Discrete Probability Distributions

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Discrete Probability Distributions

- Binomial distribution
- Multinomial distribution
- Hypergeometric distribution
- Poisson distribution
- Geometric distribution

The Bernoulli Process

1. The experiment consists of **repeated trials**.

2. Each trial results in an outcome that may be classified as a **success or failure**.

3. The probability of success, denoted by ***p*****, remains constant** from trial to trial.

4. The repeated trials are **independent**.

Binomial Distribution

- The number ***X*** of successes in ***n*** Bernoulli trials is called a **binomial random variable**.
- The probability distribution of this discrete random variable is called the **binomial distribution.**
- The probability of a success in a binomial experiment can be computed with this formula:

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \ldots, n.$$

Examples 01 – 03

- A coin is tossed 3 times. Find the probability of getting exactly two heads.
- The probability that a certain kind of component will survive a shock test is 3/4. Find the probability that exactly 2 of the next 4 components tested survive.
- The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that
- **(a)** at least 10 survive,
- **(b)** from 3 to 8 survive, and
- **(c)** exactly 5 survive?

Example # 04

- A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3% .
- **(a)** The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20?
- **(b)** Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?

Example # 05 – 07

- A coin is tossed 4 times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.
- Solve the above problem using the concept of expected values.
- A die is rolled 480 times. Find the mean, variance, and standard deviation of the number of 3s that will be rolled.

Multinomial Experiments

- Each trial in an experiment has more than two outcomes.
- the probabilities for each trial remain constant.
- the outcomes are independent for a fixed number of trials.
- Events must also be mutually exclusive.

Examples of Multinomial Experiments

- 52 playing cards.
- Rolling a dice.
- a survey might require the responses of “approve,” “disapprove,” or “no opinion.”

Multinomial Distribution

If X consists of events $E_1, E_2, E_3, \ldots, E_k$, which have corresponding probabilities $p_1, p_2, p_3, \ldots, p_k$ of occurring, and X_1 is the number of times E_1 will occur, X_2 is the number of times E_2 will occur, X_3 is the number of times E_3 will occur, etc., then the probability that X will occur is

$$P(X) = \frac{n!}{X_1! \cdot X_2! \cdot X_3! \cdots X_k!} \cdot p_1^{X_1} \cdot p_2^{X_2} \cdots p_k^{X_k}$$

where $X_1 + X_2 + X_3 + \cdots + X_k = n$ and $p_1 + p_2 + p_3 + \cdots + p_k = 1$.

Example # 08: Leisure Activity

- In a large city, 50% of the people choose a movie, 30% choose dinner and a play, and 20% choose shopping as a leisure activity. If a sample of 5 people is randomly selected, find the probability that 3 are planning to go to a movie, 1 to a play, and 1 to a shopping mall.

Example # 09: Coffee Shop Customers

- A small airport coffee shop manager found that the probabilities a customer buys 0, 1, 2, or 3 cups of coffee are 0.3, 0.5, 0.15, and 0.05, respectively. If 8 customers enter the shop, find the probability that 2 will purchase something other than coffee, 4 will purchase 1 cup of coffee, 1 will purchase 2 cups, and 1 will purchase 3 cups.

Example # 10: Arrival of delegation

The probabilities are 0.4, 0.2, 0.3, and 0.1, respectively, that a delegate to a certain convention arrived by air, bus, automobile, or train. What is the probability that among 9 delegates randomly selected at this convention, 3 arrived by air, 3 arrived by bus, 1 arrived by automobile, and 2 arrived by train?

Hypergeometric Experiment

- The result of each trial can be classified as Success or Failure.
- The probability of success changes on each trial.
- Successive trials are dependent.
- The experiment is repeated a fixed number of times.

Hypergeometric Distribution

- Given a population with only two types of objects (Success or failure), such that there are *a* items of one kind and *b* items of another kind and *a* *b* equals the total population, the probability *P*(*X*) of selecting without replacement a sample of size *n* with *X* items of type *a* and (*n* - *X)* items of type *b* is:

$$P(X) = \frac{{}_{a}C_{X} \cdot {}_{b}C_{n-X}}{{}_{a+b}C_{n}}$$

Example # 11 – 13

- **Assistant Manager Applicants:** Ten people apply for a job as assistant manager of a restaurant. Five have completed college and five have not. If the manager selects 3 applicants at random, find the probability that all 3 are college graduates.
- **House Insurance:** A recent study found that 2 out of every 10 houses in a neighborhood have no insurance. If 5 houses are selected from 10 houses, find the probability that exactly 1 will be uninsured.
- **Defective Compressor Tanks:** A lot of 12 compressor tanks is checked to see whether there are any defective tanks. Three tanks are checked for leaks. If 1 or more of the 3 is defective, the lot is rejected. Find the probability that the lot will be rejected if there are actually 3 defective tanks in the lot.

Geometric Experiment

- The outcomes of each experiment/trial can be classified into one of the two categories (Success or Failure).
- The probability of a success is the same for each experiment.
- Each experiment is independent of all the others.
- The experiment is repeated a variable number of times until the first success is obtained.

Geometric Distribution

If repeated independent trials can result in a success with probability p and a failure with probability $q = 1 - p$, then the probability distribution of the random variable X, the number of the trial on which the first success occurs, is

$$g(x; p) = pq^{x-1}, \quad x = 1, 2, 3, \ldots.$$

The mean and variance of a random variable following the geometric distribution are

$$\mu = \frac{1}{p} \text{ and } \sigma^2 = \frac{1-p}{p^2}.$$

Example 14 – 16

- **Manufacturing Process:** For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?
- **Driver's License:** The probability is 0.85 that an applicant for a driver's license will pass the road test on a given try. What is the probability that an applicant will finally pass the test on the 3^{rd} try.
- **Busy Time:** At a "busy time," a telephone exchange is very near capacity, so callers have difficulty placing their calls. It may be of interest to know the number of attempts necessary in order to make a connection. Suppose that we let p = 0.05 be the probability of a connection during a busy time. We are interested in knowing the probability that 5 attempts are necessary for a successful call.

Poisson Distribution

- Another important discrete distribution that is often used to model the frequency with which a specified event occurs during a particular period of time, is Poisson distribution.
- The Poisson probability formula is:

$$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!},$$

- Where "X" is the number of times the event occurs and λ is a parameter equal to the mean of X.

Examples # 17 – 19

- **Radioactive Particles:** During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?
- **Oil Tankers arrival:** Ten is the average number of oil tankers arriving each day at a certain port. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?
- **Typographical Errors:** If there are 200 typographical errors randomly distributed in a 500-page manuscript, find the probability that a given page contains exactly 3 errors.

Table A.2 (continued) Poisson Probability Sums $\sum_{x=0}^{r} p(x;\mu)$

	μ								
r	**10.0**	**11.0**	**12.0**	**13.0**	**14.0**	**15.0**	**16.0**	**17.0**	**18.0**
0	0.0000	0.0000	0.0000						
1	0.0005	0.0002	0.0001	0.0000	0.0000				
2	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000		
3	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000
4	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001
5	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003
6	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010
7	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029
8	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071
9	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154
10	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304
11	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549
12	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917
13	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426
14	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081
15	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867

Example # 20: Toll – Free Telephone Calls

- A sales firm receives, on average, 3 calls per hour on its toll-free number. For any given hour, find the probability that it will receive the following.
- *(a)* At most 3 calls
- *(b)* At least 3 calls
- (c) 5 or more calls

Example # 21: Left-Handed People

- The Poisson distribution can also be used to approximate the binomial distribution when the expected value $\lambda = np$ is less than 5 as shown in the example below:

- Approximately 2% of the people in a room of 200 people are left-handed, find the probability that exactly 5 people are left-handed.

Miscellaneous problems

- At university cafeteria, on the average 1.5 customers arrive per minute. Find the probabilities that:

- (a) at most 4 will arrive in a given minute.
- (b) at least 3 will arrive during an interval of 2 minutes.
- (c) exactly 15 will arrive during an interval of 6 minutes.

Miscellaneous problems (contd.)

5.3 An employee is selected from a staff of 10 to supervise a certain project by selecting a tag at random from a box containing 10 tags numbered from 1 to 10. Find the formula for the probability distribution of X representing the number on the tag that is drawn. What is the probability that the number drawn is less than 4?

5.12 A traffic control engineer reports that 75% of the vehicles passing through a checkpoint are from within the state. What is the probability that fewer than 4 of the next 9 vehicles are from out of state?

Miscellaneous problems (contd.)

The switch board of an office receives 0.9 calls per minute on the average. Find the probability that:

(a) in a given minute there will be at least one incoming call

(b) between 9:00 am and 9:02 am there will be exactly 2 incoming calls.

(c) during an interval of 4 minutes there will be at most 2 incoming calls.

Miscellaneous problems (contd.)

5.100 There are two vacancies in a certain university statistics department. Five individuals apply. Two have expertise in linear models, and one has expertise in applied probability. The search committee is instructed to choose the two applicants randomly.

(a) What is the probability that the two chosen are those with expertise in linear models?

(b) What is the probability that of the two chosen, one has expertise in linear models and one has expertise in applied probability?

Miscellaneous problems (contd.)

5.97 National security requires that defense technology be able to detect incoming projectiles or missiles. To make the defense system successful, multiple radar screens are required. Suppose that three independent screens are to be operated and the probability that any one screen will detect an incoming missile is 0.8. Obviously, if no screens detect an incoming projectile, the system is unworthy and must be improved.

(a) What is the probability that an incoming missile will not be detected by any of the three screens?

(b) What is the probability that the missile will be detected by only one screen?

(c) What is the probability that it will be detected by at least two out of three screens?

Miscellaneous problems (contd.)

- A coin is biased with probability of a head 2/3. Find the probability that a head appears on the 5^{th} trial.

End of Mid – II Syllabus