Example 3.16

A digital signal has eight levels. How many bits are needed per level? We calculate the number of bits from the formula

$$\text{Number of bits per level} = \log_2 8 = 3$$

Each signal level is represented by 3 bits.

Example 3.17

A digital signal has nine levels. How many bits are needed per level? We calculate the number of bits by using the formula. Each signal level is represented by 3.17 bits. However, this answer is not realistic. The number of bits sent per level needs to be an integer as well as a power of 2. For this example, 4 bits can represent one level.

Example 3.18

Assume we need to download text documents at the rate of 100 pages per minute. What is the required bit rate of the channel?

Solution

A page is an average of 24 lines with 80 characters in each line. If we assume that one character requires 8 bits, the bit rate is

$$100 \times 24 \times 80 \times 8 = 1{,}636{,}000 \text{ bps} = 1.636 \text{ Mbps}$$

Example 3.19

A digitized voice channel, as we will see in Chapter 4, is made by digitizing a 4-kHz bandwidth analog voice signal. We need to sample the signal at twice the highest frequency (two samples per hertz). We assume that each sample requires 8 bits. What is the required bit rate?

Solution

The bit rate can be calculated as

$$2 \times 4000 \times 8 = 64{,}000 \text{ bps} = 64 \text{ kbps}$$

Example 3.20

What is the bit rate for high-definition TV (HDTV)?

Solution

HDTV uses digital signals to broadcast high quality video signals. The HDTV Screen is normally a ratio of 16 : 9 (in contrast to 4 : 3 for regular TV), which means the screen is wider. There are 1920 by 1080 pixels per screen, and the screen is renewed 30 times per second. Twenty-four bits represents one color pixel. We can calculate the bit rate as

$$1920 \times 1080 \times 30 \times 24 = 1{,}492{,}992{,}000 \text{ or } 1.5 \text{ Gbps}$$

The TV stations reduce this rate to 20 to 40 Mbps through compression.

Bit Length

We discussed the concept of the wavelength for an analog signal: the distance one cycle occupies on the transmission medium. We can define something similar for a digital signal: the bit length. The bit length is the distance one bit occupies on the transmission medium.

$$\text{Bit length} = \text{propagation speed} \times \text{bit duration}$$

Example 3.22

What is the required bandwidth of a low-pass channel if we need to send 1 Mbps by using baseband transmission?

Solution

The answer depends on the accuracy desired.

a. The minimum bandwidth, a rough approximation, is $B =$ bit rate /2, or 500 kHz. We need a low-pass channel with frequencies between 0 and 500 kHz.
b. A better result can be achieved by using the first and the third harmonics with the required bandwidth $B = 3 \times 500$ kHz $= 1.5$ MHz.
c. Still a better result can be achieved by using the first, third, and fifth harmonics with $B = 5 \times 500$ kHz $= 2.5$ MHz.

Example 3.23

We have a low-pass channel with bandwidth 100 kHz. What is the maximum bit rate of this channel?

Solution

The maximum bit rate can be achieved if we use the first harmonic. The bit rate is 2 times the available bandwidth, or 200 kbps.

Example 3.26

Suppose a signal travels through a transmission medium and its power is reduced to one-half. This means that $P_2 = \frac{1}{2} P_1$. In this case, the attenuation (loss of power) can be calculated as

$$10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{0.5P_1}{P_1} = 10\log_{10}0.5 = 10(-0.3) = -3 \text{ dB}$$

A loss of 3 dB (-3 dB) is equivalent to losing one-half the power.

Example 3.27

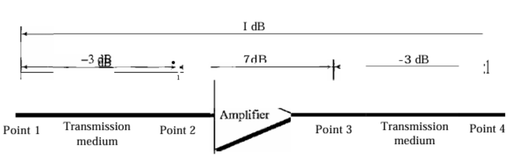
A signal travels through an amplifier, and its power is increased 10 times. This means that $P_2 = 10P_1$. In this case, the amplification (gain of power) can be calculated as

$$10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{10P_1}{P_1} = 10\log_{10}10 = 10(1) = 10 \text{ dB}$$

Example 3.28

One reason that engineers use the decibel to measure the changes in the strength of a signal is that decibel numbers can be added (or subtracted) when we are measuring several points (cascading) instead of just two. In Figure 3.27 a signal travels from point 1 to point 4. The signal is attenuated by the time it reaches point 2. Between points 2 and 3, the signal is amplified. Again, between points 3 and 4, the signal is attenuated. We can find the resultant decibel value for the signal just by adding the decibel measurements between each set of points.

Figure 3.27 *Decibels for Example 3.28*



In this case, the decibel value can be calculated as

$$\text{dB} = -3 + 7 - 3 = +1$$

The signal has gained in power.

Example 3.29

Sometimes the decibel is used to measure signal power in milliwatts. In this case, it is referred to as dB_m and is calculated as dB_m = 10 loglO *Pm'* where *Pm* is the power in milliwatts. Calculate the power of a signal if its dB_m =-30.

Solution

We can calculate the power in the signal as

$$dB_m = 10 \log_{10} Pm = -30$$

$$\text{loglO } P_m = -3 \qquad Pm = 10^{-3} \text{ mW}$$

Example 3.30

The loss in a cable is usually defined in decibels per kilometer (dB/km). If the signal at the beginning of a cable with -0.3 dBlkm has a power of 2 mW, what is the power of the signal at 5 km?

Solution

The loss in the cable in decibels is 5 x (−0.3) = -1.5 dB. We can calculate the power as

$$dB = 10 \log_{10} \frac{P_2}{P_1} = -1.5$$

$$\frac{P_2}{P_1} = 10^{-0.15} = 0.71$$

$$P_2 = 0.71 P_1 = 0.7 \times 2 = 1.4 \text{ mW}$$

Signal-to-Noise Ratio (SNR)

As we will see later, to find the theoretical bit rate limit, we need to know the ratio of the signal power to the noise power. The signal-to-noise ratio is defined as

$$SNR = \frac{\text{average signal power}}{\text{average noise power}}$$

We need to consider the average signal power and the average noise power because these may change with time. Figure 3.30 shows the idea of SNR.

SNR is actually the ratio of what is wanted (signal) to what is not wanted (noise). A high SNR means the signal is less corrupted by noise; a low SNR means the signal is more corrupted by noise.

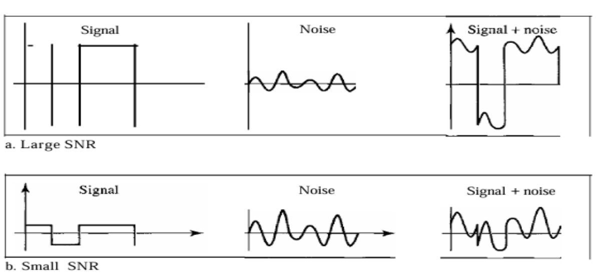
Because SNR is the ratio of two powers, it is often described in decibel units, SNR_{dB}, defined as

$$\mathbf{SNR_{dB}} = \text{lOloglo SNR}$$

Example 3.31

The power of a signal is 10 mW and the power of the noise is 1 μW; what are the values of SNR and SNR_{dB}?

Figure 3.30 *Two cases ofSNR: a high SNR and a low SNR*



Solution

The values of SNR and SNR_{dB} can be calculated as follows:

$$SNR = \frac{10{,}000 \ \mu W}{\text{ImW}} = 10{,}000$$

$$SNR_{dB} = 10 \text{ loglO } 10{,}000 = 10 \text{ loglO } 10^4 = 40$$

Example 3.32

The values of SNR and SNR_{dB} for a noiseless channel are

$$SNR = \frac{\text{signal power}}{0} = \infty$$

$$\text{SNRdB} = 10 \text{ loglO } \infty = \infty$$

We can never achieve this ratio in real life; it is an ideal.

Noiseless Channel: Nyquist Bit Rate

For a noiseless channel, the Nyquist bit rate formula defines the theoretical maximum bit rate

$$\text{BitRate} = 2 \times \text{bandwidth} \times \log_2 L$$

Example 3.33

Does the Nyquist theorem bit rate agree with the intuitive bit rate described in baseband transmission?

Solution

They match when we have only two levels. We said, in baseband transmission, the bit rate is 2 times the bandwidth if we use only the first harmonic in the worst case. However, the Nyquist formula is more general than what we derived intuitively; it can be applied to baseband transmission and modulation. Also, it can be applied when we have two or more levels of signals.

Example 3.34

Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The maximum bit rate can be calculated as

$$\text{BitRate} = 2 \times 3000 \times \log_2 2 = 6000 \text{ bps}$$

Example 3.35

Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). The maximum bit rate can be calculated as

$$\text{BitRate} = 2 \times 3000 \times \log_2 4 = 12{,}000 \text{ bps}$$

Example 3.36

We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signallevels do we need?

Solution

We can use the Nyquist formula as shown:

$$265{,}000 = 2 \times 20{,}000 \times \log_2 L$$

$$\log_2 L = 6.625 \qquad L = 2^{6.625} = 98.7 \text{ levels}$$

Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

Noisy Channel: Shannon Capacity

In reality, we cannot have a noiseless channel; the channel is always noisy. In 1944, Claude Shannon introduced a formula, called the Shannon capacity, to determine the theoretical highest data rate for a noisy channel:

$$\text{Capacity} = \text{bandwidth} \times \log_2 (1 + \text{SNR})$$

Example 3.37

Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel the capacity C is calculated as

$$C = B \log_2 (1 + \text{SNR}) = B \log_2 (1 + 0) = B \log_2 1 = B \times 0 = 0$$

This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

Example 3.38

We can calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000 Hz (300 to 3300 Hz) assigned for data communications. The signal-to-noise ratio is usually 3162. For this channel the capacity is calculated as

$$C = B \log_2 (1 + \text{SNR}) = 3000 \log_2 (1 + 3162) = 3000 \log_2 3163$$

$$= 3000 \times 11.62 = 34{,}860 \text{ bps}$$

This means that the highest bit rate for a telephone line is 34.860 kbps. If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

Example 3.41

We have a channel with a I-MHz bandwidth. The SNR for this channel is 63. What are the appropriate bit rate and signal level?

Solution

First, we use the Shannon formula to find the upper limit.

$$C = B \log_2 (1 + \text{SNR}) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 64 = 6 \text{ Mbps}$$

The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example. Then we use the Nyquist formula to find the number of signal levels.

$$4\text{Mbps} = 2 \times 1 \text{ MHz} \times \log_2 L \longrightarrow L = 4$$

The Shannon capacity gives us the upper limit;
the Nyquist formula tells us how many signal levels we need.

Example 3.44

A network with bandwidth of 10 Mbps can pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of this network?

Solution

We can calculate the throughput as

$$\text{Throughput} = \frac{12{,}000 \times 10{,}000}{60} = 2 \text{ Mbps}$$

The throughput is almost one-fifth of the bandwidth in this case.

Latency (Delay)

The latency or delay defines how long it takes for an entire message to completely arrive at the destination from the time the first bit is sent out from the source. We can say that latency is made of four components: propagation time, transmission time, queuing time and processing delay.

Latency = propagation time + transmission time + queuing time + processing delay

Propagation Time

Propagation time measures the time required for a bit to travel from the source to the destination. The propagation time is calculated by dividing the distance by the propagation speed.

$$\text{Propagation time} = \frac{\text{Distance}}{\text{Propagation speed}}$$

The propagation speed of electromagnetic signals depends on the medium and on the frequency of the signaL For example, in a vacuum, light is propagated with a speed of 3×10^8 mfs. It is lower in air; it is much lower in cable.

Example 3.45

What is the propagation time if the distance between the two points is 12,000 km? Assume the propagation speed to be 2.4×10^8 mls in cable.

Solution

We can calculate the propagation time as

$$\text{Propagation time} = \frac{12000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

The example shows that a bit can go over the Atlantic Ocean in only 50 ms if there is a direct cable between the source and the destination.

Tra/lsmissio/l Time

In data communications we don't send just 1 bit, we send a message. The first bit may take a time equal to the propagation time to reach its destination; the last bit also may take the same amount of time. However, there is a time between the first bit leaving the sender and the last bit arriving at the receiver. The first bit leaves earlier and arrives earlier; the last bit leaves later and arrives later. The time required for transmission of a message depends on the size of the message and the bandwidth of the channel.

$$\text{Transmission time} = \frac{\text{Message size}}{\text{Bandwidth}}$$

Example 3.46

What are the propagation time and the transmission time for a 2.5-kbyte message (an e-mail) if the bandwidth of the network is 1 Gbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at 2.4×10^8 *mls*.

Solution

We can calculate the propagation and transmission time as

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{2500 \times 8}{10^9} = 0.020 \text{ ms}$$

Note that in this case, because the message is short and the bandwidth is high, the dominant factor is the propagation time, not the transmission time. The transmission time can be ignored.

Example 3.47

What are the propagation time and the transmission time for a 5-Mbyte message (an image) if the bandwidth of the network is 1 Mbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at 2.4×10^8 mls.

5 M Bytes =
5×10^6 bytes

Solution

We can calculate the propagation and transmission times as

1 byte = 8 bits

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{5{,}000{,}000 \times 8}{10^6} = 40 \text{ s}$$

Note that in this case, because the message is very long and the bandwidth is not very high, the dominant factor is the transmission time, not the propagation time. The propagation time can be ignored.