

Task 1: Fibonacci Series:-

1) Simple Iterative

Dry Run:- For $n=5$

- Initially $a=0, b=1$
- Step 1: $a=1, b=1$
- Step 2: $a=1, b=2$
- Step 3: $a=2, b=3$
- Step 4: $a=3, b=5$
- Return 5.

Time Complexity:-

$O(n)$: We iterate through loop $n-1$ times, making the approach linear.

2) Simple Recursive

Dry Run:- For $n=5$

- $\text{fabo-recursive} = \text{fabo-recursive}(4) + \text{fabo-recursive}(3)$
- $\text{fabo-recursive} = \text{fabo-recursive}(3) + \text{fabo-recursive}(2)$
- Repeating until base case reached.

Time Complexity:-

- $O(2^n)$: Each function call spawns two more calls, resulting in an exponentially time complexity.

3) Dp-Memoization:-

Dry Run:- For $n=5$

- Calls $\text{fabo-memo}(5) \rightarrow$ checks memo, not found

- Calls `fibonacci(1)` → checks memo, not found
- Similarly for 3, 2 and 1. Once calculated, result store in memo.

Time Complexity:-

$O(n)$: Each fibo number is calculated once and then stored in the memo dictionary.

DP-Tabulation:-

Dry Run:- For $n=5$.

- $dp = [0, 1, 0, 0, 0, 0]$
- After step 1: $dp = [0, 1, 1, 0, 0, 0]$
- After step 2: $dp = [0, 1, 1, 2, 0, 0]$
- After step 3: $dp = [0, 1, 1, 2, 3, 0]$
- After step 4: $dp = [0, 1, 1, 2, 3, 5]$

Time Complexity:-

$O(n)$: We build a table dp from 0 to n , each value is compared in constant time.

Task 2: Minimum Coin Change:-

Simple Iterative:-

Dry Run:- For $coins = [1, 3, 4]$, $amount = 6$:

- Coin 4: $6 - 4 = 2$
- Coin 1: $2 - 1 = 1$, $1 - 1 = 0$
- Return 3 (3 coins used)

Time Complexity:

$O(\log n)$: Sorting the coins array takes $O(\log n)$.
• In worst case, it goes through all the coins for each denomination.

2) Simple Recursive:

Dry Run: For coins = [1, 3, 4], amount = 6.

min_coins_recursive(6) checks 3 branches.

• For 4, min_coins_recursive(2) → leads to using 1 and 1.

• For 3, min_coins_recursive(3) → similar process.

Time Complexity:

$O(2^n)$: The recursive tree grows exponentially since each call spawns multiple recursive calls.

3) Dp-Memoization:

Dry Run: For coins = [1, 3, 4], amount = 6.

• Calls min_coins_memoization(6) and stores results for smaller amounts in the memo, avoiding redundant recalculations.

Time Complexity:

$O(n * \text{amount})$: Each subproblem is calculated only once and stored in the memo dictionary.

4) Dp-Tabulation:-

Dry Run:- For coins = [1, 3, 4], amount = 6.

- $dp = [0, \text{inf}, \text{inf}, \text{inf}, \text{inf}, \text{inf}, \text{inf}]$
- For amount 1 using coin 1 $dp = [0, 1, \text{inf}, \text{inf}, \text{inf}, \text{inf}, \text{inf}]$
- For amount 2 using coin 1 $dp = [0, 1, 2, \text{inf}, \text{inf}, \text{inf}, \text{inf}]$
- For amount 3 using coin 3 $dp = [0, 1, 2, 1, \text{inf}, \text{inf}, \text{inf}]$
- Continue until $dp[6] = 2$.

Time Complexity:-

$O(n * \text{amount})$: We iterate through each coin for each amount, which gives us a time complexity of $O(n * \text{amount})$.

Task 3:- Longest Common Subsequence:-

1) Simple Iterative:-

This is not possible we need to check all possible subsequence, so we neglect it.

2) Simple Recursive:-

Dry Run:-

For $str1 = "ABC"$, $str2 = "AC"$, $m = 3$, $n = 2$.

- $lcs_recursive(3, 2)$ compares c and $c \rightarrow 1 + lcs_recursive(2, 1)$.

Time Complexity:-

$O(2^n)$: Each comparison branches into two recursive calls, leading to an exponential growth.

3) Dp-Memoization:-

Dry Run

For str1="ABC", str2="AC", m=3, n=2

calls `memo(3,2)` → check memo, avoids recalculating and stores values for each subproblem.

Time Complexity:-

Obv: Each subproblem (for every (m,n)) is calculated only once and stored in the memo.

4) Dp-Tabulation:-

Dry Run:-

For str1="ABC", str2="AC"

Initially `dp = [[0,0,0], [0,0,0], [0,0,0], [0,0,0]]`

After comparing each character pair and filling in table `dp[3][2] = 2` (the length of LCS is 2)

Time Complexity:-

$O(m*n)$: We iterate through all possible pairs of characters in str1 and str2, resulting in $O(m*n)$ time complexity.

Task 4: Climbing Stairs

Dry Run (For $n=5$)

Simple Iterative

- Initially $a=1, b=1$
- Step 1: $a=1, b=2$
- Step 2: $a=2, b=3$
- Step 3: $a=3, b=5$
- Step 4: $a=5, b=8$
- Return 8 (8 distinct way to climb stairs)

Time Complexity:-

$O(n)$: We compute each step's value once, resulting in linear.

2) Recursive:-

Dry Run For $n=5$

- $\text{climb_recur}(5) = \text{climb_recur}(4) + \text{climb_recur}(3)$
- $\text{climb_recur}(4) = \text{climb_recur}(3) + \text{climb_recur}(2)$
- Repeating this until base case.

Time Complexity:-

$O(2^n)$: Each function call splits into two recursive calls, leading exponential time.

3) Dp-memoization:-

Dry Run For $n=5$:

- $\text{climb_memo}(5)$ compute and store values for 4, 3, 2, and so on, avoiding recomputation.

Time Complexity:-

(n): Each subproblem is solved once, the result is stored in memo.

4) Dp-Tabulation

Dry Run:- For $n=5$

- $dp = [1, 1, 0, 0, 0, 0]$
- After step 2 $dp = [1, 1, 2, 0, 0, 0]$
- After // 3 $dp = [1, 1, 2, 3, 0, 0]$
- After // 4 $dp = [1, 1, 2, 3, 5, 0]$
- After // 5 $dp = [1, 1, 2, 3, 5, 8]$
- Return 8 (8 distinct ways to climb 5 stairs)

Time Complexity:-

(n): We return n from 2 to n; filling in table with number of ways to reach each step.

Task 5:- Knapsack Problem

1) Simple Iterative:-

Greedy approach may fail to produce optimal results.

2) Recursive:-

Dry Run:-

For weights = $[1, 3, 4, 5]$, values = $[1, 4, 5, 7]$, $W=7$, $n=4$:

knap_recur(7, 4) splits into include and exclude 4th item and so on recursively.

Time Complexity:-

(2^n) For every item, it creates branches leads to exponential.

3) Dp-Memoization:-

Dry Run:-

For weights = [1, 3, 4, 5], values = [1, 4, 5, 7], $w=7$, $n=4$
• $\text{comp_memo}(7, 4)$ computes and stores for different weights and items indices in the memo

Time Complexity:-

$O(n * W)$: Each subproblem solved only once and result is stored, so total subproblems is $O(n * W)$.

4) Dp-Tabulation:-

Dry Run:-

For weights = [1, 3, 4, 5] values = [1, 4, 5, 7], $w=7$, $n=4$:
• Initial dp table filled with 0s
• After processing each item, the table is updated based on whether we include current item or not.
• Final result is in $\text{dp}[4][7]$.

Time Complexity:-

$O(n * W)$. We fill a table of size $(n * W)$ where each entry calculated once.