Modelling Complex Q-Ball Collisions in 2D

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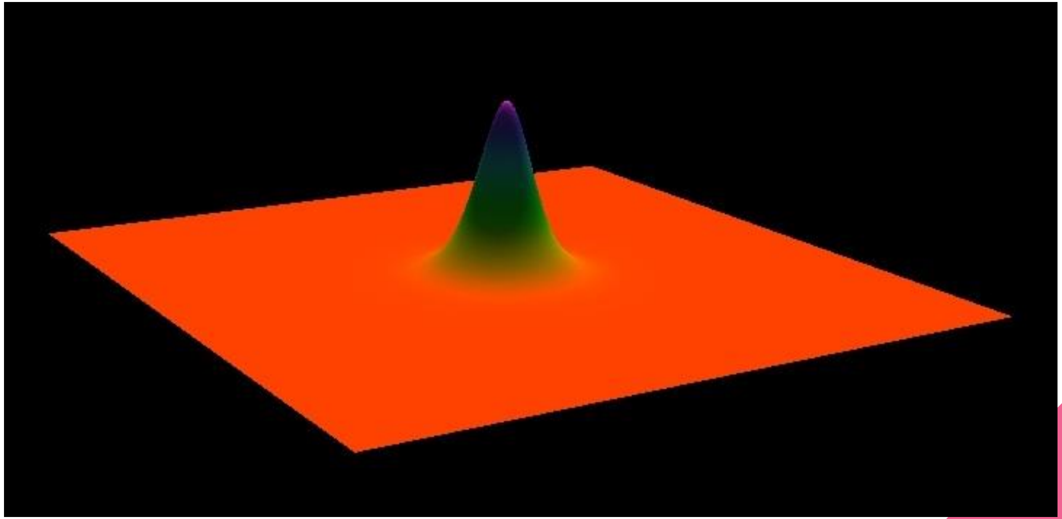
Abstract

Q-balls are characterized as stable, finite “blobs” containing a large cluster of particles. In more technical terms, they are localized spherically symmetric solutions of field equations under certain potentials. These objects are predicted by many theories and hold significant implications in the study of dark matter and particle physics, perhaps most importantly, theories which may explain matter-antimatter asymmetry and why matter exists.

The study of Q-ball behaviour, specifically Q-ball collision, is challenging as the governing equations are non-linear and thus difficult to solve analytically. Using numerical analysis, we model complex two dimensional Q-ball collisions involving multiple Q-balls and phase-shifted Q-balls. We determine some of the factors, including velocity, phase, and number of Q-balls, that would result in scattering or coalescence in a collision. Fast moving Q-balls, moving close to the speed of light, pass through each other without interactions, while the slowest Q-ball collision results in merging. At less than half the speed of light, we observe more varied interactions, from scattering to the creation of new Q-balls. Our results pave the way for the detection of Q-balls in upcoming experiments and expand on our understanding of physics.

What are Q-Balls?

Q-Balls can be thought as a type of wave that is localized, stable, and allowed to propagate. These are ‘blobs’ of matter that are spherical symmetric, and appear as solution to a certain class of field equation potentials.



Why Q-balls?

(Image credit: MARK GARLICK/SCIENCE PHOTO LIBRARY via Getty Images)

Q-balls are predicted in some supersymmetry models, including the Minimal Supersymmetric Standard Model, thus observing Q-balls may allow us to verify the prediction of such models. [1] Many has predicted that Q-balls may be part of the stuff that makes up dark matter [2].

Advancements

Q-balls lives in a complex scalar field, and for our research, we investigate the Qball equations in 2+1 dimensions (i.e. 2 dimensions + a time dimension). We use the equations calculated in [3], which propose the equation of the form:

$$\phi(r,t) = \sigma(r)e^{i\omega t}$$

For a set of positive values of omega. The paper uses the potential

$$\mathrm{U}(\phi^2) = \frac{1}{2}m|\phi|^2 - \frac{1}{3}\alpha|\phi|^3 + \frac{1}{4}\beta|\phi|^4$$

Which generates the partial differential equation governing the Q-ball.

From the basis of [3], we are able to simulate a wider class of Q balls. By a extension of their result, we use the form of equation:

$$\phi(r,t) = \sigma(r)e^{i(\omega t + k)}$$

Where k is the phase shift, and ω is allowed to be both positive and negative. Notice charge of a Q ball is given by [4]:

$$Q = w\int_{-\infty}^{\infty} \sigma^2(r)dr$$

Which means allowing ω to switch signs allows us to generate both Q-balls and anti Q-balls(Antimatter Q-ball). Furthermore, we now are able to generate Q-balls in phase and out of phase, with the extra factor of k, the phase shift.

The time evolution uses the method of finite differences, outlined in [3]. We will not go into the details of the time evolution here since it’s been introduced thoroughly in that paper.

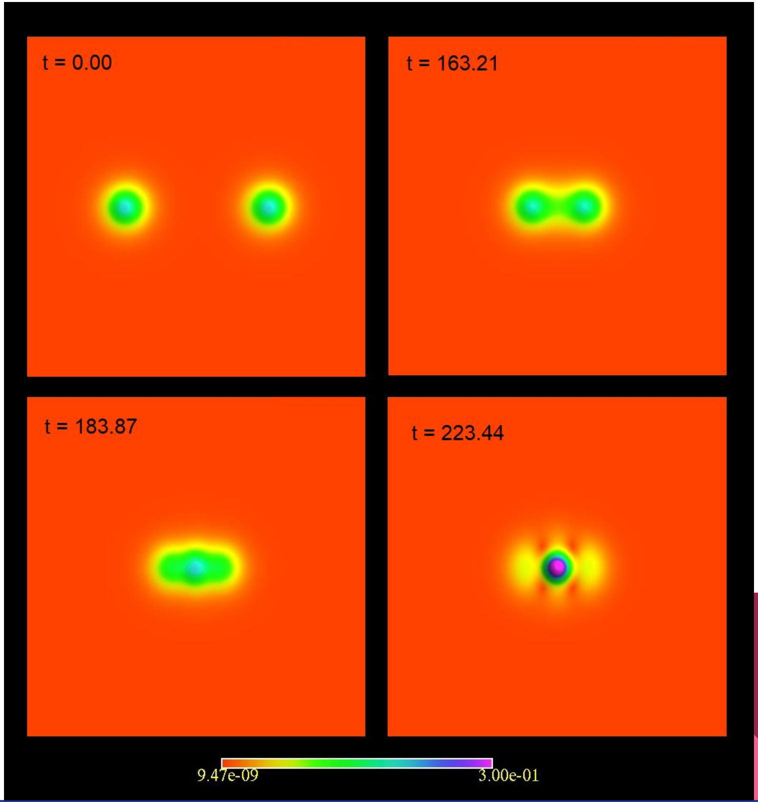
Apart from being able to generate Q-balls with these extra properties, we are also able to generate multiple Q-balls collision, at high resolutions, which gave novel results never reported in literature before, which we will be demonstrating in the following slides.

Q-ball Interactions at Low Velocities

At lower velocity, many Q-ball collisions of Q balls that are in phase generally result in the forming of new, larger Q-balls. (Coalescence of Q balls at lower velocities)

This suggests that large, slow moving Q-balls (<0.3c) may appear naturally and be stable during interactions with other Q-balls.

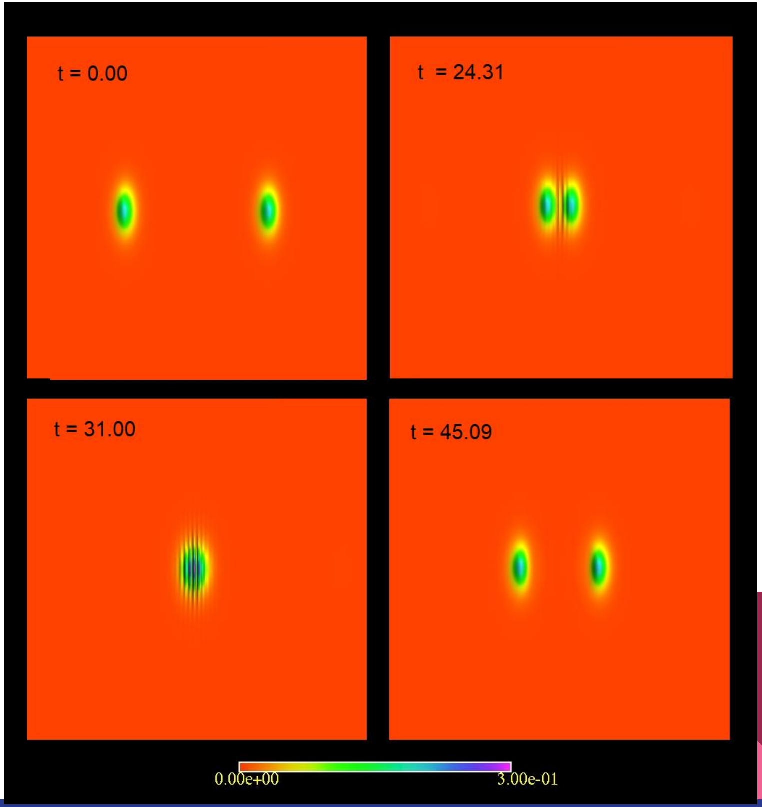
Low Velocity (v = 0.1c)



Q-ball Interactions at High Velocities

At high velocities (>0.7c), Q-balls pass through each other with little or no interaction. This may also suggest the natural existence of very high velocity Q-balls in our universe, but further investigation is needed concerning the interactions between Q-balls and other types of matter.

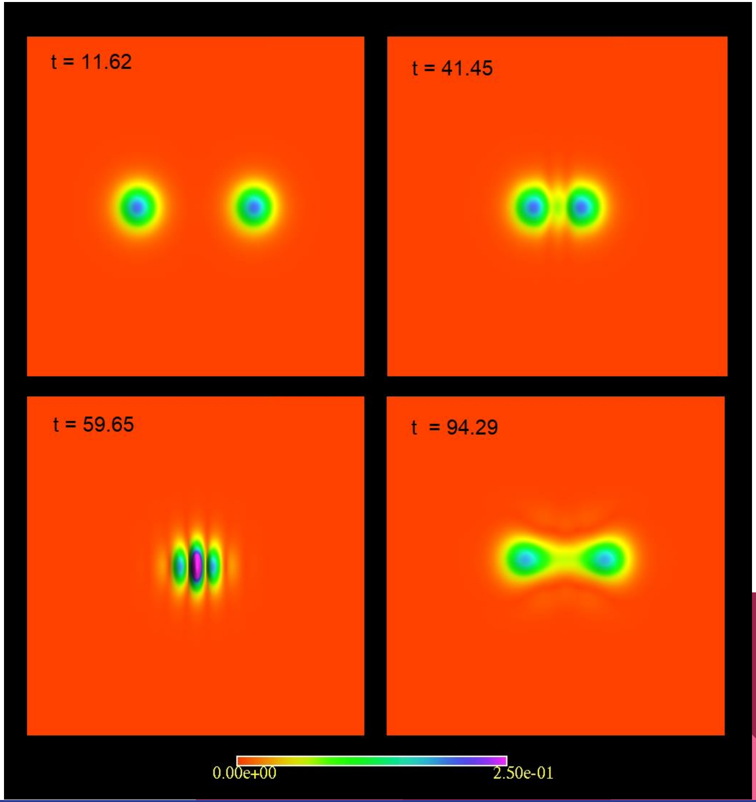
High Velocity (v = 0.9c)



Q-ball Interactions at Intermediate Velocities

Collisions at intermediate velocity ranges produces more varied results, but we generally would see some sort of energy/charge dissipation into the surrounding, which may hint at the un-likelihood of being able to observe Q-balls at such velocities naturally.

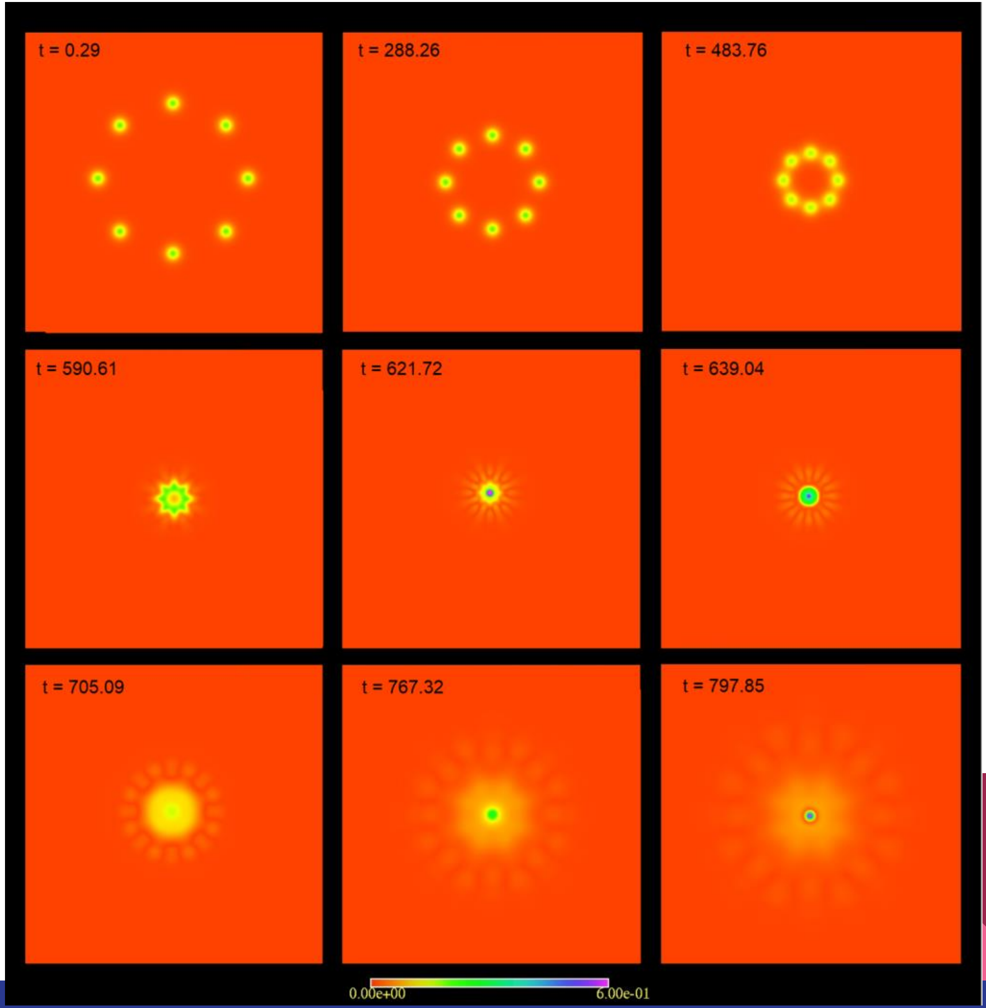
Intermediate Velocity (v = 0.4c)



Multi Q-ball Collision

One of our contributions to the current research is the study of multi Q-ball collision. Collisions function similarly to two Q-ball collisions. Here, we see many small, slow moving Q-balls come together to form a larger Q-ball.

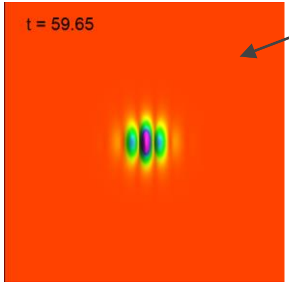
Low Velocity
(v = 0.1c)



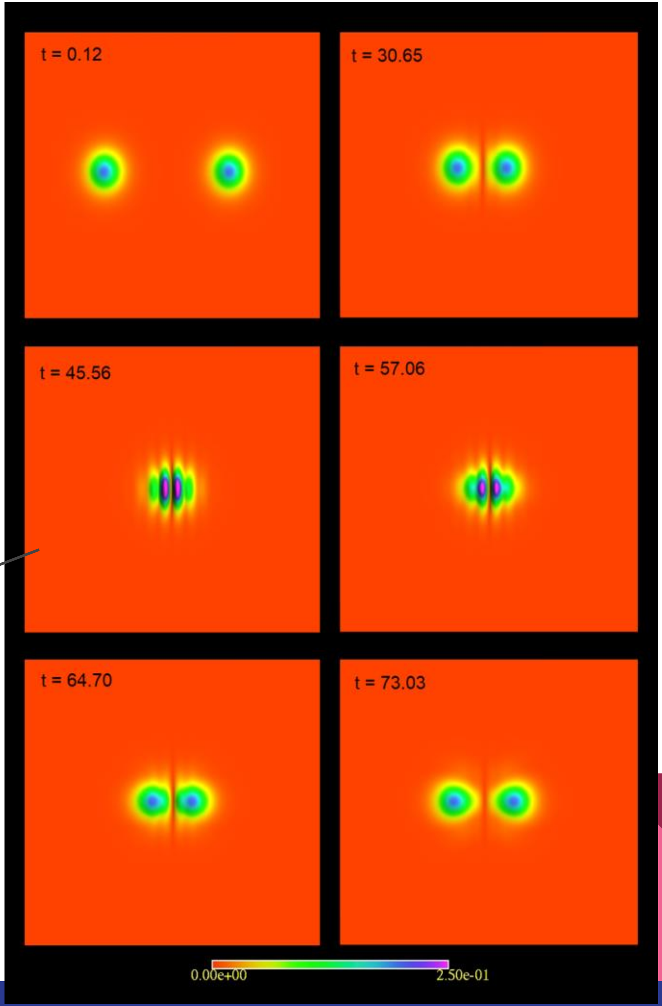
Phase-Shifted Q-ball Collision

Phase-shifted Q-ball collisions were modeled, implemented, and tested. Phase appears to affect the interference of the Q-balls during collision. The results require further verification and study.

Non-phase shifted collision



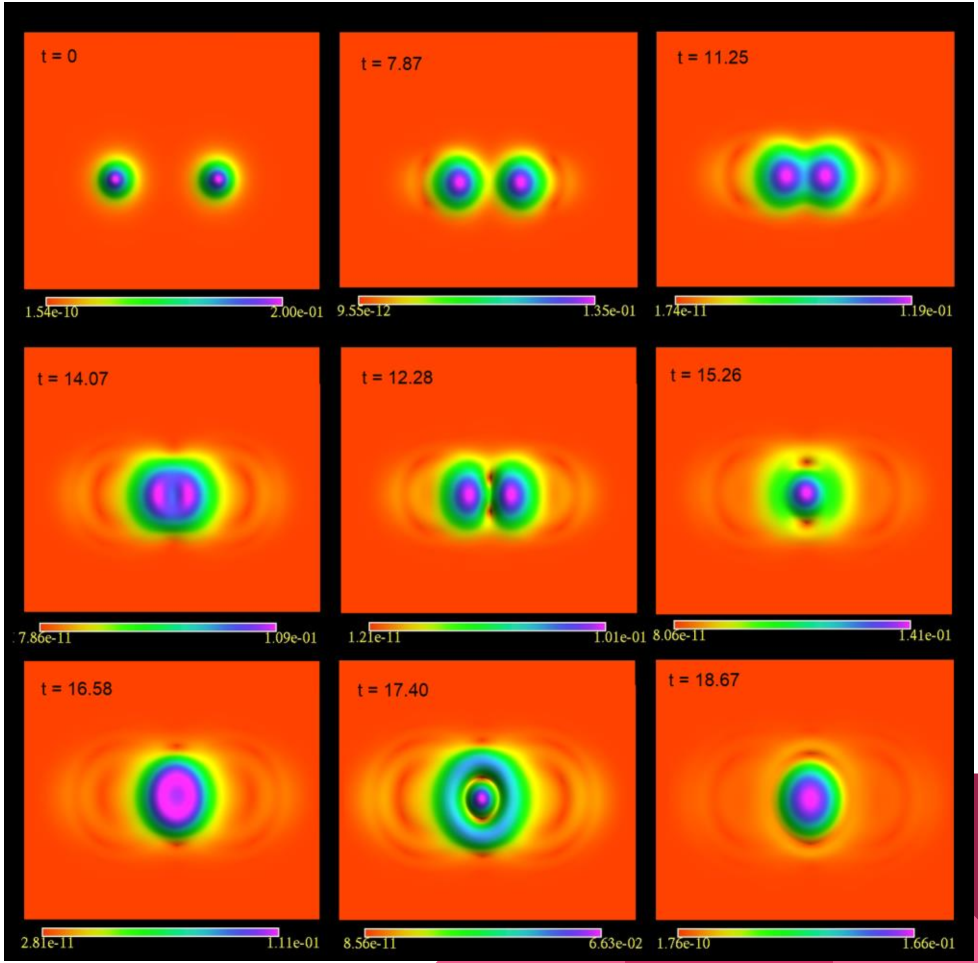
Intermediate Velocity
(v = 0.4c)



Q-ball and Anti-Q-ball Collision

We verified that Q-ball Anti Q-ball collisions generally reduces the amplitude of the Q-balls, i.e. the charge of the Q-balls, but often full annihilation does not happen. This is consistent with previous results[4].

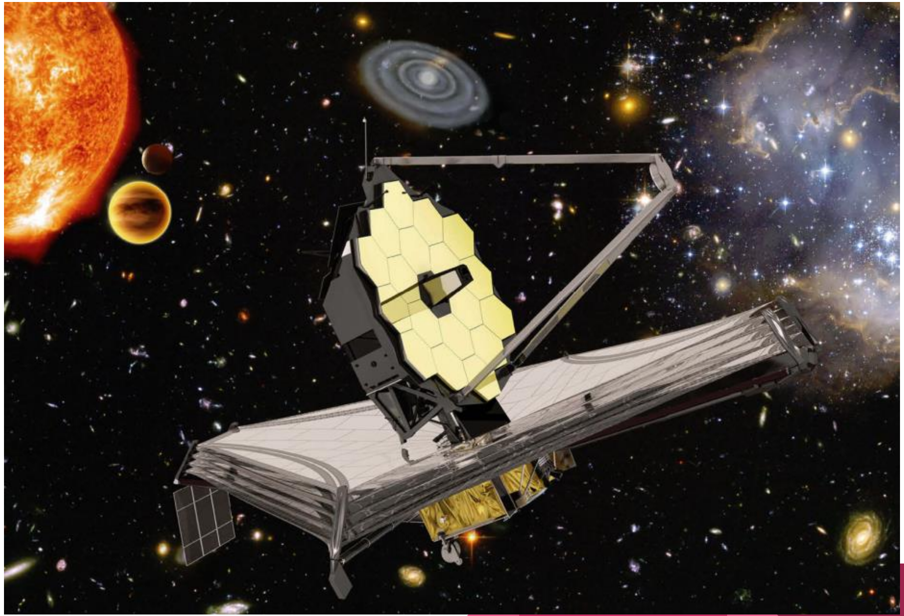
Intermediate Velocity (v = 0.4c)



Conclusion

We have added two new parameters - phase-shift and anti-Q-balls- to an existing model of complex Q-ball collision in 2D. We have compared our results against other studies of Q-ball models and we have studied the novel collision of multiple Q-balls.

Our study of Q-ball behavior will pave the way for future studies and contributes to the body of knowledge in the domain of particle physics, particularly the study of dark matter and matter-antimatter asymmetry.



(Image credit:https://www.esa.int/Science_Exploration/Space_Science/Webb_factsheet)

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References

[1] T.D. Lee and Y. Pang, Phys. Rept. 221 (1992) 25

[2] Kusenko, A., Loveridge, L. C., & Shaposhnikov, M. (2005). Supersymmetric dark-matter Q-balls and their interactions in matter. Physical Review D, 72(2). https://doi.org/10.1103/physrevd.72.025015

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