SMAI-S25-03: Linear Classifiers

C. V. Jawahar

IIIT Hyderabad

January 10, 2025

Recap

1. Actions: Project team formation and sports/game/data identification. (deadline soon)
2. Summary
   - Representation as a vector in R^d
   - Learn a function $y = f(\mathbf{W}, \mathbf{x})$ from the data.
     - Notion of Training and Testing
   - Feature Transformation as a useful trick:

$$\mathbf{x}' = \mathbf{W}\mathbf{x}$$

     - Data normalization
     - Dimensionality Reduction
   - Two Simple Classification Schemes:
     - Nearest Neighbour Algorithm (K NN)
     - Linear Classification
     - — $sign(\mathbf{w}^T\mathbf{x})$; Either +ve or -ve.

Performance Metrics

- In the classification setting:
  - Accuracy, Confusion Matrix, Precision, Recall
  - **Read Now:**
    `https://developers.google.com/machine-learning/crash-course/classification/accuracy-precision-recall`
- In the ranking/retrieval setting:
  - Precision, Recall, AP, F-score.
- Many more performance metrics see for example:
  - **Read Later:**
    `https://en.wikipedia.org/wiki/Confusion_matrix`

Discussions

Q: $\mathbf{w}^T\mathbf{x}$ is a line passing through origin. How do we characterize "general" line?

- Ans1: $\mathbf{w}^T\mathbf{x} + b$
- Ans2: $\mathbf{w}'^T\mathbf{x}'$ where $\mathbf{x}'$ is an augmented vector. i.e., $\mathbf{x}' = [\mathbf{x}^T, 1]^T$. (There is notational convenience for this.)
- (Refer the usage of "bias" in neural networks. You will appreciate this better)
- In general, when we write as $\mathbf{w}^T\mathbf{x}$, often $\mathbf{x}$ is augmented and bias is absorbed.

Problem-5

Q: We know, if we have two classes, we can classify as:

$$sign(\mathbf{w}^T\mathbf{x})$$

Assume we have three classes, how do we extend this decision making logic?
Write an algorithm to classify a sample into one of the four possible classes.

Problem-1

Consider a classification rule:
"Decide as A if $\mathbf{w}^T x \geq 0$; Else Class B"
Problem: for $\mathbf{w} = [1, 1, 1, 1]^T$, classify the following samples:

(i) $[1, 1, 1, 1]$ (ii) $[0, 1, 2, 3]^T$ (ii) $[-1, +1, -1, +1]$

Problem-2

What is the angle between the two lines characterized by

$$\mathbf{w}_1 = [1, 1]^T$$

$$\mathbf{w}_2 = [1, -1]^T$$

(a) 0 (b) 45 (c) 90 (d) 120

Problem-3

Q: There was a proposal to an airport from a supplier:
"We have a multi-sensor product that can recognize terrorists reliably. We can classify a normal person vs terrorist (two class classification problem) with an accuracy of 99.5% "

- Is this a good product for detecting terrorists? Why?
- As an SMAI student, who evaluate this product, what performance metrics you will prefer? What should be a good crieria?

Problem-4

In a problem of classification of "cricket ball vs Tennis ball" there were two features (i) size/radius measured in micometer (ii) Weight measured in Tons. Final classification will be done with KNN with Eucldean distance.

- Fact: A non-SMAI student felt that size should be in cm (1 cm $= 10^4$ mico meter) and weight in Kg (1 Ton $= 10^3$ Kg) for good performance of the algorithm she uses.
- Suggest the 2×2 matrix $\mathbf{W}$ to obtain the new feature representation from the old one.

$$\mathbf{x}' = \mathbf{W}\mathbf{x}$$