National University of Computer and Emerging Sciences
Lahore Campus

Multivariable Calculus (MT2008)

Final Exam

Date: 31 December 2024

Course Instructor
Muhammad Yaseen

Total Time (Hrs): 3
Total Marks: 100
Total Questions: 3

Roll No

Section

Student Signature

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Attempt all the questions.

CLO #1. Formulate the equation of lines, planes and surfaces.

Q1: Write standard definition of Quadric Surfaces. Identify and sketch the given surface

$$4x^2 - y^2 + 2z^2 + 4 = 0$$

[10 marks].

CLO #2. Calculate the rate of change of multivariable functions and its applications.

Q2: (a). Find $\frac{\partial z}{\partial x}$ **and** $\frac{\partial z}{\partial y}$ **if** $x^3 + y^3 + z^3 + 6xyz = 1$.

(b). Find the local maximum and minimum values and saddle points of the given function

$$f(x, y) = x^4 + y^4 - 4xy + 1.$$

(c). Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ **at the point** $(1, 1, 3)$.

[5+ 10+5 marks].

CLO #3. Evaluate the integral of a multivariable function and its applications.

Q3: (a). Evaluate the iterated integral $\int_0^1 \int_y^1 \sin x^2 \; dx \; dy$. 0.199

(b). Find the flux of the vector field $F(x, y, z) = x\,k + y\,j + z\,i$ **across the unit spere** $x^2 + y^2 + z^2 = 1$. $\frac{4\pi}{3}$

(c). Use the Divergence theorem to calculate the surface integral $\iint F.dS$; **that is, calculate the flux of** F **across S,** $F(x, y, z) = xye^z \; i + xy^2z^3 \; j - ye^z \; k$, **S is the surface of the box bounded by the coordinate planes and the planes** $x = 3, y = 2$ **and** $z = 1$. 9/2 4.5

(d). Define curvature and also find the curvature of a circle of radius a. 1/a

 0065

(e). Find a parametrization for the surface S formed by the part of the hyperbolic paraboloid $z = y^2 - x^2$, lying inside the cylinder of radius one around the z-axis and for the boundary curve C of S. Then verify stokes Theorem for S using the normal having positive k-component and the vector field $F = y\,i - x\,j + x^2\,k$. 12

(f). Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2x$. 3/9

(g). Integrate $G(x, y, z) = \sqrt{1 - x^2 - y^2}$ over the "football" surface S formed by rotating the curve $x = \cos z, y = 0, -\pi/2 \le z \le \pi/2$, around the z-axis. 0

(h). Evaluate the surface integral $\iint (x + y + z)\, dS$, where S is the part of the half cylinder $x^2 + z^2 = 1, z \le 0$, that lies between the planes $y = -2$ and $y = 0$. 0

[5+10+10+5+10+10+10+10 marks]