## DS-GA 3001.001 Special Topics in Data Science: Modeling Time Series Homework 1

Due date: February 12, by 5 pm

**Problem 1.** Consider the periodic time series (period determined by parameter  $\omega$ ) constructed as:

$$x_t = U_1 \sin(2\pi\omega t) + U_2 \cos(2\pi\omega t), \qquad (1)$$

with  $U_{1,2}$  independent random variables with zero mean and variance  $\sigma^2$ . Show that this series is weakly stationary with autocovariance function  $\gamma(h) = \sigma^2 \cos(2\pi\omega t)$ . Optional: Is this series strictly stationary?

**Problem 3.** For an MA(1),  $x_t = w_t + \theta w_{t-1}$  show that the autocorrelation function  $|\rho_x(1)| \le 0.5$ , for all  $\theta$ . For which values  $\theta$  is it maximum and minimum?

**Problem 3.** Identify the following models as ARMA(p, q):

- $x_t = 0.8x_{t-1} 0.15x_{t-2} + w_t 0.3w_{t-1}$
- $x_t = x_{t-1} 0.5x_{t-2} + w_t w_{t-1}$

Note: watch out for parameter redundancy!

**Problem 4.** Given an AR(1) model  $x_t = \phi x_{t-1} + w_t$ , determine the form of m-step ahead prediction  $x_{t+m}^t = \mathbb{E}[x_{t+m}|x_t]$  and show that the variance of this estimate has the form:

$$\mathbb{E}[\left(x_{t+m}^{t} - x_{t+m}\right)^{2}] = \sigma_{w}^{2} \frac{1 - \phi^{2m}}{1 - \phi^{2}}$$
(2)

**Problem 5.** Consider the datasets recording weekly *oil* (dollars/barrel) and *gas* (cents/gallon) provided in the data folder.

- plot data on same graph; which of the simulated series in lecture 1 do these series resemble? Do you believe they are stationary? (explain why/ why not)
- argue that the transformation  $y_t = \nabla x_t$  might be useful for the oil data. Plot  $y_t$ , estimate sample ACF and comment on result.
- construct scatterplot of  $y_t$  vs.  $y_{t+h}$  for the two datasets,  $h = \{1, 2, 3\}$ . Are these dependencies linear?