S.No. : **119** **BAS 3101**

No. of Printed Pages : 06

Following Paper ID and Roll No. to be filled in your Answer Book.		
PAPER ID : 39901	**Roll No.**	1 2 2 0 4 3 9 1 1 7

B. Tech. Examination 2022-23

(Special Carry Over Paper)

MATRICES AND CALCULUS

Time : Three Hours] ***[Maximum Marks : 60***

Note :– Attempt all questions.

SECTION – A

1. Attempt all parts of the following : $8\times1=8$

(a) If the matrix

$$A=\begin{bmatrix}1+i & 3-5i\\ 2i & 5\end{bmatrix}$$

find (A^{u}).

(b) Find latent roots of matrix :

$$A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$$

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(c) If

$$u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{x}{y}$$

then find the value of :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

(d) Find n^{th} derivative of $\log x^2$.

(e) Prove that :

$$B(m+1, n) = \frac{m}{m+n} B(m, n)$$

(f) Evaluate :

$$\int_0^1 \int_0^x e^{y/x}\, dx\, dy$$

(g) Find a unit normal vector to the surface $x^2 + 3y^2 + 2z^2 = 6$ at point (2, 0, 1).

(h) If $f = 2x^2 - 3y^2 + 4z^2$, find the value of curl (grad f).

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) State Stoke's theorem and evaluate $\oint_C \overline{F} \cdot d\overline{r}$ by Stoke's theorem, where $\overline{F} = y^2 i + x^2 j - (x + z) K$ and C is the boundary of triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0).

(b) Change order of integration and hence evaluate:

$$\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2 x^2}}\, dx\, dy$$

(c) Use the method of the Lagrange's multipliers to find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(d) If $y = \tan\text{-}1\ x$, prove that :

$$(1 + x^2)\, y_{n+2} + (2n + 1) \times y_{n+1} + n(n + 1)\, y_n = 0$$

hence, determine the values of all the derivatives of y w.r.t. x using $x = 0$.

SECTION – C

Note :- Attempt all questions. Attempt any two parts from each question. 5×8=40

3. (a) If $x^x y^y z^z = c$. Show that at :

$$x = y = z,\ \frac{\partial^2 z}{\partial x\, \partial y} = -(x \log e\, x)^{-1}$$

(b) Expand x^y in powers of $(x-1)$ and $(y-1)$ upto the third degree terms.

(c) Examine $f(x, y) = x^3 + y^3 - 3axy$ for maximum and minimum values.

4. (a) Reduce the matrix A to its normal form, when :

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

hence, find the rank of A.

(b) Tests for consistency and solve the following system of equations :

$$5x + 3y + 7z = 4$$

$$3x + 26y + 2z = 9$$

$$7x + 2y + 11z = 5$$

2. (c) Find the eigen values and the corresponding eigen vectors for the following matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{bmatrix}$$

5. (a) Prove that :

(i) $\Gamma\frac{1}{2} = \sqrt{\pi}$

(ii) $\int_0^1 \left(\log \frac{1}{y}\right)^{n-1} dy = \Gamma n$

(b) Evaluate :

$$\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)} dx\, dy$$

by changing polar co-ordinates. Hence, show that :

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

(c) Calculate the volume of the solid bounded by the surface $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$.

6. (a) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4).

(b) A fluid motion is given :

$$\bar{V} = (y + z)\, i + (z + x)\, j + (x + y)\, k$$

show that the motion is irrotational and hence find the velocity potential.

(c) Use divergence theorem to show that :

$$\int\int_S \overline{\nabla}\left(x^2+y^2+z^2\right)\cdot d\overline{s} = 6V$$

where S is any closed surface enclosing volume V.

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