# 5. Investigate the Gibbs Phenomenon

#### Observation:

From the generated plots, as more harmonics (higher N) are added, you can see ripples near sharp transitions or discontinuities.

#### **Explanation:**

This is the Gibbs Phenomenon — a characteristic overshoot (~9%) near discontinuities that does not disappear, even as  $N \to \infty$ . The overshoot becomes narrower but remains in height.

#### ■ In Your Code:

ullet The overshoot is clearly visible when N=10, becomes sharper at N=20, and more localized at N=65.

# 6. What is the Effect of Increasing N on the Plots?

## Summary:

- The approximation of x(t) improves.
- Sharp features and edges become more accurate.
- More high-frequency content is captured.
- Gibbs phenomenon is still present near discontinuities.
- The signal converges closer to the original with increasing N.

```
4. Generate Plots for Different in values
Example script to visualize:
  matlab
                                                                               ☐ Copy & Edit
  t = -2:0.0001:2;
 N_values = [5, 10, 20, 65];
     N = N_values(i);
     x = fourier_series_xt(t, N);
     subplot(2,2,i);
     plot(t, x);
     grid on;
     xlabel(['x', num2str(i), '(t)']);
     ylabel('Time(t)');
     title(['Signal with terms up to ', num2str(N)]);
This generates the exact subplot figure like you attached — four approximations of x(t) using increasing
                                             Done ↓
numbers of harmonics.
```

# ightharpoonup 3. MATLAB Function for x(t)You can create a function as:

```
function x = fourier_series_xt(t, N)
    x = zeros(size(t));
    for n = 1:2:N % Only odd harmonics
        bn = (8/(pi^2)) * (1/n^2) * sin(n*pi/2);
        x = x + bn * sin(n * pi * t);
    end
end
```

# 2. Find the Unknowns for Writing the Function

# ■ Range of Time:

The time vector in the code is:

This represents 2 full periods since the fundamental period  $T_0=rac{2\pi}{\omega_0}=rac{2\pi}{\pi}=2$ .

### ■ Input Parameters of the Function:

- Time vector t
- Number of harmonics N (odd values only)
- Fourier coefficient formula for  $b_n$

# 1. Synthesize x(t) using Fourier Series Coefficients

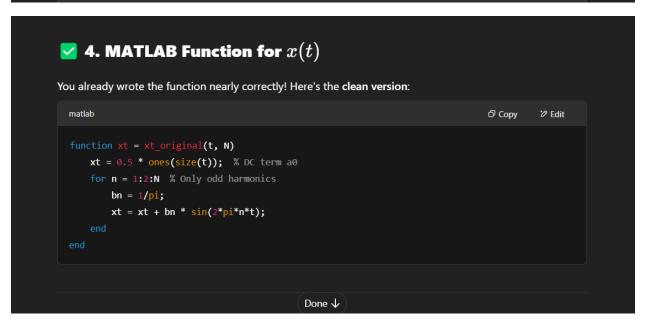
You are given:

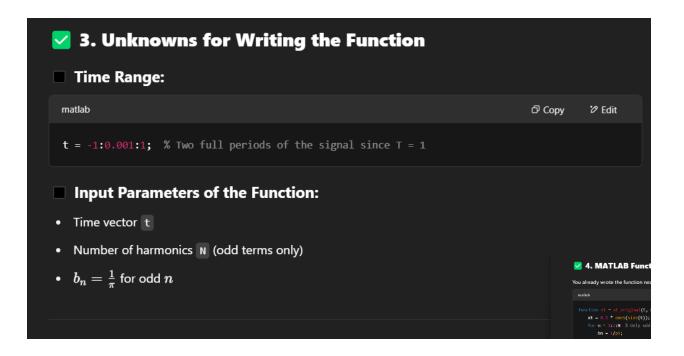
- $a_0=0$ ,  $a_n=0$
- $b_n = \frac{8}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$
- Fundamental frequency:  $\omega_0=\pi$

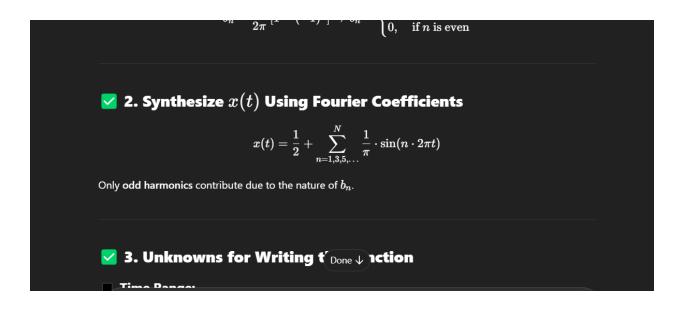
So, the signal x(t) is synthesized as:

$$x(t) = \sum_{n=1,3,5,\dots}^N rac{8}{n^2\pi^2} \sin\left(rac{n\pi}{2}
ight) \cdot \sin(n\pi t)$$

# 5. Plotting for Various N Your plotting code is correct. Just make sure the function is saved in a file named xt\_original.m or defined in the same script. Here's the whole plotting section for clarity: matlab 🗗 Сору ⊘ Edit N\_vals = [5, 10, 15, 60]; figure; for i = 1:length(N\_vals) xt = xt\_original(t, N\_vals(i)); subplot(2,2,i); plot(t, xt, 'LineWidth', 1.5); title(['x(t) with N = ', num2str(N\_vals(i))]); xlabel('Time (t)'); ylabel('x(t)');grid on; Done ↓







# 1. Given Trigonometric Fourier Coefficients

We are told:

• 
$$\omega_0=2\pi\Rightarrow T=1$$

• 
$$a_0 = \frac{1}{2}$$

• 
$$a_n = 0$$

• 
$$b_n=rac{1}{2\pi}[1-\cos(n\pi)]$$

Since  $\cos(n\pi) = (-1)^n$ , then:

$$b_n=rac{1}{2\pi}\left[1-(-1)^n
ight]\Rightarrow b_n=egin{cases} rac{1}{\pi}, & ext{if $n$ is odd} \ 0, & ext{if $n$ is even} \end{cases}$$

## 6. Gibbs Phenomenon Explanation

#### Observation from Plots:

- Near discontinuities (e.g. edges or step changes), you'll see ripples/overshoots.
- ullet These become sharper and more localized as N increases.

#### ■ What is Gibbs Phenomenon?

It's the persistent overshoot (~9%) near discontinuities when approximating a non-smooth periodic signal with a Fourier series.

Even when  $N \to \infty$ , the height of the overshoot doesn't go away, only the width narrows.

This is seen clearly in your higher-N plots like N=60.



# ▼ 7. What is the Effect of Increasing N on the Plots?

As N increases	Effect
Accuracy	Improves — the waveform resembles the actual signal more closely
Resolution	More details and sharp transitions are captured
Overshoot	Gibbs phenomenon becomes narrower but remains in height
Smoothness	The approximation follows the true function more tightly
High-frequency components	More are included, improving fidelity