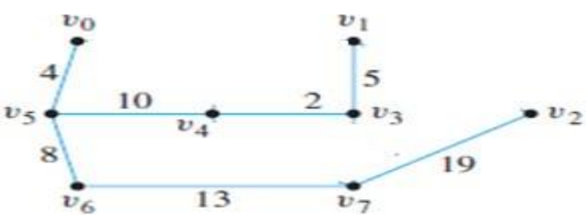
i) What is the level of n?	Level of n is 3.
ii) What is the level of a?	Level of a is 0.
iii) What is the height of this rooted tree?	Height of this rooted tree is 5
iv) What are the children of n?	u & v are the children of n.
v) What is the parent of g?	d is the parent of g.
vi) What are the siblings of j?	k & l are the siblings of j.
vii) What are the descendants of f?	m, s, t, x & y are the descendants of f.
viii) What are the internal nodes?	a, b, e, k, c, f, m, t, d, h, i, n, o & v are the internal nodes.
ix) What are the ancestors of z?	v, n, h, d & a are the ancestors of z.
x) What are the leaves?	j, l, q, r, s, x, y, g, p, u, w & z are the leaves.

18. Use Prim's algorithm to find a minimum spanning tree starting from V_0 for given graphs. Indicate the order in which edges are added to form each tree.

i) Solution: MST Cost = 61

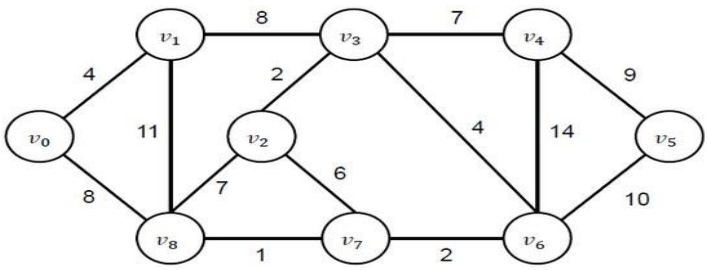
$(V_0,V_5) = 4$, $(V_5,V_6) = 8$, $(V_4,V_5) = 10$,
$(V_3,V_4) = 2$, $(V_1,V_3) = 5$, $(V_6,V_7) = 13$,
$(V_2,V_7) = 19$.

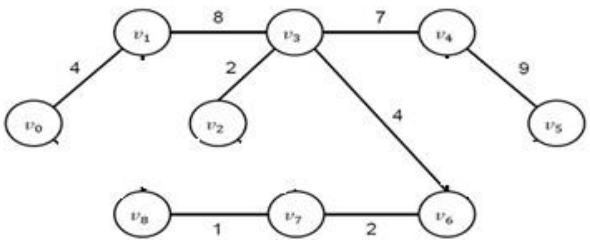


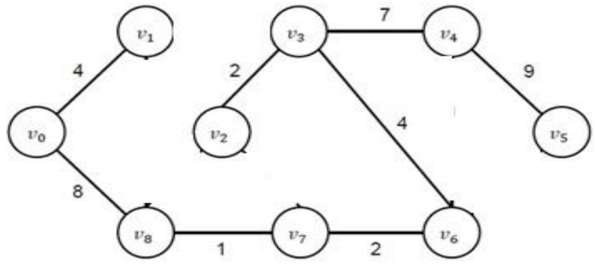


ii) Solution: MST Cost = 37

$(V_0,V_1) = 4$,	$(V_0,V_8) = 8$,	$(V_7,V_8) = 1$,
$(V_6,V_7) = 2$,	$(V_3,V_6) = 4$,	$(V_2,V_3) = 2$,
$(V_3,V_4) = 7$,	$(V_4,V_5) = 9$,	

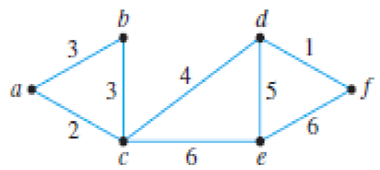




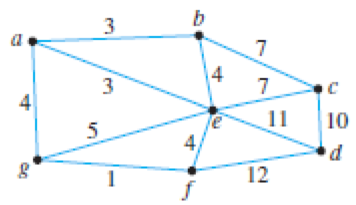


19. Use Kruskal's algorithm to find a minimum spanning tree for given graphs. Indicate the order in which edges are added to form each tree.

i)



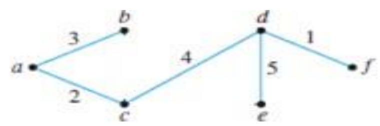
ii)



i) Solution: MST cost = 15

Order of edges added is:

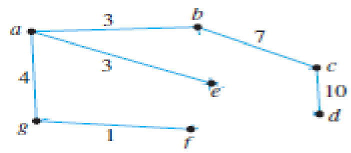
(d, f) =1,	(a, c) =2,	(a, b) =3,	~~(b, c) =3,~~
(c, d) =4,	(d, e) =5,	~~(c, e) =6,~~	~~(e, f) =6~~



ii) Solution: MST cost = 28

Order of edges added is:

(g, f) =1,	(a, b) =3,	(a, e) =3,	(a, g) =4,
~~(b, e) =4,~~	~~(e, f) =4,~~	~~(g, e) =5,~~	(b, c) =7
~~(c, e) =7,~~	(c, d) =10,	~~(d, e) =11,~~	~~(d, f) =12~~

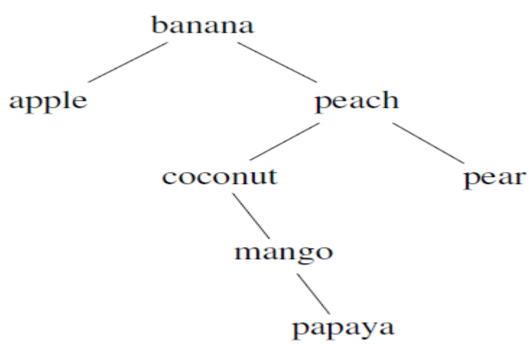


20. (a) i) Build a binary search tree for the word's banana, peach, apple, pear, coconut, mango, and papaya using alphabetical order.

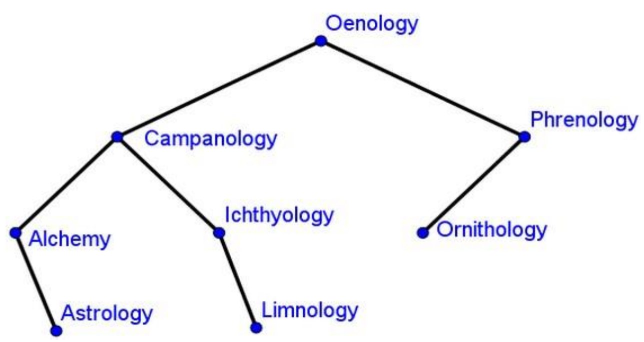
ii) Build a binary search tree for the word's oenology, phrenology, campanology, ornithology, ichthyology, limnology, alchemy, and astrology using alphabetical order.

Solution:

i)

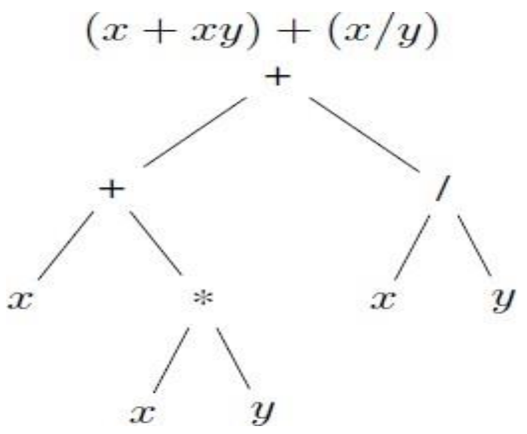


ii)

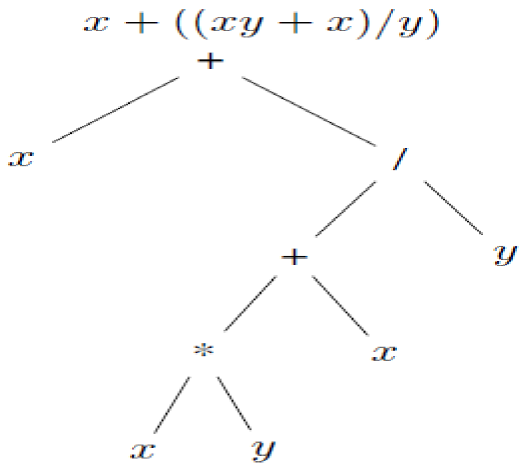


(b) Represent these expressions using binary trees.
(i) (x + xy) + (x / y)
(ii) x + ((xy + x) / y)
Solution:

i)

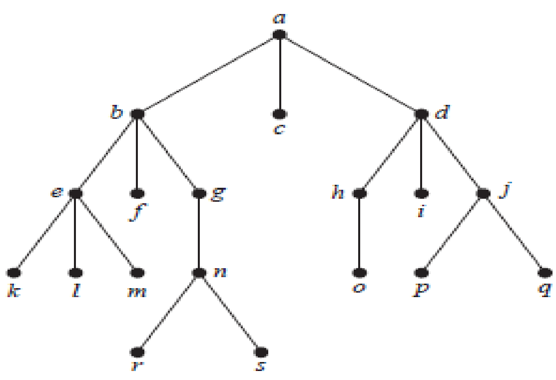


ii)

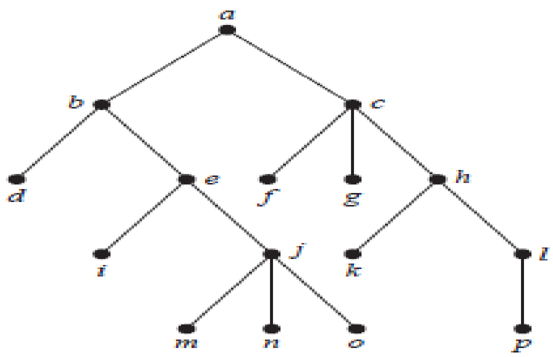


21. Determine the order in which preorder, Inorder and Postorder traversal visits the vertices of the given ordered rooted tree.

i)



ii)



Solution:

i)

Preorder: a b e k l m f g n r s c d h o I j p q
Inorder: k e l m b f r n s g a c o h d i p j q
Postorder: k l m e f r s n g b c o h I p q j d a

ii)

Preorder: a b d e i j m n o c f g h k l p
Inorder: d b i e m j n o a f c g k h p l
Postorder: d i m n o j e b f g k p l h c a

22. (a) How many edges does a tree with 10000 vertices have?
Solution:
A tree with n vertices has n – 1 edge. Hence 10000 – 1 = 9999 edges.
(b) How many edges does a full binary tree with 1000 internal vertices have?
Solution:
A full binary tree has two edges for each internal vertex. So, we'll just multiply the number of internal vertices by the number of edges. Hence 1000 * 2 = 2000 edges.

(c) **How many vertices does a full 5-ary tree with 100 internal vertices have?**

Solution:

A full $m-ary$ tree with I internal vertices has $n = mi + 1$ vertices.
From the given information, we have $m = 5$, $i = 100$
So $n = 5 \times 100 + 1 = 501$
Therefore a full 5-ary tree with 100 internal vertices has 501 vertices.

23. a) Write these expressions in Prefix and Postfix notation:

i) (x + xy) + (x / y)

Solution:

Prefix: + + x * x y / x y

Postfix: x x y * + x y / +

ii) x + ((xy + x) / y)

Solution:

Prefix: + x / + * x y x y

Postfix: x x y * x + y / +

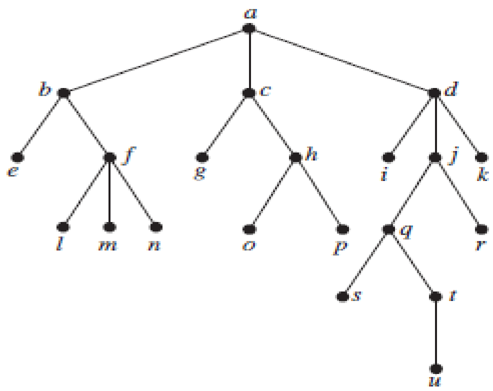
b) i) What is the value of this prefix expression + – ↑ 3 2 ↑ 2 3 / 6 – 4 2

Solution: 4

ii) What is the value of this postfix expression 4 8 + 6 5 – * 3 2 – 2 2 + * /

Solution: 3

24. Answer these questions about the rooted tree illustrated.



i) Is the rooted tree a full m-ary tree?

Solution: It is not a full m-ary tree for any m because some of its internal vertices have two children and others have three children.

ii) Is the rooted tree a balanced m-ary tree?

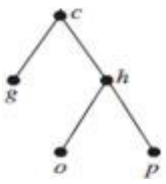
Solution: It is not balanced m-ary tree because it has leaves at levels 2, 3, 4 and 5.

iii) Draw the subtree of the tree that is rooted at

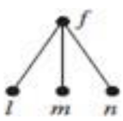
a) c. b) f. c) q.

Solution:

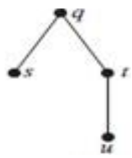
a)



b)

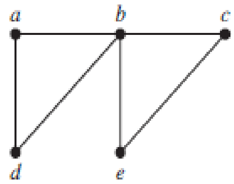


c)

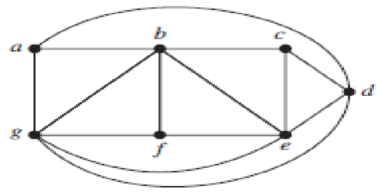


25. Find a spanning tree for the graph shown by removing edges in simple circuits. Write down the removed edges.

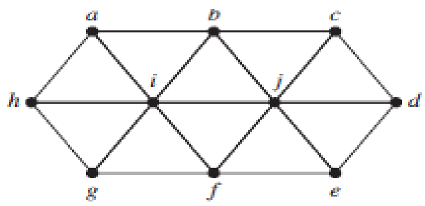
(i)



ii)

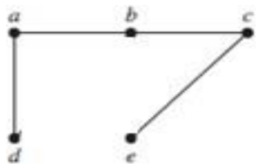


iii)

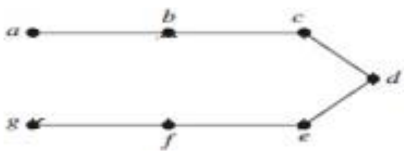


Solution:

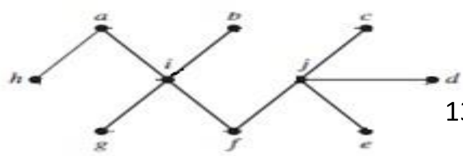
i)



ii)



iii)



26. (--------------------------)

27. Let R be the following relation defined on the set {a, b, c, d}:

R = {(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, b), (c, c), (d, b), (d, d)}

Determine whether R is:

(a) Reflexive: (b) Symmetric (c) Antisymmetric
(d) Transitive (e) Irreflexive (f) Asymmetric

Solution:

(a) R is reflexive because R contains (a, a), (b, b), (c, c), and (d, d).
(b) R is not symmetric because R contains (a, c) but not (c, a) ∈ R .
(c) R is not antisymmetric because both (b, c) ∈ R and (c, b) ∈ R , but b = c.
(d) R is not Transitive because both (a, c) ∈ R and (c, b) ∈ R , but not (a, b) ∈ R.
(e) R is not irreflexive because R contains (a, a), (b, b), (c, c), and (d, d).
(f) R is not Asymmetric because R is not Antisymmetric.

28. List the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3}, where (a, b) ∈ Rif and only if

a) a = b. b) a + b = 4. c) a > b.
d) a | b. e) gcd(a, b) = 1. f) lcm (a, b) = 2.

Solution:

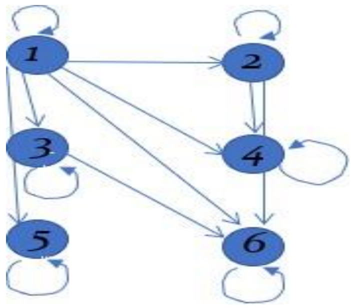
a) { (0,0), (1, 1), (2, 2), (3, 3) }
b) { (1, 3), (2, 2), (3, 1), (4, 0) }
c) { (1, 0), (2, 0), (3, 0), (4, 0), (2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3) }
d) { (1, 0), (2, 0), (3, 0), (4, 0), (1, 1), (1,2), (2,2), (1,3), (3,3) }
e) { (1,0), (0,1), (1,1), (1,2), (1,3), (2,1), (3,1), (4,1), (2,3),(3,2),(4,3) }
f) { (1,2), (2,1), (2,2) }

29. List all the ordered pairs in the relation R = {(a, b) | a divides b} on the set {1, 2, 3, 4, 5, 6}. Display this relation as Directed Graph(digraph), as well in matrix form.

mSolution:

R= { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6) }

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



30. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) { (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4) }

Solution:

a. R is not reflexive: It doesn't contain (1,1) and (4,4).
b. R is not symmetric because R contains (2, 4) but not (4, 2) ∈ R.
c. R is not antisymmetric: we have (2,3) and (3,2) but 2 ≠ 3.
d. R is Transitive because for any numbers a, b, and c, if (a, b), (b, c) ∈ R then (a, c) ∈ R.

b) { (1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4) }

Solution:

(a) R is reflexive: It contains (1,1) ,(2,2), (3,3) and (4,4).
(b) R is symmetric because (a,b) and (b,a) ∈ R.
(c) R is not antisymmetric: we have (1,2) and (2,1) but 1 ≠ 2.
(d) R is Transitive because for any numbers a, b, and c, if (a, b), (b, c) ∈ R then (a, c) ∈ R.

c) { (2, 4), (4, 2) }

Solution:

(a) R is not reflexive: It doesn't contain (1,1) ,(2,2), (3,3) and (4,4).
(b) R is symmetric because R contains (2, 4) and (4, 2) ∈ R.
(c) R is not antisymmetric: we have (2,4) and (4,2) but 2 ≠ 4.
(d) R is not Transitive because (2,4), (4, 2) ∈ R but not (2,2) ∈ R.

d) { (1, 2), (2, 3), (3, 4) }

Solution:

(a) R is not reflexive: It doesn't contain (1,1) ,(2,2), (3,3) and (4,4).
(b) R is not symmetric because (1,2) ∈ R but not (2,1) ∈ R.
(c) R is antisymmetric: we have (a,b) but not (b,a) ∈ R.
(d) R is not Transitive because (1,2), (2, 3) ∈ R but not (1,3) ∈ R.

e) {(1, 1), (2, 2), (3, 3), (4, 4)}

Solution:

(a) R is reflexive: It contains (1,1) ,(2,2), (3,3) and (4,4).
(b) R is symmetric because R contains (a,b) and (b,a) ∈ R.
(c) R is antisymmetric: we have (a,b) and (b,a) ∈ R then a = b.
(d) R is Transitive because for any numbers a, b, and c, if (a, b), (b, c) ∈ R then (a, c) ∈ R.

f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

Solution:

(a) R is not reflexive: It doesn't contain (1,1) ,(2,2), (3,3) and (4,4).
(b) R is not symmetric because (1,4) $\in$ R but not (4,1) $\in$ R.
(c) R is not antisymmetric: we have (1,3) and (3,1) $\in$ R but $1 \neq 3$.
(d) R is not Transitive because we have (1,3) and (3,1) $\in$ R but not (1,1) $\in$ R.

31. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, Asymmetric, irreflexive and/or transitive, where (a, b) $\in$ R if and only if:

a) a is taller than b.

Solution:

The relation R is **not reflexive**, because a person cannot be taller than himself/herself.

The relation R is **not symmetric**, because if person A is taller than person B, then person B is NOT taller than person A.

The relation R is **antisymmetric**, because $(a, b) \in R$ and $(b, a) \in R$ cannot occur at the same time (as one person is always taller than the other, but not the other way around).

The relation R is **transitive**, because if person A is taller than person B and if person B is taller than person C, then person A needs to be taller than person C as well.

b) a and b were born on the same day.

Solution:

The relation R is **reflexive**, because a person is born on the same day as himself/herself.

The relation R is **symmetric**, because if person A and person B are born on the same day, then person B is also born on the same day as person A.

The relation R is **not antisymmetric**, because if person A and person B are born on the same day and if person B and person A are born on the same day, then these two people are not necessarily the same person.

The relation R is **transitive**, because if person A and person B are born on the same day and if person B and person C are born on the same day, then person A and person C are also born on the same day.

c) a has the same first name as b.

Solution:

The relation R is **reflexive**, because a person has the same first name as himself/herself.

The relation R is **symmetric**, because if person A has the same first name as person B, then person B also has the same first name as person A.

The relation R is **not antisymmetric**, because if person A has the same first name as person B and if person B also has the same first name as person A, then these two people are not necessarily the same person (as there are different people with the same first name).

The relation R is **transitive**, because if person A has the same first name as person B and if person B also has the same first name as person C, then person A also has the same first name as person C.

d) a and b have a common grandparent.

Solution:

The relation R is **reflexive**, because a person has the same grandparents as himself/herself.

The relation R is **symmetric**, because if person A and person B have a common grandparent, then person B and person A also have a common grandparent.

The relation R is **not antisymmetric**, because if person A and person B have a common grandparent and if person B and person A have a common grandparent, then these two people are not necessarily the same person (as there are different people with the same grandparents).

The relation R is **not transitive**, because if person A and person B have a common grandparent and if person B and person C have a common grandparent, then person A and person C do not necessarily have a common grandparent (for example, the common grandparent of A and B can be from person B's father's side of the family, while the common grandparent of B and C can be from person B's mother's side of the family).

(a) Antisymmetric, Irreflexive, Asymmetric and Transitive
(b) Reflexive, Symmetric and Transitive
(c) Reflexive, Symmetric and Transitive
(d) Reflexive and Symmetric

32. Give an example of a relation on a set that is
a) both symmetric and antisymmetric.
Solution:
{ (1,1), (2,2), (3,3), (4,4) }

b) neither symmetric nor antisymmetric.
Solution:
{ (1,2), (2,1), (3,4) }

33. Consider these relations on the set of real numbers:
A={1,2,3}
R1 = {(a, b) ∈ R | a > b}, the "greater than" relation,
R2 = {(a, b) ∈ R | a ≥ b}, the "greater than or equal to "relation,
R3 = {(a, b) ∈ R | a < b}, the "less than" relation,
R4 = {(a, b) ∈ R | a ≤ b}, the "less than or equal to "relation,
R5 = {(a, b) ∈ R | a = b}, the "equal to" relation,
R6 = {(a, b) ∈ R | a ≠ b}, the "unequal to" relation.

Find:

a) R2 ∪ R4.	**b) R3 ∪ R6.**	**c) R3 ∩ R6.**	**d) R4 ∩ R6.**
e) R3 − R6.	**f) R6 − R3.**	**g) R2 ⊕ R6.**	**h) R3 ⊕ R5.**
i) R2 ◦ R1.	**j) R6 ◦ R6.**		

Solution:

R1= { (2,1), (3,1), (3,2) }	**R2= { (1,1), (2,2), (3,3), (2,1), (3,1), (3,2) }**
R3= { (1,2), (1,3), (2,3) }	**R4= { (1,1), (2,2), (3,3), (1,2), (1,3), (2,3) }**
R5= { (1,1), (2,2), (3,3) }	**R6= {(1,2), (1,3), (2,1), (2,3), (3,1), (3,2) }**

a) R2 ∪ R4 = { (1,1), (2,2), (3,3), (2,1), (3,1), (3,2) (1,2), (1,3), (2,3) }
b) R3 ∪ R6= {(1,2), (1,3), (2,1), (2,3), (3,1), (3,2) }
c) R3 ∩ R6 = { (1,2), (1,3), (2,3) }
d) R4 ∩ R6 = { (1,2), (1,3), (2,3) }
e) R3 − R6 = { } OR Φ

f) R6 − R3 = { (2,1), (3,1), (3,2) }
g) R2 ⊕ R6 = { (1,1), (2,2), (3,3), (1,2), (1,3), (2,3) }
h) R3 ⊕ R5 = { (1,1), (2,2), (3,3), (1,2), (1,3), (2,3) }i) R2 ◦
R1 = { (2,1), (3,1), (3,2) }
j) R6 ◦ R6 = { (1,1), (2,2), (3,3), (2,1), (3,1), (3,2) (1,2), (1,3), (2,3) }

34. (a) Represent each of these relations on { 1 , 2 , 3 } with a matrix(with the elements of this set listed in increasing order).

i) { (1 , 1), (1 , 2), (1 , 3) }

Solution: $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

ii) { (1 , 2), (2 , 1), (2 , 2) ,(3 , 3) }

Solution: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iii) { (1 , 1), (1 , 2), (1 , 3) , (2 , 2), (2 , 3) ,(3 , 3) }

Solution: $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

iv) { (1 , 3), (3 , 1) }

Solution: $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) List the ordered pairs in the relations on { 1 , 2 , 3 } corresponding to these matrices(where rows and columns correspond to the integers listed in increasing order).

(i) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ Solution: R = { (1,1), (1,3), (2,2), (3,1), (3,3) }

(ii) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ Solution: R = { (1,2), (2,2), (3,2) }

(iii) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Solution: R = { (1,1), (1,2), (1,3), (2,1), (2,3), (3,1),(3,2), (3,3) }

35. (a) Suppose that *R* is the relation on the set of strings of English letters such that *aRb* if and only if *l*(*a*) = *l*(*b*), where *l*(*x*) is the length of the string *x*. Is *R* an equivalence relation?**Solution:**
Show that all of the properties of an equivalence relation hold.

- Reflexivity: Because l(a) = l(a), it follows that aRa for all strings a.
- Symmetry: Suppose that aRb. Since l(a) = l(b), l(b) = l(a) also holds and bRa.
- Transitivity: Suppose that aRb and bRc. Since l(a) = l(b),and l(b) = l(c), l(a) = l(a) also holds and aRc.

(b) Let *m* be an integer with *m* > 1. Show that the relation *R* = {(*a*,*b*) | *a* ≡ *b* (mod *m*)} is an equivalence relation on the set of integers.

Solution:

Recall that a ≡ b (mod m) if and only if m divides a − b.

- Reflexivity: a ≡ a (mod m) since a − a = 0 is divisible by m since 0 = 0 · m.
- Symmetry: Suppose that a ≡ b (mod m). Then a − b is divisible by m, and so a − b = km, where k is an integer. It follows that b − a = (− k) m, so b ≡ a (mod m).
- Transitivity: Suppose that a ≡ b (mod m) and b ≡ c (mod m). Then m divides both a − b and b − c. Hence, there are integers k and l with a − b = km and b − c = lm. We obtain by adding the equations: a − c = (a − b) + (b − c) = km + lm = (k + l) m. Therefore, a ≡ c (mod m).

36. (a) Find the first five terms of the sequence for each of the following general terms where n >0.

(i) $2^n - 1$

Solution:1,2,4,8,16 are the first five terms of the given sequence.

(ii) $10 - \frac{3}{2}n$

Solution:

$\frac{17}{2}, 7, \frac{11}{2}, 4, \frac{5}{2}$ are the first five terms.

(iii) $\frac{(-1)^n}{n^2}$

Solution:

$-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, -\frac{1}{25}$ are the first five terms.

(iv) $\frac{3n+4}{2n-1}$

Solution: 7, $\frac{10}{3}, \frac{13}{5}, \frac{16}{7}, \frac{19}{9}$ are the first five terms.

(b) Identify the following Sequence as Arithmetic or Geometric Sequence then find the indicated term.

(i) -15, -22, -29, -36, ...; 11th term.

Solution:

Here common difference (d) = −7

$$T_n = a + (n-1)d; \quad T_{11} = -15 + (11-1)(-7) = -85$$

(ii) a - 42b, a - 39b, a - 36b, a - 33b, ….; 15th term.

Solution:

Here common difference (d) = 3b

$$T_n = a + (n-1)d\ ; \quad T_{15} = a - 42b + (15-1)(3b) = a$$

(iii) 4, 3, $\frac{9}{4}$,...... ; 17th term

Solution:

Here common ratio (r) = $\frac{3}{4}$

$$T_n = ar^{n-1}; \quad T_{17} = 4\left(\frac{3}{4}\right)^{17-1} = \frac{3^{16}}{4^{15}}$$

(iv) 32, 16, 8, ….; 9th term

Solution:

Here common ratio (r) = $\frac{1}{2}$

$$T_n = ar^{n-1}; \quad T \quad \left(\frac{1}{2}\right)^{9-1} = \frac{1}{}$$

$n \quad {}_{17} = 32(\ ) \quad 8$

37. (a) Find the G.P in which:

(i) $T_3 = 10$ and $T_5 = 2\frac{1}{2}$

Solution:

Since $T_n = ar^{n-1}$

$T_3 = ar^2 = 10$ ----(i) $\quad T_5 = ar^4 = \frac{5}{2}$ -----(ii)

Now, dividing (ii) by dividing (i), we get r = $\pm\frac{1}{2}$ and putting it in (i) we get a = 40.

Now the required G.P is 40,20,10,5, $\frac{5}{2}$,... OR 40,$-$20,10, $-$5, $\frac{5}{2}$, ...

(ii) $T_5 = 8$ and $T_8 = -\frac{64}{27}$

Solution:

Since $T_n = ar^{n-1}$

$T_5 = ar^4 = 8$ ----(i)

$T_8 = ar^7 = -\frac{64}{27}$ -------(ii)

Now, dividing (ii) by dividing (i) we get r = $-\frac{2}{3}$ and putting it in (i) we get a = $\frac{81}{2}$.

Now the required G.P is $\frac{81}{2}$, $-27, 18, -12, 8$,...

(b) Find the A.P in which:

(i) $T_4 = 7$ and $T_{16} = 31$

Solution:

Since $T_n = a + (n-1)d$;

$T_4 = a + 3d = 7$......(i) $T_{16} = a + 15d = 31$......(ii)

Now subtracting (ii) from (i), we get d = 2 and putting it in (i) we get a = 1.

Now the required A.P is 1,3,5,7,9, 11,...

(ii) $T_5 = 86$ and $T_{10} = 146$

Solution:

Since $T_n = a + (n-1)d$;

$T_5 = a + 4d = 86$......(i) $T_{10} = a + 9d = 146$......(ii)

Now subtracting (ii) from (i), we get d = 12 and putting it in (i) we get a = 38.

Now the required A.P is 38,50,62,74,86,…

38. (a) How many numbers are there between 256 and 789 that are divisible by 7. Also find their sum.

Solution:

First, we find the A.P with the common difference (d)= 7259,266,273, 280, ,....... 784

Since $T_n = a + (n-1)d$;

$784 = 259 + (n-1)(7)$;

n = 76.

Now for Sum; $S_n = \frac{n}{2}[2a + (n-1)d]$;

$S_{76} = \frac{76}{2}[2(259) + (76-1)(7)]$ = 39,634.

(b) Find the sum to n terms of an A.P whose first term is $\frac{1}{n}$ and the last term is $\frac{n^2-n+1}{n}$.

Solution:

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$-------(i)

1st we have to find "d"

Now, $T_n = a + (n-1)d$

$$\frac{n^2 - n + 1}{n} = \frac{1}{n} + (n-1)d$$

Finally, d = 1. Hence putting it in we get,

$S_n = \frac{n^2-n+2}{2}$.

39. (a) Use summation notation to express the sum of the first 100 terms of the sequence {a_j }, where $a_j = \frac{1}{j}$ for j = 1, 2, 3....

Solution:

The lower limit for the index of summation is 1, and the upper limit is 100. We write this sum as $\sum_{j=1}^{100} \frac{1}{j}$.

(b) What is the value of:

(i) $\sum_{k=4}^{8} (-1)^k$.

Solution:

$= (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8$
$= 1 + (-1) + 1 + (-1) + 1 = 1.$

(ii) $\sum_{j=1}^{5} (j)^2$.

Solution:

$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2$
$= 1 + 4 + 9 + 16 + 25 = 55.$

40. Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = -2a_{n-1}$, $a_0 = -1$

Solution:

$$a_0 = -1$$
$$a_1 = -2a_0 = -2(-1) = 2$$
$$a_2 = -2a_1 = -2(2) = -4$$
$$a_3 = -2a_2 = -2(-4) = 8$$
$$a_4 = -2a_3 = -2(8) = -16$$
$$a_5 = -2a_4 = -2(-16) = 32$$

b) $a_n = a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = -1$

Solution:

$$a_0 = 2$$
$$a_1 = -1$$
$$a_2 = a_1 - a_0 = -1 - 2 = -3$$
$$a_3 = a_2 - a_1 = -3 - (-1) = -2$$
$$a_4 = a_3 - a_2 = -2 - (-3) = 1$$
$$a_5 = a_4 - a_3 = 1 - (-2) = 3$$

c) $a_n = 3a^2_{n-1}$, $a_0 = 1$

Solution:

$$a_0 = 1$$
$$a_1 = 3a_0^2 = 3(1^2) = 3$$
$$a_2 = 3a_1^2 = 3(3^2) = 3(9) = 27$$
$$a_3 = 3a_2^2 = 3(27^2) = 3(729) = 2187$$
$$a_4 = 3a_3^2 = 3(2187^2) = 3(4782969) = 14348907$$
$$a_5 = 3a_4^2 = 3(14348907^2) = 3(205891132094649) = 617673396283947$$

d) $a_n = na_{n-1} + a^2_{n-2}$, $a_0 = -1$, $a_1 = 0$

Solution:

$$a_0 = -1$$
$$a_1 = 0$$
$$a_2 = 2a_1 + a_0^2 = 2(0) + (-1)^2 = 0 + 1 = 1$$
$$a_3 = 3a_2 + a_1^2 = 3(1) + 0^2 = 3 + 0 = 3$$
$$a_4 = 4a_3 + a_2^2 = 4(3) + 1^2 = 12 + 1 = 13$$
$$a_5 = 5a_4 + a_3^2 = 5(13) + 3^2 = 65 + 9 = 74$$

41. As we have discussed, the practical application of all the topics in the class. Now you are required to submit at least two real world applications of the following topics.

(a) Propositional Logic

State Space Search:

State-space search is the issue of testing whether a state in a change framework is reachable from at least one starting states. Change frameworks in the most fundamental cases can be identified with diagrams, and the state-space search issue for this situation is the s-t-reachability issue in charts.

Old style propositional rationale has been proposed as one response for state-space look issues for amazingly gigantic charts, due to the possibility of addressing and pondering colossal amounts of states with (by and large little) recipes.

EXAMPLE:

A∧B represents the set {1100, 1101, 1110, 1111} and A∨B represents the set {0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111}.

Questions about the relations between sets represented as formulas can be reduced to the basic logical concepts we already know, namely logical consequence, satisfiability, and validity.

1. "Is φ satisfiable?" corresponds to "Is the set represented by φ non-empty?"

2. φ |= α corresponds to "Is the set represented by φ a subset of the set represented by α?".

3. "Is φ valid?" corresponds to "Is the set represented by φ the universal set?"

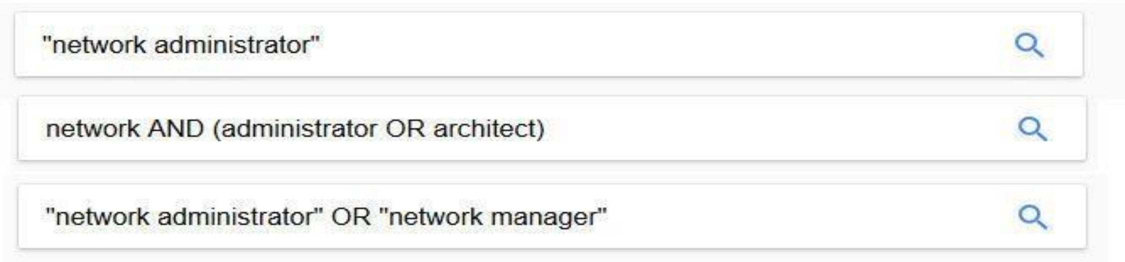
These connections allow using propositional formulas as a data structure in some applications in which conventional enumerative data structures for sets are not suitable because of the astronomic number of states. For example, if there are 100 state variables, then any formula consisting of just one atomic proposition represents a set of 2 99 = 633825300114114700748351602688 bit-vectors, which would require 7493989779944505344 TB in an explicit enumerative representation if each of the 100-bit vectors was represented with 13 bytes (wasting only 4 bits in the 13th byte.)

Boolean Searches:

Web indexes utilizing Boolean hunts utilize consistent connectives.

- AND requires records coordinate the two terms
- OR returns records that coordinate either of the terms
- NOT (or now and then AND NOT) rejects a term

Basic Boolean search commands (quotes, AND and OR) are supported in Google search, however Google defaults to AND searches automatically, so you don't need to enter AND into the search box. Google search uses additional symbols and words to refine searches such as "site:" to search a specific site or domain or use $ in front of a number to search for a price.



(b) Predicates and quantifiers

Man-Made Intelligence:

Man-made consciousness is worried about information portrayal and rationales. Data Representation is a sub zone of Artificial Intelligence stressed over getting, organizing, and executing techniques for addressing information in PCs, and to surmise new information reliant on the addressed information.
The predicate rationale is a piece of man-made brainpower which is relevant in the field of mechanical technology, medication and it is utilized in smart database so as to tackle some unpredictable issue.

EXAMPLE:

1. Mary loves everyone. [assuming D contains only humans]
   ∀x love (Mary, x)
   Note: No further parentheses are needed here, and according to the syntax on the handout, no further parentheses are possible. But “extra parentheses” are in general considered acceptable, and if you find them helpful, I have no objection. So I would also count as correct any of the following:
   ∀x (love (Mary, x)), (∀x love (Mary, x)), (∀x (love (Mary, x)))

Computer infers new conclusions in the same way using predicate logics and quantifiers.

Legitimate inferences:

Predicate Logic can be utilized to check legitimacy of a deduced conclusion. Using predicate logic, we can validate inferences.

Consider these statements, of which the first three are premises and the fourth is a valid conclusion.

“All hummingbirds are richly colored.”

"No large birds live on honey."

"Birds that do not live on honey are dull in color."

"Hummingbirds are small."

Let *P(x)*, *Q(x)*, *R(x)*, and *S(x)* be the statements "*x* is a hummingbird," "*x* is large," "*x* lives on honey," and "*x* is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and *P(x)*, *Q(x)*, *R(x)*, and *S(x)*.

Solution: We can express the statements in the argument as

$\forall x(P(x) \rightarrow S(x))$.

$\neg\exists x(Q(x) \wedge R(x))$.

$\forall x(\neg R(x) \rightarrow \neg S(x))$.

$\forall x(P(x) \rightarrow \neg Q(x))$.

(c) Sets

Clusters:

Clusters are likely the most well-known assortment type. A cluster stores an arranged assortment of qualities. As I referenced before, the qualities put away in an exhibit are of a similar sort. Sets and exhibits share a few highlights for all intents and purpose. The two of them store an assortment of estimations of a similar kind. You can include and evacuate components if the set or cluster is variable in this way exhibit is inferred for the idea of set wherein each position has interesting worth simply like sets.

EXAMPLES:

Char array [6] = {'a', 'b', 'c', 'd', 'e', 'f'}

In sets, this can be represented in the following way:

{(0,a), (1,b), (2,c), (3,d), (4,e), (5,f)}

SQL:

SQL is a domain-specific language used in programming and designed for managing data held in arelational database management system, or for stream processing in a relational data stream management system.

EXAMPLE:

1. **Union:**
   This set operator is used to combine the outputs of two or more queries into a single set of rows and columns having different records.
2. **Union All:**
   This set operator is used to join the outputs of two or more queries into a single set of rows and columns without the removal of any duplicates.
3. **Intersect:**
   This set operator is availed to retrieve the information which is common in both tables. The number of columns and data type must be same in intersect set operator.

(d) Functions

Functions in Physics:

Functions are frequently used in Physics and Mathematics.

EXAMPLE: You have given the velocity of rocket as 12000km/sec and the time required to reach the moon is 3 days. You have to compute the distance between Earth and Moon.

Formula: S(t) = V*t
After changing the given information into SI units:S(259200) =
12000000*259200
S(259200) = 3.1104e+12 meters

Functions in Programming:

Functions is the essential part of learning PC writing computer programs is tied in with taking a contribution from the client then in the wake of experiencing some work restoring a worth simply like taking an area and delivering a range.

We can make our own, “User Defined Function”.EXAMPLES:

An example of user defined function in c++ is:

```
void Print()
{
	string name;
	cin>>name;
	cout<<”Your Name is: ”<<name;
}
```