NA11

Tuesday, December 2, 2025 7:07 PM

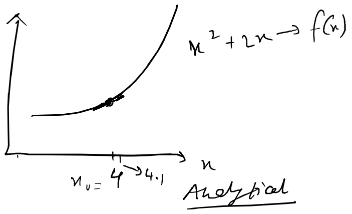
Numerical Differenciation

→ When used?

- Too complex to solve analytically
- Discrete data points given

Normal Differenciation

↳ finite differenciation



Gradient = $\frac{\text{rise}}{\text{run}}$

$m = \frac{y_1 - y_0}{x_1 - x_0}$

Forward Difference $h > 0$

- $f(x_0)' = \frac{f(x_0+h) - f(x_0)}{h}$

Numerical

$h = 0.1$

$f(4)' = \frac{f(4.1) - f(4)}{0.1} = \frac{25.01 - 24}{0.1}$

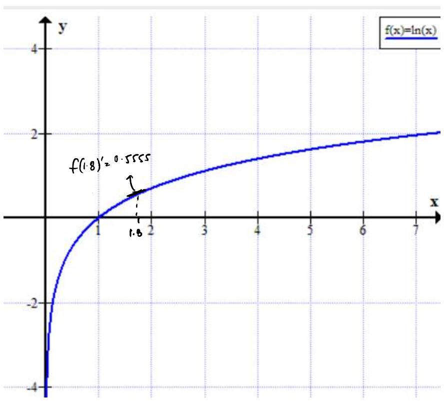
$= 10.1$

Analytical

$f(x)' = 2x + 2$

$f(4)' = 2(4) + 2$

$= 10$



① $f(x) = \ln x$ $x_0 = 1.8$

$h = 0.1$ $f(x_0)' = 0.5406722$

$h = 0.05$ $= 0.5479795$

$h = 0.01$ $= 0.5540180$

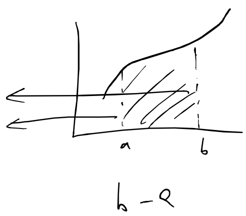
Actual → 0.5555

- Smaller values of h give more accurate result

Numerical Integration

↳ Approximates the definite integral

$\int_a^b f(x) \to F(b) - F(a)$



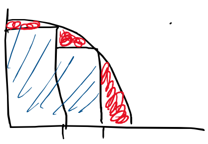
Estimation with finite sums

↳ rectangles

$n = 4$

- Greater number of sub-intervals

↳ lesser the error



Reimann Sum

- Approximates integral by dividing area under the curve into rectangles and summing the areas.

① Divide interval $[a, b]$ into equal subintervals

② Evaluate $f(x)$ at each sub-interval

③ – Width: $\Delta x_k = x_k - x_{k-1} = \frac{b-a}{n}$

② Evaluate f(x) at each sub-interval

③
- Width: $\Delta x_k = x_k - x_{k-1} = \frac{b-a}{n}$
- Height: $f(c_k)$
- Area: $f(c_k) \times \Delta x_k$

Total area of all n rectangles (Sum all rectangles)

$$\sum_{k=1}^{n} f(c_k)\Delta x_k$$

Trapezoidal Rule → Approximates using trapezium instead of rectangles

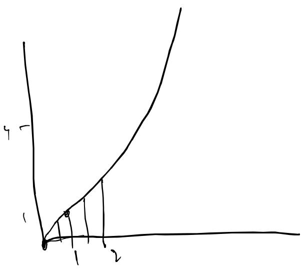
↳ More accurate than Reimann Sum

$$\int_a^b f(x)\,dx = \underbrace{\frac{h}{2}}_{\left(\frac{b-a}{n}\right)}[f(x_0) + f(x_1)]$$

Q.) $\int_0^2 x^2 \quad n=4$

$h = \frac{2-0}{4} = 0.5$

$\frac{0.5}{2}$



$\int_0^2 x^2\,dx$

$\left[\frac{x^3}{3}\right]_0^2 = \left|\frac{2^3}{3}\right| - \left|\frac{0^3}{3}\right|$

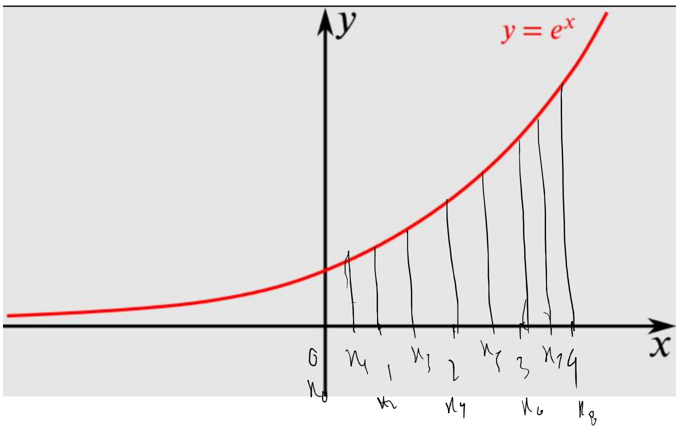
$= 2.6667$

Sum $\begin{cases} \frac{h}{2}(f(0)+f(0.5)) \\ \frac{h}{2}(f(0.5)+f(1)) \\ \frac{h}{2}(f(1)+f(1.5)) \\ \frac{h}{2}(f(1.5)+f(2)) \end{cases}$

Simpson Rule

↳ More accurate than trapezoidal rule

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] \rightarrow \text{if } n=2$$



$\int_a^b f(x)\,dx$

$= \frac{h}{3}\left[f(a) + 2\sum f(x_{\text{even middle terms}}) + 4\sum f(x_{\text{odd middle terms}}) + f(b)\right]$

$\int^4 e^x\,dx \qquad h = \frac{1}{2} \qquad h = \frac{b-a}{n}$

$0.5 = \frac{4-0}{n}$

$n = \frac{4}{0.5} = 8$

$\int_0 e^x \, dx \qquad h = \frac{1}{2} \qquad h = \frac{1}{n} \qquad n = \frac{4}{0.5} = 8$

$$\frac{0.5}{3}\left[e^0 + 2e^1 + 2e^2 + 2e^3 + 4e^{0.5} + 4e^{1.5} + 4e^{2.5} + 4e^{3.5} + e^4\right]$$

$\downarrow$

$1/6$

$= 53.6162$