Lecture 1

Friday, August 29, 2025 8:23 PM

Numerical Analysis Purpose
→ Used when solving math problems with $\boxed{\text{Analytical approach}}$ (→ Exact solution) is hard/not possible

Numerical Analysis → Approximate Solutions

$\sqrt{2} \approx 1.41421356.$ → Actual Answer → Need to make estimations near this

Goal → for iterative methods

* Previous iteration output will be input for next iteration

Problem : $x^2 - 2 = 0$

$x^2 = 2$

Analytical

$\sqrt{x \cdot x} = \sqrt{2}$

$x = \pm\sqrt{2}$

Numerical
→ Babylonian Method

$x^2 = 2$

$x \times x = 2$

$x + x = \frac{2}{x} + x$

$2x = \frac{2}{x} + x$

$x = \frac{x + \frac{2}{x}}{2}$

$x_0 = 1$ → Because 1 is perfect sq before 2

$x_1 = \frac{1 + \frac{2}{1}}{2} = \frac{3}{2} = 1.5$

$x_2 = \frac{1.5 + \frac{2}{1.5}}{2} = 1.4167$

$x_3 = \frac{1.4167 + \frac{2}{1.4167}}{2} = 1.41422$

$x_4 = \frac{1.41422 + \frac{2}{1.41422}}{2}$

$x_4 = 1.414215$

Limits

$\lim_{x \to c} f(x) = L$

e.g.; $\lim_{x \to 2} (3x+1)$

Normal Method → $x = 2$

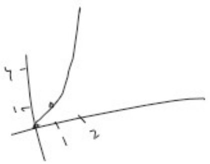
$3(2) + 1 = 7$

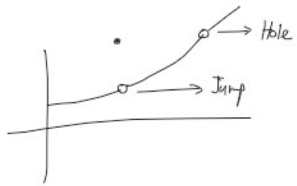
Numerical check →
$x = 1.99$ $\quad$ $x = 2.01$

$3(1.99) + 1$ $\quad$ $3(2.01) + 1$

Continuity → No jumps/holes in graph

e.g., $f(x) = x^2$ is continuous





$m = \frac{y_2 - y_1}{x_2 - x_1}$

Derivative

$f(x) = x^2 \quad f(x)' = 2x \quad f'(3) = 2(3) = 6$

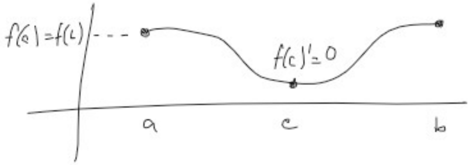
Rolle's Theorem

$f(x) = x^2 + 2$

If a function starts at $(a, f(a))$ and ends at $(b, f(b))$

Rolle's Theorem

If a function starts at $(a, f(a))$ and ends at $(b, f(b))$ such that $f(a) = f(b)$, then $f(c)' = 0$.



e.g; $f(x) = \cos x$

$(a, f(a)) \longrightarrow (x_1, y_1)$

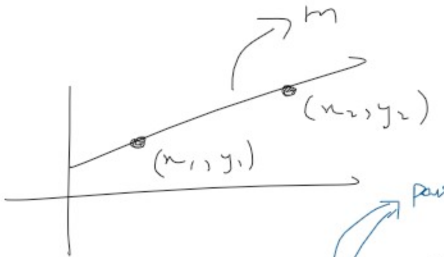
$(b, f(b)) \longrightarrow (x_2, y_2)$

Mean Value Theorem

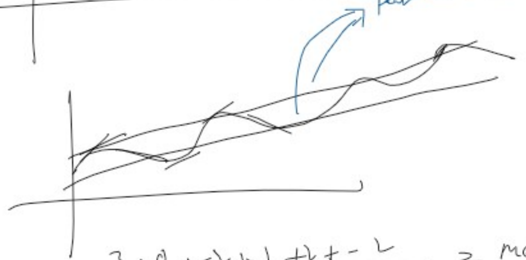
↳ Average Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$f(x)' = \frac{f(b) - f(a)}{b - a}$$



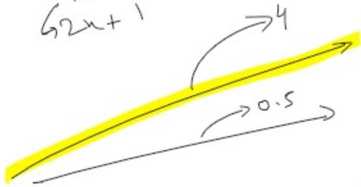
parallel lines → same gradient



eg; $\frac{3 + 0 + -2 + 1 + 2 + -2}{6}$ = mean gradient

↳ 2

forget
↳ $2x + 1$



forget
↳ $2x^2 + 1$

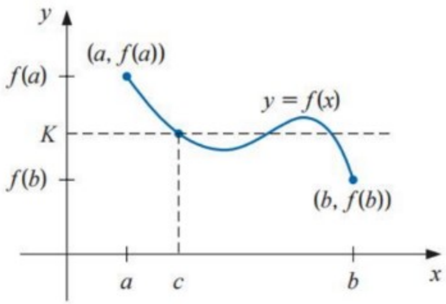
8, 0, 4, 0, 8, 4, 0, 4, 8

eg ↳ Avg = 4

Intermediate Value Theorem

Theorem

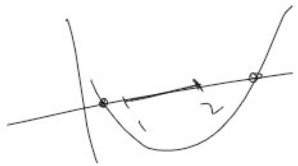
If $f \in C[a,b]$ and K is any number between $f(a)$ and $f(b)$, then there exists a number c in (a,b) for which $f(c) = K$. ■

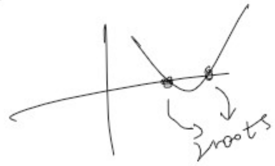


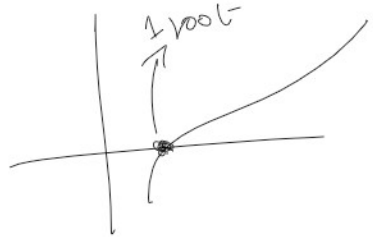
Show that the equation has a root on the given interval.

$2x^3 - 6x + 1 = 0$ on $(1, 2)$

* Signs should be different for root to exist ← y_1 y_2



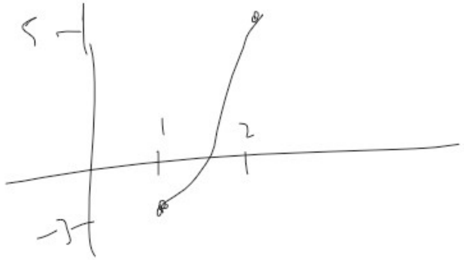




$f(1) = 2(1)^3 - 6(1) + 1 = -3$

$f(2) = 2(2)^3 - 6(2) + 1 = 5$

→ Signs different → root exists



Definite Integrals (limits given → find area under curve)

$y = 2x - 3$

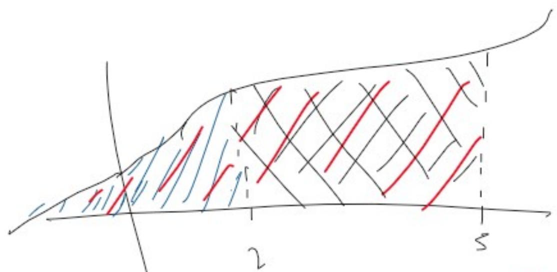
① Integrate

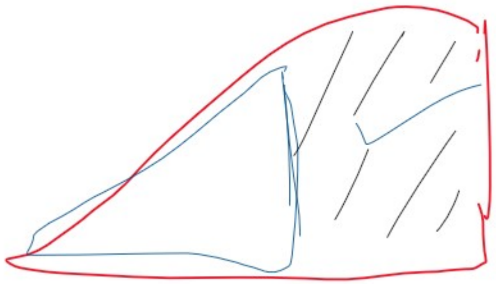
② Then Apply limits

$\int_2^5 2x - 3$

$\left| \frac{2x^2}{2} - 3x + C \right|_2^5$

$\int f(5) - \int f(2)$





$\int f(2)$

$\int f(5)$

$\int f(5) - \int f(2)$

$\int f(5) - \int f(2)$

$[(5)^2 - 3(5) + c] - [(2)^2 - 3(2) + c]$

$25 - 15 + \cancel{c} - 4 + 6 - \cancel{c} = 12$

$\int f(2)$ $\int f(5)$ $\int f(5) - \int f(2)$

Taylor Series $\rightarrow$ Approximates a function $f(x)$ near point $\underset{\substack{\uparrow \\ \text{on } x\text{-axis}}}{a}$ using its derivatives

$$f(x) = f(a) + f'(a)(x-a)^1 + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

Maclaurin Series (Special case)

$\hookrightarrow$ $a = 0 \rightarrow$ It becomes maclaurin series

$(x-a)^3 = (x-0)^3$
$= x^3$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

* Importance: Numerical methods rely on truncating these series for calculations.

$f(x) = e^x$ $\quad f'(x) = e^x \quad f''(x) = e^x \quad f'''(x) = e^x$

$x = 0$

$$f(x) = e^x = e^0 + e^0 \times x + \frac{e^0}{2!} \times x^2 + \frac{e^0}{3!} \times x^3 \dots$$

$e^0 = 1$

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$e^{0.1}$

Q.) Approximate $e^{0.1}$

$$f(0.1) = 1 + 0.1 + \frac{0.1^2}{2!} + \frac{0.1^3}{3!} = \text{Ans}$$