(S-663 Assignment 3 Question 5

Given:
$$f(n,y)$$
 is real (for property 1)
 $f(n,y)$ is real ξ even (for property 2)
To prove: $f^*(u,v) = F(-u,-v) - \text{property 1}$
 $F(u,v)$ is real ξ even $-\text{property 2}$

Solution: DFT for a 2D function
$$f(x,y)$$
 is given by $N-1$ $M-1$ $M-1$ $F(u,v) = \sum_{n=0}^{\infty} f(n,y) \cdot e^{-i2\pi(un+vy)}$ $f(x,y) \cdot e^{-i2\pi(un+vy)}$ for property 1:

Since
$$f(n,y)$$
 is sheal, $f(x,y) = f^*(n,y)$
 $f^*(u,v) = \sum_{x=0}^{N-1} f(x,y) \cdot e^{i2\pi} \left(\frac{ux}{N} + \frac{vy}{M}\right)$
 $f^*(u,v) = \sum_{x=0}^{N-1} f(x,y) \cdot e^{i2\pi} \left(\frac{ux}{N} + \frac{vy}{M}\right)$

suplacing $u \rightarrow -a$ in DFT equation.

$$F(-u,-v) = \sum_{i=1}^{N-1} \frac{M-1}{N} \frac{-i2\pi}{N} \left(\frac{-u\pi}{N} - \frac{vy}{N} \right)$$

$$F(-u,-v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y) \cdot e^{i2\pi} \left(\frac{-ux}{N} - \frac{vy}{N} \right)$$

$$F(-u,-v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x,y) \cdot e^{i2\pi} \left(\frac{ux}{N} + \frac{vy}{M} \right)$$

Hence pewued.

for property 2:

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$$f(x,y)$$
 is real $f(x,y) = f(-x,-y)$
 $f(u,v) = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} f(x,y) \cdot e^{-i2\pi} \left(\frac{ux}{N} + \frac{vy}{M}\right)$

9replace $x \to -x$ $y \to -y$ then:

 $f(u,v) = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} -f(-x,-y) \cdot e^{-i2\pi} \left(\frac{-ux}{N} - \frac{vy}{M}\right)$
 $= \sum_{x=0}^{\infty} y=0$
 $= \sum_{x=0}^{\infty} f(x,y) \cdot e^{-i2\pi} \left(\frac{-ux}{N} + \frac{-v}{M}\right) \cdot e^{-i2\pi}$
 $= \sum_{x=0}^{\infty} f(x,y) \cdot e^{-i2\pi} \left(\frac{-u}{N} + \frac{-v}{M}\right) \cdot e^{-i2\pi}$
 $= \sum_{x=0}^{\infty} f(x,y) \cdot e^{-i2\pi} \left(\frac{-u}{N} + \frac{-v}{M}\right) \cdot e^{-i2\pi}$
 $= \sum_{x=0}^{\infty} f(x,y) \cdot e^{-i2\pi} \left(\frac{-u}{N} + \frac{-v}{M}\right) \cdot e^{-i2\pi}$
 $= \sum_{x=0}^{\infty} f(x,y) \cdot e^{-i2\pi} \left(\frac{-u}{N} + \frac{-v}{M}\right) \cdot e^{-i2\pi}$

Hence peroned.