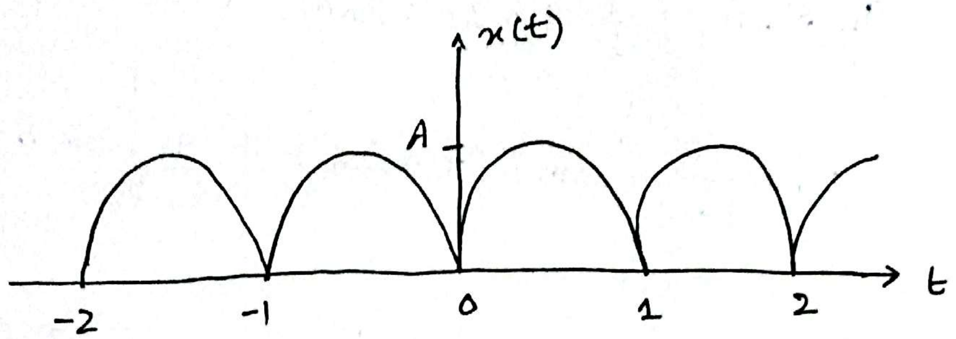
Signals and Systems Assignment #3

Name: Mudassar Hussain

Roll no: 23L-6006 , 4B

Q1 Find the compact trig. Fourier Series representation of following signals & sketch magnitude & phase spectrum.



$T_0 = 1$

$\omega = \frac{2\pi}{T_0} = 2\pi.$

Where,

$x(t) = A \sin\left(\frac{\omega_0 t}{2}\right)$

$= A \sin\left(\frac{2\pi t}{2}\right)$

$= A \sin(\pi t).$

a) $a_0 = \frac{1}{T_0} \int_{T_0} x(t)\, dt.$

$a_0 = \frac{1}{1} \int_0^1 A \sin \pi t \, dt$

$a_0 = A\left[-\frac{\cos \pi t}{\pi}\right]_0^1$

$a_0 = -\frac{A}{\pi}[\cos\pi - 1]$

$a_0 = -\frac{A}{\pi}[-1-1]$

$\boxed{a_0 = \frac{2A}{\pi}}$

b) $a_n = \frac{2}{T_0}\int_{T_0} x(t) \cdot \cos n\omega_0 t \, dt$

$$a_n = \frac{2}{1}\int_0^1 A\sin \pi t \cdot \cos n\, 2\pi t \, dt.$$

As,

$\because \sin a \cdot \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$

Therefore;

$$a_n = 2A\left[\int_0^1 \frac{1}{2} \sin(\pi t + 2n\pi t) + \sin(\pi t - 2n\pi t)\right]$$

$$= 2A\left[\frac{1}{2}\int_0^1 \sin \pi(1+2n)t \, dt + \int_0^1 \sin \pi(1-2n)t \, dt\right]$$

$$= \frac{2A}{2}\left[\left.\frac{-\cos \pi(1+2n)t}{\pi(1+2n)}\right|_0^1 + \left.\frac{-\cos \pi(1-2n)t}{\pi(1-2n)}\right|_0^1\right]$$

$$= A\left[\frac{-1}{\pi(1+2n)}\left(\cos \pi(1+2n) - 1\right) - \frac{1}{\pi(1-2n)}\left(\cos \pi(1-2n) - 1\right)\right]$$

$$= A\left[\frac{-\cos \pi(1+2n) + 1}{\pi(1+2n)} - \frac{\cos \pi(1-2n) + 1}{\pi(1-2n)}\right]$$

$$= A\left[\frac{-(-1) + 1}{\pi(1+2n)} + \frac{-(-1) + 1}{\pi(1-2n)}\right]$$

$$= A\left[\frac{2}{\pi(1+2n)} + \frac{2}{\pi(1-2n)}\right]$$

$$= A\left[\frac{2(\pi(1-2n)) + 2\pi(1+2n)}{\pi^2(1+2n)(1-2n)}\right]$$

CamScanner

$$a_n = A\left[\frac{2\pi - \cancel{4\pi n} + 2\pi + \cancel{4\pi n}}{\pi^2(1 - \cancel{2n} + \cancel{2n} - 4n^2)}\right]$$

$$= A\left[\frac{4\pi}{\pi^2(1-4n^2)}\right]$$

$$\boxed{a_n = \frac{4A}{(1-4n^2)\pi}}$$

•) $b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin \omega_0 nt \, dt$.

→ The signal is even, therefore $b_n = 0$.

→ even signal (symmetric about y axis)

$$b_n = \frac{2}{1}\int_0^1 A \sin \pi t \cdot \sin 2\pi nt \, dt.$$

<u>As</u>,

$$\because \sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

<u>Therefore</u>:

$$b_n = 2A\left[\int_0^1 \left(\frac{1}{2}\left(\cos(\pi t - 2n\pi t) - \cos(\pi t + 2n\pi t)\right)\right) dt.\right]$$

$$= \frac{2A}{2}\left[\frac{\sin(\pi t - 2n\pi t)}{\pi - 2n\pi}\Bigg|_0^1 - \frac{\sin(\pi t + 2n\pi t)}{\pi + 2n\pi}\Bigg|_0^1\right]$$

$$= A\left[\frac{\sin(\pi - 2n\pi)}{\pi - 2n\pi} - \frac{\sin(\pi + 2n\pi)}{\pi + 2n\pi}\right]$$

As, $\rightarrow \sin \pi(1-2n) = 0$, n is any integer
$\rightarrow \sin \pi(1+2n) = 0$, n is any integer.

$$b_n = A\left[\frac{0}{\pi(1-2n)} - \frac{0}{\pi(1+2n)}\right]$$

$$\boxed{b_n = 0}$$

Trignometric Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$= \frac{2A}{\pi} + \sum_{n=1}^{\infty}\left(\frac{A \cdot 4}{\pi(1-4n^2)} \cos n 2\pi t\right)$$

$$= \frac{2A}{\pi} + \frac{4A}{\pi} \sum_{n=1}^{\infty}\left(\frac{1}{1-4n^2} \cdot \cos n 2\pi t\right).$$

$$= \frac{2A}{\pi} + \frac{4A}{\pi}\left(-\frac{1}{3}\cos 2\pi t - \frac{1}{15}\cos 4\pi t - \frac{1}{35}\cos 6\pi t + \cdots\right)$$

$$x(t) = \frac{2A}{\pi} - \frac{4A}{3\pi}\cos 2\pi t - \frac{4A}{15\pi}\cos 4\pi t - \frac{4A}{35\pi}\cos 6\pi t + \cdots$$

$\rightarrow$ eq 1

Compact Trignometric Series:

for compact trignometric series, all terms must be cosine terms & amplitudes must +ve.

$\therefore$ *) $\sin kt = \cos\left(kt - \frac{\pi}{2}\right)$

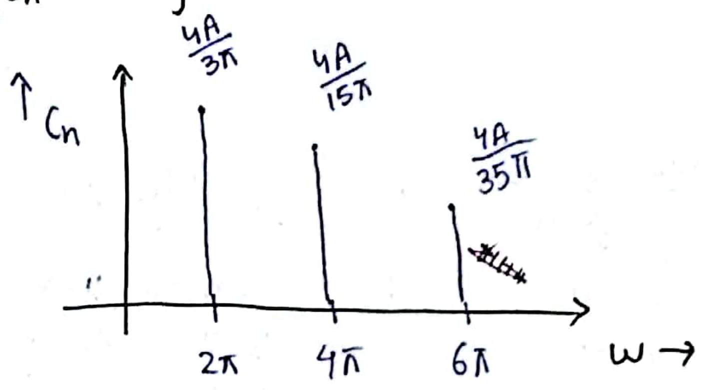
*) $-\sin kt = \cos\left(kt + \frac{\pi}{2}\right)$

*) $-\cos t = +\cos(t - \pi)$.

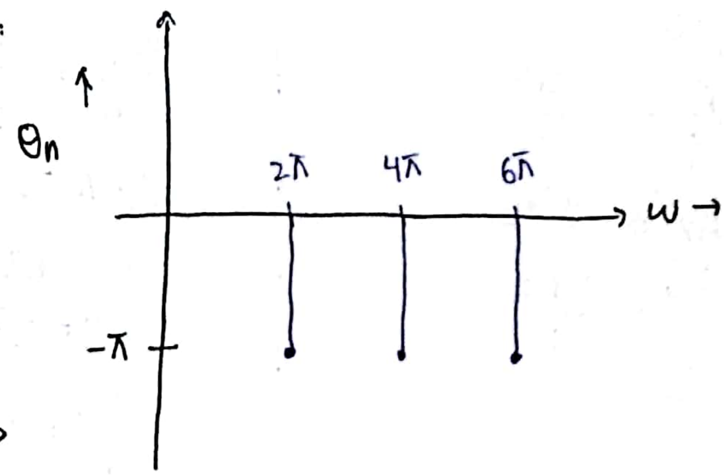
So eq (1) becomes:

$$x(t) = \frac{2A}{\pi} + \frac{4A}{3\pi}\cos(2\pi t - \pi) + \frac{4A}{15\pi}\cos(4\pi t - \pi) + \frac{4A}{35\pi}\cos(6\pi t - \pi) + \dots$$

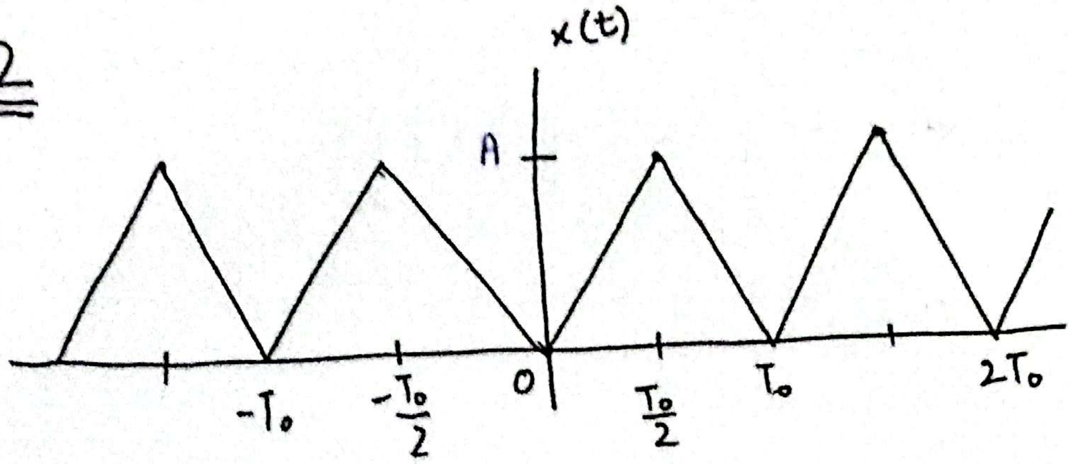
Amplitude Spectrum: $[C_n \text{ vs } \omega]$



Phase Spectrum: $[\theta_n \text{ vs } \omega]$:



Q2

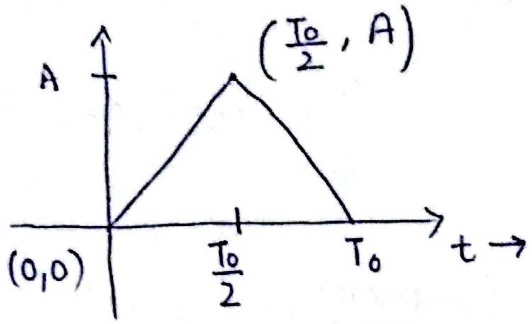


Compact Trignometric F.S.
Sketch magnitude & phase spectrum.

Consider :-

$T_0 = T_0$

$w = \frac{2\pi}{T_0}$.



Eq. of Straight line

$\rightarrow (0,0)$ to $(\frac{T_0}{2}, A)$

$$\frac{y_2 - y_1}{y - y_1} = \frac{x_2 - x_1}{x - x_1}$$

$$\frac{A - 0}{y - 0} = \frac{\frac{T_0}{2} - 0}{x - 0}$$

$$\frac{A}{y} = \frac{T_0/2}{x}$$

$$Ax = y\,T_0/2$$

$$\boxed{y = \frac{2At}{T_0}}$$

$\rightarrow (\frac{T_0}{2}, A)$ to $(T_0, 0)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - A}{0 - A} = \frac{x - T_0/2}{T_0 - T_0/2}$$

$$\frac{y - A}{-A} = \frac{2x - T_0}{2T_0 - T_0} = \frac{2x - T_0}{T_0}$$

$$(y - A)T_0 = (2x - T_0)(-A)$$

$$yT_0 - AT_0 = -2Ax + T_0 A$$

$$y = \frac{-2Ax + T_0 A + T_0 A}{T_0}$$

$$\boxed{y = -\frac{2A}{T_0}x + 2A.}$$

$$y = \begin{cases} \dfrac{2At}{T_0} & , \quad 0 < t < T_0/2 \\[2ex] 2A\left(1 - \dfrac{t}{T_0}\right) & , \quad \dfrac{T_0}{2} < t < T_0 \end{cases}$$

•) $a_0 = \dfrac{1}{T_0} \displaystyle\int_{T_0} x(t)\, dt$

$$a_0 = \frac{1}{T_0} \int_0^{T_0} \frac{2At}{T_0}\, dt$$

$$= \frac{1}{T_0} \int_0^{T_0/2} \frac{2At}{T_0}\, dt + \int_{T_0/2}^{T_0} 2A\left(1 - \frac{t}{T_0}\right) dt$$

$$a_0 = \frac{1}{T_0} \left[\frac{1}{T_0} \cdot \left(\frac{2At^2}{2} \Big|_0^{T_0/2} \right) \right] + 2A \left(\int_{\frac{T_0}{2}}^{T_0} 1\, dt - \frac{1}{T_0} \int_{T_0/2}^{T_0} t\, dt \right)$$

$$= \frac{1}{T_0} \left[\frac{A}{T_0} \left(\frac{T_0^2}{4} \right) + 2A \left(t \Big|_{T_0/2}^{T_0} - \frac{1}{T_0} \cdot \frac{t^2}{2} \Big|_{\frac{T_0}{2}}^{T_0} \right) \right.$$

$$= \frac{1}{T_0} \left[\frac{AT_0}{4} + 2A \left[\left(T_0 - \frac{T_0}{2} \right) - \frac{1}{2T_0} \left(T_0^2 - \frac{T_0^2}{4} \right) \right] \right]$$

$$= \frac{1}{T_0} \left[\frac{AT_0}{4} + 2A \left(\frac{T_0}{2} - \frac{1}{2T_0} \left(\frac{3T_0^2}{4} \right) \right) \right]$$

$$= \frac{1}{T_0} \left[\frac{AT_0}{4} + 2A \left(\frac{T_0}{2} - \frac{3T_0}{8} \right) \right]$$

$$= \frac{1}{T_0} \left[\frac{AT_0}{4} + \cancel{2}A \left(\frac{8T_0 - 6T_0}{\cancel{16}_{\,8}} \right) \right]$$

$$a_0 = \frac{1}{T_0}\left[\frac{AT_0}{4} + A\left(\frac{\cancel{2}T_0}{\cancel{8}_{4}}\right)\right]$$

$$= \frac{1}{T_0}\left[\frac{AT_0}{4} + \frac{AT_0}{4}\right]$$

$$= \frac{1}{\cancel{T_0}}\left[\frac{\cancel{2}A\cancel{T_0}}{\cancel{4}_{2}}\right]$$

$$\boxed{a_0 = \frac{A}{2}}$$

•) $a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t \, dt$

$$a_n = \frac{2}{T_0}\left[\int_0^{T_0} x(t) \cdot \cos n\frac{2\pi}{T_0}t \, dt\right]$$

$$a_n = \frac{2}{T_0}\left[\int_0^{T_0/2} \frac{2At}{T_0} \cdot \cos n\frac{2\pi}{T_0}t \, dt + \int_{T_0/2}^{T_0} 2A\left(1-\frac{t}{T_0}\right) \cdot \cos n\frac{2\pi}{T_0}t \, dt\right] \rightarrow (1)$$

Consider from eq (1)

$$\int_0^{T_0/2} \frac{2At}{T_0} \cdot \cos n\frac{2\pi}{T_0}t \, dt.$$

$$= \frac{2A}{T_0}\left[t \cdot \frac{\sin n\frac{2\pi}{T_0}t}{\frac{2n\pi}{T_0}} - \int \frac{\sin n\frac{2\pi}{T_0}t}{\frac{2n\pi}{T_0}} \, dt\right]$$

$$= \frac{2A}{T_0}\left[t.\frac{T_0}{2n\pi}. \sin \frac{n2\pi}{T_0}t - \frac{T_0}{2n\pi}\left[\frac{-\cos n\frac{2\pi}{T_0}t}{\frac{2n\pi}{T_0}}\right]\right]$$

$$= \frac{2A}{T_0}\left[t.\frac{T_0}{2n\pi}.\frac{\sin n2\pi t}{T_0} + \frac{T_0^2}{4n^2\pi^2}\cdot\cos n\frac{2\pi}{T_0}t\right]$$

Apply limits $0 \longrightarrow T_0/2$

$$= \frac{2A}{T_0}\left[t.\frac{T_0}{2n\pi} \sin n\frac{2\pi}{T_0}t + \frac{T_0^2}{4n^2\pi^2}\cdot \cos n\frac{2\pi t}{T_0}\Big|_0^{T_0/2}\right]$$

$$= \frac{2A}{T_0}\left[\frac{T_0}{2}\cdot\frac{T_0}{2n\pi} \sin n\frac{2\pi}{\cancel{T_0}}\times\frac{\cancel{T_0}}{\cancel{2}} + \frac{T_0^2}{4n^2\pi^2}\cos n\frac{2\pi}{T_0}\times\frac{T_0}{2}\right] -$$

$$\left[0\times\frac{T_0}{2n\pi}\times 0 + \frac{T_0^2}{4n^2\pi^2}\times 1\right]$$

$$= \frac{2A}{T_0}\left[\frac{T_0^2}{4n\pi} \sin n\pi + \frac{T_0^2}{4n^2\pi^2}\cos n\pi - \frac{T_0^2}{4n^2\pi^2}\right]$$

$\therefore \sin \pi n = 0$, n is any integer.

$$= \frac{2A}{T_0}\left[\frac{T_0^2}{4n^2\pi^2}\cos n\pi - \frac{T_0^2}{4n^2\pi^2}\right]$$

$$= \frac{2A}{T_0} \times \frac{T_0^2}{4n^2\pi^2}\left[-1 + \cos n\pi\right].$$

$$= \frac{2AT_0}{4n^2\pi^2}\left[\cos n\pi - 1\right] \rightarrow ⓐ$$

Similary from eq ①

$$\int_{T_0/2}^{T_0} 2A\left(1-\frac{t}{T_0}\right)\cos n\frac{2\pi}{T_0}t\ dt.$$

Let:

$$\int 2A\left(1-\frac{t}{T_0}\right)\cos n\frac{2\pi}{T_0}t\ dt.$$

$$= 2A\left[\int \cos n\frac{2\pi}{T_0}t\ dt - \frac{1}{T_0}\int t\cos n\frac{2\pi}{T_0}t\ dt\right]$$

$$= 2A\left[\frac{\sin n\frac{2\pi}{T_0}t}{\frac{2n\pi}{T_0}} - \frac{1}{T_0}\left[\int t.\cos n\frac{2\pi}{T_0}t\ dt\right]\right]$$

$$= 2A\left[\frac{T_0}{2n\pi}.\sin n\frac{2\pi}{T_0}t - \frac{1}{T_0}\left(t\frac{\sin n\frac{2\pi}{T_0}t}{\frac{n2\pi}{T_0}} - \int\frac{\sin n\frac{2\pi t}{T_0}t}{\frac{2n\pi}{T_0}}dt\right)\right]$$

$$= 2A\left[\frac{T_0}{2n\pi}.\sin n\frac{2\pi}{T_0}t - \frac{1}{T_0}\left(t\frac{T_0}{2n\pi}.\frac{\sin 2n\pi t}{T_0} - \frac{T_0}{2n\pi}\int\sin n\frac{2\pi}{T_0}t\ dt\right)\right.$$

$$= 2A\left[\frac{T_0}{2n\pi}\cdot \sin n\frac{2\pi t}{T_0} - \frac{1}{T_0}\left(\frac{t\,T_0}{2n\pi}\sin\frac{2\pi n t}{T_0} + \frac{T_0}{2n\pi}\times\frac{T_0}{2n\pi}\cos\frac{n2\pi t}{T_0}\right)\right]$$

$$= 2A\left[\frac{T_0}{2n\pi}\sin n\frac{2\pi t}{T_0} - \frac{1}{T_0}\left(t\cdot\frac{T_0}{2n\pi}\sin\frac{2\pi n t}{T_0} + \frac{T_0^2}{4n^2\pi^2}\cos\frac{2\pi n t}{T_0}\right)\right]$$

$$= 2A\left[\frac{T_0}{2n\pi}\cdot\sin\frac{2\pi n t}{T_0} - \frac{t}{2n\pi}\sin\frac{2\pi n t}{T_0} - \frac{T_0}{4n^2\pi^2}\cdot\cos\frac{2\pi n t}{T_0}\right]$$

Apply limits $\left(\frac{T_0}{2}\to T_0\right)$

$$= 2A\left[\frac{T_0}{2n\pi}\cdot\sin\frac{2n\pi t}{T_0} - \frac{t}{2n\pi}\sin\frac{2\pi n t}{T_0} - \frac{T_0}{4n^2\pi^2}\cos\frac{2\pi n t}{T_0}\right]_{\frac{T_0}{2}}^{T_0}$$

$$= 2A\left[\frac{T_0}{2n\pi}\cdot\sin\frac{2n\pi}{\cancel{T_0}}\times\cancel{T_0} - \frac{T_0}{2n\pi}\cdot\sin\frac{2\pi n}{\cancel{T_0}}\times\cancel{T_0} - \frac{T_0}{4n^2\pi^2}\cos\frac{2n\pi}{\cancel{T_0}}\times\cancel{T_0}\right]$$

$$-\left[\frac{T_0}{2n\pi}\sin\frac{2n\pi}{\cancel{T_0}}\times\frac{\cancel{T_0}}{2} - \frac{T_0/2}{2n\pi}\sin\frac{\pi n 2}{\cancel{T_0}}\times\frac{\cancel{T_0}}{2} - \frac{T_0}{4n^2\pi^2}\cos\frac{2\pi n}{\cancel{T_0}}\times\frac{\cancel{T_0}}{2}\right]$$

$$= 2A\left[\frac{T_0}{2n\pi}\sin 2n\pi - \frac{T_0}{2n\pi}\sin 2\pi n - \frac{T_0}{4n^2\pi^2}\cos 2n\pi\right]$$

$$-\left[\frac{T_0}{2n\pi}\cdot\sin n\pi - \frac{T_0}{4n\pi}\sin\pi n - \frac{T_0}{4n^2\pi^2}\cos\pi n\right].$$

$\therefore \sin \pi n = 0$

$\sin 2\pi n = 0$

$\cos 2\pi n = 1$

(for any n integer)

$$= 2A\left[\frac{-T_0}{4n^2\pi^2}\cos 2n\pi - \left(0 - 0 - \frac{T_0}{4n^2\pi^2}\cos n\pi\right)\right]$$

$$= 2A\left[\frac{-T_0}{4n^2\pi^2}\cos 2n\pi + \frac{T_0}{4n^2\pi^2}\cos n\pi\right]$$

$$= 2A\left(\frac{T_0}{4n^2\pi^2}\right)(-1 + \cos n\pi) \rightarrow (b)$$

from (a) & (b) equations (1) becomes.

$$a_n = \frac{2}{T_0}\left[\int_0^{T_0/2} \frac{2At}{T_0} \cdot \cos n\frac{2\pi t}{T_0}\,dt + \int_{T_0/2}^{T_0} 2A\left(1 - \frac{t}{T_0}\right)\cos\frac{2\pi n t}{T_0}\,dt\right]$$

$$= \frac{2}{T_0}\left[\frac{2AT_0}{4n^2\pi^2}\left[\cos n\pi - 1\right] + 2A\cdot\frac{T_0}{4n^2\pi^2}\left[-1 + \cos n\pi\right]\right]$$

$$= \frac{2}{T_0}\cdot\frac{2AT_0}{4n^2\pi^2}\left[\cos n\pi - 1 - 1 + \cos n\pi\right]$$

$$= \frac{A}{n^2\pi^2}\left[-2 + 2\cos n\pi\right]$$

$$\boxed{a_n = \frac{2A}{n^2\pi^2}\left[-1 + \cos n\pi\right]}$$

- $b_n = \frac{2}{T_0} \int_{T_0} x(t).\ \sin n\omega_0 t \ dt$

As signal $x(t)$ is even, symmetric about y-axis.

$\boxed{b_n = 0}$

Trignometric Fourier Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin \omega_0 n t].$$

$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[\frac{2A}{n^2\pi^2}(-1+\cos n\pi) \cos n\frac{2\pi}{T_0} t\right].$$

$$x(t) = \frac{A}{2} + \frac{2A}{\pi^2}\left[\frac{-2}{1^2} \cos \frac{2\pi}{T_0} t - \frac{2}{5^2} \cos \frac{10\pi t}{T_0} - \frac{2}{3^2} \cos \frac{6\pi t}{T_0} - \frac{2}{7^2} \cos \frac{14\pi}{T_0} t + \ldots\right]$$

$$= \frac{A}{2} + \frac{2A}{\pi^2}\left[-2 \cos \frac{2\pi t}{T_0} - \frac{2}{25} \cos \frac{10\pi}{T_0} t - \frac{2}{9} \cos \frac{6\pi}{T_0} t - \frac{2}{49} \cos \frac{14\pi t}{T_0} - \ldots\right]$$

$$x(t) = \frac{A}{2} - \frac{4A}{\pi^2} \cos \frac{2\pi}{T_0} t - \frac{4A}{25\pi^2} \cos \frac{10\pi}{T_0} t - \frac{4A}{9\pi^2} \cos \frac{6\pi}{T_0} t - \frac{4A}{49\pi^2} \cos \frac{14\pi t}{T_0} - \ldots$$

For Compact Trig. F.S:

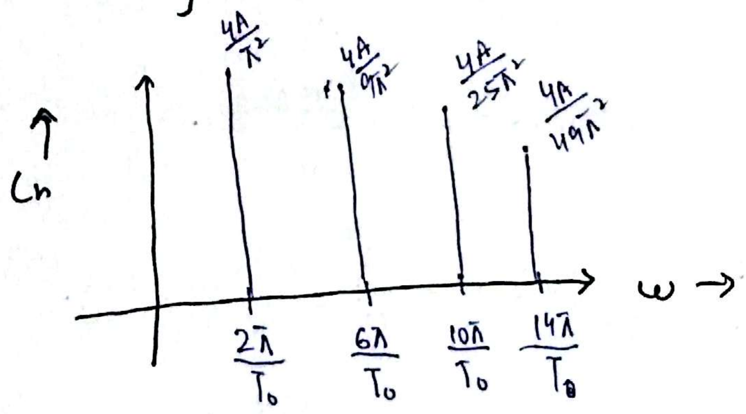
→ All terms must be cosine terms with +ve amplitude

$$-\cos\theta = +\cos(\theta-\pi)$$

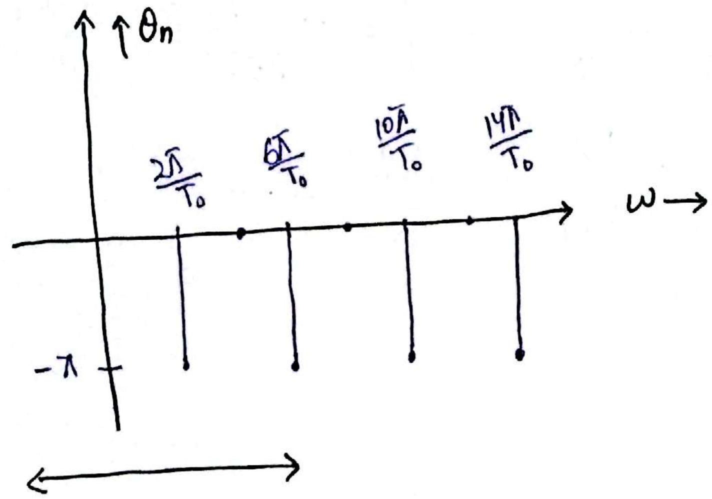
Therefore;

$$x(t) = \frac{A}{2} + \frac{4A}{\pi^2}\cos\left(\frac{2\pi t}{T_0} - \pi\right) + \frac{4A}{9\pi^2}\cos\left(\frac{6\pi t}{T_0} - \pi\right) + \frac{4A}{25\pi^2}\cos\left(\frac{10\pi t}{T_0} - \pi\right) + \frac{4A}{49\pi^2}\cos\left(\frac{14\pi t}{T_0} - \pi\right) + \cdots$$

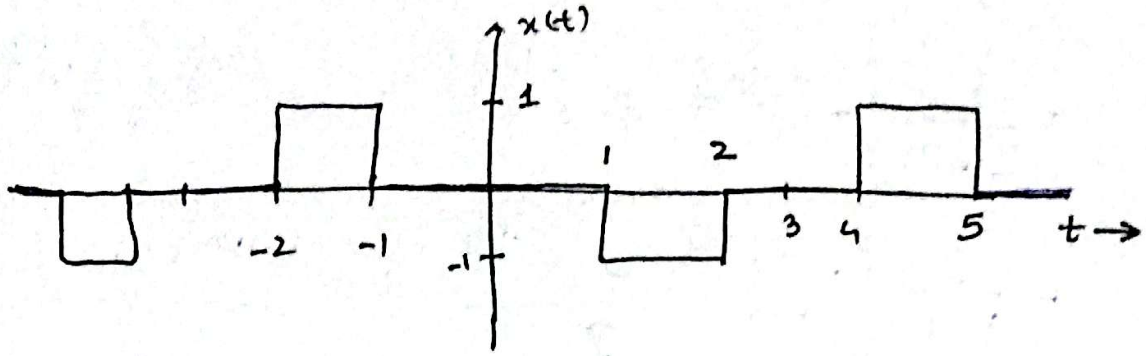
Amplitude: [C_n Vs ω]



Phase Spectrum: [θ_n Vs ω]



Q3 Find Exponetial Fourier Series.



$\rightarrow T_0 = 6$

$\rightarrow \omega_0 = \dfrac{2\pi}{T_0}$

$\omega_0 = \dfrac{\pi}{3}$.

$$\rightarrow x(t) = \sum_{n=-\infty}^{\infty} D_n \, e^{jn\omega_0 t}$$

$$\rightarrow D_n = \frac{1}{T_0} \int_{T_0} x(t)\, e^{-jn\omega_0 t} \, dt.$$

$$D_n = \frac{1}{6} \int_{-2}^{2} x(t)\, e^{-jn\omega_0 t} \, dt$$

$$= \frac{1}{6} \left[\int_{-2}^{-1} 1 \, e^{-jn\frac{\pi}{3}t} \, dt + \int_{-1}^{1} 0 \cdot e^{-jn\frac{\pi}{3}t} + \int_{1}^{2} -1 \cdot e^{-jn\frac{\pi}{3}t} \, dt \right]$$

$$= \frac{1}{6} \left[\frac{e^{-jn\frac{\pi}{3}t}}{-jn\frac{\pi}{3}} \Bigg|_{-2}^{-1} - \frac{e^{-jn\frac{\pi}{3}t}}{-jn\frac{\pi}{3}} \Bigg|_{1}^{2} \right]$$

$$= \frac{1}{6} \left[\frac{-3}{jn\pi} \left(e^{jn\frac{\pi}{3}} - e^{+j2n\frac{\pi}{3}} \right) + \frac{3}{jn\pi} \left(e^{\frac{-jn\pi 2}{3}} - e^{-jn\frac{\pi}{3}} \right) \right]$$

$$D_n = \frac{1}{\cancel{6}_2} \times \frac{\cancel{3}}{jn\pi}\left[-e^{jn\frac{\pi}{3}} + e^{j2n\frac{\pi}{3}} + e^{-j2n\frac{\pi}{3}} - e^{-jn\frac{\pi}{3}}\right]$$

$$= \frac{1}{2jn\pi}\left[-\left(e^{jn\frac{\pi}{3}} + e^{-jn\frac{\pi}{3}}\right) + \left(e^{j2n\frac{\pi}{3}} + e^{-j2n\frac{\pi}{3}}\right)\right]$$

$$= \frac{1}{jn\pi}\left[-\frac{\left(e^{jn\frac{\pi}{3}} + e^{-jn\frac{\pi}{3}}\right)}{2} + \frac{\left(e^{j2n\frac{\pi}{3}} + e^{-j2n\frac{\pi}{3}}\right)}{2}\right]$$

$$D_n = \frac{1}{jn\pi}\left[-\cos n\frac{\pi}{3} + \cos 2n\frac{\pi}{3}\right]$$

$$D_n = \frac{1}{jn\pi}\left[\cos 2n\frac{\pi}{3} - \cos n\frac{\pi}{3}\right].$$

Therefore;

The exponential Fourier Series;

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{jn\pi}\left[\cos 2n\frac{\pi}{3} - \cos n\frac{\pi}{3}\right] e^{jn\frac{\pi}{3}t}$$

$$x(t) = \frac{1}{j\pi} \sum_{n\to-\infty}^{\infty}\left[\frac{1}{n}\left(\cos 2n\frac{\pi}{3} - \cos n\frac{\pi}{3}\right) e^{jn\frac{\pi}{3}t}\right]$$

$$x(t) = \frac{1}{j\pi}\left[\frac{-1}{1} e^{j\frac{\pi}{3}t} + \frac{2}{3} e^{j\frac{3\pi}{3}t} - \frac{1}{5} e^{j\frac{5\pi}{3}t}\right.$$

$$-\frac{1}{-1} e^{-j\frac{\pi}{3}t} + \frac{2}{-3} e^{-j\frac{2\pi}{3}t} - \frac{1}{-5} e^{-j\frac{5}{3}\pi t} + \cdots \Big]$$

$$x(t) = \frac{-1}{j\pi} e^{j\frac{\pi}{3}t} + \frac{2}{j3\pi} e^{j\pi t} - \frac{1}{j5\pi} e^{j\frac{5}{3}\pi t} + \frac{1}{j\pi} e^{-j\frac{\pi}{3}t}$$

$$- \frac{2}{3j\pi} e^{-j\pi t} + \frac{1}{j5\pi} e^{-j\frac{5}{3}\pi t} + \cdots$$

$$x(t) = 0.31\, e^{j\frac{\pi}{2}} e^{j\frac{\pi}{3}t} + 0.0636\, e^{j\frac{\pi}{2}} e^{j\frac{5\pi}{3}t}$$

$$0.212\, e^{-j\pi/2} e^{j\pi t} + 0.31\, e^{-j\frac{\pi}{2}} e^{-j\frac{\pi}{3}t} + 0.212\, e^{j\pi/2} e^{-j\pi t}$$

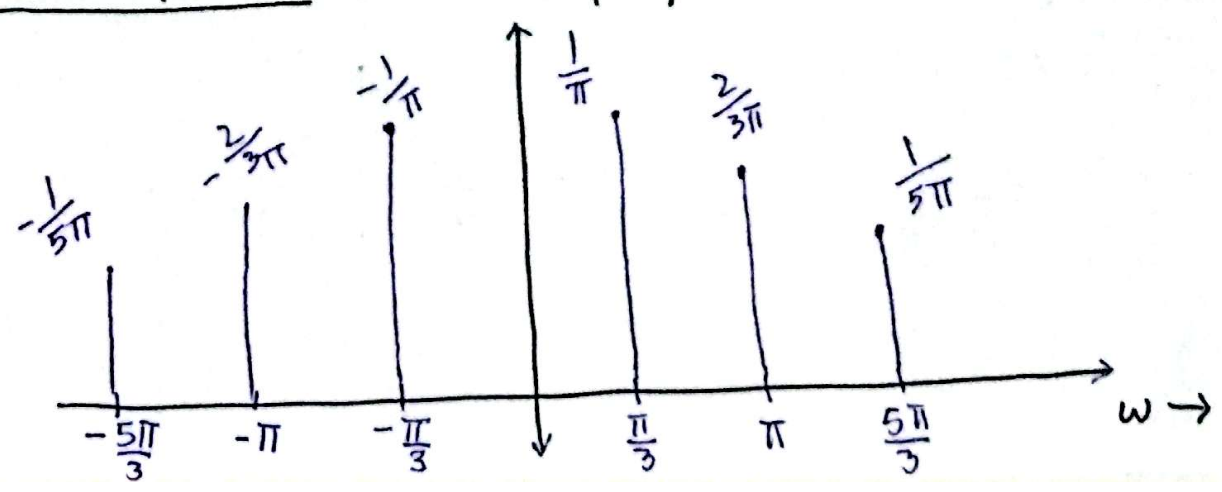
$$+ 0.0636\, e^{-j\pi/2} e^{-j\frac{5\pi}{3}t} + \cdots$$

$$x(t) = \Big[\Big(0.31\, e^{j\pi/2} e^{j\pi/3\, t} + 0.31\, e^{-j\pi/2} e^{-j\frac{\pi}{3}t}\Big) +$$

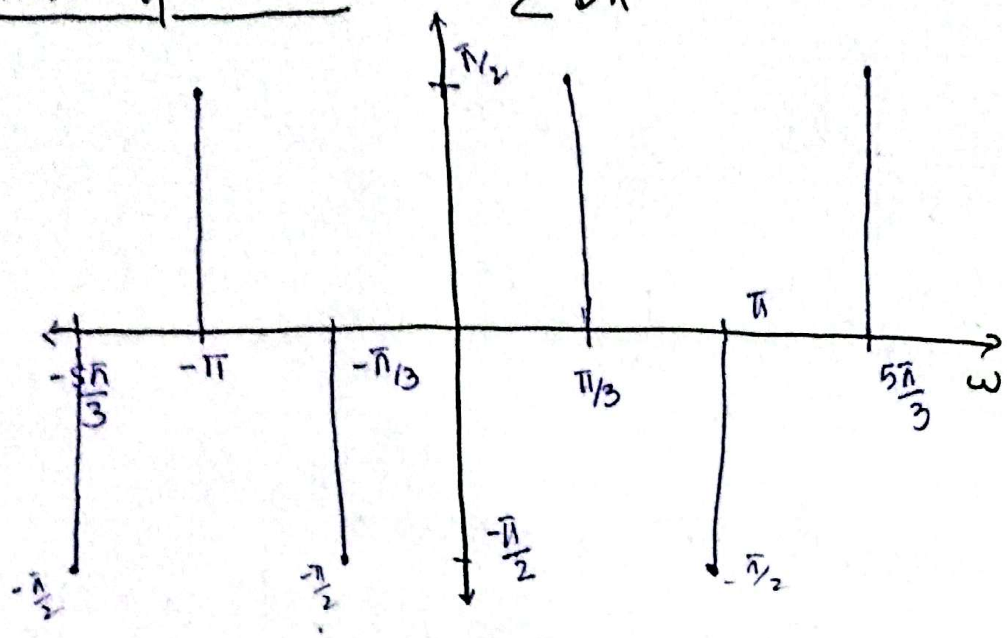
$$\Big(0.0636\, e^{j\pi/2} e^{j\frac{5\pi}{3}t} + 0.0636\, e^{-j\frac{\pi}{2}} e^{-j\frac{5\pi}{3}t}\Big) +$$

$$\Big(0.212\, e^{-j\pi/2} e^{j\pi t} + 0.212\, e^{j\pi/2} e^{-j\pi t}\Big)\Big]$$

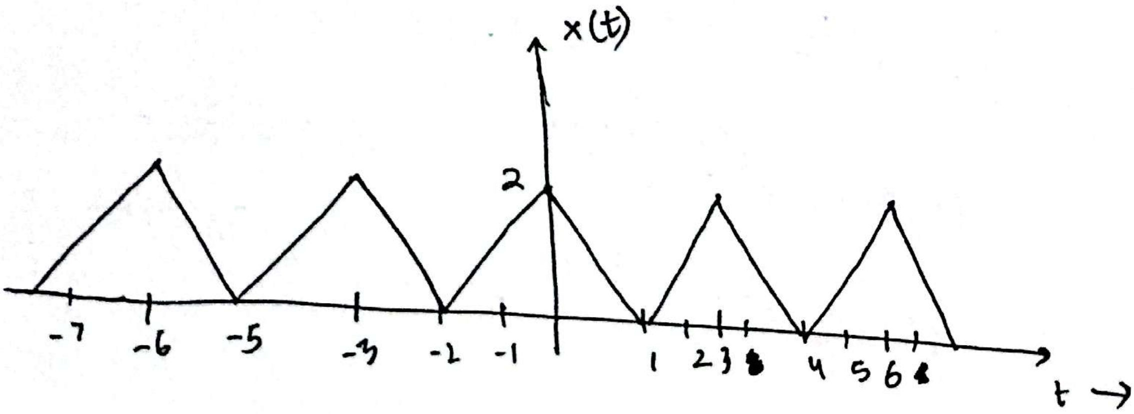
<u>Amplitude Spectrum</u>



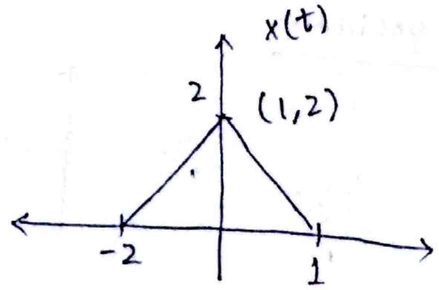
Phase spectrum:



Q4 Find Exponential Fourier Series:



Consider:



$T_0 = 1-(-2) = 3$

$w = \frac{2\pi}{T_0} = \frac{2\pi}{3}$

* Equations of Straight line:

$\rightarrow$ $(-2,0)$ to $(0,2)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-0}{2-0} = \frac{x+2}{0+2}$$

$$\boxed{y = t+2}$$

$\rightarrow$ $(0,2)$ To $(1,0)$

$$\Rightarrow \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y-2}{0-2} = \frac{x-0}{1-0}$$

$$\boxed{y = -2t+2}$$

As:

$$\rightarrow x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$\rightarrow D_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\Rightarrow D_n = \frac{1}{3}\int_{-2}^{1} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$D_n = \frac{1}{3}\left[\int_{-2}^{0}(t+2)\, e^{-jn\frac{2\pi}{3}t}\, dt + \int_{0}^{1}(-2t+2)\, e^{-jn\frac{2\pi}{3}t}\, dt\right]$$

Consider from Eq (1) $\int_{-2}^{0}(t+2)\, e^{-jn\frac{2\pi}{3}t}\, dt$

$$= \int (t+2)\, e^{-jn\frac{2\pi}{3}t}\, dt$$

$$= (t+2)\frac{e^{-jn\frac{2\pi}{3}t}}{-jn\frac{2\pi}{3}} - \int \frac{e^{-jn\frac{2\pi}{3}t}}{-jn\frac{2\pi}{3}}\, dt.$$

$$= (t+2)\left(\frac{-3}{2jn\pi}\right)e^{-jn\frac{2\pi}{3}t} + \frac{3}{2jn\pi}\int e^{-jn\frac{2\pi}{3}t}\,dt$$

$$= (t+2)\left(\frac{-3}{2jn\pi}\right)e^{-jn\frac{2\pi}{3}t} + \frac{3}{2jn\pi}\times\frac{-3}{jn2\pi}\,e^{-jn\frac{2\pi}{3}t}$$

$$= -(t+2)\left(\frac{3}{2jn\pi}\right)e^{-jn\frac{2\pi}{3}t} + \frac{9}{4n^2\pi^2}\times e^{-jn\frac{2\pi}{3}t}$$

Apply limits $(-2 \rightarrow 0)$

$$= -(t+2)\left(\frac{3}{2jn\pi}\right)e^{-jn\frac{2\pi}{3}t} + \frac{9}{4n^2\pi^2}e^{-jn\frac{2\pi}{3}t}\Bigg|_{-2}^{0}$$

$$= \left[-(0+2)\left(\frac{3}{2jn\pi}\right)(1) + \frac{9}{4n^2\pi^2}(1)\right] - \left[-(-2+2)\left(\frac{3}{jn2\pi}\right)e^{jn\frac{4\pi}{3}} - \frac{9}{4n^2\pi^2}e^{jn\frac{4\pi}{3}}\right]$$

$$= \left[\frac{-6}{j2n\pi} + \frac{9}{4n^2\pi^2}\right] - \left[0 + \frac{9}{4n^2\pi^2}e^{jn\frac{4\pi}{3}}\right]$$

$$= \frac{-6}{j2n\pi} + \frac{9}{4n^2\pi^2} - \frac{9}{4n^2\pi^2}e^{jn\frac{4\pi}{3}} \longrightarrow (a)$$

CamScanner

Similary from eq (1)

$$= \int_0^1 (-2t+2)\, e^{jn\frac{2\pi t}{3}}\, dt$$

let:

$$= \int (-2t+2)\, e^{jn\frac{2\pi}{3}t}\, dt$$

$$= (-2t+2)\frac{e^{-jn\frac{2\pi t}{3}}}{jn\frac{2\pi}{3}} - \int \frac{e^{-jn\frac{2\pi}{3}t}}{-jn\frac{2\pi}{3}}(-2)\, dt$$

$$= (-2t+2)\left(\frac{-3}{jn2\pi}\right)e^{-jn\frac{2\pi}{3}t} + \frac{3\times 2}{-jn2\pi} \cdot \frac{3}{-jn2\pi} \times e^{-jn\frac{2\pi}{3}t}$$

$$= (-2t+2)\left(\frac{-3}{jn2\pi}\right)e^{-jn\frac{2\pi}{3}t} + \frac{18}{j^2 4n^2\pi^2}\, e^{-jn\frac{2}{3}\pi t}.$$

$$= -(-2t+2)\left(\frac{3}{jn2\pi}\right)e^{-jn\frac{2\pi}{3}t} - \frac{18}{4n^2\pi^2}\, e^{-jn\frac{2\pi}{3}t}\Bigg|_0^1$$

$$= \left[-(\cancel{-2(1)+2})\left(\frac{3}{jn2\pi}\right)e^{-jn\frac{2\pi}{3}} - \frac{18}{4n^2\pi^2}\, e^{-jn\frac{2\pi}{3}}\right] -$$

$$\left[-(-2(0)+2)\left(\frac{3}{jn2\pi}\right)(1) - \frac{18}{4n^2\pi^2}\times 1\right]$$

$$= \frac{-18}{4n^2\pi^2}\cdot e^{-jn\frac{2\pi}{3}} + \frac{6}{jn2\pi} + \frac{18}{4n^2\pi^2} \longrightarrow (b)$$

Eq (a) & (b) becomes;

$$D_n = \frac{1}{3}\left[\left(\frac{-6}{j2n\pi} + \frac{9}{4n^2\pi^2} - \frac{9}{4n^2\pi^2} e^{jn\frac{4\pi}{3}}\right) + \left(-\frac{18}{4n^2\pi^2} e^{-jn\frac{2\pi}{3}} + \frac{6}{jn2\pi} + \frac{18}{4n^2\pi^2}\right)\right]$$

$$= \frac{1}{3}\left[\cancel{-\frac{6}{j2n\pi}} + \frac{9}{4n^2\pi^2} - \frac{9}{4n^2\pi^2} e^{jn\frac{4\pi}{3}} - \frac{18}{4n^2\pi^2} e^{-jn\frac{2\pi}{3}} + \cancel{\frac{6}{jn2\pi}} + \frac{18}{4n^2\pi^2}\right]$$

$$= \frac{1}{3} \times \frac{9}{4n^2\pi^2}\left[1 - 1e^{jn\frac{4\pi}{3}} - 2e^{-jn\frac{2\pi}{3}} + 2\right]$$

$$= \frac{3}{4n^2\pi^2}\left[3 - e^{jn\frac{4\pi}{3}} - 2e^{-jn\frac{2\pi}{3}}\right]$$

$$D_n = \frac{3}{4n^2\pi^2}\left[3 - \left(\cos\left(\frac{n4\pi}{3}\right) + j\sin\left(\frac{n4\pi}{3}\right)\right) - 2\left(\cos\frac{2n\pi}{3} - j\sin\left(\frac{2n\pi}{3}\right)\right)\right]$$

$n=1$

$$\rightarrow D_1 = \frac{3}{4\pi^2}\left[3 - \left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right) - 2\left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{3}{4\pi^2}\left[\frac{9}{2} + \frac{\sqrt{3}}{2}i\right]$$

$$= \frac{3}{4\pi^2}\left[5.196\, e^{j30}\right] = 0.4\, e^{j30}$$

$\rightarrow n=2$

$$D_2 = \frac{3}{4\times 4\pi^2}\left[3-\left(\frac{-1}{2}+j\frac{\sqrt{3}}{2}\right)-2\left(\frac{-1}{2}+j\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{3}{16\pi^2}\left[5.196\, e^{-j30}\right]$$

$$D_2 = 0.085\, e^{-j30}$$

$\rightarrow n=-1$

$$D_{-1} = \frac{3}{4\pi^2}\left[3-\left(\frac{-1}{2}+j\frac{\sqrt{3}}{2}\right)-2\left(\frac{-1}{2}+j\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{3}{4\pi^2}\left[5.196\, e^{-j30}\right]$$

$$D_{-1} = 0.4\, e^{-j30}$$

$\rightarrow n=-2$

$$D_{-2} = \frac{3}{4\times 4\pi^2}\left[3-\left(\frac{-1}{2}-j\frac{\sqrt{3}}{2}\right)-2\left(\frac{-1}{2}-j\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{3}{16\pi^2}\left[5.196\, e^{+j30}\right]$$

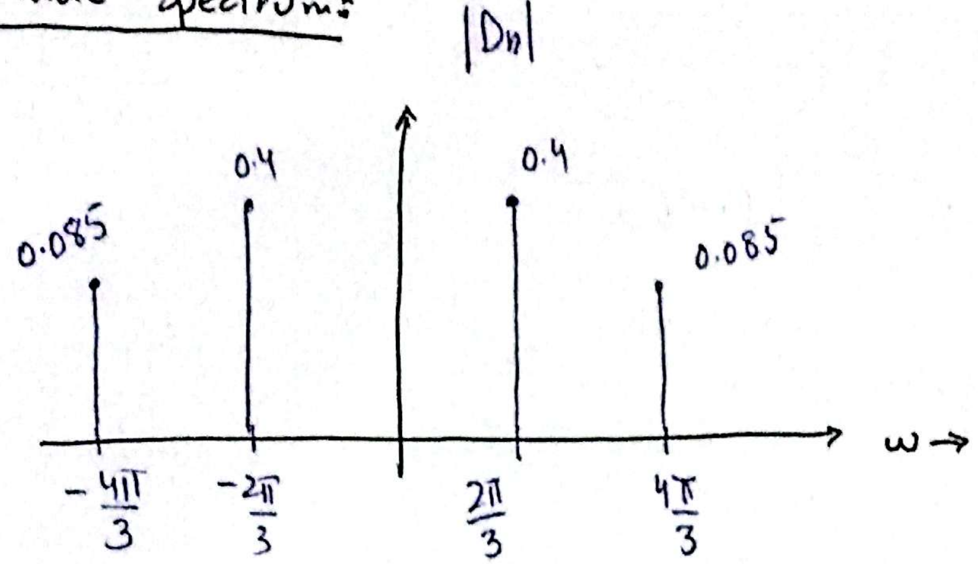
$$D_{2} = 0.085\, e^{+j30}$$

as;

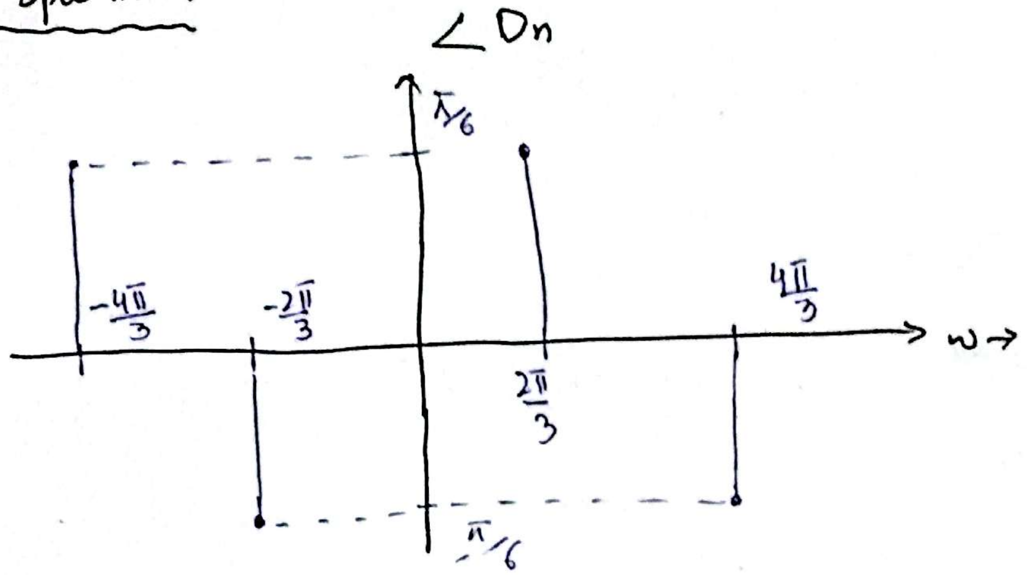
$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$x(t) = \left(0.4e^{j30}e^{j\frac{2\pi}{3}t} + 0.085\, e^{-j30}\, e^{\frac{4\pi}{3}t} + \right.$$
$$\left. 0.4\, e^{-j30}\, e^{-j\frac{2\pi}{3}t} + 0.085\, e^{+j30}\, e^{-j\frac{4\pi}{3}t} + \cdots\right)$$

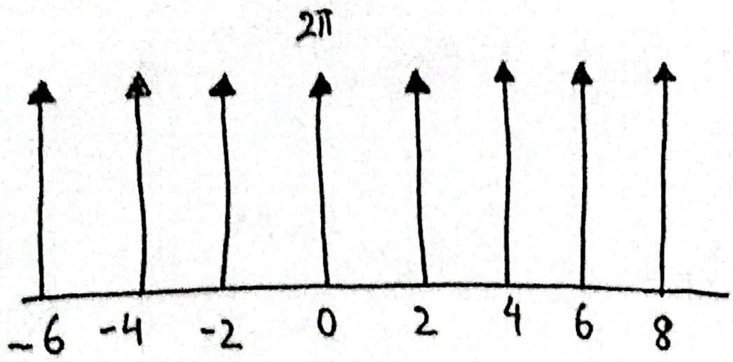
Amplitude Spectrum:



Phase Spectrum:



Q5



→ $T_0 = 2$

→ $w_0 = \frac{2\pi}{T_0}$.

$w_0 = \pi$.

The unit implse train shown can

$$\sum_{n=-\infty} \delta(t - nT_0)$$

where, $T_0 = 2$

The exponential Fourier Series is given by.

$$S_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n \, e^{jnw_0t}$$

where, $\boxed{w_0 = \pi}$

& $D_n = \frac{1}{T_0} \int_{T_0} S_{T_0}(t) \, e^{-jnw_0t} \, dt$

Choosing the interval of integration $(-1, 1)$, and recoganizing that over this interval $S_{T_0}(t) = \delta(t)$.

$$D_n = \frac{1}{2}\int_{-1}^{1} \delta(t)\, e^{-jn\pi t}\, dt$$

$$D_n = \frac{1}{2}\int_{-1}^{1} \delta(t)\, e^{-jn\pi t}\, dt$$

In this integral, the impluse is located at $\boxed{t=0}$; the value at that instant is $\boxed{2\pi.}$

Therefore; $D_n = \frac{1}{2}\int_{-1}^{1} \delta(t)\, e^{-jn\pi t}\, dt$

$$D_n = \frac{1}{2} \times 2\pi$$

$$\boxed{D_n = \pi}$$

As; $S_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n\, e^{jn\omega_0 t}$

$$S_{T_0}(t) = \sum_{n=-\infty}^{\infty} \pi\, e^{jn\omega_0 t}$$

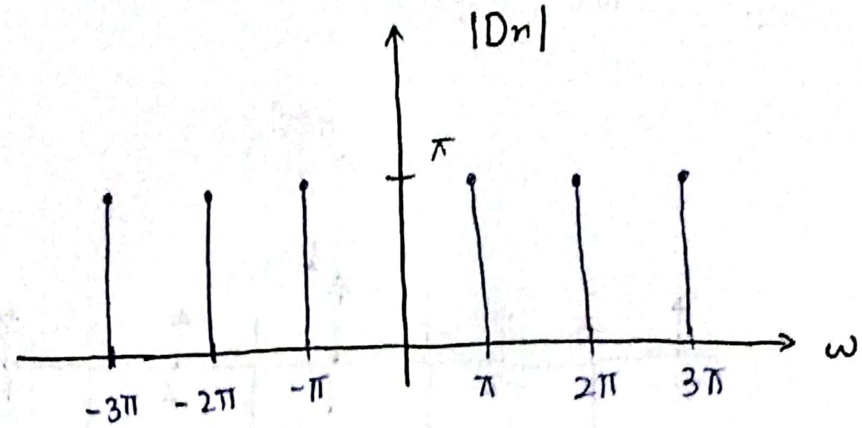
$$S_{T_0}(t) = \pi \sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}$$

$$= \pi \sum_{n=-\infty}^{\infty} e^{jn\pi t}$$

$$S_{T_0}(t) = \pi \left\{ e^{j\pi t} + e^{j2\pi t} + e^{j3\pi t} + e^{-j\pi t} + e^{-j2\pi t} + e^{-j3\pi t} + \dots \right\}$$

CamScanner

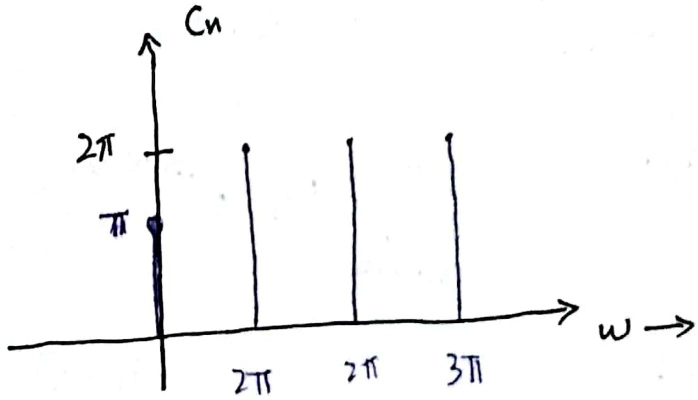
Spectra for Impluse Train:



AS;

→ $C_n = 2|D_n|$

→ $C_0 = D_0 = \pi$

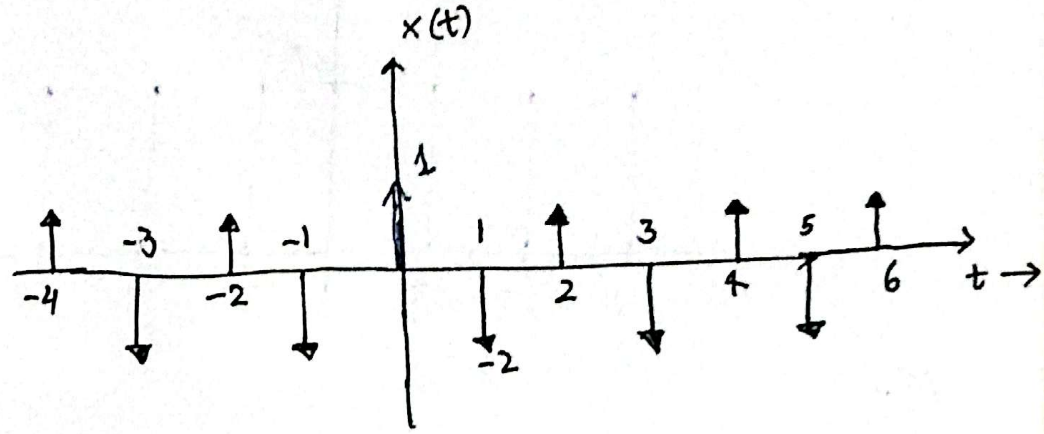


Compact Trignometric Fourier Series:

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n \omega_0 t + \theta_n);$$

$$x(t) = \pi + 2\pi \cos(\pi t) + 2\pi \cos(2\pi t) + 2\pi \cos(3\pi t) + \cdots$$

Q6 Find exponential Fourier Series.



→ $x(t) = 0$ at $t = 0$

→ $x(t) = -2$ at $t = 1$

AS;

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

Where;

$$D_n = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-jn\omega_0 t} \, dt$$

→ $\boxed{T_0 = 2}$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$\boxed{\omega_0 = \pi}$$

$$D_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$D_n = \frac{1}{2}\left[\int \delta(t)\, e^{-jn\omega_0 t}\Big|_{t=0} + \int -2\cdot\delta(t-1)\, e^{-jn\pi t}\Big|_{t=1}\right]dt$$

$$= \frac{1}{2}\left[1 \times e^{-jn\pi(0)} - 2\times 1\, e^{-jn\pi(1)}\right]$$

$$= \frac{1}{2}\left[1 - 2e^{-jn\pi}\right]$$

$$D_n = \frac{1}{2}\left[1 - 2\left[\cos(n\pi) - j\sin(n\pi)\right]\right]$$

$$= \frac{1}{2}\left[1 - 2\cos(n\pi) + 2j\sin(n\pi)\right]$$

$$= \frac{1}{2}\left[1 - 2\cos n\pi\right]$$

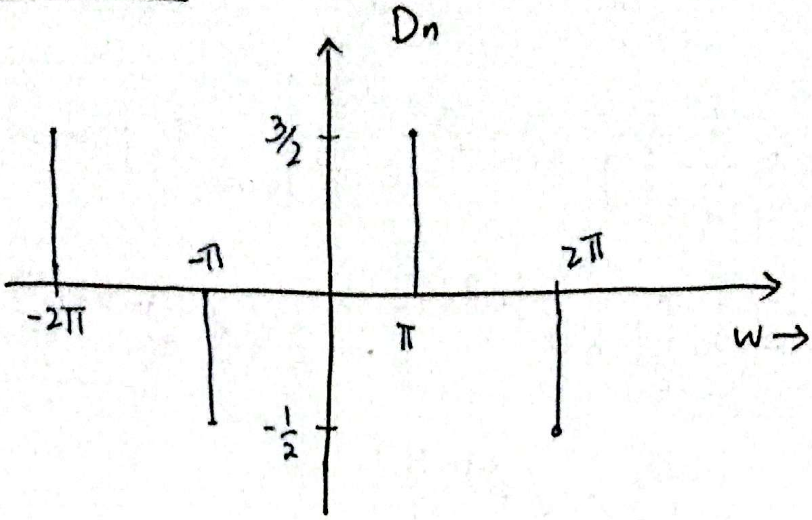
as: $$x(t) = \sum_{n=-\infty}^{\infty} D_n\, e^{jn\omega_0 t}$$

$$x(t) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(1 - 2\cos(n\pi)\right)e^{jn\pi t})$$

$$= \frac{1}{2}\left[3e^{j\pi t} - 1\,e^{j2\pi t} + 3e^{-j\pi t} - 1\,e^{-j2\pi t} + \cdots\right]$$

$$= \left[\frac{-1}{2}\left(e^{j2\pi t} + e^{-j2\pi t}\right) + \frac{3}{2}\left(e^{j\pi t} + e^{-j\pi t}\right) + \cdots\right]$$

Amplitude Spectrum:



Q7 State with reasons whether the signal are periodic or aperiodic. For periodic signal, find the period & state which of the harmonics are present in series.

a) $2\sin(3t) + 7\cos(\pi t)$.

let: $\boxed{\omega_1 = 3}$, $\boxed{\omega_2 = \pi}$

• Time period:

$$\underbrace{2\sin(3t)}_{x_1(t)} + \underbrace{7\cos(\pi t)}_{x_2(t)}$$

$$\omega_1 = \frac{2\pi}{T_1}$$

$$T_1 = \frac{2\pi}{\omega_1}$$

$$\boxed{T_1 = \frac{2\pi}{3}}$$

Similarly,

$$\omega_2 = \frac{2\pi}{T_2}$$

$$T_2 = \frac{2\pi}{\omega_2}$$

$$T_2 = \frac{2\pi}{\pi}$$

$$\boxed{T_2 = 2}$$

Taking Ratio of Time period.

$$\frac{T_1}{T_2} = \frac{2\pi/3}{2}$$

$$\frac{T_1}{T_2} = 1.047$$

So, the Ratio is irrational, therefar are signal is not periodic.

b) $7\cos(\pi t) + 5\sin(2\pi t)$

$\omega_1 = \pi$, $\omega_2 = 2\pi$

$$\underbrace{7\cos(\pi t)}_{x_1(t)} + \underbrace{5\sin(2\pi t)}_{x_2(t)}$$

1) Time Period:

- $\omega_1 = \frac{2\pi}{T_1}$

$T_1 = 2\pi/\pi$

$\boxed{T_1 = 2}$

- $\omega_2 = \frac{2\pi}{T_2}$

$T_2 = \frac{2\pi}{2\pi}$

$\boxed{T_2 = 1}$

2) Taking Ratio:

$\frac{T_2}{T_1} = \frac{2}{1}$, Ratio is in Rational, Signal is Perodic Signal.

$m = 2, \; n = 1$

Fundamental Period:

$\rightarrow nT_1 = 1 \times 2 = 2$

$\rightarrow mT_2 = 2 \times 1 = 2$

The fundamental period is 2.

c) $3\cos(\sqrt{2}t) + 5\cos(2t)$

$\omega_1 = \sqrt{2}$, $\omega_2 = 2$.

• Time Period:

$\rightarrow \omega_1 = \frac{2\pi}{T_1}$

$T_1 = \frac{2\pi}{\omega_1}$

$\boxed{T_1 = \frac{2\pi}{\sqrt{2}}}$

$\rightarrow \omega_2 = \frac{2\pi}{T_2}$

$T_2 = \frac{2\pi}{2}$

$\boxed{T_2 = \pi}$

• Ratio:

$\frac{T_1}{T_2} = \frac{2\pi/\sqrt{2}}{\pi}$

$= 1.4142.$

As, Ratio is not in rational, therefore the signal is not periodic.

d) $\sin\left(\frac{5t}{2}\right) + 3\cos\left(\frac{6t}{2}\right) + 3\sin\left(\frac{t}{7} + 30\right)$

Let $\omega_1 = \frac{5}{2}$, $\omega_2 = \frac{6}{2} = 3$, $\omega_3 = \frac{1}{7}$

• Time Period:

$\omega_1 = \frac{2\pi}{T_1}$

$\boxed{T_1 = \frac{4\pi}{5}}$

$\omega_2 = \frac{2\pi}{T_2}$

$\boxed{T_2 = \frac{2\pi}{3}}$

$\omega_3 = \frac{2\pi}{T_3}$

$\boxed{T_3 = 14\pi}$

• Taking Ratio:

$\rightarrow \frac{T_1}{T_2} = \frac{4\pi/5}{2\pi/3} = \frac{6}{5} = \frac{m}{n}$

The signal $\sin\left(\frac{5t}{2}\right) + 3\cos\left(\frac{6t}{2}\right)$ is periodic bcz ratio of their is in rational form.

Fundamental Period:

$\rightarrow n \times T_1 = 5 \times \frac{4\pi}{5} = 4\pi$

$\rightarrow m \times T_2 = 6 \times \frac{2\pi}{3} = 4\pi$

{even Period}

CamScanner

...fore; $T_F = 4\pi$ (fundamental period)

Now;

Taking Ratio of T_F to T_3

$$\frac{T_F}{T_3} = \frac{4\pi}{14\pi} = \frac{4}{14} = \frac{2}{7}.$$

As the Ratio is in Rational form, therefore the signal

$$\Rightarrow \left(\sin\left(\frac{5t}{2}\right) + 3\cos\left(\frac{6t}{2}\right)\right) + 3\sin\left(\frac{t}{7} + 30\right) \text{ is}$$

periodic.

Fundamental Period:

$\rightarrow n \times T_F = 7 \times 4\pi = 28\pi$

$\rightarrow m \times T_3 = 2 \times 14\pi = 28\pi$

It is also an even period.

Question 8

$$f(t) = 3 + \sqrt{3}\cos 2t + \sin 2t + \sin\left(5t - \frac{\pi}{6}\right) - \frac{1}{2}\cos\left(7t + \frac{\pi}{3}\right).$$

ⓐ **Amplitude & phase Spectra of Compact Trig. Series:**

$\because \sin Kt = \cos\left(Kt - \frac{\pi}{2}\right)$

$\because -\cos\theta = \cos(\theta - \pi)$

$x(t) = 2 + \frac{4}{2}\left[e^{jt} \cdot e^{j\frac{2\pi}{3}} + e \ldots\right.$

amplitude

$$f(t) = 3 + \sqrt{3}\cos 2t + \sin 2t + \sin\left(5t - \frac{\pi}{6}\right) - \frac{1}{2}\cos\left(7t + \frac{\pi}{3}\right)$$

$$= 3 + (\sqrt{3}\cos 2t + \sin 2t) + \cos\left(5t - \frac{\pi}{6} - \frac{\pi}{2}\right) + \frac{1}{2}\cos\left(7t + \frac{\pi}{3} - \pi\right)$$

$$= 3 + (\sqrt{3}\cos 2t + \sin 2t) + \cos\left(5t - \frac{2\pi}{3}\right) + \frac{1}{2}\cos\left(7t \mp \frac{2\pi}{3}\right)$$

$\hookrightarrow (1)$

Consider Eq. (1)

$$\sqrt{3}\cos 2t + \sin 2t$$

$\rightarrow a_n = \sqrt{3}$

$\rightarrow b_n = 1$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n w_0 t + \theta_n)$$

$\rightarrow C_n = \sqrt{a_n^2 + b_n^2}$

$= \sqrt{(\sqrt{3})^2 + 1}$

$\boxed{C_n = 2}$, $\boxed{w_0 = 2}$

$\rightarrow \theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right)$

$= \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$\theta_n = -\frac{\pi}{6}$

Therefore: $\sqrt{3}\cos 2t + \sin 2t = C_n \cos(w_0 t + \theta_n)$

$$= 2\cos\left(2t + -\frac{\pi}{6}\right)$$

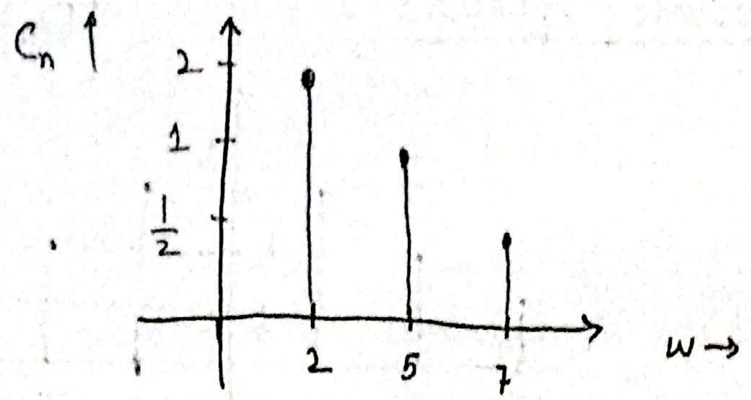
$$= 2\cos\left(2t - \frac{\pi}{6}\right)$$

Hence Eq (1) becomes,

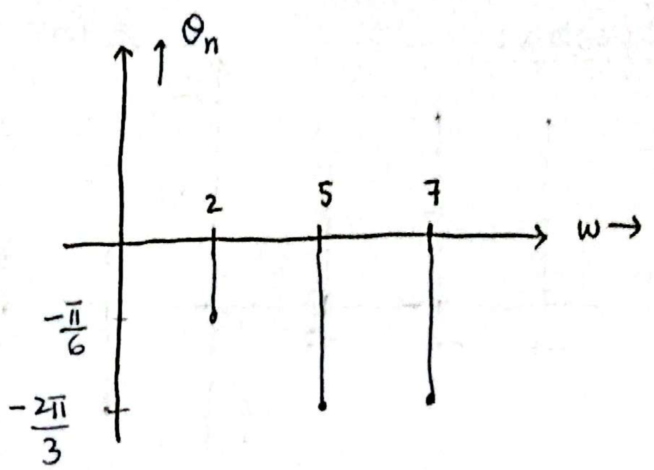
$$f(t) = 3 + 2\cos\left(2t - \frac{\pi}{6}\right) + \cos\left(5t - \frac{2\pi}{3}\right) + \frac{1}{2}\cos\left(7t - \frac{2\pi}{3}\right)$$

$\hookrightarrow$ compact Trignometric series.

Spectrum $[C_n \text{ vs } \omega]$



Phase Spectrum: $[\theta_n \text{ vs } \omega]$.



(b) Exponential fourier series spectra from part (a)

$$\rightarrow x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} dt$$

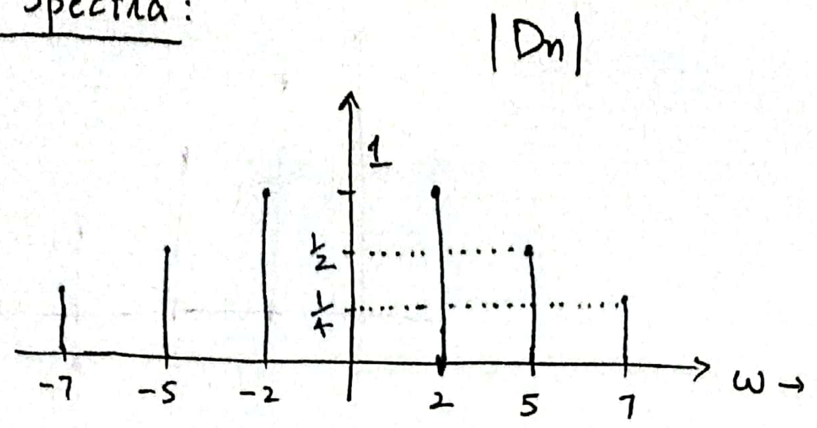
$$\rightarrow D_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} dt$$

•) $|D_n| = |D_{-n}| = \frac{C_n}{2}$

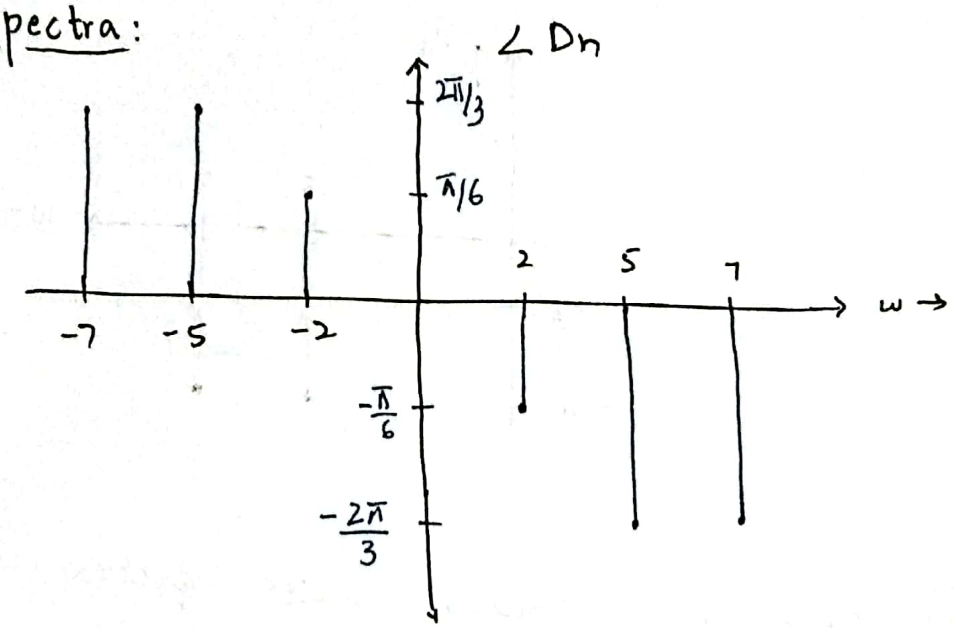
•) $\angle D_n = \theta_n$ for $+n$

$\angle D_n = -\theta_n$ for $-n$

Amplitude Spectra:



Phase Spectra:



© **Exponential Fourier Series:**

As,

$$X(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \, dt .$$

From part (b)

$$x(t) = 3 + 1\left[e^{-j\pi/6} e^{j2t} + e^{+j\pi/6} e^{-j2t}\right]$$
$$+ \frac{1}{2}\left[e^{-j\frac{2\pi}{3}} e^{+j5t} + e^{+j\frac{2\pi}{3}} e^{-j5t}\right]$$
$$+ \frac{1}{4}\left[e^{-j\frac{2\pi}{3}} e^{+j7t} + e^{j\frac{2\pi}{3}} e^{-j7t}\right]$$

$$x(t) = 3 + \left[e^{j(2t - \frac{\pi}{6})} + e^{-j(2t - \frac{\pi}{6})}\right] + \frac{1}{2}\left[e^{j(5t - \frac{2\pi}{3})} + e^{-j(5t - \frac{2\pi}{3})}\right]$$

$$+ \frac{1}{4}\left[e^{j(7t - \frac{2\pi}{3})} + e^{-j(7t - \frac{2\pi}{3})}\right].$$

(d) Calculation in part c is equal to given $f(t)$.

from part (c).

$$x(t) = 3 + \left[e^{j(2t - \frac{\pi}{6})} + e^{-j(2t - \frac{\pi}{6})}\right] + \frac{1}{2}\left[e^{j(5t - \frac{2\pi}{3})} + e^{-j(5t - \frac{2\pi}{3})}\right]$$

$$+ \frac{1}{4}\left[e^{j(7t - \frac{2\pi}{3})} + e^{-j(7t - \frac{2\pi}{3})}\right]$$

Multiplying above $x(t)$ by 2 and also dividing it by 2.

$$x(t) = 3 + 2\left[\frac{e^{j(2t - \frac{\pi}{6})} + e^{-j(2t - \frac{\pi}{6})}}{2}\right] + \frac{2}{2}\left[\frac{e^{j(5t - \frac{2\pi}{3})} + e^{-j(5t - \frac{2\pi}{3})}}{2}\right]$$

$$+ \frac{2}{4}\left[\frac{e^{j(7t - \frac{2\pi}{3})} + e^{-j(7t - \frac{2\pi}{3})}}{2}\right]$$

$$x(t) = 3 + 2\left[\cos\left(2t - \frac{\pi}{6}\right) + 1 \cos\left(5t - \frac{2\pi}{3}\right)\right] + \frac{1}{2}\cos\left(7t - \frac{2\pi}{3}\right).$$

Hence, calculation in part c is equivalent to given $f(t)$.

Q9

$$x(t) = (2+j2)e^{-j3t} + j2e^{-jt} + 3 - j2e^{jt} + (2-j2)e^{j3t}$$

b)

a) Exponential Fourier Spectrum:-

$$x(t) = 3 + 2.82\,e^{j\pi/4}\,e^{-j3t} + 2\,e^{j\pi/2}.e^{-jt} + 2\,e^{-j\pi/2}.e^{jt} +$$

$$2.82\,e^{-j\pi/4}.\,e^{j3t}.$$

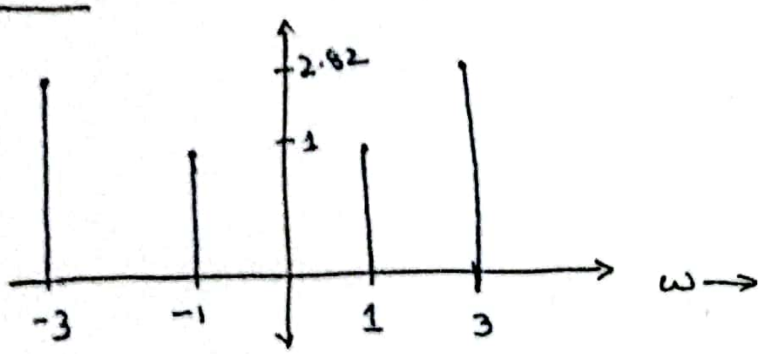
$\therefore\ |2 \pm 2j| = 2\sqrt{2}$

$\tan^{-1}\left(\pm\frac{2}{2}\right) = \pm 45^\circ$

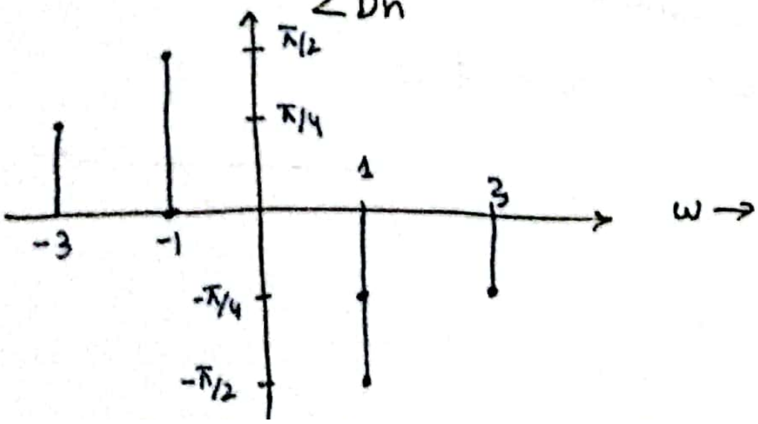
$\therefore\ |\pm j2| = 2$

$\theta = \pm\frac{\pi}{2}.$

Amplitude Spectrum: $|D_n|$



Phase Spectra: $\angle D_n$

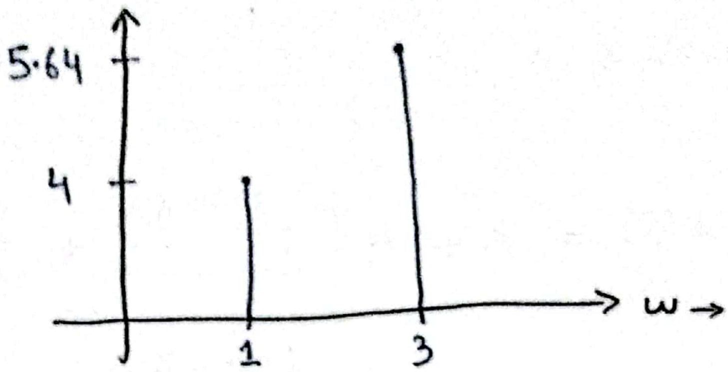


b) Trignometric Fourier Spectra from part (a)

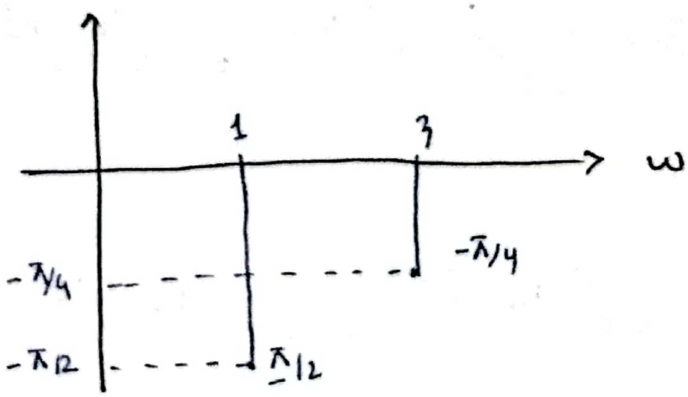
$\rightarrow C_n = 2|D_n|$

$\rightarrow +\theta_n = \angle D_n$ for $+n$.

Amplitude Spectrum: $[C_n \text{ vs } \omega]$



Phase Spectra: $[\theta_n \text{ Vs } \omega]$



Compact Trig. F.S.:

$$x(t) = 3 + 4\cos\left(1t - \frac{\pi}{2}\right) + 5.64\cos\left(3t - \frac{\pi}{4}\right).$$

c) Calculation in part (b) is equivalent to Exponential fourier Series.

→ from part (b).

$$x(t) = 3 + 4\cos\left(t - \frac{\pi}{2}\right) + 5.64\cos\left(3t - \frac{\pi}{4}\right).$$

As

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\Rightarrow x(t) = 3 + 4\left[\frac{e^{j(t-\frac{\pi}{2})} + e^{-j(t-\frac{\pi}{2})}}{2}\right] + 5.64\left[\frac{e^{j(3t-\pi/4)} + e^{-j(3t-\pi/4)}}{2}\right]$$

$$x(t) = 3 + \frac{4}{2}\left[e^{jt}e^{-j\pi/2} + e^{-jt}e^{+j\pi/2}\right] + \frac{5.64}{2}\left[e^{j3t}e^{-j\pi/4} + e^{-j3t}e^{j\frac{\pi}{4}}\right]$$

$$x(t) = 3 + 2e^{-j\pi/2}e^{jt} + 2e^{j\pi/2}e^{-jt} + 2.82\,e^{-j\pi/4}e^{j3t} + 2.82\,e^{+j\pi/4}e^{-j3t}.$$

hence, calculations in part c equivalent to exponential fourier Series.

(d) Signal band width.

The bandwidth of signal is defined as the diffeence b/w upper and Lower frequancies.

$$\text{Bandwidth} = \omega_u - \omega_l$$

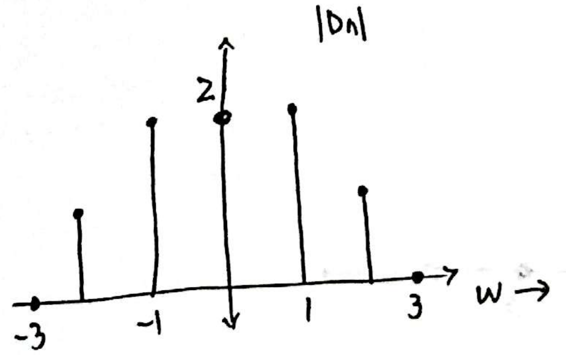
$\omega_u = 3$ (upper frequancy)

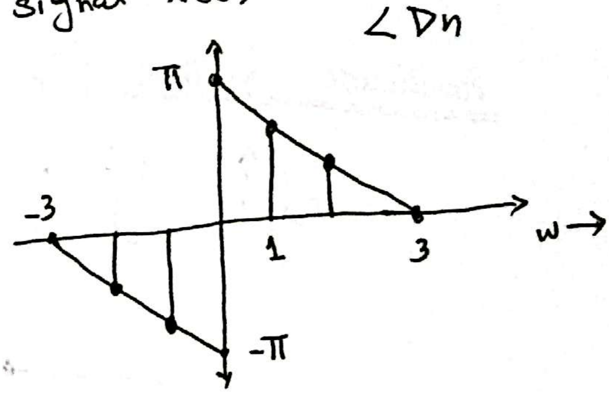
$\omega_l = 0$ (lower frequancy)

$$\text{Bandwidth} = 3 - 0$$

$$\boxed{\text{Bandwidth} = 3}$$

Q10 The Figure below shows the exponential Fourier spectra of periodic signal $x(t)$.





a) Exponential fourier Series:

⇒ Signal has special spectral components at frequancies ±1 and ±2 and ±3. The spectral components 3 is ignored bcz it is 0.

DC component is $D_0 = 2$.

The phase of the spectral component at frequencies

•) 1 is $\left(+\frac{2\pi}{3}\right)$, −1 is $\left(-\frac{2\pi}{3}\right)$

•) 2 is $\left(\pi/3\right)$, −2 is $\left(-\pi/3\right)$

± 1 and ± 2 are the frequancies at which there are exponential spectral components.

c)

Therefore;

By inspection, the Exponential Fourier Series $x(t)$ is ;

$$x(t) = 2 + \left[2e^{j\frac{2\pi}{3}} e^{+jt} + 2e^{-j\frac{2\pi}{3}} e^{-jt}\right] + \left[1 e^{j\frac{\pi}{3}} e^{+j2t} + 1 e^{-j\frac{\pi}{3}} e^{-j2t}\right].$$

$$x(t) = 2 + 2\left[e^{j(t+\frac{2\pi}{3})} + e^{-j(t+\frac{2\pi}{3})}\right] + 1\left[e^{j(2t+\frac{\pi}{3})} + e^{-j(2t+\frac{\pi}{3})}\right].$$

b) Trignometric Fourier spectra:

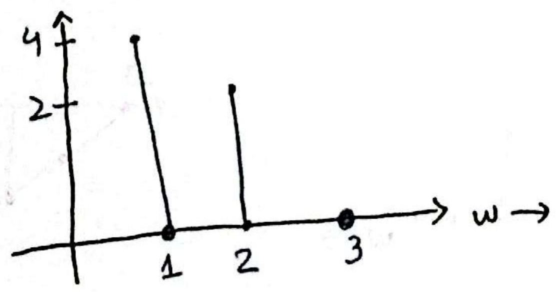
$$|D_n| = |D_{-n}| = \frac{C_n}{2}$$

$$\Rightarrow C_n = 2|D_n|$$

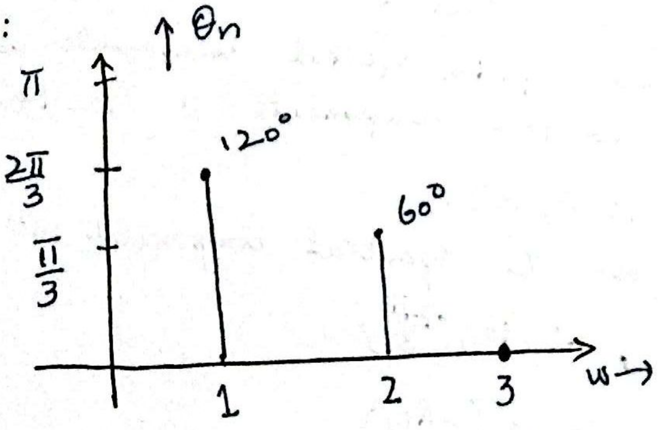
and

$$\Rightarrow \theta_n = \angle D_n \quad \text{for } +n.$$

Amplitude Spectrum: $[C_n \text{ vs } \omega]$



Phase Spectra:



c) Compact Trignometric Fourier Series $x(t)$:

→ The signal has three spectral components at frequancies 1, 2, amd 3.

The spectral components at 3 is ignored bcz it is zero.

- The phase of components at frequancy 1 is $\frac{2\pi}{3}$ and 2 is $\frac{\pi}{3}$.

- The amplitudas of components at frequancy 1 is (4) and at frequancy 2 is (2).

Therefore;

$$\rightarrow x(t) = 2 + 4\cos\left(t + \frac{2\pi}{3}\right) + 2\cos\left(2t + \frac{\pi}{3}\right)$$

d) Series found in part (a) & (c) are equivalent

from (c).

$$x(t) = 2 + 4\cos\left(t + \frac{2\pi}{3}\right) + 2\cos\left(2t + \frac{\pi}{3}\right)$$

$$\therefore \cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\Rightarrow x(t) = 2 + 4\left[\frac{e^{j\left(t+\frac{2\pi}{3}\right)} + e^{-j\left(t+\frac{2\pi}{3}\right)}}{2}\right]$$

$$+ 2\left[\frac{e^{j\left(2t+\frac{\pi}{3}\right)} + e^{-j\left(2t+\frac{\pi}{3}\right)}}{2}\right]$$

$$x(t) = 2 + \frac{4}{2}\left[e^{jt} \cdot e^{j\frac{2\pi}{3}} + e^{-jt} \cdot e^{-j\frac{2\pi}{3}}\right] + \frac{2}{2}\left[e^{j2t}\, e^{j\pi/3} + e^{-j2t}\, e^{-j\frac{\pi}{3}}\right]$$

$$x(t) = 2 + 2e^{j\frac{2\pi}{3}} e^{jt} + 2e^{-j\frac{2\pi}{3}} e^{-jt} + e^{j2t} \cdot e^{j\frac{\pi}{3}}$$

$$+ e^{-j\frac{\pi}{3}} e^{-j2t}.$$

hence, series found in part (a) & (c) are equivalent.