Q1)

Q1.

ans) Given L, K are 2 lower triangular matrices and $L, K \in R^{n \times n}$

We need to solve for $X \in R^{n \times n}$

Also, gen matrix $B \in R^{n \times n}$

Now, given

$$\underbrace{L X K}_{Y} = B$$

let $Y = XK$

$$\Rightarrow \quad LY = B$$

We perform column wise forward substitution to calculate Y; reducing to n linear systems

Now,

$$Y = XK$$

transposing both sides

$$\Rightarrow \quad (XK)^T = (Y)^T$$

$$\Rightarrow \quad K^T X^T = Y^T$$

K^T is an upper triangular matrix

Let $U = K^T$

$$\Rightarrow \quad U X^T = Y^T$$

We can solve this using row wise backward substitution to calculate X^T

X^T can easily be transformed to X

Inputs : L, K, B
Output : X

```
procedure FUNC (L, K, B):
    for i = 0, i ≤ n, i++ do
        temp ← B[i]
        for j = 0, j ≤ i-2, j++ do
            temp ← temp - L_ij * Y_j
        Y_j ← temp / L_ii
    Y ← transpose (Y)
    K ← transpose (K)
    for i = n-1, i ≥ 1, i-- do
        temp ← Y_i
        for j = i+1, j < n, j++ do
            temp ← temp - K_ij * X_j
        X_i = temp / K_ii
    X ← transpose (X)
    return X
```

Q2) c)
(i) In random matrix, the condition number increases as n increases, and so error increases; pivoting is necessary because it decreases relative error, residual does not have much effect.
(ii) In the Hilbert matrix, errors are high as compared to the other 2 matrices. This is because it is a singular matrix, so LU decomposition fails here. Rest the same as the random matrix.
(iii) In this matrix, floating-point errors(round off errors) will not happen here. Gaussian elimination with and without pivot gives the same result here so both errors and residuals are 0.

Q3) c) We get the same result in parts (a) and (b). This happens because in part (b), when we perform row equilibration, dividing each element by max of its row does not change the elementary matrix.
Also, making 1 as max element in each row, we solve the need to choose the correct pivot as in part (a) itself. Problems like 0 pivot entries, roundoff errors are easily solved by taking the 1s in each row as entry pivots, and performing gaussian elimination brings us to same result as (a)