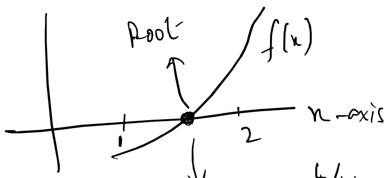
NA5

Friday, October 24, 2025 6:34 PM

Root finding → found because many equations cannot be solved analytically



b/w
$x \to [1, 2]$

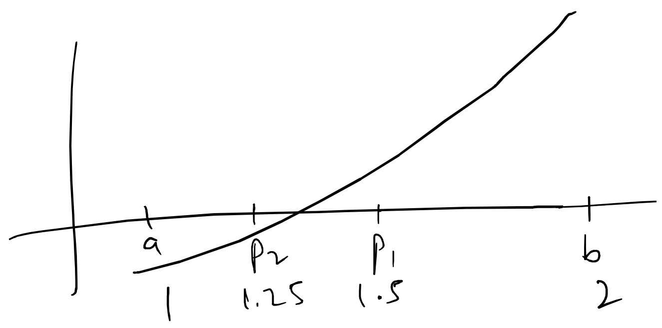
$f(x) = y \to 0$

Applications

1. Engineering
2. Physics
3. Optimization

① Bisection Method

- For finding approximate root value given intervals $[a, b]$.
- Keep halving until interval is small enough → close enough to root



$$\left.\begin{aligned} f(a) &= -ve \\ f(b) &= +ve \end{aligned}\right\} \text{Root exists in } [a, b]$$

$$\frac{a+b}{2} = p_1 \text{ (midpoint of } a, b)$$

find root is in $\overset{-ve \quad +ve}{[a, p_1]}$ or $\overset{+ve \quad +ve}{[p_1, b]}$

$f(p_1) = +ve$

$$\frac{a+p_1}{2} = p_2$$

Find root is in $\overset{-ve \quad -ve}{[a, p_2]}$ or $\overset{-ve \quad +ve}{[p_2, p_1]}$

$f(p_2) = -ve$

Root falls in b/w $[p_2, p_1]$
↓
$[1.25, 1.5]$

Q.) $f(x) = x^3 + 4x^2 - 10 = 0 \qquad [1, 2]$

$f(1) = -5 \quad (-ve)$

(+ve)

$f(1) = -5$ (-ve)

$f(2) = 14$ (+ve)

$\frac{1+2}{2} = 1.5 \longrightarrow p_1$

$f(1.5) = 2.375$ (+ve)

0.25

$\frac{1+1.5}{2} = 1.25 \longrightarrow p_2$

$f(1.25) = -1.796875$ (-ve)

0.125

$\frac{1.5+1.25}{2} = 1.375 \longrightarrow p_3$

until < 0.0001

p_n

$f(1.375) = 0.16211$ (+ve)

1×10^{-4}

$\downarrow$

$10^{-4} \longrightarrow 0.0001$

$= 0.0001$

② Fixed point iteration

- We want $f(x) = 0$
- $f(x) = x$
- $f(x) = x - g(x)$

$\cancel{x} - g(x) = x^3 + 4x^2 - 10$

$g_1(x) = x - [x^3 + 4x^2 - 10]$

$g(x) = x$

$x^3 + 4x^2 - 10 = 0$

$x^3 + 4x^2 - 10 = 0$

$x^3 + 4x^2 = 10$

$4x^2 = 10 - x^3$

$\quad) = 10$

$x + 4x$

$x(x^2 + 4x) = 10$

$x^2 + 4x = \frac{10}{x}$

$g_2(x) = x = \sqrt{\frac{10}{x} - 4x}$

$4x^2 = 10 - x^3$

$\sqrt{x^2} = \sqrt{\frac{10 - x^3}{4}}$

$g_3(x) = x = \frac{1}{2}\sqrt{10 - x^3}$

$x^3 + 4x^2 = 10$

$x^2(x + 4) = 10$

$x^2 = \frac{10}{x+4}$

$x = \sqrt{\frac{10}{x+4}} = g_4(x)$

$\sqrt[2]{\frac{10}{\cancel{1.5}+4}} = \boxed{1.348399725}$

$\downarrow$ ~~ANS~~ $p_1 p_2$ $\quad \downarrow p_1 \quad \downarrow$ ANS

$p_0 = 1.5$

$\sqrt[2]{\frac{10}{p_1 + 4}} = p_2$

$= p_3$

$= p_4$

$= p_5$

$x^2 = 2$

$g_1(x) = x = \frac{2}{x}$

$f(x) = x - g(x)$

$x^2 - 2 - x = -g(x)$

$g_2(x) = x + 2 - x^2$

$g(x) - 2 + x^2 = x$

$g_3(x) = \cancel{a}x - 2 + x^2$

(3) Newton's Method

- Based on Taylor Series of $f(x)$

$$f(x) = f(a) + f'(a)(x-a)^1 \ldots$$

Re-arrange & $x = p_n$ & $f(x) = 0$
$a = p_{n-1}$

$$* \quad p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

Q.) find root for $x^2 = 2$

$f(x) = x^2 - 2 = 0$

$f'(x) = 2x$

let $p_0 = 1$

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)}$$

$$p_1 = 1 - \frac{-1}{2} = \frac{3}{2} = 1.5$$

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)}$$

$$p_2 = 1.5 - \frac{0.25}{3} = 1.416667$$

$$p_3 = 1.416667 - \frac{1.416667^2 - 2}{2(1.416667)} = 1.414215$$

n	p_n
0	1
1	1.5
2	1.416667
3	1.414215

(4) Secant Method

- Like Newton's method, but doesn't need $f'(x)$

$$* \quad p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}.$$

Q.) $f(x) = x^2 - 2 = 0$

Q.1) $f(x) = x^2 - 2 = 0$

$p_0 = 1 \quad p_1 = 2$

$$p_2 = 2 - \frac{(2)(2-1)}{2-(-1)} = 2 - \frac{2}{3} = 1.333$$

$$p_3 = 1.333 - \frac{(-0.2222)(-0.6667)}{\cancel{0.222} - 2} = 1.400035$$

-0.2222

⋮

1.4142 ✓