National University of Computer & Emerging Sciences Lahore

EE3003 – Analog & Digital Communication (ADC)

Assignment # 01 on CLO 01

Instructor: Aroosa Umair

(CLO # 01) Explain the behavior of a communication system and the noise/distortion therein.

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Problem 1: Communication System Block Diagram

Draw and label a block diagram of a basic communication system. Explain the role of each block and discuss one example of distortion or noise that can occur in the channel.

Problem 2: Analog vs Digital Messages

Classify the following signals and complete the table given below:

- $x_1(t) = 5\cos(2000\pi t)$
- $x_2(t) = \{1, 0, 1, 1, 0, 1\}$ at 1 kbps
- $x_3(t) = 2 + 0.5\sin(100\pi t)$
- $x_4(t)$ = sequence of quantized speech samples at 8 kbps

Signal	Type	Bandwidth Requirement	Noise Immunity	Storage Method
$x_1(t)$				
$x_2(t)$				
$x_3(t)$				
$x_4(t)$				

Problem 3: Signal-to-Noise Ratio (SNR)

A digital channel has a received signal power of 5 mW and noise power of 0.05 mW.

(a) Calculate the SNR in linear scale.

(b) Calculate the SNR in decibels (dB).

(c) If the channel bandwidth is 100 kHz, compute the channel capacity using Shannon's formula.

Problem 4: Channel Bandwidth and Rate of Communication

A channel has a bandwidth of 5 kHz and a signal-to-noise ratio of 25 dB.

(a) Find the maximum data rate of this channel (bits/sec).

(b) If the required data rate is 20 kbps, determine whether this channel is suitable. Show calculations clearly.

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Problem 5: Energy Spectral Density of Exponential Signal

For the signal $x(t) = e^{-|t|}$

(a) Derive its Fourier transform.

(b) Compute the energy spectral density (ESD).

(c) Verify Parseval's theorem by showing that the total energy in time domain equals that in frequency domain.

Problem 6: Energy Spectral Density of Rectangular Pulse

Consider $x(t) = \text{rect}(t/T)$.

(a) Derive its Fourier transform.

(b) Determine its ESD and total energy.

(c) Plot the magnitude spectrum and discuss how the pulse duration T affects the bandwidth.

Problem 7: Power Spectral Density of Random Process

A wide-sense stationary process has autocorrelation $R(\tau) = \cos(100\pi\tau)$.

(a) Find its power spectral density (PSD).

(b) Identify the frequency components present in the PSD.

(c) Sketch and label the PSD spectrum.

Problem 8: PSD of Periodic Random Sequence

A random binary sequence takes values $+1$ and -1 with equal probability every T_b seconds.

(a) Derive the autocorrelation function.

(b) Find the corresponding PSD.

(c) Determine the essential bandwidth containing 90% of the signal power.

Problem 9:

The random binary signal $x(t)$ shown in Fig. P3.8-2 transmits one digit every T_b seconds. A binary **1** is transmitted by a pulse $p(t)$ of width $T_b/2$ and amplitude A; a binary **0** is transmitted by no pulse. The digits **1** and **0** are equally likely and occur randomly. Determine the autocorrelation function $\mathcal{R}_x(\tau)$ and the PSD $S_x(\omega)$.

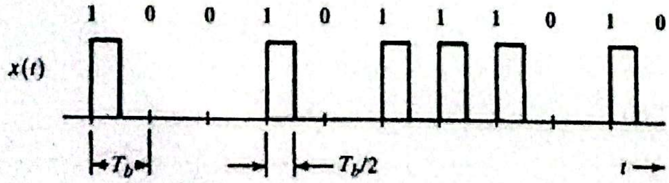
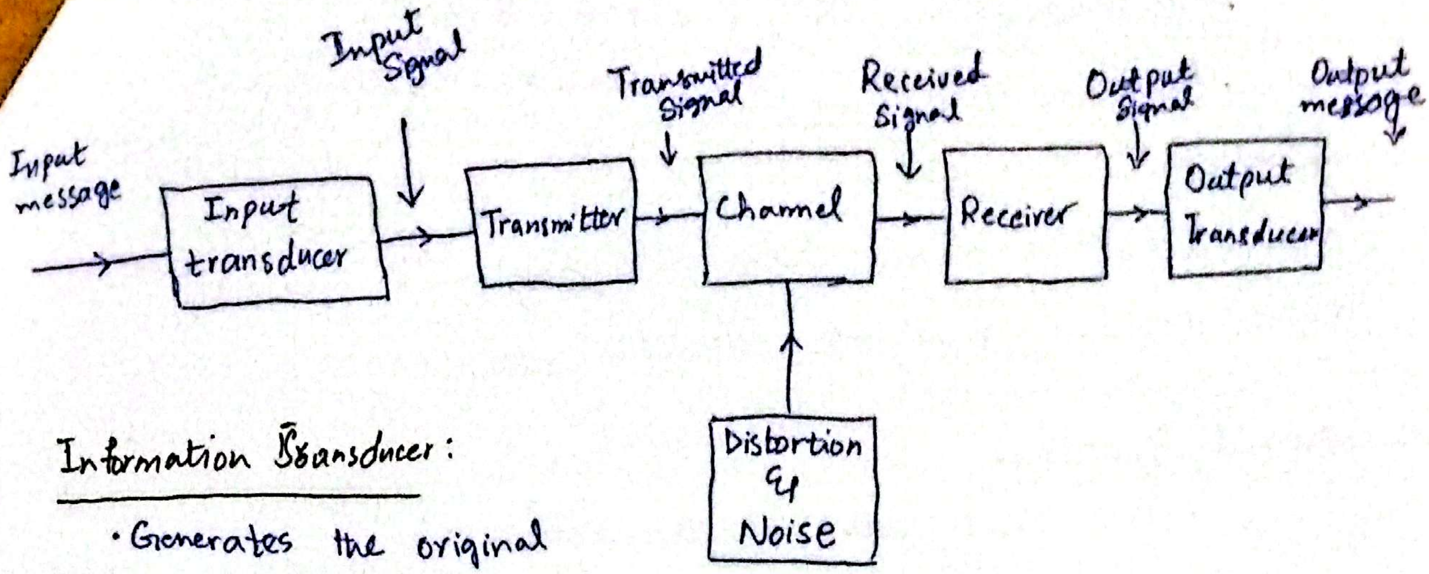


Figure P3.8-2

Problem 1:



Information Transducer:

- Generates the original message to be communicated.
- Message could be in the form of voice, text, image or text.

Example: A person speaking into a microphones.

Transmitter:

- Tasks include encoding, modulation and amplification.

Example: A mobile phone converting voice into radio waves.

Channel:

- The physical medium through which the signal travels from transmitter to receiver.
- Can be wired (cables, optical fibers) or wireless (air, satallite).
- The channel often introdues attenuation, distortion and noise.

Receiver:

- Reconstructs the original signal.
- Tasks include demodulation, decoding and filtering.

Example: A mobile phone receiving and processing radio signals back into sound.

Output transducer:

- The final user of the information.

Example: A person listening to the received noise.

Example of Noise / Distortion in Channel:

Possible distortion:

- Attenuation (weaking of signal over distance).
- Echo (delayed versions of signal overlapping)
- Interference (signals from other transmitter overlapping).

Example:

Additive White Gaussian Noise (AWGN):

→ It is random electrical noise that gets added to transmitted signal while it passes through the channel.

→ It comes from thermal motion of electrons in conductor.

* hissing sound in Radio or mobile calls during weak signals.

Problem 2:

Signal	Type	Bandwidth Requirement	Noise immunity	Storage Method
$x_1(t) = 5\cos(2000\pi t)$	Analog (continuous time & amplitude)	Ideally 0Hz. (two spectral lines at ±1kHz) A very narrow Bandwidth.	Poor (small noise visibly corrupts amplitude/phase).	Analog Recording
$x_2(t) = \{1,0,1,1,0,1\}$ at 1 kbps.	Digital (Binary Sequence)	Bandwidth ∝ Bit Rate. Nyquist minimum $= \frac{R_b}{2} = 0.5$ kHz. Practical line codes ≈ 0.5–1 kHz.	Good (robust; can use thresholds/coding)	Digital (bits in memory, disk etc.)
$x_3(t) = 2 + 0.5\sin(100\pi t)$	Analog	Ideally 0Hz around DC and ±50 Hz, effectively very low.	Poor ~~(amplitude/phase~~ .	Analog.
$x_4(t)$ = sequence of Quantized speech samples at 8 kbps	Digital (Discreate time & amplitude)	Nyquist minimum $= R_b/2 = 4$ kHz practically ≈ 4–8 kHz	Good (threshold detection, coding possible)	Digital.

Problem 3:

Received Signal Power $= P_s = 5\,mW$.

Noise Power $= P_n = 0.05\,mW$

Channel Bandwidth $B = 100\,kHz$
$= 100 \times 10^3\,Hz$.

a) SNR (Linear Scale)

$$SNR = \frac{P_s}{P_n} = \frac{5}{0.05}$$

$$SNR = 100$$

b) SNR in dB

$$SNR_{dB} = 10 \log_{10} (SNR)$$

$$= 10 \log_{10} (100)$$

$$= 20 \text{ dB}$$

c) Channel Capacity (Shannon's Formula)

$$C = B \log_2 (1 + SNR)$$

$$= 100 \times 10^3 \log_2 (1 + 100)$$

$$C = 665{,}700 \text{ bps}$$

$$= 665.7 \text{ Kbps}$$

Problem 4:

Band width $B = 5$ kHz

$SNR = 25$ dB.

Required data Rate $= R = 20$ kbps

a/

$$SNR_{dB} = 10 \log_{10} (SNR)$$

$$\frac{25}{10} = \log_{10} (SNR)$$

$$SNR = 316.2$$

$$\because C = B \log_2 (1 + SNR)$$

$$= (5 \times 10^3) \log_2 (1 + 316.2)$$

$$= 41{,}550 \text{ bps}$$

$$\boxed{C = 41.6 \text{ kbps}}$$

b/. Required Rate: $R = 20$ kbps.

. Channel Capacity $c = 41.6$ kbps

$$R < C.$$

Channel is suitable transmitting 20kbps reliably.

Problem 5

$$x(t) = e^{-|t|} \qquad G(\omega) = \frac{1}{a + j\omega}.$$

a) Fourier Transform:

$$X(\omega) = \frac{1}{a + j\omega}$$

b) ESD:

$$|X(\omega)|^2 = \frac{1}{a^2 + \omega^2}$$

c) Frequency domain:

$$E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{a^2 + \omega^2} \, d\omega$$

$$= \frac{1}{2\pi} \frac{1}{a} \tan^{-1}\left(\frac{\omega}{a}\right)\Big|_{-\infty}^{\infty}$$

$$= \frac{1}{2\pi a}\left[\tan^{-1}\left(\frac{\infty}{a}\right) + \tan^{-1}\left(\frac{\infty}{a}\right)\right]$$

$$= \frac{1}{2\pi a}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$$= \frac{1}{2a}$$

$$\Rightarrow \tan^{-1}(\infty) = 90° = \frac{\pi}{2}.$$

c) Time domain: $x(t) = e^{-at}$.

$$E_g = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} e^{-2at} \, dt \Rightarrow \int_{0}^{\infty} e^{-2at} \, dt.$$

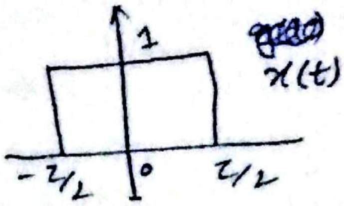
$$= \frac{e^{-2at}}{-2a}\Big|_0^{\infty} = \frac{1}{2a}.$$

$$\int \frac{1}{a^2 + \omega^2} = \frac{1}{a}\tan^{-1}\left(\frac{\omega}{a}\right).$$

Problem 6: $x(t) = \text{rect}(t/T)$

a) $\text{rect}\left(\frac{t}{\tau}\right) = \tau \,\text{sinc}\left(\frac{\omega\tau}{2}\right)$

$$\text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & |t| \le \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$$



$$X(\omega) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau}\right) e^{-j\omega t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} e^{-j\omega t}\, dt$$

$$= \frac{1}{-j\omega} e^{-j\omega t}\Big|_{-\tau/2}^{\tau/2}$$

$$= \frac{1}{-j\omega}\left[e^{-j\omega\tau/2} - e^{j\omega\tau/2}\right]$$

$$= \frac{1}{\omega}\left[2\sin\left(\frac{\omega\tau}{2}\right)\right] \Rightarrow \frac{\omega\tau}{2}\left[\frac{2\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}\right] \times \frac{1}{\omega}$$

$$= \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$$

b) ESD

$$\text{ESD} = \Psi_x(\omega) = |X(\omega)|^2$$

$$= \left|\tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)\right|^2$$

So

$$E_W = \frac{1}{2\pi}\int_{-W}^{W} \tau^2 \sin^2\left(\frac{\omega\tau}{2}\right) d\omega$$

Setting $\boxed{\omega\tau = x}$

$$d\omega = \frac{1}{\tau}\, dx.$$

$$\frac{E_W}{} = \frac{\tau}{\pi} \int_0^{W\tau} \text{sinc}^2\left(\frac{x}{2}\right) dx$$

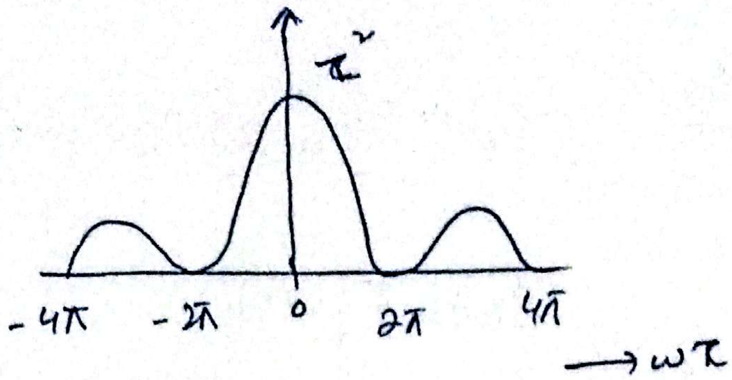
$\rightarrow \omega T = 2\pi$

$\omega = \frac{2\pi}{T}$

$\boxed{E_g = \tau}$ then.

$$\frac{E_W}{E_g} = \frac{1}{\pi} \int_0^{W\tau} \text{sinc}^2\left(\frac{x}{2}\right) dx.$$

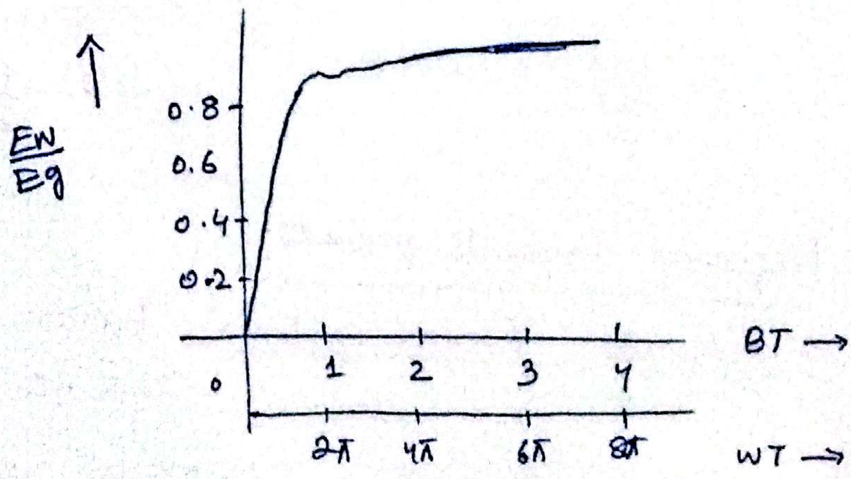
c/ Magnitude Spectrum:



So, 90.28% of total energy of pulse $x(t)$ contained within band $\omega = \frac{2\pi}{T}$ rads or $B = \frac{1}{T}$ Hz.

So, criterion, the Bandwidth of a Rectangular pulse of width T seconds is $\frac{1}{T}$ Hz.

Plot of E_W/E_g:



Problem 7: $R(\tau) = \cos(100\pi\tau)$.

a) Find PSD.

~~PSD:~~ $R_g(\tau) \Longleftrightarrow S_g(\omega)$

$$\cos(100\pi\tau) = \frac{1}{2}\left(e^{j100\pi\tau} + e^{-j100\pi\tau}\right)$$

Using: $\mathcal{F}\{e^{j\omega_0\tau}\} = 2\pi\,\delta(\omega-\omega_0)$

$$S(\omega) = \mathcal{F}\{r(\tau)\}$$

$$= \frac{1}{2}\left[2\pi\,\delta(\omega-100\pi) + 2\pi\,\delta(\omega+100\pi)\right]$$

$$S(\omega) = \pi\left[\delta(\omega-100\pi) + \delta(\omega+100\pi)\right]$$

$$f_0 = \frac{\omega}{2\pi} \qquad \omega = 100\pi$$

$$= \frac{100\pi}{2\pi} = 50\,Hz.$$

Converting PSD to f-domain:

$$\delta(\omega-\omega_0) = \frac{1}{2\pi}\,\delta(f-f_0).$$

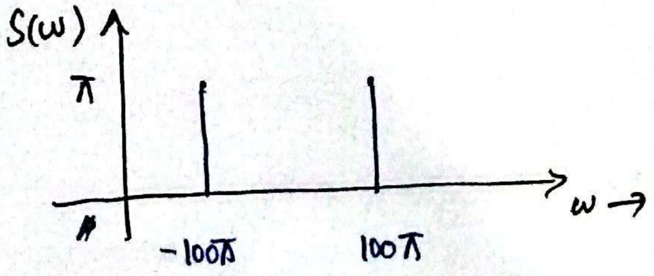
$$S_f(f) = \frac{1}{2}\left[\delta(f-50) + \delta(f+50)\right].$$

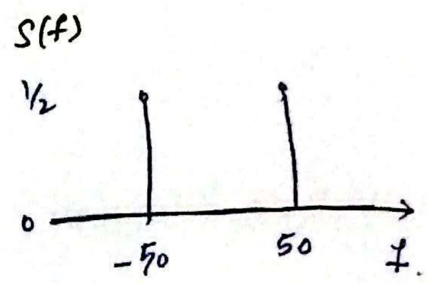
b) Frequency components presents.

- Angular Frequency Form: Impulses at $\omega = \pm 100\pi$ each with weight π.

- Ordinary frequency: $f = \pm 50\,Hz$, each with weight $1/2$.

c)





blem 08:

a)

$$g(t)\times g(t-\tau) = T_b - \tau$$

$$R_g(\tau) = \lim_{T\to+\infty} \frac{1}{T}\int_{-T/2}^{T/2} g(t)\times g(t-\tau)\,dt .$$

$T = NT_B$

$$R_g(\tau) = \lim_{N\to+\infty} \frac{1}{NT_b}\left[N(T_b-\tau)\right]$$

$$= \frac{T_b}{T_b} - \frac{\tau}{T_b}$$

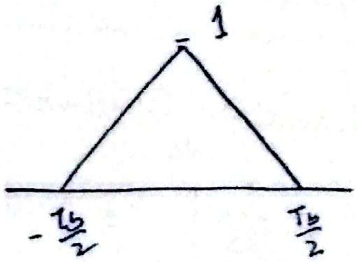
$$= 1 - \frac{\tau}{T_b}$$

$$R_g(\tau) = \begin{cases} 1-\dfrac{|\tau|}{T_b} & ; |\tau| \le T_b \\ 0 & |\tau| > T_b \end{cases}$$

$R_g(\tau)$ is an even function

$|\tau| \le T_b/2$

$R_g(\tau) = 0 \quad , |\tau| > T_b/2 .$



b)

$\cancel{S_x(f) = \frac{1}{T_b} P}$

Let $g(t)$ be unit - amplitude of ~~Rectangal~~ Rectangular pulse on $[0, T_b]$:

$$g(t) = \begin{cases} 1 , & 0 \le t < T_b/2 \\ 0 & \text{otherwise} \end{cases} .$$

2. $\cancel{G(w) = \mathcal{F}\{g(t)\}}$

$$\cancel{G(w) = \int_0^{T_b/2} e^{-jwt}\,dt = \frac{1-e^{-jwT_b/2}}{jw} = \frac{1-\left[\cos w\frac{T_b}{2} - j\sin w\frac{T_b}{2}\right]}{jw}}$$

So $S_g(w) = F\left\{ \Delta\left(\frac{t}{T_b}\right)\right\}$

$\text{tri}\left(\frac{t}{T}\right) \Longleftrightarrow T \ \text{sinc}^2\left(\frac{wT}{2}\right)$

$S_g(w) = T_b \ \text{sinc}^2\left(\frac{wT_b}{2}\right)$ convert into sin.

$= \dfrac{2 \sin\left(\frac{wT_b}{2}\right)}{w}$

$* \quad S_g(w) = \dfrac{|S_g(w)|^2}{T_b} = \dfrac{\left|\dfrac{2\sin\left(\frac{wT_b}{2}\right)}{w}\right|^2}{T_b}$

$S_g(w) = \dfrac{1}{T_b} \cdot \dfrac{4 \sin^2\left(\frac{wT_b}{2}\right)}{w^2}$

$S_g(w) = \dfrac{4\sin^2\left(\frac{wT_b}{2}\right)}{T_b w^2}.$

Convert into Since.

$$\boxed{S_g(w) = T_b \ \text{sinc}^2\left(\frac{wT_b}{2}\right)}$$

c/ Essential Bandwidth containing 90% of signal power.

$$\frac{1}{2\pi}\int_{-w_c}^{w_c} S_g(w)\, dw = 0.9$$

$$\frac{1}{2\pi} \times 2 \int_{0}^{w_c} S_g(w)\, dw = 0.9$$

$$\frac{1}{\pi}\int_{0}^{w_c} T_b \ \text{sinc}^2\left(\frac{wT_b}{2}\right) dw = 0.9$$

Let: $u = \dfrac{wT_b}{2} \longrightarrow w = \dfrac{2u}{T_b}$

$dw = \dfrac{2u}{T_b}\ du.$

CamScanner

When $\omega = \omega_c$,

uper limits becomes $u_c = \frac{\omega_c T_b}{2}$

$$= \frac{1}{\pi} T_b \int_0^{\omega_c} \operatorname{sinc}^2(u) \frac{2}{T_b} du.$$

$$= \frac{2}{\pi} \int_0^{u_c} \left(\frac{\sin u}{u}\right)^2 du$$

$$\frac{2}{\pi} \int_0^{u_c} \left(\frac{\sin u}{u}\right)^2 du = 0.9$$

multiply both sides with $\frac{\pi}{2}$.

$$\int_0^{u_c} \left(\frac{\sin u}{u}\right)^2 du = 0.45\pi$$

So $u_c \approx 2.66$ by solving above integral.

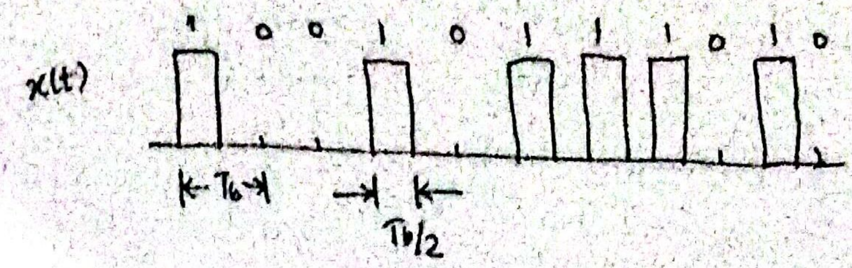
$$u_c = \frac{\omega_c T_b}{2}$$

$$\omega_{c,90\%} = \frac{2u_c}{T_b} = \frac{5.3314}{T_b} \text{ (rad/sec)}$$

$$f_{c,90\%} = \frac{\omega_{c,90\%}}{2\pi} = \frac{0.848}{T_b} \text{ (Hz)}$$

$$B_{90\%} \approx \frac{0.848}{T_b} \text{(Hz)}$$

Problem 9:



Let $T = NT_b$

On avg $N/2$ phases.

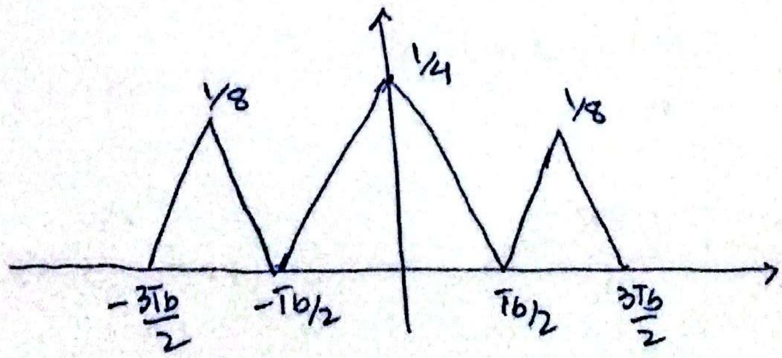
The area under the product $g(t)\,g(t-\tau)$ is $N/2$.

$$R_g(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)\cdot g(t-\tau)\,dt$$

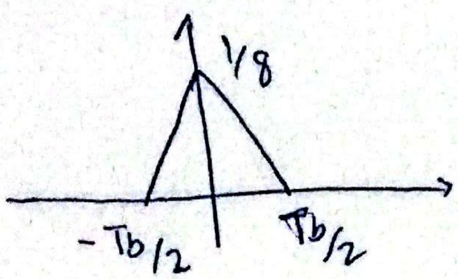
$$= \lim_{N\to\infty} \frac{1}{NT_b} \cdot \frac{N}{2}\left(\frac{T_b}{2} - \tau\right)$$

$$= \frac{1}{2}\left(\frac{1}{2} - \frac{\tau}{T_b}\right) \qquad \tau < T_b/2$$

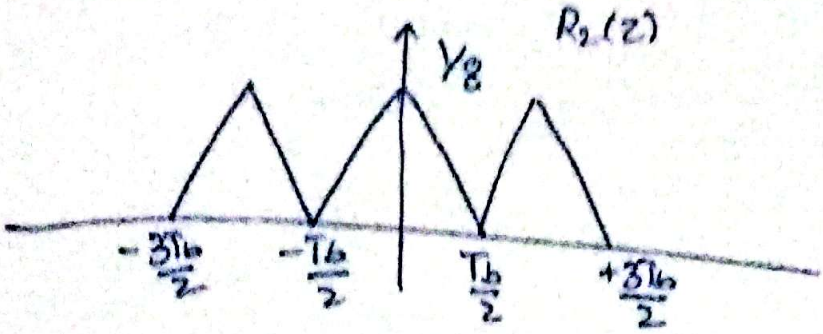
• Hence $R_g(t)$ repeats every T_b seconds with half of the magnitude.



$$R_g(\tau) = \frac{1}{8}\,\Delta\left(\frac{t}{T_b}\right)$$



We can express $R_x(t)$ as sum of two components.



The PSD is sum of two Fourier Transform $R_1(\tau)$ and $R_2(\tau)$

$$S_g(\omega) = \frac{T_b}{16}\,\text{sinc}^2\left(\frac{\omega T_b}{4}\right) + S_2(\omega)$$

For a Periodic Signal be;

$$R_2(\tau) = \sum_{n=-\infty} D_n e^{jn\omega_0 t} \qquad \because \omega_b = \frac{2\pi}{T_b}$$

$$D_n = \frac{1}{16}\,\text{sinc}^2\left(\frac{n\pi}{2}\right)$$

$$\cancel{S_2(\omega) =}\quad g(t) \Longleftrightarrow \sum_{n=-\infty}^{\infty} F\left[D_n e^{jn\omega_0 t}\right]$$

$$g(t) \Longleftrightarrow 2\pi \sum_{n=-\infty}^{\infty} D_n\,\delta(\omega - \omega_0)$$

$$\delta_{T_0}(t) = \frac{1}{T_0}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t} \qquad ; \omega_0 = \frac{2\pi}{T_0}$$

$\therefore D_n = \frac{1}{T_0}$

$$\delta_{T_0}(t) \Longleftrightarrow \frac{2\pi}{T_0}\sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$S_2(\omega) = \frac{\pi}{8}\sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n\pi}{2}\right)\delta(\omega - n\omega_0)$$

Therefore,

$$S_g(\omega) = \frac{T_b}{16} \operatorname{sinc}^2\left(\frac{\omega T_b}{4}\right) + \frac{\pi}{8} \sum_{n=-\infty}^{\infty} \operatorname{sinc}^2\left(\frac{n\pi}{2}\right) \delta(\omega - n\omega_0)$$

$$\because \omega_0 = \frac{2\pi}{T_b}$$