CS-663 Assignment 3 Question 5

Given: $f(x,y)$ is real (for property 1)
$f(x,y)$ is real & even (for property 2)

To prove: $F^*(u,v) = F(-u,-v)$ — property 1
$F(u,v)$ is real & even — property 2

Solution: DFT for a 2D function $f(x,y)$ is given by

$$F(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y) \cdot e^{-i2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)}$$

for property 1:

Since $f(x,y)$ is real, $f(x,y) = f^*(x,y)$

$$F^*(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y) \cdot e^{i2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)}$$

replacing $u \rightarrow -u$, $v \rightarrow -v$ in DFT equation.

$$F(-u,-v) = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y) \cdot e^{-i2\pi\left(\frac{-ux}{N}-\frac{vy}{M}\right)}$$

$$F(-u,-v) = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y) \cdot e^{i2\pi\left(\frac{ux}{N}+\frac{vy}{M}\right)}$$

$$= F^*(u,v)$$

$\boxed{\text{Hence proved.}}$

for property 2:

If $f(x,y)$ is real & even, $f(x,y) = f(-x,-y)$

$$F(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y) \cdot e^{-i2\pi\left(\frac{ux}{N} + \frac{vy}{M}\right)}$$

replace $x \to -x$ $y \to -y$ then:

$$F(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(-x,-y) \cdot e^{-i2\pi\left(-\frac{ux}{N} - \frac{vy}{M}\right)}$$

$$= \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y) \cdot e^{-i2\pi\left(\frac{(-u).x}{N} + \frac{(-v).y}{M}\right)}$$

$$= F(-u,-v)$$

$\boxed{\text{Hence proved}}$.