Notes for Qballs With Phaseshift

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1 ODE for Qballs

Q - balls are a class of spherically symmetric solutions in 3 + 1 dimensions arising in a family of field theories.

Here, we examine the modeling of a 2 + 1 dimension case of these such solutions. In previous results, we already know that the Q ball is governed by the following equation:

$$\ddot{\phi} - \nabla^2\phi + \phi - A|\phi|\,\phi + B\left|\phi^2\right|\phi = 0 \tag{1}$$

where $\phi = \phi(r, t)$, a radially symmetric solution that is varying with respect to time. Previously, work has been done to seek solutions in form of

$$\phi(r, t) = \sigma(r)e^{i\omega t} \tag{2}$$

and generate numerical approximations accordingly. We notice that we can add a phase shift factor, which motivates the following proposal:

$$\phi(r, t) = \sigma(r)e^{i(\omega t+k)} \tag{3}$$

where k is the phase shift. Then if we substitute equation (3) into equation (1), we still obtain the same equation governing $\sigma(r)$:

$$\nabla^2\sigma(r) + (\omega^2 - 1)\sigma(r) + A\sigma(r)^2 + B\sigma(r)^3 = 0 \tag{4}$$

This means that there are no change to the ordinary differential equations governing the Q balls with the additional phase change factor. We know that this equation is:

2 Boost

But normally, we are interested at Q balls moving at relativistic speeds, in which we need to consider the Lorentz Transformations. These are normally called boost.

We use

$$\phi'(t', x', y') = \sigma(x', y')e^{i(\omega t+k)} = \phi_1' + i\phi_2' \tag{5}$$

to denote the Q ball field in the rest frame. Notice then,

$$\phi_1'(t',x',y') = \sigma(x',y')\cos(\omega t' + k) \tag{6}$$

and

$$\phi_2'(t',x',y') = \sigma(x',y')\sin(\omega t' + k) \tag{7}$$

and the corresponding time derivatives π_1', π_2' of ϕ_1' and ϕ_2' correspondingly are

$$\pi_1'(t',x',y') = \frac{\partial \phi_1'}{\partial t'}\frac{\partial t'}{\partial t} + \frac{\partial \phi_1'}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial \phi_1'}{\partial y'}\frac{\partial y'}{\partial t} \tag{8}$$

$$\pi_2'(t',x',y') = \frac{\partial \phi_2'}{\partial t'}\frac{\partial t'}{\partial t} + \frac{\partial \phi_2'}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial \phi_2'}{\partial y'}\frac{\partial y'}{\partial t} \tag{9}$$

Now we apply the lorentz transformations $x = \gamma(x' - vt')$, $y' = y$, and $t = \gamma(t' - vx')$

$$\phi_1(t,x,y) = \sigma(\gamma(x+vt),y)\cos(w\gamma(t+vx)+k) \tag{10}$$

$$\phi_2(t,x,y) = \sigma(\gamma(x+vt),y)\sin(w\gamma(t+vx)+k) \tag{11}$$

$$\tag{12}$$

and for π_1' and π_2', we have

$$\pi_1(t,x,y) = \gamma\frac{\partial \phi_1'}{\partial t'} + \gamma v\frac{\partial \phi_2'}{\partial x'} \tag{13}$$

$$= -\omega\phi_2 + \gamma v\frac{\partial \sigma(\gamma(x+vt),y)}{\partial x}\cos(\omega\gamma(t+vx)+k) \tag{14}$$

$$\pi_2(t,x,y) = \gamma\frac{\partial \phi_1'}{\partial t'} + \gamma v\frac{\partial \phi_2'}{\partial x'} \tag{15}$$

$$= \omega\phi_1 + \gamma v\frac{\partial \sigma(\gamma(x+vt),y)}{\partial x}\sin(\omega\gamma(t+vx)+k) \tag{16}$$

At t = 0 , we then have all together:

$$\phi_1(t,x,y) = \sigma(\gamma x,y)\cos(\omega\gamma vx+k) \tag{17}$$

$$\phi_2(t,x,y) = \sigma(\gamma x,y)\sin(\omega\gamma vx+k) \tag{18}$$

$$\pi_1(t,x,y) = -\omega\phi_2 + \gamma v\frac{\partial \sigma(\gamma x,y)}{\partial x}\cos(\omega\gamma vx+k) \tag{19}$$

$$\pi_2(t,x,y) = \omega\phi_1 + \gamma v\frac{\partial \sigma(\gamma x,y)}{\partial x}\sin(\omega\gamma vx+k) \tag{20}$$

And this can be calculated numerically by the program. Notice the partial derivatives can be done using the np.grad function.

3 Anti-Qball

Now we want to consider an anti Qball, which takes the form

$$\phi(r,t) = f(r)e^{-i\omega t} \tag{21}$$

Now, with this form, we substitute into

$$\ddot{\phi} - \nabla^2\phi + \phi - A|\phi|\,\phi + B\left|\phi^2\right|\phi = 0 \tag{22}$$

And we obtain

$$\nabla^2\sigma(r) + (\omega^2 - 1)\sigma(r) + A\sigma(r)^2 + B\sigma(r)^3 = 0 \tag{23}$$

which is the same governing differential equation as before. Thus this implementation of the anti Q ball should not be too hard to complete. We will hopefully have some time to edit the code this week.