Assingment #1 , BME

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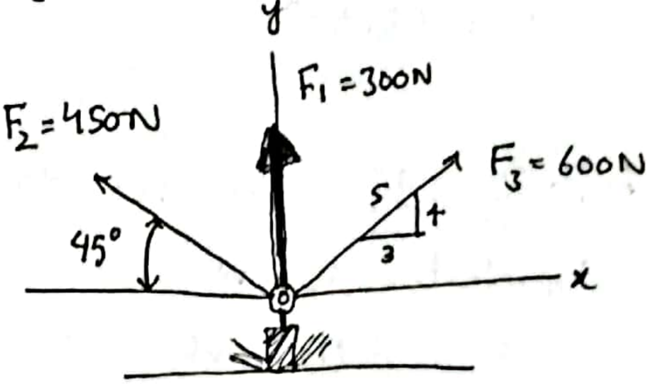
F 2-7 Reslove each force acting on the post into its x and y components.



$F_{1x} = 300 \cos(90°) = 0\ N$

$F_{1y} = 300 \sin(90°) = 300\ N$

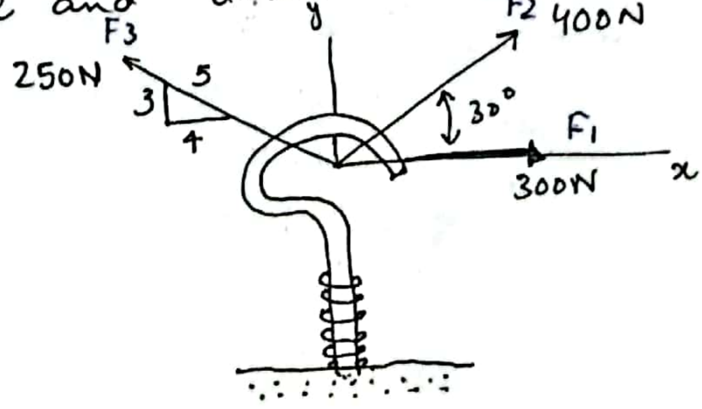
$F_1 = \begin{bmatrix} 0 \\ 300 \end{bmatrix} N.$

* $\frac{\text{Base}}{\text{hyp}} = \cos\theta$
* $\frac{\text{Prep}}{\text{hyp}} = \sin\theta$

$F_2 = \begin{bmatrix} -450 \cos 45° \\ 450 \sin 45° \end{bmatrix} = \begin{bmatrix} -318 \\ 318 \end{bmatrix} N.$

$F_3 = \begin{bmatrix} 600\left(\frac{3}{5}\right) \\ 600\left(\frac{4}{5}\right) \end{bmatrix} = \begin{bmatrix} 360 \\ 480 \end{bmatrix} N.$

F 2-8 Determine magnitude and direction of the resultant force.



$F_1 = \begin{bmatrix} 300 \cos(0) \\ 0 \end{bmatrix} = \begin{bmatrix} 300 \\ 0 \end{bmatrix} N$

$F_2 = \begin{bmatrix} 400 \cos 30° \\ 400 \sin 30° \end{bmatrix} = \begin{bmatrix} 346.41 \\ 200 \end{bmatrix} N$

$F_3 = \begin{bmatrix} -250\left(\frac{4}{5}\right) \\ 250\left(\frac{3}{5}\right) \end{bmatrix} = \begin{bmatrix} -200 \\ 150 \end{bmatrix} N$

* $\vec{F}_R = F_1 + F_2 + F_3$

$\vec{F}_R = \cancel{\begin{bmatrix} \text{[illegible]} \\ 350 \end{bmatrix}} = \begin{bmatrix} -446.41 \\ 350 \end{bmatrix}$

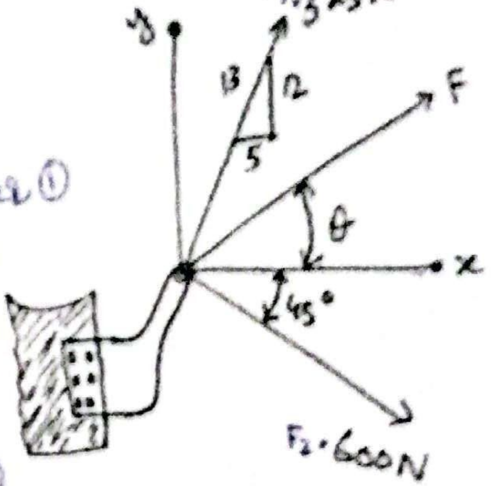
$F_R = \sqrt{(446.41)^2 + (350)^2}$

$= 567.25 \text{ N}$ ← Magnitude

$\theta = \tan^{-1}\left(\frac{350}{446.41}\right)$

$\theta = 38.09°$

F 2-10 If the resultant force acting on the bracket is to be 750 N directed along x axis, determine the magnitude of F and its direction θ.



$\vec{F}_R = \vec{F} + \vec{F}_1 + \vec{F}_2$

eq ①

$$\begin{bmatrix} 750 \\ 0 \end{bmatrix} = \begin{bmatrix} F\cos\theta + 325\left(\frac{5}{13}\right) + 600\cos 45° \\ F\sin\theta + \left(\frac{12}{13}\right)325 - 600\sin 45° \end{bmatrix}$$

eq ②

Take eq ①

$750 = F\cos\theta + 549.26$

$750 - 549.26 = F\cos\theta$

$200.74 = F\cos\theta \rightarrow$ (i)

eq ②

$0 = F\sin\theta - 124.26$

$124.26 = F\sin\theta \rightarrow$ (ii)

divide eq (ii) by (i)

$\because \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta} = \frac{124.26}{200.74}$

$\theta = \tan^{-1}\left(\frac{124.26}{200.74}\right)$

$\boxed{\theta = 31.757°}$

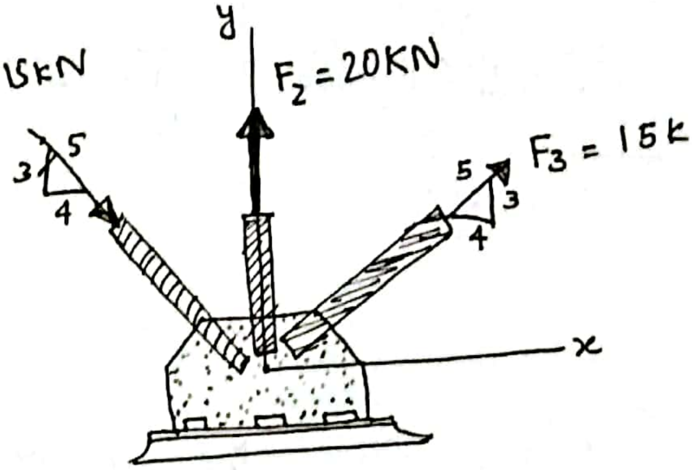
by eq ①

$F = \frac{200.74}{\cos(31.757)}$

$\boxed{F = 236.07 \text{ N}}$

F 2-12

-12 Determine the magnitude of resultant force and its direction θ measured counter clock wise from the positive x-axis.



$$F_1 = \begin{bmatrix} 15\left(\frac{4}{5}\right) \\ -15\left(\frac{3}{5}\right) \end{bmatrix} K = \begin{bmatrix} 12 \\ 9 \end{bmatrix} KN$$

$$F_2 = \begin{bmatrix} 20\cos(90^\circ) \\ 20\sin(90^\circ) \end{bmatrix} k = \begin{bmatrix} 0 \\ 20 \end{bmatrix} N$$

$$F_3 = \begin{bmatrix} 15\left(\frac{4}{5}\right) \\ +15\left(\frac{3}{5}\right) \end{bmatrix} K = \begin{bmatrix} 12 \\ 9 \end{bmatrix} KN$$

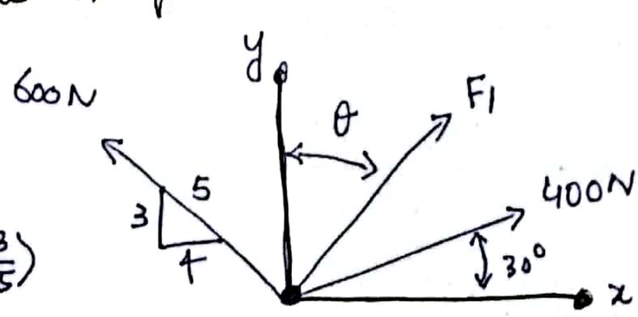
$$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

$$= \begin{bmatrix} 24 \\ 20 \end{bmatrix} KN$$

$$|F_R| = \sqrt{(24)^2 + (20)^2} = 31.2\ KN$$

$$\theta = \tan^{-1}\left(\frac{20}{24}\right) = 39.8^\circ.$$

2-39 Determine the magnitude of F_1 and its direction θ, so that resultant force is directly vertically upward and has magnitude of 800N.



y-axis:

$$800 = F_1 \cos\theta + 400 \sin 30 + 600\left(\frac{3}{5}\right)$$

$$F_1 \cos\theta = 240\ N$$

$$0 = F_1 \sin\theta + 400 \cos 30^\circ - 600\left(\frac{4}{5}\right) \quad \text{x-axis}$$

$$F_1 \sin\theta = \cancel{240\ N}\ 133.59 N.$$

$$\frac{\sin\theta}{\cos\theta} = \frac{134}{240}$$

$$\theta = \tan^{-1}\left(\frac{134}{240}\right)$$

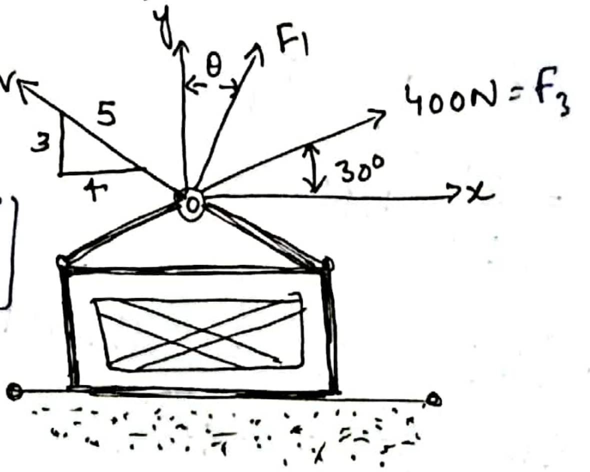
$$\boxed{\theta = 29.1^\circ}$$

$$F_1 = \frac{240}{\cos(29.1^\circ)}$$

$$\boxed{F_1 = 275N.}$$

2-40

Determine the magnitude and direction measured counter clock wise from positive x-axis, of resultant force of the three forces acting on the ring A. Take $F_1 = 500N$ & $\theta = 20^\circ$



$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= \begin{bmatrix} 500\sin 20 \\ 500\cos 20 \end{bmatrix} + \begin{bmatrix} -600\left(\frac{4}{5}\right) \\ 600\left(\frac{3}{5}\right) \end{bmatrix} + \begin{bmatrix} 400\cos 30^\circ \\ 400\sin 30^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 346.41 \\ 200 \end{bmatrix} + \begin{bmatrix} -480 \\ 360 \end{bmatrix} + \begin{bmatrix} 171.01 \\ 469.84 \end{bmatrix}$$

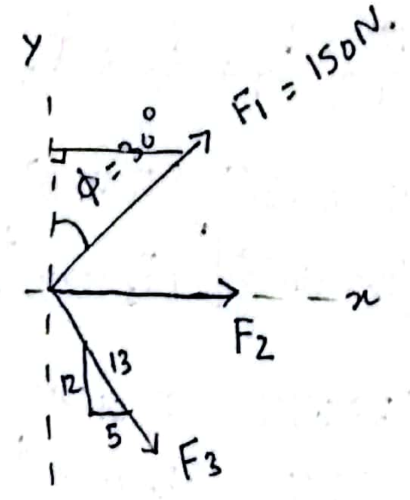
$$F_R = \begin{bmatrix} 37.42 \\ 1029.84 \end{bmatrix}$$

$$|F_R| = \sqrt{(37.42)^2 + (1029.84)^2} \Rightarrow \boxed{1030.51N = |F_R|}$$

$$\theta = \tan^{-1}\left(\frac{1029.84}{37.42}\right)$$

$$\boxed{\theta = 87.91^\circ}$$

51 If $F_1 = 150N$ and $\phi = 30°$, determine the magnitude of resultant force acting on the bracket and its direction measured clockwise from the positive x-axis.



Rectangular Components of F_1

$$F_1 = \begin{bmatrix} 150 \sin 30° \\ 150 \cos 30° \end{bmatrix} = \begin{bmatrix} 75 \\ 129.90 \end{bmatrix} N$$

Rectangular Components of F_2

$$F_2 = \begin{bmatrix} 200 \\ 0 \end{bmatrix} N$$

$$F_3 = \begin{bmatrix} 260\left(\frac{5}{13}\right) \\ -260\left(\frac{12}{13}\right) \end{bmatrix} = \begin{bmatrix} 100 \\ -240 \end{bmatrix} N$$

$$F_R = F_1 + F_2 + F_3$$

$$= \begin{bmatrix} 75 \\ 129.90 \end{bmatrix} + \begin{bmatrix} 200 \\ 0 \end{bmatrix} + \begin{bmatrix} 100 \\ -240 \end{bmatrix}$$

$$= \begin{bmatrix} 375 \\ -110.1 \end{bmatrix} N$$

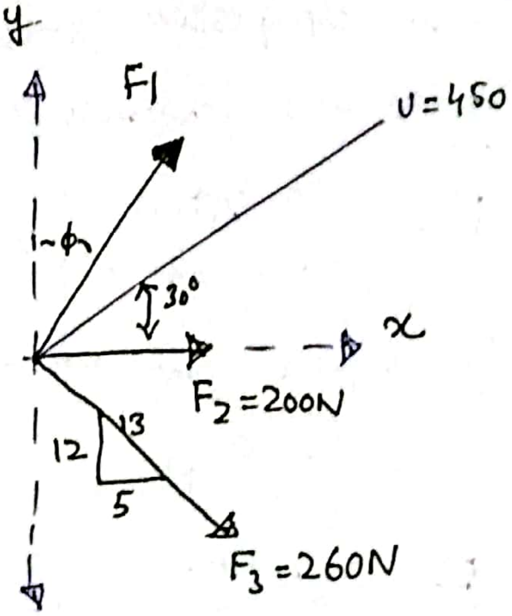
$$|F_R| = \sqrt{(375)^2 + (110.1)^2}$$

$$\boxed{= 390.83\ N}$$

$$\theta = \tan^{-1}\left(\frac{-110.1}{375}\right)$$

$$\boxed{\theta = -16.36°}$$

2-52 If magnitude of resultant force a... on the bracket is to 450N directed along the positive u-axis, determine the magnitude of F_1 and its direction ϕ.



$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\begin{bmatrix} 450\cos 30^\circ \\ 450 \sin 30^\circ \end{bmatrix} = \begin{bmatrix} F_1 \sin\phi \\ F_1 \cos\phi \end{bmatrix} + \begin{bmatrix} 200 \\ 0 \end{bmatrix} + \begin{bmatrix} 260\left(\frac{5}{13}\right) \\ -260\left(\frac{12}{13}\right) \end{bmatrix}$$

$$\begin{bmatrix} 389.711 \\ 225 \end{bmatrix} = \begin{bmatrix} F_1 \sin\phi \\ F_2 \cos\phi \end{bmatrix} + \begin{bmatrix} 300 \\ -240 \end{bmatrix}$$

$$89.71 = F_1 \sin\phi \rightarrow (i)$$

$$465 = F_2 \cos\phi \rightarrow (ii)$$

$$\frac{\sin\theta}{\cos\phi} = \frac{89.71}{465}$$

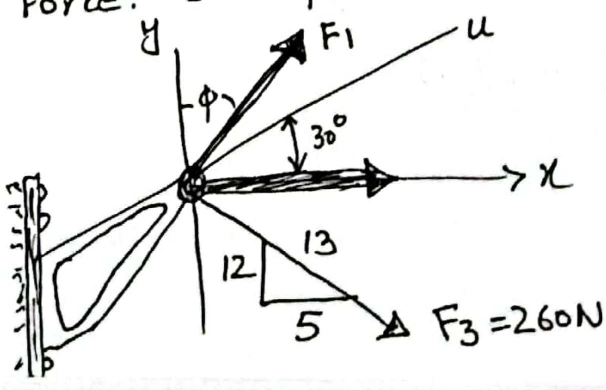
$$\phi = \tan^{-1}\left(\frac{89.71}{465}\right)$$

$$\boxed{\phi = 10.9^\circ}$$

$$|F_1| = \frac{89.71}{\sin(10.9)}$$

$$\boxed{F_1 = 474\ \text{N}}$$

2-53 If a resultant force acting on Bracket is required to be a minimum, determine the magnitudes of F_1 & resultant Force. Set $\phi = 30^\circ$



$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= \begin{bmatrix} F_1 \sin 30^\circ \\ F_1 \cos 30^\circ \end{bmatrix} + \begin{bmatrix} 200 \\ 0 \end{bmatrix} + \begin{bmatrix} 100 \\ -240 \end{bmatrix}$$

$$\begin{bmatrix} R_x \\ R_y \end{bmatrix} = \begin{bmatrix} F_1 \sin 30^\circ + 300 \\ F_1 \cos 30^\circ - 240 \end{bmatrix}$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$= \sqrt{(0.5 F_1 + 300)^2 + (0.866 F_1 - 240)^2}$$

$$R = \sqrt{F_1^2 + (-115.68 F_1) + 147600} \rightarrow ①$$

$\underline{R_{max}}$: $\frac{d}{dF_1}(F_1^2 - 115.68 F_1 + 147600) = 0$

$R = max \quad \Rightarrow 2F_1 - 115.68 = 0$

$$F_1 = \frac{115.68}{2}$$

$$\boxed{F_1 = 57.84 \text{ N}}$$

Put in eq ①

$$R = \sqrt{(57.84)^2 - (115.68)(57.84) + 147600}$$

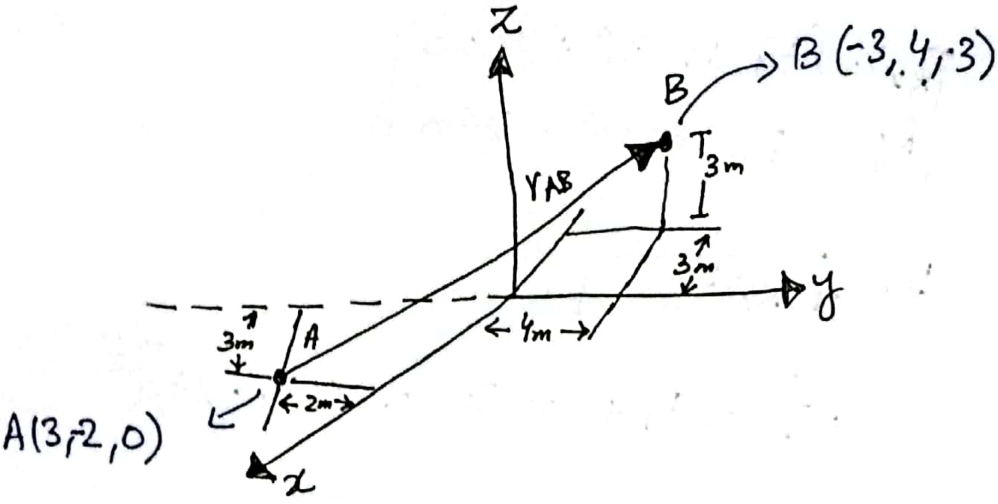
$$\boxed{R = 379.81 \text{ N}}$$

* Resultant is minimum when. $F_1 = 57.84 N$

$\longleftrightarrow$

F 2-19

Express the position vector r_{AB} in Cartesian vector form, then determine its magnitude and co-ordinate direction angles.



$$r_{AB} = r_B - r_A$$
$$= (-3, 4, 3) - (3, -2, 0)$$
$$= (-6, 6, 3)\ m$$

$$r = \sqrt{(-6)^2 + (6)^2 + (3)^2}$$
$$= \sqrt{36 + 36 + 9}$$
$$r = 9m$$

$$\frac{\vec{r}_{AB}}{|r|} = \left(-\frac{6}{9}\hat{i} + \frac{6}{9}\hat{j} + \frac{3}{9}\hat{k}\right) \Leftarrow \hat{AB}$$

$\because \alpha = \cos^{-1}\left(-\frac{6}{9}\right)$ $\quad\because \gamma = \cos^{-1}\left(\frac{3}{9}\right)$

$\boxed{\alpha = 131.81°}$ $\quad\boxed{\gamma = 70.53°}$

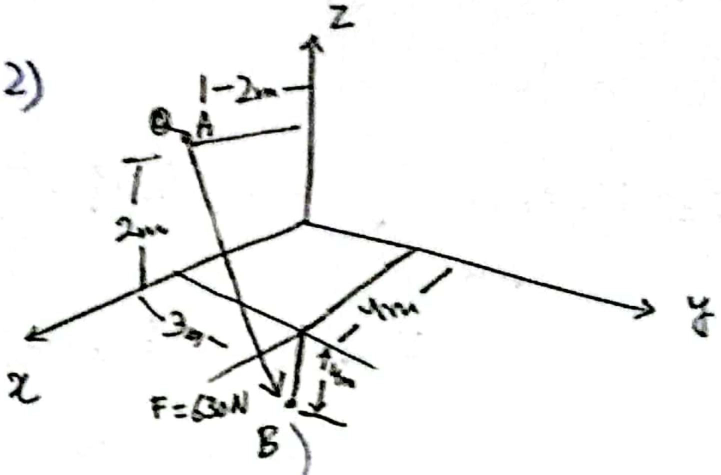
$\because \beta = \cos^{-1}\left(\frac{6}{9}\right)$

$\boxed{\beta = 48.19°}$

F2-21

2-21 Express force as a cartesian Vector

A(2,0,2)



B(4,3-4)

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$
$$= (4,3,-4) - (2,0,2)$$
$$= (2,3,-6) \text{ m}$$

$$|\vec{r}_{AB}| = \sqrt{(2)^2 + (3)^2 + (-6)^2}$$
$$= 7$$

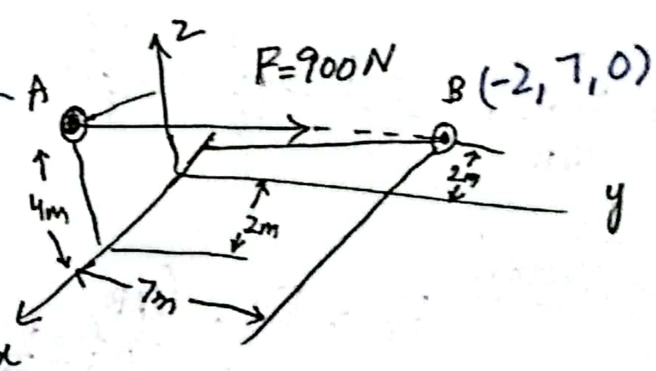
$$\hat{r}_{AB} = \frac{\vec{AB}}{|AB|} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$$

$$\vec{F}_{AB} = F \cdot \hat{AB}$$
$$= (630)\left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right)$$

$$\vec{F}_{AB} = \{180\hat{i} + 270\hat{j} - 540\hat{k}\} \text{ N.}$$

F-22 Express force as cartesian vector

A(2,0,4)



$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$
$$= (-2,7,0) - (2,0,4)$$
$$= -4\hat{i} + 7\hat{j} - 4\hat{k} \text{ m}$$

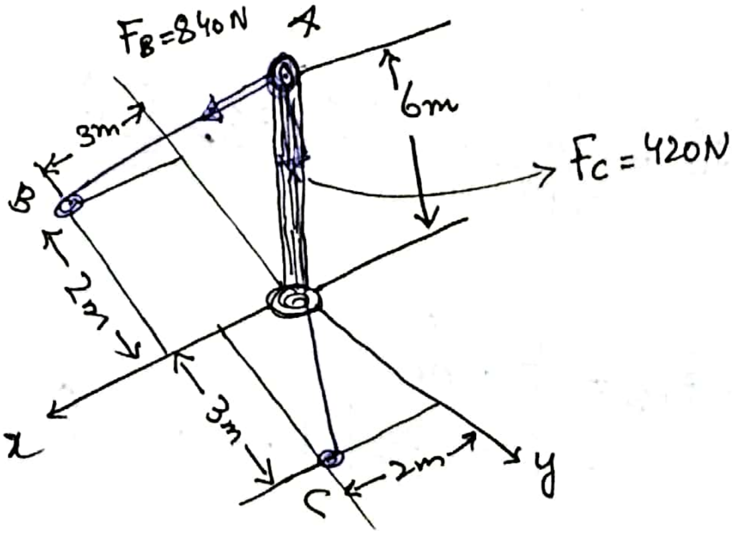
$$|r_{AB}| = \sqrt{(-4)^2 + (7)^2 + (-4)^2}$$
$$= 9m$$

$$F = F \cdot \frac{\vec{r}_{AB}}{r_{AB}} = 900 \times \left(\frac{-4\hat{i} + 7\hat{j} - 4\hat{k}}{9} \right)$$

$$F = \{-400\hat{i} + 700\hat{j} - 400k\} \text{ N}.$$

$\longleftrightarrow$

F-23 Determine the magnitude of resultant force at A.



$\boxed{\text{Unit vector along AB}}$

A → (0,0,6)

B → (3,−2,0)

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$
$$= (3,-2,0) - (0,0,6)$$
$$= (3,-2,-6)$$

$$|r_{AB}| = \sqrt{(3)^2 + (2)^2 + (6)^2} = 7m$$

$$\hat{U}_{AB} = \frac{\vec{r}_{AB}}{|r_{AB}|} = \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}$$

$\boxed{\text{Force vector}}$

$$F_B = F_B \cdot \hat{U}_{AB}$$
$$= 840 \cdot \left(\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}\right)$$
$$= \{360\hat{i} - 240\hat{j} - 720\hat{k}\} \text{ N}$$

~~unit~~ unit vector along AC

$A \rightarrow (0,0,6)$
$C \rightarrow (2,3,0)$

$$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A$$
$$= (2,3,0) - (0,0,6)$$
$$= (2,3,-6)\,m$$

$$|\vec{r}_{AC}| = \sqrt{(2)^2 + (3)^2 + (-6)^2} = 7\,m$$

$$U_{AC} = \frac{\vec{r}_{AC}}{|r_{AC}|} = \frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6}{7}\hat{k}$$

Force vector along AC

$$F_C = F_C \cdot U_{AC}$$
$$= 420 \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right)$$
$$= \{120\hat{i} + 180\hat{j} - 360\hat{k}\}\,N$$

Resultant Force at A

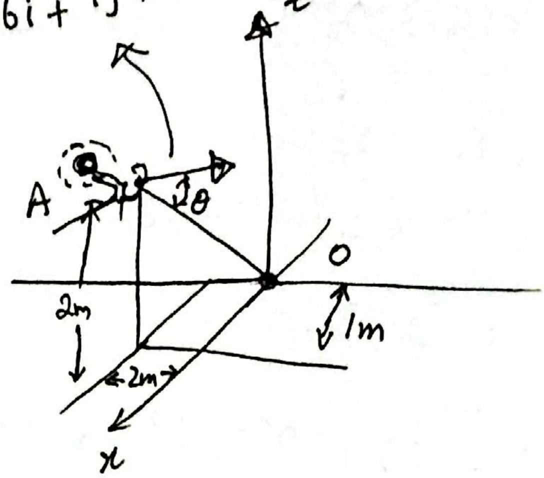
$$F = F_B + F_C$$
$$= (360, -240, -720) + (120, 180, -360)$$
$$\boxed{F = \{480\hat{i} - 60\hat{j} - 1080\hat{k}\}\,N}$$

Magnitude

$$F_R = \sqrt{(480)^2 + (60)^2 + (1080)^2}$$
$$= 1183.38\,N$$
$$\boxed{F_R = 1.18\,kN}$$

F 2-25 Determine the angle θ b/w the force and line AO.

$F = \{-6\hat{i} + 9\hat{j} + 3\hat{k}\}N$



$A = (1, -2, 2)m$

$O = (0, 0, 0)$

$\overline{AO} = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}$

$= \sqrt{(0-1)^2 + (0-(-2))^2 + (0-2)^2}$

$\overline{AO} = 3m.$

$\vec{AO} = \vec{O} - \vec{A}$
$= (0,0,0) - (1,-2,2)$
$= (-1, 2, -2)$

$|A| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$

$\vec{u}_{AO} = \left\{ \frac{x}{AO} \cdot i, \frac{y}{AO}, \frac{z \cdot k}{AO} \right\}$

$= \left\{ \frac{-1}{3} \cdot \hat{i}, \frac{2}{3} \cdot j, -\frac{2}{3} k \right\}$

$|F| = \sqrt{(6)^2 + (9)^2 + (3)^2}$

$= 11.225m$

$\hat{u}_F = \frac{F}{|F|} = \left\{ \frac{-6}{11.225} \hat{i} + \frac{9}{11.225} \hat{j} + \frac{3}{11.225} \hat{k} \right\}$

$\cancel{\cos\theta = u_{AO} \cdot u_F}$

* Dot Product

$\cos\theta = u_{AO} \cdot u_F$

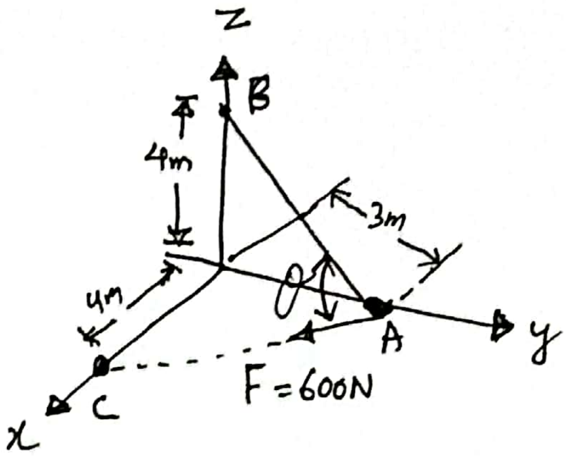
$\theta = \cos^{-1}(u_{AO} \cdot u_F)$.

$= \cos^{-1}\left(\left(-\frac{1}{3}\right)\left(\frac{-6}{11.225}\right) + \left(\frac{2}{3}\right)\left(\frac{9}{11.225}\right) + \left(\frac{-2}{3}\right)\left(\frac{3}{11.225}\right) \right)$

$\boxed{\theta = 57.7^\circ}$

F 2-26

2-26 Determine the angle b/w force and line AB.



Unit vector along AC

$A = (0,3,0)$

$C = (4,0,0)$

$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A$

$= (4,0,0) - (0,3,0)$

$= (4,-3,0)$

$|r_{AC}| = \sqrt{(4)^2 + (-3)^2 + (0)^2}$

$|r_{AC}| = 5m$

$u_{AC} = \dfrac{\vec{r}_{AC}}{|r_{AC}|} = \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j} + \frac{0}{5}\hat{k}$

Unit Vector along AB

$A = (0,3,0)$

$B = (0,0,4)$

$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

$= (0,0,4) - (0,3,0)$

$= (0,-3,4)$

$|r_{AB}| = \sqrt{0^2 + (-3)^2 + (16)} = 5m$

$u_{AB} = \dfrac{\vec{r}_{AB}}{|r_{AB}|} = \left(0i - \frac{3}{5}\hat{j} + \frac{4}{5}k\right)$

$\cos\theta = u_{AB} \cdot u_{AC}$

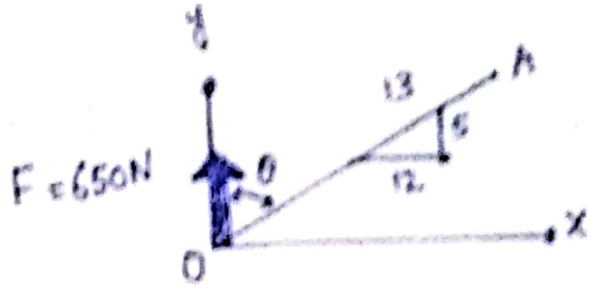
$\cos\theta = (0i - 0.6j + 0.8k) \cdot (0.8i - 0.6j + 0k)$

$\theta = \cos^{-1}(0.36)$

$\boxed{\theta = 68.89^\circ}$

F 2-27

Determine the angle θ b/w the force and the line OA



Unit Vector along OA

$O(0,0)$

$A(12,5)$

$\vec{r}_{OA} = \vec{r}_A - \vec{r}_O$

$= (12,5)$

$|r_{OA}| = \sqrt{(12)^2 + (5)^2}$

$= 13m$

$u_{OA} = \frac{\vec{r}_{OA}}{|r_{OA}|} = \frac{12}{13}\hat{i} + \frac{5}{13}\hat{j}$

Unit Force Vector

Force is along Y-axis

$j = \{0\hat{i} ; 1\hat{j}\}$

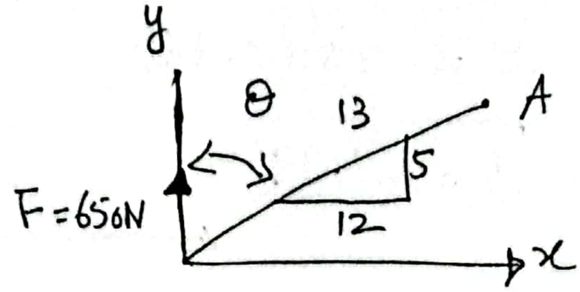
$\cos\theta = u_{OA} \cdot j$

$= \left(\frac{12}{13}\hat{i} + \frac{5}{13}\hat{j}\right) \cdot (0\hat{i} + 1\hat{j})$

$= 0.3846$

$\theta = \cos^{-1}(0.3846)$ $\Rightarrow \boxed{\theta = 67.38^\circ}$

F 2-28 Determine the component of projection of the force along the line OA.



As we calculate in previous ques, unit vector along $\overline{OA}$.

$$U_{OA} = \frac{12}{13}\hat{i} + \frac{5}{13}\hat{j}$$

Unit Force Vector

$$u_F = \{0\hat{i}, 1\hat{j}\}$$

Force Vector

$$F = F \cdot u_F$$
$$= 650 \cdot \{0\hat{i}, 1\hat{j}\}$$
$$F = \{0\hat{i} + 650\hat{j}\}\ N$$

Magnitude of Projected component:

$$F_{OA} = F \cdot u_{OA}$$
$$= \{0\hat{i} + 650\hat{j}\} \cdot \{\frac{12}{13}\hat{i} + \frac{5}{13}\hat{j}\}$$
$$F_{OA} = 250\ N$$

Vector of Projected component:

$$F_{OA} = F_{OA} \cdot U_{OA}$$
$$= 250 \cdot \left(\frac{12}{13}\hat{i} + \frac{5}{13}\hat{j}\right)$$
$$\boxed{F_{OA} = 230.78\hat{i} + 96.15\hat{j}}\ N$$