# MTH 371: Assignment II

November 8, 2021

Instructions

- Use statistical software R for your codes.
- Only basic built-in functions available in R are allowed.
- In each question, show the simulations with relevant graphs.
- Due date is November 18 (11.59 p.m. IST). No late assignments will be accepted.
- Submit all of your work which include the codes, results, graphs and reports.

1. (5 Marks) Suppose the interarrival times of a renewal process have the geometric distribution with success probability $p$. Then the interarrival times are nonnegative integers, and sum of interarrival times is $S_n = X_1 + \ldots + X_n$. Answer the following.

   (a) Simulate sum of interarrival times with $p = 0.2$.

   (b) Simulate $N_t$ and estimate $E(N_t)$ for various values of $t$. Use $p = 0.2$.

2. (10 Marks) Let there be a discrete time Markov chain with the state space $S = \{0, 1, \ldots, 7\}$. The one step transition probability is given by

$$P = \begin{bmatrix} 1/3 & 2/3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2/3 & 1/3 \end{bmatrix}$$

It is given that when the process starts the MC was in state 0. Answer the following.

(a) Simulate five times a 50 steps Markov chain. Construct a plot comparing time to the states of the process.

(b) What will be $P^{10}$, $P^{20}$, $P^{50}$. What do you observe.