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Structural Analysis

By

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QUANTUM PAGE PVT. LTD.

Ghaziabad ■ New Delhi

PUBLISHED BY : **Apram Singh**
Quantum Page Pvt. Ltd.
Plot No. 59/2/7, Site - 4, Industrial Area,
Sahibabad, Ghaziabad-201 010

Phone : 0120 - 4160479

Email : pagequantum@gmail.com **Website: www.quantumpage.co.in**

Delhi Office : 1-6590, East Rohtas Nagar, Shahdara, Delhi-110032

Structural Analysis (CE : Sem-5)
1st Edition : *2013-14*
2nd Edition : *2014-15*
3rd Edition : *2015-16*
4th Edition : *2016-17*
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Price: Rs. 90/- only

Printed Version **:** e-Book.

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KCE–502 : STRUCTURAL ANALYSIS

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KCE502 STRUCTURAL ANALYSIS **(L-T-P 3-1-0) Credit – 4**

Unit 1

Classification of Structures, Types of structural frameworks and Load transfer Mechanisms, stress resultants, degrees of freedom, Static and Kinematic Indeterminacy for beams, trusses and building frames. Analysis of cables with concentrated and continuous loadings, Effect of Temperature upon length of cable. **[8]**

Unit 2

Classification of Pin jointed determinate trusses, Analysis of determinate plane trusses (compound and complex). Method of Substitution, Method of tension coefficient for analysis of plane trusses. **[8]**

Unit 3

Strain Energy of deformable systems, Maxwell's reciprocal & Betti's theorem, Castigliano's theorems, Calculations of deflections: Strain Energy Method and unit load method for statically determinate beams, frames and trusses. Deflection of determinate beams by Conjugate beam method. **[8]**

Unit 4

Rolling loads and influence line diagrams for determinate beams and trusses, Absolute maximum bending moment and shear force. Muller-Breslau's principal & its applications for determinate structures. **[8]**

Unit 5

Arches, Types of Arches, Analysis of three hinged parabolic and circular Arches. Linear arch, Eddy's theorem, spandrel braced arch, moving load & influence lines for three hinged parabolic arch. **[8]**

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1 UNIT

Classification of Structures

PART-1

Classification of Structures, Types of Structural Frameworks and Load Transfer Mechanism, Stress Resultants, Degree of Freedom, Static and Kinematic Indeterminacy for Beams, Trusses and Building Frames.

CONCEPT OUTLINE : PART-1

Classification of Structures : The structures are classified as follows :

1. Beams and trusses.
2. Cable and arches.
3. Frames.
4. Surface structure.

Type of Structure Framework :

1. High risk structure.
2. Trusses.
3. Industrial shed.
4. Bridge deck.
5. Plate etc.

Load Transfer Mechanism :

The loads applied to a structure must be transferred to the ground by the support, *i.e.*, the support will generate reaction to counteract the action of the loads. The type and number of reactions depend on the support condition. Support may be roller, hinge or pin and fixed.

Stress Resultants :

The action of an external force on a structure due to its environment or use produces internal forces within the structure. These are called stress resultants. The most common stress resultants are tension, compression, bending, shear, torsion and bearing.

Degree of Freedom : It is a set of independent displacement that specify completely the deformed position and orientation of the body or system under loading.

Indeterminate Structure : Structures in which the reactions cannot be evaluated by the application of static equilibrium equations alone are defined as statically indeterminate or hyperstatic structures. They are also known as redundant structures.

Types of Indeterminate Structure : There are two types of indeterminate structures :

1. Externally indeterminate structure.
2. Internally indeterminate structure.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **Define the term 'Structure'. Also discuss the structural elements.**

Answer

A. Structure :

1. A structure is a system of connected parts used to support a load. Examples of structures in civil engineering are buildings, bridges and towers.
2. While designing a structure the engineer must account for its safety, aesthetic and serviceability also taking into consideration economic and environmental constraints.

B. Structural Elements : The common structural elements are as follows :

1. Tie rods 2. Beams 3. Columns

1. **Tie Rods (or Bracing Struts) :** Tie rods or bracing struts are the structural members subjected to tensile force. Tie rods are slender and are often chosen from rods, bars, angles or channels.

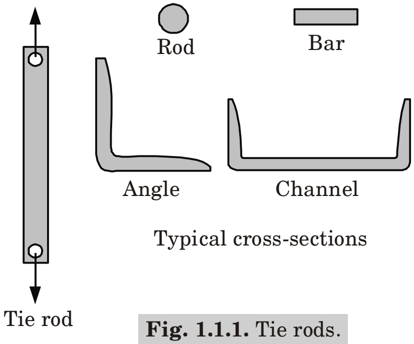


Fig. 1.1.1. Tie rods.

2. **Beams :**

i. Beams are usually straight horizontal members used primarily to carry vertical loads.
ii. When the cross-section varies, the beam is referred to as tapered or hunched. Beams are primarily designed to resist bending moment.

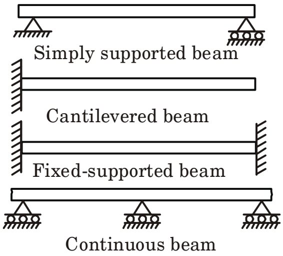


Fig. 1.1.2. Beams.

3. **Columns :** Columns are usually vertical and resist axial compressive loads. Sometimes columns are subjected to both an axial load and a bending moment, then these columns are known as beam columns.

Que 1.2. **Explain the classification of structures in detail.**

AKTU 2012-13, Marks 05

Answer

Classification of Structures : These can be classified as follows :

1. **On the Basis of Structural Forms and Shapes :**
   i. Linear forms (Skeletal structures).
   ii. Curvilinear forms (Surface structures).

i. **Linear Forms :**
   a. They are articulated structures assembled with parts consisting of linear elements, such as bars and beams, the connection between them being bolted or riveted or welded.
   b. Linear forms are preferred for residential, official, and educational purposes. The linear form is called skeletal structures.

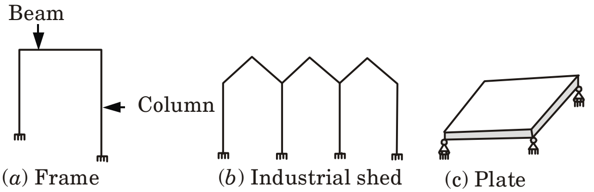


Fig. 1.2.1.

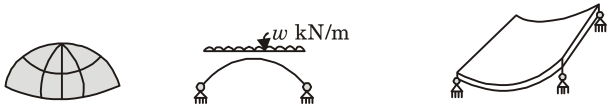
Examples : Frame, high rise structure, truss, industrial shed, bridge deck, plate, etc.

ii. **Curvilinear Forms :**
   a. Curvilinear forms as single entities mostly occupy a space.

b. These structures are idealized as continuous system for structural analysis purpose.

c. They are 3-D structures and they are also termed as surface structures.

Examples : Dome, shells, arches, cables, cylindrical members, etc.



(a) Dome (b) Arch (c) Cylindrical member

Fig. 1.2.2.

2. On the Basis of Dominant Stress Conditions :

i. Uniform stress forms. ii. Varying stress forms.

i. Uniform Stress Forms : When the stress across a section is uniform over the depth of a member or over the thickness of a panel, such a form is called uniform stress form.

Example : Cables, arches, truss member, shells etc.

ii. Varying Stress Forms : When the stress varies over the depth or thickness, from a maximum compressive stress on one surface to a maximum tensile stress on the other, such a form is called varying stress form.

Example : Beams, rigid frames, slabs, plates etc.

3. On the Basis of the Complexity of their Force Analysis :

i. Beams and trusses. ii. Cables and arches.

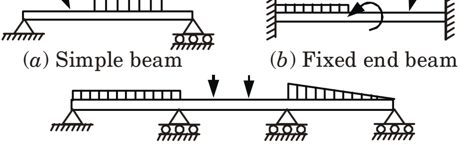
iii. Frames. iv. Surface structures.

i. Beams and Trusses :

a. Beams and trusses are a form of structures.

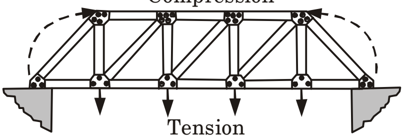
b. A simply supported beam, supported on a pin at one end and a roller at the other end is quite stable and statically determinate, and transmits the external loads to the supports mainly through shear and moment.

c. Some typically complex types of beams are fixed end beams and continuous beams. These beams are statically indeterminate and cannot be solved using equations of static equilibrium alone.



(a) Simple beam (b) Fixed end beam

(c) Continuous beam



(*d*) Truss

Fig. 1.2.3.

ii. Cables and Arches :

a. These two form of structures are used for long span.

b. Cables are usually flexible and carry their loads in tension.

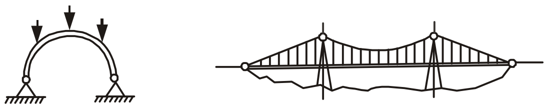
c. The external loads are not applied along the axis of the cable and consequently cable takes a form of defined sag.

d. Cables are commonly used to support bridges and building roofs.

e. Arches are characterized by high axial thrust and relatively low bending moment which result from its distinguished shape as well as the horizontal reactions that developed at the support points.

f. Arches achieve their strength in compression. Arches must be rigid in order to maintain its shape.

g. Arches are used in bridge structures, dome roofs etc.



(*a*) Arch (*b*) Cable structure

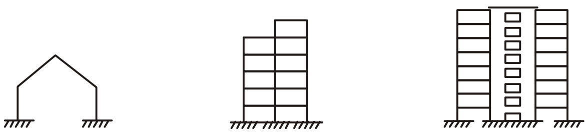
Fig. 1.2.4.

iii. Frames :

a. Frames are composed of beams and columns that are either pin or fixed connected.

b. Frames are characterized by moment resisting members at some or at all the joints.

c. If it has rigid joint connections, this structure is generally indeterminate. Frames can be two or three dimensional.



(*a*) Industrial frame (*b*) Multi storey building frame (*c*) Building frame with shear wall

Fig. 1.2.5.

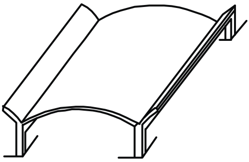
iv. Surface Structure :

a. A surface structure is made of a material having very small thickness compared to its other dimensions.

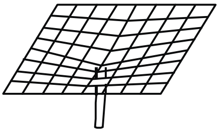
b. Surface structures may also be made of rigid material such as reinforced concrete.

c. They may be shaped as folded plates, cylinders or hyperbolic paraboloids and are referred to as thin plates or shells.

d. These structures act like cables or arches since they support loads primarily in tension or compression, with very little bending.



(*a*) Cylindrical shell



(*b*) Hyperbolic paraboloids

Fig. 1.2.6.

Que 1.3. Define the term framework and explain what are the framed structure system and continuous structure system ?

Answer

1. **Framework :** A framework is the skeleton of the complete structure and it supports all intended loads safely and economically.

2. **Framed Structure System :** Assemblage of members forming a frame to support the forces acting on it, is called the framed structure system. As frame, high-rise structure, truss etc.

3. **Continuous Structure System :** Assemblage of continuous member like shells, domes etc., are called continuous structure system. As cylindrical members, space crafts, aircrafts etc.

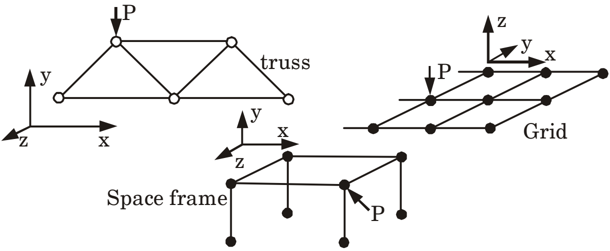


Fig. 1.3.1. Types of structural frameworks.

Que 1.4. **What do you understand by the term structural load ? Explain different types of loads.**

Answer

A. **Structural Load :** Structural load is consists of following loads :

1. **Dead Loads :**

i. Dead loads are fixed to the structure and is mostly invariant with time. Example : Self weight of the structure, furniture, etc.

ii. Dead loads reside permanently in the structure and are the basic parameter for design in all structural systems.

2. **Live Loads :** Live loads are the loads which are variable in nature, both in the time and magnitude. These loads are applied over and above the dead or fixed loads. They are generally known as imposed loads.

3. **Environmental Loads :**

i. These loads are due to wind, earthquake, floods, landslides etc., and caused by natural phenomenon.

ii. These loads are mostly non deterministic because of uncertainties associated with their occurrence, time and magnitude and terrestrial conditions.

4. **Man-made Loads :** Man-made loads are caused by human activities. These loads are like impact loads, blast loads, dynamic loads etc.

B. **Types of Loads :**

1. **Classification of Loads on the Basis of Direction in which Loads are Acting :**

i. **Gravity Loads :** Gravity loads are the dead loads, live loads, snow and ice loads because they always act vertically downwards.

ii. **Lateral Loads :** The loads which act horizontally as wind loads, earthquake loads and soil and hydrostatic pressures are known as lateral loads.

iii. **Directionless loads :** Thermal loads, loads due to misfit and blasting loads are known as directionless loads.

2. **Classification of Loads on the Basis of Their Variation with Time :**

i. **Static or Monotonic Loads :** Loads that do not vary with time and always act in the same sense are known as monotonic or static load. Example : dead loads, construction loads, snow loads and earth pressure etc.

ii. **Cyclic or Dynamic Loads :**

a. Loads that vary with time and change their sense are known as cyclic or dynamic loads.

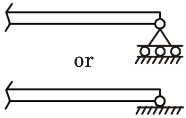
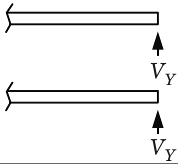
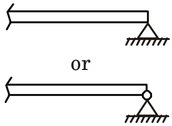
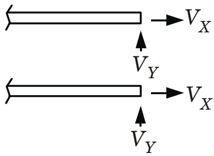
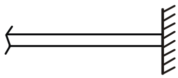
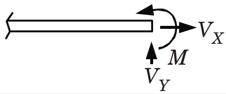
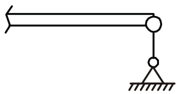
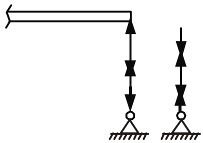
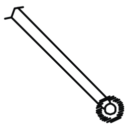
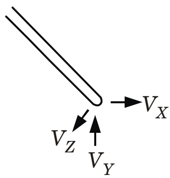
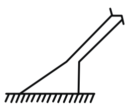
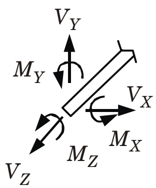
b. Example : earthquake loads, impact loads, blast loads and machine induced loads. Dynamic load sets the structure to vibrate.

c. Fatigue load is a special case of dynamic or cyclic load which is applied repeatedly on the structure for a long duration. Example : traffic and wave induced loads.

Que 1.5. **Diagrammatically show the various types of supports and reactions acting on them.**

Answer

Table 1.5.1.

Type of Supports	Diagrammatic Symbols	Reactions and Directions
Roller support		
Hinged support		
Fixed support		
Link support		
Ball and socket		
Rigid support in space		

Que 1.6. **What do you mean by degree of freedom ? Also explain the kinematic indeterminacy.**

AKTU 2012-13, Marks 05

OR

What do you understand by kinematic indeterminacy of structures ? Explain in sufficient detail.

AKTU 2013-14, Marks 05

Answer

A. Degree of Freedom :

1. When a structure composed of several members which are subjected to loads, the joints undergo displacements in the form of rotation and translation.
2. The number of independent joint displacement is called the degree of kinematic indeterminacy or the number of degree of freedom.

B. Kinematic Indeterminacy :

1. Degree of kinematic indeterminacy or number of degree of freedom is the sum of degree of freedom in rotation and translation.
2. A joint in space can have six possible movements (3 linear displacements, one along each reference axis and 3 rotations, one about each reference axis) hence it has six degree of freedom.
3. In case of a two dimensional system, a free joint has three degree of freedom [Two translation and one rotation] hence in case of two dimensional system any joint may have maximum of 3 degree of kinematic indeterminacy.
4. The total kinematic indeterminacy of a structure represents the sum of all the possible displacement that various joints of the structure can undergo.

Example :

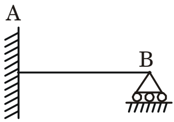


Fig. 1.6.1.

From Fig. 1.6.1,

Joint	Kinematic Indeterminacy (KI)
A	0
B	2
Total KI	2

Que 1.7. **What do you mean by static indeterminacy ? Explain giving at least two examples with reference to trusses.**

OR

What do you mean by static indeterminacy ? Explain with examples.

AKTU 2012-13, Marks 05

Answer

A. **Static Indeterminacy :** For a structure, if available equilibrium equations are less than the unknown forces then structure is known as statically indeterminate structure.

1. For a coplanar rigid frame structure if :
   i. $3m = 3j - r \Rightarrow$ Structure is statically determinate.
   ii. $3m > 3j - r \Rightarrow$ Structure is statically indeterminate.

   where, m = Number of members in a structure.
   r = Total number of forces and moment reaction components.
2. Static indeterminacy = $3m - (3j - r)$
3. Degree of static indeterminacy for truss = Total number of unknowns (external and internal) forces – Number of independent equations of equilibrium.

   If, j = Number of pin (hinge) joints connecting these members.
   i. Total number of unknown forces = $(m + r)$
   ii. Total number of independent equilibrium equations = $2j$
   iii. Degree of static indeterminacy = $(m + r) - 2j$

B. **Examples :**

1. For the truss as shown in Fig. 1.7.1, we have

$$m = 18, j = 10, \text{ and } r = 3.$$

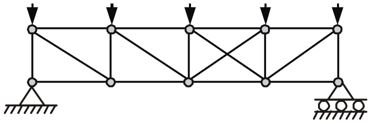


Fig. 1.7.1. Indeterminate truss.

$$\text{Degree of static indeterminacy} = (m + r) - 2j = (18 + 3) - 2 \times 10 = 21 - 20 = 1$$

So, degree of static indeterminacy = 1.

2. For the truss as shown in the Fig. 1.7.2, we have

$$m = 17, j = 10, \text{ and } r = 4.$$

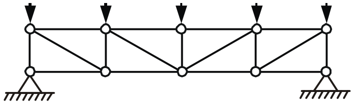


Fig. 1.7.2. (Externally) indeterminate truss.

$$\text{Degree of static indeterminacy} = (m + r) - 2j = (17 + 4) - 2 \times 10 = 21 - 20 = 1$$

So, degree of static indeterminacy = 1.

Que 1.8. **Explain external and internal indeterminacy of structure.**

Answer

A. External Indeterminacy of Structure (I_e) :

1. Let r be the total number of external reaction components at supports and e be the total number of equations of statical equilibrium available for the given structure then there may arise three different cases.

i. $r = e \rightarrow$ Structure is externally determinate.

ii. $r > e \rightarrow$ Structure is externally indeterminate.

iii. $r < e \rightarrow$ Structure is externally unstable.

Thus external indeterminacy $I_e = r - e$.

B. Internal Indeterminacy of Structure (I_i) :

i. It is defined in terms of the internal member forces that can not be determined from simple statical relations if the structure is taken as externally determinate.

ii. This type of internal indeterminacy is seen in articulated (pin joined frames) and rigid frame structure.

iii. In case of beams it is equal to zero.

1. Articulated Structures :

i. If a truss consist of m number of pin connected members (it means the truss is having m number of unknown member forces) through 'j' number of joints in the truss where 'r' number of external support reactions are developed and 'e' number of equation of statical equilibrium are available then,

ii. Total number of equations of equilibrium available at joints.

$$e = 2j \qquad [\therefore 0, \Sigma V = 0]$$

iii. Total number of unknown = Internal member forces + Reaction at supports $= m + r$

iv. Then for determinacy of the truss, $m + r = 2j$ or $m = 2j - r$

v. In case of externally determinate 2-Dimensional plane truss

$$r = 3 \qquad (r = e = 3)$$

hence $m = 2j - 3$

Total indeterminacy, $I_t = m - [2j - r]$ ...$[I_t = I_e + I_i]$

vi. For externally determinate 3 dimensional truss

$$m = 3j - r \text{ (at each joint, equations available} = 3j)$$

$$m = 3j - 6 \qquad (\because r = 6)$$

vii. Now in case of 2-D trusses :

If $m > 2j - 3$, then the truss has additional member and it is known as redundant truss.

$m = 2j - 3$, the truss is internally determinate and is a perfect truss.

$m < 2j - 3$, the truss is imperfect truss, it has deficiency of members and its configuration is unstable under certain loading condition.

Degree of internal indeterminacy, $I_i = m - (2j - 3)$

viii. In case of 3-D space trusses :

If, $m = 3j - 6$, Internally determinate space truss

$m > 3j - 6$, Internally indeterminate space truss

$m < 3j - 6$, Internally unstable truss

Degree of internal indeterminacy, $I_i = m - (3j - r) = m - (3j - 6)$,

2. Rigid Frames :

i. These frames have rigid joint and may be a 2-dimensional or 3-dimensional figure. In a member of rigid frame there exist 3 stress resultants or member forces which need to be determined.

Hence, total number of unknown = $(3m + r)$.

ii. Since at each joint of rigid frame three equations of equilibrium are available then,

Total number of equations of equilibrium = $3j$

Then for a frame to be internally determinate, $3j = 3m + r$

iii. If $3m > 3j - r$, the frame becomes internally indeterminate.

If $3m < 3j - r$, the frame becomes internally unstable or deficient.

$$I_i = 3m - (3j - r) = 3m - (3j - 3) \text{ for 2D frame.}$$

iv. Similarly in case of 3 dimensional frame total number of unknown becomes $(6m + r)$ and a total of $6j$ equations of equilibrium are available for analysis.

v. Then the frame will be internally determinate,

if $6j = 6m + r$

and if $6m > 6j - r$, the frame becomes internally indeterminate

and when $6m < 6j - r$, the frame becomes internally unstable.

$$= 6m - (6j - r) = 6m - (6j - 6) \quad \text{for 3D frame}$$

C. Total Indeterminacy of Rigid Frame Structure (I_t) :

1. A structure may be indeterminate internally or externally or both in terms of member forces or external reactions.

2. Hence, the total indeterminacy of a structure is the sum of internal and external indeterminacy,

i.e., $I_t = l_e + I_t$

$I_t = [3m - (3j - r)]$ for 2 dimensional frame.

$I_t = [6m - (6j - r)]$ for 3 dimensional frame.

Que 1.9. **What is the difference between statically determinate and statically indeterminate structure ?**

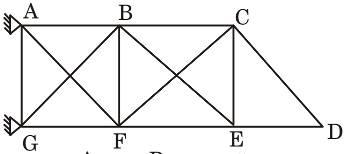
AKTU 2014-15, Marks 05

Answer

S. No.	Statically Determinate Structures	Statically Indeterminate Structures
1.	Conditions of equilibrium are sufficient to analyze the structure completely.	Conditions of equilibrium are insufficient to analyze the structure completely.
2.	The bending moment at a section or the force in any member is independent of the material of the components of the structure.	The bending moment at a section or the force in a member depends upon the material of the components of the structure.
3.	The bending moment at a section or the force in any member is independent of the cross-sectional areas of the components.	The bending moment at a section or the force in a member depends upon the cross-sectional areas of the components.
4.	No stresses are caused due to temperature changes.	Stresses are generally caused due to temperature variations.
5.	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.

Que 1.10. **Find the external and internal degree of redundancy of the structures as shown in Fig. 1.10.1.** **AKTU 2014-15, Marks 05**

A.



B.

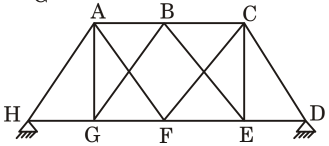


Fig. 1.10.1.

Answer

A. Unknown support reactions, $r = 4$

Number of equilibrium equations, $e = 3$

Number of members, $m = 13$

Number of joints, $j = 7$

Degree of external redundancy,

$$I_e = r - e = 4 - 3 = 1$$

Degree of internal redundancy,

$$I_i = m - [2j - 3] = 13 - [2 \times 7 - 3] = 2$$

B. Unknown support reactions, $r = 4$

Number of equilibrium equations, $e = 3$

Number of members, $m = 14$

Number of joints, $j = 8$

Degree of external redundancy, $I_e = r - e = 4 - 3 = 1$

Degree of internal redundancy,

$$I_i = m - [2j - 3] = 14 - [2 \times 8 - 3] = 1$$

PART-2

Analysis of Cable with Concentrated Load and Continuous Loadings, Effect of Temperature upon Length of Cable.

CONCEPT OUTLINE : PART-2

Cables : Cables are used as support as well as for transmission of loads from one member to the other. Cable stands for a flexible tension member.

Cable Theorem : In a cable subjected to vertical loading, the product of the horizontal component of the cable and the vertical intercept between a point on the cable and a point directly above it on the chord is equal to the moment at the corresponding point in a simple beam of the same span supporting the same loads.

Analysis of Cable : In analysis of cable, following terms are calculated,

1. **Horizontal Reaction :** $H = \dfrac{wL^2}{8d}$

2. **Tension at the Ends :** $T_A = \sqrt{V_A^2 + H^2}$

$$T_B = \sqrt{V_B^2 + H^2}$$

3. **Shape of Cable :** The cable generally found in the shape of parabola when subjected to uniformly distributed load, equation of curve,

$$y = \frac{4hx}{L^2}(L - x)$$

4. Length of the Cable :

i. When both ends at the same level :

$$l = L + \frac{8}{3}\frac{h^2}{L}$$

ii. When ends at different level :

$$\frac{L_1}{L_2} = \sqrt{\frac{h_1}{h_2}},$$

$$l_1 = 2L_1 + \frac{4}{3}\frac{h_1^2}{L_1} \text{ and } l_2 = 2L_2 + \frac{4}{3}\frac{h_2^2}{L_2}$$

$$l = l_1 + l_2$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.11. Give the analysis of a uniformly loaded cable and deduce the expressions for the following :

A. Horizontal reaction.

B. Cable tension at the ends.

C. Shape of the cable.

D. Length of the cable.

Answer

A. Expression for Horizontal Reaction :

1. Fig. 1.11.1, shows a cable supporting a uniformly distributed load of intensity w per unit length.

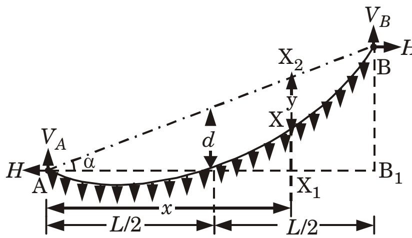


Fig. 1.11.1.

2. From the general cable theorem we have :

$$Hy = \frac{x}{L}\ \Sigma M_B - \Sigma M_x \quad ...(1.11.1)$$

where, $y = XX_2$ = Vertical ordinate between the line AB and chord at the point X.

3. $$\Sigma M_B = wL \times \frac{L}{2} = w\frac{L^2}{2}$$

4. $$\Sigma M_x = wx \times \frac{x}{2} = w\frac{x^2}{2}$$

5. From eq. (1.11.1), $Hy = \dfrac{x}{L} \times w\dfrac{L^2}{2} - w\dfrac{x^2}{2} = w\dfrac{Lx}{2} - w\dfrac{x^2}{2}$ ...(1.11.2)

6. At the mid-span, $x = L/2$ and $y = h$ = Dip of the cable.

$$\therefore \quad Hh = w\frac{L}{2}\frac{L}{2} - \frac{w}{2}\left(\frac{L}{2}\right)^2 = w\frac{L^2}{8}$$

Hence, $$H = w\frac{L^2}{8h} \quad ...(1.11.3)$$

Eq. (1.11.3) gives the expression for the horizontal reaction H and is valid whether the cable chord is inclined or horizontal.

B. Expression for Cable Tension at the Ends :

1. The cable tension T at any end is the resultant of vertical and horizontal reactions at the ends.
2. Thus,

$$T_A = \sqrt{V_A^2 + H^2} \text{ and } T_B = \sqrt{V_B^2 + H^2}$$

3. When the cable chord is horizontal, $V_A = V_B = \dfrac{wL}{2}$.

Hence, $$T_A = T_B = T = \sqrt{\left(\frac{wL}{2}\right)^2 + \left(\frac{wL^2}{8h}\right)^2} \quad \left[as\ H = \frac{wL^2}{8h}\right]$$

or $$T = \frac{wL}{2}\sqrt{1 + \frac{L^2}{16h^2}}$$

4. The inclination β of T with the vertical is given by,

$$\tan\beta = \frac{H}{V} = \frac{wL^2}{8h} \times \frac{2}{wL} = \frac{L}{4h}$$

5. It should be remembered that the horizontal component of cable tension at any point will be equal to H.

C. Shape of the Cable :

1. Let us now determine the shape of the cable under the uniformly distributed load.
2. $$\because \quad H = \frac{wL^2}{8h} \text{ and } Hy = \frac{wLx}{2} - \frac{wx^2}{2} \text{ then}$$

3. $$\left(\frac{wL^2}{8h}\right) y = \frac{wLx}{2} - \frac{wx^2}{2}$$

or, $$y = \frac{4hx}{L^2} (L - x)$$

4. This is the equation of the curve with respect to the cable chord.
5. Thus the cable, takes the form of a parabola when subjected to uniformly distributed load.

D. Length of the Cable (Both Ends at the Same Level) :

1. When both the ends of the cable are at the same level as shown in the Fig. 1.11.2.
2. The equation of the parabola can be written, with C as the origin as follows : $$y = kx^2$$
3. At A, $$x = \frac{L}{2} \text{ and } y = h$$

$\therefore$ $$k = \frac{y}{x^2} = \frac{h}{(L/2)^2} = \frac{4h}{L^2}$$

$\therefore$ $$y = \frac{4h}{L^2} x^2$$

4. $\therefore$ $$\frac{dy}{dx} = \frac{8h}{L^2} x$$

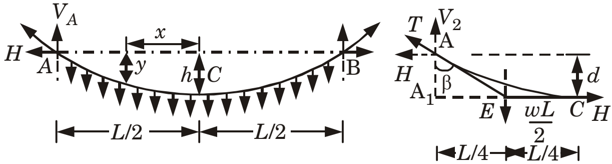


Fig. 1.11.2.

5. Consider an element of length dl of the curve, having coordinates x and y.
6. The total length of the curve is given by,

$$l = \int_0^L dl = 2\int_0^{L/2} \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} \qquad [\because\ dl = \sqrt{dx^1 + dy^2}]$$

$$l = 2 \int_0^{L/2} \left(1 + \frac{64h^2}{L^4} x^2\right)^{1/2}$$

7. Expanding $\left(1 + \frac{64h^2}{L^4} x^2\right)^{1/2}$ by Binomial theorem, and neglecting higher powers of $\frac{h^2}{L^4} x^2$, we get

$$l = 2\int_0^{L/2}\left(1+\frac{1}{2}\frac{64h^2}{L^4}x^2+.....\right)dx$$

$$= 2\left[x+\frac{32\,h^2}{3L^4}x^3\right]_0^{L/2} = 2\left[\frac{L}{2}+\frac{4}{3}\frac{h^2\,L^3}{L^4}\right]$$

$\therefore$ $$l = L+\frac{8}{3}\frac{h^2}{L}$$

E. Length of the Cable (Ends at Different Levels) :

1. Consider a cable AB with the supports A and B at different levels.
2. Let C be the lowest point of the cable, such that the horizontal equivalent of AC is L_1 and that of CB is L_2.

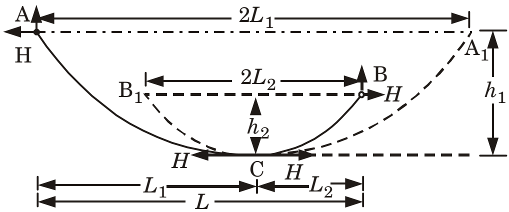


Fig. 1.11.3.

Evidently,

$$L_1 + L_2 = L \qquad ...(1.11.3)$$

3. Imagine the portion AC to be extended to a point A_1, such that A and A_1 are at the same level. Let h_1 be the dip of this hypothetical cable, below the chord $A\,A_1$. We know that,

$$H = \frac{wL^2}{8h}\text{ , where } L = 2L_1 \text{ and } h = h_1$$

$\therefore$ $$H = \frac{w}{8}\frac{(2L_1)^2}{h_1} = \frac{L_1^2}{2h_1} \qquad ...(1.11.4)$$

4. Similarly, imagine the portion BC to be extended to a point B_1 such that B and B_1 are at the same level. Let h_2 be the dip of this hypothetical cable, below the chord BB_1.

$$H = \frac{wL^2}{8h}\text{ , where } L = 2\,L_2 \text{ and } h = h_2$$

$\therefore$ $$H = \frac{w}{8}\frac{(2L_2)^2}{h_2} = \frac{L_2^2}{2h_2} \qquad ...(1.11.5)$$

5. Since H is same at C for both the portions of the cable, we get

$$\frac{wL_1^2}{2h_1} = \frac{wL_2^2}{2h_2}$$

or $$\frac{L_1}{L_2} = \sqrt{\frac{h_1}{h_2}} \quad ...(1.11.6)$$

6. Solving eq. (1.11.3) and (1.11.6), the values of L_1 and L_2 can be known in terms of L, h_1 and h_2.
7. In order to find the vertical reaction V_A at A, take moments about B. Then,

$$V_A = \frac{1}{L}\left[\frac{wL^2}{2} + H(h_1 - h_2)\right]$$

8 Where, $$H = \frac{wL_1^2}{2h_1}$$

$$V_A = \frac{w}{2L}\left[L^2 + \frac{L_1^2}{h_1}(h_1 - h_2)\right]$$

$$V_A = \frac{w}{2L}\left[L^2 + L_1^2 - L_1^2\frac{h_2}{h_1}\right] = \frac{w}{2L}\left[L^2 + L_1^2 - L_1^2\frac{L_2^2}{L_1^2}\right]$$

$$= \frac{w}{2L}[L_1^2 + L_2^2 + 2L_1 L_2 + L_1^2 - L_2^2] \qquad [\because \; L_1 + L_2 = L]$$

$$= \frac{w}{2L}[2L_1^2 + 2L_1L_2]$$

$\therefore$ $$V_A = wL_1$$

9. Similarly, $V_B = wL_2$. For the imaginary cable ACA_1, the length l_1 is given by,

$$l_1 = 2L_1 + \frac{8}{3}\left(\frac{h_1^2}{2L_1}\right) = 2L_1 + \frac{4}{3}\frac{h_1^2}{L_1}$$

10. Similarly, the length of the cable BCB_1 is given by,

$$l_2 = 2L_2 + \frac{8}{3}\left(\frac{h_2^2}{2L_2}\right) = 2L_2 + \frac{4}{3}\frac{h_2^2}{L_2}$$

11. Hence, the total length of the actual cable ABC is,

$$l = \frac{1}{2}(l_1 + l_2) = \frac{1}{2}\left\{\left(2L_1 + \frac{4}{3}\frac{h_1^2}{L_1}\right) + \left(2L_2 + \frac{4}{3}\frac{h_2^2}{L_2}\right)\right\}$$

or $$l = L_1 + \frac{2}{3}\frac{h_1^2}{L_1} + L_2 + \frac{2}{3}\frac{h_2^2}{L_2}$$

or $$l = L + \frac{2}{3}\frac{h_1^2}{L_1} + \frac{2}{3}\frac{h_2^2}{L_2}$$

Que 1.12. **A cable is used to support five equal and equidistant loads over a span of 30 m. Find the length of the cable required and**

its sectional area, if safe tensile stress is 140 N/mm². The central dip of the cable is 2.5 m and loads are 5 kN each.

Answer

Given : Span of cable, L = 30 m, Dip of cable, h = 2.5 m, Load = 5 kN, Safe tensile stress = 140 N/mm².
To Find : Length of cable, l and sectional area.

1. Let V_A and V_B be the vertical reactions at the support A and B, let H be the horizontal reaction at each support.

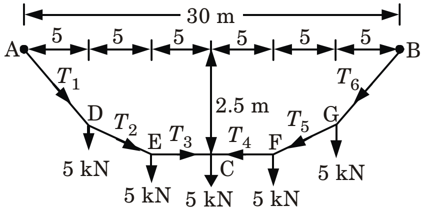


Fig. 1.12.1.

2. Taking moment about right support, B

$$\Sigma M_B = 0$$

$$V_A \times 30 = 5 \times 25 + 5 \times 20 + 5 \times 15 + 5 \times 10 + 5 \times 5$$

$$V_A = 12.5 \text{ kN}$$

3. Verticals forces $\Sigma V = 0$,

$$V_A + V_B = 25 \text{ kN}$$

$$V_B = 12.5 \text{ kN}$$

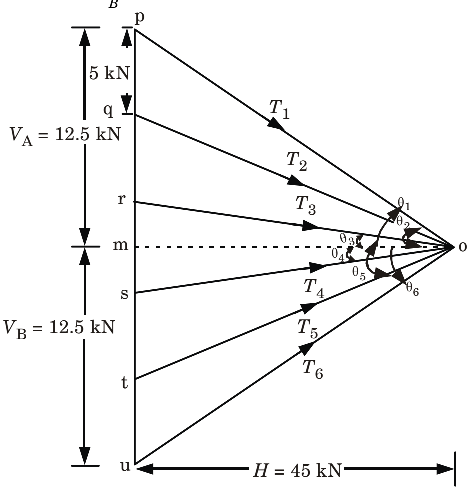


Fig. 1.12.2.

4. Taking moment about centre of chord from left portion $\Sigma M_C = 0$
$-H_A \times 2.5 + V_A \times 15 - 5 \times 10 - 5 \times 5 = 0$
$-H_A \times 2.5 + 12.5 \times 15 - 5 \times 10 - 5 \times 5 = 0$
$$H_A = 45 \text{ kN}$$
5. Draw pq, qr, rs, st and tu = 5 kN to a suitable scale.
6. Mark the point m, on this load line such that
$$mp = V_A = 12.5 \text{ kN and } mu = V_B = 12.5 \text{ kN}$$
7. Now draw mo perpendicular to the load line making $mo = H = 45$ kN. Join op, oq, or, os, ot and ou.
8. These represent the tension T_1, T_2, T_3, T_4, T_5 and T_6 in the part AD, DE, EC, CF, FG and GB.
$$T_1 = \sqrt{(12.5)^2 + 45^2} = 46.7 \text{ kN}$$
$$T_2 = \sqrt{(12.5 - 5)^2 + 45^2} = 45.6 \text{ kN}$$
$$T_3 = \sqrt{(12.5 - 10)^2 + 45^2} = 45.07 \text{ kN}$$
$T_4 = T_3$, $T_5 = T_2$ and $T_6 = T_1$ due to symmetry.
9. Let θ_1, θ_2, θ_3, θ_4, θ_5 and θ_6 are inclined with horizontal line mo.
$$\tan \theta_1 = \frac{12.5}{45} \Rightarrow \theta_1 = 15.52°$$
$$\tan \theta_2 = \frac{12.5 - 5}{45} \Rightarrow \theta_2 = 9.46°$$
$$\tan \theta_3 = \frac{12.5 - 10}{45} \Rightarrow \theta_3 = 3.18°$$
$$\theta_4 = \theta_3 = 3.18°, \theta_5 = \theta_2 = 9.46°$$
and, $$\theta_6 = \theta_1 = 15.52° \text{ due to symmetry.}$$
10. Now length of cable, $l_1 = l_6 = 5 \sec 15.52° = 5.19$ m
$$l_2 = l_5 = 5 \sec 9.46° = 5.07 \text{ m}$$
$$l_3 = l_4 = 5 \sec 3.18° = 5.008 \text{ m}$$
11. Total length of cable, $l = l_1 + l_2 + l_3 + l_4 + l_5 + l_6$
$$= 2(5.19 + 5.07 + 5.008)$$
$$l = 30.536 \text{ m}$$
12. $$\text{Area of cross-section} = \frac{\text{Total tensile force}}{\text{Tensile stress}}$$
$$= \frac{T_1 + T_2 + T_3 + T_4 + T_5 + T_6}{140}$$
$$A = \frac{274.74}{140} = 1.96 \text{ mm}^2$$

Que 1.13. **A suspension cable of span 20 m and central dip 2 m is carrying a UDL of 20 kN/m. Find the horizontal pull in the cable. Also find the maximum and minimum tensions in the cable.**

Answer

Given : Span, L = 20 m, Dip, h = 2 m, Intensity of UDL, w = 20 kN/m
To Find : Horizontal reaction, maximum and minimum tension.

1. Minimum tension in the cable,

$$T_{\min} = H = \frac{wL^2}{8h} = \frac{20 \times (20)^2}{8 \times 2} = 500 \text{ kN}$$

3. Maximum tension in the cable,

$$T_{\max} = H\sqrt{1 + \left[\frac{wL/2}{H}\right]^2} = 500\sqrt{1 + \left(\frac{20 \times 20}{2 \times 500}\right)^2} = 538.5 \text{ kN}$$

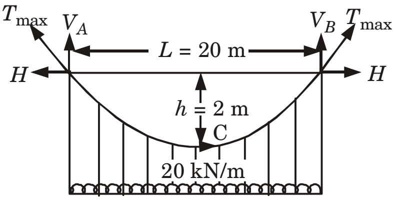


Fig. 1.13.1.

Que 1.14. **A cable is suspended between two points at the same level with a central dip of 15 m over a span of 130 m and carries a uniformly distributed load of intensity 1.5 kN/m of horizontal length. Calculate the change in the horizontal tension if the temperature rises by 30°F from the original. Take $a = 5 \times 10^{-6}$ per 1°F.**

Answer

Given : Supports are at same level (*i.e.*, $\theta = 0°$)
Span of cable, L = 130 m, Sag of cable, h = 15 m
Intensity of UDL, w = 1.5 kN/m, $\alpha = 5 \times 10^{-6}$ / °F, t = 30°
To Find : Change in horizontal tension.

1. Horizontal reaction, $H = \dfrac{wL^2}{8h} = \dfrac{1.5 \times 130^2}{8 \times 15} = 211.25$ kN

2. Change in horizontal reaction, $\dfrac{\partial H}{H} = -\dfrac{3}{16} \times \dfrac{L^2}{h^2} \times \alpha \times t$

$$= -\frac{3}{16} \times \frac{130^2}{15^2} \times 5 \times 10^{-6} \times 30 = -2.1125 \times 10^{-3}$$

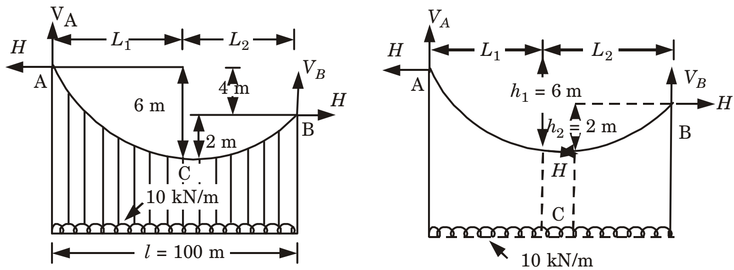
$$\delta H = -2.1125 \times 10^{-3} \times 211.25 = -0.446 \text{ kN}$$

3. There is a decrease in horizontal reaction.

Que 1.15. **A cable of uniform cross sectional area is stretched between two supports 100 m apart with one end 4 m above the other end as shown in Fig. 1.15.1. The cable is loaded with a UDL of 10 kN/m and the sag of cable measured from higher end is 6 m. Find the horizontal tension in the cable. Also find the maximum tension in the cable.**

Answer

Given : Length of span, L = 100 m
Height of lowest point of cable from highest support h_1 = 6 m
Depth of lowest point of cable from other support, h_2 = 6 – 4 = 2 m
Intensity of UDL, w = 10 kN/m
To Find : Horizontal tension and maximum tension.



(*a*) Cable supported at different levels. (*b*) Free body diagram of cable.

Fig. 1.15.1.

1. Let V_A and V_B be the vertical reaction at support A and B, respectively and H be the horizontal reaction. Let C be the lowest point.
2. The tensile force is equal to H at point C, distance between point A and C be L_1 and between C and B be L_2.
3. In part AC :

$$\Sigma M_A = 0$$

$$-H \times 6 + \frac{10 \times L_1^2}{2} = 0$$

$$H = \frac{10}{12} L_1^2 \qquad ...(1.15.1)$$

4. In Part CB :

$$\Sigma M_B = 0$$

$$H \times 2 = 10 \times \frac{L_2^2}{2}$$

$$H = \frac{10}{4} L_2^2 \qquad ...(1.15.2)$$

5. From eq. (1.15.1) and eq. (1.15.2),

$$H = \frac{10}{12} L_1^2 = \frac{10}{4} L_2^2$$

$$L_1 = L_2\sqrt{3}$$

$$L_1 + L_2 = 100$$

$$\sqrt{3}\, L_2 + L_2 = 100$$

$$L_2 = 36.6 \text{ m}$$

$$L_1 = 63.4 \text{ m}$$

$$H = \frac{10}{12} \times L_1^2 = \frac{10}{12} \times 63.4^2$$

$$= 3349.63 \text{ kN}$$

6. Considering the equilibrium of vertical forces on AC, we get

$$V_A - 10 \times L_1 = 0$$

$$V_A = 10 \times 63.4 = 634 \text{ kN}$$

7. Considering the equilibrium of vertical forces on CB, we get

$$V_B - 10 \times L_2 = 0$$

$$V_B = 10 \times 36.6 = 366 \text{ kN}$$

8. Maximum tension in the cable occurs at support A $(\because V_A > V_B)$

$$T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{634^2 + 3349.6^2}$$

$$T_{max} = 3409.07 \text{ kN}$$

Que 1.16. **A foot bridge is carried over a river of span 90 m. The supports are 3 m and 12 m higher than the lowest point of the cable. Determine the length of the cable. If the horizontal deck is loaded by UDL of 20 kN/m, find the tension in the cable.**

Answer

Given : Span, L = 90 m, Intensity of UDL, w = 20 kN/m,
To Find : Length of cable and tension in the cable.

1. Let the vertical reaction at the supports A and B be V_A and V_B and H be the horizontal reaction at each support.
3. Resolving the forces on the cable vertically, we have

$$V_A + V_B = 90 \times 20 = 1800$$

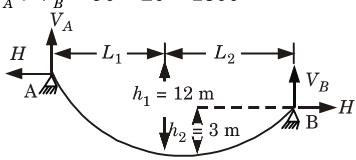


Fig. 1.16.1.

4. $$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} = \frac{90\times\sqrt{12}}{\sqrt{12}+\sqrt{3}} = 60 \text{ m},$$

$$L_2 = 90 - 60 = 30 \text{ m}$$

5. Taking moments about C of the forces on the left side of C, we have,

$$V_A \times 60 = 12\,H + \frac{20\times 60^2}{2},\ V_A = \frac{H}{5} + 600$$

6. Taking moment about point C consider the right side of C, we have,

$$V_B \times 30 = 3\,H + \frac{20\times 30^2}{2}$$

$$V_B = \frac{H}{10} + 300$$

7. $$V_A + V_B = \frac{H}{5} + 600 + \frac{H}{10} + 300 = 1800$$

8. $$\frac{3\,H}{10} = 900;\ H = 3000$$

9. $$V_B = \frac{H}{10} + 300 = \frac{3000}{10} + 300 = 600 \text{ kN}$$

10. $$V_A = \frac{H}{5} + 600 = \frac{3000}{5} + 600 = 1200 \text{ kN}$$

11. Maximum tension occurs at the higher support *i.e.*, at A,

$$T_{\max} = \sqrt{V_A^2 + H^2} = \sqrt{1200^2 + 3000^2} = 3231.1 \text{ kN}$$

12. Length of the cable, $l = L + \frac{2}{3}\frac{h_1^2}{L_1} + \frac{2}{3}\frac{h_2^2}{L_2}$

$$l = 90 + \frac{2}{3}\times\frac{12^2}{60} + \frac{2}{3}\times\frac{3^2}{30} = 91.8 \text{ m}$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain the classification of structure.

Ans. Refer Q. 1.2, Unit-1.

Q. 2. What do you understand by static and kinematic indeterminacy ?

Ans. Refer Q. 1.7 and Q. 1.6; Unit-1.

Q. 3. A cable is used to support five equal and equidistant loads over a span of 30 m. Find the length of the cable required and its sectional area, if safe tensile stress is 140 N/mm^2. The central dip of the cable is 2.5 m and loads are 5 kN each.

Ans. Refer Q. 1.12, Unit-1.

Q. 4. A cable is suspended between two points at the same level with a central dip of 15 m over a span of 130 m and carries a uniformly distributed load of intensity 1.5 kN/m of horizontal length. Calculate the change in the horizontal tension if the temperature rises by 30°F from the original. Take $\alpha = 5 \times 10^{-6}$ per 1°F.

Ans. Refer Q. 1.14, Unit-1.

☺☺☺

2
UNIT

Analysis of Trusses

PART-1

Classification of Pin Jointed Determinate Trusses, Analysis of Determinate Plane Trusses (Compound and Complex)

CONCEPT OUTLINE : PART-1

Classification of Pin Jointed Determinate Trusses : The following five criteria are the basis for the classification of trusses :
1. The shape of the upper and lower chord,
2. The type of the web,
3. The condition of the supports,
4. The purpose of the structure,
5. The level of the floor.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. Enumerate the different types of pinned jointed determinate truss with suitable example and sketches.

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OR

Explain the classification of pin-joined determinate trusses with the help of neat sketches.

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Answer

The following five criteria are the basis for the classification of trusses :

1. **According to the Shape of the Upper and Lower Chords :** The trusses can be classified into trusses with parallel chords as shown in Fig. 2.1.1, polygonal and triangular trusses or trusses with inclined chords as shown in Fig. 2.1.2.

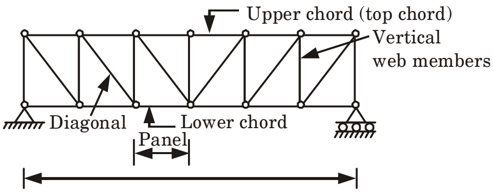

Fig. 2.1.1. A truss with parallel chords.

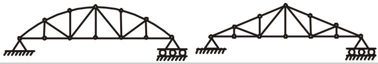

Fig. 2.1.2. Polygonal and triangular trusses.

2. **According to the Type of the Web :** It permits to subdivide the trusses into those with triangular patterns as shown in Fig. 2.1.3(a), those with quadrangular patterns as shown in Fig. 2.1.3(b) formed by vertical and diagonals, those with the web members form a letter *K* as shown in Fig. 2.1.3(c).

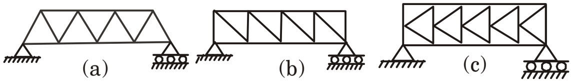

Fig. 2.1.3. Trusses according to the type of the web.

3. **According to the Conditions of the Support :** It permits to distinguish between the ordinary end-supported trusses as shown in Fig. 2.1.4(a), the cantilever trusses as shown in Fig. 2.1.4(b), the trusses cantilevering over one or both supports as shown in Fig. 2.1.4(c), and finally crescent or arched trusses as shown in Fig. 2.1.4(d) and Fig. 2.1.4(e).

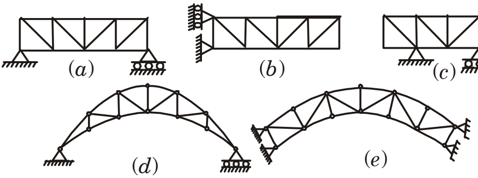

Fig. 2.1.4. Trusses-depending on the type of supports.

4. **According to their Purpose :** The trusses may be classified as roof trusses, bridge trusses, those used in crane construction.

5. **According to the Level of the Road :**

i. The trusses can be constructed so that the road is carried by the bottom chord joints as shown Fig. 2.1.5(*a*), or the upper chord joints as shown in Fig. 2.1.5(*b*).

ii. Sometimes the road (lane) is carried at some intermediate level as shown in Fig. 2.1.5(*c*).

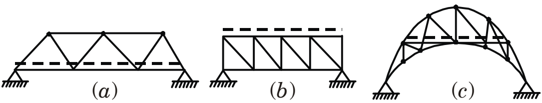

Fig. 2.1.5. Trusses depending on the level of the road.

Que 2.2. What are various types of trusses ? Explain with the help of neat sketches.

Answer

The various types of the trusses are described as follows :

A. Roof Trusses :

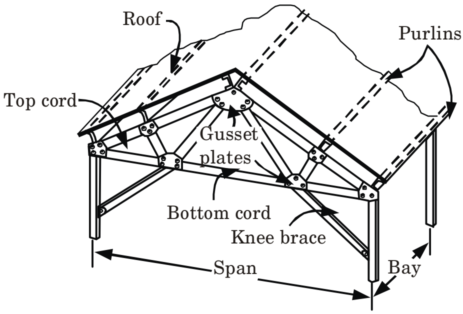

Fig. 2.2.1. Roof truss.

1. Roof trusses are often used as part of an industrial building frame, as shown in Fig. 2.2.1. Here, the roof load is transmitted to the truss at the joints by means of a series of purlins.
2. The roof truss along with its supporting columns is called as bent.
3. Ordinarily, roof trusses are supported either by columns of wood, steel, or reinforced concrete, or by masonry walls.
4. To keep the bent rigid, and thereby capable of resisting horizontal wind forces, knee braces are sometimes used at the supporting columns.
5. The space between adjacent bents is called a bay. Bays are economically spaced at about 4.6 m for spans around 18 m and about 6.1 m for spans of 30 m.
6. Bays are often tied together using diagonal bracing in order to maintain rigidity of the building's structure.
7. Trusses used to support roofs are selected on the basis of the span, the slope, and the roof material.
8. Some common types of trusses are scissors truss, the Howe and Pratt truss, Fan truss or Fink truss, Warren truss, Sawtooth truss, Bowsting truss etc.

B. Bridge Trusses :

1. The main structural elements of a typical bridge truss are shown in Fig. 2.2.2. Here it is seen that a load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses.
2. The top and bottom cords of these side trusses are connected by top and bottom lateral bracing, which serves to resist the lateral forces caused by wind and the sideway caused by moving vehicles on the bridge.
3. Additional stability is provided by the portal and sway bracing.
4. As in the case of many long-span trusses, a roller is provided at one end of a bridge truss to allow for thermal expansion.
5. A few of the typical forms of bridge truss: Pratt, Howe and Warren trusses are normally used for spans up to 61 m in length.
6. For larger spans, a truss with a polygonal upper cord, such as the Parker truss, is used for some savings in material.

7. The greatest economy of material is obtained if the diagonals have a slope between 45° and 60° with the horizontal.
8. If this rule is maintained, then for spans greater than 91 m, the depth of the truss must increase and consequently the panels will get longer.
10. This result in a heavy deck system and, to keep the weight of the deck within tolerable limits, subdivided trusses has been developed.

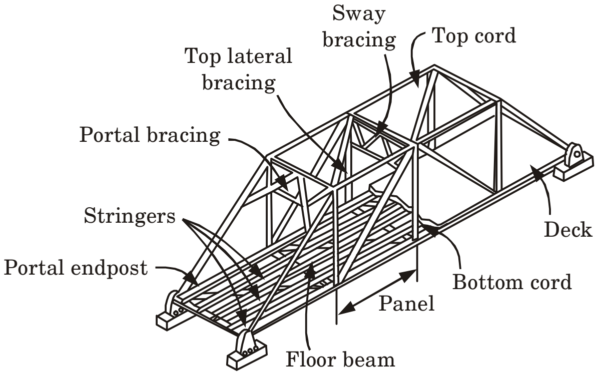

Fig. 2.2.2. Bridge truss.

Que 2.3. **What are different methods of analysis of trusses ? Explain any one in detail.**

OR

What are the methods available for the analysis of trusses ? Explain.

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Answer

Different Methods of Analysis of Trusses : To analyze a statically determinate truss we have following two methods :

1. Method of joints.
2. Method of sections.

1. Method of Joints :

i. In method of joints, the principle used is, if a truss is in equilibrium, then each of its joint must also be in equilibrium.
ii. Joint analysis should start at a joint having atleast one known force and atmost two unknown forces.
iii. If a force is pushing on the pin, member will be in compression and vice-versa.
iv. During solution of a truss always assume the unknown member force acting on the joint's free body diagram to be in tension.
v. If our assumption is correct then solution will be positive for the member.
vi. Now use the correct magnitude and direction of force on subsequent joint's free body diagram.

Procedure to Analyze :

i. First calculate the external support reactions by drawing the free body diagram of the entire truss.
ii. Draw free body diagram of a joint having at least one known force and atmost two unknown forces.
iii. Assume the unknown member force as tensile in nature *i.e.*, force is pulling the joint.
iv. Now orientation of X and Y axes are to made such that forces on the free body diagram can be easily resolved into their x and y components.
v. Apply two force equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Solve these two equations to get unknown member force and then verify their correct directional sense.
vi. Continue to analyze each of the other joints, where again it is necessary to choose a joint having atleast one known force and atmost two unknown forces.

2. Method of Sections :

i. If the forces in only a few members of a truss are to be found, then the method of sections generally used to obtain these forces.
ii. The method of section involves passing an imaginary section through the truss, thus cutting it into two parts.
iii. Each of the two parts must also be in equilibrium.
iv. Try to select the isolated portion of the truss have not more than three unknown force members.
v. Since only three independent equilibrium equations are present as :

$$\Sigma F_x = 0, \Sigma F_y = 0 \text{ and } \Sigma M = 0.$$

Procedure to Analyze :

i. Firstly cut or choose a section of truss where forces are to be determined.
ii. Selecting the section, it may first be necessary to determine the truss's external reactions.
iii. Draw the free body diagrams of the selected or sectioned truss having the least number of forces on it.
iv. Let the member force as tensile in nature, *i.e.*, pulling on the member joint pin.
v. Take summation of the moment about a point that lies at the intersection of the lines of action of two unknown forces.
vi. If two of the unknown forces are parallel, forces may be summed perpendicular to the direction of these unknowns to determine directly the third unknown force.

Que 2.4. **Discuss the concept of internal stability of a truss.**

Answer

A. Internal Stability :

1. Internal stability of a truss can be checked by inspection of the arrangement of its members.
2. If each joint is held fixed so that it cannot move in a rigid body sense with respect to the other joints, then the truss will be stable.

3. A simple truss will always be internally stable.
4. If a truss is constructed so that it does not hold its joints in a fixed position, it will be unstable or have a critical form.

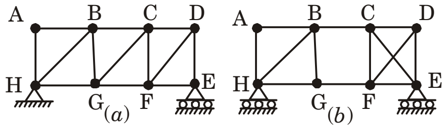

Fig. 2.4.1. (*a*) Internally stable truss. (*b*) Internally unstable truss.

Fig. 2.4.1(*a*) is internally stable. In Fig. 2.4.1(*b*) no fixity is provided between joints *B* and *F* or *G* and *C*, so the truss will collapse under load.

B. Internal Stability of a Compound Truss :

1. For checking the internal stability of a compound truss we need to identify the way in which the simple trusses are connected together.
2. In Fig. 2.4.2, when an external load is applied to joint *D*, *E* or *F* can cause the truss *DEF* to rotate slightly *i.e.*, internally unstable.

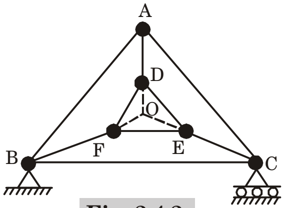

Fig. 2.4.2.

C. Internal Stability of a Complex Truss :

1. It may not be possible to tell about internal stability of a complex truss whether it is stable or unstable.
2. Remember that, however if a truss is unstable, it does not matter whether it is statically determinate or indeterminate. The use of an unstable truss is always avoided.

Que 2.5. **Explain the concept of Zero-Force Members in a truss.**

Answer

Concept of Zero-Force Member :

1. The member that supports no loading are termed as zero-force members.
2. Zero force members are used to simplify the truss analysis using the method of joints.
3. Zero force members are necessary for the stability of the truss during construction.
4. Zero force members are used to provide support if the applied loading is changed.
5. Zero force members occur in following two cases :

i. If no external load is acting at a joint and the members joined at this joint are at any angle θ then force in both of the members must be zero in order to maintain equilibrium.

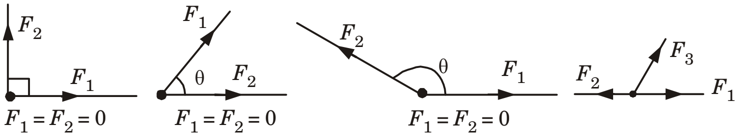

Fig. 2.5.1.

ii. If no external force is acting on a joint having three members for which two of the members are collinear, the third member is a zero force member.

Here $F_3 = 0$, as no force is acting in y-direction to balance it.

Que 2.6. **Determine the force in all the members of the truss shown in Fig. 2.6.1.**

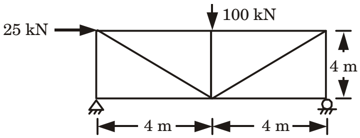

Fig. 2.6.1.

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Answer

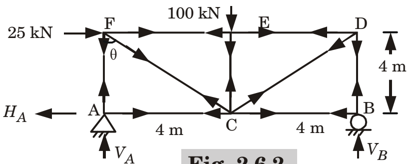

Fig. 2.6.2.

1. **Reaction at Supports :**

i.
$$\Sigma F_y = 0$$
$$V_A + V_B = 100 \text{ kN} \quad \text{...(2.6.1)}$$
$$H_A = 25 \text{ kN}$$

ii. Taking moment about A, $\Sigma M_A = 0$
$$V_B \times 8 = 100 \times 4 + 25 \times 4$$
$$V_B = 62.5 \text{ kN}$$

From eq. (2.6.1), $V_A = 37.5$ kN

2. **Joint A :**

i. Resolve the forces horizontally, $\Sigma F_x = 0$
$$F_{AC} - H_A = 0$$
$$F_{AC} = 25 \text{ kN}$$

ii. Resolve the forces vertically,
$$\Sigma F_y = 0$$
$$F_{AF} + V_A = 0$$
$$F_{AF} = -37.5 \text{ kN}$$

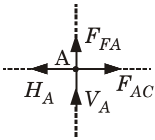

Fig. 2.6.3.

3. Joint *B* :

i. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{BC} = 0$$

ii. Resolve the forces vertically, $\Sigma F_y = 0$

$$F_{BD} + V_B = 0$$
$$F_{BD} = -62.5 \text{ kN}$$

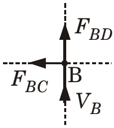

Fig. 2.6.4.

4. Joint *F* :

From Fig. 2.6.2, $\tan\theta = \dfrac{4}{4}, \theta = 45°$

i. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{FE} + F_{FC} \sin\theta + 25 = 0$$

$$F_{FE} + F_{FC} \times \frac{1}{\sqrt{2}} + 25 = 0 \qquad ...(2.6.2)$$

ii. Resolve the forces vertically, $\Sigma F_y = 0$

$$F_{FC} \cos\theta - 37.5 = 0$$

$$F_{FC} \times \frac{1}{\sqrt{2}} = 37.5$$

$$F_{FC} = 37.5 \times \sqrt{2} = 53.03 \text{ kN}$$

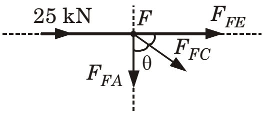

Fig. 2.6.5.

iii. From eq. (2.6.2)

$$F_{FE} + (53.03) \times \frac{1}{\sqrt{2}} = -25$$

$$F_{FE} + 37.5 = -25$$

$$F_{FE} = -62.5 \text{ kN}$$

5. Joint *E* :

i. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{ED} - F_{EF} = 0$$
$$F_{ED} + 62.5 = 0$$
$$F_{ED} = -62.5 \text{ kN}$$

ii. Resolve the forces vertically, $\Sigma F_y = 0$

$$F_{EC} + 100 = 0$$
$$F_{EC} = -100 \text{ kN}$$

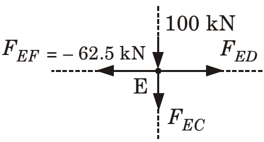

Fig. 2.6.6.

6. Joint *D* :

i. Resolve the forces horizontally,

$$\Sigma F_x = 0$$
$$F_{DE} + F_{DC} \sin 45° = 0$$

$$-62.5 + F_{DC} \times \frac{1}{\sqrt{2}} = 0$$

$$F_{DC} = 62.5 \times \sqrt{2} = 88.4 \text{ kN}$$

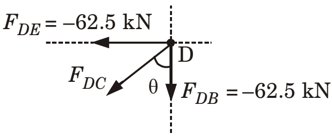

Fig. 2.6.7.

7. Forces in Similar Members :

F_{FA} = 37.5 kN (Compression), F_{AC} = 25 kN (Tension)
F_{FE} = 62.5 kN (Compression), F_{ED} = 62.5 kN (Compression)
F_{EC} = 100 kN (Compression), F_{BC} = 0
F_{BD} = 62.5 kN (Compression), F_{CD} = 88.4 kN (Tension)
F_{CF} = 53.03 kN (Tension)

Que 2.7. **Analyze the truss shown in Fig. 2.7.1.**

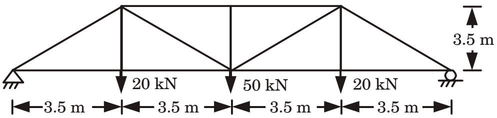

Fig. 2.7.1.

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Answer

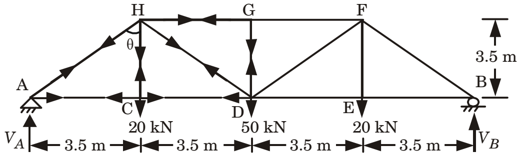

Fig. 2.7.2.

1. **Reaction at Supports :**

i.
$$\Sigma F_y = 0$$
$$V_A + V_B = 20 + 50 + 20$$
$$V_A + V_B = 90 \text{ kN} \quad \text{...(2.7.1)}$$

ii. Taking moment about A, $\Sigma M_A = 0$

$$20 \times 3.5 + 50 \times 7 + 20 \times 10.5 = V_B \times 14$$
$$V_B = 45 \text{ kN}$$

From eq. (2.7.1), $V_A = 45$ kN

From Fig. 2.7.1, $\tan \theta = \left(\frac{3.5}{3.5}\right)$

$$\theta = 45°$$

2. **Joint A :**

i. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{AC} + F_{AH} \cos 45° = 0$$
$$F_{AC} + \frac{F_{AH}}{\sqrt{2}} = 0 \quad \text{...(2.7.2)}$$

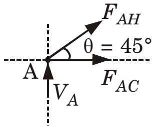

Fig. 2.7.3.

ii. Resolve the forces vertically, $\Sigma F_y = 0$

$$V_A + F_{AH} \sin 45° = 0$$
$$45 + \frac{F_{AH}}{\sqrt{2}} = 0, F_{AH} = -63.64 \text{ kN}$$

iii. From eq. (2.7.2) $F_{AC} - \dfrac{63.64}{\sqrt{2}} = 0, F_{AC} = 45$ kN

3. **Joint C :**

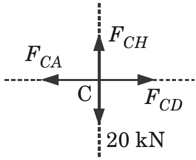

Fig. 2.7.4.

i. Resolve the forces horizontally, $\Sigma F_x = 0$

$$-F_{CA} + F_{CD} = 0$$
$$-45 + F_{CD} = 0$$
$$F_{CD} = 45 \text{ kN}$$

ii. Resolve the forces vertically, $\Sigma F_y = 0$

$$F_{CH} - 20 = 0$$
$$F_{CH} = 20 \text{ kN}$$

4. Joint *H* :

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$F_{HC} + F_{HA} \cos\theta + F_{HD} \cos\theta = 0$$

$$F_{HC} + \frac{F_{HA}}{\sqrt{2}} + \frac{F_{HD}}{\sqrt{2}} = 0$$

$$20 + \frac{F_{HD}}{\sqrt{2}} - \frac{63.64}{\sqrt{2}} = 0$$

$$F_{HD} = 35.35 \text{ kN}$$

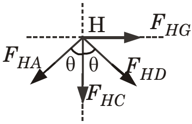

Fig. 2.7.5.

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{HG} + \frac{F_{HD}}{\sqrt{2}} - \frac{F_{HA}}{\sqrt{2}} = 0$$

$$F_{HG} + \frac{35.35}{\sqrt{2}} + \frac{63.64}{\sqrt{2}} = 0$$

$$F_{HG} = -70 \text{ kN}$$

5. It is symmetrical truss, so $F_{GD} = 0$

$$F_{AC} = F_{BE} = 45 \text{ kN (Tension)}$$
$$F_{AH} = F_{BF} = 63.64 \text{ kN (Compression)}$$
$$F_{HC} = F_{FE} = 20 \text{ kN (Tension)}$$
$$F_{CD} = F_{ED} = 45 \text{ kN (Tension)}$$
$$F_{HG} = F_{FG} = 70 \text{ kN (Compression)}$$
$$F_{HD} = F_{FD} = 35.35 \text{ kN (Tension)}$$

Que 2.8. **Determine the forces in the member marked X in the truss shown in Fig. 2.8.1.**

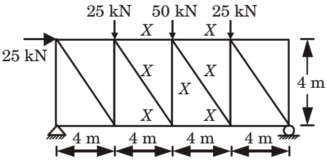

Fig. 2.8.1.

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Answer

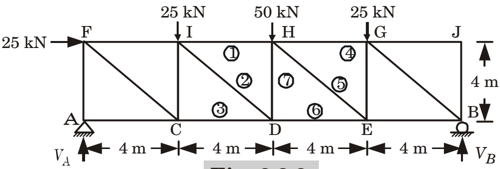


Fig. 2.8.2.

1. Let us calculate the reactions V_A and V_B

$$\Sigma F_y = 0$$

$$V_A + V_B = 25 + 50 + 25 \quad ...(2.8.1)$$

Taking moment about A,

$$V_B \times 16 = 25 \times 4 + 50 \times 8 + 25 \times 12 + 25 \times 4$$

$$V_B = 56.25 \text{ kN}$$

From eq. (2.8.1)

$$V_A = 43.75 \text{ kN}$$

2. Now draw a section line (1) – (1) cutting the members 1, 2, 3. Consider the equilibrium of the left part of the truss. Let F_1, F_2 and F_3 are the forces of members 1, 2 and 3.

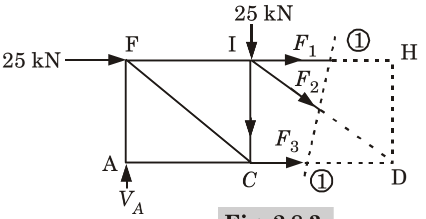


Fig. 2.8.3.

i. Taking moment about joint I,

$$43.75 \times 4 = F_3 \times 4$$

$$F_3 = 43.75 \text{ kN}$$

ii. Taking moment about joint H,

$$43.75 \times 8 - 25 \times 4 - F_3 \times 4 - F_2 \times 2.82 = 0$$

$$F_2 = \frac{43.75 \times 8 - 43.75 \times 4 - 100}{2.82}$$

$$F_2 = 26.60 \text{ kN}$$

iii. Taking moment about joint C,

$$43.75 \times 4 + 25 \times 4 + F_1 \times 4 + 26.60 \times 2.82 = 0$$

$$F_1 = -87.50 \text{ kN}$$

3. Now draw a section line (2) – (2) cutting the member 4, 5 and 6. Consider the equilibrium of right part of the truss. Let F_4, F_5 and F_6 are the forces of members 4, 5 and 6.

i. Taking moment about joint H,

$$56.25 \times 8 = 25 \times 4 + F_6 \times 4$$

$$F_6 = 87.5 \text{ kN}$$

ii. Taking moment about joint G,

$$-56.25 \times 4 + 87.5 \times 4 + F_5 \times 2.82 = 0$$

$$F_5 = -44.32 \text{ kN}$$

iii. Taking moment about joint E,

$$-56.25 \times 4 - F_4 \times 4 = 0$$

$$F_4 = -56.25 \text{ kN}$$

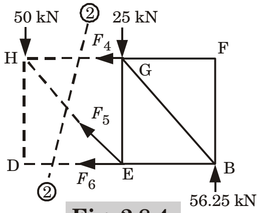


Fig. 2.8.4.

4. Now draw a section line (3) – (3) cutting the members 1, 7 and 6. Consider the equilibrium of left part of the truss. Let the unknown force is F_7.

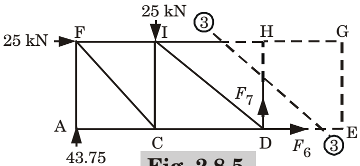


Fig. 2.8.5.

Taking moment about joint I,

$$43.75 \times 4 - F_7 \times 4 - F_6 \times 4 = 0$$

$$43.75 \times 4 - F_7 \times 4 - 87.5 \times 4 = 0$$

$$F_7 = -43.75 \text{ kN}$$

Que 2.9. **Analyze the truss shown in Fig. 2.9.1.**

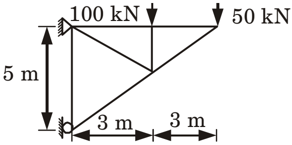


Fig. 2.9.1.

AKTU 2013-14, Marks 05

Answer

1. Joint A :

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$50 + F_{AD} \sin \theta = 0$$

$$F_{AD} = \frac{-50}{\sin \theta}$$

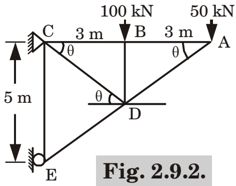


Fig. 2.9.2.

From ΔACE, $\tan \theta = \dfrac{5}{6}$, $\theta = 39.80°$

$$F_{AD} = \frac{-50}{\sin 39.80°} = -78.125 \text{ kN}$$

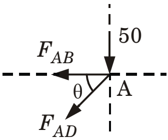


Fig. 2.9.3.

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{AB} + F_{AD} \cos 39.80° = 0$$

$$F_{AB} - 78.125 \times \cos 39.80 = 0$$

$$F_{AB} = 60.02 \text{ kN}$$

2. Joint B :

i. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{BC} = 60.02 \text{ kN}$$

ii. Resolve the forces vertically, $\Sigma F_y = 0$

$$F_{BD} = -100 \text{ kN}$$

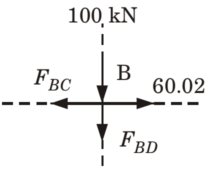


Fig. 2.9.4.

3. Joint *D* :

i. Resolve the forces horizontally, $\Sigma F_x = 0$

$F_{DC} \cos 39.80° + F_{DE} \cos 39.80$
$= F_{DA} \cos 39.80°$

$F_{DC} + F_{DE} = -78.125$ ...(2.9.1)

ii. Resolve the forces vertically, $\Sigma F_y = 0$

$F_{DA} \sin 39.80° + F_{DC} \sin 39.80° + F_{DB}$
$= F_{DE} \sin 39.80°$

$-78.125 \times \sin 39.80° + F_{DC} \sin 39.80° + (-100)$
$= F_{DE} \sin 39.80°$

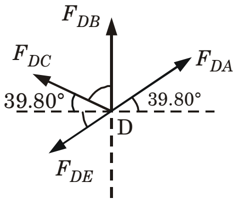


Fig. 2.9.5.

$$F_{DC} - F_{DE} = 234.35 \quad ...(2.9.2)$$

Solving the eq. (2.9.1) and (2.9.2)

$$F_{DC} = 78.11 \text{ kN}$$
$$F_{DE} = -156.24 \text{ kN}$$

4. Joint *E* :

i. Resolve the forces vertically,

$$\Sigma F_y = 0$$
$$F_{EC} + F_{DE} \sin 39.80° = 0$$
$$F_{EC} - 156.24 \times \sin 39.80° = 0$$
$$F_{EC} = 100 \text{ kN}$$

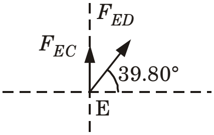


Fig. 2.9.6.

Que 2.10. **Find the forces in the members of the given truss.**

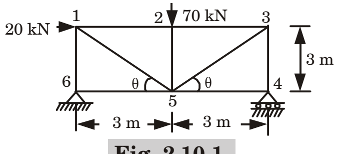


Fig. 2.10.1.

AKTU 2014-15, Marks 05

Answer

From Fig. 2.10.1, $\theta = \tan^{-1}(3/3) = 45°$

1. Reaction at Supports :

i. $\Sigma F_x = 0, -H_6 + 20 = 0$

$$H_6 = 20 \text{ kN}$$

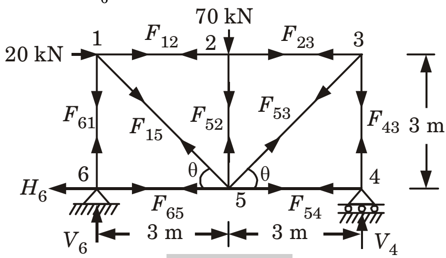


Fig. 2.10.2.

ii. $$\Sigma F_y = 0$$
$$V_6 + V_4 = 70 \quad ...(2.10.1)$$

iii Taking moment about joint 6, $\Sigma M_6 = 0$

$70 \times 3 + 20 \times 3 - V_4 \times 6 = 0$
$$210 + 60 = V_4 \times 6$$
$$V_4 \times 6 = 270$$
$$V_4 = 45 \text{ kN}$$

From eq. (2.10.1), $V_6 = 70 - V_4$
$$V_6 = 70 - 45$$
$$V_6 = 25 \text{ kN}$$

2. Joint (6) :

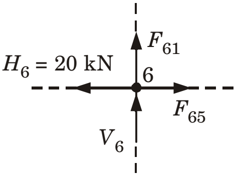


Fig. 2.10.3.

i. Resolve the forces vertically, $\Sigma F_y = 0$
$$F_{61} + V_6 = 0$$
$$F_{61} = -25 \text{ kN}$$

ii. Resolve the forces horizontally, $\Sigma F_x = 0$
$$-H_6 + F_{65} = 0$$
$$F_{65} = 20 \text{ kN}$$

3. Joint (4) :

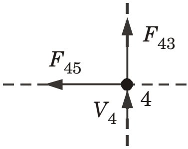


Fig. 2.10.4.

i. Resolve the forces vertically, $\Sigma F_y = 0$
$$F_{43} + \text{V}_4 = 0$$
$$F_{43} = -45 \text{ kN}$$

ii. Resolve the forces horizontally, $\Sigma F_x = 0$
$$F_{45} = 0$$

4. Joint (5) :

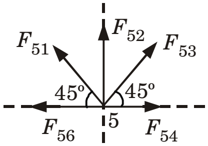


Fig. 2.10.5.

i. Resolve the forces horizontally,
$$\Sigma F_x = 0$$
$$F_{51} \cos 45° + F_{56} = F_{54} + F_{53} \cos 45°$$
$$F_{51} \cos 45° + 20 = F_{53} \cos 45° + 0$$
$$\cos 45° (F_{51} - F_{53}) = -20 \quad ...(2.10.2)$$

ii. Resolve the forces vertically,
$$\Sigma F_y = 0$$
$$F_{53} \sin 45° + F_{51} \sin 45° + F_{52} = 0 \quad ...(2.10.3)$$

5. Joint (2) :

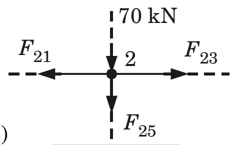


Fig. 2.10.6.

i. Resolve the forces vertically,
$$\Sigma F_y = 0$$
$$-F_{25} - 70 = 0$$
$$F_{52} = -70 \text{ kN}$$

ii. Resolve the forces horizontally,
$$\Sigma F_x = 0$$
$$F_{21} = F_{23} \quad ...(2.10.4)$$

Value of F_{52} put in eq. (2.10.3), we get
$$\sin 45°(F_{53} + F_{51}) = 70 \quad ...(2.10.5)$$

Solving eq. (2.10.2) and (2.10.5), we get
$$F_{51} = 35.36 \text{ kN}$$
$$F_{53} = 63.64 \text{ kN}$$

6. Joint (1) :

i. Resolving forces horizontally,

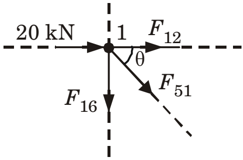


Fig. 2.10.7.

$$\Sigma F_x = 0$$
$$F_{12} + 20 + F_{15} \cos 45° = 0$$
$$F_{12} + 20 + 35.36 \times \cos 45° = 0$$
$$F_{12} = -45 \text{ kN}$$

ii. Value of F_{12} put in eq. (2.10.4),

$$F_{23} = -45 \text{ kN}$$

F_{16}= 25 kN (Compression), F_{12} = 45 kN (Compression)
F_{15} = 35.36 kN (Tension), F_{54} = 0
F_{53} = 63.64 kN (Tension), F_{52} = 70 kN (Compression)
F_{23} = 45 kN (Compression), F_{65} = 20 kN (Tension)
F_{43} = 45 kN (Compression)

PART-2

Method of Substitution and Method of Tension Coefficient for Analysis of Plane Trusses.

CONCEPT OUTLINE : PART-2

Method of Substitution : According to this, the forces in truss member is calculated by,

$$P_i = P_i' + u_i\,(P_i'/u_i)$$

Method of Tension Coefficients : This method gives member forces per unit length rather than member forces directly.

Tension coefficients, $t = \dfrac{T}{l}$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.11. **Discuss the tension coefficient method.**

AKTU 2012-13, Marks 05

Answer

A. Tension Coefficient : Tension coefficient for a member is defined as the tension per unit length of the member. It is given by,

$$t = \frac{T}{l}$$

where, t = Tension coefficient,
T = Tension or pull in the member, and
l = Length of the member.

B. Method of Tension Coefficient for Analysis of Plane Trusses :

1. All the members of the truss are initially assumed to be under tension due to load upon the truss.

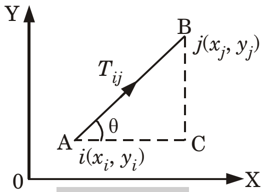


Fig. 2.11.1.

2. Let end A is at node i and end B is at node j, as shown in Fig. 2.11.1 and let the pull in the member AB is T_{ij}, then at node 'i', (at point A)

$$T_x = T_{ij} \cos \theta = T_{ij} \frac{AC}{AB} = T_{ij} \frac{(x_j - x_i)}{l_{ij}} = \frac{T_{ij} x_{ij}}{l_{ij}} = t_{ij} x_{ij} \qquad \left[\because t_{ij} = \frac{T_{ij}}{l_{ij}} \right]$$

where, l_{ij} = Length of $AB = \sqrt{x_{ij}^2 + y_{ij}^2}$

3. Similarly force at the node 'i' along Y direction, $T_y = t_{ij} \times y_{ij}$
4. Hence, if at node i there are more members and some external forces are also present then the condition of equilibrium at i can be written as,

$$\Sigma F_x = 0$$
$$\Sigma t_{ij} x_{ij} + F_x = 0$$
$$t_{ij} x_{ij} + t_{ik} x_{ik} + t_{iq} x_{iq} + \ldots + F_x = 0$$

and
$$\Sigma F_y = 0$$
$$\Sigma t_{ij} y_{ij} + F_y = 0$$
$$t_{ij} y_{ij} + t_{ik} y_{ik} + t_{iq} y_{iq} + \ldots + F_y = 0$$

Where F_x is the external load in the direction of X-axis and F_y is the external load in the direction of Y-axis.

6. In case of a space truss we have, $\Sigma t_{ij} x_{ij} + F_x = 0$

$$\Sigma t_{ij} y_{ij} + F_y = 0, \Sigma t_{ij} z_{ij} + F_z = 0$$

and
$$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2 + z_{ij}^2}$$

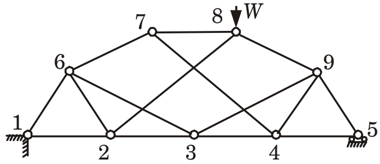
Que 2.12. **Explain in detail about method of substitution and method of tension coefficient with examples.**

AKTU 2015-16, Marks 10

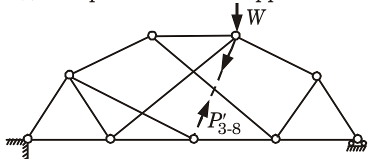
Answer

A. Method of Substitution :

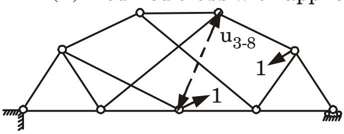
1. A complex truss, as shown in Fig. 2.12.1, has three or more connecting members at a node, all with unknown member forces.



(i) Complex truss with applied load



(ii) Modified truss with applied load



(iii) Modified truss with unit virtual load.

Fig. 2.12.1.

2. This precludes the use of the method of sections or the method of resolution at the nodes as a means of determining the forces in the truss.
3. The technique consists of removing one of the existing members at a node so that only two members with unknown forces remain and substituting another member so as to maintain the truss in stable equilibrium.
4. The forces in member 4-5 and 5-9 are obtained by resolution of forces at node 5. However, at nodes 4 and 9, three unknown member forces remain, and these cannot be determined by resolution or by the method of sections.
5. As shown in Fig. 2.12.1(ii), member 3-9 is removed, leaving only two unknown forces at node 9, which may be determined.
6. To maintain stable equilibrium, a substitute member 3-8 is added to create a modified truss, and the original applied loads are applied to the modified truss.
7. The forces P' in all the remaining members of the modified truss may now be determined. The force in member 3-8 is $P'_{3\text{-}8}$.
8. The applied loads are now removed, and unit virtual loads are applied to the modified truss along the line of action of the original member 3-9, as shown in Fig. 2.12.1(iii).
9. The forces u in the modified truss is determined; the force in member 3-8 is $-u_{3\text{-}8}$.

10. Multiplying, the forces in system (iii) by P'_{3-8}/u_{3-8} and adding them to the forces in system (ii) gives the force in member 8 as :

$$P_{3-8} = P'_{3-8} + (-u_{3-8})P'_{3-8}/u_{3-8} = 0$$

11. In fact, the substitute member 3-8 has been eliminated from the truss. Hence, by applying the principle of superposition, the final forces in the original truss are obtained from the expression :

$$P = P' + uP'_{3-8}/u_{3-8}$$

where, tensile forces are positive and compressive forces are negative.

12. The final force in member 3-8 is :

$$P_{3-8} = 1 \times P'_{3-8}/u_{3-8}$$
$$= P'_{3-8}/u_{3-8}$$

B. Method of Tension Coefficient : Refer Q. 2.11, Page 2–16C, Unit-2.

Que 2.13. **Determine the forces produced by the applied loads in members 49, 39, and 89 of the complex truss shown in Fig. 2.13.1.**

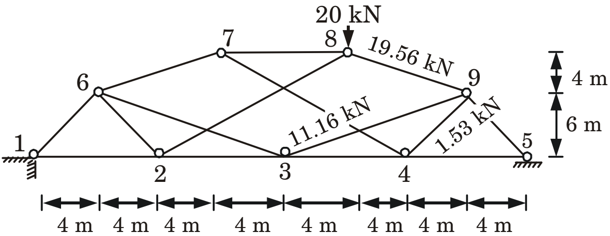


Fig. 2.13.1.

Answer

1. The modified truss shown in Fig. 2.13.2 is created by removing member 3-9, adding the substitute member 3-8, and applying the 20 kN load.
2. The member forces in the modified truss may now be determined; the values obtained are :

$$P'_{4-9} = 7.51 \text{ kN (Tension)}$$

$$P'_{8-9} = -13.89 \text{ kN (Compression)}$$

$$P'_{3-8} = 21.54 \text{ kN (Tension)}$$

3. The 20 kN load is removed from the modified truss, and unit virtual loads are applied at nodes 3 and 9 in the direction of the line of action of the force in member 3-9.

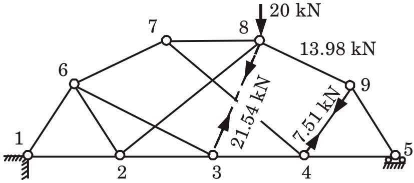


Fig. 2.13.2.

4. The member forces for this loading condition may now be determined; the values obtained are :

$$u_{4\text{-}9} = -0.81 \text{ kN (Compression)}$$
$$u_{8\text{-}9} = -0.50 \text{ kN (Compression)}$$
$$u_{3\text{-}8} = -1.93 \text{ kN (Compression)}$$

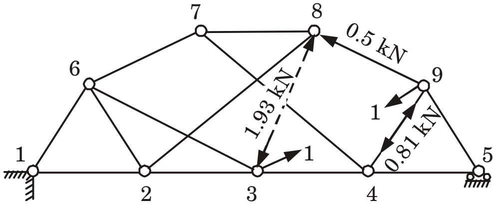


Fig. 2.13.3.

5. The multiplying ratio is given by :

$$P'_{3\text{-}8}/u_{3\text{-}8} = 21.54/1.93 = 11.16$$

6. The final member forces in the original truss are :

$$P_{4\text{-}9} = 7.51 + 11.16\,(-0.81)$$
$$= -1.53 \text{ kN (Compression)}$$
$$P_{8\text{-}9} = -13.98 + 11.16\,(-0.50)$$
$$= -19.56 \text{ kN (Compression)}$$
$$P_{3\text{-}9} = P'_{3\text{-}8}/u_{3\text{-}8} = 11.16 \text{ kN (Tension)}$$

Que 2.14. **Determine the forces in the member *OA*, *AB*, *AC* shown in Fig. 2.14.1 using tension coefficient method.**

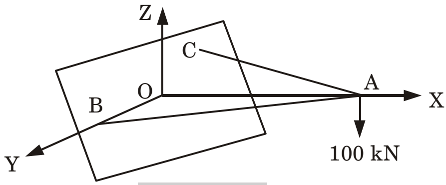


Fig. 2.14.1.

The co-ordinates of point *A*, *B* and *C* are (6, 0, 0) (0, 2.5, 0) and (0, –2, 1.5) respectively.

AKTU 2013-14, Marks 05

Answer

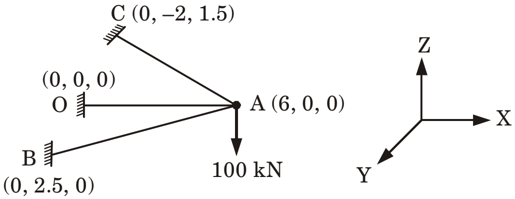


Fig. 2.14.2.

1. **Step 1 :** To fix up bar parameter, the co-ordinates of the nodal joints are indicated. The bar parameters are tabulated in table 2.14.1.

Table : 2.14.1.

S. No.	Member	x_{ij}	y_{ij}	z_{ij}	$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2 + z_{ij}^2}$	t_{ij}	F_{ij}
1	OA	6	0	0	6	120.01	720.06
2.	AB	–6	2.5	0	6.5	53.34	346.71
3	AC	–6	–2	1.5	6.5	66.67	433.35

2. **Step 2 :** Writing of equilibrium equation,

$$\Sigma F_x = 0$$

$$t_{OA} \times 6 + t_{AB} \times (-6) + t_{AC} \times (-6) = 0$$

$$t_{OA} - t_{AB} - t_{AC} = 0 \quad ...(2.14.1)$$

$$\Sigma F_y = 0$$

$$t_{OA} \times 0 + t_{AB} \times 2.5 + t_{AC} \times (-2) = 0$$

$$2.5\, t_{AB} - 2t_{AC} = 0 \quad ...(2.14.2)$$

$$\Sigma F_z = 0$$

$$t_{OA} \times 0 + t_{AB} \times 0 + t_{AC} \times 1.5 = 100$$

$$1.5\, t_{AC} = 100 \quad ...(2.14.3)$$

$$t_{AC} = 66.67$$

From eq. (2.14.2), $t_{AB} = 53.34$

From eq. (2.14.1), $t_{OA} = 120.01$

3. Force in member *OA*, *AB* and *AC* :

$$F_{OA} = l_{OA} t_{OA} = 6 \times 120.01 = 720.06 \text{ kN} \quad \text{(Tensile)}$$

$$F_{AB} = 6.5 \times 53.34 = 346.71 \text{ kN} \quad \text{(Tensile)}$$

$$F_{AC} = 6.5 \times 66.67 = 433.35 \text{ kN} \quad \text{(Tensile)}$$

Que 2.15. **Analyze the truss shown in Fig. 2.15.1 by the method of tension coefficient and determine the forces in all the members.**

AKTU 2015-16, Marks 7.5

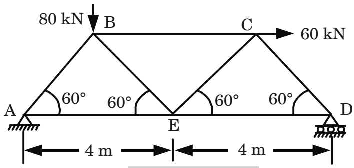


Fig. 2.15.1.

Answer

1. **Step - I :** Redraw the figure so that co-ordinate of various joints can be obtained that joint A is at origin.

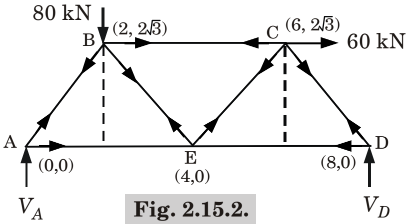


Fig. 2.15.2.

2. **Step - II :** Member parameters of truss.

S. No.	Member	x_i	x_j	$x_{ij} = (x_j - x_i)$	y_i	y_j	$y_{ij} = y_j - y_i$	$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}$
1.	*AB*	0	2	2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{2^2 + (2\sqrt{3})^2} = 4$
2.	*BC*	2	6	4	$2\sqrt{3}$	$2\sqrt{3}$	0	4
3.	*CD*	6	8	2	$2\sqrt{3}$	0	$-2\sqrt{3}$	$\sqrt{2^2 + (-2\sqrt{3})^2} = 4$
4.	*DE*	8	4	–4	0	0	0	4
5.	*EA*	4	0	–4	0	0	0	4
6.	*EB*	4	2	–2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$
7.	*EC*	4	6	2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{2^2 + (2\sqrt{3})^2} = 4$

3. **Step - III :** Calculation of tension coefficients,

i. Taking moment about A,

$$8 \times V_B = 60 \times 2\sqrt{3} + 80 \times 2$$

$$V_B = 45.98 \approx 46 \text{ kN}$$

ii. $$\Sigma F_y = 0$$

$$V_A + V_B - 80 = 0$$

$$V_A = 80 - 46$$

$$V_A = 34 \text{ kN}$$

iii. $$\Sigma F_x = 0$$

$$H_A - 60 = 0$$
$$H_A = 60 \text{ kN}$$

iv. **Joint A :**

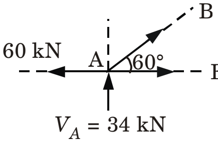


Fig. 2.15.3.

a. Resolve the forces horizontally,

$$\Sigma F_x = 0$$
$$x_{AB} \times t_{AB} + x_{AE} \times t_{AE} - 60 = 0$$
$$2 \times t_{AB} + 4 \times t_{AE} - 60 = 0$$
$$t_{AB} + 2 \times t_{AE} = 30 \quad ...(2.15.1)$$

b. Resolve the forces vertically,

$$\Sigma F_y = 0$$
$$y_{AB} \times t_{AB} + y_{AE} \times t_{AE} + 34 = 0$$
$$2\sqrt{3} \times t_{AB} + 0 = -34$$
$$t_{AB} = -9.82 \text{ kN/m}$$

c. Value of t_{AB} put in eq. (2.15.1), we get

$$-9.82 + 2\, t_{AE} = 30$$
$$2 \times t_{AE} = 30 + 9.82$$
$$t_{AE} = 19.91 \text{ kN/m}$$

v. **Joint D :**

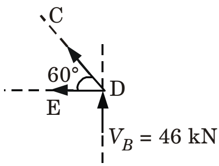


Fig. 2.15.4.

a. Resolve the forces horizontally,

$$\Sigma F_x = 0$$
$$x_{DC} \times t_{DC} + x_{DE} \times t_{DE} = 0$$
$$(-2) \times t_{DC} + (-4) \times t_{DE} = 0$$
$$t_{DC} = -2\, t_{DE} \quad ...(2.15.2)$$

b. Resolve the forces vertically,

$$\Sigma F_y = 0$$
$$y_{DC} \times t_{DC} + y_{DE} \times t_{DE} + 46 = 0$$
$$2\sqrt{3} \times t_{dc} + 0 + 46 = 0$$

$$t_{DC} = -\frac{46}{2\sqrt{3}} = -13.28 \text{ kN/m}$$

c. Value of t_{DC} put in equation (2.15.2), we get

$$-2\, t_{DE} = t_{DC}$$
$$-2\, t_{DE} = -13.28$$
$$t_{DE} = 6.64 \text{ kN/m}$$

vi. **Joint C :**

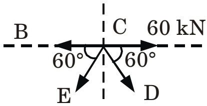


Fig. 2.15.5.

a. Resolve the forces horizontally,

$$\Sigma F_x = 0$$
$$x_{CB} \times t_{CB} + x_{CE} \times t_{CE} - x_{CD} \times t_{CD} - 60 = 0$$
$$(-4) \times t_{CB} + (-2) \times t_{CE} - 2 \times t_{CD} - 60 = 0$$
$$(-4) \times t_{CB} + (-2) \times t_{CE} - 2 \times (-13.28) - 60 = 0$$
$$2\, t_{CB} + t_{CE} = -16.72 \qquad ...(2.15.3)$$

Resolve the forces vertically, $\Sigma F_y = 0$

$$y_{CE} \times t_{CE} + y_{CD} \times t_{CD} = 0$$
$$(-2\sqrt{3})\, t_{CE} + (-2\sqrt{3}) \times (-13.28) = 0$$
$$t_{CE} = 13.28 \text{ kN/m}$$

c. Value of t_{CE} put in equation (2.15.3), we get

$$2\, t_{CB} + 13.28 = -16.72$$
$$t_{CB} = -15 \text{ kN/m}$$

vii. **Joint E :**

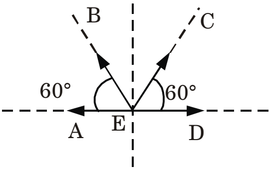


Fig. 2.15.6.

Resolve the forces vertically,

$$\Sigma F_y = 0$$
$$y_{EB} \times t_{EB} + y_{EC} \times t_{EC} = 0$$
$$(2\sqrt{3}) \times t_{EB} + (2\sqrt{3}) \times t_{EC} = 0$$
$$t_{EB} = -t_{EC}$$
$$t_{EB} = -13.28 \text{ kN/m}$$

4. **Step IV :** Final forces in members (Represented by T_{ij})

S. No.	Member	t_{ij}	l_{ij}	$T_{ij} = t_{ij} \times l_{ij}$	Nature of Force
1.	*AB*	– 9.82	4	– 39.28	Compression
2.	*BC*	– 15	4	– 60	Compression
3.	*CD*	– 13.28	4	– 53.12	Compression
4.	*DE*	6.64	4	26.56	Tension
5.	*EA*	19.91	4	79.64	Tension
6.	*BE*	– 13.28	4	– 53.12	Compression
7.	*CE*	13.28	4	53.12	Tension

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain the classification of pin jointed determinate trusses with the help of neat sketch.

Ans. Refer Q. 2.1, Unit-2.

Q. 2. What are the methods available for the analysis of trusses ?

Ans. Refer Q. 2.3, Unit-2.

Q. 3. Explain the concept of Zero-Force Members in a truss.

Ans. Refer Q. 2.5, Unit-2.

Q. 4. Discuss the tension coefficient method of truss analysis.

Ans. Refer Q. 2.11, Unit-2.

☺☺☺

3
UNIT

Strain Energy and Deflection of Beams

PART-1

Strain Energy of Deformable System, Maxwell's Reciprocal and Betti's Theorem

CONCEPT OUTLINE : PART-1

Strain Energy : When an elastic member is deformed under the action of an external loading, the member is said to have processed or stored energy which is called strain energy of the member or resilience of the member.

Maxwell's Reciprocal Deflection Theorem : In any beam or truss the deflection at any point D due to a load W at any other point C is the same as the deflection at C due to the same load W applied at point D.

Betti's Law : This law state that in any structure the material of which is elastic and follows Hooke's law and in which the supports are unyielding and the temperature is constant, the virtual work done by a system of forces P_1, P_2, P_3, ... during the distortion caused by a system of forces Q_1, Q_2, Q_3, ... is equal to the virtual work done by the system of forces Q_1, Q_2, Q_3, ... during the distortion caused by the system of forces P_1, P_2, P_3, ...

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **Define the term strain energy or resilience of the member. Deduce the strain energy expression for the following cases :**

1. **Strain energy stored in a member due to axial loading.**
2. **Strain energy stored in a member due to bending.**
3. **Strain energy stored in a beam subjected to a uniform bending moment.**

Answer

A Strain Energy :

1. When an elastic member is deformed under the action of an external loading till its elastic limit, the member has some energy stored in it, this energy is called as the strain energy of the member or the resilience of the member.

2. The strain energy of the deformed member is equal to the amount of work done by the external force to produce the deformation.

B. Expressions for Strain Energy :

1. Strain Energy in a Member due to Axial Loading :

i. Consider an elastic member of length L and cross-sectional area A subjected to an external axial load W.

ii. The extension in the member is δ, as the load is gradually applied so the magnitude of load is increased from 0 to W.

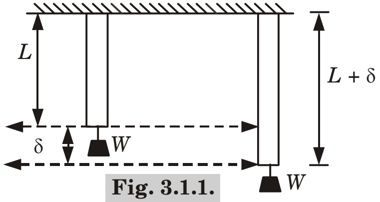


Fig. 3.1.1.

iii.
$$\text{Strain energy} = \text{External work done by the load}$$
$$= \text{Average load} \times \text{Deflection}$$
$$= \frac{1}{2}(W+0)\times\delta = \frac{1}{2}W\delta$$

iv. Now, tensile stress, $\sigma = \dfrac{\text{Load}}{\text{Area}} = \dfrac{W}{A}$

$$\text{Tensile strain, } e = \frac{\text{Tensile stress}}{\text{Modulus of elasticity}} = \frac{\sigma}{E} = \frac{W}{AE}$$

Change in length of the member, δ = Strain × Original length

$$\delta = \frac{WL}{AE}$$

v. Strain energy stored in the member $= \dfrac{1}{2}W \times \delta$

$$= \frac{1}{2}W \times \frac{WL}{AE} = \frac{1}{2}\times\frac{W^2L}{AE}$$

vi. Strain energy stored in the member per unit volume

$$= \frac{\frac{1}{2}\frac{W^2L}{AE}}{AL} = \frac{1}{2E}\left(\frac{W}{A}\right)^2 = \frac{1}{2E}\sigma^2 = \frac{\sigma^2}{2E}$$

2. Strain Energy Stored in a Member due to Bending :

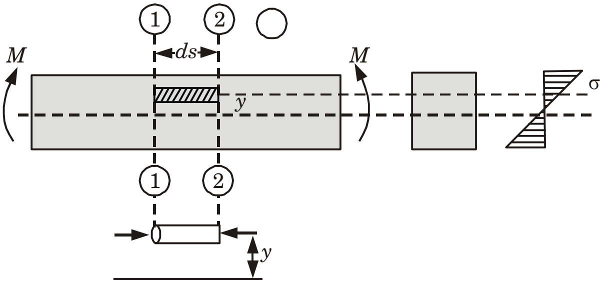


Fig. 3.1.2.

i. A beam shown in the Fig. 3.1.2 is subjected to a uniform moment M.

ii. Let an elemental length ds of the beam between two section 1-1 and 2-2 having elementary length ds.

iii. The elemental length of the beam may be considered as consisting of an infinite number of elemental cylinders each of area da and length ds.

iv. Considering a cylindrical element at y distant from the neutral layer between the sections 1-1 and 2-2.

Intensity of stress in the elemental cylinder, $\sigma = \dfrac{M}{I}\, y$

I = Moment of inertia of the entire section of beam about the neutral axis.

v. Energy stored by the elemental cylinder = Energy stored per unit volume × Volume of the cylinder.

$$= \frac{\sigma^2}{2E}\, da \times ds = \frac{1}{2E}\left(\frac{M}{I}\, y\right)^2 da \times ds = \frac{M^2}{2EI^2}\, ds \times da \times y^2$$

vi. Energy stored by the element ds = Sum of the energy stored by each elemental cylinder between the sections 1-1 and 2-2.

$$= \sum \frac{M^2}{2\,EI^2}\, ds \times da \times y^2 = \frac{M^2\, ds}{2EI^2} \sum da \times y^2$$

$\Sigma da \times y^2 = I$ = Moment of inertia of the beam section about its neutral axis.

vii. Energy stored by ds length of the beam $= \dfrac{M^2 \times ds}{2EI}$

Total energy stored by the whole beam $= \displaystyle\int \frac{M^2 ds}{2EI}$

3. Strain Energy Stored in a Beam Subjected to a Uniform Bending Moment :

i. For constant bending moment M, the strain energy

$$U_i = \int \frac{M^2 ds}{2EI} = \frac{M^2 L}{2EI}$$

ii. If the beam section is rectangular with width b and depth d and the beam is subjected to uniform bending moment M,

$$M = \frac{1}{6} \sigma \, bd^2$$

Where σ is the extreme bending stress for each section

and
$$I = \frac{bd^3}{12}$$

$$U_i = \frac{1}{2E}\left(\frac{1}{6}\sigma bd^2\right)^2 \times \frac{L}{\frac{bd^3}{12}}$$

$$U_i = \frac{\sigma^2}{6E} b \times dL = \frac{\sigma^2}{6E} \times \text{Volume of the beam}$$

Que 3.2. **State and prove Betti's law of reciprocal theorem.**

AKTU 2012-13, Marks 10

Answer

Statement : For a structure whose material is elastic and follows Hooke's law and in which the supports are unyielding and the temperature is constant, the virtual work done by a system of forces P_1, P_2, P_3, ... during the distortion caused by a system of forces Q_1, Q_2, Q_3, ... is equal to the virtual work done by the system of forces Q_1, Q_2, Q_3, ... during the distortion caused by the system of forces P_1, P_2, P_3, ...

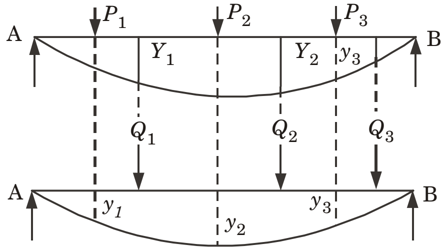


Fig. 3.2.1.

Proof :

1. Let a structure AB carries two system of forces P_1, P_2, P_3, and Q_1, Q_2, Q_3. Fig. 3.2.1 shows both the system of forces separately.

2. Let W_e = External work done on the structure when the system of forces P_1, P_2, P_3 is applied.

   W_e' = External work done on the structure when the system of forces Q_1, Q_2, Q_3 is applied.
3. Let Y_1, Y_2, Y_3 = Deflection caused by the system of forces P_1, P_2, P_3 at the points of application of the forces Q_1, Q_2, Q_3, respectively.
4. Let y_1, y_2, y_3 = Deflection caused by the system of forces Q_1, Q_2, Q_3 at the points of application of the forces P_1, P_2, P_3, respectively.
5. Let the structure be first loaded with the system of forces P_1, P_2, P_3 and work done on the structure be W_e.
6. With this system of forces acting on the structure, let the system of forces Q_1, Q_2, Q_3 be applied.

   Total work done = $W_e + W'_e + P_1 y_1 + P_2 y_2 + P_3 y_3$ ...(3.2.1)
7. Now on changing the order of loading we have the system of forces Q_1, Q_2, Q_3, acting on the structure.

   Work done on the structure = W'_e
8. Now the system of forces P_1, P_2, P_3 be applied so,

   Total work done = $W'_e + W_e + Q_1 Y_1 + Q_2 Y_2 + Q_3 Y_3$ ...(3.2.2)
9. Equating the eq. (3.2.1) and (3.2.2)

   $$W_e + W_e' + P_1 y_1 + P_2 y_2 + P_3 y_3 = W'_e + W_e + Q_1 Y_1 + Q_2 Y_2 + Q_3 Y_3$$

   $$P_1 y_1 + P_2 y_2 + P_3 y_3 = Q_1 Y_1 + Q_2 Y_2 + Q_3 Y_3$$
10. The above expression shows that the virtual work done by the system of forces P_1, P_2, P_3 due to the deflections caused by the system of forces Q_1, Q_2, Q_3 equals virtual work done by the system of forces Q_1, Q_2, Q_3 due to the deflections caused by the system of forces P_1, P_2, P_3.

Que 3.3. **State and prove the Maxwell's reciprocal theorem.**

AKTU 2015-16, Marks 10

Answer

Maxwell's Law Statement :

1. In any structure whose material is elastic and obeys Hooke's law and whose supports remain unyielding and the temperature remains unchanged, the deflection at any point D (*i.e.*, Δ_d) due to a load W acting at any other point C is equal to the deflection at any point C (*i.e.*, δ_c) due to the load W acting at the point D.

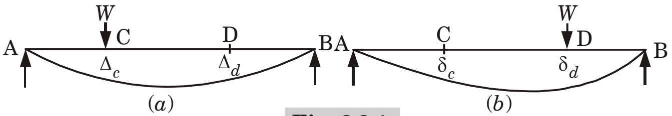


Fig. 3.3.1.

2. In Fig. 3.3.1(a) beam AB carries a load W at point C. Let the deflection at C be Δ_c and when load W is acting at point D. Let the defection at point D is δ_d then according to law of reciprocal deflections or Maxwell's reciprocal deflection theorem we have

$$\delta_d = \Delta_c$$

Proof :

1. Let initially a load W is acting at a point C and deflects the beam AB by deflection Δ_c under the load W.
2. Work done on the structure $= \dfrac{1}{2} \times W\Delta_c$
3. Let an another equal load W is acting at point D. Due to this load there will be further deflections of δ_c and δ_d at C and D.
4. So the total work done at this stage $= \dfrac{1}{2}\ W\Delta_c + \dfrac{1}{2} \times W\,\delta_d + W\delta_c$ ...(3.3.1)

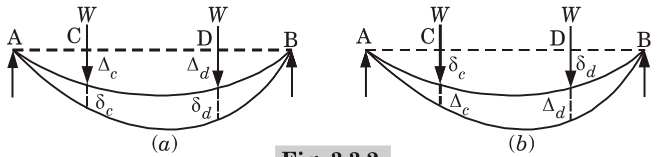


Fig. 3.3.2.

5. Now change the order of loading as shown in Fig. 3.3.2.(b). Now consider initially load W is at D. Due to this load,

$$\text{Work done} = \frac{1}{2}\ W\,\delta_d$$

6. Let another load W (same value) is applied at C so further deflections of Δ_c and Δ_d will occur at C and D respectively.

So total work done at this stage $= \dfrac{1}{2}\ W\delta_d + \dfrac{1}{2}\ W\Delta_c + W\Delta_d$ ...(3.3.2)

7. As both the eq. (3.2.1) and eq. (3.2.2) represents the same stage, hence,

$$\frac{1}{2}\ W\delta_d + \frac{1}{2}\ W\Delta_c + W\delta_c = \frac{1}{2}\ W\delta_d + \frac{1}{2}\ W\Delta_c + W\Delta_d$$

$$\delta_c = \Delta_d$$

8. The deflection at C due to the load W at D equal to the deflection at D due to the same load W at C.

Que 3.4. **Calculate the deflection under the load for truss shown in Fig. 3.4.1. All the members are have equal areas of 1250 mm^2 in cross section and E = 200 kN-m^2.**

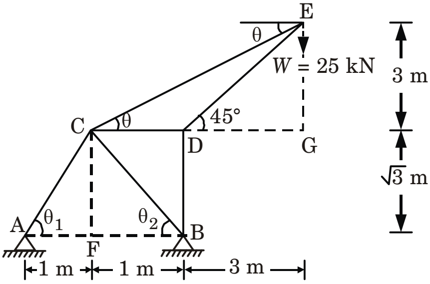


Fig. 3.4.1.

AKTU 2016-17, Marks 7.5

Answer

Given : Area of member, $A = 1250 \text{ mm}^2$
Modulus of elasticity, $E = 200 \text{ kN/m}^2$
To Find : Deflection under the load.

1. Taking moment about A,

$$W \times 5 = V_B \times 2$$

$$V_B = \frac{5}{2} W = 2.5W \text{ kN}$$

2. $\Sigma F_y = 0$,

$$V_B - V_A = W$$

$$\frac{5}{2} W - V_A = W$$

$$V_A = \frac{3}{2} W = 1.5W \,(\downarrow)\text{ kN}$$

3. In ΔACF, $\tan \theta_1 = \dfrac{\sqrt{3}}{1} = \tan 60°$

$$\theta_1 = 60°$$

Same as in ΔCBF, $\theta_2 = 60°$

4. In ΔCEG, $\tan \theta = \dfrac{3}{4} \Rightarrow \theta = 36°52'$

5. **Joint A :**

Resolve the forces vertically, $\Sigma F_y = 0$

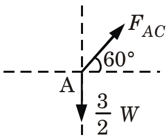


Fig. 3.4.2.

$$F_{AC} \sin 60° = \frac{3}{2} W$$

$$F_{AC} = \frac{3}{\sqrt{3}} W \text{ kN} = 1.73\, W \text{ kN}$$

6. **Joint *B* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$V_B + F_{BD} + F_{BC} \sin 60° = 0$$

$$\frac{5}{2} W + F_{BD} + F_{BC} \sin 60° = 0 \quad ...(3.4.1)$$

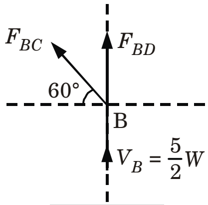


Fig. 3.4.3.

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{BC} \cos 60° = 0$$

$$F_{BC} = 0, \text{ put in eq. (3.4.1)}$$

$$\frac{5}{2} W + F_{BD} + 0 = 0$$

$$F_{BD} = -\frac{5}{2}W = -2.5\, W$$

7. **Joint *E* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$W + F_{ED} \cos 45° + F_{EC} \sin 36° 52' = 0$$

$$F_{ED} \cos 45° + F_{EC} \sin 36° 52' = -W \quad ...(3.4.2)$$

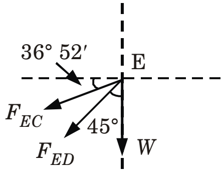


Fig. 3.4.4.

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{ED} \sin 45° + F_{EC} \cos 36° 52' = 0 \quad ...(3.4.3)$$

iii. From eq. (3.4.2) and eq. (3.4.3), we get

$$F_{ED} = -5.65\, W,\ F_{EC} = 5\, W$$

8. **Joint *D* :**

Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{DC} = F_{DE} \cos 45°$$

$$F_{DC} = -5.65\, W \times \cos 45°$$

$$= -4\, W \text{ kN}$$

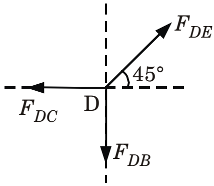


Fig. 3.4.5.

9. Strain energy stored by the structure :

$$U_i = \Sigma \frac{F^2 L}{2AE}$$

$$U_i = \frac{(1.73W)^2}{2AE} \times \sqrt{(\sqrt{3})^2 + 1^2} + 0 + \frac{(2.5\, W)^2(\sqrt{3})}{2AE}$$

$$+ \frac{(5.65\, W)^2 \times \left(\dfrac{3}{\sin 45°}\right)}{2AE} + \frac{(5W)^2 \times \left(\dfrac{3}{\sin 36°52'}\right)}{2AE} + \frac{(4W)^2 \times 1^2}{2AE}$$

$$U_i = \frac{W^2}{2AE} [5.986 + 0 + 10.82 + 135.43 + 125 + 16]$$

$$= \frac{293.236 \times W^2}{2 \times 1250 \times 10^{-6} \times 200 \times 10^6}$$

[Given unit of E is wrong. Taken value of $E = 200$ kN/mm^2]

$$U_i = 5.86 \times 10^{-4}\, W^2$$

10. $\therefore$ Vertical deflection at E

$$\Delta_E = \frac{\partial U_i}{\partial W} = 5.86 \times 10^{-4}(2W) = 5.86 \times 10^{-4} \times (2 \times 25)$$
$$\Delta_E = 0.0293 \text{ m}$$
$$\Delta_E = 29.3 \text{ mm}$$

PART-2

Castigliano's Theorems

CONCEPT OUTLINE : PART-2

Castigliano's First Theorem : "The partial derivative of the total strain energy in a structure with respect to the displacement at any one of the load points gives the value of corresponding load acting on the body in the direction of displacement"

$$P_i = \frac{\partial U}{\partial \Delta_i}$$

Castigliano's Second Theorem : In any linear elastic structure partial derivative of the strain energy with respect to load at a point is equal to the defection of the point where load is acting. The defelction being measured in the direction of load.

i.e.
$$\frac{\partial U}{\partial P_i} = \Delta_i \text{ and } \frac{\partial U}{\partial M_i} = \theta_i$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.5. **State and prove that the Castigliano's theorem.**

AKTU 2016-17, Marks 10

Answer

A. Castigliano's First Theorem :

Statement : "The partial derivative of the total strain energy in a structure with respect to the displacement at any one of the load points gives the value of corresponding load acting on the body in the direction of displacement"

$$P_i = \frac{\partial U}{\partial \Delta_i}$$

Proof :

1. Considering an elastic system subjected to a set of P ($P_1, P_2, P_3, ..., P_n$) forces, which produce displacements, $\Delta_1, \Delta_2, \Delta_3 ...\Delta_n$ respectively in the direction of the respective loads at their points of applications.
2. Then the strain energy of the system will be

$$U = \sum_{i=1}^{n} \int P_i \, d\Delta_i \text{ or } \delta U = \sum_{i=1}^{n} P_i \, d\Delta_i$$

3. If one of the displacement Δ_i is increases by $\delta\Delta_i$ in the direction of P_i keeping all other displacements unchanged then the incremental strain energy of the system will be

$$\delta U = P_i \, \delta\Delta_i \quad ...(3.5.1)$$

4. Since the other displacement were kept unchanged hence

$$\delta U = \frac{\partial U}{\partial \Delta_1} \delta\Delta_1 + \frac{\partial U}{\partial \Delta_2} \delta\Delta_2 + \frac{\partial U}{\partial \Delta_3} \delta\Delta_3 + ... + \frac{\partial U}{\partial \Delta_n} \delta\Delta_n$$

$$= \frac{\partial U}{\partial \Delta_i} \delta\Delta_i \text{ ... as other displacement are unchanged.} \quad ...(3.5.2)$$

5. Eq. (3.5.1) and (3.5.2), we get

$$P_i \, \delta\Delta_i = \frac{\partial U}{\partial \Delta_i} \delta\Delta_i$$

$$P_i = \frac{\partial U}{\partial \Delta_i} \text{ Hence proved.}$$

where P_i = External force at point 'i'.

B. Statement of Castigliano's Second Theorem :

1. In any linear elastic structure partial derivative of the strain energy with respect to load at a point is equal to the defection of the point where load is acting.
2. The deflection being measured in the direction of load.

i.e., $$\frac{\partial U}{\partial P_i} = \delta_i \text{ and } \frac{\partial U}{\partial M_i} = \theta_i$$

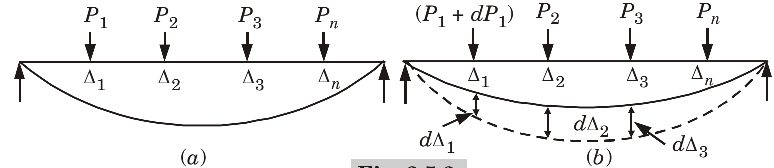


Fig. 3.5.2.

Proof :

1. Consider an linear elastic beam subjected to gradually applied forces $P_1, P_2, P_3, ..., P_n$. These forces produce deflections $\Delta_1, \Delta_2, \Delta_3$, ... at their points of application respectively.

2. So, total strain energy stored will be

$$U = \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 + ... \quad ...(3.5.3)$$

3. If the additional load dP_1 is added after P_1, P_2, P_3... were applied then additional deflection are $d\Delta_1$, $d\Delta_2$, $d\Delta_3$,.... so the increased stored strain energy,

$$dU = \frac{1}{2}dP_1 d\Delta_1 + P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3 \quad ...(3.5.4)$$

$$dU - \frac{1}{2}dP_1 d\Delta_1 = P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3$$

$$\frac{1}{2}\left(dU - \frac{1}{2}dP_1 d\Delta_1\right) = \frac{1}{2}P_1 d\Delta_1 + \frac{1}{2}P_2 d\Delta_2 + \frac{1}{2}P_3 d\Delta_3 \quad ...(3.5.5)$$

4. Adding eq. (3.5.3) and eq. (3.5.4), we get

$$\text{Total strain energy} = U + dU = \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3$$

$$+ \frac{1}{2}dP_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3 \quad ...(3.5.6)$$

5. If we assume that the $(P_1 + dP_1)$, P_2 and P_3 are being applied simultaneously then total strain energy stored will be

$$= \frac{1}{2}(P_1 + dP_1)(\Delta_1 + d\Delta_1) + \frac{1}{2}(\Delta_2 + d\Delta_2)P_2 + \frac{1}{2}(\Delta_3 + d\Delta_3)P_3 + ... \quad ...(3.5.7)$$

6. Since, total strain energy stored in both the case must be same. From eq. (3.5.6) and eq. (3.5.7), we get

$$\frac{1}{2}P_1 d\Delta_1 + \frac{1}{2}P_2 d\Delta_2 + \frac{1}{2}P_3 d\Delta_3 = \frac{1}{2}dP_1\Delta_1$$

7. From eq. (3.5.5), we have

$$\frac{1}{2}\left(dU - \frac{1}{2}dP_1 d\Delta_1\right) = \frac{1}{2}d\,P_1\Delta_1$$

Neglecting $\frac{1}{2}dP_1 d\Delta_1$ as it is too small, so

$$\frac{1}{2}dU = \frac{1}{2}d\,P_1\Delta_1$$

$$\Delta_1 = \frac{dU}{dP_1}$$

8. Similarly if the moment considered,

$$\frac{dU}{dM_1} = \theta_1$$

where, U = Total strain energy.

P_1, M_1 = Loads, moments on the structure.

δ_1, θ_1 = Displacements in the direction of the loads.

Que 3.6. **Determine the vertical deflection at point C in the frame shown in Fig. 3.6.1. Given E = 200 kN/mm^2 and I = 30 × 10^6 mm^4.**

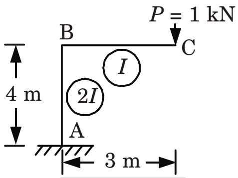


Fig. 3.6.1.

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Answer

Given : Load, P = 1 kN, E = 200 kN/mm^2, I = 30 × 10^6 mm^4.

To Find : Vertical deflection at point C.

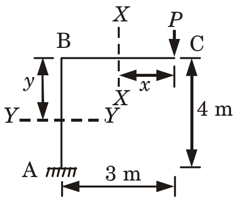

Fig. 3.6.2.

1. Flexural rigidity,

$$EI = 200 \times 30 \times 10^6 \text{ kN-mm}^2 = 6 \times 10^3 \text{ kN-m}^2$$

2. **Expression for Moment :**

Portion	***CB***	***BA***
Origin	C	B
Limit	0 – 3	0 – 4
Moment (M)	Px	$3P$

3. Strain energy stored by the frame,

W_i = Strain energy stored by CB + Strain energy stored by BA

$$W_i = \int_C^B \frac{M^2}{2EI} ds + \int_B^A \frac{M^2}{2EI} ds$$

$$W_i = \int_0^3 \frac{(Px)^2}{2EI} dx + \int_0^4 \frac{(3P)^2}{2(2EI)} dy$$

$$= \int_0^3 \frac{P^2x^2}{2EI}dx + \int_0^4 \frac{9P^2}{4EI}dy$$

4. To find the vertical deflection at point C, differentiate the total strain energy stored with respect to P, we get

$$\Delta_c = \frac{\partial U}{\partial P} = \int_0^3 \frac{2Px^2}{2EI}dx + \int_0^4 \frac{9 \times 2P}{4EI}dy$$

$$\Delta_c = \frac{P}{EI}\left[\left(\frac{x^3}{3}\right)_0^3 + \frac{9}{2} \times (y)_0^4\right] = \frac{P}{EI}[9+18] = \frac{27 \times 1}{6 \times 10^3}$$

$$\Delta_c = 4.5 \times 10^{-3} \text{ m} = 4.5 \text{ mm} \qquad [\because \quad P = 1 \text{ kN}]$$

Que 3.7. **Determine the horizontal displacement of the roller end *D* of the portal frame shown in Fig. 3.7.1. *EI* is 8000 kN-m² throughout.**

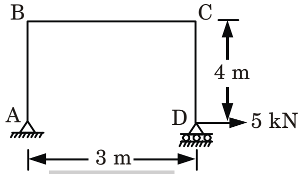

Fig. 3.7.1.

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Answer

Given : Load = 5 kN, EI = 8000 kN-m²

To Find : Horizontal displacement at D.

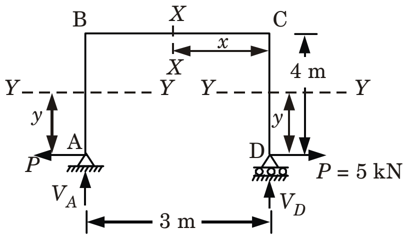

Fig. 3.7.2.

1. Horizontal reaction at $A = P\,(\leftarrow)$
2. Taking moment about point A,

$$V_D \times 3 - 5 \times 0 = 0$$
$$V_D = 0$$

3. Taking moment about support D,

$$V_A \times 3 - 5 \times 0 = 0$$

$$V_A = 0$$

4. **Expression for Moments :**

Portion	***AB***	***BC***	***CD***
Origin	A	B	D
Limit	0 – 4	0 – 3	0 – 4
Moment, (*M*)	Py	$4P$	Py

5. Strain energy stored by the frame,

$$U_i = \sum \int \frac{M^2}{2EI} ds$$

$$U_i = \int_D^C \frac{M^2}{2EI} ds + \int_C^B \frac{M^2}{2EI} ds + \int_B^A \frac{M^2}{2EI} ds$$

$$U_i = \int_0^4 \frac{(Py)^2}{2EI} dy + \int_0^3 \frac{(4P)^2}{2EI} dx + \int_0^4 \frac{(Py)^2}{2EI} dy$$

$$U_i = \int_0^4 \frac{P^2 y^2}{2EI} dy + \int_0^3 \frac{16P^2}{2EI} dx + \int_0^4 \frac{P^2 y^2}{2EI} dy$$

6. Horizontal displacement at D is given by,

$$\Delta_D = \frac{\partial U}{\partial P} = \int_0^4 \frac{2Py^2}{2EI} dy + \int_0^3 \frac{32P}{2EI} dx + \int_0^4 \frac{2Py^2}{2EI} dy$$

7. Putting $P = 5$,

$$\Delta_D = \int_0^4 \frac{5y^2}{EI} dy + \int_0^3 \frac{80}{EI} dx + \int_0^4 \frac{5y^2}{EI} dy$$

$$\Delta_D = \frac{5}{EI} \frac{[y^3]_0^4}{3} + \frac{80}{EI} [x]_0^3 + \frac{5}{EI} \frac{[y^3]_0^4}{3}$$

$$= \frac{1}{EI} \left[\frac{5}{3} \times (4^3 - 0) + 80\,(3 - 0) + \frac{5}{3} (4^3 - 0) \right]$$

$$= \frac{1}{EI} \left[\frac{320}{3} + \frac{240}{1} + \frac{320}{3} \right] = \frac{1}{EI} \left[\frac{320 + 720 + 320}{3} \right]$$

Horizontal displacement at D,

$$\Delta_D = \frac{1360}{3 \times 8000} = 0.0567 \text{ m} \approx 57 \text{ mm}$$

Que 3.8. **Determine the vertical deflection at the free end and rotation at *A* in the overhanging beam shown in Fig. 3.8.1. Assume constant *EI*. Use Castigliano's method.**

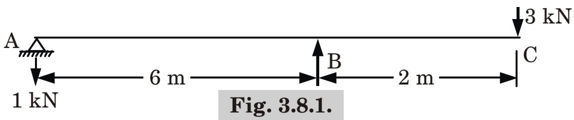

Fig. 3.8.1.

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Answer

Given : Loads = 3 kN and 1 kN, EI = Constant
To Find : Vertical deflection at C and rotation at A.

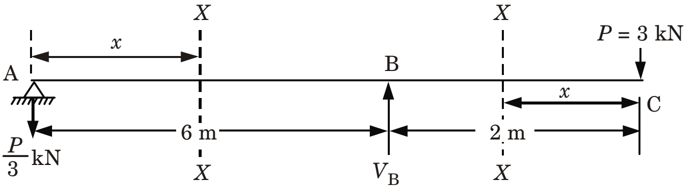

Fig. 3.8.2.

Let load P applied at end point C.

A. Vertical Deflection at Free End :

$$V_B = 1 + 3 = 4 \text{ kN}$$

1. Taking moments about A, $\Sigma M_A = 0$

$$V_B = \frac{4P}{3}(\uparrow)$$

2.
$$\Sigma F_y = 0$$
$$V'_A + V_B = 3 + 1 = 4$$
$$V_A = P - \frac{4P}{3} = -\frac{P}{3}$$

3. Expression for Moment :

Portion	***AB***	***CB***
Origin	A	C
Limit	0 – 6	0 – 2
Moment, (M)	$-\dfrac{Px}{3}$	Px

4. Strain energy stored by the beam

= Strain energy stored by AB + Strain energy stored by BC

$$U_i = \sum \int \frac{M^2 dx}{2EI} = \int_A^B \frac{M^2}{2EI} dx + \int_C^B \frac{M^2}{2EI} dx$$

$$U_i = \int_0^6 \left(\frac{-P}{3}x\right)^2 \frac{dx}{2EI} + \int_0^2 \frac{(-Px)^2}{2EI} dx$$

$$U_i = \int_0^6 \frac{P^2}{18} x^2 \frac{dx}{EI} + \int_0^2 \frac{P^2 x^2}{2EI} dx$$

5. Vertical deflection at free end,

$$\Delta_C = \frac{\partial U_i}{\partial P} = \int_0^6 \frac{2P}{18} x^2 \frac{dx}{EI} + \int_0^2 \frac{2Px^2}{2EI} dx$$

$$= \int_0^6 \frac{1}{9} Px^2 \frac{dx}{EI} + \int_0^2 \frac{Px^2}{EI} dx$$

Deflection at C, $$\Delta_C = \frac{P}{9EI}\left[\frac{x^3}{3}\right]_0^6 + \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^2$$

$$\Delta_C = \frac{P}{27EI}[6^3 + 9 \times 2^3] = \frac{32}{EI} \qquad [\because \quad P = 3 \text{ kN}]$$

B. Rotation at Support A :

1. To find the rotation at A, applied the moment M_o at A,

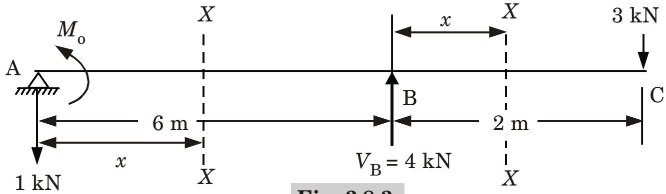

Fig. 3.8.3.

2. Strain energy stored by member,

$$U_i = \Sigma \int M^2 \frac{ds}{2EI}$$

$$M_{AB} = (M_o + 1 \times x),$$

$$M_{BC} = [M_o + (6 + x) \times 1 - 4 \times x]$$

$$U_i = \int_0^6 \frac{(M_o + 1 \times x)^2}{2EI} dx + \int_0^2 \frac{[M_o + (6 + x) - 4x]^2}{2EI} dx$$

3. Angle of rotation at the end A,

$$\theta_A = \frac{\partial U_i}{\partial M_o}$$

$$\theta_A = \int_0^6 \frac{2(M_o + x)}{2EI} dx + \int_0^2 \frac{2[M_o + 6 - 3x]}{2EI} dx$$

4. Putting $M_o = 0$,

$$\theta_A = \int_0^6 \frac{x}{EI} dx + \int_0^2 \frac{6 - 3x}{EI} dx$$

$$= \frac{1}{EI}\left[\frac{x^2}{2}\right]_0^6 + \frac{6}{EI}[x]_0^2 - \frac{3}{EI} \times \left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{1}{EI}\left[\frac{6^2}{2} - 0 + 6(2 - 0) - \frac{3}{2}(2^2 - 0)\right]$$

Rotation at A, $$\theta_A = \frac{1}{EI}[18 + 12 - 6] = \frac{24}{EI}$$

PART-3

Calculation of Deflection by Unit Load Method for Statically Determinate Beams, Trusses and Frames.

CONCEPT OUTLINE : PART-3

Unit Load Method : This method is a strain energy method and is based essentially on the conservation of energy *i.e.*, total internal strain energy stored is equal to the total external work done. It is also known as virtual work method. Deflection is given by,

$$\Delta = \int \frac{MM_1}{EI} dx$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.9. **Describe the unit load method for calculating deflection of beam, frame and truss.**

Answer

Unit Load Method :

1. Consider the body shown in Fig. 3.9.1(*a*) which is subjected to forces P_1, P_2, P_3, P_4, ..., P_n applied gradually.
2. Let displacement under load at points be Δ_1, Δ_2, Δ_3,... Δ_n, and at point C be Δ. Then,

External work done $= \frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + \frac{1}{2}\Delta_3 P_3 + ... + \frac{1}{2}\Delta_n P_n$

and strain energy stored $= \int \frac{1}{2} \sigma e \, dv$

where σ is stress and e is the strain in the element considered.

$$\therefore \quad \frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + ... + \frac{1}{2}\Delta_n P_n = \int \frac{1}{2} \sigma e \, dv \qquad ...(3.9.1)$$

4. Now, consider the same body subjected to unit load applied gradually at C when it is free of system of P forces.
5. Let the displacement at 1, 2, 3, ... n be δ_1, δ_2, δ_3, ... δ_n respectively and the displacement at C be δ. Let the stress produced in the element be p' and the strain be e'.

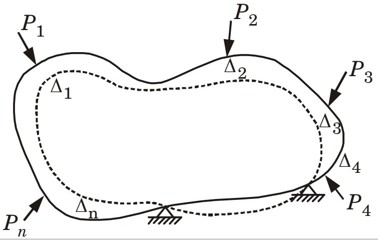

Fig. 3.9.1. A body subject to load and deformed shape.

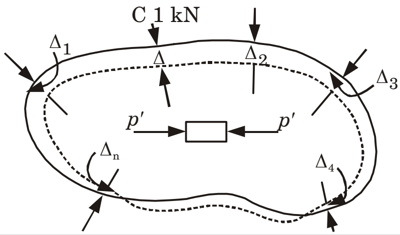

Fig. 3.9.2. System of forces applied to a body.

6. Then

$$\text{External work done} = \frac{1}{2} \times 1 \times \delta$$

$$\text{Internal work done} = \int \frac{1}{2} p'e'dv$$

$$\therefore \qquad \frac{1}{2} \times 1 \times \delta = \int \frac{1}{2} p'e'dv \qquad \text{...(3.9.2)}$$

6. Now, if P system of forces is applied to the body shown in Fig. 3.9.2(b).

$$\text{External work done} = \frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + \ldots + \frac{1}{2}\Delta_n P_n + 1 \times \Delta$$

7. Since, unit load is already acting,

Internal work done = $\int \frac{1}{2} pe\,dv + \int pe\,dv$, Since the stress p' is acting throughout the deformation. Equating internal work to external work

$$\frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + \ldots + \frac{1}{2}\Delta_n P_n + 1 \times \Delta = \int \frac{1}{2} \sigma e\,dv + \int \sigma' e\,dv \qquad \text{...(3.9.3)}$$

8. Subtracting eq. (3.9.1) from (3.9.3), we get,

$$1 \times \Delta = \int \sigma' e\,dv$$

$$\Delta = \int \sigma' e\,dv \qquad \text{...(3.9.4)}$$

where, Δ = Deflection at point where unit load is applied and is measured in the direction of unit load.

σ' = Stress in an element due to unit load.

e' = Strain in an element due to given load system.

9. The eq. (3.9.4) is the basis for the unit load method.

The Unit Load Method—Application to Beam Deflections :

1. Consider the beam shown in Fig. 3.9.3 subjected a system of P forces.
2. The stress in the element at distance y from neutral axis is

$$\sigma = \frac{M}{I}y, \text{ where } M \text{ is the moment acting at the section.}$$

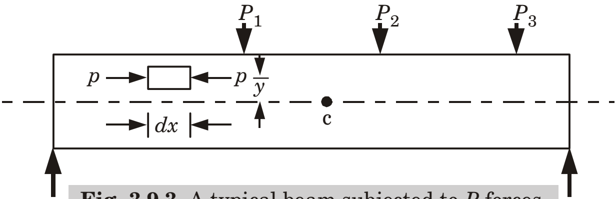

Fig. 3.9.3. A typical beam subjected to P forces.

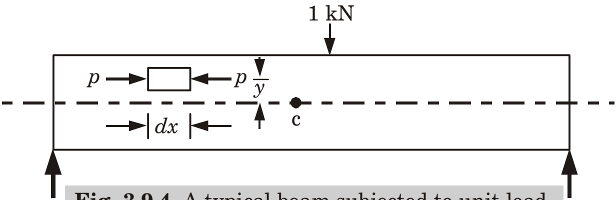

Fig. 3.9.4. A typical beam subjected to unit load.

3. Therefore, strain in the element due to given system of forces,

$$e = \frac{M}{EI}y$$

4. Let m be the moment at the section where the element is considered, due to unit load acting at C as shown in Fig. 3.9.4.
5. Then, Stress $\sigma' = \dfrac{M_1 y}{I}$

$\therefore$ From eq. (3.9.4)

$$\Delta = \int \frac{M_1 y}{I} y \times \frac{M}{EI} y \, dv$$

$$\Delta = \int_0^L \frac{MM_1}{EI^2}\left(\int_0^A y^2 \, dA\right) dx = \int_0^L \frac{MM_1}{EI^2} I dx \quad \text{Since,} \int_0^A y^2 \, dA = I$$

$$= \int_0^L \frac{MM_1}{EI} dx \qquad \text{...(3.9.5)}$$

6. From the equation (3.9.5), deflection at any point C can be found. It needs bending moment due to a given load system and unit load acting at C.
7. This procedure is applicable to rigid frames also, where only flexure effect is considered (*i.e.*, in the analysis in which the effect of axial and shear forces are neglected).

Que 3.10. **Determine the deflection and rotation at the free end of the cantilever beam shown in Fig. 3.10.1**
Use unit load method. Given $E = 2 \times 10^5$ N/mm², and I = 12 × 10⁶ mm⁴.

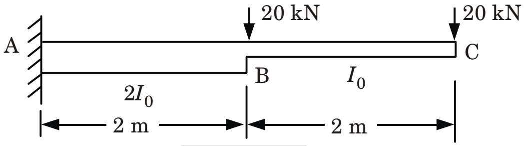

Fig. 3.10.1.

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Answer

Given : Load = 20 kN and 20 kN

$E = 2 \times 10^5$ N/mm² and $I = 12 \times 10^6$ mm⁴

To Find : Deflection and rotation at the free end.

A. Deflection at Point *C* :

1. Considering the beam into two segments *BC* and *BA* and with origin at *C* and *B* respectively and measuring *x* as positive towards left, then the expressions for BM due to external loading and due to unit load applied at *B* is shown in following table.

Portion	*CB*	*BA*
Origin	*C*	*B*
Limit	0 – 2	0 – 2
BM due to External Load, *M*	$20\,x$	$20\,(2 + x) + 20\,x = 40 + 40\,x$
BM due to Unit Load, M_1	$1x = x$	$1(2 + x) = 2 + x$

2. Deflection at *C*,

$$\Delta_C = \int_C^A \frac{MM_1}{EI} dx = \int_C^B \frac{MM_1}{EI} dx + \int_B^A \frac{MM_1}{2EI} dx$$

$$= \int_0^2 \frac{20x(x)}{EI} dx + \int_0^2 \frac{(40+40x)(2+x)}{2EI} dx$$

$$= 20\int_0^2 \frac{x^2}{EI} dx + \int_0^2 \frac{(80+120x+40x^2)}{2EI} dx$$

$$= \frac{20}{EI}\left[\frac{x^3}{3}\right]_0^2 + \frac{1}{2EI}\left[80x + \frac{120x^2}{2} + \frac{40x^3}{3}\right]_0^2$$

$$= \frac{20}{EI}\left[\frac{8}{3}\right] + \frac{1}{2EI}\left[80 \times 2 + 60 \times 4 + \frac{40}{3} \times 8\right]$$

$$= \frac{53.33}{EI} + \frac{253.33}{EI} = \frac{306.66}{24 \times 10^2} = 0.1277 \text{ m}$$

$$\Delta_C = 127.7 \text{ mm}$$

B. Slope at *C* :

1. To the find slope at C, apply a clockwise unit moment at B then the parameters are shown in following table.

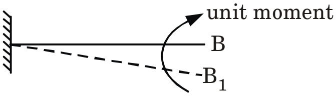


Fig. 3.10.2.

Portion	***CB***	***BA***
Origin	C	B
Limit	0 – 2	0 – 2
BM due to External Load, *M*	$20\,x$	$20\,(2 + x) + 20\,x = 40 + 40\,x$
BM due to Unit Moment, M_1	1	1

2. Slope at C,

$$\theta_C = \int_C^A \frac{MM_1}{EI} dx = \int_C^B \frac{MM_1}{EI} dx + \int_B^A \frac{MM_1}{2EI} dx$$

$$\theta_C = \int_0^2 \frac{20x(1)}{EI} dx + \int_0^2 \frac{(40 + 40x)(1)}{2EI} dx$$

$$= \int_0^2 \frac{20x}{EI} + \int_0^2 \frac{(40 + 40x)}{2EI} dx$$

$$= \frac{20}{2EI}[x^2]_0^2 + \frac{1}{2EI}\left[40x + \frac{40x^2}{2}\right]_0^2$$

Slope at C,

$$\theta_C = \frac{10}{EI}(4) + \frac{20}{EI}[2 + 2] = \frac{120}{24 \times 10^2} = 0.05 \text{ radian}$$

Que 3.11. **Determine the horizontal and vertical deflection at point *E* of the frame shown in Fig. 3.11.1. Take *EI* as constant.**

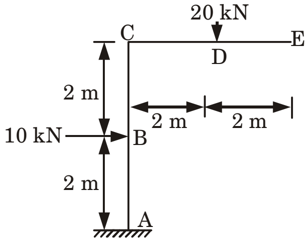


Fig. 3.11.1.

AKTU 2012-13, Marks 10

Answer

Given : Loads = 20 kN and 10 kN

To Find : Horizontal and vertical deflection at point E.

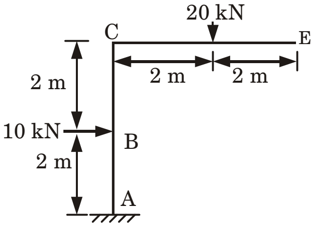


Fig. 3.11.2.

1. **Expression for Moments :**
Considering sagging moment as positive.

Portion	*ED*	*DC*	*CB*	*BA*	Remark
Origin	E	D	C	B	
Limit	0 – 2	0 – 2	0 – 2	0 – 2	
M	0	$-20x$	-20×2	$-40-10x$ $=-40-10x$	BM due to external loads
M_1	$-1 \times x$	$-(2+x) \times 1$	-4	-4	BM due to unit vertical load at E
M_2	0	0	$-1 \times x$	$-1(2+x)$	BM due to unit horizontal load at E

2. **Vertical Deflection at E :**

$$\Delta_E = \int_E^A \frac{MM_1dx}{EI} = \int_E^D \frac{MM_1dx}{EI} + \int_D^C \frac{MM_1dx}{EI} + \int_C^B \frac{MM_1dx}{EI} + \int_B^A \frac{MM_1dx}{EI}$$

$$= \int 0 + \int_0^2 20x(2+x)dx + \int_0^2 160dx + \int_0^2 (160+40x)\,dx$$

$$= \int_0^2 (40x+20x^2)dx + \int_0^2 160dx + \int_0^2 (160+40x)\,dx$$

$$= \int_0^2 (20x^2+80x+320)dx$$

$$= \left[20\frac{x^3}{3} + 80\times\frac{x^2}{2} + 320\times x\right]_0^2 = \left[\frac{20\times 2^3}{3} + \frac{80\times 2^2}{2} + 320\times 2\right]$$

Vertical deflection, $\Delta_E = \dfrac{853.33}{EI}$

3. Horizontal Deflection at *E* :

$$\Delta_{EH} = \int_E^A \frac{MM_2}{EI}\,dx$$

$$= \int_E^D \frac{MM_2}{EI}dx + \int_D^C \frac{MM_2 dx}{EI} + \int_C^B \frac{MM_2 dx}{EI} + \int_B^A \frac{MM_2 dx}{EI}$$

$$= 0 + 0 + \int_0^2 40x \times dx + \int_0^2 (40 + 10x)(x + 2)\,dx$$

$$= \int_0^2 (40x)\,dx + \int_0^2 (40x + 10x^2 + 80 + 20x)\,dx$$

$$= \int_0^2 (10x^2 + 100x + 80)\,dx = \left[\frac{10x^3}{3} + \frac{100x^2}{2} + 80x\right]_0^2$$

$$= \left[\frac{10 \times 2^3}{3} + \frac{100 \times 2^2}{2} + 80 \times 2\right]$$

Horizontal deflection, $\Delta_E = \dfrac{386.67}{EI}$

Que 3.12. **Fig. 3.12.1 shows a pin-joined truss loaded with a single load *W* is 100 kN. If the area of cross-section of all members shown in Fig. 3.12.1 is 1000 mm^2, what is the vertical deflection of point *C* ? Take *E* = 200 kN/mm^2 for all members.**

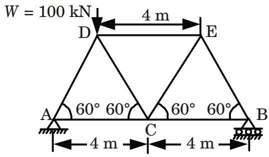


Fig. 3.12.1.

AKTU 2015-16, Marks 7.5

Answer

Given : Load, W = 100 kN, Cross section area = 1000 mm^2

E = 200 kN/mm^2

To Find : Vertical deflection at point C.

1. Taking moment about point A,

$$8 \times V_B = 100 \times 2$$

$$V_B = 25 \text{ kN}$$
$$\Sigma F_y = 0, V_A + V_B = 100 \text{ kN}$$
$$V_A = 100 - 25$$
$$V_A = 75 \text{ kN}$$

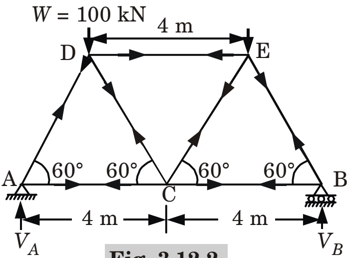


Fig. 3.12.2.

2. **Joint *A* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$V_A + P_{AD} \sin 60° = 0$$

$$P_{AD} = \frac{-75}{\sin 60°} = \frac{-150}{\sqrt{3}} \text{ kN}$$

ii. Resolve the forces horizontally,

$$\Sigma F_x = 0, \qquad P_{AC} + P_{AD} \cos 60° = 0$$

$$P_{AC} = -\left(\frac{-150}{\sqrt{3}}\right)\left(\frac{1}{2}\right) = \frac{75}{\sqrt{3}} \text{ kN}$$

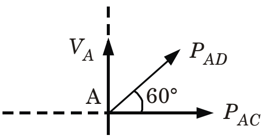


Fig. 3.12.3.

3. **Joint *D* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$-(P_{DA} \sin 60° + P_{DC} \sin 60° + 100) = 0$$

$$P_{DC} \sin 60° = +\frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{2} - 100$$

$$P_{DC} = -\frac{25}{\sin 60°}$$

$$P_{DC} = -\frac{50}{\sqrt{3}} \text{ kN}$$

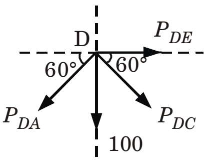


Fig. 3.12.4.

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$P_{DE} = P_{DA} \cos 60° - P_{DC} \cos 60°$$

$$= -\frac{150}{\sqrt{3}} \times \cos 60° + \frac{50}{\sqrt{3}} \times \cos 60°$$

$$P_{DE} = -\frac{75}{\sqrt{3}} + \frac{25}{\sqrt{3}} = -\frac{50}{\sqrt{3}} \text{ kN}$$

4. **Joint *C* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$P_{CE} \sin 60° + P_{CD} \sin 60° = 0$$

$$P_{CE} = \frac{50}{\sqrt{3}} \text{ kN}$$

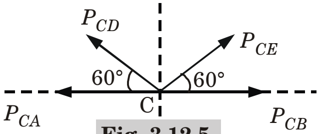


Fig. 3.12.5.

ii. Resolve the forces horizontally,

$$\Sigma F_x = 0$$

$$P_{CB} = P_{CA} + P_{CD} \cos 60° - P_{CE} \cos 60°$$

$$= \frac{75}{\sqrt{3}} - \frac{50}{\sqrt{3}} \times \frac{1}{2} - \frac{50}{\sqrt{3}} \times \frac{1}{2}$$

$$P_{CB} = \frac{75}{\sqrt{3}} - \frac{50}{\sqrt{3}} = \frac{25}{\sqrt{3}} \text{ kN}$$

5. **Joint *B* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$V_B + P_{BE} \sin 60 = 0$$

$$P_{BE} = -\frac{50}{\sqrt{3}} \text{ kN}$$

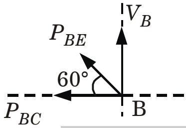


Fig. 3.12.6.

6. To find the vertical deflection at the joint *C*, remove the given load system and apply a vertical load of 1 kN at *C*.

Due to symmetry, reaction at each support = $\frac{1}{2}$ kN.

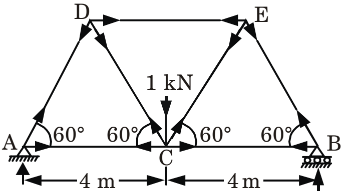


Fig. 3.12.7.

7. **Joint *A* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$K_{AD} \sin 60° + \frac{1}{2} = 0$$

$$K_{AD} = -\frac{1}{\sqrt{3}} \text{ kN}$$

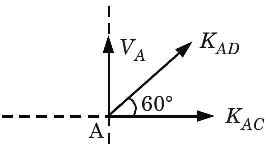


Fig. 3.12.8.

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$K_{AC} + K_{AD} \cos 60° = 0$$

$$K_{AC} = \frac{1}{\sqrt{3}} \cos 60° = \frac{1}{2\sqrt{3}} \text{ kN}$$

8. **Joint D :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$K_{DC} \sin 60° + K_{DA} \sin 60° = 0$$

$$K_{DC} = -\left(-\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} \text{ kN}$$

iii. Resolve the forces horizontally, $\Sigma F_x = 0$

$K_{DE} + K_{DC} \cos 60° = K_{DA} \cos 60°$

$$K_{DE} = \frac{-1}{\sqrt{3}} \cos 60° - \frac{1}{\sqrt{3}} \cos 60°$$

$$K_{DE} = -\frac{2}{\sqrt{3}} \cos 60° = -\frac{1}{\sqrt{3}}$$

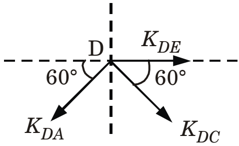


Fig. 3.12.9.

9. By symmetry, we know that the forces in the remaining members as same as before.

Vertical deflection at joint C, $\Delta_c = \sum \frac{PKL}{AE}$

Member length, $L = 4000$ mm

Member area, $A = 1000 \text{ mm}^2$

Member	P	K	PK
AD	$-150/\sqrt{3}$	$-1/\sqrt{3}$	$+ 150/3 = 50$
DE	$-50/\sqrt{3}$	$-1/\sqrt{3}$	$+ 50/3$
EB	$-50/\sqrt{3}$	$-1/\sqrt{3}$	$+ 50/3$
BC	$+25/\sqrt{3}$	$+ 1/2\sqrt{3}$	$+ 25/6$
CA	$+75/\sqrt{3}$	$+ 1/2\sqrt{3}$	$+ 75/6$
DC	$- 50/\sqrt{3}$	$+ 1/\sqrt{3}$	$- 50/3$
CE	$+50/\sqrt{3}$	$+ 1/\sqrt{3}$	$+ 50/3$
Total			100

10. Vertical deflection at the joint C, $\Delta_c = \sum \frac{PKL}{AE}$

$$= \frac{100 \times 4000}{1000 \times 200}$$

Deflection at C, $\Delta_c = 2$ mm.

Que 3.13. **A cantilever beam is of span 2 m and is subjected to a concentrated load of 20 kN at the free end. The cross section of the beam is 100 × 200 mm and E = 30 kN/mm². Calculate the slope and deflection of the beam at midspan. Use unit load method.**

AKTU 2016-17, Marks 10

Answer

Given : Span of beam, $L = 2$ m, Concentrated load, $W = 20$ kN
Cross section of beam = 100 mm × 200 mm
Modulus of elasticity, $E = 30 \text{ kN/mm}^2 = 30 \times 10^9 \text{ N/m}^2$
To Find : Slope and deflection at midspan of cantilever beam.

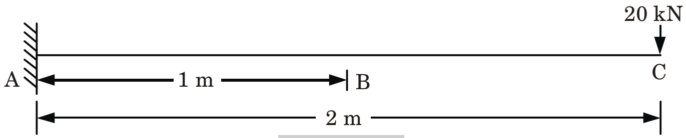


Fig. 3.13.1.

1. Moment of inertia,

$$I = \frac{100 \times 200^3}{12}$$
$$= 66.67 \times 10^6 \text{ mm}^4 = 66.67 \times 10^6 \times 10^{-12} \text{ m}^4$$
$$I = 66.67 \times 10^{-6} \text{ m}^4$$

2.

Portion	***CB***	***BA***
Origin	C	B
Limit	0 – 1	0 – 1
BM due to external load, M	$-20x$	$-20(1+x)$
BM due to unit load at midspan, M_1	0	$-1x$
BM due to unit moment at midspan, M_2	– 1	– 1

3. Deflection at B,

$$\Delta_B = \int_C^A \frac{MM_1}{EI} dx = \int_C^B \frac{MM_1}{EI} dx + \int_B^A \frac{MM_1}{EI} dx$$

$$= \int_0^1 \frac{-20x \times 0}{EI} dx + \int_0^1 \frac{20(1+x) \times x}{EI} dx$$

$$= 0 + \int_0^1 \frac{20x + 20x^2}{EI} dx = \frac{1}{EI}\left[\frac{20x^2}{2} + \frac{20}{3}x^3\right]_0^1$$

Deflection at B, $$\Delta_B = \left[\frac{20 \times 1}{2} + \frac{20 \times 1}{3}\right]\frac{1}{EI}$$

$$= \frac{16.67 \times 10^6}{30 \times 10^9 \times 66.67 \times 10^{-6}} = 8.33 \text{ mm}$$

4. Slope at B, $$\theta_B = \int_C^A \frac{MM_2}{EI} dx$$

$$= \int_0^1 \frac{-20x \times (-1)}{EI} dx + \int_0^1 \frac{-20(1+x) \times (-1)}{EI} dx$$

$$= \frac{1}{EI}\left(\frac{20\,x^2}{2}\right)_0^1 + \frac{1}{EI}\left[20x + \frac{20x^2}{2}\right]_0^1$$

$$= \frac{1}{EI}\left[20 \times 1 + 20 \times 1^2\right] = \frac{40}{EI}$$

Slope at B, $\theta_B = \dfrac{40 \times 10^3}{30 \times 10^9 \times 66.67 \times 10^{-6}} = 0.02$ radian

PART-4

Calculation of Deflection by Conjugate Beam Method for Statically Determinate Beam, Trusses and Frame.

CONCEPT OUTLINE : PART-4

Conjugate Beam : Conjugate beam is an imaginary beam of same span as the original beam loaded with $\left(\dfrac{M}{EI}\right)$ diagram of the original beam, such that the shear force and bending moment at a section will represent the rotation and deflection at the section in the original beam.

Theorem of Conjugate Beam Method :

Theorem 1 : The slope at any section of the given beam is equal to the shear force at the corresponding section of the conjugate beam.

Theorem 2 : The deflection at any section of the given beam is equal to the bending moment at the corresponding section of the conjugate beam.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.14. **Explain conjugate beam theorems.**

Answer

A. Conjugate Beam Theorems :

1. These theorems can be derived from moment area theorems and are very useful in finding deflection even if there is no point in the beam where the slope is zero.

2. Now, consider the simple supported beam shown in Fig. 3.14.1 and $\frac{M}{EI}$ diagram in Fig. 3.14.2 respectively.

Now, $\theta_C = \theta_A$ – Area of $\left(\frac{M}{EI}\right)$ diagram between A and C.

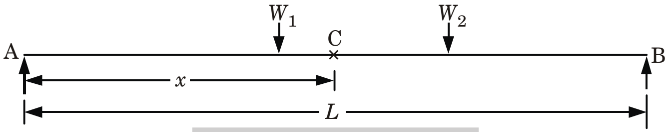


Fig. 3.14.1. A typical beam.

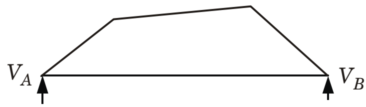


Fig. 3.14.2. *M/EI* diagram.

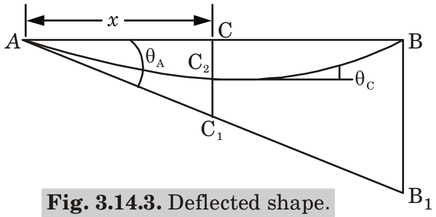


Fig. 3.14.3. Deflected shape.

$$\theta_A = \frac{BB_1}{AB}$$

$= \left(\frac{1}{L}\right)$ moment of area of $\left(\frac{M}{EI}\right)$ diagram between A and B about B.

$$\theta_C = \frac{\text{Moment of } (M/EI) \text{ diagram about } B}{L}$$

– Area of $\left(\frac{M}{EI}\right)$ diagram between A and C

3. Now, deflection at C

$= CC_2 = CC_1 - C_2C_1 = x_C\,\theta_A$ – Deflection of C w.r.t. tangent at A.

$$= \frac{x_C \times \text{Moment of } (M/EI) \text{ diagram between } A \text{ and } B \text{ about } B}{L}$$

– Area of $\left(\frac{M}{EI}\right)$ diagram between A and C about C ...(3.14.1)

4. Consider an imaginary beam of same span, loaded with $\left(\frac{M}{EI}\right)$ diagram.

5. Then, reaction at A, $V'_A = \dfrac{\text{Moment of the load about } B}{L}$

$$= \frac{\text{Moment of } (M/EI) \text{ diagram between } A \text{ and } B \text{ about } B}{L}$$

6. Shear force at C, = V'_A – load between A and C

$$= \frac{\text{Moment of } (M/EI) \text{ diagram between } A \text{ and } B \text{ about } B}{L}$$

$$- \text{Area of } \left(\frac{M}{EI}\right) \text{ diagram between } A \text{ and } C$$

7. Therefore, θ_C in the given beam is equal to the shear force in the beam loaded with $\left(\dfrac{M}{EI}\right)$ diagram.

8. Similarly, it can be observed that the deflection of C, given by eq. (3.14.1.), is equal to the bending moment in the imaginary beam loaded with $\left(\dfrac{M}{EI}\right)$ diagram.

8. The imaginary beam is called the conjugate beam and from the above discussion the following two theorems result :

   Theorem 1 : The rotation at a point in a beam is equal to the shear force in the conjugate beam.

   Theorem 2 : The deflection in a beam is equal to the bending moment in the conjugate beam.

Que 3.15. **Determine the slope and deflection at the free end of a cantilever beam of span *L* subjected to a point load *W* at the free end using any method for your choice. Take *EI* as constant.**

AKTU 2012-13, Marks 10

OR

A cantilever of span *L* is subjected to a point load *W* at the free end. Determine the slope and deflection at the free end. Take *EI* as constant.

AKTU 2013-14, Marks 05

OR

Determine the deflection at free end of a cantilever beam.

AKTU 2014-15, Marks 10

Answer

1. Consider a cantilever beam AB of length L carrying a point load W at free end.

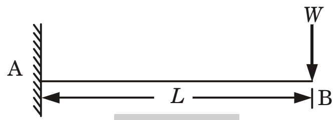


Fig. 3.15.1.

2. Bending moment diagram for cantilever is shown in Fig. 3.15.2.

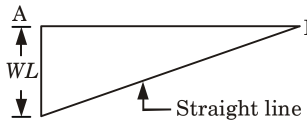


Fig. 3.15.2. BM diagram.

Corresponding conjugate beam whose load diagram is the $\frac{M}{EI}$ diagram.

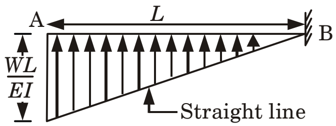


Fig. 3.15.3. Conjugate beam diagram (*M*/*EI* diagram).

3. **Slope at Free End in Cantilever Beam :**

Slope at B for the given beam = Shear force at B for conjugate beam.

$$\theta_B = \text{Area of } \frac{M}{EI} \text{ diagram}$$

$$\theta_B = \frac{1}{2} \times \frac{WL}{EI} \times L = \frac{WL^2}{2EI}$$

4. **Deflection at Free End in Cantilever Beam :**

Deflection at B for the given beam = BM at B for conjugate beam.

$$\Delta_B = \frac{1}{2} \times WL \times L \times \frac{2L}{3} = \frac{WL^3}{3}$$

Que 3.16. Determine the slope at the supports and deflection at the mid span of a simply supported beam *AB* of span *L* subjected to a point load *W* at the mid span. Take *EI* as constant. Use any method of your choice. **AKTU 2012-13, Marks 10**

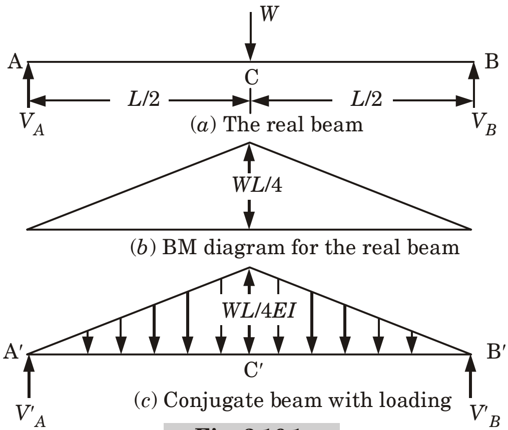
Answer

Given : Concentrated load = W, Span of beam = L.

To Find : Deflection at mid span and slope at supports.

1. Fig. 3.16.1(a) shows the real beam; BM diagram for the real beam is shown in the Fig. 3.16.1(b).

2. The $\dfrac{M}{EI}$ diagram of the real beam becomes the elastic weight or loading for the conjugate beam. The conjugate beam $A'C'B'$ (corresponding to the real beam ACB) with the loading is shown in Fig. 3.16.1(c).



(a) The real beam

(b) BM diagram for the real beam

(c) Conjugate beam with loading

Fig. 3.16.1.

3. For the conjugate beam :

$$V'_A = \frac{1}{2}\left(\frac{1}{2}L \times \frac{WL}{4EI}\right) = \frac{WL^2}{16EI}$$

4. But shear at any section of the conjugate beam is equal to the slope of the real beam.

i. Hence, θ_A = Slope at the end A of real beam $= -\dfrac{WL^2}{16EI}$

ii. Similarly, $\theta_B = S'_B = V'_B = \dfrac{WL^2}{16EI}$

5. Again, for the conjugate beam,

$$M'_C = V'_A \times \frac{L}{2} - \left(\frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4EI}\right) \times \left(\frac{1}{3} \times \frac{L}{2}\right)$$

$$= \frac{WL^2}{16EI} \times \frac{L}{2} - \frac{WL^3}{96EI} = \frac{WL^3}{48EI}$$

6. But the BM at any section of the conjugate beam is equal to the deflection of the real beam.

Hence, $\Delta_C = M'_C = \dfrac{WL^3}{48EI}$

Que 3.17. **A simply supported beam of span 8 m is subjected to a point load of 150 kN at 5 m from left support. Using the conjugate beam method, determine the slope at the supports and deflection under the load. Take *EI* as constant.** **AKTU 2013-14, Marks 10**

Answer

Given : Span of beam, $L = 8$ m, Point load, $W = 150$ kN

To Find : Slope at the supports and deflection under the load.

1. **Reaction at Supports,**

$$\Sigma F_y = 0$$

$$V_A + V_B = 150 \qquad ...(3.17.1)$$

i. Taking moment about B,

$$V_A \times 8 = 150 \times 3$$

$$V_A = 56.25 \text{ kN}$$

From eq. (3.17.1), $V_B = 93.75$ kN

ii. Draw the conjugate beam diagram.

Bending moment at distance 3 m from support B

$$= 93.75 \times 3 = 281.25 \text{ kN-m}$$

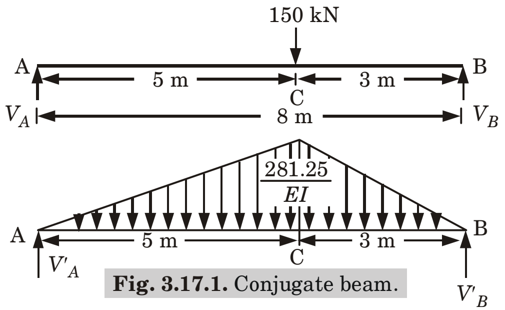


Fig. 3.17.1. Conjugate beam.

iii. Total load on the conjugate beam,

$$= \frac{1}{2} \times 8 \times \frac{281.25}{EI} = \frac{1125}{EI}$$

iv. Let V'_A and V'_B be the reactions at A and B for the conjugate beam.

$$\Sigma M_A = 0$$

$$V'_B \times 8 = \frac{1}{EI}\left\{\frac{1}{2} \times 3 \times 281.25 \times \left(5 + \frac{3}{3}\right) + \frac{1}{2} \times 5 \times 281.25 \times \left(\frac{2 \times 5}{3}\right)\right\}$$

$$V'_B \times 8 = \frac{1}{EI}\left\{\frac{1}{2} \times 281.25\left[18 + \frac{50}{3}\right]\right\}$$

$$V'_B \times 8 = \frac{4875}{EI}$$

$$V'_B = \frac{609.375}{EI}$$

$$V'_A = \frac{1125}{EI} - \frac{609.375}{EI} = \frac{515.625}{EI} \text{ kN}$$

3. Slope at Supports :

θ_A = Shear force at A for the conjugate beam.

Slope at A, $\theta_A = V'_A = \frac{515.625}{EI}$ radian

Slope at B, $\theta_B = V'_B = \frac{609.375}{EI}$ radian

4. Deflection at C :

Δ_C = BM at C for conjugate beam.

$$\Delta_C = M'_C = V'_A \times 5 - \frac{1}{2} \times 5 \times \frac{281.25}{EI} \times \frac{5}{3}$$

$$= \frac{515.625}{EI} \times 5 - \frac{1171.875}{EI} = \frac{1406.25}{EI}$$

Que 3.18. Using conjugate beam method find the deflection of a simply supported beam at point C. AB of length 10 m loaded by an UDL of intensity 20 kN per unit run. **AKTU 2014-15, Marks 10**

Answer

Given : Span = 10 m, UDL = 20 kN/m
To Find : Deflection at C.

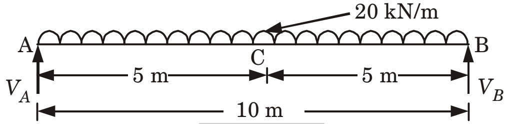


Fig. 3.18.1.

1. Reaction at Supports :

Assume 'C' point is the midpoint of the beam at a distance 5 m from each support.

Due to symmetry, $V_A = V_B = \frac{20 \times 10}{2} = 100$ kN

2. Bending Moment Diagram :

Bending moment at C, $M_C = 100 \times 5 - 20 \times 5 \times \frac{5}{2} = 250$ kN-m

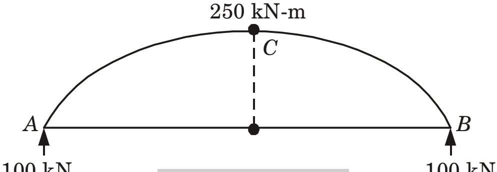


Fig. 3.18.2. BMD.

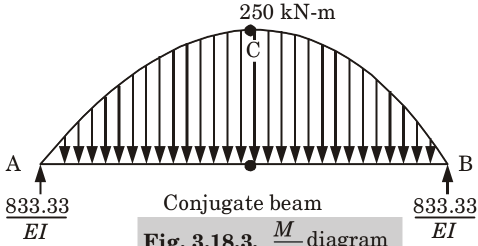


Fig. 3.18.3. $\frac{M}{EI}$ diagram

3. Total load on the conjugate beam = Area of the load diagram on the conjugate beam

$$= \frac{1}{EI} \times \frac{2}{3} \times 10 \times 250 = \frac{1666.67}{EI}$$

Reaction at each end of the conjugate beam $= \frac{1666.67}{EI} \times \frac{1}{2} = \frac{833.33}{EI}$

4. **Slope at Support :**
Slope at each end of the given beam = SF at each end of the conjugate beam = Reaction at each end of the conjugate beam.

$$\theta_A = \theta_B = \frac{833.33}{EI}$$

5. **Deflection at the Centre of the Beam :**

$$\Delta_C = \text{BM at the centre of the conjugate beam}$$

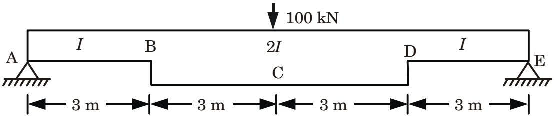
Deflection at C, $$\Delta_C = \frac{833.33}{EI} \times \frac{10}{2} - \frac{833.33}{EI} \times \frac{3}{8} \times \frac{10}{2} = \frac{2604.16}{EI}$$

Que 3.19. **A beam *ABCDE* is 12 m long and supports a load of 100 kN at *C*, simply supported at *A* and *E*. *AB* = *BC* = *CD* = *DE* = 3 m. Moment of inertia is *I* in the portion *AB* and *DE* and 2*I* in the portion *BD*. Determine the deflections at *B* and *C* by using conjugate beam method.**

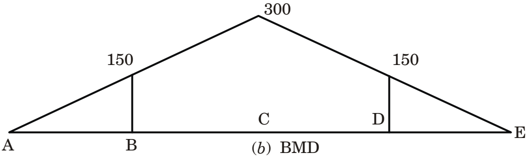
AKTU 2016-17, Marks 10

Answer

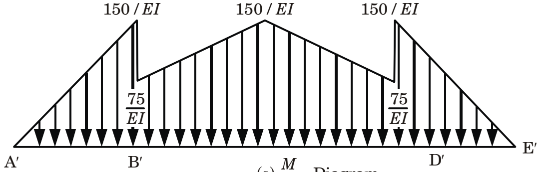
Given : Span of each part = 3 m, Concentrated load, W = 100 kN
Moment of inertia of part AB and $DE = I$
Moment of inertia of part BC and $CD = 2I$
To Find : Deflection at point B and C



(*a*) Given beam



(*b*) BMD



(*c*) $\frac{M}{EI}$ Diagram

Fig. 3.19.1.

1. Due to symmetry support reactions in beam,

$$V_A = V_E = \frac{100}{2} = 50 \text{ kN}$$

2. Moment at B, $\quad M_B = V_A \times 3 = 50 \times 3 = 150$ kN-m
   Moment at C, $\quad M_C = V_A \times 6 = 50 \times 6 = 300$ kN-m
   Fig. 3.19.1(*b*) shows the BM diagram for the given beam. Fig. 3.19.1(*c*) shows the *M/EI* diagram which is the loading on the conjugate beam.

   The thickness of this diagram is $\frac{1}{EI}$ for the parts AB and DE and $\frac{1}{2EI}$ for the part BD.

3. Total load on the conjugate beam = Area of the load diagram on the conjugate beam

$$= 2 \times \frac{1}{2} \times 3 \times \frac{150}{EI} + 2 \times 3 \times \left(\frac{75 + 150}{2}\right) \times \frac{1}{EI} = \frac{1125}{EI}$$

4. Due to symmetry reaction at supports of conjugate beam,

$$V'_A = V'_E = \frac{1125}{2\,EI} = \frac{562.5}{EI}$$

5. Deflection at point B of given beam = Bending moment at point B in conjugate beam

$$= V'_A \times 3 - \frac{1}{2} \times 3 \times \frac{150}{EI} \times \frac{3}{3}$$

$$\Delta_B = \frac{562.5}{EI} \times 3 - \frac{450}{2EI} = \frac{1462.5}{EI}$$

6. Deflection at point C in given beam = BM at point C in conjugate beam

$$= V_A' \times 6 - \frac{1}{2} \times 3 \times \frac{150}{EI} \times \left(3 + \frac{3}{3}\right) - \frac{3 \times 75}{EI} \times 1.5 - \frac{1}{2} \times 3 \times \frac{75}{EI} \times \frac{3}{3}$$

$$A_C = \frac{562.5}{EI} \times 6 - \frac{900}{EI} - \frac{337.5}{EI} - \frac{225}{2EI} = \frac{2025}{EI}$$

Que 3.20. **A simply supported beam with variable moment of inertia supports a uniformly distributed load of *w* kN/m. Estimate the maximum deflection in a beam.**

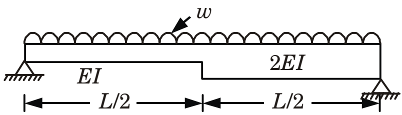


Fig. 3.20.1.

AKTU 2016-17, Marks 7.5

Answer

Given : Span of beam = L m, Intensity of UDL = w kN/m
To Find : Maximum deflection in a beam.

1. Total load on conjugate beam

$$= \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8\,EI} + \frac{2}{3}\left(\frac{L}{2}\right)\frac{wL^2}{16EI}$$

$$= \frac{wL^2}{24\,EI}\left[1 + \frac{1}{2}\right] = \frac{wL^2}{16\,EI}$$

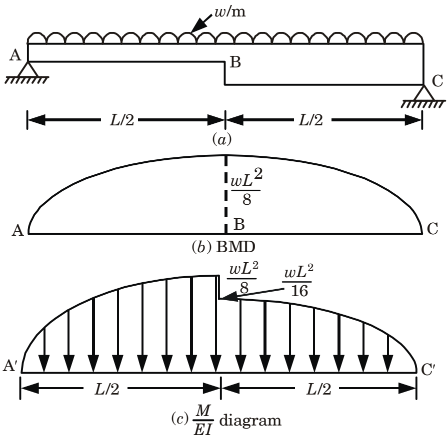


Fig. 3.20.2.

2. Take moment about point C',

$$M_C' = 0$$

$$V'_A \times L = \frac{wL^3}{24\ EI}\left(\frac{L}{2} + \frac{3}{8} \times \frac{L}{2}\right) + \frac{wL^3}{48\ EI}\left(\frac{5}{8} \times \frac{L}{2}\right)$$

$$V'_A \times L = \frac{wL^3}{24\ EI}\left(\frac{8+3}{16}\right)L + \frac{wL^3}{48\ EI}\left(\frac{5L}{16}\right)$$

$$= \frac{wL^3}{24\ EI}\left(\frac{11}{16}\right)L + \frac{wL^3}{48\ EI}\left(\frac{5L}{16}\right) = \frac{(22+5)\,wL^4}{48 \times 16\ EI}$$

$$V'_A \times L = \frac{27\,wL^4}{48 \times 16EI} = \frac{9wL^4}{256\ EI}$$

$$V'_A = \frac{9wL^3}{256EI}$$

3. Maximum deflection in beam occurs at that point, where loading is maximum in $\frac{M}{EI}$ loading diagram.

   Deflection at point B = Moment at point B in $\frac{M}{EI}$ diagram.

$$\Delta_{\max} = \left[V'_A \times \frac{L}{2} - \frac{2}{3}\left(\frac{wL^2}{8}\right) \times \frac{L}{2} \times \left(\frac{5}{8} \times \frac{L}{2}\right)\right]\frac{I}{EI}$$

$$\Delta_{\max} = \frac{9wL^3}{256EI} \times \frac{L}{2} - \frac{wL^4}{128EI}$$

$$\Delta_{\max} = \frac{wL^4}{EI}\left[\frac{9}{512} - \frac{1}{128}\right]$$

$$\Delta_{\max} = \frac{wL^4}{EI}\left[\frac{5}{512}\right]$$

Maximum deflection, $\Delta_{\max} = \dfrac{wL^4}{102.4EI}$

Que 3.21. **Determine the slopes at supports and deflection under the load for the beam shown in Fig. 3.21.1. Take Young's modulus E as 210 GPa, moment of inertia as 120 × 10^6 mm^4. Adopt conjugate beam method.**

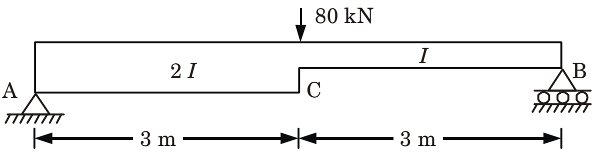


Fig. 3.21.1.

AKTU 2016-17, Marks 7.5

Answer

Given : Moment of inertia of beam, $I = 120 \times 10^6 \text{ mm}^4$
Young's modulus of beam, $E = 210$ GPa
Span of beam, $L = 6$ m
Concentrated load, $W = 80$ kN
To Find : Slope at supports and deflection under the load.

1. Due to symmetry,
Reactions at supports,
$$V_A = V_B = \frac{80}{2} = 40 \text{ kN}$$
2. BM at point $C = 40 \times 3 = 120$ kN-m
3. Fig. 3.21.2(a) shows given beam. Fig. 3.21.2(b) shows the bending moment diagram for the given beam (M-diagram) Fig. 3.21.2(c) shows the conjugate beam.
4. The load diagram for the conjugate beam is given by the $\frac{M}{EI}$ diagram.
The thickness of diagram is $\frac{1}{2EI}$ for the left and $\left(\frac{1}{EI}\right)$ for the right half part.
5. Reaction at supports :
i. $$V_A' + V_B' = \frac{1}{2} \times 3 \times \frac{60}{EI} + \frac{1}{2} \times 3 \times \frac{120}{EI} = \frac{270}{EI}$$
ii. Bending moment at point B, $\Sigma M_B = 0$
$$V_A' \times 6 = \frac{1}{2} \times 3 \times \frac{60}{EI}\left(3 + \frac{3}{3}\right) + \frac{1}{2} \times \frac{120}{EI} \times 3 \times 2 \times \frac{3}{3}$$
$$V_A' = \frac{120}{EI} \text{ and } V_B' = \frac{270}{EI} - \frac{120}{EI} = \frac{150}{EI}$$
6. Slope at support A, θ_A = Shear force at support A in conjugate beam
$$V_A' = \frac{120}{EI} = \frac{120 \times 10^6}{120 \times 10^6 \times 10} = 4.76 \times 10^{-3} \text{ radian}$$
7. Slope at support B,
$$\theta_B = V'_B = \frac{150}{EI} = \frac{150 \times 10^6}{120 \times 10^6 \times 210}$$
$$= 5.95 \times 10^{-3} \text{ radian}$$
8. Deflection under the load = BM at point C in conjugate beam
$$= \frac{120}{EI} \times 3 - \frac{1}{2} \times \frac{60}{EI} \times 3 \times \frac{3}{3}$$
Deflection at point C,
$$\Delta_C = \frac{270}{EI} = \frac{270 \times 10^9}{210 \times 120 \times 10^6} = 10.72 \text{ mm}$$

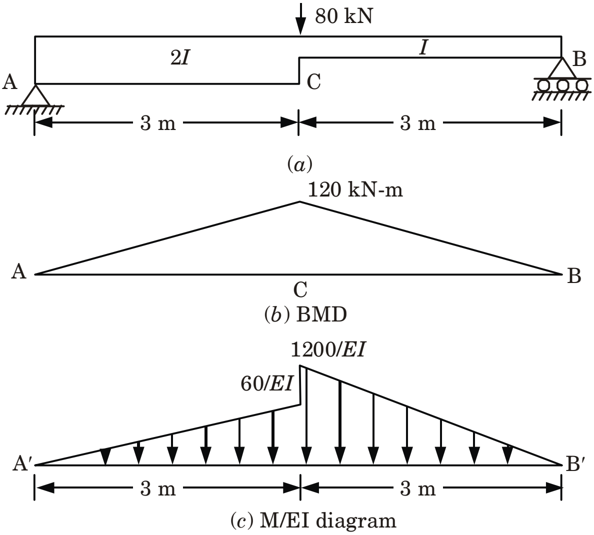


Fig. 3.21.2.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. State and prove Maxwell's reciprocal theorem.

Ans. Refer Q. 3.3, Unit-3.

Q. 2. Write down the statement of Betti's law and also prove it.

Ans. Refer Q. 3.2, Unit-3.

Q. 3. Explain the Castigliano's theorems with prove.

Ans. Refer Q. 3.5, Unit-3.

Q. 4. A cantilever of span *L* is subjected to a point load *W* at the free end. Determine the slope and deflection at the free end. Take *EI* as constant.

Ans. Refer Q. 3.15, Unit-3.

Q. 5. Determine the horizontal displacement of the roller end *D* of the portal frame shown in Fig. 3.5.1. *EI* is 8000 kN-m² throughout.

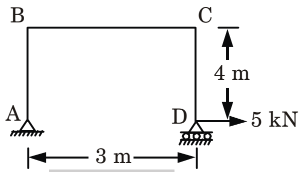


Fig. 3.5.1.

Ans. Refer Q. 3.7, Unit-3.

Q. 6. Determine the vertical deflection at the free end and rotation at *A* in the overhanging beam shown in Fig. 3.6.1. Assume constant *EI*. Use Castigliano's method.

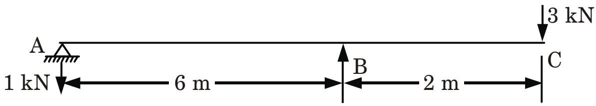


Fig. 3.6.1.

Ans. Refer Q. 3.8, Unit-3.

Q. 7. Determine the deflection and rotation at the free end of the cantilever beam shown in Fig. 3.7.1
Use unit load method. Given $E = 2 \times 10^5$ N/mm², and

$$I = 12 \times 10^6 \text{ mm}^4.$$

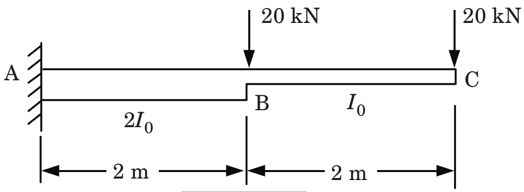


Fig. 3.7.1.

Ans. Refer Q. 3.10, Unit-3.

☺☺☺

4

UNIT

Rolling Loads and Influence Line Diagrams

PART-1

Rolling Loads and Influence Line Diagram for Beams and Trusses.

CONCEPT OUTLINE : PART-1

Rolling Loads : In actual practice, we often encounter with the loads which are moving or with positions that are liable to change.
ILD : Influence line diagram for a stress resultant is the one in which ordinate represent the value of the stress resultant for the position of unit load at the corresponding abscissa. ILD are used to identify the position of moving loads for maximum value of stress resultant and for finding its maximum value.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. What do you understand by the term rolling loads ?

Answer

1. **Rolling Loads (or Moving Loads) :** In actual practice, we often encounter with the loads which are moving or with positions that are liable to change.
2. **Example :** Axle loads of moving trucks or vehicles, wheel loads of a railway train or wheel loads of a gantry assembly on a gantry girder etc.
3. In case of rolling or moving loads it is necessary to determine the maximum shear force and bending moment at different sections as the loads traverse from one end to the other.

Que 4.2. Define influence line diagram. What are the advantages of influence line and uses of it.

Answer

A. Influence Line Diagram :

1. A curve or graph that represents the function like a reaction at support, the shear force at a section, the bending moment at a section of a structure etc., for various positions of a unit load on the span of the structure is called an influence line diagram for the function represented.

2. For statically determinate structures, the influence lines for bending moment, shear force or stress are composed of straight lines, while they are curvilinear for statically indeterminate structures.

B. Advantages of Influence Lines : Following are the advantages of influence lines :

1. If a structure is subjected to a live load or moving load, however, the variation of the shear and bending moment in the member is best described using the influence line.
2. Once the influence line is constructed, it is easy to tell at a glance where the moving load should be placed on the structure so that it creates the greatest influence at the specified point.
3. The magnitude of the associated reaction, shear, moment or deflection at the point can be calculated from the ordinates of the influence line diagram.
4. Influence lines play an important role in the design of bridges, industrial crane rails, conveyors and other structures where loads move across their span.

C. Uses of Influence Lines : Following are the uses of influence lines :

1. Influence lines are used to show the variation of shear force and bending moment in the member which is subjected to a live load or moving load.
2. Influence lines help to tell where the moving load should be placed on the structure so that it creates the greatest influence at the specified point.
3. Using the ordinates of influence line diagram, the magnitude of associated reaction, shear, moment or deflection at the point can be calculated.
4. Influence lines are used to design the structures on which the loads move across the span. The common types of structures are design of bridges, industrial crane rails and conveyors etc.

Que 4.3. **Draw the schematic ILD for reaction, SF and BM at a section when a unit load moves over a simply supported beam.**

Answer

Influence Line Diagrams for Simply Supported Beam :

Here influence line diagrams for reactions at support A, support B and shear force and bending moment at a section a distance z from the end A are drawn.

1. ILD for Reaction V_A :

Let the unit load be at a distance x from support A as shown in Fig. 4.3.1.

Now, $$V_A = \frac{1(L-x)}{L} = \left(1 - \frac{x}{L}\right), \text{ linear variation with } x$$

when $x = 0$, $V_A = 1$

when $x = L$, $V_A = 0$

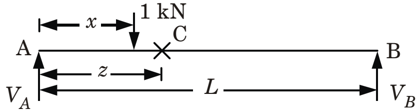


Fig. 4.3.1. Beam with unit load.

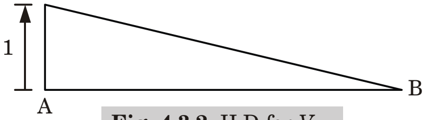


Fig. 4.3.2. ILD for V_A.

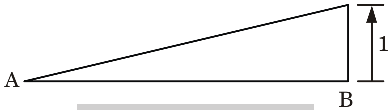


Fig. 4.3.3. ILD for V_B.

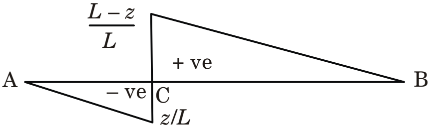


Fig. 4.3.4. ILD for S_C.

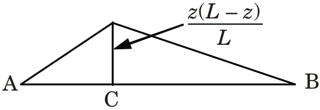


Fig. 4.3.5. ILD for M_C.

Hence, ILD for V_A is shown in Fig. 4.3.2.

2. **ILD for Reaction V_B :**

Referring to Fig. 4.3.1.

$$V_B = \frac{x}{L}, \text{ linear variation}$$

At $x = 0$, $V_B = 0$

At $x = L$, $V_B = 1$

Hence, ILD for V_B is shown in Fig. 4.3.3.

3. **ILD for Share Force at C :**

Let C be the section at a distance z from A as shown in Fig. 4.3.1.

i. When $x < z$

$$S_C = -V_B = -\frac{x}{L}, \text{ linear variation}$$

when $x = 0$, $S = 0$

$$\text{when } x = z, \quad S = -\frac{z}{L}$$

ii. When $x > z$

$$S_C = V_A = \frac{L - x}{L}, \text{ linear variation}$$

$$\text{when } x = z, \quad S_c = \frac{L - z}{L}$$

$$\text{when } x = L, \quad S_c = 0$$

Therefore, ILD for shear force at C is shown in Fig. 4.3.4.

4. **ILD for Moment M_c at C :**

i. When $x < z$

$$M_C = V_B (L - z)$$

$$= \frac{x}{L} (L - z), \text{ linear variation with } x$$

$$\text{when } x = 0, \quad M_C = 0$$

$$\text{when } x = z, \quad M_C = \frac{z(L - z)}{L}$$

ii. When $x > z$

$$M_C = V_A z = \left(\frac{L - x}{L}\right) z, \text{ linear variation with } x$$

$$\text{when } x = z, \quad M_C = \frac{z(L - z)}{L}$$

$$\text{when } x = L, \quad M_C = 0$$

Hence, ILD for moment at C is shown in Fig. 4.3.5.

Que 4.4. Discuss the maximum shear force and bending moment at a section for a single concentrated load moving over a beam.

Answer

Maximum SF and BM at a Section :

1. Let W be the moving load and the value of maximum SF and BM required be at C, which is at a distance z from A (Fig. 4.4.1). ILD for SF and BM are shown in Fig. 4.4.2 and 4.4.3 respectively.
2. From Fig. 4.4.2, it is clear that maximum negative SF occurs when the load is just to the left of section C and its value is $= \dfrac{Wz}{L}$.
3. Similarly, maximum positive SF occurs when the load is just to the right of the section and its value is $= W\left(\dfrac{L - z}{L}\right)$.
4. From ILD for moment M_C, it is clear that maximum bending moment will occur when the load is on the section itself and its value is $= \dfrac{Wz(L - z)}{L}$.

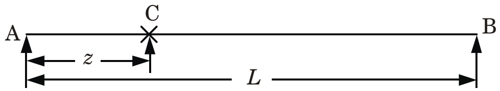


Fig. 4.4.1. Simply supported beam with given point C.

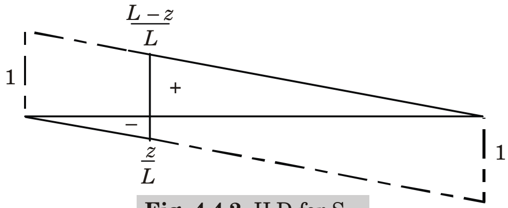


Fig. 4.4.2. ILD for S_C.

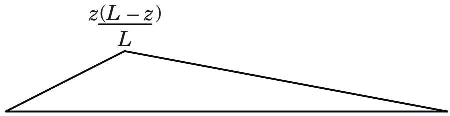


Fig. 4.4.3. ILD for M_C.

Que 4.5. **What will be the value of maximum shear force and bending moment at a section for a uniformly distributed load longer than the span ?**

OR

Draw and determine the maximum value of shear force and bending moment for a rolling UDL longer than the span of the girder.

Answer

Maximum SF and BM at given Section :

1. Let a uniformly distributed load of intensity w move from left to right. Load intensity times the area of ILD over loaded length gives the value of stress resultant (SF). Referring again to Fig. 4.4.2.
2. Negative SF is maximum, when the load covers portion AC only.
   Maximum negative SF,
$$S_C = w \times \text{Area of ILD for } S_C \text{ in length } AC$$
$$= w\left(\frac{1}{2}\right)z\left(\frac{z}{L}\right) = \frac{wz^2}{2L}$$
3. Positive SF is maximum, when the uniformly distributed load occupies the portion CB only.
   Maximum positive
$$S_C = w \times \text{Area of ILD for } S_C \text{ in length } CB$$
$$= w\left(\frac{1}{2}\right)(L-z)\left(\frac{L-z}{L}\right)$$
$$= \frac{w(L-z)^2}{2L}$$

4. From Fig. 4.4.3, it is clear that maximum moment at C will be, when the UDL covers entire span,

$$M_{C,\max} = w \times \text{Area of ILD for } M_C$$
$$= w \times \frac{1}{2} L \frac{z(L-z)}{L}$$
$$M_{C,\max} = \frac{wz(L-z)}{L}$$

Que 4.6. Discuss about the positive and negative shear force of a UDL over a simply supported beam having load shorter than the span. Also plot the corresponding shear force diagram and bending moment diagram.

OR

Plot a SFD and BMD for a UDL shorter than the span moving or rolling over the simply supported beam.

Answer

1. Let the length of uniformly distributed load w per unit length be d. Let it move from left to right over beam AB of span L.
2. When, $d < L$, position of this load for maximum shear force and bending moment at section C, shown in Fig. 4.6.2, are to be determined.

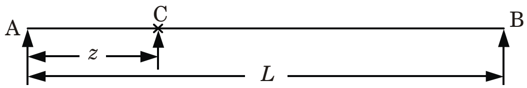


Fig. 4.6.1. Simply supported beam with given point C.

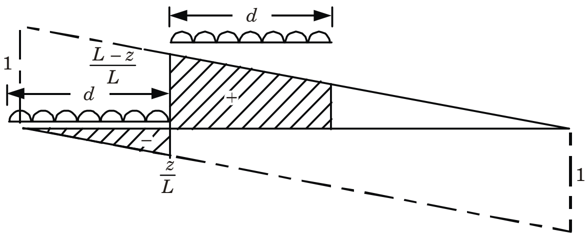


Fig. 4.6.2. Position of load for maximum +ve and –ve SF.

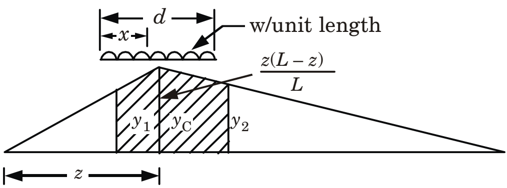


Fig. 4.6.3. Position of load for maximum moment.

3. From ILD for shear force at C, it is clear that maximum shear force will develop when the head of the load reaches the section.
4. For maximum positive shear force the tail of the UDL should reach the section.
5. Maximum bending moment will develop at C when the load is partly to the left of the section and partly to the right of section.
6. Let the position of the section be as shown in Fig. 4.6.3 from to this Fig.

$$M_C = w \times \frac{x(y_1 + y_c)}{2} + w(d - x)\frac{(y_c + y_2)}{2}$$

7. For M_C to be maximum,

$$\frac{dM_C}{dx} = 0 = \frac{w(y_1 + y_c)}{2} - \frac{w(y_c + y_2)}{2}$$

i.e., $y_1 = y_2$

8. Thus, moment at C will be maximum when the ordinates of ILD for M_C at head and tail of the UDL are equal.

Now, $y_1 = y_2$

$$\frac{(z - x)}{z} y_C = \frac{(L - z) - (d - x)}{L - z} y_C$$

$$(z - x)(L - z) = z\,(L - z - d + x)$$

$$Lz - z^2 - Lx + xz = Lz - z^2 - dz + xz$$

$$Lx = dz$$

$$\frac{x}{d} = \frac{z}{L}$$

9. Bending moment at a section is maximum when the load is so placed that the section divides the load in the same ratio as it divides the span.

Que 4.7. **A uniformly distributed load of 10 kN/m intensity covering a length of 2 m crosses a simply supported beam of span 8 m. Determine the maximum positive shear force, maximum negative shear force and maximum bending moment at a section 3 m from left support.**

AKTU 2013-14, Marks 10

Answer

Given : Intensity of UDL, w = 10 kN/m, Length of UDL = 2 m
Length of beam, L = 8 m.
To Find : $+ve\ SF_{max}$, $-ve\ SF_{max}$ and M_{Max} at Section

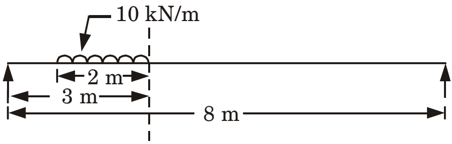


Fig. 4.7.1.

1. ILD for Shear Force :

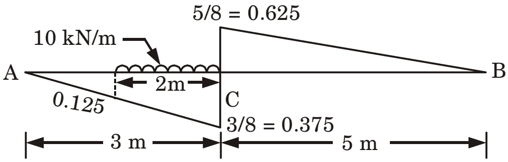


Fig. 4.7.2.

i. Maximum negative shear force occurs when the load is applied as shown in Fig. 4.7.2 on AC.

$$\text{Negative shear force} = \left(\frac{0.125 + 0.375}{2}\right) \times 2 \times 10 = 5 \text{ kN}$$

ii. Maximum positive shear force occurs, when load applied as shown in Fig. 4.7.3.

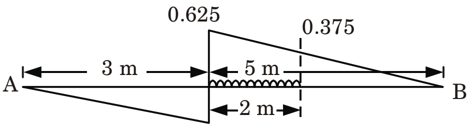


Fig. 4.7.3.

$$\text{Maximum positive shear force} = \left(\frac{0.625 + 0.375}{2}\right) \times 2 \times 10 = 10 \text{ kN}$$

2. Maximum Bending Moment :

i. The load should be so arrange that the section divides it in the same ratio as it divides the span.

Now
$$x = \frac{l}{L} b$$
$$x = \frac{2}{8} \times 5 = 1.25$$

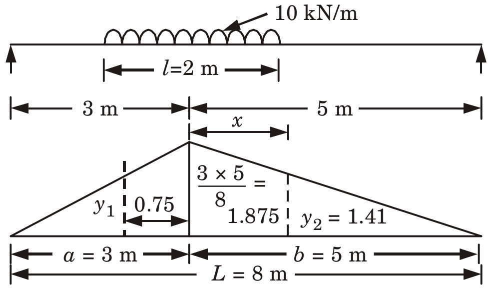


Fig. 4.7.4.

ii. From property of similar triangle :

For y_2,
$$\frac{1.875}{5} = \frac{y_2}{(5 - 1.25)}$$

$$y_2 = 1.41$$

For y_1, $$\frac{1.875}{3} = \frac{y_1}{(3-0.75)}$$

$$y_1 = 1.41$$

iii. Maximum bending moment,

$$M_{max} = \left(\frac{1.875+1.41}{2}\right) \times 1.25 \times 10 + \left(\frac{1.875+1.41}{2}\right) \times 0.75 \times 10$$

Maximum bending moment,

$$M_{max} = 32.85 \text{ kN-m}$$

Que 4.8. **A single load of 100 kN moves on a girder or span 20 m. Construct the influence line for shear force and bending moment for a section 5 m from the left support.** **AKTU 2014-15, Marks 10**

Answer

Given : Concentrated Load, w = 100 kN, Span of beam, L = 20 m
Distance of section = 5 m from left support
To Find : Make ILD for SF and BM at given section

A. Construction of ILD for Shear Force :

1. Let a unit load move along the span of a simply supported girder *AB* of span *L*.
   Let *D* be a given section. Let
   $$AD = a \text{ and } BD = b$$
2. When the unit load is between *A* and *D*
   SF at *D*, $$S_D = -V_B$$
3. But we know V_B varies from 0 to 1 as the load moves from *A* to *B*. The influence line diagram for V_B is drawn. But as long as the unit load is between *A* and *B*, $S_A = V_B$. Hence the part of the influence line diagram for V_B between *A* and *B* is also applicable to S_D.
4. Similarly, when the unit load is between *D* and *B* the SF at *D*,
   $$S_D = +V_A$$
   We know V_A varies from 1 to 0 as the load moves from *A* to *B*. The influence line diagram for V_A is drawn. The part of this diagram between *D* and *B* is applicable to S_D.
5. The part of the influence line diagram for V_B between *A* and *D* and the part of the influence line diagram for V_A between *D* and *B* will constitute the influence line diagram for S_D.
6. Maximum negative shear force,
   $$= \frac{W \times a}{L} = 100 \times \frac{5}{20} = 25 \text{ kN}$$

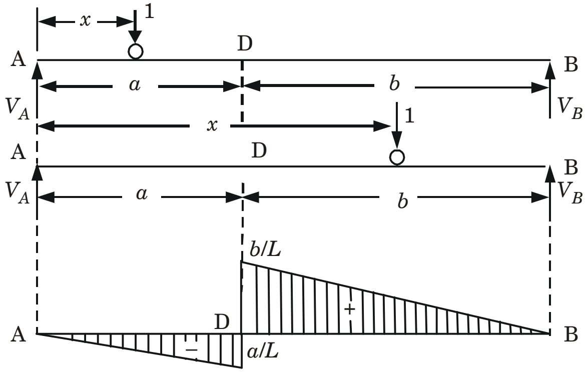


Fig. 4.8.1. ILD for V_D.

7. For part C to B :
   Maximum positive shear force

$$= \frac{W \times b}{L} = 100 \times \frac{15}{20} = 75 \text{ kN}$$

B. Construction of ILD for Bending Moment :

1. Fig. 4.8.2 shows a simply supported girder AB of span L. Let a unit load move from the end A to the end B of the girder.
2. Let D be a given section of the girder so that,

$$AD = a \text{ and } DB = L - a = b$$

3. Let the unit load be at a distance x from the left end A. The reactions at the supports A and B are given by,

$$V_A = \frac{L - x}{L}$$

and $$V_B = \frac{x}{L}$$

4. When the unit load is between A and D.
   The bending moment at D is given by,

$$M_D = V_B(L - a) = \frac{x}{L}(L - a)$$

or, $$M_D = \left[\frac{L - a}{L}\right]x$$

   The above relation is true for all load positions from A to D, *i.e.*, for values of x from $x = 0$ to $x = a$.

5. When the unit load is at A, *i.e.*, when $x = 0$,

$$M_D = 0$$

   When the unit load is at D, *i.e.*, when $x = a$

$$M_D = \left(\frac{L - a}{L}\right) a$$

Hence, as the unit load moves from A to D, the bending moment at D will vary from 0 to $\dfrac{a(L-a)}{L}$

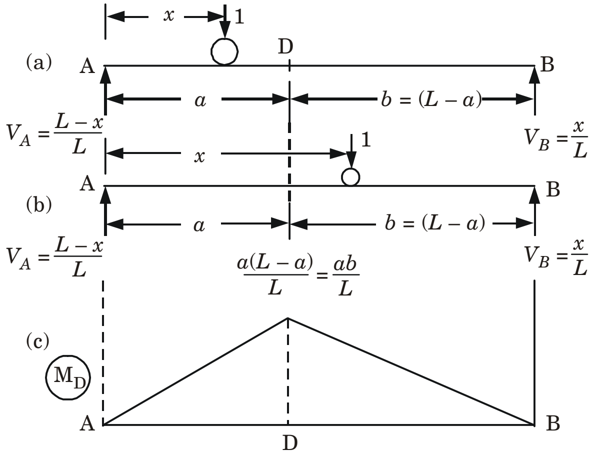


Fig. 4.8.2.

6. When the unit load is between D and B, the bending moment at D is given by, $M_D = V_A\, a$

$$M_D = \left(\frac{L-x}{L}\right) a$$

The above relation is true for all load positions of the unit load from D to B *i.e.*, for values of x from $x = a$ to $x = L$.

7. When the unit load is at D, *i.e.*, when $x = a$, $M_D = \dfrac{a}{L}(L-a)$

When the unit load is at B, *i.e.*, when $x = L$, $M_D = 0$
Hence, as the unit load moves from D to B the bending moment at D will vary from $\dfrac{a(L-a)}{L}$ to 0.

8. Ordinate of BM at section D,

$$\frac{a\,(L-a)}{L} = \frac{5 \times 15}{20} = 3.75 \text{ m}$$

Maximum BM = 100 × 3.75 = 375 kN-m
Value of BM varies from 0 to 375 kN-m and 375 kN-m to 0.

Que 4.9. **A simply supported beam has a span of 15 m. UDL of 40 kN/m and 5 m long crosses the girder from left to right. Draw the influence line diagram for shear force and bending moment at a section 6 m from left end. Use these diagrams to calculate the maximum shear force and bending moment at this section.**

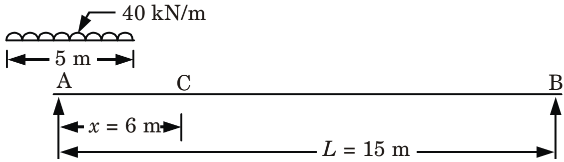


Fig. 4.9.1.

AKTU 2015-16, Marks 10

Answer

Given : Span, L = 15 m, intensity of UDL, w = 40 kN/m,
Length of UDL, l = 5 m, Distance of section, x = 6 m
To Find : Draw ILD and Calculate maximum SF and BM

1. **ILD for Shear Force and Bending Moment :**

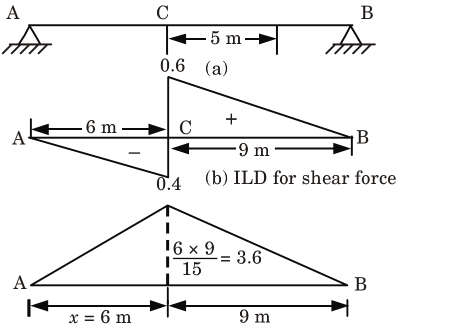


Fig. 4.9.1.

2. **Maximum Positive Shear Force at *C* :**
For this condition the tail of the UDL should be at the section, as shown in Fig. 4.9.2.

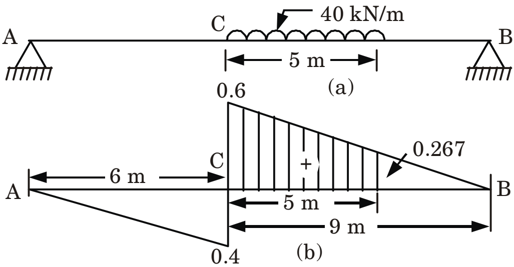


Fig. 4.9.2. ILD for shear force at given section.

Maximum positive shear force $= \left(\dfrac{0.6+0.267}{2}\right) \times 5 \times 40 = 86.7$ kN

3. **Maximum Negative Shear Force at *C* :**

Similarly for this condition the head of UDL should be at the section, as shown in Fig. 4.9.3.

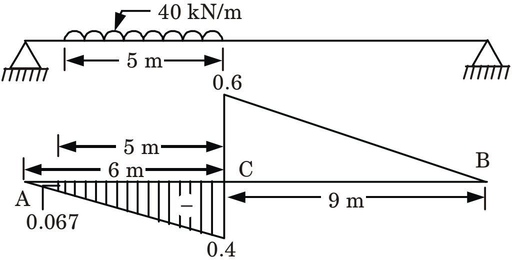


Fig. 4.9.3.

Maximum negative shear force $= \left(\dfrac{0.4+0.067}{2}\right) \times 5 \times 40 = 46.7$ kN

4. **Maximum Bending Moment at Point *C* :**

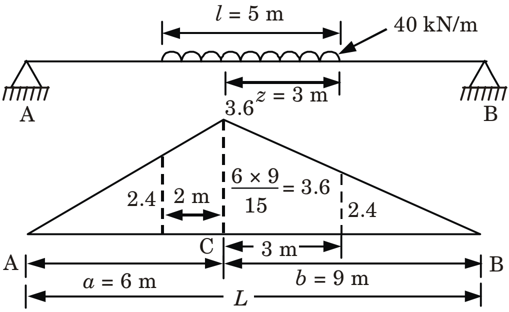


Fig. 4.9.4.

i. Condition for maximum bending moment,

$$z = \frac{l}{L} \times b$$

$$z = \frac{5}{15} \times 9$$

$$z = 3 \text{ m}$$

ii. Maximum bending moment at C,

$M_{max} = w \times$ area of the ILD covered by the load.

$$M_{max} = 40\left[\left(\frac{3.6+2.4}{2}\right) \times 3 + \left(\frac{3.6+2.4}{2}\right) \times 2\right]$$

$$M_{max} = 40\,(3 \times 3 + 3 \times 2)$$

$$M_{max} = 40 \times 15 = 600 \text{ kN-m}$$

Que 4.10. **A simply supported beam has a span of 25 m. Draw the influence line for shearing force at a section 10 m from one end and using this diagram determine the maximum shearing force due to the passage of a point load 5 kN followed immediately by uniformly distributed load of 2.4 kN/m extending over a length of 5 m.**

AKTU 2016-17, Marks 10

Answer

Given : Span of beam, L = 25 m, Distance of a section, x = 10 m
Point load, W = 5 kN, Intensity of UDL, w = 2.4 kN/m
Length of UDL = 5 m
To Find : Maximum shear force.

1. ILD for shear force at section at a distance 10 m from left support is shown in Fig. 4.10.1.

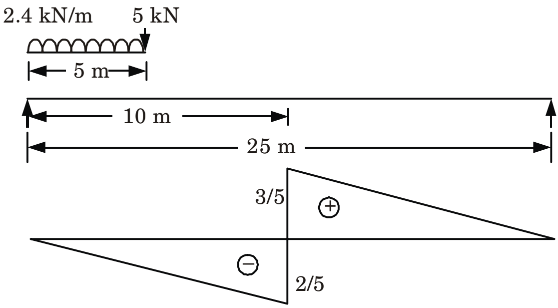


Fig. 4.10.1. ILD of shear force.

2. **Maximum Negative Shear Force :** For the maximum negative shear force placed the load in left part of section as shown in Fig. 4.10.2.

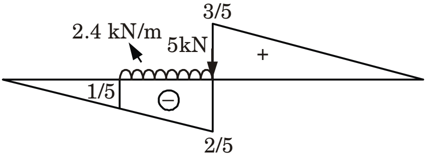


Fig. 4.10.2.

$$\text{Negative SF} = 5 \times \frac{2}{5} + 2.4 \times \left(\frac{1}{5} + \frac{2}{5}\right) \times \frac{1}{2} \times 5$$

$$= 2 + 2.4 \times 1.5 = 5.6 \text{ kN}$$

3. **Maximum Positive Shear Force :**
For the maximum positive shear force placed the load in right side of section on ILD of shear force as shown in Fig. 4.10.3.

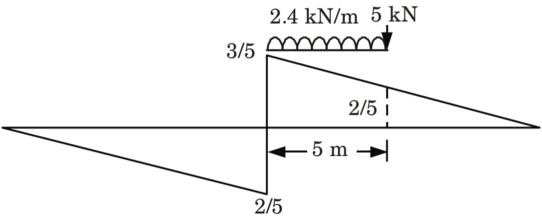


Fig. 4.10.3.

Positive $$\text{SF} = 5 \times \frac{2}{5} + 2.4 \times \left(\frac{3}{5} + \frac{2}{5}\right) \times \frac{1}{2} \times 5$$

$$= 2 + 2.4 \times 2.5 = 8 \text{ kN}$$

Que 4.11. **What do you mean by the influence lines ? Explain. Draw influence line diagram for a positive shear force, negative shear force and bending moment at a section 3 m from left support of a simply supported beam of 5 m span. Hence determine the maximum values of positive shear force, negative shear force and bending moment at the section due to two point loads 60 kN followed by 120 kN moving from left to right. The distance between the loads is 2 m.**

AKTU 2012-13, Marks 10

Answer

A. Influence Line : An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a concentrated load or forces that moves over the member.

B. Numerical :

Given : Span of beam, L = 5 m, Point loads = 60 kN and 120 kN, Distance between points load = 2 m, Distance of section = 3 m (from left).

To Find : Maximum + ve and – ve SF, BM

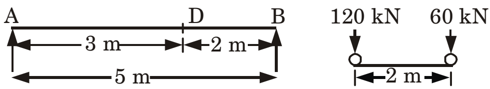


Fig. 4.11.1.

1. **ILD for Maximum Positive Shear Force at *D* :**
   The influence line diagram for the shear force at D is shown in Fig. 4.11.2.
   Maximum positive shear force at D

$$= 120 \times \frac{2}{5} + 60 \times 0 = 48 \text{ kN}$$

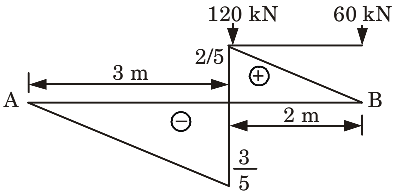


Fig. 4.11.2. ILD for positive shear force.

2. ILD for Maximum Negative Shear Force at *D* :

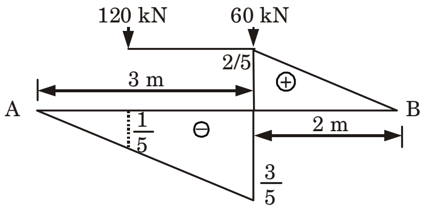


Fig. 4.11.3. ILD for negative shear force.

Maximum negative shear force at D

$$= -\left[120 \times \frac{1}{5} + 60 \times \frac{3}{5}\right] = -60 \text{ kN}$$

3. ILD for Maximum Bending Moment at *D* :

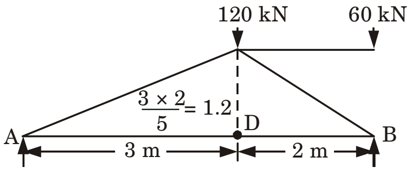


Fig. 4.11.4. ILD for bending moment at section D.

Maximum bending moment at D,

$$= 1.2 \times 120 + 0 \times 60 = 144 \text{ kN-m.}$$

Que 4.12. **Two point loads 10 kN and 20 kN spaced 3 m apart. The loads move on a simply supported beam of 20 m span. Calculate the maximum shear force and bending moment at a section 5 m from the left support.** **AKTU 2013-14, Marks 10**

Answer

Given : Span of beam, L = 20 m, Distance of section = 5 m (from A)
Point loads = 20 kN and 10 kN, Distance between point load = 3 m.
To Find : Maximum SF and BM.

1. **Maximum Positive Shear Force at *C* :**
For this condition the heavier load *i.e.*, 20 kN should be placed at just on the right side of *C*. The other load should be placed at 3 m on the right of *C* as shown in Fig. 4.12.1.

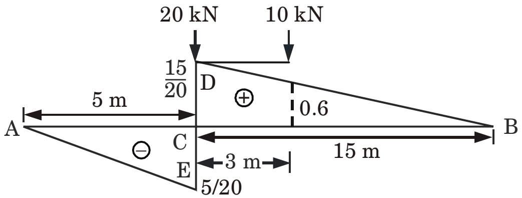


Fig. 4.12.1. The influence line diagram for shear force at *C*.

Maximum positive shear force at given section

$$= 20 \times \frac{15}{20} + 10 \times 0.6 = 21 \text{ kN}$$

2. **Maximum Negative Shear Force at *C* :**
For the maximum shear force the load should be placed as shown in Fig. 4.12.2.

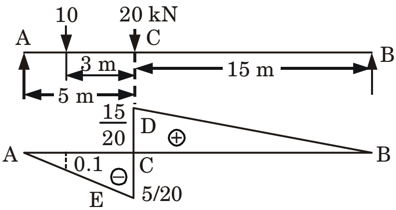


Fig. 4.12.2.

Maximum negative shear force

$$= 10 \times 0.1 + 20 \times \frac{5}{20} = 6 \text{ kN}$$

3. **Maximum Bending Moment :** ILD for bending moment is shown in Fig. 4.12.3.

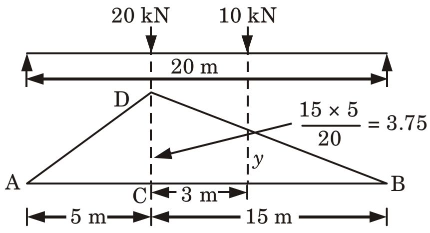


Fig. 4.12.3.

From similar triangle property,
For y,

$$\frac{3.75}{15} = \frac{y}{12}$$

$$y = 3 \text{ m}$$

Maximum bending moment,

$$M_{max} = 3.75 \times 20 + 3 \times 10 = 105 \text{ kN-m}$$

PART-2

Absolute Shear Force and Bending Moment, Muller Breslau's Principle

CONCEPT OUTLINE : PART-2

Condition for Absolute Shear Force and Bending Moment :

1. **Beam Subjected to a Single Point Load :**

   Absolute –ve $SF_{max} = W$ (at $z = L$)

   Absolute +ve $SF_{max} = W$ (at $z = 0$)

   Absolute $BM_{max} = \frac{WL}{4}$ (at $z = L/2$)

2. **Beam Subjected to a Uniformly Distributed Load (Longer than Span) :**

   Absolute –ve $SF_{max} = wL/2$ (at $z = L$)

   Absolute +ve $SF_{max} = wL/2$ (at $z = 0$)

   Absolute $BM_{max} = \frac{wL^2}{8}$ (at $z = L/2$)

3. **Muller Breslau's Principle :** According to this principle "the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function".

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.13. **How can you calculate the absolute SF and absolute BM for different load condition on a simply supported beam ?**

Answer

A. Absolute Maximum SF and BM for Single Concentrated Load :

1. For achieving this, ordinate of ILD should be maximum (Refer Fig. 4.4.2). Negative SF ordinate is maximum when $z = L$ and is equal to 1.

2. Therefore, absolute maximum negative SF = W and it occurs when the load is at $z = L$, *i.e.*, at B.

3. Ordinate of ILD for moment is maximum when $z = \frac{L}{2}$. Hence, when a load is at mid-span, absolute maximum moment occurs and its value is $\frac{WL}{4}$.

B. Absolute Maximum SF and BM for UDL (Longer than Span) :

1. Negative SF is maximum when $z = L$, *i.e.*, at B when the load occupies entire span AB (Refer Fig. 4.4.2).

$$\text{Absolute maximum SF} = w \times \frac{1}{2} \times 1 \times L = \frac{1}{2} wL$$

2. Maximum moment at any section (Refer Fig. 4.4.3)

$$M_z = \frac{1}{2} \times \frac{wz(L-z)}{L} \times L = \frac{1}{2} \times wz(L-z)$$

3. This is maximum,

when $\frac{dM_z}{dz} = 0$, $\qquad z = \frac{L}{2}$ *i.e.*, at mid-span

4. Absolute maximum moment

$$= \frac{1}{2} \times w \times \frac{L}{2} \times (L - L/2) = \frac{wL^2}{8}$$

C. Position for Absolute Maximum Moment in Case of UDL (Shorter than Span) :

1. Obviously for this y_C should be maximum (Refer Fig. 4.5.3)

Now, $$y_C = \frac{z(L-z)}{L}$$

2. For y_C to be maximum,

$$\frac{dy_C}{dz} = 0 = L - 2z$$

or $$z = \frac{L}{2}$$

i.e., Absolute maximum moment occurs at mid-span.

3. The position of the load is to be such that the section divides the load in the same ratio as it divides the span which means that absolute maximum moment CG of the load will be at the mid-span.

Que 4.14. **Uniformly distributed load of intensity 30 kN/m crosses a simply supported beam of span 60 m from left to right. The length of UDL is 15 m. Find the value of maximum bending moment for a section 20 m from left end. Find also the absolute value of maximum bending moment and shear force in the beam.**

AKTU 2015-16, Marks 10

Answer

Given : Span of beam, L = 60 m, Length of UDL, l = 15 m
Intensity of UDL, w = 30 kN/m, Distance of section, x = 20 m
To Find : BM at section, Absolute BM and shear force.

A. Maximum Bending Moment at Given Section :

1. ILD for bending moment at given section, as shown in Fig. 4.14.1.

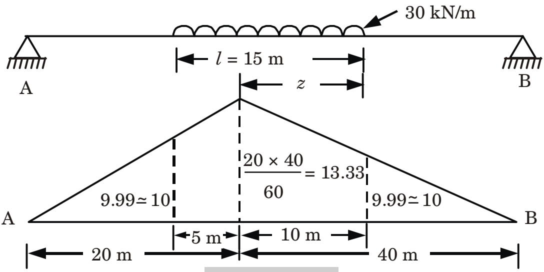


Fig. 4.14.1.

2. Condition for maximum bending moment,

$$z = \frac{l}{L} \times b = \frac{15}{60} \times 40 = 10 \text{ m.}$$

3. Maximum bending moment at given section,

$$M_{max} = w \times \text{Area of the ILD covered by the load.}$$

$$= 30 \times \left[\left(\frac{13.33+10}{2}\right) \times 5 + \left(\frac{13.33+10}{2}\right) \times 10\right]$$

$$= 30 \times 174.975$$

$$M_{max} = 5249.25 \text{ kN-m}$$

B. Absolute Maximum Bending Moment :

1. The absolute maximum bending moment occurs at the centre of the span when the loading is symmetrically placed on the span.
2. ILD for absolute value of maximum bending moment, as shown in Fig. 4.14.2.

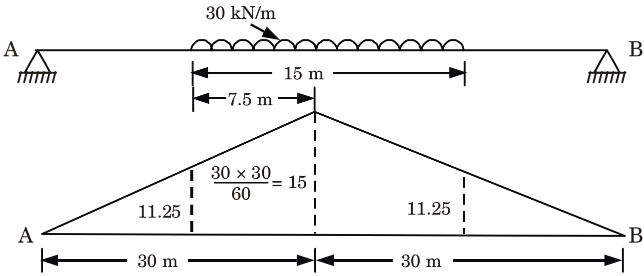


Fig. 4.14.2.

3. Maximum bending moment,

$$M_{\max} = 30 \times \left[\left(\frac{15+11.25}{2}\right) \times 7.5 \times 2\right] = 5906.25 \text{ kN-m}$$

C. Absolute Value of Shear Force :

1. Positive Shear Force :

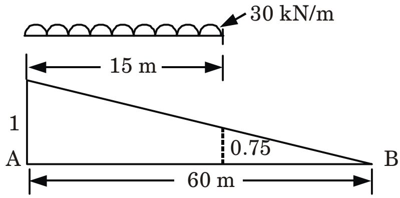


Fig. 4.14.3.

i. For maximum positive shear force the load should be placed as shown in Fig. 4.14.3.

ii. Maximum positive shear force $= 30 \times \left[\frac{1+0.75}{2}\right] \times 15$

$$\text{SF} = 393.75 \text{ kN}$$

2. Negative Shear Force :

i. For maximum negative shear force the load should be placed as shown in Fig. 4.14.4.

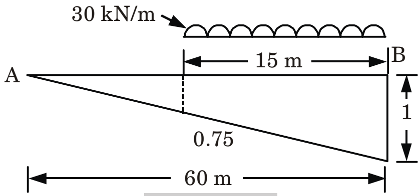


Fig. 4.14.4.

ii. Maximum negative shear force $= 30 \times \left[\frac{1+0.75}{2}\right] \times 15$

$$\text{SF} = 393.75 \text{ kN}$$

Que 4.15. **An uniformly distributed load of 40 kN/m and of length 3 metres transverse across the span of simply supported length of 18 metres. Compute the maximum bending moment at 4 m from the left support and absolute bending moment.**

AKTU 2016-17, Marks 10

Answer

Given : Span of beam, L = 18 m, Intensity of load, w = 40 kN/m
Span of UDL = 3 m
To Find : Maximum bending moment at section and absolute BM

1. Draw the ILD for BM whose maximum ordinate,

$$y = \frac{z(L-z)}{L} = \frac{4(18-4)}{18} = 3.11$$

2. For maximum moment, load position should be such that the section divides the load in the same ratio as it divides the span.

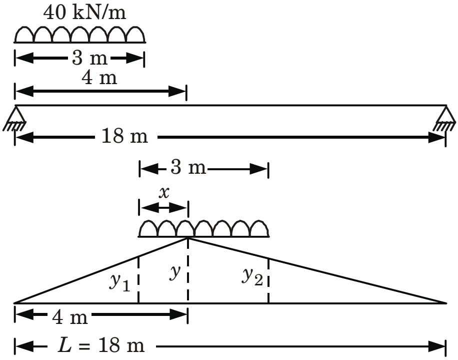


Fig. 4.15.1.

$$\frac{x}{3-x} = \frac{4}{18-4}$$

$$14x = 12 - 4x$$

$$18x = 12$$

$$x = 0.667 \text{ m}$$

3. Calculation of ordinate y_1 and y_2,

$$\frac{y_1}{4-0.667} = \frac{y}{4}$$

$$y_1 = \frac{3.11}{4} \times (4 - 0.667)$$

$$y_1 = 2.591 \text{ m}$$

$$\frac{y_2}{14-2.333} = \frac{3.11}{14}$$

$$y_2 = 2.591$$

4. Maximum moment = w × Area of ILD under the loaded length.

$$M_{max} = 40\left[\left(\frac{2.591+3.11}{2}\right) \times 0.667 + \left(\frac{3.11+2.591}{2}\right) \times 2.333\right]$$

$$M_{max} = 342.06 \text{ kN-m}$$

5. **Absolute Bending Moment :**

i. Absolute bending moment occurs at the centre of the span when the loading is symmetrically placed on the span as shown in Fig. 4.15.2.

ii. Calculation of ordinates :

$$y = \frac{9(18-9)}{18} = \frac{9}{2} = 4.5 \text{ m}$$

$$y_1 = \frac{4.5}{9} \times 7.5 = 3.75 \text{ m} = y_2$$

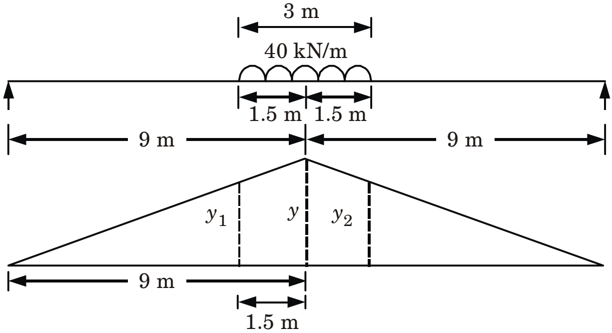


Fig. 4.15.2.

iii. Absolute bending moment = w × Area of ILD under the UDL.

$$M_{abs} = 40\left[2 \times \left(\frac{4.5 + 3.75}{2}\right) \times 1.5\right]$$

$$= 495 \text{ kN-m}$$

Que 4.16. **Discuss the method to calculate the absolute SF for a train load of concentrated loads.**

Answer

Maximum Shear Force at a Given Section of a Beam for Wheel Loads :

Consider the loads shown in Fig. 4.16.1. Let the given section be C. The ILD for S_c, the shear force at C, is shown in Fig. 4.16.2.

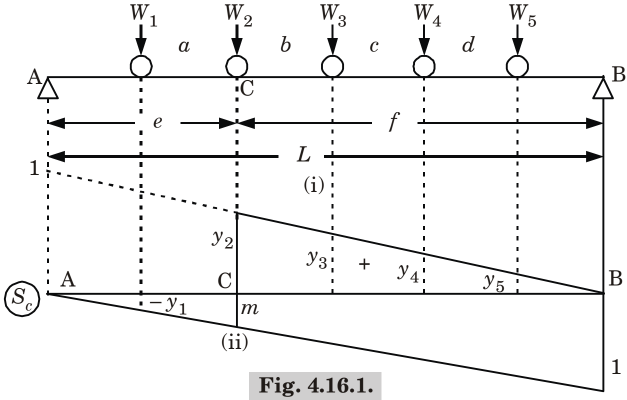


Fig. 4.16.1.

2. Let the loads move by a small distance dx towards the left from their position shown in Fig. 4.16.1.
3. The ordinate of the ILD under the load W_1 will decrease while the ordinates of the ILD under the other loads will increase, by say dy

$$\tan\theta = \frac{dy}{dx} = \frac{1}{L} \qquad \therefore dy = \frac{dx}{L}, \qquad \text{If } dx = b$$

$$\therefore \qquad dy = \frac{b}{L}$$

4. **Position 1 :** Load W_2 at C (just on the right side of C)
   For this position, the shear force at C

$$S_{c1} = -W_1y_1 + W_2y_2 + W_3y_3 + W_4y_4 + W_5y_5$$

5. **Position 2 :** Load W_3 at C (just on the right side of C)
   For this position the shear force at C,

$$S_{c2} = W_1\left(-y_1 + \frac{b}{L}\right) + W_2\left(-m + \frac{b}{L}\right) + W_3\left(y_3 + \frac{b}{L}\right) + W_4\left(y_4 + \frac{b}{L}\right) + W_5\left(y_5 + \frac{b}{L}\right)$$

6. $S_{c1} < S_{c2}$ if $W_2y_2 < -W_2m + \frac{b}{L}(W_1 + W_2 + W_3 + W_4 + W_5)$

   i.e., if $W_2(y_2 + m) < \frac{b}{L}(W_1 + W_2 + W_3 + W_4 + W_5)$, But $y_2 + m = 1$
7. For $S_{c1} < S_{c2}$,

$$\frac{W_2}{b} < \frac{W_1 + W_2 + W_3 + W_4 + W_5}{L}$$

$$\frac{W_2}{b} < \frac{\sum W}{L} \text{ i.e., } Sc_1 < S_{c2}$$

8. $$\frac{\text{Load rolled past the section}}{\text{Succeeding wheel space}} < \frac{\text{Sum of all the loads}}{\text{Span}}$$

Que 4.17. **What are the propositions used for several point loads moving over a simply supported beam ? Explain and prove any one of them.**

AKTU 2012-13, Marks 10

OR

Write down the propositions used for a number of point loads moving over a simply supported beam. Also prove at least one proposition.

AKTU 2013-14, Marks 10

Answer

A. Proposition 1 : When a system of point loads crosses a beam, simply supported at the ends, the maximum bending moment under any given wheel load occurs when this wheel load and the centre of gravity of the total wheel system are equidistance from the end of the beam.

Proof :

1. Consider a beam *AB* carrying the wheel loads as $W_1, W_2, W_3,..., W_n$. Let W_L be the resultant of all loads to the left of W_3 and W_R be the resultant of all loads to the right of W_3 (including W_3).
2. Let W be the resultant of the load system situated at distance a from W_L and b from W_R and c from W_3.

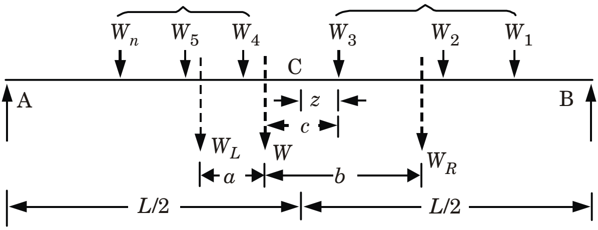


Fig. 4.17.1.

3. We have to find the maximum bending moment under the wheel load W_3.
4. To get the maximum BM under W_3, let the load W_3 be placed at a distance z from the centre C of the span.
5. It is required to find the value of the variable z.

Reaction, $$V_A = \frac{W}{L}\left[\frac{L}{2} + (c - z)\right]$$

6. Bending moment under load W_3,

$$M = V_A\left(\frac{L}{2} + z\right) - W_L\,(a + c)$$

$$= \frac{W}{L}\left[\frac{L}{2} + c - z\right]\left[\frac{L}{2} + z\right] - W_L\,(a + c)$$

$$M = \frac{W}{L}\left[\frac{L^2}{4} + cz - z^2 + \frac{cL}{2}\right] - W_L\,(a + c)$$

7. For maximum value of M, $\dfrac{dM}{dz} = 0$

$$\frac{W}{L}(c - 2z) = 0$$

$$z = \frac{c}{2}$$

Hence the bending moment under load W_3 is maximum when the centre of the span is midway between W and W_3.

B. Proposition 2 : The maximum bending moment at any given section of a simply supported beam, due to given system of point loads crossing the beam occurs when the average loading on the portion to the left of it is equal to the average loading to the right of it, *i.e.*, when the section divides the load in the same ratio as it divides the span.

Proof :

1. Consider a simply supported beam AB loaded with a system of point loads.
2. Let W_L be the resultant of all the loads left to the point C and W_R be the resultant of all loads right to the point C.
3. Let W be the resultant load located at distance y from support A.

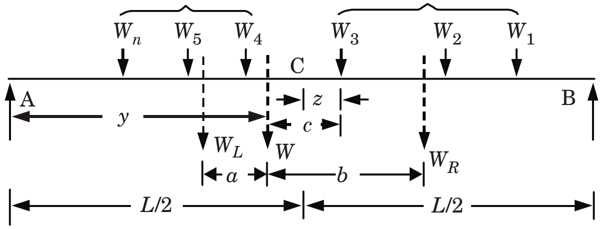


Fig. 4.17.2.

4. We have to find the load position for maximum bending moment at point C, distant x from A.
5. Reaction at support A,

$$V_A = \frac{W(L-y)}{L}$$

6. Bending moment at section x, $M_x = V_A\, x - W_L\, [x - (y - a)]$

$$M_x = \frac{W(L-y)}{L}\, x - W_L\, (x - y + a)$$

7. For maximum value of M_x, $\dfrac{dM_x}{dy} = 0$ (or should change sign)

$$\frac{dM_x}{dy} = -\frac{Wx}{L} + W_L = 0$$

$$\frac{W_L}{x} = \frac{W}{L} = \frac{W - W_L}{L - x} = \frac{W_R}{L - x}$$

$$\frac{W_L}{x} = \frac{W_R}{L - x}$$

8. The above expression shows that for maximum bending moment at section C, the average load on the portion to the left of C is equal to the average load on the portion to the right of C.

Que 4.18. **Three wheel loads 20 kN, 80 kN and 80 kN spaced 4 m apart from each other, with the 20 kN in the lead, pass over a simply supported beam of span 20 m. Determine the absolute maximum shear force and moment. Consider that loading can move in either direction.**

AKTU 2014-15, Marks 10

Answer

Given : $W_1 = 80\text{kN}$, $W_2 = 80\text{kN}$, $W_3 = 20\text{kN}$,
Span of beam, $L = 20$ m, Distance between points loads, $a = 4$ m
To Find : Absolute SF and BM.

1. **Maximum Positive Shear Force :**
i. **Ist Trial :** Place the load 80 kN at A as shown in Fig. 4.18.1.

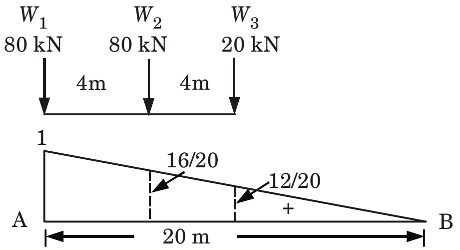


Fig. 4.18.1. ILD for maximum positive shear force.

$$\frac{W_1}{a} = \frac{80}{4} = 20$$

$$\frac{W_2 + W_3}{L} = \frac{80 + 20}{20} = 5$$

Since, $$\frac{W_1}{a} > \frac{W_2 + W_3}{L}$$

There is no need for any trial.

ii. Hence, Maximum positive shear force,

$$\text{SF} = 80 \times 1 + 80 \times \frac{16}{20} + 20 \times \frac{12}{20} = 156 \text{ kN}$$

2. **Maximum Negative Shear Force :**
i. **Ist Trial :** Place the loading as shown in Fig. 4.18.2.

$$\frac{W_3}{a} = \frac{20}{4} = 5$$

$$\frac{W_1 + W_2}{L} = \frac{80 + 80}{20} = 8$$

$$\frac{W_3}{a} < \frac{W_1 + W_2}{L}$$

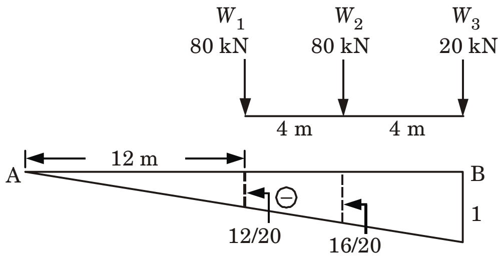


Fig. 4.18.2. ILD for maximum negative shear force.

Hence, we have to go IInd trial.

ii. **IInd trial :** Place the loading as shown in Fig. 4.18.3.

$$\frac{W_2}{a} = \frac{80}{4} = 20$$

$$\frac{W_1}{L} = \frac{80}{20} = 4$$

Since, $\frac{W_2}{a} > \frac{W_1}{L}$,

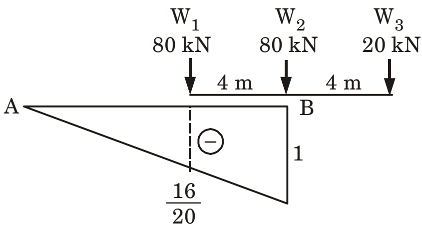


Fig. 4.18.3. ILD for maximum negative shear force.

Hence, maximum negative shear force,

$$\text{SF} = 80 \times \left(\frac{16}{20}\right) + 80 \times 1 = 144 \text{ kN}$$

3. Absolute Bending Moment :

i. To obtain absolute bending moment, firstly we have to find out the position of the resultant of given wheel loading.

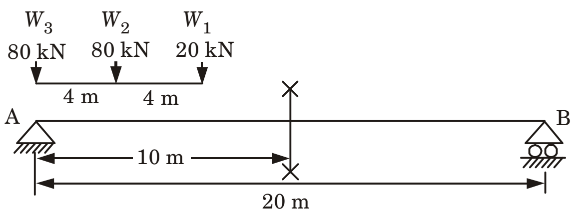


Fig. 4.18.4. Beam and the rolling loads.

ii. The absolute maximum bending moment will occur near the mid span and will occur under the W_2 (80 kN) load.

iii. Hence, for the condition of absolute maximum bending moment the load system should be so placed on the span that the resultant of all the wheel loads and the 80 kN load are equidistant from the middle point of the girder.

iv. Let us determine the position of the resultant of all the wheel loads with respect to W_1 (20 kN). Let the distance of the resultant load from the W_1 (20 kN) load be $\bar{x}$.

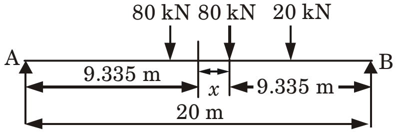
v. Taking moment about lasting 80 kN load, we have,

$$(20 + 80 + 80) \times \bar{x} = 20 \times 0 + 80 \times 4 + 80 \times 8$$

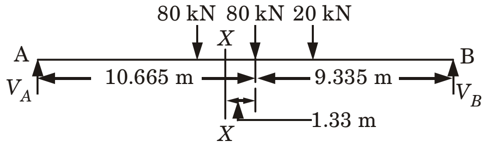
$$\bar{x} = 5.33 \text{ m}$$

vi. Distance between the resultant load and the load W_2 (80 kN) = 5.33 – 4 = 1.33 m

vii. Hence, for the condition of absolute moment, the load W_2 should be placed $\left(\frac{1.33}{2}\right)$ = 0.665 m on the right side of the centre of girder.



(a) Position of rolling load for maximum BM at section *X-X*.



(b) Position of loads for absolute maximum BM.

Fig. 4.18.5.

viii. Taking moment about the end A, we get,

$$V_B \times 20 = 180 \times 9.335$$

$$V_B = 84.015 \text{ kN}$$

and $$V_A = 180 - 84.015 = 95.985 \text{ kN}$$

$\therefore$ Absolute maximum bending moment for the girder = BM under the W_2 (80 kN) load

$$= 84.015 \times 9.335 - 20 \times 4 = 704.28 \text{ kN-m}$$

Que 4.19. **Four point loads 8, 15, 15 and 10 kN have centre to centre spacing of 2 m between consecutive loads and they traverse a girder of 30 m span from left to right with 10 kN load leading. Calculate the maximum bending moment and shear force at 8 m from the left support.**

AKTU 2015-16, Marks 15

Answer

Given : Span of beam, L = 30 m, Distance between loads = 2 m
Distance of section = 8 m (from A).
To Find : Maximum SF and BM.

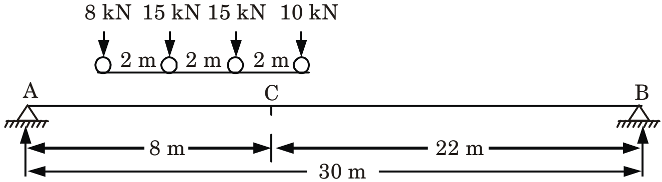


Fig. 4.19.1.

A. Maximum Bending Moment at *C* :

1. Let AB be the beam and C is the given section. Let us allow the loads to cross the given section one after another and find the average loads on AC and CB.
2. The calculation is shown in the following table.

Load Crossing the Section C	Average Load on AC	Average Load on BC	Remarks
10 kN	$\frac{38}{8}$	$\frac{10}{22}$	Average load on AC is greater than the average on CB
15 kN	$\frac{23}{8}$	$\frac{25}{22}$	Average load on AC is greater than the average on CB
15 kN	$\frac{8}{8}$	$\frac{40}{22}$	Average load on AC is less than the average on CB

3. Hence, for the maximum bending moment at C, we will place the 15 kN load exactly at C and other loads relative to this load.
4. ILD for the maximum bending moment at the section C as shown Fig. 4.19.2.

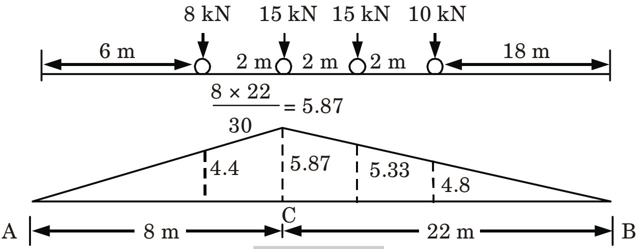


Fig. 4.19.2.

5. Maximum bending moment at C

$$= 8 \times 4.4 + 15 \times 5.87 + 15 \times 5.33 + 10 \times 4.8$$
$$= 251.2 \text{ kN-m}$$

B. Maximum Shear Force at Section :

1. In this case, $\dfrac{\Sigma W}{L} = \dfrac{8+15+15+10}{30} = \dfrac{48}{30} = 1.6$

Move LHS 8 kN from C and bring 15 kN over C, $\dfrac{8}{2} = 4 > 1.6$

Move LHS 15 kN from C and bring 15 kN over C, $\dfrac{15}{2} = 7.5 > 1.6$

2. Hence, condition $\dfrac{\text{Load rolled past the section}}{\text{Succeeding wheel space}}$

$< \dfrac{\Sigma W}{L}$ is not satisfied at any position of loads.

So 8 kN load should placed over C. For this condition, position of load and ILD shown in Fig. 4.19.3.

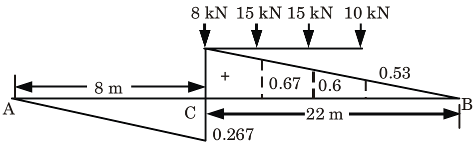


Fig. 4.19.3.

3. Maximum positive shear force at C

$$= 8 \times 0.733 + 15 \times 0.67 + 15 \times 0.6 + 10 \times 0.53$$
$$= 30.214 \text{ kN}$$

Que 4.20. A train of wheel loads, shown in Fig. 4.20.1 moves over a simply supported beam of 9 m span from left to right. Making use of the influence line diagram determine the value of maximum bending moment at a section 3 m from the left support.

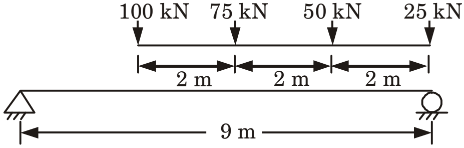


Fig. 4.20.1.

AKTU 2012-13, Marks 10

Answer

1. **Loading :**

Let AB be the beam and D is the given section.

Let us allow the loads to cross the given section one after another and find the average loads on AD and BD.

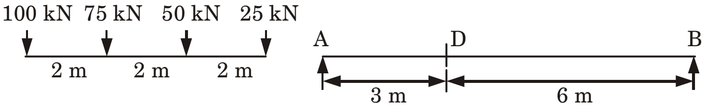


Fig. 4.20.2.

2. **Table :** Condition for maximum bending moment.

Load Crossing the Section *D*	Average Load on *AD*	Average Load on *BD*	Remark
25 kN	$\frac{225}{3}$	$\frac{25}{6}$	Average load on *AD* is greater then the average load on *BD*.
50 kN	$\frac{175}{3}$	$\frac{75}{6}$	Average load on *AD* is greater then the average load on *BD*.
75 kN	$\frac{100}{3}$	$\frac{150}{6}$	Average load on *AD* is greater then the average load on *BD*.
100 kN	$\frac{0}{3}$	$\frac{250}{6}$	Average load on *BD* is greater then the average load on *AD*.

3. Hence, for maximum BM at *D* we will place the 100 kN load exactly at *D* and the other loads relative to this load.
4. Maximum BM at $D = 100 \times 2 + 75 \times 1.33 + 50 \times 0.67 + 25 \times 0$

$$BM_{max} = 333.25 \text{ kN-m}$$

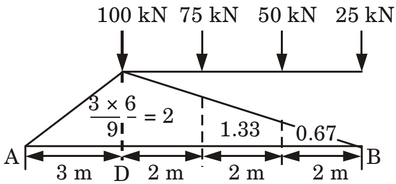


Fig. 4.20.3.

Que 4.21. Discuss Müller-Breslau's principles and its application in structural analysis.

Answer

Muller-Breslau developed a technique for rapidly constructing the shape of an influence line which is termed as Muller-Breslau's principle.

1. **Muller-Breslau's Principle :** According to this principle "the influence line for a function (reaction, shear, or moment) is to the

same scale as the deflected shape of the beam when the beam is acted upon by the function".

2. Applications in Structural Analysis :

i. Müller-Breslau's principle provides a quick method for establishing the shape of the influence line.

ii. After knowing the shape of the influence line, the ordinates at the peaks can be determined by the basic method.

iii. Using shape of influence line it is possible to locate the live load on the beam and then determine the maximum value of the function (shear or moment) by using statics.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Draw the schematic ILD for reaction, SF and BM at a section when a unit load moves over a simply supported beam.

Ans. Refer Q. 4.3.

Q. 2. A single load of 100 kN moves on a girder or span 20 m. Construct the influence line for shear force and bending moment for a section 5 m from the left support.

Ans. Refer Q. 4.8.

Q. 3. Uniformly distributed load of intensity 30 kN/m crosses a simply supported beam of span 60 m from left to right. The length of UDL is 15 m. Find the value of maximum bending moment for a section 20 m from left end. Find also the absolute value of maximum bending moment and shear force in the beam.

Ans. Refer Q. 4.14.

Q. 4. What are the propositions used for several point loads moving over a simply supported beam ? Explain and prove any one of them.

Ans. Refer Q. 4.17.

Q. 5. Discuss Müller-Breslau's principles and its application in structural analysis.

Ans. Refer Q. 4.21.

☺☺☺

5 UNIT

Analysis of Arches

PART-1

Arches, Types of Arches, Linear Arch, Eddy's Theorem.

CONCEPT OUTLINE : PART-1

Arch : It is a plane-curved beam either a bar or a rib-properly supported at its end. In this configuration the horizontal movement exists at the support.

Classification of Arches : Depending upon the number of hinges, arches may be divided into three classes :

1. Three hinged arch
2. Two hinged arch
3. Fixed arch (hingeless arch).

Linear Arch : Linear arch or theoretical arch or line of thrust is a funicular polygon structure of a two hinged or three hinged arch. It is assumed to be pin jointed at the point of action of the load.

Eddy's Theorem : The Eddy's theorem is stated as "The bending moment at any point on the arch axis is proportional to the vertical intercept between the theoretical arch and the axis of the actual arch".

Questions-Answers

Long Answer Type and Medium Answer Type Questions

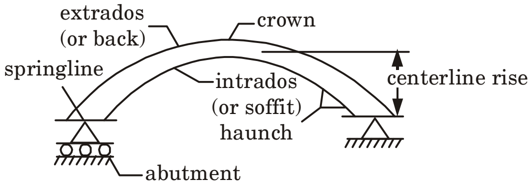
Que 5.1. **Define arch. Give the classification of arches.**

Answer

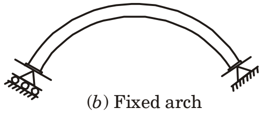
A. Arches :

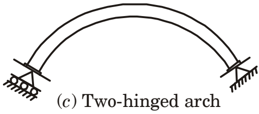
1. Arches are used to reduce the bending moments in long span structures.
2. An arch acts as an inverted cable, so it receives its load mainly in compression.
3. As arches are rigid hence they must also resist some bending and shear depending upon how it is loaded and shaped.
4. To describe an arch following nomenclature are used :

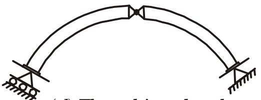
i. Abutment.
ii. Spring line.
iii. Extrados (or back).
iv. Intrados (or soffit).
v. Crown.
vi. Centerline rise.



(*a*) Arch


(*b*) Fixed arch


(*c*) Two-hinged arch


(*d*) Three-hinged arch

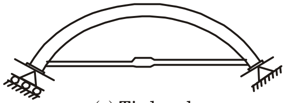

(*e*) Tied arch

Fig. 5.1.1

5. Some common type of arches are as follows :

i. Funicular Arch : If the arch of parabolic shape and subjected to a uniformly horizontally distributed vertical load then only compressive forces will be resisted by the arch. This type of arch shape is called as funicular arch because no bending or shear forces occur within the arch.

ii. Fixed Arch :

a. A fixed arch must have solid foundation abutments since it is indeterminate to the third degree, and consequently, additional stresses can be introduced into the arch due to relative settlement of its supports.

b. Fixed arch is often made from reinforced concrete and requires less material to construct than other types of arches.

iii. Two Hinged Arch :

a. It is indeterminate to the first degree. This structure can be made statically determinate by replacing one of the hinges with a roller.

b. Due to roller, capacity of the structure to resist bending along its span has been removed and it would serve as a curved beam not as an arch.

c. It is commonly made from metal or timber.

iv. Three Hinged Arch : It is statically determinate and made of metal or timber. It is not affected by settlement or temperature changes.

v. Tied Arch :

a. Tied arch allows the structure to behave as a rigid unit. Tied rod carries the horizontal component of thrust at the supports.

b. Tied arch is also unaffected by relative settlement of the supports.

B. Classification of Arches :

1. Based on the Material of Construction :

i. Steel arches.
ii. Reinforced arches.
iii. Concrete arches.
iv. Timber arches.
v. Brick arches.
vi. Stone arches.

Brick and stone arches are combinedly known as masonry arches.

2. Based on Support Conditions and Structural Behaviour :

i. Three hinged arches – Hinged at crown and abutments.
ii. Two hinged arches – Hinged at abutment only.
iii. Hingeless or fixed arches – No hinges at all.

3. Based on Shape and Structural Arrangement of the Rib :

i. Solid rib arch (also known as closed arch).
ii. Tied solid rib arch.
iii. Spandrel braced arch.
iv. Two hinged braced rib arch or crescent arch or sickle arch.
v. Two hinged braced rib arch.

Que 5.2. **What do you mean by linear arch ? Also state and prove the Eddy's theorem.**

AKTU 2012-13, Marks 10

OR

What do you understand by theoretical arch ? Also write down and prove Eddy's theorem.

AKTU 2013-14, Marks 10

Answer

A. Linear Arch (Theoretical Arch or the Line of Thrust) :

1. Consider an arch (two hinged or three hinged) subjected to a load system as point loads W_1, W_2 and W_3.
2. Let V_A and V_B be the vertical reactions at the supports A and B. Let H be the horizontal thrust.
3. Let pq, qr and rs represent the loads W_1, W_2, W_3. Let m be a point such that mp represents the vertical reaction V_A at A and sm represents the vertical reaction V_B at B. Let mo represent the horizontal thrust. Join op, oq, or and os.
4. Taking O as the pole we draw a funicular polygon $ACDEB$. In this diagram AC is parallel to op, CD is parallel to oq, DE is parallel to or and EB is parallel to os.
5. Now consider a structure $ACDEB$ consisting of members AC, CD, DE and EB which are all pin jointed and subjected to the loads W_1, W_2 and W_3 at C, D and E.

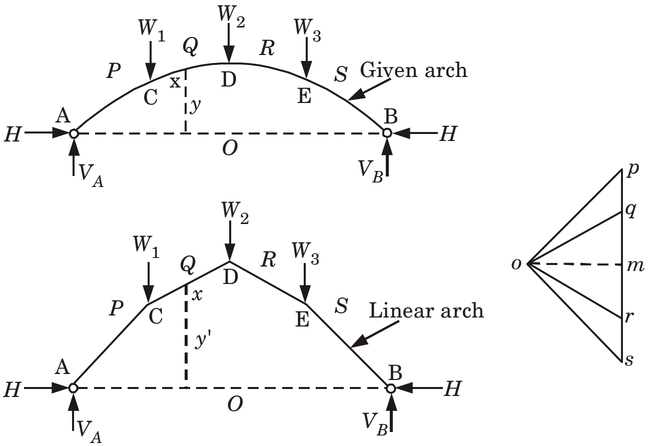


Fig. 5.2.1.

6. This structure *ACDEB* is called the linear arch or the theoretical arch.

   Definition of Linear Arch : Linear arch or theoretical arch or line of thrust is a funicular polygon structure of a two hinged or three hinged arch. It is assumed to be pin jointed at the point of action of the load. The different members of the linear arch are subjected to axial compressive forces and the joints of this linear arch are in equilibrium.

B. Eddy's Theorem :

Statements : The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch and the centre line of the actual arch.

Proof :

1. It can be easily realized that the shape of the linear arch follows the shape of the free bending moment diagram for a beam of the same span and subjected to the same loading.
2. Free BM at any section X = Polar distance $om \times y' = Hy'$

   Moment due to horizontal thrust = Hy

   where y and y' are the ordinates of the given and linear arches respectively.

   Net BM at the section $X = Hy - Hy' = H(y - y')$

   Net BM at section X is proportional to $(y - y')$.
3. Hence the BM at any section of an arch is proportional to the ordinate or the intercept between the given arch and the linear arch. This principle is called Eddy's theorem.
4. Fig. 5.2.2 shows both the given arch and the linear arch corresponding to the given load system.
5. The actual BM at the section X is proportional to the ordinate X_1X.

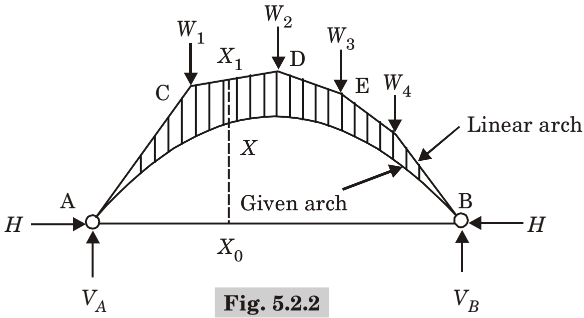
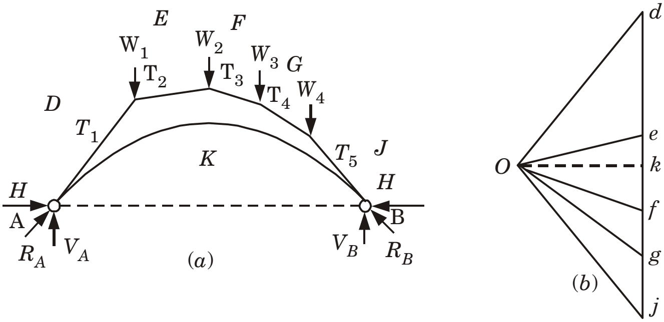


Fig. 5.2.2

Que 5.3. **Discuss what types of straining actions are sustained by an arch ?**

Answer

1. Consider a cross-section PQ of the arch Fig. 5.3.1(c). Let T be the resultant thrust acting through D along the linear arch.
2. Thrust T is neither normal to the cross-section nor does it act through the centre C of the cross-section.



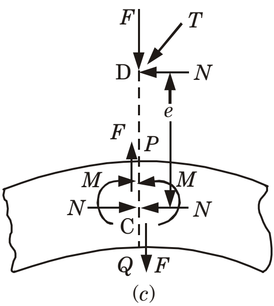


Fig. 5.3.1

3. Resultant thrust T can be resolved normal and tangential to the section PQ.
4. Let N be the normal component and F be the tangential component.
5. The tangential component F will cause shear force at the section PQ and normal component N acts eccentrically, the eccentricity being equal to CD.
6. Thus the action of N acting at D is two fold.
   i. A normal thrust N at C
   ii. A bending moment, $M = N.\, e$ at C.
7. Hence unlike beams a section of arch is subjected to three straining actions.
   i. Shear force, F
   ii. Bending moment, M
   iii. Normal thrust, N

   Sometimes shear force F is also known as radial shear.

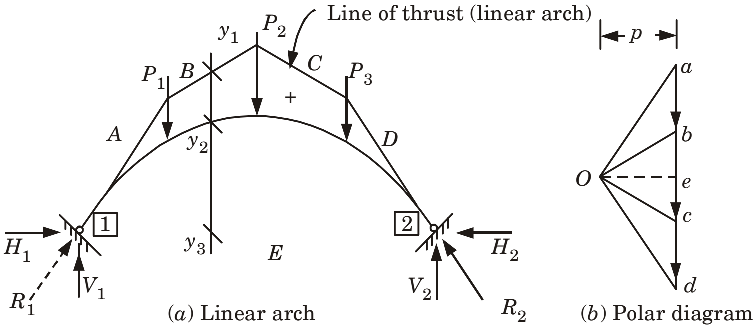
Que 5.4. **Define pressure line and explain its significance in the analysis of arches. State and prove Eddy's theorem.**

Answer

Pressure Line :

1. Fig. 5.4.1(a) indicates the funicular polygon (or force polygon) corresponding to the polar diagram of Fig. 5.4.1(b).
2. This funicular polygon indicates a system of hinged bars supported at 1 and 2.
3. The profile so formed is in equilibrium under the action of the applied loads (P_1, P_2 and P_3).
4. In cases the structure has the profile of the force polygon, the bending moment at any section will be zero.
5. The structure in such a case will be subjected only to axial compression.
6. Such a profile along the length of a beam or frame is known as pressure line or line of thrust.
7. The pressure line thus represents the profile for a given system of forces, which would induce only compressive forces.
8. The profile of a pressure line resembles an arch with linear segments, the profile is sometimes known as a linear arch.
9. It is not possible to maintain such a profile for all combinations of loads on a structure.
10. The pressure line varies from one loading case to another.
11. Thus in general a structural profile is subjected to bending moment under the action of a given loading system, the bending moment in a

structure for a given loading can be computed by making use of pressure lines.



(*a*) Linear arch (*b*) Polar diagram

Fig. 5.4.1

Eddy's Theorem : Refer Q. 5.2, Page 5–4C, Unit-5.

PART-2

Analysis of Three Hinged Parabolic Arch.

CONCEPT OUTLINE : PART-2

Three Hinged Arch : It is a structural assembly comprising two rigid reactions that are connected to each other and the abutments by pins or hinged, *i.e.*, besides hinges at the springings there is a hinge at the crown too.

Analysis of Arch : For any plane structure there are three independent equation of equilibrium which can be used conveniently.

$$\Sigma F_x = 0,$$

$$\Sigma F_y = 0,$$

$$M_A \text{ or } M_B = 0$$

For three hinge arch, the forth equation is also available

$$M_C = 0 \qquad [\because C \text{ is a hinge}]$$

Normal thrust at any section is given by,

$$N = V \sin \theta + H \cos \theta$$

Radial shear at any section is given by, $Q = H \sin \theta - V \cos \theta$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.5. **What do you understand by a three hinged arch ? Discuss its analysis.**

Answer

Three Hinged Arch :

1. A three hinged arch has a hinge at each abutment or springing and also at the crown.
2. It is statically determinate structure. There are four reaction components at each hinges as H and V.
3. To solve a three hinged arch, we have
   i. Three equilibrium equation from the static equilibrium condition.
   ii. One equation from the fact that the bending moment at the hinge and at the crown is zero.

Analysis of a Three Hinged Arch :

1. Let the three hinged arch shown in the Fig. 5.5.1 is subjected to a number of loads W_1, W_2, W_3, ...etc.

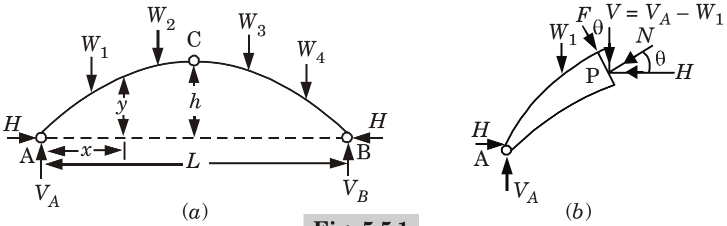


Fig. 5.5.1

2. Let the reactions at A and B be (H, V_A) and (H, V_B) respectively.

   The bending moment at crown (*i.e.*, at C) is zero, hence

$$M_c = M'_c - Hy = 0$$

$$H = \frac{M'_c}{y}$$

   V_A and V_B can be calculated by taking moments of all forces about B and A respectively.

3. At any section,

   The value of radial shear, $Q = V \cos\theta - H \sin\theta$

   The value of normal thrust, $N = V \sin\theta + H \cos\theta$

   where, $V = V_A - W_1$

B. Types of three hinged arch :

1. Three hinged parabolic arch
2. Three hinged circular arch

1. Analysis for Three Hinged Parabolic Arch :

i. From Fig. 5.5.1, considering the origin at the left hand hinge A, the equation of parabola can be considered as

$$y = kx\,(L - x) \qquad ...(5.5.1)$$

where k = Constant

ii. At $x = \dfrac{L}{2}$ let $y = h$ = Central rise

Substituting in equation (5.5.1) we get $h = k \times \dfrac{L}{2}\left(L - \dfrac{L}{2}\right) = k \times \dfrac{L^2}{4}$

$$k = \frac{4h}{L^2}$$

So $$y = \frac{4h}{L^2}\,x\,(L - x)$$

This is the equation of parabolic arch.

2. Analysis of Three Hinged Circular Arch :

i. Let us consider that the centre line of the arch to be segment of a circle of radius R, subtending an angle of 2θ at the centre.

ii. For our convenience, we take origin at D, the middle point of the span. Let at point $P\,(x, y)$ draw PC_1 parallel to AB.

iii. Then $$OP^2 = OC_1^{\,2} + PC_1^{\,2}$$

$$R^2 = \{y + (R - h)\}^2 + x^2 \qquad ...(5.5.2)$$

iv. Eq. (5.5.1), shows relation between y and x.

Also we have $h \times (2R - r) = \dfrac{L}{2} \times \dfrac{L}{2} = \dfrac{L^2}{4}$ ...(5.5.3)

v. From eq. (5.5.2) the radius R can be calculated in terms of the values of the span and the rise.

vi. The co-ordinates of $P\,(x, y)$ can also be expressed as

$$x = OP \sin\beta = R \sin\beta$$

$$y = C_1D = OC_1 - OD = R\cos\beta - R\cos\theta$$

$$y = R\,(\cos\beta - \cos\theta)$$

Que 5.6. A three hinged parabolic arch of span 50 m and a rise of 15 m is subjected to a point load of 50 kN at 12.5 m from the left support. Calculate the reactions and draw bending moment diagram. Also calculate the normal thrust and radial shear at 12.5 m from left support.

AKTU 2012-13, Marks 10

Answer

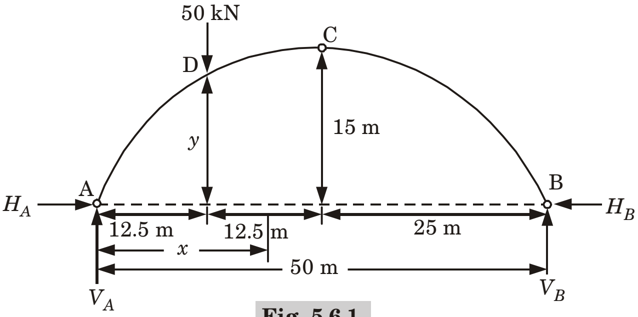


Fig. 5.6.1.

Given : Span of arch, L = 50 m, Rise of arch, h = 15 m
Point load, W = 50 kN
To Find : Support reactions, BMD, Normal thrust and radial shear.

1. **Reactions at Supports :**

i.
$$\Sigma F_y = 0$$
$$V_A + V_B = 50 \text{ kN} \quad ...(5.6.1)$$

ii. Taking moment about A,
$$\Sigma M_A = 0$$
$$V_B \times 50 = 50 \times 12.5$$
$$V_B = 12.5 \text{ kN}$$
From eq. (5.6.1), $V_A = 50 - V_B = 50 - 12.5 = 37.5$ kN

iii. Taking moment about C,
$$\Sigma M_C = 0$$
$$V_B \times 25 = H_B \times 15$$
$$12.5 \times 25 = H_B \times 15$$
$$H_B = H_A = 20.83 \text{ kN}$$

2. **Bending Moment under the Load :**

i. Equation of parabolic arch,
$$y = \frac{4hx}{L^2}(L - x)$$

ii. y at 12.5 m from left support,
$$y = \frac{4 \times 15 \times 12.5}{50^2} \times (50 - 12.5) = 11.25 \text{ m}$$

iii. Bending moment under the load,
$$M_{12.5} = V_A \times 12.5 - H_A \times y$$
$$= 37.5 \times 12.5 - 20.83 \times 11.25$$
$$= 234.41 \text{ kN-m}$$

iv. Bending moment at 12.5 m from right support.
$$M = V_B \times 12.5 - H_B \times y$$
$$= 12.5 \times 12.5 - 20.83 \times 11.25$$
$$M = -78.0875 \text{ kN-m}$$

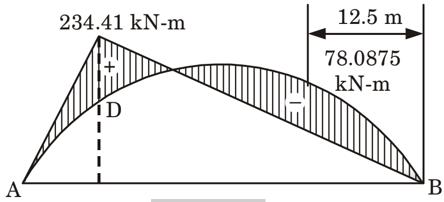


Fig. 5.6.2.

5. Normal Thrust and Radial Shear at Section *D* :

i. Let V_D and H_D be the reacting forces at D.

$$V_D = 37.5 \text{ kN}$$

and $$H_D = H_A = 20.83 \text{ kN}$$

ii. Let θ be the inclination of the tangent at D with the horizontal

$$y = \frac{4hx}{L^2}(L - x)$$

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x)$$

$$\tan\theta = \frac{4 \times 15}{50^2}(50 - 2 \times 12.5) = \frac{3}{5}$$

$$\theta = 0.6, \sin\theta = \frac{3}{\sqrt{34}}$$

$$\cos\theta = \frac{5}{\sqrt{34}}$$

iii. Normal thrust at D,

$$N = H_D \cos\theta + V_D \sin\theta$$

$$= 20.83 \times \frac{5}{\sqrt{34}} + 37.5 \times \frac{3}{\sqrt{34}}$$

Normal thrust, N $= 37.15$ kN

iv. Radial shear at D,

$$S_D = H_D \sin\theta - V_D \cos\theta$$

$$= 20.83 \times \frac{3}{\sqrt{34}} - 37.5 \times \frac{5}{\sqrt{34}}$$

Radial shear, $S_D = -21.44$ kN

Que 5.7. **Show that the parabolic shape is a funicular shape for a three hinged arch subjected to a uniformly distributed load over its entire span.** **AKTU 2015-16, Marks 10**

Answer

1. Let
L = Span of the arch,
h = Central rise, and
w = UDL applied on the arch.

2. From symmetry we have,

$$V_A = V_B = \frac{wL}{2}$$

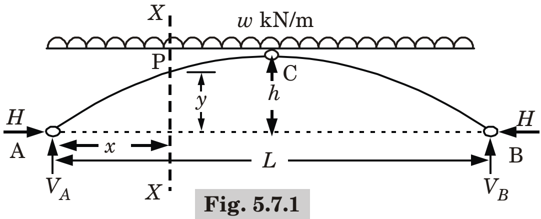


Fig. 5.7.1

3. For horizontal thrust, taking moment about C,

$$H \times h = \frac{wL}{2} \times \frac{L}{2} - \frac{wL}{2} \times \frac{L}{4} = \frac{wL^2}{8}$$

$$H = \frac{wL^2}{8h}$$

4. Let us now consider any section at distance x from A.

Equation of parabola is given by, $y = \dfrac{4h}{L^2}\, x\,(L - x)$

5. The value of bending moment at any section of the arch,

$$M_x = -Hy + V_A x - \frac{wx^2}{2}$$

$$= -\frac{wL^2}{8h} \times \frac{4h}{L^2}\, x\,(L - x) + \frac{wL}{2}x - \frac{wx^2}{2}$$

$$M_x = -\frac{wLx}{2} + \frac{wx^2}{2} + \frac{wLx}{2} - \frac{wx^2}{2} = 0$$

6. Hence a parabolic arch subjected to a UDL over its entire span has the bending moment at any section zero. That is why the parabolic shape for three hinged arch is a funicular shape.

Que 5.8. **A three hinged parabolic arch of 50 m span and a rise of 10 m is subjected to a uniformly distributed load of 20 kN/m intensity over its left half portion and point load of 100 kN at right quarter span. Calculate the bending moment, normal thrust and radial shear at a section 10 m from the left support.**

AKTU 2013-14, Marks 10

Answer

Given : Span of arch, L = 50 m, Rise of arch, h = 10 m
Intensity of UDL, w = 20 kN/m, Point load, W = 100 kN
To Find : BM, Normal thrust and Radial shear.

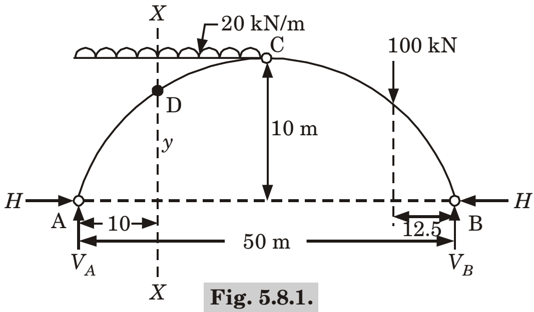


Fig. 5.8.1.

1. **Reactions at supports :**

i.
$$\Sigma F_y = 0$$
$$V_A + V_B = 20 \times 25 + 100 = 600 \text{ kN}$$

ii. Taking moment about support A,

$$V_B \times 50 = 20 \times 25 \times \frac{25}{2} + 100 \times 37.5$$
$$V_B \times 50 = 10000$$
$$V_B = 200 \text{ kN}$$
$$V_A = 600 - 200 = 400 \text{ kN}$$

iii. Taking moment about C,

$$V_A \times 25 = H \times 10 + 20 \times 25 \times \frac{25}{2}$$
$$400 \times 25 = H \times 10 + \frac{20 \times 25 \times 25}{2}$$
$$H = 375 \text{ kN}$$

2. **Bending Moment at Section *D* :**

i. Let a section X-X at a distance 10 m from A.
Rise of arch at section D,

$$\therefore \qquad y = \frac{4h}{L^2} x(L - x) = \frac{4 \times 10}{(50)^2} \times 10 \times (50 - 10)$$
$$y = 6.4 \text{ m}$$

ii. Bending moment at D,

$$\text{BM}_D = V_A \times 10 - 20 \times 10 \times 5 - H \times 6.4$$
$$= 400 \times 10 - 20 \times 10 \times 5 - 375 \times 6.4 = 600 \text{ kN-m}$$

3. **Normal Thrust and Radial Shear at Section *D* :**

i. Consider the equilibrium of the part AD of arch.
Let V_D and H_D be the reaction force at D,
and
$$H_D = 375 \text{ kN}$$
$$V_D = 400 - 20 \times 10$$
$$V_D = 200 \text{ kN}$$

ii. Let θ be the inclination of the tangent at D with the horizontal.

$$\tan \theta = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x)$$

At D, $\tan\theta = \dfrac{4\times6.4}{50\times50}(50-2\times10)$

$\theta = 17.07°$

iii. Normal thrust at D,

$$N = H_D\cos\theta + V_D\sin\theta$$
$$N = 375\times\cos 17.07° + 200\sin 17.07° = 417.18 \text{ kN}$$

iv. Radial shear at D,

$$S = H_D\sin\theta - V_D\cos\theta$$
$$S = 375\times\sin 17.07° - 200\times\cos 17.07°$$
$$= -81.11 \text{ kN}$$

Que 5.9. **A three hinged parabolic arch of span 40 m and a rise of 10 m is subjected to a triangular loading of intensity varying from 25 kN/m at supports to zero at the crown. Draw the bending moment diagram. Also determine the maximum value of the bending moment.**

AKTU 2013-14, Marks 10

Answer

Given : Span of arch, L = 40 m, Rise of arch, h = 10 m
Intensity of UVL at support = 25 kN/m
To Find : BMD, BM_{max}

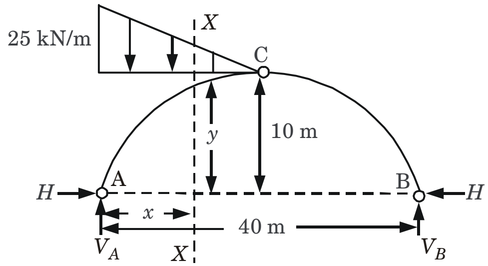


Fig. 5.9.1.

2. **Reactions at Supports :**

i. $\Sigma F_y = 0$, $V_A + V_B = \dfrac{1}{2}\times25\times20 = 250$ kN ...(5.9.1)

ii. Taking moment about A,

$$V_B\times40 = \left(\frac{1}{2}\times25\times20\right)\times\frac{20}{3}$$
$$V_B = 41.67 \text{ kN}$$

From eq. (5.9.1), V_A = 208.33 kN

iii. Taking moment about C,

$$V_B\times20 - H\times10 = 0$$

$$41.67 \times 20 = H \times 10$$
$$H = 83.34 \text{ kN}$$

3. Maximum Positive Bending Moment :

i. Let us assume a section X-X at a distance x from support A. Equation of parabolic arch as given by,

$$y = \frac{4hx(L-x)}{L^2}$$

ii. Considering the equilibrium of the part AC.

$$H_C = 83.32 \text{ kN}$$

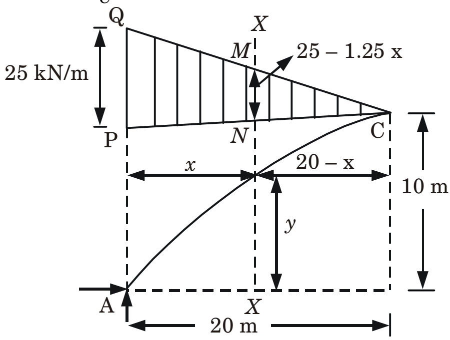


Fig. 5.9.2.

iii. From $\Delta\, PQC$ and $\Delta\, MNC$,

$$\frac{25}{20} = \frac{MN}{(20-x)}$$
$$MN = 25 - 1.25x$$

iv. Centroid of trapezoidal $MNPQ$ from side MN,

$$\left(\frac{MN + 2PQ}{PQ + MN}\right) \times \frac{x}{3} = \left(\frac{25 - 1.25x + 2 \times 25}{25 + 25 - 1.25x}\right) \times \frac{x}{3}$$

v. Bending moment at section X-X,

$$M_x = V_A x - Hy - \frac{1}{2}(25 + 25 - 1.25x) \times x \times \frac{(25 - 1.25x + 50)x}{3(25 + 25 - 1.25x)}$$

$$= 208.33x - 83.32 \times \frac{4 \times 10x(40-x)}{40^2} - \frac{75x^2 - 1.25x^3}{6}$$

vi. For BM to be maximum,

$$\frac{dM_x}{dx} = 0$$

$$208.33 - (40 - 2x) \times 2.083 - 25x + 0.625x^2 = 0$$
$$0.625x^2 - 20.834x + 125 = 0$$
$$x = 7.85 \text{ m}$$

vii. Maximum bending moment,

$$M_{max} = 208.33 \times 7.85 - 2.083 \times 7.85 \times (40 - 7.85) - 12.5 \times (7.85)^2 + 0.208\,(7.85)^3$$

$$= 440.2 \text{ kN-m}$$

4. **Maximum Negative Bending Moment :**

i. Negative bending moment will be occur in BC span.

$$y = \frac{4hx}{L^2}(L-x)$$

$$y = \frac{4 \times 10x}{40^2}(40-x)$$

$$y = \left(x - \frac{x^2}{40}\right)$$

ii. Bending moment at any section,

$$M_x = -H \times y + V_B \times x$$

$$M_x = 83.32 \times \left(x - \frac{x^2}{40}\right) + 41.67x$$

iii. For maximum BM, $\dfrac{dM}{dx} = 0$

$$-83.32\left(1 - \frac{2x}{40}\right) + 41.67 = 0$$

$$x = 10 \text{ m}$$

iv. Maximum negative bending moment,

$$= -83.32 \times \left(10 - \frac{10^2}{40}\right) + 41.67 \times 10$$

$$M_{max} = -208.2 \text{ kN-m}$$

Que 5.10. A three hinged parabolic arch of span 50 m and rise 9 m carries a load whose intensity varies 25 kN/m at the crown to 50 kN/m at the ends. Find the following at a section *D*, 10 m from left end.

1. **Bending moment.**
2. **Normal thrust.**
3. **Radial shear.**

AKTU 2014-15, Marks 10

Answer

Given : Span of arch, $L = 50$ m, Rise of arch, $h = 9$ m,
Intensity of load at crown = 25 kN/m
Intensity of load at support A and B = 50 kN/m
To Find : BM, normal thrust, radial shear.

1. $$\Sigma F_y = 0$$

$$V_A + V_B = 25 \times 50 + \frac{1}{2} \times 25 \times 25 \times 2$$

$$V_A + V_B = 1875 \text{ kN}$$

2. Taking moment about point A, $\Sigma M_A = 0$

$$-V_B \times 50 + 25 \times 50 \times 25 + \frac{1}{2} \times (25 \times 25)$$
$$\times \left(25 + \frac{2}{3} \times 25\right) + \frac{1}{2} \times 25 \times 25 \times \frac{25}{3} = 0$$

$$V_B = 937.5 \text{ kN}$$
$$V_A = 937.5 \text{ kN}$$

3. Taking moment about the crown 'C' (from left side),

$$H \times 9 + \left(25 \times 25 \times \frac{25}{2}\right) + \left(\frac{1}{2} \times 25 \times 25\right) \times \left(\frac{2}{3} \times 25\right) = 937.5 \times 25$$
$$H = 1157.4 \text{ kN}$$

4. Vertical height of a arch at a distance 10 m from support A,

$$y = \frac{4h}{L^2} x\,(L - x) = \frac{4 \times 9 \times 10}{50^2} \times (50 - 10) = 5.76 \text{ m}$$

5. Intensity of load at a distance 10 m from support $A = 15 \times \dfrac{25}{25} = 15$ kN

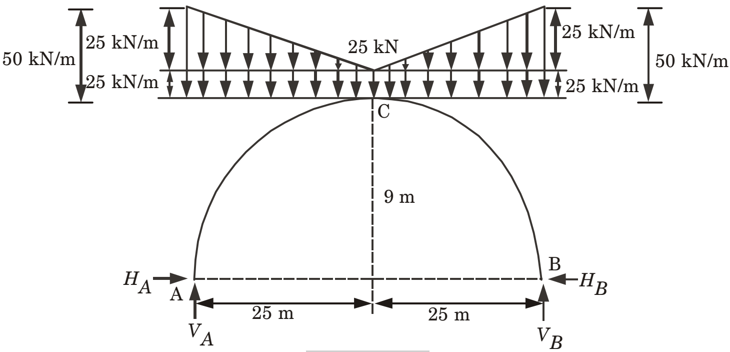


Fig. 5.10.1.

6. Bending moment at a distance 10 m from the left support,

$$M_{10} = V_A \times 10 - 40 \times 10 \times \frac{10}{2} - \frac{1}{2} \times 10 \times 10 \times \frac{2}{3} \times 10 - H \times 5.76$$

$$M_{10} = 937.5 \times 10 - 40 \times 10 \times \frac{10}{2} - \frac{1}{2} \times 10 \times 10 \times \frac{2}{3} \times 10 - 1157.4 \times 5.76$$

$$M_{10} = 375 \text{ kN-m}$$

7. Slope of arch at a section is given by,

$$\tan\theta = \frac{dy}{dx} = \frac{4h\,(L - 2x)}{L^2} = \frac{4 \times 9 \times (50 - 2 \times 10)}{50^2} = 0.432$$
$$\tan\theta = 0.432$$
$$\theta = 23.36°$$

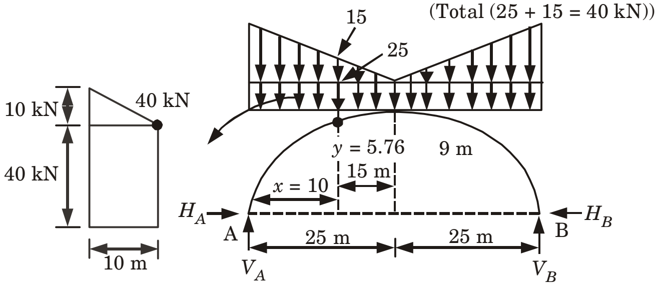


Fig. 5.10.2.

8. Vertical shear at (x =10 m) section,

Total vertical load at section $= 40 \times 10 + \frac{1}{2} \times 10 \times 10$

$$= 400 + 50 = 450 \text{ kN}$$

$$V_D = V_A - \text{Downward Load}$$

$$V_D = 937.5 - 450$$

$$V_D = 487.5 \text{ kN}$$

9. Normal thrust, $N = H \cos\theta + V_D \sin\theta$

$$N = 1157.4 \times \cos 23.36° + 487.5 \times \sin 23.36°$$

$$N = 1255.83 \text{ kN}$$

10. Radial Shear, $S = H \sin\theta - V_D \cos\theta$

$$= 1157.4 \times \sin 23.36 - 487.5 \times \cos 23.36°$$

$$S = 11.38 \text{ kN}$$

Que 5.11. **A circular arch to span 25 m with a central rise 5 m is hinged at the crown and springing. It carries a point load of 100 kN at 6 m from the left support. Calculate :**

i. The reactions at the supports.

ii. The reactions at crown.

AKTU 2015-16, Marks 10

Answer

Given : Span of arch, L = 25 m, Central rise, h = 5 m, Load = 100 kN

To Find : V_A, V_B and V_C

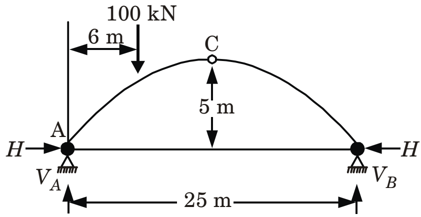


Fig. 5.11.1.

1. $\Sigma F_y = 0$, $V_A + V_B = 100$ ...(5.11.1)
2. Taking moment about A, $\Sigma M_A = 0$

$$V_B \times 25 = 100 \times 6$$
$$V_B = 24 \text{ kN}$$

From eq. (5.11.1),

$$V_A = 100 - 24 = 76 \text{ kN}$$

3. Considering right part (CB part) of arch,
Taking moment about point C.
$\Sigma M_C = 0$, $H \times 5 = V_B \times 12.5$

$$H = \frac{24 \times 12.5}{5} = 60 \text{ kN}$$

4. Shear force at crown, $V_C = 100 - 76 = 24$ kN
5. Angle between plane of section and vertical plane passing through the crown of circular arch,

$$\theta = 0°$$

7. Normal thrust at crown,

$$N = H \cos \theta + V_C \sin \theta$$
$$N = 60 \cos 0° + 24 \sin 0° = 60 \text{ kN}$$

8. Radial shear at crown, $S = H \sin \theta - V_C \cos \theta$

$$S = 60 \times \sin 0° - 24 \times \sin 0° = -24 \text{ kN}$$

Que 5.12. A three hinged semicircular arch of radius *R* carries a UDL of *w* per run over the whole span. Find

A. Horizontal thrust.

B. Location and magnitude of maximum bending moment.

AKTU 2014-15, Marks 10

Answer

A. Horizontal Thrust :

1. Let the horizontal thrust at each support be H.
2. Due to symmetry, each vertical reaction = wR.
3. We know that for a three hinged arch of span L and rise h carries a UDL of w per unit run over the whole span, the horizontal thrust at each support is $\dfrac{wL^2}{8h}$.

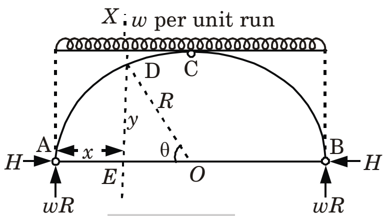


Fig. 5.12.1.

Horizontal thrust,

$$H = \frac{wL^2}{8h} = \frac{w(2R)^2}{8R} = \frac{wR}{2}$$

B. Location and Magnitude of Maximum Bending Moment :

1. The bending moment at any section X-X, the radius vector corresponding to which makes an angle θ with the horizontal is given by,

$$M_x = wRx - \frac{wx^2}{2} - Hy \qquad ...(5.12.1)$$

2. From Δ ODE, $x = R\,(1 - \cos\theta)$,

$$y = R\sin\theta$$

3. Value of x and y put in eq. (5.12.1), we get

$$M_x = wR \times R\,(1 - \cos\theta) - \frac{wR^2(1-\cos\theta)^2}{2} - \frac{wR}{2} \times R\sin\theta$$

$$\therefore \qquad M_x = wR^2\,(1 - \cos\theta) - \frac{wR^2}{2}(1-\cos\theta)^2 - \frac{wR^2}{2}\sin\theta$$

$$= \frac{wR^2}{2}(1-\cos\theta)\,[2-(1-\cos\theta)] - \frac{wR^2}{2}\sin\theta$$

$$= \frac{wR^2}{2}(1-\cos\theta)\,(1+\cos\theta) - \frac{wR^2}{2}\sin\theta$$

$$= \frac{wR^2}{2}(1-\cos^2\theta) - \frac{wR^2}{2}\sin\theta$$

$$M_x = \frac{wR^2}{2}\sin^2\theta - \frac{wR^2}{2}\sin\theta = -\frac{wR^2}{2}[\sin\theta - \sin^2\theta]$$

Since sin θ being greater than $\sin^2\theta$ the bending moment at any section of the arch is a negative or a hogging moment.

4. For M_x to be a maximum,

$$\frac{dM_x}{d\theta} = 0$$

$$-\frac{wR^2}{2}[\cos\theta - 2\sin\theta\cos\theta] = 0$$

$$\cos\theta\,(1 - 2\sin\theta) = 0$$

Either, $\cos\theta = 0$ or $1 - 2\sin\theta = 0$

$$\theta = 90° \text{ or } \theta = 30°$$

But at θ = 90° *i.e.*, at the crown the bending moment equals to zero.

5. The bending moment is maximum at θ = 30°

$$M_{\max} = -\frac{wR^2}{2}(\sin 30° - \sin^2 30°) = -\frac{wR^2}{8}$$

6. Distance of the point of maximum bending moment from the centre of semicircle,

$$OE = R\cos 30° = \frac{R\sqrt{3}}{2}$$

7. Distance from left support,

$$x = R - OE = R(1 - \cos 30°) = \left(1 - \frac{\sqrt{3}}{2}\right) = \frac{R}{2}(2 - \sqrt{3})$$

Que 5.13. **A three hinged circular arch of span 150 m and a rise of 30 m is subjected to a uniformly distributed load of 25 kN/m over the left half span. Determine the maximum bending moment in the arch. Also, calculate the normal thrust and radial shear at left quarter span.**

AKTU 2012-13, Marks 10

Answer

Given : Span of arch, L = 150 m, Rise of arch, h = 30 m
Intensity of UDL, w = 25 kN/m
To Find : Maximum BM, Normal thrust and Radial shear

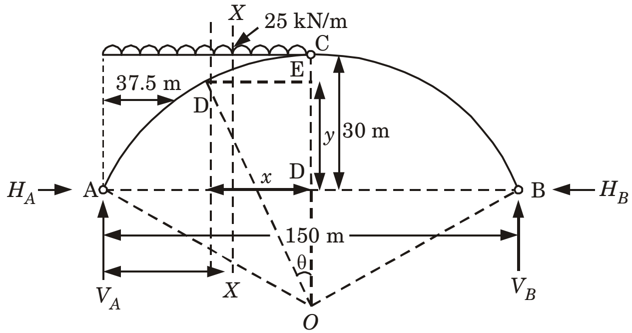


Fig. 5.13.1.

1. **Reactions at Supports :**

i. $$\Sigma F_y = 0$$

$$V_A + V_B = 25 \times \left(\frac{150}{2}\right) = 1875 \text{ kN} \quad ...(5.13.1)$$

ii. Taking moment about A, $\Sigma M_A = 0$

$$V_B \times 150 = 25 \times 75 \times \left(\frac{75}{2}\right)$$

$$V_B = 468.75 \text{ kN}$$

From eq. (5.13.1), $V_A = 1875 - 468.75 = 1406.25$ kN

iii. Taking moment about C,

$$V_B \times 75 - H \times 30 = 0$$

$$468.75 \times 75 = H \times 30$$

$$H = 1171.875 \text{ kN.}$$

2. **Maximum Bending Moment :**

i. Let radius of arch = R

$$(2R - 30) \times 30 = (75)^2$$

$$R = 108.75 \text{ m}$$

ii.
$$y = \sqrt{R^2 - x^2} - (R - 30)$$

$$y = \sqrt{(108.75^2) - x^2} - (108.75 - 30)$$

$$y = \sqrt{(108.75^2) - x^2} - 78.75$$

iii. Bending moment about X-X section,

$$M_x = V_A \times (75 - x) - H \times y - 25 \times (75 - x) \times \frac{(75 - x)}{2}$$

$$M_x = 1406.25 \times (75 - x) - 1171.875 \times [\sqrt{(108.75^2) - x^2} - 78.75] - 25 \times \frac{(75 - x)^2}{2}$$

iv. For maximum bending moment,

$$\frac{dM_x}{dx} = 0$$

$$\frac{dM_x}{dx} = -1406.25 - \frac{1171.875(-2x)}{2 \times \sqrt{(108.75)^2 - x^2}} - \frac{25}{2} \times 2\,(75 - x)(-1)$$

$$0 = -1406.25 + \frac{1171.875 \times x}{\sqrt{(108.75)^2 - x^2}} + 25\,(75 - x)$$

$$x = 34.36 \text{ m}$$

v. Maximum bending moment at a distance $(75 - 34.36) = 40.64$ m from left support.

Vertical distance of arch at section,

$$y_{(x = 34.36)} = \sqrt{(108.75)^2 - (34.36)^2} - 78.75$$

$$y_{(x = 34.36)} = 24.43 \text{ m}$$

vi. Maximum bending moment,

$$M_{max} = 1406.25 \times (75 - 34.36) - 1171.875 \times 24.43 - 25 \times \frac{(75 - 34.36)^2}{2}$$

$$M_{max} = 7875.97 \text{ kN-m}$$

3. Normal Thrust and Radial Shear at Left Quarter Span :

i.
$$\sin\theta = \left(\frac{37.5}{108.75}\right),\ \theta = 20°\ 11'$$

ii. Normal thrust at D, $N = H_D \cos\theta + V_D \sin\theta$

$$H_D = 1171.875 \text{ kN}$$

$$V_D = 1406.25 - (25 \times 37.5) = 468.75 \text{ kN}$$

$$N_D = 1171.875 \times \cos(20°\ 11') + 468.75 \times \sin(20°\ 11°)$$

Normal thrust,

$$N = 1261.65 \text{ kN}$$

iii. Radial shear at D,

$$S_D = H_D \sin\theta - V_D \cos\theta$$

$$= 1171.875 \times \sin(20°\ 11') - 468.75 \times \cos(20°\ 11')$$

$$S_D = -35.64 \text{ kN}$$

Que 5.14. **A three-hinged circular arch hinged at the springing and crown point has a span of 40 m and a central rise of 8 m. It carries a uniformly distributed load 20 kN/m over the left-half of the span together with a concentrated load of 100 kN at the right quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10 m from left support.**

AKTU 2015-16, Marks 7.5

Answer

Given : Span of arch, L = 40 m, Rise of arch, h = 8 m
Intensity of UDL, w = 20 kN/m, Point load, W = 100 kN
To Find : Reactions at the supports, Normal thrust and Radial shear.

1. $\Sigma F_y = 0$, $\quad V_A + V_B = (20 \times 20) + 100$

$$V_A + V_B = 500 \text{ kN} \quad ...(5.14.1)$$

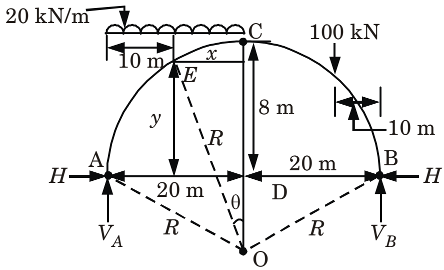


Fig. 5.14.1.

3. Taking moment about point A,

$$V_B \times 40 = 100 \times 30 + (20 \times 20) \times 10$$

From eq. (5.14.1), $\quad V_B = 175$ kN

$$V_A = 325 \text{ kN}$$

4. Taking moment about point C of the forces on the left side of C,

$$V_A \times 20 - w \times \frac{L}{2} \times \frac{L}{4} = H \times 8$$

$$325 \times 20 - 20 \times 20 \times 10 = H \times 8$$

$$H = 312.5 \text{ kN}$$

5. Let R be the radius of the arch,

$$8 \times (2R - 8) = 20 \times 20$$

$\therefore$ $\quad R = 29$ m

6. The equation of the circular arch with D as origin is,

$$y = \sqrt{R^2 - x^2} - (R - h)$$

$$y = \sqrt{29^2 - x^2} - (29 - 8)$$

$$y = \sqrt{841 - x^2} - 21$$

7. Put, $x = 10$ m

$$y = 6.221 \text{ m}$$

8. Moment at section, at a distance 10 m from A,

$$M_x = V_A \times x - Hy - w \times x \times \frac{x}{2}$$

$$M = 325 \times 10 - 312.5 \times 6.221 - 20 \times 10 \times \frac{10}{2}$$

$$M = 305.94 \text{ kN-m}$$

9. Slope at point E,

From Fig. 5.14.1, $\sin \theta = \dfrac{x}{R} = \dfrac{10}{29}$

At $x = 10$ m, $\theta = 20.17°$

10. Normal thrust at point E,

$$N_E = H \cos \theta + V_E \sin \theta$$

Shear force at point E,

$$V_E = 325 - 20 \times 10$$

$$V_E = 125 \text{ kN}$$

$$N_E = 312.5 \times \cos 20.17° + 125 \times \sin 20.17°$$

$$N_E = 336.44 \text{ kN}$$

11. Radial shear, $S_E = H \sin \theta - V_E \cos \theta$

$$= 312.5 \times \sin 20.17° - 125 \times \cos 20.17°$$

$$S_E = -9.6 \text{ kN}$$

Que 5.15. **A three hinged parabolic arch hinged at the supports and at the crown has a span of 24 m and a central rise of 4 m. It carries concentrated load of 50 kN at 18 m from the left support and UDL of 30 kN/m over the left portion. Determine the normal thrust, radial shear at a section 6 m from the left hand support.**

AKTU 2016-17, Marks 10

Answer

Given : Span of arch, $L = 24$ m, Central rise of arch, $h = 4$ m
Concentrated load, $W = 50$ kN, Intensity of UDL, $w = 30$ kN/m
Distance of section = 6 m
To Find : Normal thrust and Radial shear.

1. The arch is shown in Fig. 5.15.1. Taking moment about support B, we get

$$V_A \times 24 - 30 \times 12 \times 18 - 50 \times 6 = 0$$

$$V_A = 282.50 \text{ kN}$$

$$\Sigma F_y = 0$$

$$V_A + V_B = 30 \times 12 + 50$$

$$V_B = 30 \times 12 + 50 - 282.50 = 127.5 \text{ kN}$$

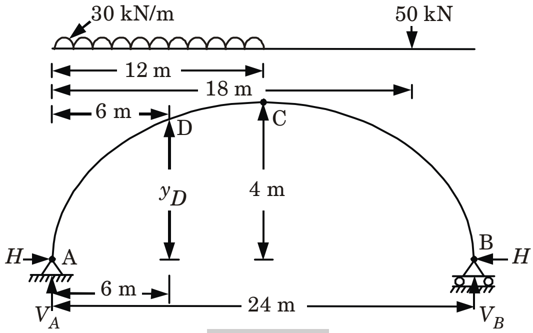


Fig. 5.15.1.

2. Taking moment about crown C (Right part),

$$V_B \times 12 - H \times 4 - 50 \times 6 = 0$$
$$127.5 \times 12 - H \times 4 - 50 \times 6 = 0$$

or $$H = 307.5 \text{ kN}$$

3. Vertical shear at point D, $V = V_A - 30 \times 6 = 282.5 - 30 \times 6 = 102.5$ kN

4. Equation of curve is given by, $y = \dfrac{4hx(L-x)}{L^2}$

$$\frac{dy}{dx} = \tan\theta = \frac{4h(L-2x)}{L^2}$$

Therefore, at $x = 6$ m,

$$\tan\theta = \frac{4 \times 4 \times (24 - 2 \times 6)}{24 \times 24}$$
$$\theta = 18.435°$$

5. Normal thrust, $N = V \sin\theta + H\cos\theta$

$= 102.5 \times \sin 18.435° + 307.5 \times \cos 18.435°$

$= 324.133$ kN

6. Radial shear, $Q = V\cos\theta - H\sin\theta$

$= 102.5 \times \cos 18.435° - 307.5 \times \sin 18.435°$

$= 0$

Que 5.16. A three hinged parabolic arch is shown in Fig. 5.16.1. Determine the normal thrust, radial shear and bending moment at quarter span and draw BMD.

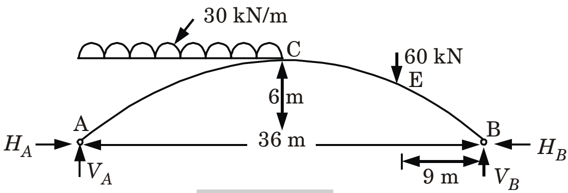


Fig. 5.16.1.

AKTU 2016-17, Marks 7.5

Answer

Given : Rise of parabolic arch, h = 6 m, Span of the arch, L = 36 m
Intensity of UDL, w = 30 kN/m, Concentrated load, W = 60 kN
To Find : Normal thrust, Radial shear and Bending moment at quarter span and Draw BMD.

1. $$\Sigma F_y = 0$$
$$V_A + V_B = 30 \times 18 + 60 = 600 \text{ kN} \quad ...(5.16.1)$$
2. Taking moment about support B, $\Sigma M_B = 0$
$$V_A \times 36 - 30 \times 18 \times 27 - 60 \times 9 = 0$$
$$V_A = 420 \text{ kN}$$
From eq. (5.16.1), $V_B = 600 - 420 = 180$ kN
3. Taking moment about hinge point C, $\Sigma M_C = 0$
$$V_A \times 18 - 30 \times 18 \times 9 = 6 \times H_A$$
$$420 \times 18 - 30 \times 18 \times 9 = 6 \times H_A$$
$$H_A = 450 \text{ kN}$$
4. Equation of parabolic arch is given by,
$$y = \frac{4hx}{L^2}(L - x)$$
Rise of arch at quarter span,
$$y = \frac{4 \times 6 \times 9}{36^2}(36 - 9) = 4.5 \text{ m}$$
5. Bending moment at quarter span,
$$M = V_A \times 9 - 30 \times 9 \times 4.5 - H_A \times y$$
$$= 420 \times 9 - 30 \times 9 \times 4.5 - 450 \times 4.5$$
$$M = 540 \text{ kN-m}$$
6. Vertical shear at quarter span,
$$V = V_A - 30 \times 9 = 420 - 30 \times 9 = 150 \text{ kN}$$
7. Slope $= \dfrac{dy}{dx} = \tan\theta = \dfrac{4h(L-2x)}{L^2} = \dfrac{4 \times 6 \times (36 - 2 \times 9)}{36^2} = 0.333$
$$\theta = 18.435°$$
8. Normal thrust at quarter span,
$$N = V \sin\theta + H \cos\theta$$
$$= 150 \times \sin 18.435° + 450 \times \cos 18.435°$$
$$N = 474.34 \text{ kN}$$
9. Radial shear at quarter span,
$$S = V\cos\theta - H\sin\theta$$
$$S = 150 \times \cos 18.435° - 450 \times \sin 18.435° = 0$$
10. Hy moment at quarter span,
$$Hy = 450 \times 4.5 = 2025 \text{ kN-m}$$

11. Beam moment at quarter span = $V_A \times 9 - 30 \times 9 \times 4.5$

$$= 420 \times 9 - 30 \times 9 \times 4.5$$

$$= 2565 \text{ kN-m}$$

12. Beam moment under concentrated load

$$= V_A \times 27 - 30 \times 18 \times 18$$

$$= 420 \times 27 - 30 \times 18 \times 18 = 1620 \text{ kN-m}$$

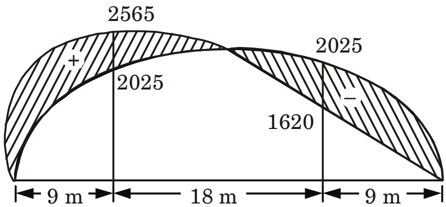


Fig. 5.16.2. BMD.

PART-3

Spandrel Arch, Moving Load and Influence Lines For Three Hinged Arch.

CONCEPT OUTLINE : PART-3

Spandrel Arch : In this type of arch the space above the arch rib and below the level of the crown is known as spandrel of the arch. This space can be closed with filling in which case the arch is called a closed spandrel arch or can be provided with truss like arrangement in which the arch is called an open spandrel arch.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.17. **With a neat sketch describe about spandrel braced arch.**

Answer

Spandrel Braced Arch :

1. A three hinged spandrel braced arch is shown in the Fig. 5.17.1.
2. In spandrel braced arch the space above the arch rib and below the level of the crown is known as spandrel of the arch.

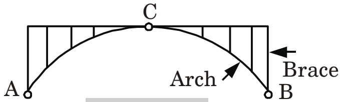


Fig. 5.17.1.

Types of Spandrel Braced Arch : The spandrel braced arch can be of following types :

1. **Closed Spandrel Arch :** In closed spandrel arch the space above the arch rib and below the level of the crown can be closed with filling.
2. **Open Spandrel Arch :** In open spandrel arch the space above the arch rib and below the level of the crown can be provided with truss-like arrangement.

Note : In the case of masonry arch bridges and short span concrete arch bridges, the spandrel is normally filled with earth.

Que 5.18. **Obtain the expression for maximum bending moment for a three hinged parabolic arch**

A. For a travelling point load.

B. For a uniformly distributed line load.

Answer

1. Consider a three-hinged parabolic arch of span L and rise h.
2. Consider any section X distant x from the left end. Fig. 5.18.1(b) shows the influence line diagram for the BM at the section X.
3. Since the arch is parabolic the ordinates XX_1 and CC_1 in the influence diagram are each equal to $\dfrac{x(L-x)}{L}$.

A. Travelling Point Load *W* (Maximum Positive BM) :

1. Let a wheel load W moves on the span of the arch. The maximum positive bending moment at a section X occurs when the load W is on the section.
2. At X,
$$M_{max} = W\frac{x(L-x)}{L} - W\frac{x}{\left(\frac{L}{2}\right)}\frac{x(L-x)}{L}$$
$$= \frac{Wx(L-x)(L-2x)}{L^2}$$
3. This relation is true for section from $x = 0$ to $x = \dfrac{L}{2}$

At, $x = 0$, $M_{max} = 0$ At $x = \dfrac{L}{2}$, $M_{max} = 0$

4. For M_{max} to have the greatest value, $\dfrac{dM_{max}}{dx} = 0$

$$\frac{dM_{max}}{dx} = \frac{W}{L^2}[(L-x)(L-4x) + x(L-2x)(-1)] = 0$$

$$6x^2 - 6Lx + L^2 = 0$$

$$x = \frac{6L \pm \sqrt{36L^2 - 24L^2}}{12} = \frac{6L \pm 2L\sqrt{3}}{12}$$

Since $x < \frac{L}{2}$

$$x = \frac{6L - 2L\sqrt{3}}{12} = \frac{L}{2} - \frac{L}{2\sqrt{3}} = 0.211L$$

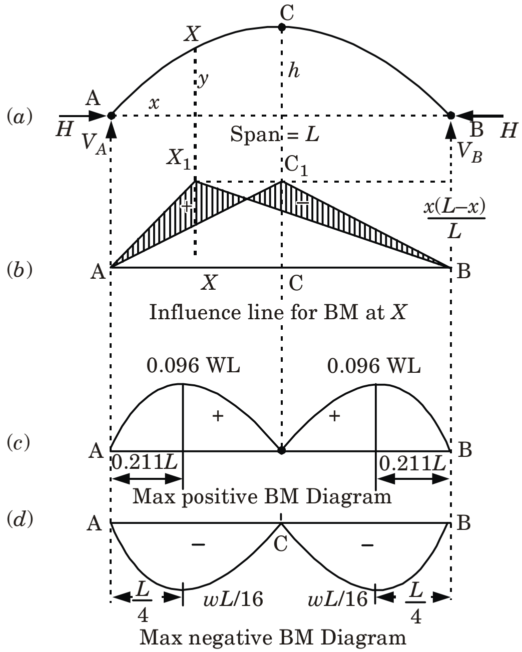


Fig. 5.18.1.

5. Substituting this value of x in the expression for M_{max}, we get M_{max}

$$= \frac{WL}{3\sqrt{3}} = 0.096\,WL$$

6. Obviously there is also another section where the greatest bending moment occurs.
7. This section is at 0.211 L from the right end. Fig. 5.18.1(c) shows the maximum positive BM diagram.

Maximum Negative BM :

1. The maximum negative bending moment at X will occur when the load is at the point C.

2. For this case the maximum negative moment at X,

$$M_{max} = -W \times \frac{x(L-x)}{L} + W \times \frac{\frac{L}{2}}{L-x} \times \frac{x(L-x)}{L}$$

$$= -W \times \frac{x(L-x)}{L} + \frac{W}{2}x = -\frac{Wx(L-2x)}{2L}$$

3. This relation is true for values of x from $x = 0$ to $x = \frac{L}{2}$

At $\quad x = 0, M_{max} = 0; \quad$ At $x = \frac{L}{2}, M_{max} = 0$

4. For the maximum negative bending moment to have the greatest value,

$$\frac{dM_{max}}{dx} = 0$$

$$-\frac{W}{2L}[L-4x] = 0; \; x = \frac{L}{4}$$

5. Substituting $x = \frac{L}{4}$ in the expression for maximum negative BM, absolute negative BM

$$= M_{max} = -\frac{WL}{16}$$

Obviously at a distance of $\frac{L}{4}$ from the right end also the absolute maximum negative BM occurs.

B. Uniformly Distributed Live Load :

1. Fig. 5.18.2(a) shows the influence line diagram of bending moment at any section X of a three-hinged parabolic arch of span L and rise h.

2. Let the section X be at a distance x form the left end $\left(x < \frac{L}{2}\right)$. The ordinates XX_1 and CC_1 are each equal to $\frac{x(L-x)}{L}$.

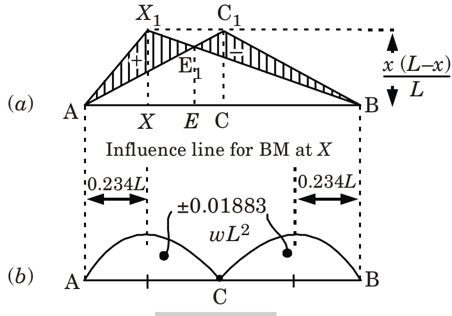


Fig. 5.18.2.

3. Let at E the ordinates of the triangles $X_1 AB$ and $C_1 AB$ meet at E_1.
Let $AE = z$
4 Since triangles AX_1B and AC_1B have the same base and the same height they are equal in area. Obviously the triangles AX_1E_1 and BC_1E_1 are also equal in area.
5. Hence the maximum positive and negative bending moment for a section is equal.
6. Studying the geometry of the influence line diagram,

$$\frac{EE_1}{CC_1} = \frac{AE}{AC} \text{ and } \frac{EE_1}{XX_1} = \frac{BE}{BX}$$

But $CC_1 = XX_1 \qquad \therefore \quad \dfrac{AE}{AC} = \dfrac{BE}{BX}$

$$\therefore \quad \frac{z}{\frac{L}{2}} = \frac{L-z}{L-x} = \frac{L}{\frac{3L}{2} - x}$$

$$\therefore \quad z = \frac{L^2}{3L-2x} \quad \text{But } EE_1 = \frac{AE}{AC} \times CC_1$$

$$\therefore \quad EE_1 = \frac{L^2}{3L-2x} \times \frac{1}{\left(\frac{L}{2}\right)} \frac{x(L-x)}{L} = \frac{2x(L-x)}{3L-2x}$$

7. Area of triangle AX_1, E_1 = Area of triangle AX_1B – Area of triangle AE_1B

$$= \frac{1}{2} L \frac{x(L-x)}{L} - \frac{1}{2} L \times \frac{2x(L-x)}{3L-2x}$$

$$= \frac{x(L-x)(L-2x)}{2(3L-2x)}$$

8. Maximum positive and negative bending moment at the section X is given by

$$M_{max} = \pm w \frac{x(L-x)(L-2x)}{2(3L-2x)}$$

9. For M_{max} to be a maximum, $\dfrac{d\,M_{max}}{dx} = 0$

$$\frac{dM_{max}}{dx} = \frac{w}{2} \frac{(3L-2x)(L^2 - 6Lx + 6x^2) - x(L-x)(L-2x)(0-2)}{(3L-2x)^2}$$

$(3L - 2x)(L^2 - 6Lx + 6x^2) + 2x(L-x)(L-2x) = 0$
$(8x^3 - 24Lx^2 + 18L^2x - 3L^3) = 0$
Putting $x = nL$, we get $8n^3 - 24n^2 + 18n - 3 = 0$
10. Solving we get $n = 0.234 \therefore x = 0.234\,L$
Substituting in the expression for M_{max} we get $M_{max} = \pm\, 0.01883\, wL^2$
11. For this condition the distance,

$$AE = z = \frac{L^2}{3L-2x} = \frac{L^2}{3L - 2 \times 0.234L} = 0.395\,L$$

Que 5.19. For a three hinged arch discuss the procedure to draw influence lines for :

A. Normal thrust. **B. Radial shear.**
C. Horizontal thrust. **D. Bending moment.**

Answer

Fig. 5.19.1(*a*) shows a three hinged arch of span L and rise h.

A. Influence line Diagrams for Horizontal Thrust :

1. Let a unit load be any section between the left end A and the crown C, at a distance x from A.
2. Obviously the vertical reactions at the supports will be,

$$V_B = \frac{x}{L} \quad \text{and} \quad V_A = \frac{L-x}{L}$$

Let H be the horizontal thrust.

3. Taking moment about the hinge C, we have

$$H \times h = \frac{x}{L} \times \frac{L}{2}, \quad H = \left(\frac{x}{2h}\right)$$

4. This is true for all position of load from A to C.

When $x = 0$, $H = 0$; When $x = \frac{L}{2}$, $H = \frac{L}{4h}$

5. Hence as the unit load moves from A to C the horizontal thrust will change from zero to $\frac{L}{4h}$.
6. Obviously as the unit load moves from C to B the horizontal thrust will change from $\frac{L}{4h}$ to zero.

B. Influence Line Diagram for Bending Moment at a given Section :

1. Let D be the given section of the three hinged arch whose coordinates are (x, y) with respect to the left support A.
2. The actual BM at the section = M_D = Beam moment at $D - Hy$.
3. Beam moment at D means the bending moment at D considering the span as that of a simply supported beam.
4. The influence line for the beam moment at D is a triangle having an altitude of $\frac{x(L-x)}{L}$.
5. The influence line for the H moment namely yH is a triangle whose altitude is $\frac{Ly}{4h}$.

6. The influence lines for beam moment at D and the H-moment at D are drawn as shown in Fig. 5.19.1(c) and thus the influence line diagram for the net or actual bending moment at D is obtained.

7. For the particular case of the parabolic arch, $y = \dfrac{4h}{L^2}x(L-x)$

$$\therefore \qquad \frac{Ly}{4h} = \frac{L}{4h} \times \frac{4h}{L^2}x(L-x) = \frac{x(L-x)}{L}$$

8. Hence for this case both the triangles $AD'B$ and $AC'B$ have the same height namely $\dfrac{x(L-x)}{L}$.

C. Influence Lines for Normal Thrust at D :

1. When the unit load is between A and D

$$N = H\cos\theta - V_B \sin\theta$$

2. When the unit load is between D and B,

$$N = H\cos\theta + V_A \sin\theta$$

3. The influence line is drawn as follows :

i. First draw the influence line for $H\cos\theta$. This is a triangle whose altitude is $\dfrac{L}{4h}\cos\theta$.

ii. On this diagram superimpose the influence line diagram for $V_B \sin\theta$ for the part AD.

iii. For the range BD, we must add the influence line diagram for $V_A \sin\theta$ to the influence line diagram for $H\cos\theta$.

D. Influence Line for Radial Shear at D :

1. When the unit load is between A and D,

$$S = H\sin\theta + V_B \cos\theta$$

2. When the unit load is between D and B,

$$S = H\sin\theta - V_A \cos\theta$$

3. He influence line is drawn as follows :

i. First draw the influence line for $H\sin\theta$. This is a triangle whose altitude is $\dfrac{L}{4h}\sin\theta$.

ii. For the range from A to D, add to the above diagram the influence line diagram for $V_B\cos\theta$.

iii. For the range from D to B, superimpose on the influence line diagram for $H\sin\theta$, the influence line diagram for $V_A\cos\theta$.

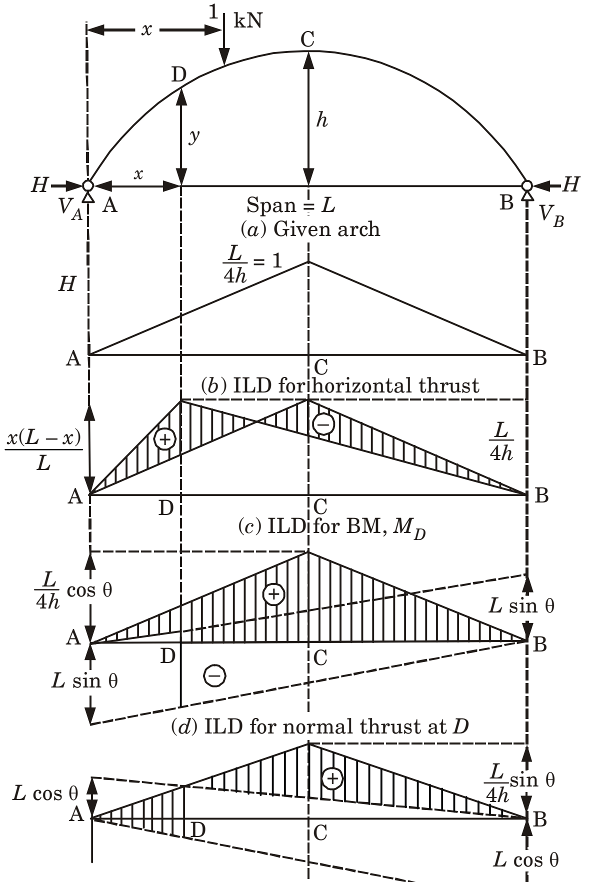


Fig. 5.19.1.

Que 5.20. **A three hinged parabolic arch has a span 40 m and rise of 10 m. Draw influence line diagram for the following :**

1. **Horizontal thrust.**
2. **BM at 8 m from the left support.**
3. **Normal thrust at the above section.**
4. **Radial shear at the above section.** **AKTU 2014-15, Marks 10**

Answer

1. Influence Line Diagram for Horizontal Thrust, *H* :

i. Let a unit load be at any section between the left end A and the crown C, at a distance x from A. Obviously the vertical reactions at the supports will be,

$$V_B = \frac{x}{L} \text{ and } V_A = \frac{L - x}{L}$$

ii. Let H be the horizontal thrust,
Taking moments about the hinge C, we get

$$H \times h = \frac{x}{L} \times \frac{L}{2}$$

$$\therefore \quad H = \left(\frac{1}{2h}\right) x$$

iii. Thus is true for all position of load from A to C.
When $x = 0, \quad H = 0$

When $x = \frac{L}{2}, \quad H = \frac{L}{4h}$

iv. Hence as the unit load moves from A to C the horizontal thrust will change from zero to $\frac{L}{4h}$.

v. Obviously as the unit load moves from C to B the horizontal thrust change from $\frac{L}{4h}$ to zero.

vi. ILD for horizontal thrust is shown in Fig. 5.20.1(*b*). This is an isosceles triangle having an latitude equal to $\frac{L}{4h} = \frac{40}{4 \times 10} = 1$ unit.

2. Influence Line for BM at the Section *D*, 8 metres from the Left End :

i. BM at D consists of :

a. The beam moment at D *i.e.*, the bending moment at D treating the span as that of a simply supported beam, and

b. The H moment at D equal to yH which is the moment at D due to the horizontal thrust.

ii. Influence line diagram for the beam moment at D is a triangle having an altitude of

$$\frac{x(L - x)}{L} = \frac{8 \times (40 - 8)}{40} = 6.4 \text{ units}$$

iii. The ordinate at D for the arch,

$$y = \frac{4h}{L^2} x(L - x) = \frac{4 \times 10}{40 \times 40} \times 8(40 - 8)$$

$$y = 6.4 \text{ m}$$

iv. The influence line diagram for the H moment at D is a triangle having an altitude of $\frac{Ly}{4h}$,

$$\frac{Ly}{4h} = \frac{40 \times 6.4}{4 \times 10} = 6.4 \text{ units}$$

v. When the arch is parabolic the altitudes of the ILD for the beam moment at D and the ILD for the H moment at D are equal.

vi. By superimposing the diagrams the net ILD for the BM at D is obtained.

3. Influence Line for Normal Thrust at D :

i. Let θ be the inclination of the tangent at D with the horizontal then,

$\tan\theta = \frac{dy}{dx}$ at D.

ii. The equation to the centre line of the arch is given by,

$$y = \frac{4h}{L^2} x(L - x)$$

iii. Slope at any section, $\frac{dy}{dx} = \frac{4h}{L^2}(L - 2x)$

iv. Slope of tangent at D (x = 8 m),

$$\tan\theta = \frac{4h}{L^2}(L - 2x) = \frac{4 \times 10}{40 \times 40}(40 - 2 \times 8)$$

$$\therefore \quad \tan\theta = \frac{3}{5}, \quad \sin\theta = \frac{3}{\sqrt{34}} = 0.515 \text{ and}$$

$$\cos\theta = \frac{5}{\sqrt{34}} = 0.858$$

v. When the unit load is between A and D,

Normal thrust at D, $N_D = H\cos\theta - V_B\sin\theta$

vi When the unit load is between D and B.

Normal thrust at D, $N_D = H\cos\theta + V_A\sin\theta$.

Influence lines for the quantities $H\cos\theta$, $V_B\sin\theta$ and $V_A\sin\theta$ are drawn.

vii. Between A and D the ILD for $H\cos\theta$ and $V_B\sin\theta$ are superimposed.

ix. Between D and B the ILD for $H\cos\theta$ and $V_A\sin\theta$ are added *i.e.*, combined.

4. Influence Line for Radial Shear at D :

i. When the unit load is between A and D, radial shear at D,

$$S = H\sin\theta + V_B\cos\theta$$

ii. When the unit load is between D and B, radial-shear at D,

$$S = H\sin\theta - V_A\cos\theta$$

iii. Influence lines for the quantities $H \sin \theta$, $V_B \cos \theta$ and $V_A \cos \theta$ are drawn.

iv. Between A and D the ILD for $H \sin \theta$ and $V_B \cos \theta$ are added *i.e.*, combined. Between D and B the ILD for $H \sin \theta$ and $V_A \cos \theta$ are superimposed.

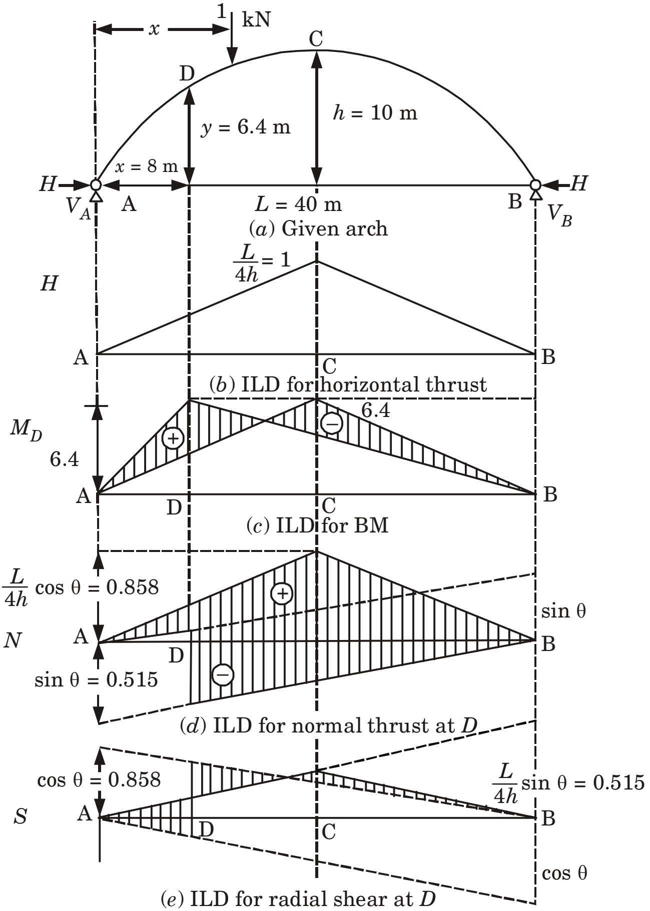


Fig. 5.20.1.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Define arch. Give the classification of arches.

Ans. Refer Q. 5.1, Unit-5.

Q. 2. What do you mean by linear arch ? Also state and prove the Eddy's theorem.

Ans. Refer Q. 5.2, Unit-5.

Q. 3. Show that the parabolic shape is a funicular shape for a three hinged arch subjected to a uniformly distributed load over its entire span.

Ans. Refer Q. 5.7, Unit-5.

Q. 4. Three hinged parabolic arch of span 50 m and rise 9 m carries a load whose intensity varies 25 kN/m at the crown to 50 kN/m at the ends. Find the following at a section D, 10 m from left end.

1. Bending moment.
2. Normal thrust.
3. Radial shear.

Ans. Refer Q. 5.10, Unit-5.

Q. 5. A three hinged parabolic arch has a span 40 m and rise of 10 m. Draw influence line diagram for the following :

1. Horizontal thrust.
2. BM at 8 m from the left support.
3. Normal thrust at the above section.
4. Radial shear at the above section.

Ans. Refer Q. 5.20, Unit-5.

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1 UNIT

Classification of Structures (2 Marks Questions)

Memory Based Questions

1.1. Define the term structure.

Ans. A structure is a system of connected parts used to support a load. Examples of structures in civil engineering are buildings, bridges and towers etc.

1.2. What are the different types of structures ?

Ans. **Classification of Structure :**

i. On the Basis of Structural Forms and Shapes :

a. Linear forms (skeletal structures), and
b. Curvilinear forms (surface structures).

ii. On the Basis of Dominant Stress Conditions :

a. Uniform stress forms, and
b. Varying stress forms.

iii. On the Basis of Complexity of their Force Analysis :

a. Beams and trusses, b. Cables and arches,
c. Frames, and d. Surface structures.

1.3. Define structural load.

Ans. Structural loads are defined as the forces, deformations or accelerations applied to a structure or its components. Loads cause stresses, deformation, and displacements in structure.

1.4. What are the different types of structural load ?

Ans. The different types of structural loads are :

i. Dead load, ii. Live load,
iii. Impact load, iv. Cyclic load,
v. Environmental loads, like :
a. Wind load, b. Snow, rain and ice loads,
c. Seismic load, d. Thermal load,
e. Ponding load, and f. Frost heaving.
vi. Other loads :
a. Foundation settlement,
b. Fire,
c. Corrosion, d. Creep or shrinkage.

1.5. Discuss the structural elements.

Ans. The common structural elements are as follows :

i. **Tie Rods or Bracing Struts :** Tie rods are the structural members subjected to tensile force.

ii. **Beams :** Beams are usually straight horizontal members used primarily to carry vertical loads.

iii. **Columns :** Columns are usually vertical and resist axial compression.

1.6. Define the term framework.

Ans. It is the skeleton of the complete structure and it supports all intended load safely and economically.

1.7. Mention any three reasons due to which sway may occur in portal frames. **AKTU 2015-16, Marks 02**

Ans. Sway in portal frames may occur due to :

i. Unsymmetry in geometry of the frame.

ii. Unsymmetry in loading.

iii. Settlement of one end of a frame.

1.8. Describe the static indeterminacy with example.

Ans. **Static Indeterminacy :** There are three conditions of static equilibrium *i.e.*, $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$. If a structure can be analyzed just by using these three conditions of equilibrium, then it is a statically determinate structure otherwise it is called as statically indeterminate structure.

Example : Fixed beam with general loading.

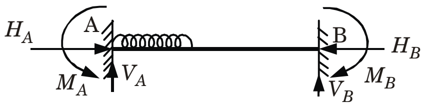

Fig. 1.8.1.

Total number of unknowns = 6

Number of equations available = 3

Static indeterminacy = 6 – 3 = 3

1.9. Describe the kinematic indeterminacy with example.

Ans. **Kinematic Indeterminacy :** When a structure is subjected to loads, each joint will undergo displacements in the form of translations and rotation. Kinematic indeterminacy of a structure means the number of unknown joint displacements in a structure.

Example :

Fig. 1.9.1.

Here support A is fixed, hence cannot have any displacement whereas support B is on rollers and hence have two degrees of freedom *i.e.*, horizontal translation and rotation.

1.10. Which method of analysis is suitable, if static indeterminacy is more than kinematic indeterminacy ?

AKTU 2016-17, Marks 02

Ans. Stiffness method is suitable, when static indeterminacy is more than kinematic indeterminacy.

1.11. State degree of freedom. **AKTU 2015-16, Marks 02**

Ans. Degree of Freedom :

i. When a structure is loaded, specified points on it, called nodes, will undergo unknown displacements. These displacements are known as degree of freedom for the structure.

ii. In 2-dimension, each node can have atmost two linear displacements and one rotational displacement.

iii. In 3-dimension, each node on frame or beam can have atmost three linear displacements and three rotational displacements.

Example : In Fig. 1.11.1

i. Number of nodes = 3 (A, B and C)

ii. Here three rotational displacements like θ_A, θ_B and θ_C.

iii. One vertical displacement and one horizontal displacement at C, as Δ_C.

iv. Total number of displacements = Degree of freedom = 5

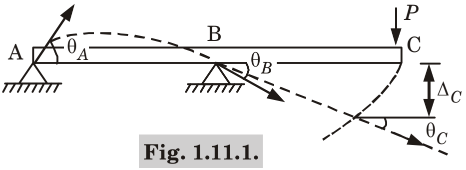

Fig. 1.11.1.

1.12. Differentiate between determinate and indeterminate structures. **AKTU 2015-16, Marks 02**

Ans.

S. No.	Determinate Structures	Indeterminate Structures
i.	Conditions of equilibrium are sufficient to analyze the structure completely.	Conditions of equilibrium are insufficient to analyze the structure completely.
ii.	No stresses are caused due to temperature changes.	Stresses are generally caused due to temperature variations.
iii.	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.

1.13. Give an example of a structure where it is externally as well as internally indeterminate.

AKTU 2016-17, Marks 02

Ans. Equilibrium conditions, $e = 3$
Total external reaction, $r = 4$
Number of member, $m = 11$
Number of joint, $j = 6$

i. External indeterminacy,
$$I_e = r - e = 4 - 3 = 1$$

ii. Internal indeterminacy,
$$I_i = m - (2j - 3) = 11 - (2 \times 6 - 3) = 2$$

iii. Total indeterminacy,
$$I = I_e + I_i = 1 + 2 = 3$$

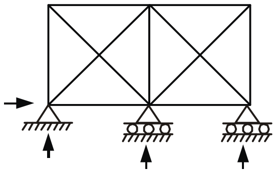

Fig. 1.13.1.

1.14. Discuss the cables.

Ans. Cables are used as support as well as for transmission of loads from one member to the other. Cable stands for a flexible tension member.

1.15. What is the expression for computing the length of a cable for horizontal span '*L*' and central dip '*h*' when both support are at same level ?

Ans. Length of cable, $l = L + \dfrac{8}{3}\dfrac{h^2}{L}$

1.16. Write down the expression for length of cable, when supports are at different level.

Ans. Length of cable in this case is given by,
$$l = l_1 + l_2$$
$$l_1 = 2L_1 + \frac{4}{3}\frac{h_1^2}{L_1}$$
and,
$$l_2 = 2L_2 + \frac{4}{3}\frac{h_2^2}{L_2}$$

1.17. Give the expression for horizontal reaction and tension at ends of suspension cable.

Ans. i. Horizontal reaction, $H = \dfrac{wL^2}{8h}$

ii. Tension in cable ends, $T = \dfrac{wL}{2}\sqrt{1 + \dfrac{L^2}{16h^2}}$

1.18. Differentiate static and kinematic indeterminacy of structure. **AKTU 2015-16, Marks 02**

Ans.

S. No.	Static Indeterminacy	Kinematic Indeterminacy
1.	Static indeterminacy of a structure is defined as the difference between number of unknown forces and number of equilibrium equations to be solved.	It is defined as the sum of all the possible displacements that various joints of the structure can undergo.
2.	Degree of static indeterminacy = $(m + r) - 2j$.	Degree of kinematic indeterminacy = Sum of degrees of freedom in rotation and translation.

1.19. What do you understand by stability of plane truss ?

Ans.

m = Number of members (or bars),
j = Total number of joints, and
r = Number of reaction components.

i. $2j = m + r \Rightarrow$ Structure is statically determinate.
ii. $2j < m + r \Rightarrow$ Structure is statically indeterminate.
iii. $2j > m + r \Rightarrow$ Structure is a mechanism and always unstable.

☺☺☺

2 UNIT

Analysis of Trusses (2 Marks Questions)

Memory Based Questions

2.1. Define truss.

Ans. Truss is a geometric arrangement of its members, the members are subjected to tensile or compressive force only. Thus, to support a given load a truss utilizes lesser materials as compared to a beam.

2.2. What are the various types of plane trusses ?

Ans.

i. On the basis of utility of trusses :
   a. Roof truss, and
   b. Bridge truss.

ii. On the basis of the geometrical configuration :
   a. Simple truss,
   b. Compound truss, and
   c. Complex truss.

2.3. Define space truss with suitable sketch.

Ans. Space trusses have members extending in three dimensions and are suitable for derricks and towers.

i. Space truss consists of members joined together at their ends to form a stable three-dimensional structure.

ii. The simplest element of a stable truss is a tetrahedron.

ii. It is formed by connecting six members together with four joints.

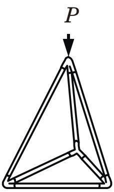

Fig. 2.3.1.

2.4. Distinguish between plane and space truss.

Ans. Plane trusses are composed of members that lie in the same plane and are frequently used for bridge and roof support whereas space trusses have members extending in three dimensions and are suitable for derricks and towers.

2.5. Discuss different types of pin jointed determinate trusses.

Ans. Following are different types of these structures :

Simple Truss : The simplest forms of the truss framework is simple truss which is one of the most stable truss.

Compound Truss : In structure, such as roof shades of factories or warehouse, it is required to connect two or more simple trusses to form one rigid framework.

Complex Truss : Those trusses which don't fulfill the requirement of a perfect truss or a compound truss but satisfy of criterion statical determinacy for the plane truss are designated as complex truss.

2.6. What do you mean by compound and complex space truss ?

Ans. **Compound Space Truss :** A compound space truss is formed by connecting two or more simple trusses together. This type of truss is used to support loads acting over a large span, because it is cheaper to construct a lighter compound truss than a heavier single simple truss.

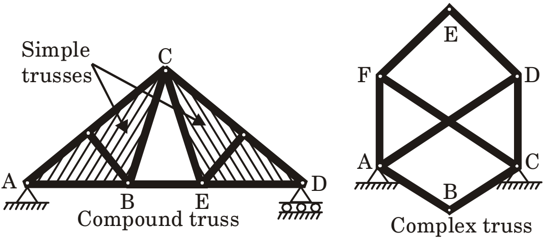

Fig. 2.6.1.

Complex Space Truss : The truss which cannot be classified as either simple or compound truss is termed as complex space truss.

2.7. Discuss the method of section.

Ans. In method of section, after determining the support reactions, a section line is drawn passing through not more than three members in which forces are not known such that frame is cut into two separate parts.

Each part should be in equilibrium under the action of loads, reactions and the forces in the members that cut by section line.

2.8. What are the limitations of method of section ?

Ans. Following are the limitations of method of section :

i. Section line cannot pass through more than three members.

ii. It cannot separate the frame into more than three parts.

2.9. Illustrate the tension coefficient method for the analysis of space truss.

Ans. Method of tension coefficient provides a relationship between the components of member forces along the coordinate axes and the member coordinates at the ends of members, this analysis results in linear simultaneous equations for the components of member forces along the coordinate axes at each joint. This method gives member force per unit length rather than member forces known as tension coefficient.

2.10. What are the various types of supports ?

Ans. The various types of supports are as follows :

i. Roller support,
ii. Hinged support,
iii. Fixed support,
iv. Link support,
v. Ball and socket support, and
vi. Rigid support in space.

2.11. What is the criteria for classification of truss ?

Ans. Classification of trusses are based on following criteria :

i. The shape of the upper and lower chords.
ii. Type of web.
iii. The condition of support.
iv. Purpose of structure.
v. The level of the floor (lane, road etc.).

2.12. What do you understand by external unstability ?

Ans. A structure is externally unstable if all of its reactions are concurrent or parallel. As shown in following Fig. 2.12.1(*a*), reactions V_1, V_2 and V_3 are concurrent and unstable. Hence, structure is externally unstable.

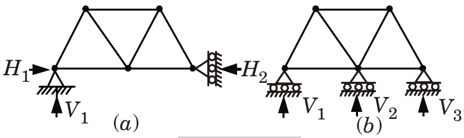

Fig. 2.12.1.

In Fig. 2.12.1(*b*), reactions V_1, V_2 and V_3 are parallel and unstable Hence, this structure is also externally unstable.

2.13. Define fatigue. **AKTU 2016-17, Marks 2.5**

Ans. A phenomenon leading to the reduced internal resistance of a material when subjected to repeated or fluctuating cyclic stresses below the tensile strength of the material.

☺☺☺

3
UNIT

Strain Energy and Deflection of Beams (2 Marks Questions)

Memory Based Questions

3.1. Define the term strain energy or resilience of the member.

Ans. When an elastic member is deformed under the action of an external loading till its elastic limit, the member has some energy stored in it, this energy is called as the strain energy of the member or the resilience of the member.

The strain energy of the deformed member is equal to the amount of work done by the external force to produce the deformation.

3.2. Write the expression of strain energy stored in a member due to axial loading.

Ans. $$\text{Strain energy} = \frac{1}{2} W \times \frac{W}{AE} \times L = \frac{1}{2} \frac{W^2 L}{AE}$$

3.3. Write the expression of strain energy stored in a member due to bending.

Ans. Total energy stored by the whole beam $= \int \dfrac{M^2 ds}{2EI}$

3.4. Write the expression of strain energy stored in a beam subjected to a uniform bending moment.

Ans. Strain energy stored by the beam,

$$U_i = \int \frac{M^2 ds}{EI} = \frac{M^2 L}{2EI} \qquad \left(\because\ I = \frac{bd^3}{12}, M = \frac{1}{6} \sigma b d^2\right)$$

$$U_i = \frac{1}{2E}\left(\frac{1}{6} \sigma\, bd^2\right)^2 \frac{L}{\frac{bd^3}{12}} = \frac{\sigma^2}{6E} bdL$$

$$U_i = \frac{\sigma^2}{6E} \times \text{Volume of the beam}$$

3.5. Write the equation in term of strain energy, which is sufficient to determine the stress in case of propped cantilever beams.

AKTU 2016-17, Marks 02

Ans. $$\sigma^2 = \frac{2E}{V} \times \text{Total strain energy}$$

where, V = Volume of the beam

Total strain energy in propped cantilever beam, $U = \int \frac{M^2 ds}{2EI}$

3.6. Write the statement of Maxwell's reciprocal theorem.

Ans. In any beam or truss, whose material is elastic and obeys Hooke's law and whose supports remain unyielding and the temperature remain unchanged, then the deflection at any point D (*i.e.*, Δ_D) due to a load W acting at any other point C is equal to the deflection at any point C (*i.e.*, δ_C) due to the load W acting at the point D.

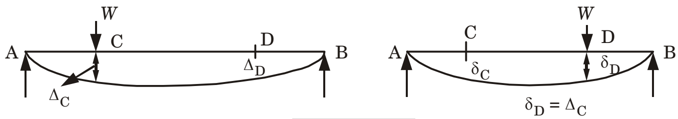

Fig. 3.6.1.

3.7. Write the statement of Betti's theorem.

Ans.

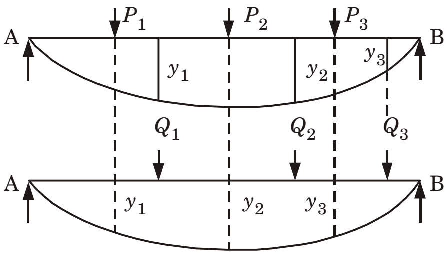

Fig. 3.7.1.

It is the generalized form of Maxwell's law. For a structure whose material is elastic and follows Hooke's law and in which the supports are unyielding and the temperature is constant, the virtual work done by a system of force P_1, P_2, P_3 ...... during the distortion caused by a system of forces Q_1, Q_2, Q_3 ...... is equal to the virtual workdone by the system of forces Q_1, Q_2, Q_3, ... during the distortion caused by the system of forces P_1, P_2, P_3......... .

3.8. Write the statement of Castigliano's first theorem.

AKTU 2016-17, Marks 02

Ans. "The partial derivative of the total strain energy in a structure with respect to the displacement at any one of the load points gives the value of corresponding load acting on the body in the direction of displacement"

$$P_i = \frac{\partial U}{\partial \Delta_i}$$

3.9. Write the statement of Castigliano's second theorem.

Ans. In any linear elastic structure, partial derivative of the strain energy with respect to load or moment at a point is equal to the deflection or slope of the point where load is acting. The deflection being measured in the direction of load.

$$\frac{\partial U}{\partial P_i} = \Delta_i \text{ and } \frac{\partial U}{\partial M_i} = \theta_i$$

3.10. Define real work method. **AKTU 2015-16, Marks 02**

Ans. The strain energy method is also known as real work method, since, work done by the actual loads are considered. From the law of conservation of energy, we can say,

Strain energy, U = Real work done by loads = $\sum_0^n \frac{1}{2} P\Delta$

3.11. What do you understand by conjugate beam ?

Ans. Conjugate beam is an imaginary beam for which the load diagram is represented by $\frac{M}{EI}$ diagram.

3.12. Give the difference between simple (or real) beam and conjugate beam.

Ans.

S. No.	Simple (Real Beam)	Conjugate Beam
i.	Actual loading diagram is the loading diagram.	Bending moment diagram is the loading diagram.
ii.	Bending moment diagram from loading diagram provides the bending moment at any section.	Bending moment diagram from loading diagram provides the deflection at any section.
iii.	Shear force diagram provides the shear stress at a section.	Shear force diagram provides the slope at a section.

3.13. State the salient features of conjugate beam method.

Ans. Following are the features of conjugate beam method :

i. The slope at any section of the given beam is equal to the shear force at the corresponding section of the conjugate beam.

ii. The deflection at any section of the given beam is equal to the bending moment at the corresponding section of the conjugate beam.

3.14. What is the advantage of conjugate beam method over other method ? **AKTU 2016-17, Marks 02**

Ans. Following are the advantages of conjugate beam method :

i. This method is applying to the beams having different moment of inertia.

ii. This method is simple, easy and consume less time.

iii. This method is useful for calculating deflection in different loading condition.

3.17. Write down the formula of deflection and slope for simply supported beam with concentrated load *W* at the midspan.

Ans.

$$\text{Slope at supports} = \frac{WL^2}{16EI}$$

$$\text{Deflection at mid span} = \frac{WL^3}{48EI}$$

3.18. Write down the formula of deflection and slope for simply supported beam having uniformly distributed load on the span *L*.

Ans.

$$\text{Slope at supports} = \frac{wL^3}{24EI}$$

$$\text{Deflection mid span} = \frac{5wL^4}{384EI}$$

3.19. A cantilever beam of length *L* subjected to a concentrated load *P* at the free end. What is the deflection at the free end ? **AKTU 2015-16, Marks 02**

Ans. Deflection at free end, $\Delta = \dfrac{PL^3}{3EI}$

3.20. Write the formulae for area and the centroid of the curve defined by $y = kx^n$. **AKTU 2016-17, Marks 02**

Ans. Area of curve, $A = \dfrac{bh}{n+1}$

Centroid of area, $\bar{x} = \left(\dfrac{n+1}{n+2}\right) b$

☺☺☺

4 UNIT

Rolling Loads and Influence Line Diagrams (2 Marks Questions)

Memory Based Questions

4.1. What do you understand by the term rolling load ?

Ans. **Rolling Load (or Moving Load) :** In actual practice, we often encounter with the loads which are moving or with positions that are liable to change.

Example : Axle loads of moving trucks or vehicles, wheel loads of a railway train or wheel loads of a gantry assembly on a gantry girder etc.

4.2. What do you understand by influence line ?

Ans. An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a concentrated load or forces that moves over the member.

4.3. What are the advantages of influence lines ?

Ans. Following are the advantages of influence lines :

i. If a structure is subjected to a live load or moving load, however the variation of the shear and bending moment in the member is best described using the influence line.

ii. The magnitude of the associated reaction, shear, moment or deflection at the point can be calculated from the ordinates of the influence line diagram.

iii. Influence lines plays an important role in the design of bridges, industrial crane rails, conveyors and other structures where loads move across their span.

4.4. Define influence line diagram (ILD).

Ans. A curve or graph that represents the function like a reaction at support, the shear force at a section, bending moment at a section of a structure etc., for various positions of a unit load on the span of the structure is called an influence line diagram for the function represented.

4.5. What is the difference between ILD and shear force or bending moment diagrams ? **AKTU 2016-17, Marks 02**

Ans.

i. The ordinate of a curve of bending moment or shear force gives the value of the bending moment or shear force at the section where the ordinate has been drawn.
In case of an influence line diagram, the ordinate at any point gives the value of bending moment or shear force only at the given section and not at the point at which ordinate has been drawn.

ii. Influence line diagram represent the effect of a moving load only at a specified point on the member, whereas shear force or bending moment diagram represent the effect of fixed loads at all points along the axis of the member.

4.6. State Muller-Breslau's principle for determinate structure.

AKTU 2015-16, Marks 02

Ans. According to this principle "the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function".

4.7. What do you understand by influence line for beam ?

Ans. If influence line for a function (reaction, shear or moment) has been constructed, it will be possible to position the live loads on the beam which will produce the maximum value of function.
There are two types of loadings to be considered :

i. For concentrated load, and
ii. For uniform load.

4.8. What do you understand by influence line for truss ?

Ans. Since, the truss members are affected only by the joint loading, we can obtain the ordinate values of the influence line for a member by loading each joint with a unit load, then use the method of joints or method of selection to calculate the force in the member. The data can be arranged in tabular form listing unit load at joint versus force in member.

4.9. Write the condition for maximum end shear of moving loads.

Ans.

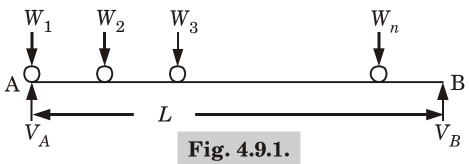

Fig. 4.9.1.

$$\frac{\text{The load rolled off}}{\text{Suceeding wheel space}} > \frac{\text{Sum of remaining loads on the Span}}{\text{Span length}}$$

Application Based Questions

4.10. Draw the ILD of the reactions for simply supported beam.

Ans.

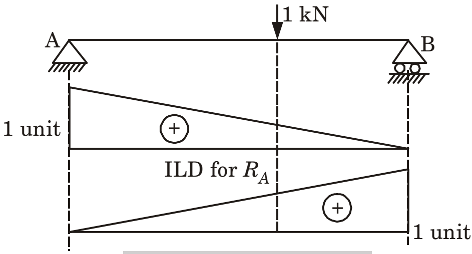

Fig. 4.10.1. ILD for R_B

4.11. Draw the ILD of shear force and bending moment for simply supported beam at any section.

Ans.

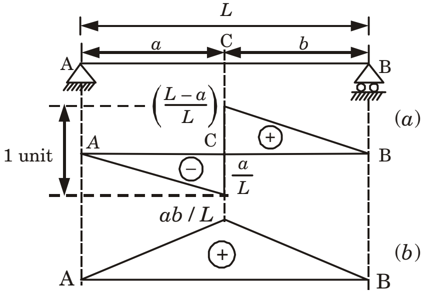

Fig. 4.11.1. ILD for (*a*) shear force and (*b*) bending moment at point *C*.

4.12. What are the uses of influence lines ?

AKTU 2016-17, Marks 02

Ans. Following are the uses of influence lines :

i. Influence lines are used to show the variation of shear force and bending moment in the member which is subjected to a live load or moving load.

ii. Influence lines help to tell where the moving load should be placed on the structure so that it creates the greatest influence at the specified point.

iii. Using the ordinates of influence line diagram, the magnitude of associated reaction, shear, moment or deflection at the point can be calculated.
iv. Influence lines are used to design the structures on which the loads move across the span. The common types of structures are design of bridges, industrial crane rails and conveyors etc.

4.13. What are the applications of Müller-Breslau's principle ?

Ans. Following are the applications of Müller-Breslau's principle :

i. Muller-Breslau principle provides a quick method for establishing the shape of the influence line.
ii. After knowing the shape of influence line, the ordinates at the peak can be determined by the basic method.
iii. Using shape of influence line, it is possible to locate the live load on the beam and then determine the maximum value of the function (shear or moment) by using statistics.

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5 UNIT

Analysis of Arches (2 Marks Questions)

Memory Based Questions

5.1. What do you understand by the term 'arch' ?

Ans.

i. An arch acts as an inverted cable, so it receives its load mainly in compression.

ii. As arches are rigid, hence they must also resist some bending and shear, depending upon, how it is loaded and shaped.

iii. These are used to reduce the bending moments in long span structures.

5.2. What are the different types of arches ?

Ans. Some common types of arches are given below :

i. Funicular arch, ii. Fixed arch,

iii. Two hinged arch, iv. Three hinged arch, and

v. Tied Arch.

5.3. State Eddy's theorem as applicable to arches.

AKTU 2015-16, Marks 02

Ans. The bending moment at any section of an arch is equal to the vertical intercept between the linear arch and the central line of the actual arch.

5.4. Define span of arch.

Ans. The horizontal distance between the lower hinge supports is called span of arch. The hinges may or may not be at the same level.

5.5. What is linear arch ?

Ans. If the shape of arch is made as per the funicular polygon and points are pin connected, then the arch will be subjected to compression only, and the load acting on arch is supported by arch without development of any bending moment anywhere. Such an arch is known as linear arch.

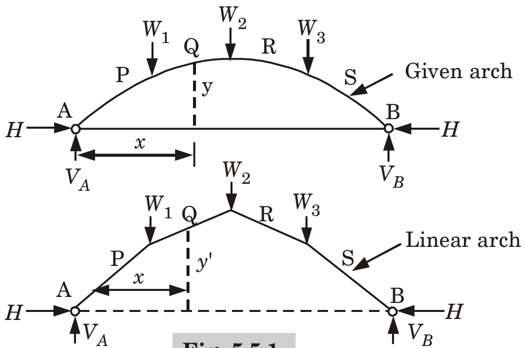

Fig. 5.5.1.

5.6. Define rise of arch.

Ans. When the supports of arch are at same level, the height of the crown (highest point of arch) above the level of the lower hinges is called rise of the arch.

5.7. Define horizontal thrust.

Ans. The horizontal component of the reaction at either lower end is called horizontal thrust at the support. It is denoted by H.

5.8. What do you understand by three hinged arch ?

Ans. A three hinged arch has a hinge at each abutment or springing and also at the crown. It is statically determinate structure.

5.9. What are the different types of three hinged arch ?

Ans. Following are the types of three hinged arch :

i. Three hinged parabolic arch, and
ii. Three hinged circular arch.

5.10. Write the equation of vertical rise in parabolic arch.

Ans. The equation of vertical rise in parabolic arch is given by,

$$y = \frac{4h}{L^2}\, x\,(L - x)$$

5.11. Give the equation for a parabolic arch whose springing is at different levels. **AKTU 2015-16, Marks 02**

Ans. The equation is given by,

$$\frac{x^2}{y} = \text{Constant}$$

5.12. Why arches are preferred than beams ?

AKTU 2016-17, Marks 02

Ans. Arches preferred than beams because :

i. The inward horizontal reactions induced by the end restraints produce hogging moments in the arch which effectively counteract the static sagging moments set up by the vertical loads.

ii. The consequent reduction in the net moments which is responsible for significantly higher load bearing capacity of an arch as compared to the corresponding beam.

5.13. What do you mean by spandrel braced arch ?

Ans. In three hinged spandrel braced arch, the space above the arch rib and below the level of crown is known as spandrel of arch.

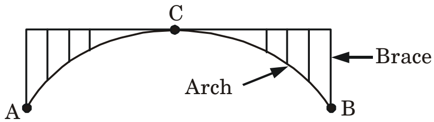

Fig. 5.13.1.

5.14. Draw a diagram for maximum positive bending moment due to movement of a single concentrated load of *W* kN in three hinged arch.

Ans.

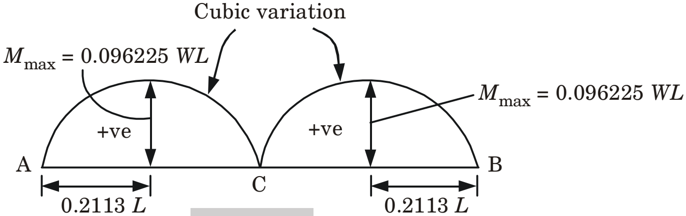

Fig. 5.14.1.

5.15. What are the different types of spandrel braced arch ?

Ans. The spandrel braced arch can be of following types :

i. Close Spandrel Arch : In closed spandrel arch, the space above the arch rib and below the level of the crown can be closed with filling.

ii. Open Spandrel Arch : In open spandrel arch, the space above the arch rib and below the level of the crown can be provided with truss-like arrangement.

5.16. Draw the diagram for maximum negative bending moment due to movement of single concentrated load of *W* kN in three hinged arch.

Ans.

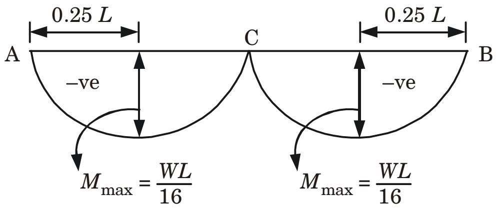

Fig. 5.16.1.

5.17. Draw the diagram for the maximum positive bending moment and maximum negative bending moment due to movement of UDL of *w* kN/m in three hinged arch.

Ans.

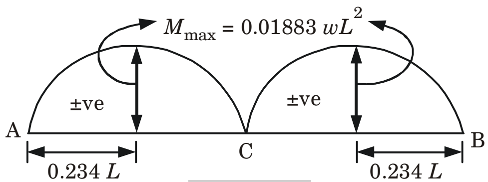

Fig. 5.17.1.

5.18. Write the effect of temperature rise on the horizontal thrust for a three hinged arch carrying a load.

Ans. While no stresses are produced in a three hinged arch due to temperature changes alone, it may be noted that, since the rise of arch is altered as a consequence of temperature change, the horizontal thrust for the arch already carrying a load will also alter.

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B. Tech.
(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2014-15
STRUCTURAL ANALYSIS–I

Time : 3 Hours **Max. Marks : 100**

Note : Attempt **all** questions.

1. Attempt any **four** of the following : **(4 × 5 = 20)**

a. What are the different methods of analysis of trusses ? Explain any one with example.

b. Find the forces in the members of the given truss.

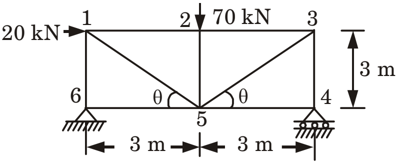

Fig. 1.

c. What is the difference between statically determinate and statically indeterminate structure ?

d. Find the external and internal degree of redundancy of the structures as shown in Fig. 2.

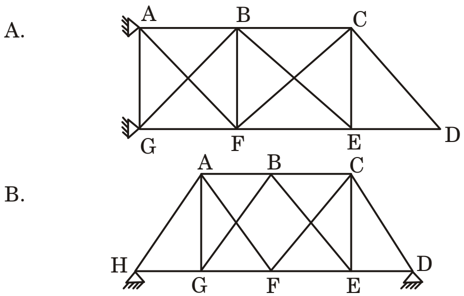

Fig. 2.

e. Enumerate the different types of pinned jointed determinate truss with suitable example and sketches.

f. Define the term tension coefficient method for plane truss with example.

2. Attempt any **two** of the following : **(2 × 10 = 20)**

a. State the Muller Breslau's principle of influence line. Draw ILD for shear at point *C*, support reactions for following given simply supported beam.

b. Three wheel loads 20 kN, 80 kN and 80 kN spaced 4 m apart from each other, with the 20 kN in the lead, pass over a simply supported beam of span 20 m. Determine the absolute maximum shear force and moment. Consider that loading can move in either direction.

c. A single load of 100 kN moves on a girder or span 20 m. Construct the influence line for shear force and bending moment for a section 5 m from the left support.

3. Attempt any **two** of the following : **(2 × 10 = 20)**

a. A three hinged parabolic arch has a span 40 m and rise of 10 m. Draw influence line diagram for the following :

1. Horizontal thrust.
2. BM at 8 m from the left support.
3. Normal thrust at the above section.
4. Radial shear at the above section.

b. A three hinged semicircular arch of radius *R* carries a UDL of *w* per run over the whole span. Find

A. Horizontal thrust.
B. Location and magnitude of maximum bending moment.

c. A three hinged parabolic arch of span 50 m and rise 9 m carries a load whose intensity varies 25 kN/m at the crown to 50 kN/m at the ends. Find the following at a section *D*, 10 m from left end.

1. Bending moment.
2. Normal thrust.
3. Radial shear.

4. Attempt any **two** of the following : **(2 × 10 = 20)**

a. Write statement of Castigliano's first theorem and Maxwell's reciprocal theorem. Prove Maxwell's theorem.

b. Determine the deflection at free end of a cantilever beam.

c. Using conjugate beam method find the deflection of a simply supported beam at point *C*. *AB* of length 10 m loaded by an UDL of intensity 20 kN per unit run.

5. Attempt any **four** of the following : **(4 × 5 = 20)**

a. A 60 mm × 40 mm × 6 mm unequal angle is placed with the longer leg vertical and is used as a simply supported at the ends. Over a span of 2 m, if it carries UDL of such magnitude so as to produce the maximum bending moment of 0.12 kN-m, determine the maximum deflection of the beam. Take E = 2.1 × 10^5 N/mm².

b. Locate the position of the shear centre for the channel section shown in Fig. 3.

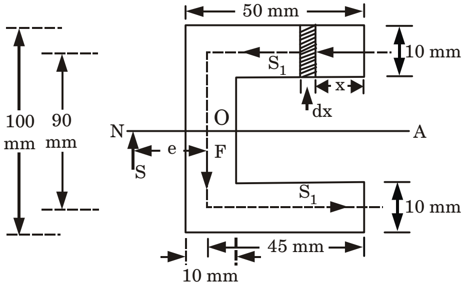

Fig. 3.

c. What do you mean by bending of curved bars ? Derive the relevant expression for the bending of curved bars with small initial curvature.

d. Define shear centre. Write down the principle of second moments of area with proof.

e. "Discuss about the Mohr's circle with respect to unsymmetrical bending".

f. Explain Winkler-Bach theory.

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SOLUTION OF PAPER (2014-15)

Note : Attempt **all** questions.

1. Attempt any **four** of the following : **(4 × 5 = 20)**

a. What are the different methods of analysis of trusses ? Explain any one with example.

Ans. Different Methods of Analysis of Trusses : To analyze a statically determinate truss we have following two methods :

1. Method of joints.
2. Method of sections.

1. Method of Joints :

i. In method of joints, the principle used is, if a truss is in equilibrium, then each of its joint must also be in equilibrium.

ii. Joint analysis should start at a joint having atleast one known force and atmost two unknown forces.

iii. If a force is pushing on the pin, member will be in compression and vice-versa.

iv. During solution of a truss always assume the unknown member force acting on the joint's free body diagram to be in tension.

v. If our assumption is correct then solution will be positive for the member.

vi. Now use the correct magnitude and direction of force on subsequent joint's free body diagram.

Procedure to Analyze :

i. First calculate the external support reactions by drawing the free body diagram of the entire truss.

ii. Draw free body diagram of a joint having at least one known force and atmost two unknown forces.

iii. Assume the unknown member force as tensile in nature *i.e.*, force is pulling the joint.

iv. Now orientation of X and Y axes are to made such that forces on the free body diagram can be easily resolved into their x and y components.

v. Apply two force equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Solve these two equations to get unknown member force and then verify their correct directional sense.

vi. Continue to analyze each of the other joints, where again it is necessary to choose a joint having atleast one known force and atmost two unknown forces.

b. Find the forces in the members of the given truss.

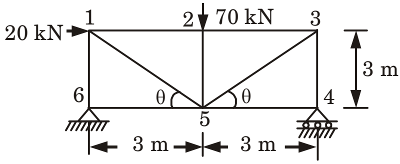


Fig. 1.

Ans.

Given : Plane truss shown in Fig. 1.
To Find : Force in member.

1. **Reaction at Supports :**

i.
$$\Sigma F_x = 0, -H_6 + 20 = 0$$
$$H_6 = 20 \text{ kN}$$

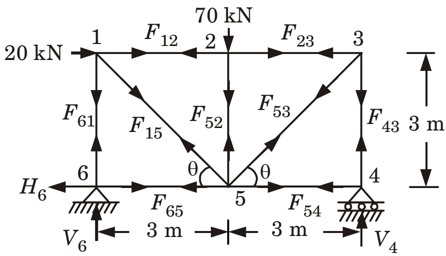


Fig. 2.

ii.
$$\Sigma F_y = 0$$
$$V_6 + V_4 = 70 \quad \text{...(1)}$$

iii Taking moment about joint 6, $\Sigma M_6 = 0$

$70 \times 3 + 20 \times 3 - V_4 \times 6 = 0$

$$210 + 60 = V_4 \times 6$$
$$V_4 \times 6 = 270$$
$$V_4 = 45 \text{ kN}$$

From eq. (1),
$$V_6 = 70 - V_4$$
$$V_6 = 70 - 45$$
$$V_6 = 25 \text{ kN}$$

2. **Joint (6) :**

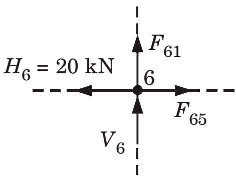


Fig. 3.

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$F_{61} + V_6 = 0$$
$$F_{61} = -25 \text{ kN}$$

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$-H_6 + F_{65} = 0$$
$$F_{65} = 20 \text{ kN}$$

3. Joint (4) :

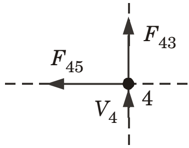


Fig. 4.

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$F_{43} + \text{V}_4 = 0$$
$$F_{43} = -45 \text{ kN}$$

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{45} = 0$$

4. Joint (5) :

i. Resolve the forces horizontally,

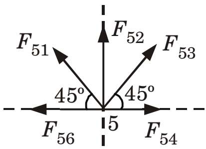


Fig. 5.

$$\Sigma F_x = 0$$
$$F_{51} \cos 45° + F_{56} = F_{54} + F_{53} \cos 45°$$
$$F_{51} \cos 45° + 20 = F_{53} \cos 45° + 0$$
$$\cos 45° (F_{51} - F_{53}) = -20 \quad \text{...(2)}$$

ii. Resolve the forces vertically,

$$\Sigma F_y = 0$$
$$F_{53} \sin 45° + F_{51} \sin 45° + F_{52} = 0 \quad \text{...(3)}$$

5. Joint (2) :

i. Resolve the forces vertically,

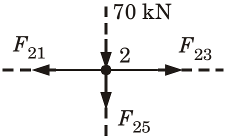


Fig. 6.

$$\Sigma F_y = 0$$

$$-F_{25} - 70 = 0$$

$$F_{52} = -70 \text{ kN}$$

ii. Resolve the forces horizontally,

$$\Sigma F_x = 0$$

$$F_{21} = F_{23} \quad \text{...(4)}$$

Value of F_{52} put in eq. (3), we get

$$\sin 45°(F_{53} + F_{51}) = 70 \quad \text{...(5)}$$

Solving eq. (2) and (5), we get

$$F_{51} = 35.36 \text{ kN}$$
$$F_{53} = 63.64 \text{ kN}$$

6. Joint (1) :

i. Resolving forces horizontally,

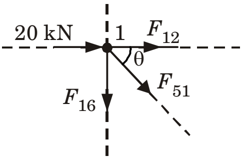


Fig. 7.

$$\Sigma F_x = 0$$

$$F_{12} + 20 + F_{15} \cos 45° = 0$$

$$F_{12} + 20 + 35.36 \times \cos 45° = 0$$

$$F_{12} = -45 \text{ kN}$$

ii. Value of F_{12} put in eq. (4),

$$F_{23} = -45 \text{ kN}$$

F_{16} = 25 kN (Compression), F_{12} = 45 kN (Compression)
F_{15} = 35.36 kN (Tension), F_{54} = 0
F_{53} = 63.64 kN (Tension), F_{52} = 70 kN (Compression)
F_{23} = 45 kN (Compression), F_{65} = 20 kN (Tension)
F_{43} = 45 kN (Compression)

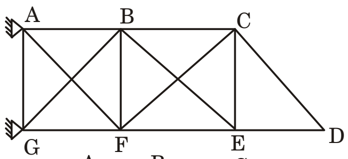
c. What is the difference between statically determinate and statically indeterminate structure ?

Ans.

S. No.	Statically Determinate Structures	Statically Indeterminate Structures
1.	Conditions of equilibrium are sufficient to analyze the structure completely.	Conditions of equilibrium are insufficient to analyze the structure completely.
2.	The bending moment at a section or the force in any member is independent of the material of the components of the structure.	The bending moment at a section or the force in a member depends upon the material of the components of the structure.
3.	The bending moment at a section or the force in any member is independent of the cross-sectional areas of the components.	The bending moment at a section or the force in a member depends upon the cross-sectional areas of the components.
4.	No stresses are caused due to temperature changes.	Stresses are generally caused due to temperature variations.
5.	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.

d. Find the external and internal degree of redundancy of the structures as shown in Fig. 8.

A.



B.

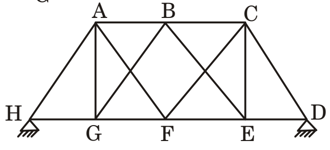


Fig. 8.

Ans.

Given : Structures shown in Fig. 8.
To Find : External and internal degree of redundancy.

A. Unknown support reactions, $r = 4$
Number of equilibrium equations, $e = 3$
Number of members, $m = 13$

Number of joints, $j = 7$
Degree of external redundancy,

$$I_e = r - e = 4 - 3 = 1$$

Degree of internal redundancy,

$$I_i = m - [2j - 3] = 13 - [2 \times 7 - 3] = 2$$

B. Unknown support reactions, $r = 4$
Number of equilibrium equations, $e = 3$
Number of members, $m = 14$
Number of joints, $j = 8$
Degree of external redundancy, $I_e = r - e = 4 - 3 = 1$
Degree of internal redundancy,

$$I_i = m - [2j - 3] = 14 - [2 \times 8 - 3] = 1$$

e. Enumerate the different types of pinned jointed determinate truss with suitable example and sketches.

Ans. The following five criteria are the basis for the classification of trusses :

1. **According to the Shape of the Upper and Lower Chords :** The trusses can be classified into trusses with parallel chords as shown in Fig. 9, polygonal and triangular trusses or trusses with inclined chords as shown in Fig. 9.

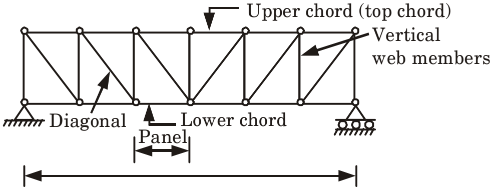


Fig. 9. A truss with parallel chords.

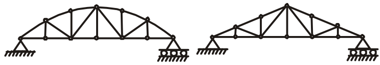


Fig. 10. Polygonal and triangular trusses.

2. **According to the Type of the Web :** It permits to subdivide the trusses into those with triangular patterns as shown in Fig. 11(a), those with quadrangular patterns as shown in Fig. 11(b) formed by vertical and diagonals, those with the web members form a letter K as shown in Fig. 11(c).

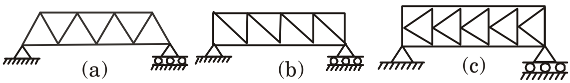


Fig. 11. Trusses according to the type of the web.

3. **According to the Conditions of the Support :** It permits to distinguish between the ordinary end-supported trusses as shown in Fig. 12(a), the cantilever trusses as shown in Fig. 12(b), the trusses cantilevering over one or both supports as shown in Fig. 12(c), and finally crescent or arched trusses as shown in Fig. 12(d) and Fig. 12(e).

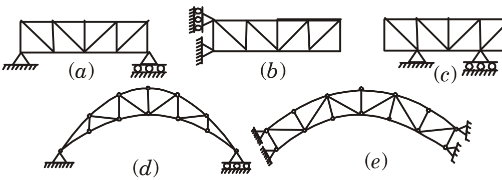


Fig. 12. Trusses-depending on the type of supports.

4. **According to their Purpose :** The trusses may be classified as roof trusses, bridge trusses, those used in crane construction.
5. **According to the Level of the Road :**
   i. The trusses can be constructed so that the road is carried by the bottom chord joints as shown Fig. 13(*a*), or the upper chord joints as shown in Fig. 13(*b*).
   ii. Sometimes the road (lane) is carried at some intermediate level as shown in Fig. 13(*c*).

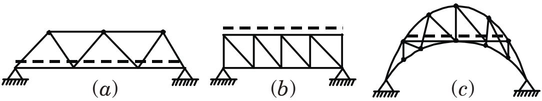


Fig. 13. Trusses depending on the level of the road.

f. Define the term tension coefficient method for plane truss with example.

Ans.

A. Tension Coefficient : Tension coefficient for a member is defined as the tension per unit length of the member. It is given by,

$$t = T/l$$

where, t = Tension coefficient,
T = Tension or pull in the member, and
l = Length of the member.

B. Method of Tension Coefficient for Analysis of Plane Trusses :

1. All the members of the truss are initially assumed to be under tension due to load upon the truss.

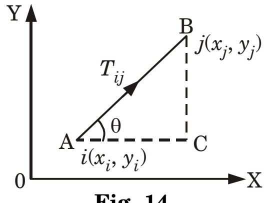


Fig. 14.

2. Let end A is at node i and end B is at node j, as shown in Fig. 14 and let the pull in the member AB is T_{ij}, then at node 'i', (at point A)

$$T_x = T_{ij} \cos \theta = T_{ij} \frac{AC}{AB} = T_{ij} \frac{(x_j - x_i)}{l_{ij}} = \frac{T_{ij} x_{ij}}{l_{ij}} = t_{ij} x_{ij} \qquad \left[\because t_{ij} = \frac{T_{ij}}{l_{ij}} \right]$$

where, l_{ij} = Length of $AB = \sqrt{x_{ij}^2 + y_{ij}^2}$

3. Similarly force at the node 'i' along Y direction, $T_y = t_{ij} \times y_{ij}$
4. Hence, if at node i there are more members and some external forces are also present then the condition of equilibrium at i can be written as,

$$\Sigma F_x = 0$$
$$\Sigma t_{ij} x_{ij} + F_x = 0$$
$$t_{ij} x_{ij} + t_{ik} x_{ik} + t_{iq} x_{iq} + ... + F_x = 0$$

and
$$\Sigma F_y = 0$$
$$\Sigma t_{ij} y_{ij} + F_y = 0$$
$$t_{ij} y_{ij} + t_{ik} y_{ik} + t_{iq} y_{iq} + ... + F_y = 0$$

Where F_x is the external load in the direction of X-axis and F_y is the external load in the direction of Y-axis.

6. In case of a space truss we have, $\Sigma t_{ij} x_{ij} + F_x = 0$

$$\Sigma t_{ij} y_{ij} + F_y = 0, \Sigma t_{ij} z_{ij} + F_z = 0$$

and
$$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2 + z_{ij}^2}$$

C. Example :

1. **Step - I :** Redraw the figure so that co-ordinate of various joints can be obtained that joint A is at origin.

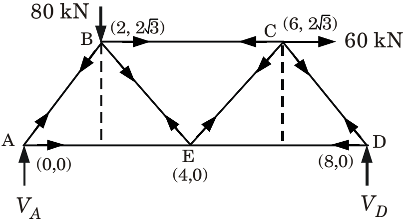


Fig. 15.

2. **Step - II :** Member parameters of truss.

S. No.	Member	x_i	x_j	$x_{ij} = (x_j - x_i)$	y_i	y_j	$y_{ij} = y_j - y_i$	$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}$
1.	*AB*	0	2	2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{2^2 + (2\sqrt{3})^2} = 4$
2.	*BC*	2	6	4	$2\sqrt{3}$	$2\sqrt{3}$	0	4
3.	*CD*	6	8	2	$2\sqrt{3}$	0	$-2\sqrt{3}$	$\sqrt{2^2 + (-2\sqrt{3})^2} = 4$
4.	*DE*	8	4	– 4	0	0	0	4
5.	*EA*	4	0	– 4	0	0	0	4
6.	*EB*	4	2	– 2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$
7.	*EC*	4	6	2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{2^2 + (2\sqrt{3})^2} = 4$

3. Step - III : Calculation of tension coefficients,

i. Taking moment about A,

$$8 \times V_B = 60 \times 2\sqrt{3} + 80 \times 2$$
$$V_B = 45.98 \approx 46 \text{ kN}$$

ii.
$$\sum F_y = 0$$
$$V_A + V_B - 80 = 0$$
$$V_A = 80 - 46$$
$$V_A = 34 \text{ kN}$$

iii.
$$\sum F_x = 0$$
$$H_A - 60 = 0$$
$$H_A = 60 \text{ kN}$$

iv. **Joint A :**

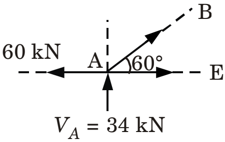


Fig. 16.

a. Resolve the forces horizontally,

$$\sum F_x = 0$$
$$x_{AB} \times t_{AB} + x_{AE} \times t_{AE} - 60 = 0$$
$$2 \times t_{AB} + 4 \times t_{AE} - 60 = 0$$
$$t_{AB} + 2 \times t_{AE} = 30 \quad ...(1)$$

b. Resolve the forces vertically,

$$\sum F_y = 0$$
$$y_{AB} \times t_{AB} + y_{AE} \times t_{AE} + 34 = 0$$
$$2\sqrt{3} \times t_{AB} + 0 = -34$$
$$t_{AB} = -9.82 \text{ kN/m}$$

c. Value of t_{AB} put in eq. (1), we get

$$-9.82 + 2\,t_{AE} = 30$$
$$2 \times t_{AE} = 30 + 9.82$$
$$t_{AE} = 19.91 \text{ kN/m}$$

v. **Joint D :**

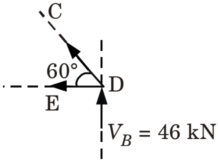


Fig. 17.

a. Resolve the forces horizontally,

$$\sum F_x = 0$$
$$x_{DC} \times t_{DC} + x_{DE} \times t_{DE} = 0$$

$$(-2) \times t_{DC} + (-4) \times t_{DE} = 0$$

$$t_{DC} = -2\, t_{DE} \quad ...(2)$$

b. Resolve the forces vertically,

$$\sum F_y = 0$$

$$y_{DC} \times t_{DC} + y_{DE} \times t_{DE} + 46 = 0$$

$$2\sqrt{3} \times t_{dc} + 0 + 46 = 0$$

$$t_{DC} = -\frac{46}{2\sqrt{3}} = -13.28 \text{ kN/m}$$

c. Value of t_{DC} put in equation (2), we get

$$-2\, t_{DE} = t_{DC}$$

$$-2\, t_{DE} = -13.28$$

$$t_{DE} = 6.64 \text{ kN/m}$$

vi. **Joint C :**

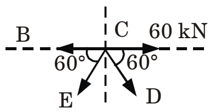


Fig. 18.

a. Resolve the forces horizontally,

$$\sum F_x = 0$$

$$x_{CB} \times t_{CB} + x_{CE} \times t_{CE} - x_{CD} \times t_{CD} - 60 = 0$$

$$(-4) \times t_{CB} + (-2) \times t_{CE} - 2 \times t_{CD} - 60 = 0$$

$$(-4) \times t_{CB} + (-2) \times t_{CE} - 2 \times (-13.28) - 60 = 0$$

$$2\, t_{CB} + t_{CE} = -16.72 \quad ...(2.15.3)$$

b. Resolve the forces vertically,

$$\sum F_y = 0$$

$$y_{CE} \times t_{CE} + y_{CD} \times t_{CD} = 0$$

$$(-2\sqrt{3})\, t_{CE} + (-2\sqrt{3}) \times (-13.28) = 0$$

$$t_{CE} = 13.28 \text{ kN/m}$$

c. Value of t_{CE} put in equation (2.15.3), we get

$$2\, t_{CB} + 13.28 = -16.72$$

$$t_{CB} = -15 \text{ kN/m}$$

vii. **Joint E :**

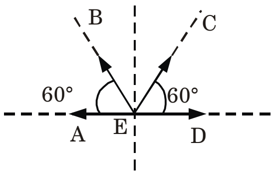


Fig. 19.

Resolve the forces vertically,

$$\sum F_y = 0$$

$$y_{EB} \times t_{EB} + y_{EC} \times t_{EC} = 0$$

$$(2\sqrt{3}) \times t_{EB} + (2\sqrt{3}) \times t_{EC} = 0$$
$$t_{EB} = -t_{EC}$$
$$t_{EB} = -13.28 \text{ kN/m}$$

4. **Step IV :** Final forces in members (Represented by T_{ij})

S. No.	Member	t_{ij}	l_{ij}	$T_{ij} = t_{ij} \times l_{ij}$	Nature of Force
1.	*AB*	– 9.82	4	– 39.28	Compression
2.	*BC*	– 15	4	– 60	Compression
3.	*CD*	– 13.28	4	– 53.12	Compression
4.	*DE*	6.64	4	26.56	Tension
5.	*EA*	19.91	4	79.64	Tension
6.	*BE*	– 13.28	4	– 53.12	Compression
7.	*CE*	13.28	4	53.12	Tension

2. Attempt any **two** of the following : **(2 × 10 = 20)**

a. **State the Muller Breslau's principle of influence line. Draw ILD for shear at point *C*, support reactions for following given simply supported beam.**

Ans.

A. **Muller Breslau's Principle :**

1. Muller-Breslau developed a technique for rapidly constructing the shape of an influence line which is termed as Muller-Breslau's principle.
2. According to this principle "the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function".

B. **Influence Line Diagrams for Simply Supported Beam :** Here influence line diagrams for reactions at support *A*, support *B* and shear force and bending moment at a section a distance *z* from the end *A* are drawn.

1. **ILD for Reaction V_A :** Let the unit load be at a distance *x* from support *A* as shown in Fig. 20.

Now, $V_A = \dfrac{1(L-x)}{L} = \left(1 - \dfrac{x}{L}\right)$, linear variation with x

when $x = 0$, $V_A = 1$

when $x = L$, $V_A = 0$

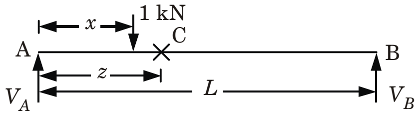


Fig. 21. Beam with unit load.

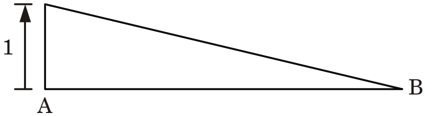


Fig. 22. ILD for V_A.

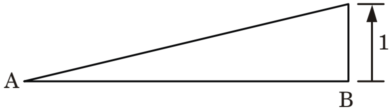


Fig. 23. ILD for V_B.

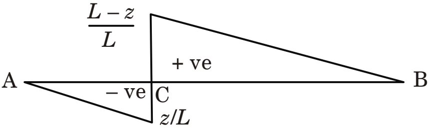


Fig. 24. ILD for S_C.

2. **ILD for Reaction V_B :**
   Referring to Fig. 21.

$$V_B = \frac{x}{L}, \text{ linear variation}$$

At $x = 0$, $V_B = 0$

At $x = L$, $V_B = 1$

Hence, ILD for V_B is shown in Fig. 23.

ILD for Share Force at C :

Let C be the section at a distance z from A as shown in Fig. 21.

i. When $x < z$

$$S_C = -V_B = -\frac{x}{L}, \text{ linear variation}$$

when $x = 0$, $S = 0$

when $x = z$, $S = -\frac{z}{L}$

ii. When $x > z$

$$S_C = V_A = \frac{L-x}{L}, \text{ linear variation}$$

when $x = z$, $S_c = \frac{L-z}{L}$

when $x = L$, $S_c = 0$

Therefore, ILD for shear force at C is shown in Fig. 24.

b. Three wheel loads 20 kN, 80 kN and 80 kN spaced 4 m apart from each other, with the 20 kN in the lead, pass over a simply supported beam of span 20 m. Determine the absolute maximum shear force and moment. Consider that loading can move in either direction.

Ans.

Given : $W_1 = 80\text{kN}$, $W_2 = 80\text{kN}$, $W_3 = 20\text{kN}$,
Span of beam, $L = 20$ m, Distance between points loads, $a = 4$ m
To Find : Absolute maximum SF and BM.

1. Maximum Positive Shear Force :

i. Ist Trial : Place the load 80 kN at A as shown in Fig. 25.

$$\frac{W_1}{a} = \frac{80}{4} = 20$$

$$\frac{W_2 + W_3}{L} = \frac{80 + 20}{20} = 5$$

Since, $\dfrac{W_1}{a} > \dfrac{W_2 + W_3}{L}$

There is no need for any trial.

ii. Hence, Maximum positive shear force,

$$\text{SF} = 80 \times 1 + 80 \times \frac{16}{20} + 20 \times \frac{12}{20} = 156 \text{ kN}$$

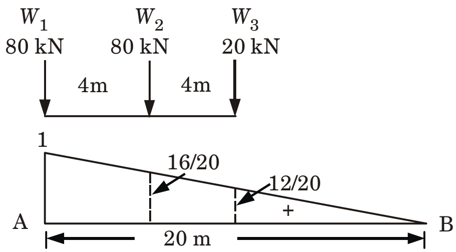


Fig. 25. ILD for maximum positive shear force.

2. Maximum Negative Shear Force :

i. Ist Trial : Place the loading as shown in Fig. 26.

$$\frac{W_3}{a} = \frac{20}{4} = 5$$

$$\frac{W_1 + W_2}{L} = \frac{80 + 80}{20} = 8$$

$$\frac{W_3}{a} < \frac{W_1 + W_2}{L}$$

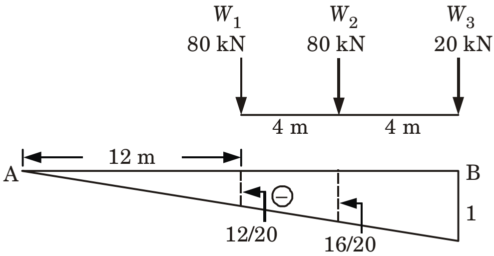


Fig. 26. ILD for maximum negative shear force.

Hence, we have to go II[nd] trial.

ii. II[nd] trial : Place the loading as shown in Fig. 27.

$$\frac{W_2}{a} = \frac{80}{4} = 20$$

$$\frac{W_1}{L} = \frac{80}{20} = 4$$

Since, $\frac{W_2}{a} > \frac{W_1}{L}$,

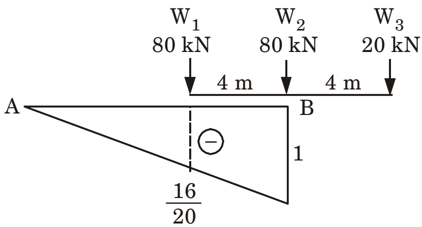


Fig. 27. ILD for maximum negative shear force.

Hence, maximum negative shear force,

$$\text{SF} = 80 \times \left(\frac{16}{20}\right) + 80 \times 1 = 144 \text{ kN}$$

3. Absolute Bending Moment :

i. To obtain absolute bending moment, firstly we have to find out the position of the resultant of given wheel loading.

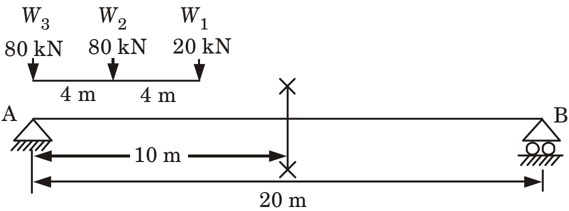


Fig. 28. Beam and the rolling loads.

ii. The absolute maximum bending moment will occur near the mid span and will occur under the W_2 (80 kN) load.

iii. Hence, for the condition of absolute maximum bending moment the load system should be so placed on the span that the resultant of all the wheel loads and the 80 kN load are equidistant from the middle point of the girder.

iv. Let us determine the position of the resultant of all the wheel loads with respect to W_1 (20 kN). Let the distance of the resultant load from the W_1 (20 kN) load be $\bar{x}$.

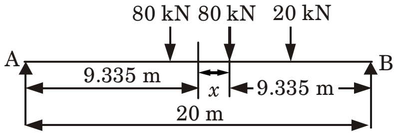
v. Taking moment about lasting 80 kN load, we have,

$$(20 + 80 + 80) \times \bar{x} = 20 \times 0 + 80 \times 4 + 80 \times 8$$

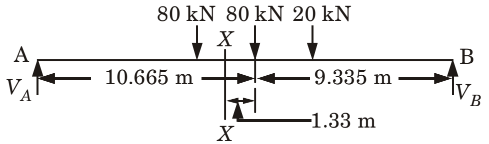
$$\bar{x} = 5.33 \text{ m}$$

vi. Distance between the resultant load and the load W_2 (80 kN) = 5.33 – 4 = 1.33 m

vii. Hence, for the condition of absolute moment, the load W_2 should be placed $\left(\frac{1.33}{2}\right)$ = 0.665 m on the right side of the centre of girder.



(a) Position of rolling load for maximum BM at section *X-X*.



(b) Position of loads for absolute maximum BM.

Fig. 29.

viii. Taking moment about the end A, we get,

$$V_B \times 20 = 180 \times 9.335$$

$$V_B = 84.015 \text{ kN}$$

and $V_A = 180 - 84.015 = 95.985$ kN

$\therefore$ Absolute maximum bending moment for the girder = BM under the W_2 (80 kN) load

$$= 84.015 \times 9.335 - 20 \times 4 = 704.28 \text{ kN-m}$$

c. A single load of 100 kN moves on a girder or span 20 m. Construct the influence line for shear force and bending moment for a section 5 m from the left support.

Ans.

Given : Concentrated Load, w = 100 kN, Span of beam, L = 20 m
Distance of section = 5 m from left support
To Find : Make ILD for SF and BM at given section.

A. Construction of ILD for Shear Force :

1. Let a unit load move along the span of a simply supported girder AB of span L.
   Let D be a given section. Let
   $$AD = a \text{ and } BD = b$$
2. When the unit load is between A and D
   SF at D, $S_D = -V_B$
3. But we know V_B varies from 0 to 1 as the load moves from A to B. The influence line diagram for V_B is drawn. But as long as the unit load is between A and B, $S_A = V_B$. Hence the part of the influence line diagram for V_B between A and B is also applicable to S_D.
4. Similarly, when the unit load is between D and B the SF at D,
   $$S_D = +V_A$$
   We know V_A varies from 1 to 0 as the load moves from A to B. The influence line diagram for V_A is drawn. The part of this diagram between D and B is applicable to S_D.
5. The part of the influence line diagram for V_B between A and D and the part of the influence line diagram for V_A between D and B will constitute the influence line diagram for S_D.
6. Maximum negative shear force,
   $$= \frac{W \times a}{L} = 100 \times \frac{5}{20} = 25 \text{ kN}$$

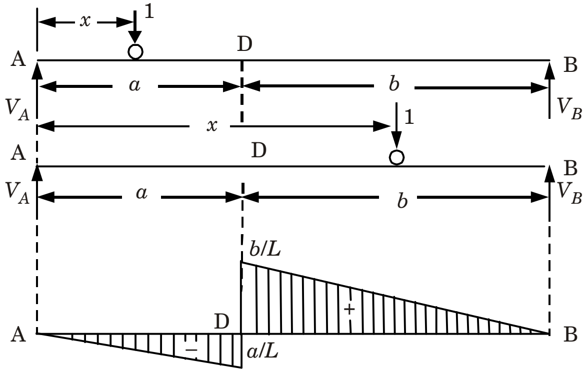


Fig. 30. ILD for V_D.

7. For part C to B :
   Maximum positive shear force
   $$= \frac{W \times b}{L} = 100 \times \frac{15}{20} = 75 \text{ kN}$$

B. Construction of ILD for Bending Moment :

1. Fig. 31 shows a simply supported girder AB of span L. Let a unit load move from the end A to the end B of the girder.
2. Let D be a given section of the girder so that,

$$AD = a \text{ and } DB = L - a = b$$

3. Let the unit load be at a distance x from the left end A. The reactions at the supports A and B are given by,

$$V_A = \frac{L-x}{L}$$

and $$V_B = \frac{x}{L}$$

4. When the unit load is between A and D.
   The bending moment at D is given by,

$$M_D = V_B(L-a) = \frac{x}{L}(L-a)$$

or, $$M_D = \left[\frac{L-a}{L}\right]x$$

   The above relation is true for all load positions from A to D, *i.e.*, for values of x from $x = 0$ to $x = a$.

5. When the unit load is at A, *i.e.*, when $x = 0$,
   $M_D = 0$
   When the unit load is at D, *i.e.*, when $x = a$

$$M_D = \left(\frac{L-a}{L}\right)a$$

   Hence, as the unit load moves from A to D, the bending moment at D will vary from 0 to $\dfrac{a(L-a)}{L}$

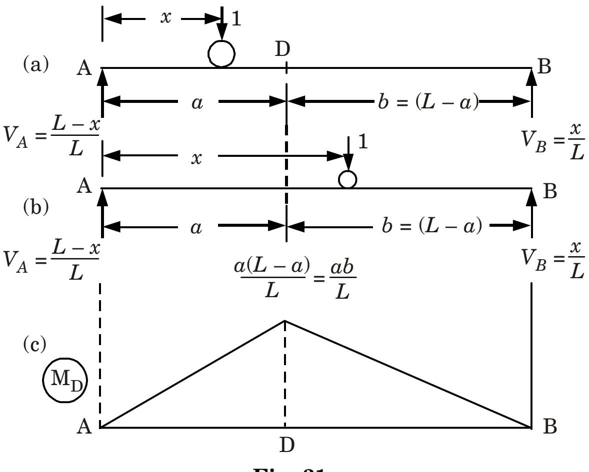


Fig. 31.

6. When the unit load is between D and B, the bending moment at D is given by, $M_D = V_A\, a$

$$M_D = \left(\frac{L-x}{L}\right) a$$

The above relation is true for all load positions of the unit load from D to B *i.e.*, for values of x from $x = a$ to $x = L$.

7. When the unit load is at D, *i.e.*, when $x = a$, $M_D = \dfrac{a}{L}(L-a)$

   When the unit load is at B, *i.e.*, when $x = L$, $M_D = 0$
   Hence, as the unit load moves from D to B the bending moment at D will vary from $\dfrac{a(L-a)}{L}$ to 0.

8. Ordinate of BM at section D,

$$\frac{a\,(L-a)}{L} = \frac{5 \times 15}{20} = 3.75 \text{ m}$$

   Maximum BM = $100 \times 3.75 = 375$ kN-m
   Value of BM varies from 0 to 375 kN-m and 375 kN-m to 0.

3. Attempt any **two** of the following : **(2 × 10 = 20)**

a. A three hinged parabolic arch has a span 40 m and rise of 10 m. Draw influence line diagram for the following :

1. Horizontal thrust.

2. BM at 8 m from the left support.

3. Normal thrust at the above section.

4. Radial shear at the above section.

Ans.

Given : Span of three hinged parabolic arch, L = 40m, rise, h = 10 m

To Find : ILD for : 1. Horizontal thrust.
2. BM at 8 m from the left support.
3. Normal thrust at the above section.
4. Radial shear at the above section.

1. Influence Line Diagram for Horizontal Thrust, *H* :

i. Let a unit load be at any section between the left end A and the crown C, at a distance x from A. Obviously the vertical reactions at the supports will be,

$$V_B = \frac{x}{L} \text{ and } V_A = \frac{L-x}{L}$$

ii. Let H be the horizontal thrust,
Taking moments about the hinge C, we get

$$H \times h = \frac{x}{L} \times \frac{L}{2}$$

$$\therefore \qquad H = \left(\frac{1}{2h}\right) x$$

iii. Thus is true for all position of load from A to C.

When $x = 0, \quad H = 0$

When $x = \frac{L}{2}, \quad H = \frac{L}{4h}$

iv. Hence as the unit load moves from A to C the horizontal thrust will change from zero to $\frac{L}{4h}$.

v. Obviously as the unit load moves from C to B the horizontal thrust change from $\frac{L}{4h}$ to zero.

vi. ILD for horizontal thrust is shown in Fig. 32(b). This is an isosceles triangle having an latitude equal to $\frac{L}{4h} = \frac{40}{4 \times 10} = 1$ unit.

2. Influence Line for BM at the Section *D*, 8 metres from the Left End :

i. BM at D consists of :

a. The beam moment at D *i.e.*, the bending moment at D treating the span as that of a simply supported beam, and

b. The H moment at D equal to yH which is the moment at D due to the horizontal thrust.

ii. Influence line diagram for the beam moment at D is a triangle having an altitude of

$$\frac{x(L-x)}{L} = \frac{8 \times (40-8)}{40} = 6.4 \text{ units}$$

iii. The ordinate at D for the arch,

$$y = \frac{4h}{L^2}\, x(L-x) = \frac{4 \times 10}{40 \times 40} \times 8(40-8)$$

$$y = 6.4 \text{ m}$$

iv. The influence line diagram for the H moment at D is a triangle having an altitude of $\frac{Ly}{4h}$,

$$\frac{Ly}{4h} = \frac{40 \times 6.4}{4 \times 10} = 6.4 \text{ units}$$

v. When the arch is parabolic the altitudes of the ILD for the beam moment at D and the ILD for the H moment at D are equal.

vi. By superimposing the diagrams the net ILD for the BM at D is obtained.

3. Influence Line for Normal Thrust at *D* :

i. Let θ be the inclination of the tangent at D with the horizontal then, $\tan \theta = \frac{dy}{dx}$ at D.

ii. The equation to the centre line of the arch is given by,

$$y = \frac{4h}{L^2}x(L-x)$$

iii. Slope at any section, $\frac{dy}{dx} = \frac{4h}{L^2}(L-2x)$

iv. Slope of tangent at D ($x = 8$ m),

$$\tan\theta = \frac{4h}{L^2}(L-2x) = \frac{4\times 10}{40\times 40}(40-2\times 8)$$

$$\therefore \quad \tan\theta = \frac{3}{5}, \quad \sin\theta = \frac{3}{\sqrt{34}} = 0.515 \text{ and}$$

$$\cos\theta = \frac{5}{\sqrt{34}} = 0.858$$

v. When the unit load is between A and D,

Normal thrust at D, $N_D = H\cos\theta - V_B\sin\theta$

vi When the unit load is between D and B.

Normal thrust at D, $N_D = H\cos\theta + V_A\sin\theta$.

Influence lines for the quantities $H\cos\theta$, $V_B\sin\theta$ and $V_A\sin\theta$ are drawn.

vii. Between A and D the ILD for $H\cos\theta$ and $V_B\sin\theta$ are superimposed.

ix. Between D and B the ILD for $H\cos\theta$ and $V_A\sin\theta$ are added *i.e.*, combined.

4. Influence Line for Radial Shear at *D* :

i. When the unit load is between A and D, radial shear at D,

$$S = H\sin\theta + V_B\cos\theta$$

ii. When the unit load is between D and B, radial-shear at D,

$$S = H\sin\theta - V_A\cos\theta$$

iii. Influence lines for the quantities $H\sin\theta$, $V_B\cos\theta$ and $V_A\cos\theta$ are drawn.

iv. Between A and D the ILD for $H\sin\theta$ and $V_B\cos\theta$ are added *i.e.*, combined. Between D and B the ILD for $H\sin\theta$ and $V_A\cos\theta$ are superimposed.

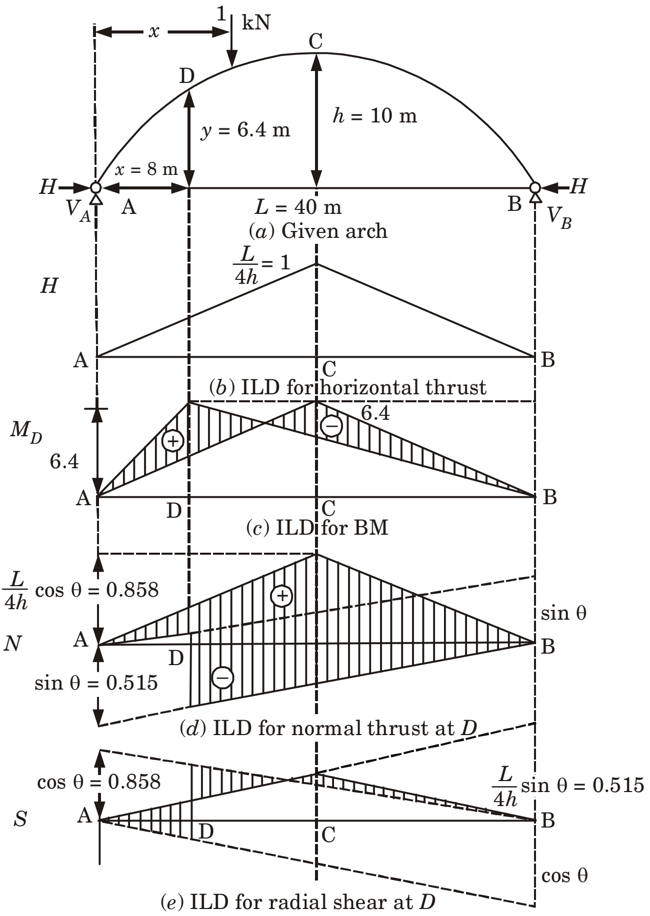


Fig. 32.

b. A three hinged semicircular arch of radius *R* carries a UDL of *w* per run over the whole span. Find

A. Horizontal thrust.

B. Location and magnitude of maximum bending moment.

Ans.

A. Horizontal Thrust :

1. Let the horizontal thrust at each support be H.
2. Due to symmetry, each vertical reaction = wR.

3. We know that for a three hinged arch of span L and rise h carries a UDL of w per unit run over the whole span, the horizontal thrust at each support is $\dfrac{wL^2}{8h}$.

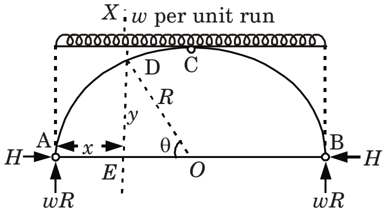


Fig. 33.

Horizontal thrust,

$$H = \frac{wL^2}{8h} = \frac{w(2R)^2}{8R} = \frac{wR}{2}$$

B. Location and Magnitude of Maximum Bending Moment :

1. The bending moment at any section X-X, the radius vector corresponding to which makes an angle θ with the horizontal is given by,

$$M_x = wRx - \frac{wx^2}{2} - Hy \qquad ...(1)$$

2. From Δ ODE, $x = R\,(1 - \cos\theta)$,
$$y = R \sin\theta$$
3. Value of x and y put in eq. (1), we get

$$M_x = wR \times R\,(1 - \cos\theta) - \frac{wR^2(1-\cos\theta)^2}{2} - \frac{wR}{2} \times R\sin\theta$$

$$\therefore M_x = wR^2\,(1 - \cos\theta) - \frac{wR^2}{2}(1 - \cos\theta)^2 - \frac{wR^2}{2}\sin\theta$$

$$= \frac{wR^2}{2}(1-\cos\theta)\,[2-(1-\cos\theta)] - \frac{wR^2}{2}\sin\theta$$

$$= \frac{wR^2}{2}(1-\cos\theta)\,(1+\cos\theta) - \frac{wR^2}{2}\sin\theta$$

$$= \frac{wR^2}{2}(1-\cos^2\theta) - \frac{wR^2}{2}\sin\theta$$

$$M_x = \frac{wR^2}{2}\sin^2\theta - \frac{wR^2}{2}\sin\theta = -\frac{wR^2}{2}[\sin\theta - \sin^2\theta]$$

Since sin θ being greater than $\sin^2\theta$ the bending moment at any section of the arch is a negative or a hogging moment.

4. For M_x to be a maximum,

$$\frac{dM_x}{d\theta} = 0$$

$$-\frac{wR^2}{2}[\cos\theta - 2\sin\theta\cos\theta] = 0$$

$\cos\theta\,(1 - 2\sin\theta) = 0$

Either, $\cos\theta = 0$ or $1 - 2\sin\theta = 0$

$\theta = 90°$ or $\theta = 30°$

But at $\theta = 90°$ *i.e.*, at the crown the bending moment equals to zero.

5. The bending moment is maximum at $\theta = 30°$

$$M_{max} = -\frac{wR^2}{2}(\sin 30° - \sin^2 30°) = -\frac{wR^2}{8}$$

6. Distance of the point of maximum bending moment from the centre of semicircle,

$$OE = R\cos 30° = \frac{R\sqrt{3}}{2}$$

7. Distance from left support,

$$x = R - OE = R(1 - \cos 30°) = R\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{R}{2}(2 - \sqrt{3})$$

c. A three hinged parabolic arch of span 50 m and rise 9 m carries a load whose intensity varies 25 kN/m at the crown to 50 kN/m at the ends. Find the following at a section *D*, 10 m from left end.

1. Bending moment.

2. Normal thrust.

3. Radial shear.

Ans.

Given : Span of arch, $L = 50$ m, Rise of arch, $h = 9$ m,
Intensity of load at crown = 25 kN/m
Intensity of load at support A and B = 50 kN/m
To Find : BM, normal thrust, radial shear.

1. $\Sigma F_y = 0$

$$V_A + V_B = 25 \times 50 + \frac{1}{2} \times 25 \times 25 \times 2$$

$$V_A + V_B = 1875 \text{ kN}$$

2. Taking moment about point A, $\Sigma M_A = 0$

$$-V_B \times 50 + 25 \times 50 \times 25 + \frac{1}{2} \times (25 \times 25) \times \left(25 + \frac{2}{3} \times 25\right) + \frac{1}{2} \times 25 \times 25 \times \frac{25}{3} = 0$$

$$V_B = 937.5 \text{ kN}$$

$$V_A = 937.5 \text{ kN}$$

3. Taking moment about the crown 'C' (from left side),

$$H \times 9 + \left(25 \times 25 \times \frac{25}{2}\right) + \left(\frac{1}{2} \times 25 \times 25\right) \times \left(\frac{2}{3} \times 25\right) = 937.5 \times 25$$

$$H = 1157.4 \text{ kN}$$

4. Vertical height of a arch at a distance 10 m from support A,

$$y = \frac{4h}{L^2} x\,(L - x) = \frac{4 \times 9 \times 10}{50^2} \times (50 - 10) = 5.76 \text{ m}$$

5. Intensity of load at a distance 10 m from support $A = 15 \times \dfrac{25}{25}$

$$= 15 \text{ kN}$$

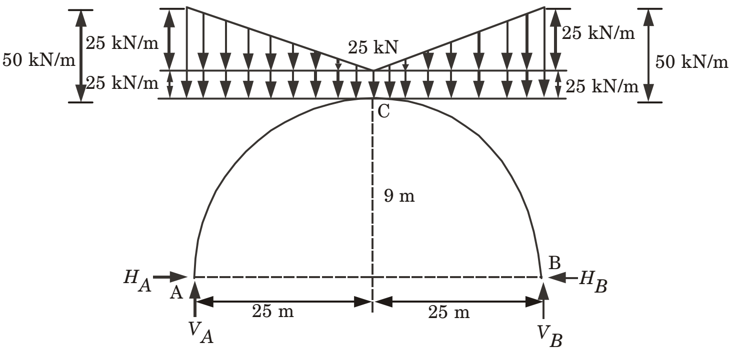


Fig. 34.

6. Bending moment at a distance 10 m from the left support,

$$M_{10} = V_A \times 10 - 40 \times 10 \times \frac{10}{2} - \frac{1}{2} \times 10 \times 10 \times \frac{2}{3} \times 10 - H \times 5.76$$

$$M_{10} = 937.5 \times 10 - 40 \times 10 \times \frac{10}{2} - \frac{1}{2} \times 10 \times 10 \times \frac{2}{3} \times 10 - 1157.4 \times 5.76$$

$$M_{10} = 375 \text{ kN-m}$$

7. Slope of arch at a section is given by,

$$\tan\theta = \frac{dy}{dx} = \frac{4h\,(L - 2x)}{L^2} = \frac{4 \times 9 \times (50 - 2 \times 10)}{50^2} = 0.432$$

$$\tan\theta = 0.432$$

$$\theta = 23.36^{\circ}$$

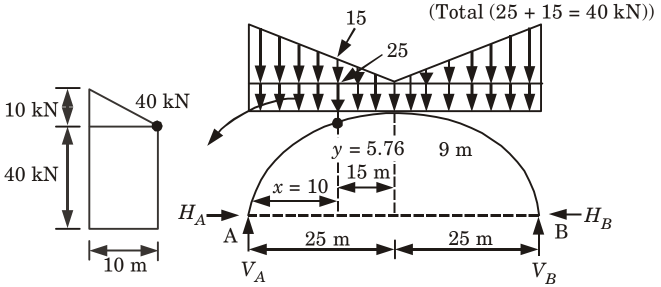


Fig. 35.

8. Vertical shear at (x = 10 m) section,

$$\text{Total vertical load at section} = 40 \times 10 + \frac{1}{2} \times 10 \times 10$$
$$= 400 + 50 = 450 \text{ kN}$$
$$V_D = V_A - \text{Downward Load}$$
$$V_D = 937.5 - 450$$
$$V_D = 487.5 \text{ kN}$$

9. Normal thrust, $N = H \cos \theta + V_D \sin \theta$
$$N = 1157.4 \times \cos 23.36° + 487.5 \times \sin 23.36°$$
$$N = 1255.83 \text{ kN}$$

10. Radial Shear, $S = H \sin \theta - V_D \cos \theta$
$$= 1157.4 \times \sin 23.36 - 487.5 \times \cos 23.36°$$
$$S = 11.38 \text{ kN}$$

4. Attempt any **two** of the following : **(2 × 10 = 20)**

a. Write statement of Castigliano's first theorem and Maxwell's reciprocal theorem. Prove Maxwell's theorem.

Ans.

A. Castigliano's First Theorem :

Statement : "The partial derivative of the total strain energy in a structure with respect to the displacement at any one of the load points gives the value of corresponding load acting on the body in the direction of displacement"

$$P_i = \frac{\partial U}{\partial \Delta_i}$$

B. Maxwell's Law Statement :

1. In any structure whose material is elastic and obeys Hooke's law and whose supports remain unyielding and the temperature remains unchanged, the deflection at any point D (*i.e.*, Δ_d) due to a load W acting at any other point C is equal to the deflection at any point C (*i.e.*, δ_c) due to the load W acting at the point D.

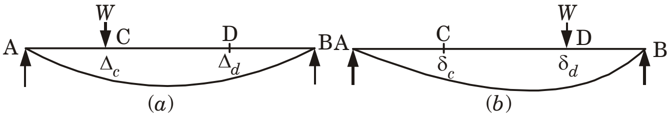


Fig. 36.

2. In Fig. 36(a) beam AB carries a load W at point C. Let the deflection at C be Δ_c and when load W is acting at point D. Let the defection at point D is δ_d then according to law of reciprocal deflections or Maxwell's reciprocal deflection theorem we have

$$\delta_d = \Delta_c$$

Proof :

1. Let initially a load W is acting at a point C and deflects the beam AB by deflection Δ_c under the load W.
2. Work done on the structure $= \frac{1}{2} \times W\Delta_c$
3. Let an another equal load W is acting at point D. Due to this load there will be further deflections of δ_c and δ_d at C and D.
4. So the total work done at this stage $= \frac{1}{2}\ W\Delta_c + \frac{1}{2} \times W\,\delta_d + W\delta_c$ ...(1)

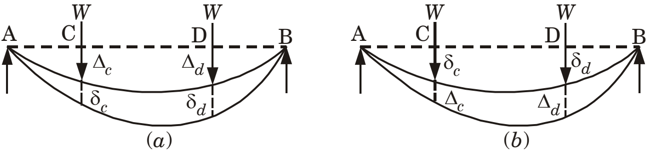


Fig. 37.

5. Now change the order of loading as shown in Fig. 37(b). Now consider initially load W is at D. Due to this load,

   Work done $= \frac{1}{2}\ W\,\delta_d$

6. Let another load W (same value) is applied at C so further deflections of Δ_c and Δ_d will occur at C and D respectively.

   So total work done at this stage $= \frac{1}{2}\ W\delta_d + \frac{1}{2}\ W\Delta_c + W\Delta_d$ ...(2)

7. As both the eq. (1) and eq. (2) represents the same stage, hence,

$$\frac{1}{2}W\delta_d + \frac{1}{2}W\Delta_c + W\delta_c = \frac{1}{2}W\delta_d + \frac{1}{2}W\Delta_c + W\Delta_d$$

$$\delta_c = \Delta_d$$

8. The deflection at C due to the load W at D equal to the deflection at D due to the same load W at C.

b. Determine the deflection at free end of a cantilever beam.

Ans.

1. Consider a cantilever beam AB of length L carrying a point load W at free end.

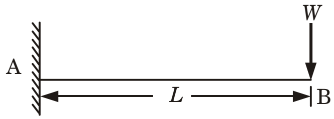


Fig. 38.

2. Bending moment diagram for cantilever is shown in Fig. 39.

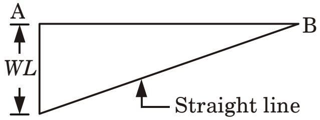


Fig. 39. BM diagram.

Corresponding conjugate beam whose load diagram is the $\frac{M}{EI}$ diagram.

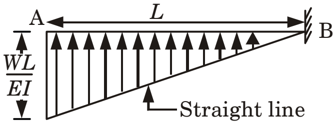


Fig. 40. Conjugate beam diagram (M/EI diagram).

3. **Deflection at Free End in Cantilever Beam :**
Deflection at B for the given beam = BM at B for conjugate beam.

$$\Delta_B = \frac{1}{2} \times WL \times L \times \frac{2L}{3} = \frac{WL^3}{3}$$

c. **Using conjugate beam method find the deflection of a simply supported beam at point *C*. *AB* of length 10 m loaded by an UDL of intensity 20 kN per unit run.**

Ans.

Given : Span = 10 m, UDL = 20 kN/m
To Find : Deflection at *C*.

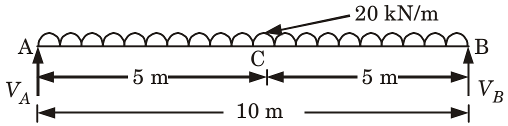


Fig. 41.

1. **Reaction at Supports :**
Assume 'C' point is the midpoint of the beam at a distance 5 m from each support.

Due to symmetry, $V_A = V_B = \frac{20 \times 10}{2} = 100$ kN

2. Bending Moment Diagram :

Bending moment at C, $M_C = 100 \times 5 - 20 \times 5 \times \frac{5}{2} = 250$ kN-m

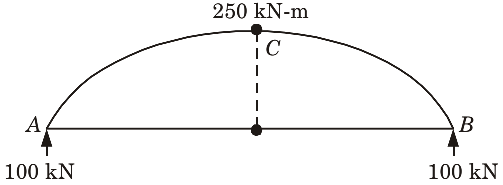


Fig. 42. BMD.

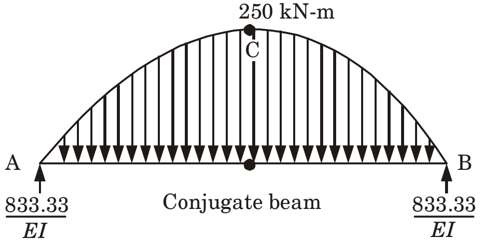


Fig. 43. M/EI diagram.

3. Total load on the conjugate beam = Area of the load diagram on the conjugate beam

$$= \frac{1}{EI} \times \frac{2}{3} \times 10 \times 250 = \frac{1666.67}{EI}$$

Reaction at each end of the conjugate beam $= \frac{1666.67}{EI} \times \frac{1}{2} = \frac{833.33}{EI}$

4. Deflection at the Centre of the Beam :

Δ_C = BM at the centre of the conjugate beam

Deflection at $C, \Delta_C = \frac{833.33}{EI} \times \frac{10}{2} - \frac{833.33}{EI} \times \frac{3}{8} \times \frac{10}{2} = \frac{2604.16}{EI}$

5. Attempt any **four** of the following : **(4 × 5 = 20)**

a. A 60 mm × 40 mm × 6 mm unequal angle is placed with the longer leg vertical and is used as a simply supported at the ends. Over a span of 2 m, if it carries UDL of such magnitude so as to produce the maximum bending moment of 0.12 kN-m, determine the maximum deflection of the beam. Take $E = 2.1 \times 10^5$ N/mm².

Ans. This question is out of syllabus from session 2017-18.

b. Locate the position of the shear centre for the channel section shown in Fig. 44.

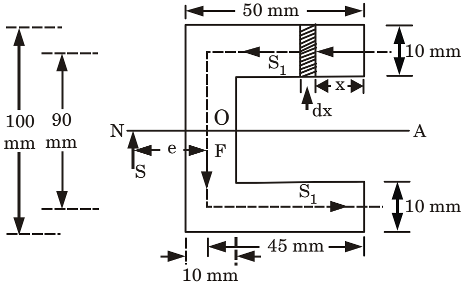


Fig. 44.

Ans. This question is out of syllabus from session 2017-18.

c. What do you mean by bending of curved bars ? Derive the relevant expression for the bending of curved bars with small initial curvature.

Ans. This question is out of syllabus from session 2017-18.

d. Define shear centre. Write down the principle of second moments of area with proof.

Ans. This question is out of syllabus from session 2017-18.

e. "Discuss about the Mohr's circle with respect to unsymmetrical bending".

Ans. This question is out of syllabus from session 2017-18.

f. Explain Winkler-Bach theory.

Ans. This question is out of syllabus from session 2017-18.

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B. Tech.
(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2015-16
STRUCTURAL ANALYSIS–I

Time : 3 Hours **Max. Marks : 100**

Section– A

1. Attempt **all** parts. All parts carry equal marks. Write answer of each part in short. **(10 × 2 = 20)**

a. **Differentiate static and kinematic indeterminacy of structure.**

b. **State degree of freedom.**

c. **Define real work method.**

d. **A cantilever beam of length *L* subjected to a concentrated load *P* at the free end. What is the deflection at the free end ?**

e. **Differentiate between determinate and indeterminate structures.**

f. **State Muller-Breslau's principle for determinate structures.**

g. **Give the equation for a parabolic arch whose springing is at different levels.**

h. **State Eddy's theorem as applicable to arches.**

i. **Mention any three reasons due to which sway may occur in portal frames.**

j. **Define shear centre.**

Section-B

2. Attempt any **five** questions from this section. **(5 × 10 = 50)**

a. **Explain in detail about method of substitution and method of tension coefficient with examples.**

b. Determine the deflection and rotation at the free end of the cantilever beam shown in Fig. 1. Use unit load method. Given $E = 2 \times 10^5$ N/mm², and $I = 12 \times 10^6$ mm⁴.

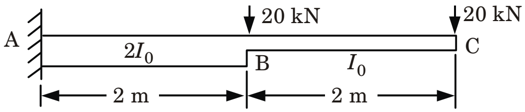

Fig. 1.

c. State and prove the Maxwell's reciprocal theorem.

d. Determine the vertical deflection at point C in the frame shown in Fig. 2. Given $E = 200$ kN/mm² and $I = 30 \times 10^6$ mm⁴.

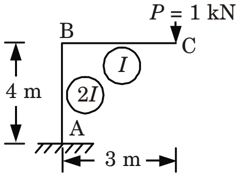

Fig. 2.

e. A simply supported beam has a span of 15 m. UDL of 40 kN/m and 5 m long crosses the girder from left to right. Draw the influence line diagram for shear force and bending moment at a section 6 m from left end. Use these diagrams to calculate the maximum shear force and bending moment at this section.

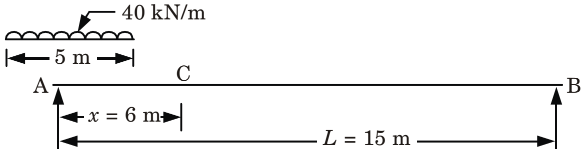

Fig. 3.

f. A circular arch to span 25 m with a central rise 5 m is hinged at the crown and springing. It carries a point load of 100 kN at 6 m from the left support. Calculate :

i. The reactions at the supports.

ii. The reactions at crown.

g. Show that the parabolic shape is a funicular shape for a three hinged arch subjected to a uniformly distributed load over its entire span.

h. Uniformly distributed load of intensity 30 kN/m crosses a simply supported beam of span 60 m from left to right. The length of UDL is 15 m. Find the value of maximum bending moment for a section 20 m from left end. Find also the absolute value of maximum bending moment and shear force in the beam.

Section-C

Note : Attempt any **two** questions from this section. **(2 × 15 = 30)**

3. a. Determine the horizontal displacement of the roller end *D* of the portal frame shown in Fig. 4. *EI* is 8000 kN-m^2 throughout.

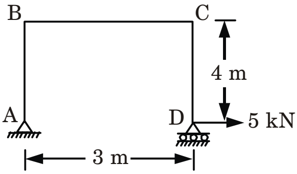

Fig. 4.

b. Fig. 5 shows a pin-joined truss loaded with a single load *W* is 100 kN. If the area of cross-section of all members shown in Fig. 5 is 1000 mm^2, what is the vertical deflection of point *C* ? Take *E* = 200 kN/mm^2 for all members.

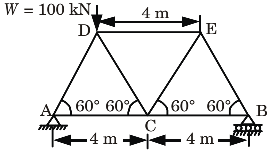

Fig. 5.

4. a. Four point loads 8, 15, 15 and 10 kN have centre to centre spacing of 2 m between consecutive loads and they traverse a girder of 30 m span from left to right with 10 kN load leading. Calculate the maximum bending moment and shear force at 8 m from the left support.

b. A three-hinged circular arch hinged at the springing and crown point has a span of 40 m and a central rise of 8 m. It carries a uniformly distributed load 20 kN/m over the left-half of the span together with a concentrated load of 100 kN at the right quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10 m from left support.

5. a. Analyze the truss shown in Fig. 6 by the method of tension coefficient and determine the forces in all the members.

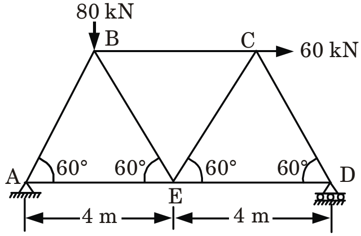

Fig. 6.

b. Determine the vertical deflection at the free end and rotation at *A* in the overhanging beam shown in Fig. 7. Assume constant *EI*. Use Castigliano's method.

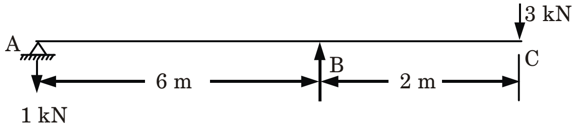

Fig. 7.

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SOLUTION OF PAPER (2015-16)

Section– A

1. Attempt **all** parts. All parts carry equal marks. Write answer of each part in short. **(10 × 2 = 20)**

a. Differentiate static and kinematic indeterminacy of structure.

Ans.

S. No.	Static Indeterminacy	Kinematic Indeterminacy
1.	Static indeterminacy of a structure is defined as the difference between number of unknown forces and number of equilibrium equations to be solved.	It is defined as the sum of all the possible displacements that various joints of the structure can undergo.
2.	Degree of static indeterminacy = $(m + r) - 2j$.	Degree of kinematic indeterminacy = Sum of degrees of freedom in rotation and translation.

b. State degree of freedom.

Ans. **Degree of Freedom :**

i. When a structure is loaded, specified points on it, called nodes, will undergo unknown displacements. These displacements are known as degree of freedom for the structure.

ii. In 2-dimension, each node can have atmost two linear displacements and one rotational displacement.

iii. In 3-dimension, each node on frame or beam can have atmost three linear displacements and three rotational displacements.

Example : In Fig. 1

i. Number of nodes = 3 (A, B and C)

ii. Here three rotational displacements like θ_A, θ_B and θ_C.

iii. One vertical displacement and one horizontal displacement at C, as Δ_C.

iv. Total number of displacements = Degree of freedom = 5

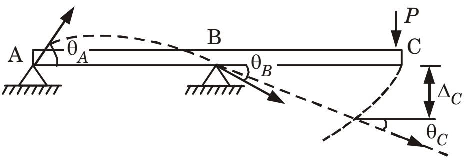

Fig. 1.

c. Define real work method.

Ans. The strain energy method is also known as real work method, since, work done by the actual loads are considered. From the law of conservation of energy, we can say,

$$\text{Strain energy, } U = \text{Real work done by loads} = \sum_{0}^{n} \frac{1}{2} P\Delta$$

d. A cantilever beam of length L subjected to a concentrated load P at the free end. What is the deflection at the free end ?

Ans. Deflection at free end, $\Delta = \dfrac{PL^3}{3EI}$

e. Differentiate between determinate and indeterminate structures.

Ans.

S. No.	Determinate Structures	Indeterminate Structures
i.	Conditions of equilibrium are sufficient to analyze the structure completely.	Conditions of equilibrium are insufficient to analyze the structure completely.
ii.	No stresses are caused due to temperature changes.	Stresses are generally caused due to temperature variations.
iii.	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.

f. State Muller-Breslau's principle for determinate structures.

Ans. According to this principle "the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function".

g. Give the equation for a parabolic arch whose springing is at different levels.

Ans. The equation is given by,

$$\frac{x^2}{y} = \text{Constant}$$

h. State Eddy's theorem as applicable to arches.

Ans. The bending moment at any section of an arch is equal to the vertical intercept between the linear arch and the central line of the actual arch.

i. Mention any three reasons due to which sway may occur in portal frames.

Ans. Sway in portal frames may occur due to :

i. Unsymmetry in geometry of the frame.
ii. Unsymmetry in loading.
iii. Settlement of one end of a frame.

j. Define shear centre.

Ans. This question is out of syllabus from session 2017-18.

Section-B

2. Attempt any **five** questions from this section. **(5 × 10 = 50)**

a. Explain in detail about method of substitution and method of tension coefficient with examples.

Ans.

A. Method of Substitution :

1. A complex truss, as shown in Fig. 2, has three or more connecting members at a node, all with unknown member forces.

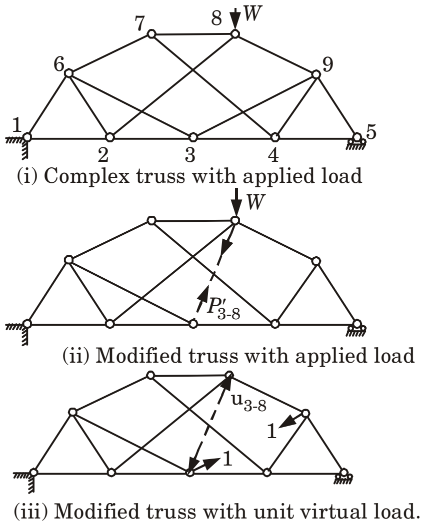

Fig. 2.

2. This precludes the use of the method of sections or the method of resolution at the nodes as a means of determining the forces in the truss.
3. The technique consists of removing one of the existing members at a node so that only two members with unknown forces remain and substituting another member so as to maintain the truss in stable equilibrium.
4. The forces in member 4-5 and 5-9 are obtained by resolution of forces at node 5. However, at nodes 4 and 9, three unknown member

forces remain, and these cannot be determined by resolution or by the method of sections.

5. As shown in Fig. 2(ii), member 3-9 is removed, leaving only two unknown forces at node 9, which may be determined.
6. To maintain stable equilibrium, a substitute member 3-8 is added to create a modified truss, and the original applied loads are applied to the modified truss.
7. The forces P' in all the remaining members of the modified truss may now be determined. The force in member 3-8 is $P'_{3\text{-}8}$.
8. The applied loads are now removed, and unit virtual loads are applied to the modified truss along the line of action of the original member 3-9, as shown in Fig. 2(iii).
9. The forces u in the modified truss is determined; the force in member 3-8 is $-u_{3\text{-}8}$.
10. Multiplying, the forces in system (iii) by P'_{3-8}/u_{3-8} and adding them to the forces in system (ii) gives the force in member 8 as :
$$P_{3-8} = P'_{3-8} + (-u_{3-8})P'_{3-8}/u_{3-8} = 0$$
11. In fact, the substitute member 3-8 has been eliminated from the truss. Hence, by applying the principle of superposition, the final forces in the original truss are obtained from the expression :
$$P = P' + uP'_{3-8}/u_{3-8}$$
where, tensile forces are positive and compressive forces are negative.
12. The final force in member 3-8 is :
$$P_{3-8} = 1 \times P'_{3-8}/u_{3-8}$$
$$= P'_{3-8}/u_{3-8}$$

Example :

1. The modified truss shown in Fig. 3 is created by removing member 3-9, adding the substitute member 3-8, and applying the 20 kN load.
2. The member forces in the modified truss may now be determined; the values obtained are :
$$P'_{4-9} = 7.51 \text{ kN (Tension)}$$
$$P'_{8-9} = -13.89 \text{ kN (Compression)}$$
$$P'_{3-8} = 21.54 \text{ kN (Tension)}$$
3. The 20 kN load is removed from the modified truss, and unit virtual loads are applied at nodes 3 and 9 in the direction of the line of action of the force in member 3-9.

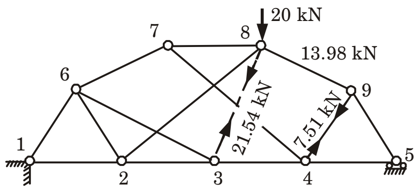

Fig. 3.

4. The member forces for this loading condition may now be determined; the values obtained are :

$$u_{4\text{-}9} = -0.81 \text{ kN (Compression)}$$
$$u_{8\text{-}9} = -0.50 \text{ kN (Compression)}$$
$$u_{3\text{-}8} = -1.93 \text{ kN (Compression)}$$

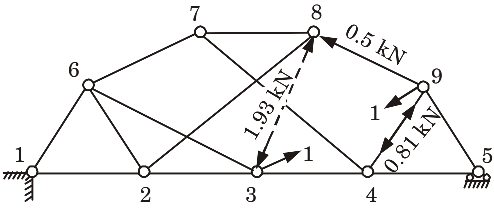

Fig. 4.

5. The multiplying ratio is given by :

$$P'_{3\text{-}8}/u_{3\text{-}8} = 21.54/1.93 = 11.16$$

6. The final member forces in the original truss are :

$$P_{4\text{-}9} = 7.51 + 11.16\,(-0.81)$$
$$= -1.53 \text{ kN (Compression)}$$
$$P_{8\text{-}9} = -13.98 + 11.16\,(-0.50)$$
$$= -19.56 \text{ kN (Compression)}$$
$$P_{3\text{-}9} = P'_{3\text{-}8}/u_{3\text{-}8} = 11.16 \text{ kN (Tension)}$$

B. Method of Tension Coefficient :

1. **Tension Coefficient :** Tension coefficient for a member is defined as the tension per unit length of the member. It is given by,

$$t = \frac{T}{l}$$

where, t = Tension coefficient,
T = Tension or pull in the member, and
l = Length of the member.

2. **Method of Tension Coefficient for Analysis of Plane Trusses :**

i. All the members of the truss are initially assumed to be under tension due to load upon the truss.

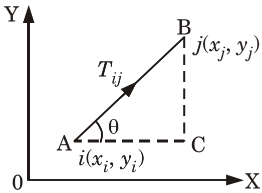

Fig. 5.

ii. Let end A is at node i and end B is at node j, as shown in Fig. 5 and let the pull in the member AB is T_{ij}, then at node 'i', (at point A)

$$T_x = T_{ij} \cos\theta = T_{ij}\,\frac{AC}{AB} = T_{ij}\,\frac{(x_j - x_i)}{l_{ij}} = \frac{T_{ij}x_{ij}}{l_{ij}} = t_{ij}x_{ij} \quad \left[\because\ t_{ij} = \frac{T_{ij}}{l_{ij}}\right]$$

where, l_{ij} = Length of $AB = \sqrt{x_{ij}^2 + y_{ij}^2}$

iii. Similarly force at the node 'i' along Y direction, $T_y = t_{ij} \times y_{ij}$

iv. Hence, if at node i there are more members and some external forces are also present then the condition of equilibrium at i can be written as,

$$\Sigma F_x = 0$$

$$\Sigma t_{ij} x_{ij} + F_x = 0$$

$$t_{ij} x_{ij} + t_{ik} x_{ik} + t_{iq} x_{iq} + \ldots + F_x = 0$$

v. And

$$\Sigma F_y = 0$$

$$\Sigma t_{ij} y_{ij} + F_y = 0$$

$$t_{ij} y_{ij} + t_{ik} y_{ik} + t_{iq} y_{iq} + \ldots + F_y = 0$$

Where F_x is the external load in the direction of X-axis and F_y is the external load in the direction of Y-axis.

vi. In case of a space truss we have, $\Sigma t_{ij} x_{ij} + F_x = 0$

$$\Sigma t_{ij} y_{ij} + F_y = 0,\ \Sigma t_{ij} z_{ij} + F_z = 0$$

and $$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2 + z_{ij}^2}$$

Example :

1. **Step - I :** Redraw the figure so that co-ordinate of various joints can be obtained that joint A is at origin.

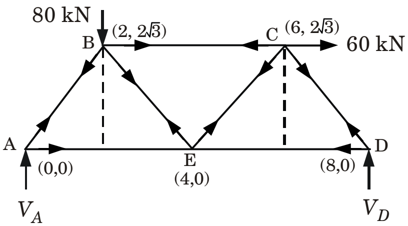

Fig. 6.

2. **Step - II :** Member parameters of truss.

S. No.	Member	x_i	x_j	$x_{ij} = (x_j - x_i)$	y_i	y_j	$y_{ij} = y_j - y_i$	$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}$
1.	*AB*	0	2	2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{2^2 + (2\sqrt{3})^2} = 4$
2.	*BC*	2	6	4	$2\sqrt{3}$	$2\sqrt{3}$	0	4
3.	*CD*	6	8	2	$2\sqrt{3}$	0	$-2\sqrt{3}$	$\sqrt{2^2 + (-2\sqrt{3})^2} = 4$
4.	*DE*	8	4	– 4	0	0	0	4
5.	*EA*	4	0	– 4	0	0	0	4
6.	*EB*	4	2	– 2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$
7.	*EC*	4	6	2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{2^2 + (2\sqrt{3})^2} = 4$

3. Step - III : Calculation of tension coefficients,

i. Taking moment about A, $M_A = 0$

$$8 \times V_B = 60 \times 2\sqrt{3} + 80 \times 2$$
$$V_B = 45.98 \approx 46 \text{ kN}$$

ii. Resolve the forces vertically,

$$\sum F_y = 0$$
$$V_A + V_B - 80 = 0$$
$$V_A = 80 - 46$$
$$V_A = 34 \text{ kN}$$

iii. Resolve the forces horizontally

$$\sum F_x = 0$$
$$H_A - 60 = 0$$
$$H_A = 60 \text{ kN}$$

iv. **Joint A :**

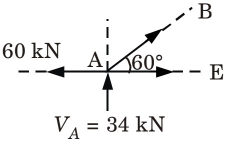

Fig. 7.

a. Resolve the forces horizontally,

$$\sum F_x = 0$$
$$x_{AB} \times t_{AB} + x_{AE} \times t_{AE} - 60 = 0$$
$$2 \times t_{AB} + 4 \times t_{AE} - 60 = 0$$
$$t_{AB} + 2 \times t_{AE} = 30 \qquad \ldots(1)$$

b. Resolve the forces vertically,

$$\sum F_y = 0$$
$$y_{AB} \times t_{AB} + y_{AE} \times t_{AE} + 34 = 0$$
$$2\sqrt{3} \times t_{AB} + 0 = -34$$
$$t_{AB} = -9.82 \text{ kN/m}$$

c. Value of t_{AB} put in eq. (1), we get

$$-9.82 + 2\, t_{AE} = 30$$
$$2 \times t_{AE} = 30 + 9.82$$
$$t_{AE} = 19.91 \text{ kN/m}$$

v. **Joint D :**

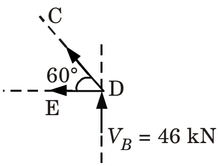

Fig. 8.

a. Resolve the forces horizontally,

$$\sum F_x = 0$$
$$x_{DC} \times t_{DC} + x_{DE} \times t_{DE} = 0$$
$$(-2) \times t_{DC} + (-4) \times t_{DE} = 0$$
$$t_{DC} = -2\, t_{DE} \quad \text{...(2)}$$

b. Resolve the forces vertically,

$$\sum F_y = 0$$
$$y_{DC} \times t_{DC} + y_{DE} \times t_{DE} + 46 = 0$$
$$2\sqrt{3} \times t_{dc} + 0 + 46 = 0$$
$$t_{DC} = -\frac{46}{2\sqrt{3}} = -13.28 \text{ kN/m}$$

c. Value of t_{DC} put in equation (2), we get

$$-2\, t_{DE} = t_{DC}$$
$$-2\, t_{DE} = -13.28$$
$$t_{DE} = 6.64 \text{ kN/m}$$

vi. **Joint C :**

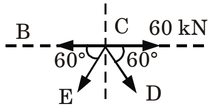

Fig. 9.

a. Resolve the forces horizontally,

$$\sum F_x = 0$$
$$x_{CB} \times t_{CB} + x_{CE} \times t_{CE} - x_{CD} \times t_{CD} - 60 = 0$$
$$(-4) \times t_{CB} + (-2) \times t_{CE} - 2 \times t_{CD} - 60 = 0$$
$$(-4) \times t_{CB} + (-2) \times t_{CE} - 2 \times (-13.28) - 60 = 0$$
$$2\, t_{CB} + t_{CE} = -16.72 \quad \text{...(2.15.3)}$$

Resolve the forces vertically,

$$\sum F_y = 0$$
$$y_{CE} \times t_{CE} + y_{CD} \times t_{CD} = 0$$
$$(-2\sqrt{3})\, t_{CE} + (-2\sqrt{3}) \times (-13.28) = 0$$
$$t_{CE} = 13.28 \text{ kN/m}$$

c. Value of t_{CE} put in equation (2.15.3), we get

$$2\, t_{CB} + 13.28 = -16.72$$
$$t_{CB} = -15 \text{ kN/m}$$

vii. **Joint E :**

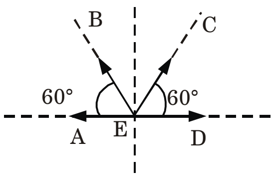

Fig. 10.

a. Resolve the forces vertically,

$$\sum F_y = 0$$

$$y_{EB} \times t_{EB} + y_{EC} \times t_{EC} = 0$$
$$(2\sqrt{3}) \times t_{EB} + (2\sqrt{3}) \times t_{EC} = 0$$
$$t_{EB} = -\, t_{EC}$$
$$t_{EB} = -\, 13.28 \text{ kN/m}$$

4. **Step IV :** Final forces in members (Represented by T_{ij})

S. No.	Member	t_{ij}	l_{ij}	$T_{ij} = t_{ij} \times l_{ij}$	Nature of Force
1.	*AB*	– 9.82	4	– 39.28	Compression
2.	*BC*	– 15	4	– 60	Compression
3.	*CD*	– 13.28	4	– 53.12	Compression
4.	*DE*	6.64	4	26.56	Tension
5.	*EA*	19.91	4	79.64	Tension
6.	*BE*	– 13.28	4	– 53.12	Compression
7.	*CE*	13.28	4	53.12	Tension

b. **Determine the deflection and rotation at the free end of the cantilever beam shown in Fig. 11. Use unit load method. Given $E = 2 \times 10^5$ N/mm², and $I = 12 \times 10^6$ mm⁴.**

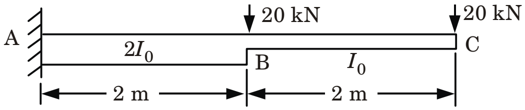

Fig. 11.

Ans.

Given : Load = 20 kN and 20 kN
$E = 2 \times 10^5$ N/mm² and $I = 12 \times 10^6$ mm⁴
To Find : Deflection and rotation at the free end.

A. **Deflection at Point *C* :**

1. Considering the beam into two segments *BC* and *BA* and with origin at *C* and *B* respectively and measuring *x* as positive towards left, then the expressions for BM due to external loading and due to unit load applied at *B* is shown in following table.

Portion	***CB***	***BA***
Origin	*C*	*B*
Limit	0 – 2	0 – 2
BM due to External Load, *M*	$20\,x$	$20\,(2 + x) + 20\,x = 40 + 40\,x$
BM due to Unit Load, M_1	$1x = x$	$1(2 + x) = 2 + x$

2. Deflection at C,

$$\Delta_C = \int_C^A \frac{MM_1}{EI} dx = \int_C^B \frac{MM_1}{EI} dx + \int_B^A \frac{MM_1}{2EI} dx$$

$$= \int_0^2 \frac{20x(x)}{EI} dx + \int_0^2 \frac{(40+40x)(2+x)}{2EI} dx$$

$$= 20\int_0^2 \frac{x^2}{EI} dx + \int_0^2 \frac{(80+120x+40x^2)}{2EI} dx$$

$$= \frac{20}{EI}\left[\frac{x^3}{3}\right]_0^2 + \frac{1}{2EI}\left[80x + \frac{120x^2}{2} + \frac{40x^3}{3}\right]_0^2$$

$$= \frac{20}{EI}\left[\frac{8}{3}\right] + \frac{1}{2EI}\left[80\times 2 + 60\times 4 + \frac{40}{3}\times 8\right]$$

$$= \frac{53.33}{EI} + \frac{253.33}{EI} = \frac{306.66}{24\times 10^2} = 0.1277 \text{ m}$$

$\Delta_C = 127.7$ mm

B. Slope at *C* :

1. To the find slope at C, apply a clockwise unit moment at B then the parameters are shown in following table.

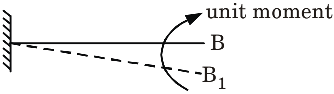

Fig. 12.

Portion	***CB***	***BA***
Origin	C	B
Limit	0 – 2	0 – 2
BM due to External Load, *M*	$20\,x$	$20\,(2 + x) + 20\,x = 40 + 40\,x$
BM due to Unit Moment, M_1	1	1

2. Slope at C, $\theta_C = \int_C^A \frac{MM_1}{EI} dx = \int_C^B \frac{MM_1}{EI} dx + \int_B^A \frac{MM_1}{2EI} dx$

$$\theta_C = \int_0^2 \frac{20x(1)}{EI} dx + \int_0^2 \frac{(40+40x)(1)}{2EI} dx$$

$$= \int_0^2 \frac{20x}{EI} + \int_0^2 \frac{(40+40x)}{2EI} dx$$

$$= \frac{20}{2EI}\left[x^2\right]_0^2 + \frac{1}{2EI}\left[40x + \frac{40x^2}{2}\right]_0^2$$

Slope at C, $\theta_C = \frac{10}{EI}(4) + \frac{20}{EI}[2+2] = \frac{120}{24\times 10^2} = 0.05$ radian

c. State and prove the Maxwell's reciprocal theorem.

Ans.

A. Maxwell's Law Statement :

1. In any structure whose material is elastic and obeys Hooke's law and whose supports remain unyielding and the temperature remains unchanged, the deflection at any point D (*i.e.*, Δ_d) due to a load W acting at any other point C is equal to the deflection at any point C (*i.e.*, δ_c) due to the load W acting at the point D.

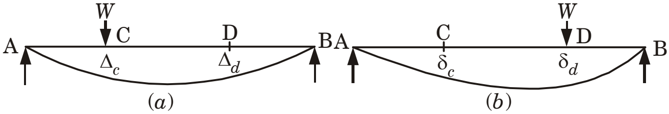

Fig. 13.

2. In Fig. 13(*a*) beam AB carries a load W at point C. Let the deflection at C be Δ_c and when load W is acting at point D. Let the defection at point D is δ_d then according to law of reciprocal deflections or Maxwell's reciprocal deflection theorem we have

$$\delta_d = \Delta_c$$

B. Proof :

1. Let initially a load W is acting at a point C and deflects the beam AB by deflection Δ_c under the load W.
2. Work done on the structure $= \dfrac{1}{2} \times W\Delta_c$
3. Let an another equal load W is acting at point D. Due to this load there will be further deflections of δ_c and δ_d at C and D.
4. So the total work done at this stage $= \dfrac{1}{2}\ W\Delta_c + \dfrac{1}{2} \times W\,\delta_d + W\delta_c$ ...(1)

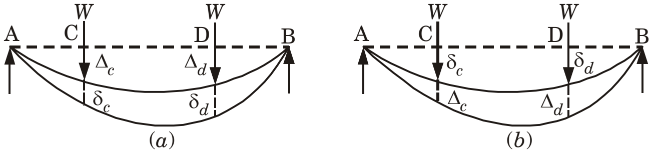

Fig. 14.

5. Now change the order of loading as shown in Fig. 14(*b*). Now consider initially load W is at D. Due to this load,

$$\text{Work done} = \frac{1}{2}\ W\,\delta_d$$

6. Let another load W (same value) is applied at C so further deflections of Δ_c and Δ_d will occur at C and D respectively.

   So total work done at this stage $= \dfrac{1}{2}\ W\delta_d + \dfrac{1}{2}\ W\Delta_c + W\Delta_d$ ...(2)

7. As both the eq. (1) and eq. (2) represents the same stage, hence,

$$\frac{1}{2}\ W\delta_d + \frac{1}{2}\ W\Delta_c + W\delta_c = \frac{1}{2}\ W\delta_d + \frac{1}{2}\ W\Delta_c + W\Delta_d$$

$$\delta_c = \Delta_d$$

8. The deflection at C due to the load W at D equal to the deflection at D due to the same load W at C.

d. Determine the vertical deflection at point C in the frame shown in Fig. 15. Given $E = 200$ kN/mm^2 and $I = 30 \times 10^6$ mm^4.

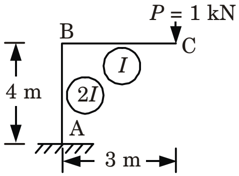

Fig. 15.

Ans.

Given : Load, $P = 1$ kN, $E = 200$ kN/mm^2, $I = 30 \times 10^6$ mm^4.
To Find : Vertical deflection at point C.

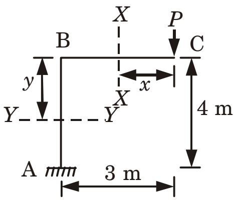

Fig. 16.

1. Flexural rigidity,

$$EI = 200 \times 30 \times 10^6 \text{ kN-mm}^2 = 6 \times 10^3 \text{ kN-m}^2$$

2. **Expression for Moment :**

Portion	***CB***	***BA***
Origin	C	B
Limit	0 – 3	0 – 4
Moment (M)	Px	$3P$

3. Strain energy stored by the frame,

W_i = Strain energy stored by CB + Strain energy stored by BA

$$W_i = \int_C^B \frac{M^2}{2EI} ds + \int_B^A \frac{M^2}{2EI} ds$$

$$W_i = \int_0^3 \frac{(Px)^2}{2EI} dx + \int_0^4 \frac{(3P)^2}{2(2EI)} dy$$

$$= \int_0^3 \frac{P^2 x^2}{2EI} dx + \int_0^4 \frac{9P^2}{4EI} dy$$

4. To find the vertical deflection at point C, differentiate the total strain energy stored with respect to P, we get

$$\Delta_c = \frac{\partial U}{\partial P} = \int_0^3 \frac{2Px^2}{2EI}dx + \int_0^4 \frac{9 \times 2P}{4EI}dy$$

$$\Delta_c = \frac{P}{EI}\left[\left(\frac{x^3}{3}\right)_0^3 + \frac{9}{2} \times (y)_0^4\right] = \frac{P}{EI}[9+18] = \frac{27 \times 1}{6 \times 10^3}$$

$$\Delta_c = 4.5 \times 10^{-3} \text{ m} = 4.5 \text{ mm} \qquad [\because \quad P = 1 \text{ kN}]$$

e. A simply supported beam has a span of 15 m. UDL of 40 kN/m and 5 m long crosses the girder from left to right. Draw the influence line diagram for shear force and bending moment at a section 6 m from left end. Use these diagrams to calculate the maximum shear force and bending moment at this section.

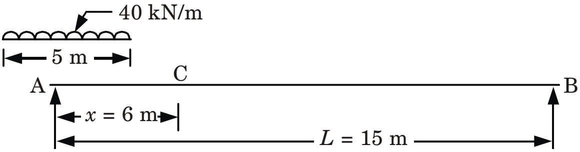

Fig. 17.

Ans.

Given : Span, L = 15 m, intensity of UDL, w = 40 kN/m, Length of UDL, l = 5 m, Distance of section, x = 6 m
To Find : Draw ILD and Calculate maximum SF and BM

1. ILD for Shear Force and Bending Moment :

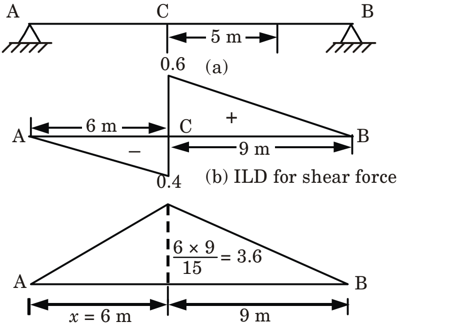

Fig. 18.

2. Maximum Positive Shear Force at C :
For this condition the tail of the UDL should be at the section, as shown in Fig. 19.

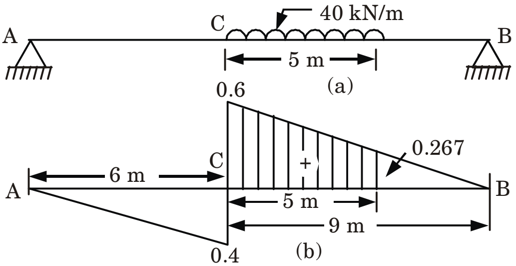

Fig. 19. ILD for shear force at given section.

$$\text{Maximum positive shear force} = \left(\frac{0.6 + 0.267}{2}\right) \times 5 \times 40 = 86.7 \text{ kN}$$

3. Maximum Negative Shear Force at *C* :

Similarly for this condition the head of UDL should be at the section, as shown in Fig. 20.

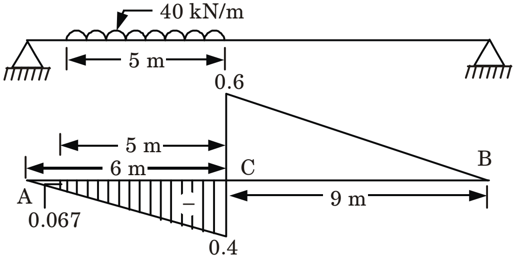

Fig. 20.

$$\text{Maximum negative shear force} = \left(\frac{0.4 + 0.067}{2}\right) \times 5 \times 40 = 46.7 \text{ kN}$$

4. Maximum Bending Moment at Point *C* :

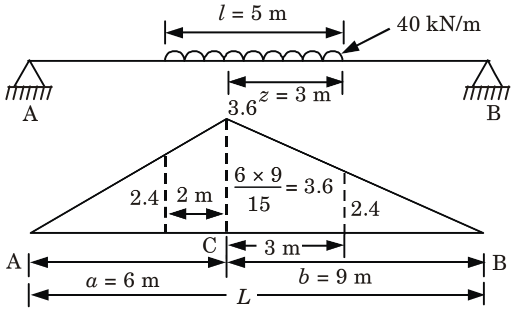

Fig. 21.

i. Condition for maximum bending moment,

$$z = \frac{l}{L} \times b$$

$$z = \frac{5}{15} \times 9$$
$$z = 3 \text{ m}$$

ii. Maximum bending moment at C,

$$M_{max} = w \times \text{area of the ILD covered by the load.}$$
$$M_{max} = 40\left[\left(\frac{3.6+2.4}{2}\right)\times 3+\left(\frac{3.6+2.4}{2}\right)\times 2\right]$$
$$M_{max} = 40\,(3 \times 3 + 3 \times 2)$$
$$M_{max} = 40 \times 15 = 600 \text{ kN-m}$$

f. A circular arch to span 25 m with a central rise 5 m is hinged at the crown and springing. It carries a point load of 100 kN at 6 m from the left support. Calculate :

i. The reactions at the supports.

ii. The reactions at crown.

Ans.

Given : Span of arch, $L = 25$ m, Central rise, $h = 5$ m, Load = 100 kN
To Find : V_A, V_B and V_C

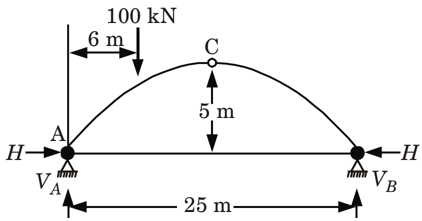

Fig. 22.

1. $\Sigma F_y = 0,\ V_A + V_B = 100$ ...(1)
2. Taking moment about A, $\Sigma M_A = 0$

$$V_B \times 25 = 100 \times 6$$
$$V_B = 24 \text{ kN}$$

From eq. (1),

$$V_A = 100 - 24 = 76 \text{ kN}$$

3. Considering right part (CB part) of arch,
Taking moment about point C.
$\Sigma M_C = 0, \quad H \times 5 = V_B \times 12.5$

$$H = \frac{24 \times 12.5}{5} = 60 \text{ kN}$$

4. Shear force at crown,

$$V_C = 100 - 76 = 24 \text{ kN}$$

5. Angle between plane of section and vertical plane passing through the crown of circular arch,

$$\theta = 0°$$

7. Normal thrust at crown,
$$N = H \cos \theta + V_C \sin \theta$$
$$N = 60 \cos 0° + 24 \sin 0° = 60 \text{ kN}$$
8. Radial shear at crown,
$$S = H \sin \theta - V_C \cos \theta$$
$$S = 60 \times \sin 0° - 24 \times \sin 0° = -24 \text{ kN}$$

g. Show that the parabolic shape is a funicular shape for a three hinged arch subjected to a uniformly distributed load over its entire span.

Ans.

1. Let
$$L = \text{Span of the arch,}$$
$$h = \text{Central rise, and}$$
$$w = \text{UDL applied on the arch.}$$
2. From symmetry we have,
$$V_A = V_B = \frac{wL}{2}$$

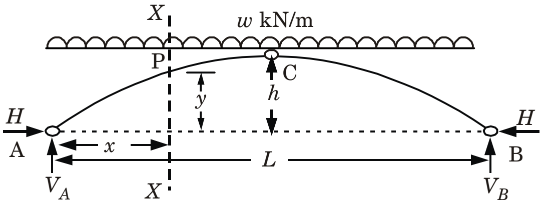

Fig. 23.

3. For horizontal thrust, taking moment about C,
$$H \times h = \frac{wL}{2} \times \frac{L}{2} - \frac{wL}{2} \times \frac{L}{4} = \frac{wL^2}{8}$$
$$H = \frac{wL^2}{8h}$$
4. Let us now consider any section at distance x from A.

Equation of parabola is given by, $y = \dfrac{4h}{L^2}\, x\,(L - x)$

5. The value of bending moment at any section of the arch,
$$M_x = -Hy + V_A x - \frac{wx^2}{2}$$
$$= -\frac{wL^2}{8h} \times \frac{4h}{L^2} x\,(L - x) + \frac{wL}{2} x - \frac{wx^2}{2}$$
$$M_x = -\frac{wLx}{2} + \frac{wx^2}{2} + \frac{wLx}{2} - \frac{wx^2}{2} = 0$$
6. Hence a parabolic arch subjected to a UDL over its entire span has the bending moment at any section zero. That is why the parabolic shape for three hinged arch is a funicular shape.

h. Uniformly distributed load of intensity 30 kN/m crosses a simply supported beam of span 60 m from left to right. The length of UDL is 15 m. Find the value of maximum bending moment for a section 20 m from left end. Find also the absolute value of maximum bending moment and shear force in the beam.

Ans.

Given : Span of beam, L = 60 m, Length of UDL, l = 15 m
Intensity of UDL, w = 30 kN/m, Distance of section, x = 20 m
To Find : BM at section, Absolute BM and shear force.

A. Maximum Bending Moment at Given Section :

1. ILD for bending moment at given section, as shown in Fig. 24.

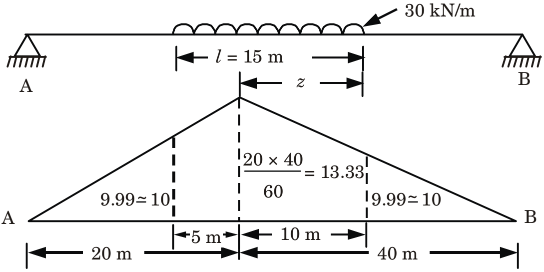

Fig. 24.

2. Condition for maximum bending moment,

$$z = \frac{l}{L} \times b = \frac{15}{60} \times 40 = 10 \text{ m.}$$

3. Maximum bending moment at given section,

$$M_{max} = w \times \text{Area of the ILD covered by the load.}$$

$$= 30 \times \left[\left(\frac{13.33 + 10}{2}\right) \times 5 + \left(\frac{13.33 + 10}{2}\right) \times 10\right]$$

$$= 30 \times 174.975$$

$$M_{max} = 5249.25 \text{ kN-m}$$

B. Absolute Maximum Bending Moment :

1. The absolute maximum bending moment occurs at the centre of the span when the loading is symmetrically placed on the span.
2. ILD for absolute value of maximum bending moment, as shown in Fig. 25.
3. Maximum bending moment,

$$M_{max} = 30 \times \left[\left(\frac{15 + 11.25}{2}\right) \times 7.5 \times 2\right] = 5906.25 \text{ kN-m}$$

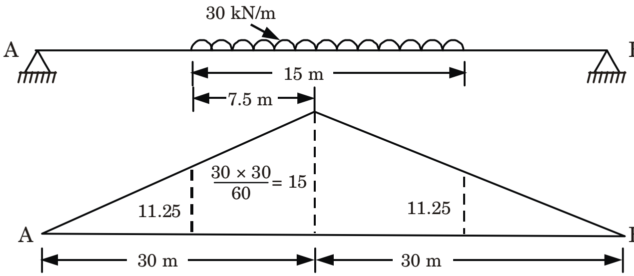

Fig. 25.

C. Absolute Value of Shear Force :

1. Positive Shear Force :

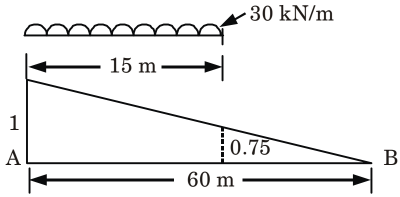

Fig. 26.

i. For maximum positive shear force the load should be placed as shown in Fig. 26.

ii. Maximum positive shear force = $30 \times \left[\frac{1+0.75}{2}\right] \times 15$

SF = 393.75 kN

2. Negative Shear Force :

i. For maximum negative shear force the load should be placed as shown in Fig. 27.

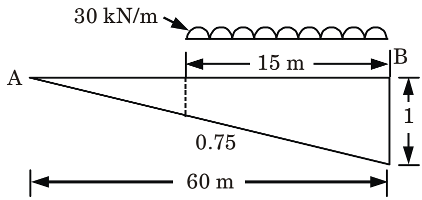

Fig. 27.

ii. Maximum negative shear force = $30 \times \left[\frac{1+0.75}{2}\right] \times 15$

SF = 393.75 kN

Section-C

Note : Attempt any **two** questions from this section. **(2 × 15 = 30)**

3. a. Determine the horizontal displacement of the roller end *D* of the portal frame shown in Fig. 28. *EI* is 8000 kN-m^2 throughout.

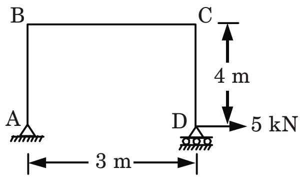

Fig. 28.

Ans.

Given : Load = 5 kN, EI = 8000 kN-m^2
To Find : Horizontal displacement at D.

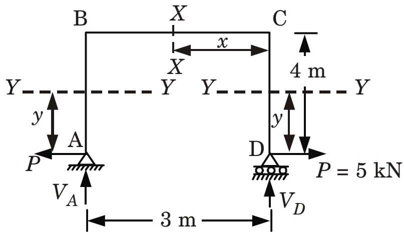

Fig. 29.

1. Horizontal reaction at $A = P$ (←)
2. Taking moment about point A,

$$V_D \times 3 - 5 \times 0 = 0$$
$$V_D = 0$$

3. Taking moment about support D,

$$V_A \times 3 - 5 \times 0 = 0$$
$$V_A = 0$$

4. **Expression for Moments :**

Portion	***AB***	***BC***	***CD***
Origin	A	B	D
Limit	0 – 4	0 – 3	0 – 4
Moment, (*M*)	Py	$4P$	Py

5. Strain energy stored by the frame,

$$U_i = \sum \int \frac{M^2}{2EI} ds$$

$$U_i = \int_D^C \frac{M^2}{2EI} ds + \int_C^B \frac{M^2}{2EI} ds + \int_B^A \frac{M^2}{2EI} ds$$

$$U_i = \int_0^4 \frac{(Py)^2}{2EI} dy + \int_0^3 \frac{(4P)^2}{2EI} dx + \int_0^4 \frac{(Py)^2}{2EI} dy$$

$$U_i = \int_0^4 \frac{P^2 y^2}{2EI} dy + \int_0^3 \frac{16P^2}{2EI} dx + \int_0^4 \frac{P^2 y^2}{2EI} dy$$

6. Horizontal displacement at D is given by,

$$\Delta_D = \frac{\partial U}{\partial P} = \int_0^4 \frac{2Py^2}{2EI} dy + \int_0^3 \frac{32P}{2EI} dx + \int_0^4 \frac{2Py^2}{2EI} dy$$

7. Putting $P = 5$,

$$\Delta_D = \int_0^4 \frac{5y^2}{EI} dy + \int_0^3 \frac{80}{EI} dx + \int_0^4 \frac{5y^2}{EI} dy$$

$$\Delta_D = \frac{5}{EI} \frac{[y^3]_0^4}{3} + \frac{80}{EI}[x]_0^3 + \frac{5}{EI} \frac{[y^3]_0^4}{3}$$

$$= \frac{1}{EI}\left[\frac{5}{3} \times (4^3 - 0) + 80\,(3 - 0) + \frac{5}{3}(4^3 - 0)\right]$$

$$= \frac{1}{EI}\left[\frac{320}{3} + \frac{240}{1} + \frac{320}{3}\right] = \frac{1}{EI}\left[\frac{320 + 720 + 320}{3}\right]$$

Horizontal displacement at D,

$$\Delta_D = \frac{1360}{3 \times 8000} = 0.0567 \text{ m} \approx 57 \text{ mm}$$

b. Fig. 30 shows a pin-joined truss loaded with a single load W is 100 kN. If the area of cross-section of all members shown in Fig. 30 is 1000 mm², what is the vertical deflection of point C ? Take E = 200 kN/mm² for all members.

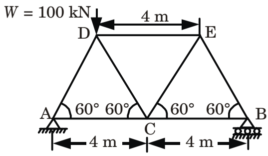

Fig. 30.

Ans.

Given : Load, W = 100 kN, Cross section area = 1000 mm²
E = 200 kN/mm²
To Find : Vertical deflection at point C.

1. Taking moment about point A, $\Sigma M_A = 0$

$$8 \times V_B = 100 \times 2$$
$$V_B = 25 \text{ kN}$$

2. $\Sigma F_y = 0$, $V_A + V_B = 100$ kN

$$V_A = 100 - 25$$
$$V_A = 75 \text{ kN}$$

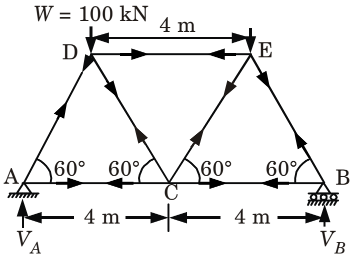

Fig. 31.

3. **Joint *A* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$V_A + P_{AD} \sin 60° = 0$$

$$P_{AD} = \frac{-75}{\sin 60°} = \frac{-150}{\sqrt{3}} \text{ kN}$$

ii. Resolve the forces horizontally,

$\Sigma F_x = 0,$ $\quad P_{AC} + P_{AD} \cos 60° = 0$

$$P_{AC} = -\left(\frac{-150}{\sqrt{3}}\right)\left(\frac{1}{2}\right) = \frac{75}{\sqrt{3}} \text{ kN}$$

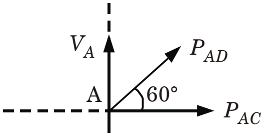

Fig. 32.

4. **Joint *D* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$-(P_{DA} \sin 60° + P_{DC} \sin 60° + 100) = 0$$

$$P_{DC} \sin 60° = +\frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{2} - 100$$

$$P_{DC} = -\frac{25}{\sin 60°}$$

$$P_{DC} = -\frac{50}{\sqrt{3}} \text{ kN}$$

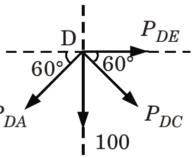

Fig. 33.

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$P_{DE} = P_{DA} \cos 60° - P_{DC} \cos 60°$$

$$= -\frac{150}{\sqrt{3}} \times \cos 60° + \frac{50}{\sqrt{3}} \times \cos 60°$$

$$P_{DE} = -\frac{75}{\sqrt{3}} + \frac{25}{\sqrt{3}} = -\frac{50}{\sqrt{3}} \text{ kN}$$

5. **Joint *C* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$P_{CE} \sin 60° + P_{CD} \sin 60° = 0$

$$P_{CE} = \frac{50}{\sqrt{3}} \text{ kN}$$

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

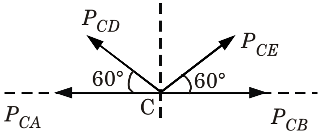

Fig. 34.

$P_{CB} = P_{CA} + P_{CD} \cos 60° - P_{CE} \cos 60°$

$$= \frac{75}{\sqrt{3}} - \frac{50}{\sqrt{3}} \times \frac{1}{2} - \frac{50}{\sqrt{3}} \times \frac{1}{2}$$

$$P_{CB} = \frac{75}{\sqrt{3}} - \frac{50}{\sqrt{3}} = \frac{25}{\sqrt{3}} \text{ kN}$$

6. **Joint *B* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$V_B + P_{BE} \sin 60 = 0$

$$P_{BE} = -\frac{50}{\sqrt{3}} \text{ kN}$$

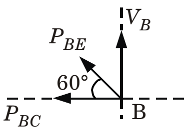

Fig. 35.

7. To find the vertical deflection at the joint *C*, remove the given load system and apply a vertical load of 1 kN at *C*.

Due to symmetry, reaction at each support = $\frac{1}{2}$ kN.

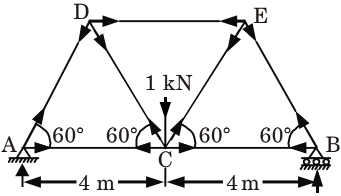

Fig. 36.

8. **Joint *A* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$$K_{AD} \sin 60° + \frac{1}{2} = 0$$

$$K_{AD} = -\frac{1}{\sqrt{3}} \text{ kN}$$

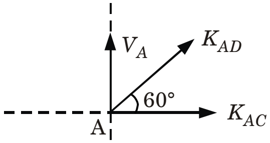

Fig. 37.

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$K_{AC} + K_{AD} \cos 60° = 0$

$$K_{AC} = \frac{1}{\sqrt{3}} \cos 60° = \frac{1}{2\sqrt{3}} \text{ kN}$$

9. **Joint D :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

$K_{DC} \sin 60° + K_{DA} \sin 60° = 0$

$$K_{DC} = -\left(-\frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} \text{ kN}$$

iii. Resolve the forces horizontally, $\Sigma F_x = 0$

$K_{DE} + K_{DC} \cos 60° = K_{DA} \cos 60°$

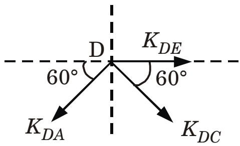

Fig. 38.

$$K_{DE} = \frac{-1}{\sqrt{3}} \cos 60° - \frac{1}{\sqrt{3}} \cos 60°$$

$$K_{DE} = -\frac{2}{\sqrt{3}} \cos 60° = -\frac{1}{\sqrt{3}}$$

10. By symmetry, we know that the forces in the remaining members as same as before.

Vertical deflection at joint C, $\Delta_c = \sum \frac{PKL}{AE}$

Member length, L = 4000 mm
Member area, A =1000 mm^2

Member	***P***	***K***	***PK***
AD	$-150/\sqrt{3}$	$-1/\sqrt{3}$	+ 150/3 = 50
DE	$-50/\sqrt{3}$	$-1/\sqrt{3}$	+ 50/3
EB	$-50/\sqrt{3}$	$-1/\sqrt{3}$	+ 50/3
BC	$+25/\sqrt{3}$	$+\ 1/2\sqrt{3}$	+ 25/6
CA	$+75/\sqrt{3}$	$+\ 1/2\sqrt{3}$	+ 75 / 6
DC	$-\ 50/\sqrt{3}$	$+\ 1/\sqrt{3}$	– 50 / 3
CE	$+50/\sqrt{3}$	$+\ 1/\sqrt{3}$	+ 50 / 3
Total			100

11. Vertical deflection at the joint C, $\Delta_c = \sum \frac{PKL}{AE}$

$$= \frac{100 \times 4000}{1000 \times 200}$$

Deflection at C, Δ_c = 2 mm.

4. a. Four point loads 8, 15, 15 and 10 kN have centre to centre spacing of 2 m between consecutive loads and they traverse a girder of 30 m span from left to right with 10 kN load leading. Calculate the maximum bending moment and shear force at 8 m from the left support.

Ans.

Given : Span of beam, L = 30 m, Distance between loads = 2 m
Distance of section = 8 m (from A).
To Find : Maximum SF and BM.

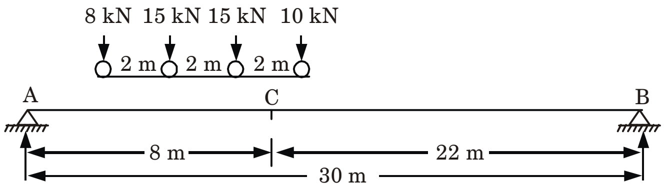

Fig. 39.

A. Maximum Bending Moment at *C* :

1. Let *AB* be the beam and *C* is the given section. Let us allow the loads to cross the given section one after another and find the average loads on *AC* and *CB*.
2. The calculation is shown in the following table.

Load Crossing the Section *C*	Average Load on *AC*	Average Load on *BC*	Remarks
10 kN	$\frac{38}{8}$	$\frac{10}{22}$	Average load on *AC* is greater than the average on *CB*
15 kN	$\frac{23}{8}$	$\frac{25}{22}$	Average load on *AC* is greater than the average on *CB*
15 kN	$\frac{8}{8}$	$\frac{40}{22}$	Average load on *AC* is less than the average on *CB*

3. Hence, for the maximum bending moment at *C*, we will place the 15 kN load exactly at *C* and other loads relative to this load.
4. ILD for the maximum bending moment at the section *C* as shown Fig. 40.

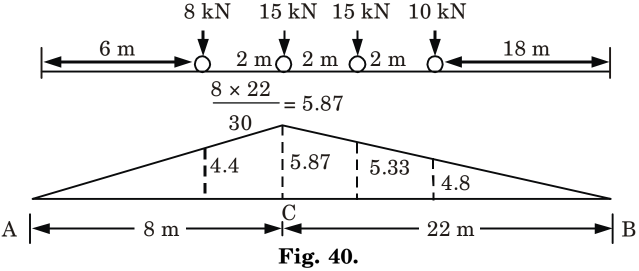

Fig. 40.

5. Maximum bending moment at *C*

$$= 8 \times 4.4 + 15 \times 5.87 + 15 \times 5.33 + 10 \times 4.8$$
$$= 251.2 \text{ kN-m}$$

B. Maximum Shear Force at Section :

1. In this case, $\frac{\Sigma W}{L} = \frac{8 + 15 + 15 + 10}{30} = \frac{48}{30} = 1.6$

   Move LHS 8 kN from *C* and bring 15 kN over *C*, $\frac{8}{2} = 4 > 1.6$

   Move LHS 15 kN from *C* and bring 15 kN over *C*, $\frac{15}{2} = 7.5 > 1.6$

2. Hence, condition $\frac{\text{Load rolled past the section}}{\text{Succeeding wheel space}}$

   $< \frac{\Sigma W}{L}$ is not satisfied at any position of loads.

So 8 kN load should placed over C. For this condition, position of load and ILD shown in Fig. 41.

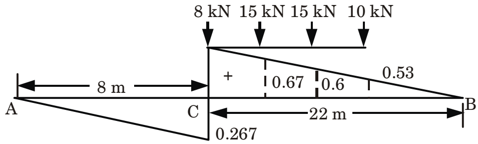

Fig. 41.

3. Maximum positive shear force at C

$$= 8 \times 0.733 + 15 \times 0.67 + 15 \times 0.6 + 10 \times 0.53$$
$$= 30.214 \text{ kN}$$

b. A three-hinged circular arch hinged at the springing and crown point has a span of 40 m and a central rise of 8 m. It carries a uniformly distributed load 20 kN/m over the left-half of the span together with a concentrated load of 100 kN at the right quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10 m from left support.

Ans.

Given : Span of arch, L = 40 m, Rise of arch, h = 8 m
Intensity of UDL, w = 20 kN/m, Point load, W = 100 kN
To Find : Reactions at the supports, Normal thrust and Radial shear.

1. $\Sigma F_y = 0$, $V_A + V_B = (20 \times 20) + 100$

$$V_A + V_B = 500 \text{ kN} \quad ...(1)$$

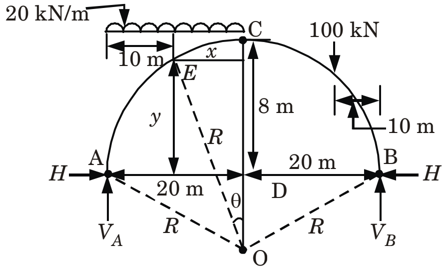

Fig. 42.

3. Taking moment about point A,

$$V_B \times 40 = 100 \times 30 + (20 \times 20) \times 10$$

From eq. (1), $V_B = 175$ kN

$$V_A = 325 \text{ kN}$$

4. Taking moment about point C of the forces on the left side of C,

$$V_A \times 20 - w \times \frac{L}{2} \times \frac{L}{4} = H \times 8$$

$$325 \times 20 - 20 \times 20 \times 10 = H \times 8$$
$$H = 312.5 \text{ kN}$$

5. Let R be the radius of the arch,
$$8 \times (2R - 8) = 20 \times 20$$
$$\therefore \quad R = 29 \text{ m}$$

6. The equation of the circular arch with D as origin is,
$$y = \sqrt{R^2 - x^2} - (R - h)$$
$$y = \sqrt{29^2 - x^2} - (29 - 8)$$
$$y = \sqrt{841 - x^2} - 21$$

7. Put, $x = 10$ m
$$y = 6.221 \text{ m}$$

8. Moment at section, at a distance 10 m from A,
$$M_x = V_A \times x - Hy - w \times x \times \frac{x}{2}$$
$$M = 325 \times 10 - 312.5 \times 6.221 - 20 \times 10 \times \frac{10}{2}$$
$$M = 305.94 \text{ kN-m}$$

9. Slope at point E,

From Fig. 42, $\sin \theta = \dfrac{x}{R} = \dfrac{10}{29}$

At $x = 10$ m, $\quad \theta = 20.17°$

10. Normal thrust at point E,
$$N_E = H \cos \theta + V_E \sin \theta$$
Shear force at point E,
$$V_E = 325 - 20 \times 10$$
$$V_E = 125 \text{ kN}$$
$$N_E = 312.5 \times \cos 20.17° + 125 \times \sin 20.17°$$
$$N_E = 336.44 \text{ kN}$$

11. Radial shear, $\quad S_E = H \sin \theta - V_E \cos \theta$
$$= 312.5 \times \sin 20.17° - 125 \times \cos 20.17°$$
$$S_E = -9.6 \text{ kN}$$

5. a. Analyze the truss shown in Fig. 43 by the method of tension coefficient and determine the forces in all the members.

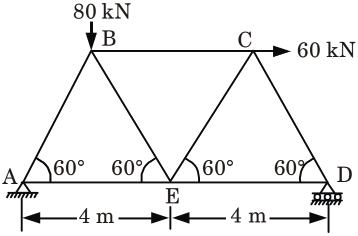

Fig. 43.

Ans.

1. **Step - I :** Redraw the figure so that co-ordinate of various joints can be obtained that joint A is at origin.

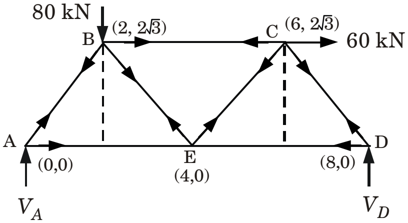

Fig. 44.

2. **Step - II :** Member parameters of truss.

S. No.	Member	x_i	x_j	$x_{ij} = (x_j - x_i)$	y_i	y_j	$y_{ij} = y_j - y_i$	$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}$
1.	AB	0	2	2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{2^2 + (2\sqrt{3})^2} = 4$
2.	BC	2	6	4	$2\sqrt{3}$	$2\sqrt{3}$	0	4
3.	CD	6	8	2	$2\sqrt{3}$	0	$-2\sqrt{3}$	$\sqrt{2^2 + (-2\sqrt{3})^2} = 4$
4.	DE	8	4	–4	0	0	0	4
5.	EA	4	0	–4	0	0	0	4
6.	EB	4	2	–2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$
7.	EC	4	6	2	0	$2\sqrt{3}$	$2\sqrt{3}$	$\sqrt{2^2 + (2\sqrt{3})^2} = 4$

3. **Step - III :** Calculation of tension coefficients,

i. Taking moment about A, $\sum M_A = 0$

$$8 \times V_B = 60 \times 2\sqrt{3} + 80 \times 2$$
$$V_B = 45.98 \approx 46 \text{ kN}$$

ii.
$$\sum F_y = 0$$
$$V_A + V_B - 80 = 0$$
$$V_A = 80 - 46 = 34 \text{ kN}$$

iii.
$$\sum F_x = 0$$
$$H_A - 60 = 0$$
$$H_A = 60 \text{ kN}$$

iv. **Joint A :**

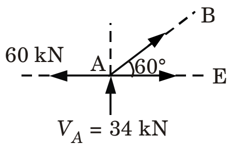

Fig. 45.

a. Resolve the forces horizontally,

$$\sum F_x = 0$$

$$x_{AB} \times t_{AB} + x_{AE} \times t_{AE} - 60 = 0$$

$$2 \times t_{AB} + 4 \times t_{AE} - 60 = 0$$

$$t_{AB} + 2 \times t_{AE} = 30 \quad ...(1)$$

b. Resolve the forces vertically,

$$\sum F_y = 0$$

$$y_{AB} \times t_{AB} + y_{AE} \times t_{AE} + 34 = 0$$

$$2\sqrt{3} \times t_{AB} + 0 = -34$$

$$t_{AB} = -9.82 \text{ kN/m}$$

c. Value of t_{AB} put in eq. (1), we get

$$-9.82 + 2\, t_{AE} = 30$$

$$2 \times t_{AE} = 30 + 9.82$$

$$t_{AE} = 19.91 \text{ kN/m}$$

v. **Joint D :**

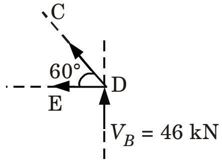

Fig. 46.

a.Resolve the forces horizontally,

$$\sum F_x = 0$$

$$x_{DC} \times t_{DC} + x_{DE} \times t_{DE} = 0$$

$$(-2) \times t_{DC} + (-4) \times t_{DE} = 0$$

$$t_{DC} = -2\, t_{DE} \quad ...(2)$$

b. Resolve the forces vertically,

$$\sum F_y = 0$$

$$y_{DC} \times t_{DC} + y_{DE} \times t_{DE} + 46 = 0$$

$$2\sqrt{3} \times t_{dc} + 0 + 46 = 0$$

$$t_{DC} = -\frac{46}{2\sqrt{3}} = -13.28 \text{ kN/m}$$

c. Value of t_{DC} put in equation (2), we get

$$-2\, t_{DE} = t_{DC}$$

$$-2\, t_{DE} = -13.28$$

$$t_{DE} = 6.64 \text{ kN/m}$$

vi. **Joint C :**

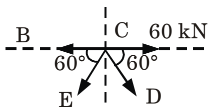

Fig. 47.

a. Resolve the forces horizontally, $\sum F_x = 0$

$$x_{CB} \times t_{CB} + x_{CE} \times t_{CE} - x_{CD} \times t_{CD} - 60 = 0$$
$$(-4) \times t_{CB} + (-2) \times t_{CE} - 2 \times t_{CD} - 60 = 0$$
$$(-4) \times t_{CB} + (-2) \times t_{CE} - 2 \times (-13.28) - 60 = 0$$
$$2\, t_{CB} + t_{CE} = -16.72 \qquad ...(3)$$

b. Resolve the forces vertically, $\sum F_y = 0$

$$y_{CE} \times t_{CE} + y_{CD} \times t_{CD} = 0$$
$$(-2\sqrt{3})\, t_{CE} + (-2\sqrt{3}) \times (-13.28) = 0$$
$$t_{CE} = 13.28 \text{ kN/m}$$

c. Value of t_{CE} put in equation (3), we get

$$2\, t_{CB} + 13.28 = -16.72$$
$$t_{CB} = -15 \text{ kN/m}$$

vii. **Joint E :**

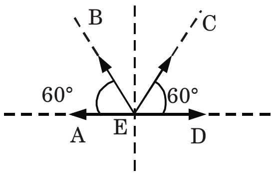

Fig. 48.

Resolve the forces vertically, $\sum F_y = 0$

$$y_{EB} \times t_{EB} + y_{EC} \times t_{EC} = 0$$
$$(2\sqrt{3}) \times t_{EB} + (2\sqrt{3}) \times t_{EC} = 0$$
$$t_{EB} = -t_{EC}$$
$$t_{EB} = -13.28 \text{ kN/m}$$

4. **Step IV :** Final forces in members (Represented by T_{ij})

S. No.	Member	t_{ij}	l_{ij}	$T_{ij} = t_{ij} \times l_{ij}$	Nature of Force
1.	*AB*	– 9.82	4	– 39.28	Compression
2.	*BC*	– 15	4	– 60	Compression
3.	*CD*	– 13.28	4	– 53.12	Compression
4.	*DE*	6.64	4	26.56	Tension
5.	*EA*	19.91	4	79.64	Tension
6.	*BE*	– 13.28	4	– 53.12	Compression
7.	*CE*	13.28	4	53.12	Tension

b. Determine the vertical deflection at the free end and rotation at *A* in the overhanging beam shown in Fig. 49. Assume constant *EI*. Use Castigliano's method.

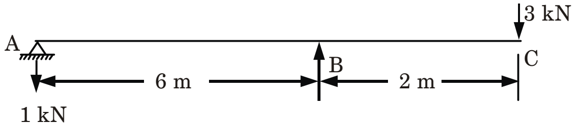

Fig. 49.

Ans.

Given : Loads = 3 kN and 1 kN, EI = Constant
To Find : Vertical deflection at C and rotation at A.

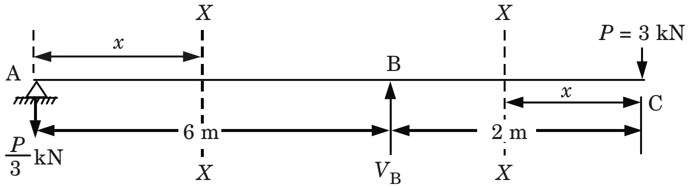

Fig. 50.

Let load P applied at end point C.

A. Vertical Deflection at Free End :

$$V_B = 1 + 3 = 4 \text{ kN}$$

1. Taking moments about A, $\Sigma M_A = 0$

$$V_B = \frac{4P}{3} (\uparrow)$$

2. $$\Sigma F_y = 0$$

$$V'_A + V_B = 3 + 1 = 4$$

$$V_A = P - \frac{4P}{3} = -\frac{P}{3}$$

3. **Expression for Moment :**

Portion	***AB***	***CB***
Origin	A	C
Limit	0 – 6	0 – 2
Moment, (*M*)	$-\frac{Px}{3}$	Px

4. Strain energy stored by the beam

= Strain energy stored by AB + Strain energy stored by BC

$$U_i = \sum \int \frac{M^2 dx}{2EI} = \int_A^B \frac{M^2}{2EI} dx + \int_C^B \frac{M^2}{2EI} dx$$

$$U_i = \int_0^6 \left(\frac{-P}{3}x\right)^2 \frac{dx}{2EI} + \int_0^2 \frac{(-Px)^2}{2EI} dx$$

$$U_i = \int_0^6 \frac{P^2}{18} x^2 \frac{dx}{EI} + \int_0^2 \frac{P^2 x^2}{2EI} dx$$

5. Vertical deflection at free end,

$$\Delta_C = \frac{\partial U_i}{\partial P} = \int_0^6 \frac{2P}{18} x^2 \frac{dx}{EI} + \int_0^2 \frac{2Px^2}{2EI} dx$$

$$= \int_0^6 \frac{1}{9} Px^2 \frac{dx}{EI} + \int_0^2 \frac{Px^2}{EI} dx$$

Deflection at C,

$$\Delta_C = \frac{P}{9EI}\left[\frac{x^3}{3}\right]_0^6 + \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^2$$

$$\Delta_C = \frac{P}{27EI}[6^3 + 9 \times 2^3] = \frac{32}{EI} \qquad [\because \quad P = 3 \text{ kN}]$$

B. Rotation at Support *A* :

1. To find the rotation at A, applied the moment M_o at A,

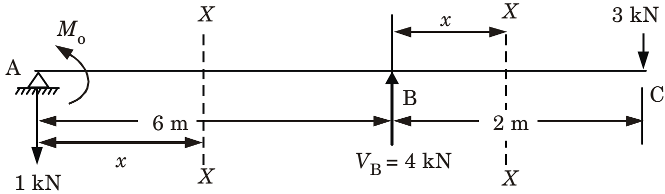

Fig. 51.

2. Strain energy stored by member,

$$U_i = \Sigma \int M^2 \frac{ds}{2EI}$$

$$M_{AB} = (M_o + 1 \times x),$$

$$M_{BC} = [M_o + (6 + x) \times 1 - 4 \times x]$$

$$U_i = \int_0^6 \frac{(M_o + 1 \times x)^2}{2EI} dx + \int_0^2 \frac{[M_o + (6 + x) - 4x]^2}{2EI} dx$$

3. Angle of rotation at the end A,

$$\theta_A = \frac{\partial U_i}{\partial M_o}$$

$$\theta_A = \int_0^6 \frac{2(M_o + x)}{2EI} dx + \int_0^2 \frac{2[M_o + 6 - 3x]}{2EI} dx$$

4. Putting $\qquad M_o = 0,$

$$\theta_A = \int_0^6 \frac{x}{EI} dx + \int_0^2 \frac{6 - 3x}{EI} dx$$

$$= \frac{1}{EI}\left[\frac{x^2}{2}\right]_0^6 + \frac{6}{EI}[x]_0^2 - \frac{3}{EI} \times \left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{1}{EI}\left[\frac{6^2}{2} - 0 + 6(2-0) - \frac{3}{2}(2^2 - 0)\right]$$

Rotation at A, $\theta_A = \frac{1}{EI}[18 + 12 - 6] = \frac{24}{EI}$

☺☺☺

B. Tech.
(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2016-17
STRUCTURAL ANALYSIS-I

Time : 3 Hours **Max. Marks : 100**

SECTION-A

1. Explain the following : **($10 \times 2 = 20$)**

a. Give an example of a structure where it is externally as well as internally indeterminate.

b. Which method of analysis is suitable, if static indeterminacy is more than kinematic indeterminacy ?

c. What are the uses of influence lines ?

d. Distinguish between influence line diagram and bending moment diagram.

e. Classify the arches based on materials, shapes and structural systems.

f. Why arches are preferred than beams ?

g. Write the formulae for area the centroid of the curve defined by $y = kx^n$.

h. What is the advantage of conjugate beam method over other method ?

i. State Castigliano's first theorem ?

j. Write the equation in term of strain energy, which is sufficient to determine the stress in case of propped cantilever beams.

SECTION-B

2. Attempt any **five** of the following questions : **($5 \times 10 = 50$)**

a. A simply supported beam has a span of 25 m. Draw the influence line for shearing force at a section 10 m from one end and using this diagram, determine the maximum shearing force due to the passage of a point load 5 kN followed immediately by uniformly distributed load of 2.4 kN/m extending over a length of 5 m ?

b. An uniformly distributed load of 40 kN/m and of length 3 metres transverse across the span of simply supported length of 18 metres. Compute the maximum bending moment at 4 m from the left support and absolute bending moment.

c. A three hinged parabolic arch hinged at the supports and at the crown has a span of 24 m and a central rise of 4 m. It carries concentrated load of 50 kN at 18 m from the left support and UDL of 30 kN/m over the left portion. Determine the normal thrust, radial shear at a section 6 m from the left hand support.

d. Find the slope and deflection at the free end of a cantilever shown in Fig. 1 by moment area method. Moment area of *AC* is twice the inertia of *BC*.

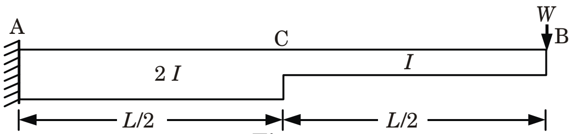

Fig. 1.

e. A beam *ABCDE* is 12 m long and supports a load of 100 kN at *C*, simply supported at *A* and *E*. $AB = BC = CD = DE = 3$ m. Moment of inertia is *I* in the portion *AB* and *DE* and 2*I* in the portion *BD*. Determine the deflections at *B* and *C* by using conjugate beam method.

f. A cantilever beam is of span 2 m and is subjected to a concentrated load of 20 kN at the free end. The cross section of the beam is 100 × 200 mm and $E = 30$ kN/mm^2. Calculate the slope and deflection of the beam at midspan. Use unit load method.

g. State and prove that the Castigliano's theorem.

h. i. Define fatigue.

ii. What is the polar moment of inertia ?

iii. What is unsymmetrical bending ?

iv. What are the reasons for unsymmetrical bending occurring in the beams ?

SECTION-C

Attempt any **two** of the following questions : **(2 × 15 = 30)**

3. a. A simply supported beam with variable moment of inertia supports a uniformly distributed load of w kN/m. Estimate the maximum deflection in a beam.

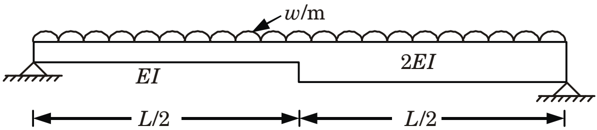

Fig. 2.

b. Determine the slopes at supports and deflection under the load for the beam shown in Fig. 3. Take Young's modulus E as 210 GPa, moment of inertia as 120×10^6 mm^4. Adopt conjugate beam method.

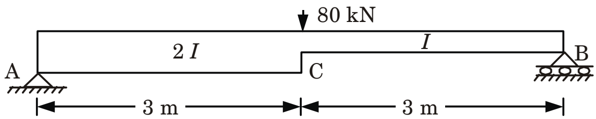

Fig. 3.

4. a. Calculate the deflection under the load for truss shown in Fig. 4. All the members are have equal areas of 1250 mm^2 in cross section and E = 200 kNm^2.

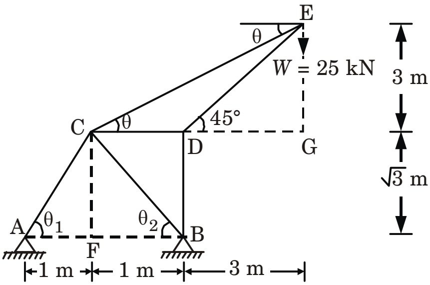

Fig. 4.

b. A three hinged parabolic arch is shown in Fig. 5. Determine the normal thrust, radial shear and bending moment at quarter span and draw BMD.

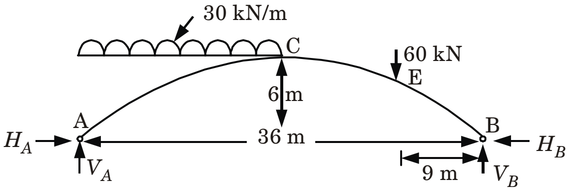

Fig. 5.

5. Fig. 6 shows a frame subjected to a load of 3.4 kN. Find the resultant stress at *A* and *B*.

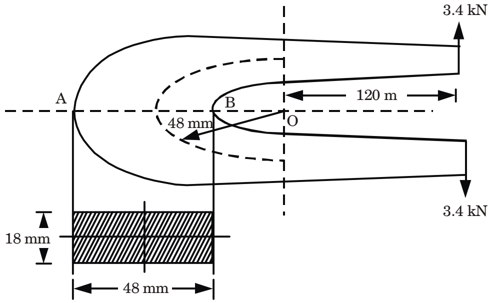

Fig. 6.

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SOLUTION OF PAPER (2016-17)

SECTION-A

1. Explain the following : (10 × 2 = 20)

a. Give an example of a structure where it is externally as well as internally indeterminate.

Ans. Equilibrium conditions, $e = 3$
Total external reaction, $r = 4$
Number of member, $m = 11$
Number of joint, $j = 6$

i. External indeterminacy,
$$I_e = r - e = 4 - 3 = 1$$
ii. Internal indeterminacy,
$$I_i = m - (2j - 3) = 11 - (2 \times 6 - 3) = 2$$
iii. Total indeterminacy,
$$I = I_e + I_i = 1 + 2 = 3$$

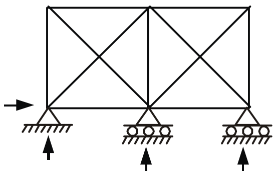


Fig. 1.

b. Which method of analysis is suitable, if static indeterminacy is more than kinematic indeterminacy ?

Ans. Stiffness method is suitable, when static indeterminacy is more than kinematic indeterminacy.

c. What are the uses of influence lines ?

Ans. Following are the uses of influence lines :

i. Influence lines are used to show the variation of shear force and bending moment in the member which is subjected to a live load or moving load.
ii. Influence lines help to tell where the moving load should be placed on the structure so that it creates the greatest influence at the specified point.
iii. Using the ordinates of influence line diagram, the magnitude of associated reaction, shear, moment or deflection at the point can be calculated.
iv. Influence lines are used to design the structures on which the loads move across the span. The common types of structures are design of bridges, industrial crane rails and conveyors etc.

d. Distinguish between influence line diagram and bending moment diagram.

Ans.

i. The ordinate of a curve of bending moment or shear force gives the value of the bending moment or shear force at the section where the ordinate has been drawn.
In case of an influence line diagram, the ordinate at any point gives the value of bending moment or shear force only at the given section and not at the point at which ordinate has been drawn.

ii. Influence line diagram represent the effect of a moving load only at a specified point on the member, whereas shear force or bending moment diagram represent the effect of fixed loads at all points along the axis of the member.

e. Classify the arches based on materials, shapes and structural systems.

Ans. **Classification of Arches :**

1. Based on the Material of Construction :

i. Steel arches. ii. Reinforced arches.
iii. Concrete arches. iv. Timber arches.
v. Brick arches. vi. Stone arches.

Brick and stone arches are combinedly known as masonry arches.

2. Based on Support Conditions and Structural Behaviour :

i. Three hinged arches – Hinged at crown and abutments.
ii. Two hinged arches – Hinged at abutment only.
iii. Hingeless or fixed arches – No hinges at all.

3. Based on Shape and Structural Arrangement of the Rib :

i. Solid rib arch (also known as closed arch).
ii. Tied solid rib arch.
iii. Spandrel braced arch.
iv. Two hinged braced rib arch or crescent arch or sickle arch.
v. Two hinged braced rib arch.

f. Why arches are preferred than beams ?

Ans. Arches preferred than beams because :

i. The inward horizontal reactions induced by the end restraints produce hogging moments in the arch which effectively counteract the static sagging moments set up by the vertical loads.

ii. The consequent reduction in the net moments which is responsible for significantly higher load bearing capacity of an arch as compared to the corresponding beam.

g. Write the formulae for area the centroid of the curve defined by $y = kx^n$.

Ans. Area of curve, $$A = \frac{bh}{n+1}$$

Centroid of area, $\bar{x} = \left(\dfrac{n+1}{n+2}\right) b$

h. What is the advantage of conjugate beam method over other method ?

Ans. Following are the advantages of conjugate beam method :

i. This method is applying to the beams having different moment of inertia.
ii. This method is simple, easy and consume less time.
iii. This method is useful for calculating deflection in different loading condition.

i. State Castigliano's first theorem ?

Ans. "The partial derivative of the total strain energy in a structure with respect to the displacement at any one of the load points gives the value of corresponding load acting on the body in the direction of displacement"

$$P_i = \frac{\partial U}{\partial \Delta_i}$$

j. Write the equation in term of strain energy, which is sufficient to determine the stress in case of propped cantilever beams.

Ans.
$$\sigma^2 = \frac{2E}{V} \times \text{Total strain energy}$$

where, V = Volume of the beam

Total strain energy in propped cantilever beam, $U = \int \dfrac{M^2 ds}{2EI}$

SECTION-B

2. Attempt any **five** of the following questions : **(5 × 10 = 50)**

a. A simply supported beam has a span of 25 m. Draw the influence line for shearing force at a section 10 m from one end and using this diagram, determine the maximum shearing force due to the passage of a point load 5 kN followed immediately by uniformly distributed load of 2.4 kN/m extending over a length of 5 m ?

Ans.

Given : Span of beam, L = 25 m, Distance of a section, x = 10 m
Point load, W = 5 kN, Intensity of UDL, w = 2.4 kN/m
Length of UDL = 5 m
To Find : Maximum shear force.

1. ILD for shear force at section at a distance 10 m from left support is shown in Fig. 2.

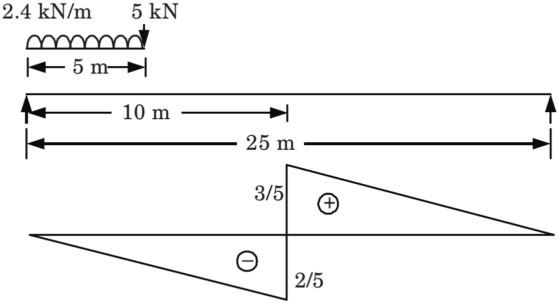


Fig. 2. ILD of shear force.

2. **Maximum Negative Shear Force :** For the maximum negative shear force placed the load in left part of section as shown in Fig. 3.

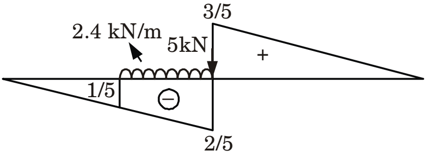


Fig. 3.

$$\text{Negative SF} = 5 \times \frac{2}{5} + 2.4 \times \left(\frac{1}{5} + \frac{2}{5}\right) \times \frac{1}{2} \times 5$$
$$= 2 + 2.4 \times 1.5 = 5.6 \text{ kN}$$

3. **Maximum Positive Shear Force :** For the maximum positive shear force placed the load in right side of section on ILD of shear force as shown in Fig. 4.

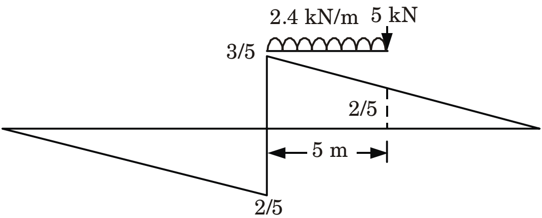


Fig. 4.

$$\text{Positive} \qquad \text{SF} = 5 \times \frac{2}{5} + 2.4 \times \left(\frac{3}{5} + \frac{2}{5}\right) \times \frac{1}{2} \times 5$$
$$= 2 + 2.4 \times 2.5 = 8 \text{ kN}$$

b. An uniformly distributed load of 40 kN/m and of length 3 metres transverse across the span of simply supported length of 18 metres. Compute the maximum bending moment at 4 m from the left support and absolute bending moment.

Ans.

Given : Span of beam, L = 18 m, Intensity of load, w = 40 kN/m
Span of UDL = 3 m
To Find : Maximum bending moment at section and absolute BM

1. Draw the ILD for BM whose maximum ordinate,

$$y = \frac{z(L-z)}{L} = \frac{4(18-4)}{18} = 3.11$$

2. For maximum moment, load position should be such that the section divides the load in the same ratio as it divides the span.

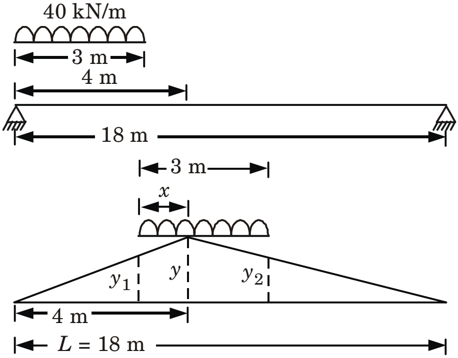


Fig. 5.

$$\frac{x}{3-x} = \frac{4}{18-4}$$

$$14x = 12 - 4x$$

$$18x = 12$$

$$x = 0.667 \text{ m}$$

3. Calculation of ordinate y_1 and y_2,

$$\frac{y_1}{4-0.667} = \frac{y}{4}$$

$$y_1 = \frac{3.11}{4} \times (4-0.667)$$

$$y_1 = 2.591 \text{ m}$$

$$\frac{y_2}{14-2.333} = \frac{3.11}{14}$$

$$y_2 = 2.591$$

4. Maximum moment = w × Area of ILD under the loaded length.

$$M_{max} = 40\left[\left(\frac{2.591 + 3.11}{2}\right) \times 0.667 + \left(\frac{3.11 + 2.591}{2}\right) \times 2.333\right]$$

$$M_{max} = 342.06 \text{ kN-m}$$

5. **Absolute Bending Moment :**

i. Absolute bending moment occurs at the centre of the span when the loading is symmetrically placed on the span as shown in Fig. 6.

ii. Calculation of ordinates :

$$y = \frac{9(18-9)}{18} = \frac{9}{2} = 4.5 \text{ m}$$

$$y_1 = \frac{4.5}{9} \times 7.5 = 3.75 \text{ m} = y_2$$

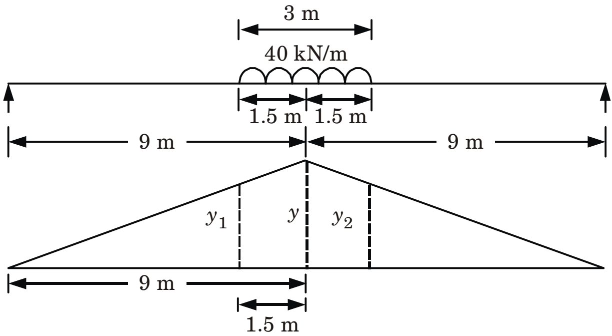


Fig. 6.

iii. Absolute bending moment = w × Area of ILD under the UDL.

$$M_{abs} = 40\left[2 \times \left(\frac{4.5 + 3.75}{2}\right) \times 1.5\right]$$

$$= 495 \text{ kN-m}$$

c. **A three hinged parabolic arch hinged at the supports and at the crown has a span of 24 m and a central rise of 4 m. It carries concentrated load of 50 kN at 18 m from the left support and UDL of 30 kN/m over the left portion. Determine the normal thrust, radial shear at a section 6 m from the left hand support.**

Ans.

Given : Span of arch, L = 24 m, Central rise of arch, h = 4 m
Concentrated load, W = 50 kN, Intensity of UDL, w = 30 kN/m
Distance of section = 6 m
To Find : Normal thrust and Radial shear.

1. The arch is shown in Fig. 7. Taking moment about support B, we get

$$V_A \times 24 - 30 \times 12 \times 18 - 50 \times 6 = 0$$
$$V_A = 282.50 \text{ kN}$$
$$\Sigma F_y = 0$$
$$V_A + V_B = 30 \times 12 + 50$$
$$V_B = 30 \times 12 + 50 - 282.50 = 127.5 \text{ kN}$$

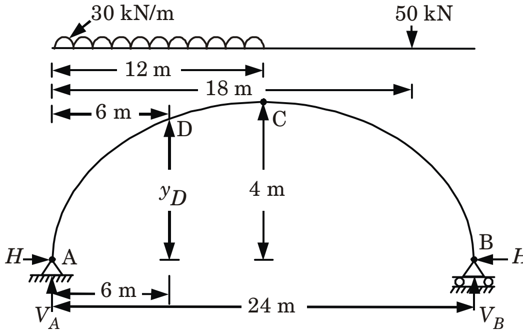


Fig. 7.

2. Taking moment about crown C (Right part),

$$V_B \times 12 - H \times 4 - 50 \times 6 = 0$$
$$127.5 \times 12 - H \times 4 - 50 \times 6 = 0$$

or $H = 307.5$ kN

3. Vertical shear at point D, $V = V_A - 30 \times 6 = 282.5 - 30 \times 6 = 102.5$ kN

4. Equation of curve is given by, $y = \dfrac{4hx(L - x)}{L^2}$

$$\frac{dy}{dx} = \tan\theta = \frac{4h(L - 2x)}{L^2}$$

Therefore, at $x = 6$ m,

$$\tan\theta = \frac{4 \times 4 \times (24 - 2 \times 6)}{24 \times 24}$$
$$\theta = 18.435°$$

5. Normal thrust, $N = V \sin\theta + H \cos\theta$

$$= 102.5 \times \sin 18.435° + 307.5 \times \cos 18.435°$$
$$= 324.133 \text{ kN}$$

6. Radial shear, $Q = V \cos\theta - H \sin\theta$

$$= 102.5 \times \cos 18.435° - 307.5 \times \sin 18.435° = 0$$

d. Find the slope and deflection at the free end of a cantilever shown in Fig. 8 by moment area method. Moment area of *AC* is twice the inertia of *BC*.

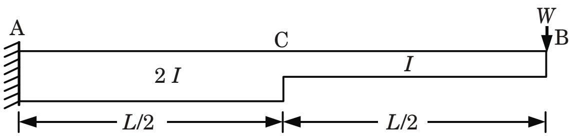


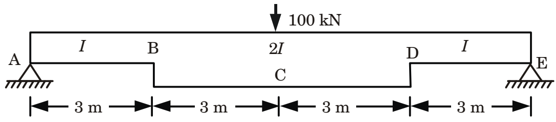
Fig. 8.

Ans. This question is out of syllabus from session 2017-18.

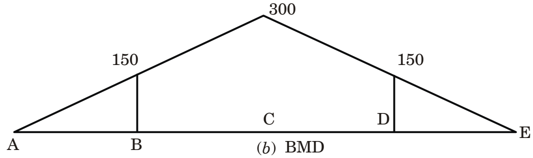
e. A beam *ABCDE* is 12 m long and supports a load of 100 kN at *C*, simply supported at *A* and *E*. *AB* = *BC*= *CD* = *DE* = 3 m. Moment of inertia is *I* in the portion *AB* and *DE* and 2*I* in the portion *BD*. Determine the deflections at *B* and *C* by using conjugate beam method.

Ans.

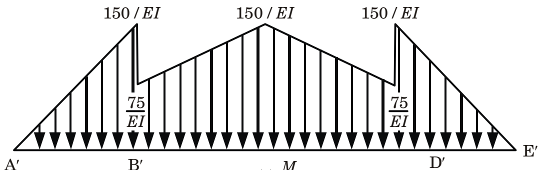
Given : Span of each part = 3 m, Concentrated load, W = 100 kN
Moment of inertia of part AB and $DE = I$
Moment of inertia of part BC and $CD = 2I$
To Find : Deflection at point B and C



(a) Given beam



(b) BMD



(c) $\frac{M}{EI}$ Diagram

Fig. 9.

1. Due to symmetry support reactions in beam,

$$V_A = V_E = \frac{100}{2} = 50 \text{ kN}$$

2. Moment at B, $M_B = V_A \times 3 = 50 \times 3 = 150$ kN-m
Moment at C, $M_C = V_A \times 6 = 50 \times 6 = 300$ kN-m
Fig. 9(*b*) shows the BM diagram for the given beam. Fig. 9(*c*) shows the *M*/*EI* diagram which is the loading on the conjugate beam. The thickness of this diagram is $\frac{1}{EI}$ for the parts AB and DE and $\frac{1}{2EI}$ for the part BD.
3. Total load on the conjugate beam = Area of the load diagram on the conjugate beam

$$= 2 \times \frac{1}{2} \times 3 \times \frac{150}{EI} + 2 \times 3 \times \left(\frac{75 + 150}{2}\right) \times \frac{1}{EI} = \frac{1125}{EI}$$

4. Due to symmetry reaction at supports of conjugate beam,

$$V'_A = V'_E = \frac{1125}{2\,EI} = \frac{562.5}{EI}$$

5. Deflection at point B of given beam = Bending moment at point B in conjugate beam

$$= V'_A \times 3 - \frac{1}{2} \times 3 \times \frac{150}{EI} \times \frac{3}{3}$$

$$\Delta_B = \frac{562.5}{EI} \times 3 - \frac{450}{2EI} = \frac{1462.5}{EI}$$

6. Deflection at point C in given beam = BM at point C in conjugate beam

$$= V'_A \times 6 - \frac{1}{2} \times 3 \times \frac{150}{EI} \times \left(3 + \frac{3}{3}\right) - \frac{3 \times 75}{EI} \times 1.5 - \frac{1}{2} \times 3 \times \frac{75}{EI} \times \frac{3}{3}$$

$$A_C = \frac{562.5}{EI} \times 6 - \frac{900}{EI} - \frac{337.5}{EI} - \frac{225}{2EI} = \frac{2025}{EI}$$

f. A cantilever beam is of span 2 m and is subjected to a concentrated load of 20 kN at the free end. The cross section of the beam is 100 × 200 mm and E = 30 kN/mm^2. Calculate the slope and deflection of the beam at midspan. Use unit load method.

Ans.

Given : Span of beam, L = 2 m, Concentrated load, W = 20 kN
Cross section of beam = 100 mm × 200 mm
Modulus of elasticity, E = 30 kN/mm^2 = 30 × 10^9 N/m^2
To Find : Slope and deflection at midspan of cantilever beam.

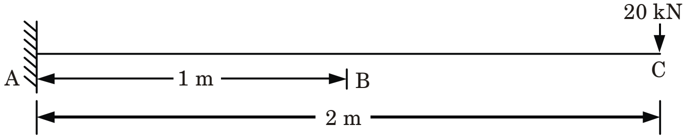


Fig. 10.

1. Moment of inertia,

$$I = \frac{100 \times 200^3}{12}$$

$$= 66.67 \times 10^6 \text{ mm}^4 = 66.67 \times 10^6 \times 10^{-12} \text{ m}^4$$

$$I = 66.67 \times 10^{-6} \text{ m}^4$$

2.

Portion	***CB***	***BA***
Origin	C	B
Limit	0 – 1	0 – 1
BM due to external load, M	$-20x$	$-20(1+x)$
BM due to unit load at midspan, M_1	0	$-1x$
BM due to unit moment at midspan, M_2	– 1	– 1

3. Deflection at B,

$$\Delta_B = \int_C^A \frac{MM_1}{EI}dx = \int_C^B \frac{MM_1}{EI}dx + \int_B^A \frac{MM_1}{EI}dx$$

$$= \int_0^1 \frac{-20x \times 0}{EI}dx + \int_0^1 \frac{20(1+x)\times x}{EI}dx$$

$$= 0 + \int_0^1 \frac{20x + 20x^2}{EI}dx = \frac{1}{EI}\left[\frac{20x^2}{2} + \frac{20}{3}x^3\right]_0^1$$

Deflection at B, $\Delta_B = \left[\frac{20\times 1}{2} + \frac{20\times 1}{3}\right]\frac{1}{EI}$

$$= \frac{16.67\times 10^6}{30\times 10^9 \times 66.67\times 10^{-6}} = 8.33 \text{ mm}$$

4. Slope at B, $\theta_B = \int_C^A \frac{MM_2}{EI}dx$

$$= \int_0^1 \frac{-20x\times(-1)}{EI}dx + \int_0^1 \frac{-20(1+x)\times(-1)}{EI}dx$$

$$= \frac{1}{EI}\left(\frac{20\,x^2}{2}\right)_0^1 + \frac{1}{EI}\left[20x + \frac{20x^2}{2}\right]_0^1$$

$$= \frac{1}{EI}\left[20\times 1 + 20\times 1^2\right] = \frac{40}{EI}$$

Slope at B, $\theta_B = \frac{40\times 10^3}{30\times 10^9 \times 66.67\times 10^{-6}} = 0.02$ radian

g. State and prove that the Castigliano's theorem.

Ans.

A. Castigliano's First Theorem :

Statement : "The partial derivative of the total strain energy in a structure with respect to the displacement at any one of the load points gives the value of corresponding load acting on the body in the direction of displacement"

$$P_i = \frac{\partial U}{\partial \Delta_i}$$

Proof :

1. Considering an elastic system subjected to a set of P ($P_1, P_2, P_3, ...,$ P_n) forces, which produce displacements, $\Delta_1, \Delta_2, \Delta_3 ...\Delta_n$ respectively in the direction of the respective loads at their points of applications.
2. Then the strain energy of the system will be

$$U = \sum_{i=1}^{n} \int P_i \, d\Delta_i \ or \ \delta U = \sum_{i=1}^{n} P_i \, d\Delta_i$$

3. If one of the displacement Δ_i is increases by $\delta\Delta_i$ in the direction of P_i keeping all other displacements unchanged then the incremental strain energy of the system will be

$$\delta U = P_i \, \delta\Delta_i \quad ...(1)$$

4. Since the other displacement were kept unchanged hence

$$\delta U = \frac{\partial U}{\partial \Delta_1} \delta\Delta_1 + \frac{\partial U}{\partial \Delta_2} \delta\Delta_2 + \frac{\partial U}{\partial \Delta_3} \delta\Delta_3 + ... + \frac{\partial U}{\partial \Delta_n} \delta\Delta_n$$

$$= \frac{\partial U}{\partial \Delta_i} \delta\Delta_i \text{ ... as other displacement are unchanged.} \quad ...(2)$$

5. Eq. (1) and (2), we get

$$P_i \, \delta\Delta_i = \frac{\partial U}{\partial \Delta_i} \delta\Delta_i$$

$$P_i = \frac{\partial U}{\partial \Delta_i} \text{ Hence proved.}$$

where P_i = External force at point 'i'.

B. Statement of Castigliano's Second Theorem :

1. In any linear elastic structure partial derivative of the strain energy with respect to load at a point is equal to the defection of the point where load is acting.
2. The deflection being measured in the direction of load.

i.e., $$\frac{\partial U}{\partial P_i} = \delta_i \text{ and } \frac{\partial U}{\partial M_i} = \theta_i$$

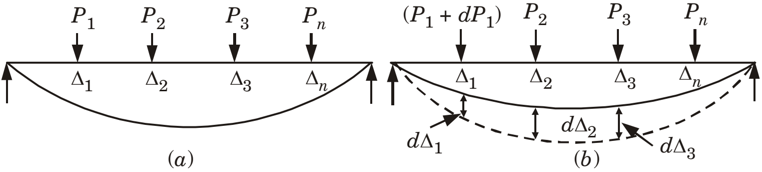


Fig. 11.

Proof :

1. Consider an linear elastic beam subjected to gradually applied forces $P_1, P_2, P_3, ..., P_n$. These forces produce deflections $\Delta_1, \Delta_2, \Delta_3,$ .... at their points of application respectively.

2. So, total strain energy stored will be

$$U = \frac{1}{2} P_1\Delta_1 + \frac{1}{2} P_2\Delta_2 + \frac{1}{2} P_3\Delta_3 + ... \quad ...(3)$$

3. If the additional load dP_1 is added after P_1, P_2, P_3... were applied then additional deflection are $d\Delta_1$, $d\Delta_2$, $d\Delta_3$,.... so the increased stored strain energy,

$$dU = \frac{1}{2} dP_1 d\Delta_1 + P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3 \quad ...(4)$$

$$dU - \frac{1}{2} dP_1 d\Delta_1 = P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3$$

$$\frac{1}{2}\left(dU - \frac{1}{2} dP_1 d\Delta_1\right) = \frac{1}{2} P_1 d\Delta_1 + \frac{1}{2} P_2 d\Delta_2 + \frac{1}{2} P_3 d\Delta_3 \quad ...(5)$$

4. Adding eq. (3) and eq. (4), we get

$$\text{Total strain energy} = U + dU = \frac{1}{2} P_1\Delta_1 + \frac{1}{2} P_2\Delta_2 + \frac{1}{2} P_3\Delta_3$$

$$+ \frac{1}{2} dP_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3 \quad ...(6)$$

5. If we assume that the $(P_1 + dP_1)$, P_2 and P_3 are being applied simultaneously then total strain energy stored will be

$$= \frac{1}{2} (P_1 + dP_1)(\Delta_1 + d\Delta_1) + \frac{1}{2} (\Delta_2 + d\Delta_2) P_2 + \frac{1}{2} (\Delta_3 + d\Delta_3) P_3 + ... \quad ...(7)$$

6. Since, total strain energy stored in both the case must be same. From eq. (6) and eq. (7), we get

$$\frac{1}{2} P_1 d\Delta_1 + \frac{1}{2} P_2 d\Delta_2 + \frac{1}{2} P_3 d\Delta_3 = \frac{1}{2} dP_1\Delta_1$$

7. From eq. (5), we have

$$\frac{1}{2}\left(dU - \frac{1}{2} dP_1 d\Delta_1\right) = \frac{1}{2} d\,P_1\Delta_1$$

Neglecting $\frac{1}{2} dP_1 d\Delta_1$ as it is too small, so

$$\frac{1}{2} dU = \frac{1}{2} d\,P_1\Delta_1$$

$$\Delta_1 = \frac{dU}{dP_1}$$

8. Similarly if the moment considered,

$$\frac{dU}{dM_1} = \theta_1$$

where, U = Total strain energy.

P_1, M_1 = Loads, moments on the structure.

δ_1, θ_1 = Displacements in the direction of the loads.

h. i. Define fatigue.
ii. What is the polar moment of inertia ?
iii. What is unsymmetrical bending ?
iv. What are the reasons for unsymmetrical bending occurring in the beams ?

Ans.

i. A phenomenon leading to the reduced internal resistance of a material when subjected to repeated or fluctuating cyclic stresses below the tensile strength of the material.
ii. This question is out of syllabus from session 2017-18.
iii. This question is out of syllabus from session 2017-18.
iv. This question is out of syllabus from session 2017-18.

SECTION-C

Attempt any **two** of the following questions : **(2 × 15 = 30)**

3. a. A simply supported beam with variable moment of inertia supports a uniformly distributed load of *w* kN/m. Estimate the maximum deflection in a beam.

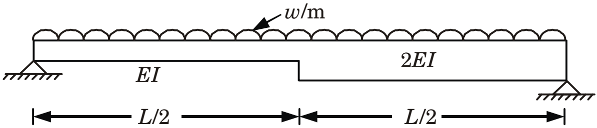


Fig. 12.

Ans.

Given : Span of beam = L m, Intensity of UDL = w kN/m
To Find : Maximum deflection in a beam.

1. Total load on conjugate beam

$$= \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8\,EI} + \frac{2}{3}\left(\frac{L}{2}\right)\frac{wL^2}{16EI}$$

$$= \frac{wL^2}{24\,EI}\left[1 + \frac{1}{2}\right] = \frac{wL^2}{16\,EI}$$

2. Take moment about point C',

$$M_C' = 0$$

$$V'_A \times L = \frac{wL^3}{24\,EI}\left(\frac{L}{2} + \frac{3}{8} \times \frac{L}{2}\right) + \frac{wL^3}{48\,EI}\left(\frac{5}{8} \times \frac{L}{2}\right)$$

$$V'_A \times L = \frac{wL^3}{24\,EI}\left(\frac{8+3}{16}\right)L + \frac{wL^3}{48\,EI}\left(\frac{5L}{16}\right)$$

$$= \frac{wL^3}{24\,EI}\left(\frac{11}{16}\right)L + \frac{wL^3}{48\,EI}\left(\frac{5L}{16}\right) = \frac{(22+5)\,wL^4}{48 \times 16\,EI}$$

$$V'_A \times L = \frac{27\,wL^4}{48 \times 16EI} = \frac{9wL^4}{256\,EI}$$

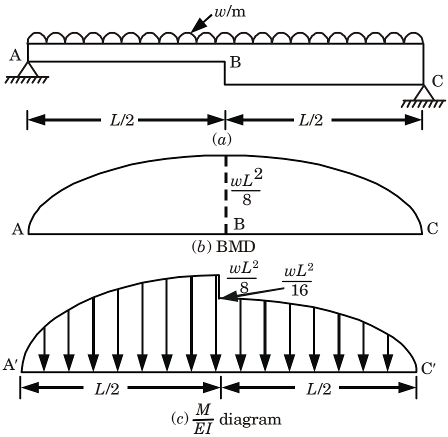


Fig. 13.

$$V'_A = \frac{9wL^3}{256EI}$$

3. Maximum deflection in beam occurs at that point, where loading is maximum in $\frac{M}{EI}$ loading diagram.

   Deflection at point B = Moment at point B in $\frac{M}{EI}$ diagram.

$$\Delta_{max} = \left[V'_A \times \frac{L}{2} - \frac{2}{3}\left(\frac{wL^2}{8}\right) \times \frac{L}{2} \times \left(\frac{5}{8} \times \frac{L}{2}\right)\right]\frac{I}{EI}$$

$$\Delta_{max} = \frac{9wL^3}{256EI} \times \frac{L}{2} - \frac{wL^4}{128EI}$$

$$\Delta_{max} = \frac{wL^4}{EI}\left[\frac{9}{512} - \frac{1}{128}\right]$$

$$\Delta_{max} = \frac{wL^4}{EI}\left[\frac{5}{512}\right]$$

   Maximum deflection, $\Delta_{max} = \frac{wL^4}{102.4EI}$

b. Determine the slopes at supports and deflection under the load for the beam shown in Fig. 14. Take Young's modulus E as 210 GPa, moment of inertia as 120 × 10⁶ mm⁴. Adopt conjugate beam method.

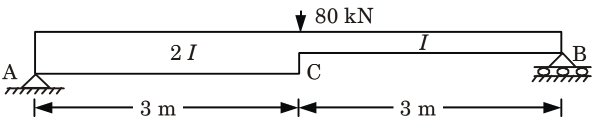


Fig. 14.

Ans.

Given : Moment of inertia of beam, $I = 120 \times 10^6$ mm^4
Young's modulus of beam, $E = 210$ GPa
Span of beam, $L = 6$ m
Concentrated load, $W = 80$ kN
To Find : Slope at supports and deflection under the load.

1. Due to symmetry,
   Reactions at supports,

$$V_A = V_B = \frac{80}{2} = 40 \text{ kN}$$

2. BM at point $C = 40 \times 3 = 120$ kN-m
3. Fig. 15(a) shows given beam. Fig. 15(b) shows the bending moment diagram for the given beam (M-diagram) Fig. 15(c) shows the conjugate beam.
4. The load diagram for the conjugate beam is given by the $\frac{M}{EI}$ diagram. The thickness of diagram is $\frac{1}{2EI}$ for the left and $\left(\frac{1}{EI}\right)$ for the right half part.
5. Reaction at supports :

i. $$V'_A + V'_B = \frac{1}{2} \times 3 \times \frac{60}{EI} + \frac{1}{2} \times 3 \times \frac{120}{EI} = \frac{270}{EI}$$

ii. Bending moment at point B, $\Sigma M_B = 0$

$$V'_A \times 6 = \frac{1}{2} \times 3 \times \frac{60}{EI}\left(3 + \frac{3}{3}\right) + \frac{1}{2} \times \frac{120}{EI} \times 3 \times 2 \times \frac{3}{3}$$

$$V'_A = \frac{120}{EI} \text{ and } V'_B = \frac{270}{EI} - \frac{120}{EI} = \frac{150}{EI}$$

6. Slope at support A, θ_A = Shear force at support A in conjugate beam

$$V'_A = \frac{120}{EI} = \frac{120 \times 10^6}{120 \times 10^6 \times 10} = 4.76 \times 10^{-3} \text{ radian}$$

7. Slope at support B,

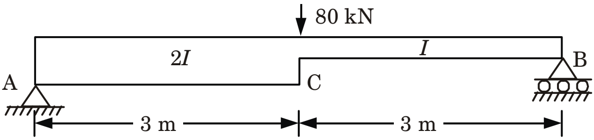
$$\theta_B = V'_B = \frac{150}{EI} = \frac{150 \times 10^6}{120 \times 10^6 \times 210}$$
$$= 5.95 \times 10^{-3} \text{ radian}$$

8. Deflection under the load = BM at point C in conjugate beam

$$= \frac{120}{EI} \times 3 - \frac{1}{2} \times \frac{60}{EI} \times 3 \times \frac{3}{3}$$

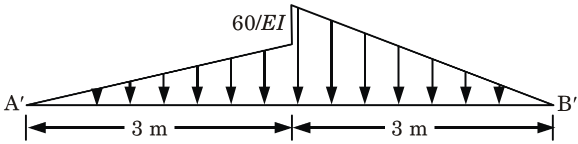
Deflection at point C,

$$\Delta_C = \frac{270}{EI} = \frac{270 \times 10^9}{210 \times 120 \times 10^6} = 10.72 \text{ mm}$$



(a)

(b) BMD



(c) M/EI diagram

Fig. 15.

4. a. Calculate the deflection under the load for truss shown in Fig. 16. All the members are have equal areas of 1250 mm² in cross section and E = 200 kNm².

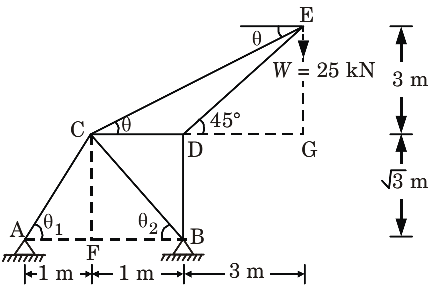


Fig. 16.

Ans.

Given : Area of member, A = 1250 mm²
Modulus of elasticity, E = 200 kN/m²
To Find : Deflection under the load.

1. Taking moment about A,

$$W \times 5 = V_B \times 2$$

$$V_B = \frac{5}{2}W = 2.5W \text{ kN}$$

2. $\Sigma F_y = 0$,

$$V_B - V_A = W$$

$$\frac{5}{2}W - V_A = W$$

$$V_A = \frac{3}{2}W = 1.5W\ (\downarrow) \text{ kN}$$

3. In ΔACF, $\tan \theta_1 = \frac{\sqrt{3}}{1} = \tan 60°$

$$\theta_1 = 60°$$

Same as in $\Delta CBF, \theta_2 = 60°$

4. In ΔCEG, $\tan \theta = \frac{3}{4} \Rightarrow \theta = 36°52'$

5. **Joint *A* :**

Resolve the forces vertically, $\Sigma F_y = 0$

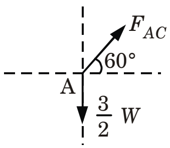


Fig. 17.

$$F_{AC} \sin 60° = \frac{3}{2}W$$

$$F_{AC} = \frac{3}{\sqrt{3}}W \text{ kN} = 1.73\ W \text{ kN}$$

6. **Joint *B* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

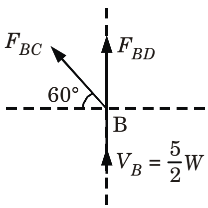


Fig. 18.

$$V_B + F_{BD} + F_{BC} \sin 60° = 0$$

$$\frac{5}{2}W + F_{BD} + F_{BC} \sin 60° = 0 \qquad \text{...(1)}$$

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{BC} \cos 60° = 0$$
$$F_{BC} = 0, \text{ put in eq. (3.4.1)}$$
$$\frac{5}{2} W + F_{BD} + 0 = 0$$
$$F_{BD} = -\frac{5}{2} W = -2.5\, W$$

7. **Joint *E* :**

i. Resolve the forces vertically, $\Sigma F_y = 0$

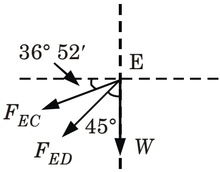


Fig. 19.

$$W + F_{ED} \cos 45° + F_{EC} \sin 36° 52' = 0$$
$$F_{ED} \cos 45° + F_{EC} \sin 36° 52' = -W \quad ...(2)$$

ii. Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{ED} \sin 45° + F_{EC} \cos 36° 52' = 0 \quad ...(3)$$

iii. From eq. (2) and eq. (3), we get

$$F_{ED} = -5.65\, W, F_{EC} = 5\, W$$

8. **Joint *D* :**

Resolve the forces horizontally, $\Sigma F_x = 0$

$$F_{DC} = F_{DE} \cos 45°$$
$$F_{DC} = -5.65\, W \times \cos 45°$$
$$= -4\, W \text{ kN}$$

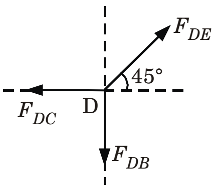


Fig. 20.

9. Strain energy stored by the structure :

$$U_i = \Sigma \frac{F^2 L}{2AE}$$

$$U_i = \frac{(1.73W)^2}{2AE} \times \sqrt{(\sqrt{3})^2 + 1^2} + 0 + \frac{(2.5\, W)^2(\sqrt{3})}{2AE}$$

$$+ \frac{(5.65\, W)^2 \times \left(\dfrac{3}{\sin 45°}\right)}{2AE} + \frac{(5W)^2 \times \left(\dfrac{3}{\sin 36°52'}\right)}{2AE} + \frac{(4W)^2 \times 1^2}{2AE}$$

$$U_i = \frac{W^2}{2AE}[5.986 + 0 + 10.82 + 135.43 + 125 + 16]$$

$$= \frac{293.236 \times W^2}{2 \times 1250 \times 10^{-6} \times 200 \times 10^6}$$

[Given unit of E is wrong. Taken value of E = 200 kN/mm^2]

$$U_i = 5.86 \times 10^{-4} W^2$$

10. $\therefore$ Vertical deflection at E

$$\Delta_E = \frac{\partial U_i}{\partial W} = 5.86 \times 10^{-4}(2W) = 5.86 \times 10^{-4} \times (2 \times 25)$$

$$\Delta_E = 0.0293 \text{ m} = 29.3 \text{ mm}$$

b. A three hinged parabolic arch is shown in Fig. 21. Determine the normal thrust, radial shear and bending moment at quarter span and draw BMD.

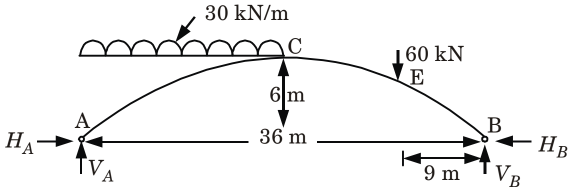


Fig. 21.

Ans.

Given : Rise of parabolic arch, h = 6 m, Span of the arch, L = 36 m
Intensity of UDL, w = 30 kN/m, Concentrated load, W = 60 kN
To Find : Normal thrust, Radial shear and Bending moment at quarter span and Draw BMD.

1. $$\Sigma F_y = 0$$
$$V_A + V_B = 30 \times 18 + 60 = 600 \text{ kN} \quad ...(1)$$
2. Taking moment about support B, $\Sigma M_B = 0$
$V_A \times 36 - 30 \times 18 \times 27 - 60 \times 9 = 0$
$$V_A = 420 \text{ kN}$$
From eq. (1), $V_B = 600 - 420 = 180$ kN
3. Taking moment about hinge point C, $\Sigma M_C = 0$
$V_A \times 18 - 30 \times 18 \times 9 = 6 \times H_A$
$420 \times 18 - 30 \times 18 \times 9 = 6 \times H_A$
$$H_A = 450 \text{ kN}$$
4. Equation of parabolic arch is given by,
$$y = \frac{4hx}{L^2}(L - x)$$
Rise of arch at quarter span,
$$y = \frac{4 \times 6 \times 9}{36^2}(36 - 9) = 4.5 \text{ m}$$
5. Bending moment at quarter span,

$$M = V_A \times 9 - 30 \times 9 \times 4.5 - H_A \times y$$
$$= 420 \times 9 - 30 \times 9 \times 4.5 - 450 \times 4.5$$
$$M = 540 \text{ kN-m}$$

6. Vertical shear at quarter span,

$$V = V_A - 30 \times 9 = 420 - 30 \times 9 = 150 \text{ kN}$$

7. Slope $= \dfrac{dy}{dx} = \tan\theta = \dfrac{4h\,(L - 2x)}{L^2} = \dfrac{4 \times 6 \times (36 - 2 \times 9)}{36^2} = 0.333$

$$\theta = 18.435°$$

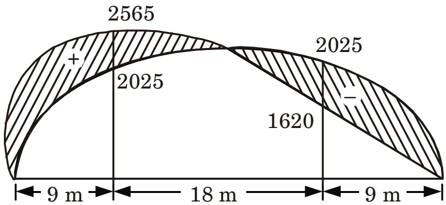


Fig. 22.

8. Normal thrust at quarter span,

$$N = V \sin\theta + H \cos\theta$$
$$= 150 \times \sin 18.435° + 450 \times \cos 18.435°$$
$$N = 474.34 \text{ kN}$$

9. Radial shear at quarter span,

$$S = V \cos\theta - H \sin\theta$$
$$S = 150 \times \cos 18.435° - 450 \times \sin 18.435° = 0$$

10. *Hy* moment at quarter span,

$$Hy = 450 \times 4.5 = 2025 \text{ kN-m}$$

11. Beam moment at quarter span $= V_A \times 9 - 30 \times 9 \times 4.5$

$$= 420 \times 9 - 30 \times 9 \times 4.5$$
$$= 2565 \text{ kN-m}$$

12. Beam moment under concentrated load

$$= V_A \times 27 - 30 \times 18 \times 18$$
$$= 420 \times 27 - 30 \times 18 \times 18 = 1620 \text{ kN-m}$$

5. Fig. 23 shows a frame subjected to a load of 3.4 kN. Find the resultant stress at *A* and *B*.

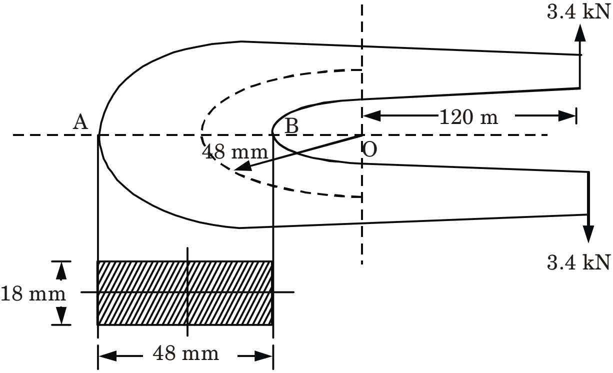


Fig. 23.

Ans. This question is out of syllabus from session 2017-18.

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B. Tech.
(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2017-18
STRUCTURAL ANALYSIS-I

Time : 3 Hours **Max. Marks : 70**

Note. 1. Attempt all sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt **all** questions in brief : **(7 × 2 = 14)**

a. What do you mean by static and kinematic indeterminacy of a structure ?

b. Define space truss with suitable example.

c. Draw the influence line diagram of bending moment for a simply supported beam at a section D.

d. State the Eddy's theorem.

e. Three hinged arch is a determinate structure. Why ?

f. Write statement of Castigliano's first and second theorem.

g. What do you mean by principal axes ?

SECTION-B

2. Attempt any **three** of the following : **(3 × 7 = 21)**

a. The loading and support condition of a plane truss is shown in Fig. 1. Find the forces in member *AB* and *BF*.

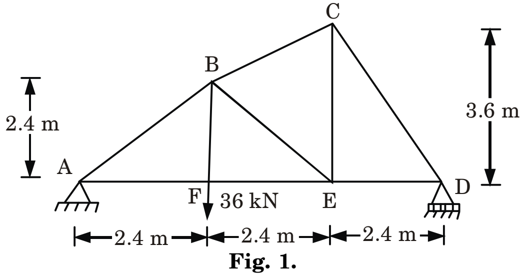

Fig. 1.

b. A uniformly distributed load of 50 kN/m longer than the span rolls over a girder of 30 m span. Determine the maximum SF and BM at a section 12 m from left hand support.

c. A three hinged parabolic arch of span 40 m and rise 10 m carries concentrated loads of 20 kN and 70 kN at a distance 8 m and 16 m from the left and a uniformly distributed load of 50 kN/m on the right half of the span. Find the horizontal thrust.

d. Determine the slope and deflection at free end of cantilever beam of span L, and uniformly loaded with load 'w'. EI = Constant.

e. A channel section has overall depth of 250 mm, flange width of 125 mm, and flange thickness of 20 mm and also web thickness of 20 mm. Find the location of shear center.

SECTION-C

3. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. Determine the static indeterminacy (D_s) and Kinematic indeterminacy (D_s) for a given frame.

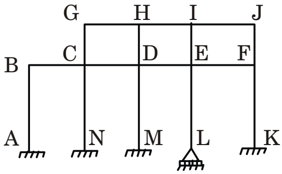

Fig. 2.

b. A braced cantilever is loaded as shown in Fig. 3. All the members are of same cross sectional area. Find the force in *BE*.

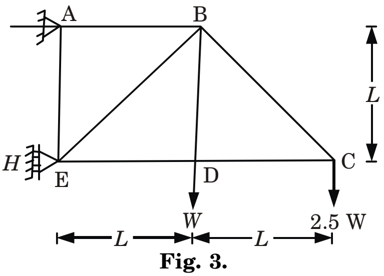

Fig. 3.

4. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. Two wheel loads 160 kN and 100 kN, spaced 4 m apart, are moving over a simply supported beam of 12 m span. Determine the maximum shear force and maximum bending moment that may be developed anywhere on the beam.

b. The load system shown in Fig. 4 moves from left to right on a girder of span 10 m. Find the absolute maximum bending moment for the girder.

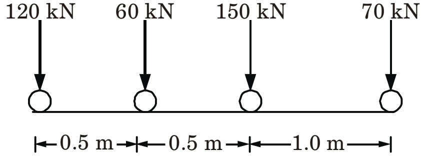

Fig. 4.

5. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. Draw the influence line diagram for normal thrust of a three hinged parabolic arch at a section *D*.

b. Proof that bending moment at any section of a three hinged parabolic arch having a UDL over its whole span will be zero.

6. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. A simply supported beam of uniform cross section subjected to concentrated load *W* at mid span. If span of the beam is 10 m, calculate slope at its end and also calculate the deflection at mid span. Use conjugate beam method.

b. Determine the horizontal deflection at a free end of the frame as shown in Fig. 5, by using unit load method.

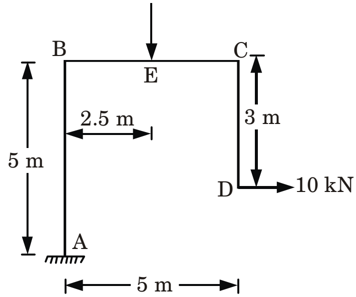

Fig. 5.

7. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. A beam of rectangular section 80 mm wide and 120 mm deep is subjected to a bending moment of 12 kN-m the trace of the plane loading is inclined at 45° to the YY-axis of the section. Calculate the maximum bending stress induced in the beam.

b. A cast iron beam of T-section as shown in Fig. 6. The beam is simply supported on a span of 8 m, the beam carries a UDL of 1.5 kN/m length on the entire span. Determine the maximum tensile and compressive stresses.

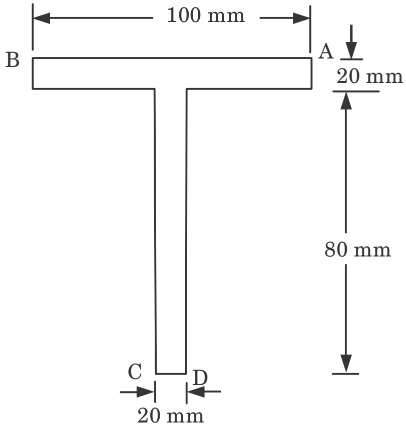

Fig. 6.

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SOLUTION OF PAPER (2017-18)

Note. 1. Attempt all sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt **all** questions in brief : **(7 × 2 = 14)**

a. What do you mean by static and kinematic indeterminacy of a structure ?

Ans. Static Indeterminacy : Static indeterminacy of a structure is defined as the difference between number of unknown forces and number of equilibrium equations to be solved. Degree of static indeterminacy = $(m + r) - 2j$.

Kinematic Indeterminacy : It is defined as the sum of all the possible displacements that various joints of the structure can undergo. Degree of kinematic indeterminacy = Sum of degrees of freedom in rotation and translation.

b. Define space truss with suitable example.

Ans. Space trusses have members extending in three dimensions and are suitable for derricks and towers.

i. Space truss consists of members joined together at their ends to form a stable three-dimensional structure.

ii. The simplest element of a stable truss is a tetrahedron.

ii. It is formed by connecting six members together with four joints.

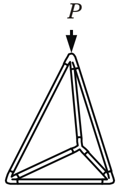


Fig. 1.

c. Draw the influence line diagram of bending moment for a simply supported beam at a section *D*.

Ans.

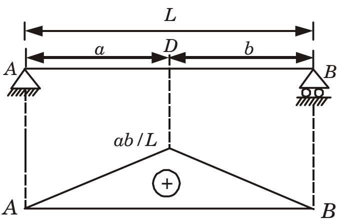


Fig. 2. ILD for bending moment at point *D*.

d. State the Eddy's theorem.

Ans. The bending moment at any section of an arch is equal to the vertical intercept between the linear arch and the central line of the actual arch.

e. Three hinged arch is a determinate structure. Why ?

Ans. Yes, three hinged arch is a determinate structure, due to total unknown reaction (4) equal to total known reaction (4).

f. Write statement of Castigliano's first and second theorem.

Ans. **Castigliano's First Theorem :** "The partial derivative of the total strain energy in a structure with respect to the displacement at any one of the load points gives the value of corresponding load acting on the body in the direction of displacement"

$$P_i = \frac{\partial U}{\partial \Delta_i}$$

Castigliano's Second Theorem : In any linear elastic structure, partial derivative of the strain energy with respect to load or moment at a point is equal to the deflection or slope of the point where load is acting. The deflection being measured in the direction of load.

$$\frac{\partial U}{\partial P_i} = \Delta_i \text{ and } \frac{\partial U}{\partial M_i} = \theta_i$$

g. What do you mean by principal axes ?

Ans. The two axes for which product of inertia (I_{xy}) is zero for a section are known as principal axes or principal centroidal axes of the cross-section.

SECTION-B

2. Attempt any **three** of the following : **(3 × 7 = 21)**

a. The loading and support condition of a plane truss is shown in Fig. 3. Find the forces in member *AB* and *BF*.

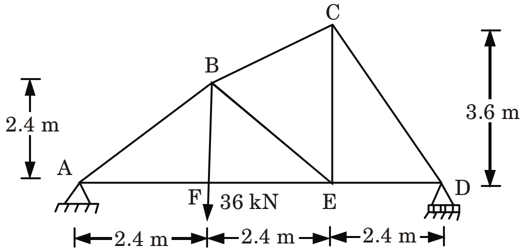


Fig. 3.

Ans.

Given : Plane truss as shown Fig. 3.
To Find : Forces in members *AB* and *BF*.

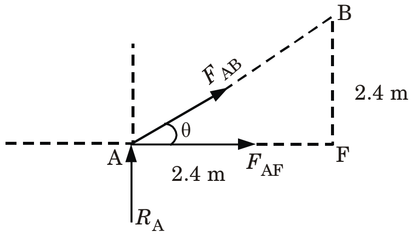


Fig. 4.

1. **Vertical Reactions at Supports :**

i. $$\Sigma F_y = 0$$
$$V_A + V_D = 36 \text{ kN} \quad ...(1)$$

ii. Take moment about point *D*, we get
$$\Sigma M_D = 0$$
$$V_A \times 7.2 - 36 \times 4.8 = 0$$
$$V_A = 24 \text{ kN}$$

iii. $$V_D = 36 - R_A = 36 - 24 = 12 \text{ kN}$$

2. **Consider Joint A :**

i. From ΔAFB, $$\tan\theta = \frac{BF}{AF} = \frac{2.4}{2.4} = 1$$
$$\theta = 45°$$

ii. Horizontally resolve the, $\Sigma F_x = 0$
$$F_{AF} + F_{AB}\cos\theta = 0$$
$$F_{AF} + F_{AB}\cos 45° = 0, \qquad F_{AF} = -\frac{F_{AB}}{\sqrt{2}} \quad ...(2)$$

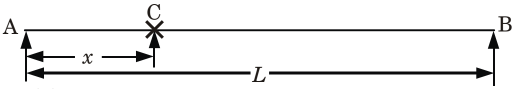
iii. Vertically resolve the forces, $\Sigma F_y = 0$
$$V_A + F_{AB}\sin\theta = 0$$
$$24 + F_{AB}\sin 45° = 0$$
$$F_{AB} = \frac{-24}{\sin 45°} = -24\sqrt{2} \text{ kN}$$

The value of F_{AB} is put into eq. (2), we get
$$F_{AF} = \frac{-F_{AB}}{\sqrt{2}} = \frac{-(-24\sqrt{2})}{\sqrt{2}} = 24 \text{ kN}$$

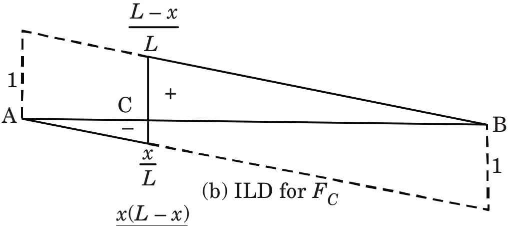
b. A uniformly distributed load of 50 kN/m longer than the span rolls over a girder of 30 m span. Determine the maximum SF and BM at a section 12 m from left hand support.

Ans.

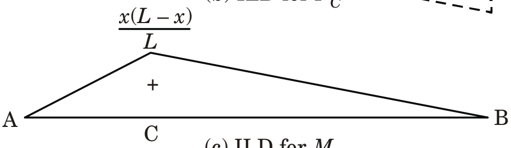
Given : Intensity of UDL, w = 50 kN/m, Distance of section, x = 12 m, Span of beam, L = 30 m
To Find : Maximum shear force and bending moment at section.



(a) Simply supported beam with given point C



(b) ILD for F_C



(c) ILD for M_C

Fig. 5.

1. Negative SF is maximum, when the load covers portion AC only.
2. Maximum negative SF = w × Area of ILD for SF in length AC

$$= -w \times \frac{1}{2} x \times \frac{x}{L} = 50 \times \frac{1}{2} \times 12 \times \frac{12}{30} = -120 \text{ kN}$$

3. Positive SF is maximum when the uniformly distributed load occupies the portion CB only.
4. Maximum positive SF = w × Area of ILD for SF in length CB

$$= w \times \frac{1}{2} \times (L-x) \times \left(\frac{L-x}{L}\right)$$

$$= 50 \times \frac{1}{2} \times (30-12) \left(\frac{30-12}{30}\right) = 270 \text{ kN}$$

5. From Fig. 5(c) it is called that maximum moment at C will be, when the UDL covers entire span,

$$M_{C,\max} = w \times \text{Area of ILD for } M_C$$

$$= w \times \frac{1}{2} \times L \times x\left(\frac{L-x}{L}\right)$$

$$= 50 \times \frac{30}{2} \times 12 \times \left(\frac{30-12}{30}\right) = 5400 \text{ kN-m}$$

c. A three hinged parabolic arch of span 40 m and rise 10 m carries concentrated loads of 20 kN and 70 kN at a distance 8 m and 16 m from the left and a uniformly distributed load of 50 kN/m on the right half of the span. Find the horizontal thrust.

Ans.

Given : Span of arch = 40 m, Rise = 10 m, Loads = 20 kN and 70 kN, Intensity of UDL, w = 50 kN/m
To Find : Horizontal thrust.

1. Vertical reactions,

$$\Sigma V = 0$$
$$V_A + V_B = 20 + 70 + 20 \times 50 = 1090 \text{ kN}$$

2. Taking moment about point 'B',

$$\Sigma M_B = 0$$
$$V_A \times 40 - 20 \times 32 - 70 \times 24 - 50 \times 20 \times 10 = 0$$
$$V_A = 308 \text{ kN}$$
$$V_B = 1090 - 308 = 782 \text{ kN}$$

3. Taking moment about point 'C',

$$\Sigma M_C = 0 \text{ (Consider left part)}$$
$$V_A \times 20 - H \times 10 - 20 \times 12 - 70 \times 4 = 0$$
$$308 \times 20 - H \times 10 - 12 \times 20 - 70 \times 4 = 0$$
$$H = 564 \text{ kN}$$

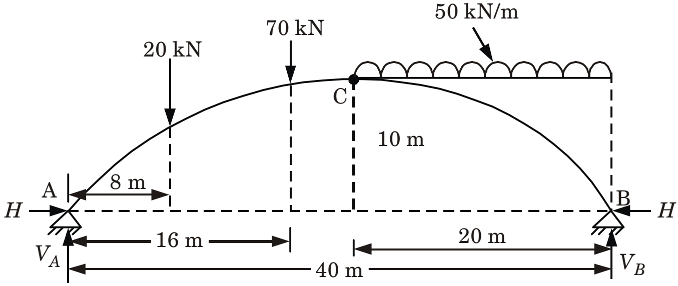


Fig. 6.

d. Determine the slope and deflection at free end of cantilever beam of span L, and uniformly loaded with load 'w'. EI = Constant.

Ans.

Given : Span of beam = L, Intensity of UDL = w
To Find : Slope and deflection at free end.

1. Fig. 7(a) shows a cantilever beam AB of length L carrying a uniformly distributed load of w per unit run over the whole length.

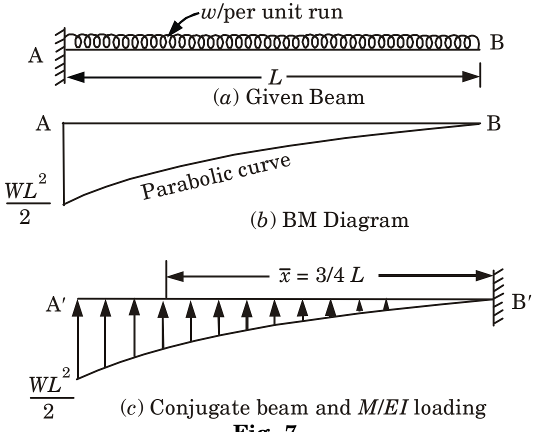
2. Fig. 7(*b*) shows the bending moment diagram for the cantilever.
3. Fig. 7(*c*) shows the corresponding conjugate beam $A'B'$ with M/EI loading.
4. Slope at B for the given beam = Shear force at B' for the conjugate beam

$$= \text{Volume of load diagram.}$$
$$= \text{Area} \times \text{thickness.}$$
$$= \frac{1}{3} \times \text{Base} \times \text{Height} \times \text{Thickness}$$

Slope at B for the given beam $= \dfrac{1}{3} \times L \times \dfrac{wL^2}{2} \times \dfrac{1}{EI} = \dfrac{wL^3}{6\,EI}$

5. Deflection at B for the given beam = BM at B for conjugate beam

$$= \frac{wL^3}{6\,EI} \times \frac{3}{4} L = \frac{wL^4}{8\,EI} \text{ (Downward)}$$



(*a*) Given Beam

(*b*) BM Diagram

(*c*) Conjugate beam and *M/EI* loading

Fig. 7.

e. A channel section has overall depth of 250 mm, flange width of 125 mm, and flange thickness of 20 mm and also web thickness of 20 mm. Find the location of shear center.

Ans.

Given : Depth of web = 250 mm, Flange width = 125 mm
Flange thickness = 20 mm, Web thickness = 20 mm
To Find : Location of shear center.

1. From Fig. 8, $b = 125 - \dfrac{20}{2} = 115$ mm

$$t_1 = t_2 = 20 \text{ mm}$$
$$h = 250 - 20 = 230 \text{ mm}$$

2.
$$S_1 = \int_0^b \frac{S(A\bar{y})}{I_x \times t} dA = \int_0^b \frac{Sxth}{2I_x t} t dx$$

$$S_1 = 2 \times \int_0^{115} \frac{S \times (x \times 20) \times 115 \times (20 \times dx)}{2 \times I_x \times 20}$$

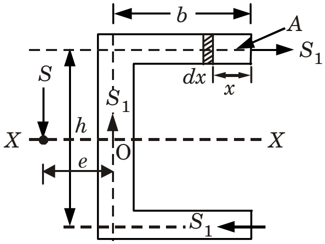


Fig. 8.

$$S_1 = \int_0^{115} \frac{S \times x \times 2300 \times dx}{I_x}$$

$$S_1 = \frac{2300 \times S}{I_x} \left[\frac{x^2}{2}\right]_0^{115} = \frac{2300 \times S \times (115)^2}{2I_x}$$

3. Taking moments of shear forces about the centre O of the web, we get,

$$S \times e = S_1 \times h$$

$$e = \frac{S_1 \times h}{S} = \frac{2300 \times S \times (115)^2}{S \times 2I_x} \times h$$

$$I_x = \frac{h^2 t}{12}(h + 6b) = \frac{230^2 \times 20}{12} \times (230 + 6 \times 115)$$

$$I_x = 81.113 \times 10^6 \text{ mm}^4$$

4. Shear centre, $e = \dfrac{2300 \times (115)^2 \times 230}{2 \times 81.113 \times 10^6}$

$$e = 43.125 \text{ mm}$$

5. **Check :** $e = \dfrac{3b}{6 + \dfrac{A_w}{A_f}} = \dfrac{3 \times 115}{6 + \dfrac{20 \times 230}{20 \times 115}}$ $\left[\because \begin{array}{l} A_w = h \times t \\ A_f = bt \end{array}\right]$

$$e = 43.125 \text{ mm}$$

SECTION-C

3. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. Determine the static indeterminacy (D_s) and Kinematic indeterminacy (D_s) for a given frame.

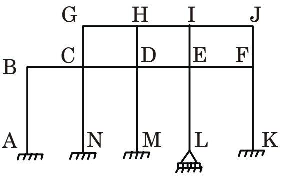


Fig. 9.

Ans.

1. **Static Indeterminacy :**

i. Static indeterminacy of rigid frame structure :

$$I_t = I_e + I_i$$
$$I_t = (3\,m + r) - 3j$$

Where, m = Number of member.
r = Number of reaction.
j = Number of joint.

ii. From Fig. 9,

$$m = 16, r = 13, j = 14$$
$$D_S = I_t = (3\,m + r) - 3j = 3 \times 16 + 13 - 3 \times 14 = 19$$

2. **Kinematic Indeterminacy :**

i. Degree of freedom at supports :
   a. At point A, N, M and $K = 0$ (Fixed support)
   b. At point $L = 02$ (hinged point)

ii. Degree of freedom at B, C, D, E and F
   a. Possible rotations at B, C, D, E and $F = 01$ rotation at each
      Total rotations at B, C, D, E and $F = 05$
   b. Possible translation at B or $F = 01$

iii. Degree of freedom at point G, H, I and J.
   a. Rotations at point G, H, I and $J = 01$ rotation on each
      Total at point G, H, I and $J = 01 \times 4 = 04$.
   b. Translation at point G, or $J = 01$

iv. Hence total degree of freedom = 2 + 5 + 1 + 4 + 1 = 13.

b. A braced cantilever is loaded as shown in Fig. 10. All the members are of same cross sectional area. Find the force in *BE*.

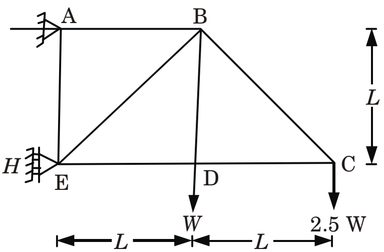


Fig. 10.

Ans.

Given : A braced cantilever as shown in Fig. 10.
To Find : The force in BE.

1. **Reaction at Supports :**

i. From ΔEDB

$$\tan\theta = \frac{BD}{DE} = \frac{L}{L} = \theta = 45°$$

ii. $$\Sigma F_x = 0$$

$$H_A + H_E = 0 \qquad ...(1)$$

iii. $$\Sigma F_y = 0$$

$$V_A = 2.5\ W + W = 3.5\ W$$

iv. Take moment about joint E, we get

$$H_A \times L + W \times L + 2.5\ W \times 2L = 0$$

$$H_A = -6\ W$$

v. From eq. (1), we get

$$H_E = 6\ W$$

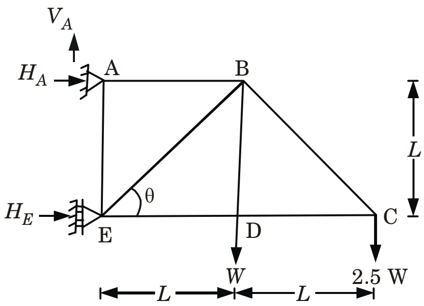


Fig. 11.

2. **Joint 'A' :**

i. Resolves the forces vertically, $\Sigma F_y = 0$

$$V_A = F_{AE}, \qquad F_{AE} = 3.5\ W$$

ii. Resolves the forces horizontally,

$$\Sigma F_x = 0$$

$$H_A + F_{AB} = 0$$

$$F_{AB} = -H_A = -(-6W)$$

$$F_{AB} = 6\ W$$

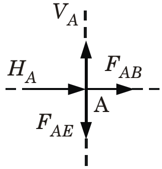


Fig. 12.

3. Point '*E*'

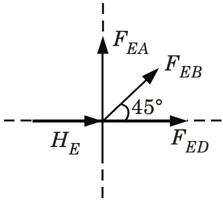


Fig. 13.

i. Resolves the forces vertically, $\Sigma F_y = 0$

$$F_{EA} + F_{EB} \sin 45° = 0$$
$$3.5\,W + F_{EB} \sin 45° = 0$$
$$F_{EB} \sin 45° = -3.5\,W$$
$$F_{EB} \times \frac{1}{\sqrt{2}} = -3.5\,W$$

Force in member *BE*, $F_{EB} = -3.5 \times \sqrt{2}\,W$

4. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. Two wheel loads 160 kN and 100 kN, spaced 4 m apart, are moving over a simply supported beam of 12 m span. Determine the maximum shear force and maximum bending moment that may be developed anywhere on the beam.

Ans.

Given : $W_1 = 160$ kN, $W_2 = 100$ kN,
Distance between two load = 4 m, Span of beam, $L = 12$ m.
To Find : Maximum shear force and Maximum bending moment.

1. Maximum Shear Force :

i. For absolute maximum positive shear, the load position is as shown in Fig. 14.

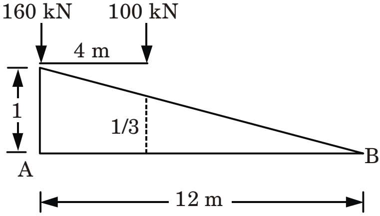


Fig. 14.

ii. Maximum positive SF $= 160 \times 1 + 100 \times \dfrac{1}{3} = 193.33$ kN

iii. For absolute maximum negative shear, the load position is as shown in Fig. 15.

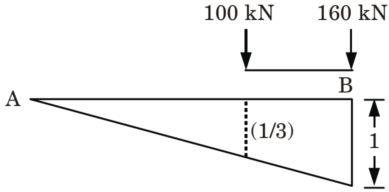


Fig. 15.

iv. Maximum negative SF = $160 \times 1 + 100 \times \frac{1}{3}$ = 193.33 kN.

2. Maximum Bending Moment :

i. Taking moment about leading load 160 kN

$$\bar{x} = \frac{100 \times 3 + 160 \times 0}{160 + 100} = 1.154 \text{ m}$$

ii. Distance between the resultant load and the load (160 kN)

$= 1.154$ m.

iii. Hence, for the condition of absolute moment, the load 160 kN should be placed $\frac{1.154}{2} = 0.577$ m on the right side of the centre of beam.

iv. For maximum bending moment this load should be at

$= 6 + 0.577 = 6.577$ m

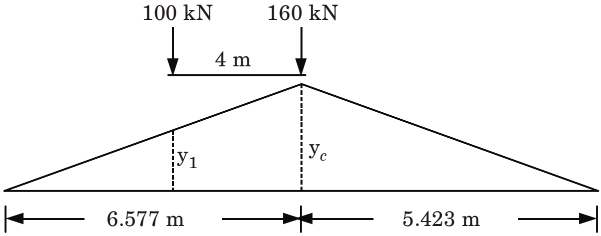


Fig. 16.

$$y_c = \frac{6.577 \times 5.423}{12} = 2.973$$

v. Maximum bending moment

$= 100 \times y_1 + 160 \times y_c$

$= 100 \times \frac{2.9773}{6.577} \times 2.577 + 160 \times 2.973$

$= 592.17$ kN-m

b. The load system shown in Fig. 17 moves from left to right on a girder of span 10 m. Find the absolute maximum bending moment for the girder.

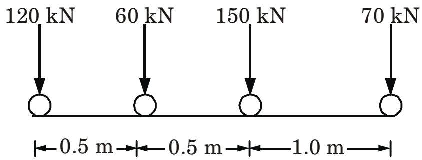


Fig. 17.

Ans.

Given : Span of beam, L = 10 m, Loads are given in Fig. 17.
To Find : Absolute bending moment.

1. To obtain absolute bending moment, firstly we have to find out the position of the resultant of given wheel loading. Let the distance of the resultant load from the 70 kN load be $\bar{x}$

$$\bar{x} = \frac{70 \times 0 + 150 \times 1 + 60 \times 1.5 + 120 \times 2}{70 + 150 + 60 + 120} = 1.2 \text{ m}$$

2. It is nearer to 150 kN load and this load is heavier than another nearer load of other load. Hence maximum moment will occurs under 150 kN load.
3. Distance between this load and the resultant,

$$d = 1.2 - 1 = 0.2 \text{ m}$$

4. Position of 150 kN load for maximum moment

$$= \frac{10}{2} + \frac{0.2}{2} = 5.1 \text{ m from } A$$

5. ILD ordinate for a section at 5.1 m is moment

$$y_c = \frac{x(L-x)}{L} = \frac{5.1 \times (10 - 5.1)}{10} = 2.499 \text{ m } = 2.5 \text{ m}$$

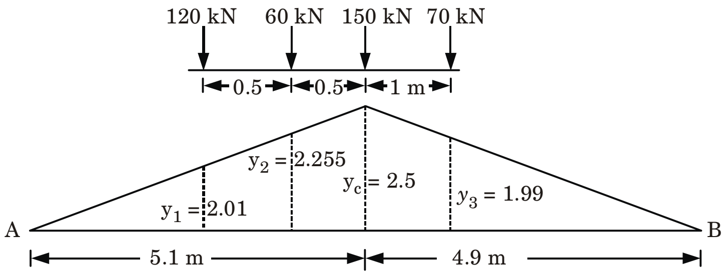


Fig. 18.

6. Absolute maximum bending moment

$$= 120 \times y_1 + 60 \times y_2 + 150 \times y_c + 70 \times y_3$$
$$= 120 \times 2.01 + 60 \times 2.255 + 150 \times 2.5 + 70 \times 1.99 = 890.8 \text{ kN-m}$$

5. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. Draw the influence line diagram for normal thrust of a three hinged parabolic arch at a section *D*.

Ans. Influence line Diagrams for Normal Thrust at Section *D* :
Fig. 19 (*c*) shows a three hinged arch of span *L* and rise *h*.

1. Let a unit load be any section between the left end *A* and the crown *C*, at a distance *x* from *A*.
2. Obviously the vertical reactions at the supports will be,

$$V_B = \frac{x}{L} \quad \text{and} \quad V_A = \frac{L - x}{L}$$

Let *H* be the horizontal thrust.

3. Taking moment about the hinge *C*, we have

$$H \times h = \frac{x}{L} \times \frac{L}{2}, \quad H = \left(\frac{x}{2h}\right)$$

4. This is true for all position of load from *A* to *C*.

When $x = 0$, $H = 0$; When $x = \frac{L}{2}$, $H = \frac{L}{4h}$

5. Hence as the unit load moves from *A* to *C* the horizontal thrust will change from zero to $\frac{L}{4h}$.
6. Obviously as the unit load moves from *C* to *B* the horizontal thrust will change from $\frac{L}{4h}$ to zero.

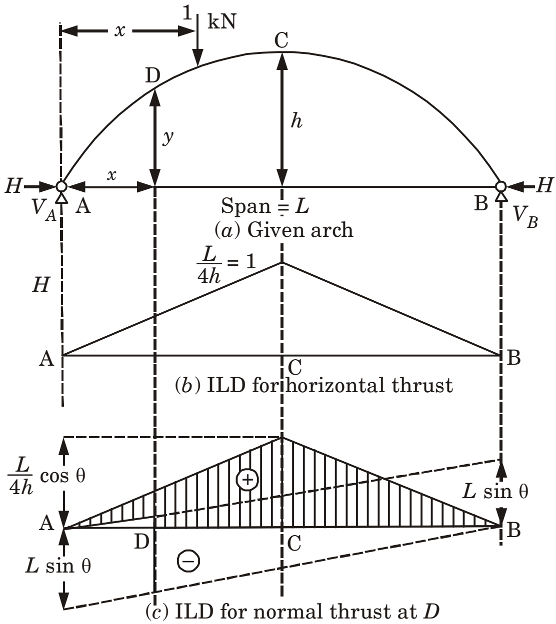


Fig. 19.

7. When the unit load is between A and D

$$N = H \cos \theta - V_B \sin \theta$$

8. When the unit load is between D and B,

$$N = H \cos \theta + V_A \sin \theta$$

9. The influence line is drawn as follows :
   i. First draw the influence line for $H \cos \theta$. This is a triangle whose altitude is $\frac{L}{4h} \cos \theta$.
   ii. On this diagram superimpose the influence line diagram for $V_B \sin \theta$ for the part AD.
   iii. For the range BD, we must add the influence line diagram for $V_A \sin \theta$ to the influence line diagram for $H \cos \theta$.

b. Proof that bending moment at any section of a three hinged parabolic arch having a UDL over its whole span will be zero.

Ans.

1. Let

$$L = \text{Span of the arch,}$$
$$h = \text{Central rise, and}$$
$$w = \text{UDL applied on the arch.}$$

2. From symmetry we have,

$$V_A = V_B = \frac{wL}{2}$$

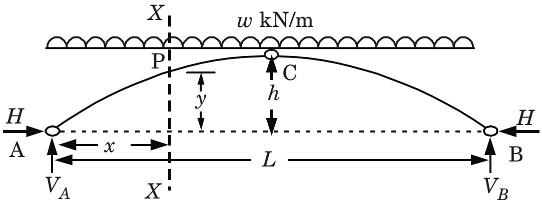


Fig. 20.

3. For horizontal thrust, taking moment about C,

$$H \times h = \frac{wL}{2} \times \frac{L}{2} - \frac{wL}{2} \times \frac{L}{4} = \frac{wL^2}{8}$$

$$H = \frac{wL^2}{8h}$$

4. Let us now consider any section at distance x from A.

Equation of parabola is given by, $y = \frac{4h}{L^2}\, x\,(L - x)$

5. The value of bending moment at any section of the arch,

$$M_x = -Hy + V_A x - \frac{wx^2}{2}$$

$$= -\frac{wL^2}{8h} \times \frac{4h}{L^2} x (L - x) + \frac{wL}{2} x - \frac{wx^2}{2}$$

$$M_x = -\frac{wLx}{2} + \frac{wx^2}{2} + \frac{wLx}{2} - \frac{wx^2}{2} = 0$$

6. Hence a parabolic arch subjected to a UDL over its entire span has the bending moment at any section zero. That is why the parabolic shape for three hinged arch is a funicular shape.

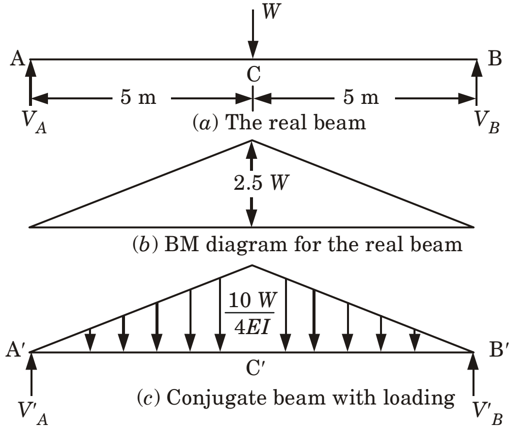
6. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. A simply supported beam of uniform cross section subjected to concentrated load *W* at mid span. If span of the beam is 10 m, calculate slope at its end and also calculate the deflection at mid span. Use conjugate beam method.

Ans.

Given : Concentrated load = W, Span of beam = 10 m.
To Find : Deflection at mid span and slope at supports.

1. Fig. 21(a) shows the real beam, BM diagram for the real beam is shown in the Fig. 21(b).
2. The $\frac{M}{EI}$ diagram of the real beam becomes the elastic weight or loading for the conjugate beam. The conjugate beam $A'C'B'$ (corresponding to the real beam ACB) with the loading is shown in Fig. 21(c).



(a) The real beam

(b) BM diagram for the real beam

(c) Conjugate beam with loading

Fig. 21.

3. For the conjugate beam :

$$V'_A = \frac{1}{2}\left(\frac{1}{2}\times 10 \times \frac{10\,W}{4EI}\right) = \frac{10^2\,W}{16EI} = \frac{6.25\,W}{EI}$$

4. But shear at any section of the conjugate beam is equal to the slope of the real beam.

i. Hence, θ_A = Slope at the end A of real beam $= -\dfrac{6.25\,W}{EI}$

ii. Similarly, $\theta_B = S'_B = V'_B = \dfrac{6.25\,W}{EI}$

5. Again, for the conjugate beam,

$$M'_C = V'_A \times \frac{10}{2} - \left(\frac{1}{2}\times\frac{10}{2}\times\frac{W10}{4EI}\right)\times\left(\frac{1}{3}\times\frac{10}{2}\right)$$

$$= \frac{W10^2}{16EI}\times\frac{10}{2} - \frac{W10^3}{96EI} = \frac{W10^3}{48EI} = \frac{20.833\,W}{EI}$$

6. But the BM at any section of the conjugate beam is equal to the deflection of the real beam.

Hence, $\Delta_C = M'_C = \dfrac{20.833\,W}{EI}$

b. Determine the horizontal deflection at a free end of the frame as shown in Fig. 22, by using unit load method.

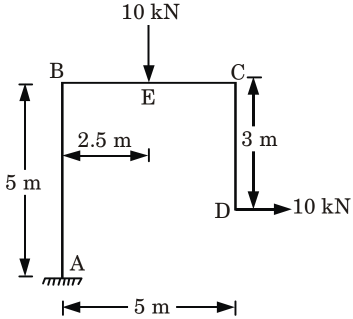


Fig. 22.

Ans.

Given : A frame ABCD as shown in Fig. 22.
To Find : Horizontal deflection at a free end.

1. **Expression for moments :**

Portion	***DC***	***CE***	***EB***	***BA***	**Remarks**
Origin	D	C	B		
limit	$0 - 3$	$0 - 2.5$	$0 - 2.5$	$0 - 5$	
M	$10x$	$10 \times 3 = 30$	$30 - 10x$	$30 - 25 - 10x = 5 - 10x$	BM due to external load
M_1	$1x = x$	$1 \times 3 = 3$	3	$3 - x$	BM due to unit horizontal load at D

2. **Horizontal deflection at *D* :**

$$\Delta_{DH} = \int_D^A \frac{MM_1}{EI}\,dx = \int_0^3 \frac{10x \times x}{EI}\,dx + \int_0^{2.5} \frac{30 \times 3dx}{EI} + \int_0^{2.5} \frac{(30 - 10x)\,3dx}{EI} + \int_0^5 \frac{(5 - 10x)(3 - x)\,dx}{EI}$$

$$= \frac{1}{EI}\left[10\left[\frac{x^3}{3}\right]_0^3 + 90\,[x]_0^{2.5} + 90\,[x]_0^{2.5} - 30\left[\frac{x^2}{2}\right]_0^{2.5} + 15\,[x]_0^5 - 35\left[\frac{x^2}{2}\right]_0^5 + 10\left[\frac{x^3}{3}\right]_0^5\right]$$

$$= \frac{1}{EI}\left[\frac{10}{3}[27 - 0] + 90\,[2.5 - 0] + 90\,[2.5 - 0] - \frac{30}{2}[2.5^2 - 0] + 15\,[5 - 0] - \frac{35}{2}[25 - 0] + \frac{10}{3}[125 - 0]\right]$$

$$\Delta_{DH} = \frac{500.416}{EI} \text{ unit}$$

7. Attempt any **one** part of the following : **(1 × 7 = 7)**

a. **A beam of rectangular section 80 mm wide and 120 mm deep is subjected to a bending moment of 12 kN-m the trace of the plane loading is inclined at 45° to the YY-axis of the section. Calculate the maximum bending stress induced in the beam.**

Ans.

Given : Width of beam, $b = 80$ mm, Depth of beam, $d = 120$ mm
Bending moment, $M = 12$ kN-m, Inclination of load, $\theta = 45°$
To Find : Maximum bending stress.

1. Let the plane of loading be inclined at an angle θ with Y-Y axis and the neutral axis be inclined at β with the X-X axis.

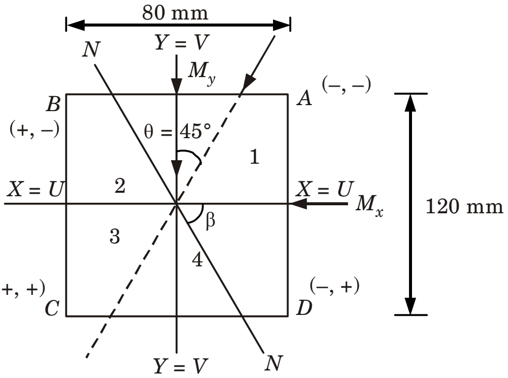


Fig. 23.

2. Moment of Inertia :

$$I_x = I_u = 1/12 \times 80 \times 120^3 = 11.52 \times 10^6 \text{ mm}^4$$

$$I_y = I_v = 1/12 \times 120 \times 80^3 = 5.12 \times 10^6 \text{ mm}^4$$

3. Bending Moment Components :

Bending moment in Y-direction,

$$M_y = 12 \times 10^6 \times \cos 45° = 8.5 \times 10^6 \text{ N-mm}$$

Bending moment in X-direction,

$$M_x = 12 \times 10^6 \times \sin 45° = 8.5 \times 10^6 \text{ N-mm}$$

4. Bending Stress :

i. The bending stress at any point (x, y) in the section consists of two points. Once due to bending about the axis X-X and the other due to the bending about Y-Y *i.e.*,

$$\sigma = \frac{M_y x}{I_x} + \frac{M_x y}{I_y}$$

ii. As both the components are to give tensile stress in the 3rd quadrant, x and y both can be assumed positive in this quadrant.
Thus assume the following for positive and negative direction for x and y;

a. x positive towards left of O and negative towards right of O.

b. y positive downward from O and negative upward from O.

iii. Stress at corner A, $x_A = -40$, $y_A = -60$

$$\sigma_A = \frac{M_y}{I_x} y_A + \frac{M_x}{I_y} x_A$$

$$\sigma_A = \frac{8.5 \times 10^6}{11.52 \times 10^6} \times (-60) + \frac{8.5 \times 10^6}{5.12 \times 10^6} \times (-40)$$

$$\sigma_A = 0.74 \times (-60) + 1.7 \times (-40)$$

$$\sigma_A = 112.4 \text{ N/mm}^2 \text{ (Compressive)}$$

Similarly,

iv. Stress at corner B, $\sigma_B = 0.74 \times (60) + 1.7 \times (-40)$
$\sigma_B = 23.6 \text{ N/mm}^2$ (Compressive)

v. Stress at corner C, $\sigma_C = 0.74 \times (60) + 1.7 \times (40)$
$\sigma_C = 112.4 \text{ N/mm}^2$ (Tensile)

vi. Stress at corner D, $\sigma_D = 0.74 \times (-60) + 1.7 \times (40)$
$\sigma_D = 23.6 \text{ N/mm}^2$ (Tensile)

5. Location of Neutral Axis :

$$\tan \beta = \frac{I_u}{I_v} \tan \theta = \frac{11.52 \times 10^6}{5.12 \times 10^6} \times \tan 45°$$

$$\tan \beta = 2.25, \quad \beta = 66°$$

b. A cast iron beam of T-section as shown in Fig. 24. The beam is simply supported on a span of 8 m, the beam carries a UDL of 1.5 kN/m length on the entire span. Determine the maximum tensile and compressive stresses.

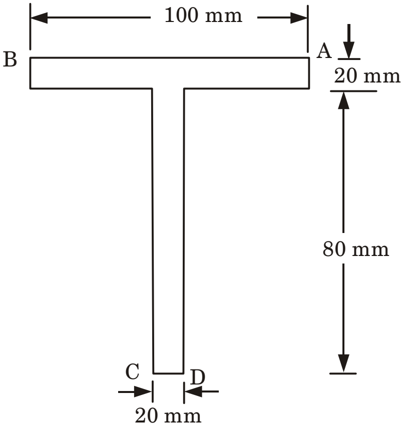


Fig. 24.

Ans.

Given : A cost iron beam of T-section as shown in Fig. 24. Span, $L = 8$ m, Intensity of $UDL = 1.5$ kN/m

To Find : Maximum tensile and compressive stresses.

1. Total area = $100 \times 20 + 80 \times 20 = 3600 \text{ m}^2$
2. Centroid of T-section from top,

$$\bar{y} = \frac{100 \times 20 \times 10 + 80 \times 20 \times (40 + 20)}{100 \times 20 + 80 \times 20}$$

$$\bar{y} = 32.22 \text{ mm}$$

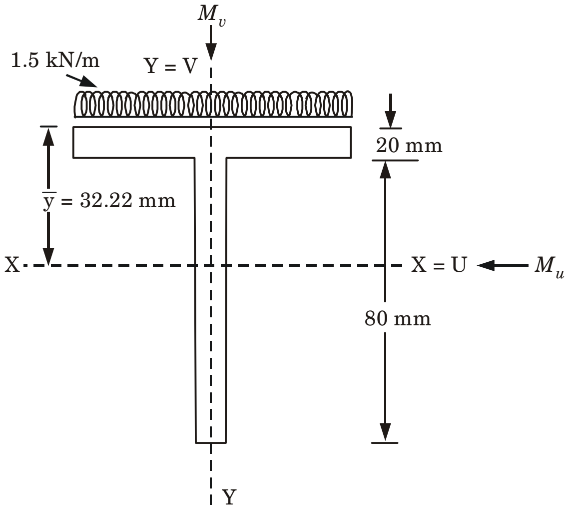


Fig. 25.

3. Moment of inertia about X axis,

$$I_x = \frac{100 \times (20)^3}{12} + 100 \times 20 \times (32.22 - 10)^2 + \frac{20 \times 80^3}{12} + 20 \times 80 \times (60 - 32.22)^2$$

$$I_x = Iv = 314.22 \times 10^4 \text{ mm}^4$$

4. Moment of inertia about Y axis,

$$I_y = I_v = \frac{20 \times 100^3}{12} + \frac{80 \times 20^3}{12} = 172 \times 10^4 \text{ mm}^4$$

5. Moment, $M_u = \dfrac{wl^2}{8} = \dfrac{1.5 \times 8^2}{8} = 12 \text{ kN-m} = 12 \times 10^6 \text{ N-mm}$

$M_v = 0$

6. Maximum compression stress,

$$\sigma_A = \frac{M_u}{I_u} \times v_A + \frac{M_u}{I_v} u_A$$

$$\sigma_A = \frac{12 \times 10^6 \times 32.22}{314.22 \times 10^4} + 0 = 123.05 \text{ N/mm}^2$$

7. Maximum tensile stress,

$$\sigma_D = \frac{M_u}{I_u} v_D + \frac{M}{I_v} u_D$$

$$\sigma_D = \frac{12 \times 10^6 \times (100 - 32.22)}{314.22 \times 10^4} + 0 = 258.9 \text{ N/mm}^2$$

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B. Tech.
(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2018-19
STRUCTURAL ANALYSIS

Time : 3 Hours **Max. Marks : 70**

Note. 1. Attempt all sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt **all** questions in brief : **(7 × 2 = 14)**

a. Explain degree of freedom of a structure.

b. Give an example for a structure which is externally as well as internally indeterminate.

c. State Maxwell's law of reciprocal deflections.

d. Give an expression for strain energy stored in a beam due to bending.

e. List the assumptions made in truss analysis.

f. What is the shape of the influence line diagram for maximum bending moment in a simply supported beam ?

g. State Eddy's theorem.

SECTION-B

2. Attempt any **three** of the following : **(7 × 3 = 21)**

a. Determine the forces in the members by method of joints. See Fig. 1.

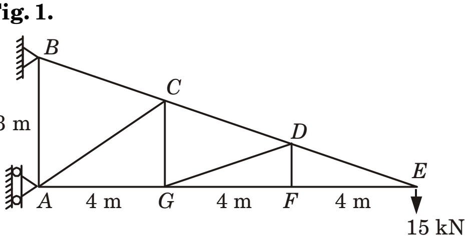

Fig. 1.

b. Three hinged parabolic arch of span 10 m and central rise 2.5 m supports a point load of 100 kN at left quarter span and a UDL of 20 kN/m over the right half of the span. See Fig. 2.

i. Draw the influence line diagram.

ii. Determine the reactions, normal thrust and radial shear at right quarter span point.

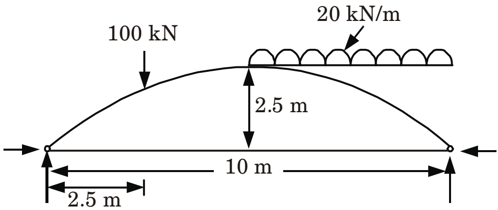

Fig. 2.

c. A train of four concentrated loads crosses a simply supported girder of 10 m span with 30 kN leading. See Fig. 3. Determine

i. The maximum bending moment at 6 m from the left support.

ii. Absolute maximum bending moment anywhere in the girder.

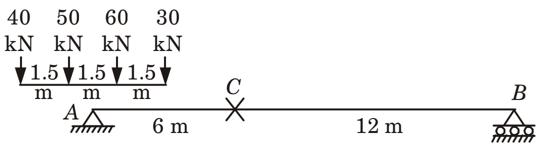

Fig. 3.

d. State and prove Betti's law.

e. A cord supported at its ends 60 m apart carries loads of 30 kN, 15 kN, 18 kN at 15, 30 and 45 m respectively from the left end. If the point on the cord where the 15 kN load is supported is 15 m below the level of end supports determine.

i. The reactions at the supports.

ii. Tension in different parts of the cord.

iii. The total length of the cord.

SECTION-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Find static indeterminacy and kinematic indeterminacy of the given structure. See Fig. 4.

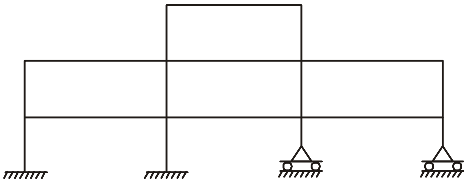

Fig. 4.

b. Find static indeterminacy and kinematic indeterminacy of the given structure. See Fig. 5.

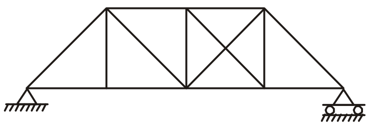

Fig. 5.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Determine the forces in all the members using method of substitution, for the given Fig. 6.

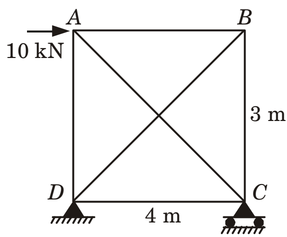

Fig. 6.

b. The Fig. 7 shows a warren type cantilever truss along with the imposed loads. Determine the forces in all the members using tension coefficients.

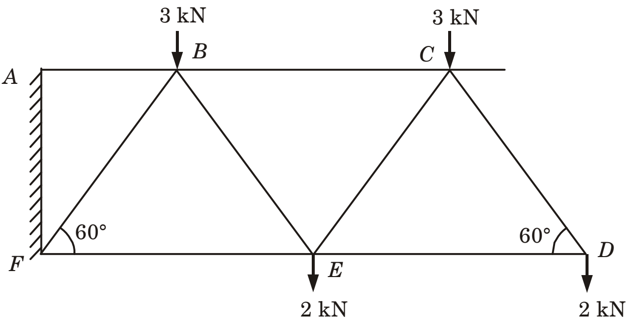

Fig. 7.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A beam *ABCD* is simply supported at its ends *A* and *D* over a span of 30 m. It is made three portions *AB*, *BC* and *CD* each 10 m in length. The moment of inertia of the section of these lines are *I*, 3*I* and 2*I* respectively. The beam carries a point load 150 kN at *B* and point load of 300 kN at *C*. Neglecting the weight of the beam calculate the slopes and deflection at *A*, *B*, *C* and *D*

Where E = 200 kN/mm^2, $I = 2 \times 10^{10}$ mm^4.

b. Find the horizontal movement of the roller end *D* of the portal frame shown in Fig. 8. Take $E = 2 \times 10^8$ kN/m^2 and $I = 3 \times 10^4$ m^4. The moment of inertia of the column section is *I* while that of beam is 2*I*.

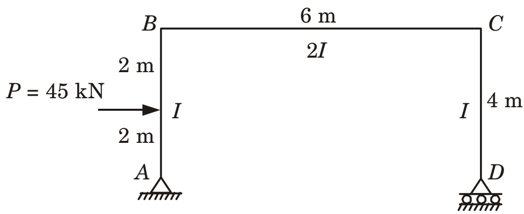

Fig. 8.

6. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. By using Muller-Breslau's principle construct influence line diagram for the beam as shown in Fig. 9.

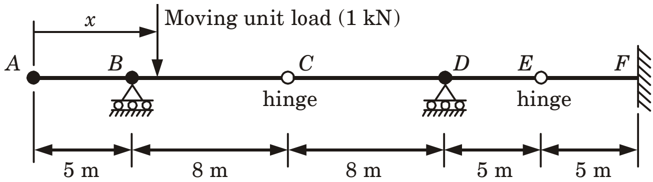

Fig. 9.

b. Assume a unit point load is rolling along the bridge deck from points *A* to *D* in a simple truss as shown in Fig. 10. The distance *X* is the distance of the moving load from point *A*. Construct the influence line diagram for the three members *EF*, *BF* and *BC*.

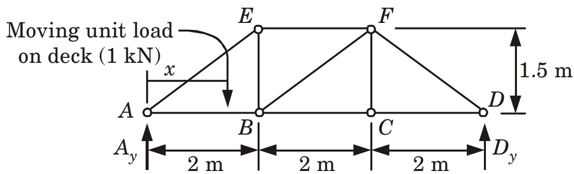

Fig. 10.

7. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A three hinged parabolic arch of span 30 m and central rise of 5 m. It is subjected to a concentrated load of 40 kN at 6 span. Calculate the normal thrust, shear force and bending moment at 6 m from the left support Fig. 11.

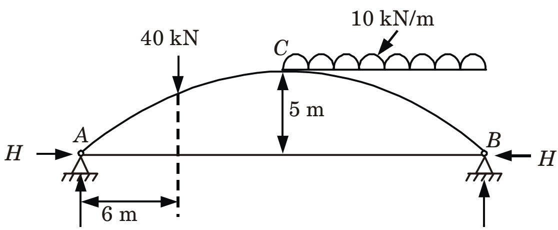

Fig. 11.

b. Classify the arches based on materials, shapes and structural systems with the help of neat sketch. Also, distinguish between two hinged and three hinged arches.

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SOLUTION OF PAPER (2018-19)

Note. 1. Attempt all sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt **all** questions in brief : **(7 × 2 = 14)**

a. Explain degree of freedom of a structure.

Ans. Degree of Freedom :

i. When a structure is loaded, specified points on it, called nodes, will undergo unknown displacements. These displacements are known as degree of freedom for the structure.

ii. In 2-dimension, each node can have atmost two linear displacements and one rotational displacement.

iii. In 3-dimension, each node on frame or beam can have atmost three linear displacements and three rotational displacements.

Example : In Fig. 1

i. Number of nodes = 3 (A, B and C)

ii. Here three rotational displacements like θ_A, θ_B and θ_C.

iii. One vertical displacement and one horizontal displacement at C, as Δ_C.

iv. Total number of displacements = Degree of freedom = 5

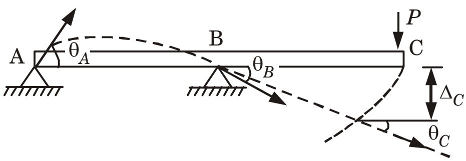


Fig. 1.

b. Give an example for a structure which is externally as well as internally indeterminate.

Ans. Equilibrium conditions, $e = 3$

Total external reaction, $r = 4$

Number of member, $m = 11$

Number of joint, $j = 6$

i. External indeterminacy,

$$I_e = r - e = 4 - 3 = 1$$

ii. Internal indeterminacy,

$$I_i = m - (2j - 3) = 11 - (2 \times 6 - 3) = 2$$

iii. Total indeterminacy,

$$I = I_e + I_i = 1 + 2 = 3$$

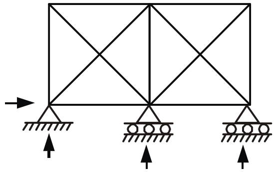


Fig. 2.

c. State Maxwell's law of reciprocal deflections.

Ans. In any beam or truss, whose material is elastic and obeys Hooke's law and whose supports remain unyielding and the temperature remain unchanged, then the deflection at any point D (*i.e.*, Δ_D) due to a load W acting at any other point C is equal to the deflection at any point C (*i.e.*, δ_C) due to the load W acting at the point D.

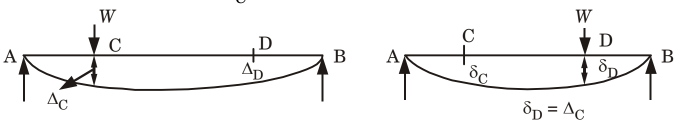


Fig. 3.

d. Give an expression for strain energy stored in a beam due to bending.

Ans. Total energy stored by the whole beam = $\int \frac{M^2 ds}{2EI}$

e. List the assumptions made in truss analysis.

Ans. Following are the assumptions made in truss analysis :

1. The centroidal axis of each member coincides with the line connecting the centers of the adjacent members and the members only carry axial force.
2. All members are connected only at their ends by frictionless hinges in plane trusses.
3. All loads and support reactions are applied only at the joints.

f. What is the shape of the influence line diagram for maximum bending moment in a simply supported beam ?

Ans. The maximum bending moment for any position of load occur under the load. So, the equation for maximum bending moment is

$$M_{max} = x(L - x)/L$$

Thus, the ILD for maximum bending moment is parabolic. This is also called envelop of maximum bending moment.

g. State Eddy's theorem.

Ans. The bending moment at any section of an arch is equal to the vertical intercept between the linear arch and the central line of the actual arch.

SECTION-B

2. Attempt any **three** of the following : **(7 × 3 = 21)**

a. Determine the forces in the members by method of joints. See Fig. 4.

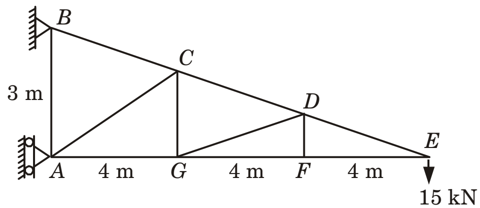


Fig. 4.

Ans.

Given : Truss shown in Fig. 4.
To Find : The forces in the members by method of joints.

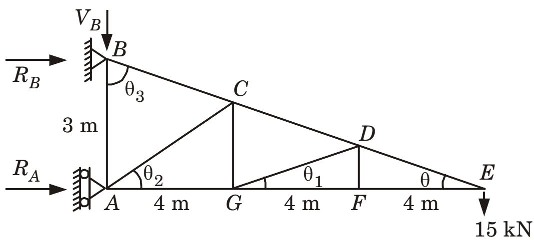


Fig. 5.

1. Let the forces in all the members are tensile in nature.
2. **Calculation for Support Reactions :**

i. Taking moment about point B,

$$\Sigma M_B = 0 \quad \Rightarrow \quad 15 \times 12 - R_A \times 3 = 0$$

$$R_A = 60 \text{ kN}$$

ii. $\Sigma F_x = 0,\ R_A + R_B = 0 \quad \Rightarrow \quad R_B = -60$ kN

iii. $\Sigma F_y = 0,\ \ V_B + 15 = 0 \quad \Rightarrow \quad V_B = -15$ kN

3. **Considering the Joint E :**

i. From ΔAEB (Fig. 5), $\tan \theta = 3/12$

$$\theta = \tan^{-1}(3/12) = 14.03°$$

ii. Resolving the forces vertically, $\Sigma F_y = 0 \quad \Rightarrow \quad F_{ED} \sin \theta - 15 = 0$

$$F_{ED} = \frac{15}{\sin \theta} = \frac{15}{\sin 14.03°} = 61.87 \text{ kN}$$

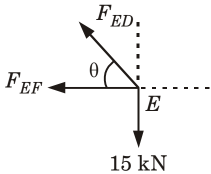


Fig. 6.

iii. Resolving the forces horizontally, $\Sigma F_x = 0 \Rightarrow -F_{ED} \cos\theta - F_{EF} = 0$

$$F_{EF} = -F_{ED} \cos\theta = -61.87 \cos 14.03°$$

$$F_{EF} = -60.02 \text{ kN} \simeq -60 \text{ kN}$$

4. Considering the Joint *F* :

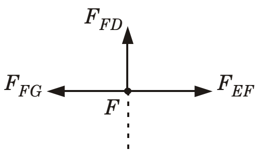


Fig. 7.

i. Resolving the forces vertically, $\Sigma F_y = 0 \quad \Rightarrow \quad F_{FD} = 0$ kN

ii. Resolving the forces horizontally, $\Sigma F_x = 0 \quad \Rightarrow \quad F_{EF} - F_{FG} = 0$

$$F_{FG} = F_{EF} = -60 \text{ kN}$$

5. Considering the Joint *D* :

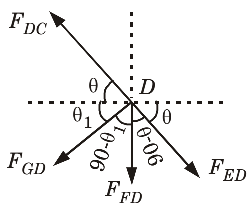


Fig. 8.

i. From geometry of Fig. 5, $\theta = \theta_1 = 14.03°$

ii. Resolving the forces horizontally, $\Sigma F_x = 0$

$$F_{ED} \cos\theta - F_{GD} \cos\theta_1 - F_{DC} \cos\theta = 0$$

$$61.87 \cos 14.03° - F_{GD} \cos 14.03° - F_{DC} \cos 14.03° = 0$$

$$F_{GD} + F_{DC} = 61.87 \quad \text{...(1)}$$

iii. Resolving the forces vertically, $\Sigma F_y = 0$

$$F_{DC} \sin\theta - F_{GD} \sin\theta_1 - F_{FD} - F_{ED} \sin\theta = 0$$

$$F_{DC} \sin 14.03° - F_{GD} \sin 14.03° - 0 - 61.87 \sin 14.03° = 0$$

$$F_{DC} - F_{GD} = 61.87 \quad \text{...(2)}$$

iv. From eq. (1) and eq. (2), we get

$$F_{DC} = 61.87 \text{ kN}, F_{GD} = 0 \text{ kN}$$

6. Considering the Joint *G* :

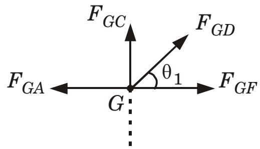


Fig. 9.

i. Resolving the forces vertically, $\Sigma F_y = 0 \Rightarrow F_{GC} + F_{GD} \sin \theta_1 = 0$

$$F_{GC} + 0 = 0 \Rightarrow F_{GC} = 0 \text{ kN}$$

ii. Resolving the forces horizontally, $\Sigma F_x = 0$

$$F_{GF} + F_{GD} \cos \theta_1 - F_{GA} = 0$$

$$-60 + 0 - F_{GA} = 0$$

$$F_{GA} = -60 \text{ kN}$$

7. Considering the Joint *A* :

i. From ΔABE and ΔGCE (Fig. 2),

$$\frac{AB}{AE} = \frac{GC}{GE} \Rightarrow \frac{3}{12} = \frac{GC}{8}$$

$$GC = 2 \text{ m}$$

So, $\tan \theta_2 = \dfrac{GC}{AG} \Rightarrow \tan \theta_2 = \dfrac{2}{4} \Rightarrow \theta_2 = 26.57°$

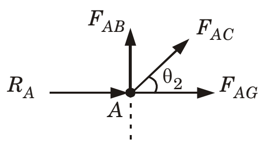


Fig. 10.

ii. Resolving the forces horizontally, $\Sigma F_x = 0$

$$F_{AG} + F_{AC} \cos \theta_2 + R_A = 0$$

$$-60 + F_{AC} \cos 26.57° + 60 = 0$$

$$F_{AC} = 0 \text{ kN}$$

iii. Resolving the forces vertically, $\Sigma F_y = 0 \Rightarrow F_{AB} + F_{AC} \sin \theta_2 = 0$

$$F_{AB} + 0 \times \sin \theta_2 = 0$$

$$F_{AB} = 0 \text{ kN}$$

8. Considering the Joint *B* :

i. From ΔABE (Fig. 5), $\theta_3 = 90° - 14.03° = 75.97°$

ii. Resolving the forces horizontally, $\Sigma F_x = 0 \Rightarrow R_B + F_{BC} \sin \theta_3 = 0$

$$-60 + F_{BC} \sin 75.97° = 0$$

$$F_{BC} = 61.84 \text{ kN}$$

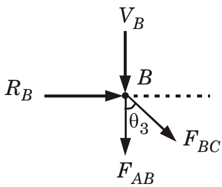


Fig. 11.

Members	Force in Members (kN)	Nature of Force
AB	F_{AB} = 0 kN	Neutral
BC	F_{BC} = 61.84 kN	Tensile
AC	F_{AC} = 0 kN	Neutral
AG	F_{GA} = – 60 kN	Compressive
GC	F_{GC} = 0 kN	Neutral
GD	F_{GD} = 0 kN	Neutral
GF	F_{FG} = – 60 kN	Compressive
CD	F_{DC} = 61.87 kN	Tensile
FD	F_{FD} = 0 kN	Neutral
FE	F_{EF} = – 60 kN	Compressive
DE	F_{ED} = 61.87 lN	Tensile

b. Three hinged parabolic arch of span 10 m and central rise 2.5 m supports a point load of 100 kN at left quarter span and a UDL of 20 kN/m over the right half of the span. See Fig. 12.

i. Draw the influence line diagram.

ii. Determine the reactions, normal thrust and radial shear at right quarter span point.

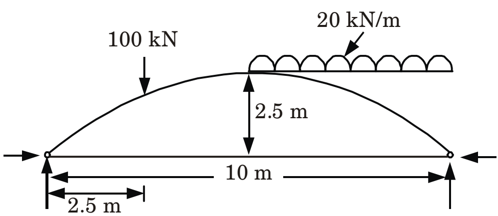


Fig. 12.

Ans.

Given : Span of arch, $L = 10$ m, Rise of arch, $h = 2.5$ m, Point load = 100 kN, Intensity of UDL, $w = 20$ kN/m

To Find : i. Draw the influence line diagram.

ii. Calculate the reactions, normal thrust and radial shear at right quarter span point.

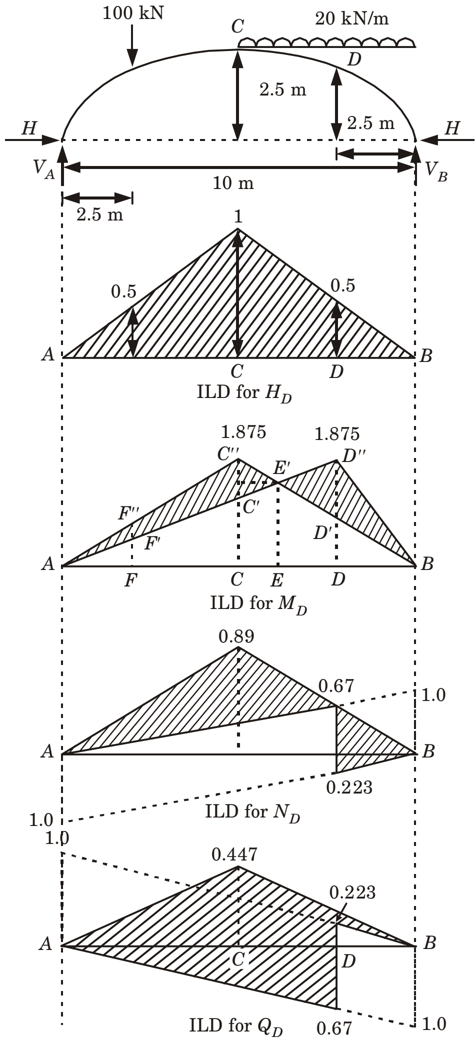


Fig. 13.

1. $$\Sigma M_A = 0$$

$$\Rightarrow V_B \times 10 - 100 \times 2.5 - 20 \times 5 \times 7.5 = 0$$

$$V_B = 100 \text{ kN}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad V_A + V_B = 100 + 20 \times 5$$

$$100 + V_B = 200$$

$$\Rightarrow \quad V_B = 100 \text{ kN}$$

2. **ILD for *H* :**

Maximum ordinate of ILD for $H = \dfrac{L}{4h} = \dfrac{10}{4 \times 2.5} = 1$

$$H_D = 0.5 \times 100 + 1/2 \times 1 \times 5 \times 20 = 100 \text{ kN}$$

3. **ILD for M_D :**

Ordinate at C = Ordinate at D

$$= \frac{Z(L-Z)}{L} = \frac{2.5(10-2.5)}{10} = 1.875$$

$$EE' = 1.875 \times \frac{3.75}{5} = 1.40$$

$$F'F'' = FF'' - FF'$$

$$= 1.875 \times \frac{2.5}{5} - 1.4 \times \frac{2.5}{6.25}$$

$$= 0.3775$$

$$C'C'' = CC'' - CC'$$

$$= 1.875 - 1.4 \times \frac{5}{6.25} = 0.755$$

$$M_D = -100 \times 0.3775 - 20 \times \left(\frac{1}{2} \times \frac{2.5}{2} \times 0.755\right) + 20 \times \left(\frac{1}{2} \times 1.875 \times 10 - \frac{1}{2} \times 1.4 \times 10\right)$$

$$= -37.75 - 9.4375 + 47.5$$

$$M_D = 0.3125 \text{ kN-m}$$

4. **Normal Thrust :** Vertical shear at quarter right support,

$$V_D = 100 - 20 \times 2.5 = 50 \text{ kN}$$

Horizontal force, $H = 100$ kN

$$y = \frac{4hx}{L^2}(L-x)$$

$$\frac{dy}{dx} = \frac{4h}{L^2}(L-2x) = \frac{4 \times 2.5}{10^2}(10 - 2 \times 2.5) = 0.5$$

$$\tan\theta = 0.5$$
$$\theta = 26.56°$$
$$N = V_D \sin\theta + H\cos\theta$$
$$= 50 \times \sin 26.56° + 100 \times \cos 26.56° = 111.8 \text{ kN}$$

$$N_C = \frac{L}{4h}\cos\theta = \frac{10}{4\times 2.5}\cos 26.56° = 0.894$$

$$N_D(+) = \frac{Z}{L}\cos\theta = \frac{7.5}{10}\cos 26.56° = 0.670$$

$$N_D(-) = \frac{L-Z}{L}\cos\theta = \frac{(10-7.5)}{10}\cos 26.56° = 0.223$$

5. **Radial Shear :**

$$Q = V_D\cos\theta - H\sin\theta$$
$$= 50\times\cos 26.56° - 100\times\sin 26.56° = 9.8\times 10^{-3}\text{ kN}$$

$$Q_C = \frac{L}{4h}\sin\theta = \frac{10}{4\times 2.5}\sin 26.56° = 0.447$$

$$Q_D(+) = \frac{L-Z}{L}\cos\theta = 0.223$$

$$Q_D(-) = \frac{Z}{L}\cos\theta = 0.67$$

c. A train of four concentrated loads crosses a simply supported girder of 10 m span with 30 kN leading. See Fig. 14. Determine

i. The maximum bending moment at 6 m from the left support.

ii. Absolute maximum bending moment anywhere in the girder.

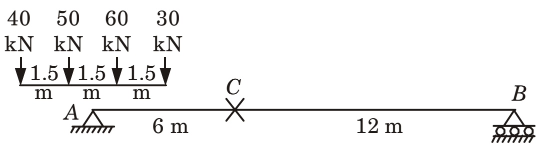


Fig. 14.

Ans.

Given : Beam shown in Fig. 14.
To Find : Determine :

i. Maximum bending moment at 6 m from the left support.
ii. Absolute maximum bending moment anywhere in the girder.

1. ILD for Maximum Bending Moment :

i. To find the load position for maximum moment, average load on portion AC and CB are to be found as loads crosses section C one after another.

Table 1 : Calculation to find load position for maximum M_C

Load Crossing	Average Load		Remarks
	AC	BC	
	W_{AC}	W_{BC}	
30 kN	150/6	30/12	$W_{AC} > W_{BC}$
60 kN	90/6	90/12	$W_{AC} > W_{BC}$
50 kN	40/6	140/12	$W_{AC} < W_{CB}$

ii. Hence, load position for maximum moment at C is when 50 kN load is on C.

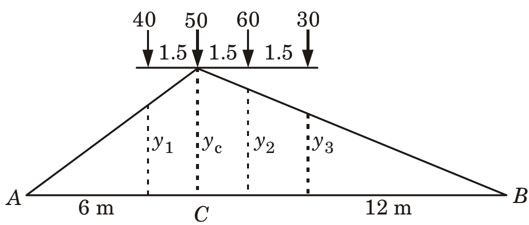


Fig. 15.

iii. The maximum ordinate, $y_c = \dfrac{z(l-z)}{l} = \dfrac{6 \times 12}{18} = 4$

iv. Maximum moment, $M_C = 40\,y_1 + 50y_c + 60y_2 + 30y_3$

$$= 40\left(\frac{6-1.5}{6}\right)y_c + 50y_c + 60\left(\frac{12-1.5}{12}\right)y_c + 30\left(\frac{12-1.5-1.5}{12}\right)y_c$$

$= (30 + 50 + 52.5 + 22.5)y_c$

$= 155 \times 4 = 620$ kN-m

2. ILD for Absolute Maximum Bending Moment :

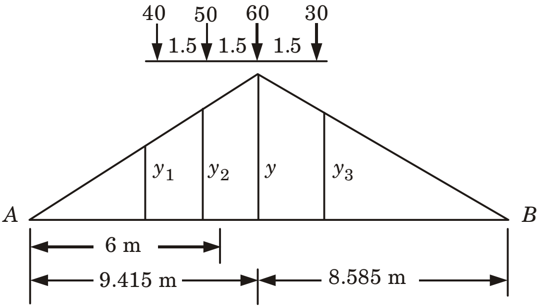


Fig. 16.

i. For finding position for absolute maximum moment, position of CG of load system is to be located. Distance of CG for the leading load of 30 kN

$$a = \frac{60 \times 1.5 + 50 \times 3 + 40 \times 4.5}{40 + 50 + 60 + 30} = 2.33 \text{ m}$$

ii. It is nearer to 60 kN load and this load is heavier than another nearer load of 50 kN. Hence, maximum moment will occur under 60 kN load. Distance between this load and the resultant,

$$d = 2.33 - 1.5 = 0.83$$

$\therefore$ Position of 60 kN load for maximum moment

$$= \frac{18}{2} + \frac{0.83}{2} = 9.415 \text{ m from } A$$

iii. For the section at 9.415 m from A, ILD ordinate for moment,

$$y = \frac{z(l - z)}{l}$$

$$y = \frac{9.415\,(18 - 9.415)}{18} = 4.49 \text{ m}$$

iv. Absolute maximum moment

$$= 40y_1 + 50y_2 + 60y + 30y_3$$

$$= \left[40 \times \frac{6.415}{9.415} + 50 \times \frac{7.915}{9.415} + 60 + 30 \times \frac{7.085}{8.585}\right] 4.49 = 691.67 \text{ kN-m}$$

d. State and prove Betti's law.

Ans. **Statement :** For a structure whose material is elastic and follows Hooke's law and in which the supports are unyielding and the temperature is constant, the virtual work done by a system of forces P_1, P_2, P_3, ... during the distortion caused by a system of forces Q_1, Q_2, Q_3, ... is equal to the virtual work done by the system of forces Q_1, Q_2, Q_3, ... during the distortion caused by the system of forces P_1, P_2, P_3, ...

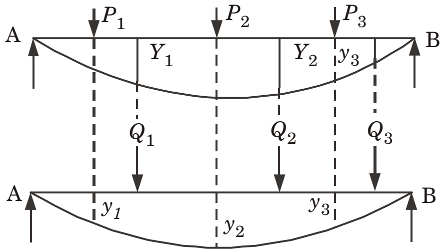


Fig. 17.

Proof :

1. Let a structure *AB* carries two system of forces P_1, P_2, P_3, and Q_1, Q_2, Q_3. Fig. 17 shows both the system of forces separately.
2. Let W_e = External work done on the structure when the system of forces P_1, P_2, P_3 is applied.
   W_e' = External work done on the structure when the system of forces Q_1, Q_2, Q_3 is applied.
3. Let Y_1, Y_2, Y_3 = Deflection caused by the system of forces P_1, P_2, P_3 at the points of application of the forces Q_1, Q_2, Q_3, respectively.
4. Let y_1, y_2, y_3 = Deflection caused by the system of forces Q_1, Q_2, Q_3 at the points of application of the forces P_1, P_2, P_3, respectively.
5. Let the structure be first loaded with the system of forces P_1, P_2, P_3 and work done on the structure be W_e.
6. With this system of forces acting on the structure, let the system of forces Q_1, Q_2, Q_3 be applied.
   Total work done = $W_e + W'_e + P_1 y_1 + P_2 y_2 + P_3 y_3$ ...(1)
7. Now on changing the order of loading we have the system of forces Q_1, Q_2, Q_3, acting on the structure.
   Work done on the structure = W'_e
8. Now the system of forces P_1, P_2, P_3 be applied so,
   Total work done = $W'_e + W_e + Q_1 Y_1 + Q_2 Y_2 + Q_3 Y_3$ ...(2)
9. Equating the eq. (1) and (2)
   $W_e + W_e' + P_1 y_1 + P_2 y_2 + P_3 y_3 = W'_e + W_e + Q_1 Y_1 + Q_2 Y_2 + Q_3 Y_3$
   $P_1 y_1 + P_2 y_2 + P_3 y_3 = Q_1 Y_1 + Q_2 Y_2 + Q_3 Y_3$
10. The above expression shows that the virtual work done by the system of forces P_1, P_2, P_3 due to the deflections caused by the system of forces Q_1, Q_2, Q_3 equals virtual work done by the system of forces Q_1, Q_2, Q_3 due to the deflections caused by the system of forces P_1, P_2, P_3.

e. A cord supported at its ends 60 m apart carries loads of 30 kN, 15 kN, 18 kN at 15, 30 and 45 m respectively from the left end. If the point on the cord where the 15 kN load is supported is 15 m below the level of end supports determine.

i. The reactions at the supports.

ii. Tension in different parts of the cord.

iii. The total length of the cord.

Ans.

Given : Span of cord = 60 m, Concentrated loads on cord = 30 kN, 15 kN, and 18 kN at 15, 30 and 45 m respectively from the left end dip of cord at the position of 15 kN = 15 m.

To Find : Determine :

i. Reaction at the support.

ii. Tension in different part.

iii. Length of the cord.

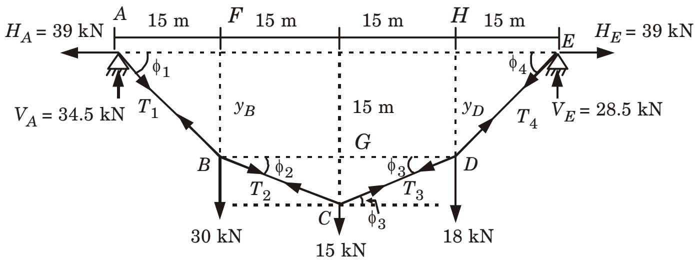


Fig. 18.

1. $\Sigma F_x = 0$

$$-H_A + H_E = 0 \quad \Rightarrow \quad H_A = H_E \qquad ...(1)$$

2. $$\Sigma F_y = 0 \quad \Rightarrow \quad V_A + V_E = 63 \text{ kN} \qquad ...(2)$$

3. Taking moment about point A, $\Sigma M_A = 0$

$$30 \times 15 + 15 \times 30 + 18 \times 45 - V_E \times 60 = 0$$

$$V_E = 28.5 \text{ kN}$$

4. From eq. (2), we get $V_A = 34.5$ kN

5. Taking moment about point C, $\Sigma M_c = 0$

(Always take BM about the point for which sag is given)

$$-H_A \times 15 - 30 \times 15 + 34.5 \times 30 = 0$$

$$H_A = 39 \text{ kN} = H_E$$

6. Considering the Point A :

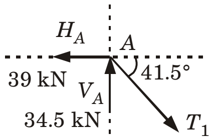


Fig. 19.

i. Taking moment about point B (Left part of Fig. 14), $\Sigma M_B = 0$

$34.5 \times 15 - 39 \times y_B = 0$

$$y_B = 13.27 \text{ m}$$

ii. From ΔABF, $\quad \phi_1 = \tan^{-1}\left(\frac{13.27}{15}\right)$

$$\phi_1 = 41.5°$$

iii. Resolving the forces horizontally, $\Sigma F_x = 0$

$T_1 \cos 41.5° - 39 = 0$

$$T_1 = 52.07 \text{ kN}$$

7. Considering the Point *B* :

i. From ΔBCG, $\tan \phi_2 = \frac{(15 - 13.27)}{15}$

$$\phi_2 = 6.58°$$

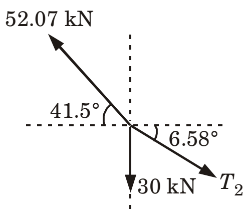


Fig. 20.

ii. Resolving the forces horizontally, $\Sigma F_x = 0$

$T_2 \cos 6.58° - 52.07 + \cos 41.5° = 0$

$$T_2 = 39.25 \text{ kN}$$

8. Considering the Point *C* :

i. From ΔCGD, $\tan \phi_3 = \frac{(15 - 10.96)}{15}$

$$\phi_3 = 15.07°$$

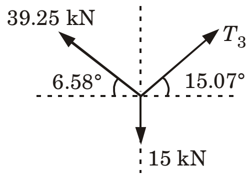


Fig. 21.

ii. Taking moment about point D (right part of Fig. 14), $\Sigma M_D = 0$

$$-39 \times y_D + 28.5 \times 15 = 0$$

$$y_D = 10.96 \text{ m}$$

iii. Resolving the forces horizontally, $\Sigma F_x = 0$

$$T_3 \cos 15.07° - 39.25 \cos 6.58 = 0$$

$$T_3 = 40.83 \text{ kN}$$

9. Considering the Point *E* :

i. From ΔDEH, $\tan \phi_4 = 10.96/ 15$

$$\phi_4 = 36.15°$$

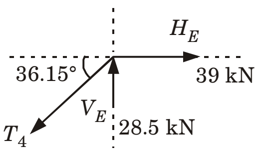


Fig. 22.

ii. Resolving the forces horizontally, $\Sigma F_x = 0$

$$39 - T_4 \cos 36.15° = 0$$

$$T_4 = 48.29 \text{ kN}$$

10. Length of the chord = $\overline{AB} + \overline{BC} + \overline{CD} + \overline{DE}$

i. $$\cos \phi_1 = \frac{15}{\overline{AB}} \Rightarrow \overline{AB} = \frac{15}{\cos 41.5°}$$

$$\overline{AB} = 20.03 \text{ m}$$

ii. $$\cos \phi_2 = \frac{15}{\overline{BC}} \Rightarrow \overline{BC} = \frac{15}{\cos 6.58°}$$

$$\overline{BC} = 15.10 \text{ m}$$

iii. $$\cos \phi_3 = \frac{15}{\overline{CD}} \Rightarrow \overline{CD} = \frac{15}{\cos 15.07°}$$

$$\overline{CD} = 15.53 \text{ m}$$

iv. $$\cos \phi_4 = \frac{15}{\overline{DE}} \Rightarrow \overline{DE} = \frac{15}{\cos 36.15°}$$

$$\overline{DE} = 18.57 \text{ m}$$

vi. Total length of chord

$$= 20.03 + 15.10 + 15.53 + 18.57$$
$$= 69.23 \text{ m}$$

SECTION-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Find static indeterminacy and kinematic indeterminacy of the given structure. See Fig. 23.

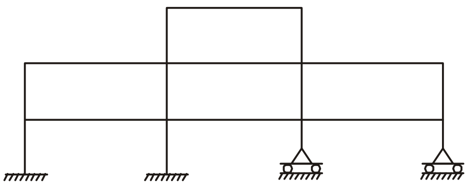


Fig. 23.

Ans.

Given : Frame shown in Fig. 23.
To Find : Static indeterminacy and kinematic indeterminacy.

1. Static Indeterminacy :

For 2D Rigid frame

$$D_s = 3\text{ m} + r_e - 3j$$

where, m = Number of members.

r_e = Number of reactions.

j = Number of joints.

$$m = 17,\ r_e = 8,\ j = 14$$
$$D_s = 3 \times 17 + 8 - 3 \times 14$$
$$= 59 - 42 = 17$$

2. Kinematic Indeterminacy :

For 2D rigid frame,

i. Members are axially extensible,

$$D_K = 3j - r_e = 3 \times 14 - 8 = 34$$

ii. Members are axially rigid,

$$D_K = 3j - r_e - m$$
$$= 3 \times 14 - 8 - 17$$
$$= 42 - 25 = 17$$

b. Find static indeterminacy and kinematic indeterminacy of the given structure. See Fig. 24.

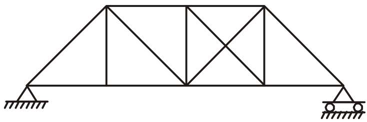


Fig. 24.

Ans.

Given : Truss shown in Fig. 24.
To Find : Static indeterminacy and kinematic indeterminacy.

1. Static Indeterminacy :

For 2D truss,

$$D_S = m + r_e - 2j$$
$$m = 14,\ r_e = 3,\ j = 8$$
$$D_S = 14 + 3 - 16 = 1$$

2. Kinematic Indeterminacy :

For 2D pin jointed truss,

$$D_K = 2j - r_e - m$$
$$D_K = 2 \times 8 - 3 - 14$$
$$= 16 - 17$$
$$= -1 \text{ (Kinematically unstable)}$$

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Determine the forces in all the members using method of substitution, for the given Fig. 25.

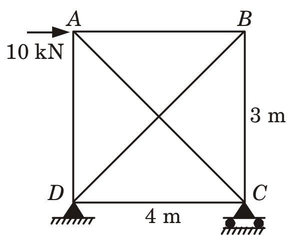


Fig. 25.

Ans. In substitution method, we identify certain member forces of the truss which is replaceable and substitute it by another member at another convenient location.

But in the given truss, we are not able to find a place for the replacement of substitute member.

Hence the given question cannot be solved.

b. The Fig. 26 shows a warren type cantilever truss along with the imposed loads. Determine the forces in all the members using tension coefficients.

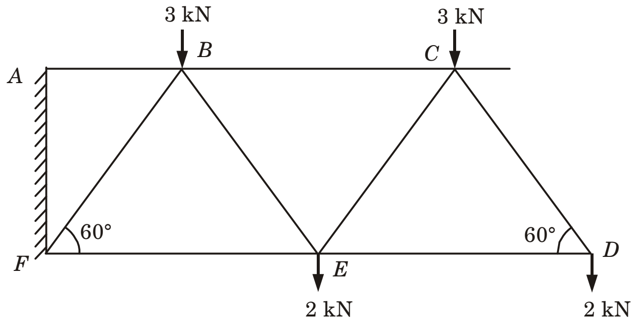


Fig. 26.

Ans.

Given : Truss shown in Fig. 26.
To Find : The forces in all the members using tension coefficients.

1. Let, the length of members of equilateral triangles are same, *i.e.*, L. Taking joint F as the origin, the co-ordinates of various joints are :

$$F(0, 0), E(L, 0), D(2L, 0),\ A\left(0, \frac{\sqrt{3}L}{2}\right), B\left(\frac{L}{2}, \frac{\sqrt{3}L}{2}\right), C\left(\frac{3L}{2}, \frac{\sqrt{3}L}{2}\right)$$

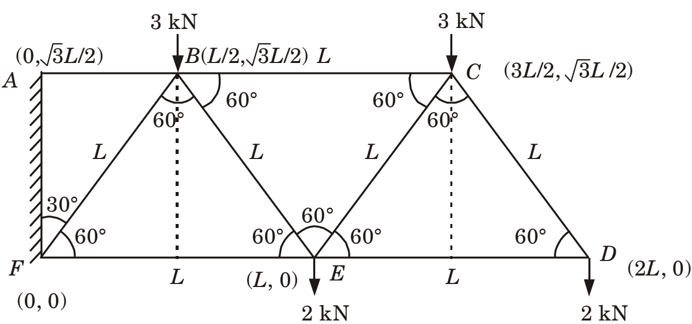


Fig. 27.

2. **Considering the Joint D :**

i. Equation using for tension coefficient is given by,

$$t_{DC}(x_C - x_D) + t_{DE}(x_E - x_D) = 0$$

ii. Substituting the value of x, y coordinates, we get

$$t_{DC}\left(\frac{3L}{2} - 2L\right) + t_{DE}(L - 2L) = 0$$

$$t_{DC} + 2t_{DE} = 0 \quad \text{...(1)}$$

iii. And $t_{DC}(y_C - y_D) + t_{DE}(y_E - y_D) - 2 = 0$

iv. Substituting the value of x, y coordinates, we get

$$t_{DC}\left(\frac{\sqrt{3}L}{2} - 0\right) + t_{DE}(0 - 0) - 2 = 0$$

$$\frac{\sqrt{3}Lt_{DC}}{2} = 2 \quad \Rightarrow \quad t_{DC} = \frac{4}{\sqrt{3}L} \text{ kN/m} \quad \text{...(2)}$$

v. From eq. (1) and (2), we get

$$t_{DC} = \frac{4}{\sqrt{3}L} \text{ kN/m}, \; t_{DE} = \frac{-2}{\sqrt{3}L} \text{ kN/m}$$

vi. $$F_{DC} = t_{DC} \times L = \frac{4}{\sqrt{3}L} \times L = \frac{4}{\sqrt{3}} \text{ kN (Tensile)}$$

vii. $$F_{DE} = t_{DE} \times L$$

$$= \frac{-2}{\sqrt{3}L} \times L = \frac{-2}{\sqrt{3}} \text{ kN} = \frac{2}{\sqrt{3}} \text{ kN (Compressive)}$$

3. Considering the Joint *C* :

i. $t_{CD}(x_D - x_C) + t_{CE}(x_E - x_C) + t_{CB}(x_B - x_C) = 0$

ii. Substituting the value of x, y coordinates, we get

$$\frac{4}{\sqrt{3}L}\left(2L - \frac{3L}{2}\right) + t_{CE}\left(L - \frac{3L}{2}\right) + t_{CB}\left(\frac{L}{2} - \frac{3L}{2}\right) = 0$$

$$t_{CE}\left(\frac{-L}{2}\right) + t_{CB}(-L) = \frac{-2}{\sqrt{3}}$$

$$t_{CE} + 2t_{CB} = \frac{4}{\sqrt{3}L} \quad \text{...(3)}$$

iii. And $t_{CD}(y_D - y_C) + t_{CE}(y_E - y_C) + t_{CB}(y_B - y_C) - 3 = 0$

iv. Substituting the value of x, y coordinates, we get

$$\frac{4}{\sqrt{3}L}\left(0 - \frac{\sqrt{3}L}{2}\right) + t_{CE}\left(0 - \frac{\sqrt{3}L}{2}\right) - 3 = 0 \Rightarrow t_{CE} = \frac{-10}{\sqrt{3}L} \quad \text{...(4)}$$

v. From eq. (3) and eq. (4), we get

$$t_{CE} = \frac{-10}{\sqrt{3}L} \text{ kN/ m}, \; t_{CB} = \frac{7}{\sqrt{3}L} \text{ kN/ m}$$

vi. $$F_{CE} = \frac{-10}{\sqrt{3}L} \times L = \frac{-10}{\sqrt{3}} \text{ kN} = \frac{10}{\sqrt{3}} \text{ kN (Compressive)}$$

vii. $$F_{CB} = \frac{7}{\sqrt{3}L} \times L = \frac{7}{\sqrt{3}} \text{ kN (Tensile)}$$

4. Considering the Joint *E* :

i. $t_{ED}(x_D - x_E) + t_{EC}(x_C - x_E) + t_{EB}(x_B - x_E) + t_{EF}(x_F - x_E) = 0$

ii. Substituting the value of x, y coordinates, we get

$$\frac{-2}{\sqrt{3}L}(2L - L) + \left(\frac{-10}{\sqrt{3}L}\right)\left(\frac{3L}{2} - L\right) + t_{EB}\left(\frac{L}{2} - L\right) + t_{EF}(0 - L) = 0$$

$$t_{EB}\left(\frac{-L}{2}\right) + t_{EF}(-L) = \frac{7}{\sqrt{3}}$$

$$t_{EB} + 2t_{EF} = -\frac{14}{\sqrt{3}L} \qquad ...(5)$$

iii. And $t_{ED}(y_D - y_E) + t_{EC}(y_C - y_E) + t_{EB}(y_B - y_E) + t_{EF}(y_F - y_E) - 2 = 0$

iv. Substituting the value of x, y coordinates, we get

$$\frac{-2}{\sqrt{3}L}(0 - 0) + \left(\frac{-10}{\sqrt{3}L}\right)\left(\frac{\sqrt{3}L}{2} - 0\right) + t_{EB}\left(\frac{\sqrt{3}L}{2} - 0\right) + t_{EF}(0 - 0) - 2 = 0$$

$$-5 + t_{EB}\left(\frac{\sqrt{3}L}{2}\right) - 2 = 0 \Rightarrow t_{EB} = \frac{14}{\sqrt{3}L} \qquad ...(6)$$

v. From eq. (5) and (6), we get

$$t_{EB} = \frac{14}{\sqrt{3}L} \text{ kN/m}, \; t_{EF} = \frac{-14}{\sqrt{3}L} \text{ kN/m}$$

vi. $$F_{EB} = \frac{14}{\sqrt{3}L} \times L = \frac{14}{\sqrt{3}} \text{ kN (Tensile)}$$

vii. $$F_{EF} = \frac{-14}{\sqrt{3}} \times L = \frac{-14}{\sqrt{3}} \text{ kN} = \frac{14}{\sqrt{3}} \text{ kN (Compressive)}$$

5. Considering the Joint *B* :

i. $t_{BA}(x_A - x_B) + t_{BF}(x_F - x_B) + t_{BE}(x_E - x_B) + t_{BC}(x_C - x_B) = 0$

ii. Substituting the value of x, y coordinates, we get

$$t_{BA}\left(0 - \frac{L}{2}\right) + t_{BF}\left(0 - \frac{L}{2}\right) + \frac{14}{\sqrt{3}L}\left(L - \frac{L}{2}\right) + \frac{7}{\sqrt{3}L}\left(\frac{3L}{2} - \frac{L}{2}\right) = 0$$

$$t_{BA} + t_{BF} = \frac{-28}{\sqrt{3}L} \qquad ...(7)$$

iii. And $t_{BA}(y_A - y_B) + t_{BF}(y_F - y_B) + t_{BE}(y_E - y_B) + t_{BC}(y_C - y_B) - 3 = 0$

iv. Substituting the value of x, y coordinates, we get

$$t_{BA}\left(\frac{\sqrt{3}L}{2} - \frac{\sqrt{3}L}{2}\right) + t_{BF}\left(0 - \frac{\sqrt{3}L}{2}\right) + \frac{14}{\sqrt{3}L}\left(0 - \frac{\sqrt{3}L}{2}\right) + \frac{7}{\sqrt{3}L}$$

$$\left(\frac{\sqrt{3}L}{2} - \frac{\sqrt{3}L}{2}\right) - 3 = 0$$

$$t_{BF} = \frac{-20}{\sqrt{3}L} \qquad ...(8)$$

v. From eq. (7) and eq. (8), we get

$$t_{BF} = \frac{-20}{\sqrt{3}L}\ \text{kN/m},\ t_{BA} = \frac{-8}{\sqrt{3}L}\ \text{kN/m}$$

vi. $F_{BF} = \dfrac{-20}{\sqrt{3}L} \times L = \dfrac{-20}{\sqrt{3}}$ kN $= \dfrac{20}{\sqrt{3}}$ kN (Compressive)

vii. $F_{BA} = \dfrac{-8}{\sqrt{3}} \times \dfrac{L}{2} = \dfrac{-4}{\sqrt{3}}$ kN $= \dfrac{4}{\sqrt{3}}$ kN (Compressive)

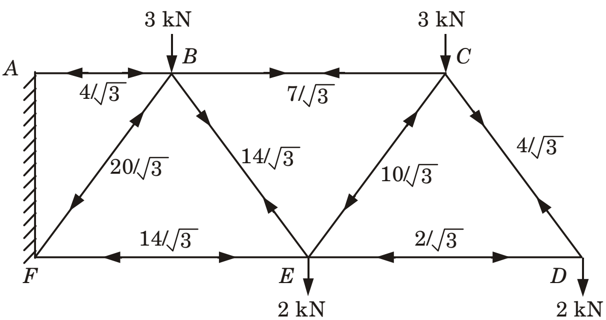


Fig. 28.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A beam *ABCD* is simply supported at its ends *A* and *D* over a span of 30 m. It is made three portions *AB*, *BC* and *CD* each 10 m in length. The moment of inertia of the section of these lines are *I*, 3*I* and 2*I* respectively. The beam carries a point load 150 kN at *B* and point load of 300 kN at *C*. Neglecting the weight of the beam calculate the slopes and deflection at *A*, *B*, *C* and *D*

Where $E = 200$ kN/mm^2, $I = 2 \times 10^{10}$ mm^4.

Ans.

Given : Span of beam AD = 30 m, Part AB = Part BC = Part CD = 10 m, Point load at the B = 150 kN, Point load at C = 300 kN, $E = 200\ \text{kN/mm}^2$, $I = 2 \times 10^{10}\ \text{mm}^4$
To Find : Slopes and deflection at A, B, C and D.

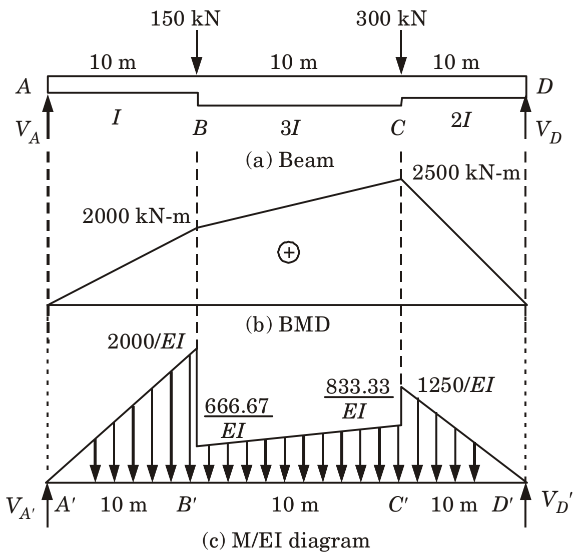


Fig. 29.

1. To Compute Reactions in Real Beam :

i. $\Sigma F_x = 0,\ V_A + V_D = 450$ kN ...(1)

ii. Taking moment about point D, $\Sigma M_D = 0$

$V_A \times 30 - 150 \times 20 - 300 \times 10 = 0$

$$V_A = \frac{6000}{30} = 200 \text{ kN}$$

iii. From eq. (1), we get

$$V_D = 450 - 200 = 250 \text{ kN}$$

2. Calculate Moment at all Point :

i. $$M_A = 0$$

ii. $$M_B = V_A \times 10 = 200 \times 10 = 2000 \text{ kN-m}$$

iii. $M_C = V_D \times 10 = 250 \times 10 = 2500$ kN-m

iv. $M_D = 0$

3. Slope at Support *A* :

Taking moment about point D' in M/EI diagram, $\Sigma M_{D'} = 0$

$$V_{A'} \times 30 = \left[\frac{1}{2} \times 2000 \times 10\left(20 + \frac{10}{3}\right) + 666.67 \times 10 \times (10+5)\right.$$

$$\left. + \frac{1}{2}(833.33 - 666.67) \times 10 \quad \times\left(10 + \frac{10}{3}\right) + \frac{1}{2} \times 1250 \times 10 \times \left(10 \times \frac{2}{3}\right)\right]$$

$$V_{A'} = 12870.37 \text{ kN}$$

Slope at A, $$\theta_A = \frac{12870.37}{EI} = \frac{12870.87}{200 \times 2 \times 10^{10} \times 10^{-6}}$$

$$= 3.2176 \times 10^{-3} \text{ rad}$$

4. Slope at Support *D* :

Taking moment about point A' in M/EI diagram, $\Sigma M_{A'} = 0$

$$V_{D'} \times 30 = \frac{1}{2} \times 2000 \times 10 \times 10 \times \frac{2}{3} + 666.67 \times 10 \times (10 + 5)$$

$$+ \frac{1}{2}(833.33 - 666.67) \times 10 \quad \times\left(10 + 10 \times \frac{2}{3}\right) + \frac{1}{2} \times 1250 \times 10 \times \left(20 + \frac{10}{3}\right)$$

$$V_{D'} = 10879.63 \text{ kN}$$

Slope at D, $$\theta_D = \frac{10879.63}{EI} = \frac{10879.63}{4 \times 10^{12} \times 10^{-6}} = 2.72 \times 10^{-3} \text{ rad}$$

5. Slop at Point *B* :

$$\theta_B = \frac{12870.37}{EI} - \frac{1}{2} \times \frac{2000 \times 10}{EI}$$

$$= \frac{2870.37}{EI} = \frac{2870.37}{4 \times 10^6} = 7.176 \times 10^{-4} \text{ rad}$$

6. Slope at Point *C* :

$$\theta_C = \frac{10879.63}{EI} - \frac{1}{2} \times 1250 \times 10$$

$$= \frac{4629.63}{EI} = \frac{4629.63}{4 \times 10^6} = 1.16 \times 10^{-3} \text{ rad}$$

7. **Deflection at Support A :** $\delta_A = 0$ (Due to simple support)
8. **Deflection at Support D :** $\delta_D = 0$ (Due to simple support)
9. **Deflection at Point B :** $\delta_B = \dfrac{12870.37}{EI} \times 10 - \dfrac{1}{2} \times \dfrac{2000 \times 10}{EI} \times \dfrac{10}{3}$

$$= \frac{95370.367}{EI} = \frac{95370.367}{4 \times 10^6} = 0.02384 \text{ m}$$

10. **Deflection at Point C :** $\delta_C = \dfrac{10879.63}{EI} \times 10 - \dfrac{1}{2} \times \dfrac{1250}{EI} \times 10 \times \dfrac{10}{3}$

$$= \frac{87962.67}{EI} = \frac{87962.67}{4 \times 10^6} = 0.022 \text{ m}$$

b. Find the horizontal movement of the roller end D of the portal frame shown in Fig. 30. Take $E = 2 \times 10^8$ kN/m² and $I = 3 \times 10^4$ m⁴. The moment of inertia of the column section is I while that of beam is $2I$.

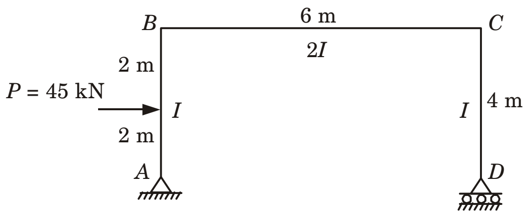
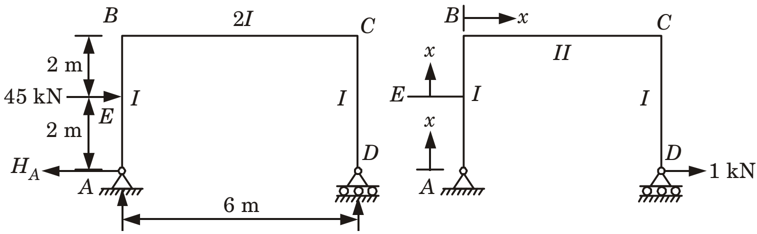


Fig. 30.

Ans.

Given : Frame shown in Fig. 30, $E = 2 \times 10^8$ kN/m², $I = 3 \times 10^4$ m⁴
To Find : The horizontal movement of the roller end D.
Note : In this question the value of I is incorrect so we assume $I = 3 \times 10^{-4}$ m⁴.

1. The deflection analysis is the same as that followed. The total internal virtual work for the frame is the sum of the internal virtual work of all the sections of the frame.
2. The unit force (1 kN) is applied at support D in the direction of the desired displacement Fig. 31(*b*). The origin and the positive direction of x for each section is indicated in Fig. 31(*b*).
3. Vertical reaction, $V_A = V_D = 0$
4. Horizontal reaction, $H_A = 45$ kN
5. The expressions for m_x and M_x for each member are determined as usual from statics. The resulting expressions for m_x and M_x for all sections are tabulated in Table. 2.



(a) Frame and the loading. (b) Unit virtual force at D.

Fig. 31.

Section	Origin for x	Limits for x	M_x	m_x	$\int_0^L m_x \frac{Mdx}{EI}$
AE	A	$0-2$	$45x$	$1 \times x$	$\frac{1}{EI}\int_0^2 45x^2 dx$
EB	E	$0-2$	$45(2+x) - 45x = 90$	$1 \times (2+x)$	$\frac{1}{EI}\int_0^2 90(2+x)dx$
BC	B	$0-6$	$45 \times 4 - 45 \times 2 = 90$	1×4	$\frac{1}{2EI}\int_0^6 4 \times 90\, dx$
DC	D	$0-4$	0	$1 \times x$	0

6. $$\Delta H_D = \int \frac{m\,Mdx}{EI} = \frac{1}{EI}\int_0^2 45x^2 dx + \frac{1}{EI}\int_0^2 90(2+x)dx + \frac{1}{2EI}\int_0^6 4 \times 90\, dx$$

$$= \frac{1}{EI}\left[\frac{45}{3}[x^3]_0^2 + 90\left[2x + \frac{x^2}{2}\right]_0^2 + \frac{360}{2}[x]_0^6\right]$$

$$= \frac{1}{EI}\left[\frac{45}{3} \times 8 + 90 \times 6 + 180 \times 6\right] = \frac{1}{EI}[120 + 540 + 1080] = \frac{1740}{EI}$$

$$\Delta H_D = \frac{1740}{2 \times 10^8 \times 3 \times 10^{-4}} = 0.029 \text{ m}$$

$$\Delta H_D = 29 \text{ mm}$$

6. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. By using Muller-Breslau's principle construct influence line diagram for the beam as shown in Fig. 32.

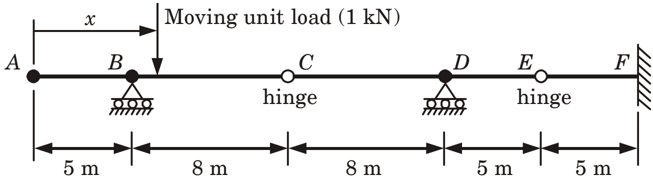


Fig. 32.

Ans.

Given : Beam shown in Fig. 32.
To Find : ILD for the given beam.

ILD by Muller-Breslau's Principle :

1. For a vertical or horizontal reaction component, the influence line for that reaction is constructed by removing the reaction component and then displacing the structure by 1.0 in the location of the reaction component and in the same direction as that component. This process is illustrated in Fig. 33.

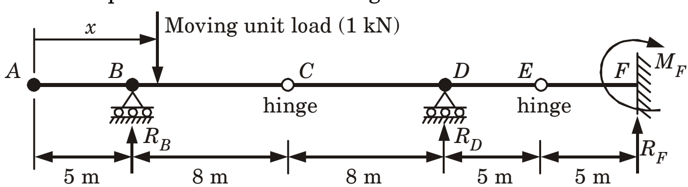
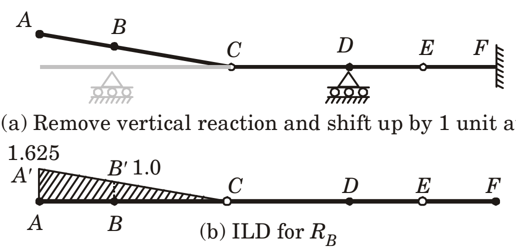


Fig. 33.

i. ILD for R_B :



(a) Remove vertical reaction and shift up by 1 unit at *B*

(b) ILD for R_B

Fig. 34.

Ordinate AA' :

$$\frac{AA'}{BB'} = \frac{AC}{BC} \Rightarrow \frac{AA'}{1} = \frac{13}{8}$$

$$AA' = 13/8 = 1.625$$

ii. ILD for R_D :

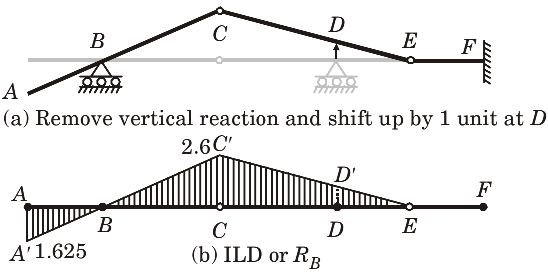


Fig. 35.

a. Ordinate CC' :

$$\frac{CC'}{DD'} = \frac{CE}{DE} \Rightarrow \frac{CC'}{1} = \frac{13}{5}$$

$$CC' = 2.6$$

b. Ordinate AA' :

$$\frac{AA'}{CC'} = \frac{AB}{BC} \Rightarrow \frac{AA'}{2.6} = \frac{5}{8}$$

$$AA' = 1.625$$

iii. ILD for R_F :

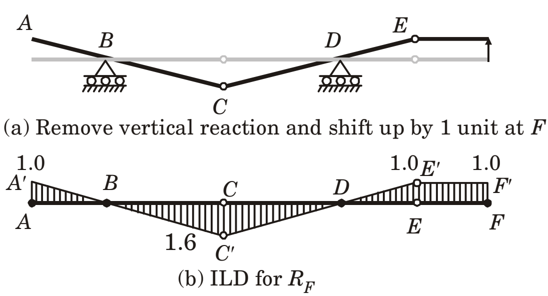


Fig. 36.

a. Ordinate EE' and $FF' = 1$

b. Ordinate CC' :

$$\frac{CC'}{EE'} = \frac{CD}{ED} \Rightarrow \frac{CC'}{1} = \frac{8}{5}$$

$$CC' = 1.6$$

c. Ordinate AA' :

$$\frac{AA'}{CC'} = \frac{AB}{BC} \Rightarrow \frac{AA'}{1.6} = \frac{5}{8}$$

$$AA' = 1$$

2. ILD for M_F :

i. To find the influence line for a moment reaction, the process is very similar to that for the vertical reaction forces described above except that the moment restraint at the reaction location is removed (instead of the vertical reaction restraint). This process is illustrated in Fig. 37.

ii. This is a counter clockwise rotation because we are considering counter clockwise rotations and moments to be positive. Again, all of the beam segment must remain straight and rigid as they move.

iii. This results in the displaced structure shows in Fig. 37(b), with rotations in the internal hinges at points C and E, and with the beam being held down at the roller and pin supports at points B, D and F.

iv. This displaced shape results in the equivalent influence line shown in Fig. 37(c), where the slope of the right side at point F is 1.0. Using this slope we can directly find the value of influence line at point E.

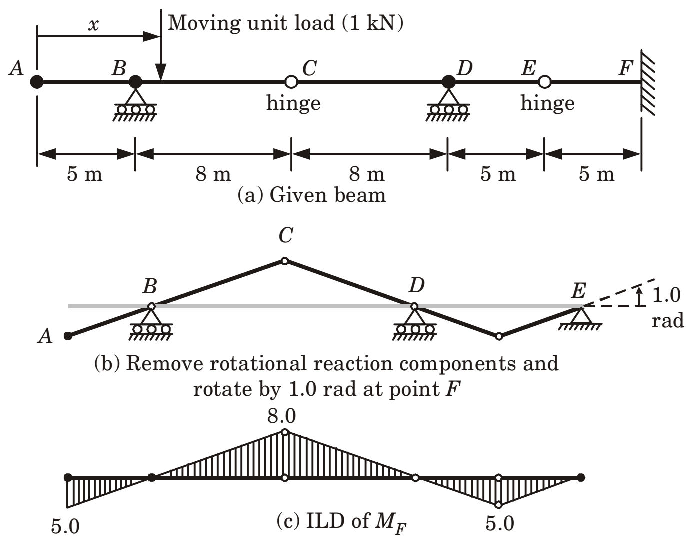


Fig. 37.

v. To find the influence line for the moment reaction at point F, the rotational reaction component is removed from the fixed support. This again causes the determinate structure to become unstable, leaving a pin at point F. Once the rotational support is removed, the beam is rotated at point F by 1.0 rad.

vi. Slope of IL of M_F (at $x = 26$) = rise run = slope × run $= 1.0 \times 5 = 5.0$. This value means that when a point unit load (1 kN) is placed at point E, the moment reaction at point F, $M_F = 5$ kN-m. The remainder of the values on the influence line may be found using similar triangles.

b. Assume a unit point load is rolling along the bridge deck from points *A* to *D* in a simple truss as shown in Fig. 38. The distance *X* is the distance of the moving load from point *A*. Construct the influence line diagram for the three members *EF*, *BF* and *BC*.

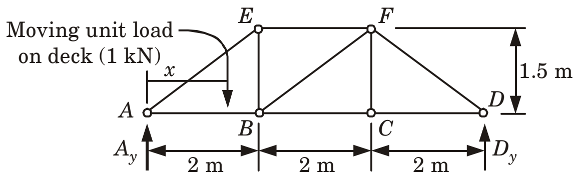


Fig. 38.

Ans.

Given : Truss shown in Fig. 38.
To Find : Construct the ILD for the three members *EF*, *BF* and *BC*.

1. Calculation of Support Reactions :

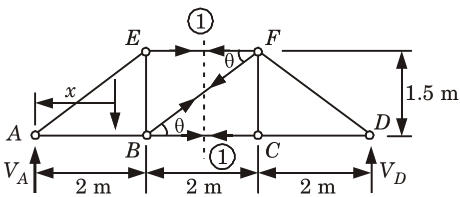


Fig. 39.

i. Taking moment about point, $\Sigma M_A = 0$

$$V_D \times 6 - 1 \times x = 0$$

$$V_D = \frac{x}{6} \text{ kN}$$

ii. $\Sigma F_y \Rightarrow V_A + V_D = 1 \text{ kN}$

$$V_A = 1 - V_D = 1 - \frac{x}{6} = \frac{(6-x)}{6} \text{ kN}$$

2. ILD for *EF* :

i. Consider the section 1-1 as shown in the Fig. 40. When the load is in portion *AC* taking moment about *C*, we get

$$P_{EF} = \frac{V_D \times 2}{1.5} = \frac{(x/6) \times 2}{(3/2)}$$

$$P_{EF} = \frac{2x}{9}$$

ii. Therefore, when unit load at *C*,

$$P_{EF} = \frac{2 \times 4}{9} = \frac{8}{9} \text{ kN (compressive)} \qquad (\because \quad x = 4 \text{ m})$$

and, when load is at *A*, $P_{EF} = 0$. $(\because \quad x = 0 \text{ m})$

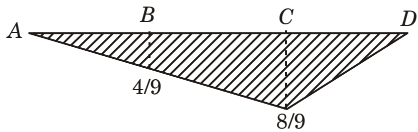


Fig. 40. ILD for P_{EF}.

iii. When load is in portion *C* to *D*, taking moment about *C*, we get

$$P_{EF} \times 1.5 = V_A \times 4 \Rightarrow P_{EF} = \frac{\left(\frac{6-x}{6}\right) \times 4}{1.5}$$

$$P_{EF} = \frac{4(6-x)}{9} \text{ (Compressive)}$$

When load is at *C*, $P_{EF} = \frac{4 \times 2}{9} = \frac{8}{9}$ $(\because x = 4 \text{ m})$

and when load is at *D*, $P_{EF} = 0$ $(\because x = 6 \text{ m})$

3. ILD for *BC* :

i. Consider section 1-1, when load is in portion *AB*.
Taking moment about point *E*, $\Sigma M_E = 0$,

$$P_{BC} \times 1.5 = V_D \times 4$$

$$P_{BC} = \frac{(x/6) \times 4}{1.5} = \frac{4x}{9} \text{ (Tensile)}$$

When load is at *A*, $P_{BC} = 0$ $(\because x = 0)$

When load is at *B*, $P_{BC} = 8/9$ kN $(\because x = 2 \text{ m})$

ii. When load is in portion *C* to *D*, moment equilibrium at *E* gives,

$$P_{BC} \times 1.5 = V_A \times 2 = \left(\frac{6-x}{6}\right) \times 2$$

$$P_{BC} = \frac{2(6-x)}{9}$$

When load is at C, $P_{BC} = 4/9$ kN $(\because x = 4 \text{ m})$

When load is at D, $P_{BC} = 0$ $(\because x = 0)$

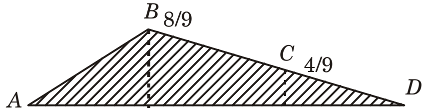


Fig. 41. ILD for P_{BC}.

4. **ILD for *BF* :**

i. From ΔBFC,

$$\therefore \qquad \tan\theta = \frac{1.5}{2} \Rightarrow \theta = 36.87°$$

ii. Consider section 1-1 considering the equilibrium of right side portion, when the load is in portion AB.

$P_{BF} \times \sin\theta = V_D \Rightarrow P_{BF} = (x/6)$ coses θ (Compressive)

When load is at A, $P_{BF} = 0$ $(\because x = 0)$

When load is at B,

$$P_{BF} = (2/6)\text{ cosec }\theta = (1/3)\text{ cosec }\theta = \frac{1}{3}\text{ cosec }(36.87°)$$

$$= 0.56 \qquad (\because x = 2 \text{ m})$$

iii. When load is in portion C to D, considering left side portion

$\Sigma F_y = 0$, gives, $P_{BF} \sin\theta = -V_A$

$$P_{BF} = \left(\frac{6-x}{6}\right)\text{cosec }\theta = \left(\frac{6-x}{6}\right)\text{ cosec }\theta \text{ (Tensile)}$$

When load is at D, $P_{BF} = 0$ $(\because x = 6 \text{ m})$

When load is at C, $P_{BF} = 2/6$ cosec $\theta = 1/3$ cosec θ

$$= \frac{1}{3}\text{ cosec }(36.87°) = 0.56$$

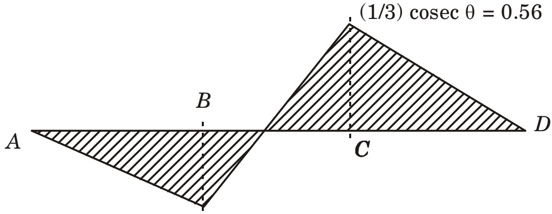


Fig. 42. ILD for P_{BF}.

7. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A three hinged parabolic arch of span 30 m and central rise of 5 m. It is subjected to a concentrated load of 40 kN at 6 span. Calculate the normal thrust, shear force and bending moment at 6 m from the left support Fig. 43.

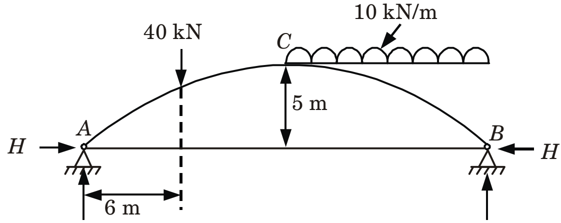


Fig. 43.

Ans.

Given : Span of arch, $L = 30$ m, Rise of arch = 5 m, Concentrated load = 40 kN, Intensity of IDL, $w = 10$ kN/m
To Find : The normal thrust, shear force and bending moment at 6m from the left support.

1. The arch is shown in Fig. 44. Taking moment about B, we get

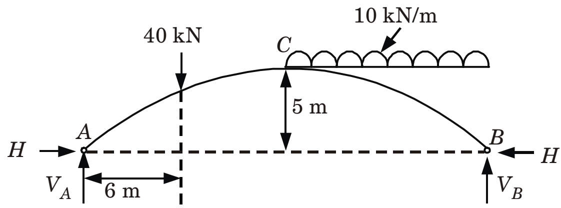


Fig. 44.

$$V_A \times 30 - 10 \times 15 \times 7.5 - 40 \times 24 = 0$$
$$V_A = 69.5 \text{ kN}$$
$$V_B = 10 \times 15 + 40 - 69.5 = 120.5 \text{ kN}$$

2. Taking moment about crown C, $\Sigma M_C = 0$

$$V_A \times 15 - H \times 5 - 40 \times 9 = 0$$
$$69.5 \times 15 - H \times 5 - 40 \times 9 = 0$$
or $$H = 136.5 \text{ kN}$$

3. Bending moment at 6 m from the left support :

In the parabolic arch, $y = \dfrac{4hx(L-x)}{L^2}$

Therefore, at $x = 6$ m

$$y_D = \frac{4 \times 5 \times 6(30-6)}{30^2} = 3.2 \text{ m}$$

$$M = V_A \times 6 - H \times y_D \quad ...(1)$$
$$M = 69.5 \times 6 - 136.5 \times 3.2 = -19.8 \text{ kN-m}$$

4. Vertical shear at D,
$$V_D = V_A = 69.5 \text{ kN}$$

5. Curve is given by,
$$y = \frac{4hx(L-x)}{L^2}$$

6. $$\frac{dy}{dx} = \tan\theta = \frac{4h(L-2x)}{L^2}$$

Therefore, at $x = 6$ m,
$$\tan\theta = \frac{4 \times 5(30 - 2 \times 6)}{30^2}$$
$$\therefore \qquad \theta = 21.80°$$

7. Normal thrust, $N = V_D \sin\theta + H\cos\theta$
$$= 69.5 \times \sin 21.80° + 136.5 \times \cos 21.80°$$
$$= 152.54 \text{ kN}$$

8. Radial shear,
$$Q = V_D \cos\theta - H\sin\theta$$
$$= 69.5 \times \cos 21.80° - 136.5 \times \sin 21.80° = 13.83 \text{ kN}$$

b. Classify the arches based on materials, shapes and structural systems with the help of neat sketch. Also, distinguish between two hinged and three hinged arches.

Ans. **Classification of Arches :**

1. Some common type of arches are as follows :

i. Funicular Arch : If the arch of parabolic shape and subjected to a uniformly horizontally distributed vertical load then only compressive forces will be resisted by the arch. This type of arch shape is called as funicular arch because no bending or shear forces occur within the arch.

ii. Fixed Arch :

a. A fixed arch must have solid foundation abutments since it is indeterminate to the third degree, and consequently, additional stresses can be introduced into the arch due to relative settlement of its supports.

b. Fixed arch is often made from reinforced concrete and requires less material to construct than other types of arches.

iii. Two Hinged Arch :

a. It is indeterminate to the first degree. This structure can be made statically determinate by replacing one of the hinges with a roller.

b. Due to roller, capacity of the structure to resist bending along its span has been removed and it would serve as a curved beam not as an arch.

c. It is commonly made from metal or timber.

iv. Three Hinged Arch : It is statically determinate and made of metal or timber. It is not affected by settlement or temperature changes.

v. Tied Arch :

a. Tied arch allows the structure to behave as a rigid unit. Tied rod carries the horizontal component of thrust at the supports.

b. Tied arch is also unaffected by relative settlement of the supports.

2. Based on the Material of Construction :

i. Steel arches.
ii. Reinforced arches.
iii. Concrete arches.
iv. Timber arches.
v. Brick arches.
vi. Stone arches.

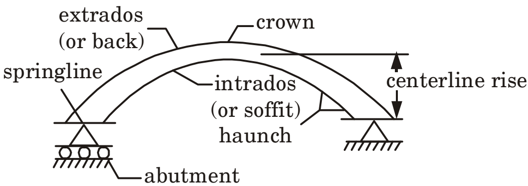
Brick and stone arches are combinedly known as masonry arches.

3. Based on Support Conditions and Structural Behaviour :

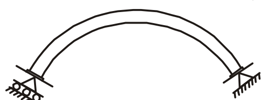
i. Three hinged arches – Hinged at crown and abutments.
ii. Two hinged arches – Hinged at abutment only.
iii. Hingeless or fixed arches – No hinges at all.

4. Based on Shape and Structural Arrangement of the Rib :

i. Solid rib arch (also known as closed arch).
ii. Tied solid rib arch.
iii. Spandrel braced arch.
iv. Two hinged braced rib arch or crescent arch or sickle arch.
v. Two hinged braced rib arch.

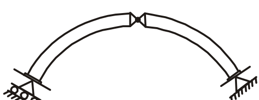


(*a*) Arch

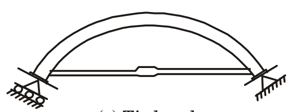


(*b*) Fixed arch

(*c*) Two-hinged arch



(*d*) Three-hinged arch



(*e*) Tied arch

Fig. 45.

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