284428991 Electromagnetics Drill Solution Hayt8e Chapter 1to5

Engineering Electromagnetics by William Hyatt-8th Edition (Ghulam Ishaq Khan Institute of Engineering Sciences and Technology)

ELECTROMAGNETICS HAYT 8 EDITION

DRILL SOLUTION FROM CHAPTERS 1 TO 5

CHAPTER 1

D1.1. Given points $M(-1, 2, 1)$, $N(3, -3, 0)$, and $P(-2, -3, -4)$, find: (*a*) $\mathbf{R}_{MN}$; (*b*) $\mathbf{R}_{MN} + \mathbf{R}_{MP}$; (*c*) $|\mathbf{r}_M|$; (*d*) $\mathbf{a}_{MP}$; (*e*) $|2\mathbf{r}_P - 3\mathbf{r}_N|$.

(*a*) $\mathbf{R}_{MN} = (3-(-1))\mathbf{a}_x + (-3-2)\mathbf{a}_y + (0-1)\mathbf{a}_z = 4\mathbf{a}_x - 5\mathbf{a}_y - \mathbf{a}_z$

(*b*) $\mathbf{R}_{MN} + \mathbf{R}_{MP} = \mathbf{R}_{MN} + (-2-(-1))\mathbf{a}_x + (-3-2)\mathbf{a}_y + (-4-1)\mathbf{a}_z$

$= \mathbf{R}_{MN} - \mathbf{a}_x - 5\mathbf{a}_y - 5\mathbf{a}_z = 4\mathbf{a}_x - 5\mathbf{a}_y - \mathbf{a}_z - \mathbf{a}_x - 5\mathbf{a}_y - 5\mathbf{a}_z$

$= 3\mathbf{a}_x - 10\mathbf{a}_y - 6\mathbf{a}_z$

(*c*) $|\mathbf{r}_M| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6} = 2.45$

(*d*) $$\mathbf{a}_{MP} = \frac{-\mathbf{a}_x - 5\mathbf{a}_y - 5\mathbf{a}_z}{\sqrt{(-1)^2 + (-5)^2 + (-5)^2}} = \frac{-\mathbf{a}_x - 5\mathbf{a}_y - 5\mathbf{a}_z}{\sqrt{51}} = -0.14\mathbf{a}_x - 0.7\mathbf{a}_y - 0.7\mathbf{a}_z$$

(*e*) $2\mathbf{r}_P - 3\mathbf{r}_N = 2(-2\mathbf{a}_x - 3\mathbf{a}_y - 4\mathbf{a}_z) - 3(3\mathbf{a}_x - 3\mathbf{a}_y) = -4\mathbf{a}_x - 6\mathbf{a}_y - 8\mathbf{a}_z - 9\mathbf{a}_x + 9\mathbf{a}_y$

$= -13\mathbf{a}_x + 3\mathbf{a}_y - 8\mathbf{a}_z$

$|2\mathbf{r}_P - 3\mathbf{r}_N| = \sqrt{(-13)^2 + 3^2 + (-8)^2} = 11\sqrt{2} = 15.56$

D1.2. A vector field $\mathbf{S}$ is expressed in rectangular coordinates as $\mathbf{S} = \{125/[(x-1)^2 + (y-2)^2 + (z+1)^2]\}\{(x-1)\mathbf{a}_x + (y-2)\mathbf{a}_y + (z+1)\mathbf{a}_z\}$. (*a*) Evaluate $\mathbf{S}$ at $P(2, 4, 3)$. (*b*) Determine a unit vector that gives the direction of $\mathbf{S}$ at P. (*c*) Specify the surface $f(x, y, z)$ on which $|\mathbf{S}| = 1$.

(*a*) $$\mathbf{S} = \frac{125}{(2-1)^2 + (4-2)^2 + (3+1)^2}\{(2-1)\mathbf{a}_x + (4-2)\mathbf{a}_y + (3+1)\mathbf{a}_z\}$$

$$= \frac{125}{1^2 + 2^2 + 4^2}(\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z) = \frac{125}{21}(\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z)$$

$= 5.95\mathbf{a}_x + 11.9\mathbf{a}_y + 23.8\mathbf{a}_z$

(*b*) $$\mathbf{a}_S = \frac{5.95\mathbf{a}_x + 11.9\mathbf{a}_y + 23.8\mathbf{a}_z}{\sqrt{5.95^2 + 11.9^2 + 23.8^2}} = \frac{5.95\mathbf{a}_x + 11.9\mathbf{a}_y + 23.8\mathbf{a}_z}{27.27}$$

$= 0.218\mathbf{a}_x + 0.436\mathbf{a}_y + 0.873\mathbf{a}_z$

(*c*) $1 = |\mathbf{S}| =$

$$\sqrt{\left(\frac{125(x-1)}{(x-1)^2 + (y-2)^2 + (z+1)^2}\right)^2 + \left(\frac{125(y-2)}{(x-1)^2 + (y-2)^2 + (z+1)^2}\right)^2 + \left(\frac{125(z+1)}{(x-1)^2 + (y-2)^2 + (z+1)^2}\right)^2}$$

$$1 = \sqrt{\frac{125^2[(x-1)^2 + (y-2)^2 + (z+1)^2}{[(x-1)^2 + (y-2)^2 + (z+1)^2]^2}} = \frac{125\sqrt{(x-1)^2 + (y-2)^2 + (z+1)^2}}{(x-1)^2 + (y-2)^2 + (z+1)^2}$$

Transposing,

$$\frac{(x-1)^2 + (y-2)^2 + (z+1)^2}{\sqrt{(x-1)^2 + (y-2)^2 + (z+1)^2}} = 125$$

Conjugating,

$$\frac{(x-1)^2 + (y-2)^2 + (z+1)^2}{\sqrt{(x-1)^2 + (y-2)^2 + (z+1)^2}} \bullet \frac{\sqrt{(x-1)^2 + (y-2)^2 + (z+1)^2}}{\sqrt{(x-1)^2 + (y-2)^2 + (z+1)^2}} = 125$$

$$\sqrt{(x-1)^2 + (y-2)^2 + (z+1)^2} = 125$$

D1.3. The three vertices of a triangle are located at $A(6, -1, 2)$, $B(-2, 3, -4)$, and $C(-3, 1, 5)$. Find: (*a*) $\mathbf{R}_{AB}$; (*b*) $\mathbf{R}_{AC}$; (*c*) the angle θ_{BAC} at vertex A; (*d*) the (vector) projection of $\mathbf{R}_{AB}$ on $\mathbf{R}_{AC}$.

(*a*) $\mathbf{R}_{AB} = (-2-6)\mathbf{a}_x + (3-(-1))\mathbf{a}_y + (-4-2)\mathbf{a}_z$

$= -8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z$

(*b*) $\mathbf{R}_{AC} = (-3-6)\mathbf{a}_x + (1-(-1))\mathbf{a}_y + (5-2)\mathbf{a}_z$

$= -9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$

(*c*) Locating angle θ_{BAC},

$\mathbf{R}_{AB} \bullet \mathbf{R}_{AC} = (-8)(-9) + 4(2) + (-6)3 = 72 + 8 - 18 = 62$

Using dot product,

$\mathbf{R}_{AB} \bullet \mathbf{R}_{AC} = |\mathbf{R}_{AB}||\mathbf{R}_{AC}| \cos\theta_{BAC}$

$$\theta_{BAC} = \cos^{-1}\frac{\mathbf{R}_{AB}\bullet\mathbf{R}_{AC}}{|\mathbf{R}_{AB}||\mathbf{R}_{AC}|} = \cos^{-1}\frac{62}{\left(\sqrt{(-8)^2+4^2+(-6)^3}\right)\left(\sqrt{(-9)^2+2^2+3^3}\right)}$$

$$= 53.6°$$

(*d*) Analyzing the projection,

$$|\mathbf{R}_{AB\,(\mathrm{L})}| = \mathbf{R}_{AB}\bullet\mathbf{a}_{AC} = \mathbf{R}_{AB}\bullet\frac{\mathbf{R}_{AC}}{|\mathbf{R}_{AC}|} = \frac{62}{\sqrt{(-9)^2+2^2+3^2}} = 6.395$$

$$\mathbf{R}_{AB\,(\mathrm{L})} = |\mathbf{R}_{AB\,(\mathrm{L})}|\,\mathbf{a}_{AC} = |\mathbf{R}_{AB\,(\mathrm{L})}|\left(\frac{\mathbf{R}_{AC}}{|\mathbf{R}_{AC}|}\right) = (6.395)\frac{-9\mathbf{a}_x+2\mathbf{a}_y+3\mathbf{a}_z}{\sqrt{(-9)^2+2^2+3^2}}$$

$$= -5.94\mathbf{a}_x + 1.319\mathbf{a}_y + 1.979\mathbf{a}_z$$

D1.4. The three vertices of a triangle are located at $A(6, -1, 2)$, $B(-2, 3, -4)$, and $C(-3, 1, 5)$. Find: (*a*) $\mathbf{R}_{AB}\times\mathbf{R}_{AC}$; (*b*) the area of the triangle; (*c*) a unit vector perpendicular to the plane in which the triangle is located.

(*a*) $$\mathbf{R}_{AB}\times\mathbf{R}_{AC} = \begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ -8 & 4 & -6\\ -9 & 2 & 3\end{vmatrix} = [4(3)-(-6)(2)]\mathbf{a}_x + [(-6)(-9)-(-8)(3)]\mathbf{a}_y + [(-8)(2)-4(-9)]\mathbf{a}_z$$

$$= 24\mathbf{a}_x + 78\mathbf{a}_y + 20\mathbf{a}_z$$

(*b*) $$A_\triangle = \frac{1}{2}\text{base}\times\text{height} = \frac{1}{2}|\mathbf{R}_{AB}\times\mathbf{R}_{BC}|$$

Finding the direction of vector $\mathbf{R}_{BC}$

$$\mathbf{R}_{BC} = (-3-(-2))\mathbf{a}_x + (1-3)\mathbf{a}_y + (5-(-4))\mathbf{a}_z = -\mathbf{a}_x - 2\mathbf{a}_y + 9\mathbf{a}_z$$

$$\mathbf{R}_{AB}\times\mathbf{R}_{BC} = \begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ -8 & 4 & -6\\ -1 & -2 & 9\end{vmatrix} = 24\mathbf{a}_x + 78\mathbf{a}_y + 20\mathbf{a}_z$$

$$|\mathbf{R}_{AB}\times\mathbf{R}_{BC}| = \sqrt{24^2+78^2+20^2} = 84$$

$$A_\triangle = (1/2)(84) = 42$$

(*c*) $$\mathbf{a}_{R_{AB}\times R_{AC}} = \frac{24\mathbf{a}_x+78\mathbf{a}_y+20\mathbf{a}_z}{84} = 0.286\mathbf{a}_x + 0.928\mathbf{a}_y + 0.238\mathbf{a}_z$$

D1.5. (*a*) Give the rectangular coordinates of the point $C(\rho = 4.4, \phi = -115°, z = 2)$. (*b*) Give the cylindrical coordinates of the point $D(x = -3.1, y = 2.6, z = -3)$. (*c*) Specify the distance from C to D.

(*a*) Converting cylindrical to rectangular coordinates

Table 1 Cylindrical to rectangular coordinate systems
$x = \rho\cos\phi$
$y = \rho\sin\phi$
$z = z$

$$x = 4.4\cos -115° = -1.86$$
$$y = 4.4\sin -115° = -3.99$$
$$z = 2$$
$$C(x = -1.86, y = -3.99, z = 2)$$

(b) Converting rectangular to cylindrical coordinates

Table 2 Rectangular to cylindrical coordinate systems
$\rho = \sqrt{x^2+y^2}$
$\phi = \tan^{-1} y/x$
$z = z$

$$\rho = \sqrt{(-3.1)^2+2.6^2} = 4.05$$

(ϕ is added with 180° which it lies on the second quadrant.)

$$\phi = \tan^{-1} y/x + 180° = \tan^{-1}(2.6/-3.1) + 180° = 140°$$
$$z = -3$$
$$D(\rho = 4.05, \phi = 140°, z = -3)$$

(c) $$\mathbf{CD} = (-3.1-(-1.86))\mathbf{a}_x + (2.6-(-3.99))\mathbf{a}_y + (-3-2)\mathbf{a}_z$$
$$= -1.24\mathbf{a}_x + 6.59\mathbf{a}_y - 5\mathbf{a}_z$$

$|\mathbf{CD}| = \sqrt{(-1.24)^2 + 6.59^2 + (-5)^2} = 8.36$

D1.6. Transform to cylindrical coordinates: (*a*) $\mathbf{F} = 10\mathbf{a}_x - 8\mathbf{a}_y + 6\mathbf{a}_z$ at point $P(10, -8, 6)$; (*b*) $\mathbf{G} = (2x + y)\mathbf{a}_x - (y - 4x)\mathbf{a}_y$ at point $Q(\rho, \phi, z)$. (*c*) Give the rectangular components of the vector $\mathbf{H} = 20\mathbf{a}_\rho - 10\mathbf{a}_\phi + 3\mathbf{a}_z$ at $P(x = 5, y = 2, z = -1)$.

(a) Taking note that the dot product of the unit vectors in cylindrical to rectangular coordinate systems and vice versa

Table 3 Dot products of the unit vectors in cylindrical and rectangular coordinate systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1

$\mathbf{F} = (10\mathbf{a}_x - 8\mathbf{a}_y + 6\mathbf{a}_z)\cdot\mathbf{a}_\rho + (10\mathbf{a}_x - 8\mathbf{a}_y + 6\mathbf{a}_z)\cdot\mathbf{a}_\phi + (10\mathbf{a}_x - 8\mathbf{a}_y + 6\mathbf{a}_z)\cdot\mathbf{a}_z$

$= (10\cos\phi - 8\sin\phi + 0)\mathbf{a}_\rho + [(10(-\sin\phi) + (-8)(\cos\phi) + 0]\mathbf{a}_\phi + (0 + 0 + 6)\mathbf{a}_z$

Since $\phi = (\tan^{-1}(-8/10) + 180°) = 141.43°$

$= (10\cos 141.43° - 8\sin 141.43°)\mathbf{a}_\rho + [(10(-\sin 141.43°) + (-8)(\cos 141.43°)]\mathbf{a}_\phi + 6\mathbf{a}_z$

$= 12.81\mathbf{a}_\rho + 6\mathbf{a}_z$

(*b*) $\mathbf{G} = [(2x + y)\mathbf{a}_x - (y - 4x)\mathbf{a}_y]\cdot\mathbf{a}_\rho + [(2x + y)\mathbf{a}_x - (y - 4x)\mathbf{a}_y]\cdot\mathbf{a}_\phi$

Segregating the unit vectors to avoid confusion,

$G_\rho = [(2x + y)\mathbf{a}_x - (y - 4x)\mathbf{a}_y]\cdot\mathbf{a}_\rho$

Converting also the variables of rectangular to cylindrical coordinates,

$= (2\rho\cos\phi + \rho\sin\phi)\cos\phi - (\rho\sin\phi - 4\rho\cos\phi)\sin\phi$

$= 2\rho\cos^2\phi + \rho\sin\phi\cos\phi - \rho\sin^2\phi + 4\rho\sin\phi\cos\phi$

$= 2\rho\cos^2\phi - \rho\sin^2\phi + 5\rho\sin\phi\cos\phi$

$G_\phi = [(2x + y)\mathbf{a}_x - (y - 4x)\mathbf{a}_y]\cdot\mathbf{a}_\phi$

$= (2\rho\cos\phi + \rho\sin\phi)(-\sin\phi) - (\rho\sin\phi - 4\rho\cos\phi)\cos\phi$

$= -2\rho\sin\phi\cos\phi - \rho\sin^2\phi - \rho\sin\phi\cos\phi + 4\rho\cos^2\phi$

$= -2\rho\sin\phi\cos\phi - \rho\sin^2\phi - \rho\sin\phi\cos\phi + 4\rho\cos^2\phi$

$= 4\rho\cos^2\phi - \rho\sin^2\phi - 3\rho\sin\phi\cos\phi$

Adding together,

$\mathbf{G} = (2\rho\cos^2\phi - \rho\sin^2\phi + 5\rho\sin\phi\cos\phi)\,\mathbf{a}_\rho + (4\rho\cos^2\phi - \rho\sin^2\phi - 3\rho\sin\phi\cos\phi)\,\mathbf{a}_\rho$

(*c*) Applying the dot product from Table 3,

$\mathbf{H} = (20\mathbf{a}_\rho - 10\mathbf{a}_\phi + 3\mathbf{a}_z)\cdot\mathbf{a}_x + (20\mathbf{a}_\rho - 10\mathbf{a}_\phi + 3\mathbf{a}_z)\cdot\mathbf{a}_y + (20\mathbf{a}_\rho - 10\mathbf{a}_\phi + 3\mathbf{a}_z)\cdot\mathbf{a}_z$

Since $\phi = (\tan^{-1}(2/5) + 180°) = 201.8°$

$H_x = 20\cos\phi + 10\sin\phi + 0 = 20\cos 201.8° + 10\sin 201.8° = 22.3$

$H_y = 20\sin\phi - 10\cos\phi + 0 = 20\sin 201.8° - 10\cos 201.8° = -1.86$

$H_z = 0 + 0 + 3 = 3$

D1.7. Given the two points, $C(-3, 2, 1)$ and $D(r = 5, \theta = 20°, \phi = -70°)$, find: (*a*) the spherical coordinates of C; (*b*) the rectangular coordinates of D; (*c*) the distance from C to D.

(*a*) Converting rectangular to spherical coordinates,

Table 4 Rectangular to spherical coordinate systems
$r = \sqrt{x^2 + y^2 + z^2}$
$\theta = \cos^{-1}\dfrac{z}{r}$
$\phi = \tan^{-1} y/x$

$r = \sqrt{(-3)^2 + 2^2 + 1^2} = 3.74$

$\theta = \cos^{-1}(1/3.74) = 74.5°$

$\phi = \tan^{-1}(2/-3) + 180° = 146.3°$

$C(r = 3.74, \theta = 74.5°, \phi = 146.3°)$

(*b*) Converting spherical to rectangular coordinates,

Table 5 Spherical to rectangular coordinate systems
$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

$x = 5 \sin 20° \cos -70° = 0.585$
$y = 5 \sin 20° \sin -70° = -1.607$
$z = 5 \cos 20° = 4.7$
$D(x = 0.585, y = -1.607, z = 4.7)$

(*c*) Using rectangular coordinates to find the distance

$$|\mathbf{CD}| = \sqrt{(0.585-(-3))^2+(-1.607-2)^2+(4.7-1)^2} = 6.29$$

D1.8. Transform the following vectors to spherical coordinates at the points given: (*a*) $10\mathbf{a}_x$ at $P(x = -3, y = 2, z = 4)$; (*b*) $10\mathbf{a}_y$ at $Q(\rho = 5, \phi = 30°, z = 4)$; (*c*) $10\mathbf{a}_z$ at $M(r = 4, \theta = 110°, \phi = 120°)$.

(*a*) Taking note that the dot product of the unit vectors in spherical to rectangular coordinate systems and vice versa

Table 6 Dot products of the unit vectors in spherical and rectangular coordinate systems

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \bullet$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \bullet$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \bullet$	$\cos\theta$	$-\sin\theta$	0

$10\mathbf{a}_x = 10(\sin\theta\cos\phi\,\mathbf{a}_r + \cos\theta\cos\phi\,\mathbf{a}_\theta - \sin\phi\,\mathbf{a}_\phi)$

From Table 4, we calculate and have $\theta = 42.03°$ and $\phi = 146.31°$. Substituting,

$= 10 \sin 42.03° \cos 146.31°\,\mathbf{a}_r + 10 \cos 42.03° \cos 146.31°\,\mathbf{a}_\theta - 10 \sin 146.31°\,\mathbf{a}_\phi$

$= -5.57\mathbf{a}_r - 6.18\mathbf{a}_\theta - 5.55\mathbf{a}_\phi$

(*b*) $10\mathbf{a}_y = 10(\sin\theta\sin\phi\,\mathbf{a}_r + \cos\theta\sin\phi\,\mathbf{a}_\theta + \cos\phi\,\mathbf{a}_\phi)$

We manipulate to convert in cylindrical to spherical coordinates. From Table 1, we calculate and have $x = 4.33$, $y = 2.5$ and $z = 4$. Then from Table 4, we convert rectangular to spherical coordinates. We calculate and have $\theta = 51.34°$ and $\phi = 30°$. Substituting,

$= 10 \sin 51.34° \sin 30°\,\mathbf{a}_r + 10 \cos 51.34° \sin 30°\,\mathbf{a}_\theta + 10 \cos 30°\,\mathbf{a}_\phi$

$= 3.9\mathbf{a}_r - 3.12\mathbf{a}_\theta - 8.66\mathbf{a}_\phi$

(*c*) $10\mathbf{a}_z = 10(\cos\theta\,\mathbf{a}_r + (-\sin\theta)\,\mathbf{a}_\theta)$

$= 10 \cos 110°\,\mathbf{a}_r - 10 \sin 110°\,\mathbf{a}_\theta = -3.42\mathbf{a}_r - 9.4\mathbf{a}_\theta$

CHAPTER 2

D2.1. A charge $Q_A = -20$ μC is located at $A(-6, 4, 7)$, and a charge $Q_B = 50$ μC is at $B(5, 8, -2)$ in free space. If distances are given in meters, find: (*a*) $\mathbf{R}_{AB}$; (*b*) R_{AB}. Determine the vector force exerted on Q_A by Q_B if $\epsilon_0 =$ (*c*) $10^{-9}/(36\pi)$ F/m; (*d*) 8.854×10^{-12} F/m.

(*a*) $\mathbf{R}_{AB} = (5-(-6))\mathbf{a}_x + (8-4)\mathbf{a}_y + (-2-7)\mathbf{a}_z$ m $= 11\mathbf{a}_x + 4\mathbf{a}_y - 9\mathbf{a}_z$ m

(*b*) $R_{AB} = \sqrt{11^2+4^2+(-9)^2}$ m $= 14.76$ m

(*c*) $$\mathbf{F}_{AB} = \frac{Q_1Q_2}{4\pi\epsilon_0 R_{AB}^2}\mathbf{a}_{AB} = \frac{Q_1Q_2}{4\pi\epsilon_0 R_{AB}^2}\frac{\mathbf{R}_{AB}}{R_{AB}} = \frac{Q_1Q_2}{4\pi\epsilon_0 R_{AB}^3}\mathbf{R}_{AB}$$

There is an ambiguous value of $\mathbf{F}_{AB}$ with regards from the answer of the textbook. Trying in the higher decimal number of $R_{AB} = 14.7648$,

$$= \frac{(-20\times10^{-6})(50\times10^{-6})}{4\pi(10^{-9}/36\pi)(14.7648)^3}(11\mathbf{a}_x+4\mathbf{a}_y-9\mathbf{a}_z) = (-2.7961\times10^{-3})(11\mathbf{a}_x + 4\mathbf{a}_y - 9\mathbf{a}_z)$$

$= 30.76\mathbf{a}_x + 11.184\mathbf{a}_y - 25.16\mathbf{a}_z$ mN

(*d*) $$\mathbf{F}_{AB} = \frac{Q_1Q_2}{4\pi\epsilon_0 R_{AB}^3}\mathbf{R}_{AB} = \frac{(-20\times10^{-6})(50\times10^{-6})}{4\pi(8.854\times10^{-12})(14.7648)^3}(11\mathbf{a}_x+4\mathbf{a}_y-9\mathbf{a}_z)$$

$= (-2.7923\times10^{-3})(11\mathbf{a}_x + 4\mathbf{a}_y - 9\mathbf{a}_z) = 30.72\mathbf{a}_x + 11.169\mathbf{a}_y - 25.13\mathbf{a}_z$ mN

D2.2. A charge of -0.3 μC is located at $A(25, -30, 15)$ (in cm), and a second charge of 0.5 μC is at $B(-10, 8, 12)$ cm. Find $\mathbf{E}$ at: (*a*) the origin; (*b*) $P(15, 20, 50)$ cm.

(*a*) Solving first the vector and magnitude from charge A to the origin O,

$\mathbf{R}_{AO} = (0-25)\mathbf{a}_x + (0-(-30))\mathbf{a}_y + (0-15)\mathbf{a}_z$ cm $= -25\mathbf{a}_x + 30\mathbf{a}_y - 15\mathbf{a}_z$ cm

$= -0.25\mathbf{a}_x + 0.3\mathbf{a}_y - 0.15\mathbf{a}_z$ m

$|\mathbf{R}_{AO}| = \sqrt{(-0.25)^2+0.3^2+(-0.15)^2}$ m $= 0.4183$ m

$$\mathbf{E}_A = \frac{Q_A}{4\pi\epsilon_0 R_{AO}^2}\mathbf{a}_{AO} = \frac{Q_A}{4\pi\epsilon_0 R_{AO}^2}\frac{\mathbf{R}_{AO}}{R_{AO}} = \frac{Q_A}{4\pi\epsilon_0 R_{AO}^3}\mathbf{R}_{AO}$$

$$= \frac{-0.3\times10^{-6}}{4\pi\epsilon_0(0.4183)^3}(-0.25\mathbf{a}_x + 0.3\mathbf{a}_y - 0.15\mathbf{a}_z)\ \text{V/m}$$

$$= 9.21\mathbf{a}_x - 11.05\mathbf{a}_y + 5.53\mathbf{a}_z\ \text{kV/m}$$

For the charge B to the origin,

$$\mathbf{R}_{BO} = (0-(-10))\mathbf{a}_x + (0-8)\mathbf{a}_y + (0-12)\mathbf{a}_z \text{ cm} = 10\mathbf{a}_x - 8\mathbf{a}_y - 12\mathbf{a}_z \text{ cm}$$

$$= 0.1\mathbf{a}_x - 0.08\mathbf{a}_y - 0.12\mathbf{a}_z \text{ m}$$

$$|\mathbf{R}_{BO}| = \sqrt{0.1^2 + (-0.08)^2 + (-0.12)^2} \text{ m} = 0.1755 \text{ m}$$

$$\mathbf{E}_B = \frac{Q_B}{4\pi\epsilon_0 R_{BO}^3}\mathbf{R}_{BO} = \frac{0.5\times10^{-6}}{4\pi\epsilon_0(0.1755)^3}(0.1\mathbf{a}_x - 0.08\mathbf{a}_y - 0.12\mathbf{a}_z)\ \text{V/m}$$

$$= 83.13\mathbf{a}_x - 66.51\mathbf{a}_y - 99.76\mathbf{a}_z\ \text{kV/m}$$

Combining $\mathbf{E}_A$ and $\mathbf{E}_B$,

$$\mathbf{E} = 92.3\mathbf{a}_x - 77.6\mathbf{a}_y - 94.2\mathbf{a}_z \text{ kV/m}$$

(*b*) Same concept in (*a*)

$$\mathbf{R}_{AP} = (15-25)\mathbf{a}_x + (20-(-30))\mathbf{a}_y + (50-15)\mathbf{a}_z \text{ cm} = -0.1\mathbf{a}_x + 0.5\mathbf{a}_y + 0.35\mathbf{a}_z \text{ m}$$

$$|\mathbf{R}_{AP}| = 0.6185 \text{ m}$$

$$\mathbf{E}_A = 1.14\mathbf{a}_x - 5.7\mathbf{a}_y - 3.99\mathbf{a}_z\ \text{kV/m}$$

$$\mathbf{R}_{BP} = (15-(-10))\mathbf{a}_x + (20-8)\mathbf{a}_y + (50-12)\mathbf{a}_z \text{ cm} = 0.25\mathbf{a}_x + 0.12\mathbf{a}_y + 0.38\mathbf{a}_z \text{ m}$$

$$|\mathbf{R}_{BP}| = 0.4704 \text{ m}$$

$$\mathbf{E}_B = 10.79\mathbf{a}_x + 5.18\mathbf{a}_y + 16.41\mathbf{a}_z\ \text{kV/m}$$

$$\mathbf{E} = 11.9\mathbf{a}_x - 0.52\mathbf{a}_y - 12.4\mathbf{a}_z \text{ kV/m}$$

D2.3. Evaluate the sums: (*a*) $\sum_{m=0}^{5}\frac{1+(-1)^m}{m^2+1}$; (*b*) $\sum_{m=1}^{4}\frac{(0.1)^m+1}{(4+m^2)^{1.5}}$.

(*a*) $$\frac{1+(-1)^0}{0^2+1} + \frac{1+(-1)^1}{1^2+1} + \frac{1+(-1)^2}{2^2+1} + \frac{1+(-1)^3}{3^2+1} + \frac{1+(-1)^4}{4^2+1} + \frac{1+(-1)^5}{5^2+1} = 2.52$$

(*b*) $$\frac{(0.1)^1+1}{(4+1^2)^{1.5}} + \frac{(0.1)^2+1}{(4+2^2)^{1.5}} + \frac{(0.1)^3+1}{(4+3^2)^{1.5}} + \frac{(0.1)^4+1}{(4+4^2)^{1.5}} = 0.176$$

D2.4. Calculate the total charge within each of the indicated volumes: (*a*) $0.1 \le |x|, |y|, |z| \le 0.2$: $\rho_v = 1/(x^3y^3z^3)$; (*b*) $0 \le \rho \le 0.1$, $0 \le \phi \le \pi$, $2 \le z \le 4$; $\rho_v = \rho^2 z^2 \sin 0.6\phi$; (*c*) universe: $\rho_v = e^{-2r}/r^2$.

(*a*) The differential volume of the rectangular coordinates is $dv = dx\,dy\,dz$. Using the formula,

$$Q = \int_{vol}\rho_v dv = \int_{0.1}^{0.2}\int_{0.1}^{0.2}\int_{0.1}^{0.2}\frac{1}{x^3y^3z^3}dx\,dy\,dz = \int_{0.1}^{0.2}x^{-3}dx\int_{0.1}^{0.2}y^{-3}dy\int_{0.1}^{0.2}z^{-3}dz$$

$$= \left.\frac{x^{-2}}{-2}\right|_{0.1}^{0.2}\left.\frac{y^{-2}}{-2}\right|_{0.1}^{0.2}\left.\frac{z^{-2}}{-2}\right|_{0.1}^{0.2} = 112.5 \text{ C}$$

Notice that the charge Q is a large number. Neglecting the rate of change of the cubical volume, we have $xyz = (0.1)^3$ and $xyz = (0.2)^3$ which are very small; then the volume Δv is nearest to 0. Yielding,

$$Q = \rho_v\Delta v = [1/(x^3y^3z^3)]\,0 = 0$$

(*b*) The differential volume of the cylindrical coordinates is $dv = \rho\,d\rho\,d\phi\,dz$. Using the formula,

$$Q = \int_0^{0.1}\int_0^{\pi}\int_2^4(\rho^2z^2\sin 0.6\phi)\rho\,d\rho\,d\phi\,dz = \int_0^{0.1}\rho^3\,d\rho\int_0^{\pi}\sin 0.6\phi\,d\phi\int_2^4 z^2\,dz$$

$$= \left(\left.\frac{\rho^4}{4}\right|_0^{0.1}\right)\left(-\left.\frac{\cos 0.6\phi}{0.6}\right|_0^{\pi}\right)\left(\left.\frac{z^3}{3}\right|_2^4\right) = \left(\frac{0.1^4}{4}-0\right)\left(\frac{-\cos 0.6(180°)}{0.6} - \frac{-\cos 0.6(0°)}{0.6}\right)\left(\frac{4^3}{3}-\frac{2^3}{3}\right)$$

$$= 1.018 \text{ mC}$$

(*c*) The differential volume of the spherical coordinates is $dv = r^2 \sin\theta\, d\theta\, d\phi\, dr$. Assuming this universe to be a perfect sphere, we have limits as $0 \le r \le \infty$, $0 \le \phi \le 2\pi$, $0 \le \theta \le \pi$. Using the formula,

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_2^{\infty}(e^{-2r}/r^2)(r^2\sin\theta\,d\theta\,d\phi\,dr) = \int_0^{2\pi}d\phi\int_0^{\pi}\sin\theta\,d\theta\int_2^{\infty}e^{-2r}\,dr$$

$$= \left(\phi\Big|_0^{2\pi}\right)\left(-\cos\theta\Big|_0^{\pi}\right)\left(-e^{-2r}/2\Big|_0^{\infty}\right) = (2\pi - 0)(-\cos 180° - (-\cos 0°))(-e^{-\infty}/2 - (-e^{0}/2))$$

$= 6.28$ C

D2.5. Infinite uniform line charges of 5 nC/m lie along the (positive and negative) x and y axes in free space. Find **E** at: (*a*) $P_A(0, 0, 4)$; (*b*) $P_B(0, 3, 4)$.

(*a*) $\mathbf{E} = \dfrac{\rho_L}{2\pi\epsilon_0\rho}\mathbf{a}_\rho$ where ρ is just a radical distance R, then

$$= \frac{\rho_L}{2\pi\epsilon_0 R}\frac{\mathbf{R}}{R} = \frac{\rho_L}{2\pi\epsilon_0 R^2}\mathbf{R}$$

Finding $\mathbf{E}_x$ and $\mathbf{E}_y$ at Point A since infinite uniform line charges lie along x and y axes,

$$\mathbf{R}_x = 4\mathbf{a}_z$$

$$\mathbf{E}_x = \frac{5\times10^{-9}}{2\pi\epsilon_0 4^2}(4\mathbf{a}_z) = 22.469\mathbf{a}_z \text{ V/m}$$

$$\mathbf{R}_y = 4\mathbf{a}_z$$

$$\mathbf{E}_y = \frac{5\times10^{-9}}{2\pi\epsilon_0 4^2}(4\mathbf{a}_z) = 22.469\mathbf{a}_z \text{ V/m}$$

Adding together,

$$\mathbf{E} = 22.469\mathbf{a}_z + 22.469\mathbf{a}_z \text{ V/m} = 45\mathbf{a}_z \text{ V/m}$$

(*b*) Finding **R** and **E** at Point B along x and y axes,

$$\mathbf{R}_x = 3\mathbf{a}_y + 4\mathbf{a}_z$$

$$\mathbf{R}_y = 4\mathbf{a}_z$$

$$\mathbf{E}_x = \frac{5\times10^{-9}}{2\pi\epsilon_0\left(\sqrt{3^2+4^2}\right)^2}(3\mathbf{a}_y + 4\mathbf{a}_z) = 10.785\mathbf{a}_y + 14.38\mathbf{a}_z \text{ V/m}$$

$$\mathbf{E}_y = \frac{5\times10^{-9}}{2\pi\epsilon_0\left(\sqrt{4^2}\right)^2}(4\mathbf{a}_z) = 22.469\ \mathbf{a}_z \text{ V/m}$$

$$\mathbf{E} = 10.785\mathbf{a}_y + (14.38 + 22.469)\mathbf{a}_z \text{ V/m} = 10.8\mathbf{a}_y + 36.9\mathbf{a}_z \text{ V/m}$$

D2.6. Three infinite uniform sheets of charge are located in free space as follows: 3 nC/m^2 at $z = -4$, 6 nC/m^2 at $z = 1$, and -8 nC/m^2 at $z = 4$. Find **E** at the point: (*a*) $P_A(2, 5, -5)$; (b) $P_B(4, 2, -3)$; (*c*) $P_C(-1, -5, 2)$; (*d*) $P_D(-2, 4, 5)$.

(*a*) Obtaining first the electric field due to charges,

$$\mathbf{E}_{z=-4} = \frac{\rho_s}{2\epsilon_0}\mathbf{a}_N = \frac{3\times10^{-9}}{2\epsilon_0}\mathbf{a}_z = 169.41\mathbf{a}_z \text{ V/m}$$

$$\mathbf{E}_{z=1} = \frac{6\times10^{-9}}{2\epsilon_0}\mathbf{a}_z = 338.82\mathbf{a}_z \text{ V/m}$$

$$\mathbf{E}_{z=4} = \frac{-8\times10^{-9}}{2\epsilon_0}\mathbf{a}_z = -451.76\mathbf{a}_z \text{ V/m}$$

Since the point A is located below all the sheets of charge, all directions of electric field **E** are negative $\mathbf{a}_z$ direction with relative to normal, which is $\mathbf{a}_N$, of P_A. Combining together,

$$\mathbf{E} = -\mathbf{E}_1 - \mathbf{E}_2 - \mathbf{E}_3 = -169.41\mathbf{a}_z - 338.82\mathbf{a}_z - (-451.76\mathbf{a}_z) \text{ V/m} = -56.5\mathbf{a}_z \text{ V/m}$$

(*b*) The location of point B suggests that the electric field **E** contributed by the surface charge at $z = -4$ will be in positive $\mathbf{a}_z$ direction while others are negative. Combining together,

$$\mathbf{E} = \mathbf{E}_1 - \mathbf{E}_2 - \mathbf{E}_3 = 282.3\mathbf{a}_z \text{ V/m}$$

(*c*) Same arguments in (*b*), only $z = 4$ will be in negative $\mathbf{a}_z$ direction. Combining together,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 - \mathbf{E}_3 = 960\mathbf{a}_z \text{ V/m}$$

(*d*) We know that the point D is located above all the sheets of charge and all electric fields **E** are positive $\mathbf{a}_z$ direction. Combining together,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = 56.5\mathbf{a}_z \text{ V/m}$$

D2.7. Find the equation of that streamline that passes through the point $P(1, 4, -2)$ in the field $\mathbf{E} =$ (*a*) $\dfrac{-8x}{y}\mathbf{a}_x + \dfrac{4x^2}{y^2}\mathbf{a}_y$; (*b*) $2e^{5x}[y(5x+1)\mathbf{a}_x + x\mathbf{a}_y]$.

(*a*) $$\frac{dy}{dx} = \frac{\mathbf{E}_y}{\mathbf{E}_x} = \frac{4x^2/y^2}{-8x/y} = -\frac{x}{2y}$$

$$2ydy = -xdx$$

$$\int 2ydy = \int -xdx$$

$$y^2 = -\frac{x^2}{2}+\frac{C^2}{2}$$

For $P(x = 1, y = 4, z = -2)$,

$$4^2 = -\frac{1^2}{2}+\frac{C^2}{2};\ C^2 = 33$$

$x^2 + 2y^2 = 33$

(*b*) $$\frac{dy}{dx} = \frac{x}{y\,(5x+1)}$$

$$\int ydy = \int \frac{x}{5x+1}dx$$

Letting $u = 5x + 1$ and $du = 5\ dx$, and then $x = (u - 1)/5$,

$$\frac{y^2}{2} = \int \frac{u-1}{5}\left(\frac{1}{u}\right)\left(\frac{du}{5}\right) = \int \frac{u-1}{25u}du = \int \frac{1}{25}du - \int \frac{1}{25u}du = \frac{1}{25}(u - \ln|u|)$$

$$\frac{y^2}{2} = \frac{1}{25}[(5x+1) - \ln|5x+1|] + \frac{C^2}{2}$$

For $P(x = 1, y = 4, z = -2)$,

$$\frac{4^2}{2} = \frac{1}{25}[(5(1)+1) - \ln|5(1)+1|] + \frac{C^2}{2};\ C^2 = 15.66$$

$$\frac{y^2}{2} = 0.04(5x+1) - 0.04\ln|5x+1| + \frac{15.66}{2}$$

$y^2 = 15.7 + 0.4x - 0.08\ln|5x + 1|$

CHAPTER 3

D3.1. Given a 60-μC point charge located at the origin, find the total electric flux passing through: (*a*) that portion of the sphere $r = 26$ cm bounded by $0 < \theta < \pi/2$ and $0 < \phi < \pi/2$; (*b*) the closed surface defined by $\rho = 26$ cm and $z = \pm 26$ cm; (*c*) the plane $z = 26$ cm.

(*a*) $$\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r$$

$$d\Psi = \frac{Q}{4\pi r^2}\mathbf{a}_r(r^2\sin\theta\,d\theta\,d\phi)\mathbf{a}_r = \frac{Q}{4\pi}\sin\theta\,d\theta\,d\phi$$

$$\Psi = \frac{Q}{4\pi}\int_0^{\pi/2}\int_0^{\pi/2}\sin\theta\,d\theta\,d\phi = \frac{Q}{4\pi}\int_0^{\pi/2}\sin\theta\,d\theta\int_0^{\pi/2}d\phi = \frac{Q}{4\pi}\left(-\cos\theta\Big|_0^{\pi/2}\right)\left(\phi\Big|_0^{\pi/2}\right)$$

$$= \frac{60\times10^{-6}}{4\pi}(-\cos 90° - (-\cos 0°))((\pi/2) - 0) = 7.5\ \mu\text{C}$$

(*b*) Deriving Gauss's law of the cylinder,

$$Q = \oint \mathbf{D}_S \cdot d\mathbf{S} = D_S\oint \rho\,d\phi\,dz = D_S\rho\int_0^{\pi}d\phi\int_0^{L}dz = D_S\rho\,\phi\Big|_0^{2\pi}z\Big|_0^{L} = D_S 2\pi\rho L$$

$$D_S = D_\rho = \frac{Q}{2\pi\rho L} \quad \text{or} \quad \mathbf{D}_\rho = \frac{Q}{2\pi\rho L}\mathbf{a}_\rho$$

Solving Ψ,

$$d\Psi = \frac{Q}{2\pi\rho L}\mathbf{a}_\rho(\rho\,d\phi\,dz)\mathbf{a}_\rho = \frac{Q}{2\pi\rho L}\rho\,d\phi\,dz = \frac{Q}{2\pi\rho L}\rho\int_0^{2\pi}d\phi\int_0^{L}dz = \frac{Q}{2\pi\rho L}(2\pi\rho L)$$

$\Psi = Q = 60\ \mu\text{C}$

(*c*) Getting the equation in (*b*) with changing the limits of ϕ as $0 \le \phi \le \pi$ because only the flux lines pass in half through the plane,

$$d\Psi = \frac{Q}{2\pi\rho L}\rho\int_0^{\pi}d\phi\int_0^{L}dz = \frac{Q}{2\pi\rho L}(\pi\rho L)$$

$\Psi = Q/2 = 30\ \mu\text{C}$

D3.2. Calculate **D** in rectangular coordinates at point $P(2, -3, 6)$ produced by: (*a*) a point charge $Q_A = 55$ mC at $Q(-2, 3, -6)$; (*b*) a uniform line charge $\rho_{LB} = 20$ mC/m on the axis; (*c*) a uniform surface charge density $\rho_{SC} = 120\ \mu\text{C/m}^2$ on the plane $z = -5$ m.

(*a*) $\mathbf{D} = \epsilon_0\mathbf{E} = \dfrac{Q}{4\pi R^2}\mathbf{a}_R = \dfrac{Q}{4\pi R^3}\mathbf{R}$

$\mathbf{R}_{QP} = (2-(-2))\mathbf{a}_x + (-3-3)\mathbf{a}_y + (6-(-6))\mathbf{a}_z = 4\mathbf{a}_x - 6\mathbf{a}_y + 12\mathbf{a}_z$

$R_{QP} = 14$

$$\mathbf{D}_{QP} = \frac{Q}{4\pi R_{QP}^3}\mathbf{R}_{QP} = \frac{55\times10^{-3}}{4\pi 14^3}(4\mathbf{a}_x - 6\mathbf{a}_y + 12\mathbf{a}_z) = 6.38\mathbf{a}_x - 9.57\mathbf{a}_y + 19.14\mathbf{a}_z\ \mu\text{C/m}^2$$

(*b*) $\mathbf{D} = \epsilon_0\mathbf{E} = \dfrac{\rho_L}{2\pi R}\mathbf{a}_R = \dfrac{\rho_L}{2\pi R^2}\mathbf{R}$

$\mathbf{R}_x = (2-x)\mathbf{a}_x - 3\mathbf{a}_y + 6\mathbf{a}_z$

Since the infinite line charge density is along x axis, the electric field E at point P is having only y and z components present. Canceling x component due to symmetry,

$\mathbf{R}_x = -3\mathbf{a}_y + 6\mathbf{a}_z$

$R_x = \sqrt{45}$

$$\mathbf{D}_x = \frac{\rho_L}{2\pi R_x^2}\mathbf{R}_x = \frac{20\times10^{-3}}{2\pi\left(\sqrt{45}\right)^2}(-3\mathbf{a}_y + 6\mathbf{a}_z) = -212\mathbf{a}_y + 424\mathbf{a}_z\ \mu\text{C/m}^2$$

(*c*) $\mathbf{D} = \epsilon_0\mathbf{E} = \dfrac{\rho_S}{2}\mathbf{a}_R$

The infinite surface change density is an infinite x-y plane located at $z = -5$ and the charge is spread on that plane. Going back the formula,

$\mathbf{D} = (\rho_S/2)\mathbf{a}_z = (120/2)\ \mu\mathbf{a}_z = 60\mathbf{a}_z\ \mu\text{C/m}^2$

D3.3. Given the electric flux density, $\mathbf{D} = 0.3r^2\mathbf{a}_r$ nC/m^2 in free space: (*a*) find **E** at point $P(r = 2, \theta = 25°, \phi = 90°)$; (*b*) find the total charge within the sphere $r = 3$; (*c*) find the total electric flux leaving the sphere $r = 4$.

(*a*) $\mathbf{E} = \mathbf{D}/\epsilon_0 = (0.3\times10^{-9}r^2\mathbf{a}_r)/\epsilon_0 = (0.3\times10^{-9}(2^2)\mathbf{a}_r)/\epsilon_0 = 135.5\mathbf{a}_r$ V/m

(*b*) $Q = \oint \mathbf{D}\cdot d\mathbf{S} = 0.3\times10^{-9}r^2\mathbf{a}_r\int_0^{2\pi}\int_0^{\pi} r^2\sin\theta\, d\theta\, d\phi\ \mathbf{a}_r = 0.3\times10^{-9}r^2(4\pi r^2)$

$= 0.3\times10^{-9}(4\pi r^4) = 0.3\times10^{-9}(4\pi(3^4)) = 305$ nC

(*c*) Same procedure in (*b*),

$\Psi = Q = 1.2\times10^{-9}\pi r^4 = 1.2\times10^{-9}\pi(4)^4 = 965$ nC

D3.4. Calculate the total electric flux leaving the cubical surface formed by the six planes $x, y, z = \pm5$ if the charge distribution is: (*a*) two point charges, 0.1 μC at $(1, -2, 3)$ and 1/7 μC at $(-1, 2, -2)$; (*b*) a uniform line charge of π μC/m at $x = -2$, $y = 3$; (*c*) a uniform surface charge of 0.1 μC/m^2 on the plane $y = 3x$.

(*a*) Both given charges are enclosed by the cubical volume according to Gauss's law. Adding,

$\Psi = Q_1 + Q_2 = 0.1\ \mu\text{C} + (1/7)\ \mu\text{C} = 0.243\ \mu\text{C}$

(*b*) The line charge distribution passes through $x = -2$ and $y = 3$; it is parallel to z axis. The total length of that charge distribution enclosed by the given cubical volume is 10 units as $z = \pm5$. Applying Gauss's law of line charge,

$\Psi = \rho_L L = \pi(10)\ \mu\text{C} = 31.4\ \mu\text{C}$

(*c*) A straight line equation, $y = 3x$, passes through the origin. We need to find the length of that line which is enclosed by the given volume. The length is moving up and down along z axis as $z = \pm5$ to a form a plane. Putting $y = 5$,

$5 = 3; x = 5/3$

Finding the length of that line on the plane formed by positive x and y axes,

$\sqrt{5^2 + (5/3)^2} = 5.27$

The same length we get on the plane formed by negative x and y axes. Adding two lengths,

$5.27 + 5.27 = 10.54$

That straight line moves between $z = \pm5$ to form a plane. Knowing the area,

$10(10.54) = 105.4$

Applying Gauss's law of surface charge,

$\Psi = \rho_S S = (0.1\ \mu\text{C})105.4 = 10.54\ \mu\text{C}$

D3.5. A point charge of 0.25 μC is located at $r = 0$, and uniform surface charge densities are located as follows: 2 mC/m^2 at $r = 1$ cm, and –0.6 mC/m^2 at $r = 1.8$ cm. Calculate **D** at: (*a*) $r = 0.5$ cm; (*b*) $r = 1.5$ cm; (*c*) $r = 2.5$ cm. (*d*) What uniform charge density should be established at $r = 3$ cm to cause **D** = 0 at $r = 3.5$ cm?

Note: Always remember the radius r within the charge Q and charge density D are different. Assume them as $r_{(Q)}$ and $r_{(D)}$.

(*a*) $\mathbf{D} = \dfrac{Q_1}{4\pi r_{(D)}^2}\mathbf{a}_r = \dfrac{0.25\times10^{-6}}{4\pi(0.5\times10^{-2})^2}\mathbf{a}_r = 796\mathbf{a}_r\ \mu\text{C}$

(*b*) Finding the second charge Q_2,

$Q_2 = \rho_S S = \rho_S(4\pi r_{(Q)}^2) = 2\times10^{-3}(4\pi(1\times10^{-2})^2) = 2.513\times10^{-6}$ C

$$\mathbf{D} = \frac{Q_1+Q_2}{4\pi r_{(D)}^2}\mathbf{a}_r = \frac{0.25\times10^{-6}+2.513\times10^{-6}}{4\pi(1.5\times10^{-2})^2}\mathbf{a}_r = 977\mathbf{a}_r\ \mu\text{C}$$

(*c*) Finding the third charge Q_3,

$Q_3 = \rho_S(4\pi r_{(Q)}^2) = -0.6\times10^{-3}(4\pi(1.8\times10^{-2})^2) = -2.443\times10^{-6}$ C

$$\mathbf{D} = \frac{Q_1+Q_2+Q_3}{4\pi r_{(D)}^2}\mathbf{a}_r = \frac{0.25\times10^{-6}+2.513\times10^{-6}-2.443\times10^{-6}}{4\pi(2.5\times10^{-2})^2}\mathbf{a}_r = 40.8\mathbf{a}_r\ \mu\text{C}$$

(*d*) $$0 = \mathbf{D} = \frac{Q_1+Q_2+Q_3+Q_4}{4\pi r_{(D)}^2}\mathbf{a}_r = \frac{Q_1+Q_2+Q_3+\rho_S S}{4\pi r_{(D)}^2}\mathbf{a}_r = \frac{0.32\times10^{-6}+\rho_S 4\pi r_{(Q)}^2}{4\pi r_{(D)}^2}\mathbf{a}_r$$

$0 = (0.32\times10^{-6}+4\rho_S\pi r_{(Q)}^2)\mathbf{a}_r$

$-0.32\times10^{-6} = 4\rho_S\pi r_{(Q)}^2$

$$\rho_S = -\frac{0.32\times10^{-6}}{4\pi r_{(Q)}^2} = -\frac{0.32\times10^{-6}}{4\pi(3\times10^{-2})^2} = -28.3\ \mu\text{C/m}^2$$

D3.6. In free space, let $\mathbf{D} = 8xyz^4\mathbf{a}_x + 4x^2z^4\mathbf{a}_y + 16x^2yz^3\mathbf{a}_z$ pC/m^2. (*a*) Find the total electric flux passing through the rectangular surface $z = 2$, $0 < x < 2$, $1 < y < 3$, in the $\mathbf{a}_z$ direction. (*b*) Find **E** at $P(2, -1, 3)$. (*c*) Find an approximate value for the total charge contained in an incremental sphere located at $P(2, -1, 3)$ and having a volume of 10^{-12} m^3.

(*a*) $\Psi = \oint \mathbf{D}\bullet d\mathbf{S}$ where $d\mathbf{S} = dx\,dy\,\mathbf{a}_z$

$$= \int_1^3\int_0^2 (8xyz^4\mathbf{a}_x + 4x^2z^4\mathbf{a}_y + 16x^2yz^3\mathbf{a}_x)\bullet dxdy\mathbf{a}_z = \int_1^3\int_0^2 16x^2yz^3\,dxdy$$

$$= z^3\int_1^3 y\,dy\int_0^2 16x^2\,dx = z^3(y^2/2)\Big|_1^3(16x^3/3)\Big|_0^2 = [16z^3(4)(8/3)]_{z=2} = 1365\text{ pC}$$

(*b*) $\mathbf{E} = \mathbf{D}/\epsilon_0 = [((8xyz^4\mathbf{a}_x + 4x^2z^4\mathbf{a}_y + 16x^2yz^3\mathbf{a}_z)\times10^{-12})/\epsilon_0]_{x=2,\,y=-1,\,z=3}$

$= -146.4\mathbf{a}_x + 146.4\mathbf{a}_y - 195.2\mathbf{a}_z$ V/m

(*c*) Q = Charge enclosed in volume $\Delta v \doteq \left(\dfrac{\partial D_x}{\partial x}+\dfrac{\partial D_y}{\partial y}+\dfrac{\partial D_z}{\partial z}\right)\times\Delta v$

Finding the partial derivatives of each term,

$\partial D_x/\partial x = \partial(8xyz^4)/\partial x = (8yz^4)_{x=2,\,y=-1,\,z=3} = -648$

$\partial D_y/\partial y = \partial(4x^2z^4)/\partial y = 0$

$\partial D_z/\partial z = \partial(16x^2yz^3)/\partial z = (48x^2yz^2)_{x=2,\,y=-1,\,z=3} = -1728$

$Q = (-648-1728)\times10^{-12}\,\Delta v = [(-648-1728)\times10^{-12}](10^{-12}) = -2.38\times10^{-21}$ C

D3.7. In each of the following parts, find a numerical value for div **D** at the point specified: (*a*) $\mathbf{D} = (2xyz - y^2)\mathbf{a}_x + (x^2z - 2xy)\mathbf{a}_y + x^2y\mathbf{a}_z$ C/m^2 at $P_A(2, 3, -1)$; (*b*) $\mathbf{D} = 2\rho z^2\sin^2\phi\,\mathbf{a}_\rho + \rho z^2\sin 2\phi\,\mathbf{a}_\phi + 2\rho^2 z\sin^2\phi\,\mathbf{a}_z$ C/m^2 at $P_B(\rho = 2, \phi = 110°, z = -1)$; (*c*) $\mathbf{D} = 2r\sin\theta\cos\phi\,\mathbf{a}_r + r\cos\theta\cos\phi\,\mathbf{a}_\theta - r\sin\phi\,\mathbf{a}_\phi$ C/m^2 at $P_C(r = 1.5, \theta = 30°, \phi = 50°)$.

(*a*) div $\mathbf{D} = \partial D_x/\partial x + \partial D_y/\partial y + \partial D_z/\partial z$

Finding the partial derivatives of each term,

$\partial D_x/\partial x = \partial(2xyz - y^2)/\partial x = 2yz$

$$\partial D_y/\partial y \;=\; \partial(x^2z-2xy)/\partial y \;=\; -2x$$

$$\partial D_z/\partial z \;=\; \partial(x^2y)/\partial z \;=\; 0$$

$$\text{div}\,\mathbf{D} \;=\; (2yz-2x)_{x=2,\,y=3,\,z=-1} \;=\; -10$$

(*b*) $$\text{div}\,\mathbf{D} \;=\; \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial\rho}+\frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi}+\frac{\partial D_z}{\partial z}$$

Finding the partial derivatives of each term,

$$\frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial\rho} = \frac{1}{\rho}\frac{\partial(2\rho^2z^2\sin^2\phi)}{\partial\rho} = (4z^2\sin^2\phi)_{\rho=2,\phi=110°,z=-1} = 3.532$$

$$\frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} = \frac{1}{\rho}\frac{\partial(\rho z^2\sin 2\phi)}{\partial\phi} = (2z^2\cos 2\phi)_{\rho=2,\phi=110°,z=-1} = -1.532$$

$$\frac{\partial D_z}{\partial z} = \frac{\partial(2\rho^2z\sin^2\phi)}{\partial\rho} = (2\rho^2\sin^2\phi)_{\rho=2,\phi=110°,z=-1} = 7.064$$

$$\text{div}\,\mathbf{D} \;=\; 3.532+(-1.532)+7.064 \;=\; 9.06$$

(*c*) $$\text{div}\,\mathbf{D} \;=\; \frac{1}{r^2}\frac{\partial(r^2D_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(\sin\theta D_\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$\frac{1}{r^2}\frac{\partial(r^2D_r)}{\partial r} = \frac{1}{r^2}\frac{\partial(2r^3\sin\theta\cos\phi)}{\partial r} = (6\sin\theta\cos\phi)_{r=1.5,\theta=30°,\phi=50°} = 1.928$$

$$\frac{1}{r\sin\theta}\frac{\partial(\sin\theta D_\theta)}{\partial\theta} = \frac{1}{r\sin\theta}\frac{\partial(r\sin\theta\cos\theta\cos\phi D_\theta)}{\partial\theta}$$

Applying product rule of the derivative of $\sin\theta\cos\theta$,

$$= \frac{\cos\phi}{\sin\theta}(\cos^2\theta-\sin^2\theta)_{r=1.5,\theta=30°,\phi=50°} = 0.643$$

$$\frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi} = \frac{1}{r\sin\theta}\frac{\partial(-r\sin\phi)}{\partial\phi} = (-\cos\phi/\sin\theta)_{r=1.5,\theta=30°,\phi=50°} = -1.286$$

$$\text{div}\,\mathbf{D} \;=\; 1.928+0.643-1.286 \;=\; 1.29$$

D3.8. Determine an expression for the volume charge density associated with each $\mathbf{D}$ field: (*a*) $\mathbf{D} = \frac{4xy}{z}\,\mathbf{a}_x+\frac{2x^2}{z}\,\mathbf{a}_y-\frac{2x^2y}{z^2}\,\mathbf{a}_z$; (*b*) $\mathbf{D} = z\sin\phi\,\mathbf{a}_\rho + z\cos\phi\,\mathbf{a}_\phi + \rho\sin\phi\,\mathbf{a}_z$; (*c*) $\mathbf{D} = \sin\theta\sin\phi\,\mathbf{a}_r + \cos\theta\sin\phi\,\mathbf{a}_\theta + \cos\phi\,\mathbf{a}_\phi$.

Note: The volume charge density ρ_v is equal to div $\mathbf{D}$.

(*a*) $$\partial D_x/\partial x \;=\; \partial(4xy/z)/\partial x \;=\; 4y/z$$

$$\partial D_y/\partial y \;=\; \partial(2x^2/z)/\partial y \;=\; 0$$

$$\partial D_z/\partial z \;=\; \partial(-2x^2y/z^2)/\partial z \;=\; 4x^2y/z^3$$

$$\rho_v \;=\; 4y/z \;+\; 4x^2y/z^3 \;=\; (4y/z^3)(x^2+z^2)$$

(*b*) $$(1/\rho)\partial(\rho D_\rho)/\partial\rho \;=\; (1/\rho)\,\partial(\rho z\sin\phi)/\partial\rho \;=\; (z\sin\phi)/\rho$$

$$(1/\rho)\partial D_\phi/\partial\phi \;=\; (1/\rho)\,\partial(z\cos\phi)/\partial\phi \;=\; (-z\sin\phi)/\rho$$

$$\partial D_z/\partial z \;=\; \partial(\rho\sin\phi)/\partial z \;=\; 0$$

$$\rho_v \;=\; (z\sin\phi)/\rho \;+\; (-z\sin\phi)/\rho \;=\; 0$$

(*c*) $$(1/r^2)\partial(r^2D_r)/\partial r \;=\; (1/r^2)\,\partial(r^2\sin\theta\sin\phi)/\partial r \;=\; (2\sin\theta\sin\phi)/r$$

$$(1/r\sin\theta)\partial(\sin\theta D_\theta)/\partial\theta \;=\; (1/r\sin\theta)\partial(\sin\theta\cos\theta\sin\phi)/\partial\theta$$

$$=\; (\sin\phi/r\sin\theta)(\cos^2\theta-\sin^2\theta)$$

$$(1/r\sin\theta)\partial D_\phi/\partial\phi \;=\; (1/r\sin\theta)\,\partial(\cos\phi)/\partial\phi \;=\; -\sin\phi/(r\sin\theta)$$

$$\rho_v \;=\; (2\sin\theta\sin\phi)/r \;+\; (\sin\phi/r\sin\theta)(\cos^2\theta-\sin^2\theta) \;+\; (-\sin\phi/(r\sin\theta))$$

$$= \frac{2\sin^2\theta\sin\phi+\cos^2\theta\sin\phi-\sin^2\theta\sin\phi-\sin\phi}{r\sin\theta} = \frac{\sin^2\theta\sin\phi+\cos^2\theta\sin\phi-\sin\phi}{r\sin\theta}$$

Since $\sin^2\theta+\cos^2\theta \;=\; 1$,

$$= \frac{\sin\phi(\sin^2\theta+\cos^2\theta)-\sin\phi}{r\sin\theta} = \frac{0}{r\sin\theta} = 0$$

D3.9. Given the field $\mathbf{D} = 6\rho \sin(1/2)\phi\ \mathbf{a}_\rho + 1.5\rho \cos(1/2)\phi\ \mathbf{a}_\phi$ C/m^2, evaluate both sides of the divergence theorem for the region bounded by $\rho = 2$, $\phi = 0$, $\phi = \pi$, $z = 0$, and $z = 5$.

Using the divergence theorem with the formula: $\oint_S \mathbf{D} \bullet d\mathbf{S}$,

$$\begin{aligned}
\oint_S \mathbf{D} \bullet d\mathbf{S} &= \oint_S (6\rho \sin(\phi/2)\mathbf{a}_\rho + 1.5\rho\cos(\phi/2)\mathbf{a}_\phi \bullet (\rho\, d\phi\, dz\mathbf{a}_\rho - d\rho\, dz\mathbf{a}_\phi) \\
&= \oint_S 6\rho\sin(\phi/2)\,\rho\, d\phi\, dz - \oint_S 1.5\rho\cos(\phi/2)\, d\rho\, dz \\
&= \int_0^\pi\int_0^5 6\rho\sin(\phi/2)\,\rho\, d\phi\, dz - \int_0^2\int_0^5 1.5\rho\cos(\phi/2)\, d\rho\, dz \\
&= 6\rho^2\int_0^\pi \sin(\phi/2)\, d\phi\int_0^5 dz - (1.5\cos(\phi/2))\int_0^2 \rho\, d\rho\int_0^5 dz \\
&= 24\left(-2\cos\phi/2\Big|_0^\pi\right)\left(z\Big|_0^5\right) - (1.5\cos\phi/2)\left(\rho^2/2\Big|_0^2\right)\left(z\Big|_0^5\right)
\end{aligned}$$

Since the second surface lies at $\phi = 0°$,

$$= 24(2)(5) - (1.5\cos\phi/2)(2)(5) = 240 - 15 = 225$$

Using another divergence theorem with the formula: $\int_{\text{vol}} \Delta \bullet \mathbf{D}\, dv$ but solving first the $\nabla \bullet \mathbf{D}$,

$$\begin{aligned}
\nabla \bullet \mathbf{D} &= \text{div}\,\mathbf{D} = (1/\rho)\partial(\rho D_\rho)/\partial\rho + (1/\rho)\partial D_\phi/\partial\phi + \partial D_z/\partial z \\
&= (1/\rho)\partial(6\rho^2\sin\phi/2)/\partial\rho + (1/\rho)\partial(1.5\rho\cos\phi/2)/\partial\phi + 0 \\
&= 12\sin\phi/2 + (-1.5/2)\sin\phi/2 = 11.25\sin\phi/2
\end{aligned}$$

$$\begin{aligned}
\int_{\text{vol}} \nabla \bullet \mathbf{D}\, dv &= \int_0^2\int_0^\pi\int_0^5 (11.25\sin\phi/2)\rho\, d\rho\, d\phi\, dz = 11.25\int_0^2 \rho\, d\rho\int_0^\pi (\sin\phi/2)\, d\phi\int_0^5 dz \\
&= 11.25\left(\rho^2/2\Big|_0^2\right)\left(-2\cos\phi/2\Big|_0^\pi\right)\left(z\Big|_0^5\right) = 11.25(2)(2)(5) = 225
\end{aligned}$$

CHAPTER 4

D4.1. Given the electric field $\mathbf{E} = (1/z^2)(8xyz\mathbf{a}_x + 4x^2z\mathbf{a}_y - 4x^2y\mathbf{a}_z)$ V/m, find the differential amount of work done in moving a 6-nC charge a distance of 2 μm, starting at $P(2, -2, 3)$ and proceeding in the direction $\mathbf{a}_L = (a)\ -6/7\mathbf{a}_x + 3/7\mathbf{a}_y + 2/7\mathbf{a}_z$; $(b)\ 6/7\mathbf{a}_x - 3/7\mathbf{a}_y - 2/7\mathbf{a}_z$; $(c)\ 3/7\mathbf{a}_x + 6/7\mathbf{a}_y$.

(*a*) $dW = -Q\mathbf{E} \bullet d\mathbf{L}$

Finding first the differential length $d\mathbf{L}$,

$$\begin{aligned}
d\mathbf{L} &= \mathbf{a}_L \bullet dL = (-6/7\mathbf{a}_x + 3/7\mathbf{a}_y + 2/7\mathbf{a}_z)(2\times10^{-6}) = (-12/7\mathbf{a}_x + 6/7\mathbf{a}_y + 4/7\mathbf{a}_z)(1\times10^{-6}) \\
dW &= -(6\times10^{-9})(1/z^2)(8xyz\mathbf{a}_x + 4x^2z\mathbf{a}_y - 4x^2y\mathbf{a}_z) \bullet (-12/7\mathbf{a}_x + 6/7\mathbf{a}_y + 4/7\mathbf{a}_z)(1\times10^{-6}) \\
&= -6\times10^{-15}[(1/z^2)((-96/7)xyz + (24/7)x^2z - (16/7)x^2y)]_{x=2,\,y=-2,\,z=3} = -6\times10^{-15}(224/9) \\
&= -149.3 \text{ fJ}
\end{aligned}$$

(*b*) Same procedure in (*a*),

$$\begin{aligned}
d\mathbf{L} &= (12/7\mathbf{a}_x - 6/7\mathbf{a}_y + 4/7\mathbf{a}_z)(1\times10^{-6}) \\
dW &= -(6\times10^{-9})(1/z^2)(8xyz\mathbf{a}_x + 4x^2z\mathbf{a}_y - 4x^2y\mathbf{a}_z) \bullet (12/7\mathbf{a}_x - 6/7\mathbf{a}_y + 4/7\mathbf{a}_z)(1\times10^{-6}) \\
&= -6\times10^{-15}[(1/z^2)((96/7)xyz - (24/7)x^2z - (16/7)x^2y)]_{x=2,\,y=-2,\,z=3} = -6\times10^{-15}(-224/9) \\
&= 149.3 \text{ fJ}
\end{aligned}$$

(*c*) Same procedure in (*a*) and (*b*),

$$\begin{aligned}
d\mathbf{L} &= (6/7\mathbf{a}_x + 12/7\mathbf{a}_y + 0\mathbf{a}_z)(1\times10^{-6}) \\
dW &= -(6\times10^{-9})(1/z^2)(8xyz\mathbf{a}_x + 4x^2z\mathbf{a}_y - 4x^2y\mathbf{a}_z) \bullet (6/7\mathbf{a}_x + 12/7\mathbf{a}_y + 0\mathbf{a}_z)(1\times10^{-6}) \\
&= -6\times10^{-15}[(1/z^2)((48/7)xyz + (48/7)x^2z]_{x=2,\,y=-2,\,z=3} = -6\times10^{-15}(0) \\
&= 0
\end{aligned}$$

D4.2. Calculate the work done in moving a 4-C charge from $B(1, 0, 0)$ to $A(0, 2, 0)$ along the path $y = 2 - 2x$, $z = 0$ in the field $\mathbf{E} = (a)\ 5\mathbf{a}_x$ V/m; $(b)\ 5x\mathbf{a}_x$ V/m; $(c)\ 5x\mathbf{a}_x + 5y\mathbf{a}_y$ V/m.

(*a*) $W = -Q\int_B^A \mathbf{E} \bullet d\mathbf{L}$ where $d\mathbf{L} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$

$$= -4\int_1^0 (5\mathbf{a}_x + 0\mathbf{a}_y + 0\mathbf{a}_z) \bullet (dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z) = -4\int_1^0 5\, dx = -20x\Big|_1^0 = -20(-1) = 20 \text{ J}$$

(*b*) Same procedure in (*a*),

$$W = -4\int_1^0 5x\, dx = -20(x^2/2)\Big|_1^0 = -20(-0.5) = 10 \text{ J}$$

(*c*) Same procedure in (*a*) and (*b*),

$$W = -4\int_{(1,0)}^{(0,2)}(5x\,dx+5y\,dy) = -20\left(\frac{x^2}{2}+\frac{y^2}{2}\Bigg|_{(1,0)}^{(0,2)}\right) = -20(1.5) = -30\text{ J}$$

D4.3. We will see later that a time-varying **E** field need not be conservative. (If it is not conservative, the work expressed by equation: $W=-Q\int_{\text{initial}}^{\text{final}}\mathbf{E}\bullet d\mathbf{L}$ may be a function of the path used.) Let $\mathbf{E}=y\mathbf{a}_x$ V/m at a certain instant of time, and calculate the work required to move a 3-C charge from (1, 3, 5) to (2, 0, 3) along the straight-line segments joining: (*a*) (1, 3, 5) to (2, 3, 5) to (2, 0, 5) to (2, 0, 3); (*b*) (1, 2, 5) to (1, 3, 3) to (1, 0, 3) to (2, 0, 3).

(*a*) Use the formula in D4.2 (*a*). One differential unit vector of each line segments is used. For segment (1, 3, 5) to (2, 3, 5),

$$W_1 = -3\int_1^2(y\,\mathbf{a}_x+0\mathbf{a}_y+0\mathbf{a}_z)\bullet dx\,\mathbf{a}_x = -3y\int_1^3 dx = -3y\left(x\Big|_1^2\right)_{y=3} = -9\text{ J}$$

For segment (2, 3, 5) to (2, 0, 5),

$$W_2 = -3\int_1^2(y\,\mathbf{a}_x+0\mathbf{a}_y+0\mathbf{a}_z)\bullet dy\,\mathbf{a}_y = 0$$

For segment (2, 0, 5) to (2, 0, 3),

$$W_3 = -3\int_1^2(y\,\mathbf{a}_x+0\mathbf{a}_y+0\mathbf{a}_z)\bullet dz\,\mathbf{a}_z = 0$$

Adding the total work, $W = W_1 + W_2 + W_3 = -9$ J

(*b*) Same procedure in (*a*). For segment (1, 2, 5) to (1, 3, 3),

$$W_1 = -3y\int_1^1 dx = -3y\left(x\Big|_1^2\right)_{y=3} = 0$$

Both W_2 and W_3 are zeroes no matter what the line segments are. So adding the total work, $W=0$.

D4.4. An electric field is expressed in rectangular coordinates by $\mathbf{E} = 6x^2\mathbf{a}_x + 6y\mathbf{a}_y + 4\mathbf{a}_z$ V/m. Find: (*a*) V_{MN} if points M and N are specified by $M(2, 6, -1)$ and $N(-3, -3, 2)$; (*b*) V_M if $V = 0$ at $Q(4, -2, -35)$; (*c*) V_N if $V = 2$ at $P(1, 2, -4)$.

$$(a)\ V_{MN} = -\int_N^M\mathbf{E}\bullet d\mathbf{L} = -\int_{(-3,-3,2)}^{(2,6,-1)}(6x^2\,dx+6y\,dy+4dz) = -(2x^2+3y^2+4z)\Big|_{(-3,-3,2)}^{(2,6,-1)}$$

$$= -(70+81-12) = -139\text{ V}$$

$$(b)\ V_{MQ} = -\int_Q^M\mathbf{E}\bullet d\mathbf{L} = -(2x^2+3y^2+4z)\Big|_{(4,-2,-35)}^{(2,6,-1)} = -(-112+96+136) = -120\text{ V}$$

$$V_{MQ} = V_M - V_Q$$
$$V_M = V_{MQ} + V_Q = -120\text{ V} + 0 = -120\text{ V}$$

$$(c)\ V_{NP} = -\int_P^N\mathbf{E}\bullet d\mathbf{L} = -(2x^2+3y^2+4z)\Big|_{(1,2,-4)}^{(-3,-3,2)} = -(-56+15+24) = 17\text{ V}$$

$$V_{NP} = V_N - V_Q$$
$$V_N = V_{NP} + V_P = 17\text{ V} + 2\text{ V} = 19\text{ V}$$

D4.5. A 15-nC point charge is at the origin in free space. Calculate V_1 if point P_1 is located at $P_1(-2, 3, -1)$ and (a) $V = 0$ at (6, 5, 4); (*b*) $V = 0$ at infinity; (*c*) $V = 5$ V at (2, 0, 4).

(*a*) Assume that second point is M. Applying the potential difference V_{AB}, or V_{PM} in the problem,

$$V_{PM} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_P}-\frac{1}{R_M}\right)$$

$$R_P = \sqrt{(-2)^2+3^2+(-1)^2} = \sqrt{14}$$

$$R_M = \sqrt{6^2+5^2+4^2} = \sqrt{77}$$

$$V_{PM} = \frac{15\times10^{-9}}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{14}}-\frac{1}{\sqrt{77}}\right) = 20.67\text{ V}$$

$$V_{PM} = V_P - V_M$$
$$V_P = V_{PM} + V_M = 20.67\text{ V} + 0 = 20.67\text{ V}$$

(*b*) Same procedure in (*a*),

$$R_P = \sqrt{14}$$
$$R_M = \infty$$

$$V_{PM} = \frac{15\times10^{-9}}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{14}}-\frac{1}{\infty}\right) = 36\text{ V}$$

$$V_P = V_{PM} + V_M = 36 \text{ V} + 0 = 36 \text{ V}$$

(*c*) Same procedure in (*a*) and (*b*),

$$R_P = \sqrt{14}$$

$$R_M = \sqrt{2^2 + 4^2} = 20$$

$$V_{PM} = \frac{15\times10^{-9}}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{14}} - \frac{1}{\sqrt{20}}\right) = 5.89 \text{ V}$$

$$V_P = V_{PM} + V_M = 5.89 \text{ V} + 5 \text{ V} = 10.89 \text{ V}$$

D4.6. If we take the zero reference for potential at infinity, find the potential at (0, 0, 2) caused by this charge configuration in free space (*a*) 12 nC/m on the line $\rho = 2.5$ m, $z = 0$; (*b*) point charge of 18 nC at (1, 2, –1); (*c*) 12 nC/m on the line $y = 2.5$, $z = 0$, $-1.0 < x < 1.0$.

(*a*) $V(\mathbf{r}) = \int \frac{\rho_L(\mathbf{r}')\, dL'}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|}$ where $dL' = \rho\, d\phi$

Since the distance is converted in rectangular coordinates, we can mentally solve it where $\phi = 0°$.

$$|\mathbf{r} - \mathbf{r}'| = \sqrt{(0-2.5)^2 + (2-0)^2} = 3.20$$

$$V(\mathbf{r}) = \int_0^{2\pi} \frac{12\times10^{-9}\rho\, d\phi}{4\pi\epsilon_0(3.20)} = \frac{12\times10^{-9}\rho}{4\pi\epsilon_0(3.20)}\int_0^{2\pi} d\phi = \frac{12\times10^{-9}\rho}{4\pi\epsilon_0(3.20)}\left(\phi\Big|_0^{2\pi}\right)_{\rho=2.5} = 529 \text{ V}$$

(*b*) $V(\mathbf{r}) = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_1|}$

$$|\mathbf{r} - \mathbf{r}'| = \sqrt{(0-1)^2 + (0-2)^2 + (2-(-1))^2} = \sqrt{14}$$

$$V(\mathbf{r}) = \frac{18\times10^{-9}}{4\pi\epsilon_0\sqrt{14}} = 43.2 \text{ V}$$

(*c*) Same formula in (*a*) but now $dL' = dx$. Solving the distance $|\mathbf{r} - \mathbf{r}'|$ where x is symmetric,

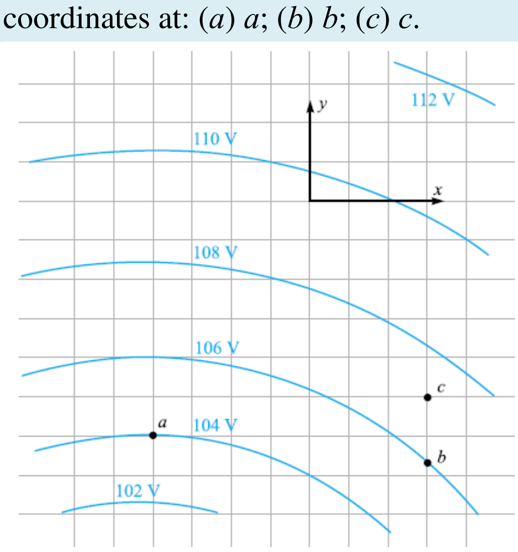
$$|\mathbf{r} - \mathbf{r}'| = \sqrt{(0-x)^2 + (0-2.5)^2 + (2-0)^2} = \sqrt{x^2 + 10.25}$$

$$V(\mathbf{r}) = \int_{-1}^{1} \frac{12\times10^{-9}}{4\pi\epsilon_0\sqrt{x^2+10.25}}\, dx = \frac{12\times10^{-9}}{4\pi\epsilon_0}\int_{-1}^{1}\frac{1}{\sqrt{x^2+10.25}}\, dx$$

Integrating $1/\sqrt{x^2+10.25}$ is solved by trigonometric substitution and yielding,

$$= \frac{12\times10^{-9}}{4\pi\epsilon_0}\left(\ln\left|\frac{x+\sqrt{x^2+10.25}}{\sqrt{10.25}}\right|_{-1}^{1}\right) = \frac{12\times10^{-9}}{4\pi\epsilon_0}(0.615) = 66.3 \text{ V}$$

D4.7. A portion of a two-dimensional ($E_z = 0$) potential field is shown the figure below. The grid lines are 1mm apart in the actual field. Determine approximate values for **E** in rectangular coordinates at: (*a*) *a*; (*b*) *b*; (*c*) *c*.



Requires graphical solution.

D4.8. Given the potential field in cylindrical coordinates, $V = [100/(z^2 + 1)]\rho\cos\phi$ V, and point P at $\rho = 3$ m, $\phi = 60°$, $z = 2$ m, find values at P for (*a*) V; (*b*) **E**; (*c*) E; (*d*) dV/dN; (*e*) $\mathbf{a}_N$; (*f*) ρ_v in free space.

(*a*) $V = [100/(z^2+1)]\rho\cos\phi = [100/(2^2+1)]3\cos 60° = 30$ V

(b) $\mathbf{E} = -\nabla V = -\left(\dfrac{\partial V}{\partial \rho}\mathbf{a}_\rho + \dfrac{1}{\rho}\dfrac{\partial V}{\partial \phi}\mathbf{a}_\phi + \dfrac{\partial V}{\partial \rho}\mathbf{a}_z\right)$

$$= -\left(\frac{100}{z^2+1}\cos\phi\ \mathbf{a}_\rho - \frac{100}{z^2+1}\sin\phi\ \mathbf{a}_\phi - \frac{200z}{(z^2+1)^2}\rho\cos\phi\ \mathbf{a}_z\right) = -(10\mathbf{a}_\rho - 17.32\mathbf{a}_\phi - 24\mathbf{a}_z)$$

$$= -10\mathbf{a}_\rho + 17.32\mathbf{a}_\phi + 24\mathbf{a}_z \text{ V/m}$$

(c) $E = \sqrt{(-10)^2 + 17.32^2 + 24^2} = 31.2$ V/m

(d) $dV/dN = E = 31.2$ V/m

(e) $\mathbf{E} = -(dV/dN)\mathbf{a}_N$

$$\mathbf{a}_N = -\frac{\mathbf{E}}{dV/dN} = -\left(\frac{-10\mathbf{a}_\rho + 17.32\mathbf{a}_\phi + 24\mathbf{a}_z}{31.2}\right) = 0.32\mathbf{a}_\rho - 0.55\mathbf{a}_\phi - 0.77\mathbf{a}_z$$

(f) $\mathbf{D} = \epsilon_0\mathbf{E} = \epsilon_0\left(-\dfrac{100}{z^2+1}\cos\phi\ \mathbf{a}_\rho + \dfrac{100}{z^2+1}\sin\phi\ \mathbf{a}_\phi + \dfrac{200z}{(z^2+1)^2}\rho\cos\phi\ \mathbf{a}_z\right)$

$$\rho_v = \nabla\bullet\mathbf{D} = \text{div}\,\mathbf{D} = (1/\rho)\partial(\rho D_\rho)/\partial\rho + (1/\rho)\partial D_\phi/\partial\phi + \partial D_z/\partial z$$

$$= 0 + 0 + 200\epsilon_0\,\rho\cos\phi\frac{\partial[z/(z^2+1)^2]}{\partial z}$$

Applying product rule of the derivative of $z/(z^2+1)^2$ and yielding,

$$= 200\epsilon_0\,\rho\cos\phi\left[\frac{1}{(z^2+1)^2} - \frac{4z^2}{(z^2+1)^3}\right]_{\rho=3,\ \phi=60°,\ z=2} = -234 \text{ pC/m}^3$$

D4.9. An electric dipole located at the origin in free space has a moment $\mathbf{p} = 3\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$ nC · m. (a) Find V at $P_A(2, 3, 4)$. (b) Find V at $r = 2.5$, $\theta = 30°$, $\phi = 40°$.

(a) $V = \dfrac{1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^2}\mathbf{p}\bullet\dfrac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} = \dfrac{1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^3}\mathbf{p}\bullet\mathbf{r}-\mathbf{r}'$

$$\mathbf{r}-\mathbf{r}' = (2-0)\mathbf{a}_x + (3-0)\mathbf{a}_y + (4-0)\mathbf{a}_z = 2\mathbf{a}_x + 3\mathbf{a}_y + 4\mathbf{a}_z$$

$$|\mathbf{r}-\mathbf{r}'| = \sqrt{2^2+3^2+4^2} = \sqrt{29}$$

$$V = \frac{1}{4\pi\epsilon_0\left(\sqrt{29}\right)^3}(3\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)(1\times10^{-9})\bullet(2\mathbf{a}_x + 3\mathbf{a}_y + 4\mathbf{a}_z) = \frac{4\times10^{-9}}{4\pi\epsilon_0\left(\sqrt{29}\right)^3} = 0.23 \text{ V}$$

(b) Using Table 5 from Chapter 1 to convert the point P_A to rectangular coordinates,

$x = 0.958$, $y = 0.803$, $z = 2.165$

$$\mathbf{r}-\mathbf{r}' = (0.958-0)\mathbf{a}_x + (0.803-0)\mathbf{a}_y + (2.165-0)\mathbf{a}_z = 0.958\mathbf{a}_x + 0.803\mathbf{a}_y + 2.165\mathbf{a}_z$$

$$|\mathbf{r}-\mathbf{r}'| = \sqrt{0.958^2 + 0.803^2 + 2.165^2} = 2.5$$

$$V = \frac{(3\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)(1\times10^{-9})}{4\pi\epsilon_0 2.5^3}\bullet(0.958\mathbf{a}_x + 0.803\mathbf{a}_y + 2.165\mathbf{a}_z) = \frac{3.43\times10^{-9}}{4\pi\epsilon_0 2.5^3} = 1.97 \text{ V}$$

D4.10. A dipole of moment $\mathbf{p} = 6\mathbf{a}_z$ nC · m is located at the origin in free space. (a) Find V at $P(r = 4, \theta = 20°, \phi = 0°)$. (b) Find $\mathbf{E}$ at P.

(a) Same formula in D4.9(a). Converting the point P to rectangular coordinates,

$x = 1.368$, $y = 0$, $z = 3.759$

Since the procedure is very similar with D4.9(a) and (b) where $\mathbf{p}$ is at the origin,

$\mathbf{r}-\mathbf{r}' = 1.386\mathbf{a}_x + 3.759\mathbf{a}_z$ and $|\mathbf{r}-\mathbf{r}'| = 4$

$$V = \frac{(1.368\mathbf{a}_x + 3.759\mathbf{a}_z)(1\times10^{-9})}{4\pi\epsilon_0 4^3}\bullet 6\mathbf{a}_z = \frac{22.55\times10^{-9}}{4^4\pi\epsilon_0} = 3.167 \text{ V} = 3.17 \text{ V}$$

(b) $\mathbf{E} = \dfrac{Qd}{4\pi\epsilon_0 r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)$

Since there is no charge given, we have to derive with the help of the voltage V since we have the answer in (a). Deriving,

$$V = \frac{Qd\cos\theta}{4\pi\epsilon_0 r^2};\ Qd = \frac{4\pi\epsilon_0 r^2 V}{\cos\theta}$$

Substituting,

$$\mathbf{E} = \frac{(4\pi\epsilon_0 r^2)/\cos\theta}{4\pi\epsilon_0 r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta) = \frac{V}{r}(2\mathbf{a}_r + \tan\theta\,\mathbf{a}_\theta)$$

All variables are given in (*a*). Solving,

$$= \frac{3.167}{4}(2\mathbf{a}_r + \tan 20°\ \mathbf{a}_\theta) = 1.58\mathbf{a}_r + 0.29\mathbf{a}_\theta \text{ V/m}$$

D4.11. Find the energy stored in free space for the region 2 mm < r < 3 mm, 0 < θ < 90°, 0 < ϕ < 90°, given the potential field V = : (*a*) $200/r$ V; (*b*) $(300 \cos\theta)/r^2$ V.

(*a*) $\mathbf{E} = -\nabla V = -\{(\partial V/\partial r)\mathbf{a}_r + (1/r)(\partial V/\partial\theta)\mathbf{a}_\theta + (1/r\sin\theta)(\partial V/\partial\phi)\mathbf{a}_\phi\}$

$$= -[(\partial(200/r)/\partial r)\mathbf{a}_r + 0\mathbf{a}_\theta + 0\mathbf{a}_\phi] = (200/r^2)\mathbf{a}_r$$

$$E = \sqrt{(200/r^2)^2} = 200/r^2$$

$$0.5\int_{\text{vol}}\epsilon_0 E^2 dv = 0.5\int_{\text{vol}}\epsilon_0\left(200/r^2\right)^2(r^2\sin\theta\, d\theta\, d\phi\, dr)$$

$$= 20\,000\epsilon_0\int_{0.002}^{0.003}(1/r^2)\,dr\int_0^{\pi/2}\sin\theta\, d\theta\int_0^{\pi/2}d\phi$$

$$= 20\,000\epsilon_0\left(-1/r\Big|_{0.002}^{0.003}\right)\left(-\cos\theta\Big|_0^{\pi/2}\right)\left(\phi\Big|_0^{\pi/2}\right) = 20\,000\epsilon_0(500/3)(\pi/2) = 46.4\ \mu\text{J}$$

(*b*) Same procedure in (*a*),

$$\mathbf{E} = -\{\partial[(300\cos\theta)/r^2]/\partial r\,\mathbf{a}_r + (1/r)\partial[(300\cos\theta)/r^2]/\partial\theta\,\mathbf{a}_\theta + 0\mathbf{a}_\phi\}$$

$$= (600\cos\theta)/r^3\,\mathbf{a}_r + (300\sin\theta)/r^3\,\mathbf{a}_\theta$$

$$E = \sqrt{[(600\cos\theta)/r^3]^2 + [(300\sin\theta)/r^3]^2} = \sqrt{(360\,000\cos^2\theta)/r^6 + (90\,000\sin^2\theta)/r^6}$$

$$0.5\int_{\text{vol}}\epsilon_0 E^2 dv = 0.5\int_{\text{vol}}\epsilon_0\{(360\,000\cos^2\theta)/r^6 + (90\,000\sin^2\theta)/r^6\}(r^2\sin\theta\, d\theta\, d\phi\, dr)$$

$$= 45\,000\epsilon_0\int_{\text{vol}}(1/r^4)(4\sin\theta\cos^2\theta + \sin^3\theta)\ d\theta\, d\phi\, dr$$

$$= 45\,000\epsilon_0\int_{0.002}^{0.003}(1/r^4)\,dr\int_0^{\pi/2}(4\sin\theta\cos^2\theta + \sin^3\theta)d\theta\int_0^{\pi/2}d\phi$$

Integrating $(4\sin\theta\cos^2\theta + \sin^3\theta)$ is solved by 'integration by substitution' and trigonometric identity yielding,

$$= 45\,000\epsilon_0\left(-\frac{1}{3r^3}\Bigg|_{0.002}^{0.003}\right)\left(-\cos^3\theta - \cos\theta\Big|_0^{\pi/2}\right)\left(\phi\Big|_0^{\pi/2}\right)$$

$$= 45\,000\epsilon_0[-1/(3(0.003)^3) + 1/(3(0.002)^3)](2)(\pi/2) = 36.7\text{ J}$$

CHAPTER 5

D5.1. Given the vector current density $\mathbf{J} = 10\rho^2 z\mathbf{a}_\rho - 4\rho\cos^2\phi\ \mathbf{a}_\phi$ mA/m^2: (*a*) find the current density at $P(\rho = 3, \phi = 30°, z = 2)$; (*b*) determine the total current flowing outward through the circular band $\rho = 3$, $0 < \phi < 2\pi$, $2 < z < 2.8$.

(*a*) $\mathbf{J} = 10(3^2)(2)\ \mathbf{a}_\rho - 4(3)(\cos 30°)^2\ \mathbf{a}_\phi$ mA/m^2 = $180\mathbf{a}_\rho - 9\ \mathbf{a}_\phi$ mA/m^2

(*b*) $I = \oint_S \mathbf{J}\cdot d\mathbf{S}$ where $d\mathbf{S} = \rho\ d\phi\ dz\ \mathbf{a}_\rho$

$$= (1\times10^{-3})\oint_S(10\rho^2 z\mathbf{a}_\rho - 4\rho cos^2\phi\mathbf{a}_\phi)\cdot(\rho\, d\phi\, dz\,\mathbf{a}_\rho + 0\mathbf{a}_\phi) = (1\times10^{-3})10\rho^3\int_0^{2\pi}d\phi\int_2^{2.8}z\,dz$$

$$= 0.01\rho^3\left(\phi\Big|_0^{2\pi}\right)\left(z^2/2\Big|_2^{2.8}\right)_{\rho=3} = 0.01(27)(2\pi)(1.92) = 3.26\text{ A}$$

D5.2. Current density is given in cylindrical coordinates as $\mathbf{J} = -10^6 z^{1.5}\mathbf{a}_z$ A/m^2 in the region $0 \le \rho \le 20\ \mu$m, $\mathbf{J} = 0$. (*a*) Find the total current crossing the surface $z = 0.1$ m in the $\mathbf{a}_z$ direction. (*b*) If the charge velocity is 2×10^6 m/s at $z = 0.1$ m, find ρ_v there. (*c*) If the volume charge density at $z = 0.15$ m is -2000 C/m^3, find the charge velocity.

(*a*) $I = \oint_S \mathbf{J}\cdot d\mathbf{S}$ where $d\mathbf{S} = \rho\ d\phi\ d\rho\ \mathbf{a}_z$

$$= \oint_S -10^6 z^{1.5}\mathbf{a}_z\cdot\ \rho\ d\phi\ dz\ \mathbf{a}_z = -10^6 z^{1.5}\int_0^{20\times10^{-6}}\rho\ d\rho\int_0^{2\pi}d\phi$$

$$= -10^6(0.1^{1.5})(2\times10^{-10})(2\pi) = -39.7\ \mu\text{A}$$

(*b*) $\mathbf{J} = \rho_v\mathbf{v}$; $\rho_v = J_z/v_z$ where they represent as z component.

$\rho_v = [-10^6(0.1^{1.5})]/(2\times10^6) = -15.8\ \text{mC/m}^3$

(*c*) Same formula in (*b*),

$v_z = J_z/\rho_v = [-10^6(0.15^{1.5})]/(-2000) = 29.0\ \text{m/s}$

D5.3. Find the magnitude of the current density in a sample of silver for which $\sigma = 6.17 \times 10^7$ S/m and $\mu_e = 0.0056\ \text{m}^2/\text{V}\cdot\text{s}$ if (*a*) the drift velocity is 1.5 μm/s; (*b*) the electric field intensity is 1 mV/m; (*c*) the sample is a cube 2.5 mm on a side having a voltage of 0.4 mV between opposite faces; (*d*) the sample is a cube 2.5 mm on a side carrying a total current of 0.5 A.

(*a*) $\mathbf{J} = \sigma\mathbf{E}$

Since $\mathbf{v}_d = -\mu_e\mathbf{E}$, then $\mathbf{J} = -\sigma\mathbf{v}_d/\mu_e$.

Since we only find the value of the magnitude of the current density J, which we know that vectors are denoted by bold letters, we consider **J** and other vectors as the same dot unit vector. The negative is canceled because they refer to the direction.

$J = \sigma v_d/\mu_e = [(6.17\times10^7)(1.5\times10^{-6})]/0.0056 = 16.5\ \text{kA/m}^2$

(*b*) $J = \sigma E = (6.17 \times 10^7)(1 \times 10^{-3}) = 61.7\ \text{kA/m}^2$

(*c*) $J = \sigma V/L = [(6.17\times10^7)(0.4\times10^{-3})]/(2.5\times10^{-3}) = 9.9\ \text{MA/m}^2$

(*d*) $J = I/S = 0.5/(2.5\times10^{-3})^2 = 80\ \text{kA/m}^2$

D5.4. A copper conductor has a diameter of 0.6 in. and it is 1200 ft long. Assume that it carries a total dc current of 50 A. (*a*) Find the total resistance of the conductor. (*b*) What current density exists in it? (*c*) What is the dc voltage between the conductor ends? (*d*) How much power is dissipated in the wire?

(*a*) We must convert the English units to metric systems. A 1200 feet is about 365.76 m. A 0.6 in is about 0.01524 m. The radius is half of the diameter; so $r = 7.62 \times 10^{-3}$ m.

The conductivity of the copper is 5.8×10^7 S/m.

The area of the circle is πr^2.

$R = L/\sigma S = 365.76/[(5.8\times10^7)\pi(7.62 \times 10^{-3})^2] = 0.035\ \Omega$

(*b*) $J = I/S = 50/[\pi(7.62\times10^{-3})^2] = 2.74 \times 10^5\ \text{A/m}^2$

(*c*) $V = IR = 50(0.035) = 1.75$ V

With regards from the answer of the textbook, $V = 50(0.03457) = 1.73$ V

(*d*) $P = VI = 1.75(50) = 87.5$ W

With regards from the answer of the textbook, $P = 1.7285(50) = 86.4$ W

D5.5. Given the potential field in free space, $V = 100 \sinh 5x \sin 5y$ V, and a point $P(0.1, 0.2, 0.3)$, find at P: (*a*) V; (*b*) **E**; (*c*) $|\mathbf{E}|$; (*d*) $|\rho_S|$ if it is known that P lies on a conductor surface.

(*a*) The function sinh(*x*) is kind of a hyperbolic trigonometry where $\sinh(x) = (e^x - e^{-x})/2$. Be careful that the points are in radians, not degrees.

$V = 100 \sinh 5(0.1) \sin 5(0.2) = 43.8$ V

(*b*) Using back the formula in Chapter 4,

$$\mathbf{E} = -\nabla V = -\{(\partial V/\partial x)\mathbf{a}_x + (\partial V/\partial y)\mathbf{a}_y + (\partial V/\partial z)\mathbf{a}_z\}$$

$$= -\{(\partial(100\sinh 5x \sin 5y)/\partial x)\mathbf{a}_x + (\partial(100\sinh 5x\sin 5y)/\partial y)\mathbf{a}_y + 0\mathbf{a}_z\}$$

The derivative of sinh x is cosh x or $(e^x + e^{-x})/2$. But sinh $5x\ dx$ is 5 cosh $5x$.

$$= -500\cosh 5x \sin 5y\ \mathbf{a}_x - 500\sinh 5x\cos 5y\ \mathbf{a}_y$$

$$= -500\cosh 5(0.1)\sin 5(0.2)\ \mathbf{a}_x - 500\sinh 5(0.1)\cos 5(0.2)\ \mathbf{a}_y$$

$$= -474\mathbf{a}_x - 140.8\mathbf{a}_y\ \text{V/m}$$

(*c*) $E = \sqrt{(-474)^2 + (-140.8)^2} = 495$ V/m

(*d*) $\rho_S = \epsilon_0 E_N = \epsilon_0(495) = 4.38\ \text{nC/m}^2$

D5.6. A perfectly conducting plane is located in free space at $x = 4$, and a uniform infinite line charge of 40 nC/m lies along the line $x = 6$, $y = 3$. Let $V = 0$ at the conducting plane. At $P(7, -1, 5)$ find: (*a*) V; (*b*) **E**.

It is really good if we refer to answer first in (*b*) because it is difficult to solve a voltage with lack of given variables like the charge Q and the electric field **E**. In (*b*), a method of image is required. We are going to mirror the infinite line charge ρ_L including the distance (or radial vector) **R** without the conducting plane. For more details, see Section 5.5. The book is explained that the radial vector from positive (or negative if ever) line charge to point P is symmetric where the plane is located. For example, if the line charge (not the plane) is at $z = 3$ and the plane is at $z = 0$, then we put an image line charge at $z = -3$ which the plane is removed.

(*b*) Locating the radial vector **R** from the given location of line charge and P,

$\mathbf{R}_+ = \mathbf{a}_x - 4\mathbf{a}_y$ and

$\mathbf{R}_- = -5\mathbf{a}_x - 4\mathbf{a}_y$

How did we obtain them? Well, draw a two-dimensional rectangular coordinates. Locate P at $x = 7$ and $y = -1$. Notice that no z axis is included because the conducting plane activates on x axis and the line charge activates on x and y axes. The plane is at $x = 4$ while the line charge is $x = 6$ and $y = 3$. Therefore, the *image* line charge is symmetrically at $x = 2$ and $y = 3$. We draw the plane, line charge and image line charge with horizontal line.

We can now find the radial vector **R**. For $\mathbf{R}_+$, the line charge is one unit away from $P(x = 7)$, so $7 - 6 = 1$; the line charge is four units away from $P(y = 3)$, so $-1 - 3 = -4$. The image line charge reacts on negative distance because we deal with directions no matter where plane is placed. For $\mathbf{R}_-$, the image line charge is five units away from $P(x = 7)$, so $-(7 - 2) = -5$; the image line charge is four units away from $P(y = 3)$ is $-(-1 - 3) = 4$. Finding their magnitudes, $R_+ = \sqrt{17} = 4.123$ and $R_- = \sqrt{41} = 6.403$.

Knowing each electric fields,

$$\mathbf{E}_+ = \frac{\rho_L}{2\pi\epsilon_0 R_+}\mathbf{a}_{R_+} = \frac{40\times10^{-9}}{2\pi\epsilon_0\left(\sqrt{17}\right)^2}(\mathbf{a}_x - 4\mathbf{a}_y) = 42.294\mathbf{a}_x - 169.177\mathbf{a}_y$$

$$\mathbf{E}_- = \frac{\rho_L}{2\pi\epsilon_0 R_-}\mathbf{a}_{R_-} = \frac{40\times10^{-9}}{2\pi\epsilon_0\left(\sqrt{41}\right)^2}(-5\mathbf{a}_x + 4\mathbf{a}_y) = -87.683\mathbf{a}_x + 70.147\mathbf{a}_y$$

We cannot get the exact answer from the textbook. Adding together,

$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = -45.39\mathbf{a}_x - 99.03\mathbf{a}_y$ V/m

(*a*) We know that the plane stands at x axis. Finding the voltage V where only the electric field E and length for P are extracted at x direction,

$V = E_x L_x = \sqrt{(-45.39)^2}(7) = 317.73$ V

D5.7. Using the values given in this section for the electron and hole mobilities in silicon at 300 K, and assuming hole and electron charge densities are 0.0029 C/m^3 and –0.0029 C/m^3, respectively, find: (*a*) the component of the conductivity due to holes; (*b*) the component of the conductivity due to electrons; (*c*) the conductivity.

The electron and hole mobilities of pure silicon is 0.12 and 0.025.

(*a*) $\sigma_h = \rho_h\mu_h = (0.0029)0.025 = 72.5\ \mu$S/m

(*b*) $\sigma_e = -\rho_e\mu_e = -(-0.0029)0.12 = 348\ \mu$S/m

(*c*) $\sigma = \sigma_h + \sigma_e = 421\ \mu$S/m

D5.8. A slab of dielectric material has a relative dielectric constant of 3.8 and contains a uniform electric flux density of 8 nC/m^2. If the material is lossless, find: (*a*) E; (*b*) P; (*c*) the average number of dipoles per cubic meter if the average dipole moment is 10^{-29} C · m.

(*a*) $\mathbf{D} = \epsilon\mathbf{E}$ where $\epsilon = \epsilon_0\epsilon_r$

$E = D/\epsilon_0\epsilon_r = (8\times10^{-9})/3.8\epsilon_0 = 238$ V/m

(*b*) $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$; $P = D - \epsilon_0 E$

$P = (8\times10^{-9}) - 238\epsilon_0 = 5.89$ nC/m^2

(*c*) $$\mathbf{P} = \lim_{\Delta v\to 0}\frac{1}{\Delta v}\sum_{i=1}^{n\Delta v}\mathbf{p}_i$$

We have one dipole moment **p** and no limit of the average volume Δv. Generalizing the formula,

$\mathbf{P} = \mathbf{p}/\Delta v$; $1/\Delta v = P/p = 5.89\times10^{-9}/10^{-29} = 5.89\times10^{20}$ m^{-3}

D5.9. Let Region 1 ($z < 0$) be composed of a uniform dielectric material for which $\epsilon_r = 3.2$, while Region 2 ($z > 0$) is characterized by $\epsilon_r = 2$. Let $\mathbf{D}_1 = -30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z$ nC/m^2 and find: (*a*) D_{N1}; (b) $\mathbf{D}_{t1}$; (*c*) D_{t1}; (*d*) D_1; (*e*) θ_1; (*f*) $\mathbf{P}_1$.

(*a*) Since the Region 1 affects the z axis, only D_z is referred.

$D_{N1} = D_{1z} = \sqrt{70^2}$ nC/m^2 = 70 nC/m^2

(b) While the normal component D_N refers to the mentioned region where it locates, then the rest which does not affect the region is D_{t1}.

$\mathbf{D}_{t1} = \mathbf{D}_1 - \mathbf{D}_{N1} = [(-30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z) - 70\mathbf{a}_z]$ nC/m^2 = $-30\mathbf{a}_x + 50\mathbf{a}_y$ nC/m^2

(c) $D_{t1} = \sqrt{(-30)^2 + 50^2}$ = 58.31 nC/m^2

(d) $D_1 = \sqrt{(-30)^2 + 50^2 + 70^2}$ = 91.1 nC/m^2

(e) $D_{N1} = D_1 \cos\theta_1$

$\theta_1 = \cos^{-1} D_{N1}/D_1 = \cos^{-1}(70\times10^{-9}/91.1\times10^{-9}) = 39.8°$

(f) $\mathbf{D}_1 = \epsilon_0\mathbf{E}_1 + \mathbf{P}_1$ where $\mathbf{D} = \epsilon_0\epsilon_r\mathbf{E}$

$\mathbf{P}_1 = \mathbf{D}_1 - \epsilon_0\mathbf{E}_1 = \mathbf{D}_1(1-(1/\epsilon_{r1})) = (-30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z)(1-(1/3.2))$ nC/m^2

$= -20.6\mathbf{a}_x + 34.4\mathbf{a}_y + 48.1\mathbf{a}_z$ nC/m^2

D5.10. Continue Problem D5.9 by finding: (a) $\mathbf{D}_{N2}$; (b) $\mathbf{D}_{t2}$; (c) $\mathbf{D}_2$; (d) $\mathbf{P}_2$; (e) θ_2.

(a) $\mathbf{D}_{N1} = \mathbf{D}_{N2} = 70\mathbf{a}_z$ nC/m^2

(b) $\mathbf{D}_{t1}$ is not equal to $\mathbf{D}_{t2}$ because of their relative permittivity ϵ_r.

$\mathbf{D}_{t1}/\mathbf{D}_{t2} = \epsilon_{r1}/\epsilon_{r2}$

$\mathbf{D}_{t2} = \mathbf{D}_{t1}\epsilon_{r2}/\epsilon_{r1} = [(-30\mathbf{a}_x + 50\mathbf{a}_y)2]/3.2$ nC/m^2 = $-18.75\mathbf{a}_x + 31.25\mathbf{a}_y$ nC/m^2

(c) $\mathbf{D}_1$ is not equal to $\mathbf{D}_2$ because $\mathbf{D}_2$ is the cause of the refraction.

$\mathbf{D}_2 = \mathbf{D}_{t2} + \mathbf{D}_{N2} = -18.75\mathbf{a}_x + 31.25\mathbf{a}_y + 70\mathbf{a}_z$ nC/m^2

(d) $\mathbf{P}_2 = \mathbf{D}_2(1-(1/\epsilon_{r2})) = (-18.75\mathbf{a}_x + 31.25\mathbf{a}_y + 70\mathbf{a}_z)(1-(1/2))$nC/m^2

$= -9.38\mathbf{a}_x + 15.63\mathbf{a}_y + 35\mathbf{a}_z$ nC/m^2

(e) $\tan\theta_1/\tan\theta_2 = \epsilon_{r1}/\epsilon_{r2}$

$\theta_2 = \tan^{-1}[(\epsilon_{r2}\tan\theta_1)/\epsilon_{r1}] = \tan^{-1}[(2\tan 39.8°)/3.2] = 27.5°$

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