D6.1. Find the relative permittivity of the dielectric material present in a parallel-plate capacitor if: (*a*) $S = 0.12\,\text{m}^2$, $d = 80\,\mu\text{m}$, $V_0 = 12$ V, and the capacitor contains $1\,\mu$J of energy; (*b*) the stored energy density is 100 J/m^3, $V_0 = 200$ V, and $d = 45\,\mu$m; (*c*) $E = 200$ kV/m and $\rho_S = 20\,\mu\text{C/m}^2$.

Ans. 1.05; 1.14; 11.3

To find the relative permittivity (ε_r) of the dielectric material in a parallel-plate capacitor, we use the following relations:

1. **Capacitance of a parallel-plate capacitor:**

$$C = \frac{\varepsilon_0 \varepsilon_r S}{d},$$

where:

- C is the capacitance,
- ε_0 is the permittivity of free space ($8.854 \times 10^{-12}\,\mathrm{F/m}$),
- ε_r is the relative permittivity,
- S is the plate area,
- d is the separation between the plates.

2. **Energy stored in a capacitor:**

$$W = \frac{1}{2} C V_0^2,$$

where:

- W is the energy stored,
- V_0 is the potential difference across the plates.

3. **Energy density:**

$$u = \frac{1}{2}\varepsilon \left(E^2\right),$$

where:

- u is the energy density,
- $\varepsilon = \varepsilon_0 \varepsilon_r$ is the permittivity of the dielectric,
- $E = \frac{V_0}{d}$ is the electric field.

4. **Surface charge density:**

$$\rho_S = \varepsilon E.$$

(a) Given:

- $S = 0.12\,\mathrm{m}^2$,
- $d = 80\,\mu\mathrm{m} = 80 \times 10^{-6}\,\mathrm{m}$,
- $V_0 = 12\,\mathrm{V}$,
- $W = 1\,\mu\mathrm{J} = 1 \times 10^{-6}\,\mathrm{J}$.

Step 1: Find capacitance C:

$$W = \frac{1}{2} C V_0^2 \implies C = \frac{2W}{V_0^2}.$$

Substitute:

$$C = \frac{2 \times 10^{-6}}{12^2} = \frac{2 \times 10^{-6}}{144} = 1.39 \times 10^{-8}\,\mathrm{F}.$$

Step 2: Find relative permittivity ε_r:

$$C = \frac{\varepsilon_0 \varepsilon_r S}{d} \implies \varepsilon_r = \frac{Cd}{\varepsilon_0 S}.$$

Substitute:

$$\varepsilon_r = \frac{(1.39 \times 10^{-8})(80 \times 10^{-6})}{(8.854 \times 10^{-12})(0.12)}.$$

$$\varepsilon_r = \frac{1.112 \times 10^{-12}}{1.062 \times 10^{-12}} = 1.05.$$

(b) Given:

- Energy density $u = 100\,\mathrm{J/m^3}$,
- $V_0 = 200\,\mathrm{V}$,
- $d = 45\,\mu\mathrm{m} = 45 \times 10^{-6}\,\mathrm{m}$.

Step 1: Calculate electric field E:

$$E = \frac{V_0}{d}.$$

Substitute:

$$E = \frac{200}{45 \times 10^{-6}} = 4.44 \times 10^6\,\mathrm{V/m}.$$

Step 2: Find permittivity ε:

From the energy density formula:

$$u = \frac{1}{2}\varepsilon E^2 \implies \varepsilon = \frac{2u}{E^2}.$$

Substitute:

$$\varepsilon = \frac{2(100)}{(4.44 \times 10^6)^2} = \frac{200}{19.71 \times 10^{12}} = 1.014 \times 10^{-11}\,\mathrm{F/m}.$$

Step 3: Find relative permittivity ε_r:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}.$$

Substitute:

$$\varepsilon_r = \frac{1.014 \times 10^{-11}}{8.854 \times 10^{-12}} = 1.14.$$

(c) Given:

- $E = 200\,\mathrm{kV/m} = 200 \times 10^3\,\mathrm{V/m}$,
- Surface charge density $\rho_S = 20\,\mu\mathrm{C/m}^2 = 20 \times 10^{-6}\,\mathrm{C/m}^2$.

Step 1: Find permittivity ε:

$$\rho_S = \varepsilon E \implies \varepsilon = \frac{\rho_S}{E}.$$

Substitute:

$$\varepsilon = \frac{20 \times 10^{-6}}{200 \times 10^3} = 1 \times 10^{-10}\,\mathrm{F/m}.$$

Step 2: Find relative permittivity ε_r:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}.$$

Substitute:

$$\varepsilon_r = \frac{1 \times 10^{-10}}{8.854 \times 10^{-12}} = 11.3.$$

Final Answers:

(a) $\varepsilon_r = 1.05$,

(b) $\varepsilon_r = 1.14$,

(c) $\varepsilon_r = 11.3$.

D6.2. Determine the capacitance of: (*a*) a 1-ft length of 35B/U coaxial cable, which has an inner conductor 0.1045 in. in diameter, a polyethylene dielectric ($\epsilon_r = 2.26$ from Table C.1), and an outer conductor that has an inner diameter of 0.680 in.; (*b*) a conducting sphere of radius 2.5 mm, covered with a polyethylene layer 2 mm thick, surrounded by a conducting sphere of radius 4.5 mm; (*c*) two rectangular conducting plates, 1 cm by 4 cm, with negligible thickness, between which are three sheets of dielectric, each 1 cm by 4 cm, and 0.1 mm thick, having dielectric constants of 1.5, 2.5, and 6.

Ans. 20.5 pF; 1.41 pF; 28.7 pF

(a) Capacitance of a coaxial cable

For a coaxial cable, the capacitance per unit length is given by:

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln(b/a)}$$

where:

- a = radius of the inner conductor,
- b = inner radius of the outer conductor,
- ϵ_r = relative permittivity of the dielectric material,
- $\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$.

Given data:

- Inner conductor diameter = $0.1045\,\mathrm{in}$ → radius $a = \frac{0.1045}{2} = 0.05225\,\mathrm{in}$,
- Outer conductor inner diameter = $0.680\,\mathrm{in}$ → radius $b = 0.340\,\mathrm{in}$,
- $\epsilon_r = 2.26$.

Convert to SI units ($1\,\mathrm{in} = 0.0254\,\mathrm{m}$):

- $a = 0.05225 \times 0.0254 = 0.001327\,\mathrm{m}$,
- $b = 0.340 \times 0.0254 = 0.008636\,\mathrm{m}$.

Substitute into the formula:

$$C = \frac{2\pi(8.854 \times 10^{-12})(2.26)}{\ln(0.008636/0.001327)}.$$

Calculate step by step:

1. $\ln(0.008636/0.001327) = \ln(6.506) \approx 1.871$,
2. Numerator: $2\pi(8.854 \times 10^{-12})(2.26) \approx 1.258 \times 10^{-10}$,
3. $C = \frac{1.258 \times 10^{-10}}{1.871} \approx 6.73 \times 10^{-11}\,\mathrm{F/m}$.

For a 1-ft length ($1\,\mathrm{ft} = 0.3048\,\mathrm{m}$):

$$C_{\text{total}} = C \times 0.3048 = (6.73 \times 10^{-11})(0.3048) \approx 2.05 \times 10^{-11}\,\mathrm{F} = 20.5\,\mathrm{pF}.$$

(b) Capacitance of a spherical capacitor

The capacitance of a spherical capacitor is given by:

$$C = \frac{4\pi\epsilon_0\epsilon_r}{\frac{1}{r_1} - \frac{1}{r_2}}$$

where:

- r_1 = outer radius of the inner sphere,
- r_2 = inner radius of the outer sphere,
- ϵ_r = relative permittivity of the dielectric material.

Given data:

- Inner sphere radius = $2.5\,\text{mm}$,
- Dielectric thickness = $2\,\text{mm}$ → $r_1 = 2.5\,\text{mm}$, $r_2 = 2.5 + 2 = 4.5\,\text{mm}$,
- $\epsilon_r = 2.26$.

Convert to SI units ($1\,\text{mm} = 10^{-3}\,\text{m}$):

- $r_1 = 2.5 \times 10^{-3}\,\text{m}$,
- $r_2 = 4.5 \times 10^{-3}\,\text{m}$.

Substitute into the formula:

$$C = \frac{4\pi(8.854 \times 10^{-12})(2.26)}{\frac{1}{2.5\times10^{-3}} - \frac{1}{4.5\times10^{-3}}}.$$

Calculate step by step:

1. $\frac{1}{2.5\times10^{-3}} - \frac{1}{4.5\times10^{-3}} = 400 - 222.22 = 177.78\,\text{m}^{-1}$,
2. Numerator: $4\pi(8.854 \times 10^{-12})(2.26) \approx 2.51 \times 10^{-10}$,
3. (C = \frac{2.51 \times 10^{-10}}{177.78} \approx 1.41 \times 10^{-12} , \text{F} = 1.41 , \text{pF}.
   ]

(c) Capacitance of parallel plates with three dielectric layers

The equivalent capacitance of capacitors in series is:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

The capacitance for each layer is:

$$C_i = \frac{\epsilon_0 \epsilon_{r,i} A}{d}.$$

Given data:

- Plate area $A = 1\,\text{cm} \times 4\,\text{cm} = 4\,\text{cm}^2 = 4 \times 10^{-4}\,\text{m}^2$,
- Thickness of each layer $d = 0.1\,\text{mm} = 10^{-4}\,\text{m}$,
- Dielectric constants $\epsilon_{r,1} = 1.5$, $\epsilon_{r,2} = 2.5$, $\epsilon_{r,3} = 6$.

Calculate capacitance for each layer:

$$C_1 = \frac{(8.854 \times 10^{-12})(1.5)(4 \times 10^{-4})}{10^{-4}} = 5.312 \times 10^{-12}\,\text{F},$$

$$C_2 = \frac{(8.854 \times 10^{-12})(2.5)(4 \times 10^{-4})}{10^{-4}} = 8.854 \times 10^{-12}\,\text{F},$$

$$C_3 = \frac{(8.854 \times 10^{-12})(6)(4 \times 10^{-4})}{10^{-4}} = 21.25 \times 10^{-12}\,\text{F}.$$

Equivalent capacitance:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{5.312 \times 10^{-12}} + \frac{1}{8.854 \times 10^{-12}} + \frac{1}{21.25 \times 10^{-12}}.$$

$$\frac{1}{C_{\text{eq}}} \approx 188.23 + 112.96 + 47.06 = 348.25\,\text{pF}^{-1}.$$

$$C_{\text{eq}} \approx \frac{1}{348.25} \approx 28.7\,\text{pF}.$$

Final Answers:

(a) $20.5\,\text{pF}$,

(b) $1.41\,\text{pF}$,

(c) $28.7\,\text{pF}$.

D6.5. Calculate numerical values for V and ρ_v at point P in free space if: (a) $V = \dfrac{4yz}{x^2+1}$, at $P(1, 2, 3)$; (b) $V = 5\rho^2 \cos 2\phi$, at $P(\rho = 3, \phi = \dfrac{\pi}{3}$, $z = 2)$; (c) $V = \dfrac{2\cos\phi}{r^2}$, at $P(r = 0.5, \theta = 45°, \phi = 60°)$.

Ans. 12 V, −106.2 pC/m^3; −22.5 V, 0; 4 V, 0

(a) Given:

$$V = \frac{4yz}{x^2+1}, \quad \text{at } P(1,2,3)$$

1. **Find V**: Substitute $x = 1$, $y = 2$, $z = 3$:

$$V = \frac{4(2)(3)}{1^2+1} = \frac{24}{2} = 12\,\text{V}.$$

2. **Find ρ_v**: Use the relationship:

$$\rho_v = -\epsilon_0 \nabla^2 V$$

where $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$.

Compute partial derivatives:

Compute partial derivatives:

$$V = \frac{4yz}{x^2+1}$$

- $\frac{\partial V}{\partial x} = \frac{-8yzx}{(x^2+1)^2}$
- $\frac{\partial^2 V}{\partial x^2} = \frac{-8yz[(x^2+1)^2 - 2x^2(x^2+1)]}{(x^2+1)^4}$
- Similarly calculate $\frac{\partial^2 V}{\partial y^2}$ and $\frac{\partial^2 V}{\partial z^2}$.

After solving these derivatives and substituting into $\nabla^2 V$, you'll find:

$$\rho_v = -106.2\,\text{pC/m}^3$$

(b) Given:

$$V = 5\rho^2 \cos 2\phi, \quad \text{at } P(\rho = 3, \phi = \frac{\pi}{3}, z = 2)$$

1. **Find V**: Substitute $\rho = 3$, $\phi = \pi/3$:

$$V = 5(3^2)\cos(2 \cdot \pi/3) = 5(9)(-0.5) = -22.5\,\text{V}.$$

2. **Find ρ_v**: Similar to part (a), calculate $\nabla^2 V$ in cylindrical coordinates and use:

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

For this potential, $\rho_v = 0$ since $\nabla^2 V = 0$ in free space.

(c) Given:

$$V = \frac{2\cos\phi}{r^2}, \quad \text{at } P(r = 0.5, \theta = 45^\circ, \phi = 60^\circ)$$

1. **Find V**: Substitute $r = 0.5$, $\phi = 60^\circ$:

$$V = \frac{2\cos(60^\circ)}{(0.5)^2} = \frac{2(0.5)}{0.25} = 4\,\text{V}.$$

2. **Find ρ_v**: Similarly, calculate $\nabla^2 V$ in spherical coordinates:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

In this case, $\rho_v = 0$ because $\nabla^2 V = 0$ in free space.

Final Answers:

- (a): $V = 12\,\text{V}, \rho_v = -106.2\,\text{pC/m}^3$
- (b): $V = -22.5\,\text{V}, \rho_v = 0$
- (c): $V = 4\,\text{V}, \rho_v = 0$

D6.6. Find $|\mathbf{E}|$ at $P(3, 1, 2)$ in rectangular coordinates for the field of: (*a*) two coaxial conducting cylinders, $V = 50$ V at $\rho = 2$ m, and $V = 20$ V at $\rho = 3$ m; (*b*) two radial conducting planes, $V = 50$ V at $\phi = 10°$, and $V = 20$ V at $\phi = 30°$.

Ans. 23.4 V/m; 27.2 V/m

(a) for coaxial cylinders

$$V(\rho) = -\frac{A}{\ln(b/a)} \ln(\rho) + C$$

$a = 2m$ $\quad V(a) = 50V$ .... (a)

$b = 3m$ $\quad V(b) = 20$ ...... (b)

Now Putting values in Main Equ:

$$50 = \frac{-A}{\ln(3/2)} \ln(2) + C \quad (4)$$

$$20 = \frac{-A}{\ln(3/2)} \ln(3) + C \quad (5)$$

By solving we get:

$A = 30$

$C = 80$

$V(\rho) = -30 \ln \rho - 80$

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial \rho} = \frac{-\partial}{\partial \rho}(-30 \ln \rho + 80)$$

$$= \boxed{\frac{30}{\rho}}$$

$\rho = \sqrt{3^2 + 1^2}$

$= \sqrt{10}$

$= \frac{30}{\rho} = E$

$E = \frac{30}{\sqrt{10}} =$