

TIS – Računske naloge: Medsebojna informacija – rešitve

1. $U \setminus P \begin{matrix} DA \\ NE \end{matrix}$

DA	0.092	0.258	0.3
NE	0.008	0.692	0.7
	0.05	0.95	1

$H(U|P=NE) = ?$

$H(U|P=NE) = H\left(\frac{0.258}{0.95}, \frac{0.692}{0.95}\right) = 0.846 \text{ bit}$

$$I(X;Y) = H(X) - H(X|Y)$$

$$= H(Y) - H(Y|X)$$

$$= H(X) + H(Y) - H(X,Y)$$

$H(Y|X) = \sum_i P(x_i) \cdot H(Y|x_i)$
 $H(Y|x_i) = -\sum_j P(y_j|x_i) \log P(y_j|x_i)$

Diagram showing Venn diagram for joint entropy $H(X,Y)$ and conditional entropies $H(X|Y)$ and $H(Y|X)$.

2. $S \setminus B \begin{matrix} \bar{c} \\ \bar{o} \end{matrix}$

M	0.45	0.05	0.5
Z	0.15	0.35	0.5
	0.6	0.4	1

$3x+x=0.6$
 $x=0.15$

$H(B|S) = ?$

$H(B|S=M) = H\left(\frac{0.45}{0.5}, \frac{0.05}{0.5}\right) = 0.47 \text{ bit}$

$H(B|S=Z) = H\left(\frac{0.15}{0.5}, \frac{0.35}{0.5}\right) = 0.88 \text{ bit}$

$H(B|S) = P_M \cdot H(B|S=M) + P_Z \cdot H(B|S=Z)$
 $= 0.5 \cdot 0.47 + 0.5 \cdot 0.88 = 0.686 \text{ bit}$

3. $S \setminus R \begin{matrix} 0 \\ 1 \end{matrix}$

0	0.599	0.006	0.6
1	0.008	0.392	0.4
	0.602	0.398	1

$I(R;S) = ?$

$I(R;S) = H(R) - H(R|S) = 0.8648 \text{ bit}$

$H(R) = H(0.602, 0.398) = 0.9638 \text{ bit}$

$H(R|S=0) = H(0.99, 0.01) = 0.0908$

$H(R|S=1) = H(0.99, 0.02) = 0.1919$

$H(R|S) = 0.6 \cdot 0.0908 + 0.4 \cdot 0.1919 = 0.105$

4. $S \setminus N \begin{matrix} D_N \\ \bar{D}_N \end{matrix}$

D_S	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{3}{16}$
\bar{D}_S	$\frac{3}{16}$	$\frac{10}{16}$	$\frac{13}{16}$
	$\frac{5}{16}$	$\frac{11}{16}$	1

$I(S;N) = H(S) + H(N) - H(S,N)$

$= H\left(\frac{3}{16}, \frac{13}{16}\right) + H\left(\frac{5}{16}, \frac{11}{16}\right) - H\left(\frac{3}{16}, \frac{1}{16}, \frac{3}{16}, \frac{10}{16}\right)$
 $= 0.6962 + 0.8960 - 1.5016 = 0.0906 \text{ bit}$

5. $S \setminus N \begin{matrix} D_N \\ \bar{D}_N \end{matrix}$

D_S	0	$\frac{3}{16}$	$\frac{3}{16}$
\bar{D}_S	0	$\frac{13}{16}$	$\frac{13}{16}$

$I(S;N) = H\left(\frac{3}{16}, \frac{13}{16}\right) + H(0,1)$
 $- H\left(\frac{3}{16}, \frac{13}{16}, 0, 0\right) = 0.6 \text{ bit}$

5. $X \dots \text{radio}; Y \dots \text{TV}$

$$H(X, Y) = 4 \text{ bit}$$

$$H(X) = 3,5 \text{ bit}$$

$$H(Y) = 3 \text{ bit}$$

$$I(X; Y) = H(X) + H(Y) - H(X, Y)$$

$$I(X; Y) = 3 + 3,5 - 4 = \underline{2,5 \text{ bit}}$$

6.

$$P_i = \begin{bmatrix} 0,95 & 0,05 \\ 0,05 & 0,95 \end{bmatrix}$$

↓
BSK

$$C = 1 - H(P, 1-P) = 1 - H(0,95, 0,05)$$

$$= 1 - 0,2864 = \underline{0,713 \text{ bit}}$$