Monotonost:

- naraščajoče: $a_{n+1} \geq a_n$
- padajoče: $a_{n+1} \leq a_n$

Malo pravil...:

- $\lim_{n\to\infty} (a_n)^{\alpha} = (\lim_{n\to\infty} a_n)^{\alpha}$
- $\lim_{n\to\infty} a_n^{b_n} = \lim_{n\to\infty} a_n^{\lim b_n}$

soda: $f(-x) = f(x)$

liha: $f(-x) = -f(x)$

injektivna: vse točke so preslikane v različne točke

vse vrednost Z_f $(Z_f = \mathbb{R})$

bijektivna = injektivna + surjektivna

leva limita: $x \nearrow a$

desna limita: $x \searrow a$

$\lim_{x\searrow a} f(x) = f(a) = \lim_{x\nearrow a} f(x)$

$\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$

L'HOSPITALOVO P.

Če $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ ali pa $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = \pm\infty$:

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

MONOTONOST

- če $f'(x) \geq 0$ za $x \in (a,b)$ $\Rightarrow f$ naraščajoča na (a,b)
- če $f'(x) \leq 0$ za $x \in (a,b)$ $\Rightarrow f$ padajoča na (a,b)

KONVEKSNOST IN KONKAVNOST

- $f''(x) \geq$ za $x \in (a,b)$ $\Rightarrow f$ konveksna na (a,b)
- $f''(x) \leq$ za $x \in (a,b)$ $\Rightarrow f$ konkavna na (a,b)

ODVODI

tangenta: $y - f(x_0) = f'(x_0)(x - x_0)$

$k = f'(x_0)$

$f'(x_0) = 0$ - stacionarne t.

$f''(x_0) > 0$ - l. minimum

$f''(x_0) < 0$ - l. maximum

$x^{\alpha} \Rightarrow \alpha \cdot x^{\alpha-1}$

$e^x \Rightarrow e^x$

$\log x \Rightarrow \frac{1}{x}$

$\log_a x \Rightarrow \frac{1}{x} \cdot \log a$

$a^x; a>0 \Rightarrow a^x \cdot \log a$

$\sin x \Rightarrow \cos x$

$\cos x \Rightarrow -\sin x$

$\tan x \Rightarrow \frac{1}{\cos^2 x}$

$\mathrm{ctan} \Rightarrow -\frac{1}{\sin^2 x}$

$\mathrm{arctg} \Rightarrow \frac{1}{1+x^2}$

$\arcsin \Rightarrow \frac{1}{\sqrt{1-x^2}}$

ISKANJE NIČEL

BISEKCIJA:

$$x_n = \frac{a+b}{2} \quad [a,b] := \begin{cases} [a, x_n], \text{ če je } f(a)f(x_n) < 0 \\ [x_n, b], \text{ če je } f(x_n)f(b) < 0 \end{cases}$$

SEKANTNA METODA:

$$x_n = x_{n-1} - \frac{f(x_{n-1})(x_{n-1} - x_{n-2})}{f(x_{n-1}) - f(x_{n-2})}$$

REGULA FALSI:

$$x_n = b - \frac{f(b)(b-a)}{f(b) - f(a)} \quad [a,b] = \begin{cases} [a, x_n], \text{ će } f(a)f(x_n) < 0 \\ [x_n, b], \text{ če je } f(x_n)f(b) < 0 \end{cases}$$

NAVADNA ITERACIJA

$$x_n = g(x_{n-1})$$

FUNKCIJE VEČ SPREMENLJIVK

- gradient: $(\mathrm{grad}\, f)(x,y) = (f_x(x,y), f_y(x,y))$
- stacionarne t.: $f_x(x,y) = 0$, $f_y(x,y) = 0$
- smerni odvod: $f_{\vec a}(x_0, y_0) = \frac{(\mathrm{grad}\, f)(x_0,y_0) \cdot \vec a}{|\vec a|}$

(oz. kar $(\mathrm{grad}\, f)(x_0,y_0) \cdot \vec e$)

- nivojnice: $f(x,y) = c$

HESSEJEVA MATRIKA IN KLASIFIKACIJA EKSTREMOV

$$D(x,y) = f_{xx}(x,y) f_{yy}(x,y) - f_{xy}(x,y)^2$$

- če je $D(x_0,y_0) > 0$ in $f_{xx}(x_0,y_0) > 0 \Rightarrow (x_0,y_0)$ lokalni min
- če je $D(x_0,y_0) > 0$ in $f_{xx}(x_0,y_0) < 0 \Rightarrow (x_0,y_0)$ lokalni max
- če je $D(x_0,y_0) < 0 \Rightarrow (x_0,y_0)$ sedlo
- če je $D(x_0,y_0) \Rightarrow$ ne vemo

INTEGRALI

- nova spremenljivka:

$$\int f(t(x))t'(x)\,dx = \int f(t)\,dt$$

- po delih (per partes):

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int v(x)u'(x)\,dx$$

- Newton-Leibnizova formula:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

- ploščina pod grafom:

$$S = \int_a^b f(x)\,dx$$

- volumen vrtenine:

$$V = \pi \int_a^b (f(x))^2\,dx$$

Funkcija	Odvod
c	0
x	1
x^n	nx^{n-1}
$\frac{1}{x}$	$\frac{1}{x^2}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\sqrt[n]{x}$	$\frac{1}{n\sqrt[n]{x^{n-1}}}$
$\sin x$	$\cos x$
$\sin(ax)$	$a\cos(ax)$
$\cos x$	$\sin x$
$\cos(ax)$	$a\sin(ax)$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$\frac{1}{\sin^2 x}$
e^x	e^x
e^{kx}	ke^{kx}
a^x	$a^x \ln a$
x^x	$x^x(1+\ln x)$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x\ln a}$
$\arcsin x$	$\frac{1}{\sqrt{1 \quad x^2}}$
$\arccos x$	$\frac{1}{\sqrt{1 \quad x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\mathrm{arccot}\, x$	$\frac{1}{1+x^2}$

α	0^0	$30^0 = \frac{\pi}{6}$	$45^0 = \frac{\pi}{4}$	$60^0 = \frac{\pi}{3}$	$90^0 = \frac{\pi}{2}$	$180^0 = \pi$	$270^0 = \frac{2\pi}{3}$	$360^0 = 2\pi$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$tg\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	0	$-\infty$	0
$ctg\alpha$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\infty$	0	∞

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \text{pri } n \neq -1$$

$$\int x^{-1}\,dx = \int \frac{dx}{x} = \ln|x| + C$$

$$\int \sqrt{x}\,dx = \frac{2}{3}x^{3/2} + C$$

$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

$$\int \frac{1}{\sqrt{a-x^2}}\,dx = \arctan\frac{x}{\sqrt{a-x^2}} + C$$

$$\int \frac{x}{\sqrt{x^2-a}}\,dx = \sqrt{x^2-a} + C$$

$$\int e^x\,dx = e^x + C$$

$$\int e^{cx}\,dx = \frac{1}{c}e^{cx} + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\int xe^x\,dx = e^x(x-1) + C$$

$$\int \frac{dx}{e^x} = -\frac{1}{e^x} + C$$

$$\int \frac{x}{e^x}\,dx = -\frac{x+1}{e^x} + C$$

$$\int \frac{e^x}{x}\,dx = -\mathrm{Ei}(-x) + C \qquad \text{Opom}$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int \log_a x\,dx = x\log_a x - \frac{x}{\ln a} + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \cos(nx)\,dx = \frac{\sin(nx)}{n} + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sin(nx)\,dx = -\frac{\cos(nx)}{n} + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \frac{dx}{\cos^2 x} = \int \sec^2 x\,dx = \tan x + C$$

$$\int \frac{dx}{\sin^2 x} = \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sin^2 x\,dx = \frac{2x - \sin 2x}{4} + C = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$\int \cos^2 x\,dx = \frac{2x + \sin 2x}{4} + C = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\int (ax+b)\,dx = \frac{ax^2}{2} + bx + C$$

$$\int (ax^2+bx+c)\,dx = \frac{a}{3}x^3 + \frac{b}{2}x^2 + cx + C$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$$

$$\int \frac{1}{x^2+1}\,dx = \arctan x + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

Glede na stopnji polinomov p in q ločimo pri funkciji $f(x) = \frac{p(x)}{q(x)}$ tri primere.

1. Če je stopnja polinoma p manjša od stopnje polinoma q, potem je premica $y = 0$ **vodoravna asimptota** grafa funkcije f.
2. Če sta stopnji polinomov enaki, potem ima graf funkcije f **vodoravno asimptoto** $y = c$, kjer je c kvocient vodilnih koeficientov polinomov.
3. Če je stopnja polinoma p večja od stopnje polinoma q, potem je asimptota polinom stopnje vsaj 1. Ko je ta asimptota linearna funkcija, pravimo, da ima graf funkcije f **poševno asimptoto**.