Complex Variables and Transforms
Assignment No. 1

Instructions

Answer all the questions. Show all your workings clearly. The use of external resources is allowed, but you must cite all sources appropriately.

Submission Deadline: Submit your completed assignment in hard copy by 11th September 2024. Late submissions will not be accepted without prior approval.

Questions

1. Find and graph all roots in the complex plane:
   (a) $\sqrt[3]{1+i}$
   (b) $\sqrt[3]{3+4i}$
   (c) $\sqrt[4]{-4}$
2. Differentiate the following functions and find the value at the specified points:
   (a) $\frac{z-i}{z+i}$ at $z = i$
   (b) $\frac{z^3}{(z+i)^3}$ at $z = i$
3. Are the following functions analytic? Use either the Cauchy-Riemann equations or the definition of analyticity to justify your answer:
   (a) $f(z) = e^x(\cos y - i \sin y)$
   (b) $f(z) = \frac{1}{(z-\bar{z}^5)}$
   (c) $f(z) = \ln|z| + i \arg z$

4. Determine whether the following functions are harmonic. If your answer is yes, find a corresponding analytic function $f(z) = u(x, y) + iv(x, y)$:

   (a) $u = x^2 + y^2$

   (b) $u = x/(x^2 + y^2)$