Assignment #04 — B&S

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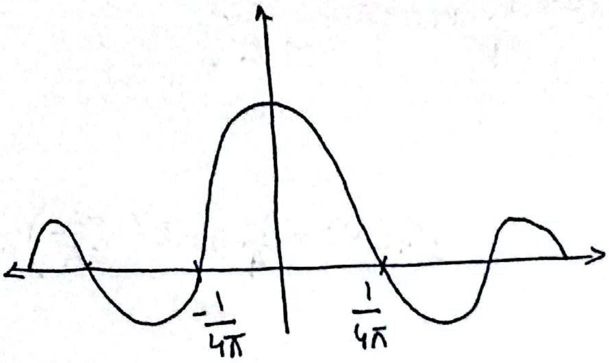
Question # 01

$x_1(t) = \text{sinc } 4\pi t$; $x_2(t) = \cos 2\pi t$

a) $x_1(t)$



$X_1(w) = ?$

$x_1(t) = \text{sinc}(4\pi t)$

As;

$\because \frac{W}{\pi} \text{sinc}(Wt) \Longleftrightarrow \text{rect}\left(\frac{w}{2W}\right)$

$W = 4\pi$

$$\text{sinc}(4\pi t) = \frac{W}{\pi} \text{sinc}(wt)$$

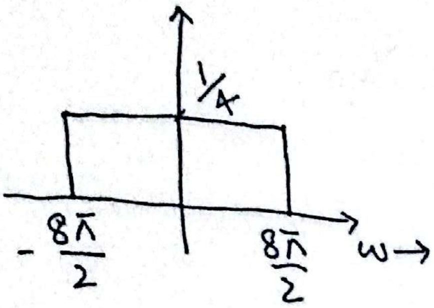
$$\text{sinc}(4\pi t) = \frac{4\pi}{\pi} \text{sinc}(4\pi t)$$

$$\text{sinc}(4\pi t) = 4 \text{ sinc}(4\pi t)$$

Therefore;

$$\text{Sinc}\,(4\pi t) = 4\,\text{sinc}\,(4\pi t) \Longleftrightarrow \text{rect}\left(\frac{\omega}{2\times 4\pi}\right)$$

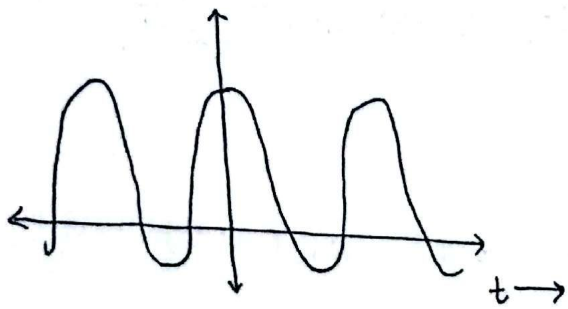
$$\text{Sinc}\,(4\pi t) \Longleftrightarrow \frac{1}{4}\,\text{rect}\left(\frac{\omega}{8\pi}\right)$$



$$X_1(\omega) = \frac{1}{4}\,\text{rect}\left(\frac{\omega}{8\pi}\right).$$

b)

$$x_2(t) = \cos 2\pi t.$$



$X_2(\omega) = ?$

$$\cos 2\pi t = \frac{e^{j2\pi t} + e^{-j2\pi t}}{2}$$

$$\cos 2\pi t = \frac{1}{2}\left[e^{j2\pi t} + e^{-j2\pi t}\right]. \longrightarrow (1)$$

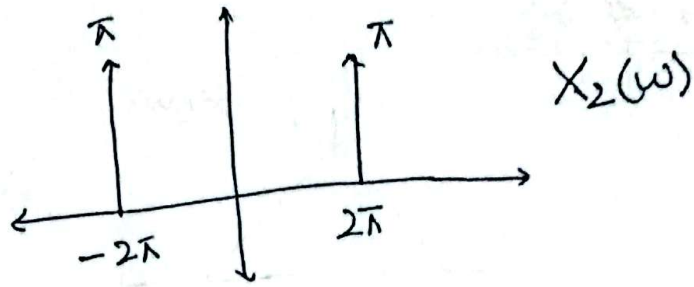
As we know,

$$e^{j\omega_0 t} = 2\pi\,\delta(\omega-\omega_0)$$

$$e^{-j\omega_0 t} = 2\pi\,\delta(\omega+\omega_0)$$

$$\cos 2\pi t \Longleftrightarrow \frac{1}{2}\left[2\pi\,\delta(\omega-\omega_0) + 2\pi\,\delta(\omega+\omega_0)\right]$$

$$\cos 2\pi t \Longleftrightarrow \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$



c) $Y(\omega)$

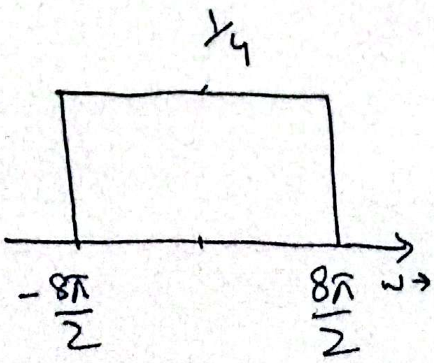
As $y(t) = x_1(t) * y_2(t)$

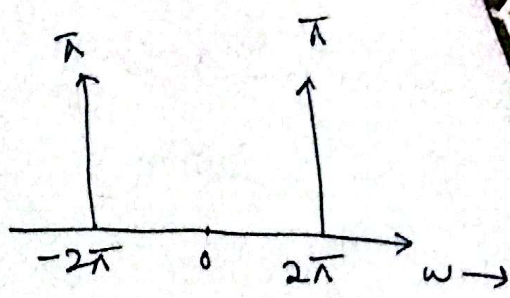
From time convulution property.

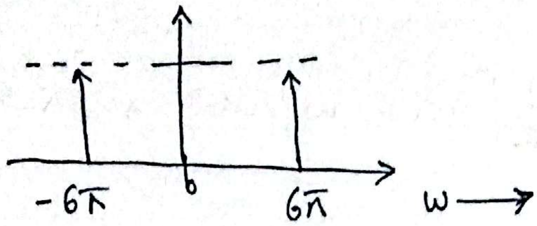
$$x_1(t) * x_2(t) \Longleftrightarrow X_1(\omega)\; X_2(\omega)$$

$$\text{Sinc}(4\pi t) * \cos(2\pi t) \Longleftrightarrow \left[\frac{1}{4}\,\text{rect}\left(\frac{\omega}{8\pi}\right)\right] \times \left[\pi\,\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)\right]$$

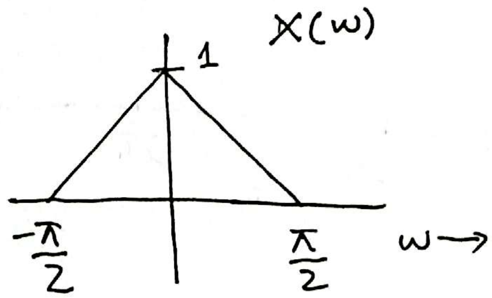
$$\text{Sinc}(4\pi t) * \cos(2\pi t) \Leftrightarrow \left[\frac{1}{4}\,\text{rect}\left(\frac{\omega}{8\pi}\right)\right] \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right].$$







Question #02



i) $x(3t)$

By using Scaling Property

$$g(at) \Longleftrightarrow \frac{1}{|a|} G\left(\frac{\omega}{a}\right)$$

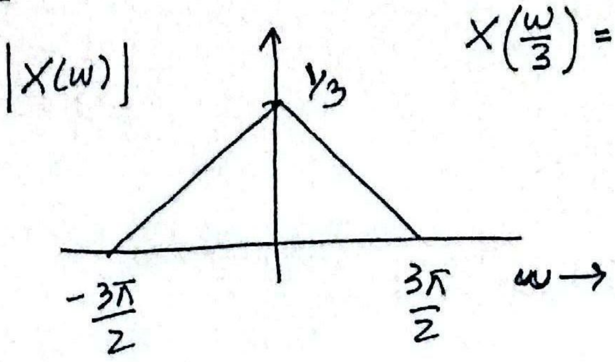
- $g(at)$ represensts that function compressed in time.
- $G\left(\frac{\omega}{a}\right)$ shows that function expanded in frequancy.

Therefore;

$$x(3t) \Longleftrightarrow \frac{1}{3} X\left(\frac{\omega}{3}\right).$$

Magnitude Spectrum:

$$X\left(\frac{\omega}{3}\right) = X(\omega).$$



Where the phase spectrum is zero.

ii) $x(t-5)$

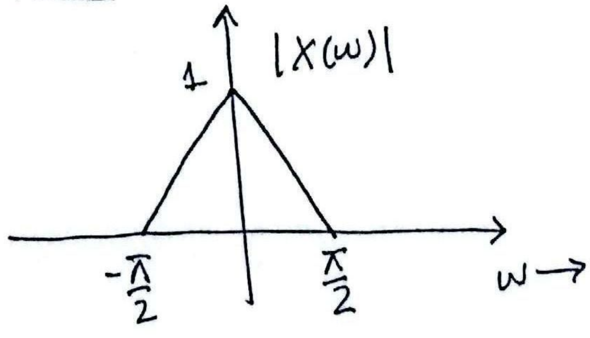
property

$$x(t-t_0) \Longleftrightarrow X(\omega)\, e^{-j\omega_0 t_0}$$

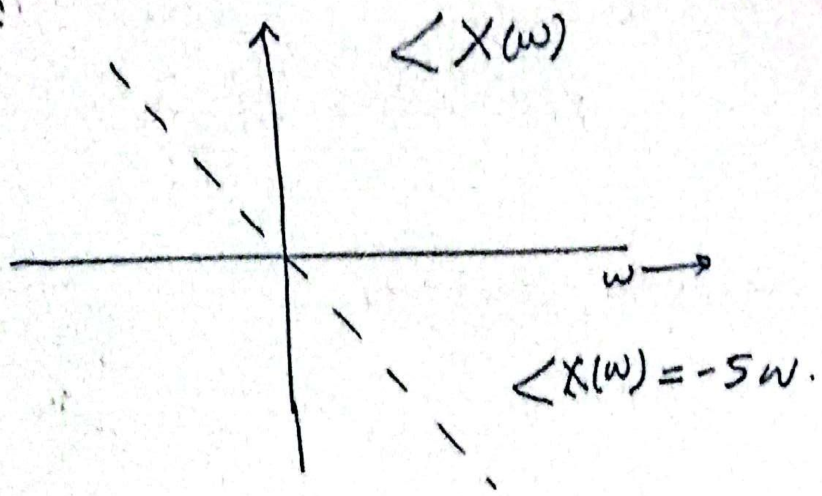
$$x(t-5) \Longleftrightarrow X(\omega)\, e^{-j5\omega}$$

So time shifting property amplitude spectrum doesnot change. Only phase spectrum will change by ωt_0.

Amplitude/Magnitude Spectrum:



Phase Spectrum:

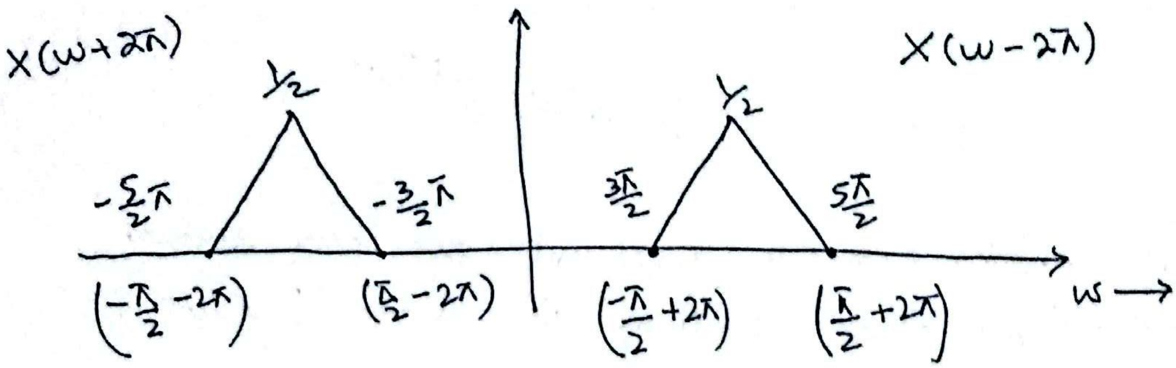


iii) $x(t)\cos(2\pi t)$

$$x(t)\cos\omega_0 t \Leftrightarrow \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

$$x(t)\cos(2\pi t) \Leftrightarrow \frac{1}{2}\left[X(\omega-2\pi) + X(\omega+2\pi)\right]$$

Magnitude Spectrum:



The phase spectrum of (iii) will be 0.

iv) $x(t) * \operatorname{sinc}\left(\frac{\pi t}{4}\right)$

using convulution property.

$$x_1(t) * x_2(t) \Longleftrightarrow X_1(\omega)\, X_2(\omega)$$

Therfore;

$$x(t) * \operatorname{sinc}\left(\frac{\pi}{4} t\right) \Longleftrightarrow [X_1(\omega)][X_2(\omega)]$$

$$\rightarrow x(t) \Longleftrightarrow X(\omega)$$

$$\rightarrow \operatorname{sinc}\left(\frac{\pi}{4} t\right) = \,?$$

As,
$$\frac{W}{\pi} \operatorname{sinc}(Wt) \Longleftrightarrow \operatorname{rect}\left(\frac{\omega}{2W}\right)$$

$$\Rightarrow \operatorname{sinc}\left(\frac{\pi}{4} t\right) = \frac{W}{\pi} \operatorname{sinc}(Wt)$$

$$\operatorname{sinc}\left(\frac{\pi}{4} t\right) = \frac{\pi/4}{\pi} \operatorname{sinc}\left(\frac{\pi}{4} t\right) \qquad \because W = \frac{\pi}{4}$$

$$\operatorname{sinc}\left(\frac{\pi}{4} t\right) = \frac{1}{4} \operatorname{sinc}\left(\frac{\pi}{4} t\right)$$

Therefore;

$$\frac{1}{4} \operatorname{sinc}\left(\frac{\pi}{4} t\right) \Longleftrightarrow \operatorname{rect}\left(\frac{\omega}{2 \times \frac{\pi}{4}}\right)$$

$$\frac{1}{4}\,\mathrm{sinc}\left(\frac{\pi}{4}t\right) \Longleftrightarrow \mathrm{rect}\left(\frac{\omega}{\pi/2}\right)$$

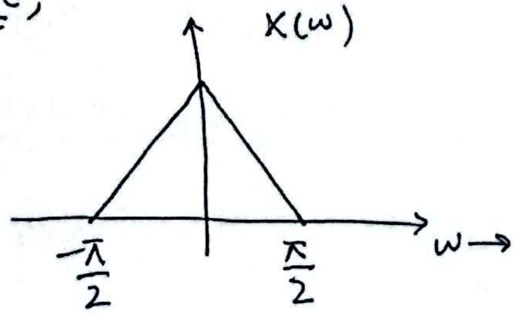
$$\mathrm{sinc}\left(\frac{\pi}{4}t\right) \doteq \frac{1}{4}\,\mathrm{sinc}\left(\frac{\pi}{4}t\right) \Longleftrightarrow 4\,\mathrm{rect}\left(\frac{\omega}{\pi/2}\right)$$

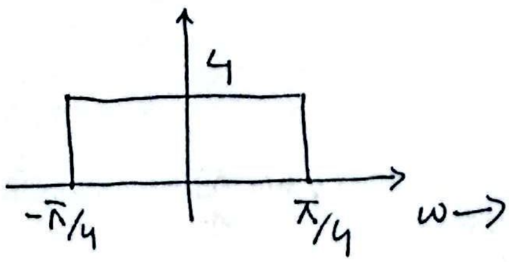
So,

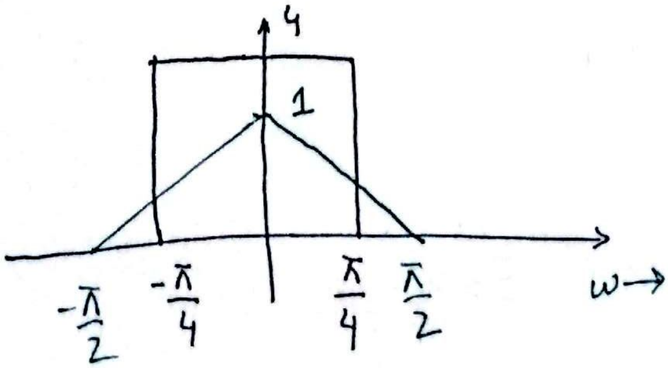
$$x(t) * \mathrm{sinc}\left(\frac{\pi}{4}t\right) = \left(X(\omega)\right)\left(4\,\mathrm{rect}\left(\frac{\omega}{\pi/2}\right)\right)$$

$$x(t) * \mathrm{sinc}\left(\frac{\pi}{4}t\right) = X(\omega) * 4\,\mathrm{rect}\left(\frac{\omega}{\pi/2}\right)$$

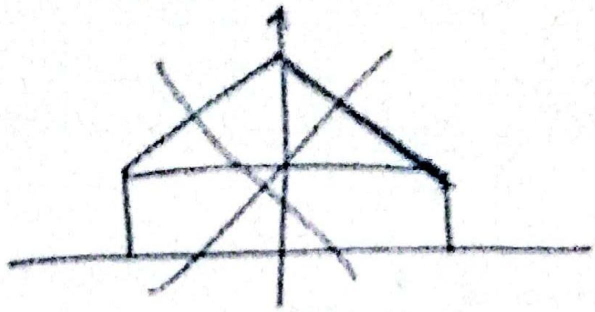
where;

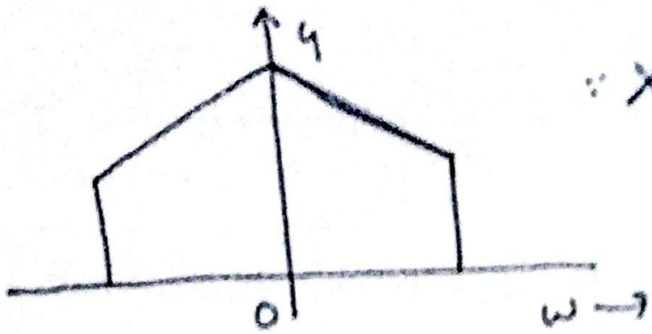






Amplitude Spectrum:

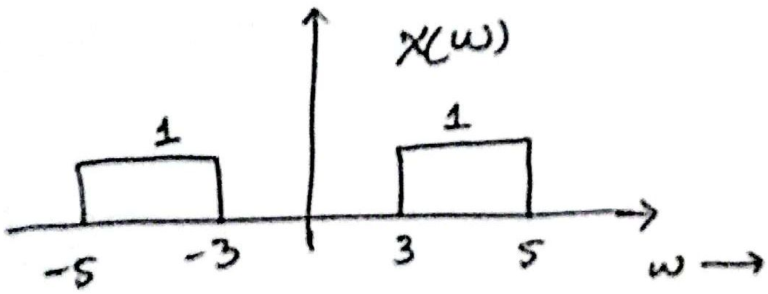




$\therefore X(\omega) \times 4\,\mathrm{rec}\left(\frac{\omega}{\pi/2}\right)$

The phase spectrum will be zero.

Q3 Use frequency shifting property to find IFT of following signal.



Frequency Shifting Property:

$$g(t) \Longleftrightarrow G(\omega)$$

* $g(t)\,e^{j\omega_0 t} \Longleftrightarrow G(\omega - \omega_0)$

* $g(t)\,e^{-j\omega_0 t} \Longleftrightarrow G(\omega + \omega_0)$

$$x(t)e^{j\omega_0 t} + x(t)\,e^{-j\omega_0 t} \Longleftrightarrow X(\omega-\omega_0) + X(\omega+\omega_0)$$

$$x(t)\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] \Longleftrightarrow X(\omega-\omega_0) + X(\omega+\omega_0). \rightarrow (1)$$

dividing eq (1) by "2".

$$x(t)\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] \Longleftrightarrow \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

$$x(t)\cos(\omega_0 t) \Longleftrightarrow \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

$\because W_0 = 4$

$$x(t)\cos(4t) \Longleftrightarrow \frac{1}{2}\left[X(\omega-4) + X(\omega+4)\right] \rightarrow (2)$$

In our case $X(\omega) = rect(\omega)$

Therefore;

$$x(t)\cos(4t) \Longleftrightarrow \frac{1}{2}\left[rect(\omega-4) + rect(\omega+4)\right]$$

$$x(t) \Longleftrightarrow \frac{1}{2} \times \frac{1}{\cos(4t)}\left[rect(\omega-4) + rect(\omega+4)\right]$$

$$x(t) \Longleftrightarrow \frac{1}{2} \times \frac{1}{\cos(4t)}\left[\frac{1}{\pi} sinc(t-4) + \frac{1}{\pi} sinc(t+4)\right]$$

Question # 04

$$X(\omega) = \frac{1}{(a+j\omega)^2}$$

Convolution Property:

$$x_1(t) * x_2(t) \Longleftrightarrow X_1(\omega) * X_2(\omega)$$

$$\cancel{X}(\omega) = \frac{1}{(a+j\omega)} \times \frac{1}{(a+j\omega)}$$

where;

$$e^{-at} \cdot u(t) \longleftrightarrow \frac{1}{a+j\omega}$$

Therefore, from time convulution property.

$$e^{-at}u(t) * e^{-at}u(t) \Longleftrightarrow \frac{1}{(a+j\omega)} \times \frac{1}{(a+j\omega)}$$

As;

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\; x_2(t-\tau)\; d\tau$$

$$e^{-at}u(t) * e^{-at}u(t) = \int_{-\infty}^{\infty} e^{-a\tau}\; u(\tau)\; e^{-a(t-\tau)}\; u(t-\tau)\, d\tau$$

$$x(t) = \int_0^t \left(e^{-a\tau} \cdot e^{-at} \cdot e^{a\tau}\right)\; d\tau$$

$$= e^{-at}\int_0^t e^{-a\tau} \cdot e^{a\tau}\; d\tau$$

$$x(t) = e^{-at} \int_0^t \left(e^{(-a+a)}\right)^{\tau} d\tau$$

$$= e^{-at} \int_0^t e^0 \, d\tau$$

$$= e^{-at} \; \tau \Big|_0^t$$

$$\boxed{x(t) = t \,.\, e^{-at}}$$

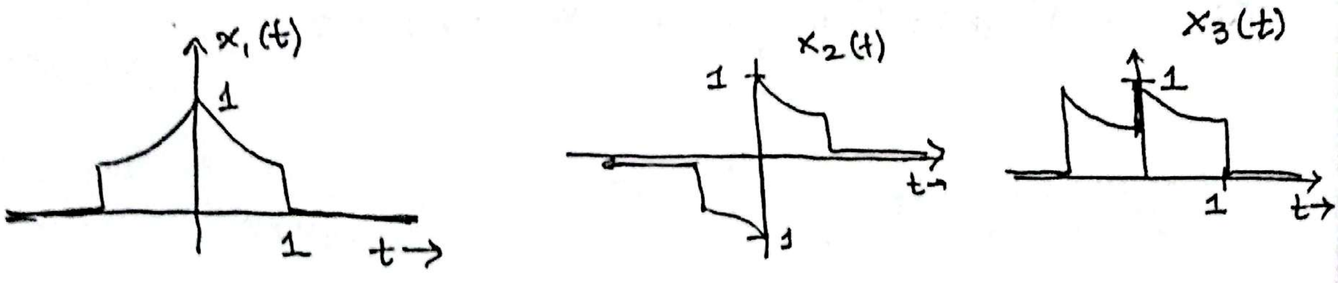
Therefore:

(IFT)

$$x(t) = \left[t \cdot e^{-at}\right] u(t)$$

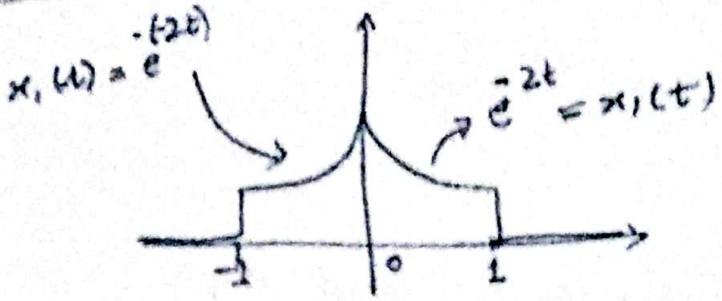
Question #5

$$x(t) = \begin{cases} e^{-2t} & , \; 0 \le t \le 1 \\ 0 & , \; \text{otherwise} \end{cases}$$



a) Write $x_1(t)$, $x_2(t)$, $x_3(t)$ in terms of $x(t)$.

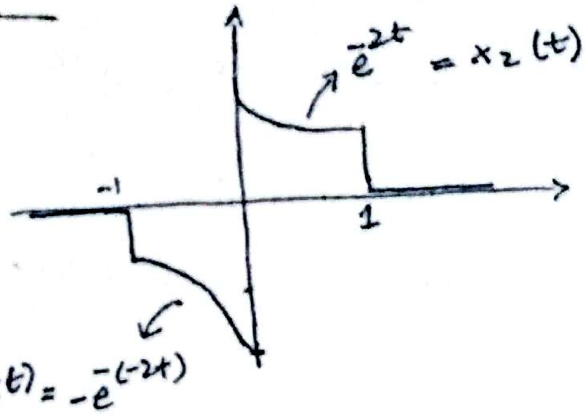
) terms of $x(t)$



So,

$$x(t) = x_1(t) + x_2(t)$$
$$= e^{-2t} + e^{2t}.$$

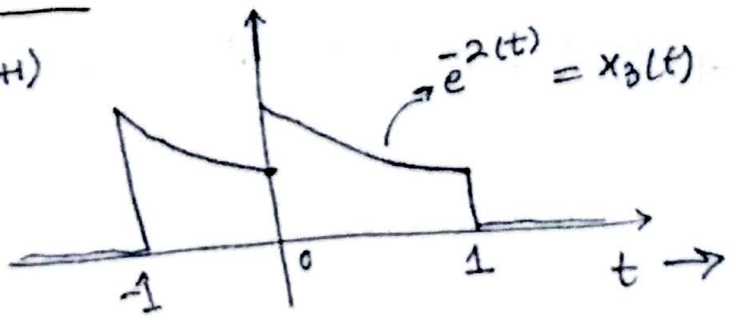
$x_2(t)$ in terms of $x(t)$:



$$x(t) = x_2(t) + (-x_2(-t))$$
$$= x_2(t) - x_2(-t)$$
$$= e^{-2t} - e^{2t}.$$

$x_3(t)$ in terms of $x(t)$:

$$x_3(t+1) = e^{-2(t+1)}$$



So;

$$x(t) = x_3(t) + x_3(t+1)$$

$$x(t) = e^{-2t} + e^{-2(t+1)}$$

b) Write $X_1(\omega)$, $X_2(\omega)$ and $X_3(\omega)$ in terms of $X(\omega)$

$X_1(\omega)$ in terms of $X(\omega)$:

As, $x(t) = x_1(t) + x_1(-t)$

$$x(t) = e^{-2t} + e^{+2t}$$

$$x(t) \Longleftrightarrow X(\omega)$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$X_2(\omega)$ in terms of $X(\omega)$:

As;

$$x(t) = x_2(t) - x_2(-t)$$

$$x(t) = e^{-2t} - e^{+2t}$$

$$\Rightarrow X(\omega) = X_2(\omega) + X_2(-\omega)$$

$X_3(w)$ -in terms of $X(w)$

$$x(t) = x_3(t) + x_3(t+1)$$

As:

- $x(t - t_0) \Longleftrightarrow X(\omega)\, e^{-j\omega t_0}$
- $x(t + t_0) \Longleftrightarrow X(\omega)\, e^{+j\omega t_0}$

Therefore;

$$X(w) = X_3(w) + X(\omega)\, e^{+j\omega_0 t_0}$$

$$X(\omega) = X_3(\omega) + X(\omega)\, e^{j\omega}.$$