National University of Computer & Emerging Sciences, Karachi
Fall/Spring/Summer-2011 CS-Department
MidTerm 1
23rd February 2017, 10:30 am – 11:30 am

Course Code: CS211	**Course Name: Discrete Structures**
Instructor Name : Jalaluddin Qureshi	
Student Roll No:	**Group:**

Instructions:

- Return the question paper.Read each question completely before answering it. There are **5questions and 1 page.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Invigilators/ instructor can not assist you in understanding the question.
- All the answers must be solved according to the sequence given in the question paper.
- Marks will be awarded iff justifications has been provided.

Time: 60 minutes. **Max Marks**: 10mark/question x 5questions = 50 marks

Part A (Set Theory)

Question 1:

Using the following relationship: |AUB|=|A|+|B|- |A∩B| (1),
Show that the following can be derived:
|AUBUC|=|A|+|B|+|C|- |A∩B|- |A∩C|- |C∩B|+|A∩B∩C| (2).
Based on this exercise, and using Equation (1) and/or (2) derive the formula for |A U B U C U D|.

Solution:

Use substitution method, let, T=BUC, so we have,
|AUT|=|A|+|T|- |A∩T|
|AUBUC|=|A|+|BUC|- |A∩(BUC)|
Since |BUC|=|B|+|C|- |B∩C|,
|AUBUC|=|A|+|B|+|C|- |B∩C|- |A∩(BUC)|
|AUBUC|=|A|+|B|+|C|- |B∩C|- |(A∩B) U (A∩C)|
|AUBUC|=|A|+|B|+|C|- |B∩C|- (|(A∩B)|+|(A∩C)| - |A∩B∩C |)
|AUBUC|=|A|+|B|+|C|- |A∩B|- |A∩C|- |C∩B|+|A∩B∩C|

The same approach of substitution can be used to show that,
|A U B U C U D|= |A|+|B|+|C| +|D| - |A∩B|- |A∩C|- |C∩B| - |A∩D| - |B∩D| -
|C∩D|+|A∩B∩C|+|A∩B∩D|+|D∩B∩C|+|A∩C∩D| - |A∩B∩C∩D |

Question 2:

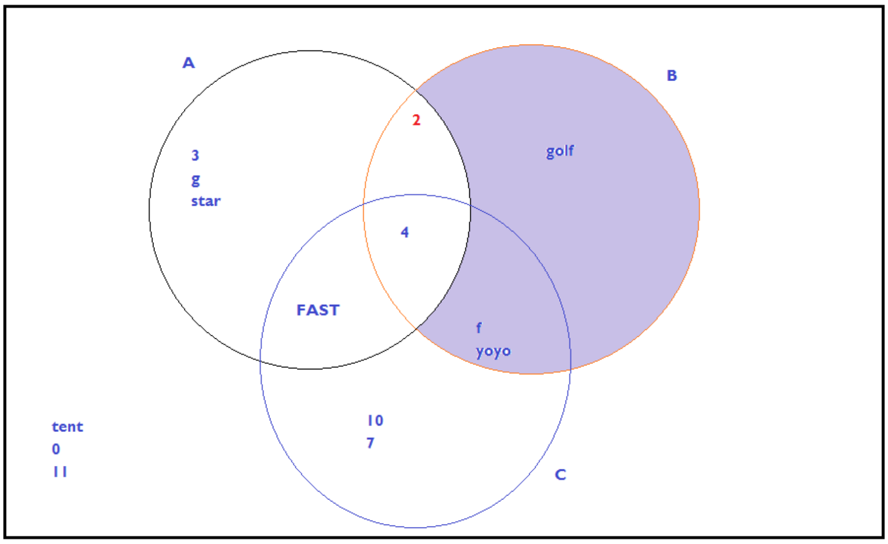
If A={2,3,4,g,FAST,star}, B={4,f,yoyo, 2, golf, 2}, C={yoyo, 10, 7, f, 4, FAST},
$\mathcal{U}$= A U B U C U {3, golf, 7, tent, 0, 11}, (Hint: $\mathcal{U}$ is the universal set). Find the following, and plot these on a Venn Diagram (by shading appropriate region).

(a) A^c∩B (b) ($\mathcal{U}$\B)∩ C^c
(c) (A \ $\mathcal{U}$) U (C\B) (d) $(A∩C)^c$ \ B

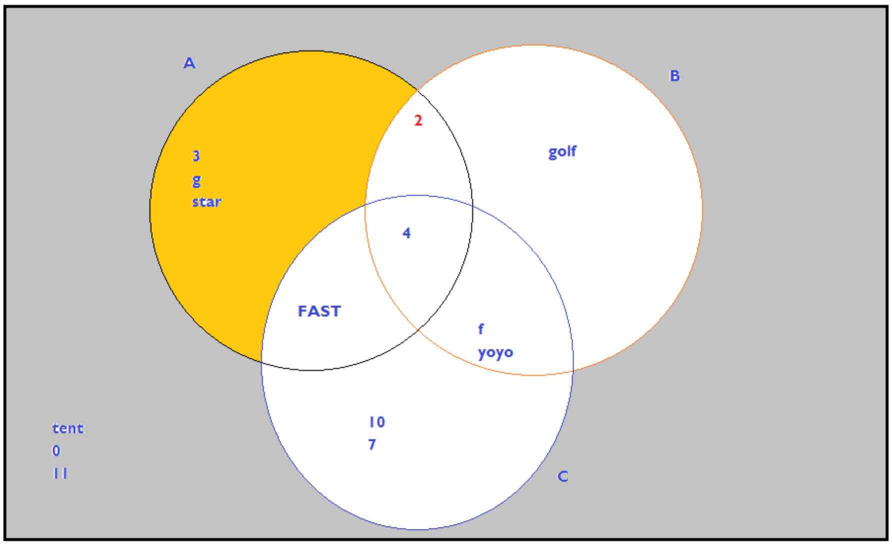
Solution:

The solution is given by the elements in the common region.

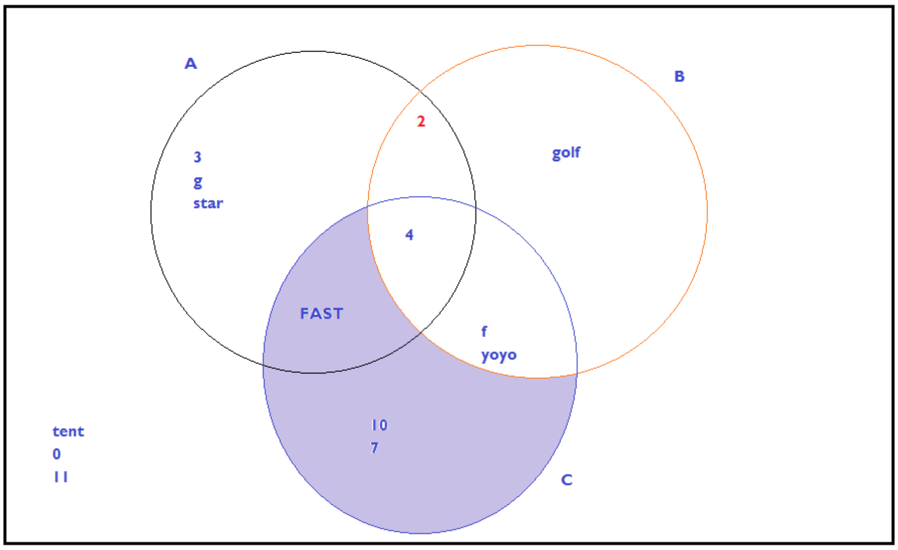
(a)


A
B
2
3
g
star
golf
4
FAST
f
yoyo
tent
0
11
10
7
C

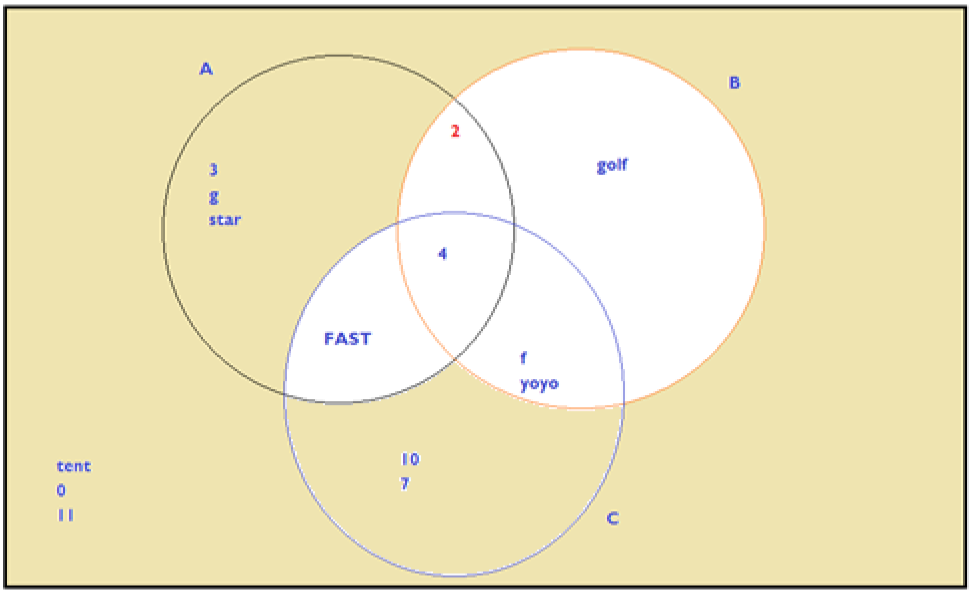
(b)


A
B
2
3
g
star
golf
4
FAST
f
yoyo
tent
0
11
10
7
C

(c)


A
B
2
3
g
star
golf
4
FAST
f
yoyo
tent
0
11
10
7
C

(d)



<u>Part B (Logic Theory)</u>

<u>Question 3:</u>

Determine (using appropriate technique) whether the following relationship is correct/ incorrect:

$P \rightarrow (Q \vee R) \equiv (P \rightarrow Q) \wedge (\neg P \rightarrow R)$.

<u>Solution:</u>

	P	¬P	Q	R	Q∨R	P→ (Q∨R)	M (P→Q)	N (¬P→R)	M∧N
1	T	F	T	T	T	T	T	T	T
2	T	F	T	F	T	T	T	T	T
3	T	F	F	F	F	F	F	T	F
4	T	F	F	T	T	T	F	T	F
5	F	T	T	F	T	T	T	F	F
6	F	T	F	T	T	T	T	T	T
7	F	T	F	F	F	T	T	F	F
8	F	T	T	T	T	T	T	T	T

As the two truth tables are not same, the equivalence relationship is false.

As there are two tables (with total 16 entries), at the discretion of the examiner, you will lose 1 marks for a wrong entry.

<u>Question 4:</u>

Determine whether the assignment c←c+1 (read as, c is assigned the value given by c plus one) will be executed by the if-statement, where x←5, y←3, z←7.

(a) If ¬{(x<y) ∧ (y≤z)} then c←c+1 (b) If ¬{(x=z) ∧ ((x≤z)} then c←c+1

(c) If ¬{(x≥y) ∨ (x<z)} then c←c+1 (d) If ¬{(x≤z) ∨ (y=z)} then c←c+1

<u>Solution:</u>

(a) ¬{(x<y) ∧ (y≤z)} then c←c+1 TRUE

(x<y) is false, (y≤z) is true, their conjunction is false, and negation of this is true.

(b) ¬{(x=z) ∧ ((x≤z)} then c←c+1 TRUE

(x=z) is false, (x≤z) is true, their conjunction is false, and negation of this is true.

(c) ¬{(x≥y) ∨ (x<z)} then c⇐c+1 FALSE

(x≥y) is true, (x<z) is also true, their disjunction is true, and its negation is false.

(d) ¬{(x≤z) ∨ (y=z)} then c⇐c+1 FALSE

(x≤z) is true, (y=z) is false, their disjunction is true, and its negation is false.

Question 5:

Using the algebra of Propositions, simplify the following:

(x∨¬y∨z) ∧ (¬x∨¬y∨z) ∨ (w∧¬x)

After the simplication, draw its truth table.

Solution

$(x \vee \bar{y} \vee z) \wedge (\bar{x} \vee \bar{y} \vee z) \vee (w \wedge \bar{x})$

Conjunction has the highest precedence so,

$= \{[x \wedge (\bar{x} \vee \bar{y} \vee z)] \vee [\bar{y} \wedge (\bar{x} \vee \bar{y} \vee z)] \vee [z \wedge (\bar{x} \vee \bar{y} \vee z)]\} \vee (w \wedge \bar{x})$

$= \{[\cancelto{F}{(x \wedge \bar{x})} \vee (x \wedge \bar{y}) \vee (x \wedge z)] \vee [(\bar{y} \wedge \bar{x}) \vee \cancelto{\bar{y}}{(\bar{y} \wedge \bar{y})} \vee (\bar{y} \wedge z)] \vee [(z \wedge \bar{x}) \vee (z \wedge \bar{y}) \vee \cancelto{F}{(z \wedge z)}]\} \vee (w \wedge \bar{x})$

$= \cancelto{\bar{y}}{(x \wedge \bar{y})} \vee (x \wedge z) \vee \cancelto{\bar{y}}{(\bar{y} \wedge \bar{x})} \vee \bar{y} \vee \cancelto{\bar{y}}{(\bar{y} \wedge z)} \vee (z \wedge \bar{x}) \vee (w \wedge \bar{x})$

$= \cancelto{z}{(x \wedge z)} \vee \bar{y} \vee \cancelto{z}{(z \wedge \bar{x})} \vee (w \wedge \bar{x}) = \bar{y} \vee z \vee (w \wedge \bar{x})$

It is possible to simplify it further, but the above solution is sufficient, given time limitation in exam.

Based on the above reduction, a truth table for the above can be easily drawn.