0068

National University of Computer and Emerging Sciences
Lahore Campus

Electromagnetic Theory (EE3005)

Final Exam

Date: December 17th 2025

Course Instructor(s)

Dr. Afzal Ahmed (Course Moderator)

Mr. Mohsin Yousuf

Total Time (Hrs):	3
Total Marks:	100
Total Questions:	5

Roll No ________ Section ________ Student Signature ________

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Attempt all the questions.

CLO 1: Demonstrate the use of 3D orthogonal coordinate system and vector analysis tools in problem solving.

[20 Marks]

Q1:

a) **Determine** the distance in meters between $P\left(\overset{r}{1.5m}, \overset{\theta}{\frac{\pi}{2}}, \overset{\phi}{20^{\circ}}\right)$ and $Q\left(\overset{r}{3.5m}, \overset{\theta}{\frac{3\pi}{4}}, \overset{\phi}{115^{\circ}}\right)$, where the points are given in spherical coordinate system (SCS). [5]

b) **Convert** the vector field $H = \left(\frac{A}{\rho}\right) a_{\varphi}$, where A is a constant, from cylindrical to spherical coordinates. [5]

CCS → SCS ⇒ $(\rho, \phi, z) \rightarrow (r, \theta, \phi)$

c) The surface $r = 1m$ and $3m$, $\theta = 10^{\circ}$ and 60°, and $\varphi = 0^{\circ}$ and 90°, identify a closed surface. **Draw** the diagram and **calculate** the following: [10]

i. The enclosed volume.

ii. The total area of the enclosing surface.

iii. Total precise length of the twelve edges of the surface.

CLO 2: Analyze electrostatic fields and/or its properties governed by Coulomb's / Gauss's law for a given charge distribution in free space and/or dielectrics.

[20 Marks]

Q2:

a) Charge density within a region of free space is given as $\rho_v = \frac{\rho_0 r \cos\theta}{a}\ C/_{m^3}$, where ρ_0 and a are constants. **Find** the total charge **Q** lying within: [10]

i. The sphere, $r \leq a$;

ii. The cone, $r \leq a$, $0 \leq \theta \leq 0.1\pi$;

iii. The region, $r \leq a$, $0 \leq \theta \leq 0.1\pi$, and $0 \leq \varphi \leq 0.2\pi$.

b) **Calculate** $\nabla . D$ at the point specified if [10]

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i. $D = \left(\frac{1}{z^2}\right)\left[10xyza_x + 5x^2za_y + (2z^3 - 5x^2y)a_z\right]$ at $P(-2,3,5)$;

ii. $D = 5z^2a_\rho + 10\rho za_z$ at $P(3,-45^\circ,5)$;

iii. $D = 2r\sin\theta\sin\varphi a_r + r\cos\theta\sin\varphi a_\theta + r\cos\varphi a_\varphi$ at $P(3,45^\circ,-45^\circ)$.

CLO 2: Analyze electrostatic fields and/or its properties governed by Coulomb's / Gauss's law for a given charge distribution in free space and/or dielectrics.

Q3: **[20 Marks]**

a) A copper sphere of radius 4 cm carries a uniformly distributed total charge of $10\ \mu C$ in free space. [10]

i. **Find** the total energy stored in the electrostatic field.

ii. **Calculate** the capacitance of the isolated sphere.

b) Given the potential field, $V = \frac{100}{z^2+1}\rho\cos\varphi$ V, and point P at $\rho = 3m$, $\varphi = 60^\circ$, $z = 2m$, **find** values at P for: [10]

i. The potential; V. And the electric flux intensity; E.

ii. dV/dN, and ρ_v in free space.

CLO 3: Calculate the capacitance with one dimensional potential variation using direct integration.

Q4: **[20 Marks]**

a) **Define** capacitance with mathematical formulae. Using this formula, **calculate** the capacitance of a parallel-plate capacitor with proper diagram. The plates are filled with dielectric having permittivity ε and are oriented along xz plane at $y = 0$ and $y = d$. [8]

*Hint: You can find **D** by applying boundary conditions at any one of the plates.*

b) **Derive** Laplace's equation from point form of Gauss's law and **calculate** the capacitance of a co-axial capacitor of inner radius a, outer radius b, and length L by direct integration method. [12]

CLO 4: Explain reflection, magnetic fields and its effects for a given distribution of moving charges using laws of magnetostatics.

Q5: **[20 Marks]**

a) There is a region with cylindrical symmetry in which the conductivity is given by $\sigma = 1.5e^{-150\rho}\ kS/m$. An electric field of $30a_z\ V/m$ is present. [10]

i. **Calculate** current density **J**.

ii. Also **find** the total current crossing the surface $\rho < \rho_0$, $z = 0$, all φ.

iii. Make use of Ampere's circuital law to **find H**.

b) **State** Faraday's Law and **derive** Maxwell's equations $\nabla \times E$ and $\nabla \times H$ for time varying fields. [10]

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Formula Sheet

$$\mathbf{F}_1 = -\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{21} = -\frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12} \qquad \mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2}\mathbf{a}_R \qquad \mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n}\frac{Q_m}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_m|^2}\mathbf{a}_m \qquad \mathbf{E} = \frac{\rho_S}{2\epsilon_0}\mathbf{a}_N$$

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\mathbf{a}_\rho \qquad \mathbf{D} = \int_{vol}\frac{\rho_v dv}{4\pi R^2}\mathbf{a}_R \qquad \Psi = \oint_S \mathbf{D}_S \cdot d\mathbf{S} = \text{charge enclosed} = Q \qquad \mathbf{J} = \rho_v \mathbf{v} \qquad \text{div}\,\mathbf{D} = \rho_v$$

$$\mathbf{E} = -\nabla V \qquad \nabla^2 V = 0 \qquad (\nabla \cdot \mathbf{J}) = -\frac{\partial \rho_v}{\partial t} \qquad V(\mathbf{r}) = \int_{vol}\frac{\rho_v(\mathbf{r}')\,dv'}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|}$$

$$\text{div}\,\mathbf{D} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \qquad \text{div}\,\mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} \quad \text{(cylindrical)}$$

$$\text{div}\,\mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi} \quad \text{(spherical)} \qquad W = -Q\int_{init}^{final}\mathbf{E}\cdot d\mathbf{L}$$

$$d\mathbf{L} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z \quad \text{(rectangular)}$$
$$d\mathbf{L} = d\rho\,\mathbf{a}_\rho + \rho\,d\phi\,\mathbf{a}_\phi + dz\,\mathbf{a}_z \quad \text{(cylindrical)}$$
$$d\mathbf{L} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi \quad \text{(spherical)}$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z \qquad \nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi \qquad I = \int_S \mathbf{J}\cdot d\mathbf{S}$$

$$\mathbf{E}\times\mathbf{n}\big|_s = 0$$

$$W_E = \tfrac{1}{2}\int_{vol}\mathbf{D}\cdot\mathbf{E}\,dv = \tfrac{1}{2}\int_{vol}\epsilon_0 E^2\,dv \qquad W_E = \tfrac{1}{2}\int_{vol}\rho_v V\,dv \qquad \mathbf{J} = \sigma\mathbf{E} \qquad \sigma = -\rho_e\mu_e \qquad \mathbf{D}\cdot\mathbf{n}\big|_s = \rho_s$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2} \qquad \mathbf{P} = \lim_{\Delta v \to 0}\frac{1}{\Delta v}\sum_{i=1}^{n\Delta v}\mathbf{p}_i \qquad \mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} \qquad \mathbf{P} = \chi_e\epsilon_0\mathbf{E} \qquad (\mathbf{D}_1 - \mathbf{D}_2)\cdot\mathbf{n} = \rho_s \qquad (\mathbf{E}_1 - \mathbf{E}_2)\times\mathbf{n} = 0$$

$$\mathbf{H} = \oint\frac{I\,d\mathbf{L}\times\mathbf{a}_R}{4\pi R^2} \qquad W_E = \tfrac{1}{2}CV_0^2 = \tfrac{1}{2}QV_0 = \tfrac{1}{2}\frac{Q^2}{C} \qquad \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon} \qquad V = \frac{Q}{4\pi\epsilon_0 r}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2} \qquad \oint \mathbf{H}\cdot d\mathbf{L} = I \qquad \mathbf{F} = Q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad \Phi = \int_S \mathbf{B}\cdot d\mathbf{S}$$

$$\mathbf{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\phi \qquad \mathbf{B} = \mu_0\mathbf{H} \qquad \oint\mathbf{H}\cdot d\mathbf{L} = \int_S(\nabla\times\mathbf{H})\cdot d\mathbf{S} \qquad I\,d\mathbf{L} = \mathbf{K}\,dS = \mathbf{J}\,dv$$

$$\nabla\times\mathbf{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial\phi} - \frac{\partial H_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial\rho}\right)\mathbf{a}_\phi + \left(\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial\phi}\right)\mathbf{a}_z \quad \text{(cylindrical)}$$

$$\frac{dW_E}{dv} = \tfrac{1}{2}\mathbf{D}\cdot\mathbf{E}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} \qquad \mathbf{F} = \oint I\,d\mathbf{L}\times\mathbf{B}$$

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z\cdot$	$\cos\theta$	$-\sin\theta$	0

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla\times\mathbf{H} = \frac{1}{r\sin\theta}\left(\frac{\partial(H_\phi\sin\theta)}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(rH_\phi)}{\partial r}\right)\mathbf{a}_\theta + \frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta}\right)\mathbf{a}_\phi \quad \text{(spherical)}$$

$dS_\rho = \rho d\varphi dz\, \boldsymbol{a}_\rho, dS_\varphi = d\rho dz\, \boldsymbol{a}_\varphi, dS_z = \rho d\varphi d\rho\, \boldsymbol{a}_z, dv = \rho d\rho d\varphi dz, dS_r = r^2\sin\theta\, d\theta d\varphi\, \boldsymbol{a}_r, dS_\theta = r\sin\theta\, dr d\varphi\, \boldsymbol{a}_\theta, dS_\varphi = r dr d\theta\, \boldsymbol{a}_\varphi, dv = r^2\sin\theta\, dr d\theta d\varphi$

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Electromagnetic Theory (EE3005)

Sessional-I Exam

Date: September 20, 2025

Course Instructor(s)

1. Dr. Afzal Ahmed (Course Moderator)
2. Mohsin Yousuf

Total Time (Hrs):	**1**
Total Marks:	**40**
Total Questions:	**2**

Roll No ____________ Section ____________ Student Signature ____________

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1. ***Attempt all questions and remember to solve parts of the same question together.***
2. ***Final answers should be correct up to two decimal places with proper SI units.***
3. ***Show all the steps with the help of diagrams and equations.***

CLO # 01: Demonstrate the use of 3D orthogonal coordinate system and vector analysis tools in problem solving

Q1: A vector field is given in rectangular coordinate system (RCS) as: ***[20 marks]***

$$Q_R = \frac{\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2}}[(x-y)a_x + (x+y)a_y]$$

Zunaira likes eating ice-cream cone, help her in answering the following questions using Figure 1.

(a) **Convert** the above vector field Q_R in spherical coordinate system (SCS) and name it Q_S. **[7]**

(b) **Compute** the line integral, $\int_L Q_S \cdot dL$ where, L is the circular edge of the ice-cream cone by specifying the dL element first. **[3]**

(c) **Compute** the surface integral $\int_{S_1} Q_S \cdot dS_1$ where, S_1 is the top surface. Also, find $\int_{S_2} Q_S \cdot dS_2$ where, S_2 is the slanting conical surface of the shape. **[6]**

(d) **Determine** the total volume of the ice-cream cone using dv in SCS. **[4]**

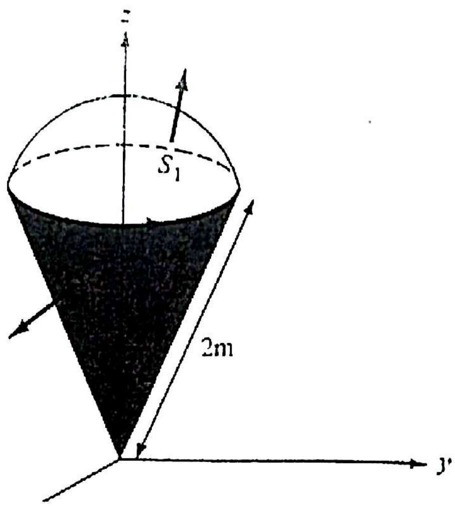

Figure 1. Volume in the form of ice cream cone

Hint Q1:

$$dL_{SCS} = dr\, a_r + r d\theta\, a_\theta + r \sin\theta\, d\phi\, a_\phi$$

$$dS_r = r^2 \sin\theta\, d\theta d\phi\, a_r,\ dS_\theta = r\sin\theta\, dr d\phi\, a_\theta,\ dS_\phi = r dr d\theta\, a_\phi,\ dv = r^2 \sin\theta\, dr d\theta d\phi$$

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$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

$r = \sqrt{x^2+y^2+z^2}$, $(r \geq 0)$
$\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$, $(0° \leq \theta \leq 180°)$
$\phi = \tan^{-1}\frac{y}{x}$

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z\cdot$	$\cos\theta$	$-\sin\theta$	0

CLO # 02: Analyze electrostatic fields and/or its properties governed by Coulomb's / Gauss's law for a given charge distribution in free space and / or dielectrics.

Q2: **[20 marks]**

(a) A uniform volume charge density, $\rho_v = 0.2$ $[\mu C/m^3]$ is present throughout the spherical shell extending from $r = 3$ cm to $r = 5$ cm. It is given that $\rho_v = 0$ elsewhere. **[8]**

(i) **Find** the total electric flux leaving the shell Ψ_{shell}.

(ii) Also **find** r_1 if half of the total charge computed in part **(i)** is located in the region: $3 < r < r_1$ [cm].

Hint Q2(a): The total electric flux leaving the shell is the amount of charge present in the shell.

(b) Three infinite sheets of charge are located in free space as follows: ***[12]***

1. $\rho_{S1} = 10\, nC/m^2$ at $y = -4$
2. $\rho_{S2} = 6\, nC/m^2$ at $y = 1$
3. $\rho_{S3} = -10\, nC/m^2$ at $y = 4$.

Answer the following questions:

(i) **Draw** the charge distribution in RCS.

(ii) **Find** electric field intensity E everywhere.

$E = \frac{\rho_s}{2\epsilon_0}$

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0059

Electromagnetic Theory (EE3005)

Sessional-II Exam

Date: November 01, 2025

Course Instructor(s)

1. Dr. Afzal Ahmed (Course Moderator)
2. Mohsin Yousuf

Total Time (Hrs):	1
Total Marks:	40
Total Questions:	2

Roll No ____________ Section ____________ Student Signature ____________

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1. *Attempt all questions and remember to solve parts of the same question together.*
2. *Final answers should be correct up to two decimal places with proper SI units.*
3. *Show all the steps with the help of diagrams and equations.*

CLO # 02: Analyze electrostatic fields and/or its properties governed by Coulomb's / Gauss's law for a given charge distribution in free space and / or dielectrics.

Q1: [20 marks]

(a) Given the field, [10]

$$D = 6\,\rho \sin\frac{1}{2}\varphi\, a_\rho + 1.5\,\rho\cos\frac{1}{2}\varphi\, a_\varphi \;\; [C/m^2]$$

Examine both sides of the *divergence theorem* for the region bounded by $\rho = 2$, $\varphi = 0$, $\varphi = \pi$, $z = 0$, and $z = 5$.

Hint:

$$\nabla \cdot \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

(b) Four point-charges $Q_1 = +q, Q_2 = -q, Q_3 = -q,$ and $Q_4 = +q$ are located at $(0,0), (a,0), (0,a),$ and (a,a) respectively in yz-plane as shown in Figure 1, where $q = 0.8\,nC$ and $a = 4\,cm$. Find the total potential energy stored in this *quadrupole*. [10]

Hint: $W_E = \frac{1}{2}\sum Q_i V_i$

$W_{1,all} = 2\frac{KQ^2}{a^2} + \frac{KQ^2}{a\sqrt{2}}$

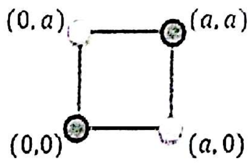

Figure 1. Quadrupole Charges

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CLO # 02: Analyze electrostatic fields and/or its properties governed by Coulomb's / Gauss's law for a given charge distribution in free space and / or dielectrics.

Q2: **[20 marks]**

(a) Current charge density is defined as $\mathbf{J} = -10^6\ z^{5/2}\ \mathbf{a}_z\ [A/m^2]$ in CCS within the region $0 \le \rho \le 30\ \mu m$ and is zero ($\mathbf{J} = 0$), elsewhere. Answer the following illustrating the linear relationship between *current density* and *charge density* as well as *velocity*: **[9]**

- **(i)** **Find** the total current 'I' crossing the surface $z = 0.1\ m$ in the $\mathbf{a}_z$ direction.
- **(ii)** If the charge velocity 'v' is $3 \times 10^6\ m/s$ at $z = 0.1\ m$, **find** ρ_v there.
- **(iii)** If the volume charge density 'ρ_v' at $z = 0.15\ m$ is $-3000\ [C/m^3]$, **find** the charge velocity there.

(b) Two extensive homogeneous isotropic dielectrics meet on plane $z = 0$. Let Region 1 ($z \ge 0$) have relative permittivity $\varepsilon_{r1} = 3$, while Region 2 ($z \le 0$) is characterized by $\varepsilon_{r2} = 5$. A uniform electric field $E_1 = 3\mathbf{a}_x - 2\mathbf{a}_y + 5\mathbf{a}_z$ V/m exists for $z \ge 0$. **[11]**

Extrapolate (i) E_{N1}, (ii) E_{t1}, (iii) E_{t1}, (iv) E_1, (v) E_{t2}, (vi) E_{N2}, (vii) E_2, (viii) P_1, (ix) P_2, (x) D_1 and (xi) D_2.

__Hint Part (b):__ $z = const.$ is an xy-plane, 'z' component of any vector field corresponds to the *normal* component, whereas, 'x' and 'y' components are *tangential*. Use boundary conditions for dielectric-to-dielectric interface.