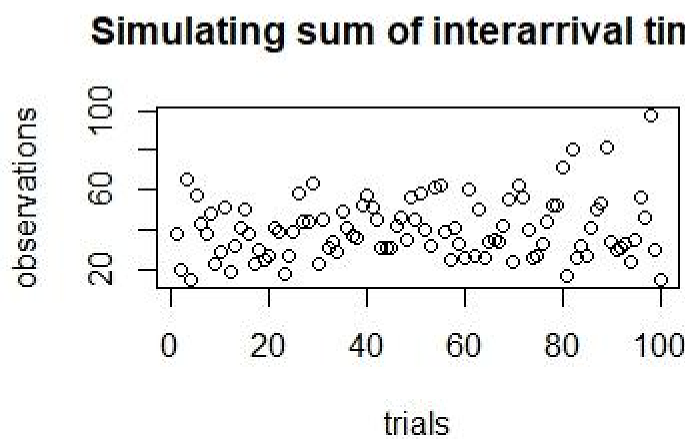
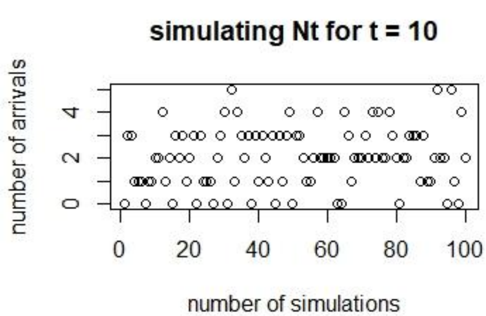
Q1

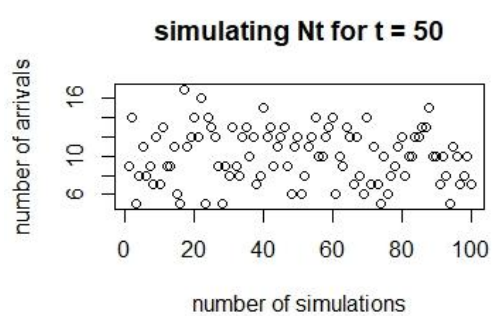
ans) Given the interarrival times of a renewal process have geometric distribution with success probability p = 0.2

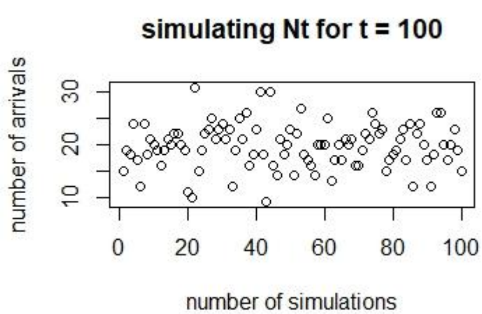
(a) Here I have simulated the sum of interarrival times with p = 0.2. We see that the sum of multiple geometric distributions S_n results in a pascal distribution represented by function rnbinom (negative binomial). We have assumed N = 100 and size = 10 for below plot -

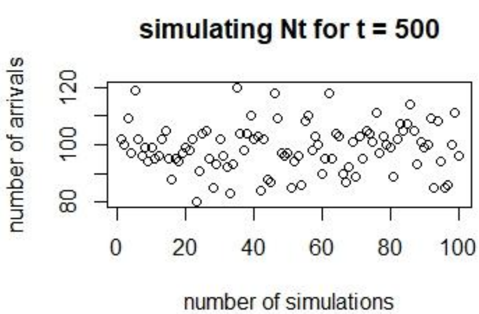


(b) In this part we simulate N_t and estimate $E(N_t)$ for t = 10,50,100,500. We have taken N = 100. $E(N_t)$ have been calculated using the mean function as in the code.





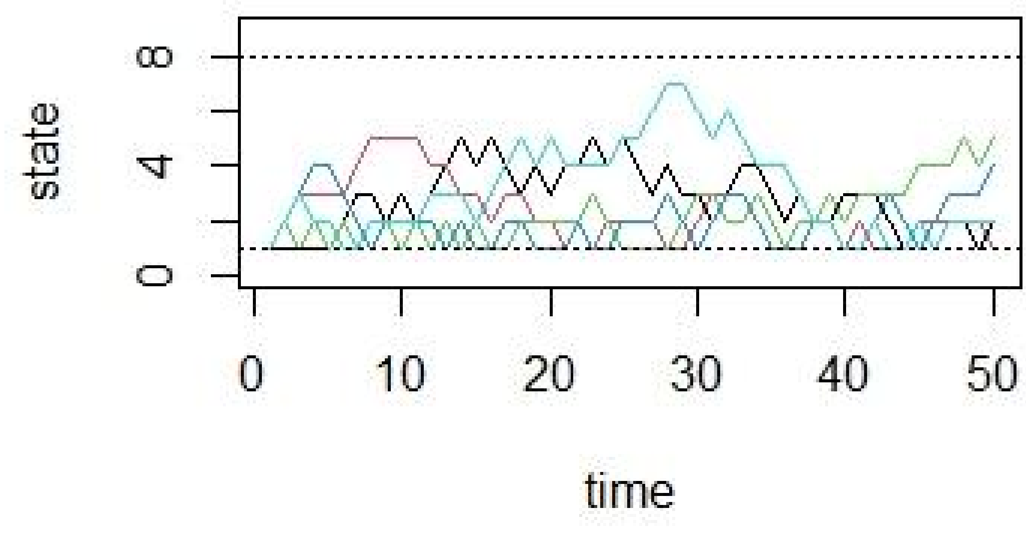




Q2

ans) State Space of Matrix S = {0,1,2,3,4,5,6,7}

(a) My code begins with writing a function to simulate discrete Markov chains according to the transformation matrix provided. The function includes storing the number of states and their respective values. Then for each iteration, we determine X_{t+1} from a multinomial distribution and return them. Finally, we call this function to simulate the Markovmarkov chain, and a plot of state vs time is displayed.



(b) We run a loop to calculate P^{10}, P^{20}, and P^{50}. The values are represented below -

```
[1] "P 10"
            [,1]       [,2]       [,3]       [,4]       [,5]       [,6]       [,7]        [,8]
[1,] 0.151619841 0.28281597 0.22913174 0.16054463 0.09656387 0.04951820 0.02269302 0.007112737
[2,] 0.141407983 0.26618571 0.22168030 0.16284780 0.10503141 0.05962844 0.03187184 0.011346509
[3,] 0.114565869 0.22168030 0.19990178 0.16616708 0.12591238 0.08738505 0.05962844 0.024759098
[4,] 0.080272316 0.16284780 0.16616708 0.16296635 0.14852072 0.12591238 0.10503141 0.048281935
[5,] 0.048281935 0.10503141 0.12591238 0.14852072 0.16296635 0.16616708 0.16284780 0.080272316
[6,] 0.024759098 0.05962844 0.08738505 0.12591238 0.16616708 0.19990178 0.22168030 0.114565869
[7,] 0.011346509 0.03187184 0.05962844 0.10503141 0.16284780 0.22168030 0.26618571 0.141407983
[8,] 0.007112737 0.02269302 0.04951820 0.09656387 0.16054463 0.22913174 0.28281597 0.151619841
```

```
[1] "P 20"
           [,1]       [,2]       [,3]      [,4]      [,5]       [,6]       [,7]       [,8]
[1,] 0.10831537 0.20908002 0.18810782 0.1582822 0.1258427 0.09721392 0.07773415 0.03542381
[2,] 0.10454001 0.20236928 0.18368113 0.1569752 0.1277481 0.10178841 0.08403078 0.03886708
[3,] 0.09405391 0.18368113 0.17123670 0.1531470 0.1329210 0.11456493 0.10178841 0.04860696
[4,] 0.07914112 0.15697524 0.15314697 0.1471824 0.1399638 0.13292098 0.12774808 0.06292133
[5,] 0.06292133 0.12774808 0.13292098 0.1399638 0.1471824 0.15314697 0.15697524 0.07914112
[6,] 0.04860696 0.10178841 0.11456493 0.1329210 0.1531470 0.17123670 0.18368113 0.09405391
[7,] 0.03886708 0.08403078 0.10178841 0.1277481 0.1569752 0.18368113 0.20236928 0.10454001
[8,] 0.03542381 0.07773415 0.09721392 0.1258427 0.1582822 0.18810782 0.20908002 0.10831537
```

```
[1] "P 50"
           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]       [,8]
[1,] 0.07645703 0.1519181 0.1491274 0.1450948 0.1406191 0.1365868 0.1337965 0.06640032
[2,] 0.07595903 0.1510207 0.1485064 0.1448732 0.1408408 0.1372078 0.1346937 0.06689824
[3,] 0.07456368 0.1485064 0.1467666 0.1442524 0.1414619 0.1389477 0.1372078 0.06829342
[4,] 0.07254739 0.1448732 0.1442524 0.1433553 0.1423593 0.1414619 0.1408408 0.07030957
[5,] 0.07030957 0.1408408 0.1414619 0.1423593 0.1433553 0.1442524 0.1448732 0.07254739
[6,] 0.06829342 0.1372078 0.1389477 0.1414619 0.1442524 0.1467666 0.1485064 0.07456368
[7,] 0.06689824 0.1346937 0.1372078 0.1408408 0.1448732 0.1485064 0.1510207 0.07595903
[8,] 0.06640032 0.1337965 0.1365868 0.1406191 0.1450948 0.1491274 0.1519181 0.07645703
```

We observe that the above Markov chain is irreducible(every state is accessible from every state). This also means that the above Markov chain is aperiodic too(Every state is aperiodic and the state space is a recurrent communicating class.