National University of Computer and Emerging Sciences
Lahore Campus

Complex Variables and Transforms (MT2003)

Sessional-II Exam

Total Time (Hrs): 1
Total Marks: 30
Total Questions: 3

Date: November 05 2024

Course Instructor(s)
Mr. Tasaduque Hussain Shah

Solution Key

Roll No ______ Section ______ Student Name & Signature ______

Instructions: Attempt all questions and write answers in the provided space. Rough work may be done on separate sheets but will not be attached or graded. No marks will be given for steps if the final answer is incorrect.

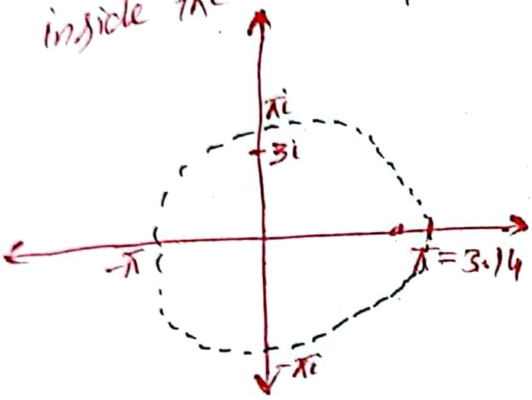
[CLO-3] Q1: Match each concept in the left column with the most suitable description in the right column. Write your chosen option in the center. ***[10]***

Concept	Ans.	Description
Cauchy Integral Theorem	f	a) Expresses periodic functions as sums of sine and cosine components.
Parameterization	e	b) Required the function to be analytic in the numerator within the contour.
Pole	g	c) Sources of infinite values
Fourier Series	a	d) Integral over unbounded domains
Even/Odd Functions	j	e) Expresses complex paths with simpler functions
Residue Integration	i	f) Ensures zero integral of the analytic function within the simple closed path.
Cauchy Integral Formula	b	g) A specific point where the function is undefined.
Improper Integral	d	h) Lacks differentiability at certain points
Non-Analytic Behavior	h	i) Evaluates the integral around a closed path by identifying poles and their residues.
Singularities	c	j) Allows certain Fourier's coefficients to be zero

[CLO-3] Q2 (a): Evaluate the integral. Does Cauchy's theorem apply? Show details. **[5]**

$$\oint_C \frac{dz}{z-3i},\ C \text{ the circle } |z| = \pi \text{ counterclockwise.}$$

No, because $f(z)$ is not analytic inside the simple closed path.



$f(z) = 1$

$z_0 = 3i \Rightarrow f(z_0) = 1$

$$\oint_C \frac{f(z)}{z-z_0}\,dz = 2\pi i\,(f(z_0))$$

$$= 2\pi i\,(1)$$

$$= 2\pi i$$

[CLO-4] Q2 (b): Find the Fourier series of the given function. **[5]**

$$f(x) = \begin{cases} -k & \text{if} \quad -\pi < x < 0 \\ k & \text{if} \quad 0 < x < \pi \end{cases}$$

Since $f(x)$ is an odd function. therefore $a_0 = a_n = 0$.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$\Rightarrow b_n = \frac{1}{\pi}\int_{-\pi}^{0} -k\sin(nx)\,dx + \frac{1}{\pi}\int_{0}^{\pi} k\sin(nx)\,dx$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\frac{k\cos(nx)}{n}\Big|_{-\pi}^{0}\right] + \frac{1}{\pi}\left[\frac{-k\cos(nx)}{n}\Big|_{0}^{\pi}\right]$$

$$\Rightarrow b_n = \frac{k}{n\pi}\left[\cos(0) - \cos(-n\pi) - \cos(n\pi) + \cos(0)\right]$$

$$b_n = \frac{k}{n\pi}\left[2\cos(0) - 2\cos(n\pi)\right]$$

$$b_n = \frac{2k}{n\pi}\left[1 - \cos(n\pi)\right]$$

$b_2 = b_4 = \dots = 0$

for $n =$ odd

$$b_n = \frac{4k}{n\pi}$$

$$\int_{-\infty}^{\infty} \frac{\sin x}{(x-1)(x^2+4)}\,dx$$

$$(x-1)(x^2+4) = 0$$

$$x = 1, \quad x = \pm 2i$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{\sin x}{(x-1)(x-2i)(x+2i)}\,dx$$

$n=1, x_0 = 2i$:

$$\operatorname*{Res}_{x=x_0} f(x) = \frac{1}{(n-1)!} \lim_{x \to x_0} \left\{ \frac{d^{n-1}}{dx^{n-1}} (x-x_0)^n f(x) \right\}$$

$$= \lim_{x \to 2i} \left\{ \cancel{(x-2i)} \frac{\sin x}{(x-1)\cancel{(x-2i)}(x+2i)} \right\}$$

$$= \frac{\sin(2i)}{(2i-1)(4i)}$$