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Assignment # 2

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Ex 4.44:

f(x,y)		X				
		0	1	2	3	
Y	0	0	3/70	9/70	3/70	3/14
	1	1/35	9/35	9/35	1/35	4/7
	2	3/70	9/70	3/70	0	3/17
		1/14	3/7	3/7	1/14	1

To find: $\sigma_{xy}, \sigma_y, \sigma_{xy}, P_{xy}$

$$\sigma_x^2 = E(X)^2 - \mu_x^2$$
$$= \sum_{x=0}^{3} X^2 f(x) - \left(\sum_{x=0}^{3} x f(x)\right)^2$$
$$= \frac{39}{14} - \left(\frac{3}{2}\right)^2$$
$$= \frac{15}{28}$$
$$\boxed{\sigma_x = \sqrt{\frac{15}{28}}}$$

$$\sigma_y^2 = E(Y^2) - \mu_y^2$$
$$= \sum_{y=0}^{2} Y^2 f(y) - \left(\sum_{y=0}^{2} y f(y)\right)^2$$
$$= \frac{10}{7} - 1^2$$
$$\sigma_y^2 = \frac{3}{7} \Rightarrow \boxed{\sigma_y = \sqrt{\frac{3}{7}}}$$

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$\sigma_{xy} = E(XY) - \mu_x \mu_y$

$= \sum_x \sum_y (xy) f(x,y) - \sum_x x f(x) \sum_y y f(y)$

$= \frac{9}{7} - \left(\frac{3}{2}\right)(1)$

$\boxed{\sigma_{xy} = \frac{-3}{14}}$

$\rho_{xy} = \frac{-3/14}{(\sqrt{3/7})(\sqrt{15/28})}$

$\boxed{\rho_{xy} = -0.4472}$

Ex 4.45:

f(x,y)		X			
		1	2	3	
y	1	0.05	0.05	0.10	0.20
	3	0.05	0.10	0.35	0.50
	5	0.00	0.20	0.10	0.30
		0.10	0.35	0.55	1

$\sigma_x^2 = E(X^2) - (\mu_x)^2$

$= \sum_{x=1}^{3} x^2 f(x) - \left(\sum_{x=1} x f(x)\right)^2$

$= 6.45 - (2.45)^2$

$\sigma_x^2 = 0.4475$

$\boxed{\sigma_x = 0.6689}$

$$\sigma_y^2 = E(Y^2) - (\mu_y)^2$$
$$= \sum_{y}^{1,3,5} y^2 f(x) - \left(\sum_{y}^{1,3,5} y f(x)\right)^2$$
$$= 12.2 - (3.2)^2$$
$$\sigma_y^2 = 1.96$$
$$\boxed{\sigma_y = 1.4}$$

$$\sigma_{xy} = E(X.Y) - \mu_x \mu_y$$
$$= 7.85 - (3.2)(2.45)$$
$$\boxed{\sigma_{xy} = 0.01}$$

$$P_{xy} = \frac{0.01}{(1.4)(0.6689)}$$
$$\boxed{P_{xy} = 0.01067}$$

Ex 4.46:

$$f(x) = \begin{cases} \frac{3}{392} \times 10^{-4}(x^2+y^2) & ; 30 \leq x \leq 50, 30 \leq y \leq 50 \\ 0 & ; \text{else where} \end{cases}$$

$$g(x) = \frac{3 \times 10^{-4}}{392} \int_{30}^{50} (x^2+y^2)\, dy$$

$$= \frac{3}{392} \times 10^{-4} \left[x^2 y + \frac{y^3}{3} \right]_{30}^{50}$$

$$= \frac{3}{392} \times 10^{-4} \left[50x^2 + \frac{50^3}{3} - 30x^2 - \frac{30^3}{3} \right]$$

$$= \frac{3}{392} \times 10^{-4} \left[20x^2 + \frac{98000}{3} \right]$$

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$$\because \mu_x = K\int_{30}^{50} x\, g(x)\, dx$$

$$= \frac{3}{392}\times10^{-4}\left[\frac{20x^4}{4} + \frac{98000x^2}{6}\right]_{30}^{50}$$

$$= 40.8163$$

$$g(y) = \frac{3}{392}\times10^{-4}\int_{30}^{50}(x^2+y^2)\,dx$$

$$= \frac{3}{392}\times10^{-4}\left[\frac{x^3}{3} + y^2x\right]_{30}^{50}$$

$$= \frac{3}{392}\times10^{-4}\left[\frac{125000}{3} + 50y^2 - \frac{27000}{3} - 30y^2\right] = \frac{3}{392}\times10^{-4}\left[20y^2 - \frac{98000}{3}\right]$$

$$\because \mu_y = K\int_{30}^{50} y\, g(y)\, dy$$

$$\cancel{= \frac{3}{392}\times10^{-4}\int_{30}^{50}}$$

$$= \frac{3}{392}\times10^{-4}\left[\frac{20y^4}{2} + \frac{98000y^2}{6}\right]_{30}^{50}$$

$$\boxed{\mu_y = 40.8163}$$

$$\sigma_{xy} = \int_{30}^{50}\int_{30}^{50}(x-\mu_x)(y-\mu_y)f(x,y)\,dx\,dy$$

$$= \frac{3}{392}\times10^{-4}\int_{30}^{50}\int_{30}^{50}(xy - 40.8163y - 40.8163x + 1665.8)(x^2+y^2)\,dx\,dy$$

$$\boxed{\sigma_{xy} = -0.6642}$$

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Ex 4.47:

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 \leq y \leq 1, 0 \leq x \leq 1 \end{cases}$$

$$g(x) = \frac{2}{3}\int_0^1 (x+2y)\,dy$$

$$= \frac{2}{3}\left[xy + y^2\right]_0^1$$

$$\boxed{g(x) = \frac{2}{3}\left[x+1\right]}$$

$$g(y) = \frac{2}{3}\int_0^1 (x+2y)\,dx$$

$$g(y) = \frac{2}{3}\left[\frac{x^2}{2} + 2xy\right]_0^1$$

$$\boxed{g(y) = \frac{2}{3}\left[\frac{1}{2} + 2y\right]}$$

$$\mu_x = E(X) = \frac{2}{3}\int (x^2+x)\,dx$$

$$= \frac{2}{3}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$$

$$= \frac{2}{3}\left[\frac{1}{3} + \frac{1}{2}\right]$$

$$\boxed{E(X) = \frac{5}{9}}$$

(d)

$$\mu_y = E(Y) = \frac{2}{3}\int_0^1 \left(\frac{y}{2} + 2y^2\right) dy$$

$$= \frac{2}{3}\left[\frac{y^2}{2\cdot 2} + \frac{2y^3}{3}\right]_0^1$$

$$= \frac{2}{3}\left[\frac{1}{4} + \frac{2}{3}\right]$$

$$\boxed{E(Y) = \frac{11}{18}}$$

$$E(X^2) = \frac{2}{3}\int_0^1 (x^3 + x^2)\, dx$$

$$= \frac{2}{3}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1$$

$$= \frac{2}{3}\left[\frac{1}{4} + \frac{1}{3}\right]$$

$$\boxed{E(X^2) = \frac{7}{18}}$$

$$E(Y^2) = \frac{2}{3}\int_0^1 \left(\frac{y^2}{2} + 2y^3\right) dy$$

$$= \frac{2}{3}\left[\frac{y^3}{6} + \frac{2y^4}{4}\right]_0^1$$

$$= \frac{2}{3}\left[\frac{1}{6} + \frac{1}{2}\right]$$

$$\boxed{E(Y^2) = \frac{4}{9}}$$

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$\because \sigma_x^2 = E(x^2) - (\mu x)^2$

$= \frac{7}{18} - \left(\frac{5}{9}\right)^2$

$\sigma_x^2 = \frac{13}{162}$

$\boxed{\sigma_x = 0.283}$

$\sigma_y^2 = E(Y^2) - (\mu_y)^2$

$= \frac{4}{9} - \frac{11}{18}$

$\sigma_y^2 = \frac{23}{324}$

$\boxed{\sigma_y = 0.2664}$

$E(XY) = \frac{2}{3}\int_0^1\int_0^1 (xy)(x+2y)\,dx\,dy$

$= \frac{2}{3}\int_0^1\int_0^1 (x^2y + 2y^2x)\,dx\,dy$

$= \frac{2}{3}\int_0^1 \left[\frac{x^3y}{3} + 2\frac{y^2x^2}{2}\right]_0^1 dy$

$= \frac{2}{3}\int_0^1 \left(\frac{y}{3} + y^2\right) dy$

$= \frac{2}{3}\left[\frac{y^2}{6} + \frac{y^3}{3}\right]_0^1$

$= \frac{2}{3}\left[\frac{1}{6} + \frac{1}{3}\right]$

$\boxed{E(XY) = \cancel{\frac{1}{6}}\ \frac{1}{3}}$

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$\because \sigma_{xy} = \frac{1}{3} - \left(\frac{5}{9} \times \frac{11}{18}\right)$

$\boxed{\sigma_{xy} = \frac{-1}{162}}$

$\because P_{xy} = \frac{-1/162}{(0.2664)(0.283)}$

$\boxed{P_{xy} = -0.0818}$