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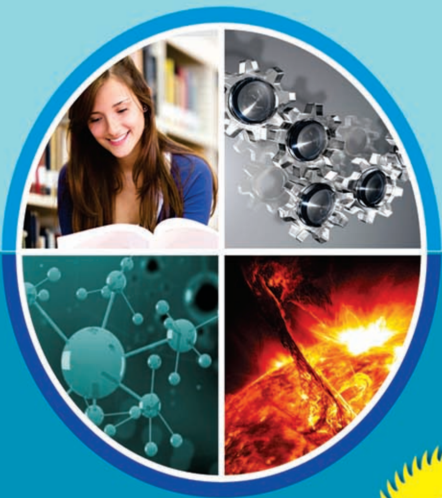

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Fluid Mechanics & Fluid Machines

By

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UNIT-2 : FLUID FLOW & CONTINUITY EQUATION (2–1 A to 2–35 A)

Continuum & free molecular flows. Steady and unsteady, uniform and non-uniform, laminar and turbulent flows, rotational and irrotational flows, compressible and incompressible flows, subsonic, sonic and supersonic flows, sub-critical, critical and supercritical flows, one, two- and threedimensional flows, streamlines, continuity equation for 3D and 1D flows, circulation, stream function and velocity potential. Buckingham's Pi theorem, important dimensionless numbers and their significance.

UNIT-3 : FLOW THROUGH PIPES (3–1 A to 3–49 A)

Equation of motion for laminar flow through pipes, turbulent flow, isotropic, homogenous turbulence, scale and intensity of turbulence, measurement of turbulence, eddy viscosity, resistance to flow, minor losses, pipe in series and parallel, power transmission through a pipe, siphon, water hammer, three reservoir problems and pipe networks. Boundary layer thickness, boundary layer over a flat plate, laminar boundary layer, application of momentum equation, turbulent boundary layer, laminar sublayer, separation and its control, Drag and lift, drag on a sphere, a two-dimensional cylinder, and an aerofoil, Magnus effect.

UNIT-4 : IMPACT OF JET (4–1 A to 4–54 A)

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UNIT-5 : CENTRIFUGAL & RECIPROCATING PUMPS (5–1 A to 5–51 A)

Classifications of centrifugal pumps, Vector diagram, Work done by impellor, Efficiencies of centrifugal pumps, Specific speed, Cavitation & separation, Performance characteristics. Reciprocating pump theory, Slip, Indicator diagram, Effect of acceleration, air vessels, Comparison of centrifugal and reciprocating pumps, Performance characteristics.

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FLUID MECHANICS AND FLUID MACHINES

L-T-P
3-1-0

Objectives:

- To learn about the application of mass and momentum conservation laws for fluid flows.
- To understand the importance of dimensional analysis.
- To obtain the velocity and pressure variations in various types of simple flows.
- To analyze the flow in water pumps and turbines.

UNIT-I

Definition of fluid, Newton's law of viscosity, Units and dimensions-Properties of fluids,mass density, specific volume, specific gravity, viscosity, compressibility and surfacetension, Incompressible flow, Bernoulli's equation and its applications - Pitot tube, orifice meter, venturi meter and bend meter, notches and weirs, momentum equation and its application to pipe bends.

UNIT-II

Continuum & free molecular flows. Steady and unsteady, uniform and non-uniform, laminar and turbulent flows, rotational and irrotational flows, compressible and incompressible flows, subsonic, sonic and supersonic flows, sub-critical, critical and supercritical flows, one, two- and three-dimensional flows, streamlines, continuity equation for 3D and 1D flows, circulation, stream function and velocity potential. Buckingham's Pi theorem, important dimensionless numbers and their significance.

UNIT-III

Equation of motion for laminar flow through pipes, turbulent flow, isotropic, homogenous turbulence, scale and intensity of turbulence, measurement of turbulence, eddy viscosity, resistance to flow, minor losses, pipe in series and parallel, power transmission through a pipe, siphon, water hammer, three reservoir problems and pipe networks.

Boundary layer thickness, boundary layer over a flat plate, laminar boundary layer, application of momentum equation, turbulent boundary layer, laminar sublayer, separation and its control, Drag and lift, drag on a sphere, a two-dimensional cylinder, and an aerofoil, Magnus effect.

UNIT-IV

Introduction to hydrodynamic thrust of jet on a fixed and moving surface, Classification of turbines, Impulse turbines, Constructional details, Velocity triangles, Power and efficiency calculations, Governing of Pelton wheel.

Francis and Kaplan turbines, Constructional details, Velocity triangles, Power and efficiency Principles of similarity, Unit and specific speed, Performance characteristics, Selection of water turbines.

UNIT-V

Classifications of centrifugal pumps, Vector diagram, Work done by impellor, Efficiencies of centrifugal pumps, Specific speed, Cavitation & separation, Performance characteristics.

Reciprocating pump theory, Slip, Indicator diagram, Effect of acceleration, air vessels, Comparison of centrifugal and reciprocating pumps, Performance characteristics.

Course Outcomes:

- Upon completion of this course, students will be able to mathematically analyze simple flow situations.
- They will be able to evaluate the performance of pumps and turbines.

Books and References:
1. Introduction to fluid mechanics and Fluid machines by S.K Som, Gautam Biswas, S Chakraborty.
2. Fluid mechanics and machines by R.K Bansal.
3. F. M. White, Fluid Mechanics, 6th Ed., Tata McGraw-Hill, 2008.
4. Fluid Mechanics and Its Applications byV.K.Gupta et.al.
5. Fluid Mechanics byYunusCengel.
6. Batchelor, G. K. (1999). Introduction to fluid dynamics. New Delhi, India: Cambridge University Press.
7. Acheson, D. J. (1990). Elementary fluid dynamics. New York, USA: Oxford UniversityPress.
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1
UNIT

Fluid and Bernoulli's Equation

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Part-2 : Units and Dimensions, Properties of Fluids, Mass Density, Specific Volume, Specific Gravity, Viscosity .... **1–5A to 1–6A**

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Part-4 : Bernoulli's Equation and its Applications-Pitot Tube, Orifice Meter, Venturimeter and Bend Meter .... **1–9A to 1–21A**

Part-5 : Notches and Weirs .... **1–21A to 1–26A**

Part-6 : Momentum Equation and its Application to Pipe Bends .... **1–26A to 1–32A**

PART-1

Definition of Fluid, Newton's Law of Viscosity.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **What is fluid ? State Newton's law of viscosity and derive the same. What are its applications ?**

Answer

A. Fluid :

1. A fluid is a substance which deforms continuously when subjected to external shearing force.

B. Newton's Law of Viscosity :

1. This law states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.

Mathematically, $$\tau = \mu \frac{du}{dy}$$

C. Derivation :

1. From Fig. 1.1.1, let two layers of fluid at a distance 'dy' apart, move one over the other at different velocities u and $u + du$.
2. The viscosity together with relative velocity causes shear stress acting between fluid layers.
3. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by τ.

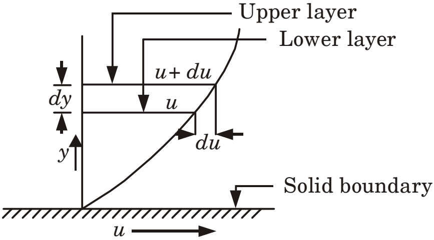

Fig. 1.1.1. Velocity variation near a solid boundary.

Mathematically, $\tau \, \alpha \, \frac{du}{dy}$ or $\tau = \mu \frac{du}{dy}$

Where, μ = Constant of proportionality and is known as coefficient of dynamic viscosity or viscosity.

$\frac{du}{dy}$ = Rate of shear deformation or velocity gradient.

D. Applications :

1. Lubrication in bearings.
2. Relative movement between two plates.

Que 1.2. **An oil of viscosity 5 poise is used for lubrication between a shaft and sleeve. The diameter of the shaft is 0.5 m and it rotates at 200 rpm. Calculate the power lost in oil for the sleeve length of 100 mm, the thickness of the oil film is 1 mm.**

AKTU 2014-15, Marks 05

Answer

Given : $\mu = 5$ poise $= \frac{5}{10} = 0.5$ N-s/m^2, $D = 0.5$ m, $N = 200$ rpm, $L = 100$ mm $= 0.1$ m, $t = 1.0$ mm $= 1 \times 10^{-3}$ m
To Find : Power lost in the oil.

1. Tangential velocity of shaft is given as,

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.5 \times 200}{60} = 5.236 \text{ m/s}$$

2. Using the relation, $\tau = \mu \frac{du}{dy}$

Where, du = Change in velocity $= u - 0 = u = 5.236$ m/s

dy = Change in distance $= t = 1 \times 10^{-3}$ m

$$\therefore \quad \tau = \frac{0.5 \times 5.236}{1 \times 10^{-3}} = 2618 \text{ N/m}^2$$

3. Shear force on the shaft,

$$F = \tau A = \tau \times \pi DL \qquad (\because \; A = \pi DL)$$

$$= 2618 \times \pi \times 0.5 \times 0.1 = 411.23 \text{ N}$$

4. Torque on the shaft,

$$T = F \times \frac{D}{2} = 411.23 \times \frac{0.5}{2} = 102.81 \text{ N-m}$$

5. $$\text{Power lost} = T\omega = T \times \frac{2\pi N}{60}$$

$$= 102.81 \times \frac{2\pi \times 200}{60} = 2153 \text{ W} = 2.15 \text{ kW}$$

Que 1.3. A square plate 50 cm × 50 cm weighing 200 N slides down an inclined plane of slope 1 vertical : 2.5 horizontal with a uniform velocity of 0.40 m/s. If a thin layer of oil of thickness 0.5 cm fills the space between the plate and the inclined plane. Determine the coefficient of viscosity of oil.

Answer

Given : $A = 50 \times 50 = 2500 \text{ cm}^2 = 0.25 \text{ m}^2$, $u = 0.40$ m/s
$dy = 0.5 \text{ cm} = 5 \times 10^{-3}$ m
To Find : Coefficient of viscosity of oil.

1. Component of weight along the plane = $W \sin \theta$

Where, $$\sin \theta = \frac{BC}{AC} = \frac{1}{\sqrt{2.5^2 + 1^2}} = \frac{1}{2.693}$$

$\therefore$ $$F = W \sin \theta = 200 \times \frac{1}{2.693} = 74.27 \text{ N}$$

2. Now $$\tau = \mu \frac{du}{dy} \qquad \text{...(1.3.1)}$$

Where, $du = u - 0 = 0.4$ m/s and $dy = 5 \times 10^{-3}$ m

4. We also know, $$\tau = \frac{F}{A} \qquad \text{...(1.3.2)}$$

5. Equating eq. (1.3.1) and eq. (1.3.2), we get

$$\frac{F}{A} = \mu \frac{du}{dy}$$

$\therefore$ $$\mu = \frac{F}{A} \times \frac{dy}{du}$$

$$\mu = \frac{74.27}{0.25} \times \frac{5 \times 10^{-3}}{0.40} = 3.7135 \text{ Pa-s or } 37.135 \text{ Poise}$$

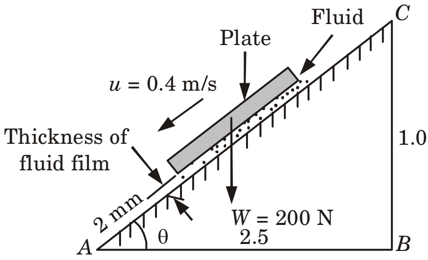

Fig. 1.3.1.

PART-2

Units and Dimensions, Properties of Fluids, Mass Density, Specific Volume, Specific Gravity, Viscosity.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 1.4. Discuss some physical properties of fluids in brief.

AKTU 2014-15, Marks 05

Answer

Some physical properties of fluids are as follows :

a. Density or Mass Density :

1. It may be defined as the mass per unit volume at a standard temperature and pressure. It is also known as specific mass. It is denoted by ρ and its unit is kg/m³.

Mathematically, $$\rho = \frac{m}{V}$$

Where, m = Mass (kg), and

V = Volume (m³).

b. Weight Density :

1. It can be defined as the weight per unit volume at the standard temperature and pressure. It is also known as specific weight. It is denoted by W and its unit is N/m³.

Mathematically, $W = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{mg}{V} = \rho g \qquad \left(\because \dfrac{m}{V} = \rho\right)$

c. Specific Volume :

1. It is defined as the volume per unit mass of fluid.

Mathematically, $v = \dfrac{V}{m} = \dfrac{1}{\rho}$

d. Specific Gravity :

1. It is the ratio of the specific weight of the given fluid to the specific weight of a standard fluid.

$$S = \frac{\text{Specific weight of given fluid}}{\text{Specific weight of standard fluid}}$$

2. For liquids, standard fluid is pure water at 4 °C and air is standard fluid for gases.

e. Viscosity :

1. It is defined as the property of a fluid which determines its resistance to shearing stresses. Its SI unit is Pa-s and CGS unit is poise.
2. An ideal fluid has no viscosity.
3. Viscosity of fluids is due to cohesion and adhesion.

Que 1.5. What is the difference between dynamic viscosity and kinematic viscosity ?

Answer

S. No.	Dynamic Viscosity	Kinematic Viscosity
1.	It is defined as the property of a fluid which determines its resistance to shearing stresses.	It is defined as the ratio between the dynamic viscosity and density of fluid.
2.	It is denoted by μ.	It is denoted by ν.
3.	Mathematically, $\mu = \dfrac{\tau}{\left[\dfrac{du}{dy}\right]}$	Mathematically, $\nu = \dfrac{\mu}{\rho}$
4.	The unit of μ is Ns/m^2.	The unit of ν is m^2/s.

PART-3

Compressibility, Surface Tension and Incompressible Flow.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.6. **Explain the following :**

a. Compressibility,
b. Surface tension, and
c. Incompressible flow.

Answer

a. Compressibility :

1. The property by virtue of which fluids undergo a change in volume under the action of external pressure is known as compressibility.
2. It is the reciprocal of bulk modulus of elasticity which is defined as the ratio of compressive stress to volumetric strain.
3. Let, V = Volume of gas enclosed in the cylinder, and
   p = Pressure of gas when volume is V.
4. If the pressure is increased to $p + dp$, the volume of gas decreases from V to $V - dV$.

$$\therefore \quad \text{Volumetric strain} = -\frac{dV}{V}$$

5. Bulk modulus, $K = \dfrac{\text{Increase of pressure}}{\text{Volumetric strain}} = \dfrac{dp}{-dV/V}$

   And, compressibility $= \dfrac{1}{K}$

b. Surface Tension :

1. It is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids.
2. It is denoted by sigma (σ) and its SI unit is N/m.
3. This occurs due to the force of cohesion at the free surface as shown in Fig. 1.6.1.

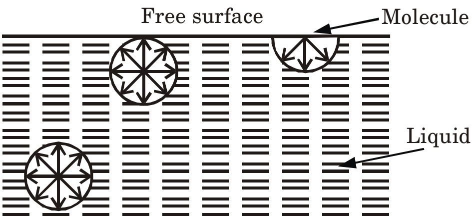

Fig. 1.6.1.

4. Consider a liquid molecule in the interior of liquid mass, surrounded by other molecules all around and is in equilibrium.
5. At the free surface of the liquid, there are no liquid molecules above the surface to balance the force of the molecules below it.
6. As a result, there is a net inward force on the molecule and this force is normal to the surface.
7. Thus at the free surface a thin layer of molecules is formed which acts as membrane because of which a thin small needle can float on the free surface.

c. Incompressible Flow :

1. It is that type of flow in which the density is constant for the fluid flow.

   i.e., ρ = Constant
2. Incompressible flow is also known as isochoric flow which means same area or space.
3. These type of flow are easy to model as temperature and pressure of liquid and gases do not vary in incompressible flow.

Que 1.7. Determine the bulk modulus of elasticity and compressibility of a liquid. If the pressure of liquid is increased from 70 N/cm^2 to 130 N/cm^2. The volume of liquid decreases by 0.15 %.

AKTU 2018-19, Marks 07

Answer

Given : $dp = 130 - 70 = 60$ N/cm^2, $dV = 0.15$ %
To Find : Bulk modulus of elasticity and compressibility of liquid.

1. Bulk modulus, $$K = \frac{dp}{dV} = \frac{60}{\frac{0.15}{100}} = 4 \times 10^4 \text{ N/cm}^2$$
2. Compressibility of liquid $$= \frac{1}{\text{Bulk modulus}} = \frac{1}{4 \times 10^4} = 2.5 \times 10^{-5} \text{ cm}^2\text{/N}$$

Que 1.8. **What should be the diameter of a droplet of water, if the pressure inside is to be 0.0018 kg(f)/cm^2 greater than the outside ? Given the value of surface tension of water in contact with air at 20 °C as 0.0075 kg(f)/m.** **AKTU 2015-16, Marks 05**

Answer

Given : p = 0.0018 kg(f)/cm^2, σ = 0.0075 kg(f)/m = 7.5 × 10^{-5} kg(f)/cm

To Find : Diameter of droplet of water.

1. For water droplet,

$$p = \frac{4\sigma}{d}$$

$$\therefore \quad d = \frac{4\sigma}{p}$$

$$d = \frac{4 \times 7.5 \times 10^{-5}}{0.0018} = 0.1667 \text{ cm}$$

PART-4

Bernoulli's Equation and its Applications-Pitot Tube, Orifice Meter, Venturimeter and Bend Meter.

CONCEPT OUTLINE

Continuity Equation : It states that the discharge throughout the flow remains constant.

Mathematically, $Q = Av =$ Constant

Venturimeter : It is a device used for measuring the rate of flow of a fluid flowing through a pipe.

Pitot Tube : It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.9. **State Bernoulli's theorem for steady flow of an incompressible fluid.**

Answer

1. Bernoulli's theorem states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant.
2. It can be mathematically stated as given below,

   Pressure energy + Kinetic energy + Potential energy = Constant

$$\frac{p}{\rho g} + \frac{\text{v}^2}{2g} + z = \text{Constant}$$

3. Bernoulli's equation for real fluids is,

$$\frac{p_1}{\rho g} + \frac{\text{v}_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\text{v}_2^2}{2g} + z_2 + h_L$$

   Where, h_L = Loss of energy.

Que 1.10. **How will you obtain Bernoulli's equation from Euler's equation of motion along a streamline ? Write assumptions of Bernoulli's equation.**

Answer

A. Assumptions :

1. The fluid should be ideal, *i.e.*, viscosity is zero.
2. The flow should be steady.
3. The flow should be incompressible.
4. The flow should be irrotational.

B. Bernoulli's Equation from Euler's Equation :

1. Euler's equation of motion is given by,

$$\frac{\partial p}{\rho} + g dz + \text{v}\, d\text{v} = 0$$

2. On integrating, we have

$$\int \frac{dp}{\rho} + \int g dz + \int \text{v}\, d\text{v} = \text{Constant}$$

$$\frac{p}{\rho} + gz + \frac{\text{v}^2}{2} = \text{Constant}$$

$$\frac{p}{\rho g} + z + \frac{\text{v}^2}{2g} = \text{Constant} \qquad ...(1.10.1)$$

   Where, $\frac{p}{\rho g}$ = Pressure head,

$$\frac{v^2}{2g} = \text{Kinetic head, and}$$

$$z = \text{Potential head.}$$

Eq. (1.10.1) is known as Bernoulli's equation.

Que 1.11. **Water flows through a 0.9 m diameter pipe at the end of which there is a reducer connecting to a 0.6 m diameter pipe. If the gauge pressure at the entrance to the reducer is 412.02 kN/m² and the velocity is 2 m/s, determine the resultant thrust on the reducer, assuming that the frictional loss of head in the reducer is 1.5 m.**

Answer

Given : d_1 = 0.9 m, d_2 = 0.6 m, p_1 = 412.02 kN/m², v_1 = 2 m/s, h_f = 1.5 m

To Find : Resultant thrust.

1. From continuity equation,

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{v_1 A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 \times v_1 = \left(\frac{0.9}{0.6}\right)^2 \times 2 = 4.5 \text{ m/s}$$

2. Applying Bernoulli's theorem at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

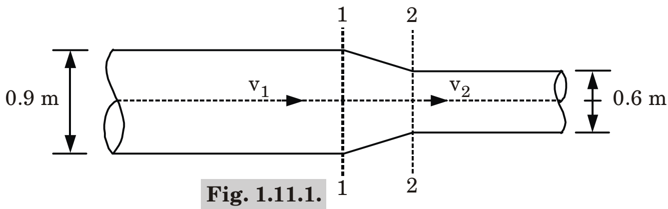

Fig. 1.11.1.

As it is horizontal pipe, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h_f$$

$$\frac{412.02 \times 10^3}{1000 \times 9.81} + \frac{2^2}{2 \times 9.81} = \frac{p_2}{1000 \times 9.18} + \frac{(4.5)^2}{2 \times 9.18} + 1.5$$

$$p_2 = 389.18 \text{ kN/m}^2$$

Hence, resultant thrust on the reducer = 389.18 kN/m²

Que 1.12. **Suggest the device used for the measurement of fluid flow through ducts or pipes. Explain them.**

OR

What are the various applications of Bernoulli's equation ? Explain them.

Answer

Some of the simple applications of Bernoulli's equation are as follows :

A. Venturimeter :

1. A venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe.
2. It consists of three parts, as given below :
   i. A short converging part,
   ii. Throat, and
   iii. Diverging part.
3. It works on the principle of Bernoulli's theorem.
4. As shown in Fig. 1.12.1, a venturimeter is fitted in a horizontal pipe through which a fluid is flowing.

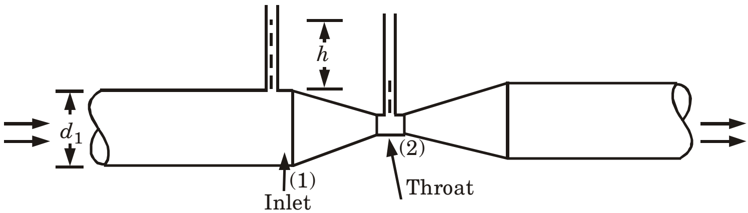

Fig. 1.12.1.

5. Let,

d_1 = Diameter of pipe at section (1),

p_1 = Pressure at section (1),

v_1 = Velocity of fluid at section (1), and

a_1 = Area at section (1) = $\frac{\pi}{4} d_1^2$.

d_2, p_2, v_2, a_2 = Corresponding values at section (2).

6. By applying Bernoulli's theorem at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad ..(1.12.1)$$

7. As it is horizontal pipe, hence $z_1 = z_2$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad ...(1.12.2)$$

8. But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections (1) and (2) and it is equal to h,

$$\frac{p_1 - p_2}{\rho g} = h$$

$$\therefore \quad h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad ...(1.12.3)$$

9. Applying continuity equation at sections (1) and (2), we have

$$a_1 v_1 = a_2 v_2 \text{ or } v_1 = \frac{a_2 v_2}{a_1}$$

10. Substituting this value of v_1 in eq. (1.12.3), we get

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

or $$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore \quad v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

11. Discharge, $$Q = a_2 v_2$$

$$Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad ...(1.12.4)$$

12. Eq. (1.12.4) gives the discharge under ideal conditions called as theoretical discharge whereas actual discharge will be less than theoretical discharge and is given by,

$$\therefore \quad Q_{act} = \frac{C_d \, a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}} \qquad \left(\because C_d = \frac{Q_{act}}{Q_{theo}}\right)$$

Where C_d is the coefficient of discharge for venturimeter and its value is less than unity.

13. It the liquid flowing in pipe and liquid in U-tube manometer have different specific gravity, following cases may be considered to obtain the level of difference of two liquids :

Case I :

1. If pipe is horizontal ($i.e., z_1 = z_2$) and the differential manometer contains liquid heavier than the liquid flowing through the pipe.

Then, $$h = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

Where, S_h = Specific gravity of the heavier liquid,

S_o = Specific gravity of the liquid flowing through pipe, and

x = Difference of the heavier liquid column in U-tube.

Case II :

1. If pipe is horizontal ($i.e., z_1 = z_2$) and the differential manometer contains liquid lighter than the liquid flowing through the pipe.

Then, $$h = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) = x\left[1 - \frac{S_l}{S_o}\right]$$

Where, S_l = Specific gravity of lighter liquid in U-tube, and

x = Difference of the lighter liquid columns in U-tube.

Case III :

1. If pipe is inclined and the differential manometer contains liquid heavier than the liquid flowing through the pipe.

Then, $$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

Case IV :

1. If pipe is inclined and the differential manometer contains liquid lighter than the liquid flowing through the pipe.

Then, $$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_1}{S_o}\right]$$

B. Orifice Meter :

1. It works on the Bernoulli's principle and is a device used for measuring the rate of flow of a fluid flowing through a pipe.
2. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe.

3. It is cheaper device as compared to venturimeter.

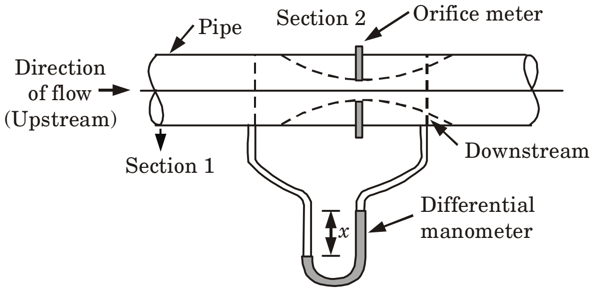

Fig. 1.12.2. Orifice Meter.

3. As shown in Fig. 1.12.2, let,

p_1 = Pressure at section (1),

v_1 = Velocity of flow at section (1),

a_1 = Area of pipe at section (1), and

p_2, v_2, a_2 = Corresponding values at section (2).

4. Applying Bernoulli's equation at section (1) and (2), we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1 = \frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

$$\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right) = \frac{v_2^2}{2g}-\frac{v_1^2}{2g}$$

Where, $\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right) = h$ = Differential head

$$h = \frac{v_2^2}{2g}-\frac{v_1^2}{2g}$$

$$v_2 = \sqrt{2gh+v_1^2} \qquad \text{...(1.12.5)}$$

5. Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta.

6. If a_0 is the area of orifice then,

$$C_C = \frac{a_2}{a_0}$$

$$a_2 = a_0\, C_C \qquad \text{...(1.12.6)}$$

Where, C_C = Coefficient of contraction.

7. By continuity equation,

$$a_1 \mathrm{v}_1 = a_2 \mathrm{v}_2$$

$$\mathrm{v}_1 = \frac{a_0 C_C}{a_1} \mathrm{v}_2 \quad ...(1.12.7)$$

8. Substituting the value of v_1 in eq. (1.12.5), we get

$$\mathrm{v}_2 = \sqrt{2gh + \left(\frac{a_0 C_C}{a_1}\right)^2 \mathrm{v}_2^2}$$

$$\mathrm{v}_2{}^2 = 2gh + \left(\frac{a_0 C_C}{a_1}\right)^2 \mathrm{v}_2^2$$

$$\mathrm{v}_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_C^2\right] = 2gh$$

$$\therefore \qquad \mathrm{v}_2 = \frac{\sqrt{2\, gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_C^2}}$$

9. Discharge, $Q = \mathrm{v}_2 a_2 = \mathrm{v}_2 \times a_0\, C_C$ $\quad (\because\ a_2 = a_0\, C_C)$

i.e.,

$$Q = \frac{a_0 C_C \sqrt{2\, gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad ...(1.12.8)$$

10. The above expression can be simplified by using,

$$C_d = C_C \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_C^2}}$$

$$C_C = C_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_C^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

11. Substituting the value of C_C in eq. (1.12.8), we get

$$Q = a_0 \times C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_C^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_C^2}}$$

$$= \frac{C_d a_0 \sqrt{2\,gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2\,gh}}{\sqrt{a_1^2 - a_0^2}}$$

Where, C_d = Coefficient of discharge for orifice meter.

12. The coefficient of discharge for orifice meter is much smaller than that for a venturimeter.

Que 1.13. **A horizontal venturimeter with a discharge coefficient of 0.98 is being used to measure the flow rate of a liquid of density 1030 kg/m³. The pipe diameter at entry to the venturi is 75 mm and the venturi throat has an area of 1000 mm². If the flow rate is 0.011 m³/s. Determine the height difference recorded on a *U*-tube manometer connecting the throat to the upstream pipe. Take the relative density of mercury to be 13.6.** **AKTU 2016-17, Marks 05**

Answer

Given : $C_d = 0.98$, $\rho_l = 1030$ kg/m³, $S_l = \frac{1030}{1000} = 1.03$ kg/m³,
$S_{Hg} = 13.6$, $d_1 = 75$ mm $= 0.075$ m, $Q = 0.011$ m³/s, $a_2 = 1000$ mm² $= 1 \times 10^{-3}$ m²

To find : Height difference.

1. As we know, flow rate, $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (.075)^2 = 4.4178 \times 10^{-3}\ \text{m}^2$$

$$0.011 = 0.98 \times \frac{4.4178 \times 10^{-3} \times 1 \times 10^{-3}}{\sqrt{(4.4178 \times 10^{-3})^2 - (1 \times 10^{-3})^2}} \times \sqrt{2 \times 9.81 \times h}$$

$$h = 6.092\ \text{m}$$

2. We know that, $$h = x\left[\frac{S_{Hg}}{S_l} - 1\right]$$

$$6.092 = x\left[\frac{13.6}{1.03} - 1\right]$$

$$6.092 = x \times \frac{12.57}{1.03}$$

$$x = 0.4992 \text{ m}$$

Que 1.14. **In a vertical conveying oil of specific gravity 0.8, two pressure gauges have been installed at *A* and *B* where the diameters are 16 cm and 8 cm respectively. *A* is 2 m above *B*. The pressure gauge readings have shown that the pressure at *B* is greater than at *A* by 0.981 N/cm^2. Neglecting all losses, calculate the flow rate. If the gauges at *A* and *B* are replaced by tubes filled with the same liquid and connected to a *U*-tube containing mercury. Calculate the difference of level of mercury in the two limbs of *U*-tube.**

AKTU 2014-15, Marks 05

Answer

Given : $S_o = 0.8$, $\rho_o = 0.8 \times 1000 = 800$ kg/m^3,

$D_A = 16$ cm $= 0.16$ m, $D_B = 8$ cm $= 0.08$ m, $z_A - z_B = 2$ m

To Find : i. Flow rate.

ii. Difference in the level of mercury.

1. Area at *A*, $$A_A = \frac{\pi}{4}(0.16)^2 = 0.0201 \text{ m}^2$$

Area at *B*, $$A_B = \frac{\pi}{4}(0.08)^2 = 0.005026 \text{ m}^2$$

2. Difference of pressures,

$$p_B - p_A = 0.981 \text{ N/cm}^2 = 0.981 \times 10^4 \text{ N/m}^2 = 9810 \text{ N/m}^2$$

3. Pressure head, $$= \frac{p_B - p_A}{\rho g} = \frac{9810}{800 \times 9.81} = 1.25 \text{ m}$$

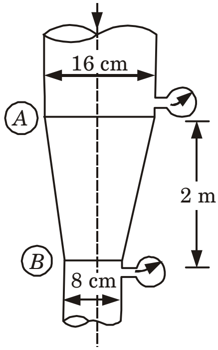

Fig. 1.14.1.

4. Applying Bernoulli's theorem at A and B taking the reference line passing through section B,

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$

$$\frac{p_A}{\rho g} - \frac{p_B}{\rho g} + z_A - z_B = \frac{v_B^2}{2g} - \frac{v_A^2}{2g}$$

$$-1.25 + 2.0 = \frac{v_B^2}{2g} - \frac{v_A^2}{2g} \qquad \left(\because \frac{p_B - p_A}{\rho g} = 1.25\right)$$

$$0.75 = \frac{v_B^2}{2g} - \frac{v_A^2}{2g} \qquad \text{... (1.14.1)}$$

5. Now applying continuity equation at A and B, we get

$$v_A A_A = v_B A_B$$

$$v_B = \frac{v_A A_A}{A_B} = \frac{v_A \times 0.0201}{0.005026} = 4v_A$$

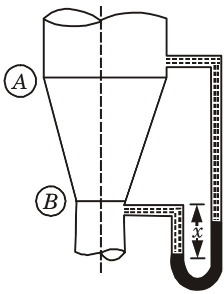

Fig. 1.14.2.

6. Substituting the value of v_B in eq. (1.14.1), we get

$$0.75 = \frac{16 v_A^2}{2g} - \frac{v_A^2}{2g} = \frac{15 v_A^2}{2g}$$

$$\therefore \quad v_A = \sqrt{\frac{0.75 \times 2 \times 9.81}{15}} = = 0.99 \text{ m/s}$$

7. Rate of flow, $Q = v_A A_A = 0.99 \times 0.0201 = 0.01989 \text{ m}^3/\text{s}$

8. We know that, $h = x \left(\frac{S_g}{S_o} - 1 \right)$

Where, $h = \left(\frac{p_A}{\rho g} + z_A \right) - \left(\frac{p_B}{\rho g} + z_B \right) = \frac{p_A - p_B}{\rho g} + z_A - z_B$

$$= -1.25 + 2.0 = 0.75 \qquad \left(\because \frac{p_B - p_A}{\rho g} = 1.25 \right)$$

$$\therefore \quad 0.75 = x \left[\frac{13.6}{0.8} - 1 \right] = x \times 16$$

Difference of level of mercury in the U-tube,

$$x = 0.04687 \text{ m} = 4.687 \text{ cm}$$

Que 1.15. **Explain Elbow meter with neat sketch and give its application.**

Answer

A. Elbow Meter :

1. When a liquid flows in a pipe bend, there exists a difference of pressure between the outside and inside of the bend. This difference of pressure is used to measure the discharge in pipeline.
2. In a pipe bend, the pressure at the outer wall of bend is more than that at the inner wall.
3. Now from Fig. 1.15.1, we see a pipe bend with two pressure p_o at outside wall and p_i at inside wall of the pipe. These two points are connected to the limbs of U-tube manometer.
4. From the relation between velocity and pressure difference, we have

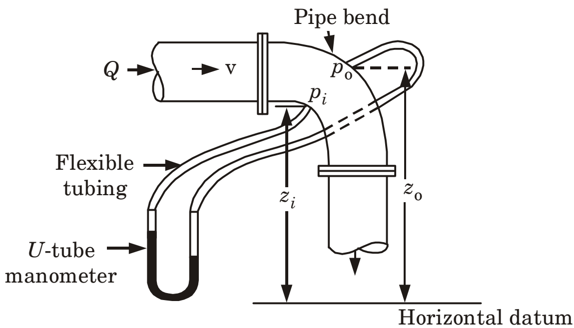

Fig. 1.15.1. Elbow meter.

$$K\frac{v^2}{2g} = \left(\frac{p_o}{w}+z_o\right)-\left(\frac{p_i}{w}+z_i\right)$$

$$v = \frac{1}{\sqrt{K}} \times \sqrt{2g} \times \sqrt{\left(\frac{p_o}{w}+z_o\right)-\left(\frac{p_i}{w}+z_i\right)}$$

Where, K = Constant (1.3 to 3.2 depends upon size and shape of the bend).

v = Velocity of flow.

5. Discharge, $Q = Av = C_d A\sqrt{2g\left[\left(\frac{p_o}{w}+z_o\right)-\left(\frac{p_i}{w}+z_i\right)\right]}$

Where, C_d = Coefficient of discharge = $\frac{1}{\sqrt{K}}$ $(0.56 \le C_d \le 0.88)$

B. Applications :

1. An elbow meter can be used for the measurement of discharge in pipes which are fitted with bends or elbow.

PART-5

Notches and Weirs.

CONCEPT OUTLINE

Notch : A notch is a device used for the measurement of the rate of flow of a liquid through a small channel or tank.
Weir : A weir is a concrete or masonry structure placed in an open channel over which the flow occurs.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.16. What is notch ? What are the different types of notches ?

Answer

A. Notch :

1. A notch is a device used for the measurement of the rate of flow of a liquid through a small channel or a tank.
2. It may also be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

B. Types of Notches : The different types of notches are as follows :

i. Rectangular notch,
ii. Triangular notch,
iii. Trapezoidal notch, and
iv. Stepped notch.

Que 1.17. Derive the expression for discharge over the following notches :

A. Rectangular, and
B. Triangular.

Answer

A. Discharge over Rectangular Notch :

1. As shown in Fig. 1.17.1, consider a rectangular notch provided in a channel carrying water.
2. Let, H = Head of water over the crest, and
L = Length of notch.
3. In order to find the discharge of water flowing over the notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water.

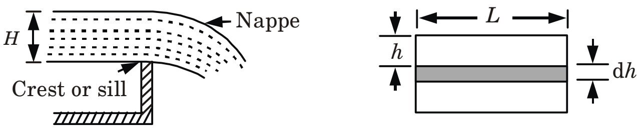

Fig. 1.17.1.

4. The area of strip = $L \times dh$

Theoretical velocity of water flowing through strip = $\sqrt{2gh}$

5. The discharge dQ through strip is,

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d L\, dh\, \sqrt{2gh} \quad ...(1.17.1)$$

6. The total discharge, Q for the whole notch is determined by integrating the eq. (1.17.1) between the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d L\sqrt{2gh}\, dh = C_d L\sqrt{2g}\int_0^H h^{1/2} dh$$

$$= C_d L\sqrt{2g}\left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$Q = \frac{2}{3} C_d L\sqrt{2g}\, H^{3/2}$$

B. Discharge Over Triangular Notch :

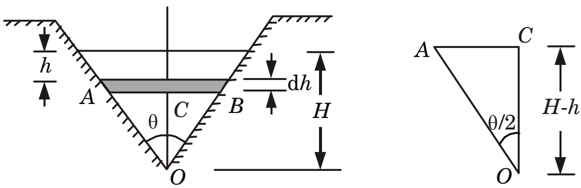

Fig. 1.17.2.

1. Let, H = Head of water above the notch, and

θ = Angle of notch.

2. Consider a horizontal strip of water having thickness dh at a depth of h from the free surface of water.

3. From the geometry of notch,

$$\tan \theta/2 = \frac{AC}{OC} = \frac{AC}{H-h}$$

$$\therefore \quad AC = (H-h)\tan\left(\frac{\theta}{2}\right)$$

$$\text{Width of strip} = AB = 2 \times AC = 2\,(H-h)\tan\left(\frac{\theta}{2}\right)$$

4. The theoretical velocity of water through strip = $\sqrt{2gh}$

5. Discharge through the strip is,

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity(theoretical)}$$

$$= C_d\, 2\,(H-h)\tan\left(\frac{\theta}{2}\right)\, dh\, \sqrt{2gh}$$

$$= 2\, C_d\,(H-h)\tan\left(\frac{\theta}{2}\right)\sqrt{2gh}\, dh$$

6. Total discharge is,

$$Q = \int_0^H 2C_d(H-h)\tan\left(\frac{\theta}{2}\right)\times\sqrt{2gh}\; dh$$

$$= 2C_d\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\int_0^H (H-h)\, h^{1/2}dh$$

$$= 2C_d\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\int_0^H (Hh^{1/2}-h^{3/2})\, dh$$

$$= 2C_d\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\left[\frac{Hh^{3/2}}{3/2}-\frac{h^{5/2}}{5/2}\right]_0^H$$

$$Q = \frac{8}{15}C_d\tan\left(\frac{\theta}{2}\right)\sqrt{2g}\; H^{5/2}$$

Que 1.18. Define weir and give its classification. Differentiate between notch and weir.

Answer

A. Weir :

1. It is any regular obstruction in an open stream over which the flow takes place.

B. Classification of Weirs :

a. On the Basis of Shape :

1. Rectangular weir, and
2. Cipoletti weir.

b. On the Basis of Nature of Discharge :

1. Ordinary weir, and
2. Submerged weir.

c. On the Basis of the Width of Crest :

1. Narrow crested, and
2. Broad crested.

d. According to the Nature of Crest :

1. Sharp crested weir, and
2. Ogee weir.

e. On the Basis of the Effect of Sides on the Emerging Nappe :

1. Weir with end contraction, and
2. Weir without end contraction.

C. Difference between Notch and Weir :

S. No.	Notch	Weir
1.	The size of notch is very small.	The size of weir is large.
2.	It is used to determine the flow through small tanks or pipes.	It is used to measure the flow of rivers.

Que 1.19. **Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.4 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion = 0.62 while for triangular portion = 0.60.**

AKTU 2017-18, Marks 10

Answer

Given : $AE = 1$ m, $CD = L = 0.4$ m, $h = 0.3$ m, $H = 0.20$ m, For rectangular portion, $C_{d1} = 0.62$, For triangular portion, $C_{d2} = 0.60$
To Find : Discharge through trapezoidal notch.

1. From ΔABC, we have

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{(AE - CD)/2}{BC} = \frac{(1.0 - 0.4)/2}{0.3} = \frac{0.6/2}{0.3} = 1$$

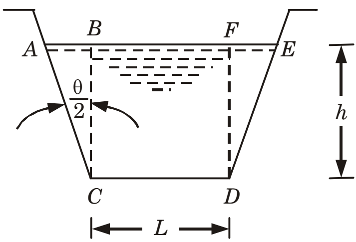

Fig. 1.19.1.

2. Discharge through trapezoidal notch is given as,

$$Q = \frac{2}{3} C_{d1} L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_{d2} \tan\frac{\theta}{2} \sqrt{2g}\, H^{5/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times (0.2)^{2/3} + \frac{8}{15} \times 0.60 \times 1 \times \sqrt{2 \times 9.81} \times (0.2)^{5/2}$$

$$= 0.0655 + 0.02535 = 0.09085 \text{ m}^3/\text{s} = 90.85 \text{ litres/s}$$

PART-6

Momentum Equation and its Application to Pipe Bend.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.20. Describe momentum equation. Where this equation is used ?

Answer

A. Momentum Equation :

1. This equation is based on the law of conservation of momentum or on the momentum principle.
2. According to law of conservation of momentum, the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in the direction of force.
3. According to Newton's second law of motion,

$$F = ma$$

Where, m = Mass of fluid,

a = Acceleration in direction of force, and

F = Force acting on fluid.

$$F = m\frac{d\text{v}}{dt} \qquad \left(\because\ a = \frac{d\text{v}}{dt}\right)$$

$$F = \frac{d(m\text{v})}{dt} \qquad (\because\ m \text{ is constant})$$

This is known as the momentum principle or momentum equation.

B. Uses :

1. This equation is used to determine the force or impulse acting at the bend in the bend pipes, reducers, moving vanes and jet propulsion etc.
2. This equation is used to determine the characteristics of flow in sudden enlargement in a pipe.

Que 1.21. Derive an expression for the force exerted by a flowing fluid on a pipe bend.

Answer

1. Consider section x-x and y-y in a bend pipe having pressure, cross section area and velocity as p_1, A_1, v_1 at x-x section and p_2, A_2, v_2 at y-y section.

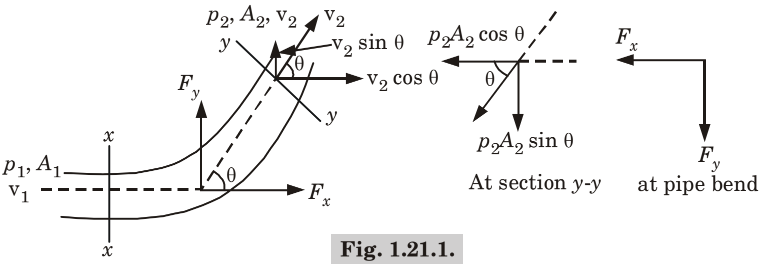

Fig. 1.21.1.

2. Forces F_x and F_y are acting on the pipe bend due to fluid flow but force exerted by the pipe bend F_x and F_y are acting in opposite direction.
3. Using impulse momentum equation in X-direction,

$$p_1A_1 - p_2A_2 \cos\theta - F_x = \frac{d}{dt}(mv)$$

$$p_1A_1 - p_2A_2 \cos\theta - F_x = \rho Q\,(v_2 \cos\theta - v_1)$$

Where, ρQ = Mass of fluid flowing per second, and

$v_2 \cos\theta - v_1$ = Change in velocity in X-direction.

$$\therefore \quad F_x = \rho Q\,(v_1 - v_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta$$

4. Now, using impulse momentum equation in Y-direction,

$$0 - p_2A_2 \sin\theta - F_y = \rho Q\,(v_2 \sin\theta - 0)$$

$$\therefore \quad F_y = -\rho Q\, v_2 \sin\theta - p_2A_2 \sin\theta$$

5. Now the resultant force F_R acting on the bend,

$$F_R = \sqrt{F_x^2 + F_y^2}$$

6. Direction of resultant force,

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Que 1.22. Water is flowing in a 300 mm pipeline fitted with a 45° bend in the vertical plane. The diameter at the outlet of the bend is 150 mm. The pipe axis at the inlet is horizontal and the outlet is 1.5 m above the inlet. If the flow through the bend is 0.4 m³/s and a head loss of 0.5 m occurs in the bend, calculate the magnitude and direction of the resultant force the bend support must withstand. The volume of the bend is 0.075 m³ and the pressure at the inlet is 300 kN/m².

AKTU 2016-17, Marks 05

Answer

Given : d_1 = 300 mm, d_2 = 150 mm, $\theta_1 = 0°$, $\theta_2 = 45°$, Q = 0.4 m³/s
h_f = 0.5 m, V = 0.075 m³, p_1 = 300 kN/m², $z_1 = 0$, z_2 = 1.5 m
To Find : Magnitude and direction of resultant force.

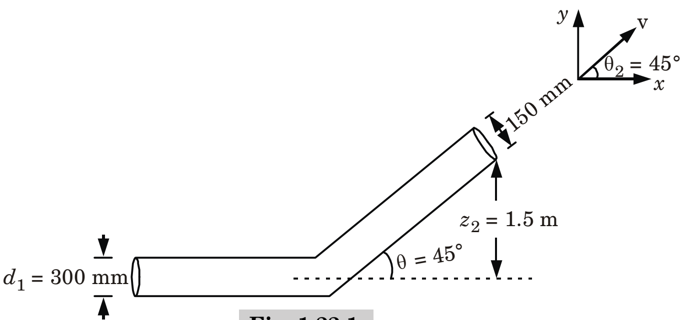

Fig. 1.22.1.

1. For continuity of flow,

$$Q = A_1 v_1 = A_2 v_2$$

$$0.4 = \frac{\pi}{4}(0.3)^2 v_1 = \frac{\pi}{4}(0.15)^2 v_2$$

$$v_1 = 5.66 \text{ m/s}$$

$$v_2 = 22.64 \text{ m/s}$$

2. By applying Bernoulli's equation,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

$$\frac{300 \times 10^3}{1000 \times 9.81} + \frac{(5.66)^2}{2 \times 9.81} + 0 = \frac{p_2}{1000 \times 9.81} + \frac{(22.64)^2}{2 \times 9.81} + 1.5 + 0.5$$

$$p_2 = 40113.09 \text{ N/m}^2 = 40.113 \text{ kN/m}^2$$

3. Now applying the impulse momentum equation in both X and Y direction.
   i. For X-direction :

$$p_1A_1 \cos \theta_1 - p_2A_2 \cos \theta_2 - F_x = \rho Q\,(v_2 \cos \theta_2 - v_1 \cos \theta_1)$$

$$300 \times 10^3 \times \frac{\pi}{4}(0.3)^2 \times \cos 0° - 40.113 \times 10^3 \times \frac{\pi}{4} \times (0.15)^2 \times \cos 45° - F_x$$

$$= 1000 \times 0.4\,(22.64 \cos 45° - 5.64 \cos 0°)$$

$$F_x = 16.556 \text{ kN}$$

   ii. For Y-direction :

$$p_1A_1 \sin \theta_1 - p_2A_2 \sin \theta_2 + F_y + W = \rho Q\,(v_2 \sin \theta_2 + v_1 \sin \theta_1)$$

$$F_y - 40.113 \times 10^3 \times \frac{\pi}{4}(0.15)^2 \sin 45° + (0.075 \times 9810)$$

$$(\because\ \theta_1 = 0°)$$

$$= 1000 \times 0.4\,(22.64 \sin 45°)$$

$$F_y = 6.169 \text{ kN}$$

4. Resultant Force, $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(16.556)^2 + (6.169)^2}$

$$F_R = 17.67 \text{ kN}$$

5. We know that, $\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$$\alpha = \tan^{-1}\left(\frac{6.169}{16.556}\right) = 20.436°$$

Thus, force of 17.67 kN acts on the bend at an angle of 20.436° from inlet axis.

Que 1.23. In a 45° bend a rectangular air duct of 1 m^2 cross-sectional area is gradually reduces to 0.5 m^2 area. Find the magnitude and direction of the force required to hold the duct in position if the velocity of flow at the 1 m^2 section is 10 m/s and pressure is 2.943 N/cm^2. Take density of air as 1.16 kg/m^3.

AKTU 2014-15, Marks 05

Answer

Given : $A_1 = 1\ m^2, A_2 = 0.5\ m^2$, $v_1 = 10$ m/s, $p_1 = 2.943\ N/cm^2$ $= 2.943 \times 10^4\ N/m^2$, $\rho = 1.16\ kg/m^3$
To Find : Magnitude and direction of the force.

1. Applying continuity equation at sections (1) and (2), we have

$$A_1 v_1 = A_2 v_2$$

$$\therefore \quad v_2 = \frac{A_1 v_1}{A_2} = \frac{1}{0.5} \times 10 = 20 \text{ m/s}$$

2. Discharge, $Q = A_1 v_1 = 1 \times 10 = 10\ m^3/s$

3. Applying Bernoulli's equation at (1) and (2), we have

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \qquad (\because \quad z_1 = z_2)$$

$$\frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{10^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{20^2}{2 \times 9.81}$$

$$\therefore \quad \frac{p_2}{\rho g} = \frac{2.943 \times 10^4}{1.16 \times 9.81} + \frac{10^2}{2 \times 9.81} - \frac{20^2}{2 \times 9.81}$$

$$= 2586.2 + 5.0968 - 20.387 = 2570.90 \text{ m}$$

$$\therefore \quad p_2 = 2570.90 \times 1.16 \times 9.81 = 29255.8 \text{ N}$$

5. Force along X-axis, $F_x = \rho Q\ [v_{1x} - v_{2x}] + (p_1 A_1)_x + (p_2 A_2)_x$

Where, $v_{1x} = 10$ m/s, $v_{2x} = v_2 \cos 45° = 20 \times 0.7071$

$$(p_1 A_1)_x = p_1 A_1 = 29430 \times 1 = 29430 \text{ N}$$

and $(p_2 A_2)_x = -p_2 A_2 \cos 45°$

$$= -29255.8 \times 0.5 \times 0.7071$$

$$\therefore \quad F_x = 1.16 \times 10 \times [10 - 20 \times 0.7071] + 29430 \times 1 - 29255.8 \times 0.5 \times 0.7071$$

$$= -48.05 + 29430 - 10343.39$$

$$= 19038.56 \text{ N}$$

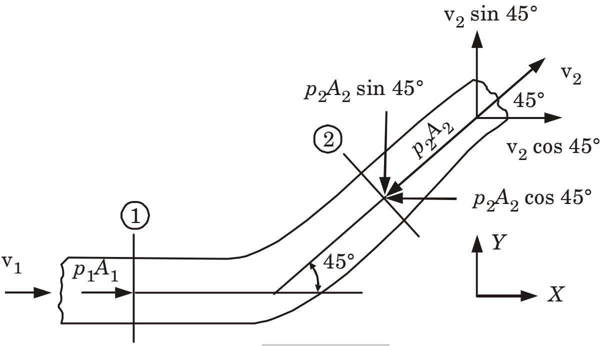

Fig. 1.23.1.

6. Similarly force along Y-axis, $F_y = \rho Q[v_{1y} - v_{2y}] + (p_1A_1)_y + (p_2A_2)_y$

Where, $v_{1y} = 0$, $v_{2y} = v_2 \sin 45° = 20 \times 0.7071 = 14.142$

$$(p_1A_1)_y = 0,\ (p_2A_2)_y = -p_2A_2 \sin 45° = -29255.8 \times 0.5 \times 0.7071$$
$$= -10343.39$$
$$F_y = 1.16 \times 10[0 - 14.142] + 0 - 10343.39$$
$$= -164.05 - 10343.39 = -10507.44 \text{ N}$$

7. Resultant force, $F_R = \sqrt{F_x^{\ 2} + F_y^{\ 2}} = \sqrt{(19038.56)^2 + (10507.44)^2}$

$$= 21746.65 \text{ N}$$

8. The direction of F_R with X-axis is given as,

$$\tan \theta = \frac{F_y}{F_x} = \frac{10507.44}{19038.56} = 0.5519$$

$\therefore$ $$\theta = \tan^{-1} 0.5519 = 28°\ 53'40''.$$

Hence the force required to hold the duct in position is equal to 21746.65 N but it is acting in the opposite direction of F_R.

Que 1.24. A 30 cm diameter horizontal pipe terminates in a nozzle with the exit diameter of 7.5 cm. If the water flows through the pipe at a rate of 0.15 m³/s. What force will be exerted by the fluid on the nozzle ?

AKTU 2018-19, Marks 07

Answer

Given : $d_1 = 30$ cm $= 0.3$ m, $d_2 = 7.5$ cm $= 0.075$ m, $Q = 0.15\ \text{m}^3/\text{s}$
To Find : Force exerted by the fluid on the nozzle.

1. Area of pipe, $A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.071 \text{ m}^2$

2. Area of nozzle, $A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.075)^2 = 4.418 \times 10^{-3} \text{ m}^2$

3. Applying continuity equation,

$$A_1 \text{v}_1 = A_2 \text{v}_2 = Q$$

$$\text{v}_1 = \frac{Q}{A_1} = \frac{0.15}{0.071} = 2.112 \text{ m/s}$$

$$\text{v}_2 = \frac{Q}{A_2} = \frac{0.15}{4.418 \times 10^{-3}} = 33.95 \text{ m/s}$$

4. We know that,

$$\frac{p_1}{\rho g} + \frac{\text{v}_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\text{v}_2^2}{2g} + z_2$$

$$\frac{p_1}{\rho g} + \frac{\text{v}_1^2}{2g} = \frac{\text{v}_2^2}{2g} \quad \left[\because \frac{p_2}{\rho g} = \text{Atmospheric pressure} = 0, z_1 = z_2\right]$$

$$\frac{p_1}{\rho g} = \frac{\text{v}_2^2 - \text{v}_1^2}{2g}$$

$$p_1 = \left[\frac{(33.95)^2 - (2.112)^2}{2}\right] \times 1000 = 574070.978 \text{ N/m}^2$$

5. Net force in direction of x, F_x = Rate of change of momentum in direction x.

$$p_1 A_1 - p_2 A_2 + F_n = \rho Q (\text{v}_2 - \text{v}_1)$$

Where, F_n = Force exerted by fluid on nozzle.

$$574070.978 \times 0.071 - 0 + F_n = 1000 \times 0.15 \times (33.95 - 2.112)$$

$$40759.04 + F_n = 4775.7$$

$$F_n = -35983.34 \text{ N}$$

Here negative sign indicates that the force exerted by the nozzle on water is acting from right to left.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Discuss some physical properties of fluids in brief.

Ans. Refer Q. 1.4, Unit-1.

Q. 2. Explain the following :

a. Compressibility,
b. Surface tension, and
c. Incompressible flow.

Ans. Refer Q. 1.6, Unit-1.

Q. 3. Determine the bulk modulus of elasticity and compressibility of a liquid. If the pressure of liquid is increased from 70 N/cm^2 to 130 N/cm^2. The volume of liquid decreases by 0.15 %.

Ans. Refer Q. 1.7, Unit-1.

Q. 4. Suggest the device used for the measurement of fluid flow through ducts or pipes. Explain them.

Ans. Refer Q. 1.12, Unit-1.

Q. 5. A horizontal venturimeter with a discharge coefficient of 0.98 is being used to measure the flow rate of a liquid of density 1030 kg/m^3. The pipe diameter at entry to the venturi is 75 mm and the venturi throat has an area of 1000 mm^2. If the flow rate is 0.011 m^3/s. Determine the height difference recorded on a *U*-tube manometer connecting the throat to the upstream pipe. Take the relative density of mercury to be 13.6.

Ans. Refer Q. 1.13, Unit-1.

Q. 6. Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.4 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion = 0.62 while for triangular portion = 0.60.

Ans. Refer Q. 1.19, Unit-1.

Q. 7. Water is flowing in a 300 mm pipeline fitted with a 45° bend in the vertical plane. The diameter at the outlet of the bend is 150 mm. The pipe axis at the inlet is horizontal and the outlet is 1.5 m above the inlet. If the flow through the bend is 0.4 m^3/s and a head-loss of 0.5 m occurs in the bend, calculate the magnitude and direction of the resultant force the bend support must withstand. The volume of the bend is 0.075 m^3 and the pressure at the inlet is 300 kN/m^2.

Ans. Refer Q. 1.22, Unit-1.

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2
UNIT

Types of Fluid Flow and Continuity Equation

CONTENTS

Part-1 : Continuum .... **2–2A to 2–2A**

Part-2 : Free Molecular Flows – Steady .... **2–2A to 2–4A**
and Unsteady Flows, Uniform and Non-Uniform Flows, Laminar and Turbulent Flows, Rotational and Irrotational Flows, Compressible and Incompressible Flows

Part-3 : Subsonic, Sonic and Supersonic .... **2–4A to 2–5A**
Flows, Subcritical, Critical and Super Critical Flows, One, Two and Three Dimensional Flows

Part-4 : Streamlines .... **2–5A to 2–6A**

Part-5 : Continuity Equation for 3D and .... **2–6A to 2–11A**
1D Flows

Part-6 : Circulation .... **2–11A to 2–13A**

Part-7 : Stream Function .... **2–14A to 2–19A**

Part-8 : Velocity Potential .... **2–19A to 2–24A**

Part-9 : Buckingham's-Pi Theorem .... **2–24A to 2–30A**

Part-10 : Important Dimensionless Numbers .... **2–30A to 2–34A**
and their Significance

PART-1

Continuum.

CONCEPT OUTLINE

Continuum : A continuous and homogeneous medium is called 'continuum'.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **Discuss continuum in brief.**

Answer

1. Continuum can be defined as a continuous and homogeneous medium.
2. The continuum concept helps to study the overall behaviour and properties of fluids without any reference to atomic and molecular structure.
3. In continuum approach, fluid properties such as density, viscosity, thermal conductivity, temperature, etc. can be expressed as continuous functions of space and time.
4. There are factors which are to be considered with great importance in determining the validity of continuum model. One such factor is the distance between molecules which is a function of molecular density.
5. The other factor which checks the validity of continuum is the elapsed time between collisions.

PART-2

Free Molecular Flows – Steady and Unsteady Flows, Uniform and Non-Uniform Flows, Laminar and Turbulent Flows, Rotational and Irrotational Flows, Compressible and Incompressible Flows.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.2. **Explain the following with example :**

a. Steady and unsteady flows,
b. Laminar and turbulent flows,
c. Rotational and irrotational flows,
d. Compressible and incompressible flows, and
e. Uniform and non-uniform flows.

Answer

a. Steady and Unsteady Flows :

1. Steady flow is that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time.

Mathematically,

$$\left(\frac{\partial \text{v}}{\partial t}\right)_{\text{at fixed point}} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{\text{at fixed point}} = 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{\text{at fixed point}} = 0$$

Example : Flow of liquid through a long pipe of constant diameter at a constant rate.

2. Unsteady flow is that type of flow in which the velocity, pressure, density, etc, at a point changes with respect to time.

Mathematically,

$$\left(\frac{\partial \text{v}}{\partial t}\right)_{\text{at fixed point}} \neq 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{\text{at fixed point}} \neq 0, \quad \left(\frac{\partial p}{\partial t}\right)_{\text{at fixed point}} \neq 0$$

Example : Flow of liquid through a long pipe of constant diameter at either increasing or decreasing rate.

b. Laminar and Turbulent Flows :

1. Laminar flow is one in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel.

Example : Flow through a capillary tube.

2. Turbulent flow is that type of flow in which the particles move in a zig-zag way.

Example : Flow in natural streams, artificial channels, sewers etc.

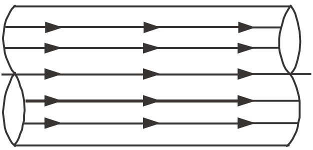

Fig. 2.2.1. Laminar flow.

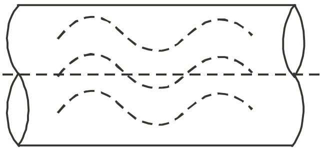

Fig. 2.2.2. Turbulent flow.

c. Rotational and Irrotational Flows :

1. Rotational flow is that type of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

   Example : Flow of liquid in the rotating tanks.

2. If the fluid particles while flowing along stream lines, do not rotate about their own axis that type of flow is called irrotational flow.

   Example : Flow over a drain hole of a stationary tank or a wash basin.

d. Compressible and Incompressible Flows :

1. Compressible flow is that type of flow in which the density of the fluid changes from point to point.

   Mathematically, $\rho \neq$ Constant

   Examples : Flow of gases through orifices nozzles, gas turbines etc.

2. Incompressible flow is that type of flow in which the density is constant for the fluid flow.

   Mathematically, $\rho =$ Constant

   Examples : Subsonic, aerodynamics.

e. Uniform and Non-uniform Flows :

1. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space.

   Mathematically, $$\left(\frac{\partial \text{v}}{\partial S}\right)_{t\,=\,\text{Constant}} = 0$$

   Where, ∂v = Change of velocity, and

   ∂S = Length of flow in the direction S.

   Example : Flow through a straight pipe of constant diameter.

2. Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space.

   Mathematically, $$\left(\frac{\partial \text{v}}{\partial S}\right)_{t\,=\,\text{Constant}} \neq 0$$

   Example : Flow around a uniform diameter pipe bend or a canal bend and flow through a non-prismatic pipe or channel.

PART-3

Subsonic, Sonic and Supersonic Flows, Subcritical, Critical and Super Critical Flows, One, Two and Three Dimensional Flows.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.3. **Write short note on :**

a. Subsonic, sonic and supersonic flows.
b. Subcritical, critical and supercritical flows.
c. One, two and three dimensional flows.

Answer

a. Subsonic, Sonic and Supersonic Flows :

1. When Mach number is less than 1 ($M < 1$), flow is subsonic flow.
2. When Mach number is equal to 1 ($M = 1$), flow is sonic flow.
3. When Mach number is greater than 1 ($M > 1$), flow is supersonic flow.

b. Subcritical, Critical and Supercritical Flows :

1. When Froude number is less than one ($Fe < 1$), the flow is subcritical flow.
2. When Froude number is equal to one ($Fe = 1$), the flow is critical flow.
3. When Froude number is greater than one ($Fe > 1$), the flow is supercritical flow.

c. One, Two and Three Dimensional Flows :

1. One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only.
   Mathematically, $u = f(x)$, $v = 0$ and $w = 0$
   Where u, v and w are velocity components in x, y and z directions respectively.
2. Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates.
   Mathematically, $u = f_1(x, y)$, $v = f_2(x, y)$ and $w = 0$
3. Three-dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions.
   Mathematically, $u = f_1(x, y, z)$, $v = f_2(x, y, z)$ and $w = f_3(x, y, z)$

PART-4

Streamlines.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.4. **Define the following :**

i. Streamlines,
ii. Path line, and
iii. Streak line.

Answer

i. **Streamlines :** A streamline may be defined as an imaginary line within the flow so that the tangent at any point on it indicates the velocity at that point.

ii. **Path Line :** A path line is the path followed by a fluid particle in motion. A path line shows the direction of particle as it moves ahead.

iii. **Streak Line :** The streak line is a curve which gives an instantaneous picture of the location of the fluid particles, which have passed through a given point.

PART-5

Continuity Equation for 3D and 1D Flow.

CONCEPT OUTLINE

Continuity Equation : It is based on the principle of 'conservation of mass'. It states that, if no fluid is added or removed from the pipe in any length then the mass passing across different section shall be same.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.5. **Derive the continuity equation for 1-D fluid flow through a pipe.**

Answer

1. Consider two cross-section of a pipe as shown in Fig. 2.5.1.

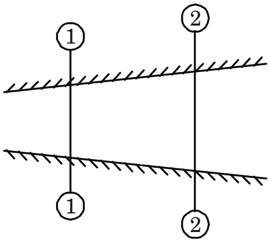

Fig. 2.5.1. Fluid flow through a pipe.

2. Let, A_1 = Area of the pipe at section 1–1,

   v_1 = Velocity of the fluid at section 1–1,

   ρ_1 = Density of the fluid at section 1–1,

   A_2, v_2, ρ_2 = Corresponding values at section 2–2.

3. The total quantity of fluid passing through section 1–1 = $\rho_1 A_1 v_1$

   The total quantity of fluid passing through section 2–2 = $\rho_2 A_2 v_2$

3. From the law of conservation of matter (theorem of continuity), we have

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \text{...(2.5.1)}$$

4. Eq. (2.5.1) is applicable to the compressible as well as incompressible fluids and is called continuity equation. In case of incompressible fluids, $\rho_1 = \rho_2$ and the continuity eq. (2.5.1) reduces to

$$A_1 v_1 = A_2 v_2 \qquad \text{...(2.5.2)}$$

Que 2.6. **Derive continuity equation for a 3-D steady or unsteady flow in a cartesian co-ordinate system.** **AKTU 2015-16, Marks 05**

Answer

1. Consider an elementary rectangular parallelopiped with sides of length δx, δy and δz as shown in Fig. 2.6.1.

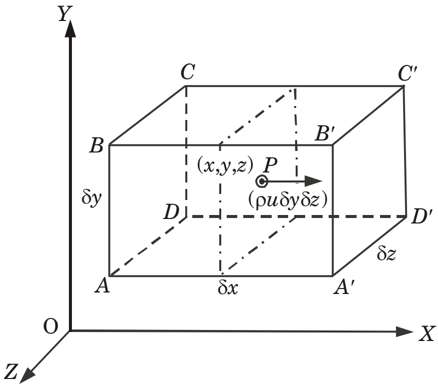

Fig. 2.6.1. Elementary rectangular parallelopiped.

2. Let the centre of the parallelopiped be at a point $P(x, y, z)$ where the velocity components in the x, y and z directions are u, v and w respectively and ρ be the mass density of the fluid.

3. The mass of fluid passing per unit time through the face of area $\delta y \delta z$ normal to the X-axis through point P is,

$$(\rho u \delta y \delta z)$$

4. Then the mass of fluid flowing per unit time into the parallelopiped through the face $ABCD$ is,

$$(\rho u\ \delta y\ \delta z) + \frac{\partial}{\partial x}(\rho u\ \delta y\ \delta z)\left(-\frac{\delta x}{2}\right) \qquad ...(2.6.1)$$

5. Similarly the mass of fluid per unit time out of the parallelopiped through the face $A'B'C'D'$ is,

$$(\rho u\ \delta y\ \delta z) + \frac{\partial}{\partial x}(\rho u\ \delta y\ \delta z)\left(\frac{\delta x}{2}\right) \qquad ...(2.6.2)$$

6. Therefore, the net mass of fluid from eq. (2.6.1) and eq. (2.6.2),

$$\left[(\rho u\ \delta y\ \delta z) - \frac{\partial}{\partial x}(\rho u\ \delta y\ \delta z)\frac{\delta x}{2}\right] - \left[(\rho u\ \delta y\ \delta z) + \frac{\partial}{\partial x}(\rho u\ \delta y\ \delta z)\frac{\delta x}{2}\right]$$

$$= -\frac{\partial}{\partial x}(\rho u)\ \delta x\ \delta y\ \delta z$$

7. Similarly the net mass of fluid that remains in the parallelopiped per unit time

$= -\dfrac{\partial}{\partial y}(\rho \mathrm{v})\ \delta x\ \delta y\ \delta z$, through pair of faces $AA'D'D$ and $BB'C'C$

$= -\dfrac{\partial}{\partial z}(\rho w)\ \delta x\ \delta y\ \delta z$, through pair of faces $DD'C'C$ and $AA'B'B$

8. By adding all these expressions the net total mass of fluid that has remained in the parallelopiped per unit time is obtained as

$$-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho \mathrm{v})}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]\delta x\ \delta y\ \delta z \qquad ...(2.6.3)$$

9. The mass of the fluid in the paralleopiped is $(\rho\ \delta x\ \delta y\ \delta z)$ and its rate of increase with time is

$$\frac{\partial}{\partial t}(\rho\ \delta x\ \delta y\ \delta z) = \frac{\partial \rho}{\partial t}(\delta x\ \delta y\ \delta z) \qquad ...(2.6.4)$$

10. According to law of conservation of mass, equating the eq. (2.6.3) and eq. (2.6.4), we get

$$-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho \mathrm{v})}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right](\delta x\ \delta y\ \delta z) = \frac{\partial \rho}{\partial t}(\delta x\ \delta y\ \delta z)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho \mathrm{v})}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \qquad ...(2.6.5)$$

11. Eq. (2.6.5) represents the continuity equation in cartesian coordinates in its most general form which is applicable for steady as well as unsteady flow, uniform and non-uniform flow, and compressible as well as incompressible fluids.

12. For steady flow since, $\dfrac{\partial \rho}{\partial t} = 0$, eq. (2.6.5) reduces to

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \qquad ...(2.6.6)$$

13. For an incompressible fluid, ρ = constant, then,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Que 2.7. **A 500 mm diameter pipe carrying water at rate 0.5 m³/sec branches into two pipes of 200 mm and 400 mm diameters. If the rate of flow of water through small diameter pipe is 0.2 m³/sec. Determine velocity of flow in each pipe.** **AKTU 2017-18, Marks 10**

Answer

Given : d = 500 mm = 0.5 m, Q = 0.5 m³/sec, d_1 = 200 mm = 0.2 m
d_2 = 400 mm = 0.4 m, Q_1 = 0.2 m³/sec
To Find : Velocity of flow in each pipe.

1. We know, $Q = Q_1 + Q_2$

$$Q_2 = Q - Q_1 = 0.5 - 0.2 = 0.3 \text{ m}^3/\text{sec}$$

2. Now, Q = Area of main pipe × Velocity

$$Q = \frac{\pi}{4} d^2 v$$

$$0.5 = \frac{\pi}{4} \times (0.5)^2 \times v$$

$$v = 2.5 \text{ m/sec}$$

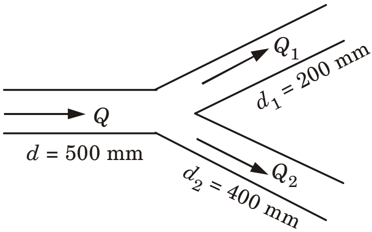

Fig. 2.7.2.

3. Similarly, Q_1 = Area of pipe 1 × Velocity

$$Q_1 = \frac{\pi}{4} d_1^2 v_1$$

$$0.2 = \frac{\pi}{4} \times (0.2)^2 \times v_1$$

$$v_1 = 6.36 \text{ m/sec}$$

4. Similarly, Q_2 = Area of pipe 2 × Velocity

$$Q_2 = \frac{\pi}{4} d_2^2 v_2$$

$$0.3 = \frac{\pi}{4} \times (0.4)^2 \times v_2$$

$$v_2 = 2.38 \text{ m/sec}$$

Que 2.8. **A jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, what will be the diameter at a point 4.5 m above the nozzle, if the velocity with which the jet leaves the nozzle is 12 m/s ?**

AKTU 2014-15, Marks 05

Answer

Given : D_1 = 25 mm = 0.025 m, v_1 = 12 m/s, h = 4.5 m
To Find : Diameter at a point 4.5 m above the nozzle.

1. Consider the vertical motion of the jet from the outlet of the nozzle to the point A (neglecting any loss of energy)

   Initial velocity, $u = v_1 = 12$ m/s

   Final velocity, $v = v_2$

3. Using,

$$v^2 - u^2 = 2gh$$

$$v_2^2 - 12^2 = 2 \times (-9.81) \times 4.5$$

$$v_2 = \sqrt{12^2 - 2 \times 9.81 \times 4.5} = \sqrt{144 - 88.29}$$

$$= 7.46 \text{ m/s}$$

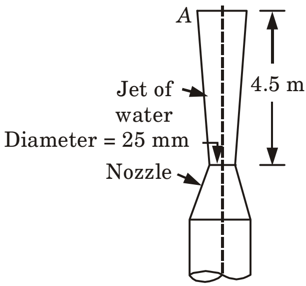

Fig. 2.8.1.

4. Now applying continuity equation to the outlet of nozzle and at point A,

$$A_1 v_1 = A_2 v_2$$

$$A_2 = \frac{A_1 v_1}{v_2} = \frac{\frac{\pi}{4} D_1^2 \times v_1}{v_2} = \frac{\pi \times (0.025)^2 \times 12}{4 \times 7.46} = 0.0007896 \text{ m}^2$$

5. Let D_2 = Diameter of jet at point A.

   Then $A_2 = (\pi/2)\, D_2^2$

$$0.0007896 = (\pi/4) \times D_2^2$$

$$\therefore \quad D_2 = \sqrt{\frac{0.0007896 \times 4}{\pi}} = 0.0317 \text{ m} = 31.7 \text{ mm}$$

Que 2.9. **Two velocity components are given in the following equations, find the third component such that they satisfy the continuity equation :**

$$\boldsymbol{u = x^3 + y^2 + 2z^2, \text{ v} = -x^2 y - yz - xy}$$

AKTU 2015-16, Marks 05

Answer

Given : $u = x^3 + y^2 + 2z^2$, v $= -x^2y - yz - xy$
To Find : Third component of velocity that satisfy the continuity equation.

1. From continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial \text{v}}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{...(2.9.1)}$$

2. $$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^3 + y^2 + 2z^2) = 3x^2 + 0 + 0 = 3x^2$$

3. $$\frac{\partial \text{v}}{\partial y} = \frac{\partial}{\partial y}(-x^2y - yz - xy) = -x^2 - z - x$$

4. Putting the value of $\frac{\partial u}{\partial x}$ and $\frac{\partial \text{v}}{\partial y}$ in eq. (2.9.1), we get

$$3x^2 + (-x^2 - z - x) + \frac{\partial w}{\partial z} = 0$$

$$2x^2 - z - x + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} = -2x^2 + z + x \quad \text{...(2.9.2)}$$

5. On integration of eq. (2.9.2) w.r.t z, we get

$$w = -2x^2z + xz + \frac{z^2}{2}$$

PART-6

Circulation

CONCEPT OUTLINE

Circulation : The flow along a closed curve is called circulation.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.10. **Write a short note on circulation.**

Answer

1. Let us consider a closed curve in a two-dimensional flow field as shown in Fig. 2.10.1, the curve being cut by the streamlines.
2. Let P be the point of intersection of the curve with one streamline, θ be the angle which the streamline makes with the curve.
3. The component of velocity along the closed curve at the point of intersection is v cos θ.
4. Circulation Γ is defined mathematically as the line integral of the tangential velocity about a closed path (contour).

Thus, $\Gamma = \oint \text{v} \cos\theta \, ds$

Where, v = Velocity in the flow field at the element ds, and

θ = Angle between v and tangent to the path (in the positive anticlockwise direction along the path) at the point.

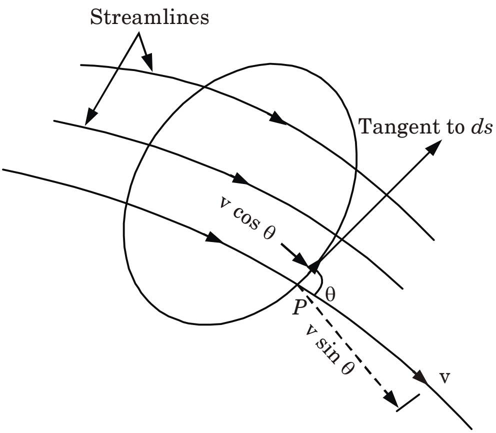

Fig. 2.10.1. Circulation in a two-deimensinal flow.

Que 2.11. **If the velocity field is given by $u = (16y - 8x)$, v = $(8y - 7x)$ find the circulation around the closed curve defined by $x = 2$, $y = 1$, $x = 4$, $y = 4$.**

Answer

Given : $u = 16y - 8x$, $v = 8y - 7x$
Closed curve defined by $x = 2$, $y = 1$, $x = 4$, $y = 4$
To Find : Circulation around the closed curve.

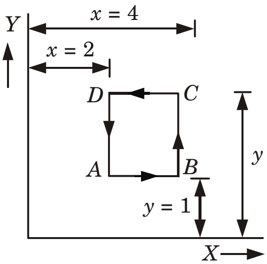

Fig. 2.11.1.

1. Circulation $\Gamma_{ABCD} = \int_{ABCD} (udx + vdy)$

$$= \int_{AB} (udx + vdy) + \int_{BC} (udx + vdy) + \int_{CD} (udx + vdy) + \int_{DA} (udx + vdy)$$

$$\Gamma_{ABCD} = \int_2^4 (16y - 8x)dx + \int_1^4 (8y - 7x)dy + \int_4^2 (16y - 8x)dx + \int_4^1 (8y - 7x)dy$$

$$= \underset{(i)}{[16yx - 4x^2]_2^4} + \underset{(ii)}{[4y^2 - 7xy]_1^4} + \underset{(iii)}{[16yx - 4x^2]_4^2} + \underset{(iv)}{[4y^2 - 7xy]_4^1}$$

2. For (*i*) Integral : $y = 1$
For (*ii*) Integral : $x = 4$
For (*iii*) Integral : $y = 4$
For (*iv*) Integral : $x = 2$
Using the above values we have,

$$\Gamma_{ABCD} = [(16 \times 1 \times 4) - (4 \times 4^2) - (16 \times 1 \times 2) + (4 \times 2^2)] + [4 \times 4^2 - (7 \times 4 \times 4) - (4 \times 1^2) + (7 \times 4 \times 1)] + [(16 \times 4 \times 2) - (4 \times 2^2) - (16 \times 4 \times 4) + (4 \times 4^2)] + [(4 \times 1^2) - (7 \times 2 \times 1) - (4 \times 4^2) + (7 \times 2 \times 4)]$$

$$= [64 - 64 - 32 + 16] + [64 - 112 - 4 + 28] + [128 - 16 - 256 + 64] + [4 - 14 - 64 + 56]$$

$$= -16 - 24 - 80 - 18 = -138$$

3. Area of the curve $ABCD = (4 - 2) \times (4 - 1) = 6$ square unit

$\therefore$ Circulation per unit area $= \dfrac{-138}{6} = -23$ unit

PART-7

Stream Function.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.12. **What is stream function ? Give its properties.**

Answer

A. Stream Function :

1. Stream function is the scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angle to that direction. It is denoted by ψ and defined only for two dimensional flow.
2. Mathematically, for steady clockwise flow,
$$\psi = f(x, y) \text{ such that}$$
$$\frac{\partial \psi}{\partial x} = -\text{v} \quad \text{and} \quad \frac{\partial \psi}{\partial y} = u \qquad ...(2.12.1)$$
3. The continuity equation for two-dimensional flow is,
$$\frac{\partial u}{\partial x} + \frac{\partial \text{v}}{\partial y} = 0 \qquad ...(2.12.2)$$
4. On substituting the values of u and v from eq. (2.12.1) in eq. (2.12.2), we have
$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0$$
$$\frac{\partial^2 \psi}{\partial x\, \partial y} - \frac{\partial^2 \psi}{\partial x\, \partial y} = 0$$
The flow may be rotational or irrotational.
5. The rotational component is given by,
$$\omega_z = \frac{1}{2}\left(\frac{\partial \text{v}}{\partial x} - \frac{\partial u}{\partial y}\right)$$
6. Substituting the values of u and v from eq. (2.12.1) in the above rotational component,
$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)\right] = -\frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{d^2 \psi}{\partial y^2}\right]$$

7. For irrotational flow $\omega_z = 0$. Hence the above equation becomes

$$\frac{\partial^2\psi}{\partial x^2} + \frac{d^2\psi}{\partial y^2} = 0$$

This is Laplace equation for ψ.

B. Properties of Stream Function :

1. Existence of stream function (ψ) represents a possible case of fluid flow which may be rotational or irrotational.
2. In case it satisfies the Laplace equation, it is a possible case of an irrotational flow.

Que 2.13. Derive the equation of a streamline for a 2-D flow. Prove that the discharge between two streamlines is the difference in their stream function values.

Answer

A. Equation of a Streamline for 2-D Flow :

1. For constant stream function, $d\psi = 0$

i.e., $$\frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy = 0$$

$$-\text{v}dx + udy = 0 \qquad \left\{\because \frac{\partial\psi}{\partial x} = -\text{v}, \frac{\partial\psi}{\partial y} = u\right\}$$

$$\text{v}dx = udy$$

$$\frac{dx}{u} = \frac{dy}{\text{v}}$$

2. Above equation represent the equation of a streamline in x-y plane.

B. Discharge between Two Streamlines :

1. Let $\psi(x, y)$ represent the streamline L. The adjacent streamline M has stream function $\psi + d\psi$.
2. Let the velocity vector V perpendicular to the line AB has components u and v in the direction of X and Y axes respectively.
3. From continuity equation,

Flow across AB = Flow across AO + Flow across OB

$$Vds = -\text{v}dx + udy$$

Negative sign shows that the v is acting in downward direction.

5. Putting $\text{v} = -\dfrac{\partial\psi}{\partial x}$, $u = \dfrac{\partial\psi}{\partial y}$ and $Vds = dq$, we get

$$dq = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy$$

$$dq = d\psi$$

6. Hence discharge between two streamlines is the difference in their stream function values.

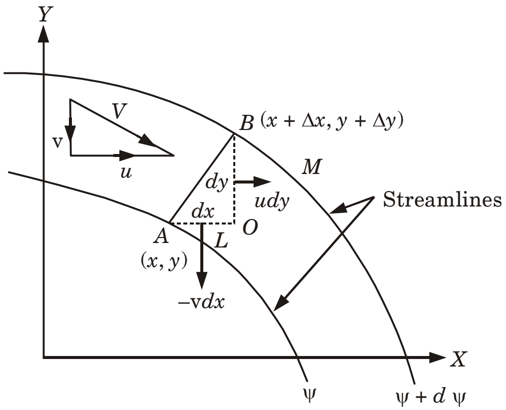

Fig. 2.13.1. Flow between two points and its relation to stream function.

Que 2.14. **What is the relationship between equipotential line and line of constant stream function at the point of intersection ?**

OR

Prove that stream function (ψ) and potential function (ϕ) are orthogonal to each other.

Answer

1. For equipotential line, $d\phi = 0$

$$\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$$

$$-udx + (-\text{v})dy = 0 \qquad \left\{ \because \frac{\partial \phi}{\partial x} = -u \text{ and } \frac{\partial \phi}{\partial y} = -\text{v} \right\}$$

$$\frac{dy}{dx} = -\frac{u}{\text{v}} = \text{Slope of equipotential line}$$

2. For constant stream function, $d\psi = 0$

$$\frac{\partial \Psi}{\partial x} dx + \frac{\partial \Psi}{\partial y} dy = 0$$

$$-\text{v}dx + udy = 0 \qquad \left\{ \because \frac{\partial \Psi}{\partial x} = -\text{v}, \ \frac{\partial \Psi}{\partial y} = u \right\}$$

$$\frac{dy}{dx} = \frac{\text{v}}{u} = \text{Slope of streamline}$$

3. Now, slope of streamline × slope of equipotential line

$$= \left(\frac{v}{u}\right) \times \left(-\frac{u}{v}\right) = -1$$

4. The product of the slope of the equipotential line and the slope of the stream line at the point of intersection is equal to – 1. Thus the equipotential lines are orthogonal to the streamlines at all points of intersection.

Que 2.15. **Sketch the streamlines represented by $\psi = x^2 + y^2$. Also find out the velocity and its direction at point (1, 2).**

AKTU 2014-15, Marks 10

Answer

Given : $\psi = x^2 + y^2$
To Find : i. Sketch of streamlines.
ii. Velocity and its direction at point (1, 2).

1. Streamlines given by, $\psi = x^2 + y^2$
2. Let ψ = 1, 2, 3 and so on.
   Then, we have $1 = x^2 + y^2$
   $2 = x^2 + y^2$
   $3 = x^2 + y^2$ and so on.
3. Each equation is an equation of a circle. Thus we shall get concentric circles of different diameters shown in Fig. 2.15.1.
4. The velocity components u and v are,

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(x^2 + y^2) = 2y$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(x^2 + y^2) = -2x$$

5. At the point (1, 2), the velocity components are,
   $u = 2 \times 2 = 4$ units/s
   $v = -2 \times 1 = -2$ units/s

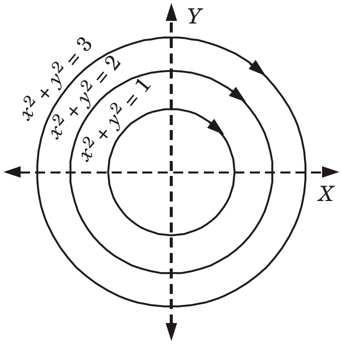

Fig. 2.15.1. Streamlines.

6. Resultant velocity $= \sqrt{u^2 + v^2} = \sqrt{4^2 + (-2)^2}$

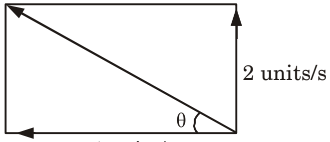

Fig. 2.15.2.

$= \sqrt{20} = 4.47$ units/s

7. We know that, $\tan\theta = \dfrac{v}{u} = \dfrac{2}{4} = \dfrac{1}{2}$

$\therefore \qquad \theta = \tan^{-1} 0.5 = 26° \, 34'$

Thus resultant velocity makes an angle of 26° 34′ with x-axis in clockwise direction.

Que 2.16. If for a 2-D potential flow, the velocity potential is given by $\phi = x(2y - 1)$. Determine the velocity at the point $P(4, 5)$. Determine also the value of stream function at the point P.

AKTU 2014-15, Marks 10

Answer

Given : $\phi = x(2y - 1)$ $P(4, 5)$

To Find : i. Velocity at point P.
ii. Stream function at point P.

1. The velocity components in the direction of x and y are,

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}[x(2y-1)] = -[2y-1] = 1 - 2y$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}[x(2y-1)] = -[2x] = -2x$$

2. At point $P(4, 5)$, *i.e.*, at $x = 4$, $y = 5$

$$u = 1 - 2 \times 5 = -9 \text{ units/s}$$
$$v = -2 \times 4 = -8 \text{ units/s}$$

3. Resultant velocity at P $= \sqrt{(-9)^2 + (-8)^2} = \sqrt{81 + 64}$
$= 12.04$ units/s

4. We know that, $$\frac{\partial \psi}{\partial y} = u = 1 - 2y \qquad ...(2.16.1)$$

and $$\frac{\partial \psi}{\partial x} = -v = 2x \qquad ...(2.16.2)$$

5. Integrating eq. (2.16.1) w.r.t 'y', we get

$$\int d\psi = \int (1 - 2y)\, dy$$

$$\psi = y - \frac{2y^2}{2} + K$$

$$\psi = y - y^2 + K \qquad ...(2.16.3)$$

The constant of integration K is not a function of y but it can be a function of x.

6. Differentiating the eq. (2.16.3) w.r.t x,

$$\frac{\partial \psi}{\partial x} = \frac{\partial K}{\partial x}$$

But from eq. (2.16.2)

$$\frac{\partial \psi}{\partial x} = 2x$$

7. Equating the value of $\frac{\partial \psi}{\partial x}$, we get

$$\frac{\partial K}{\partial x} = 2x$$

Integrating this equation,

$$K = \int 2x dx = \frac{2x^2}{2} = x^2$$

8. Substituting this value of K in eq. (2.16.3), we get

$$\psi = y - y^2 + x^2.$$

9. Stream function ψ at P (4, 5) = $5 - 5^2 + 4^2 = 5 - 25 + 16 = -4$ units

PART-8

Velocity Potential.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.17. **What is velocity potential ? Also derive the Laplace equation for velocity potential.**

Answer

A. Velocity Potential :

1. The velocity potential is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is denoted by ϕ (phi).

2. Thus mathematically the velocity potential is defined as :

$$\phi = f(x, y, z, t) \quad \text{... for unsteady flow}$$

and

$$\phi = f(x, y, z) \quad \text{... for steady flow}$$

such that

$$\left.\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ \text{v} &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}\right\} \quad ...(2.17.1)$$

Where u, v, and w are the components of velocity in the x, y and z directions respectively.

3. The negative sign signifies that ϕ decreases with an increase in the values of x, y and z. In other words it indicates that the flow is always in the direction of decreasing ϕ.

B. Laplace Equation for Velocity Potential :

1. For an incompressible steady flow the continuity equation is,

$$\frac{\partial u}{\partial x} + \frac{\partial \text{v}}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad ...(2.17.2)$$

2. By substituting the values of u, v and w from eq. (2.17.1) in eq. (2.17.2), we get

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\frac{d^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

This equation is known as Laplace equation for velocity potential.

Que 2.18. What is the relation between stream function and velocity potential function ?

Answer

1. From velocity potential function,

$$u = -\frac{\partial \phi}{\partial x} \text{ and v} = -\frac{\partial \phi}{\partial y}$$

2. Stream function gives, $u = \dfrac{\partial \psi}{\partial y}$ and $\text{v} = -\dfrac{\partial \psi}{\partial x}$

$$\therefore \quad u = -\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \text{ and v} = -\frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}$$

3. Hence, $\dfrac{\partial \phi}{\partial x} = -\dfrac{\partial \psi}{\partial y}$ and $\dfrac{\partial \phi}{\partial y} = \dfrac{\partial \psi}{\partial x}$

Que 2.19. **The velocity components in a two-dimensional flow field for an incompressible fluid are expressed as**

$$u = \frac{y^3}{3} + 2x - x^2y \; ; \; v = xy^2 - 2y - \frac{x^3}{3}$$

a. Show that these functions represent a possible case of an irrotational flow.

b. Obtain an expression for stream function ψ.

c. Obtain an expression for velocity potential ϕ.

AKTU 2015-16, Marks 15

Answer

Given : $u = y^3/3 + 2x - x^2y$, $v = xy^2 - 2y - x^3/3$

To Find :
i. Prove given velocity functions represent a possible case of an irrotational flow.
ii. Expression for stream function ψ.
iii. Expression for velocity potential ϕ.

1. The rotational component is given by,

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \quad ...(2.19.1)$$

2. Now,

$$\frac{\partial v}{\partial x} = y^2 - x^2$$

$$\frac{\partial u}{\partial y} = y^2 - x^2$$

3. Putting the values of $\frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y}$ in eq. (2.19.1), we get

$$\omega_z = \frac{1}{2}\,[y^2 - x^2 - (y^2 - x^2)] = 0$$

Since ω_z is zero therefore these functions represent a possible case of an irrotational flow.

4. The velocity components in terms of stream function are,

$$\frac{\partial \psi}{\partial x} = -v = -(xy^2 - 2y - x^3/3) \quad ...(2.19.2)$$

$$\frac{\partial \psi}{\partial y} = u = y^3/3 + 2x - x^2y \quad ...(2.19.3)$$

5. Integrating eq. (2.19.2) w.r.t x, we get

$$\psi = \int (-xy^2 + 2y + x^3/3)\, dx$$

$$\psi = -\frac{x^2 y^2}{2} + 2xy + \frac{x^4}{4 \times 3} + K \quad ...(2.19.4)$$

Where K is a constant of integration which is independent of x but can be a function of y.

6. Differentiating eq. (2.19.4) w.r.t to y, we get

$$\frac{\partial \psi}{\partial y} = -\frac{2x^2 y}{2} + 2x + \frac{\partial K}{\partial y} = -x^2 y + 2x + \frac{\partial K}{\partial y} \quad ...(2.19.5)$$

7. From eq. (2.19.3) and eq. (2.19.5), we have

$$-x^2y + 2x + \frac{\partial K}{\partial y} = y^3/3 + 2x - x^2y$$

$$\therefore \quad \frac{\partial K}{\partial y} = y^3/3$$

8. On integrating, we get

$$K = \int (y^3/3)\, dy = \frac{y^4}{4 \times 3} = \frac{y^4}{12}$$

9. Substituting this value of K in eq. (2.19.4), we get

$$\psi = -\frac{x^2 y^2}{2} + 2xy + \frac{x^4}{12} + \frac{y^4}{12}$$

10. We know that, $\dfrac{d\phi}{dx} = -u$ and $\dfrac{d\phi}{dy} = -\text{v}$

11. Therefore,

$$\frac{d\phi}{dx} = -\frac{y^3}{3} - 2x + x^2y \quad ...(2.19.6)$$

$$\frac{\partial \phi}{\partial y} = -xy^2 + 2y + \frac{x^3}{3} \quad ...(2.19.7)$$

12. Integrating eq. (2.19.6) w.r.t x, we get

$$\phi = -\frac{y^3 x}{3} - x^2 + \frac{x^3 y}{3} + C \quad ...(2.19.8)$$

Where, C is a constant of integration which is independent of x but can be function of y.

13. Differentiating eq. (2.19.8) wrt y, we get

$$\frac{\partial \phi}{\partial y} = -y^2x + \frac{x^3}{3} + \frac{\partial C}{\partial y} \quad ...(2.19.9)$$

14. Comparing the values of $\dfrac{\partial \phi}{\partial y}$ from eq. (2.19.7) and eq. (2.19.9), we get

$$\frac{\partial C}{\partial y} = 2y$$

$$C = y^2$$

15. Substituting this value of C in eq. (2.19.8), we get

$$\phi = \frac{x^3 y}{3} - \frac{xy^3}{3} - x^2 + y^2$$

Que 2.20. **Velocity field in fluid medium is given by :**

$$\mathbf{v} = 10x^2y\,\hat{i} + 15xy\,\hat{j} + (25t - 3xy)\,\hat{k}$$

Find acceleration at (1, 2, – 1) m and t = 0.5 sec.

AKTU 2017-18, Marks 07

Answer

Given : $\text{v} = 10x^2y\,\hat{i} + 15xy\,\hat{j} + (25t - 3xy)\,\hat{k}$

To Find : Acceleration at (1, 2, – 1) m and t = 0.5 sec.

1. The velocity components u, v and w are,

$$u = 10x^2y,\ \text{v} = 15xy,\ w = 25t - 3xy$$

2. Acceleration is given by,

$$a_x = u\frac{\partial u}{\partial x} + \text{v}\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \quad ...(2.20.1)$$

$$a_y = u\frac{\partial \text{v}}{\partial x} + \text{v}\frac{\partial \text{v}}{\partial y} + w\frac{\partial \text{v}}{\partial z} + \frac{\partial \text{v}}{\partial t} \quad ...(2.20.2)$$

$$a_z = u\frac{\partial w}{\partial x} + \text{v}\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \quad ...(2.20.3)$$

3. Now from velocity component, we get

$$\frac{\partial u}{\partial x} = 20xy,\ \frac{\partial u}{\partial y} = 10x^2,\ \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial \text{v}}{\partial x} = 15y,\ \frac{\partial \text{v}}{\partial y} = 15x,\ \frac{\partial \text{v}}{\partial z} = 0 \text{ and } \frac{\partial \text{v}}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = -3y,\ \frac{\partial w}{\partial y} = -3x,\ \frac{\partial w}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial t} = 25$$

4. Substituting these values in eq. (2.20.1), eq. (2.20.2) and eq. (2.20.3), we get

$$a_x = 10x^2y\,(20xy) + 15xy\,(10x^2) + (25t - 3xy)\,(0) + 0$$
$$= 200x^3y^2 + 150x^3y$$
$$a_y = 10x^2y\,(15y) + 15xy\,(15x) + (25t - 3xy)\,(0) + 0$$
$$= 150x^2y^2 + 225\,x^2y$$
$$a_z = 10x^2y(-3y) + 15xy\,(-3x) + (25t - 3xy)\,(0) + 25$$
$$= -30x^2y^2 - 45x^2y + 25$$

5. Acceleration component at (1, 2, – 1) m and t = 0.5 sec,

$$a_x = 200\,(1)^3\,(2)^2 + 150\,(1)^3\,(2) = 800 + 300 = 1100$$
$$a_y = 150\,(1)^2\,(2)^2 + 225\,(1)^2\,(2) = 600 + 450 = 1050$$
$$a_z = -30\,(1)^2\,(2)^2 - 45\,(1)^2\,(2) + 25 = -120 - 90 + 25 = -185$$

6. Acceleration is given as,

$$A = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$A = 1100 \hat{i} + 1050 \hat{j} - 185 \hat{k}$$

7. Resultant,

$$A = \sqrt{(1100)^2 + (1050)^2 + (-185)^2}$$

$$A = 1531.9 \text{ m/sec}^2$$

PART-9

Buckingham's-Pi Theorem.

CONCEPT OUTLINE

Dimensional Analysis : It is a mathematical technique which makes use of the study of dimensions for solving several engineering problems.

π-terms : The dimensionless terms used in Buckingham's-pi theorem are called π-terms.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.21. Give Buckingham-pi theorem and explain Buckingham's-pi method.

Answer

A. Buckingham-Pi Theorem :

1. This theorem states that if there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into $(n - m)$ dimensionless terms.

B. Buckingham's-Pi Method :

1. If there are n variables (both independent and dependent) in a physical phenomenon and the variables contain m fundamental dimensions (M, L, T), then the variables are arranged into $(n - m)$ dimensionless terms. Each term is called a π-term.
2. Let V_d, V_1, V_2, V_3 .... V_n are the variables involved in a physical problem.
3. Let V_d be the dependent variable and V_1, V_2 ... V_n are the independent variables on which V_d depends. Then V_d is a function of V_1, V_2 ... V_n and mathematically it is expressed as,

$$V_d = f(V_1, V_2 \ldots\ldots V_n) \quad \ldots(2.21.1)$$

4. Eq. (2.21.1) can also be written as,

$$f_1 (V_d, V_1, V_2 \ldots V_n) = 0 \quad \ldots(2.21.2)$$

Eq. (2.21.2) is a dimensionally homogeneous equation and it contains n variables.

5. If there are m fundamental dimensions (*i.e.*, M, L, T), then according to Buckingham's-pi theorem eq. (2.21.2) can be written in terms of dimensionless groups or π-terms in which the number of π-terms is equal to $(n - m)$. Hence eq. (2.21.2) becomes

$$f(\pi_1 \; \pi_2 \ldots \pi_{(n-m)}) = 0 \quad \ldots(2.21.3)$$

Each of π-term is dimensionless and is independent of the systems.

6. Each π-term contains $m + 1$ number of variables where m is the number of fundamental dimensions and is also called as repeating variable.

7. Let in the above case V_1, V_2 and V_3 be the repeating variables. If fundamental dimension is $m\,(M, L, T) = 3$, then each π-term is written as

$$\left.\begin{aligned} \pi_1 &= V_1^{a_1}\, V_2^{b_1}\, V_3^{c_1}\, V_4 \\ \pi_2 &= V_1^{a_2}\, V_2^{b_2}\, V_3^{c_2}\, V_5 \\ \pi_{n-m} &= V_1^{a_{n-m}}\, V_2^{b_{n-m}}\, V_3^{c_{n-m}}\, V_x \end{aligned}\right\} \quad \ldots(2.21.4)$$

8. Each equation is solved by the principle of dimensionless homogeneity and values of a_1, b_1, c_1 etc., are obtained.

9. These values are substituted in the eq. (2.21.4) and values of π_1, $\pi_2 \ldots \pi_{n-m}$ are obtained.

10. These values of π's are substituted in eq. (2.21.3) and the final equation for the phenomenon is obtained by expressing any one of the π-terms as a function of others as

$$\pi_1 = \phi[\pi_2, \pi_3 \ldots \pi_{n-m}]$$

or

$$\pi_2 = \phi[\pi_1, \pi_3 \ldots \pi_{n-m}]$$

Que 2.22. **Using Buckingham's π theorem, show that the discharge, Q consumed by an oil ring is given by,**

$$Q = (Nd^3)\, f\,[\mu / (\rho N d^2), \sigma / (\rho N^2 d^3), \omega / (\rho N^2 d)]$$

Where, d is internal diameter of ring, N is rotational speed, ρ is density, μ is viscosity, σ is surface tension and ω is the specific weight of oil.

AKTU 2014-15, Marks 10

Answer

Given : Discharge is a function of $d, N, \rho, \mu, \sigma, \omega$

$Q = f(d, N, \rho, \mu, \sigma, \omega)$ or $f_1(Q, d, N, \rho, \mu, \sigma, \omega) = 0$

To Prove : $Q = (Nd^3)\, f\,[\mu / (\rho N d^2), \sigma / (\rho N^2 d^3), \omega / (\rho N^2 d)]$

1. Total number of variables, $n = 7$
2. Dimensions of each variable are,

$$Q = L^3T^{-1},\ d = L,\ N = T^{-1},$$
$$\rho = ML^{-3},\ \mu = ML^{-1}T^{-1},\ \sigma = MT^{-2}$$

and
$$\omega = ML^{-2}T^{-2}$$

3. Total number of fundamental dimensions, $m = 3$
4. Total number of π-terms $= n - m = 7 - 3 = 4$
5. Now discharge function can be written as,

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0 \quad ...(2.22.1)$$

6. Choosing d, N, ρ as repeating variables, the π-terms are,

$$\pi_1 = d^{a_1}\ N^{b_1}\ \rho^{c_1}\ Q$$
$$\pi_2 = d^{a_2}\ N^{b_2}\ \rho^{c_2}\ \mu$$
$$\pi_3 = d^{a_3}\ N^{b_3}\ \rho^{c_3}\ \sigma$$
$$\pi_4 = d^{a_4}\ N^{b_4}\ \rho^{c_4}\ \omega$$

7. **First π-term :** $\pi_1 = d^{a_1}\ N^{b_1}\ \rho^{c_1}\ Q$

Substituting dimensions on both sides,

$$M^0L^0T^0 = [L]^{a_1}\ [T^{-1}]^{b_1}\ [ML^{-3}]^{c_1}\ [L^3T^{-1}]$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1$, $\therefore\ c_1 = 0$

Power of L, $0 = a_1 - 3c_1 + 3$, $\therefore\ a_1 = 3c_1 - 3 = 0 - 3 = -3$

Power of T, $0 = -b_1 - 1$, $\therefore\ b_1 = -1$

Substituting a_1, b_1, c_1 in π_1, we have

$$\pi_1 = d^{-3}\ N^{-1}\ \rho^0\ Q = \frac{Q}{d^3N}$$

8. **Second π-term :** $\pi_2 = d^{a_2}\ N^{b_2}\ \rho^{c_2}\ \mu$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a_2}\ [T^{-1}]^{b_2}\ [ML^{-3}]^{c_2}\ [ML^{-1}T^{-1}]$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_2 + 1$, $\therefore\ c_2 = -1$

Power of T, $0 = -b_2 - 1$, $\therefore\ b_2 = -1$

Power of L, $0 = a_2 - 3c_2 - 1$, $\therefore\ a_2 = -2$

Substituting the values of a_2, b_2, c_2 in π_2, we have

$$\pi_2 = d^{-2}\ N^{-1}\ \rho^{-1}\ \mu = \frac{\mu}{d^2N\rho} \text{ or } \frac{\mu}{\rho N d^2}$$

9. **Third π-term :** $\pi_3 = d^{a_3}\ N^{b_3}\ \rho^{c_3}\ \sigma$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a_3} [T^{-1}]^{b_3} [ML^{-3}]^{c_3} [MT^{-2}]$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_3 + 1$, $\therefore c_3 = -1$

Power of L, $0 = a_3 - 3c_3$, $\therefore a_3 = 3c_3 = -3$

Power of T, $0 = -b_3 - 2$, $\therefore b_3 = -2$

Substituting the values of a_3, b_3, c_3 in π_3, we have

$$\pi_3 = d^{-3} N^{-2} \rho^{-1} \sigma = \frac{\sigma}{d^3 N^2 \rho}$$

10. **Fourth π-term :** $\pi_4 = d^{a_4} N^{b_4} \rho^{c_4} \omega$

Substituting dimensions on both sides,

$$M^0L^0T^0 = [L]^{a_4} [T^{-1}]^{b_4} [ML^{-3}]^{c_4} [ML^{-2}T^{-2}]$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_4 + 1$, $\therefore c_4 = -1$

Power of L, $0 = a_4 - 3c_4 - 2$, $\therefore a_4 = 3c_4 + 2 = -3 + 2 = -1$

Power of T, $0 = -b_4 - 2$, $\therefore b_4 = -2$

Substituting the values of a_4, b_4, c_4 in π_4, we have

$$\pi_4 = d^{-1} N^{-2} \rho^{-1} \omega = \frac{\omega}{dN^2\rho}$$

11. Now substituting the values of $\pi_1, \pi_2, \pi_3, \pi_4$ in eq. (2.22.1), we get

$$f\left(\frac{Q}{d^3N}, \frac{\mu}{\rho N d^2}, \frac{\sigma}{d^3N^2\rho}, \frac{\omega}{dN^2\rho}\right) = 0$$

or
$$\frac{Q}{d^3N} = f_1\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{d^3N^2\rho}, \frac{\omega}{dN^2\rho}\right]$$

$$Q = d^3N\phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{d^3N^2\rho}, \frac{\omega}{dN^2\rho}\right]$$

Que 2.23. **Find the form of equation for discharge *Q* through a sharp edged triangular notch; assuming *Q* depends upon the central angle α of the notch, head *H*, gravitational acceleration '*g*' and on the mass density ρ, viscosity μ and surface tension σ of the fluid.**

AKTU 2017-18, Marks 10

Answer

Given : Discharge Q is a function of $H, g, \alpha, \rho, \mu, \sigma$

$$f_1 = (Q, H, g, \alpha, \rho, \mu, \sigma)$$

To Find : Discharge equation.

1. Total number of variables, $n = 7$
2. Dimensions of each variable are,

$$Q = L^3T^{-1}, g = LT^{-2}, \rho = ML^{-3}, \mu = ML^{-1}T^{-1}$$
$$\sigma = MT^{-2}, H = L$$

3. Number of fundamental dimensions, $m = 3$
4. Number of π-terms = 7 – 3 = 4
5. Now, discharge function can be written as,

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0 \qquad ...(2.23.1)$$

6. Choosing g, H, ρ as the repeating variables, the π-terms are

$$\pi_1 = H^{a1}\, g^{b1}\, \rho^{c1}\, Q$$
$$\pi_2 = H^{a2}\, g^{b2}\, \rho^{c2}\, \alpha$$
$$\pi_3 = H^{a3}\, g^{b3}\, \rho^{c3}\, \mu$$
$$\pi_4 = H^{a4}\, g^{b4}\, \rho^{c4}\, \sigma$$

7. **π_1-term :** $\pi_1 = H^{a1}\, g^{b1}\, \rho^{c1}Q$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a1}\, [LT^{-2}]^{b1}\, [ML^{-3}]^{c1}\, [L^3T^{-1}]$$
$$M^0L^0T^0 = M^{c1}L^{(a_1 + b_1 - 3c_1 + 3)}\, T^{(-2b_1 - 1)}$$

Equating the powers of M, L, T on both sides,

Power of M, $\quad 0 = c_1, \quad \therefore c_1 = 0$

Power of T, $\quad 0 = -2b_1 - 1, \quad \therefore b_1 = -1/2$

Power of L, $\quad 0 = a_1 + b_1 - 3c_1 + 3, \quad \therefore a_1 = -5/2$

On substituting the values of a_1, b_1 and c_1 in π_1 term, we have

$$\pi_1 = H^{-\frac{5}{2}}\; g^{-1/2}\rho^0 Q$$
$$\pi_1 = \frac{Q}{H^{5/2}g^{1/2}}$$

8. **π_2-term :** $\pi_2 = H^{a2}\, g^{b2}\, \rho^{c2}\, \alpha$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a2}\, [LT^{-2}]^{b2}\, [ML^{-3}]^{c2}\, [M^0L^0T^0]$$
$$M^0L^0T^0 = M^{c2}L^{(a_2 + b_2 - 3c_2)}\, T^{-2b_2}$$

Equating the powers of M, L, T on both sides,

Power of M, $\quad 0 = c_2, \quad \therefore c_2 = 0$

Power of T, $\quad 0 = -2b_2, \quad \therefore b_2 = 0$

Power of L, $\quad 0 = a_2 + b_2 - 3c_2, \quad \therefore a_2 = 0$

On substituting the values of a_2, b_2 and c_2 in π_2 term, we have

$$\pi_2 = \alpha$$

9. **π_3-term :** $\pi_3 = H^{a3}\, g^{b3}\, \rho^{c3}\, \mu$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a3}\, [LT^{-2}]^{b3}\, [ML^{-3}]^{c3}\, [ML^{-1}T^{-1}]$$

$$M^0L^0T^0 = M^{(c3 + 1)}L^{(a3 + b3 - 3c3 - 1)}\, T^{(-2b3 - 1)}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_3 + 1$, $\therefore c_3 = -1$

Power of T, $0 = -2b_3 - 1$, $\therefore b_3 = -1/2$

Power of L, $0 = a_3 + b_3 - 3c_3 - 1$, $\therefore a_3 = -3/2$

On substituting the values of a_3, b_3 and c_3 in π_3 term, we have

$$\pi_3 = H^{-3/2} g^{-1/2} \rho^{-1} \mu$$

$$\pi_3 = \frac{\mu}{H\rho\sqrt{gH}}$$

10. **π_4-term :** $\pi_4 = H^{a4}\, g^{b4}\, \rho^{c4}\, \sigma$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a4}\, [LT^{-2}]^{b4}\, [ML^{-3}]^{c4}\, [MT^{-2}]$$

$$M^0L^0T^0 = M^{(c4 + 1)}L^{(a4 + b4 - 3c4)}\, T^{(-2b4 - 2)}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_4 + 1$, $\therefore c_4 = -1$

Power of T, $0 = -2b_4 - 2$, $\therefore b_4 = -1$

Power of L, $0 = a_4 + b_4 - 3c_4$, $\therefore a_4 = -2$

On substituting the values of a_4, b_4 and c_4 in π_4 term, we have

$$\pi_4 = H^{-2} g^{-1} \rho^{-1} \sigma$$

$$\pi_4 = \frac{\sigma}{H^2 g\rho}$$

11. Substituting the values of π_1, π_2, π_3 and π_4 in eq. (2.23.1), we have

$$f_1\left(\frac{Q}{H^{5/2}g^{1/2}}, \alpha, \frac{\mu}{H\rho\sqrt{gH}}, \frac{\sigma}{H^2 g\rho}\right) = 0$$

$$Q = H^{5/2}g^{1/2}\, f\left(\alpha, \frac{\mu}{H\rho\sqrt{gH}}, \frac{\sigma}{H^2 g\rho}\right)$$

Que 2.24. Assuming the drag force, *F* exerted on a body is a function of the following :
Fluid density ρ, Fluid viscosity μ, Diameter *d*, Velocity *u*
Show the drag force can be expressed as,

$$F = d^2 u^2 \rho \, \phi \, (R_e)$$

Where φ is some unknown function and *Re* is Reynolds number.

AKTU 2016-17, Marks 10

Answer

Same as Q. 2.22, Page 2–25A, Unit-2.

Que 2.25. The pressure drop 'Δ*p*' in a pipe of diameter '*D*' and length '*L*' due to viscous flow depends on the velocity 'v', dynamic viscosity 'μ', average height '*k*' and mass density 'ρ' using Buckingham's theorem obtain expression for 'Δ*p*'.

AKTU 2015-16, Marks 05

Answer

Same as Q. 2.23, Page 2–27A, Unit-2.

$$\left(\text{Answer : } \Delta p = \rho \mathrm{v}^2 \phi \left[\frac{l}{D}, \frac{\mu}{\rho \mathrm{v} D}, \frac{k}{D} \right] \right)$$

PART-10

Important Dimensional Numbers and their Significance.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.26. Define the term dimensionless numbers and discuss some important dimensionless numbers and their significance and applications.

Answer

A. Dimensionless Numbers : Dimensionless numbers are the ratio of inertia force and a force, which may be a viscous force, gravity force, pressure force, surface tension force or elastic force.

B. Important Dimensionless Numbers :

a. Reynold's Number :

1. It is defined as the ratio of the inertia force to the viscous force.

Reynold's number, $(R_e) = \dfrac{\text{Inertia force}}{\text{Viscous force}}$

2. $$\text{Inertia force} = \text{Mass} \times \text{Acceleration}$$
$$= \text{Density} \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}}$$
$$= \rho\, A\text{v} \times \text{v} = \rho\, A\text{v}^2$$
$\{\because$ Volume per time = Area × Velocity = Av$\}$

$$\text{Viscous force } (F_v) = \text{Shear stress} \times \text{Area} = \tau \times A$$
$$= \mu \frac{du}{dy} \times A = \frac{\mu \text{v}}{L} \text{A}$$

3. So, $$\text{Reynold's number} = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho A\text{v}^2}{\left(\dfrac{\mu \text{v} A}{L}\right)}$$

$$Re = \frac{\rho \text{v} L}{\mu} \text{ or } \frac{\rho \text{v} d}{\mu} \qquad \text{(for pipe flow)}$$

i. Significance :

1. Reynold's number is used to determine whether the flow is laminar or turbulent.
2. Reynold's number signifies the relative predominance of the inertia to the viscous forces occurring in the flow systems.

ii. Applications :

1. Motion of submarine completely under water.
2. Incompressible flow through pipes of smaller size.
3. Flow through low speed turbo machines.

b. Froude's Number :

1. It is the square root of the ratio of inertia force to the gravity force of a flowing fluid. It is denoted by F_e.

Mathematically, $F_e = \sqrt{\dfrac{F_i}{F_g}}$

2. Inertia force, $F_i = \rho\, A\text{v}^2$

Gravity force, $F_g = mg = \rho\, ALg$

$$\therefore \qquad F_e = \sqrt{\frac{\rho A v^2}{\rho A L g}}$$

$$F_e = \frac{v}{\sqrt{Lg}}$$

i. Significance :

1. It signifies the dynamic similarity of the flow situation where gravitational force (F_g) is most significant.
2. Froude number differentiates the super critical, subcritical and critical flow.

ii. Applications :

1. Flow over notches and weir.
2. Flow over the spillway of a dam.
3. Flow through open channels.
4. Motion of ship in rough and turbulent sea.

c. Euler's Number (E_u) :

1. It is the square root of the ratio of the inertia force to the pressure force of a flowing fluid.

Mathematically, $E_u = \sqrt{\dfrac{F_i}{F_p}}$

2. Pressure force (F_p) = Pressure × Area

$$= p \times A$$

Inertia force (F_i) = $\rho A v^2$

$$\therefore \qquad E_u = \sqrt{\frac{\rho A v^2}{pA}} = \frac{v}{\sqrt{p/\rho}}$$

i. Significance :

1. It signifies those flow problems or situations in which pressure gradient exists.

ii. Applications :

1. Discharge through orifice and mouth piece.
2. Pressure rise due to sudden closure of valves.
3. Flow through pipes.
4. Water hammer created in penstocks.

d. Weber's Number (W_e) :

1. It is the square root of the ratio of inertia force of a flowing fluid to the surface tension force of a flowing fluid.

Mathematically, $W_e = \sqrt{\dfrac{F_i}{F_s}}$

2. Inertia force (F_i) = $\rho A v^2$

Surface tension force (F_s) = Surface tension per unit length × length = σL

$$\therefore \quad W_e = \sqrt{\frac{\rho A v^2}{\sigma L}} \qquad \{\because A = L^2\}$$

$$= \sqrt{\frac{\rho L^2 v^2}{\sigma L}} = \sqrt{\frac{\rho L v^2}{\sigma}} = \sqrt{\frac{v^2}{\sigma / \rho L}} = \frac{v}{\sqrt{\sigma / (\rho L)}}$$

i. Significance :

1. It signifies those flow problems in which surface tension force is dominant.

ii. Applications : It is applicable in following situations :

1. Capillary movement.
2. Flow of blood in veins and arteries.
3. Liquid atomization.

e. Mach Number (M) :

1. It is defined as the square root of the ratio of inertia force to the elastic force.

Mathematically, $M = \sqrt{\dfrac{F_i}{F_e}}$

2. Inertia force (F_i) = $\rho A v^2$

Elastic force (F_e) = $KA = KL^2$ ($\because$ Area = L^2)

$$\therefore \quad M = \sqrt{\frac{\rho A v^2}{KL^2}} = \frac{v}{\sqrt{K/\rho}} = \frac{v}{C}$$

Where, $\sqrt{\dfrac{K}{\rho}} = C$ (Velocity of sound in the fluid)

i. Significance :

1. Mach number is used to differentiate the flow as subsonic flow, sonic flow and supersonic flow.

ii. Applications :

1. High velocity flow in pipes.
2. Motion of missiles or high speed projectiles.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain the following with example :
a. Steady and unsteady flows,
b. Laminar and turbulent flows,
c. Rotational and irrotational flows,
d. Compressible and incompressible flows, and
e. Uniform and non-uniform flows.

Ans. Refer Q. 2.2, Unit-2.

Q. 2. Derive continuity equation for a 3-D steady or unsteady flow in a cartesian coordinate system.

Ans. Refer Q. 2.6, Unit-2.

Q. 3. A 500 mm diameter pipe carrying water at rate 0.5 m^3/sec branches into two pipes of 200 mm and 400 mm diameters. If the rate of flow of water through small diameter pipe is 0.2 m^3/sec. Determine velocity of flow in each pipe.

Ans. Refer Q. 2.7, Unit-2.

Q. 4. Two velocity components are given in the following equations, find the third component such that they satisfy the continuity equation :
$u = x^3 + y^2 + 2z^2$, $v = -x^2 y - yz - xy$

Ans. Refer Q. 2.9, Unit-2.

Q. 5. Sketch the streamlines represented by $\Psi = x^2 + y^2$. Also find out the velocity and its direction at point (1, 2).

Ans. Refer Q. 2.15, Unit-2.

Q. 6. The velocity components in a two-dimensional flow field for an incompressible fluid are expressed as

$$u = \frac{y^3}{3} + 2x - x^2y \; ; \; v = xy^2 - 2y - \frac{x^3}{3}$$

a. Show that these functions represent a possible case of an irrotational flow.

b. Obtain an expression for stream function ψ.

c. Obtain an expression for velocity potential ϕ.

Ans. Refer Q. 2.19, Unit-2.

Q. 7. Using Buckingham's π theorem, show that the discharge, Q consumed by an oil ring is given by

$Q = (Nd^3)\, f\,[\mu / (\rho N d^2), \sigma / (\rho N^2 d^3), \omega / (\rho N^2 d)]$

Where, d is internal diameter of ring, N is rotational speed, ρ is density, μ is viscosity, σ is surface tension and ω is the specific weight of oil.

Ans. Refer Q. 2.22, Unit-2.

☺☺☺

3 UNIT

Flow Through Pipes, Boundary Layer Thickness

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Part-11 : Three reservoir Problems and Pipe Networks .... 3–23A to 3–26A

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Part-13 : Boundary Layer over a Flat Plate, Laminar Boundary Layer, Application of Momentum Equation, Turbulent Boundary Layer, Laminar Sub-Layer ...... **3–33A to 3–37A**

Part-14 : Separation and its Control .... **3–37A to 3–40A**

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Part-18 : Magnus Effect .... **3–47A to 3–47A**

PART-1

Equation of Motion for Laminar Flow through Pipes.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **What are the characteristics of a laminar flow ? Derive the expression for the velocity distribution for viscous flow through a circular pipe. Also sketch the distribution of velocity and shear stress across a section of pipe.** **AKTU 2016-17, Marks 15**

Answer

A. Characteristics of Laminar Flow :

1. Laminar flow obeys Newton's law of viscosity.
2. The laminar flow is rotational.
3. No slip will occur at the boundary of laminar flow.
4. There will be no mixing of layers occur in laminar flow.
5. For laminar flow, Reynold's number < 2000.

B. Derivation for Velocity and Shear Stress Distribution :

1. Let us consider a horizontal pipe having diameter d and radius R.
2. Direction of fluid is shown in Fig. 3.1.1.
3. Take a fluid element in between the radius r and $r + dr$ and length of the fluid element be Δx.
4. If p is the pressure on the face AB, then pressure on face CD will be $p + \frac{\partial p}{\partial x}\Delta x$.

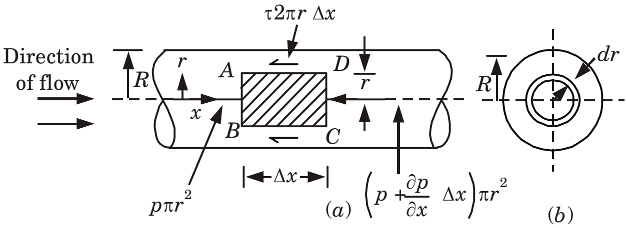


Fig. 3.1.1.

5. Total pressure force = Pressure force at face AB – Pressure force at face CD

$$= p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 = -\frac{\partial p}{\partial x}\Delta x \pi r^2$$

6. The shear force acting on the surface AD and BC

$$= -\tau 2\pi r \Delta x \text{ (opposite to the direction of flow)}$$

a. For Shear Stress Distribution :

1. Now, $\Sigma F = 0$

$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau 2\pi r \Delta x = 0$$

$$\frac{\partial p}{\partial x} r = -2\tau$$

Shear stress, $\tau = -\dfrac{r}{2}\dfrac{\partial p}{\partial x}$ ...(3.1.1)

2. At $r = R$,

Wall shear stress, $\tau_w = \dfrac{-R}{2}\dfrac{\partial p}{\partial x}$

3. As $\dfrac{\partial p}{\partial x}$ = Constant, so $\tau \propto r$

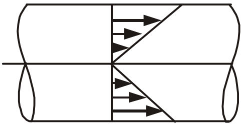


Fig. 3.1.2. Shear stress distribution.

b. For Velocity Distribution :

1. According to Newton's law of viscosity,

$\tau = \mu \dfrac{du}{dy}$, where y is measured from pipe wall.

2. So, $y = R - r$

Differentiating both the sides,

$dy = -dr$ $\{dR = 0$, as R is constant$\}$

3. Therefore, $\tau = -\mu \dfrac{du}{dr}$ ...(3.1.2)

4. From eq. (3.1.1) and eq. (3.1.2), we get

$$\tau = -\mu\frac{du}{dr} = -\frac{r}{2}\frac{\partial p}{\partial x}$$

$$\therefore \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x} r$$

$$du = \frac{1}{2\mu}\frac{\partial p}{\partial x} r dr \qquad \left(\because \frac{\partial p}{\partial x} \text{ and } \frac{1}{2\mu} \text{ are constants}\right)$$

6. On integrating both the sides,

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2$$

7. For $r = R$, $u = 0$ and $r = r$, $u = u$

Now,
$$[u]_u^0 = \frac{1}{4\mu}\frac{\partial p}{\partial x}\left[r^2\right]_r^R$$

$$-u = \frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$$

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$$

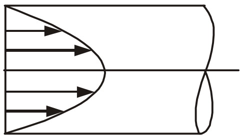


Fig. 3.1.3. Velocity distribution.

Hence velocity distribution is parabolic in nature.

8. When $r = 0$,

$$u_{\text{max.}} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

Que 3.2. **Prove that the maximum velocity in a circular pipe for viscous flow is equal to two times the average velocity of flow.**

Answer

1. We know that, $u_{\text{max.}} = -\dfrac{1}{4\mu}\dfrac{\partial p}{\partial x} R^2$

2. Discharge through an elemental ring of radius r,

$$dQ = \text{Velocity at a radius } r \times \text{Area of ring element}$$
$$= u \times 2\pi r\, dr$$
$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times (2\pi r\, dr)$$

3. Total discharge, $Q = \displaystyle\int dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times (2\pi r\, dr)$

$$= -\frac{2\pi}{4\mu}\frac{\partial p}{\partial x}\int_0^R r(R^2 - r^2)dr = -\frac{2\pi}{4\mu}\frac{\partial p}{\partial x}\left[R^2\frac{r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= -\frac{2\pi}{4\mu}\frac{\partial p}{\partial x}\frac{R^4}{4}$$

4. Now, $$\bar{u} = \frac{Q}{\pi R^2} = \frac{\left(-\frac{2\pi}{4\mu}\frac{\partial p}{\partial x}\frac{R^4}{4}\right)}{\pi R^2} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

5. Ratio of maximum velocity to average velocity,

$$\frac{u_{max.}}{\bar{u}} = \frac{\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2} = 2$$

So, ratio of maximum velocity to average velocity will be equal to 2.

Que 3.3. **Prove that for laminar flow through a circular pipe, energy correction factor (α) = 2.**

Answer

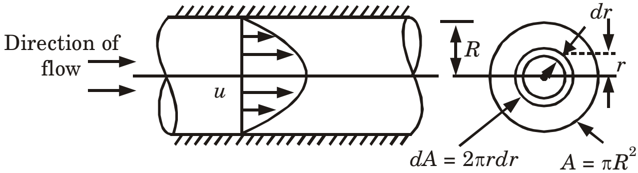


Fig. 3.3.1.

1. Kinetic energy per second of the fluid flowing through an elementary ring of radius r and of width dr,

$$KE = \frac{1}{2} \times \text{Mass per second} \times u^2$$

$$= \frac{1}{2}\rho dQ u^2 \qquad [\because \text{Mass per second} = \rho dQ]$$

$$= \frac{1}{2}\rho\,(u \times 2\pi r\,dr)\,u^2 \qquad [\because dQ = u \times 2\pi r dr]$$

$$= \pi\rho\, r u^3\, dr$$

2. Total actual kinetic energy of flow per sec

$$= \int_0^R \pi\rho\, r\, u^3\, dr$$

3. On putting, $u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)$, we have

Total KE per second,

$$= \int_0^R \pi\rho\left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2 - r^2)\right]^3 r\,dr$$

$$= \pi\rho\left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\right]^3 \int_0^R [R^2 - r^2]^3\, r\,dr$$

$$= \pi\rho\left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\right]^3 \int_0^R (R^6 r - r^7 - 3R^4 r^3 + 3R^2 r^5)\,dr$$

$$= \pi\rho\left[\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\right]^3 \left[R^6\frac{r^2}{2} - \frac{r^8}{8} - 3R^4\frac{r^4}{4} + 3R^2\frac{r^6}{6}\right]_0^R$$

$$= \frac{\pi\rho}{64\,\mu^3}\left(\frac{-\partial p}{\partial x}\right)^3 \left[\frac{R^8}{2} - \frac{R^8}{8} - \frac{3R^8}{4} + \frac{3R^6}{6}\right]$$

$$= \frac{\pi\rho}{64\,\mu^3}\left(\frac{-\partial p}{\partial x}\right)^3 R^8\left[\frac{12 - 3 - 18 + 12}{24}\right] = -\frac{\pi\rho}{64\mu^3}\left(\frac{\partial p}{\partial x}\right)^3 \frac{R^8}{8}$$

4. Kinetic energy of the flow for average velocity per second

$$= \frac{1}{2}\times\left(\frac{\text{Mass}}{\text{Sec}}\right)\times\bar{u}^2 = \frac{1}{2}\times(\rho A\bar{u})\times\bar{u}^2$$

(Mass per second = Area × Density × Average velocity)

$$= \frac{1}{2}\rho A\bar{u}^3$$

5. On putting, $A = \pi R^2$, and $\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$, we have

KE of the flow per second for average velocity

$$= \frac{1}{2}\rho\pi R^2\left[\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2\right]^3 = -\frac{1}{2}\left[\frac{\pi\rho}{64\mu^3}\left(\frac{\partial p}{\partial x}\right)^3\frac{R^8}{8}\right]$$

6. Energy correction factor,

$$\alpha = \frac{\text{KE of flow / s for actual velocity}}{\text{KE of flow / s for average velocity}}$$

$$\alpha = \frac{-\frac{\pi\rho}{64\mu^3}\left(\frac{\partial p}{\partial x}\right)^3\frac{R^8}{8}}{-\frac{1}{2}\left[\frac{\pi\rho}{64\mu^3}\left(\frac{\partial p}{\partial x}\right)^3\frac{R^8}{8}\right]} = 2$$

Que 3.4. **For laminar flow of an oil having dynamic viscosity μ = 1.766 Pa-s in a 0.3 m diameter pipe, the velocity distribution is parabolic with a maximum point velocity of 3 m/s at the centre of the pipe. Calculate the shear stresses at the pipe wall and within the fluid 50 mm from the pipe wall.**

AKTU 2015-16, Marks 05

Answer

Given : μ = 1.766 Pa-s, D = 0.3 m, u_{max} = 3 m/s
To Find : Shear stresses at the pipe wall and within the fluid 50 mm from the pipe wall.

1. Average velocity of flow,

$$\bar{u} = \frac{1}{2} u_{max} = \frac{3}{2} = 1.5 \text{ m/s}$$

2. Now,

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{p_1 - p_2}{L} = \frac{32\mu\bar{u}}{D^2}$$

Thus,

$$\left(-\frac{\partial p}{\partial x}\right) = \frac{32 \times 1.766 \times 1.5}{(0.3)^2} = 941.87 \text{ Pa/m}$$

3. The shear stress at the pipe wall,

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{r}{2} = \frac{(941.87 \times 0.3)}{2 \times 2} = 70.64 \text{ Pa}$$

4. The shear stress at 50 mm from the pipe wall is,

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{r}{2} = 941.87 \times \frac{(0.15 - 0.05)}{2} = 47.09 \text{ Pa}$$

Que 3.5. **Find the loss of head due to friction and power required to pump an oil of specific gravity 0.85 and absolute viscosity 1.5 poise through a 25 cm diameter and 10 km long pipe laid at a slope of 1 in 200. The rate of flow of oil is 0.022 m³/s.**

Answer

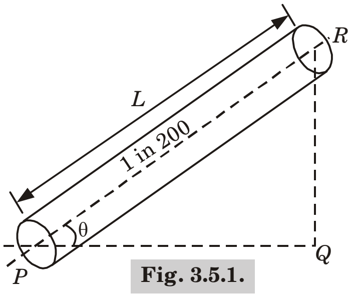


Fig. 3.5.1.

Given : Specific gravity of oil = 0.85, $\rho_o = 0.85 \times 1000 = 850 \text{ kg/m}^3$

$\mu_o = 1.5 \text{ poise} = \dfrac{1.5}{10} = 0.15 \text{ N-s/m}^2$, $d = 25 \text{ cm} = 0.25 \text{ m}$

$L = 10 \text{ km} = 10000 \text{ m}$, $\tan\theta = \dfrac{1}{200}$, $Q = 0.022 \text{ m}^3/\text{s}$

To Find : i. Loss of head due to friction.
ii. Power required to pump the oil.

1. Velocity of flow,

$$\text{v} = \frac{Q}{A} = \frac{0.022}{\frac{\pi}{4}d^2} = \frac{0.022 \times 4}{\pi (0.25)^2}$$

$$\text{v} = 0.448 \text{ m/s}$$

2. Reynold's number, $R_e = \dfrac{\rho_o \text{v} d}{\mu}$

$$R_e = 850 \times 0.448 \times \frac{0.25}{0.15} = 634.67$$

This value is less than 2000. Hence, flow is laminar.

3. From ΔPQR, $\tan\theta = \dfrac{1}{200} \Rightarrow \sin\theta = \dfrac{1}{200}$

(As θ is very small so $\sin\theta = \tan\theta$)

$$\sin\theta = \frac{RQ}{PR}$$

$$RQ = PR \sin\theta = 10000 \times \frac{1}{200} = 50 \text{ m}$$

4. Loss of head due to friction in pipe is given as,

$$p_1 - p_2 = \frac{32\mu_o \bar{u} L}{d^2} + \rho_o g h \qquad (\because h = RQ \text{ and } \bar{u} = \text{v})$$

$$= \frac{32 \times 0.15 \times 0.448 \times 10000}{(0.25)^2} + (850 \times 9.81 \times 50)$$

$$p_1 - p_2 = 344064 + 416925 = 760989 \text{ N/m}^2$$

5. Head loss $= \dfrac{p_1 - p_2}{\rho_o g}$

$$h_f = \frac{760989}{850 \times 9.81} = 91.26 \text{ m}$$

6. Weight of oil flowing per second,

$$w = \rho_o g Q$$
$$= 850 \times 9.81 \times 0.022 = 183.447 \text{ N/s}$$

7. Power required to pump the oil $= w\, h_f$

$$= 91.26 \times 183.447 = 16741.37 \text{ W}$$

PART-2

Turbulent Flow.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.6. What is turbulent flow ? Write down the various types of turbulence.

Answer

A. Turbulent Flow :

1. In a pipe, turbulent flow occurs when $R_e > 4000$.
2. In a turbulent flow, the fluid motion is irregular and there is complete mixing of fluid due to collision of fluid masses with one another.
3. As the fluid masses in adjacent layers have different velocities, interchange of fluid masses between the adjacent layers is accompanied by a transfer of momentum which causes additional shear stresses of high magnitude between adjacent layers.
4. The contribution of fluid viscosity to total shear is small and is usually neglected.

B. Types of Turbulence : The turbulence can be classified as follows :

i. **Wall Turbulence :** It occurs in immediate vicinity of solid surfaces and in the boundary layer flows, where the fluid has a negligible mean acceleration.

ii. **Free Turbulence :** It occurs in jets, wakes, mixing layers etc.

iii. **Convective Turbulence :** It takes place where there is conversion of PE into KE by the process of mixing.

PART-3

Isotropic, Homogeneous Turbulence.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.7. Write short note on isotropic and homogeneous turbulence.

Answer

A. Homogeneous Turbulence :

1. If the turbulence has the same structure quantitatively in all parts of the flow field, the turbulence is said to be homogeneous.
2. The term homogeneous turbulence implies that the velocity fluctuations in the system are random. The average turbulent characteristics are independent of the position in the fluid, *i.e.*, invariant to axis translation.

B. Isotropic Turbulence :

1. Turbulence is called isotropic if its statistical features have no directional preference and perfect disorder persists. Its velocity fluctuations are independent of the axis of reference, *i.e*. invariant to axis rotation and reflection.
2. In isotropic turbulence, fluctuations are independent of the direction of reference.

PART-4

Scale and Intensity of Turbulence.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.8. Describe turbulence length scale and turbulence intensity in short.

Answer

A. Turbulence Length Scale :

1. The turbulence length scale, l is a physical quantity describing the size of the large energy containing eddies in a turbulent flow.
2. The turbulent length scale is often used to estimate the turbulent properties of the given problem.
3. The turbulent length scale should normally not be larger than the dimension of the problem, since that would mean that the turbulent eddies are larger than the problem size.

B. Turbulence Intensity :

1. Turbulence intensity is a scale characterizing turbulence expressed as a percent.

2. An idealized flow of air with absolutely no fluctuations in air speed or direction would have a turbulence intensity value of 0 %. This idealized case never occurs on earth.
3. However, due to how turbulence intensity is calculated, values greater than 100 % are possible. This can happen, for example, when the average air speed is small and there are large fluctuations present.

PART-5

Measurement of Turbulence.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.9. **Describe the measurement of turbulence with the help of hot-wire anemometer.**

Answer

1. A hot-wire anemometer is an instrument which is commonly used for measuring the velocity of flow of a compressible fluid such as gas.
2. The anemometer consists of a platinum, nickel or tungsten wire of about 5×10^{-3} to 8×10^{-3} mm diameter and 16 mm length.
3. The wire is mounted on the ends of two pointed prongs.
4. In the arrangement shown in Fig. 3.9.1(*a*), constant current is passed through wire by keeping the voltage across the bridge.
5. As the air or gas flows the hot-wire cools, its resistance changes and the galvanometer deflects.
6. The galvanometer deflection is correlated with the velocity of flow of air or gas by calibration. It is then termed as constant current hot-wire anemometer.
7. Fig. 3.9.1(*b*) illustrates another arrangement for hot-wire anemometer which is termed as constant temperature (or constant-resistance) hot-wire anemometer.
8. Initially when there is no flow and hot-wire is in contact with air or gas at rest, a small current is passed through hot-wire.
9. As the air or gas flows past hot-wire, its temperature and hence its resistance will vary, which will cause the galvanometer needle to deflect from zero reading.
10. Now by adjusting the variable resistance B the current passing through hot-wire is suitably adjusted so that its temperature and hence the resistance is maintained constant and the galvanometer reading is brought back to zero.

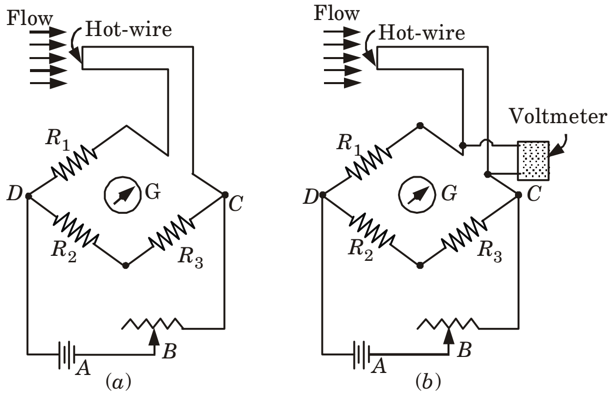


Fig. 3.9.1. (*a*) Constant-current hot-wire anemometer, (*b*) Constant-temperature hot-wire anemometer.

11. The reading of the voltmeter connected across hot-wire will change which may be noted.

PART-6

Eddy Viscosity.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.10. **What is eddy viscosity ?**

Answer

1. The viscosity which accounts for momentum transport by turbulence eddies is known as eddy viscosity.
2. Similar to the expression for viscous shear, turbulent shear in mathematical form is expressed as,

$$\tau_t = \eta \frac{d\bar{u}}{dy}$$

Where, τ_t = Shear stress due to turbulence,
η = Eddy viscosity, and
$\bar{u}$ = Average velocity at a distance y from boundary.

3. The ratio of η (eddy viscosity) and ρ (mass density) is known as kinematic eddy viscosity and is denoted by ε (epsilon).
Mathematically, $\varepsilon = \eta / \rho$

PART-7

Resistance to Flow, Minor Losses.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.11. **Briefly explain resistance to flow ? Discuss major losses and minor losses in detail.**

Answer

A. Resistance to Flow :

1. When water flows in a pipe, it experiences some resistance to its motion, due to which its velocity and ultimately the head of water available reduced. This resistance is known as resistance to flow.
2. These resistance are due to :

i. Friction,
ii. Sudden enlargement of pipe,
iii. Sudden contraction of pipe,
iv. Bend of pipe,
v. An obstruction in pipe, and
vi. Pipe fittings.

B. Major Losses or Loss of Energy or Head due to Friction :

a. Darcy - Weisbach Formula for Head Loss due to Friction :

1. The equation is, $h_f = \dfrac{4fLv^2}{2g \times d}$

Where, h_f = Loss of head due to friction,

f = Coefficient of friction and it is a function of Reynold's number

$= \dfrac{16}{R_e}$ for $R_e < 2000$ (laminar flow)

$= \dfrac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = Length of pipe,

v = Mean velocity of flow, and

d = Diameter of pipe.

b. Chezy's Formula for Loss of Head due to Friction in Pipes :

1. The equation is, $h_f = \dfrac{f'}{\rho g} \times \dfrac{P}{A} \times L \times v^2$

Where, P = Wetted perimeter of pipe, and

A = Area of cross section of pipe.

2. The ratio of $\frac{A}{P} = \frac{\text{Area of flow}}{\text{Perimeter (wetted)}}$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

C. Minor Energy or Head Losses :

1. The loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy.
2. The minor loss of energy includes the following :

a. Loss of Head due to Sudden Enlargement :

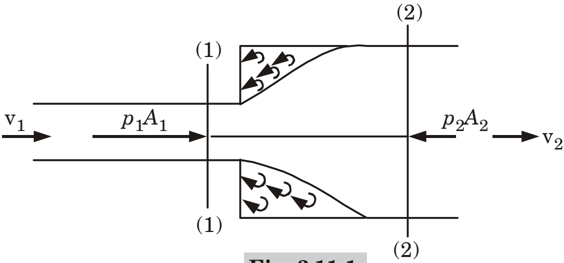


Fig. 3.11.1.

$$\therefore \qquad h_e = \frac{(v_1 - v_2)^2}{2g}$$

b. Loss of Head due to Sudden Contraction :

$$h_c = \frac{v_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = K\frac{v_2^2}{2g} = 0.5\frac{v_2^2}{2g}$$

Where, $$K = \left[\frac{1}{C_c} - 1\right]^2$$

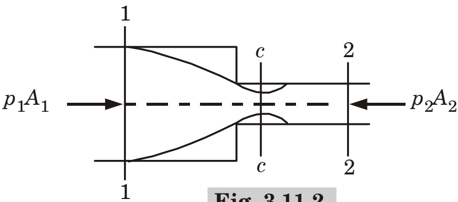


Fig. 3.11.2.

c. Loss of Head at the Entrance of a Pipe :

1. This type of loss occurs when a liquid enters a pipe which is connected to a large tank or reservoir.
2. Loss of head at the entrance (or inlet) of a pipe with sharp cornered entrance is taken as $0.5\frac{v^2}{2g}$.

   Where, v = Velocity of liquid in pipe.

d. Loss of Head at the Exit of Pipe :

$$h_o = \frac{v^2}{2g}, \text{ Where, v = Velocity at outlet of pipe.}$$

e. Loss of Head due to an Obstruction in a Pipe :

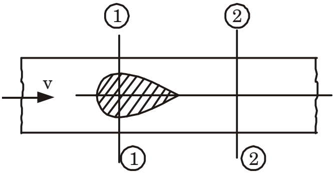


Fig. 3.11.3.

$$h_o = \frac{v^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$

Where, a = Maximum area of obstruction,

A = Area of pipe, and

C_c = Coefficient of contraction.

f. Loss of Head in Pipe due to Bend :

$$h_b = \frac{Kv^2}{2g}$$

Where, h_b = Loss of head due to bend,

v = Velocity of flow, and

K = Coefficient of bend.

Que 3.12. **If 300 mm length of 200 mm diameter pipe with friction factor 0.018 is to be replaced by 150 mm diameter pipe with friction factor 0.02 to carry the same discharge, what length will have to be provided ?**

Answer

Given : $L_1 = 0.3$ m, $d_1 = 0.2$ m, $f_1 = 0.018$, $d_2 = 0.15$ m, $f_2 = 0.02$

To Find : Length of the pipe.

1. Here, $h_{f1} = h_{f2}$

$$\frac{4f_1L_1 v_1^2}{2gd_1} = \frac{4f_2L_2 v_2^2}{2gd_2}$$

$$\frac{0.018 \times 0.3 \times v_1^2}{0.2} = \frac{0.02 \times L_2 \times v_2^2}{0.15}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{0.02 \times L_2 \times 0.2}{0.018 \times 0.3 \times 0.15}} = \sqrt{4.94 \times L_2} \quad ...(3.12.1)$$

2. Now for same discharge

$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{0.15}{0.20}\right)^2 = 0.5625 \quad ...(3.12.2)$$

3. From eq. (3.12.1) and eq. (3.12.2), we get

$$\sqrt{4.94 \times L_2} = 0.5625$$

$$L_2 = 0.064 \text{ m}$$

So, the length of the pipe to be provided = 0.064 m

PART-8

Pipes in Series and Parallel.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.13. **Derive the head loss expressions for the pipes in series and parallel.**

Answer

A. Pipes in Series :

1. Let, L_1, L_2, L_3 = Length of pipes 1, 2, and 3 respectively,

d_1, d_2, d_3 = Diameter of pipes 1, 2, and 3 respectively,

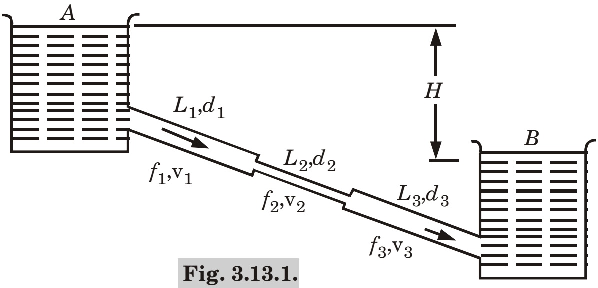


Fig. 3.13.1.

v_1, v_2, v_3 = Velocity of flow through pipes 1, 2, and 3,

f_1, f_2, f_3 = Coefficient of friction for pipes 1, 2, and 3,

H = Difference of water level in two tanks.

2. Pipes are in series, as shown in Fig. 3.13.1, hence the discharge passing through each pipe is same.

$$\therefore \quad Q = A_1 v_1 = A_2 v_2 = A_3 v_3$$

3. The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$\therefore H = \frac{0.5v_1^2}{2g} + \frac{4f_1L_1v_1^2}{d_1 \times 2g} + \frac{0.5v_2^2}{2g} + \frac{4f_2L_2v_2^2}{d_2 \times 2g} + \frac{(v_2 - v_3)^2}{2g} + \frac{4f_3L_3v_3^2}{d_3 \times 2g} + \frac{v_2^2}{2g} \quad ...(3.13.1)$$

4. If minor losses are neglected, then the eq. (3.13.1) becomes,

$$H = \frac{4f_1L_1v_1^2}{d_1 \times 2g} + \frac{4f_2L_2v_2^2}{d_2 \times 2g} + \frac{4f_3L_3v_3^2}{d_3 \times 2g} \quad ...(3.13.2)$$

5. If the coefficient of friction is same for all pipes, *i.e.*, $f_1 = f_2 = f_3 = f$, then eq. (3.13.2) becomes as,

$$H = \frac{4f}{2g}\left[\frac{L_1v_1^2}{d_1} + \frac{L_2v_2^2}{d_2} + \frac{L_3v_3^2}{d_3}\right] \quad ...(3.13.3)$$

B. Pipes in Parallel :

1. The pipes are said to be in parallel (Fig. 3.13.2) when a main line divides into two or more parallel pipes which again join together downstream and continues as a main line.
2. It may be seen from Fig. 3.13.2, the rate of discharge in the main line is equal to the sum of rate of flow through branch pipes.

Thus, $$Q = Q_1 + Q_2 \quad ...(3.13.4)$$

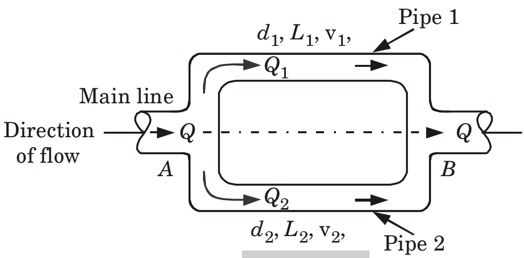


Fig. 3.13.2.

3. When the pipes are arranged in parallel, the loss of head in each pipe (branch) is same.

$\therefore$ Loss of head in pipe 1 = Loss of head in pipe 2

or $$h_f = \frac{4f_1L_1v_1^2}{d_1 \times 2g} = \frac{4f_2L_2v_2^2}{d_2 \times 2g}$$

When, $f_1 = f_2$, then ...(3.13.5)

$$\frac{L_1v_1^2}{d_1} = \frac{L_2v_2^2}{d_2} \quad ...(3.13.6)$$

Que 3.14. **A compound piping system consists of 1800 m of 0.50 m, 1200 m of 0.40 m and 600 m of 0.30 m new cast iron pipes connected in series. Convert the system to (*a*) an equivalent length of 0.40 m pipe, and (*b*) equivalent size pipe of 3600 m long.** **AKTU 2015-16, Marks 05**

Answer

Given : $L_1 = 1800$ m, $d_1 = 0.50$ m, $L_2 = 1200$ m, $d_2 = 0.40$ m
$L_3 = 600$ m, $d_3 = 0.30$ m
To Find : a. Equivalent length of 0.40 m pipe.
b. Equivalent size pipe of 3600 m long.

1. From equivalent pipe size equation,

$$\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} = \frac{L}{d^5}$$

$$\frac{L}{d^5} = \frac{1800}{0.50^5} + \frac{1200}{0.40^5} + \frac{600}{0.30^5}$$

$$\frac{L}{d^5} = 421701.08 \quad \text{...(3.14.1)}$$

2. Equivalent length of 0.40 m pipe,
Putting $d = 0.40$ m in eq. (3.14.1), we get
$L = 4318.22$ m
3. Equivalent size of 3600 m long pipe,
Putting $L = 3600$ m in eq. (3.14.1), we get
$d = 0.3857$ m

PART-9

Power Transmission through a Pipe.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.15. **Derive an expression for power transmission through pipes.**

Answer

1. Consider a tank and pipe connected system as shown in Fig. 3.15.1.
2. Let, H = Head of water at inlet of pipe,
L = Length of pipe,
d = Diameter of pipe,
v = Velocity of water in pipe,
f = Coefficient of friction, and

h_f = Head loss in pipe due to friction.

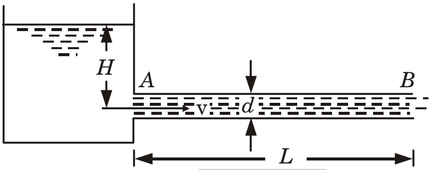


Fig. 3.15.1.

3. The head available at outlet of pipe

$$= \text{Total head at inlet} - \text{Head loss due to friction in pipe}$$

$$= H - h_f = H - \frac{4fLv^2}{d \times 2g} \qquad \left(\because h_f = \frac{4fLv^2}{2dg}\right)$$

4. Weight of water flowing through pipe per second,

$$W = \rho g \times \text{Volume of water per sec}$$

$$= \rho g \times \text{Area} \times \text{Velocity} = \rho g \ \frac{\pi}{4} d^2 v$$

5. Power transmitted at outlet of pipe

$$= \text{Weight of water per sec} \times \text{head at outlet}$$

$$= \rho g \frac{\pi}{4} \ d^2 v \left(H - \frac{4fLv^2}{d \times 2g}\right) \text{Watts}$$

Que 3.16. **A pipe of diameter 300 mm and length 3500 m is used for the transmission of power by water. The total head at the inlet of the pipe is 500 m. Find the maximum power available at the outlet, if the value of f = 0.006.** **AKTU 2014-15, Marks 10**

Answer

Given : d = 300 mm = 0.3 m, L = 3500 m, H_1 = 500 m, f = 0.006
To Find : Maximum power available at the outlet.

1. By the condition of maximum power transmitted,

$$h_f = \frac{H_1}{3} = \frac{500}{3}$$

$$H_2 = H_1 - h_f = 500 - \frac{500}{3} = \frac{1000}{3} \text{ m}$$

2. We know that, $h_f = \dfrac{4fLv^2}{2dg}$

$$\frac{500}{3} = \frac{4 \times 0.006 \times 3500 \times v^2}{2 \times 0.3 \times 9.81}$$

$$v = 3.42 \text{ m/s}$$

3. Discharge, $Q = vA = 3.42 \times \frac{\pi}{4} \times (0.3)^2 = 0.242 \text{ m}^3/\text{s}$

4. Maximum power transmitted = $\rho g Q H_2$

$$= 1000 \times 9.81 \times 0.242 \times \frac{1000}{3} = 791.34 \text{ kW}$$

PART-10

Syphon, Water Hammer.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.17. **What is meant by syphon ?**

Answer

1. Syphon is a long bent pipe employed for carrying water from a reservoir at a higher elevation to another reservoir to a lower elevation when the two reservoirs are separated by a hill or high level ground in between as shown in Fig. 3.17.1

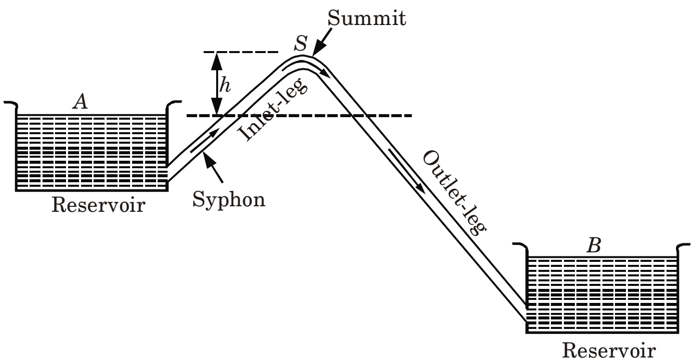


Fig. 3.17.1.

2. The highest point (S) of the syphon is called the summit.
3. The pressure at the point S is less than atmospheric pressure (Since S lies above the free water surface in the tank A).
4. When the pressure at S becomes less than 2.7 m of water absolute, the dissolved air and other gases would come out from water and collect at

the summit. Therefore syphon should be so laid that no section of the pipe will be more than 7.6 m above the hydraulic gradient at the section.

5. Moreover, in order to limit the reduction of the pressure at the summit the length of the inlet-leg (rising portion of the syphon) of the syphon is also required to be limited.

Que 3.18. **What is meant by water hammer ? Give expression for the rise of pressure due to water hammer.**

Answer

A. Water Hammer :

1. In a long pipe, when flowing water is suddenly brought to rest by closing the valve or by any similar cause, there will be a sudden rise in pressure due to the momentum of water being destroyed. This phenomenon of sudden rise in pressure is known as water hammer or hammer blow.
2. A sudden rise in pressure has the effect of hammering action on the walls of the pipe.

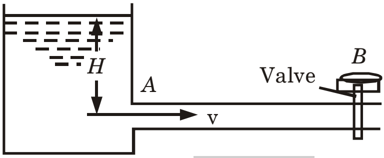


Fig. 3.18.1.

B. Expression for the Rise of Pressure :

The following cases of water hammer in pipes will be considered :

a. Gradual Closure of Valve :

1. Let, A = Area of cross section of the pipe AB,
   L = Length of pipe,
   v = Velocity of flow of water through pipe,
   t = Time (in second) required to close the valve, and
   p = Intensity of pressure wave produced.
2. Mass of water in pipe AB = ρ × Volume of water = ρAL
3. The valve is gradually closed in time 't' seconds and hence the water is brought from initial velocity v to zero velocity in time 't' seconds.
4. Retardation of water

$$= \frac{\text{Change of velocity}}{\text{Time}} = \frac{\text{v} - 0}{t} = \frac{\text{v}}{t}$$

5. Retarding force = Mass × Retardation = $\rho AL \dfrac{\text{v}}{t}$ ...(3.18.1)
6. If p is the intensity of pressure wave produced due to closure of the valve, the force due to pressure wave

$$= p \times \text{Area of pipe} = pA \quad ...(3.18.2)$$

7. Equating the two forces given by eq. (3.18.1) and eq. (3.18.2), we get

$$\rho AL \frac{v}{t} = pA$$

$$p = \frac{\rho L v}{t} \quad ...(3.18.3)$$

8. Head of pressure, $H = \dfrac{p}{\rho g} = \dfrac{\rho L v}{\rho g \times t} = \dfrac{Lv}{gt}$ ...(3.18.4)

9. The valve closure is said to gradual if $t > \dfrac{2L}{C}$ and sudden if $t < \dfrac{2L}{C}$.

Where, t = Time in second, and

C = Velocity of pressure wave.

b. Sudden Closure of Valve and Pipe is Rigid :

1. Let the pipe is rigid and valve fitted at the end B is closed suddenly.
2. Let, K = Bulk modulus of water.
3. When the valve is closed suddenly, the kinetic energy of the flowing water is converted into strain energy of water if the effect of friction is neglected and pipe wall is assumed perfectly rigid.
4. Loss of kinetic energy = $\dfrac{1}{2} \times$ Mass of water in pipe $\times v^2$

$$= \frac{1}{2} \rho AL v^2$$

5. Gain of strain energy

$$= \frac{1}{2}\left(\frac{p^2}{K}\right) \times \text{Volume} = \frac{1}{2}\frac{p^2}{K} AL$$

6. On equating loss of kinetic energy to gain of strain energy, we have

$$\frac{1}{2}\rho AL v^2 = \frac{1}{2}\frac{p^2}{K} AL$$

$$p^2 = \rho K v^2$$

$$p = v\sqrt{K\rho} = v\sqrt{\frac{K\rho^2}{\rho}} = \rho v C \qquad \left(\because \sqrt{K/\rho} = C\right)$$

PART-11

Pipe Networks and Three Reservoir Problems.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.19. **What is pipe network ? Give the necessary conditions of pipe network.**

Answer

A. Pipe Network :

1. A pipe network is a system in which many pipes are interconnected and they form several loops or circuits of pipes.
2. Example :
   a. Water supply system in a city is the very commonly used pipe network system.
   b. Supply of steam from boiler to other machineries is done by pipe network system, etc.

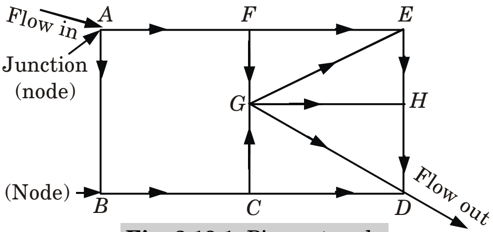


Fig. 3.19.1. Pipe network.

3. In such system, it is required to determine the distribution of flow through the various pipes of the network.

B. Necessary Conditions for any Pipe Network :

1. The system should follow the continuity equation *i.e.*,

   Flow into the junction = Flow out of the junction
2. For a loop of pipe circuit, $\Sigma h_f = 0$
3. The head loss in each pipe is expressed as, $h_f = r\, Q^n$

   Where, r = Constant, and n = 2 for turbulent flow.

Que 3.20. **Three reservoirs, *A*, *B* and *C* are connected by a pipe system having lengths 700 m, 1200 m and 500 m and diameters 400 mm, 300 mm and 200 mm respectively. The water levels in reservoir *A* and *B* from a datum line are 50 m and 45 m respectively. The level of water in reservoir *C* is below the level of water in reservoir *B*. Find the discharge into or from the reservoirs *B* and *C*. If the rate of flow from reservoir *A* is 150 litre/s, find the height of water level in the reservoir *C*. (Take *f* = 0.005, for all pipes)**

Answer

Given : L_1 = 700 m, d_1 = 400 mm = 0.4 m, Q_1 = 150 litre/s
L_2 = 1200 mm, d_2 = 300 mm = 0.3 m, L_3 = 500 mm, d_3 = 200 mm = 0.2 m,
z_A = 50 m, z_B = 45 m
To Find : i. Discharge into or from the reservoirs B and C.
ii. Height of water level in the reservoir C.

1. Applying Bernoulli's equation to point E and D,

$$z_A = z_D + \frac{p_D}{\rho g} + h_{f1}$$

Where,
$$v_1 = \frac{Q_1}{\text{Area}} = \frac{(150 \times 10^{-3})}{\frac{\pi}{4}(0.4)^2} = 1.194 \text{ m/s}$$

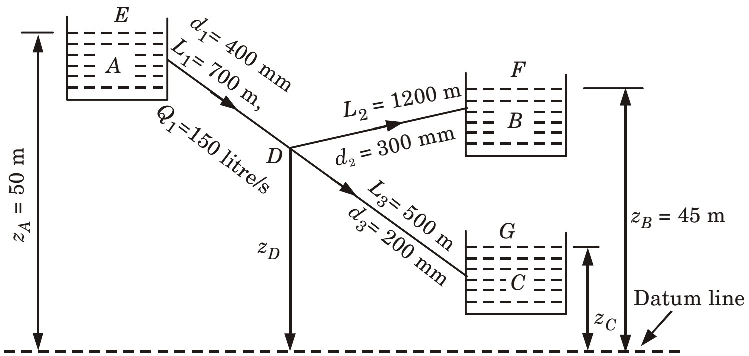


Fig. 3.20.1.

$$h_{f1} = \frac{4fL_1v_1^2}{2d_1g} = \frac{4 \times 0.005 \times 700(1.19)^2}{2 \times 0.4 \times 9.81} = 2.54 \text{ m}$$

So,
$$z_D + \frac{p_D}{\rho g} = z_A - h_{f1} = 50 - 2.54 = 47.46 \text{ m}$$

Here, piezometric head at D = 47.46 m. But z_B = 45 m, hence water flows from D to B.

2. Applying Bernoulli's equation to point B and D,

$$z_D + \frac{p_D}{\rho g} = z_B + h_{f2}$$
$$47.46 = 45 + h_{f2}$$
$$h_{f2} = 2.46 \text{ m}$$

3.
$$h_{f2} = \frac{4f\ L_2v_2^2}{d_2 \times 2g} \Rightarrow v_2 = \sqrt{\frac{2gh_{f2}\ d_2}{4f\ L_2}}$$

$$v_2 = \sqrt{\frac{2 \times 9.81 \times 2.46 \times 0.3}{4 \times 0.005 \times 1200}} = 0.777 \text{ m/s}$$

4. Discharge in pipe (2), $Q_2 = A_2 \, v_2$

$$= \frac{\pi}{4} d_2^2 \, v_2 = \frac{\pi}{4} \times 0.3^2 \times 0.777$$

$$Q_2 = 0.055 \text{ m}^3/\text{s} = 55 \text{ litres/s}$$

5. Apply Bernoulli's equation to D and C,

$$z_D + \frac{p_D}{\rho g} = z_C + h_{f3}$$

$$47.46 = z_C + \frac{4f \, L_3 v_3^2}{2 d_3 g} \quad \text{...(3.20.1)}$$

6. From continuity equation, $Q_1 = Q_2 + Q_3$

$$150 \times 10^{-3} = 0.055 + Q_3$$

Discharge in pipe C, $Q_3 = 0.15 - 0.055 = 0.095 \text{ m}^3/\text{s}$

7. Velocity in pipe DC, $v_3 = \dfrac{Q}{\frac{\pi}{4} d_3^2} = \dfrac{0.095}{\frac{\pi}{4}(0.2)^2} = 3.024$ m/s

8. Putting value of v_3 in eq. (3.20.1), we get

$$47.46 = z_C + \frac{4 \times 0.005 \times 500 \times (3.024)^2}{2 \times 0.2 \times 9.81}$$

$$z_C = 24.156 \text{ m}$$

PART-12

Boundary Layer Thickness.

CONCEPT OUTLINE

Boundary Layer Thickness : It is defined as the distance from the boundary in which the velocity reaches 99 percent of the velocity of free stream *i.e.*, $u = 0.99\,U$. It is denoted by δ.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.21. **What is a boundary layer ? Explain with a sketch development of boundary layer over a smooth plate.**

Answer

1. When a real fluid flow over a solid wall, the fluid particles closed to the boundary get adhered to the boundary and as a result of this condition no slip occurs.
2. In other words the velocity of fluid close to the boundary will be the same as that of the boundary.
3. As we move farther away from the boundary, the velocity will be higher and as a result of this variation of velocity, the velocity gradient $\frac{du}{dy}$ will exist.
4. Thus the velocity of fluid increases from zero velocity on the stationary boundary to free-stream velocity (U) of the fluid in the direction normal to the boundary.
5. The variation of velocity from zero to free stream velocity in the direction normal to the boundary takes place in a narrow region in the vicinity of solid boundary.
6. This narrow region of the fluid is called boundary layer.

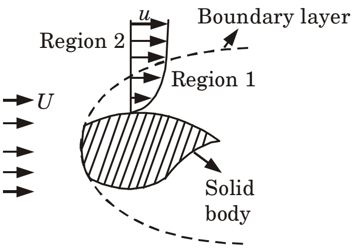


Fig. 3.21.1.

7. Hence the flow of fluid in the neighbourhood of the solid boundary may be divided into following two regions :

a. **Region 1 :**

1 A very thin layer of the fluid called the boundary layer, in the immediate neighbourhood of the solid boundary, where the variation of velocity from zero at the solid boundary to the free stream velocity in the direction normal to the boundary takes place.

2. In this region, the velocity gradient $\frac{du}{dy}$ exists and hence the fluid exerts a shear stress on the wall (wall shear) in the direction of motion.

3. The value of shear stress is given by,

$$\tau = \mu \frac{du}{dy}$$

b. Region 2 :

1. The remaining fluid, which is outside the boundary layer. The velocity outside the boundary layer is constant and equal to free stream velocity.
2. As there is no variation of velocity in this region the velocity gradient $\frac{du}{dy}$ becomes zero. As a result of this the shear stress is zero.

Que 3.22. **What do you understand by momentum thickness, displacement thickness and energy thickness ?**

AKTU 2014-15, Marks 10

OR

Explain the displacement thickness, momentum thickness related to boundary layer.

AKTU 2018-19, Marks 07

Answer

a. Displacement Thickness :

1. It can be defined as the distance, measured perpendicular to the boundary by which the main/free stream is displaced on account of formation of boundary layer. It is denoted by δ^*.

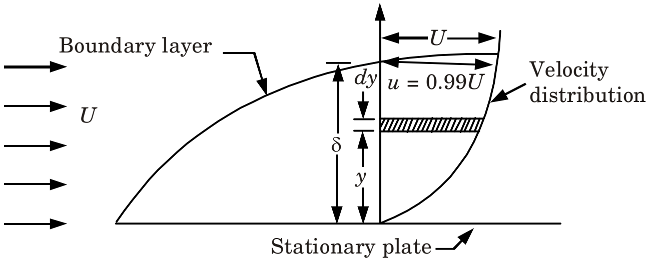


Fig. 3.22.1. Displacement thickness.

2. Let fluid of density ρ flow past a stationary plate with velocity U as shown in Fig. 3.22.1.
3. Consider an elementary strip of thickness dy at a distance y from the plate.
4. Mass flow per second through the elementary strip = $\rho u dy$
5. Mass flow per second through elementary strip, if the plate was not there = $\rho U dy$
6. Reduction of mass flow rate through elementary strip
$$= \rho(U - u)\, dy$$
7. Total reduction of mass flow rate due to introduction of plate

$$= \int_0^{\delta} \rho(U - u)dy \quad ...(3.22.1)$$

8. Let the plate is displaced by a distance δ^* and velocity of flow for the distance δ^* is equal to the main/free stream velocity. Then, loss of mass of fluid/sec flowing through the distance δ^*

$$= \rho U \delta^* \quad ...(3.22.2)$$

9. On equating eq. (3.22.1) and eq. (3.22.2), we get

$$\rho U \delta^* = \int_0^{\delta} \rho(U - u)dy$$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

b. Momentum Thickness :

1. It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation. It is denoted by θ.
2. Mass of flow per second through elementary strip = $\rho u dy$
3. Momentum/sec of this fluid inside the boundary layer

$$= \rho u dy \times u = \rho u^2 dy$$

4. Momentum/sec of the same mass of fluid before entering the boundary layer = $\rho u U dy$
5. Loss of momentum/sec = $\rho u U dy - \rho u^2 dy = \rho u(U - u)\, dy$
6. Total loss of momentum/sec = $\int_0^{\delta} \rho u(U - u)dy$ ...(3.22.3)
7. Let θ be the distance by which plate is displaced when fluid is flowing with a constant velocity U. Then, loss of momentum/sec of fluid flowing through distance θ with a velocity U

$$= \rho \theta U^2 \quad ...(3.22.4)$$

8. On equating eq. (3.22.3) and eq. (3.22.4), we get

$$\rho \theta U^2 = \int_0^{\delta} \rho u(U - u)dy$$

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

c. Energy Thickness :

1. It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation. It is denoted by δ_e. or δ^{**}
2. Mass of flow per second through elementary strip = $\rho u dy$

3. KE of this fluid inside the boundary layer

$$= \frac{1}{2} mu^2 = \frac{1}{2} (\rho u dy) u^2$$

4. KE of same mass of fluid before entering the boundary layer

$$= \frac{1}{2} (\rho u dy) U^2$$

5. Loss of KE through elementary strip

$$= \frac{1}{2} (\rho u dy)\, U^2 - \frac{1}{2} (\rho u dy) u^2 = \frac{1}{2} \rho u\, (U^2 - u^2) dy$$

6. So, total loss of KE of fluid $= \int_0^{\delta} \frac{1}{2} \rho u (U^2 - u^2) dy$ ...(3.22.5)

7. Let δ_e be the distance by which plate is displaced to compensate for reduction in KE. Then, loss of KE through δ_e of fluid flowing with velocity U

$$= \frac{1}{2} (\rho U \delta_e)\, U^2 \qquad ...(3.22.6)$$

8. On equating eq. (3.22.5) and (3.22.6), we have

$$\frac{1}{2} (\rho U \delta_e)\, U^2 = \int_0^{\delta} \frac{1}{2} \rho u (U^2 - u^2) dy$$

$$\delta_e = \int_0^{\delta} \frac{u}{U} \left(1 - \frac{u^2}{U^2}\right) dy$$

Que 3.23. **For the velocity distribution $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$, find the energy thickness δ**.**

AKTU 2015-16, Marks 05

Answer

Given : $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

To Find : Energy thickness δ**

1. Energy thickness δ** is given as,

$$\delta^{**} = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy = \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)^2\right] dy$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right)\right] dy$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{4y^2}{\delta^2} - \frac{y^4}{\delta^4} + \frac{4y^3}{\delta^3}\right) dy$$

$$= \int_0^{\delta}\left(\frac{2y}{\delta} - \frac{8y^3}{\delta^3} - \frac{2y^5}{\delta^5} + \frac{8y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5}\right) dy$$

$$= \int_0^{\delta}\left[\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{8y^3}{\delta^3} + \frac{12y^4}{\delta^4} - \frac{6y^5}{\delta^5} + \frac{y^6}{\delta^6}\right] dy$$

$$= \left[\frac{2y^2}{2\delta} - \frac{y^3}{3\delta^2} - \frac{8y^4}{4\delta^3} + \frac{12y^5}{5\delta^4} - \frac{6y^6}{6\delta^5} + \frac{y^7}{7\delta^6}\right]_0^{\delta}$$

$$= \frac{\delta^2}{\delta} - \frac{\delta^3}{3\delta^2} - \frac{2\delta^4}{\delta^3} + \frac{12\delta^5}{5\delta^4} - \frac{\delta^6}{\delta^5} + \frac{\delta^7}{7\delta^6} = \delta - \frac{\delta}{3} - 2\,\delta + \frac{12}{5}\delta - \delta + \frac{\delta}{7}$$

$$= -2\,\delta - \frac{\delta}{3} + \frac{12}{5}\delta + \frac{\delta}{7} = \frac{-210\,\delta - 35\,\delta + 252\,\delta + 15\,\delta}{105} = \frac{22\,\delta}{105}$$

Que 3.24. **The velocity distribution in the boundary layer is given by,**

$$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

Find displacement thickness and momentum thickness.

AKTU 2017-18, Marks 10

Answer

Given : $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$

To Find : i. Displacement thickness.
ii. Momentum thickness.

1. Displacement thickness, δ^* :

$$\delta^* = \int_0^{\delta}\left(1 - \frac{u}{U}\right) dy$$

$$\delta^* = \int_0^{\delta}\left[1 - \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] dy$$

$$\delta^* = \left[y + \frac{\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2} \times \frac{1}{\delta}}\right]_0^{\delta}$$

$$\delta^* = \left[\delta - \frac{2\delta}{\pi}\right]$$

$$\delta^* = \left[\frac{\pi - 2}{\pi}\right]\delta$$

2. Momentum thickness, θ :

$$\theta = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u}{U}\right] dy$$

$$\theta = \int_0^{\delta} \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\left[1 - \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] dy$$

$$= \int_0^{\delta} \left[\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - \sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right] dy$$

$$= \int_0^{\delta} \left[\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right) - \left\{\frac{1}{2} - \frac{\cos\{\pi(y/\delta)\}}{2}\right\}\right] dy$$

$$= \left[\frac{-\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2\delta}} - \frac{1}{2}y + \frac{\sin\left(\pi\frac{y}{\delta}\right)}{\frac{2\pi}{\delta}}\right]_0^{\delta}$$

$$= \left[\frac{-\cos\left(\frac{\pi}{2}\right)}{\frac{\pi}{2\delta}} - \frac{1}{2}\delta + \frac{\sin(\pi)}{\frac{2\pi}{\delta}}\right] - \left[\frac{-\cos(0)}{\frac{\pi}{2\delta}} - 0 + \frac{\sin(0)}{\frac{2\times\pi}{\delta}}\right]$$

$$= \left[\frac{-\delta}{2}\right] - \left[\frac{-2\delta}{\pi}\right] = \frac{2\delta}{\pi} - \frac{\delta}{2} = \left[\frac{2}{\pi} - \frac{1}{2}\right]\delta$$

Que 3.25. **Find the displacement thickness for velocity distribution in the boundary layer given by,**

AKTU 2018-19, Marks 07

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Answer

Given : $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

To Find : Displacement thickness.

1. Displacement thickness δ^* is given by,

$$\delta^* = \int_0^{\delta}\left(1 - \frac{u}{U}\right) dy$$

$$\delta^* = \int_0^{\delta}\left[1 - \left\{2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right\}\right] dy$$

$$= \int_0^{\delta}\left[1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right] dy = \left[y - \frac{2y^2}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^{\delta}$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} = \delta - \delta + \frac{\delta}{3} = \frac{\delta}{3}$$

PART-13

Boundary Layer over a Flat Plate, Laminar Boundary Layer, Application of Momentum Equation, Turbulent Boundary Layer, Laminar Sub-Layer.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.26. **Explain laminar boundary layer, turbulent boundary layer and laminar sub-layer with sketch in case of flow over a plate.**

Answer

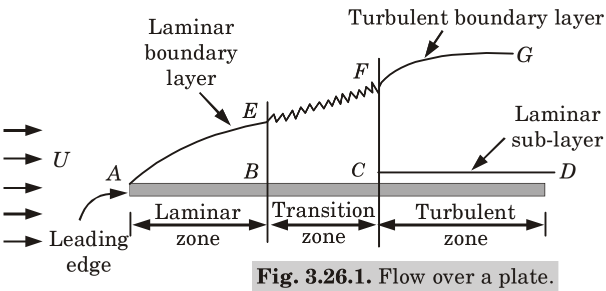


Fig. 3.26.1. Flow over a plate.

A. Laminar Boundary Layer :

1. Consider the flow of fluid, having free-stream velocity (U), over a smooth thin plate which is flat and placed parallel to the direction of free stream of fluid.
2. Considering the flow with zero pressure gradient on one side of the plate, which is stationary.
3. The velocity of fluid on the surface of the plate should be equal to the velocity of the plate.
4. As the plate is stationary and hence velocity of fluid on the surface of the plate is zero, but at a distance away from the plate, the fluid is having certain velocity.
5. Thus a velocity gradient sets up in the fluid near the surface of the plate. This velocity gradient develops shear resistance which retards the fluid.

6. The fluid with a uniform free stream velocity (U) is retarded in the vicinity of the solid surface of the plate and the boundary layer region begins at the sharp leading edge.
7. At subsequent points downstream the leading edge, the boundary layer region increases because the retarded fluid is further retarded. This is also referred as the growth of boundary layer.
8. Near the leading edge of the surface of the plate, where the thickness is small, the flow in the boundary layer is laminar though the main flow is turbulent.
9. This layer of the fluid is said to be laminar boundary layer shown by *AE* in Fig. 3.26.1. The length of the plate from the leading edge, up to which laminar boundary layer exists is called laminar zone shown by distance *AB*.
10. The distance of *B* from leading edge is obtained from Reynolds number equal to 5×10^5 for a plate.

B. Turbulent Boundary Layer :

1. If the length of the plate is further increased, the thickness of boundary layer goes on increasing in the downstream direction.
2. Then the laminar boundary layer becomes unstable and motion of fluid within it, is disturbed and irregular which leads to a transition from laminar to turbulent boundary layer.
3. This short length over which the boundary layer flow changes from laminar to turbulent is called transition zone which is shown by distance *BC*.
4. Further downstream the transition zone, the boundary layer is turbulent and continues to grow in thickness.
5. This layer of boundary is called turbulent boundary layer, which is shown by the portion *FG* in Fig. 3.26.1.

C. Laminar Sub-Layer :

1. This is the region in the turbulent boundary layer zone, adjacent to the solid surface of the plate.
2. In this zone, the velocity variation is influenced only by viscous effects.
3. Though the velocity distribution would be a parabolic curve in the laminar sub-layer zone, but in view of the very small thickness one can reasonably assume that the velocity variation is linear and thus the velocity gradient can also be taken as constant.
4. Hence, the shear stress in the laminar sub-layer would be constant and equal to the boundary shear stress τ_o.

   So, shear stress in the sub-layer is $\tau_o = \mu \left(\dfrac{\partial u}{\partial y}\right)_{y=0} = \mu \dfrac{U}{y}$ (As linear variation occurs).

Que 3.27. **Derive momentum integral equation for the boundary layer (Von-Karman).**

Answer

1. Let us consider a thin smooth flat plate with boundary layer as shown in Fig. 3.27.1.
2. Let the free stream velocity of flow be U over the plate.
3. Consider a small strip of dx at a distance x from one end of plate.
4. Now take the element $ABCD$, τ_o is the wall shear stress acting on the plate and velocity variation in strip $ABCD$ is also shown in the Fig. 3.27.1.

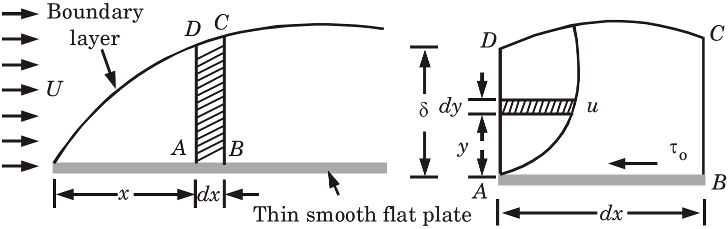


Fig. 3.27.1.

5. In strip $ABCD$, CD is the outer edge of the boundary layer.
6. Consider unit width of plate perpendicular to the direction of flow.
7. Mass rate of fluid entering through AD = $\int_0^\delta \rho u dy$
8. Mass rate of fluid leaving through BC = $\int_0^\delta \rho u dy + \frac{d}{dx}\left[\int_o^\delta \rho u dy\right] dx$
9. Mass rate of fluid entering the control volume $ABCD$, through the surface CD = Mass rate of fluid through BC – Mass rate of fluid through AD

$$= \int_0^\delta \rho u dy + \frac{d}{dx}\left[\int_0^\delta \rho u dy\right] dx - \int_0^\delta \rho u dy = \frac{d}{dx}\left[\int_0^\delta \rho u dy\right] dx$$

10. The entering fluid through DC has uniform velocity U.
11. Momentum rate of fluid entering in $ABCD$ through AD (in x-direction)

$$P_{AD} = \int_0^\delta \rho u^2 dy$$

12. Momentum rate of fluid entering in $ABCD$ through BC (in x-direction)

$$P_{BC} = \int_0^{\delta} \rho\, u^2 dy + \frac{d}{dx}\left[\int_0^{\delta} \rho u^2 dy\right] dx$$

13. Momentum rate of fluid entering the *ABCD* through *CD* (in *x*-direction)

$$P_{CD} = \text{Mass} \times \text{Velocity}$$

$$= \frac{d}{dx}\left[\int_0^{\delta} \rho u dy\right] dx U$$

$$P_{CD} = \frac{d}{dx}\left[\int_0^{\delta} \rho u U dy\right] dx$$

14. Rate of change of momentum = Momentum rate of fluid through *BC*. – Momentum rate of fluid through *AD* – Momentum rate of fluid through *DC*.

$= P_{BC} - P_{AD} - P_{DC}$

$$= \int_0^{\delta} \rho\, u^2 dy + \frac{d}{dx}\left[\int_0^{\delta} \rho\, u^2 dy\right] dx \;\; - \int_0^{\delta} \rho u^2 dy - \frac{d}{dx}\left[\int_0^{\delta} \rho u U dy\right] dx$$

$$= \frac{d}{dx}\left[\int_0^{\delta} \rho u^2 dy - \int_0^{\delta} \rho u U dy\right] dx$$

15. Rate of change of momentum

$$= \rho \frac{d}{dx}\left[\int_0^{\delta} \left(u^2 - u\, U\right) dy\right] dx \qquad \text{...(3.27.1)}$$

16. According to momentum principle,

Force = Rate of change of momentum

17. In control volume *ABCD* only a shear force is acting on the side *AB* in the direction *B* to *A*.

18. So, drag force, $\Delta F_D = \tau_o\, dx$, which is opposite to the direction of motion of fluid.

19. Thus, the total external force in the direction of rate of change of momentum $= -\tau_o \times dx$ (Negative sign indicates opposite direction) ...(3.27.2)

20. Now equating eq. (3.27.1) and eq. (3.27.2), we have

$$\rho \frac{d}{dx}\left[\int_0^{\delta} (u^2 - uU)\, dy\right] dx = -\tau_o \times dx$$

$$\tau_o = -\rho \frac{d}{dx}\left[\int_0^{\delta} (u^2 - uU)\, dy\right]$$

$$\tau_o = -\rho \frac{d}{dx}\left[\int_0^{\delta} U^2 \left\{\left(\frac{u}{U}\right)^2 - \left(\frac{u}{U}\right)\right\} dy\right]$$

Hence,

$$\tau_o = \rho U^2 \frac{d}{dx}\left[\int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy\right]$$

Here, $\int_o^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$ = Momentum thickness (θ).

$$\therefore \quad \tau_o = \rho U^2 \frac{d\theta}{dx}$$

$$\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx}$$

The above equation is known as Von-Karman momentum equation for boundary layer flow.

Que 3.28. **Oil with density 900 kg/m^3 and kinematic viscosity 10^{-5} m^2/sec is flowing over a plate of 3 m long and 2 m wide with a velocity of 3 m/sec parallel to 3 m side. Find the boundary layer thickness at the point of transition and at the end of plate.**

AKTU 2017-18, Marks 10

Answer

Given : $\rho = 900$ kg/m^3, $U = 3$ m/s, $\nu = 10^{-5}$ m^2/s, $L = 3$ m, $b = 2$ m,

To Find : Boundary layer thickness at the point of transition and at the end of the plate.

1. Reynolds Number, $R_e = \frac{UL}{\nu} = \frac{3 \times 3}{10^{-5}} = 9 \times 10^5 > 5 \times 10^5$

   Hence, upto a certain distance flow will be laminar then changes to turbulent flow.

2. Let, x be the distance up to which flow is laminar. Hence,

$$R_e = \frac{Ux}{\nu}$$

$$5 \times 10^5 = \frac{3x}{10^{-5}}$$

$$x = \frac{5}{3}\text{ m} = 1.67 \text{ m}$$

3. Boundary layer thickness at $x = 1.67$ or at transition equals to,

$$\delta = \frac{4.91x}{\sqrt{R_e}} = \frac{4.91 \times 1.67}{\sqrt{5 \times 10^5}} = 0.0116 \text{ m}$$

4. Now, boundary layer thickness at the end of plate (*i.e.*, at $x = 3$ m)

$$\delta = \frac{4.91 \times 3}{\sqrt{9 \times 10^5}} = 0.0155 \text{ m}$$

PART-14

Separation and its Control.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.29. What is boundary layer separation ? Explain with neat sketches, the necessary conditions for boundary layer separation. What are common methods to control boundary layer separation ?

AKTU 2016-17, Marks 10

Answer

A. Boundary Layer Separation :

1. When a solid body is kept or immersed in a flowing fluid, boundary layer is formed adjacent to the solid body.
2. Within this thin layer of fluid, the velocity varies from zero to free stream velocity in the direction normal to the solid body.
3. Along the length of the solid body, the thickness of the boundary layer increases.
4. The fluid layer adjacent to the solid surface has to do work against surface friction at the expense of its kinetic energy.
5. This loss of the kinetic energy is recovered from the immediate fluid layer in contact with the layer adjacent to solid surface through momentum exchange process. Thus the velocity of layer goes on decreasing.

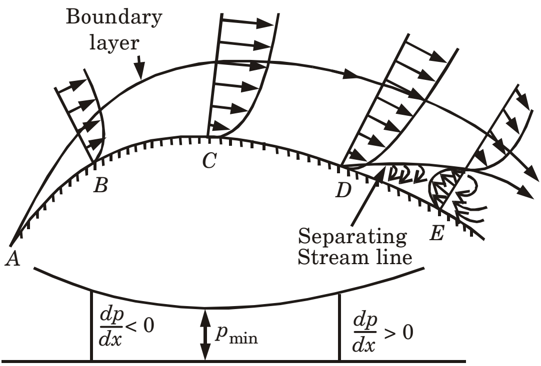


Fig. 3.29.1.

6. Along the length of the solid body, at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body if it cannot provide kinetic energy to overcome the resistance

offered by the solid body. Thus, in other words, the boundary layer will get separated from the surface. This phenomenon is called the boundary layer separation.

7. The point on the body at which the boundary layer is on the verge of separation from the surface is called point of separation.

B. Necessary Conditions for Boundary Layer Separation : For boundary layer separation, pressure gradient should be positive in the direction of flow $\left(\frac{\partial p}{\partial x} > 0\right)$ *i.e.*, the pressure should be in increasing manner in the direction of flow (Fig. 3.29.1).

C. Methods to Control Boundary Layer Separation :

i. Streamlined Body Shape :

1. Using streamlined body shape, the transition point of boundary layer (from laminar to turbulent) can be moved downstream which results in the reduction of the skin friction drag. Hence, separation of layers may be eliminated.

ii. Acceleration of Fluid in the Boundary Layer :

1. In this method, we supply additional energy to the particles of fluid which are being retarded in the boundary layer.
2. Energy can be transferred by :
   a. Injecting the fluid into the region of boundary layer with the help of some device.
   b. Diverting a portion of fluid from high pressure region to the retarded region of boundary layer through a slot provided in the body.

iii. By sucking the retarded flow.

iv. By providing slots near the leading edge.

v. Energising the flow by introducing optimum amount of swirl in the incoming flow.

vi. Remove the retarded or slow moving fluid particles in the boundary layer by suction through a porous surface.

Que 3.30. Discuss the effect of pressure gradient on boundary layer separation.

AKTU 2014-15, Marks 05

Answer

The effect of pressure gradient $\left(\frac{\partial p}{\partial x}\right)$ on boundary layer separation can be explained by considering the flow over a curved surface *ABCDE* as shown in Fig. 3.29.1.

a. Region *ABC* of the Curved Surface :

1. In this region, the area of flow decreases and hence velocity increases. This means that flow gets accelerated in this region.
2. Due to increase of the velocity the pressure decreases in the direction of the flow and hence pressure gradient $\frac{dp}{dx}$ is negative in this region.
3. As long as $\frac{dp}{dx} < 0$, the entire boundary layer moves in forward direction.

b. Region *CDE* of the Curved Surface :

1. The pressure is minimum at point C.
2. Along this region, the area of flow increases and hence velocity of flow along the direction of fluid decreases.
3. Due to decrease of velocity, the pressure increases in the direction of flow and hence pressure gradient $\frac{dp}{dx}$ is positive $\left(\frac{dp}{dx} > 0\right)$.
4. Thus in the region *CDE*, the pressure gradient is positive and velocity of fluid layers along the direction of flow decreases.
5. As explained in the Fig. 3.29.1, the velocity of the layer adjacent to the solid surface along the length of the solid surface goes on decreasing as the kinetic energy of the layer is used to overcome the frictional resistance of the surface.
6. Thus the combined effect of positive pressure gradient and surface resistance reduce the momentum of the fluid which is unable to overcome the surface resistance.
7. A stage comes, when the momentum of the fluid is unable to overcome the surface resistance and the boundary layer starts separating from the surface at the point D.
8. Downstream the point D, the flow is reversed and the velocity gradient becomes negative.
9. Thus the positive pressure gradient helps in separating the boundary layers.

PART-15

Drag and Lift.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.31. **Explain the force exerted by a flowing fluid on a stationary body.**

Answer

1. As shown in Fig. 3.31.1, consider a body held stationary in a real fluid, which is flowing at a uniform velocity U.
2. The fluid exerts a force on the stationary body.
3. The total force (F_R) exerted by the fluid on the body is perpendicular to the surface of the body. Thus the total force is inclined to the direction of motion.
4. The component of total force (F_R) in the direction of flow is called drag and is given as,

$$F_D = C_D \frac{A\rho v^2}{2}$$

Where, C_D = Coefficient of drag.

5. The component of total force (F_R) in the direction perpendicular to the direction of flow is known as lift and is given as,

$$F_L = C_L \frac{A\rho v^2}{2}$$

Where, C_L = Coefficient of lift.

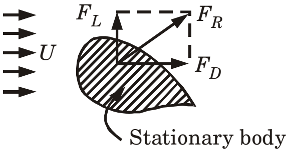


Fig. 3.31.1.

Que 3.32. **Define coefficient of lift and coefficient of drag.**

Answer

A. Coefficient of Lift : It is defined as the ratio of the total lift force to the quantity $\frac{1}{2}\rho AU^2$.

Mathematically, $$C_L = \frac{F_L}{\frac{1}{2}\rho AU^2}$$

B. Coefficient of Drag : Average coefficient of drag is defined as the ratio of the total drag force to the quantity $\frac{1}{2}\rho AU^2$. It is also called coefficient of drag and is denoted by C_D.

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}$$

Que 3.33. **A square plate of side 2 m is moved in a stationary air of density 1.2 kg/m^3 with a velocity of 50 km/hr. If the coefficient of drag and lift are 0.2 and 0.8 respectively, determine the drag force, lift force and resultant force.**

AKTU 2017-18, Marks 07

Answer

Given : $A = 2 \times 2 = 4$ m^2, v = 50 km/hr = 13.89 m/s, $\rho = 1.2$ kg/m^3, $C_D = 0.2$, $C_L = 0.8$

To Find : i. Drag force.
ii. Lift force.
iii. Resultant force.

1. Drag force, $F_D = C_D A \dfrac{\rho v^2}{2} = 0.2 \times 4 \times \dfrac{1.2 \times (13.89^2)}{2} = 92.6$ N

2. Lift force, $F_L = C_L A \dfrac{\rho v^2}{2} = 0.8 \times 4 \times \dfrac{1.2 \times (13.89^2)}{2} = 370.43$ N

3. Resultant force, $F_R = \sqrt{F_D{}^2 + F_L{}^2} = \sqrt{(92.6)^2 + (370.43)^2} = 381.83$ N

Que 3.34. **A kite 60 cm × 60 cm in size weighing 3 N makes an angle of 10° with the horizontal. The thread attached to it makes an angle of 45° to the horizontal and pull on the string is 25 N. The wind is flowing over the kite at 15 m/s. Find C_L and C_D for the kite.**

AKTU 2018-19, Marks 07

Answer

Given : $A = 0.6 \times 0.6 = 0.36$ m^2, $W = 3$ N, $U = 15$ m/sec, $\theta_1 = 10°$, $P = 25$ N, $\theta_2 = 45°$

To Find : C_L and C_D for kite.

Data Assumed : $\rho_{air} = 1.25$ kg/m^3

1. Drag force, F_D = Force exerted by wind in the direction of motion (*i.e.*, in *x-x* direction)

$F_D = P \cos 45° = 25 \cos 45° = 17.68$ N

And lift force, F_L = Component of P in vertically downward direction + weight of kite (W)

$= 25 \sin 45 + 3 = 20.68$ N

2. Lift force, $$F_L = C_L A\rho \frac{U^2}{2}$$

$$C_L = \frac{2F_L}{A\rho U^2} = \frac{2 \times 20.68}{0.36 \times 1.25 \times 15^2} = 0.4085$$

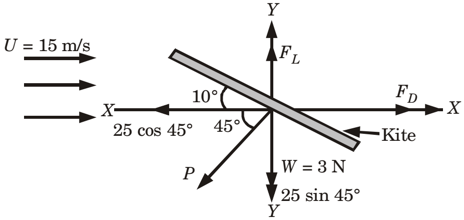


Fig. 3.34.1.

3. Drag force, $$F_D = C_D A\rho \frac{U^2}{2}$$

$$C_D = \frac{2F_D}{A\rho U^2} = \frac{2 \times 17.68}{0.36 \times 1.25 \times 15^2} = 0.349$$

PART-16

Drag on Sphere, Two Dimensional Cylinder.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.35. **Explain drag on a sphere.**

Answer

1. Consider a flow of real fluid passing with velocity U over a sphere having diameter d, density ρ and μ.
2. Stokes developed a mathematical equation to determine the total drag acting on a sphere which is immersed in a fluid having Reynold's number less than 0.2.
3. According to stokes for Reynold's number less than 0.2, all the inertia forces acting on the fluid are assumed to be negligible; only viscous forces are to be considered.

4. According to Stokes,

   Total drag acting on sphere, $F_D = 3\pi\mu\, dU$

5. In this total drag, two-third portion is contributed by skin-friction and the rest portion is contributed by pressure drag.

6. Hence, pressure drag, $F_{DP} = \dfrac{1}{3} F_D = \pi\mu dU$

   And skin friction drag, $F_{DF} = \dfrac{2}{3} F_D = 2\pi\mu dU$

Que 3.36. Discuss about the effect of Reynold's number on coefficient of drag for cylinder.

Answer

1. Let us consider a cylinder having diameter, d and length, L is placed in a real fluid having kinematic viscosity ν and free flow velocity U.
2. If the Reynold's number for the flow is less than 0.2, the inertia forces are negligible as compared to viscous force.
3. If Reynold's number increases, inertia force also increases and flow pattern becomes unsymmetrical with respect to the axis perpendicular to flow direction.
4. From experiment following observations are to be made :
   i. If $R_e < 1$,

      Drag force $F_D \propto$ Velocity (U) and $C_D \propto \dfrac{1}{R_e}$

   ii. If $1 < R_e < 2000$

      C_D will decreases and at $R_e = 2000$, $C_D \approx 0.95$

   iii. If $2000 < R_e < 3 \times 10^4$

      C_D will start to increase up to a maximum value of 1.2 at $R_e = 3 \times 10^4$

   iv. If $3 \times 10^4 < R_e < 3 \times 10^5$

      C_D again decreases and $C_D \approx 0.3$ at $R_e = 3 \times 10^5$

   v. If $R_e > 3 \times 10^5$

      C_D again increases and attains a maximum value of 0.7.

PART-17

Aerofoil.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.37. What do you understand by coefficient of lift, coefficient of drag and aerofoil ?

AKTU 2014-15, Marks 10

Answer

A. Coefficient of Lift : Refer Q. 3.32, Page 3–41A, Unit-3.

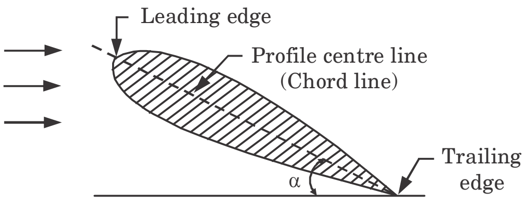
B. Coefficient of Drag : Refer Q. 3.32, Page 3–41A, Unit-3.

C. Aerofoil :

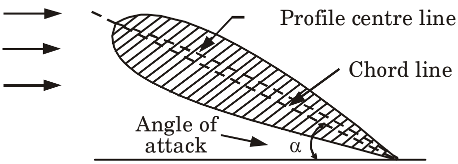
1. An aerofoil or airfoil is a streamlined body which may be either symmetrical or unsymmetrical.
2. Following are the necessary and important definitions related to aerofoil :

a. Chord Line : It is the line joining the leading and trailing edges of the aerofoil. The length of the line is known as chord of aerofoil. It is denoted by C.

b. Profile Centre Line : It is the line joining the midpoints of the profile.



(*a*) Symmetrical



(*b*) Unsymmetrical

Fig. 3.37.1. Airfoil.

c. **Angle of Attack :** The angle between the chord line and direction of the fluid stream is known as angle of attack. It is denoted by α.

d. **Camber :** It is the curvature of an airfoil.

e. **Stall :**

1. This is the condition when angle of attack (α) is greater than the angle of attack at maximum lift.
2. At stall the air separates from the airfoil or wing and eddies are formed as a consequence of which there is a considerable increase in the drag coefficient.

f. **Aspect Ratio (AR) :** The ratio of span of the wing to its mean chord is called the aspect ratio of a wing.

$$AR = \frac{L}{C}$$

Where, L = Span of the wing, and

C = Mean chord.

Que 3.38. Discuss the development of lift on an airfoil.

Answer

1. Airfoils are streamline bodies, it may or may not be symmetrical in shapes.
2. There is negative pressure created on the upper part of airfoil due to which there is a lift force act on the airfoil.
3. The drag force acting on airfoil is very small due to the design of the shape of the body (because shape of airfoil is streamlined).
4. Circulation Γ developed on the airfoil so that the streamline at the trailing edge of the airfoil is tangential to the airfoil is given as,

$$\Gamma = \pi CU \sin \alpha$$

5. Lift force acting on airfoil, $F_L = \rho UL\Gamma$

$$= \rho UL (\pi CU \sin \alpha)$$

$$= \pi \rho CU^2 L \sin \alpha \quad ...(3.38.1)$$

6. Lift force acting on airfoil in terms of coefficient of lift is given by,

$$F_L = \frac{1}{2} C_L \rho A U^2 \quad ...(3.38.2)$$

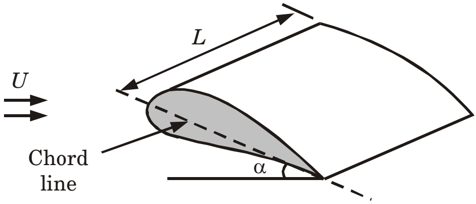


Fig. 3.38.1. Airfoil.

7. On equating the eq. (3.38.1) and eq. (3.38.2), we get coefficient of lift, C_L as,

$$C_L = 2\pi \sin\alpha \quad ...(3.38.3)$$

8. Eq. (3.38.3) shows that coefficient of lift depends upon the angle of attack.

PART-18

Magnus Effect.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.39. Explain the Magnus effect with an example.

AKTU 2017-18, Marks 10

Answer

A. Magnus Effect :

1. When a cylinder is rotated in a uniform flow, a lift force is produced on a cylinder.
2. This phenomenon of the lift force produced by a rotating cylinder in a uniform flow is known as Magnus effect.

B. Example :

1. This effect has been successfully employed in the propulsion of ships.
2. The Magnus effect may also be used with advantage in the games like table tennis, golf, cricket etc.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What are the characteristics of a laminar flow ? Derive the expression for the velocity distribution for viscous flow through a circular pipe. Also sketch the distribution of velocity and shear stress across a section of pipe.

Ans. Refer Q. 3.1, Unit-3.

Q. 2. A compound piping system consists of 1800 m of 0.50 m, 1200 m of 0.40 m and 600 m of 0.30 m new cast iron pipes connected in series. Convert the system to (*a*) an equivalent length of 0.40 m pipe, and (*b*) equivalent size pipe of 3600 m long.

Ans. Refer Q. 3.14, Unit-3.

Q. 3. What do you understand by momentum thickness, displacement thickness and energy thickness ?

Ans. Refer Q. 3.22, Unit-3.

Q. 4. For the velocity distribution $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$, find the energy thickness δ^{**}.

Ans. Refer Q. 3.23, Unit-3.

Q. 5. Oil with density 900 kg/m^3 and kinematic viscosity 10^{-5} m^2/sec is flowing over a plate of 3 m long and 2 m wide with a velocity of 3 m/sec parallel to 3 m side. Find the boundary layer thickness at the point of transition and at the end of plate.

Ans. Refer Q. 3.28, Unit-3.

Q. 6. What is boundary layer separation ? Explain with neat sketches, the necessary conditions for boundary layer separation. What are common methods to control boundary layer separation ?

Ans. Refer Q. 3.29, Unit-3.

Q. 7. A kite 60 cm × 60 cm in size weighing 3 N makes an angle of 10° with the horizontal. The thread attached to it makes an angle of 45° to the horizontal and pull on the string is 25 N. The wind is flowing over the kite at 15 m/s. Find C_L and C_D for the kite.

Ans. Refer Q. 3.34, Unit-3.

Q. 8. What do you understand by coefficient of lift, coefficient of drag and aerofoil ?

Ans. Refer Q. 3.37, Unit-3.

Q. 9. Explain the Magnus effect with an example.

Ans. Refer Q. 3.39, Unit-3.

☺☺☺

4 UNIT

Impact of Jet, Impulse Turbine and Reaction Turbines

CONTENTS

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Part-2 : Classification of Turbines .... **4–16A to 4–19A**

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PART-1

Introduction to Hydrodynamic Thrust of Jet on a Fixed and Moving Surface.

CONCEPT OUTLINE

Impact of Jet : If a plate, which may be fixed or moving is placed in the path of jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of motion and known as impact of jet.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. Derive the formula for dynamic force exerted by fluid jet on stationary plate for the following cases :

i. When plate is normal to jet.

ii. Flat plate inclined to jet.

iii. When plate is curved and jet impinges at the center of plate.

iv. When plate is unsymmetrical and curved and jet impinges at one end.

Answer

Following notation are used in driving the formula for dynamic force for given cases :

$$\begin{aligned} v &= \text{Velocity of jet,} \\ d &= \text{Diameter of jet,} \\ a &= \text{Area of cross-section of jet,} \\ \theta &= \text{Angle between the jet and plate,} \\ \rho &= \text{Density of water, and} \\ Q &= \text{Discharge of water (m}^3\text{/s).} \end{aligned}$$

i. When the Plate is Normal to Jet :

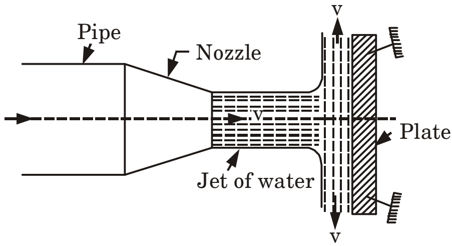


Fig. 4.1.1. Force exerted by jet on vertical plate.

1. Consider a jet of water coming out from the nozzle, striking a flat vertical plate as shown in Fig. 4.1.1.
2. Plate is at 90° to the jet and jet after striking will move along the plate. So, velocity component of water after strike, in direction of jet will be zero.
3. Dynamic force exerted by the jet on the plate in direction of jet is calculated as,

F_x = Rate of change of momentum in the direction of force
= Mass striking the plate/sec × Change in velocity in direction of jet

$$= \frac{\text{Mass}}{\text{Time}} \text{ [Initial velocity – Final velocity]}$$

$$= \frac{m}{t} [\text{v} - 0]$$

$$= \rho Q \; [\text{v} - 0] \qquad \left(\because \; \frac{m}{t} = \rho Q\right)$$

$$= \rho a\text{v} \times \text{v} \qquad (\because \; Q = a\text{v})$$

$$F_x = \rho a\text{v}^2$$

ii. Flat Plate Inclined to Jet :

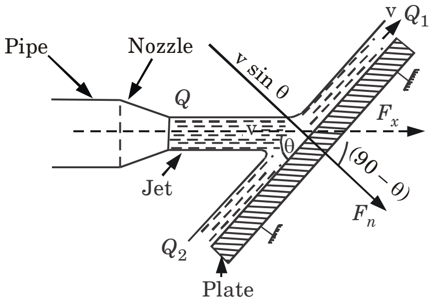


Fig. 4.1.2. Jet striking stationary inclined plate.

1. If plate is smooth and there is no loss of energy, then jet will move over the plate with a velocity (v) as shown in Fig. 4.1.2.
2. Now, normal force is calculated as,

F_n = Mass of jet striking the plate/sec
× Change in velocity in normal direction to the plate

$$= \frac{\text{Mass}}{\text{Time}} \text{ [Initial velocity in normal direction to plate – Final velocity in normal direction to plate]}$$

$$= \frac{m}{t} [\text{v} \sin\theta - 0] = \rho Q \; [\text{v} \sin\theta]$$

$$= \rho a\text{v} \; [\text{v} \sin\theta] \qquad (\because \; Q = a\text{v})$$

$$F_n = \rho a\text{v}^2 \sin\theta$$

3. Horizontal component of force,

$$F_x = F_n \sin\theta$$
$$= \rho a v^2 \sin\theta \times \sin\theta = \rho a v^2 \sin^2\theta$$

4. Vertical component of force,

$$F_y = F_n \cos\theta = \rho a v^2 \sin\theta \times \cos\theta = \rho a v^2 \sin\theta \cos\theta$$

iii. When Plate is Curved and Jet Impinges at the Center of Plate :

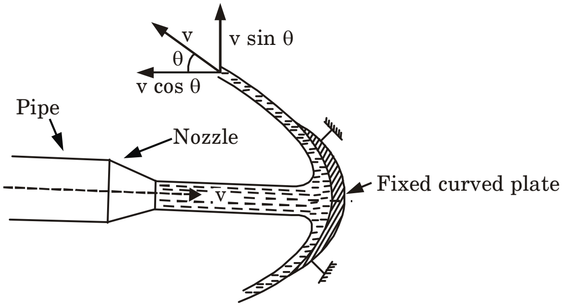


Fig. 4.1.3. Jet striking a fixed curved plate at center.

1. As shown in Fig. 4.1.3,

Component of velocity in direction of jet = – v cos θ
Component of velocity in perpendicular direction of jet

$$= v \sin\theta$$

2. Dynamic force exerted by the jet in X-direction,

F_x = Mass of jet striking the plate/sec × Change in velocity in direction of the plate

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$
$$= \frac{m}{t} [v - (-v \cos\theta)]$$
$$= \rho Q [v + v \cos\theta] = \rho a v [v + v \cos\theta]$$
$$= \rho a v^2 (1 + \cos\theta)$$

3. Dynamic force exerted by the jet in direction perpendicular to jet,

F_y = Mass of jet striking/sec × Change in velocity in normal direction to plate

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$
$$= \frac{m}{t} [0 - (v \sin\theta)]$$
$$= \rho Q (-v \sin\theta) = \rho a v (-v \sin\theta)$$
$$F_y = -\rho a v^2 \sin\theta$$

(Negative sign indicate that force acting in downward direction.)

iv. When Plate is Unsymmetrical and Curved and Jet Impinges at One End :

1. Let, θ = Angle made by jet with X-axis at inlet tip of the curved plate, and

   ϕ = Angle made by jet with X-axis at outlet tip of the curved plate.

2. Components of velocity resolved at inlet of curved plate,

   In X-direction = v cos θ
   In Y-direction = v sin θ

3. Similarly, at outlet of curved plate,

   In X-direction = – v cos ϕ
   In Y-direction = v sin ϕ

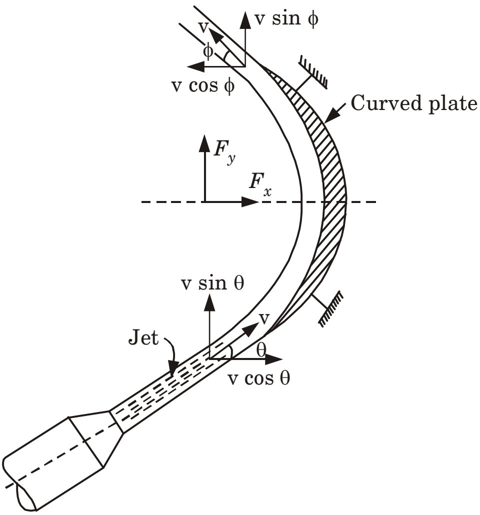


Fig. 4.1.4. Jet striking curved fixed plate at one end.

4. Now, force exerted by jet of water in X-direction,

$$F_x = \text{Mass of jet striking the plate/sec} \times \text{Change in velocity in } X\text{-direction}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

$$= \frac{m}{t} [\text{v} \cos \theta - (- \text{v} \cos \phi)]$$

$$= \rho Q[\text{v} \cos \theta + \text{v} \cos \phi] = \rho a\text{v} (\text{v} \cos \theta + \text{v} \cos \phi)$$

$$F_x = \rho a\text{v}^2 (\cos \theta + \cos \phi)$$

5. Force exerted by jet of water in Y-direction,

$$F_y = \text{Mass of jet striking the plate/sec} \times \text{Change in velocity in } Y\text{-direction}$$

$$= \frac{\text{Mass}}{\text{Time}} \times [\text{Initial velocity} - \text{Final velocity}]$$

$$= \frac{m}{t} [\text{v} \sin \theta - \text{v} \sin \phi] = \rho a\text{v} [\text{v} \sin \theta - \text{v} \sin \phi]$$

$$= \rho a\text{v}^2 (\sin \theta - \sin \phi)$$

6. When the plate is symmetrical, then $\theta = \phi$

So, $F_x = 2 \rho a\text{v}^2 \cos \theta$, and $F_y = 0$

Que 4.2. Derive the formula for dynamic force exerted by fluid jet on moving plate for the following cases :

i. When plate is normal to jet.

ii. Flat plate inclined to jet.

iii. When plate is curved and jet impinges at the center of plate.

iv. When plate is curved and jet impinges at one end.

OR

Derive an expression for force exerted by a jet on a fixed inclined plate. Also give an expression for a force exerted by jet on flat moving plate in the direction of jet.

AKTU 2014-15, Marks 05

Answer

A. Expression for Force Exerted by Jet on a Fixed Inclined Plate : Refer Q. 4.1, Page 4–2A, Unit-4.

B. Expression for Force Exerted by Fluid Jet on Moving Plate : The notations used are same as used in case of fixed plate. The one extra notation is of 'u' representing velocity of plate.

i. When the Plate is Normal to Jet :

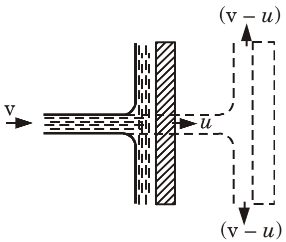


Fig. 4.2.1. Jet striking a flat vertical moving plate.

1. Relative velocity of jet with respect to plate = $(\text{v} - u)$
2. Mass of water striking the plate/sec

$$= \rho \times \text{Area of jet} \times \text{Relative velocity}$$

$$= \rho a(\text{v} - u) \qquad ...(4.2.1)$$

3. Force exerted by the jet on the moving plate in direction of the jet,

$$F_x = \text{Mass of water striking per sec} \times \text{Change in velocity of jet}$$

$$= \frac{\text{Mass}}{\text{Time}} \times [\text{Initial velocity with which water strikes} - \text{Final velocity}]$$

$$= \rho a(v - u)\,[(v - u) - 0] = \rho a(v - u)^2$$

ii. Flat Plate Inclined to the Jet :

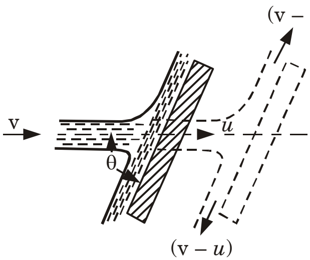


Fig. 4.2.2. Jet striking an inclined moving plate.

1. Mass of water striking the plate per sec,

$$= \rho \times \text{Area of jet} \times \text{Relative velocity}$$

$$= \rho a(v - u)$$

2. Component of relative velocity of jet striking normal to the plate,

$$= (v - u) \sin \theta$$

3. Force exerted by the jet in normal direction of plate,

$$F_n = \text{Mass of water striking per sec} \times \text{Change in velocity normal to plate}$$

$$= \text{Mass of water striking per sec} \times [\text{Initial velocity in normal direction with which jet strikes} - \text{Final velocity}]$$

$$= \rho a(v - u)\,[(v - u) \sin \theta - 0]$$

$$F_n = \rho a(v - u)^2 \sin \theta$$

4. Force exerted in X-direction by the jet,

$$F_x = F_n \sin \theta$$

$$= \rho a(v - u)^2 \sin \theta \times \sin \theta = \rho a(v - u)^2 \sin^2\theta$$

5. Force exerted in Y-direction by the jet,

$$F_y = F_n \cos \theta$$

$$= \rho a(v - u)^2 \sin \theta \times \cos \theta$$

$$= \rho a(v - u)^2 \sin \theta \cos \theta$$

iii. When the Plate is Curved and Jet Impinges at the Center of Plate :

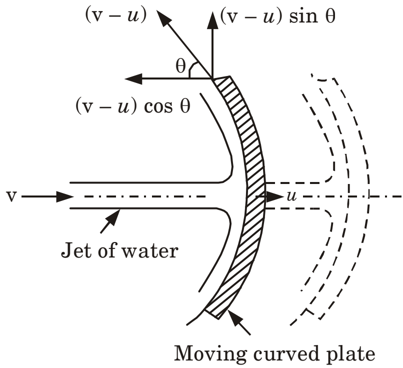


Fig. 4.2.3. Jet striking a curved moving plate.

1. If the plate is smooth and there is no loss of energy due to impact of jet, then the jet will leave the plate with same velocity by which jet strike the plate. So, velocity of jet leaving the plate = $(v - u)$
2. Now, component of velocity in direction of jet = $(v - u) \cos \theta$
3. Component of velocity perpendicular to the direction of jet = $(v - u) \sin \theta$
4. Mass of water striking the plate per sec
   = ρ × Area of jet × Relative velocity by which jet strike plate = $\rho a(v - u)$
5. Force exerted by the jet of water on the curved plate in direction of jet,

$$\begin{aligned} F_x &= \text{Mass of water striking the plate/sec} \times \\ &\quad \text{Change in velocity in direction of jet} \\ &= \rho a(v-u)\,[\text{Initial velocity} - \text{Final velocity}] \\ &= \rho a(v-u)\,[(v-u) - \{-(v-u)\cos\theta)\}] \\ &= \rho a(v-u)\,[(v-u) + (v-u)\cos\theta] \\ &= \rho a(v-u)^2\,(1+\cos\theta) \end{aligned}$$

6. Force exerted by the jet of water in perpendicular direction of jet,

$$\begin{aligned} F_y &= \text{Mass of water strike the plate/sec} \times \text{Change in} \\ &\quad \text{velocity in perpendicular direction to the jet} \\ &= \rho a(v-u)\,[0 - (v-u)\sin\theta] \\ &= -\rho a(v-u)^2 \sin\theta \end{aligned}$$

iv. When the Plate is Curved and Jet Impinges at One End :

1. As shown in Fig. 4.2.4, at inlet of the plate following terms are represented as,
   v_1 (represent by AB) = Velocity of jet at inlet,
   u_1 (represent by AC) = Velocity of plate,
   v_{r1} (represent by CB) = Relative velocity of jet and plate,
   v_{w1} (represent by AD) = Component of velocity of jet v_1 in X-direction also known as velocity of whirl at inlet,
   v_{f1} (represent by BD) = Component of velocity of jet v_1 in Y-direction, also known as velocity of flow at inlet,

α = Angle between the direction of jet and direction of motion of plate also known as guide blade angle,

θ = Angle made by relative velocity (v_{r1}) with direction of motion at inlet, also known as vane angle at inlet,

v_2, u_2, v_{r2}, v_{w2} and v_{f2} = Corresponding values at outlet,

β = Angle made by velocity of jet v_2, and

ϕ = Angle made by relative velocity v_{r2}.

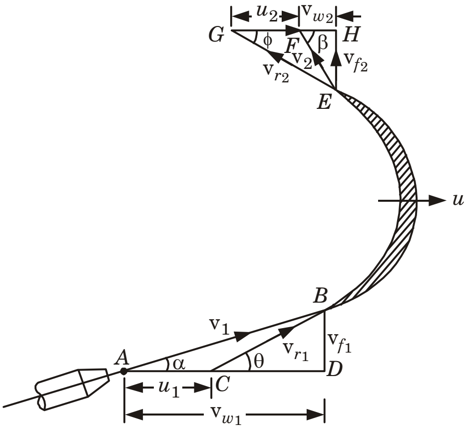


Fig. 4.2.4. Jet striking a moving curved vane at one of the tips.

2. ΔABD and ΔEGH are the velocity triangles at inlet and outlet.
3. Mass of water strike the plate /sec

$$= \rho \times \text{Area of jet} \times \text{Relative velocity}$$
$$= \rho a v_{r1}$$

4. Relative velocity at inlet in X-direction by which jet of water striking

$$= (v_{w1} - u_1)$$

And, relative velocity at outlet in X-direction by which jet leaving

$$= -(v_{w2} + u_2)$$

5. Force exerted by the jet in the direction of motion,

$$F_x = \text{Mass of water striking/sec} \times \text{Change in velocity in } X\text{-direction}$$

$$F_x = \text{Mass of water striking/sec} \times [\text{Relative velocity at inlet} - \text{Relative velocity at outlet}]$$

$$= \rho a v_{r1}[(v_{w1} - u_1) - \{-(u_2 + v_{w2})\}]$$

$$= \rho a v_{r1}[(v_{w1} - u_1 + v_{w2} + u_2)]$$

$$F_x = \rho a\, v_{r1}\,(v_{w1} + v_{w2}) \qquad [\because \; u_1 = u_2]$$

Que 4.3. **A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which inclined at 45° to the axis of the jet. Find the normal pressure on the plate;**

i. When the plate is stationary, and

ii. When the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.

AKTU 2017-18, Marks 10

Answer

Given : d = 7.5 cm = 0.075 m, v = 30 m/s, θ = 90° – 45° = 45°, u = 15 m/s.

To Find :
1. Normal pressure on the plate :
   i. When plate is stationary, and
   ii. When plate is moving with a velocity of 15 m/s.
2. Power and efficiency of jet when the plate is moving.

1. Area,
$$a = \frac{\pi}{4}(0.075)^2 = 0.004417 \text{ m}^2$$

2. When the plate is stationary, the normal force on the plate is given as,
$$F_n = \rho a \text{v}^2 \sin\theta$$
$$= 1000 \times 0.004417 \times 30^2 \times \sin 45° = 2810.96 \text{ N}$$

3. When the plate is moving with a velocity 15 m/s and away from the jet, the normal force on the plate is given as,
$$F_n = \rho a(\text{v} - u)^2 \sin\theta$$
$$= 1000 \times 0.004417 \times (30 - 15)^2 \times \sin 45° = 702.74 \text{ N}$$

4. Force in the direction of jet is given as,
$$F_x = F_n \sin\theta = 702.74 \times \sin 45° = 496.9 \text{ N}$$

5. Work done per second by the jet

= Force in the direction of jet × Distance moved by the plate in the direction of jet/sec

$= F_x u$

$= 496.9 \times 15 = 7453.5$ Nm/s

6.
$$\text{Power (in kW)} = \frac{\text{Work done per second}}{1000} = \frac{7453.5}{1000} = 7.453 \text{ kW}$$

7.
$$\text{Efficiency of the jet} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second}}{\text{Kinetic energy of the jet}}$$
$$= \frac{7453.5}{\frac{1}{2}(\rho a \text{v})\text{v}^2} = \frac{7453.5}{\frac{1}{2}\rho a \text{v}^3}$$

$$= \frac{7453.5}{\frac{1}{2} \times 1000 \times 0.004417 \times 30^3}$$

$$= 0.1249 \simeq 0.125 = 12.5\ \%$$

Que 4.4. **A jet of water of diameter 50 mm having a velocity of 20 m/s strikes a curved vane which is moving with a velocity of 10 m/s in the direction of jet. The jet leaves the vane at an angle of 60 degree to the direction of motion of vane at outlet. Determine**

i. Force exerted by the jet on the vane in the direction of motion.

ii. Work done per second by the jet.

AKTU 2015-16, 2016-17; Marks 10

Answer

Given : $v_1 = 20$ m/s, $d = 50$ mm $= 0.05$ m, $u_1 = 10$ m/s, $\theta = 60°$

To Find : i. Force exerted by the jet.
ii. Work done per second.

1. As jet and vane are moving in the same direction,

$\therefore$ $$\alpha = 0$$

2. Angle made by the leaving jet, with the direction of motion $= 60°$

$\therefore$ $$\beta = 180° - 60° = 120°$$

3. For this problem, we have

$$u_1 = u_2 = u = 10 \text{ m/s}$$

$$v_{r1} = v_{r2}$$

4. From Fig. 4.4.1, we have

$$v_{r1} = AB - AC = v_1 - u_1$$
$$= 20 - 10 = 10 \text{ m/s}$$

$$v_{w1} = v_1 = 20 \text{ m/s}$$

$\therefore$ $$v_{r2} = v_{r1} = 10 \text{ m/s}$$

5. Now in ΔEFG, $$EG = v_{r2} = 10 \text{ m/s}$$

$$GF = u_2 = 10 \text{ m/s}$$

$$\angle GEF = 180° - (60° + \phi) = (120° - \phi)$$

6. From sine rule, we have

$$\frac{EG}{\sin 60°} = \frac{FG}{\sin(120° - \phi)} \quad \text{or} \quad \frac{10}{\sin 60°} = \frac{10}{\sin(120° - \phi)}$$

or $$\sin 60° = \sin(120° - \phi)$$

$\therefore$ $$60° = 120° - \phi \quad \text{or} \quad \phi = 120° - 60° = 60°$$

7. Now, $$v_{w2} = HF = GF - GH$$
$$= u_2 - v_{r2} \cos\phi = 10 - 10 \times \cos 60°$$
$$= 10 - 5 = 5 \text{ m/s}$$

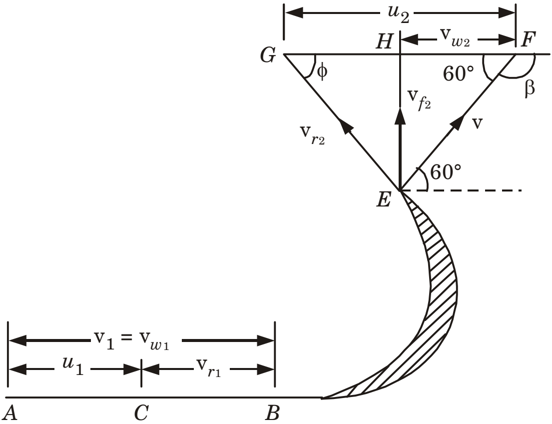


Fig. 4.4.1.

8. The force exerted by the jet on the vane in the direction of motion is given as,

$$F_x = \rho a v_{r1} [v_{w1} - v_{w2}]$$

(–ve sign is taken as β is an obtuse angle)

$$= 1000 \times 0.001963 \times 10 [20 - 5] \text{ N} = 294.45 \text{ N}$$

9. Work done per second by the jet

$$= F_x u = 294.45 \times 10 = 2944.5 \text{ N m/s}$$

$$= 2944.5 \text{ W}$$

Que 4.5. **A jet of water moving at 12 m/s impinges on a concave shaped vane to deflect the jet through 120° when stationary. The vane is moving at 5 m/s. Assuming the vane is smooth, find**

i. The angle of jet so that there is no shock at inlet.

ii. The absolute velocity of the jet at exit both in magnitude and direction.

iii. The work done per second per N of water.

Answer

Given : v_1 = 12 m/s, Angle of deflection = 120°, $u_1 = u_2 = u$ = 5 m/s

To Find : i. The angle of jet so that there is no shock at inlet.
ii. The absolute velocity of the jet at exit both in magnitude and direction.
iii. The work done per second per N of water.

1. Assuming vane to be symmetrical, we have $\theta = \phi$

Now, $120° = 180° - (\theta + \phi)$

$$\theta + \phi = (180° - 120°) = 60°$$

So, $$\theta = \phi = 30°$$

2. Applying sine rule to ΔABC, we have

$$\frac{AB}{\sin(180° - \theta)} = \frac{AC}{\sin(30° - \alpha)} \quad \text{or} \quad \frac{v_1}{\sin\theta} = \frac{u_1}{\sin(30° - \alpha)}$$

$$\frac{12}{\sin 30°} = \frac{5}{\sin(30° - \alpha)}$$

$$\sin(30° - \alpha) = \frac{5 \times \sin 30°}{12} = 0.2083$$

$\therefore$ $$30° - \alpha = \sin^{-1} 0.2083 = 12°$$

$$\alpha = 30° - 12° = 18°$$

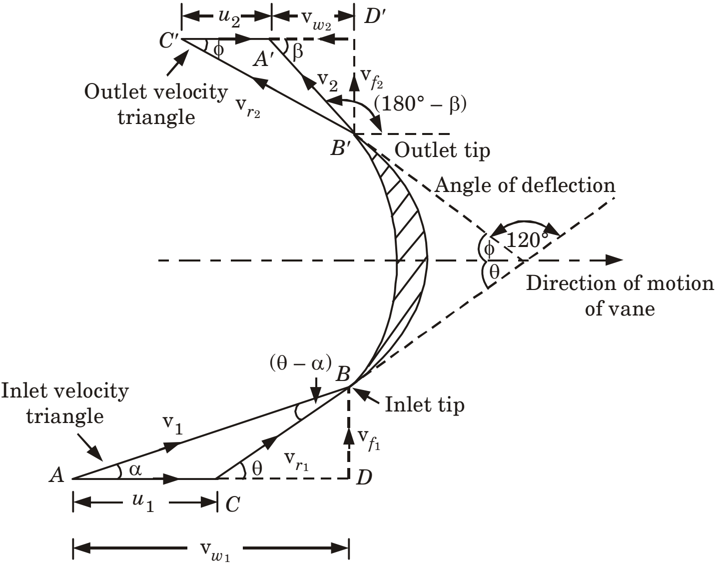


Fig. 4.5.1.

3. Again applying sine rule to ΔABC, we have

$$\frac{v_1}{\sin(180° - \theta)} = \frac{v_{r1}}{\sin\alpha}$$

or, $$\frac{12}{\sin\theta} = \frac{v_{r_1}}{\sin 18°}$$

$\therefore$ $$v_{r_1} = \frac{12 \times \sin 18°}{\sin\theta} = \frac{12 \times \sin 18°}{\sin 30°} = 7.42 \text{ m/s}$$

4. In ΔABD,

$$v_{w_1} = v_1 \cos\alpha = 12 \cos 18° = 11.41 \text{ m/s}$$

5. Now, since the vane is smooth, therefore,

$$v_{r_2} = v_{r1} = 7.42 \text{ m/s}$$

6. At outlet, from $\Delta B'C'D'$, we have

$$v_{r_2} \cos \phi = u_2 + v_{w_2}$$

$$\therefore \quad v_{w_2} = v_{r_2} \cos \phi - u_2 = 7.42 \cos 30° - 5 = 1.42 \text{ m/s}$$

7. Also, $$v_{f_2} = v_{r_2} \sin \phi = 7.42 \sin 30° = 3.71 \text{ m/s}$$

8. Now, $$\tan \beta = \frac{v_{f_2}}{v_{w_2}} = \frac{3.71}{1.42} = 2.613$$

$\therefore$ Angle of jet at outlet, $\beta = \tan^{-1} 2.613 = 69.06°$

9. Hence, angle made by v_2 at outlet with direction of motion of vane is

$$= 180° - \beta = 180° - 69.06° = 110.94°$$

10. Absolute velocity of jet at exit,

$$v_2 = \sqrt{v_{w_2}^2 + v_{f2}^2}$$

$$= \sqrt{(1.42)^2 + (3.71)^2} = 3.97 \text{ m/s}$$

11. The work done per second per N of water

$$= \frac{1}{g}(v_{w_1} u_1 + v_{w_2} u_2) = \frac{1}{g}(v_{w_1} + v_{w_2}) \times u$$

$$= \frac{1}{9.81}(11.41 + 1.42) \times 5 = 6.539 \text{ Nm}$$

Que 4.6. **Show that in case of jet striking the series of flat plates mounted on wheel periphery, the efficiency will be maximum when tangential velocity of wheel is half of the jet.**

Answer

1. As shown in Fig. 4.6.1, a large number of plates are mounted on the circumference of a wheel at a fixed distance.
2. The jet strikes on the plate and due to the force exerted by jet on the plate, the wheel starts moving at a constant speed.
3. Let, u = Velocity of vane.
4. Mass of water per second striking the series of plate = $\rho a v$

   and, jet of water strikes the plate with a velocity = $(v - u)$
5. After striking, the jet moves tangential to plate and hence velocity component in the direction of motion of plate is equal to zero.
6. Force exerted by the jet in the direction of motion of plate,

$$F_x = \text{Mass per sec [Initial velocity – Final velocity]}$$
$$= \rho a v[(v - u) - 0]$$
$$= \rho a v(v - u)$$

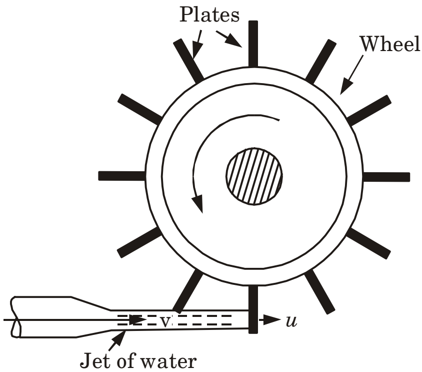


Fig. 4.6.1.

7. Work done by the jet on the series of plates per second,

$$= \text{Force} \times \text{Velocity} = F_x u = \rho a \text{v}(\text{v} - u)u$$

8. Kinetic energy of jet per second $= \dfrac{1}{2} m\text{v}^2 = \dfrac{1}{2}(\rho a \text{v})\,\text{v}^2 = \dfrac{1}{2}\rho a \text{v}^3$

9. Efficiency,

$$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}}$$

$$\eta = \frac{\rho a \text{v}(\text{v} - u)u}{\frac{1}{2}\rho a \text{v}^3}$$

$$\eta = \frac{2u(\text{v} - u)}{\text{v}^2}$$

10. For maximum efficiency, $\dfrac{d\eta}{du} = 0$

$$\frac{d}{du}\left(\frac{2u(\text{v} - u)}{\text{v}^2}\right) = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u\text{v} - 2u^2}{\text{v}^2}\right] = 0$$

$$\frac{2\text{v} - 2 \times 2u}{\text{v}^2} = 0 \quad \text{or} \quad 2\text{v} - 4u = 0$$

$$u = \frac{\text{v}}{2} \quad \text{or} \quad \text{v} = 2u$$

Hence, the efficiency is maximum when tangential velocity of wheel is half of the velocity of jet.

Que 4.7. **What is the difference between the force of jet when it impinges on a single moving flat plate and the force of jet when it strikes on a series of moving plates ?**

AKTU 2014-15, Marks 05

Answer

1. The force of jet when it impinges on a single moving flat plate is given by,

$$F_1 = \rho a(v - u)^2$$

2. The force when jet impinges on a series of moving plate is given by,

$$F_2 = \rho a(v - u)v$$

3. The difference between the two forces is given as,

$$= F_1 - F_2$$
$$= \rho a(v - u)\ [v - u - v]$$
$$= -\ \rho a(v - u)u$$

PART-2

Classification of Turbines.

CONCEPT OUTLINE

Turbines : These are defined as the hydraulic machines which convert hydraulic energy into mechanical energy.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.8. **Discuss the classification of hydraulic turbines.**

AKTU 2016-17, Marks 10

OR

Classify hydraulic turbines in detail. **AKTU 2017-18, Marks 10**

Answer

Hydraulic turbines are classified as follows :

a. **According to the Type of Energy Available at Inlet :**

i. **Impulse Turbine :** In an impulse turbine, all the available energy of water is converted into kinetic energy or velocity head.
Example : Pelton wheel turbine.

ii. **Reaction Turbine :** In a reaction turbine, at the entrance to the runner, only a part of the available energy of water is converted into kinetic energy and a substantial part remains in the form of pressure energy.

Example : Francis turbine, Kaplan turbine.

b. According to the Direction of Flow through Runner :

i. Tangential Flow Turbine : In this turbine, the water flows along the tangent to the path of rotation of the runner.

Example : Pelton wheel turbine.

ii. Radial Flow Turbine : In this turbine, the water flows along the radial direction.

Example : Francis turbine.

iii. Axial Flow Turbine : In this turbine, water flows through the runner wholly and mainly along the direction parallel to the axis of rotation of the runner.

Example : Kaplan turbine.

iv. Mixed Flow Turbine : In this turbine, water enters the runner at the outer periphery in radial direction and leaves it at the centre in the direction parallel to the axis of rotation of the runner.

Example : Modern Francis turbine.

c. According to the Head at Inlet of Turbine :

i. High Head Turbine : These are the turbines which are capable of working under very high head ranging more than 250 m.

Example : Pelton wheel turbine.

ii. Medium Head Turbine : These are the turbines which are capable of working under head ranging from 60 m to 250 m.

Example : Francis turbine.

iii. Low Head Turbine : These are the turbines which are capable of working under head less than 60 m.

Example : Kaplan turbine.

d. According to the Specific Speed of the Turbine :

i. Low Specific Speed Turbine : Low specific speed ranging less than 60.

Example : Pelton wheel turbine.

ii. Medium Specific Speed Turbine : Ranging between 60 to 300.

Example : Francis turbine.

iii. High Specific Speed Turbine : Ranging between 300 to 1000.
Example : Kaplan turbine.

Que 4.9. Explain different types of head used in hydroelectric power plant and also draw the schematic hydroelectric power plant.

Answer

A. Different Heads used in Hydroelectric Power Plant :

i. Gross Head :

1. The difference between the head race level and tail race level when no water is flowing is known as gross head.

2. It is denoted by H_g.

ii. Net Head or Effective Head :

1. It is the head available at the entrance to the turbine.
2. It is obtained by subtracting all the losses of head from gross head.
3. Net head is given by,

$$H = H_g - h_f$$

Where, H_g = Gross head, and
h_f = Total loss of head between the head race and the entrance of the turbine.

B. Layout of a Hydroelectric Power Plant :

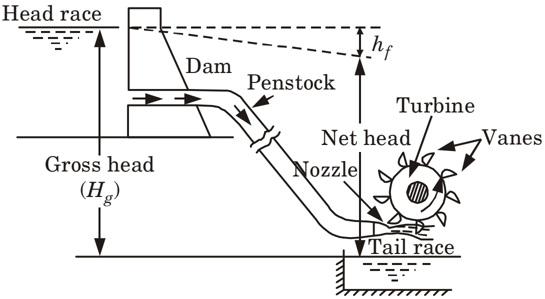


Fig. 4.9.1. Layout of a hydroelectric power plant.

Que 4.10. **Explain different types of efficiency of turbine.**

Answer

The following are the important efficiencies of a turbine :

i. Hydraulic Efficiency : It is defined as the ratio of power given by water to the runner of a turbine to power supplied by the water at the inlet of the turbine.

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}}$$

$$= \frac{\text{RP(Runner power)}}{\text{WP(Water power or hydraulic power)}}$$

ii. Mechanical Efficiency :

1. The ratio of the power available at the shaft of the turbine to the power delivered to the runner is defined as mechanical efficiency.

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the turbine}} = \frac{\text{SP}}{\text{RP}}$$

2. The power delivered by water to the runner of a turbine is transmitted to the shaft of the turbine. Due to mechanical losses, power available at the shaft of the turbine is less than the power delivered to the runner of a turbine.

iii. **Volumetric Efficiency :** The ratio of the volume of the water actually striking the runner to the volume of water supplied to the turbine is defined as volumetric efficiency.

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}}$$

iv. **Overall Efficiency :** It is defined as the ratio of power available at the shaft of the turbine to the power supplied by the water at the inlet of the turbine.

$$\eta_o = \frac{\text{Power available at the shaft of the turbine}}{\text{Power available at the inlet of the turbine}} = \frac{SP}{WP} = \frac{SP}{RP} \times \frac{RP}{WP}$$

$$\eta_o = \eta_m \eta_h$$

PART-3

Impulse Turbines, Constructional Details, Velocity Triangles, Power and Efficiency Calculations.

CONCEPT OUTLINE

Impulse Turbine : If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as impulse turbine.
Pelton Wheel : It is a tangential flow impulse turbine. The pressure at the inlet and outlet of the turbine is atmospheric. This turbine is used for high heads.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.11. Describe briefly the function of various main components of Pelton wheel turbine with neat sketches.

OR

Explain construction and working of Pelton wheel turbine.

Answer

A. Construction of Pelton Wheel Turbine :

i. Nozzle and Flow Regulating Arrangement :

1. The amount of water striking the buckets of runner is controlled by providing a spear in the nozzle.
2. Spear has the streamlined head which is fixed to end of the rod.

3. The spear is push forward in the nozzle to reduce the water flow and is push backward to increase the water flow.

ii. Runner Reduce Gap with Buckets :

1. Runner consists of a circular disc with a number of buckets evenly spaced around its periphery.
2. Each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter.
3. The jet of water impinges on the splitter which divides the jet into two equal portions.

iii. Casing :

1. Fig. 4.11.1 shows the casing of a Pelton wheel turbine.
2. The function of the casing is to prevent the splashing of the water and to discharge water to tail race.
3. The casing of the Pelton wheel does not perform any hydraulic function.

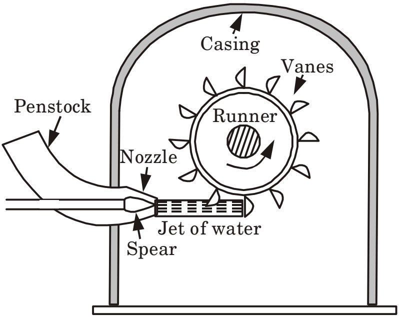


Fig. 4.11.1. Pelton turbine.

iv. Breaking Jet :

1. When the nozzle is completely closed by the motion of spear in forward direction, the amount of water striking the runner reduces to zero.
2. But due to inertia, runner goes on revolving. Therefore a jet from back of vane is used to stop the wheel known as breaking jet.

B. Working of Pelton Wheel Turbine :

1. The water stored at high head is made to flow through the penstock and reaches the nozzle of the Pelton turbine.
2. The nozzle increases the kinetic energy of the water and directs the water in the form of jet.
3. The jet of water from the nozzle strikes the bucket (vanes) of the runner. This made the runner to rotate at very high speed.
4. The quantity of water striking the vanes or buckets is controlled by the spear present inside the nozzle.

5. The generator is attached to the shaft of the runner which converts the mechanical energy of the runner into electrical energy.

Que 4.12. **Prove that the work done per second per unit weight of water in Pelton turbine is given as $\frac{1}{g}(v_{w1} + v_{w2})\,u$.**

OR

Draw inlet and outlet velocity triangles for a Pelton wheel and indicate the direction of velocities.

AKTU 2015-16, Marks 05

Answer

1. Fig. 4.12.1 shows the inlet and outlet velocity triangles.

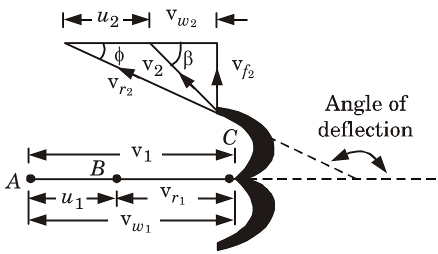


Fig. 4.12.1.

2. Let, v_1 and v_2 = Absolute velocity at inlet and velocity of jet at outlet respectively,
   v_{r_1} and v_{r_2} = Relative velocity of jet at inlet and outlet respectively,
   v_{f_1} and v_{f_2} = Velocity of flow at inlet and outlet respectively, and
   v_{w_1} and v_{w_2} = Velocity of whirl at inlet and outlet respectively.
3. Velocity triangle at inlet will be straight line where,

$$v_{r1} = v_1 - u_1 = v_1 - u$$
$$v_{w1} = v_1$$
$$\alpha = 0°, \quad \theta = 0°$$

4. Velocity triangle at outlet,

$$v_{r2} = v_{r1}$$
$$v_{w2} = v_{r2} \cos \phi - u_2$$

5. Force exerted by the jet of water in the direction of motion,

$$F_x = \rho a v_1 [v_{w1} + v_{w2}]$$

6. Work done by the jet on runner per second = $F_x u$

$$= \rho a v_1 [v_{w1} + v_{w2}] u$$

7. Work done/sec per unit weight of water striking

$$= \frac{\text{Work done by jet on runner per second}}{\text{Weight of water striking per second}}$$

$$= \frac{\rho a \mathrm{v}_1 [\mathrm{v}_{w1} + \mathrm{v}_{w2}] u}{\rho a \mathrm{v}_1 g} = \frac{1}{g} [\mathrm{v}_{w1} + \mathrm{v}_{w2}] u$$

Que 4.13. **Prove that hydraulic efficiency for Pelton wheel turbine is given by,**

$$\eta_h = \frac{2(\mathrm{v}_1 - u)(1 + \cos \phi) u}{\mathrm{v}_1^2}$$

Also find out the condition for maximum efficiency for Pelton wheel turbine.

Answer

1. Work done per second = $\rho a \mathrm{v}_1 [\mathrm{v}_{w1} + \mathrm{v}_{w2}] u$
2. KE of jet per second = $\frac{1}{2} m \mathrm{v}_1^2 = \frac{1}{2} (\rho a \mathrm{v}_1) \mathrm{v}_1^2$
3. For Pelton wheel turbine, hydraulic efficiency is given by,

$$\eta_h = \frac{\text{Work done per second}}{\text{KE of jet per second}}$$

$$\eta_h = \frac{\rho a \mathrm{v}_1 [\mathrm{v}_{w1} + \mathrm{v}_{w2}] u}{\frac{1}{2} (\rho a \mathrm{v}_1) \mathrm{v}_1^2} \quad \text{...(4.13.1)}$$

4. For a Pelton wheel, we have

$$\mathrm{v}_{w1} = \mathrm{v}_1, \mathrm{v}_{r1} = \mathrm{v}_1 - u_1 = \mathrm{v}_1 - u$$
$$\mathrm{v}_{r2} = \mathrm{v}_1 - u$$
$$\mathrm{v}_{w2} = \mathrm{v}_{r2} \cos \phi - u_2 = \mathrm{v}_{r2} \cos \phi - u = (\mathrm{v}_1 - u) \cos \phi - u$$

5. Substituting the value of v_{w1} and v_{w2} in eq. (4.13.1), we get

$$\eta_h = \frac{2[\mathrm{v}_1 + (\mathrm{v}_1 - u) \cos \phi - u] u}{\mathrm{v}_1^2}$$

$$= \frac{2[\mathrm{v}_1 - u + (\mathrm{v}_1 - u) \cos \phi] u}{\mathrm{v}_1^2}$$

$$= \frac{2(\mathrm{v}_1 - u)(1 + \cos \phi) u}{\mathrm{v}_1^2}$$

6. For maximum efficiency,

$$\frac{d}{du}(\eta_h) = 0$$

$$\frac{d}{du}\left[\frac{2(\mathrm{v}_1 - u)(1 + \cos \phi) u}{\mathrm{v}_1^2}\right] = 0$$

$$\frac{(1+\cos\phi)}{v_1^2}\frac{d}{du}[2(v_1-u)u] = 0$$

$$\frac{d}{du}[2v_1 u - 2u^2] = 0$$

$$2v_1 - 4u = 0$$

$$u = \frac{v_1}{2}$$

7. So, hydraulic efficiency of a Pelton wheel turbine will be maximum when the velocity of wheel is half the velocity of jet of water at inlet.

Que 4.14. **What are the design aspects of Pelton wheel ?**

Answer

The following points should be considered while designing a Pelton wheel :

i. **Velocity of Jet at Inlet :**

Where, $v_1 = C_v\sqrt{2gH}$

C_v = Coefficient of velocity, and

= 0.98 or 0.99

H = Net head available.

ii. **Velocity of Wheel :** It is given by,

$$u = \phi\sqrt{2gH}$$

Where, ϕ = Speed ratio (varies from 0.43 to 0.48)

iii. **Angle of Deflection :** It is taken as 165°, if no angle is given.

iv. **Mean Diameter of Wheel :** It is given by,

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi N}$$

v. **Jet Ratio (*m*) :** It is defined as the ratio of the pitch diameter of the Pelton wheel to the diameter of the jet. It is usually taken between 11 and 15.

$$m = \frac{\text{Diameter of pitch circle }(D)}{\text{Diameter of jet }(d)}$$

vi. **Number of Bucket on Runner (Z) :** It is given by, $Z = 15 + m/2$. It is usually taken 20 to 25.

vii. **Number of Jets :** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

$$\text{Number of jet} = \frac{\text{Total flow}}{\text{Flow through one jet}}$$

Number of jet practically should not be greater than 6.

Que 4.15. **A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at a rate of 700 lit/s under a head of 30 m. The**

bucket deflects the jet through an angle of 160 degree. Calculate power and hydraulic efficiency. **AKTU 2015-16, Marks 05**

Answer

Given : $u_1 = u_2 = u = 10$ m/s, $Q = 700$ lit/s $= 0.7$ m^3/s, $H = 30$ m, $\phi = 180° - 160° = 20°$

To Find : i. Power.
ii. Hydraulic efficiency.

Data Assumed : $C_v = 0.98$.

1. The velocity of jet, $v_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s

$\therefore$ $$v_{r1} = v_1 - u_1 = 23.77 - 10 = 13.77 \text{ m/s}$$
$$v_{w1} = v_1 = 23.77 \text{ m/s}$$

2. From outlet velocity triangle,
$$v_{r2} = v_{r1} = 13.77 \text{ m/s}$$
$$v_{w2} = v_{r2}\cos\phi - u_2$$
$$= 13.77 \cos 20° - 10 = 2.94 \text{ m/s}$$

3. Work done by the jet per second on the runner is given as
$$= \rho a v_1 [v_{w1} + v_{w2}]u$$
$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$$
$(\because \ a v_1 = Q = 0.7 \text{ m}^3/\text{s})$
$$= 186970 \text{ Nm/s}$$

4. Power given to turbine $= \dfrac{186970}{1000} = 186.97$ kW

5. The hydraulic efficiency of the turbine is given as,
$$\eta_h = \frac{2[v_{w_1} + v_{w_2}]u}{v_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$
$$= 0.9454 \quad \text{or} \quad 94.54\ \%$$

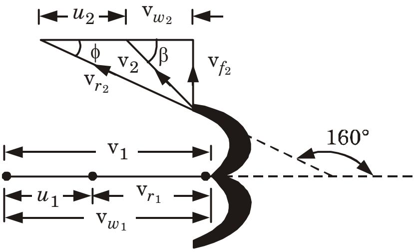


Fig. 4.15.1.

Que 4.16. **Determine the power given by the jet of water to the runner of a Pelton wheel which is having tangential velocity as 20 m/s. The net head on the turbine is 50 m and discharge through**

the jet water is 0.03 m³/s. The side clearance angle is 15° and take C_v = 0.975.

AKTU 2015-16, Marks 7.5

Answer

Given : u = 20 m/s, H = 50 m, Q = 0.03 m³/s, ϕ = 15°, C_v = 0.975
To Find : Power given by jet to the runner.

1. Velocity of the jet, $v_1 = C_v\sqrt{2gH} = 0.975\sqrt{2 \times 9.81 \times 50} = 30.54$ m/s
2. From inlet velocity triangle of Pelton wheel, we have

$$v_{w_1} = v_1 = 30.54 \text{ m/s}$$
$$v_{r_1} = v_{w_1} - u_1 = 30.54 - 20.0 = 10.54 \text{ m/s}$$

3. From outlet velocity triangle, we have

$$v_{r_2} = v_{r_1} = 10.54 \text{ m/s}$$
$$v_{r_2} \cos \phi = 10.54 \cos 15° = 10.18 \text{ m/s}$$

4. As $v_{r_2} \cos \phi$ is less than u_2, the velocity triangle at outlet will be as shown in Fig. 4.16.1.

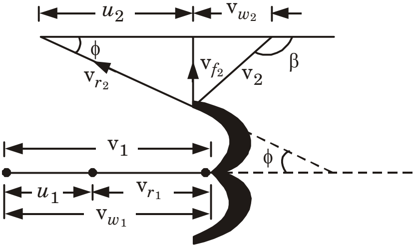


Fig. 4.16.1.

$$\therefore \quad v_{w_2} = u_2 - v_{r_2} \cos \phi = 20 - 10.18 = 9.82 \text{ m/s}$$

5. Also as β is an obtuse angle, the work done per second on the runner,

$$= \rho a v_1 [v_{w_1} - v_{w_2}]u = \rho Q [v_{w_1} - v_{w_2}]u$$
$$= 1000 \times 0.03 \times [30.54 - 9.82] \times 20 = 12432 \text{ Nm/s}$$

6. Power given to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{12432}{1000} = 12.432 \text{ kW}$$

Que 4.17. **A Pelton wheel turbine has following specifications :**

Shaft power = 12000 kW
Head = 400 meters
Speed = 750 rpm
Overall efficiency = 0.85

and the ratio of jet diameter to the wheel diameter is 1/6. Determine :

i. The wheel diameter.
ii. Diameter of the jet and number of jets required.

Take C_v = 0.98 and ϕ = 0.45.

Answer

Given : P = 12000 kW, H = 400 m, N = 750 rpm, η_o = 0.85, d/D = 1/6, C_v = 0.98, ϕ = 0.45

To Find :
i. Wheel diameter.
ii. Jet diameter.
iii. Number of jets required.

1. Velocity of jet, $v_1 = C_v \sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 400} = 86.82$ m/s

2. Velocity of wheel, $u = \phi\sqrt{2gH}$

$$= 0.45\sqrt{2 \times 9.81 \times 400} = 39.86 \text{ m/s}$$

3. We know that, $u = \dfrac{\pi DN}{60}$

$$\therefore \quad D = \frac{u \times 60}{\pi \times N} = \frac{39.86 \times 60}{3.14 \times 750} = 1.01 \text{ m}$$

4. Now, $\dfrac{d}{D} = \dfrac{1}{6}$

$$d = \frac{D}{6} = \frac{1.01}{6} = 0.168 \text{ m}$$

5. Discharge of one jet,

$$q = \text{Area of jet} \times \text{Velocity of jet}$$

$$= \frac{\pi}{4} d^2 v_1 = \frac{\pi}{4}(0.168)^2 \times 86.82 = 1.92 \text{ m}^3/\text{s}$$

6. Overall efficiency is given as,

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}}$$

$$\eta_o = \frac{P}{\dfrac{\rho g Q H}{1000}}$$

$$0.85 = \frac{12000 \times 1000}{1000 \times 9.81 \times Q \times 400}$$

$$Q = 3.60 \text{ m}^3/\text{s}$$

7. $$\text{Number of jets} = \frac{\text{Total discharge }(Q)}{\text{Discharge of one jet }(q)}$$

$$= \frac{3.60}{1.92} = 1.875 \approx 2$$

Number of jets = 2

PART-4

Governing of Pelton Wheel.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.18. **Explain the governing of a Pelton turbine. Use neat sketch.** **AKTU 2016-17, Marks 10**

OR

What do you understand by term governing of turbine ? Explain governing mechanism of Pelton wheel. **AKTU 2017-18, Marks 10**

OR

Explain construction and working of Pelton wheel turbine. Also how the speed of Pelton wheel govern ? **AKTU 2014-15, Marks 05**

Answer

A. Construction and Working of Pelton Wheel Turbine : Refer Q. 4.11, Page 4–19A, Unit-4.

B. Governing of Turbines :

1. It is defined as the operation by which the speed of the turbine is kept constant under all conditions.
2. It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load conditions on the turbine.

C. Governing Mechanism of Pelton Turbine :

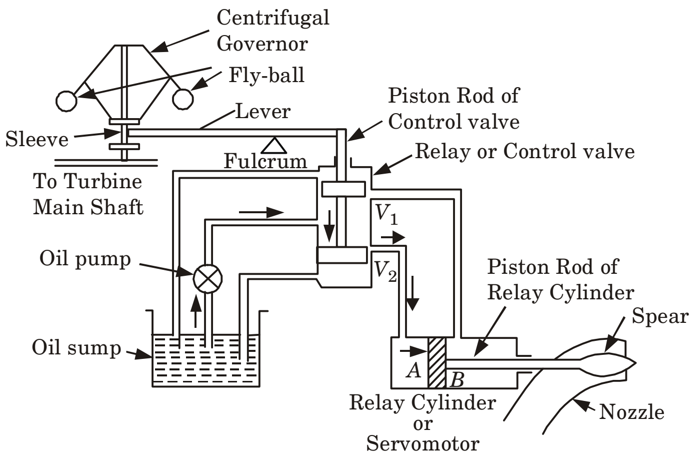


Fig. 4.18.1. Governing of Pelton turbine.

i. When the Load on Generator Decreases :

1. When the load on the generator decreases, the speed of generator and hence the turbine increases beyond normal speed.
2. The fly-balls of the centrifugal governor move outward due to the increased centrifugal force on them.
3. Due to the outward movement of the fly-balls, the sleeve moves up. As a consequence the portion of the lever to the right of the fulcrum moves down pushing the piston rod of the control valve downwards.
4. This closes the valve V_1 and opens the valve V_2.
5. A gear pump pumps oil from the oil sump to the relay valve or control valve. Oil flows through valve V_2 and exerts force on the face A of the piston of the relay cylinder.
6. The piston rod along with the spear moves to the right. This decreases the area of flow of the nozzle and hence, the rate of water flows to the turbine.
7. Consequently the speed of the turbine decreases till it becomes normal.

ii. When the Load on Generator Increases :

1. When the load on the generator increases, the speed of generator and hence the turbine decreases beyond normal speed.
2. The fly-balls of the centrifugal governor move inward due to the decreased centrifugal force on them.
3. Due to the inward movement of the fly-balls, the sleeve moves down and the piston rod of control valve goes up.
4. This closes the valve V_2 and opens the valve V_1.
5. Oil flows through valve V_1 and exerts force on the face B of the piston of the relay cylinder.
6. The piston rod along with the spear moves to the left. This increases the area of flow of the nozzle and hence, the rate of water flows to the turbine.
7. As a consequence, the speed of the turbine increases till it becomes normal.

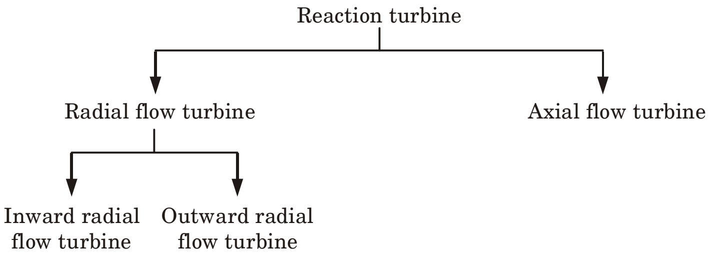
PART-5

Francis and Kaplan Turbines, Constructional Details, Velocity Triangles, Power and Efficiency.

CONCEPT OUTLINE

Reaction Turbine : If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as reaction turbine.

Classification of Reaction Turbine :



Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.19. Give the comparison between impulse and reaction turbine.

Answer

S. No.	Impulse Turbine	Reaction Turbine
1.	The available fluid energy is converted into KE by a nozzle.	The energy of the fluid is partly transformed into KE before it enters the runner of the turbine.
2.	The pressure remains same throughout the action of water on the runner.	After entering the runner with an excess pressure, water undergoes changes both in velocity and pressure while passing through the runner.
3.	Water may be allowed to enter a part or whole of the wheel circumference.	Water is admitted over the circumference of the wheel.
4.	Water tight casing is required.	Water tight casing is not necessary.
5.	The wheel/turbine does not run full and air has a free access to the buckets.	Water completely fills all the passages between the blades while flowing between inlet outlet sections does work on the blades.
6.	Always installed above the tail race. No draft tube is used.	Unit may be installed above or below the tail race. Use of a draft tube is made.

7.	Relative velocity of water either remains constant or reduces slightly due to friction.	Due to continuous drop in pressure during flow through the blade, the relative velocity increases.

Que 4.20. **State the differences between inward and outward radial flow reaction turbine.**

Answer

S. No.	Inward Flow Reaction Turbine	Outward Flow Reaction Turbine
1.	Water enters at the outer periphery, flows inward and discharge at the inner periphery.	Water enters at the inner periphery flows outward and discharges at the outer periphery.
2.	Negative centrifugal head reduces the relative velocity of water at the outlet.	Positive centrifugal head increases the relative velocity of water at the outlet.
3.	Discharge does not increase.	The discharge increases.
4.	Easy and effective speed control.	Speed control is very difficult.
5.	The turbine adjusts the speed by itself.	The turbine cannot adjust the speed by itself.

Que 4.21. **Describe briefly the function of various components of radial flow reaction turbine or Francis turbine with neat sketch.**

OR

Explain the functions of the following parts of reaction turbine :
i. Scroll casing,
ii. Draft tube,
iii. Guide blades, and
iv. Runner.

AKTU 2015-16, Marks 7.5

Answer

The main parts of a radial flow reaction turbine are as follows :

i. **Scroll Casing :** The spiral casing around the runner of the turbine is known as the volute casing or scroll casing.

ii. **Draft Tube :**

1. The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure.

2. Therefore, it is not safe to discharge (water) directly into atmosphere because it may results into evaporation.
3. To avoid this problem, draft tube is employed which supplies water from exit of the turbine to the tail race.

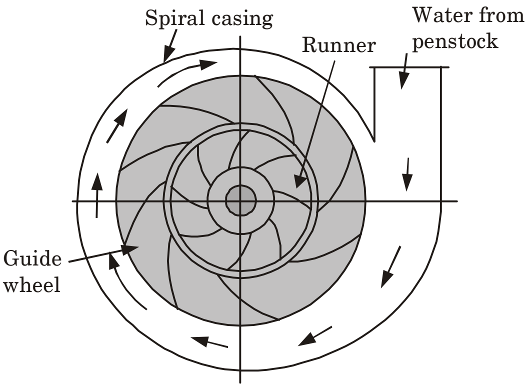


Fig. 4.21.1. Main parts of a radial flow reaction turbine.

iii. Guide Blade :

1. It consists of a stationary circular wheel all around the runner of the turbine.
2. The stationary guide vanes are permanently fixed to the casing which guides the water to enter into the runner. These are non rotating unit.

iv. Runner :

1. It is a circular wheel on which a series of radial curved vanes are fixed. Surface of vane is very smooth to avoid friction losses.
2. Runner is a rotating unit. It is the main part of turbine.

v. Penstock : It is a large size conduit which conveys water from the upstream of the dam to the turbine runner.

Que 4.22. **Show that the work done per second per unit weight of water in reaction turbine is given as**

$$= \frac{1}{g}(v_{w_1} u_1)$$

Where, v_{w_1} **= Velocity of whirl at inlet, and**
u_1 **= Tangential velocity of wheel at inlet.**

Answer

1. Consider a series of radial curved vanes mounted on a wheel as shown in Fig. 4.22.1. Jet of water strikes the vanes and wheel starts rotating at constant angular speed.

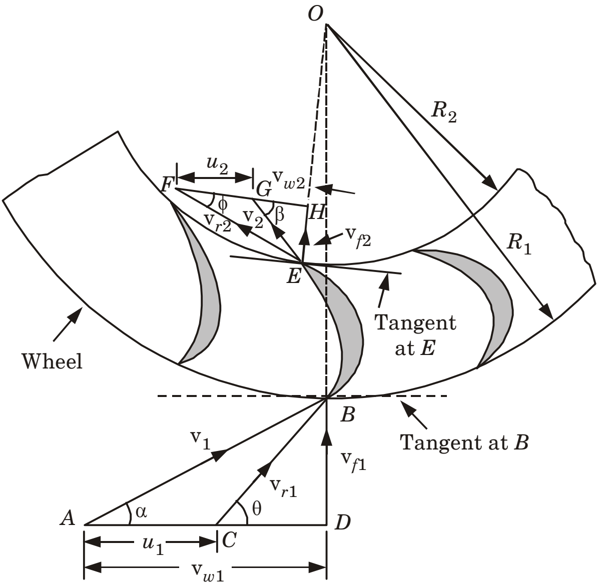


Fig. 4.22.1. Series of radial curved vanes mounted on a wheel.

2. Let, u_1 and u_2 = Tangential velocity of wheel at inlet and outlet,

R_1 and R_2 = Radius of wheel at inlet and outlet,

ω = Angular speed of wheel,

v_{w_1} = Velocity of whirl at inlet = $v_1 \cos \alpha$, and

v_{w_2} = Velocity of whirl at outlet = $v_2 \cos \beta$.

3. Momentum of water at inlet = Mass of water coming out from nozzle × Component of v_1 in tangential direction

$= \rho a v_1 v_{w_1}$

4. Similarly, momentum of water at outlet,

$= \rho a v_1 \times (-v_{w_2})$

(Negative sign shows direction of flow is in opposite direction)

5. Angular momentum per second at inlet,

= Momentum at inlet × Radius at inlet

$= \rho a v_1 v_{w_1} R_1$

6. Angular momentum per second at outlet,

= Momentum at outlet × Radius at outlet

$= -\rho a v_1 v_{w2} R_2$

7. Torque exerted by water on wheel,

T = (Initial angular momentum per second) – (Final angular momentum per second)

$= (\rho a v_1 v_{w_1} R_1) - (-\rho a v_1 v_{w2} R_2)$

$$= \rho a v_1 [v_{w_1} R_1 + v_{w2} R_2]$$

8. Now, work done per second on wheel

$$= \text{Torque} \times \text{Angular velocity}$$

$$= \rho a v_1 [v_{w_1} R_1 + v_{w_2} R_2] \times \omega$$

$$= \rho a v_1 [v_{w_1} u_1 + v_{w_2} u_2]$$

$$(\because \quad u_1 = \omega R_1 \text{ and } u_2 = \omega R_2)$$

$$= \rho Q [v_{w_1} u_1 + v_{w_2} u_2] \qquad (\because \quad a v_1 = Q)$$

9. Weight of water striking per second = $\rho Q g$

10. Work done per second per unit weight of water

$$= \frac{\text{Work done per second}}{\text{Weight of water striking per second}}$$

$$= \frac{\rho Q [v_{w_1} u_1 + v_{w_2} u_2]}{\rho Q g}$$

$$= \frac{1}{g} [v_{w_1} u_1 + v_{w_2} u_2]$$

11. If discharge is radial, then $v_{w_2} = 0$

$\therefore$ Work done per second per unit weight of water = $\frac{1}{g} (v_{w_1} u_1)$

Que 4.23. **Following data pertain to a Francis turbine :**

Net head = 60 m, speed = 650 rpm, shaft power = 275 kW, ratio of outer diameter to inner diameter = 2, ratio of wheel width to wheel diameter = 0.1, flow ratio = 0.17, $\eta_{hydraulic}$ = 0.95 and $\eta_{overall}$ = 0.85. The flow velocity remains constant and the discharge is radial. Find out wheel width, diameter and blade angles at inlet and outlet.

Answer

Given : H = 60 m, N = 650 rpm, SP = 275 kW, B_1/D_1 = 0.1, D_1/D_2 = 2, Flow ratio = 0.17, η_h = 0.95, η_0 = 0.85

To Find :
- i. Width of wheel.
- ii. Diameter of blade at inlet and outlet.
- iii. Blade angle at inlet and outlet.

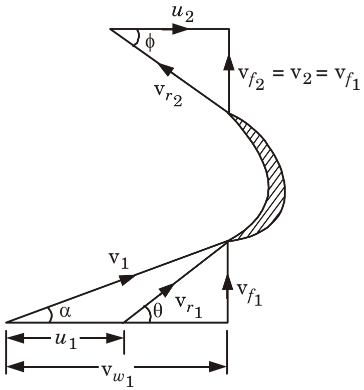


Fig. 4.23.1.

1. Flow ratio, $$0.17 = \frac{v_{f_1}}{\sqrt{2gH}}$$

$$v_{f_1} = 0.17 \times \sqrt{2 \times 9.81 \times 60}$$
$$= 5.83 \text{ m/s}$$

Since velocity of flow is constant, so

$$v_{f_1} = v_{f_2} = 5.83 \text{ m/s}$$

2. Discharge at outlet is radial, so

$$v_{w_2} = 0, \text{ and } v_{f_2} = v_2$$

3. Now, overall efficiency,

$$\eta_o = \frac{SP}{WP}$$

$$0.85 = \frac{275}{WP}$$

$$WP = 323.53 \text{ kW}$$

4. Water power is also given as,

$$WP = \frac{\rho g Q H}{1000}$$

$$323.53 = \frac{1000 \times 9.81 \times Q \times 60}{1000}$$

$$Q = 0.550 \text{ m}^3/\text{s}$$

5. Discharge, $$Q = \text{Actual area of flow} \times v_{f_1}$$

$$Q = \pi D_1 B_1 v_{f_1}$$

$$0.550 = \pi \times D_1 \times 0.1 \times D_1 \times 5.83$$

$$D_1^2 = 0.30029$$

$$D_1 = 0.548 \text{ m}$$

6. It is given that, $\dfrac{B_1}{D_1} = 0.1$

$$B_1 = 0.548 \times 0.1 \simeq 0.055 \text{ m}$$

7. Tangential speed of turbine at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.548 \times 650}{60} = 18.65 \text{ m/s}$$

8. Hydraulic efficiency is given as,

$$\eta_h = \frac{\text{v}_{w_1} u_1}{gH}$$

$$0.95 = \frac{\text{v}_{w_1} \times 18.65}{9.81 \times 60}$$

$$\text{v}_{w_1} = 30 \text{ m/s}$$

9. From inlet velocity triangle,

$$\tan \alpha = \frac{\text{v}_{f_1}}{\text{v}_{w_1}} = \frac{5.83}{30}$$

$$\alpha = 11°$$

10. Again from inlet velocity triangle,

$$\tan \theta = \frac{\text{v}_{f_1}}{\text{v}_{w_1} - u_1} = \frac{5.83}{30 - 18.65}$$

$$\theta = 27.18°$$

11. Diameter of runner at outlet,

$$D_2 = D_1/2$$

$$= 0.548/2 = 0.274$$

12. We know, $u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi \times 0.274 \times 650}{60}$

$$= 9.325 \text{ m/s}$$

13. From outlet velocity triangle,

$$\tan \phi = \frac{\text{v}_{f_2}}{u_2} = \frac{\text{v}_{f_1}}{u_2} = \frac{5.83}{9.325}$$

$$\phi = 32.013°$$

Que 4.24. **Explain governing mechanism of Francis turbine.**

AKTU 2014-15, Marks 05

Answer

1. Governing of Francis turbines is usually done by altering the position of the guide vanes and thus controlling the flow rate by changing the gate openings to the runner.
2. The guide blades of a reaction turbine as shown in Fig. 4.24.1 are pivoted and connected by levers and links to the regulating ring.
3. Two long regulating rods connects the regulating ring and regulating lever.
4. The regulating lever is attached to a regulating shaft which is controlled by a servomotor piston of the oil pressure governor.
5. The penstock feeding the turbine inlet has a relief valve.
6. When the guide vanes have to be closed, the relief valve opens and diverts the water to the tail race.
7. Thus the double regulation, which is the simultaneous operation of two elements, is accomplished by moving the guide vanes and relief valve in Francis turbine by the governor.

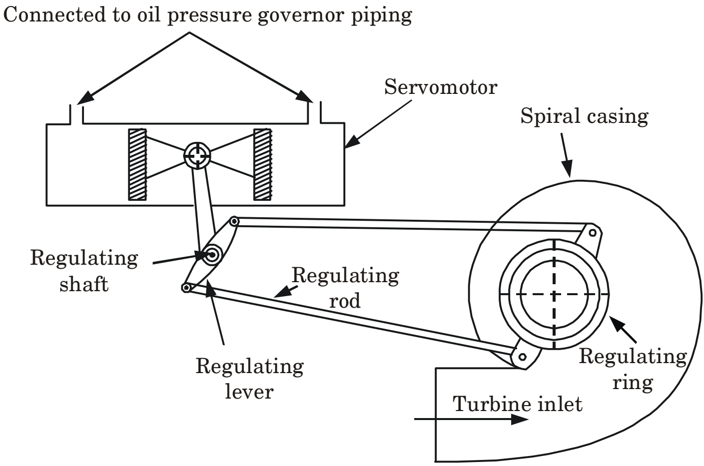


Fig. 4.24.1. Governing of Francis turbine.

Que 4.25. **Write a short note on Kaplan turbine.**

Answer

1. The Kaplan turbine is a propeller type water turbine which has adjustable blades that can be rotated about pivots fixed to the boss of the runner.
2. The blades are adjusted automatically by servo mechanism so that at all loads the flow enters them without shock. Thus, a high efficiency is

maintained even at part load. The servomotor cylinder is usually accommodated in the hub.

3. The Kaplan turbine has purely axial flow. It behaves like a propeller turbine at full load conditions.

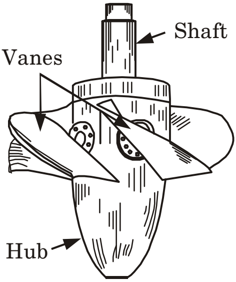


Fig. 4.25.1. Kaplan turbine runner.

4. Kaplan turbine, like every propeller turbine, is a high speed turbine and is used for smaller heads; as the speed is high, the number of runner vanes is small.

5. Discharge for Kaplan turbine is given as,

$$Q = \frac{\pi}{4}(D_o^2 - D_b^2)\, v_{f_1}$$

Where, D_o = Outer diameter of runner,

D_b = Diameter of hub, and

v_{f_1} = Velocity of flow at inlet.

Que 4.26. **Differentiate between Francis and Kaplan turbine.**

Answer

S. No.	Francis Turbine	Kaplan Turbine
1.	Radially inward or mixed flow turbine.	Partially axial flow turbine.
2.	Number of vanes varies from 16 to 24 blades.	Number of vanes varies from 3 to 8 blades.
3.	Horizontal or vertical position of shaft.	Only vertical position of shaft.
4.	Runner vanes are not adjustable.	Runner vanes are adjustable.
5.	Medium flow rate.	Large flow rate.

Que 4.27. **The hub diameter of a Kaplan turbine, working under a head of 12 m is 0.35 times the diameter of the runner. The turbine is running at 100 rpm. If the vane angle of the runner at the outlet is 15° and flow ratio is 0.6, find :**

i. Diameter of the runner,

ii. Diameter of the boss, and

iii. Discharge through the runner.

The whirl component of velocity at the outlet is zero.

Answer

Given : $H = 12$ m, $D_b = 0.35\ D_o$, $N = 100$ rpm, $\phi = 15°$, $v_{w2} = 0$

Flow ratio = 0.6

To Find :
i. Runner diameter.
ii. Boss diameter.
iii. Discharge through runner.

1. Flow ratio, $$0.6 = \frac{v_{f1}}{\sqrt{2gH}}$$

$$v_{f_1} = 0.6 \times \sqrt{2gH}$$
$$= 0.6 \times \sqrt{2 \times 9.81 \times 12}$$
$$= 9.2 \text{ m/s}$$

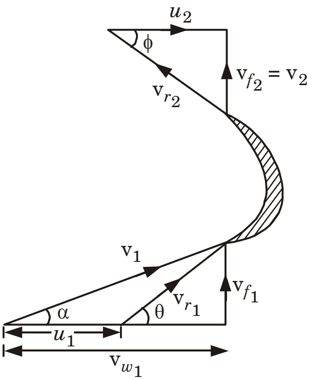


Fig. 4.27.1.

2. From outlet velocity triangle, $\tan \phi = \dfrac{v_{f2}}{u_2} = \dfrac{v_{f1}}{u_2}$

$$\tan 15° = \frac{9.2}{u_2}$$

$$u_2 = 34.33 \text{ m/s}$$

3. For Kaplan turbine, $u_1 = u_2 = 34.33$ m/s

4. Now,
$$u_1 = \frac{\pi D_o N}{60}$$
$$34.33 = \frac{\pi \times D_o \times 100}{60}$$
$$D_o = 6.55 \text{ m}$$
$\therefore$
$$D_b = 0.35 \times 6.55 = 2.3 \text{ m}$$

5. Discharge through runner,
$$Q = \frac{\pi}{4}[D_o^2 - D_b^2] v_{f1}$$
$$= \frac{\pi}{4}[6.55^2 - 2.3^2] \times 9.2$$
$$= 271.77 \text{ m}^3/\text{s}$$

PART-6

Principles of Similarity.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.28. What are the different types of similarities that exist between model and prototype ?

Answer

Following are the three types of similarities that exist between model and prototype :

i. Geometric Similarity :

1. The geometric similarity is said to exist between the model and the prototype if the ratio of all corresponding linear dimension in the model and prototype are equal.

ii. Kinematic Similarity :

1. It means the similarity of motion between model and prototype.
2. If at the corresponding points in the model and in the prototype, the velocity or acceleration ratios are same (both in magnitude and direction), the two flows are said to be kinematically similar.

iii. Dynamic Similarity :

1. Dynamic similarity means the similarity of force (both in magnitude and in direction) in model and in prototype.

2. Dynamic similarity is said to exist between the model and the prototype if the ratios of the corresponding forces acting at the corresponding points are equal.

PART-7

Unit and Specific Speed.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.29. **Define the term unit power, unit speed and unit discharge with reference to a hydraulic turbine.**

OR

Show that in a given turbine :

$$u \propto \sqrt{H}\ ,\ Q \propto \sqrt{H}\ \text{ and } P \propto H^{3/2}$$

where u is peripheral speed, Q is discharge rate, P is power and H is the available head.

Answer

A. Unit Speed :

1. It is defined as the speed of a turbine working under a unit head (*i.e.*, under a head of 1 m). It is denoted by N_u.
2. Let, N = Speed of turbine under a head H,

   H = Head under which a turbine is working, and

   u = Tangential velocity.
3. The tangential velocity, absolute velocity of water and head on the turbine are related as,

$$u \propto \text{v}$$

but, $$\text{v} \propto \sqrt{H}$$

$\therefore$ $$u \propto \sqrt{H} \quad \text{...(4.29.1)}$$

4. Tangential velocity is also given as,

$$u = \frac{\pi DN}{60} \quad [D = \text{Diameter of turbine}]$$

5. For a given turbine, diameter (D) is constant,

$\therefore$ $$u \propto N$$
$$N \propto u$$

So, $$N \propto \sqrt{H} \quad \text{[From eq. (4.29.1)]}$$

$$N = K_1 \sqrt{H} \quad \text{...(4.29.2)}$$

Where, K_1 = Constant of proportionality.

6. If head on turbine is unity, speed becomes unit speed.

$$N_u = K_1 \sqrt{1}$$

$$K_1 = N_u$$

7. Putting value of K_1 in eq. (4.29.2), we get

$$N = N_u \sqrt{H}$$

$$N_u = \frac{N}{\sqrt{H}}$$

B. Unit Discharge :

1. It is defined as the discharge passing through a turbine, which is working under a unit head. It is denoted by Q_u.
2. Let, Q = Discharge passing through turbine when head is H, and

   a = Area of flow of water.
3. Discharge passing through a given turbine under a head H is given by,

$$Q = \text{Area of flow} \times \text{Velocity} \quad ...(4.29.3)$$

4. But for a turbine, area of flow is constant and given as

$$\text{Velocity} \propto \sqrt{H} \quad ...(4.29.4)$$

5. So from eq. (4.29.3) and eq. (4.29.4)

$$Q \propto \sqrt{H}$$

$$Q = K_2 \sqrt{H} \quad ...(4.29.5)$$

Where, K_2 = Constant of proportionality.

6. For unit head, $H = 1$, $Q = Q_u$

So, $Q_u = K_2 \sqrt{1}$

$$Q_u = K_2$$

7. Putting the value of K_2 in eq. (4.29.5), we get

$$Q = Q_u \sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$

C. Unit Power :

1. It is defined as the power developed by a turbine, working under a unit head. It is represented by symbol P_u.
2. Let, P = Power developed by the turbine under head H.
3. Overall efficiency is given as,

$$\eta_o = \frac{\text{Power developed}}{\text{Water power}} = \frac{P}{\dfrac{\rho g Q H}{1000}} \quad ...(4.29.6)$$

$$P \propto QH$$

$$\propto \sqrt{H} \times H \qquad (\because \quad Q \propto \sqrt{H})$$

$$\propto H^{3/2}$$

$$P = K_3 H^{3/2} \qquad ...(4.29.7)$$

Where, K_3 = Constant of proportionality.

4. For unit head, $H = 1, P = P_u$

So, $P_u = K_3 (1)^{3/2}$

$$P_u = K_3$$

5. Putting the value of K_3 in eq. (4.29.7), we get

$$P = P_u H^{3/2}$$

$$P_u = \frac{P}{H^{3/2}}$$

Que 4.30. **Define specific speed of a turbine and derive an expression for the same.**

OR

Deduce an expression for the specific speed of a hydraulic turbine and explain how it is useful in practice. **AKTU 2014-15, Marks 05**

Answer

A. Specific Speed :

1. It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles, gate openings, etc., with the actual turbine but of such a size that it will develop unit power when working under unit head. It is denoted by symbol N_s.

B. Derivation of the Specific Speed :

1. Let, D = Diameter of actual turbine,

   N = Speed of actual turbine,

   u = Tangential velocity of turbine,

   N_s = Specific speed of the turbine, and

   v = Absolute velocity of water.

2. The overall efficiency of any turbine is given as,

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{P}{\dfrac{\rho g Q H}{1000}} \qquad ...(4.30.1)$$

$$P = \eta_o \frac{\rho g Q H}{1000}$$

$$P \propto Q\,H \text{ [as } \eta_o \text{ and } \rho, g \text{ are constants]} \qquad ...(4.30.2)$$

3. We know that, $u \propto \sqrt{H}$ ...(4.30.3)

4. Tangential velocity is given as,

$$u = \frac{\pi DN}{60}$$

$$u \propto DN \quad \text{...(4.30.4)}$$

5. From eq. (4.30.3) and eq. (4.30.4),

$$\sqrt{H} \propto DN$$

$$D \propto \frac{\sqrt{H}}{N} \quad \text{...(4.30.5)}$$

6. Discharge through turbine is given by

$$Q = \text{Area} \times \text{Velocity}$$

$\because$ $\text{Area} \propto BD$ [Where, B = Width, $B \propto D$]

$$\propto D^2$$

$\because$ $\text{Velocity} \propto \sqrt{H}$

$\therefore$ $Q \propto D^2 \sqrt{H} \propto \left(\frac{\sqrt{H}}{N}\right)^2 \sqrt{H}$ [From eq. (4.30.5)]

$$Q \propto \frac{H^{3/2}}{N^2} \quad \text{...(4.30.6)}$$

7. Substitute the value of eq. (4.30.6) in eq. (4.30.2),

$$P \propto \frac{H^{3/2}}{N^2} H \propto \frac{H^{5/2}}{N^2}$$

$$P = K \frac{H^{5/2}}{N^2} \quad \text{...(4.30.7)}$$

Where, K = Constant of proportionality

8. If $P = 1$, $H = 1$, the speed $N = N_s$,

So, $1 = K \frac{(1)^{5/2}}{N_s^2}$

$$K = N_s^2$$

9. Putting the value of K in eq. (4.30.7), we get

$$P = N_s^2 \frac{H^{5/2}}{N^2}$$

$$N_s^2 = \frac{N^2 P}{H^{5/2}}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

C. Usefulness or Significance of Specific Speed :

1. Specific speed plays an important role for selecting the turbine.
2. The performance of a turbine can be predicted by knowing the specific speed of the turbine.

Que 4.31. **A Kaplan turbine has the following specification Rated discharge = 260 m³/sec, head = 10 m, speed = 80 rpm, runner hub diameter = 2.3 m, runner vane tip diameter = 6.7 m, power produced = 18000 kW, hydraulic efficiency = 85 %. Find the flow ratio, overall efficiency, specific speed and the degree of reaction.**

AKTU 2014-15, Marks 05

Answer

Given : $Q = 260\ \text{m}^3/\text{s}$, $H = 10$ m, $N = 80$ rpm, $D_b = 2.3$ m, $D_0 = 6.7$ m, $P = 18000$ kW, $\eta_h = 85\ \% = 0.85$

To Find :
i. Flow ratio.
ii. Overall efficiency.
iii. Specific speed.
iv. Degree of reaction.

1. Discharge,

$$Q = \frac{\pi}{4}(D_0^2 - D_b^2)\mathrm{v}_{f1}$$

$$260 = \frac{\pi}{4}[(6.7)^2 - (2.3)^2]\mathrm{v}_{f1}$$

$$\mathrm{v}_{f1} = \frac{260 \times 4}{\pi \times 39.6}$$

$$\mathrm{v}_{f1} = 8.36\ \text{m/s}$$

2. We know that,

$$\text{Flow ratio} = \frac{\mathrm{v}_{f1}}{\sqrt{2gH}} = \frac{8.36}{\sqrt{2 \times 9.81 \times 10}}$$

$$\text{Flow ratio} = 0.597$$

3. Overall efficiency,

$$\eta_0 = \frac{P \times 1000}{\rho g Q H} = \frac{18000 \times 1000}{1000 \times 9.81 \times 260 \times 10}$$

$$\eta_0 = 0.7057 \text{ or } 70.57\ \%$$

4. Specific speed,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{80\sqrt{18000}}{(10)^{5/4}} = \frac{10733.126}{17.783}$$

$$N_s = 603.56\ \text{rpm}$$

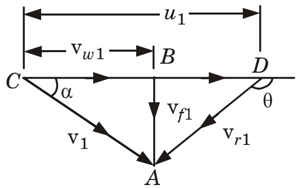


Fig. 4.31.1. Inlet velocity triangle.

5. For Kaplan turbine,

$$u_1 = u_2 = \frac{\pi D_0 N}{60} = \frac{\pi \times 6.7 \times 80}{60} = 28.06 \text{ m/s}$$

6. Hydraulic efficiency,

$$\eta_h = \frac{v_{w1} u_1}{gH}$$

$$v_{w1} = \frac{\eta_h g H}{u_1} = \frac{0.85 \times 9.81 \times 10}{28.06}$$

$$v_{w1} = 2.97 \text{ m/s}$$

7. From ΔABC, $\tan \alpha = \dfrac{v_{f1}}{v_{w1}} = \dfrac{8.36}{2.97}$

$$\alpha = 70.44°$$

8. Also from ΔABD,

$$\tan (180° - \theta) = \frac{v_{f1}}{u_1 - v_{w1}} = \frac{8.36}{28.06 - 2.97}$$

So, $\theta = 161.57°$

9. Degree of reaction,

$$R = 1 - \frac{\cot \alpha}{2(\cot \alpha - \cot \theta)}$$

$$= 1 - \frac{\cot 70.44°}{2\,(\cot 70.44° - \cot 161.57°)} = 0.947$$

Que 4.32. **A Kaplan turbine runner is to be designed to develop 9100 kW. The net available head is 5.6 m. If the speed ratio = 2.09, flow ratio = 0.68, overall efficiency = 86 % and the diameter of the boss is 1/3 the diameter of the runner. Find the diameter of the runner, its speed and the specific speed of the turbine.**

AKTU 2015-16, Marks 10

Answer

Given : $P = 9100$ kW, $H = 5.6$ m, Speed ratio = 2.09,
Flow ratio = 0.68, $\eta_o = 86\ \% = 0.86$, $D_b = (1/3)\, D_0$

To Find :
i. Runner diameter.
ii. Speed of turbine.
iii. Specific speed of turbine.

1.
$$\text{Speed ratio} = \frac{u_1}{\sqrt{2gH}}$$
$$u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.6} = 21.90 \text{ m/s}$$

2.
$$\text{Flow ratio} = \frac{\mathrm{v}_{f_1}}{\sqrt{2gH}}$$
$$\mathrm{v}_{f_1} = 0.68 \times \sqrt{2 \times 9.81 \times 5.6} = 7.13 \text{ m/s}$$

3. Overall efficiency,
$$\eta_o = \frac{P \times 1000}{\rho g Q H}$$
$$Q = \frac{P \times 1000}{\rho g H \eta_o} = \frac{9100 \times 1000}{1000 \times 9.81 \times 5.6 \times 0.86}$$
$$= 192.61 \text{ m}^3/\text{s}$$

4. The discharge through a Kaplan turbine is given by
$$Q = \frac{\pi}{4}[D_o^2 - D_b^2]\mathrm{v}_{f_1}$$
$$192.61 = \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.13 \qquad \left(\because\ D_b = \frac{D_o}{3}\right)$$
$$= \frac{\pi}{4}\left[1 - \frac{1}{9}\right] D_o^2 \times 7.13$$
$$D_o = \sqrt{\frac{4 \times 192.61 \times 9}{\pi \times 8 \times 7.13}} = 6.22 \text{ m}$$

5. The speed of turbine is given by, $u_1 = \dfrac{\pi D_o N}{60}$
$$\therefore \qquad N = \frac{60 u_1}{\pi D_o} = \frac{60 \times 21.90}{\pi \times 6.22} = 67.24 \text{ rpm}$$

6. Specific speed,
$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{67.24\sqrt{9100}}{(5.6)^{5/4}} = 745 \text{ rpm}$$

Que 4.33. **A Kaplan turbine develops 9000 kW under a net head of 7.5 m. Overall efficiency of the turbine is 86 %. The speed ratio based on the outer diameter is 2.2 and the flow ratio is 0.66. Diameter of the**

boss is 0.35 times the external diameter of the wheel. Determine the diameter of runner and the specific speed of the runner.

AKTU 2017-18, Marks 10

Answer

Same as Q. 4.32, Page 4–45A, Unit-4 .
(**Answer :** Diameter of runner = 5.08 m and specific speed = 764 rpm)

Que 4.34. **A model turbine, diameter of runner 380 mm develops 9 kW at a speed of 1500 rpm under a head of 7.6 m. A geometrically similar turbine 1.9 m runner diameter has to operate with same efficiency under a head of 15 m. What speed and power would be expected ?**

AKTU 2014-15, Marks 05

Answer

Given : D_m = 380 mm = 0.38 m, P_m = 9 kW, N_m = 1500 rpm, H_m = 7.6 m, D_p = 1.9 m, H_p = 15 m
To Find : Speed and power of turbine.

1. Using relation,

$$\left(\frac{H}{N^2D^2}\right)_m = \left(\frac{H}{N^2D^2}\right)_p \Rightarrow \frac{H_m}{N_m^{\ 2}D_m^{\ 2}} = \frac{H_p}{N_p^{\ 2}D_p^{\ 2}}$$

$$N_p^{\ 2} = N_m^{\ 2}\frac{D_m^{\ 2}}{D_p^{\ 2}}\frac{H_p}{H_m}$$

∴ Speed of second turbine,

$$N_p = N_m\frac{D_m}{D_p}\left(\frac{H_p}{H_m}\right)^{1/2}$$

$$= 1500 \times \left(\frac{0.38}{1.9}\right) \times \left(\frac{15}{7.6}\right)^{1/2} = 421.46 \text{ rpm}$$

2. Using relation,

$$\left(\frac{P}{N^3D^5}\right)_m = \left(\frac{P}{N^3D^5}\right)_p \Rightarrow \frac{P_m}{N_m^{\ 3}D_m^{\ 5}} = \frac{P_p}{N_p^{\ 3}D_p^{\ 5}}$$

∴ Power produced by turbine,

$$P_p = P_m\left(\frac{D_p}{D_m}\right)^5\left(\frac{N_p}{N_m}\right)^3$$

$$= 9 \times \left(\frac{1.9}{0.38}\right)^5\left(\frac{421.46}{1500}\right)^3 = 623.86 \text{ kW}$$

Que 4.35. **A reaction turbine is revolving at a speed of 200 rpm and develops 5886 kW SP when working under a head of 200 m with an overall efficiency of 80 %. Determine unit speed, unit discharge and unit power. The speed ratio for the turbine is given as 0.48. Find the speed, discharge and power when this turbine is working under a head of 150 m.**

AKTU 2017-18, Marks 10

Answer

Given : N_1 = 200 rpm, P_1 = 5886 kW, H_1 = 200 m, η_0 = 80 % = 0.8, H_2 = 150 m, ϕ = Speed ratio = 0.48

To Find :
i. Unit speed.
ii. Unit discharge.
iii. Unit power.
iv. Speed, discharge, power when turbine is working under a head of 150 m.

1. Unit speed,
$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{200}{\sqrt{200}} = 14.142 \text{ rpm}$$

2. Overall efficiency is given by,
$$\eta_0 = \frac{P_1 \times 1000}{\rho g Q_1 H_1}$$
$$0.8 = \frac{5886 \times 1000}{1000 \times 9.81 \times Q_1 \times 200}$$
$$0.8 = \frac{3}{Q_1} \quad \text{or} \quad Q_1 = 3.75 \text{ m}^3/\text{s}$$

3. Unit discharge,
$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{3.75}{\sqrt{200}} = 0.265 \text{ m}^3/\text{s}$$

4. Unit power,
$$P_u = \frac{P_1}{H_1^{3/2}} = \frac{5886}{(200)^{3/2}} = 2.08 \text{ kW}$$

5. When head = 150 m,

i. For speed, $\dfrac{N_1}{\sqrt{H_1}} = \dfrac{N_2}{\sqrt{H_2}}$
$$N_2 = \frac{N_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{200 \times \sqrt{150}}{\sqrt{200}} = 173.2 \text{ rpm}$$

ii. For discharge, $\dfrac{Q_1}{\sqrt{H_1}} = \dfrac{Q_2}{\sqrt{H_2}}$
$$Q_2 = \frac{Q_1\sqrt{H_2}}{\sqrt{H_1}} = \frac{3.75 \times \sqrt{150}}{\sqrt{200}} = 3.25 \text{ m}^3/\text{s}$$

iii. For power, $\dfrac{P_1}{H_1^{3/2}} = \dfrac{P_2}{H_2^{3/2}}$

$$P_2 = \frac{P_1 H_2^{3/2}}{H_1^{3/2}} = \frac{5886 \times (150)^{3/2}}{(200)^{3/2}} = 3823 \text{ kW}$$

PART-8

Performance Characteristics.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.36. Discuss performance characteristics of a hydraulic turbine.

AKTU 2016-17, Marks 10

OR

Discuss the various characteristics curves of hydraulic turbines in details.

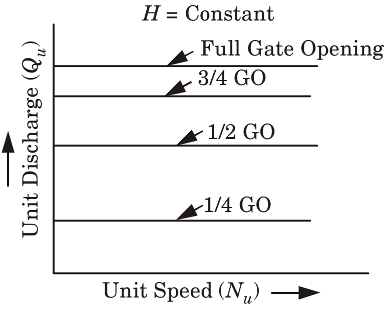
AKTU 2017-18, Marks 10

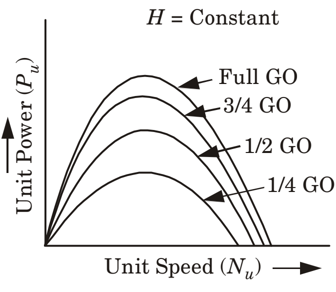
Answer

1. Characteristic curves are defined as the curves, with the help of which the exact behaviours and performance of the turbine under different working conditions can be known.
2. Following are the important characteristic curves for the turbine :

i. Main Characteristic Curves or Constant Head Curves :

1. These are obtained by maintaining a constant head and a constant gate opening (GO) on the turbine.
2. Main characteristics curve for Pelton wheel and various reaction turbines are shown in Fig. 4.36.1 and Fig. 4.36.2 respectively.





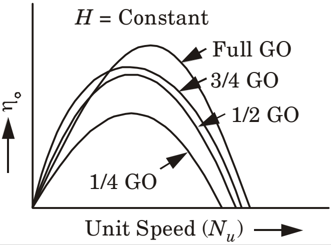
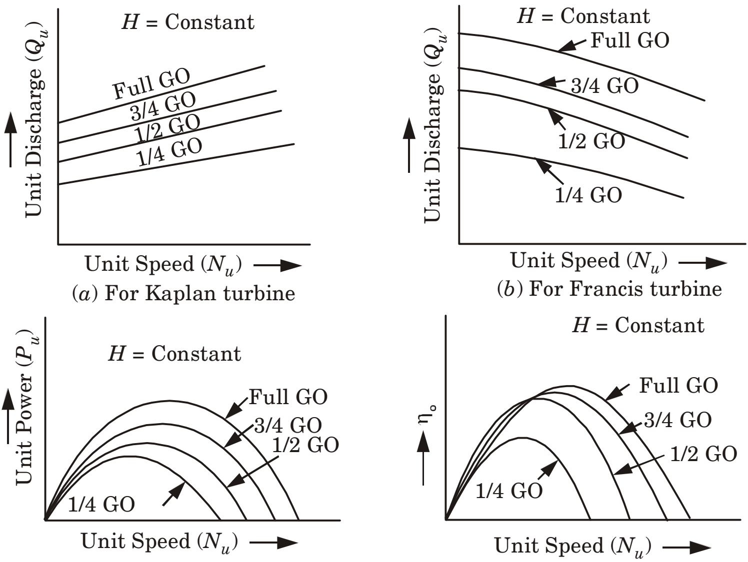


Fig. 4.36.1. Main characteristic curves for a Pelton wheel.



(*a*) For Kaplan turbine

(*b*) For Francis turbine

(*c*)

(*d*)

Fig. 4.36.2. Main characteristic curves for reaction turbine.

ii. Operating Characteristic Curves or Constant Speed Curves :

1. These curves are plotted when the speed on the turbine is constant.
2. For operating characteristics, N and H are constant and hence the variation of power and efficiency with respect to discharge Q are plotted. These variations are shown in Fig. 4.36.3.

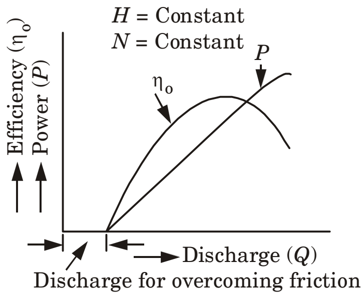


Fig. 4.36.3. Operating characteristic curves.

iii. Constant Efficiency Curves or Muschel Curves or Iso-Efficiency Curves :

1. These curves are obtained from the speed v/s efficiency and speed v/s discharge curves for different gate openings.
2. A constant efficiency curve is shown in Fig. 4.36.4.

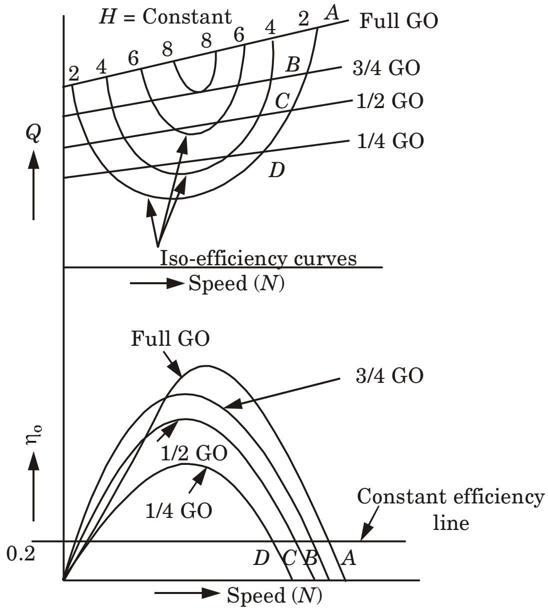


Fig. 4.36.4. Constant efficiency curves.

PART-9

Selection of Water Turbines.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.37. What points should be considered while selection of hydraulic turbines ?

Answer

The following points should be considered while selection of hydraulic turbines :

a. Specific Speed :

1. High specific speed is essential where head is low and output is large, otherwise the rotational speed will be low which may leads to high cost of turbo generator and power house.

b. Rotational Speed :

1. Rotational speed depends on specific speed.
2. Also the rotational speed of an electrical generator with which the turbine is to be directly coupled, depends on the frequency and number of pair of poles.

c. Efficiency :

1. The turbine selected should be such that it gives the highest overall efficiency for various operating conditions.

d. Partload Operation :

1. In general, the efficiency at partloads and overloads is less than normal. For the sake of economy the turbine should always run with maximum possible efficiency to get more revenue.

e. Cavitation :

1. The installation of water turbines of reaction type over the tail race is affected by cavitation.
2. The critical value of cavitation factor must be obtained to see that the turbine works in safe zone. Such a value of cavitation factor also affects the design of turbine, especially of Kaplan, propeller and bulb types.

f. Disposition of Turbine Shaft :

1. Vertical shaft arrangement is better for large sized reaction turbines.
2. In case of large size impulse turbines, horizontal shaft arrangement is mostly employed.

g. Head :

i. **Very High Heads :** For heads greater than 350 m, Pelton turbine is generally employed.

ii. **High Heads :** In this range (150 m to 350 m), either Pelton or Francis turbine may be employed.

iii. **Medium Heads :** A Francis turbine is usually employed in this range (60 m to 150 m).

iv. **Low Heads :** Between 30 m and 60 m heads, both Francis and Kaplan turbine may be used.
v. **Very Low Heads :** For very low heads (2 m to 15 m), bulb turbines are employed.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Derive the formula for dynamic force exerted by fluid jet on moving plate for the following cases :
i. When plate is normal to jet.
ii. Flat plate inclined to jet.
iii. When plate is curved and jet impinges at the center of plate.
iv. When plate is curved and jet impinges at one end.

Ans. Refer Q. 4.2, Unit-4.

Q. 2. A jet of water of diameter 50 mm having a velocity of 20 m/s strikes a curved vane which is moving with a velocity of 10 m/s in the direction of jet. The jet leaves the vane at an angle of 60 degree to the direction of motion of vane at outlet. Determine
i. Force exerted by the jet on the vane in the direction of motion.
ii. Work done per second by the jet.

Ans. Refer Q. 4.4, Unit-4.

Q. 3. Discuss the classification of hydraulic turbines.

Ans. Refer Q. 4.8, Unit-4.

Q. 4. A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at a rate of 700 lit/s under a head of 30 m. The bucket deflects the jet through an angle of 160 degree. Calculate power and hydraulic efficiency.

Ans. Refer Q. 4.15, Unit-4.

Q. 5. Explain the governing of a Pelton turbine. Use neat sketch.

Ans. Refer Q. 4.18, Unit-4.

Q. 6. Define specific speed of a turbine and derive an expression for the same.

Ans. Refer Q. 4.30, Unit-4.

Q. 7. A Kaplan turbine runner is to be designed to develop 9100 kW. The net available head is 5.6 m. If the speed ratio = 2.09, flow ratio = 0.68, overall efficiency = 86 % and the diameter of the boss is 1/3 the diameter of the runner. Find the diameter of the runner, its speed and the specific speed of the turbine.

Ans. Refer Q. 4.32, Unit-4.

Q. 8. Discuss performance characteristics of a hydraulic turbine.

Ans. Refer Q. 4.36, Unit-4.

☺☺☺

5 UNIT

Centrifugal and Reciprocating Pumps

CONTENTS

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Work Done by Impeller
Efficiencies of Centrifugal Pumps

Part-3 : Specific Speed .... **5–16A to 5–18A**

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Part-8 : Indicator Diagram .... **5–31A to 5–33A**

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Part-10 : Air Vessels .... **5–44A to 5–47A**

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Part-12 : Performance Characteristics .... **5–48A to 5–49A**

PART-1

Classifications of Centrifugal Pumps.

CONCEPT OUTLINE

Pumps : The hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **Define a centrifugal pump. Give its classification.**

Answer

A. Centrifugal Pump :

1. If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, then the hydraulic machine is called centrifugal pump.
2. It is a radial outward flow machine. It acts as the reverse of an inward radial flow reaction turbine.
3. It works on the principle of forced vortex flow.

B. Classification of Centrifugal Pump : On the basis of characteristic features, the centrifugal pumps are classified as follows :

i. Type of Casing :
1. Volute pumps.
2. Turbine pump or diffusion pump.

ii. Working Head :
1. Low lift centrifugal pumps.
2. Medium lift centrifugal pumps.
3. High lift centrifugal pumps.

iii. Liquid Handled :
1. Closed impeller pump.
2. Semi-open impeller pump.
3. Open impeller pump.

iv. Number of Impellers per Shaft :
1. Single stage centrifugal pump.
2. Multi-stage centrifugal pump.

v. Number of Entrances to the Impeller :
1. Single entry or single suction pump.
2. Double entry or double suction pump.

vi. Relative Direction of Flow through Impeller :

1. Radial flow pump.
2. Axial flow pump.
3. Mixed flow pump.

PART-2

Vector Diagram, Work Done by Impeller, Efficiencies of Centrifugal Pumps.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.2. **Give the constructional details of a centrifugal pump. Also explain its working.**

Answer

A. Construction : Main parts of a centrifugal pump are :

a. Impeller :

1. An impeller is a wheel (or rotor) with a series of backward curved vanes (or blades).
2. It is mounted on a shaft which is coupled to an electric motor.
3. The impellers are of following three types :
   - i. Shrouded or closed impeller.
   - ii. Semi-open impeller.
   - iii. Open impeller.

b. Casing :

1. The casing is an air tight chamber surrounding the pump impeller.
2. The following three types of casing are commonly employed :

   i. Volute Casing :

   1. In this type of casing the area of flow gradually increases from the impeller outlet to the delivery pipe so as to reduce the velocity of flow.
   2. Thus the increase in pressure occurs in volute casing.

   ii. Vortex Casing :

   1. If a circular chamber is provided between the impeller and the volute chamber, the casing is known as vortex casing.
   2. The circular chamber is known as vortex or whirlpool chamber and such a pump is known as volute pump with vortex chamber.

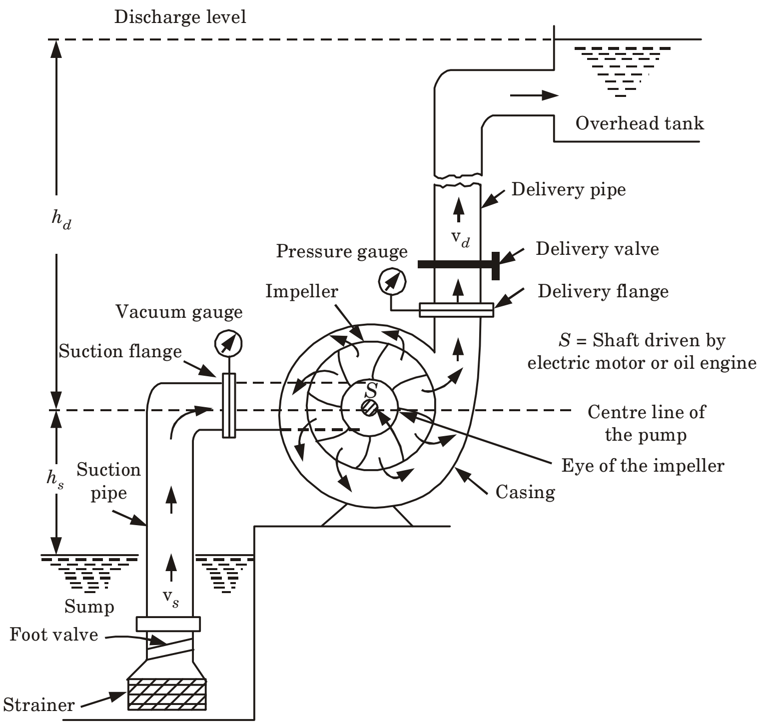

Fig. 5.2.1. Main parts of a centrifugal pump.

3. The vortex chamber converts some of the kinetic energy into the pressure energy.

iii. Casing with Guide Blades :

1. In this type of casing, impeller is surrounded by a series of guide blades (or vanes) mounted on a ring which is known as a diffuser.
2. Machines with diffuser blades have maximum efficiency but are less satisfactory when a wide range of operating conditions is required.

c. Suction Pipe :

1. The pipe which connects the centre/eye of the impeller to sump from which liquid is to be lifted is known as suction pipe.
2. To prevent the entry of solid particles, debris etc., the suction pipe is provided with a strainer at its lower end.

d. Delivery Pipe :

1. The pipe which is connected at its lower end to the outlet of the pump and delivers the liquid to the required height is known as delivery pipe.

2. A delivery valve is provided on the delivery pipe to regulate the supply of water.

B. Working of Centrifugal Pump :

1. The first step in the operation of a centrifugal pump is priming.
2. Priming is the operation in which the suction pipe, casing of the pump and the portion of the delivery pipe upto the delivery valve are completely filled with the liquid which is to be pumped, so that no air pocket is left.
3. After the pump is primed, the delivery valve is kept closed and the electric motor is started to rotate the impeller.
4. The rotation of the impeller in the casing full of liquid produces a forced vortex which imparts a centrifugal head to the liquid and thus results in an increase of pressure throughout the liquid mass.
5. Now as long the delivery valve is closed and the impeller is rotating, it just churns the liquid in the casing.
6. When the delivery valve is opened the liquid is made to flow in an outward radial direction thereby leaving the vanes of the impeller at the outer circumference with high velocity and pressure.

Que 5.3. **Define the terms :**

a. Suction head,
b. Delivery head,
c. Static head, and
d. Manometric head.

Answer

a. **Suction Head (h_s) :** It is the vertical height of the centre line of the centrifugal pump above the water surface in sump from which water is to be lifted.

b. **Delivery Head (h_d) :** It is the vertical height between the centre line of the pump and the water surface in the tank to which water is delivered.

c. **Static Head (H_s) :** It is total vertical height through which water has to be lifted. It is given as,

$$H_s = h_s + h_d$$

d. **Manometric Head (H_m) :** It is defined as the head against which a centrifugal pump has to work.

Que 5.4. **Obtain an expression for the work done by an impeller of a centrifugal pump on water.**

Answer

1. The absolute velocity of water at inlet makes an angle 90° with the direction of motion of the impeller at inlet. Hence angle α = 90° and $v_{w1} = 0$.
2. Fig. 5.4.1 shows the velocity triangles at the inlet and outlet.

3. Let, N = Speed of impeller,

D_1 and D_2 = Diameter of impeller at inlet and outlet,

u_1 and u_2 = Tangential velocity of impeller at inlet and outlet,

v_1 and v_2 = Absolute velocity at inlet and outlet,

v_{r1} and v_{r_2} = Relative velocity at inlet and outlet,

v_{w_1} and v_{w_2} = Whirl velocity at inlet and outlet, and

v_{f_1} and v_{f_2} = Flow velocity at inlet and outlet.

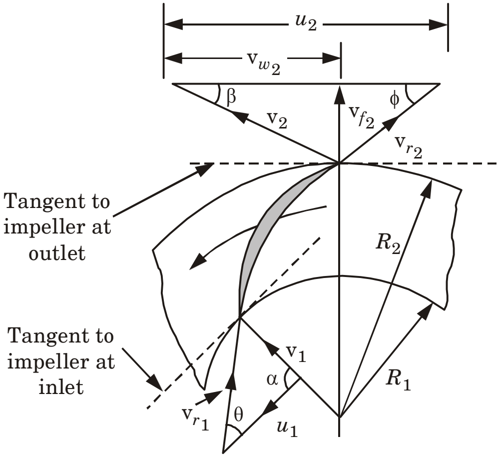

Fig. 5.4.1. Velocity triangles at inlet and outlet.

4. A centrifugal pump is reverse of a radially inward flow reaction turbine. So, work done by the impeller on the unit weight of water striking per second

$$= -\text{ [Work done in case of turbine]}$$

$$= -\left[\frac{1}{g}(v_{w_1} u_1 - v_{w_2} u_2)\right] = \frac{1}{g}[(v_{w_2} u_2 - v_{w_1} u_1)]$$

$$= \frac{v_{w_2} u_2}{g} \qquad (\because \; v_{w_1} = 0)$$

5. Work done by impeller on water per second

$$= \frac{W}{g} v_{w_2} u_2$$

Where, W = Weight of water = $\rho g Q$

Q = Volume of water

Que 5.5. **What do you mean by manometric efficiency, mechanical efficiency and overall efficiency of centrifugal pump ?**

Answer

a. Manometric Efficiency (η_{mano}) : It is defined as the ratio of the manometric head developed by the pump to the head imparted by the impeller to the liquid.

$$\eta_{mano} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to liquid}}$$

$$= \frac{H_m}{\left(\dfrac{v_{w_2} u_2}{g}\right)} = \frac{gH_m}{v_{w2}\, u_2}$$

b. Mechanical Efficiency (η_m) :

1. It is defined as the ratio of the power delivered by the impeller to the power input to the pump shaft.

$$\eta_m = \frac{\text{Power delivered at impeller}}{\text{Power input to the shaft}}$$

2. Power delivered at impeller in kW

$$= \frac{\text{Work done by impeller per second}}{1000}$$

$$= \frac{W}{g} \times \frac{v_{w_2} u_2}{1000}$$

$$\therefore \qquad \eta_m = \frac{\dfrac{W}{g}\left(\dfrac{v_{w_2} u_2}{1000}\right)}{\text{SP}}$$

c. Overall Efficiency (η_o) :

1. The overall efficiency of the pump is defined as the ratio of the power output from the pump to the power input from the prime mover driving the pump.

$$\eta_o = \frac{\text{Power output}}{\text{Power input}}$$

2. $$\text{Power output} = \frac{\text{Weight of water lifted} \times H_m}{1000}$$

$$\text{Power input} = \text{Shaft power}$$

$$\therefore \qquad \eta_o = \frac{WH_m / 1000}{\text{SP}}$$

or $$\eta_o = \eta_{mano} \times \eta_m$$

Que 5.6. **Show that pressure rise in the impeller of a centrifugal pump is given by :**

$$\frac{1}{2g}[v_{f1}^2 + u_2^2 - v_{f2}^2 \operatorname{cosec}^2 \phi]$$

Neglect all frictional losses and assume that the blades of the impeller are curved back through angle ϕ at outlet. Notations used have usual meaning.

Answer

1. Apply Bernoulli's equation at inlet and outlet of the impeller and neglecting losses from inlet to outlet.

Total energy at inlet = Total energy at outlet – Work done by impeller on water

$$\left(\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1\right)=\left(\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2\right)-\frac{v_{w2}\,u_2}{g} \qquad ...(5.6.1)$$

2. If inlet and outlet of the impeller at the same height

$$\left(\frac{p_1}{\rho g}+\frac{v_1^2}{2g}\right)=\left(\frac{p_2}{\rho g}+\frac{v_2^2}{2g}\right)-\frac{v_{w2}\,u_2}{g} \qquad (\because z_1=z_2)$$

$$\left(\frac{p_2}{\rho g}-\frac{p_1}{\rho g}\right)=\frac{v_1^2}{2g}-\frac{v_2^2}{2g}+\frac{v_{w2}\,u_2}{g} \qquad ...(5.6.2)$$

Where, $\left(\frac{p_2}{\rho g}-\frac{p_1}{\rho g}\right)$ = Pressure rise in impeller

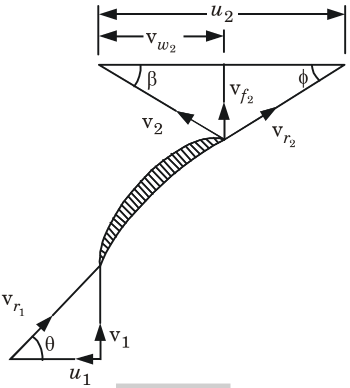

Fig. 5.6.1.

3. From inlet velocity triangle,

$$v_1 = v_{f1} \qquad ...(5.6.3)$$

4. From outlet velocity triangle,

$$\tan\phi=\frac{v_{f2}}{u_2-v_{w2}}$$

$$u_2-v_{w_2}=\frac{v_{f2}}{\tan\phi}$$

$$v_{w_2} = u_2 - \frac{v_{f2}}{\tan\phi} = u_2 - v_{f_2}\cot\phi \quad \text{...(5.6.4)}$$

5. Again from outlet velocity triangle,

$$V_2^2 = v_{f_2}^{\ 2} + v_{w_2}^{\ 2} = v_{f_2}^{\ 2} + (u_2 - v_{f_2}\cot\phi)^2$$
$$= v_{f_2}^{\ 2} + u_2^2 + v_{f_2}^{\ 2}\cot^2\phi - 2u_2 v_{f_2}\cot\phi$$
$$= v_{f_2}^{\ 2}(1+\cot^2\phi) + u_2^2 - 2u_2 v_{f_2}\cot\phi$$
$$= v_{f_2}^{\ 2}\operatorname{cosec}^2\phi + u_2^2 - 2u_2 v_{f_2}\cot\phi \quad (\because\ 1+\cot^2\phi = \operatorname{cosec}^2\phi) \quad \text{...(5.6.5)}$$

6. Now put the values of v_1 from eq. (5.6.3), v_{w_2} from eq. (5.6.4) and v_2^2 from eq. (5.6.5) in eq. (5.6.2), we get
Pressure rise in impeller

$$= \frac{v_{f1}^2}{2g} - \frac{1}{2g}(v_{f2}^2\operatorname{cosec}^2\phi + u_2^2 - 2u_2 v_{f2}\cot\phi) + \frac{(u_2 - v_{f2}\cot\phi)u_2}{g}$$

$$= \frac{1}{2g}[v_{f1}^2 - v_{f2}^2\operatorname{cosec}^2\phi - u_2^2 + 2u_2 v_{f2}\cot\phi + 2u_2^2 - 2u_2 v_{f2}\cot\phi]$$

$$= \frac{1}{2g}[v_{f1}^2 + u_2^2 - v_{f2}^2\operatorname{cosec}^2\phi]$$

Que 5.7. A centrifugal pump having outer dia. equal to two times of inner dia. and running at 1000 rpm works against a total head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s, the vanes are set back at an angle 40° at outlet. If the outer dia. of the impeller is 500 mm and width at outlet is 50 mm. Determine :

i. Vane angle at inlet

ii. Work done by the impeller on water per sec

iii. Manometric efficiency

AKTU 2014-15, Marks 10

Answer

Given : N = 1000 rpm, H_m = 40 m, $v_{f1} = v_{f2}$ = 2.5 m/s, ϕ = 40°,
D_2 = 500 mm = 0.50 m, $D_1 = D_2/2$ = 0.50/2 = 0.25 m,
B_2 = 50 mm = 0.05 m

To Find : i. Vane angle at inlet.
ii. Work done by impeller on water per second.
iii. Manometric efficiency.

1. Tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

and

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18 \text{ m/s}$$

2. Discharge is given by,

$$Q = \pi D_2 B_2 \, v_{f2} = \pi \times 0.50 \times 0.05 \times 2.5 = 0.1963 \text{ m}^3/\text{s}$$

3. From inlet velocity triangle,

$$\tan \theta = \frac{v_{f1}}{u_1} = \frac{2.5}{13.09} = 0.191$$

$$\therefore \quad \theta = \tan^{-1} 0.191 = 10.81°$$

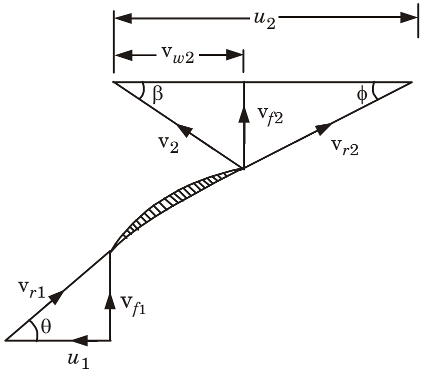

Fig. 5.7.1.

4. Work done by impeller on water per second is given by

$$= \frac{W}{g} v_{w2} u_2 = \frac{\rho g Q}{g} v_{w2} u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times v_{w2} \times 26.18 \quad \text{...(5.7.1)}$$

5. But from outlet velocity triangle, we have

$$\tan \phi = \frac{v_{f2}}{u_2 - v_{w2}} = \frac{2.5}{(26.18 - v_{w2})}$$

$$\therefore \quad 26.18 - v_{w2} = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 40°} = 2.979$$

$$\therefore \quad v_{w2} = 26.18 - 2.979 = 23.2 \text{ m/s}$$

6. Substituting this value of v_{w2} in eq. (5.7.1), we get the work done by impeller as

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$$= 119227.9 \text{ Nm/s}$$

7. Now, manometric efficiency,

$$\eta_{mano} = \frac{gH_m}{v_{w2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = 64.6 \%$$

Que 5.8. **A centrifugal pump runs at 950 rpm, its outer and inner diameters are 500 mm and 250 mm. The vanes are set back at 35° to the wheel rim. If the radial velocity of water through the impeller is constant at 4 m/s, find (a) The angle of vane at the inlet. (b) The velocity of water at exit. (c) The direction of water at the outlet. (d) The work done by the impeller per kg of water. Assume entry of water at inlet is radial.**

AKTU 2015-16, Marks 7.5

Answer

Given : N = 950 rpm, D_1 = 250 mm = 0.25 m, D_2 = 500 mm = 0.5 m, ϕ = 35°, $v_{f1} = v_{f2}$ = 4 m/s

To Find :
i. The angle of vane at inlet.
ii. Velocity of water at exit.
iii. Direction of water at outlet.
iv. Work done by the impeller per kg of water.

1. Tangential velocity of impeller at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 950}{60} = 12.43 \text{ m/s}$$

2. From velocity triangle at inlet, we have

$$\tan\theta = \frac{v_{f_1}}{u_1} \quad \text{or} \quad \tan\theta = \frac{4}{12.43} = 0.322$$

$$\theta = \tan^{-1} 0.322 = 17.85°$$

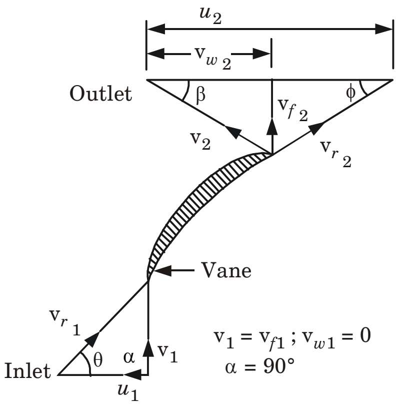

Fig. 5.8.1.

3. Velocity of water at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 950}{60} = 24.87 \text{ m/s}$$

4. From velocity triangle at outlet, we have

$$v_{w2} = u_2 - \frac{v_{f2}}{\tan \phi}$$

$$= 24.87 - \frac{4}{\tan 35°} = 19.157 \text{ m/s}$$

5. Now,
$$\tan \beta = \frac{v_{f2}}{v_{w2}} = \frac{4}{19.157} = 0.208$$
$$\beta = \tan^{-1} 0.208 = 11.75°$$

6. Work done by impeller per kg of water,
$$\text{Work done} = \frac{v_{w2} u_2}{g} = \frac{19.157 \times 24.87}{9.81} = 48.56 \text{ J/kg}$$

Que 5.9. **A centrifugal pump delivers 1.27 m³ of water per minute at 1200 rpm. The impeller diameter is 350 mm and breadth at outlet 12.7 mm. The pressure difference between inlet and outlet of pump casing is 272 kN/m². Assuming manometric efficiency as 63 %, calculate the impeller exit blade angle.** **AKTU 2016-17, Marks 10**

Answer

Given : $Q = 1.27$ m³/min, $N = 1200$ rpm, $D_2 = 350$ mm, $B_2 = 12.7$ mm, $\Delta p = p_o - p_i = 272$ kN/m², $\eta_{mano} = 63$ %
To Find : Impeller exit blade angle.

1. Tangential velocity of impeller at outlet,
$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.350 \times 1200}{60}$$
$$u_2 = 21.99 \text{ m/s}$$

2. We know that,
$$Q = \pi D_2 B_2 v_{f2}$$
$$\frac{1.27}{60} = \pi \times 0.350 \times 0.0127 \times v_{f2}$$
$$v_{f2} = 1.516 \text{ m/s}$$

3. Manometric head is given as
$$H_m = \frac{p_o - p_i}{\rho g} = \frac{272 \times 1000}{1000 \times 9.81} = 27.72 \text{ m}$$

4. Manometric efficiency,
$$\eta_{mano} = \frac{g H_m}{v_{w_2} u_2}$$
$$0.63 = \frac{9.81 \times 27.72}{v_{w_2} \times 21.99}$$
$$v_{w_2} = 19.63 \text{ m/s}$$

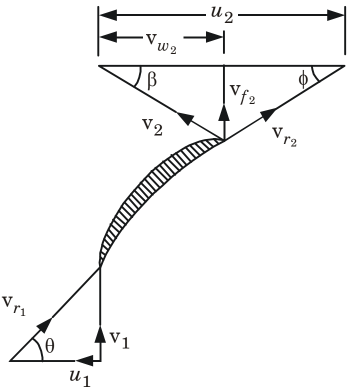

Fig. 5.9.1.

5. From outlet velocity triangle we have,

$$\tan\phi = \frac{v_{f_2}}{u_2 - v_{w_2}}$$

$$\tan\phi = \frac{1.516}{21.99 - 19.63}$$

$$\phi = \tan^{-1}\left(\frac{1.516}{21.99 - 19.63}\right)$$

$$\phi = 32.72°$$

Que 5.10. **A centrifugal pump running at 700 rpm is supplying 9 m³/min of water against a head of 19.6 m. The blade angle at the blade exit is 135° with the direction of motion of the blade tip. Assume the entry of water at the inlet of vane is radial. The velocity of flow at inlet and outlet is constant at 1.8 m/s. Determine the necessary diameter and width of the impeller at exit allowing 8 % for vanes thickness and 40 % of energy corresponding to the velocity at exit from the impeller is recovered.** **AKTU 2016-17, Marks 15**

Answer

Given : $Q = 9\ \text{m}^3/\text{min} = 0.15\ \text{m}^3/\text{s}$, $N = 700$ rpm, $H_m = 19.6$ m, $v_{f1} = v_{f2} = 1.8$ m/s, $\phi = 180° - 135° = 45°$

To Find : i. Diameter of impeller.
ii. Width of the impeller at exit.

1. We know that,

$$H_m = \frac{v_{w2}u_2}{g}$$

$$19.6 = \frac{v_{w2}u_2}{9.81}$$

$$v_{w2}u_2 = 192.27 \qquad ...(5.10.1)$$

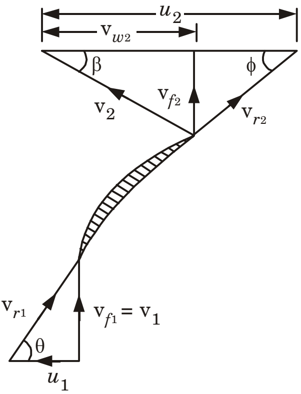

Fig. 5.10.1.

2. Now,
$$\tan \phi = \frac{v_{f2}}{u_2 - v_{w2}}$$
$$\tan 45° = \frac{1.8}{u_2 - v_{w2}}$$
$$u_2 - v_{w2} = 1.8 \qquad ...(5.10.2)$$

3. From eq. (5.10.1) and eq. (5.10.2), we have
$$u_2 - \frac{192.27}{u_2} = 1.8$$
$$u_2^2 - 1.8\, u_2 - 192.27 = 0$$
$$u_2 = 14.79 \text{ m/s}$$

4. We know that,
$$u_2 = \frac{\pi D_2 N}{60}$$
$$14.79 = \frac{\pi \times D_2 \times 700}{60}$$
$$D_2 = 0.40 \text{ m}$$

5. Discharge,
$$Q = \pi D_2 B_2 v_{f2}$$
$$0.15 = \pi \times 0.40 \times B_2 \times 1.8$$
$$B_2 = 0.066 \text{ m} = 6.6 \text{ cm}$$

Que 5.11. **Derive an expression for minimum starting speed of centrifugal pump.**

Answer

1. Head due to pressure rise in impeller $= \dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g}$

2. The flow of water will commence only if head due to pressure rise in impeller $\geq H_m$

$$\therefore \qquad \frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m$$

3. For minimum speed, we must have $\dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g} = H_m$ ...(5.11.1)

4. Manometric efficiency is given as,

$$\eta_{\text{mano}} = \frac{gH_m}{\mathrm{v}_{w_2} u_2}$$

$$\therefore \qquad H_m = \eta_{\text{mano}} \frac{\mathrm{v}_{w_2} u_2}{g}$$

5. Substituting this value of H_m in eq. (5.11.1), we have

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{\text{mano}} \frac{\mathrm{v}_{w_2} u_2}{g}$$

$$\frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{\text{mano}} \frac{\mathrm{v}_{w_2} \pi D_2 N}{60g} \qquad \text{...(5.11.2)}$$

$$\left(\because \; u_2 = \frac{\pi D_2 N}{60} \text{ and } u_1 = \frac{\pi D_1 N}{60}\right)$$

6. Dividing eq. (5.11.2) by $\dfrac{\pi N}{60g}$, we get

$$\frac{\pi N D_2^2}{120} - \frac{\pi N D_1^2}{120} = \eta_{\text{mano}} \mathrm{v}_{w_2} D_2$$

$$\text{or} \qquad \frac{\pi N}{120}[D_2^2 - D_1^2] = \eta_{\text{mano}} \mathrm{v}_{w_2} D_2$$

$$\therefore \qquad N = \frac{120 \eta_{\text{mano}} \mathrm{v}_{w_2} D_2}{\pi [D_2^2 - D_1^2]} \qquad \text{...(5.11.3)}$$

Eq. (5.11.3) gives the minimum starting speed of the centrifugal pump.

Que 5.12. A centrifugal pump with 1.2 m diameter runs at 200 rpm and discharges 1900 liters water per second, the average lift being 6 m. The angle which the vanes make at exit with the tangent to the impeller is 26° and the radial velocity of flow is 2.5 m/s. The inner diameter of the impeller is 0.6 m. Determine :
The power required to drive the pump, the manometric efficiency and the minimum rpm to start pumping against a head of 6 m.

AKTU 2017-18, Marks 10

Answer

Given : $D_2 = 1.2$ m, $N = 200$ rpm, $Q = 1900$ lit/s $= 1.9$ m^3/s,
$H_m = 6$ m, $\phi = 26°$, $v_{f_2} = 2.5$ m/s, $D_1 = 0.6$ m

To Find : i. Power required to drive the pump.
ii. Manometric efficiency.
iii. Minimum rpm to start pumping against a head of 6 m.

1. Power required to drive the pump,

$$P = \frac{\rho g Q H_m}{1000} = \frac{1000 \times 9.81 \times 1.9 \times 6}{1000} = 111.83 \text{ kW}$$

2. Tangential velocity at outlet is given as,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.56 \text{ m/s}$$

3. Also, we know that,

$$\tan \phi = \frac{v_{f_2}}{u_2 - v_{w_2}}$$

$$\tan 26° = \frac{2.5}{12.56 - v_{w_2}}$$

$$v_{w_2} = 7.43 \text{ m/s}$$

4. Manometric efficiency is given as,

$$\eta_{\text{mano}} = \frac{g H_m}{v_{w_2} u_2} = \frac{9.81 \times 6}{7.43 \times 12.56} = 0.63 = 63 \%$$

5. Minimum rpm to start pump is given as,

$$N = \frac{120 \eta_{\text{mano}} v_{w_2} D_2}{\pi [D_2^2 - D_1^2]} = \frac{120 \times 0.63 \times 7.43 \times 1.2}{\pi [1.2^2 - 0.6^2]}$$

$$N = 198.66 \text{ rpm}$$

PART-3

Specific Speed.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.13. **Define specific speed of a centrifugal pump and derive the equation for the same.**

AKTU 2015-16, Marks 10

Answer

A. Specific Speed :

1. It is defined as the speed of a geometrically similar pump which would deliver one cubic meter of liquid per second against a head of one meter.
2. It is denoted by N_s.

B. Expression for Specific Speed :

1. Discharge Q for a centrifugal pump is given as,
$$Q = \text{Area} \times \text{Velocity of flow}$$
$$= \pi D B \mathrm{v}_f \qquad \text{...(5.13.1)}$$
2. We know that, $B \propto D$, then from eq. (5.13.1),
$$Q \propto D^2 \mathrm{v}_f \qquad \text{...(5.13.2)}$$
3. Tangential velocity is given as,
$$u = \frac{\pi D N}{60} \quad \text{or} \quad u \propto DN \qquad \text{...(5.13.3)}$$
4. Tangential velocity (u) and velocity of flow (v_f) are related to the manometric head (H_m) as,
$$u \propto \mathrm{v}_f \propto \sqrt{H_m} \qquad \text{...(5.13.4)}$$
5. From eq. (5.13.3) and eq. (5.13.4), we get
$$\sqrt{H_m} \propto DN$$
$$D \propto \frac{\sqrt{H_m}}{N}$$
6. Putting the value of D in eq. (5.13.2), we get
$$Q \propto \frac{H_m}{N^2}\mathrm{v}_f \propto \frac{H_m}{N^2}\sqrt{H_m} \qquad \left(\because \ \mathrm{v}_f \propto \sqrt{H_m}\right)$$
$$Q \propto \frac{H_m^{3/2}}{N^2}$$
$$Q = K\frac{H_m^{3/2}}{N^2} \qquad \text{...(5.13.5)}$$
Where, K = Constant of proportionality.
7. If $H_m = 1$ m, $Q = 1$ m^3/s, so $N = N_s$, then from eq. (5.13.5), we get
$$1 = K\frac{(1)^{3/2}}{N_s^2}$$
$$N_s^2 = K$$
8. Putting the value of K in eq. (5.13.5), we get
$$Q = N_s^2 \frac{H_m^{3/2}}{N^2}$$

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

This expression is showing the specific speed of pump.

Que 5.14. **Two geometrically similar pumps are running at the same speed of 1000 rpm. One pump has an impeller diameter of 0.30 metre and lifts water at the rate of 20 litres per second against a head of 15 metres. Determine the head and impeller diameter of the other pump to deliver half the discharge.**

Answer

Given : N_1 = 1000 rpm, D_1 = 0.30 m, Q_1 = 20 lit/s = 0.020 m^3/s

H_{m_1} = 15 m, N_2 = 1000 rpm, $Q_2 = \frac{Q_1}{2} = \frac{20}{2}$ = 10 lit/s = 0.01 m^3/s

To Find : i. Diameter of impeller (D_2).

ii. Head developed (H_{m_2}).

1. We know,

$$\frac{N_1\sqrt{Q_1}}{H_{m_1}^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_{m_2}^{3/4}}$$

$$\therefore \quad \frac{1000\times\sqrt{0.02}}{15^{3/4}} = \frac{1000\times\sqrt{0.01}}{H_{m_2}^{3/4}}$$

$$H_{m_2}^{3/4} = \frac{1000\times\sqrt{0.01}\times 15^{3/4}}{1000\times\sqrt{0.02}} = \sqrt{\frac{0.01}{0.02}}\times 7.622 = 5.389$$

$$\therefore \quad H_{m_2} = (5.389)^{4/3} = 9.45 \text{ m}$$

2. Now,

$$\left(\frac{\sqrt{H_m}}{DN}\right)_1 = \left(\frac{\sqrt{H_m}}{DN}\right)_2$$

$$\frac{\sqrt{H_{m_1}}}{D_1N_1} = \frac{\sqrt{H_{m_2}}}{D_2N_2}$$

$$\frac{\sqrt{15}}{0.3\times1000} = \frac{\sqrt{9.45}}{D_2\times1000}$$

$$\therefore \quad D_2 = \frac{\sqrt{9.45}\times0.3}{\sqrt{15}} = 0.238 \text{ m} = 238 \text{ mm}$$

PART-4

Cavitation and Separation.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.15. **What is priming in centrifugal pump ? Why it is done ? What is self-priming pump ? Explain.** **AKTU 2017-18, Marks 10**

Answer

A. Priming and its Necessity :

1. It is an operation in which suction pipe, casing of the pump and a portion of delivery pipe is completely filled with water by an outside source before starting the pump to remove air, gas or vapour from these parts.
2. The work done by impeller per unit weight of liquid per second is known as head generated by the pump. This means that when pump is running in air, the head generated is in terms of meter of air.
3. If pump is primed with water, then head will generate in term of meter of water.
4. But as density of air is low, so head generated by pump is also low even negligible and hence water may not be sucked by the pump.
5. To avoid this difficulty priming of centrifugal pump is necessary.

B. Self-priming Pump :

1. The internal construction of some pumps is such that special arrangements containing a supply of liquid are provided in the suction pipe due to which automatic priming of the pump occurs, such pumps are known as 'self-priming pumps'.
2. Self-priming pumps are designed with a large reservoir surrounding the pump casing.
3. The advantage associated with self-priming pump is being portable in nature.
4. These are commonly used in sewage lift stations, where raw sewage is pumped into a treatment facility.

Que 5.16. **What is net positive suction head (NPSH) ?**

Answer

1. Net positive suction head (NPSH) is defined as the absolute pressure head at the inlet to the pump minus the vapour pressure head (in absolute units) plus the velocity head.

NPSH = Absolute pressure head at inlet of the pump – Vapour pressure head + Velocity head

$$= \frac{p_1}{\rho g} - \frac{p_v}{\rho g} + \frac{v_s^2}{2g} \quad ...(5.16.1)$$

2. But, the absolute pressure head at inlet of the pump is given by,

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left[\frac{v_s^2}{2g} + h_s + h_{f_s}\right] \quad ...(5.16.2)$$

3. Substituting the value of eq. (5.16.2) in eq. (5.16.1), we get

$$\text{NPSH} = \left(\frac{p_a}{\rho g} - \left[\frac{v_s^2}{2g} + h_s + h_{f_s}\right]\right) - \frac{p_v}{\rho g} + \frac{v_s^2}{2g}$$

$$= \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{f_s}$$

$$\text{NPSH} = H_a - H_v - h_s - h_{fs} \quad \left(\because \frac{p_a}{\rho g} = H_a \text{ and } \frac{p_v}{\rho g} = H_v\right)$$

Que 5.17. **Write a short note on cavitation in centrifugal pump.**

Answer

1. Cavitation begins to appear in centrifugal pumps when the pressure at the suction falls below the vapour pressure of the liquid.
2. The intensity of cavitation increases with the decrease in value of NPSH.
3. As in the case of turbines, for pumps also, Thoma's cavitation factor is used to indicate the onset of cavitation. For pumps Thoma's cavitation factor is defined as :

$$\sigma = \frac{H_a - H_s - H_v}{H_m} = \frac{H_{sv}}{H_m}$$

Where, H_a = Atmospheric pressure head,
H_v = Vapour pressure head,
H_s = Total suction head $\left(= h_s + h_{fs} + \frac{v_s^2}{2g}\right)$, and
H_{sv} = Net positive suction head (NPSH).

4. The cavitation will occur if the value of σ is less than the critical value, σ_c at which the cavitation just begins. The cavitation parameter σ is a function of specific speed, efficiency of the pump, and number of vanes.
5. The harmful effects of cavitation are :
   i. Pitting and erosion of surface (due to continuous hammering action of collapsing bubbles).
   ii. Sudden drop in head, efficiency and the power delivered to the fluid.
   iii. Noise and vibration (produced by the collapse of bubbles).

Que 5.18. **A centrifugal pump discharges 5 m³/s under a head of 130 m running at 600 rpm. Outer diameter of impeller is 2 m and has a positive suction lift of 3.2 m including velocity head and friction losses in suction pipe. Experiments were conducted on a geometrically similar model of 0.4 m outer diameter of impeller under a head of 90 m. Vapour pressure of liquid is equal to 0.35 m of head. Calculate the discharge, speed and suction lift for the model. Assume atmospheric pressure head = 10.2 m of water.**

AKTU 2017-18, Marks 10

Answer

Given : $Q_1 = 5$ m³/s, $H_{m_1} = 130$ m, $N_1 = 600$ rpm, $D_1 = 2$ m, $H_s = 3.2$ m, $D_2 = 0.4$ m, $H_{m_2} = 90$ m, $H_v = 0.35$ m of water, $H_a = 10.2$ m of water

To Find : i. Discharge.
ii. Speed.
iii. Suction lift.

1. From the relation,

$$\frac{H_{m_1}}{D_1^2 N_1^2} = \frac{H_{m_2}}{D_2^2 N_2^2}$$

$$N_2 = \sqrt{\frac{H_{m_2} D_1^2 N_1^2}{H_{m_1} D_2^2}} = \sqrt{\frac{90 \times (2)^2 \times (600)^2}{130 \times (0.4)^2}}$$

$$= 2496.2 \text{ rpm}$$

2. From the relation,

$$\frac{Q_1}{D_1^3 N_1} = \frac{Q_2}{D_2^3 N_2}$$

$$Q_2 = \frac{Q_1 D_2^3 N_2}{D_1^3 N_1} = \frac{5 \times (0.4)^3 \times 2496.2}{(2)^3 \times (600)} = 0.166 \text{ m}^3/\text{s}$$

3. Positive suction lift,

$$\sigma_p = \frac{H_a - H_s - H_v}{H_{m_1}} = \frac{10.2 - 3.2 - 0.35}{130} = 0.051$$

4. For cavitation similarity, $\sigma_m = \sigma_p$

$$\therefore \qquad \sigma_m = \frac{10.2 - H_s - 0.35}{90} = 0.051$$

$$H_s = 5.26 \text{ m}$$

PART-5

Performance Characteristics.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.19. **Discuss the characteristics curves for the centrifugal pump.**

Answer

1. Characteristics curves of a centrifugal pump are defined as those curves which are plotted from the results of a number of tests on the centrifugal pump.
2. These curves are necessary to predict the behaviour and performance of the pump when the pump is working under different flow rates, heads and speeds.
3. Followings are the important characteristic curve for pumps :

a. Main Characteristics Curves :

1. The main characteristic curve of a centrifugal pump consists of variation of manometric head, (H_m), power and discharge with respect to speed.
2. Fig. 5.19.1 shows main characteristic curves of a pump.

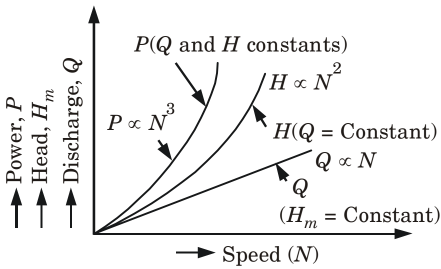

Fig. 5.19.1.

b. Operating Characteristics Curves :

1. If the speed is kept constant, the variation of manometric head, power and efficiency with respect to discharge gives the operating characteristics of pump.
2. Different operating characteristics curves are shown in Fig. 5.19.2.

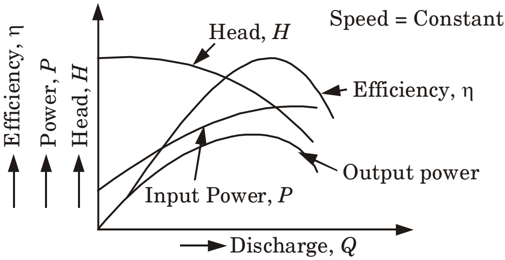

Fig. 5.19.2.

c. Constant Efficiency Curves or Muschel Curves :

1. For obtaining constant efficiency curves for a pump, the head versus discharge curves and efficiency versus discharge curves for different speed are used.
2. Fig. 5.19.3(*a*) shows the head versus discharge curves for different speeds.

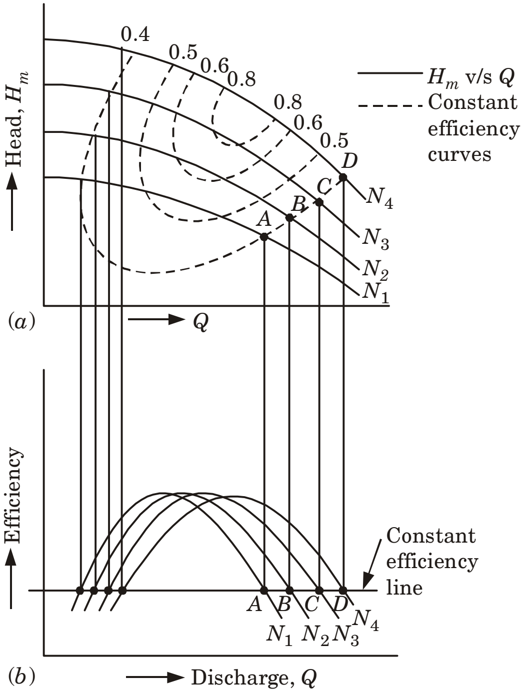

Fig. 5.19.3.

3. The efficiency versus discharge curves for the different speeds is shown in Fig. 5.19.3(*b*).
4. By combining these curves (*H*-*Q* curves and η-*Q* curves), constant efficiency curves are obtained as shown in Fig. 5.19.3(*a*).

PART-6

Reciprocating Pump Theory.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.20. Define and classify the reciprocating pump. Also differentiate between single acting and double acting reciprocating pump.

Answer

A. Reciprocating Pump :

1. If mechanical energy is converted into pressure energy by means of reciprocating motion of a piston into a cylinder, then pump is known as reciprocating pump.
2. It is a positive displacement pump as it sucks and raises the liquid by actually displacing it with a piston that executes a reciprocating motion in a closely fitted cylinder.
3. The amount of liquid pumped is equal to the volume displaced by the piston.

B. Classification of Reciprocating Pump :

a. According to the Water being in Contact with Piston :

1. Single acting pump.
2. Double acting pump.

b. According to Number of Cylinders Provided :

1. Single cylinder pump.
2. Double cylinder pump.
3. Triple cylinder pump.

C. Difference between Single Acting and Double Acting Reciprocating Pump :

S. No.	Single Acting Pump	Double Acting Pump
1.	The liquid being pumped is in contact with one side of piston/plunger of pump.	The liquid being pumped is in contact with both side of piston/ plunger of pump.
2.	It has only one delivery stroke for one complete revolution of the crank.	It has two delivery strokes for one complete revolution of the crank.
3.	Discharge is less.	Discharge is more as compared to single acting.
4.	Power required is less.	Power required is more.
5.	Work saved by fitting air vessel is 84.8 %.	Work saved by fitting air vessel is 39.2 %.

Que 5.21. With the help of a neat sketch explain the construction and working principle of reciprocating pump.

OR

With the help of a neat sketch explain the working principle of reciprocating pump.

AKTU 2015-16, Marks 10

Answer

A. Construction of Reciprocating Pump :

1. A reciprocating pump consists of a piston or a plunger inside a cylinder as shown in Fig. 5.21.1.
2. Piston is connected to the crankshaft through piston rod and connecting rod. The crankshaft is rotated by means of electric motor.
3. Suction and delivery pipes are connected to the cylinder with non-return suction and delivery valves.
4. Non-return valves are one way valves which allow the liquid to flow in one direction only.
5. Here, suction valve allows liquid to flow from the suction pipe to the cylinder, while delivery valve allows liquid to flow from the cylinder to the delivery pipe.

B. Working of Reciprocating Pump :

1. A reciprocating pump consists of a piston or a plunger executing reciprocating motion inside a cylinder as shown in Fig. 5.21.1.

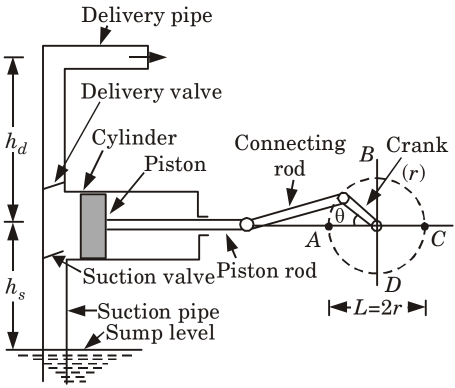

Fig. 5.21.1. Main parts of a reciprocating pump.

2. As the crank moves outwards (from A to C), the piston moves towards right in the cylinder causing a vacuum in the cylinder.
3. Due to the pressure difference between the sump and the cylinder, liquid is drawn into the cylinder through the non-return suction valve.
4. During this outward stroke, the delivery valve remains closed.
5. During the return stroke of the crank (from C to A), the piston moves towards the left causing an increase in pressure in the cylinder which opens the delivery valve and closes the inlet valve.
6. The liquid is forced into the delivery pipe and is raised to a required height.

Que 5.22. **Derive the expression for discharge, work done and power of single acting reciprocating pump.**

Answer

1. Let,
D = Diameter of cylinder,
A = Cross-sectional area of the piston,
r = Radius of crank,
L = Length of stroke,
h_s = Suction head,
h_d = Delivery head, and
N = Speed of crank.
2. Discharge of water in one revolution of crank = Area × Stroke length
$= AL$
3. Discharge of pump per second,
Q = Discharge in one revolution × Number of revolutions per second

$$= AL\frac{N}{60} = \frac{ALN}{60}$$

4. Weight of water delivered per second,

$$w = \rho g Q$$

$$w = \frac{\rho g ALN}{60}$$

5. Work done per second = Weight of water lifted per second × Total height through which liquid is lifted

$$= w(h_s + h_g)$$

$$= \frac{\rho g ALN}{60}(h_s + h_g)$$

6. Power required to drive the pump,

$$P = \frac{\text{Work done per second}}{1000}$$

$$P = \frac{\rho g ALN(h_s + h_d)}{60000}\ \text{kW}$$

Que 5.23. Derive an expression for discharge, work done and power for double acting pump.

Answer

1. Let, D = Diameter of piston, and
   d = Diameter of piston rod.
2. Area on one side of piston,

$$A = \frac{\pi}{4}D^2$$

3. Area on other side where piston rod is connected,

$$A_1 = \frac{\pi}{4}D^2 - \frac{\pi}{4}d^2 = \frac{\pi}{4}(D^2 - d^2)$$

4. Volume of water discharge in one revolution of crank

$$= A \times \text{Stroke length} + A_1 \times \text{Stroke length}$$

$$= AL + A_1L$$

$$= (A + A_1)L$$

$$= \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}(D^2 - d^2)\right] \times L$$

5. Discharge of pump per second = Volume of water discharge × Number of revolutions per second

$$Q = \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}(D^2 - d^2)\right] L \times \frac{N}{60}$$

6. If $d << D$, then d can be neglected.

$$\therefore \quad Q = 2 \times \frac{\pi}{4} D^2 \times \frac{LN}{60} = \frac{2ALN}{60} \qquad \left(\because A = \frac{\pi}{4} D^2\right)$$

7. Weight of water delivered per second,

$$w = \rho g Q$$

$$= \frac{2\rho g\ ALN}{60}$$

8. Work done per second = Weight of water delivered per second × Total height

$$= w\ (h_s + h_d)$$

$$= \frac{2\rho g\ ALN}{60}\ (h_s + h_d)$$

9. Power required to drive the pump,

$$P = \frac{\text{Work done per second}}{1000}$$

$$P = \frac{2\rho g ALN\ (h_s + h_d)}{60000}\ \text{kW}$$

Que 5.24. **Give the curve showing the variation of discharge with crank angle for double acting reciprocating pump.**

AKTU 2014-15, Marks 10

Answer

1. When the crank rotates from θ = 0° to θ = 180°, Fig. 5.24.1, the piston or plunger which is initially at its extreme left position move to its extreme right position.
2. During the outward movement of the piston or plunger a partial vacuum is created in the cylinder, which enables the atmospheric pressure acting on the liquid surface in the well or sump below, to force the liquid up the suction pipe and fill the cylinder by forcing open the suction valve.
3. At the end of the suction stroke the piston or plunger is at its extreme right position, the crank is, at θ = 180°, the cylinder is full of liquid, the suction valve is closed and the delivery valve is just at the point of opening.
4. When the crank rotates from θ = 180° to θ = 360° the piston or plunger moves inward from its extreme right position towards left. The inward movement of the piston or plunger causes the pressure of the liquid in the cylinder to rise above atmospheric pressure, due to which the suction valve closes and the delivery valve opens.
5. At the end of the delivery stroke the piston or plunger is at extreme left position, the crank is at θ = 0° or 360° so that it has completed one full revolution, and both the suction and the delivery valves are closed.

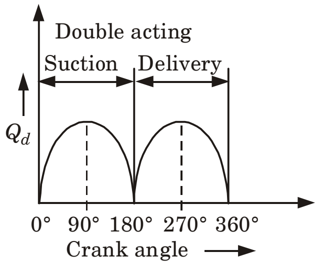

Fig. 5.24.1.

PART-7

Slip.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.25. **Define the following terms :**

i. Coefficient of discharge,

ii. Slip of pump, and

iii. Negative slip.

Answer

i. Coefficient of Discharge (C_d) :

1. It is defined as the ratio of actual discharge to the theoretical discharge. It is given as,

$$C_d = \frac{Q_{\text{act}}}{Q_{\text{th}}}$$

$$= \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= C_v C_c$$

ii. Slip of a Pump :

1. It is defined as the difference between the theoretical discharge and actual discharge.

$$\text{Slip} = Q_{\text{th}} - Q_{\text{act}}$$

2. The slip is mostly expressed as percentage slip which is given by,

$$\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$= \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100 = (1 - C_d) \times 100$$

3. For most of the reciprocating pumps the actual discharge Q_{act} is less than the theoretical discharge Q_{th}, C_d is less than one and the slip of the pump is positive.

iii. Negative Slip :

1. If actual discharge of the pump is more than the theoretical discharge, the slip will be negative, which is known as negative slip.
2. Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

Que 5.26. **A single acting reciprocating pump, running at 50 rpm delivers 0.00736 m³/s of water. The diameter of the piston is 200 mm and stroke length 300 mm. The suction and delivery heads are 3.5 m and 11.5 m respectively. Determine :**

i. Theoretical discharge

ii. Coefficient of discharge

iii. Percentage slip of the pump

iv. Power required to run the pump

AKTU 2014-15, Marks 10

Answer

Given : Q_{act} = 0.00736 m³/s, N = 50 rpm, D = 200 mm = 0.2 m, L = 300 mm = 0.3 m, h_s = 3.5 m, h_d = 11.5 m

To Find :
i. Theoretical discharge.
ii. Coefficient of discharge.
iii. Percentage slip of the pump.
iv. Power required to run the pump.

1. Area of cylinder, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.2)^2 = 0.0314 \text{ m}^2$
2. Theoretical discharge,

$$Q_{th} = \frac{ALN}{60} = \frac{0.0314 \times 0.3 \times 50}{60} = 0.00785 \text{ m}^3/\text{s}$$

3. Coefficient of discharge,

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.00736}{0.00785} = 0.937$$

4. Percentage slip of the pump $= \dfrac{Q_{th} - Q_{act}}{Q_{th}} \times 100$

$$= \frac{0.00785 - 0.00736}{0.00785} \times 100 = 6.24\ \%$$

5. Power required to run the pump,

$$P = \rho g \frac{ALN}{60} \frac{(h_s + h_d)}{1000}$$

$$= \frac{1000 \times 9.81 \times 0.0314}{60} \times 0.3 \times 50 \left(\frac{3.5 + 11.5}{1000}\right)$$

$$= 1.155 \text{ kW}$$

Que 5.27. **A single acting reciprocating pump running at 50 rpm delivers 0.01 m³/s of water. The diameter of the piston is 200 mm and stroke length 400 mm. Determine (i) The theoretical discharge of the pump. (ii) Coefficient of discharge. (iii) Slip and percentage slip of the pump.**

AKTU 2015-16, Marks 7.5

Answer

Same as Q. 5.26, Page 5–30A, Unit-5.

[**Answer :**

i. Theoretical discharge = 0.01046 m³/s
ii. Coefficient of discharge = 0.9560
iii. Slip = $Q_{th} - Q_{act}$ = 4.6×10^{-4} m³/s
iv. Percentage slip = 4.4 %]

PART-8

Indicator Diagram.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.28. **What do you understand by an indicator diagram ? Explain ideal indicator diagram.**

AKTU 2017-18, Marks 10

Answer

A. Indicator Diagram :

1. It is defined as the graph between the pressure head in the cylinder and distance travelled by piston from inner dead center for one complete revolution of crank.

2. In reciprocating pump, maximum distance travelled by piston is stroke length. So indicator diagram is a graph between pressure head and stroke length.
3. Pressure head is taken as ordinate and stroke length is taken as abscissa.

B. Ideal Indicator Diagram :

1. The graph between the pressure head and stroke length of the piston for one complete revolution of the crank under ideal conditions is known as ideal indicator diagram.

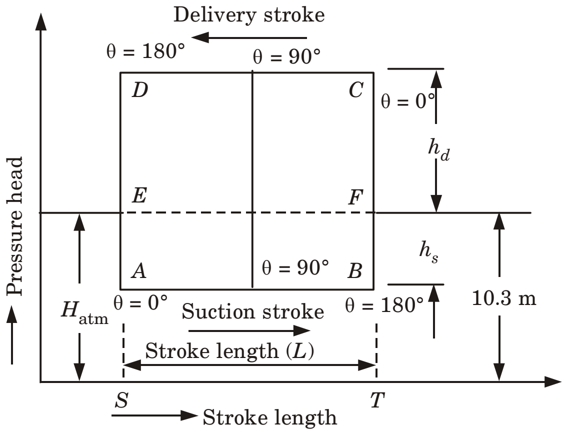

Fig. 5.28.1. Ideal indicator diagram.

2. As shown in Fig. 5.28.1, different notations are taken as

$$H_{atm} = \text{Atmospheric pressure head,}$$
$$= 10.3 \text{ m of water}$$
$$L = \text{Length of stroke,}$$
$$h_s = \text{Suction head, and}$$
$$h_d = \text{Delivery head.}$$

3. During suction stroke, the pressure head in the cylinder is constant and equal to suction head (h_s) which is below the atmospheric pressure head (H_{atm}) by a height of h_s.
4. This pressure head during suction stroke is represented by a horizontal line *AB* which is below the line *EF* by the height h_s (suction head).
5. During the delivery stroke, pressure head in cylinder is constant and equal to delivery head (h_d) this is represented by line *CD*. This line *CD* is above the line *EF* (atmospheric pressure) by a height of h_d.

Que 5.29. **Prove that work done by the pump is proportional to the area of indicator diagram. What do you know about slip both positive and negative in a reciprocating pump ?**

AKTU 2014-15, Marks 10

Answer

A. Proof :

1. From the indicator diagram (Refer Fig. 5.28.1), area of diagram is given as,

$$\text{Area} = AB \times BC = AB \times (BF + FC)$$
$$= L \times (h_s + h_d)$$

2. Work done by the reciprocating pump per second is given as,

$$W = \frac{\rho g A L N}{60} (h_s + h_d)$$
$$= KL(h_s + h_d)$$

$\therefore$ $$W \propto L(h_s + h_d) \quad \left[K = \frac{\rho g A N}{60} \text{ (constant)}\right]$$

$\therefore$ Work done by pump $\propto$ Area of indicator diagram

B. Slip : Refer Q. 5.25, Page 5–29A, Unit-5.

PART-9

Effect of Acceleration.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.30. Derive an expression for accelerating head in reciprocating pump assuming piston motion by SHM.

AKTU 2016-17, Marks 10

Answer

1. Let,
ω = Angular speed of the crank (rad/s),
A = Area of the cylinder,
a = Area of the pipe,
l = Length of pipe (suction or delivery),
r = Radius of crank, and
θ = Angle turned by crank in radian in time t.
$\theta = \omega t$

2. Let x is the distance travelled by the piston as shown in Fig. 5.30.1.

$$x = \text{Distance } AF = AO - FO = r - r\cos\theta$$
$$[\because \quad AO = r,\ FO = r\cos\theta]$$
$$x = r - r\cos\omega t \quad \ldots(5.30.1) \quad [\because \theta = \omega t]$$

3. Differentiate eq. (5.30.1) with respect to t, which gives the velocity of piston. So,

$$V = \frac{dx}{dt} = \frac{d}{dt} [r - r \cos \omega t]$$
$$= 0 - r [- \sin \omega t] \omega = r\omega \sin \omega t \qquad ...(5.30.2)$$

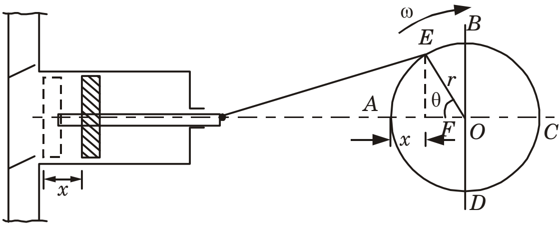

Fig. 5.30.1. Velocity and acceleration of piston.

4. As per continuity equation, the volume of water flowing into cylinder per second is equal to the volume of water flowing from the pipe per second. So,

Velocity of water in cylinder × Area of cylinder
= Velocity of water in pipe × Area of pipe

$VA = va$

[∵ Velocity of water in cylinder = Velocity of piston = V]

v = Velocity of water in pipe.

$$v = \frac{VA}{a} = \frac{A}{a} V$$

$$v = \frac{A}{a} r\omega \sin \omega t \qquad ...(5.30.3)$$

5. Acceleration of water in pipe is obtained by differentiating eq. (5.30.3) with respect to t. So acceleration of water in pipe

$$= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{A}{a} r\omega \sin \omega t\right)$$

$$= \frac{A}{a} r\omega^2 \cos \omega t \qquad ...(5.30.4)$$

6. Mass of water in pipe = ρ × Volume of water in pipe
= ρ × [Area of pipe × Length of pipe] = $\rho[al] = \rho al$

7. Force required to accelerate the water in pipe
= Mass of water in pipe × Acceleration of water in pipe

$$= \rho al \frac{A}{a} r\omega^2 \cos \omega t$$

8. Now, intensity of pressure due to acceleration,

$$= \frac{\text{Force required to accelerate the water}}{\text{Area of pipe}}$$

$$= \frac{\rho al \frac{A}{a} r\omega^2 \cos \omega t}{a} = \rho l \frac{A}{a} r\omega^2 \cos \theta \qquad [\because \omega t = \theta]$$

9. Pressure head (h_a) due to acceleration,

$$h_a = \frac{\text{Intensity of pressure due to acceleration}}{\text{Weight density of liquid}}$$

$$= \frac{\rho l \dfrac{A}{a} r\omega^2 \cos\theta}{\rho g}$$

$$h_a = \frac{l}{g}\frac{A}{a} r\omega^2 \cos\theta \qquad ...(5.30.5)$$

10. The pressure head due to acceleration in suction and delivery pipe is obtained from eq. (5.30.5) by using subscripts '*s*' and '*d*' is given below :

$$h_{as} = \frac{l_s}{g}\frac{A}{a_s} r\omega^2 \cos\theta$$

$$h_{ad} = \frac{l_d}{g}\frac{A}{a_d} r\omega^2 \cos\theta$$

11. For maximum pressure head,

$\cos\theta = 1$

$$(h_a)_{max} = \frac{l}{g}\frac{A}{a} r\omega^2$$

Que 5.31. **What is the effect of acceleration in suction and delivery pipes on indicator diagram ?**

Answer

A. Effect of Acceleration in the Suction Pipe : Let l_s and a_s are length and cross-sectional area of the suction pipe respectively.

i. At the Beginning of the Suction Stroke :

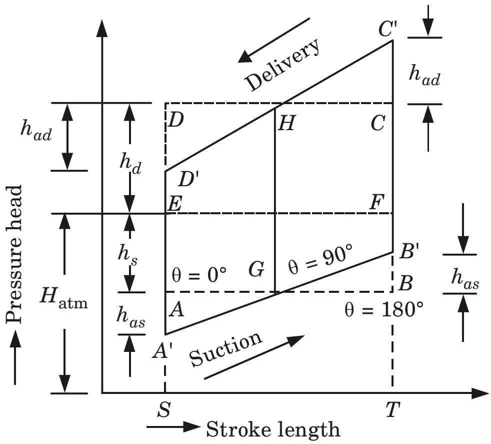

Fig. 5.31.1. Effect of acceleration on indicator diagram.

The accelerating head, $h_{as} = \frac{l_s}{g}\frac{A}{a_s}\omega^2 r$

Negative pressure (vacuum) head, $h_s + h_{as} = h_s + \frac{l_s}{g}\frac{A}{a_s}\omega^2 r$

Absolute pressure head $= H_{\text{atm}} - \left(h_s + \frac{l_s}{g}\frac{A}{a_s}\omega^2 r\right)$

ii. At the Middle of the Suction Stroke :

The acceleration head, $h_{as} = 0$

Negative pressure (vacuum) head $= h_s$

Absolute pressure head $= H_{\text{atm}} - h_s$

iii. At the End of the Suction Stroke :

The acceleration head, $h_{as} = -\frac{l_s}{g}\frac{A}{a_s}\omega^2 r$

Negative pressure (vacuum) head $= h_s + h_{as} = h_s - \frac{l_s}{g}\frac{A}{a_s}\omega^2 r$

Absolute pressure head $= H_{\text{atm}} - \left(h_s - \frac{l_s}{g}\frac{A}{a_s}\omega^2 r\right)$

B. Effect of Acceleration in the Delivery Pipe :

1. In the beginning of delivery stroke the liquid in the delivery pipe is accelerated, while at the end of delivery stroke the liquid is retarded.
2. Let l_d and a_d are the length and cross-sectional area of the delivery pipe respectively.

i. At the Beginning of the Delivery Stroke :

Pressure (gauge) head, $h_d + h_{ad} = h_d + \frac{l_d}{g}\frac{A}{a_d}\omega^2 r$

ii. At the Middle of the Delivery Stroke :

Pressure (gauge) head $= h_d$ $(\because \; h_{ad} = 0)$

iii. At the End of the Delivery Stroke :

Pressure (gauge) head $= h_d - \frac{l_d}{g}\frac{A}{a_d}\omega^2 r$

Absolute pressure head $= H_{\text{atm}} + h_d - \frac{l_d}{g}\frac{A}{a_d}\omega^2 r$

Que 5.32. Find the expression for the head lost due to friction in suction and delivery pipes. Also discuss its effect on indicator diagram.

Answer

A. Expression for Head Lost due to Friction :

1. Velocity of water in suction and delivery pipes is given as,

$$v = \frac{A}{a} r\omega \sin\theta \qquad ...(5.32.1)$$

Where, A = Area of the piston in the cylinder,
a = Area of the pipe (delivery or suction), and
r = Crank radius.

2. Loss of head due to friction in pipe is given as,

$$h_f = \frac{4flv^2}{2gd} \qquad ...(5.32.2)$$

3. Substituting the value of v from eq. (5.32.1) into eq. (5.32.2), we get

$$h_f = \frac{4fl}{2gd}\left[\frac{A}{a} r\omega \sin\theta\right]^2$$

4. Loss of head due to friction in suction pipe is given as,

$$h_{fs} = \frac{4fl_s}{2gd_s}\left[\frac{A}{a_s} r\omega \sin\theta\right]^2 \qquad ...(5.32.3)$$

5. Loss of head due to friction in delivery pipe is given as,

$$h_{fd} = \frac{4fl_d}{2gd_d}\left[\frac{A}{a_d} r\omega \sin\theta\right]^2 \qquad ...(5.32.4)$$

B. Effect of Friction in Suction and Delivery Pipes on Indicator Diagram :

1. From the eq. (5.32.3) and eq. (5.32.4), it is evident that the variation of h_{fs} or h_{fd} with θ is parabolic :
   i. At the beginning of suction or delivery stroke : $\theta = 0°$, $\sin\theta = 0$ and therefore $h_{fs} = 0$, $h_{fd} = 0$ *i.e.*, there is no loss of head due to friction.

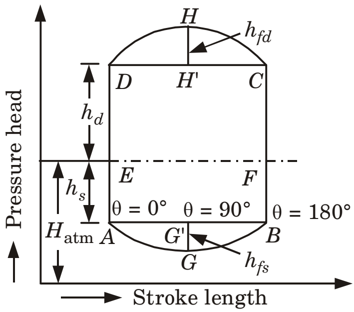

Fig. 5.32.1. Effect of friction on indicator diagram.

   ii. At the middle of the suction or delivery stroke : $\theta = 90°$, and $\sin\theta = 1$.

$$\therefore \qquad h_{fs} = \frac{4fl_s}{2gd_s}\left(\frac{A}{a_s}\omega r\right)^2$$

and $$h_{fd} = \frac{4fl_d}{2gd_d}\left(\frac{A}{a_d}\omega r\right)^2$$

iii. At the end of suction or delivery stroke : θ = 180°, sin θ = 0 and therefore h_{fs} and h_{fd} = 0.

2. These results, evidently, indicate that frictional losses are zero at the beginning and end of the strokes and maximum at the mid of the strokes.
3. Fig. 5.32.1 shows the effect of friction on the indicator diagram.

Que 5.33. **Draw the indicator diagram for reciprocating pump considering acceleration and friction head in suction and delivery pipes and find expression for the work done for a single pump.**

Answer

A. Effect of Acceleration and Friction Head in Suction and Delivery Pipes on Indicator Diagram : The acceleration head (h_a) and friction head (h_f) at any instant of flow in the suction and delivery pipes of a reciprocating pump are given as :

$$h_a = \frac{l}{g}\frac{A}{a}\omega^2 r\cos\theta;\ h_f = \frac{4fl}{2gd}\left(\frac{A}{a}\omega r\sin\theta\right)^2$$

a. During Suction Stroke : The pressure head on the piston during suction stroke for any angle θ of the crank = ($h_s + h_{as} + h_{fs}$)

i. At the beginning of the suction stroke, θ = 0° and we have

$$h_{as} = \frac{l_s}{g}\frac{A}{a_a}\omega^2 r \text{ and } h_{fs} = 0$$

∴ Pressure head in the cylinder = ($h_s + h_{as}$) below atmospheric head

$$= H_{\text{atm}} - (h_s + h_{as}) \text{ absolute}$$

ii. At middle of suction stroke, θ = 90° and we have

$$h_{as} = 0,\ h_{fs} = \frac{4fl_s}{2gd_s}\left(\frac{A}{a_s}\omega r\right)^2$$

∴ Pressure head in the cylinder = ($h_s + h_{fs}$) below atmospheric head

$$= H_{\text{atm}} - (h_s + h_{fs}) \text{ absolute}$$

iii. At the end of suction stroke, θ = 180° and we have

$$h_{as} = -\frac{l_s}{g}\frac{A}{a_s}\omega^2 r \text{ and } h_{fs} = 0$$

∴ Pressure head in the cylinder = ($h_s - h_{as}$) below atmospheric head

$= H_{atm} - (h_s - h_{as})$ absolute

b. During Delivery Stroke : The pressure head on the piston during delivery stroke for any angle θ of the crank = $(h_d + h_{ad} + h_{fd})$

i. At the beginning of delivery stroke, θ = 0° and we have

$$h_{ad} = \frac{l_d}{g}\frac{A}{a_d}\,\omega^2 r \text{ and } h_{fd} = 0$$

∴ Pressure head in the cylinder = $(h_d + h_{ad})$ above atmospheric head

$= H_{atm} + (h_d + h_{ad})$ absolute

ii. At middle of delivery stroke, θ = 90° and we have

$$h_{ad} = 0,\ h_{fd} = \frac{4fl_d}{2gd_d}\left(\frac{A}{a_d}\,\omega r\right)^2$$

∴ Pressure head in the cylinder = $(h_d + h_{fd})$ above atmospheric head

$= H_{atm} + (h_d + h_{fd})$ absolute

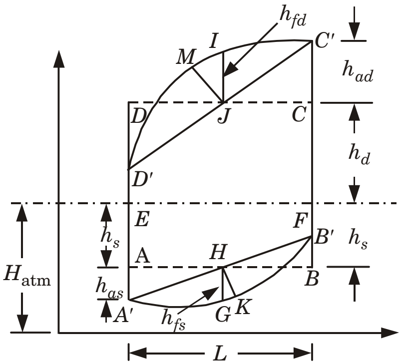

Fig. 5.33.1.

iii. At the end of delivery stroke, θ = 180° and we have

$$h_{ad} = -\frac{l_d}{g}\frac{A}{a_d}\,\omega^2 r \text{ and } h_{fd} = 0$$

∴ Pressure head in the cylinder = $(h_d - h_{ad})$ above atmospheric head

$= H_{atm} + (h_d - h_{ad})$ absolute

B. Expression for the Work Done for a Single Acting Pump :

1. Fig. 5.33.1 shows a complete indicator diagram including the effects of acceleration and friction.
2. Area $A'HB'C'JD'$ = Area $ABCD = (h_s + h_d)L$
3. Area of parabola $A'GB' = A'B' \times \frac{2}{3}\,HK$

$$= \frac{2}{3} (A'B' \times HK)$$

$$= \frac{2}{3} (AB \times GH) = \frac{2}{3} Lh_{fs}$$

4. Similarly, area of parabola $C'ID'$

$$= C'D' \times \frac{2}{3} JM = \frac{2}{3} (C'D' \times JM)$$

$$= \frac{2}{3} (CD \times JI) = \frac{2}{3} Lh_{fd}$$

5. Area of indicator diagram

$$A'GB'C'ID' = \text{Area } A'HB'C'JD' + \text{Area of parabola } A'GB' + \text{Area of parabola } C'ID'$$

$$= (h_s + h_d) L + \frac{2}{3} Lh_{fs} + \frac{2}{3} Lh_{fd}$$

$$= \left(h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd}\right) L$$

6. As the area of the indicator diagram is proportional to work done by the pump, therefore,

$$\text{Work done by pump per second} \propto \left(h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd}\right) L$$

$$= K\left(h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd}\right) L$$

Where, K = Constant of proportionality.

7. Hence, the work done per second by a single-acting pump

$$= \frac{wALN}{60}\left(h_s + h_d + \frac{2}{3} h_{fs} + \frac{2}{3} h_{fd}\right) \qquad \left(\because K = \frac{wAN}{60}\right)$$

Que 5.34. **What is the maximum speed of a reciprocating pump during suction stroke and delivery stroke ?**

Answer

a. Maximum Speed during Suction Stroke :

1. Absolute pressure head during suction stroke is minimum at the beginning of stroke and will be equal to separation pressure head (h_{sep}).

So,
$$h_{sep} = H_{atm} - (h_s + h_{as})$$

$$h_{as} = H_{atm} - h_s - h_{sep} \qquad ...(5.34.1)$$

2. The value of h_{as} is also given as,

$$h_{as} = \frac{l_s}{g} \frac{A}{a_s} r\omega^2 \qquad ...(5.34.2)$$

3. From eq. (5.34.1) and eq. (5.34.2), we get

$$\frac{l_s}{g}\frac{A}{a_s}r\omega^2 = H_{atm} - h_s - h_{sep}$$

This equation will give maximum value of ω during suction stroke without separation.

b. Maximum Speed during Delivery Stroke :

1. During delivery stroke, the probability of separation is only at the end of delivery stroke. The pressure head in the cylinder at the end of delivery stroke

$$= (H_{atm} + h_d) - h_{ad}$$

2. If separation is to be avoided, the pressure head should be more than separation pressure. In limiting case,

$$h_{sep} = (H_{atm} + h_d) - h_{ad}$$

$$h_{ad} = (H_{atm} + h_d) - h_{sep} \quad ...(5.34.3)$$

3. But pressure head due to acceleration at the end of delivery stroke is given as,

$$h_{ad} = \frac{l_d}{g}\frac{A}{a_d}r\omega^2 \quad ...(5.34.4)$$

4. From eq. (5.34.3) and eq. (5.34.4), we get

$$\frac{l_d}{g}\frac{A}{a_d}r\omega^2 = (H_{atm} + h_d) - h_{sep}$$

From this equation we can get maximum value of ω during delivery stroke without separation.

Que 5.35. **A single acting reciprocating pump of 12 cm diameter and 24 cm stroke is delivering water to the tank which is 10 m above the center of pump. The pump is located 5 m above the center of sump. The diameter and the length of the suction pipe are 5 cm and 5 m respectively, and diameter and length of delivery pipe are 4 cm and 20 m respectively. Find the maximum speed of the pump to avoid separation either in suction pipe or delivery pipe. Take atmospheric pressure head 10.33 m of water and separation occurs at 80 kN/m² below atmospheric pressure.**

AKTU 2017-18, Marks 10

Answer

Given : D = 12 cm = 0.12 m, L = 24 cm = 0.24 m, h_s = 5 m, h_d = 10 m, d_s = 5 cm = 0.05 m, l_s = 5 m, d_d = 4 cm = 0.04 m, l_d = 20 m, H_{atm} = 10.33 m, $p_{sep} = 80 \times 10^3$ N/m²

To Find : Maximum speed of the pump.

1. Separation pressure head,

$$h_{sep} = \frac{p_{sep}}{\rho g}$$

$$= \frac{80 \times 10^3}{1000 \times 10} \quad \text{(Assuming, } g = 10 \text{ m/s}^2\text{)}$$

$= 8$ m below atmosphere

$= (H_{atm} - 8)$ absolute

$= (10.33 - 8) = 2.33$ m (absolute)

2. The maximum speed during suction stroke is given by,

$$H_{atm} - h_s - h_{sep} = \frac{l_s}{g} \frac{A}{a_s} \omega^2 r$$

$$10.33 - 5 - 2.33 = \frac{5}{9.81} \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_s^{\,2}} \times \omega^2 \times 0.12 \quad [\because \quad r = L/2 = 0.12]$$

$$3 = \frac{5}{9.81} \times \left(\frac{0.12}{0.05}\right)^2 \times \omega^2 \times 0.12 = 0.3523\omega^2$$

$$\therefore \qquad \omega = \sqrt{\frac{3}{0.3523}} = 2.92 \text{ rad/s}$$

$$\frac{2\pi N}{60} = 2.92 \qquad \left(\because \ \omega = \frac{2\pi N}{60}\right)$$

$$N = 27.88 \text{ rpm}$$

3. The maximum speed during delivery stroke is given by,

$$H_{atm} + h_d - h_{sep} = \frac{l_d}{g} \frac{A}{a_d} \omega^2 r$$

$$10.33 + 10 - 2.33 = \frac{20}{9.81} \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d_d^{\,2}} \omega^2 r$$

$$18 = \frac{20}{9.81} \times \left(\frac{0.12}{0.04}\right)^2 \times \omega^2 \times 0.12 = 2.2\omega^2$$

$$\omega = \sqrt{\frac{18}{2.2}} = 2.86 \text{ rad/s}$$

$$\frac{2\pi N}{60} = 2.86 \qquad \left(\because \ \omega = \frac{2\pi N}{60}\right)$$

$$N = 27.31 \text{ rpm}$$

4. Thus, the maximum speed of the pump without separation during suction and delivery stroke is the minimum of these two speeds, *i.e.*, minimum of 27.88 and 27.31 rpm

$\therefore$ Maximum speed = 27.31 rpm

Que 5.36. **A single acting reciprocating pump running at 60 rpm has its piston area of 80 cm² and stroke length 150 mm. The area of suction pipe is 60 cm². The suction head is 3 m. Assuming a friction factor of 0.04, find the pressure head on the piston at the beginning, middle and at the end of the suction stroke if the length of suction pipe is 6 m.**
Assume motion of piston as SHM. Can cavitation take place if the working liquid is water ?

AKTU 2016-17, Marks 15

Answer

Given : $N = 60$ rpm, $A = 80\ \text{cm}^2 = 80 \times 10^{-4}\ \text{m}^2$, $L = 150$ mm $= 0.15$ m, $a_s = 60\ \text{cm}^2 = 60 \times 10^{-4}\ \text{m}^2$, $h_s = 3$ m, $f = 0.04$, $l_s = 6$ m

To Find : Pressure head on piston :

i. At the beginning of suction stroke,
ii. At the middle of suction stroke, and
iii. At the end of suction stroke.

1. Angular speed, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 60}{60} = 6.28$ rad/s
2. The pressure head due to acceleration in suction pipe is given by,

$$h_{as} = \frac{l_s}{g}\frac{A}{a_s}\omega^2 r \cos\theta$$

$$= \frac{l_s}{g}\frac{A}{a_s}\omega^2 \frac{L}{2}\cos\theta \qquad \left(\because\ r = \frac{L}{2}\right)$$

$$h_{as} = \frac{6}{9.81} \times \frac{80}{60} \times (6.28)^2 \times \frac{0.15}{2} \times \cos\theta = 2.4\cos\theta$$

3. The loss of head due to friction in suction pipe is given as,

$$h_{fs} = \frac{4fl_s}{2gd_s}\left(\frac{A}{a_s}\omega r \sin\theta\right)^2$$

$$= \frac{4 \times 0.04 \times 6}{0.087 \times 2 \times 9.81} \times \left(\frac{80}{60} \times 6.28 \times 0.075 \sin\theta\right)^2$$

$$\left(\because\ d_s = \sqrt{\frac{4a_s}{\pi}} = \sqrt{\frac{4 \times 60 \times 10^{-4}}{\pi}} = 0.087\ \text{m}\right)$$

$$= 0.22\sin^2\theta$$

4. At the beginning of suction stroke, $\theta = 0°$

So, pressure head $= h_s + 2.4\cos 0° + 0.22\sin^2 0°$
$= 3 + 2.4 + 0 = 5.4$ m

5. At the middle of suction stroke, $\theta = 90°$

So, pressure head $= 3 + 2.4 \cos 90° + 0.22 \sin^2 90° = 3 + 0 + 0.22$
$= 3.22$ m

6. At the end of suction stroke, $\theta = 180°$
So, pressure head $= 3 + 2.4 \cos 180° + 0.22 \sin^2 180° = 3 - 2.4 + 0$
$= 0.6$ m

7. Since, initial pressure head is less than length of suction pipe so cavitation can takes place.

PART-10

Air Vessels.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.37. What is air vessel ? Describe the function of air vessel with the help of neat sketch. **AKTU 2013-14, Marks 05**

OR

Explain the air vessel in the reciprocating pump and its advantages.

Answer

A. Air Vessel :

1. It is a closed chamber containing compressed air in the top portion and liquid at bottom of chamber.
2. One air vessel is fixed on the suction pipe just near the suction valve and one is fixed on the delivery pipe near the delivery valve.
3. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out the vessel, the air will expand in the chamber.

B. Function of Air Vessel :

1. A single acting reciprocating pump is shown in Fig. 5.37.1 with air vessels on suction side and delivery side. Air vessel works like an intermediate reservoir.
2. During first half of suction stroke, discharge of water entering the cylinder is more than the mean discharge, this excess quantity of discharge is supplied by the air vessel.
3. And during second half, the discharge entering the cylinder is less than the mean discharge. This excess quantity of water is stored in air vessel.
4. In case of delivery stroke, the function of air vessel get reversed *i.e.*, it stores water during first half of stroke and delivers water during second half.

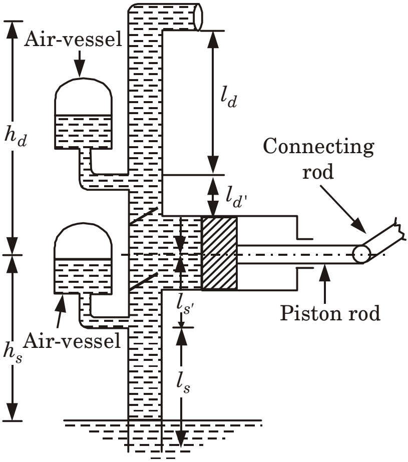

Fig. 5.37.1

C. Advantages of Air Vessel :

1. It gives a continuous supply of liquid at a uniform rate.
2. Pump runs at high speed without separation.
3. Power required to drive the pump is reduced.

Que 5.38. Show that the work saved in overcoming friction in the pipelines by fitting air vessels is 84.8 % for a single acting reciprocating pump. **AKTU 2015-16, Marks 7.5**

Answer

1. Work done against friction without air vessels :
   i. Loss of head due to friction is given as,

$$h_f = \frac{4fl}{2gd}\left[\frac{A}{a} r\omega \sin\theta\right]^2$$

   ii. Variation of h_f with θ is parabolic so indicator diagram for loss of head due to friction in pipe will be a parabola.
   iii. The work done by pump against friction per stroke is equal to the area of indicator diagram due to friction.
   iv. Work done by the pump per stroke against friction,

$$W_1 = \text{Area of parabola}$$

$$= \frac{2}{3} \times \text{Base} \times \text{Height}$$

$$= \frac{2}{3} L \left[\frac{4fl}{2gd} \left(\frac{A}{a} r\omega \right)^2 \right] \qquad \left(\text{Height, } h_f \text{ at } \theta = 90° = \frac{4fl}{2gd} \left(\frac{A}{a} r\omega \right)^2 \right)$$

2. Work done against friction with air vessels :

i. When air vessel is fitted, mean velocity of flow is given as,

$$\bar{v} = \frac{A}{a} \frac{\omega r}{\pi}$$

ii. Loss of head due to friction is given as,

$$= \frac{4fl}{2gd} \bar{v}^2 = \frac{4fl}{2gd} \left(\frac{A}{a} \frac{\omega r}{\pi} \right)^2$$

iii. This is independent of θ, so indicator diagram will be rectangle.

iv. Work done by pump per stroke against friction,

W_2 =Area of rectangle = Base × Height

$$= L \frac{4fl}{2gd} \left(\frac{A}{a} \frac{\omega r}{\pi} \right)^2$$

$$= \frac{1}{\pi^2} L \frac{4fl}{2gd} \left(\frac{A}{a} r\omega \right)^2$$

3. Ratio of W_2 and W_1 is given as,

$$\frac{W_2}{W_1} = \frac{\frac{1}{\pi^2} \times L \times \frac{4fl}{2gd} \times \left(\frac{A}{a} r\omega \right)^2}{\frac{2}{3} \times L \times \frac{4fl}{2gd} \times \left(\frac{A}{a} r\omega \right)^2} = \frac{3}{2\pi^2} = 0.15198$$

4. Work saved is given as,

$$= W_1 - W_2$$

$$= L \frac{4fl}{2gd} \left(\frac{A}{a} r\omega \right)^2 \left[\frac{2}{3} - \frac{1}{\pi^2} \right]$$

5. Percentage of work saved per stroke,

$$= \frac{W_1 - W_2}{W_1} = 1 - \frac{W_2}{W_1}$$

$$= 1 - 0.15198 = 0.848$$

$$= 84.8 \%$$

Que 5.39. **Show that work saved against friction in the delivery pipe of a double acting reciprocating pump by fitting an air vessel is 39.20 %.**

Answer

1. Work lost in friction per stroke for double acting reciprocating pump is same as single acting pump, so

$$W_1 = \frac{2}{3} L \left[\frac{4fl}{2gd}\left(\frac{A}{a} r\omega\right)^2\right]$$

2. When the air vessel is fitted to pipe, mean velocity of flow, for double-acting pump is

$$\bar{v} = \frac{\text{Discharge}}{\text{Area of pipe}} = \frac{Q}{a}$$

$$= \frac{2ALN}{60a} \qquad \left(\because Q = \frac{2ALN}{60}\right)$$

$$= \frac{2A \times 2r \times 60\omega}{60a \times 2\pi} \qquad \left(\because N = \frac{60\omega}{2\pi} \text{ and } L = 2r\right)$$

$$= \frac{2A}{a} \frac{r\omega}{\pi}$$

3. Loss of head due to friction for double acting pump,

$$h_f = \frac{4fl}{2gd}\bar{v}^2 = \frac{4fl}{2gd}\left(\frac{2A}{a}\frac{r\omega}{\pi}\right)^2$$

4. Work lost against friction per stroke,

$$W_2 = \text{Area of rectangle}$$
$$= \text{Base} \times \text{Height}$$
$$= L \times \frac{4fl}{2gd}\left(\frac{2A}{a}\frac{r\omega}{\pi}\right)^2$$
$$= \frac{4}{\pi^2} L \left[\frac{4fl}{2gd}\left(\frac{A}{a} r\omega\right)^2\right]$$

5. Work saved per stroke is given as,

$$= \frac{W_1 - W_2}{W_1}$$

$$= \frac{\frac{2}{3} L \left[\frac{4fl}{2gd}\left(\frac{A}{a} r\omega\right)^2\right] - \frac{4}{\pi^2} L \left[\frac{4fl}{2gd} \times \left(\frac{A}{a} r\omega\right)^2\right]}{\frac{2}{3} L \left[\frac{4fl}{2gd}\left(\frac{A}{a} r\omega\right)^2\right]}$$

$$= \frac{\left(\frac{2}{3}\right) - \left(\frac{4}{\pi^2}\right)}{\frac{2}{3}} = 0.392 = 39.20\ \%$$

PART-1 1

Comparison of Centrifugal and Reciprocating Pumps.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.40. Differentiate between centrifugal and reciprocating pump. Also derive an expression for starting speed of a centrifugal pump.

AKTU 2014-15, Marks 10

Answer

A. Difference between Centrifugal and Reciprocating Pump :

S. No.	Centrifugal Pump	Reciprocating Pump
1.	It gives large discharge and less head.	It gives small discharge and high head.
2.	Priming is needed.	It is self primed.
3.	It is simple in construction.	Complicated construction.
4.	Maintenance cost is low.	Maintenance cost is high.
5.	Flywheel is not used.	Flywheel is used.
6.	Handle highly viscous fluid.	It can handle low viscous fluid.
7.	Installation cost is low.	High installation cost.
8.	Efficiency is high.	Low efficiency.
9.	Starting torque is more.	Low starting torque.
10.	It needs no air vessel.	Air vessel is used.

B. Starting Speed of Centrifugal Pump : Refer Q. 5.11, Page 5–14A, Unit-5.

PART-1 2

Performance Characteristics.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.41. **Discuss the operating characteristics curves for a reciprocating pump.**

Answer

1. The operating characteristic curves indicating the performance of a reciprocating pump are shown in Fig. 5.41.1.

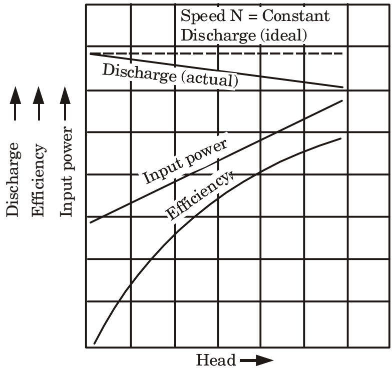

Fig. 5.41.1. Operating characteristic curves of a reciprocating pump.

2. These curves are obtained by plotting discharge, power input and overall efficiency against the head developed by the pump when it is operating at a constant speed.
3. As shown in Fig. 5.41.1, under ideal conditions the discharge of a reciprocating pump operating at constant speed is independent of the head developed by the pump.
4. However, in actual practice it is observed that the discharge of a reciprocating pump slightly decreases as the head developed by the pump increases.
5. Further the input power for a reciprocating pump increases almost linearly beyond a certain minimum value with the increase in the head developed by the pump.
6. The overall efficiency of a reciprocating pump also increases with the increase in the head developed by the pump as shown in Fig. 5.41.1.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Give the constructional details of a centrifugal pump. Also explain its working.

Ans. Refer Q. 5.2, Unit-5.

Q. 2. A centrifugal pump runs at 950 rpm, its outer and inner diameters are 500 mm and 250 mm. The vanes are set back at 35° to the wheel rim. If the radial velocity of water through the impeller is constant at 4 m/s, find (a) The angle of vane at the inlet. (b) The velocity of water at exit. (c) The direction of water at the outlet. (d) The work done by the impeller per kg of water. Assume entry of water at inlet is radial.

Ans. Refer Q. 5.8, Unit-5.

Q. 3. Define specific speed of a centrifugal pump and derive the equation for the same.

Ans. Refer Q. 5.13, Unit-5.

Q. 4. What is priming in centrifugal pump ? Why it is done ? What is self-priming pump ? Explain.

Ans. Refer Q. 5.15, Unit-5.

Q. 5. With the help of a neat sketch explain the construction and working principle of reciprocating pump.

Ans. Refer Q. 5.21, Unit-5.

Q. 6. A single acting reciprocating pump, running at 50 rpm delivers 0.00736 m^3/s of water. The diameter of the piston is 200 mm and stroke length 300 mm. The suction and delivery heads are 3.5 m and 11.5 m respectively. Determine :

i. Theoretical discharge
ii. Coefficient of discharge
iii. Percentage slip of the pump
iv. Power required to run the pump

Ans. Refer Q. 5.26, Unit-5.

Q. 7. What do you understand by an indicator diagram ? Explain ideal indicator diagram.

Ans. Refer Q. 5.28, Unit-5.

Q. 8. Derive an expression for accelerating head in reciprocating pump assuming piston motion by SHM.

Ans. Refer Q. 5.30, Unit-5.

Q. 9. Show that the work saved in overcoming friction in the pipelines by fitting air vessels is 84.8 % for a single acting reciprocating pump.

Ans. Refer Q. 5.38, Unit-5.

Q. 10. Differentiate between centrifugal and reciprocating pump. Also derive an expression for starting speed of a centrifugal pump.

Ans. Refer Q. 5.40, Unit-5.

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1 UNIT

Fluid and Bernoulli's Equation (2 Marks Questions)

1.1. Define the term fluid.

Ans. A fluid is a substance which deforms continuously when subjected to external shearing force.

1.2. Define control volume. **AKTU 2016-17, Marks 02**

Ans. For applying basic principles of fluid flow usually control volume approach is adopted, in which a definite volume with fixed boundary shape is chosen in space along the fluid flow passage. This definite volume is called the control volume.

1.3. Enumerate some important properties of liquid.

Ans. Some important properties of liquid are :

1. Density,
2. Viscosity,
3. Adhesion,
4. Specific gravity,
5. Cohesion, and
6. Surface tension.

1.4. Define viscosity.

Ans. Viscosity may be defined as the property of a fluid which determines its resistance to shearing stresses. It is a measure of the internal fluid friction which causes resistance to flow.

1.5. What is kinematic viscosity ?

Ans. Kinematic viscosity is defined as the ratio of the dynamic viscosity to the density of fluid. It is denoted by ν.

1.6. State the Newton's law of viscosity.

Ans. Newton's law of viscosity states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient of viscosity. Mathematically,

$$\tau = \mu \frac{du}{dy}$$

1.7. Draw the figure of shear stress v/s rate of deformation.

AKTU 2018-19, Marks 02

Ans.

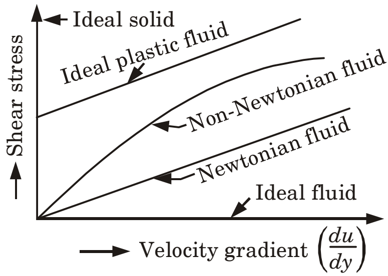

Fig. 1.7.1.

1.8. Define surface tension. **AKTU 2016-17, Marks 02**

Ans. Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

1.9. Define the term cohesion and adhesion.

AKTU 2017-18, Marks 02

Ans. **Cohesion :** It is the molecular attraction between similar types of molecules.

Adhesion : It is the molecular attraction between dissimilar types of molecules.

1.10. Describe the assumptions of Bernoulli's equation.

Ans. Assumptions of Bernoulli's equation are as follows :

1. Fluid is ideal *i.e.*, viscosity is zero.
2. Flow is incompressible.
3. Flow is steady.
4. Flow is irrotational.

1.11. State the Bernoulli's theorem.

Ans. Bernoulli's theorem states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant.

1.12. What is venturimeter ?

Ans. A venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe.

1.13. Write short note on pitot static tube.

AKTU 2017-18, Marks 02

Ans. Pitot tube is a device used for measuring the velocity of flow at any point or a channel. It works on the principle of Bernoulli's theorem.

1.14. What is stagnation point ?

Ans. Stagnation point is a point where the velocity of the fluid becomes zero.

1.15. Write the advantages of triangular notch or weir over rectangular notch or weir.

Ans. Advantages of triangular notch or weir over rectangular notch or weir are as follows :

1. Ventilation of a triangular notch or weir is not necessary.
2. For measuring low discharge, a triangular notch or weir is preferred.

1.16. What is coefficient of discharge ?

Ans. Coefficient of discharge is defined as the ratio of the actual discharge to the theoretical discharge of flow.

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2 UNIT

Types of Fluid Flow and Continuity Equation (2 Marks Questions)

2.1. Differentiate between steady and unsteady flow.

AKTU 2015-16, Marks 02

Ans. **Steady Flow :** The type of flow in which the fluid properties like velocity, pressure, density, etc. at a point do not change with time is called steady flow.

Unsteady Flow : Unsteady flow is that type of flow in which the velocity, pressure or density at a point change with respect to time.

2.2. Define laminar and turbulent flow.

Ans. **Laminar Flow :** A flow in which paths taken by individual particle do not cross one another and move along well defined paths is known as laminar flow.

Turbulent Flow : A turbulent flow is that flow in which fluid particles move in a zig-zag way.

2.3. Explain the rotational and irrotational flow.

AKTU 2017-18, Marks 02

OR

Define rotational and irrotational flow.

AKTU 2016-17, Marks 02

Ans. **Rotational Flow :** A flow is said to be rotational if the fluid particles while moving in the direction of flow rotate about their mass centers.

Irrotational Flow : A flow is said to be irrotational if the fluid particles while moving in the direction of flow do not rotate about their mass centers.

2.4. Define the continuity equation.

Ans. The equation based on the principle of conservation of mass is called continuity equation.

2.5. What do you understand by circulation ?

AKTU 2015-16, Marks 02

Ans. Circulation is defined as the line integral of the tangential velocity about a closed path (contour). Circulation around regular curves can be obtained by integration.

2.6. Write down the definition of stream function.

AKTU 2015-16, Marks 02

Ans. Stream function is defined as a function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to this direction. It is denoted by ψ.

$$u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}$$

2.7. The velocity distribution between two parallel plate is given by $u = (a^2 - y^2)$, where u is the velocity at a distance y from the middle of the two plates. Find the expression for stream function.

AKTU 2018-19, Marks 02

Ans.

Given : $u = (a^2 - y^2)$
To Find : Expression for stream function.

$$\frac{\partial \psi}{\partial y} = -u$$

$$\partial \psi = -(a^2 - y^2)\, dy = (y^2 - a^2)\, dy$$

On integration, $$\psi = \frac{y^3}{3} - ya^2 + c$$

2.8. Discuss velocity potential function.

Ans. Velocity potential function is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is denoted by ϕ.

$$u = -\frac{\partial \phi}{\partial x}, v = -\frac{\partial \phi}{\partial y}, w = -\frac{\partial \phi}{\partial z}$$

2.9. What do you understand by Reynolds number ?

Ans. Reynolds number is defined as the ratio of inertia force of a flowing fluid to the viscous force of the fluid. Thus,

$$R_e = \frac{v d}{\nu} \text{ or } \frac{\rho v d}{\mu}$$

2.10. What do you understand by dimensional homogeneity ?

AKTU 2018-19, Marks 02

Ans. Dimensional homogeneity means the dimensions of each term in an equation on both sides are equal.

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3 UNIT

Flow Through Pipes, Boundary Layer Thickness (2 Marks Questions)

3.1. Write the characteristics of laminar flow.

Ans. Characteristics of laminar flow are as follows :
1. Flow is irrotational.
2. No slip will occur at the boundary.
3. Each fluid layer flows separately.

3.2. What do you understand by kinetic energy correction factor ? **AKTU 2015-16, 2016-17, Marks 02**

Ans. Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy of the flow per second based on average velocity across the same section.

3.3. When will a laminar flow change to turbulent flow ?

Ans. A laminar flow may change to turbulent flow when :
1. There is an increased velocity of flow.
2. There is an increased diameter of pipe.
3. The viscosity of fluid is decreased.

3.4. Give some examples of turbulent flow.

Ans. Some examples of turbulent flow are as follows :
1. Smoke rising from a cigarette.
2. Flow over a golf ball.
3. The mixing of warm and cold air in the atmosphere.

3.5. Define eddy viscosity.

Ans. The viscosity which accounts for momentum transport by turbulent eddies is known as eddy viscosity.

3.6. What does Hagen Poiseuille equation refer to ? What is Hagen Poiseuille's formula ? **AKTU 2016-17, Marks 02**

Ans. Hagen Poiseuille equation refers to loss of pressure head.

Hagen Poiseuille's formula is given by :

For circular plate = $\dfrac{32\,\mu \bar{u} L}{\rho\, g d^2}$

For parallel plate = $\dfrac{12\,\mu \bar{u} L}{\rho\, g b^2}$

3.7. Define surface loss. **AKTU 2018-19, Marks 02**

Ans. Surface loss is loss of pressure or head that occurs in pipe flow due to effect of the fluid's viscosity near the surface of pipe.

3.8. What do you understand by TEL and HGL ?

AKTU 2015-16, 2016-17, Marks 02

Ans. **TEL :** Total energy line (TEL) is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

HGL : Hydraulic gradient line is defined as the line which gives the sum of pressure head and datum head of a flowing fluid with respect to some reference line.

3.9. In which cases syphon is used ?

Ans. Syphon is used in the following cases :

1. To carry water from one reservoir to another reservoir separated by a hill or ridge.
2. To empty a channel not provided with any outlet sluice.

3.10. Define water hammer in pipes.

Ans. The wave of high pressure has the effect of hammering action on the walls of the pipe. This phenomenon is known as water hammer in pipes.

3.11. What are the necessary conditions for a pipe network ?

Ans. Followings are the necessary conditions for any network of pipes :

1. Flow into each junction must be equal to the flow out of the junction.
2. The algebraic sum of head losses around each loop must be zero.

3.12. What do you understand by displacement thickness ?

AKTU 2015-16, Marks 02

OR

What is displacement thickness ?

AKTU 2016-17, Marks 02

Ans. Displacement thickness is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow on account of boundary layer formation.

3.13. What do you understand by shape factor ?

AKTU 2017-18, Marks 02

Ans. The ratio of momentum thickness (θ) to displacement thickness (δ*) is known as shape factor.

3.14. Define turbulent boundary layer and laminar sub layer.

Ans. **Turbulent Boundary :** Downstream to the transition zone, the boundary layer is turbulent and continues to grow in thickness. This layer of boundary is called turbulent boundary layer.

Laminar Sub Layer : This is the region in boundary layer zone, adjacent to solid surface of the plate.

3.15. Explain the drag and lift. **AKTU 2017-18, Marks 02**

Ans. **Drag :** The component of the total force (F_R) in the direction of motion is called drag. This component is denoted by F_D.

Lift : The component of the total force (F_R) in the direction perpendicular to the direction of motion is known as lift. This is denoted by F_L.

3.16. What is average coefficient of drag ?

Ans. Average coefficient of drag is defined as the ratio of the total drag force to the quantity $\frac{1}{2}\rho A U^2$. It is also called coefficient of drag and is denoted by C_D.

3.17. Define aerofoil.

Ans. An aerofoil is a streamlined body which may be either symmetrical or unsymmetrical.

3.18. Discuss Magnus effect.

Ans. When a cylinder is rotated in a uniform flow, a lift force is produced on the cylinder. This phenomenon is known as Magnus effect.

3.19. Define impulse momentum equation.

AKTU 2016-17, Marks 02

Ans. It states that the impulse of force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum $d(m\text{v})$ in direction of force.

Mathematically,

$$F dt = d(m\text{v})$$

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4 UNIT

Impact of Jet, Impulse Turbine and Reaction Turbines (2 Marks Questions)

4.1. What are fluid machines or hydraulic machines ?

AKTU 2015-16, 2017-18; Marks 02

Ans. Hydraulic machines are defined as the machines which convert either hydraulic energy into mechanical energy or mechanical energy into hydraulic energy.

4.2. Define the term turbines.

Ans. Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy.
Example : Pelton wheel, Francis turbine, Kaplan turbine, etc.

4.3. Define overall efficiency.

Ans. Overall efficiency is defined as the ratio of power available at the shaft of the turbine to the power supplied by the water at the inlet of the turbine.

4.4. Define penstocks.

Ans. Water from storage reservoir is carried through pipes of large diameters usually made of steel or reinforced concrete to the turbine, these pipes are known as penstocks.

4.5. What is the function of the nozzle in an impulse turbine ?

AKTU 2016-17, Marks 02

Ans. A nozzle is pipe of varying cross-sectional area used to direct or modify the fluid flow. It converts the pressure energy of fluid into kinetic energy.

4.6. State the function of breaking jet in Pelton wheel turbine.

AKTU 2017-18, Marks 02

Ans. When the nozzle is completely closed by the motion of spear in forward direction, the amount of water striking the runner reduces to zero. But due to inertia, runner goes on revolving. Therefore a breaking jet is used to stop this revolving wheel.

4.7. Define runaway speed of turbine.

AKTU 2015-16, Marks 02

Ans. Runaway speed is the maximum speed at which a turbine would run when there is no external load but operating under design head and discharge.

4.8. Why is the shape of the bucket of a Pelton wheel like two spoons ?

Ans. The advantage of having two spoons lies in the fact that axial forces neutralize each other, being equal and opposite and hence the bearings support the wheel shaft are not subjected to any axial thrust.

4.9. What is the function of needle spear in Pelton wheel ?

Ans. A needle spear moving inside the nozzle controls the water flow through the nozzle and at the same time provides a smooth flow with negligible energy loss.

4.10. Why is governing of a turbine necessary ?

Ans. Governing of a turbine is necessary as a turbine is directly coupled to an electric generator, which is required to run at constant speed under all fluctuating loads conditions. This is possible only when the speed of turbine is constant.

4.11. Differentiate between impulse turbine and a reaction turbine.

AKTU 2015-16, Marks 02

Ans.

S. No.	Impulse Turbine	Reaction Turbine
1.	If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as impulse turbine.	If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as reaction turbine.
2.	Water may be allowed to enter a part or whole of the wheel circumference.	Water is admitted over the circumference of the wheel.

4.12. What do you mean by radial flow turbine ?

AKTU 2016-17, Marks 02

Ans. Radial flow turbines are those turbines in which the water flows in the radial direction. The water may flow radially from outwards to inwards direction or from inwards to outwards direction.

4.13. Why spiral casing of varying area is employed in reaction turbine ?

AKTU 2017-18, Marks 02

Ans. Spiral casing of varying area is employed in reaction turbine, so that flow velocity can be kept constant throughout the circumference.

4.14. What is the significance of specific speed ?

Ans. Specific speed plays an important role for selecting the type of the turbine. Also the performance of a turbine can be predicted by knowing the specific speed of the turbine.

4.15. Define unit speed.

AKTU 2016-17, Marks 02

Ans. Unit speed is defined as the speed of a turbine working under a unit head (*i.e.*, under a head of 1 m). It is denoted by N_u.

$$N_u = \frac{N}{\sqrt{H}}$$

4.16. List the characteristic curves of hydraulic turbine.

AKTU 2015-16, Marks 02

Ans. Following are the important characteristic curves for the hydraulic turbine :

1. Main characteristic curves or constant head curves,
2. Operating characteristic curves or constant speed curves, and
3. Constant efficiency curves or iso-efficiency curves.

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5 UNIT

Centrifugal and Reciprocating Pumps (2 Marks Questions)

5.1. What is centrifugal pump ?

Ans. The hydraulic machine which converts the mechanical energy into pressure energy by means of centrifugal force acting on the fluid is known as centrifugal pump.

5.2. Differentiate between volute and vortex casing of a centrifugal pump.

AKTU 2017-18, Marks 02

Ans.

S. No.	Volute Casing	Vortex Casing
1.	It has less efficiency.	It has more efficiency.
2.	More eddy currents are present.	Less eddy currents are presents.

5.3. What is meant by manometric head for centrifugal pump ?

AKTU 2017-18, Marks 02

Ans. Manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by H_m.

5.4. Differentiate between static head and manometric head.

AKTU 2015-16, Marks 02

Ans.

S. No.	Static Head	Manometric Head
1.	The sum of suction head and delivery head is known as static head.	It is defined as the head against which a centrifugal pump has to work.
2.	$H_s = h_s + h_d$	$H_m = \left(Z_d + \frac{p_d}{\rho g} + \frac{v_d^2}{2g}\right) - \left(Z_s + \frac{p_s}{\rho g} + \frac{v_d^2}{2g}\right)$

5.5. Define manometric efficiency. **AKTU 2016-17, Marks 02**

Ans. Manometric efficiency is defined as the ratio of the manometric head developed by the pump to the head imparted by the impeller to the liquid.

5.6. Define the specific speed of a centrifugal pump.

Ans. Specific speed is defined as the speed of a geometrically similar pump which would deliver one cubic meter of liquid per second against a head of one meter. It is denoted by N_s.

5.7. What are the advantages of model testing ?

AKTU 2016-17, Marks 02

Ans. Advantages of model testing are as follows :

1. Model tests are economical and convenient.
2. The performance and efficiency of a hydraulic machine or structure can be predicted in advance by model testing.
3. To know about the safety and reliability of the parts, which cannot be exactly checked by analytical methods, model testing is required.

5.8. What is the purpose of priming of a centrifugal pump ?

AKTU 2015-16, Marks 02

Ans. The density of air is low, so head generated by pump is also low even negligible and hence water may not be sucked by the pump. To avoid this difficulty priming of centrifugal pump is necessary.

5.9. What is NPSH ? **AKTU 2017-18, Marks 02**

Ans. Net positive suction head (NPSH) is defined as the absolute pressure head at the inlet to the pump minus the vapour pressure head plus the velocity head.

5.10. What is the significance of characteristic curves ?

Ans. Characteristic curves are necessary to predict the behaviour and performance of the pump when the pump is working under different flow rate, head and speed.

5.11. What is meant by positive displacement pump ?

AKTU 2017-18, Marks 02

Ans. Pumps in which the liquid is sucked and then it is actually pushed or displaced due to the thrust exerted on it by a moving member are known as positive displacement pump.

5.12. Define the term slip of reciprocating pump.

AKTU 2017-18, Marks 02

Ans. Slip is defined as the difference between the theoretical discharge and actual discharge of the pump.

5.13. Define negative slip. When it occur in reciprocating pump ?

Ans. If the actual discharge of the pump is more than the theoretical discharge, the slip will be negative, which is known as negative slip. Negative slip occurs when delivery pipe is short, suction pipe is long and pipe is running at high speed.

5.14. Define ideal indicator diagram.

Ans. The graph between pressure head in the cylinder and stroke length of the piston for one complete revolution of the crank under ideal condition is known as ideal indicator diagram.

5.15. What do you mean by maximum speed of a reciprocating pump ?

AKTU 2015-16, Marks 02

Ans. Maximum speed of a reciprocating pump is defined as the speed without separation or the speed before which separation of liquid takes place.

5.16. What is the cause of acceleration head ?

AKTU 2016-17, Marks 02

Ans. The velocity of flow of water in the suction and delivery pipe is not uniform, being zero at beginning and end of the stroke and maximum at the center of stroke. This is the main cause of acceleration head.

5.17. What is the purpose of an air vessel fitted in the pump ?

AKTU 2015-16, Marks 02

Ans. The air vessel is used for the following purposes :

1. To obtain a continuous supply of liquid at a uniform rate.
2. To save the power required to drive the pump.
3. To run the pump at high speed without separation.

5.18. What will be the total % work saved by fitting the air vessel ? Explain.

AKTU 2015-16, Marks 02

Ans. Work saved in single acting reciprocating pump is 84.8 % while in double acting reciprocating pump the work saved is 39.2 %.

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B.Tech.

(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2019-20

FLUID MECHANICS AND FLUID MACHINES

Time : 3 Hours **Max. Marks : 70**

Note : Attempt **all** sections. If require any missing data; then choose suitably.

Section-A

1. Attempt **all** questions in brief. **(2 × 10 = 20)**

a. **2 liter petrol weighs 14 N. Calculate the specific weight, mass density, specific volume and specific gravity of petrol with respect to water.**

b. **Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.**

c. **What do you understand by Euler's number ?**

d. **State Bernoulli's theorem.**

e. **What is water hammering ?**

f. **A square flat plate of dimension 1.5 m moves at 50 km/hr in stationary air of density 1.15 kg/m^3. If the coefficient of drag and lift are 0.15 and 0.75 respectively, determine the lift and drag force.**

g. **Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with a velocity of 20 m/s.**

h. **Differentiate between turbine and pump.**

i. **How will you classify the turbines ?**

j. **Define slip, percentage slip and negative slip of a reciprocating pump.**

Section-B

2. Attempt any **three** of the following : **(10 × 3 = 30)**

a. Develop a formula for capillary rise of a fluid having surface tension σ and contact angle θ between :

i. Two concentric glass tubes of radii r_0 and r_i, and

ii. Two vertical glass plates set parallel to each other having a gap t between them.

b. The velocity potential for a two dimensional flow is $\phi = x(2y - 1)$. Determine the velocity at the point $P(4, 5)$. Also obtain the value of stream function at P.

c. Determine the displacement thickness, momentum thickness, shape factor and energy thickness of the following velocity profiles in the boundary layer on a flat plate. $\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$ where u is the velocity at a height y above the surface and U_0 is the free stream velocity.

d. Define the term governing of turbine. Describe with neat sketch the working of an oil pressure governor.

e. What do you mean by manometric efficiency, mechanical efficiency and overall efficiency of a centrifugal pump ?

Section-C

3. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A 30 cm diameter pipe conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

b. What is pitot tube ? How will you determine the velocity at any point with the help of pitot tube ?

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. If the velocity field is given by $u = x + y$ and $v = x^3 - y$. Find the circulation around a closed contour defined by $x = 1$, $y = 0$, $y = 1$ and $x = 0$.

b. The pressure difference Δp in a pipe of diameter D and length l due to viscous flow depends on the velocity v, viscosity μ and density ρ. Using Buckingham π theorem, obtain the expression for Δp.

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A fluid of viscosity 0.7 N-s/m^2 and specific gravity 1.3 is flowing through circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as 196.2 N/m^2 find :

i. Pressure gradient,

ii. Average velocity, and

iii. Reynolds number of the flow.

b. Describe the phenomenon of boundary layer formation over a smooth flat plate.

6. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 liters/s under the head of 30 meters. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume coefficient of velocity as 0.98.

b. With the help of neat sketch explain the working of Kaplan turbine.

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Define specific speed of a centrifugal pump. Derive an expression for the same.

b. Discuss the effect of acceleration in suction and delivery pipes on indicator diagram.

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SOLUTION OF PAPER (2019-20)

Note : Attempt **all** sections. If require any missing data; then choose suitably.

Section-A

1. Attempt **all** questions in brief. **(2 × 10 = 20)**

a. 2 liter petrol weighs 14 N. Calculate the specific weight, mass density, specific volume and specific gravity of petrol with respect to water.

Ans.

Given : $V = 2 \text{ ltr} = 2 \times 10^{-3} \text{ m}^3$, $W = 14$ N.

To Find :
- i. Specific weight (w),
- ii. Mass density (ρ),
- iii. Specific volume (v), and
- iv. Specific gravity (S).

1. Specific weight, $$w = \frac{W}{V} = \frac{14}{2 \times 10^{-3}} = 7000 \text{ N/m}^3$$

2. Mass density, $$\rho = \frac{m}{V} = \frac{w}{g} = \frac{7000}{9.81} = 713.5576 \text{ kg/m}^3$$

3. Specific volume, $$v = \frac{1}{\rho} = \frac{1}{713.5576} = 1.401 \times 10^{-3} \text{ m}^3/\text{kg}$$

4. Specific gravity, $$S = \frac{\text{Density of petrol}}{\text{Density of water}} = \frac{713.5576}{1000} = 0.714$$

b. Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.

Ans.

Given : $d = 40 \text{ mm} = 40 \times 10^{-3}$ m, $p = 2.5 \text{ N/m}^2$
To Find : Surface Tension (σ).

1. For soap bubble, $$p = \frac{8\sigma}{d}$$

2. Surface tension, $$\sigma = \frac{pd}{8} = \frac{2.5 \times 40 \times 10^{-3}}{8} = 0.0125 \text{ N/m}$$

c. What do you understand by Euler's number ?

Ans.

1. It is the square root of the ratio of the inertia force to the pressure force of a flowing fluid.

Mathematically, $$E_u = \sqrt{\frac{F_i}{F_p}}$$

2. $$\text{Pressure force } (F_p) = \text{Pressure} \times \text{Area} = p \times A$$

$$\text{Inertia force } (F_i) = \rho A v^2$$

$\therefore$ $$E_u = \sqrt{\frac{\rho A v^2}{pA}} = \frac{v}{\sqrt{p/\rho}}$$

i. Significance :

1. It signifies those flow problems or situations in which pressure gradient exists.

ii. Applications :

1. Discharge through orifice and mouth piece.
2. Pressure rise due to sudden closure of valves.
3. Flow through pipes.
4. Water hammer created in penstocks.

d. State Bernoulli's theorem.

Ans.

1. Bernoulli's theorem states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant.
2. It can be mathematically stated as given below,

Pressure energy + Kinetic energy + Potential energy = Constant

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant}$$

3. Bernoulli's equation for real fluids is,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

Where, h_L = Loss of energy.

e. What is water hammering ?

Ans. In a long pipe, when flowing water is suddenly brought to rest by closing the valve or by any similar cause, there will be a sudden rise in pressure due to the momentum of water being destroyed. This phenomenon of sudden rise in pressure is known as water hammer or hammer blow.

f. A square flat plate of dimension 1.5 m moves at 50 km/hr in stationary air of density 1.15 kg/m³. If the coefficient of drag and lift are 0.15 and 0.75 respectively, determine the lift and drag force.

Ans.

Given : Area of plate, $A = 1.5 \times 1.5 = 2.25\ m^2$, v = 50 km/hr = 13.89 m/s, Density of air, $\rho = 1.15\ kg/m^3$, Coefficient of drag, $C_D = 0.15$, Coefficient of lift, $C_L = 0.75$.
To Find : i. Lift force, and
ii. Drag force.

1. Lift force,

$$F_L = C_L A \frac{\rho v^2}{2}$$

$$= 0.75 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} = 187.2\ N$$

2. Drag force, $F_D = C_D A \frac{\rho v^2}{2}$

$$= 0.15 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} = 37.44\ N$$

g. Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with a velocity of 20 m/s.

Ans.

Given : Diameter of jet, d = 75 mm = 0.075 m, v = 20 m/s
To Find : Force exerted by a jet of water.

1. Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.075)^2 = 0.004417\ m^2$
2. The forced exerted by the jet of water on a stationary vertical plate is given by,

$$F = \rho a v^2$$
$$= 1000 \times 0.004417 \times 20^2 \quad (\because \rho = 1000\ kg/m^3)$$
$$= 1766.8\ N$$

h. Differentiate between turbine and pump.

Ans.

S. No.	Turbine	Pump
1.	It converts hydraulic energy into mechanical energy.	It converts mechanical energy into hydraulic energy.
2.	It decreases the energy of the fluid.	It increases the energy of the fluid.

i. How will you classify the turbines ?

Ans. Hydraulic turbines are classified as follows :

i. According to the Type of Energy Available at Inlet :

1. Impulse turbine, and
2. Reaction turbine.

ii. According to the Direction of Flow through Runner :

1. Tangential flow turbine,
2. Radial flow turbine,
3. Axial flow turbine, and
4. Mixed flow turbine.

iii. According to the Head at Inlet of Turbine :

1. High head turbine,
2. Medium head turbine, and
3. Low head turbine.

iv. According to the Specific Speed of the Turbine :

1. Low specific speed turbine,
2. Medium specific speed turbine, and
3. High specific speed turbine.

j. Define slip, percentage slip and negative slip of a reciprocating pump.

Ans.

i. Slip and Percentage Slip of a Pump :

1. It is defined as the difference between the theoretical discharge and actual discharge.

$$\text{Slip} = Q_{\text{th}} - Q_{\text{act}}$$

2. The slip is mostly expressed as percentage slip which is given by,

$$\text{Percentage slip} = \frac{Q_{\text{th}} - Q_{\text{act}}}{Q_{\text{th}}} \times 100$$

$$= \left(1 - \frac{Q_{\text{act}}}{Q_{\text{th}}}\right) \times 100 = (1 - C_d) \times 100$$

3. For most of the reciprocating pumps the actual discharge Q_{act} is less than the theoretical discharge Q_{th}, C_d is less than one and the slip of the pump is positive.

ii. Negative Slip :

1. If actual discharge of the pump is more than the theoretical discharge, the slip will be negative, which is known as negative slip.
2. Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

Section-B

2. Attempt any **three** of the following : **(10 × 3 = 30)**

a. Develop a formula for capillary rise of a fluid having surface tension σ and contact angle θ between :

i. Two concentric glass tubes of radii r_0 and r_i, and

ii. Two vertical glass plates set parallel to each other having a gap t between them.

Ans.

i. Capillary Rise when Two Concentric Glass Tubes :

1. $$T \cos \theta = \pi (r^2_0 - r_i^2)\, h\rho g \quad ...(1)$$
But $$T = \sigma \pi(r_0 + r_i)$$
2. Substituting value of T in eq. (1), we get
$$\sigma\pi(r_0 + r_i) \cos \theta = \pi(r^2_0 - r_i^2)\, h\rho g$$
$$\sigma\pi (r_0 + r_i) \cos \theta = \pi(r_0 + r_i)(r_0 - r_i)\, h\rho g$$
3. Capillary rise, $$h = \frac{\sigma \cos \theta}{(r_0 - r_i)\, \rho g}$$

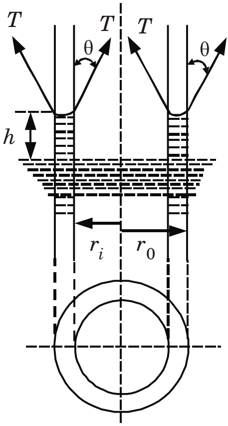

Fig. 1.

B. Capillary Rise when Two Vertical Glass Plates Set Parallel :

1. Let, σ = Surface tension,
θ = Contact angle.
h = Height of liquid between plates above general liquid surface.
2. The weight of liquid of height h is balanced by the force between the plates = Volume of liquid of height h between the plates × w
$$= t \times L \times h \times w \quad ...(2)$$
Where, L = Length of plate, and
w = Weight density of the liquid.
3. Vertical component of surface tensile force
$$= (\sigma \times \text{circumference}) \times \cos \theta$$
$$= \sigma \times 2L \times \cos \theta \quad ...(3)$$
4. For equilibrium, eq. (2) and eq. (3) must balance.
$$t \times L \times h \times w = \sigma \times 2L \times \cos \theta$$
or $$h = \frac{2\sigma \cos \theta}{t \times w}$$

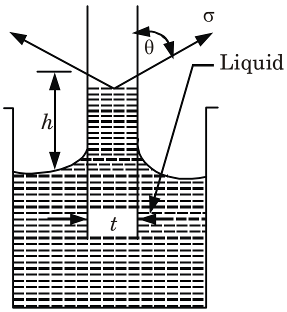

Fig. 2.

ii. The velocity potential for a two dimensional flow is $\phi = x(2y - 1)$. Determine the velocity at the point $P(4, 5)$. Also obtain the value of stream function at P.

Ans.

Given : $\phi = x(2y - 1)$, $P(4, 5)$
To Find : i. Velocity at point P.
ii. Stream function at point P.

1. The velocity components in the direction of x and y are,

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}[x(2y-1)] = -[2y-1] = 1 - 2y$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}[x(2y-1)] = -[2x] = -2x$$

2. At point $P(4, 5)$, *i.e.*, at $x = 4$, $y = 5$

$$u = 1 - 2 \times 5 = -9 \text{ units/s}$$
$$v = -2 \times 4 = -8 \text{ units/s}$$

3. Resultant velocity at $P = \sqrt{(-9)^2 + (-8)^2} = \sqrt{81 + 64}$

$$= 12.04 \text{ units/s}$$

4. We know that, $\frac{\partial \psi}{\partial y} = u = 1 - 2y$ ...(1)

and $\frac{\partial \psi}{\partial x} = -v = 2x$ ...(2)

5. Integrating eq. (1) w.r.t 'y', we get

$$\int d\psi = \int (1 - 2y)\, dy$$

$$\psi = y - \frac{2y^2}{2} + K$$

$$\psi = y - y^2 + K \quad ...(3)$$

The constant of integration K is not a function of y but it can be a function of x.

6. Differentiating the eq. (3) w.r.t x,

$$\frac{\partial \psi}{\partial x} = \frac{\partial K}{\partial x}$$

But from eq. (2)

$$\frac{\partial \psi}{\partial x} = 2x$$

7. Equating the value of $\frac{\partial \psi}{\partial x}$, we get

$$\frac{\partial K}{\partial x} = 2x$$

Integrating this equation,

$$K = \int 2x dx = \frac{2x^2}{2} = x^2$$

8. Substituting this value of K in eq. (3), we get

$$\psi = y - y^2 + x^2.$$

9. Stream function ψ at P (4, 5) = $5 - 5^2 + 4^2 = 5 - 25 + 16 = -4$ units

c. Determine the displacement thickness, momentum thickness, shape factor and energy thickness of the following velocity profiles in the boundary layer on a flat plate. $\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$ where u is the velocity at a height y above the surface and U_0 is the free stream velocity.

Ans.

Given : $\frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}$

To Find : i. Displacement thickness.
ii. Momentum thickness.
iii. Shape factor.
iv. Energy thickness.

1. The displacement thickness δ^* is given by,

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_0}\right) dy$$

$$= \int_0^\delta \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy \qquad \left[\because \frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}\right]$$

$$= \int_0^\delta \left[1 - \frac{y^{1/7}}{\delta^{1/7}}\right] dy$$

$$= \left[y - \frac{y^{1/7+1}}{\frac{8}{7}\delta^{1/7}} \right]_0^{\delta}$$

$$= \left[\delta - \frac{7}{8}\frac{\delta^{8/7}}{\delta^{1/7}} \right] = \delta - \frac{7}{8}\delta = \frac{\delta}{8}$$

2. The momentum thickness θ is given by,

$$\theta = \int_0^{\delta} \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy$$

$$= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{1/7} \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy \qquad \left[\because \frac{u}{U_0} = \left(\frac{y}{\delta}\right)^{1/7}\right]$$

$$= \int_0^{\delta} \left(\frac{y^{1/7}}{\delta^{1/7}} - \frac{y^{2/7}}{\delta^{2/7}}\right) dy$$

$$= \left[\frac{y^{1/7+1}}{\frac{8}{7}\delta^{1/7}} - \frac{y^{2/7+1}}{\frac{9}{7}\delta^{2/7}} \right]_0^{\delta} = \left[\frac{7}{8}\frac{\delta^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{\delta^{9/7}}{\delta^{2/7}} \right]$$

$$= \left[\frac{7}{8}\delta - \frac{7}{9}\delta \right] = \left[\frac{63 - 56}{72} \right]\delta = \frac{7}{72}\delta$$

3. Shape factor $= \dfrac{\theta}{\delta^*} = \dfrac{7}{72}\delta \times \dfrac{8}{\delta} = \dfrac{7}{9}$

4. Energy thickness δ_e is given by,

$$\delta_e = \int_0^{\delta} \frac{u}{U_0}\left(1 - \frac{u^2}{U_0^{\,2}}\right) dy$$

$$= \int_0^{\delta} \left(\frac{y}{\delta}\right)^{1/7} \left[1 - \left(\frac{y}{\delta}\right)^{2/7}\right] dy$$

$$= \int_0^{\delta} \left(\frac{y^{1/7}}{\delta^{1/7}} - \frac{y^{3/7}}{\delta^{3/7}}\right) dy$$

$$= \left[\frac{y^{1/7+1}}{\frac{8}{7}\delta^{1/7}} - \frac{y^{3/7+1}}{\frac{10}{7}\delta^{3/7}} \right]_0^{\delta}$$

$$= \left[\frac{7}{8}\frac{\delta^{8/7}}{\delta^{1/7}} - \frac{7}{10}\frac{\delta^{10/7}}{\delta^{3/7}} \right]$$

$$= \left[\frac{7}{8}\delta - \frac{7}{10}\delta \right]$$

$$= \frac{7}{40}\delta$$

d. Define the term governing of turbine. Describe with neat sketch the working of an oil pressure governor.

Ans.

A. Governing of Turbines :

1. It is defined as the operation by which the speed of the turbine is kept constant under all conditions.
2. It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load conditions on the turbine.

B. Working of Oil Pressure Governing :

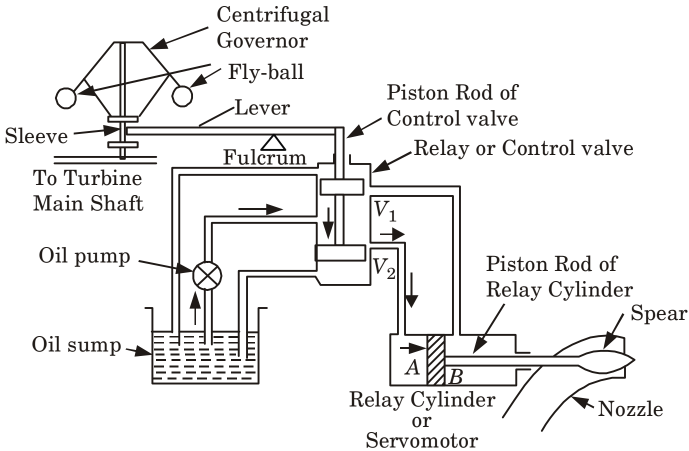

Fig. 3. Oil Pressure Governor.

i. When the Load on Generator Decreases :

1. When the load on the generator decreases, the speed of generator and hence the turbine increases beyond normal speed.
2. The fly-balls of the centrifugal governor move outward due to the increased centrifugal force on them.
3. Due to the outward movement of the fly-balls, the sleeve moves up. As a consequence the portion of the lever to the right of the fulcrum moves down pushing the piston rod of the control valve downwards.
4. This closes the valve V_1 and opens the valve V_2.
5. A gear pump pumps oil from the oil sump to the relay valve or control valve. Oil flows through valve V_2 and exerts force on the face A of the piston of the relay cylinder.
6. The piston rod along with the spear moves to the right. This decreases the area of flow of the nozzle and hence, the rate of water flows to the turbine.
7. Consequently the speed of the turbine decreases till it becomes normal.

ii. When the Load on Generator Increases :

1. When the load on the generator increases, the speed of generator and hence the turbine decreases beyond normal speed.
2. The fly-balls of the centrifugal governor move inward due to the decreased centrifugal force on them.
3. Due to the inward movement of the fly-balls, the sleeve moves down and the piston rod of control valve goes up.
4. This closes the valve V_2 and opens the valve V_1.
5. Oil flows through valve V_1 and exerts force on the face B of the piston of the relay cylinder.
6. The piston rod along with the spear moves to the left. This increases the area of flow of the nozzle and hence, the rate of water flows to the turbine.
7. As a consequence, the speed of the turbine increases till it becomes normal.

e. What do you mean by manometric efficiency, mechanical efficiency and overall efficiency of a centrifugal pump ?

Ans.

i. Manometric Efficiency (η_{mano}) : It is defined as the ratio of the manometric head developed by the pump to the head imparted by the impeller to the liquid.

$$\eta_{mano} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to liquid}}$$

$$= \frac{H_m}{\left(\dfrac{v_{w_2} u_2}{g}\right)} = \frac{gH_m}{v_{w2}\, u_2}$$

ii. Mechanical Efficiency (η_m) :

1. It is defined as the ratio of the power delivered by the impeller to the power input to the pump shaft.

$$\eta_m = \frac{\text{Power delivered at impeller}}{\text{Power input to the shaft}}$$

2. Power delivered at impeller in kW

$$= \frac{\text{Work done by impeller per second}}{1000}$$

$$= \frac{W}{g} \times \frac{v_{w_2} u_2}{1000}$$

$$\therefore \qquad \eta_m = \frac{\dfrac{W}{g}\left(\dfrac{v_{w_2} u_2}{1000}\right)}{SP}$$

iii. Overall Efficiency (η_o) :

1. The overall efficiency of the pump is defined as the ratio of the power output from the pump to the power input from the prime mover driving the pump.

$$\eta_o = \frac{\text{Power output}}{\text{Power input}}$$

2. $$\text{Power output} = \frac{\text{Weight of water lifted} \times H_m}{1000}$$

$$\text{Power input} = \text{Shaft power}$$

$$\therefore \quad \eta_o = \frac{WH_m / 1000}{\text{SP}}$$

or $$\eta_o = \eta_{mano} \times \eta_m$$

Section-C

3. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A 30 cm diameter pipe conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

Ans.

Given : d_1 = 30 cm, v_1 = 2.5 m/s, d_2 = 20 cm, v_2 = 2 m/s, d_3 = 15 cm
To Find : i. Discharge in pipe (1), Q_1.
ii. Velocity in pipe (3), v_3.

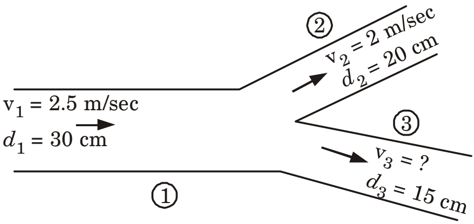

Fig. 4.

1. Area of pipe (1),

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 \text{ m}^2$$

$$v_1 = 2.5 \text{ m/s}$$

2. Area of pipe (2),

$$A_2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$$

$$v_2 = 2 \text{ m/s}$$

3. Area of pipe (3),

$$A_3 = \frac{\pi}{4}(0.15)^2 = 0.01767 \text{ m}^2$$

4. Let Q_1, Q_2 and Q_3 are discharge in pipe 1, 2 and 3 respectively. Then according to continuity equation,

$$Q_1 = Q_2 + Q_3 \quad \text{...(1)}$$

5. The discharge Q_1 in pipe 1 is given by,

$$Q_1 = A_1 v_1 = 0.07068 \times 2.5 = 0.1767 \text{ m}^3\text{/s}$$

6. The discharge Q_2 in pipe 2 is given by,

$$Q_2 = A_2 v_2 = .0314 \times 2.0 = 0.0628 \text{ m}^3\text{/s}$$

7. Substituting the values of Q_1 and Q_2 in eq. (1),

$$0.1767 = 0.0628 + Q_3$$

$$\therefore \quad Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3\text{/s}$$

8. We know that, $Q_3 = A_3 v_3 = 0.01767 \times v_3$

$$0.1139 = 0.01767 \times v_3$$

$$v_3 = \frac{0.1139}{0.01767} = 6.446 \text{ m/s}$$

b. What is pitot tube ? How will you determine the velocity at any point with the help of pitot tube ?

Ans.

A. Pitot Tube : It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

B. Expression of Velocity at any Point :

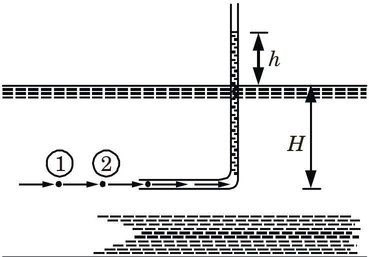

Fig. 5. Pitot-tube.

1. Let

p_1 = Intensity of pressure at point (1),
v_1 = Velocity of flow at (1),
p_2 = Pressure at point (2),
v_2 = Velocity at point (2),
H = Depth of tube in the liquid, and

h = Rise of liquid in the tube above the free surface.

2. Applying Bernoulli's eq. at point (1) and (2),

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad ...(1)$$

3. Since $z_1 = z_2$ as point (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{Pressure head at (1)} = H \quad ...(2)$$

$$\frac{p_2}{\rho g} = \text{Pressure head at (2)} = (h + H) \quad ...(3)$$

4. Substituting values of eq. (2) and eq. (3) in eq. (1), we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H)$$

$$h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

$\because$ This is theoretical velocity.

5. Actual velocity is given by,

$$(v_1)_{act} = C_v\sqrt{2gh}$$

Where, C_v = Coefficient of pitot tube.

6. Velocity at any point,

$$v = C_v\sqrt{2gh}$$

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. If the velocity field is given by $u = x + y$ and $v = x^3 - y$. Find the circulation around a closed contour defined by $x = 1$, $y = 0$, $y = 1$ and $x = 0$.

Ans.

Given : $u = x + y$, $v = x^3 - y$
Closed curve defined by $x = 1$, $y = 0$, $y = 1$, $x = 0$
To Find : Circulation around the closed curve.

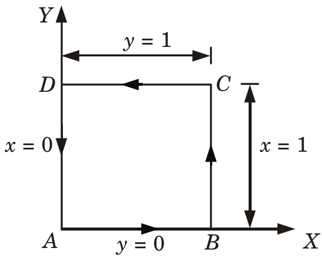

Fig. 6.

1. Circulation,

$$\Gamma_{ABCD} = \int_{ABCD}(udx + \text{v}dy)$$
$$= \int_{AB}(udx + \text{v}dy) + \int_{BC}(udx + \text{v}dy) + \int_{CD}(udx + \text{v}dy) + \int_{DA}(udx + \text{v}dy)$$
$$= \int_0^1 (x + y)dx + \int_0^1 (x^3 - y)dy + \int_0^1 (x + y)dx + \int_0^1 (x^3 - y)dy$$
$$= \left[\frac{x^2}{2} + xy\right]_0^1 + \left[x^3y - \frac{y^2}{2}\right]_0^1 + \left[\frac{x^2}{2} + xy\right]_1^0 + \left[x^3y - \frac{y^2}{2}\right]_1^0$$
$$= \left[\left(\frac{1^2}{2} + 1 \times 0\right) - \left(\frac{0^2}{2} + 0 \times 0\right)\right] + \left[\left(1^3 \times 1 - \frac{1^2}{2}\right) - \left(1^3 \times 0 - \frac{0^2}{2}\right)\right]$$
$$+ \left[\left(\frac{0^2}{2} + 0 \times 1\right) - \left(\frac{1^2}{2} + 1 \times 1\right)\right] + \left[\left(0^3 \times 0 - \frac{0^2}{2}\right) - \left(0^3 \times 1 - \frac{1^2}{2}\right)\right]$$
$$= 0$$

$\therefore$ Circulation per unit area = 0

b. The pressure difference Δp in a pipe of diameter D and length l due to viscous flow depends on the velocity v, viscosity μ and density ρ. Using Buckingham π theorem, obtain the expression for Δp.

Ans.

Given : Δp is a function of $D, l, \text{v}, \mu, \rho, k$,

$$f_1 = (\Delta p, D, l, \text{v}, \mu, \rho, k)$$

To Find : Expression for Δp.

Data Assume : k = Roughness

1. Total number of variables, $n = 7$
2. Dimensions of each variable are,

$$\Delta p = ML^{-1}T^{-2},\ D = L,\ l = L,\ \text{v} = LT^{-1},$$
$$\mu = ML^{-1}T^{-1},\ \rho = ML^{-3},\ k = L$$

3. Number of fundamental dimensions, $m = 3$
4. Number of π-terms = 7 – 3 = 4
5. Now, Δp function can be written as,

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0 \quad ...(1)$$

6. Choosing D, v, ρ as the repeating variables, the π-terms are

$$\pi_1 = D^{a_1}\, \text{v}^{b_1}\, \rho^{c_1}\, \Delta p$$
$$\pi_2 = D^{a_2}\, \text{v}^{b_2}\, \rho^{c_2}\, l$$
$$\pi_3 = D^{a_3}\, \text{v}^{b_3}\, \rho^{c_3}\, \mu$$

$$\pi_4 = D^{a4}\, v^{b4}\, \rho^{c4}\, k$$

7. π_1-term : $\pi_1 = D^{a1}\, v^{b1}\, \rho^{c1}\, \Delta p$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a1}\, [LT^{-1}]^{b1}\, [ML^{-3}]^{c1}\, [ML^{-1}T^{-2}]$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1 + 1$ $\therefore c_1 = -1$

Power of L, $0 = a_1 + b_1 - 3c_1 - 1,$ $\therefore a_1 = -b_1 + 3c_1 + 1 = 2 - 3 + 1 = 0$

Power of T, $0 = -b_1 + 2,$ $\therefore b_1 = -2$

On substituting the values of a_1, b_1 and c_1 in π_1 term, we have

$$\pi_1 = D^0\, v^{-2}\, \rho^{-1}\, \Delta p = \frac{\Delta p}{\rho v^2}$$

8. π_2-term : $\pi_2 = D^{a2}\, v^{b2}\, \rho^{c2}\, l$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a2}\, [LT^{-1}]^{b2}\, [ML^{-3}]^{c2}\, L$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_2,$ $\therefore c_2 = 0$

Power of L, $0 = a_2 - b_2 - 3c_2 + 1,$ $\therefore a_2 = b_2 + 3c_2 - 1 = -1$

Power of T, $0 = -b_2,$ $\therefore b_2 = 0$

On substituting the values of a_2, b_2 and c_2 in π_2 term, we have

$$\pi_2 = D^{-1}\, v^0\, \rho^0\, l = \frac{l}{D}$$

9. π_3-term : $\pi_3 = D^{a3}\, v^{b3}\, \rho^{c3}\, \mu$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a3}\, [LT^{-1}]^{b3}\, [ML^{-3}]^{c3}\, [ML^{-1}T^{-1}]$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_3 + 1,$ $\therefore c_3 = -1$

Power of L, $0 = a_3 + b_3 - 3c_3 - 1,$ $\therefore a_3 = -b_3 + 3c_3 + 1 = 1 - 3 + 1 = -1$

Power of T, $0 = -b_3 - 1,$ $\therefore b_3 = -1$

On substituting the values of a_3, b_3 and c_3 in π_3 term, we have

$$\pi_3 = D^{-1}\, v^{-1} \rho^{-1}\, \mu = \mu / Dv\rho$$

10. π_4-term : $\pi_4 = D^{a4}\, v^{b4}\, \rho^{c4}\, k$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = [L]^{a4}\,[LT^{-1}]^{b4}\,[ML^{-3}]^{c4}\,L$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_4$, $\therefore c_4 = 0$

Power of L, $0 = a_4 - b_4 - 3c_4 + 1$, $\therefore a_4 = b_4 + 3c_4 - 1 = -1$

Power of T, $0 = -b_4$, $\therefore b_4 = 0$

On substituting the values of a_4, b_4 and c_4 in π_4 term, we have

$$\pi_4 = D^{-1}\,\mathrm{v}^0\,\rho^0\,k = \frac{k}{D}$$

11. Substituting the values of π_1, π_2, π_3 and π_4 in eq. (1), we have

$$f_1\left(\frac{\Delta p}{\rho \mathrm{v}^2}, \frac{l}{D}, \frac{\mu}{D\mathrm{v}\rho}, \frac{k}{D}\right) = 0$$

or

$$\frac{\Delta p}{\rho \mathrm{v}^2} = \phi\left[\frac{l}{D}, \frac{\mu}{\rho \mathrm{v} D}, \frac{k}{D}\right]$$

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A fluid of viscosity 0.7 N-s/m^2 and specific gravity 1.3 is flowing through circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as 196.2 N/m^2 find :

i. Pressure gradient,

ii. Average velocity, and

iii. Reynolds number of the flow.

Ans.

Given : $\mu = 0.7$ N-s/m^2, Specific gravity = 1.3, $\rho = 1.3 \times 1000 = 1300$ kg/m^3, $d = 100$ mm = 0.1 m, $\tau_o = 196.2$ N/m^2

1. The maximum shear stress (τ_o) is given by,

$$\tau_o = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$$196.2 = -\frac{\partial p}{\partial x} \times \frac{d}{4} = -\frac{\partial p}{\partial x} \times \frac{0.1}{4}$$

2. So, pressure gradient,

$$-\frac{\partial p}{\partial x} = \frac{196.2 \times 4}{0.1} = 7848 \text{ N/m}^2 \text{ per m}$$

3. Average velocity,

$$\bar{u} = \frac{1}{2}u_{\max} = \frac{1}{2}\left[-\frac{1}{4\mu}\frac{\partial p}{\partial x}r^2\right]$$

$$\left\{\because \quad u_{\max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}r^2\right\}$$

$$= \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)r^2$$

$$= \frac{1}{8 \times 0.7} \times (7848) \times (0.05)^2$$

$$\left\{\because \quad r = \frac{d}{2} = \frac{1}{2} = 0.05\right\}$$

$$= 3.50 \text{ m/s}$$

4. Reynolds number,

$$R_e = \frac{\rho \bar{u} d}{\mu}$$

$$= 1300 \times \frac{3.50 \times 0.1}{0.7} = 650$$

b. Describe the phenomenon of boundary layer formation over a smooth flat plate.

Ans.

1. When a real fluid flow over a solid wall, the fluid particles closed to the boundary get adhered to the boundary and as a result of this condition no slip occurs.
2. In other words the velocity of fluid close to the boundary will be the same as that of the boundary.
3. As we move farther away from the boundary, the velocity will be higher and as a result of this variation of velocity, the velocity gradient $\frac{du}{dy}$ will exist.
4. Thus the velocity of fluid increases from zero velocity on the stationary boundary to free-stream velocity (U) of the fluid in the direction normal to the boundary.
5. The variation of velocity from zero to free stream velocity in the direction normal to the boundary takes place in a narrow region in the vicinity of solid boundary.
6. This narrow region of the fluid is called boundary layer.

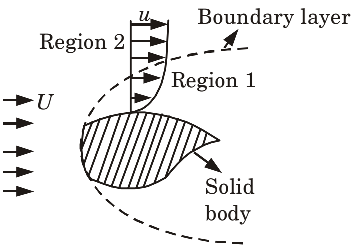

Fig. 7.

7. Hence the flow of fluid in the neighbourhood of the solid boundary may be divided into following two regions :

i. Region 1 :

1. A very thin layer of the fluid called the boundary layer, in the immediate neighbourhood of the solid boundary, where the variation of velocity from zero at the solid boundary to the free stream velocity in the direction normal to the boundary takes place.
2. In this region, the velocity gradient $\frac{du}{dy}$ exists and hence the fluid exerts a shear stress on the wall (wall shear) in the direction of motion.
3. The value of shear stress is given by,

$$\tau = \mu \frac{du}{dy}$$

ii. Region 2 :

1. The remaining fluid, which is outside the boundary layer. The velocity outside the boundary layer is constant and equal to free stream velocity.
2. As there is no variation of velocity in this region the velocity gradient $\frac{du}{dy}$ becomes zero. As a result of this the shear stress is zero.

6. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 liters/s under the head of 30 meters. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume coefficient of velocity as 0.98.

Ans.

Given : $u_1 = u_2 = u = 10$ m/s, $Q = 700$ lit/s $= 0.7$ m^3/s, $H = 30$ m, $\phi = 180° - 160° = 20°$

To Find : i. Power.
ii. Hydraulic efficiency.

Data Assumed : $C_v = 0.98$.

1. The velocity of jet,

$$v_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77 \text{ m/s}$$

$$\therefore \quad v_{r1} = v_1 - u_1 = 23.77 - 10 = 13.77 \text{ m/s}$$

$$v_{w1} = v_1 = 23.77 \text{ m/s}$$

2. From outlet velocity triangle,

$$v_{r2} = v_{r1} = 13.77 \text{ m/s}$$
$$v_{w2} = v_{r2} \cos \phi - u_2$$
$$= 13.77 \cos 20° - 10 = 2.94 \text{ m/s}$$

3. Work done by the jet per second on the runner is given as

$$= \rho a v_1 [v_{w1} + v_{w2}] u$$
$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$$
$$(\because \quad a v_1 = Q = 0.7 \text{ m}^3/\text{s})$$
$$= 186970 \text{ Nm/s}$$

4. Power given to turbine = $\dfrac{186970}{1000}$ = 186.97 kW

5. The hydraulic efficiency of the turbine is given as,

$$\eta_h = \frac{2[v_{w_1} + v_{w_2}]u}{{v_1}^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$
$$= 0.9454 \quad \text{or} \quad 94.54\,\%$$

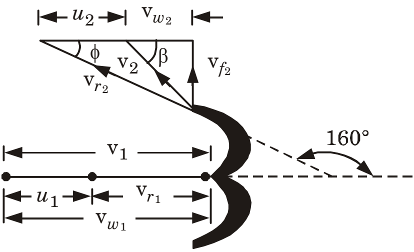

Fig. 8.

b. With the help of neat sketch explain the working of Kaplan turbine.

Ans.

1. The working head of water is low so large flow rates are allowed in the Kaplan turbine.
2. The water enters the turbine through the guide vanes which are aligned such as to give the flow a suitable degree of swirl determined according to the rotor of the turbine.
3. The flow from guide vanes pass through the curved passage which forces the radial flow to axial direction with the initial swirl imparted by the inlet guide vanes which is now in the form of free vortex.
4. The axial flow of water with a component of swirl applies force on the blades of the rotor and loses its momentum, both linear and angular, producing torque and rotation in the shaft.

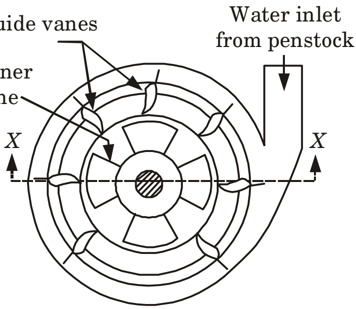

Fig. 9.

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Define specific speed of a centrifugal pump. Derive an expression for the same.

Ans.

A. Specific Speed :

1. It is defined as the speed of a geometrically similar pump which would deliver one cubic meter of liquid per second against a head of one meter.
2. It is denoted by N_s.

B. Expression for Specific Speed :

1. Discharge Q for a centrifugal pump is given as,

$$Q = \text{Area} \times \text{Velocity of flow}$$
$$= \pi D B \mathrm{v}_f \quad ...(1)$$

2. We know that, $B \propto D$, then from eq. (1),

$$Q \propto D^2 \mathrm{v}_f \quad ...(2)$$

3. Tangential velocity is given as,

$$u = \frac{\pi DN}{60} \text{ or } u \propto DN \quad ...(3)$$

4. Tangential velocity (u) and velocity of flow (v_f) are related to the manometric head (H_m) as,

$$u \propto \mathrm{v}_f \propto \sqrt{H_m} \quad ...(4)$$

5. From eq. (3) and eq. (4), we get

$$\sqrt{H_m} \propto DN$$

$$D \propto \frac{\sqrt{H_m}}{N}$$

6. Putting the value of D in eq. (2), we get

$$Q \propto \frac{H_m}{N^2} \mathrm{v}_f \propto \frac{H_m}{N^2}\sqrt{H_m} \qquad \left(\because \mathrm{v}_f \propto \sqrt{H_m}\right)$$

$$Q \propto \frac{H_m^{\ 3/2}}{N^2}$$

$$Q = K \frac{H_m^{\ 3/2}}{N^2} \qquad \text{...(5)}$$

Where, K = Constant of proportionality.

7. If H_m = 1 m, Q = 1 m³/s, so $N = N_s$, then from eq. (5), we get

$$1 = K \frac{(1)^{3/2}}{N_s^{\ 2}}$$

$$N_s^{\ 2} = K$$

8. Putting the value of K in eq. (5), we get

$$Q = N_s^{\ 2} \frac{H_m^{\ 3/2}}{N^2}$$

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

This expression is showing the specific speed of pump.

b. Discuss the effect of acceleration in suction and delivery pipes on indicator diagram.

Ans.

A. Effect of Acceleration in the Suction Pipe : Let l_s and a_s are length and cross-sectional area of the suction pipe respectively.

i. At the Beginning of the Suction Stroke :

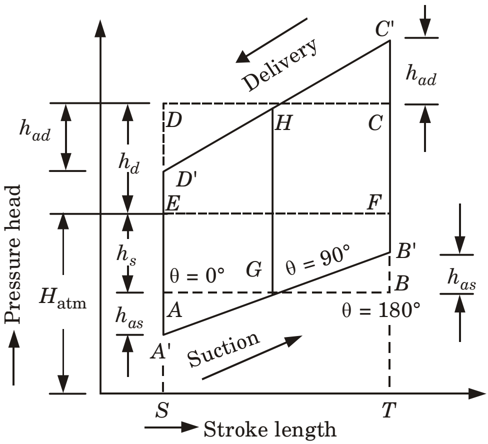

Fig. 10. Effect of acceleration on indicator diagram.

The accelerating head, $h_{as} = \frac{l_s}{g}\frac{A}{a_s}\omega^2 r$

Negative pressure (vacuum) head, $h_s + h_{as} = h_s + \frac{l_s}{g}\frac{A}{a_s}\omega^2 r$

Absolute pressure head $= H_{\text{atm}} - \left(h_s + \frac{l_s}{g}\frac{A}{a_s}\omega^2 r\right)$

ii. At the Middle of the Suction Stroke :

The acceleration head, $h_{as} = 0$

Negative pressure (vacuum) head $= h_s$

Absolute pressure head $= H_{\text{atm}} - h_s$

iii. At the End of the Suction Stroke :

The acceleration head, $h_{as} = -\frac{l_s}{g}\frac{A}{a_s}\omega^2 r$

Negative pressure (vacuum) head $= h_s + h_{as} = h_s - \frac{l_s}{g}\frac{A}{a_s}\omega^2 r$

Absolute pressure head $= H_{\text{atm}} - \left(h_s - \frac{l_s}{g}\frac{A}{a_s}\omega^2 r\right)$

B. Effect of Acceleration in the Delivery Pipe :

1. In the beginning of delivery stroke the liquid in the delivery pipe is accelerated, while at the end of delivery stroke the liquid is retarded.
2. Let l_d and a_d are the length and cross-sectional area of the delivery pipe respectively.

i. At the Beginning of the Delivery Stroke :

Pressure (gauge) head, $h_d + h_{ad} = h_d + \frac{l_d}{g}\frac{A}{a_d}\omega^2 r$

ii. At the Middle of the Delivery Stroke :

Pressure (gauge) head $= h_d$ $\quad (\because \ h_{ad} = 0)$

iii. At the End of the Delivery Stroke :

Pressure (gauge) head $= h_d - \frac{l_d}{g}\frac{A}{a_d}\omega^2 r$

Absolute pressure head $= H_{\text{atm}} + h_d - \frac{l_d}{g}\frac{A}{a_d}\omega^2 r$

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