ADC - A2

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Question #1

(a) $m(t) = \frac{\sin(2\pi t)}{t}$

$= \frac{\sin(2\pi t)}{\pi t} \cdot \pi$

$s(t) = m(t) \cos(2\pi f_c t)$

$= 2\frac{\sin(2\pi t)}{2t} \times \cos(2\pi f_c t)$

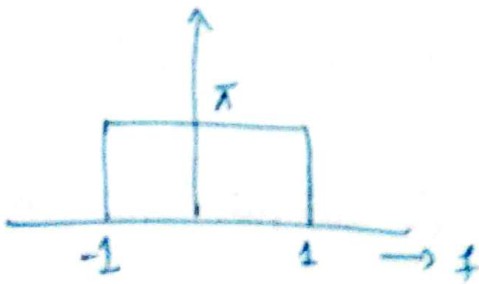
$* \sin a \cos b = \frac{\sin(a+b) + \cos(a-b)}{2}$

$s(t) = \frac{1}{2t}\left[\sin(2\pi(1+f_c)t + \sin(2\pi(1-f_c)t\right].$

$M(\omega) = \pi * rect\left(\frac{\omega}{4\pi}\right)$

$M(f) = \pi \, rect\left(\frac{2\pi f}{4\pi}\right)$

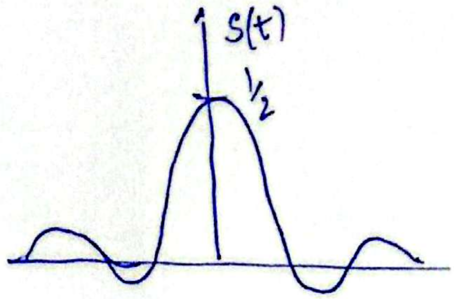
$M(f) = \pi \, rect\left(\frac{f}{2}\right)$

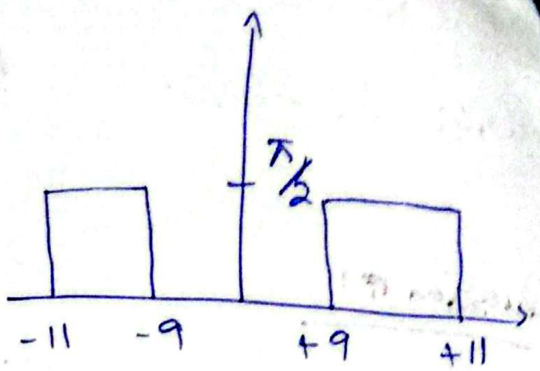


$S(f) = M(f) * \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$

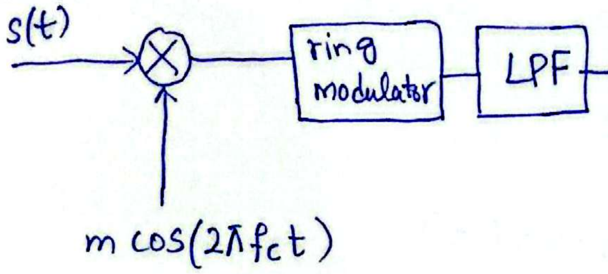
$= \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right]$

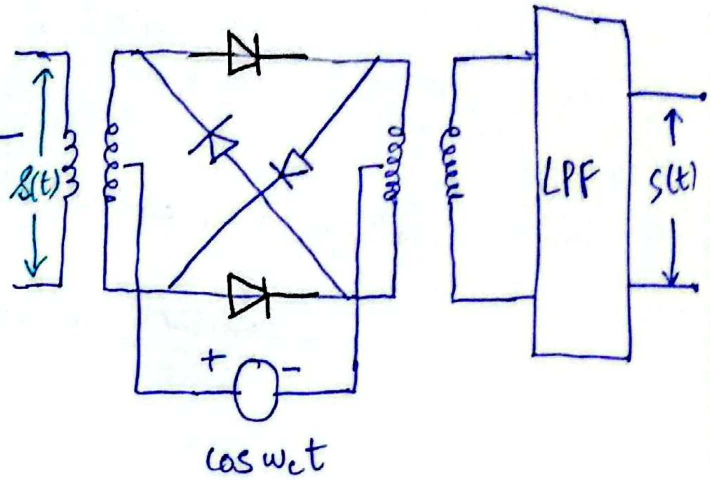
$$S(f) = \frac{1}{2}\left[M(f-10) + M(f+10)\right]$$





b)





Ring modulator

input Signal ⇒ $m(t) \cdot \cos 2\pi f_c t$

-"1" carrier causes periodic switching on and off of the input signal.

∴ output is $m(t)\cos(2\pi f_c t) \cdot x(t)$

$$= m(t)\cos(w_c t)\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos w_c t - \frac{1}{3}\cos 3w_c t + \cdots\right)\right]$$

$$= \frac{1}{2} m(t)\cos w_c t + \frac{2}{\pi} m(t)\cos 2w_c t + \cdots\cdots$$

$$= \frac{2}{\pi}\left[\frac{1+\cos 2w_c t}{2}\right] m(t) + \cdots$$

$$\pm \frac{1}{\pi} m(t) + \underbrace{\frac{m(t)\cos 2w_c t}{\pi}}_{\text{eliminated LPF.}}$$

Question #2 multi-tone modulation.

modulation Signal: $m(t) = \cos w_m t + 2 \sin 3w_m t.$

carrier is: $c(t) = 3\cos(w_c t)$; $w_c >> w_m$

Geonral Expression:

$$\phi_{AM}(t) = \cancel{A_c[1 + k\,m(t)]}$$

$$= [A + m(t)]\cos w_c t.$$

$$= A\cos w_c t + m(t)\cos w_c t$$

$$\boxed{A = 3}$$

$$= [3 + \cos w_m t + 2\sin 3w_m t]\cos(w_c t).$$

$$= 3\left[1 + \frac{1}{3}\cos w_m t + \frac{2}{3}\sin 3w_m t\right]\cos w_c t$$

$$m_1 = \frac{1}{3} \quad , \quad m_2 = \frac{2}{3}$$

<u>total modulation index:</u>

$$m = \sqrt{m_1^2 + m_2^2}$$

$$= \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} \Rightarrow \boxed{0.74 = m}$$

therefore, the AM signal φ_{AM} satisfy the detection critera.

<u>Now we</u>

$$\varphi_{Am}(t) = 3\cos w_c t + (\cos w_m t \times \cos w_c t) + (2\sin 3w_m t) \times (\cos w_c t)$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right].$$

$$\mathcal{F}\{\cos(w_o t)\} = \pi\left[\delta(w - w_o) + \delta(w + w_o)\right]$$

$$\mathcal{F}\{\sin(w_o t)\} = \frac{\pi}{j}\left[\delta(w - w_o) + \delta(w + w_o)\right].$$

$$\cos w_m \times \cos w_c t = \frac{1}{2}\left[\cos(w_m + w_c)t + \cos(w_m - w_c t)\right]$$

$$2\sin 3w_m t \times \cos w_c t = \sin(3w_m + w_c)t + \sin(3w_m - w_c t)$$

$$\varphi_{Am}(t) = 3\cos w_c t + \frac{1}{2}\left[\cos(w_m + w_c)t + \cos(w_c + w_m)t\right] + \left[\sin(3w_m + w_c)t + \sin(3w_m - w_c)t\right]$$

$$\cancel{\varphi_{Am}(t) = 3\pi\left[\delta(w - w_c) + \delta(w + w_c)\right] + \frac{\pi}{2}\left[\cos(w_m + w_c) + \cos(w_m - w_c)\right]}$$

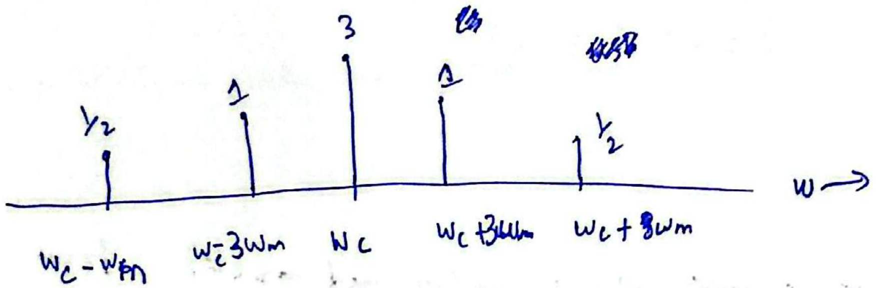
$+ \frac{\pi}{j}[\delta(\omega)$

$$\varphi_{Am}(\omega) = 3\pi\left[\delta(\omega_0 - \omega_c) + \delta(\omega_0 + \omega_c)\right] + \frac{\pi}{2}\left[\delta(\omega_0 + (\omega_m + \omega_c)) + \right.$$

$$\left.\delta(\omega_0 - (\omega_m \mp \omega_c)\right] + \frac{\pi}{j}\left[\delta(\omega_0 \mp (3\omega_m + \omega_c)) + \delta(\omega_0 + 3\omega_m - \omega_c)\right]$$

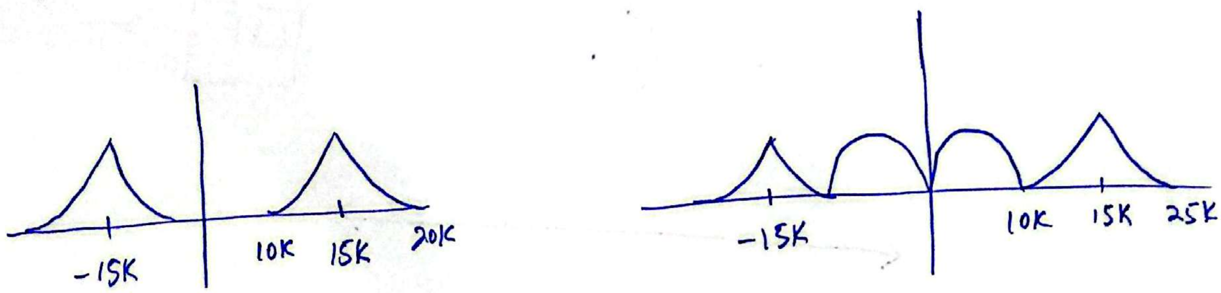
Compact form:

$$\Phi_{Am}(\omega) = 3\delta(\omega \pm \omega_c) + \frac{1}{2}\left[\delta(\omega \pm (\omega_c \pm \omega_m))\right] + \left[\delta(\omega \pm (\omega_c \pm 3\omega_m)\right]$$



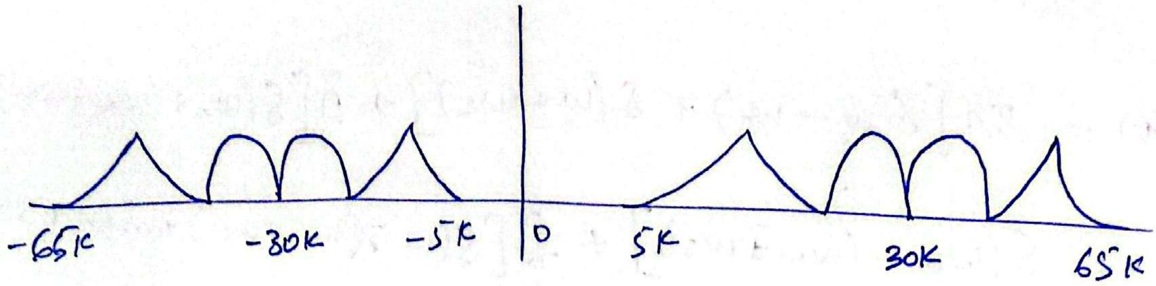
Question # 3

a) at ⓐ at ⓑ



$\begin{array}{r}30\\-25\\\hline 5\end{array}$ $\begin{array}{r}30\\25\\\hline 65\end{array}$

at (c)

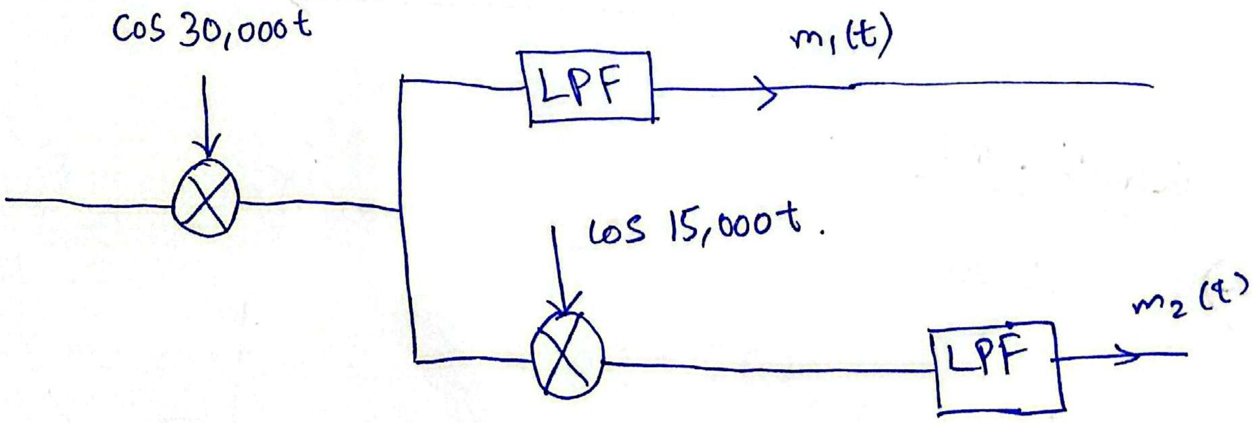


b) Bandwidth of channel:

$$\text{Bandwidth} = 2 \times \frac{(65 - 5)}{2} = 60\text{K rad/s}$$

$$\boxed{B = 60\text{K rad/s}}$$

c) Receiver to recover signal $m_1(t)$ and $m_2(t)$ from received modulated signal.



Question # 04

$$\cos^2\theta = \frac{1+2\cos 2\theta}{2}$$

$$\cos^3\theta = \frac{1}{4}(3\cos\theta + \cos 3\theta)$$

$$\left(\frac{1+\cos 2\theta}{2}\right)^2 = \frac{1}{4}(1+2\cos 2\theta + \cos^2 2\theta)$$

$$\cos^5\theta = (\cos^2\theta)^2 \cos\theta$$

$$= \left(\frac{1+2\cos 2\theta}{2}\right)^2 \cos\theta$$

$$\cos^5\theta = \frac{1}{4}(\cos\theta + 2\cos\theta\cos 2\theta + \cos\theta\cos^2 2\theta) \rightarrow (1)$$

$K\, m(t) \cos w_c t \rightarrow$ modulated signal

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)].$$

For $2\cos\theta\cos 2\theta = \frac{1}{2}\times 2\,[\cos(3\theta) + \cos(\theta)] = \cos(3\theta) + \cos(\theta)$

For $\cos\theta . \cos^2 2\theta$

$$* \cos^2\theta = \frac{1+2\cos 2\theta}{2}$$

$$* \cos^2 2\theta = \frac{1+2\cos 4\theta}{2}$$

$$\Rightarrow \cos\theta\left(\frac{1+\cancel{\cos\theta}\ \cos 4\theta}{2}\right)$$

$$= \frac{1}{2}[\cos\theta] + \frac{1}{2}[\cos\theta \cos 4\theta].$$

For $\cos\theta \cos 4\theta = \frac{1}{2}\left[\cos(5\theta) + \cos(3\theta)\right]$.

$$= \frac{1}{2}(\cos\theta) + \frac{1}{2}\cdot\frac{1}{2}\left[\cos(5\theta) + \cos(3\theta)\right]$$

$$= \frac{1}{2}\cos\theta + \frac{1}{4}\left[\cos(3\theta)\right] + \frac{1}{4}\left[\cos(5\theta)\right] = \cos\theta\cos^2 2\theta.$$

put eq ①

$$\cancel{\frac{1}{4}\cos\theta + \frac{1}{4}\cos(3\theta) + \frac{1}{4}\cos 5\theta}$$

$$= \frac{1}{4}\left[\cos\theta + (\cos(3\theta) + \cos\theta) + \frac{1}{2}\cos\theta + \frac{1}{4}\cos(3\theta) + \frac{1}{4}\cos 5\theta\right]$$

$$= \frac{1}{4}\left[\frac{5}{2}\cos\theta + \frac{5}{4}\cos 3\theta + \frac{1}{4}\cos 5\theta\right].$$

- $\cos\theta$ terms: $1+1+\frac{1}{2} = \frac{5}{2}\cos\theta$

$$= \frac{5}{8}\cos\theta + \frac{5}{16}\cos 3\theta + \frac{1}{16}\cos 5\theta$$

- $\cos 3\theta$ term $= 1 + \frac{1}{4} = \frac{5}{4}\cos 3\theta$
- $\cos 5\theta$ term $= \frac{1}{4}\cos 5\theta$.

$$\cos^5\theta = \frac{1}{16}\left(10\cos\theta + 5\cos 3\theta + \cos 5\theta\right)$$

2) Time domain at point b

$m(t)$ by $\cos^5(w_c t)$

$$s_b(t) = m(t)\cos^5(w_c t)$$

$$= \frac{1}{16}\left[10\, m(t)\cos w_c t + 5 m(t)\cos 3 w_c t + m(t)\cos 5 w_c t\right]$$

$$= \frac{5}{8} m(t) \cos \omega_c t + \frac{5}{16} m(t) \cos 3\omega_c t + \frac{1}{16} m(t) \cos 5\omega_c t.$$

DSB-SC components created at $\pm \omega_c$, $\pm 3\omega_c$, $\pm 5\omega_c$

time domain amplitude $5/8, 5/16, 1/16$

Desired output at c :

After filter: $S_c(t) = K\, m(t) \cos \omega_c t$

Time domain

$K = 5/8$

frequency Domain $S_c(\omega) = \frac{K}{2}\left[M(\omega - \omega_c) + M(\omega + \omega_c)\right]$

b) signal spectra at b, c

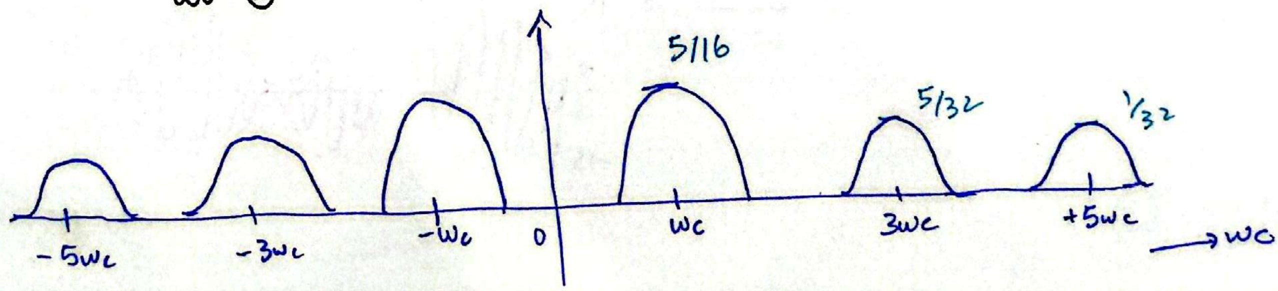
$M(\omega) = 0$ for $|\omega| > 2\pi B$

the spectrum at point b.

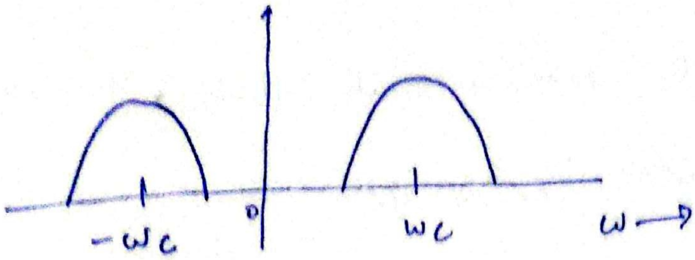
$$S_b(\omega) = \frac{5}{16}\left[M(\omega - \omega_c) + M(\omega + \omega_c)\right] + \frac{5}{32}\left[M(\omega - 3\omega_c) + M(\omega + 3\omega_c)\right] + \frac{1}{32}\left[M(\omega - 5\omega_c) + M(\omega + 5\omega_c)\right].$$

$$\mathcal{F}\{m(t) \cos(n\omega_c t)\} = \frac{1}{2}\left[M(\omega - n\omega_c) + M(\omega + n\omega_c)\right].$$

at (b)



at C



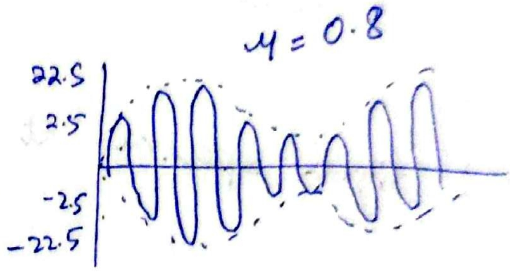
Question 5

$|m(t)| = m_p = 10$.

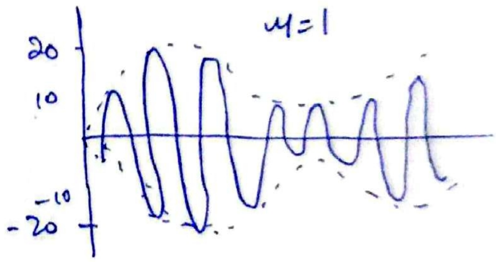
a)

$$\boxed{\mu = \frac{m_p}{A}}$$

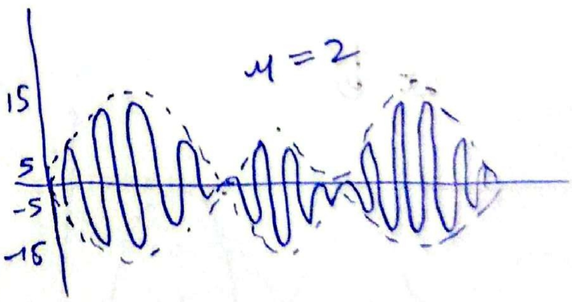
a/ $\mu = 0.8 \rightarrow A = \frac{10}{0.8} = 12.5 \rightarrow$



b/ $\mu = 1 \rightarrow A = \frac{10}{1} = 10$



c/ $\mu = 2 \rightarrow A = \frac{10}{2} = 5$



Given $0.8 = \mu$

$\mu = \frac{m_p}{A} \Rightarrow 0.8 = \frac{10}{A}$

$\boxed{A = 12.5V}$

P_c = carrier power

$= \frac{A_c^2}{2} = \frac{(12.5)^2}{2}$

$\boxed{P_c = 78.125W}$

b) $P_s = \frac{\overline{m^2(t)}}{2}$

$= \frac{100}{2}$

$\boxed{P_s = 50}$

$\eta = \frac{\mu^2}{2+\mu^2}$

$= 0.2424$

$= 24.24\%$

$\eta = \frac{P_s}{P_c+P_s} = \frac{50}{78.12+50}$

$= 0.39025$

$= 39\%$

Question #6

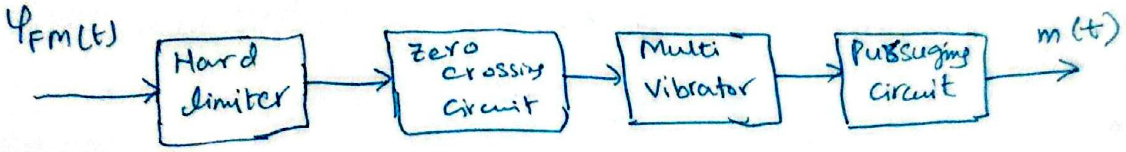
(a) FM demodulation:

(i) Slope-detection Method:

- Converts frequancy variations of FM into amplitude variations using a tuned circuit set slightly off resonance.
- Then an envelope extracts the message signal.

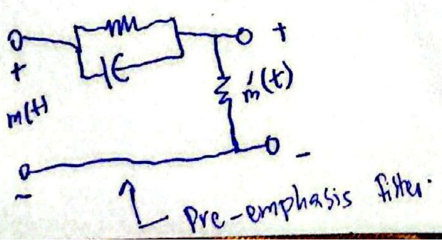
(ii) Zero-Crossing detector:

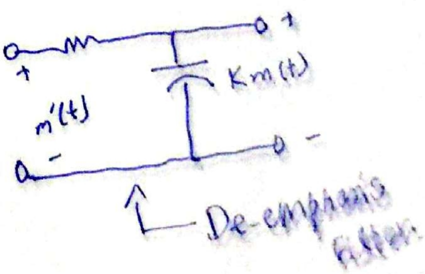
- Converts a waveforms into a digital pulse train by generating a pulse everytime the signal crosses the zero voltage.



(b) Pre-emphasis and De-emphasis:

- Pre-emphasis (Transmitter): Boots high-frequancy components before transmission to improve signal-to-noise (SNR) ratio.
- De-emphasis (Receiver): Reduces those boosted frequancies back to normal and also suppresses high-frequancy noise.
- Together they improve overall FM audio quality.





Question: 7

$w_c = 2\pi \times 10^6$ rad/s

$\varphi_{FM}(t) = 5\cos(w_c t + 10\cos 2000\pi t)$

a) Find power of modulated signal:

$$P = \frac{(A)^2}{2} = \frac{25}{2} = \boxed{12.5 \text{ W}}$$

b) Find the approximate band of frequancies occupied by FM waveform.

$w = 2\pi B$

$B = \frac{2000\pi}{2\pi}$

$\boxed{B = 1000\pi}$

$\Rightarrow \frac{d}{dt}\left[+10 \times 2000\pi \sin(2000\pi t)\right)$

$\Rightarrow -20000\pi \sin(2000\pi t)$.

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{20000\pi}{2\pi} = \boxed{10{,}000 \text{ Hz} = \Delta f}$$

$$B_T = 2(\Delta f + B) = 2(10 + 1) = \boxed{22 \text{ kHz}.}$$

So Band $\approx 1000{,}000$ Hz $\pm$ 11,000 Hz

$\approx 989{,}000 - 1{,}011{,}000$ Hz

c) Phase deviation $\Delta\phi$

$$\boxed{\Delta\phi = 10.}$$

d) Estimate Bandwidth: $\phi_{FM}(t)$

$f_m = \frac{2000\pi}{2\pi}$

$= 1000$ Hz

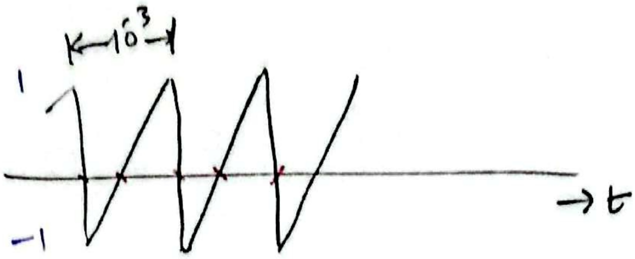
$$\text{Bandwidth} = 2(\Delta f + f_m)$$

$$= 2(10{,}000 + 1{,}000)$$

$$= 22 \text{ kHz}.$$

Question # 8 $\quad w_c = 2\pi \times 10^6$

$K_f = 2000\pi \quad , \quad K_p = \frac{\pi}{2}$



(a) Equations for PM & FM :

PM:

$$\varphi_{pm}(t) = w_c t + k_p\, m(t)$$

$$\varphi_{pm}(t) = w_c t + \frac{\pi}{2} m(t).$$

FM:

$$\varphi_{FM}(t) = w_c t + K_f \int_0^t m(\tau)\, d\tau$$

$$m(\tau) = -1 + 2000\, t$$

$$\int_0^t m(\tau)\, d\tau = -t + 1000 t^2$$

$\rightarrow$ Find intercept.

$m(0) = -1$

$-1 = a(0) \pm b$

$\boxed{b = -1}$

$a = \frac{1-(-1)}{2\times 10^{-3}} = 2000$.

$$\varphi_{FM}(t) = w_c t + K_f(-t + 1000t^2) \quad 0 \leq t < T'$$

(b) Frequancy deviation Δf :

PM :

$$\Delta f = \frac{K_p \dot{m}_p}{2\pi} = \frac{(\pi/2) \times (2000)}{2\pi}$$

$$\dot{m}_p = \frac{1-(-1)}{10^{-3}-0} = 2 \times 10^{+3}.$$

$$\boxed{\Delta f = 500 \text{ Hz}}$$

FM: $\Delta f = \dfrac{k_f\, m_p}{2\pi} = \dfrac{(2000\pi)(1)}{2\pi}$

$$\boxed{\Delta f = 1000\ Hz}$$

(C) Bandwidth: $B = 5 \times 1000 = 5000\,Hz.$

@ message assumed Bandwith at fith harmonic

PM:

$$B_{pm} = 2(\Delta f + B)$$

$$= 2(500 + 5000)$$

$$\boxed{B_{pm} = 11\ kHz}$$

FM:

$$B_{FM} = 2(\Delta f + B)$$

$$= 2(1000 + 5000)$$

$$\boxed{B_{FM} = 12\ kHz}$$

d) sketch:

$$f_i = \frac{2\pi \times 10^6}{2\pi} + \frac{k_F \cdot m(t)}{2\pi}$$

$$= 1\times10^6 + (2000\pi)(1)$$

$f_{max} = 1\times10^6 + 2000\pi = 1.006\ M\ Hz$

$f_{min} = 1\times10^6 - 2000\pi = 0.993\ M\ Hz$

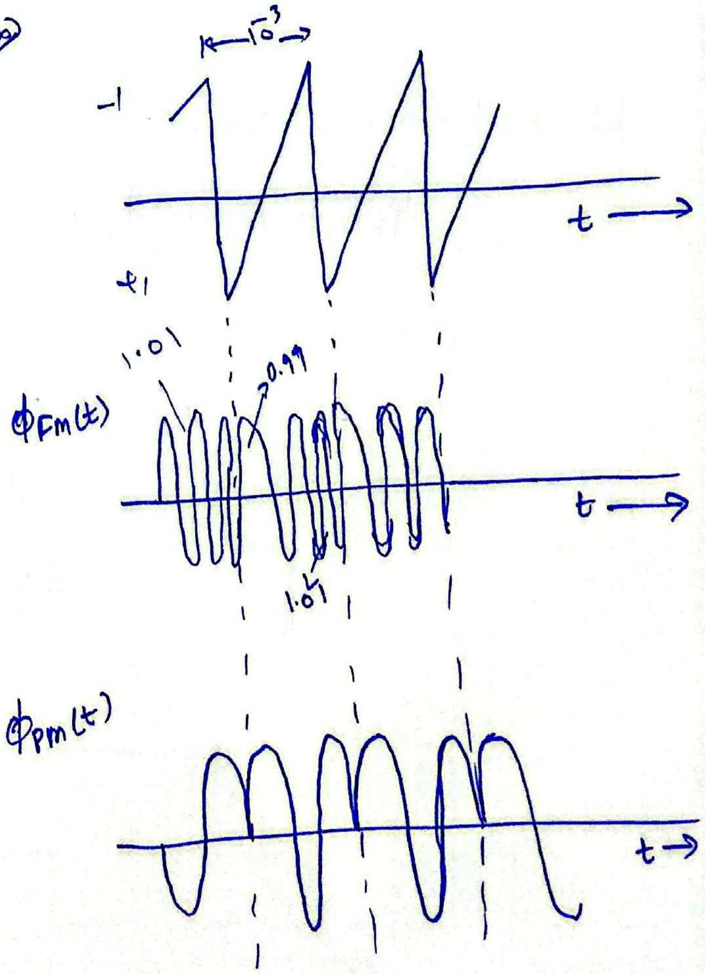
$$f_i = \frac{2\pi\times10^6}{2\pi} + \frac{k_p \cdot \dot{m}(t)}{2\pi}$$

$$= \frac{2\pi\times10^6}{2\pi} + \left(\frac{\pi}{2}\right)(2000)$$

$$= 1\times10^6 + 1000\pi$$

$f_{max} = 1\times10^6 + 1000\pi = 1.0031\ M.Hz$

$f_{min} = 1\times10^6 - 1000\pi = 0.99\ M \cdot Hz$



Question #9

$\varphi_{em}(t) = 10\cos(15{,}000\pi t)$

$w_c = 10{,}000\pi$

(a) PM signal, $k_p = 100$.

$\varphi_{PM} = A\cos[w_c t + k_p m(t)]$.

$= 10\cos[10{,}000\pi t + 100\, m(t)]$.

$\boxed{m(t) = 50\pi t}$

(b) FM signal, $k_f = 100$.

$$\varphi_{FM} = A\cos\left[w_c t + k_f \int_0^t m(\alpha)\, d\alpha\right]$$

$$= 10\cos\left[10{,}000\pi t + 100 \int_0^t m(\alpha)\, d\alpha\right].$$

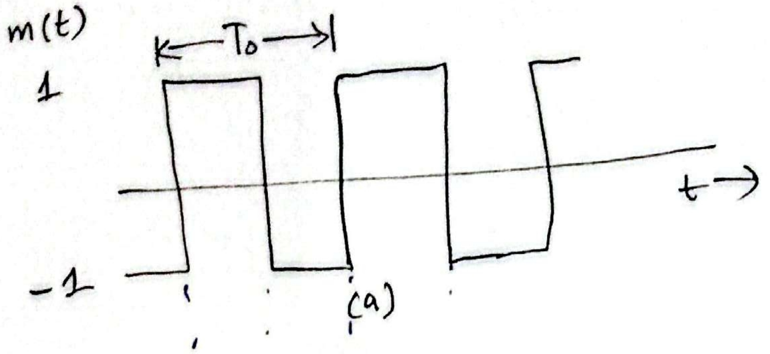
$\boxed{m(t) = 50\pi}$

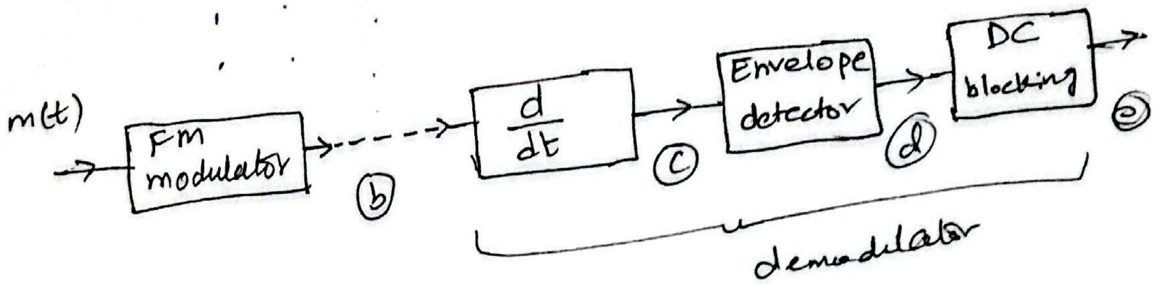
Question # 10

$f_c = 20\,kHz$

$\Delta f = 2\,kHz.$

Sketch waveform b, c, d, and e.





Solution.

FM