What Randomness Tells Us About Risk?

1. The Tension Between Drift and Diffusion

The Geometric Brownian Motion (GBM) formula encapsulates the fundamental battle in finance: the tension between **expectation** (Drift) and **uncertainty** (Diffusion).

- **The Drift (μ)** represents our rational hope. It is the deterministic component—the slow, steady upward march of the economy or a company's value over time. If risk were zero ($\sigma = 0$), finance would be simple algebra.
- **The Randomness (σZ_t)** represents reality. It is the "noise" that disrupts the signal. In our code, Z_t is a standard normal variable, representing the millions of unpredictable shocks—news, geopolitical events, sentiment shifts—that hit the market daily.

Reflection: Risk is not merely the possibility of losing money; it is the **dominance of noise over signal** in the short term. The simulation shows that over short time intervals (small Δt), the random shock ($\sigma\sqrt{\Delta t}Z_t$) often mathematically overpowers the drift trend. This explains why predicting stock prices day-to-day is gambling, while investing decade-to-decade is planning.

2. The Cone of Uncertainty (Time Scales Risk)

When we plot the price paths, we see them start at a single point (S_0) and fan out widely as time progresses. This "trumpet" or "cone" shape is a visual representation of how risk accumulates.

- Mathematically, volatility scales with the **square root of time ($\sqrt{t}$)**.
- This tells us that **uncertainty is a function of time**. The further into the future we try to look, the wider the range of possible outcomes becomes. Our simulation demonstrates that risk is not a static number; it is a dynamic path. A 1% drop today changes the starting point for tomorrow, meaning "bad luck" can compound just as easily as "good luck."

3. The Shape of Risk: Asymmetry and the Log-Normal Distribution

The histogram of final prices provides the most profound insight. Even though the input shocks (Z_t) were normally distributed (symmetric "Bell Curve"), the output prices are **Log-Normally distributed** (skewed to the right).

- **The Insight:** Risk in equity markets is asymmetric. A stock can only go down to \$0 (100% loss), but it can go up to infinity. This skew results in a "long tail" to the right.
- **The "Volatility Drag":** Notice the term $-0.5\sigma^2$ in the drift part of the formula. This is the "penalty" of volatility. It mathematically proves that **volatility eats into returns**. If a stock drops 50% and then gains 50%, you are not back to where you started (you are down 25%). The simulation shows that highly volatile assets require significantly higher returns just to break even mathematically.

Conclusion

Building a GBM simulator reveals that **risk is not an anomaly; it is a feature of the system.** The "randomness" we code is not just chaos; it is the fuel for opportunity. Without the random component (σ), there would be no risk, but there would also be no risk premium—no reason for stocks to return more than safe government bonds.

The simulation teaches us that we cannot predict *the* future (a single path), but we can map the *probability* of the future (the histogram). We accept randomness not by trying to eliminate it, but by structuring our exposure (through diversification or hedging) so that we can survive the "left tail" of the histogram long enough to benefit from the drift.