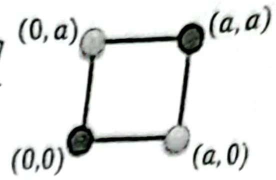
Q1 Answer the following: [20 marks]

(a) Define electric dipole and dipole moment **p** with the help of diagram. [4]

(b) A dipole of moment $\mathbf{p} = 3\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$ [n C · m] is located at the origin in free space. Find V at $P(r = 3.5\,m, \theta = 30°, \varphi = 60°)$. [6]

Hint, $V = \dfrac{\mathbf{p}\cdot\mathbf{a}_r}{4\pi\varepsilon_0 r^2}$

(c) Four point-charges $Q_1 = +q, Q_2 = -q, Q_3 = -q, and\ Q_4 = +q$ are located at $(0,0), (a,0), (0,a)$, and (a,a) respectively in yz-plane. **Formulate** for the potential energy stored in this *quadrupole*. [10]

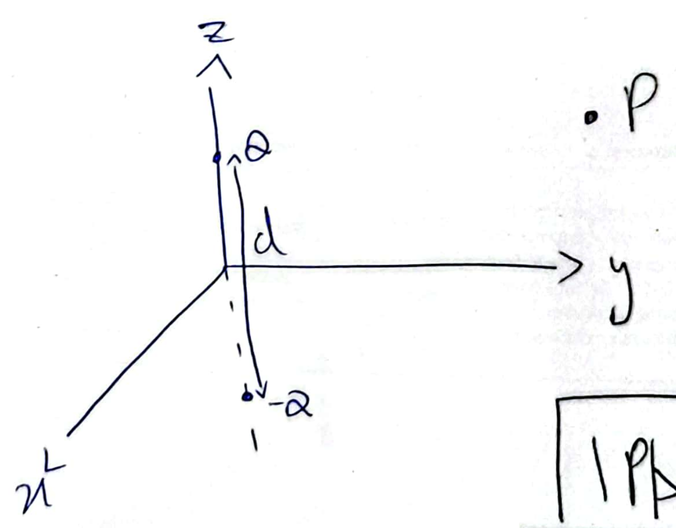


Hint, $W_E = \dfrac{1}{2}\sum Q_i V_i$

CLO # 02

Q1(a)

An electric dipole consists of two charges of equal magnitude but oppossite sign separated by a distance d, which is small in comparison to the point P, where we want to know the potential.



$\boxed{|P| > d}$

The dipole moment is simply one of the charge of the dipole times the seperation between them.

$\vec{p} = Q\vec{d}$

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Dipole moment (p) is a vector with the units $\boxed{C \cdot m}$.

$r^2 =$

(b) $V = \frac{1}{4\pi\epsilon_0 r^2} \times p \cdot a_r$

$P(r = 3.5m, \theta = 30°, \phi = 60°)$

$x = r\sin\theta\cos\phi = 3.5\sin30°\cos60° = 0.875$

$y = r\sin\theta\sin\phi = 3.5\sin30°\sin60° = 1.516$

$z = r\cos\theta = 3.5\cos30° = 3.03$

$P(0.875, 1.516, 3.03)$
$\quad x \quad y \quad z$

$r = \sqrt{0.875^2 + 1.516^2 + 3.03^2} = 3.5$

$r^2 = 12.24$

$a_r = \frac{1}{3.5}(0.875a_x + 1.516a_y + 3.03a_z) = 0.25a_x + 0.433a_y + 0.866a_z$

$\frac{1}{4\pi\epsilon_0 r^2} = \frac{1}{4\times\pi\times8.85\times10^{-12}\times12.24} = 734624566.5$

$V = \frac{1}{4\pi\epsilon_0 r^2} \times p \cdot a_r$

$p \cdot a_r$

$(3\times10^{-9}a_x - 2\times10^{-9}a_y + 1\times10^{-9}a_z) \cdot (0.25a_x + 0.433a_y + 0.866a_z)$

7.5×10^{-10}

$V = 734624566.5 \times 7.5\times10^{-10}$

$\boxed{V = 0.55}$

V at P.

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$(\overset{x_1}{0}, \overset{y_1}{a})$ $(\overset{x_2}{a}, \overset{y_2}{0})$ $(\,,\,)$

$\sqrt{a^2+a^2} = \sqrt{2a^2} = \sqrt{2}a$ $\sqrt{a^2+a^2}$ $\sqrt{2a^2}$

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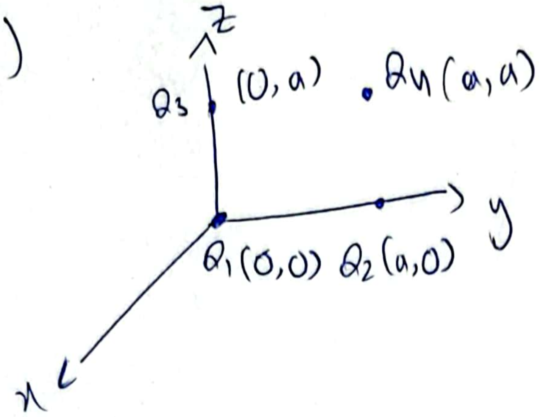
(c)



$Q_1 = +q$
$Q_2 = -q$
$Q_3 = -q$
$Q_4 = +q$

$(\overset{x_1}{a}, \overset{y_1}{a})$
$(\overset{x_2}{0}, 0)$
$\sqrt{(0-a)^2 + (a-a)^2}$
$\sqrt{a^2} = a$
$(\overset{x_1}{0}, \overset{y_1}{a})$ $(\overset{x_2}{a}, \overset{y_2}{0})$
$\sqrt{a^2+a^2} = \sqrt{2}a$
$\sqrt{(a-a)^2 + (a-0)^2}$
$\sqrt{a^2} = a$

$$W_E = \frac{1}{2}\sum Q_i V_i$$

$$W_E = \frac{1}{2}\left(\cancel{Q_1V_1 + Q_2} V_2 + Q_3V_3 + Q_4V_4\right)$$

For point Charge: $V = \frac{q}{4\pi\epsilon_0 r}$

$$W_E = \frac{1}{2}\Big[Q_1V_{1,2} + Q_1V_{1,3} + Q_1V_{1,4} + Q_2V_{2,1} + Q_2V_{2,3} + Q_2V_{2,4} + Q_3V_{3,1} + Q_3V_{3,2} + Q_3V_{3,4} + Q_4V_{4,1} + Q_4V_{4,2} + Q_4V_{4,3}\Big]$$

$Q_1V_{1,2} = q \times \frac{q}{4\pi\epsilon_0 a} = \frac{q^2}{4\pi\epsilon_0 a}$

$Q_1V_{1,3} = q \times \frac{q}{4\pi\epsilon_0 a} = \frac{q^2}{4\pi\epsilon_0 a}$

$Q_1V_{1,4} = q \times \frac{q}{4\pi\epsilon_0\sqrt{2}a} = \frac{q^2}{4\pi\epsilon_0\sqrt{2}a}$

$Q_2V_{2,1} = -q \times \frac{-q}{4\pi\epsilon_0 a} = \frac{q^2}{4\pi\epsilon_0 a}$

$Q_2V_{2,3} = -q \times \frac{-q}{4\pi\epsilon_0\sqrt{2}a} = \frac{q^2}{4\pi\epsilon_0\sqrt{2}a}$

$Q_2V_{2,4} = -q \times \frac{-q}{4\pi\epsilon_0 a} = \frac{q^2}{4\pi\epsilon_0 a}$

$Q_3V_{3,1} = -q \times \frac{-q}{4\pi\epsilon_0 a} = \frac{q^2}{4\pi\epsilon_0 a}$

$Q_3V_{3,2} = -q \times \frac{-q}{4\pi\epsilon_0\sqrt{2}a} = \frac{q^2}{4\pi\epsilon_0\sqrt{2}a}$

$Q_3V_{3,4} = -q \times \frac{-q}{4\pi\epsilon_0 a}$

$Q_3V_{3,4} = \frac{q^2}{4\pi\epsilon_0 a}$

$Q_3V_{4,1} = q \times \frac{q}{4\pi\epsilon_0\sqrt{2}a} = \frac{q^2}{4\pi\epsilon_0\sqrt{2}a}$

$Q_4V_{4,2} = q \times \frac{q}{4\pi\epsilon_0 a} = \frac{q^2}{4\pi\epsilon_0 a}$

$Q_4V_{4,3} = q \times \frac{q}{4\pi\epsilon_0 a} = \frac{q^2}{4\pi\epsilon_0 a}$

$$W_E = \frac{1}{2}\frac{q^2}{4\pi\epsilon_0}\left(\frac{1}{a} + \frac{1}{a} + \frac{1}{\sqrt{2}a} + \frac{1}{a} + \frac{1}{\sqrt{2}a} + \frac{1}{a} + \frac{1}{a} + \frac{1}{\sqrt{2}a} + \frac{1}{a} + \frac{1}{\sqrt{2}a} + \frac{1}{a} + \frac{1}{a}\right)$$

$$\boxed{W_E = \frac{q^2}{8\pi\epsilon_0}\left(\frac{8}{a} + \frac{4}{\sqrt{2}a}\right)}$$ ✓

$q \; \frac{q}{4\pi\epsilon_0 r^2}$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

$\frac{2}{2} + \frac{2}{4} = 2 + \frac{1}{2}$

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$= 3(a_x \cdot$

$(\cos$

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$\frac{\rho_s}{2\pi\rho} a_\rho \quad \frac{C}{m^2}$

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Q2 Electric flux density is defined in the free space as following, **[20 marks]**

$$D = z\rho \cos^2 \varphi \; \boldsymbol{a_z} \; [\mathrm{C/m^2}]$$

Determine the following:

(a) Volume charge density ρ_v at point $P(1, \pi/4, 3)$. **[6]**

(b) Total charge enclosed Q_{encl} by a cylinder of radius $\rho = 1\mathrm{m}$ with height $-2 \le z \le 2\mathrm{m}$ using *two* different methods. Draw the diagram as well. **[6+8]**

CLO # 02

(a) $\operatorname{div} D = \rho_V = \nabla \cdot D$

$(\rho, \phi, z) \Rightarrow CCS$

$\nabla \Rightarrow$ only a_z component needed

$\boxed{\nabla \cdot D = \frac{\partial}{\partial z} D_z}$

$\hookrightarrow$ All components are zero.

$\nabla \cdot D = \rho_V = \frac{\partial \, z\rho\cos^2\phi}{\partial z}$

$\rho_V = \rho\cos^2\phi$

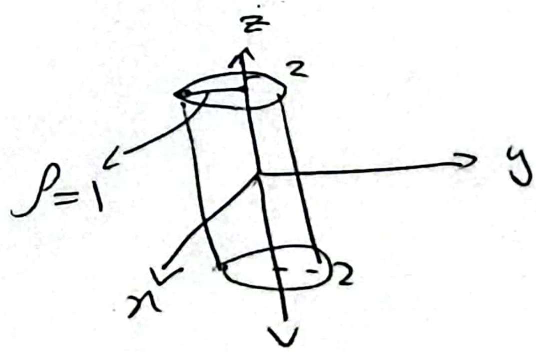
ρ_V at $P(\overset{\rho}{1}, \overset{\phi}{\pi/4}, \overset{z}{3})$

ρ_V at $P = 1 \cos^2 \pi/4$

$\boxed{\rho_V = \frac{1}{2} \; C/m^3}$

(b) $Q_{encl} = \underbrace{\oint_S D \cdot ds}_{\text{Method 1}} = \underbrace{\int_{vol} \rho_V \, dV}_{\text{Method 2}} \Rightarrow$ Gauss's law

$\rho = 1\mathrm{m}$ height $-2 \le z \le 2$



Method 1:—

$dS_z = \rho d\rho d\phi \, a_z$

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$Q_{enc} = \oint_S D \cdot ds$

$\cancel{D \cdot ds = z\rho\cos^2\phi}$

$D = z\rho\cos^2\phi \, a_z$

$ds_z = \rho \, d\rho \, d\phi \, a_z$

$D \cdot ds = z\rho^2\cos^2\phi \, d\rho \, d\phi$

$Q_{enc} = z \iint \rho^2 \cos^2\phi \, d\rho \, d\phi$

$Q_{enc} = z \int_0^1 \int_0^{2\pi} \rho^2 \cos^2\phi \, d\rho \, d\phi$

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$\cos 2\theta = \cos^2\theta - \sin^2\theta$

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$\cos 2\theta = \cos^2\theta - (1-\cos^2\theta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$\frac{\cos 2\theta + 1}{2}$

Method 2

$Q_{enc} = \int_{vol} \rho_v \, dV \qquad \rho_v = \rho \cos^2\phi$

$\cancel{\iiint \rho \cos^2\phi \, \rho \, d\rho \, d\phi \, dz}$

$\int_{-2}^{2}\int_{0}^{2\pi}\int_{0}^{1} \rho^2 (\cos\phi)^2 \, d\rho \, d\phi \, dz$

$\left| \frac{\rho^3}{3} \right|_0^1 \times (2-(-2)) \times \frac{1}{2}\int_0^{2\pi} \cos 2\phi + 1 \, d\phi$

$\frac{1}{3} \times 4 \times \frac{1}{2} \left| \frac{\sin 2\phi}{2} + \phi \right|_0^{2\pi}$

$\frac{1}{3} \times 4 \times \frac{1}{2} (2\pi)$

$\frac{1}{3} \times 4 \times \pi = \boxed{\frac{4\pi}{3} \, C}$

National University of Computer and Emerging Sciences, Lahore Campus

	Course:	Electromagnetic Theory	Course Code:	EE3005
	Program:	BS (Electrical Engineering)	Semester:	Fall 2023
	Duration:	60 Minutes	Total Marks:	[40]
	Paper Date:	Friday, November 10, 2023	Weightage:	15%
	Section:	BEE – 5A	Page(s):	7
	Exam:	Midterm-2	Questions:	2

Name: Ahmad Sheraz **Roll. No:** 21L-5720

Instruction/Notes:

- Closed book, closed notes exam.
- Attempt all questions. Programmable calculators are not allowed.
- Exchange of anything, especially calculators, is strictly prohibited.
- Your answers should be correct up to two **(2) decimals** with proper **SI** units.
- Efficiently use the space provided **(No Additional Sheets allowed)**.
- **Draw diagrams** where necessary.

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Problems ↓	Direct Assessment of CLOs										Part (a)	Part (b)	Part (c)	Total Score
	CLO # 2					CLO # 2								
	E 5	P 4	D 3	B 2	N 1	E 5	P 4	D 3	B 2	N 1				
1											3 / 4	4 / 6	4 / 10	11 / 20
2											6 / 6	2 / 6	8 / 8	16 / 20

CLO	Statement ↓ Score →	Exemplary (5)	Proficient (4)	Developing (3)	Beginning (2)	Novice (1)	AT
02	Formulate electrostatic fields and/or its properties governed by Coulomb's / Gauss's law for a given charge distribution in free space and / or dielectrics.	Complete formulation of the electrostatic model with correct solution	Formulate the electrostatic model with minor mistakes	Complete sketch of the problem with partial formulation of electrostatic model	Identify the correct Law along with partial sketch of the problem at hand	Cannot identify the correct Law to solve the problem	3

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Hint, $V = \dfrac{\mathbf{p}\cdot\mathbf{a}_r}{4\pi\varepsilon_0 r^2}$

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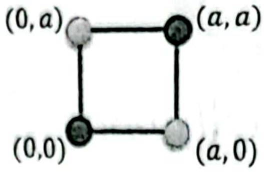
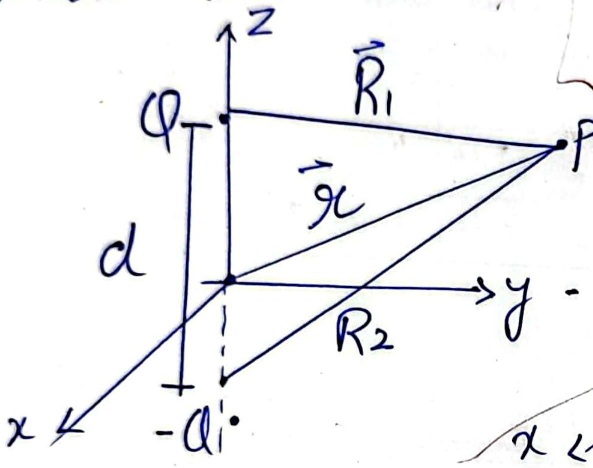
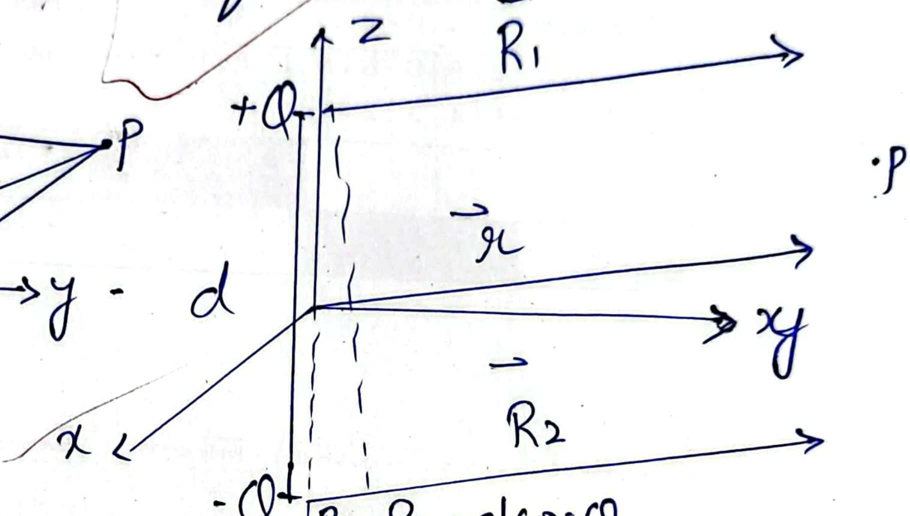


Hint, $W_E = \frac{1}{2}\sum Q_i V_i$

CLO # 02

a) The electric dipole or simply dipole is the sum of two charges of equal magnitude but of differenent sign separated by distance "d", smaller than the distance of point "P" where we have to find $\vec{E}$ and $\vec{V}$.





$R_2 - R_1 = d\cos\theta$

$V = \dfrac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}$

$V = \dfrac{Q\vec{d}\cdot\hat{a}_r}{4\pi\varepsilon_0 r^2}$

b) $V = \dfrac{\vec{P}\cdot\hat{a}_r}{4\pi\varepsilon_0 r^2} = \dfrac{1}{4\pi\varepsilon_0|r-r'|^2}\ \vec{P}\cdot\dfrac{r-r'}{|r-r'|}$

at origin $r'(0,0,0)$

$r = P$ $\Rightarrow$ •) $x = \rho\cos\phi$

$x = r\sin\theta\cos\phi$

$x = 0.875\,m$

•) $y = \rho\sin\phi$

$= r\sin\theta\sin\phi$

$y = 1.515\,m$

•) $z = r\cos\theta$

$z = 3.5\cos(30°)$

$z = 3.03$

$r = (0.875, 1.515, 3.03)$

$r - r' = (0.875, 1.515, 3.03)$

$|r - r'| = 3.498\,m$

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Now;

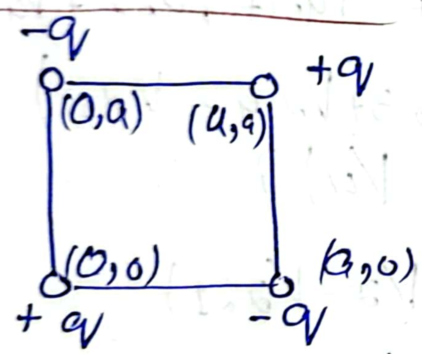
$$V = \frac{1}{4\pi\epsilon_0 |r-r'|}\vec{P}\cdot\frac{r-r'}{|r-r'|} \Rightarrow \frac{1}{4\pi\epsilon_0 (3.498)^2}\vec{P}\cdot\frac{(0.875a_x + 1.515a_y + 3.03a_z)}{3.498}$$

$$V = \frac{1}{4\pi \times 8.85\times10^{-12}(3.498)^2}\times(3\hat{a}_x - 2\hat{a}_y + a_z)\cdot\frac{(0.875\hat{a}_x + 1.515\hat{a}_y + 3.03\hat{a}_z)}{3.498}$$

$$V = \frac{(2.65 - 2.302 + 3.03)/3.498}{1.3607\times10^{-9}}$$

$$V = \frac{(3.378)/(3.498)}{1.361\times10^{-9}}$$

$$V = 70.9\times10^{7}\,V$$

c) 

Q_2 [C] :-

$\Rightarrow Q_2 V_{2,1}$
$-q V_{2,1}$

Q_3 [C] :-

$\Rightarrow Q_3 V_{3,1} + Q_3 V_{3,2}$
$-q V_{3,1} - q V_{3,2}$

Q_4 [C] :-

$\Rightarrow Q_4 V_{4,1} + Q_4 V_{4,2} + Q_4 V_{4,3}$

$\Rightarrow +q V_{4,1} + q V_{4,2} + q V_{4,3}$

Total Work.

$W_E = -q V_{2,1} - q V_{3,1} - q V_{3,2}$
$+ q V_{4,1} + q V_{4,2} + q V_{4,3}$

$$W_E = q(-V_{2,1} - V_{3,1} - V_{3,2} + V_{4,1} + V_{4,2} + V_{4,3}) \rightarrow a$$

find W_E for Q_1 after $-q_2, -q_3, +q_4$ are placed:-

$$W_E = q(-V_{1,2} - V_{1,3} - V_{2,3} + V_{1,4} + V_{2,4} + V_{3,4}) \rightarrow b$$

Summing (a) and (b)

$$2W_E = q(-V_{2,1} - V_{3,1} - V_{3,2} + V_{4,1} + V_{4,2} + V_{4,3}) + q(-V_{1,2} - V_{1,3} - V_{2,3} + V_{1,4} + V_{2,4} + V_{3,4})$$

$$W_E = \frac{q}{2}\left[(-V_{1,2} - V_{1,3} + V_{1,4}) + (V_{2,3} + V_{2,4} - V_{2,1}) + (-V_{3,1} - V_{3,2} + V_{3,4}) + (V_{4,1} + V_{4,2} + V_{4,3})\right]$$

$$W_E = \frac{q}{2}\left[(-V_{1,2} - V_{1,3} + V_{1,4}) + (V_{2,3} + V_{2,4} - V_{2,1}) + (-V_{3,1} - V_{3,2} + V_{3,4}) + (V_4)\right]$$

$$W_E = \frac{q}{2}\left[(-(V_{1,2} + V_{1,3} - V_{1,4}) + (V_2 - V_{2,1}) - (V_{3,1} + V_{3,2} - V_{3,4}) + (V_4)\right]$$

$$W_E = \frac{q}{2}\left[-(V_1 - V_{1,4}) + (V_2 - V_{2,1}) - (V_3 - V_{3,4}) + (V_4)\right]$$

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Q2 Electric flux density is defined in the free space as following, **[20 marks]**

$$D = z\rho \cos^2\varphi \; \boldsymbol{a}_z \; [\mathrm{C/m^2}]$$

Determine the following:

(a) Volume charge density ρ_v at point $P(1, \pi/4, 3)$. [6]

(b) Total charge enclosed Q_{encl} by a cylinder of radius $\rho = 1$m with height $-2 \le z \le 2$m using two different methods. Draw the diagram as well. [6+8]

CLO # 02

a) $\vec{D} = z\rho \cos^2\phi \, \hat{a}_z$

$\vec{\nabla}\cdot\vec{D} = \rho_v$

$\mathrm{div}\,\bar{D} = \rho_v$

$\rho_v = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$

only $\vec{D}$ changing with "z"

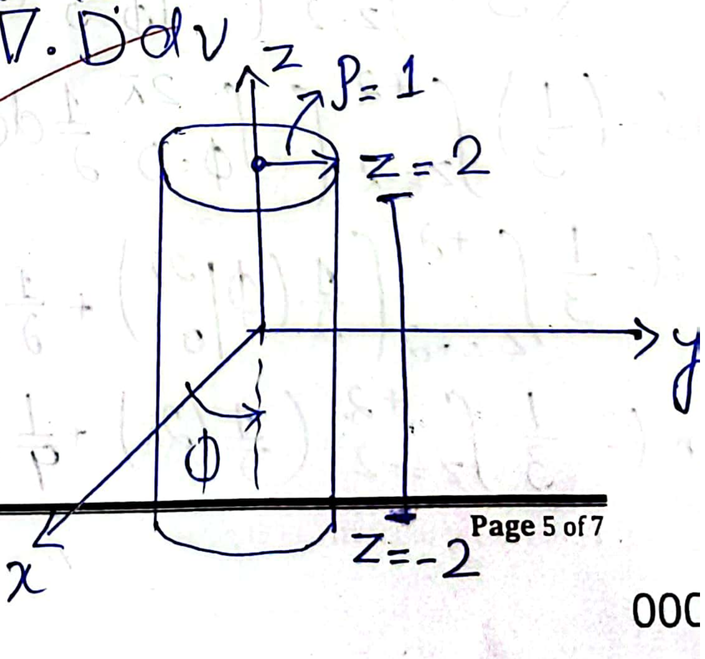
$\rho_v = \frac{\partial}{\partial z}(z\rho\cos^2\phi)$

$\rho_v = \rho\cos^2\phi$

$\rho_v = 1 \times \left(\cos\left(\frac{\pi}{4}\right)\right)^2$

$\rho_v = 0.5 \frac{C}{m^3}$ at $P(1, \pi/4, 3)$

b) $\oint_s \vec{D}_s \cdot d\vec{s} = \int_{vol} \vec{\nabla}\cdot\vec{D}\,dv$



Method 1

$\int_{vol} \vec{\nabla}\cdot\vec{D}\,dv = Q_{encl}$

$\vec{\nabla}\cdot\vec{D} = \rho$

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$$\vec{\nabla}\cdot\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D\phi}{\partial\phi} + \frac{\partial D_z}{\partial D_z}$$

$$\vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial z}(z\rho\cos^2\phi)$$

$$\vec{\nabla}\cdot\vec{D} = \rho\cos^2\phi \leftarrow$$

$$\Rightarrow Q_{encl} = \int_{vol} \vec{\nabla}\cdot\vec{D}\,dv$$

$$Q_{encl} = \int_{z=-2}^{+2}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{1} \rho\cos^2\phi\,\rho\,d\rho\,d\phi\,dz$$

$$Q_{encl} = \int_{z=-2}^{+2}\int_{\phi=0}^{2\pi}\left[\int_0^1 \rho^2\,d\rho\right]d\phi\,dz$$

$$Q_{encl} = \frac{\rho^3}{3}\Big|_0^1 \int_{z=-2}^{+2}\left[\int_{\phi=0}^{2\pi}\cos^2\phi\,d\phi\right]dz$$

$$Q_{encl} = \left(\frac{1}{3}\right)\int_{z-2}^{+2}\left[\int_{\phi=0}^{2\pi}\left(\frac{1+\cos2\phi}{2}\right)d\phi\right]dz$$

$$Q_{encl} = \left(\frac{1}{3}\right)\int_{z=-2}^{+2}\left[\int_{\phi=0}^{2\pi}\frac{1}{2}d\phi + \int_{\phi=0}^{2\pi}\frac{\cos2\phi}{2}d\phi\right]dz$$

$$Q_{encl} = \frac{1}{3}\int_{z=-2}^{+2}\left[\frac{1}{2}\left(\phi\Big|_0^{2\pi}\right) + \frac{1}{2}\left(\frac{\sin2\phi}{2}\right)\Big|_0^{2\pi}\right]dz$$

$$Q_{encl} = \frac{1}{3}\int_{z=-2}^{+2}\left(\frac{1}{2}(2\pi) + \frac{1}{4}(\sin(4\pi) - \sin(0))\right]dz$$

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$$Q_{encl} = \frac{1}{3}\int_{-2}^{+2}\left[\pi + \frac{1}{4}(0-0)\right] dz$$

$$Q_{encl} = \frac{1}{3}\pi\left(z\Big|_{-2}^{+2}\right)$$

$$Q_{encl} = \frac{1}{3}\pi(2+2) = \frac{4}{3}\pi \; C$$

Method 2 :-

$$Q_{encl} = \oint D_s \cdot d\vec{s} \qquad dS_z = \rho\, d\rho\, d\phi\, \hat{a}_z$$

$$= \oint z\rho\cos^2\phi\, a_z \cdot dS_z \cancel{\hat{a}_z}$$

$$= \int_{\phi=-0}^{2\pi}\int_{\rho=0}^{1} z\rho\cos^2\phi\, \hat{a}_z \cdot \rho\, d\rho\, d\phi\, \hat{a}_z$$

$$\rightarrow \cos^2\phi$$

$$= z \times \int_{0}^{1}\rho^2\, d\rho \times \int_0^{2\pi}\frac{1+\cos 2\phi}{2}\, d\phi$$

$$= 3\times\frac{\rho^3}{3}\Big|_0^1 \times\left(\int_0^{2\pi}\frac{1}{2}\, d\phi + \frac{1}{2}\int_0^{2\pi}\cos 2\phi\, d\phi\right)$$

$$= 3\times\left(\frac{1}{3}\right)\times\left(\frac{1}{2}(2\pi-0) + \frac{1}{2}\left(\cancel{2\pi} + \frac{1}{2}\sin 2\phi\Big|_0^{2\pi}\right)\right)$$

$$= 1\times\left(\pi + \frac{1}{2}(0)\right)$$

$$= \pi \; C$$

$$\Rightarrow \pi C$$

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