- 1D convolution mask : $m = (w_0, w_1, \ldots w_6)$

- image vector f : $[f_0, f_1, f_2 \ldots f_{n-1}]$

output of the convolution of m with f →

$$m * f = [w_0 f_0,\ w_0 f_1 + w_1 f_0,\ w_0 f_2 + w_1 f_1 + w_2 f_0 \ldots w_6 f_{n-1}]$$

We have to represent the vector m as a matrix

Now, f is a $n \times 1$ dimensional matrix

$$\underbrace{\begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}}_{M = ?} \times \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_{n-1} \end{bmatrix} = \begin{bmatrix} w_0 \\ w_0 f_1 + w_1 f_0 \\ w_0 f_2 + w_1 f_1 + w_2 f_0 \\ \vdots \\ w_6 f_{n-1} \end{bmatrix}$$

$\therefore$ the matrix M is :

$$
\begin{bmatrix}
w_0 & 0 & 0 & 0 & \cdots & & & & 0 \\
w_1 & w_0 & 0 & 0 & \cdots & & & & 0 \\
w_2 & w_1 & w_0 & 0 & \cdots & & & & 0 \\
\vdots & & & & & & & & \\
w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & \cdots & 0 \\
0 & w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 \cdots & 0 \\
& & & \vdots & & & & & \\
0 & 0 & \cdots & 0 & w_6 & w_5 & w_4 & w_3 & w_2 \; w_1 \; w_0 \\
& & & & \vdots & & & & \\
0 & 0 & \cdots & & & & 0 & 0 \; 0 & w_6
\end{bmatrix}
$$

$\uparrow$

Toeplitz matrix $\therefore$ each descending diagonal is constant.

Applications :

→ Toeplitz matrices can model systems that have shift invariant properties

→ Applications also include the Discrete Fourier transform.
This is related to the Fourier series as the multiplication operator by a trignometric polynomial, compressed to a finite dimensional space can be represented by such a matrix.

→ They commute asymtotically. ∴ They diagonalize in the same basis when the row and column dimension → ∞

→ ∵ It is in matrix form with most elements 0, it is fast and effecient for larger masks