MTH 373/573:
Scientific Computing

Rubric for HW 1

IIIT-Delhi, Monsoon 2021
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General Notes:

- Any minor mistakes like calculation error, sign error or like shall not be penalized.
- On the contrary, any final result or statement with *no* intermediate steps ("pulling out of thin air") will lead to zero credit for that particular problem or portion of a problem.
- After TAs grade and return your solutions (target: 5-6 days from date of release of this solution), please carefully consult both the provided solution and this rubric before contacting us for a rebuttal of your grading.
  - You must post your query for any grading related questions on the discord server's private `#grading` channel. This is the *preferred* mode of raising your queries and one which will be readily visible to both the instructor and all TAs.
  - As a discouraged and less preferred option, you cand send a mail to the instructor including all TAs about your grading concerns.
  - Any attempt to privately contact a strict subset of {TAs, instructor} for grading queries will be ignored!

Problem 1: Condition Number — 5 + 5 = 10 points

If solutions to both parts are correct, then full credit will be given. If not, the following are points for some intermediate steps.

- 3 points for simply stating or deriving the condition number for the problem of abstract evaluation of a function $f : \mathbb{R} \longrightarrow \mathbb{R}$.
- 2 points for each part (4 points total) for correctly writing each given problem as a function evaluation $f(x) = y = \ldots$ as in the provided solution.

Problem 2: Vector Norms are Lipschitz Continuous — 30 points

The provided solution is probably "optimal". Any effort which closely emulates this shall be given full credit. However, other slightly "inefficient" yet acceptable solutions include doing a case analysis.

For an incorrect or incomplete solution, 5 points for stating triangle inequality (anywhere in the solution of this problem), 5 points for stating any properties of norms (anywhere in the solution). And up to 10 points for providing details.

If a solution is provided for a specific norm (say, 1-, 2- or ∞- norms), there will be a penalty of -10 points even if the solution for each (or any) of the cases is correct. A solution which is correct but for a general p-norm will lead to a penalty of -5 points. The stated property is true for *any* vector norm, in fact *any* norm (not only for vectors) since it depends only on the triangle inequality

which is a defining property for any norm.

Problem 3: p-norms and Tensor Product — 15 + 5 = 20 points

Full credit of 15 + 5 = 20 points for a correct solution to each part.

Any alternative solution in which a solution is provided separately for each of the vector p-norms, $p = 1, 2, \infty$ is okay too and will be given full credit. A partial or incorrect solution will be given up to 10 points (out of 15) depending on how an argument is provided.

An incorrect conjecture for part (b) will lead to deduction of at least 1 point and up to 3 points. (2 points minimum credit for providing a solution to part (b).)

Problem 4: Nilpotent Matrix is Singular — 10 points

−2 points for any solution that is correct but *does not* use linear independence of columns (rows) of the matrix as required. For example, for an argument about the singularity of A using determinant, eigenvalues, singular values, characteristic/minimal polynomial, and like. The stated problem requirement means there isn't much room for any alternative "optimal" proof for this problem.

An alternative solution, however, which argues that $A^2 = AA = 0$ as being that A multiplied by the first column of itself leads to a 0 column, *mutatis mutandis* for other columns is correct. Such a solution will also receive full credit.

Problem 5: Condition Number of Matrices — 5 points

A penalty of at least −1 point for an incorrect solution depending on details. Some laxity, like not stating submultiplicative property when invoking it, is allowed and shall not be penalized.

Problem 6: Exploring IEEE Double Precision using Python — 15 points

For part (a), critical to mention *underflow* (UFL) to receive full credit. Likewise, for parts (b) and (c), the key phrases/words are *machine epsilon* (ε_M) and *overflow* (OFL), respectively.

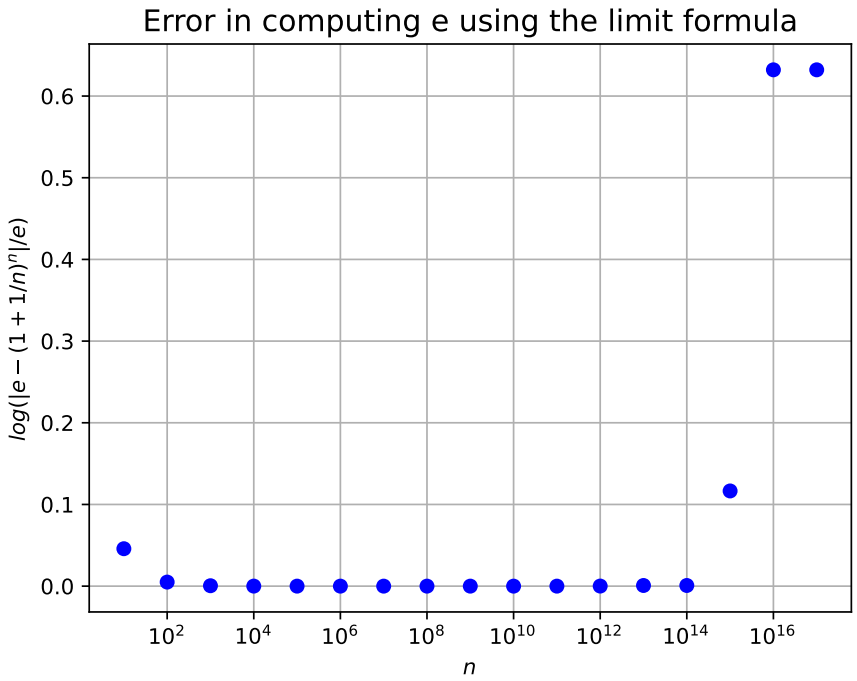
Not providing the key words or phrase in any explanation for a given part will lead to a penalty of −1 point for it.

Any explanation which exceeds 6 sentences (not lines) will be penalized −0.5 points even if correct and references the key words. Similarly, any explanation that stretches to one page will be penalized −1 points. These penalties will be additional to the −1 point for an incorrect solution as appropriate.

Problem 7: Understanding Round off Error — 20 + 15 = 35 points

Important: The provided solution for both parts are pedagogical and meant to teach you something. I expect *no one* in this class to detail an analysis as in the provided solution. However, if someone has done so to explain why the error falls and then rises, the TAs shall flag it for me. Alternatively, you can directly let me know, say, by mail. I will award you a *bonus 1 point for the course total*. This is because providing such an analysis is not expected and will be highly deserving of a bonus point accolade!

Note: For part (a), there was a SNAFU on my part! I should have asked for a `loglog` plot as in my provided solution instead of the `semilogx` plot in the posted HW. (Mistakes happen!) No points shall be taken off for providing this `semilogx` plot.



In part (a), an explanation that touches upon the round off error or references the machine epsilon ε_M as a cause for the error to reduce and then increase shall be awarded the full credit, and the total of 20 points for this part if the plot is correct.

For part (b), any reference to using ε_M for the stopping criterion or to comparing two successive computed sums is acceptable and shall get full credit. Otherwise, if the stopping criterion is incorrect of if computations are performed for longer than 20 iterations, there will be a penalty of -5 points. Reporting that the relative error for this computation is around ε or less than it or even 0 (!) is okay, and full credit shall be awarded provided the stopping criterion is correct. Finally, note that the number of iterations needed for the sum to converge not be mentioned since it was not asked.