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A cellular automata model for forest fire spread prediction: The case of the wildfire that swept through Spetses Island in 1990

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ABSTRACT

We present and illustrate the simulation results of a cellular automata model describing the dynamics of a forest fire spread on a mountainous landscape taking into account factors such as the type and density of vegetation, the wind speed and direction and the spotting phenomenon. The model is used to simulate the wildfire that broke up on Spetses in August of 1990 and destroyed a major part of the Island's forest. We used a black-box non-linear optimization approach to fine-tune some of the model's parameters based on a geographical information system incorporating available data from the real forest fire. The comparison between the simulation and the actual-observed results showed that the proposed model predicts in a quite adequate manner the evolution characteristics in space and time of the real incident and as such could be potentially used to develop a fire risk-management tool for heterogeneous landscapes.

1. Introduction

Forest fires have been the cause of numerous and irreversible damages with deep ecological and socio-economic impacts. Entire ecosystems thriving with – also rare-animal and plant life have been wiped off while rural properties, villages and civil infrastructures lying along the verge of forests have also suffered from the ravages of wildfires which quite often result in the loss of human lives. What happened during the last days of August of 2007 in Peloponnesus was the worst damage of the last 100 years in Greece: a total of 2000 km^2 of forest and agricultural areas, mostly olive groves, burned in a few days by eight major wildfires with several smaller ignition points. The fires not only destroyed a major part of the southwestern Greek forest area and agricultural structure, but also threatened the archeological area of Olympia, the birthplace of the Olympic Games and took a high death toll of 76 people plunging the country into mourning; most of them were locals and firefighters struggling to battle the blazes [1].

It is easily understood that the need for designing and developing effective ways of dealing with forest wildfires is constantly increasing as such phenomena appear ever more often. The fire suppression policies can be generally categorized into preventive and operational ones. Preventive policies are more strategic in principle, trying to minimize the likelihood of a burst of a fire by organizing the available resources and constructing anti-fire zones to hold up the spread of possible fire outbreaks. When a wildfire breaks out, drastic operational interventions are needed at a more tactical level, such as the efficient allocation of the defense mechanisms and the rapid evacuation of villages. In both cases, the strategic and tactic combat against the wildfires has much to benefit from mathematical models that could be used to predict the spread of the fire in

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space and time. Moreover, such models would be also useful in the case of controllable fires which are often used as an important rangeland management tool in areas like Savanna [2] or in Australia [3].

A model for predicting a wildfire spread would have to take into account external environmental factors like meteorological conditions as well as specific characteristics of the terrain. According to [4], the most important factors that affect the rate of spread and shape of a forest fire front are the fuel type (type of vegetation) and humidity, wind speed and direction, forest topography (slope and natural barriers), fuel continuity (vegetation thickness), and spotting which is a phenomenon where burning material is transferred by the wind or other reasons such as the fling of flaming pinecones to areas that are not adjacent to the fire front.

Building a mathematical model that could predict the spread of a wildfire, taking into account all the aforementioned factors is not an easy task as between them there are complex interactions, often poorly understood. The subject has been tackled very often in the literature, where a lot of interesting models have been proposed. One of the most important among them is the pioneering work of Rothermel [5,6] who based on laboratory experiments has identified the dynamic equations that characterize the maximum fire spread rate. Rothermel's equations have subsequently been applied in a variety of approaches which in terms of spatial representation can be categorized in two types. The first type consists of models based on continuous planes [7], where it is assumed that the fire front travels on a continuous landscape, advancing in an elliptical shape. Solutions to these models are usually obtained by solving a system of partial differential equations which can be rather computationally demanding. The second type, which is simpler and computationally faster, consists of the models that are based on a grid [8]. A quite different approach compared to these two types, but still very efficient is presented in [9,10], where the spread of fire is modeled using a fuzzy/neural model.

The grid-based models can also be divided into two subcategories. The first one is based on the Bond–Percolation method where fire spreads from one grid division to another according to a specific probability estimated based on historical data [11]. Although these models can be used to describe in an efficient manner the shape of the burned area, they are limited in describing the dynamics of the fire spread [12]. The second subcategory of grid models is based on the cellular automata (CA) methodology which resolves the above problem. More specifically a finite grid is applied to the macroscopic scale splitting it to a large number of cells. Each cell is usually described by several variables whose states, being usually discrete, evolve in discrete time, according to a set of rules and the states of the neighboring cells. CA have proven to be powerful in predicting emergent complex macroscopic dynamics from simple rules defining the physics at the microscopic-atomistic scale [13]. This property, together with the fact that CA can easily be combined with digital data from geographical information systems (GIS) or other sources makes them attractive candidates for modeling the complex behavior of wildfire spread.

Not surprisingly, many researchers have proposed CA-based approaches for modeling the spread of forest wildfires: in [14] the authors present a CA-based model that can predict the spread of a fire in both homogeneous and inhomogeneous forests and can easily incorporate weather conditions and land topography. A modification of this approach resulting to a more realistic model was presented in [15] where a more accurate factor of propagation from diagonal neighbor cells is introduced and a detailed form of the rate of spread is presented. In [16] the authors introduce a model based on CA being able to incorporate a number of environmental factors. In [12] it is proposed a CA approach to expand Rothermel's surface fire behavior model to multiple dynamic dimensions. The study presented in [17] integrates GIS and CA techniques to develop a fire spreading model with a flexible and user-friendly interface. In [18] it is presented a CA model that is capable of predicting fire spread in spatially heterogeneous Savanna-type landscapes. In [19] the authors propose a model that can deal with the process of fire spread in areas with buildings. In [20] a hexagonal lattice was used to develop a CA model which was integrated with Rothermel's model to simulate wildfire-spreading processes both spatially and temporally. Another attempt to integrate Rothermel's equations with a CA model is made in [21] where the results are compared with a real fire case study.

This work presents an enhanced methodology for predicting the spread of a wildfire that is based on the CA framework. Special care has been taken to formulate the rules that define the interactions between the adjacent cells, so that all the major factors that affect the fire spread are taken into account while the methodology also incorporates an attempt to model the spotting phenomenon. The resulting model was applied to simulate the evolution of a wildfire that swept through Spetses in 1990 and destroyed almost the half of the island's forest. The island of Spetses overlooks the east coastline of Peloponnesus and is of major historical importance as well as resort destination of the international touristic circuit (Fig. 1a and b). It should be noted that the specific characteristics of the particular terrain with steep changes in the altitude enable to draw useful conclusions regarding the effect of the slope on the fire spread while case studies of many other fire models that have been proposed in the literature were applied mostly on relatively flat terrains. Also the comparison between the real fire front data and the simulation was used to fine-tune some of the model parameters: a non-linear optimization problem was wrapped around our simulator in order to determine the values of those parameters that minimize the differences between the real-case and the simulated results.

The rest of this article is organized as follows: in the next section we present the proposed CA model while in the first part of the third section we describe how the wildfire evolved and what left behind. In the second part of the third section we illustrate the fine-tuning procedure of the parameters using a non-linear optimization approach and we demonstrate the simulation results followed by a comparison with the actual situation. Finally, the main conclusions of our results are summarized in section four where we also set some directions for future research.

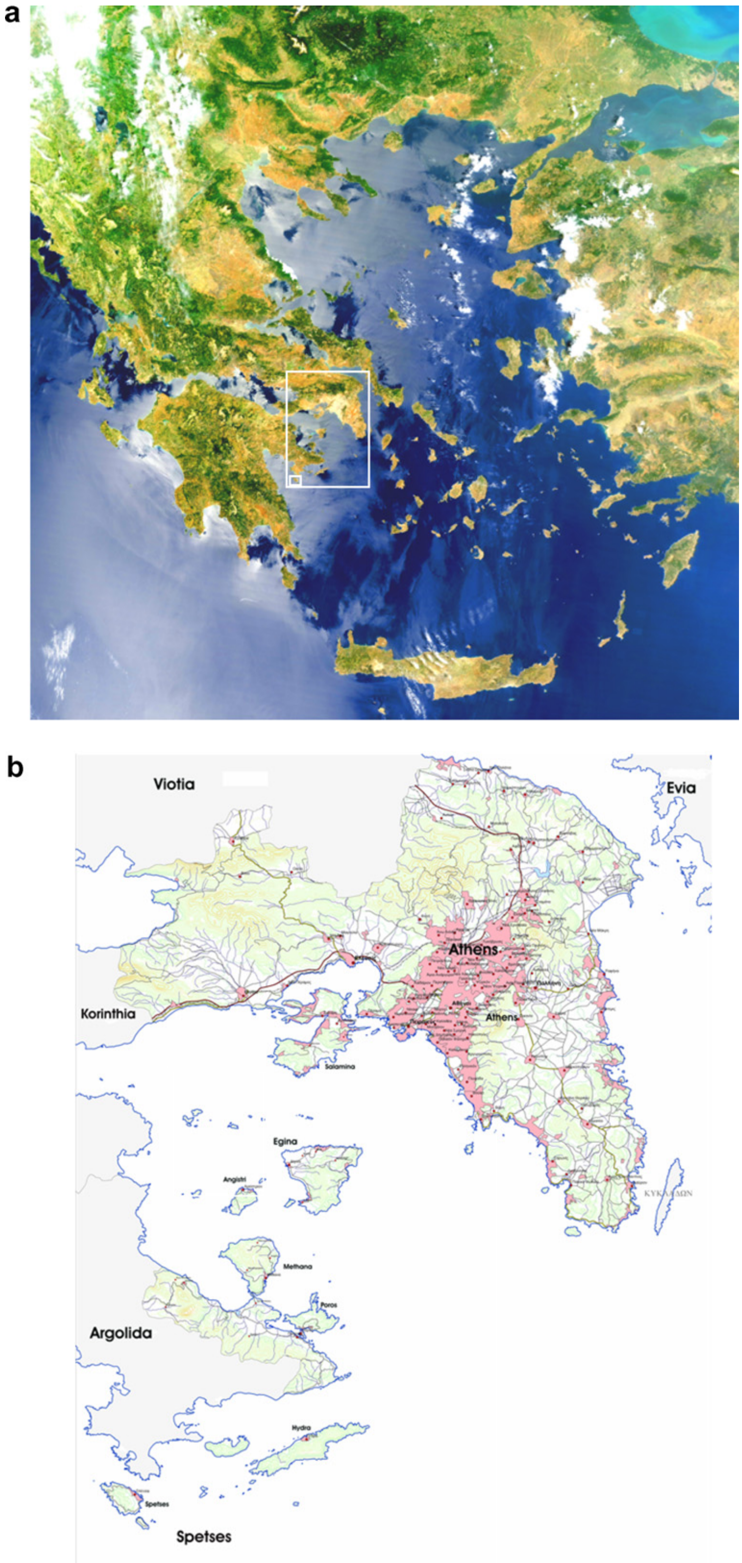

Fig. 1. (a) A satellite picture showing Greece. The location of the Island of Spetses is marked by the small white rectangle. (b) A map of the region of Attica (Athens) with Spetses, corresponding to the bigger white rectangle marked in (a).

2. CA-based methodology for wildfire prediction

2.1. Grid definition

The methodology uses a two-dimensional grid tessellating the forest area to a number of cells. Actually each cell represents a small patch of land and its shape has been chosen to be square, thus offering eight possible directions of fire spread (see Fig. 2). Other researchers (see for example [20,21]) are utilizing hexagonal cells which indeed can describe more accurately the spatial behavior of the fire, however this comes at the cost of significantly increasing the computational complexity of the resulting model. Though the proposed methodology could be easily modified to take into account hexagonal cells, for the purposes of this paper the square grid description is preferred as it simplifies considerably the calculations.

2.2. State of cells

Each cell is characterized by a finite number of states which evolve in discrete time. The possible states are the following:

State = *1*: The cell contains no forest fuel. This state may describe the cells corresponding to parts of the city with no vegetation, rural areas with no vegetation etc. We assume that cells that are in this state cannot be burned.
State = *2*: The cell contains forest fuel that has not ignited.
State = *3*: The cell contains forest fuel that is burning.
State = *4*: The cell contained forest fuel that has been burned down.

The state of each cell is then coded as an element of a matrix **S** which from now on will be called the state matrix. Fig. 3 shows an example of how an area of 16 cells with random number of states is coded in a matrix form.

2.3. Rules of evolution

At each discrete time step t of the simulation, the following rules are applied to the elements i, j of the state matrix **S** (and thus to all the cells):

Rule 1: IF state $(i,j,t) = 1$ THEN state $(i,j,t+1) = 1$.
This rule implies that the state of a cell with no forest fuel (empty cell) remains the same and thus it cannot catch fire.
Rule 2: IF state $(i,j,t) = 3$ THEN state $(i,j,t+1) = 4$.
This rule implies that a burning cell at the current time step will be burned down at the next time step.
Rule 3: IF state $(i,j,t) = 4$ THEN state $(i,j,t+1) = 4$.
This rule implies that the state of an empty cell that has been burned down in the previous step stays the same.
Rule 4: IF state $(i,j,t) = 3$ THEN state $(i \pm 1, j \pm 1, t+1) = 3$ with a probability p_{burn}.

This rule implies that when a cell catches fire at the current time step, the fire can be propagated to the neighboring cells at the next time step with a probability p_{burn}. This probability is a function of various parameters that affect the fire spread and will be analyzed in the following paragraphs. It should be noted that due to the square grid we have assumed that the fire can be propagated to the neighboring cells $i \pm 1, j \pm 1$; these are the eight cells depicted in Fig. 2.

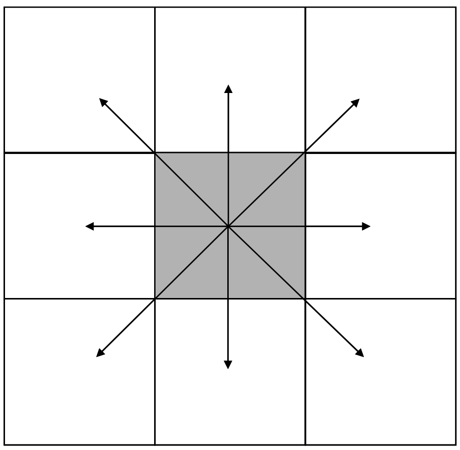

Fig. 2. Possible directions of fire propagation on a square grid.

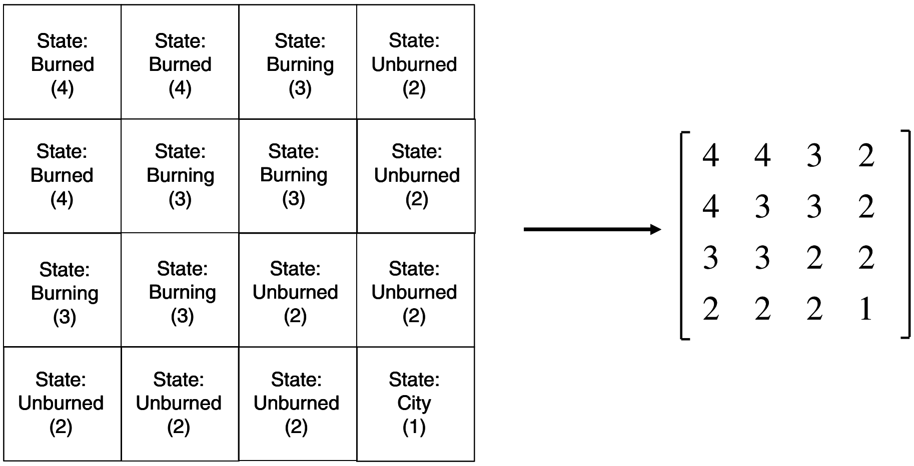

Fig. 3. Coding the state of cells in the state matrix **S**.

2.4. Variables that affect the spread of fire

The proposed methodology takes into account the following variables that can affect both the shape of the front and the spread rate of a forest fire: the type of vegetation, the density of vegetation, the wind speed and direction, the ground elevation and the spotting effect. The variables that are terrain-specific, i.e. the type and density of vegetation and the ground elevation are also coded in matrices similar to the state matrix. The probability p_{burn} is calculated by

$$p_{\mathrm{burn}} = p_{\mathrm{h}}(1 + p_{\mathrm{veg}})(1 + p_{\mathrm{den}})p_{\mathrm{w}}p_{\mathrm{s}}, \tag{1}$$

where p_{h} denotes the constant probability that a cell adjacent to a burning cell containing a given type of vegetation and density will catch fire at the next time step under no wind and flat terrain; $p_{\mathrm{den}}, p_{\mathrm{veg}}, p_{\mathrm{w}}, p_{\mathrm{s}}$ are the fire propagation probabilities that depend on the density of vegetation, the type of vegetation, the wind speed and the slope, respectively. Notice that these probabilities are multiplied by the constant probability p_{h} to obtain the corrected probability that takes into account all the aforementioned factors.

2.5. Effect of the type and density of vegetation

The effects of the type and density of vegetation are represented by the probabilities p_{veg} and p_{den}, respectively. More specifically the type and the density of vegetation in the area are split into a number of discrete categories. Here, the type of vegetation has been clustered into three categories numbered from 1 to 3 where 1 is for agricultural areas, 2 is for shrubs and 3 is for pine trees, while the density of vegetation has been scaled into three categories numbered from 1 to 3, where 1 is for sparse vegetation and 3 is for dense vegetation. These are organized into two matrices, the vegetation matrix and the density matrix where for each of the respective categories a constant value for the probabilities p_{veg} and p_{den} is assigned.

2.6. Effect of the wind speed and direction

Many empirical relationships have been suggested in the literature to model the effect of the wind on the rate of the fire spread. A commonly used one can be found in [20,22] and reads:

$$R_{\mathrm{w}} = R_{0\mathrm{w}} \exp(\beta\theta_{\mathrm{f}}), \tag{2a}$$

R_{w} denotes the spread rate in the presence of wind, $R_{0\mathrm{w}}$ is the spread rate when the wind speed is equal to zero, θ_{f} is the flame angle measured from the direction perpendicular to the direction of the fire spread and β is a constant that is usually obtained through least squares linear regression from experimental data [2]. When the direction of the fire spread is the same as the wind direction then the flame angle is set to θ_{f} while when they are in opposite directions the flame angle is set to $-\theta_{\mathrm{f}}$; in all other directions θ_{f} is set equal to zero. In [23] the following empirical equation is suggested to calculate the fire angle:

$$tg(\theta_{\mathrm{f}}) = 0.4226V, \tag{2b}$$

where V denotes the wind speed.

In this study we employ a more flexible empirical wind-effect relation where the probability that contains the effect of wind velocity and direction is calculated by the following equations:

$$p_{\mathrm{w}} = \exp(c_1 V)f_t, \quad f_t = \exp(Vc_2(\cos(\theta) - 1)), \tag{3}$$

where c_1, c_2 are constants to be determined and θ is the angle between the direction of the fire propagation and the direction of the wind. Notice that using this formula the wind direction can receive any continuous value between 0 and 360° in contrast with the most of the aforementioned techniques where the wind direction can only receive a few discrete values. Fig. 4 illustrates the general form of the probability p_w as a function of the angle θ for some arbitrary values of the constant parameters c_1, c_2 and wind velocity V.

2.7. Effect of the ground elevation

The effect of ground elevation (slope) is modeled using the following equation, introduced in [22]:

$$R_s = R_{0s} \exp(a\theta_s), \tag{4a}$$

where R_{0s} is the spread rate when the slope is equal to zero, θ_s is the slope angle of the patch and a is a constant that can be adjusted from experimental data. According to Eq. (4a), the probability that models the effect of the patch-slope reads:

$$p_s = \exp(a\theta_s). \tag{4b}$$

It should be noted that due to the square grid, the slope angle is calculated in a different way depending on whether the two neighboring cells are adjacent or diagonal to the burning cell. More specifically for adjacent cells the slope angle reads:

$$\theta_s = \tan^{-1}\left(\frac{E_1 - E_2}{l}\right), \tag{5a}$$

where E_1 and E_2 are the altitude of the two cells and l is the length of the square side, whereas for diagonal cells the formula becomes:

$$\theta_s = \tan^{-1}\left(\frac{E_1 - E_2}{l\sqrt{2}}\right). \tag{5b}$$

2.8. Spotting (non-local) effect

Spotting is a phenomenon where burning material like small boughs and twigs is transported by the wind or by some other reasons to spots that are not adjacent to the fire front. The burning material may then ignite a new fire depending on the type and density of vegetation at the spots.

A classical type of such firebrands that are responsible for the spotting phenomenon are the pop-corn-like flinging flaming pinecones, which are usually too heavy to get carried away by the wind at long distances, but are shot in the air as far as one hundred meters. Obviously the probabilistic nature of this phenomenon makes it rather difficult to create a reliable deterministic mathematical model for it. The proposed model in this work incorporates this long-range action trying to approximate the spotting process of the pinecones which is a major mechanism for the spread of a fire in pine trees forests – a most common type in Greece. The proposed approach can be easily extended to take into account different kinds of spotting phenomena.

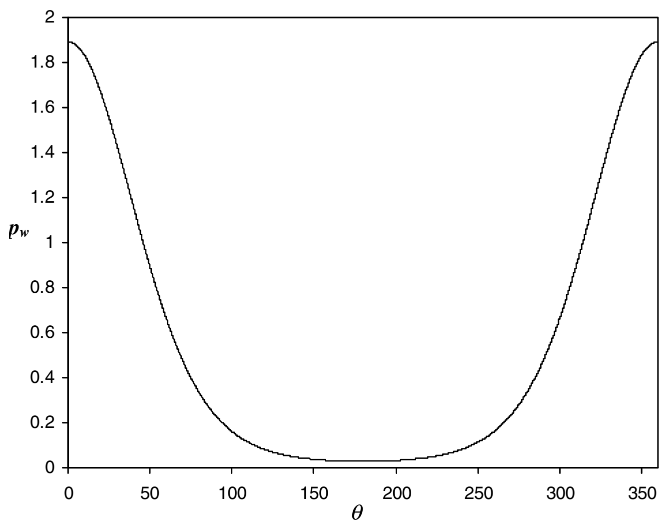

Fig. 4. Effect of angle θ on wind probability p_w.

In the case of a pine tree forest it is assumed that each burning cell shoots in the air a number of flaming pinecones denoted by N_p. This number is drawn from a Poisson distribution with a mean value λ. The direction towards which each burning pinecone goes is then drawn from a uniform distribution. The next step is to calculate the traveling distance d_p for each pinecone. This distance is a function of two components: the first component represents the thrust of a pinecone and can be modeled only in a probabilistic manner; the second component is the wind direction and velocity. The combined effect of these two components can be modeled using the following formula:

$$d_p = r_n \exp(Vc_2(\cos(\theta_p) - 1)), \tag{6}$$

where r_n is a random number drawn from a normal distribution with a given mean and standard deviation and θ_p is the angle between the direction of the wind and the direction of the pinecone blasting. Knowing the angle of blasting and the distance, the cells where the burning cones will land can be easily found. Whether these cells will ignite or not, is a function of both the vegetation density and type. Thus we define a constant probability p_{c0} that a cell will catch fire due to spotting and we correct this probability by a factor of p_{cd} where p_{cd} receives constant values depending on the vegetation density and type of the cell as:

$$p_c = p_{c0}(1 + p_{cd}). \tag{7}$$

In this case, we add one more evolution rule that reads:

Rule 5: IF state $(i,j,t) = 3$ THEN state $(i \pm i_c, j \pm j_c, t+1) = 3$ with a probability p_c where i_c and j_c are pointers that depend on the distance d_p and the direction of pinecone fling.

3. The case study: the wildfire that swept through Spetses island in 1990

The proposed methodology was applied for the prediction of the spread of a wildfire that swept through Spetses on August 1, 1990. The fire cropped up due to unknown causes somewhere near the middle of the Island's forest and was quickly spread fanned to the south by moderate to strong north winds. The fire was extinguished around 11 h later, after having burned down a forest area of almost 6 km^2, almost the one third of the total area of the island. Fig. 5 provides a stereoscopic map of the island depicting the burned area (encircled by the white contour).

We chose to model the particular fire incident for two main reasons: the first is that most of the results of the particular wildfire (e.g. total burned area, time to extinction) were well documented by the authorities and the second one is that the specific terrain of the island with steep changes in the altitude provides a suitable environment for testing the effectiveness of the proposed CA-based model.

The first step in applying our proposed methodology was to generate the states of the altitude, vegetation density and vegetation type matrices based on digital geographical data. In order to do so we employed a geographical information system (GIS), namely the Arc GIS 9.2 by ESRI. First we created a shape-file coding the type and the density of the Island's vegetation. This file was created by the digitalization of photomaps that were taken before the wildfire incident. The second step was to create a shape-file coding the altitude data, based on a digital model of the ground configuration of Spetses. Finally, one more shape-file coding the shape of the burned area was created. Based on these digital files, a vector data file was built, containing the values of all the aforementioned variables of interest, overlaid on the grid. The side of the square grid was selected to be equal to five meters. This selection was sufficient enough to give a crisp image and representation of the geographical data. Based on the vector data file the altitude, vegetation density and vegetation type matrices, together with a

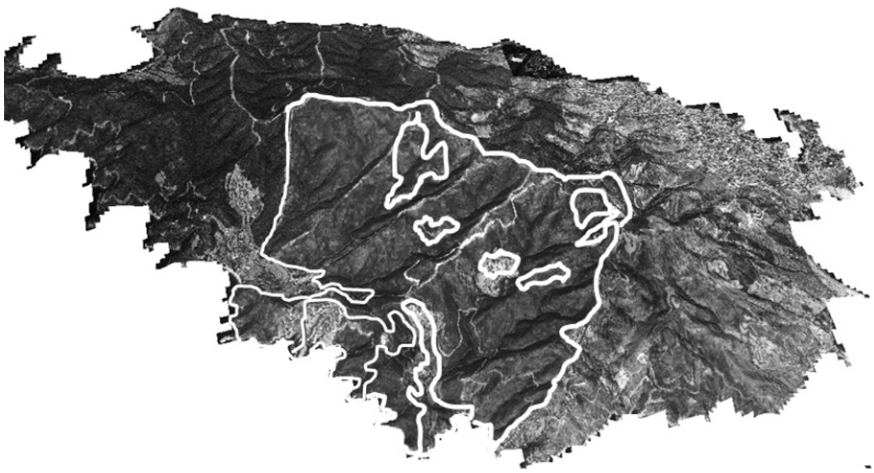

Fig. 5. Stereoscopic map of the Spetses island after the fire took place; the burned area is also marked by the white contours.

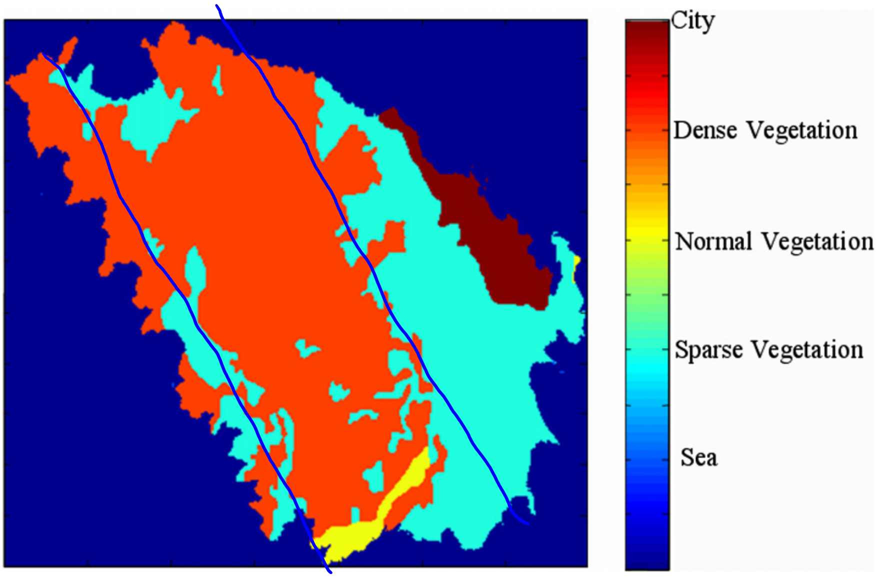

Fig. 6. Density of vegetation map of Spetses Island; the density is color-coded.

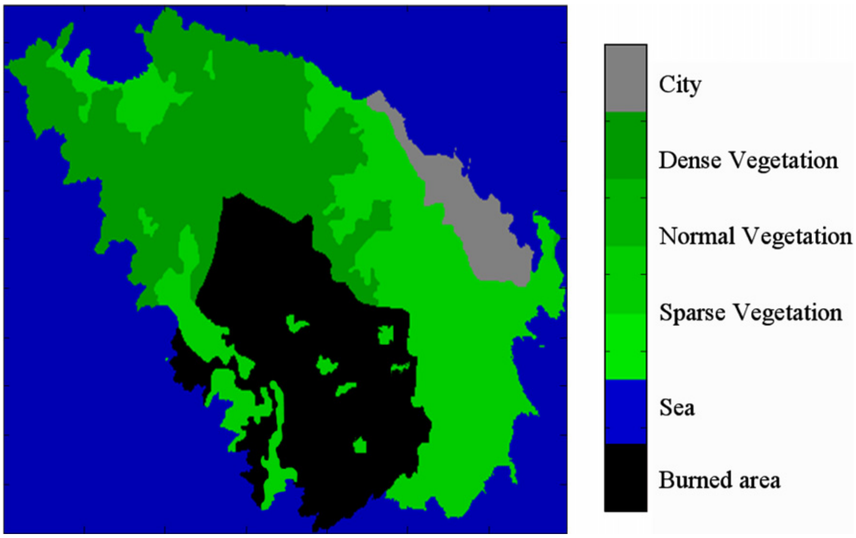

Fig. 7. Real burned area (recreated by photomap).

matrix containing the burned area were created in Matlab which was our simulation environment. Figs. 6 and 7 visualize the vegetation density of the island, and the total burned area. There are three types of vegetation, namely haleppo-pine trees, shrubs and agricultural areas while the vegetation density was classified into three discrete categories: sparse, normal and dense.

Table 1
Meteorological conditions at Spetses Island on August, 1, 1990

Variable	Value
Wind direction	North
Wind speed	8–10 m/s (~5 Beaufort)
Average temperature	30 °C
Humidity	36%

3.1. Simulation results and discussion

The meteorological conditions for the particular day in the area are given in Table 1. The values for the probabilities p_{den} and p_{veg} depending on the density and type of vegetation were chosen empirically and are given in Tables 2 and 3, respectively. The rest of the parameters in the CA model, namely the constant probability of fire propagation p_h, the slope coefficient a and the wind coefficients c_1, c_2, were determined by wrapping around the simulator a non-linear optimization technique, setting as objective the minimization of the difference between the number of burned cells predicted by the simulation and those that were burned in reality. Here we used the Nelder–Mead optimization algorithm [24]. At each iteration of the optimization procedure the entire fire incident was simulated until the fire stopped (no burning cells). The values of the parameters obtained by the optimization procedure are given in Table 4. It should be noted that the values of the

Table 2
Values for the probability p_{den} depending on the density of vegetation

Category	Density	p_{den}
1	Sparse	−0.4
2	Normal	0
3	Dense	0.3

Table 3
Values for the probability p_{veg} depending on the type of vegetation

Category	Type	p_{veg}
1	Agricultural	−0.3
2	Thickets	0
3	Hallepo-pine	0.4

Table 4
Optimized values for the CA algorithm operational parameters

Parameter	Value
p_h	0.58
a	0.078
c_1	0.045
c_2	0.131

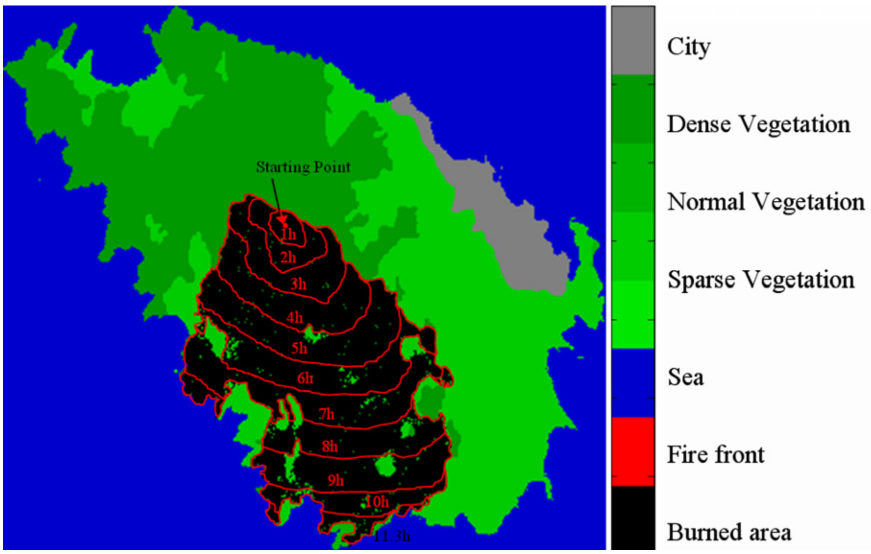

Fig. 8. Burned area predicted by the simulation; it is shown the starting point while the fire front evolution is also marked by the contours depicting one-hour intervals.

parameters c_1 and α appearing in Eqs. (3) and (4) are in close agreement with the ones that have been reported in the bibliography for relevant parameters [2,20], which is an indication of the robustness of the method.

The simulation results of the fire spread are shown in Fig. 8. With black color is depicted the final burned area while the red contours show the evolution of the fire in time-intervals of 1 h. In our simulations the fire burned a total of 5.4 km^2 in 11.3 h. The actual forest fire burned a total of 5.9 km^2 in 11 h. A comparison between Figs. 7 and 8 shows that the burned area predicted by the simulation is quite close to the actual one. It should be noted that the parameter values obtained from the optimization procedure do not, most likely, provide the global minimum; however forcing the burned area produced by the simulation to be exactly the same with the one corresponding to the real incident is not always a strict objective. One should take into account that most of the times the digitization of the areas is obtained by image processing of photomaps or satellite images. Due to limited spatial resolution the procedure introduces a vagueness which is not the only one: uncertainty may be also attributed to several factors such as meteorological (e.g. lack of accurate information about the exact direction and speed of the wind, presence of sudden gusts of wind, variation of the temperature and humidity during the fire that affect the fire propagation probability), lack of modeling of certain phenomena (e.g. vortices and turbulent flow dynamics of the air due to the morphology of the terrain) and lack of information about the results of the firefighting suppression efforts.

4. Conclusions

This paper presents an improved CA methodology for the dynamical prediction of the spread of a wildfire. The methodology has the ability to take into account several factors that affect the fire spread including meteorological as well as spatio-geographical parameters. It also introduces a new technique for incorporating the spotting phenomenon which is usually not encountered by most of the models suggested in the literature but depending on the situation may have a significant impact on the evolution of the fire spread.

The proposed methodology has been applied to simulate the dynamics of a real wildfire that burst on a famous Greek island destroying almost the half of its forest. The special nature of the terrain, including steep changes in the altitude and the type and density of vegetation offers a good benchmark for evaluating the proposed approach. The values for some of the model coefficients have been optimized using a non-linear optimization technique, which was wrapped around the simulator, trying to minimize the difference between the actual burned area and the one predicted by the model. The simulation results are very close to the actual ones thus confirming that the proposed methodology holds promising in predicting in a satisfactory manner the spread of a forest fire in heterogeneous terrains. However, the robustness of the produced model will be further validated by applying it to the prediction of different large-scale, real-world fire incidents. Still, we believe that further improvements in the model may contribute to even better predictions. Currently we are working on the combination of the CA-based model with computational fluid dynamics (CFD) tools. It is well known that airflow patterns near the surface may have a big impact on the evolution of the fire front. The coupling of such CA-based models with CFD will allow the accurate simulation of the flow field near the terrain thus increasing the fire spread prediction efficiency.

References

[1] Associated Press, Firefighters Gain Control of Greek Wildfire, 2007, 30 August.
[2] S.G. Berjak, J.H. Hearne, An improved cellular automaton model for simulating fire in a spatially heterogeneous Savanna system, Ecol. Modell. 148 (2002) 133–151.
[3] M.A. McCarthy, H.P. Possingham, A.M. Gill, Using stochastic dynamic programming to determine optimal fire management for *Banksia ornata*, J. Appl. Ecol. 38 (2001) 585–592.
[4] W. Fons, Analysis of fire spread in light forest fuels, J. Agric. Res. 72 (1946) 93–121.
[5] R. Rothermel, A mathematical Model for predicting fire spread in wildland fuels, Res. Pap. INT-115, Ogden, UT, US, Dept. of Agric., Forest Service, Intermountain Forest and Range Experiment Station, 1972.
[6] R. Rothermel, How to predict the spread and intensity of forest fire and range fires, Gen. Tech. Rep. INT-143, Ogden, UT, US, Dept. of Agric., Forest Service, Intermountain Forest and Range Experiment Station, 1983.
[7] G.D. Richards, A general mathematical framework for modeling two-dimensional wildland fire spread, Int. J. Wildland Fire 56 (1995) 3–72.
[8] E. Pastor, L. Zarate, E. Planas, J. Arnaldos, Mathematical models and calculation systems for the study of wildland fire behaviour, Prog. Energy Combust. Sci. 29 (2003) 139–153.
[9] D. Vakalis, H. Sarimveis, C. Kiranoudis, A. Alexandridis, G.V. Bafas, A GIS based operational system for wildland fire crisis management I. Mathematical modelling and simulation, Appl. Math. Modell. 28 (4) (2004) 389–410.
[10] D. Vakalis, H. Sarimveis, C. Kiranoudis, A. Alexandridis, G. Bafas, A GIS based operational system for wildland fire crisis management II. System architecture and case studies, Appl. Math. Modell. 28 (4) (2004) 411–425.
[11] M.A. McCarthy, Fire modeling and biodiversity, in: N. Klomp, I. Lunt (Eds.), Frontiers in Ecology, Oxford, Elsevier, 1997, pp. 79–88.
[12] Z. Yongzhong, E. Youhao, H. Tao, Z. Songbing, W. Jihe, A CA-based information system for surface fire spreading simulation, in: IEEE, IGARSS'05 Proceedings, 2005, pp. 3484–3487.
[13] J. Von Neumann, The Theory of Self-reproducing Automata, University of Illinois Press, Urbana IL, 1966.
[14] I. Karafyllidis, A. Thanailakis, A model for predicting forest fire spreading using cellular automata, Ecol. Modell. 99 (1) (1997) 87–97.
[15] A.H. Encinas, L.H. Encinas, S.H. White, A.M. del Rey, G.R. Sanchez, Simulation of forest fire fronts using cellular automata, Adv. Eng. Software 38 (2007) 372–378.
[16] X. Li, W. Magill, Modeling fire spread under environmental influence using a cellular automaton approach, Complex. Int. 8 (2001) 1–14.
[17] S. Yassemi, S. Dragicevic, M. Schmidt, Design and implementation of an integrated GIS-based cellular automata model to characterize forest fire, Ecol. Modell. 210 (2008) 71–84.
[18] S. Berjak, J. Hearne, An improved cellular automaton model for simulating fire in a spatially heterogeneous Savanna system, Ecol. Modell. 148 (2002) 133–151.

[19] A. Ohgai, Y. Gohnai, S. Ikaruga, M. Murakami, K. Watanabe, Cellular automata modeling for fire spreading as a tool to aid community-based planning for disaster mitigation, in: J.P. Van Leeuwen, H.J.P. Timmermans (Eds.), Recent Advances in Design and Decision Support Systems in Architecture and Urban Planning, Kluwer Academic Publishers, Dordrecht, 2004, pp. 193–209.
[20] Z. Yongzhong, Z.D. Feng, H. Tao, W. Liyu, L. Kegong, D. Xin, Simulating wildfire spreading processes in spatially heterogeneous landscapes using an improved cellular automaton model, in: IGARSS'04, Proceedings of the 2004 IEEE International, 2004, pp. 3371–3374.
[21] G.A. Trunfio, Predicting wildfire spreading through a hexagonal cellular automata model, in: P.M.A. Sloot, B. Chopard, A.G. Hoekstra (Eds.), ACRI 2004, LNCS 3305, Springer-Verlag, Berlin, Heidelberg, 2004, pp. 385–394.
[22] D.R. Weise, G.S. Biging, Effects of wind velocity and slope on flame properties, Can. J. Forest Res. 26 (1996) 1849–1858.
[23] N.P. Cheney, Fire behaviour, in: A.M. Gill, G.H. Groves, I.R. Noble (Eds.), Fire and the Australian Biota, Australian Academy of Science, Canberra, 1981, pp. 157–175.
[24] C.T. Kelley, Iterative Methods for Optimization, SIAM, Philadelphia, 1995.