CLASSMATE
Date:
Page: 1

- NAME :- MUDASIR - LATEEF
- Roll No :- MCA-21-62 [62]
- CLASS :- MCA 1St SEM
- REG NO:- IUST0121015444.
- Subject:- DISCRETE MATHEMATICS.
- ASSIGNMENT ON:- Proposition, logic, truth table, propositional equivalence, logical equivalence, Predicate and QUANTIFIER.

Proposition:-

A proposition or statement is a declarative sentence that is either True or FAlse, But not both for example "three plus tree equals six" and three plus tree equals seven are both statements the first statement is true the second It is FALSE.

eg

(A) A. The sun rises in the west.

(b) B. $2+4=6$

(C) C. $(5,6) \subset (7,6,5)$

(D) D. Do you speak Hindi

(E) $4-x^{6}=8$

⇒ A, b and C are statements the first is False. Second and third are true.

⇒ D is a Question NOT a declarative sentence. Hence It is not a statement

⇒ E is a declarative sentence but not a statement since It is true or False depends on the value of x.

Truth TAbles:-

A truth-table is a table that shows the truth value of a compound proposition for all possible cases. each statement is true or False in a truth table, each statement is typically represented by a letter or variable like P, q, or r and each statement also has its own corresponding column in the truth table.

Input values:-

let's take the statement It is raining outside this statement which we can represent with the variable p is either true or FALSE.

p = It is raining outside.

If it's raining then p is true. If it isn't raining, then p is FALSE.

The negation of a statement, called not p, is the statement that contradicts p and has the opposite truth value.

not p = It is not raining outside.

If it is raining outside then not p is false
If it is not raining outside then not p is true

Negation	
p	~p
T	F
F	T

Conjuction:- A conjunction is a compound statement representing the word and for example we have the following statements.

p = It is raining outside

q = the football game is cancelled

the conjunction of p and q is It is raining outside and the football game is cancelled.

this statement will only be true if both p and q are true; If p and q is FALSE then the conjunction is FALSE.

CONJUNCTION		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION:- A disjunction is a compound statement representing the word Or in order for a disjunction to be true. One or Both of the original statements has to be true the disjunction of the above statements p or q. It is raining. Outside or the football game is cancelled, this statement is true. If p or q or both statements are true.

DISJUNCTION		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Propositional Equivalence:-

If p and q are statements the compound statement p if and only if q denoted by $p \Leftrightarrow q$ is called equivalence. the connective if and only if is denoted by symbol $\Leftrightarrow$

classmate

Page 5

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautologies and Contradictions:-

A compound proposition that is always true for all possible truth value of its variables or in other words contain only T in the last colum of its truth table. is called Tautology.

A compound proposition that is always FAlSE for all possible values. of its variables or in other words contain only F. in the last column of its truth table is called Contradiction.

Tautologies $(p \lor q)$

p	q	$p \lor q$	$p \to (p \lor q)$
T	T	T	T
T	F	T	T
F	T	T	T
T	T	F.	T

Contradictions

p	q	$\sim q$	$q \land \sim p$	$p \land (q \land \sim p)$
T	T	F	F	F.
T	F	F	F	F.
F	T	T	T	F.
F	F	T	F	F

Logically Equivalent:-

If two proposition $P(p,q)$ and $Q(p,q,..)$ where p,q are propositional variables have the same truth values in every possible case or $P \leftrightarrow Q$ is a tautology then the propositions are called logically equivalent or simply equivalent and denoted by

$P(p,q) \equiv Q(p,q,....)$ or $P(p,q,..) \Leftrightarrow Q(p,q,..)$

It is always permissible and sometime desirable to replace a given proposition by an equivalent one.

To test whether two propositions ~~P or Q~~ P and Q are logically equivalent the following steps are followed.

1. Construct the truth table for P.
2. Construct the truth table for Q using the same propositional variables.
3. Check each combination of truth values of the propositional variables to see whether the value of P is the same as the truth value of Q if in each row the truth value of P is the same as the truth value of Q then P and Q are logically equivalent.

⇒ **Conditional Proposition:-** is a statement

A Conditional Statements ~~are~~ ~~those~~ statements ~~where a hypothesis that~~ can be written in the form "If P then Q" where P and Q are sentences. For this conditional statement P is called the hypothesis and Q is called the conclusion. Intuitively "If P then Q.

is called the conclusion. Intuitively if p then q means that q must be true whenever p is true.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi Conditional Statement :-

A statement p if and only if q such statement are said to be bi-conditional statement and denoted by $p \Leftrightarrow q$ or $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(I) **Converse** the proposition $q \rightarrow p$ is called the converse of $p \rightarrow q$.

(II) **Inverse**:- the proposition $\sim p \rightarrow \sim q$ is called the Inverse of $p \rightarrow q$.

(III) **Contrapositive**:- the proposition $\sim q \rightarrow \sim p$ is called Contrapositive of $p \rightarrow q$.

Scanned with CamScanner

Show $p \to q$ and its contrapositive $\sim q \to \sim p$ are logically equivalent

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	T	T

Predicate logic.

A predicate is an expression of one or more variables determined on some specific domain. A predicate with variables can be made a proposition by either authorizing a value to the variable or by quantifying the variable.

⇒ Examples of predicates

1. Consider $E(x,y)$ denote $x = y$.
2. Consider $X(a,b,c)$ denoted $a+b+c=0$
3. Consider $M(x,y)$ denoted x is married to y.

Quantifier:-

The variables of ~~Qua~~ predicates is quantified by quantifiers. There are two types of quantifier in predicate logic Existential quantifier and Universal quantifier.

1. **Existential Quantifier:-** If $P(x)$ is a proposition over the universe U then it is denoted as

classmate
Page 9

$\exists x\ P(x)$ and read as there exists at least one variable x such that P(x) is true the quantifier $\exists$ is called the existential quantifier

UNIVERSAL QUANTIFIER:-

If P(x) is a proposition over the universe U then it is denoted as $\forall x, P(x)$ and read as For every $x \in U$ P(x) is true the quantifier $\forall$ is called universal quantifier.