19k-0273 (BCS-5A)

Assignment#2 (1)

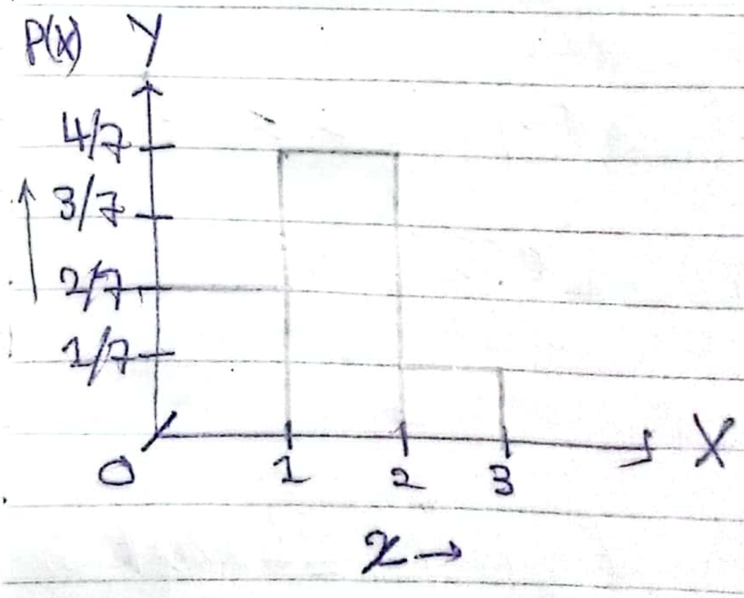
3.1) X: Discrete, Y: Continuous, M: Continuous [Pg#91]
N: Discrete, P: Discrete, Q: Continuous

3.11) X: no. of defective sets, total sets = 7, defective 2, non-defective = 7-2 = 5, $n(S) = \binom{7}{3}$,
random-size = 3

So,
$$P(X=x) = f(x) = \frac{\overset{D}{\binom{2}{x}}\overset{ND}{\binom{5}{3-x}}}{\binom{7}{3}} \quad ; x = 0,1,2$$

Hence,

x	0	1	2
$P(X=x)$	2/7	4/7	1/7



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3.9)

a, $P(0<X<1) = \int_0^1 \frac{2(x+2)}{5}\,dx = \left|\frac{(x+2)^2}{5}\right|_0^1 = 1$ Ans

b, $P(1/4<X<1/2) = \int_{1/4}^{1/2} \frac{2(x+2)}{5}\,dx = \left|\frac{(x+2)^2}{5}\right|_{1/4}^{1/2} = 19/80$ Ans

3.20) $F(x) = \frac{2}{27}\int_2^{\infty}(1+t)\,dt = \frac{2}{27}\left|\left(t+\frac{t^2}{2}\right)\right|_2^{\infty} = \frac{(x+4)(x-2)}{27}$ Ans

Now, $P(3<X<4) = F(4)-F(3) = \frac{(8)(2)}{27} - \frac{(7)(1)}{27} = 1/3$ Ans

3.13) C.D.F of X is,

$$F(x) = \begin{cases} 0 & , x<0 \\ 0.41 & , 0\leq x<1 \\ 0.78 & , 1\leq x<2 \\ 0.94 & , 2\leq x<3 \\ 0.99 & , 3\leq x<4 \\ 1 & , x\geq 4 \end{cases}$$ Ans

3.39) Given: [Pg#104]

X: no. of oranges, Y: no. of apples, random-size=4,
oranges=3, apples=2, bananas=3, total fruits=3+2+3=8
$n(S)=\binom{8}{4}$

a. $\therefore P(X=x, Y=y) = f(x,y) = \dfrac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}$ Ans

where, $x=0,1,2,3$; $y=0,1,2$; $1\leq x+y\leq 4$

b. $P(X+Y\leq 2) = f(0,1)+f(1,0)+f(1,1)+f(2,0)+f(0,2)$
$= \frac{1}{35}+\frac{3}{70}+\frac{9}{35}+\frac{9}{70}+\frac{3}{70}$
$=1/2$ Ans

3.43)
a. $P(0\leq X\leq 1/2, 1/4\leq Y\leq 1/2) = \int_0^{1/2}\int_{1/4}^{1/2} 4xy\,dy\,dx$
$= \frac{3}{8}\int_0^{1/2} x\,dx$
$=3/64$ Ans

b. $P(X<Y) = \int_0^1\int_0^y 4xy\,dx\,dy$
$=2\int_0^1 y^3\,dy$
$=1/2$ Ans

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3.57)

(a) $1 = k\int_0^2\int_0^1\int_0^1 xy^2z\,dx\,dy\,dz$

$= 2k\int_0^1\int_0^1 y^2z\,dy\,dz$

$= \frac{2k}{3}\int_0^1 z\,dz$

$= \frac{k}{3} z^2\Big|_0^1$

$\Rightarrow 1 = k/3 \Rightarrow \boxed{k=3}$ Ans

(b) $P(X<1/4, Y>1/2, 1<Z<2) = 2k\int_1^2\int_{1/2}^1\int_0^{1/4} xy^2z\,dx\,dy\,dz$

$= \frac{9}{2}\int_0^{1/4}\int_{1/2}^1 y^2z\,dy\,dz$

$= \frac{21}{16}\int_1^2 z\,dz$

$= 21/512$ Ans

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3.62)

a.

x	1	3	5	7
$f(x)$	0.4	0.2	0.2	0.2

b. $P(4<X\leq 7) = P(X\leq 7) - P(X\leq 4)$
$= F(7) - F(4)$
$= 1-0.6$
$= 0.4$ Ans

3.64)

a. $P(X\leq 1/2, Y\leq 1/2) = \frac{3}{2}\int_0^{1/2}\int_0^{1/2}(x^2+y^2)\,dy\,dx$

$= \frac{3}{2}\int_0^{1/2}\left(x^2y+\frac{y^3}{3}\right)\Big|_0^{1/2}dx$

$= \frac{3}{4}\int_0^{1/2}\left(x^2+\frac{1}{12}\right)dx$

$= 1/16$ Ans

b. $P(X\geq 3/4) = \frac{3}{2}\int_{3/4}^{1}\left(x^2+\frac{1}{3}\right)dx$

$= 53/128$ Ans

3.49) a.

x	1	2	3
$g(x)$	0.1	0.35	0.55

b.

y	1	2	3
$h(y)$	0.2	0.5	0.3

c. $P(Y=3|X=2) = 0.1/(0.05+0.1+0.2) = 0.2857$ Ans

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4.1) $\mu = E(X) = (0)(0.41) + (1)(0.37) + (2)(0.16) + (3)(0.05) + (4)(0.01)$

$= 0.88$ Ans

4.7) Expected Gain $= E(X) = (4000)(0.3) + (-1000)(0.7)$

$\Rightarrow E(X) = 500$ Ans

~~4.8)~~

4.10) $\because \mu_X = \Sigma x g(x) = (1)(0.17) + (2)(0.5) + (3)(0.33) = 2.16$

$\because \mu_Y = \Sigma y h(y) = (1)(0.23) + (2)(0.5) + (3)(0.27) = 2.04$

Ans

4.12) $E(X) = \int_0^1 2x(1-x)\,dx$

$= 1/3$

So,

$(1/3)(5000) = 1667.67$ dollars Ans

4.20) $E[g(X)] = E(e^{2X/3})$

$= \int_0^\infty e^{2x/3} \cdot e^{-x}\,dx$

$= \int_0^\infty e^{-x/3}\,dx$

$= 3$ Ans

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4.33) $\because \sigma^2 = E[(X-\mu)^2]$

$= \sum_x (x-\mu)^2 f(x)$ [here, $\mu = 500$]

$= (-1500)^2(0.7) + (3500)^2(0.3)$

$= 5250000$ Ans

4.34) $\because \mu = (-2)(0.3) + (3)(0.2) + (5)(0.5) = 2.5$

So, $E(X^2) = (-2)^2(0.3) + (3)^2(0.2) + (5)^2(0.5) = 15.5$

Hence, $\sigma^2 = E(X^2) - \mu^2 \Rightarrow \sigma^2 = 9.25 \Rightarrow \sigma = 3.041$

Ans

4.45) $\mu_X = \sum x g(x) = 2.45$, $\mu_Y = \sum y h(y) = 3.20$,

$\because E(XY) = \sum_x \sum_y xy f(x,y)$

$= \{(1)(0.05) + (2)(0.05) + (3)(0.1)\} + \{(2)(0.05) + (4)(0.1) + (6)(0.35)\} + \{(3)(0) + (6)(0.2) + (9)(0.1)\}$

$= 7.85$

So, $\sigma_{XY} = E(XY) - \mu_X\mu_Y$

$= 7.85 - (2.45)(3.20)$

$\Rightarrow \boxed{\sigma_{XY} = 0.01}$ Ans

4.58) $\because E(X) = \int_0^1 x^2 dx + \int_1^2 x(2-x) dx = 1$

Also, $E(X^2) = \int_0^1 x^3 dx + \int_1^2 x^2(2-x) dx = 7/6$

$\therefore E(Y) = 60E(X^2) + 39E(X)$
$= 60(7/6) + 39(1)$
$\Rightarrow \boxed{E(Y) = 109 \text{ kilowatt hrs.}}$ Ans

[Pg#131]

5.3) $\because$ uniform distribution: $f(x) = 1/10$; when $x = 1, 2, \ldots, 10$
$f(x) = 0$; elsewhere

$\therefore P(X<4) = \sum_{x=1}^{3} f(x) = 3/10$ Ans

5.12) From Table A.1:
$n = 9$, $p = 0.25$

So, $P(X<4) = 0.8343$ Ans

5.26) Given: $n = 8$, $p = 0.60$
a, $P(X=6) = \binom{8}{6}(0.6)^6(0.4)^2 = 0.209$ Ans

b, $P(X=6) = P(X \leq 6) - P(X \leq 5) = 0.8936 - 0.6846 = 0.209$
Ans

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5.30) $\therefore P(X \geq 1) = 1 - P(X=0)$

$= 1 - h(0; 15, 3, 6)$

$= 1 - \dfrac{\binom{6}{0}\binom{9}{3}}{\binom{15}{3}}$

$= 53/65$ Ans

5.33) By hypergeometric distribution;

a) $\dfrac{\binom{12}{2}\binom{40}{5}}{\binom{52}{7}} = 0.3246$ Ans

b) $1 - \dfrac{\binom{48}{7}}{\binom{52}{7}} = 0.4496$ Ans

5.34) $\therefore h(2; 9, 5, 4) = \dfrac{\binom{4}{2}\binom{5}{3}}{\binom{9}{5}} = 10/21$ Ans

[Pg#165]

5.50) a) By -ve binomial distribution,

$b^*(7; 3, 1/2) = \binom{6}{2}(1/2)^7 = 0.1172$ Ans

b) By geometric distribution:

$g(4; 1/2) = (1/2)(1/2)^3 = 1/16$ Ans

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5.55) By geometric dist:

(a) $P(X=3) = g(3;0.7) = (0.7)(0.3)^2 = 0.063$ Ans

(b) $P(X<4) = \sum_{x=1}^{3} g(x;0.7) = \sum_{x=1}^{3} (0.7)(0.3)^{x-1} = 0.973$ Ans

5.57)

(a) $P(X\geq 4) = 1-P(X\leq 3) = 0.1429$ Ans

(b) $P(X=0) = p(0;2) = 0.1353$ Ans

[Pg#185]

6.8)

(a) $z=(17-30)/6 = -2.17$, $\Rightarrow$ Area $= 1-0.015 = 0.985$ Ans

~~(b) $z=-0.76$, $k=(2.5)(-0.76)+18=16.1$ Ans~~

(c) $z_1 = (32-30)/6 = 0.33$, $z_2 = (41-30)/6 = 1.83 \Rightarrow$ Area $= 0.3371$ Ans

(d) $z=0.84$; $\therefore x = 30+(6)(0.84) = 35.04$ Ans

(e) $z_1 = -1.15$, $z_2 = 1.15 \Rightarrow x_1 = 30+(6)(-1.15) = 23.1$, $x_2 = 30+(6)(1.15) = 36.9$ Ans

(b) $z = (22-30)/6 = -1.33$, $\therefore$ Area $= 0.0918$ Ans

6.11)

(a) $z = (224-200)/15 = 1.6$, $\therefore P(Z>1.6) = 0.0548$ Ans

(b) $z_1 = (191-200)/15 = -0.6$, $z_2 = (209-200)/15 = 0.6$

$\therefore P(191<X<209) = P(-0.6<Z<0.6) = 0.7257-0.2743 = 0.4514$ Ans

(c) $z = (230-200)/15 = 2$; $P(X>230) = P(Z>2) = 0.0228$

$\therefore (1000)(0.0228) = 22.8 \approx 23$ cups will overflow. Ans

(d) $z = -0.67$, $x = (15)(-0.67)+200 = 189.95$ millimeters. Ans

6.14)

a) $Z = (10.075 - 10.0)/0.03 = 2.5$
$\therefore P(X > 10.075) = P(Z > 2.5) = 0.0062 = 62\%$ Ans

b) $Z_1 = (9.97 - 10)/0.03 = -1.0$, $Z_2 = (10.03 - 10)/0.03 = 1.0$
$\therefore P(9.97 < X < 10.03) = P(-1 < Z < 1) = 0.8413 - 0.1587 = 0.6826$ Ans

c) $Z = -1.04$, $x = 10 + (0.03)(-1.04) = 9.969$ cm Ans

6.15)

a) similar to previous ones

b) $Z = (15 - 24)/3.8 = -2.37$; $P(X > 15) = P(Z > -2.37) = 0.9911 = 99.11\%$ Ans

c) similar to "a"

d) $Z = 1.04$, $x = (3.8)(1.04) + 24 = 27.952$ mins. Ans

e) By binomial dist. with $p = 0.0571$, so:
$b(2; 3, 0.0571) = \binom{3}{2}(0.0571)^2(0.9429)$ Ans
$= 0.0092$

6.19) Given: $\mu = 15.9$, $\sigma = 1.5$

a) $\because P(13.75 < X < 16.22) = P\left(\frac{13.745 - 15.9}{1.5} < Z < \frac{16.22 - 15.9}{1.5}\right)$
$= P(-1.437 < Z < 0.217) = 0.5871 - 0.0749 = 0.5122 \approx 51\%$

b) $\because P(Z > 1.645) = 0.05$; $x = (1.645)(1.5) + 15.9 + 0.005 = 18.37$
Ans

6.21)

a) $Z = (10175 - 10000)/100 = 1.75$; $\therefore P(X > 10175) = P(Z > 1.75)$
$= 0.0401$

b) $Z_1 = (9775 - 10000)/100 = -2.25$, $Z_2 = (10225 - 10000)/100 = 2.25$
$\therefore P(X < 9775) + P(X > 10225) = P(Z < -2.25) + P(Z > 2.25)$
$= 2P(Z < -2.25)$
$= 0.0244$ Ans