Q2 Evaluate the following integrals:

(a) $\int_{-\infty}^{\infty} (t^2 + \cos \pi t)\, \delta(t-1)\, dt.$

$t - 1 = 0$

$t = 1$

$= (1)^2 + \cos \pi (1)$

$= 1 + \cos \pi$

$= 1 + (-1)$

$= 0.$

(b) $\int_{-\infty}^{\infty} (e^{-t})\, \delta(2t-2)\, dt$

$2t - 2 = 0$

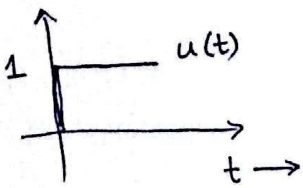
$t = 1$

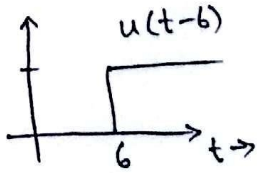
$= e^{-1}$

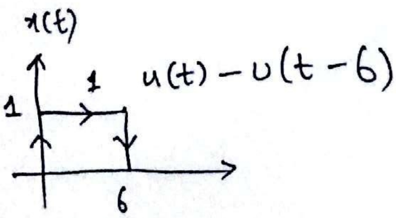
$= \frac{1}{e}$

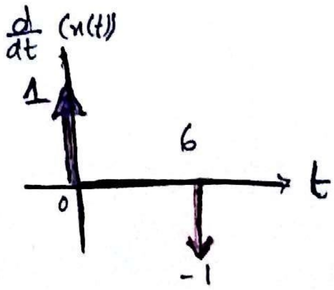
Q3

a) $x(t) = u(t) - u(t-6)$



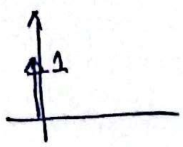






$\Rightarrow \frac{d}{dt}(1) = 0$

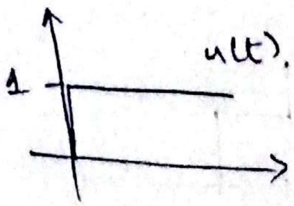
$\Rightarrow \frac{d}{dt} x(t) = 1 \longrightarrow$

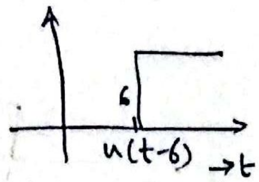


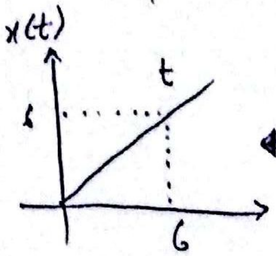
$\Rightarrow \frac{d}{dt}(x(t)) = \phi$

$= +\phi 6$

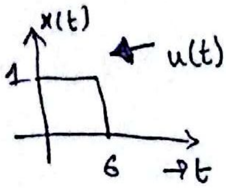
$$x(t) = t\,[u(t) - u(t-6)]$$







$\leftarrow x(t) = t\,[u(t) - u(t-6)]$

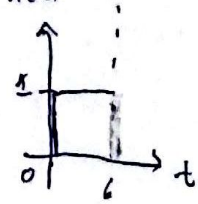


$\leftarrow u(t) - u(t-6)$

$6 = 6 \cdot x$

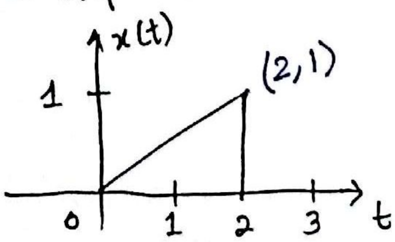
$\boxed{x = 1}$

$\frac{d}{dt} x(t)$



Q4 Express the signals shown below in terms of unit step function

(a)



in terms of x_1, y_1 x_2, y_2

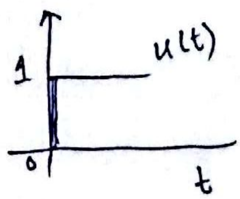
$(0,0) \rightarrow (2,1)$

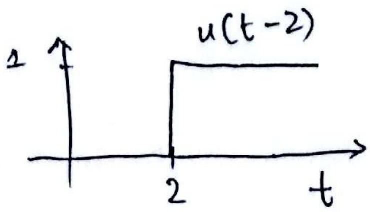
$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

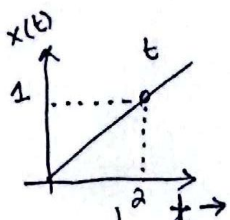
$$\frac{y - 0}{1 - 0} = \frac{x - 0}{2 - 0}$$

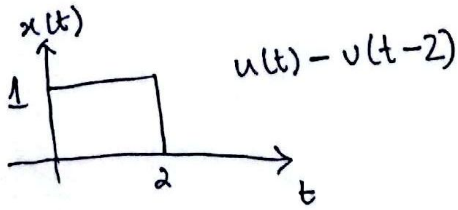
$$\frac{y}{1} = \frac{x}{2}$$

$y = t/2 \Leftarrow$ Ramp



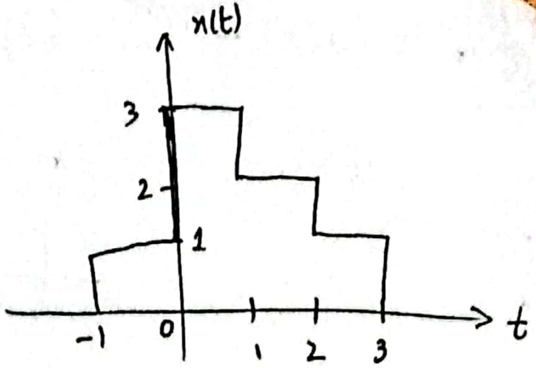




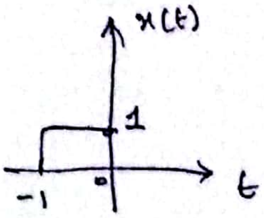


$\hookrightarrow x(t) = \frac{t}{2}[u(t) - u(t-2)]$.

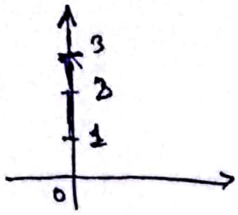
(b)



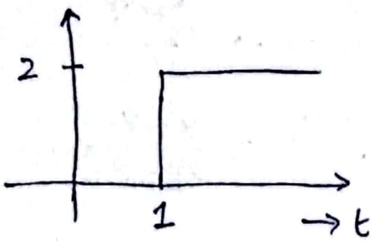
at $t=-1$ $u(t+1)$



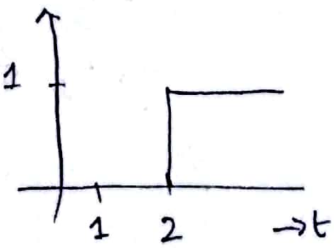
at $t=0$, $2u(t)$



at $t=1$, $u(t-1)$



at $t=2$, $u(t-2)$



at $t=3$, $u(t-3)$

Result:

$$x(t) = u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3).$$

Q-3

~~1/3~~ Find and sketch the odd and even component of $\sin w_0 t \ u(t)$

$x(t) = \sin w_0 t \ u(t)$

$x(-t) = \sin w_0 (-t) \ u(-t)$

$x(-t) = \sin w_0 (-t)$

$u(-t)$ is 1 for $t < 0$;

$u(-t)$ is 0 for $t \geq 0$;

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

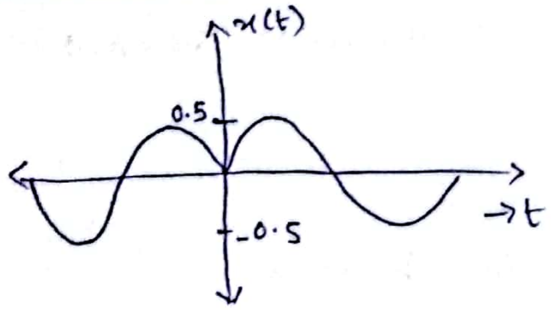
~~$t \geq 0$~~

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

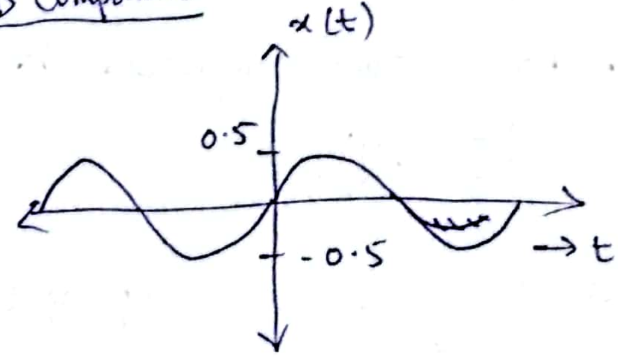
$$x(t) = \frac{x(t) + x(-t)}{2} + \frac{x(t) - x(-t)}{2}$$

$$= \underbrace{\frac{1}{2}\left[\frac{\sin(w_0 t u(t)) + \sin(-w_0 t u(-t))}{}\right]}_{\text{Even}} + \underbrace{\left[\frac{\sin(w_0 t u(t)) - \sin(-w_0 t u(t))}{2}\right]}_{\text{odd}}$$

Even Component:

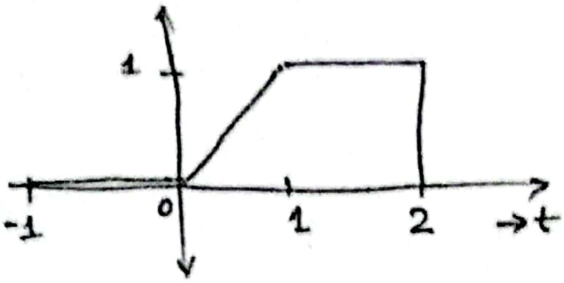


ODD Component:



Q7 Consider the Signal $y(t) = x(-2t \mp 3)$ shown fig.

Determine and carefully sketch the original signal $x(t)$



1) <u>Time Shifting</u>:

$$\phi(t) = x(t-T) \begin{cases} \phi(t) \text{ is delayed by } T \text{ seconds if } T>0. \\ \phi(t) \text{ is advanced by } T \text{ seconds if } T<0. \end{cases}$$

2) <u>Time Scaling</u>: $(a>1)$

Time compression: $\phi(t) = x(at)$

Time Expansion: $\phi(t) = x\left(\frac{t}{a}\right)$

3) <u>Time reversal</u>: $\phi(t) = x(-t)$.

$\boxed{\because y(t) = x(-2t \mp 3)}$

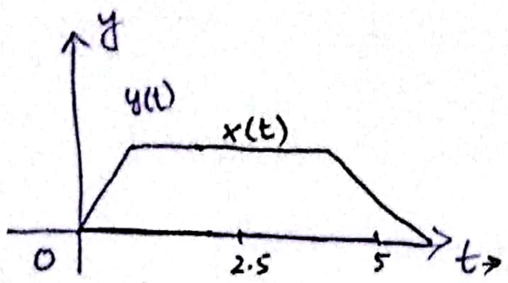
- Apply time shift by 3, the co-ordinates becomes $(t+3, y)$
- Apply time compression by -2, the co-ordinates becomes $\left(\frac{t+3}{-2}, y\right)$.

Let t' be abscissa of $y(t)$ determine t as a function t'.

$$t' = \frac{t+3}{-2} \Rightarrow t = -2t' - 3$$

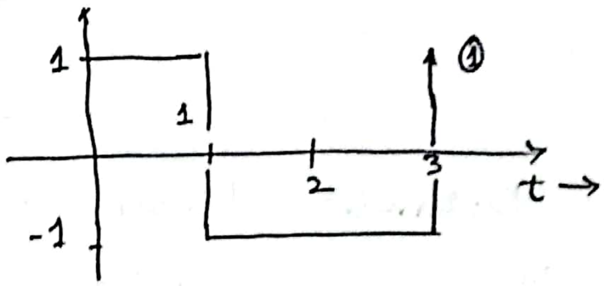
The generic co-ordinates of $x(t)$ can be written as $(-2t'-3, y')$ corresponding time shift by 3 and time expansion by -2 relative $y(t)$:

$$x(t) = y\left(-\frac{1}{2}t + 3\right)$$



Q8 Find and sketch $\int_{-\infty}^{t} x(t)\,dt$ for the signal $x(t)$ illustrated below.

(a)



@ $t \in [0,1]$

$x_1(t) = u(t) - u(t-1)$

@ $t \in [1,3]$

$x_2(t) = -u(t-1) + u(t-3)$

@ $t = 3$

$x_3(t) = u(t)\,\delta(t-3)$

$= u(3)\,\delta(t-3)$

$= \delta(t-3)$

So $x(t) = x_1(t) + x_2(t) + x_3(t)$.

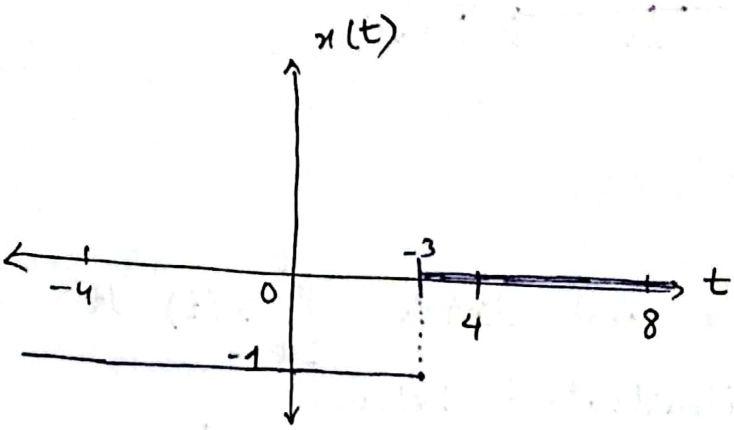
$= u(t) - 2u(t-1) + u(t-3) + \delta(t-3)$

$$= \int_{-\infty}^{t} u(\tau) - 2u(\tau - 1) + u(\tau - 3) + \delta(\tau - 3)\, d\tau$$

$$= \int_{0}^{t} d\tau - 2\int_{1}^{t} d\tau + \int_{3}^{t} d\tau + \int_{-\infty}^{t} \delta(\tau - 3)\, d\tau$$

$$= t - 2(t-1) + (t-3) + u(t-3)$$

$$= -1 + u(t-3).$$



Q9 Determine Linear & non-Linear

(a) $\dfrac{dy}{dt} + 2y(t) = x^2(t)$

$x_1^2(t) \longrightarrow y_1^2(t)$

$x_2^2(t) \longrightarrow y_2^2(t)$

$$\frac{dy_1^2}{dt} + 2y_1^2(t) = x_1^2(t) \longrightarrow (1)$$

$$\frac{dy_2^2}{dt} + 2y_2^2(t) = x_2^2(t) \longrightarrow (2)$$

multiply (1) by K_1 and (2) by K_2 & add them.

$$\frac{d}{dt}\left[K_1 y_1^*(t) + K_2 y_2(t)\right] + 2\left[K_1 y_1(t) + K_2 y_2(t)\right] =$$

$$K_1 x_1^2(t) + K_2 x_2^2(t). \rightarrow (2)$$

when input is $K_1 x_1^2(t) + K_2 x_1^2(t)$ The System response is $K_1 y_1(t) + K_2 y_2(t)$.

the system is non-linear.

(b) $$\frac{dy}{dt} + 3t\, y(t) = t^2 x(t)$$

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

$$\frac{dy_1}{dt} + 3t\, y_1(t) = t^2 x_1(t) \rightarrow (A.1)$$

$$\frac{dy_2}{dt} + 3t\, y_2(t) = t^2 x_2(t) \rightarrow (A.2)$$

multiply eq (A.1) by K1 and (A.2) by K2 & add them.

$$\frac{d}{dt}\left[K_1 Y_1(t) + K_2 Y_2(t)\right] + 3t\left[K_1 Y_1(t) + K_2 Y_2(t)\right]$$

$$= t^2\left[K_1 x_1(t) + K_2 x_2(t)\right] \rightarrow (2)$$

when input is $K_1 x_1(t) + K_2 x_2(t)$ the system response is $K_1 Y_1(t) + K_2 Y_2(t)$ as eq (2).

So System is Linear.

Q10

Input $x(t)$ and output $y(t)$ determine if the system is time varying or time invariant.

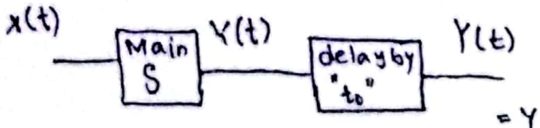
ⓐ $y(t) = \sin(x(t)) \cos t$.

- If we delay the input $x(t)$ by "t_0",
  $x(t)$ delayed by "t_0" will be $x(t-t_0)$

  then output is,

$$y(t, t_0) = [\sin(x(t-t_0))] \cos t \rightarrow ①$$

- Delay output $y(t)$ by "t_0" (i.e. replace t by $t-t_0$) and calculate output.



$= Y(t-t_0)$ {time invariant}
$\neq Y(t-t_0)$ {not time invariant}

$$Y(t) = \sin(x(t)) \cos t$$

$$Y(t-t_0) = \sin(x(t-t_0)) \cos(t-t_0) \rightarrow ②$$

$$eq (1) \neq eq (2)$$

So, the System is not time invarient.

Q-6 Which of the following signals are power signals & which are energy signals? which are neither? Justify answers?

a) $u(t) + 5u(t-1) - 2u(t-2)$.

$$u(t) = \begin{cases} 1 & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$

CASE 1: $t<0$

$x(t) = 0 + 0 + 0$
$= 0$

CASE 2 $0 \leq t < 1$

- $u(t) = 1$, $u(t-1) = 0$, $u(t-2) = 0$

$x(t) = 1$

CASE 3:

$1 \leq t < 2$

$u(t) = 1$, $u(t-1) = 1$, $u(t-2) = 0$

$x(t) = 1 + 5(1) + 0 = 6$

case 4: $t \geq 2$

- $u(t) = 1$, $u(t-1) = 1$, $u(t-2) = 01$

$x(t) = 1 + 5(1) - 2(1)$
$= 4$

Piece-wise function:

$$x(t) = \begin{cases} 0 & , t < 0 \\ 1 & , 0 \leq t < 1 \\ 6 & , 1 \leq t < 2 \\ 4 & , t \geq 2 \end{cases}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E = \int_0^1 1^2 \, dt + \int_1^2 6^2 \, dt + \int_2^{\infty} 4^2 \, dt$$

$$= \int_0^1 1 \, dt + \int_1^2 36 \, dt + \int_2^{\infty} 16 \, dt$$

$$E = (1-0) + 36(2-1) + 16(\infty - 2)$$

$$= 1 + 36 + \infty = \infty$$

$\boxed{\text{Energy is infinite}}$

Power:

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\int_0^1 1 \, dt + \int_1^2 36 \, dt \int_2^T 16 \, dt \right)$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(1 + 36 + 16(T - \cancel{3}2) \right)$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(1 + 36 + 16T - 32 \right)$$

$$= \lim_{T\to\infty} \frac{1}{2T} (16T + 5)$$

$$= \lim_{T\to\infty} \left(8 + \frac{5}{2T} \right)$$

$\boxed{P = 8} \rightarrow$ finite

The Given Signal is Power signal.

$(1+e^{-5t})\,u(t)$

$$E = \lim_{T\to\infty} \int_{-T}^{T} \left|(u(t))(1+e^{-5t})\right|^2 dt$$

$$u(t) = \begin{cases} 1 & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$

$$= \lim_{T\to\infty} \left[\int_{-T}^{0} \underbrace{|(1+e^{-5t})\,u(t)|^2}_{0} dt + \int_{0}^{T} |\cancel{e^{-t}}^{(1+e^{-5t})^2} (u(t))|^2 dt \right]$$

$$= \lim_{T\to\infty} \int_{0}^{T} (1+e^{-5t})^2 \, dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} (1 + 2e^{-5t} + e^{-10t}) \, dt$$

$$= \lim_{T\to\infty} \left(t + \frac{2e^{-5t}}{-5} + \frac{e^{-10t}}{-10} \right) \Bigg|_{0}^{T}$$

$$= \lim_{T\to\infty} \left(T + \frac{2}{-5} e^{-5T} + \frac{e^{-10T}}{-10} \right) \qquad \Rightarrow e^{-\infty} = 0$$

$$\boxed{E = \infty}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$\Rightarrow$ (As we solve at above)

$$= \lim_{T\to\infty} \frac{1}{2T} \left(T + \frac{2}{-5} e^{-5T} + \frac{e^{-10T}}{-10} \right)$$

$$= \frac{1}{2T} (T + 0 + 0)$$

$$\boxed{P = \frac{1}{2}}$$

So, Given Signal is a Power Signal.