

$$\psi(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} x^i y^j$$

$$\begin{aligned} \therefore \psi(x, y) = & a_{00} + a_{01}y + a_{02}y^2 + a_{03}y^3 \\ & + a_{10}x + a_{11}xy + a_{12}xy^2 + a_{13}xy^3 \\ & + a_{20}x^2 + a_{21}x^2y + a_{22}x^2y^2 + a_{23}x^2y^3 \\ & + a_{30}x^3 + a_{31}x^3y + a_{32}x^3y^2 + a_{33}x^3y^3 \end{aligned}$$

$$\therefore \psi(x, y) = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{bmatrix} a_{00} & a_{01} & \dots & a_{03} \\ \vdots & \vdots & \vdots & \vdots \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ y \\ y^2 \\ y^3 \end{bmatrix}$$

(Matrix representation)

$$\begin{array}{c} \therefore \\ \psi(x, y) = \end{array} \begin{array}{c} \uparrow \\ \text{I} \end{array} \begin{bmatrix} 1 & y & y^2 & y^3 & x & xy & \dots & x^3y & x^3y^2 & x^3y^3 \end{bmatrix} \begin{array}{c} \uparrow \\ \text{A} \end{array} \begin{bmatrix} a_{00} \\ a_{01} \\ \vdots \\ a_{33} \end{bmatrix} \begin{array}{c} \uparrow \\ \text{B} \end{array}$$

∴ for each pixel in the neighbourhood

(16 neighbours) we get 1 equation each.

∴ 16 equations :

$$I_1 = A_1 B$$

$$I_2 = A_2 B$$

⋮

$$I_{16} = A_{16} B$$

$$\begin{matrix} \therefore \\ \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{16} \end{bmatrix} \end{matrix} = \begin{matrix} \begin{bmatrix} 1 & y_1 & y_1^2 & \dots & x_1^3 y_1^3 \\ 1 & y_2 & y_2^2 & \dots & x_2^3 y_2^3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & y_{16} & y_{16}^2 & \dots & x_{16}^3 y_{16}^3 \end{bmatrix} \end{matrix} \begin{matrix} \begin{bmatrix} a_{00} \\ a_{01} \\ \vdots \\ a_{33} \end{bmatrix} \end{matrix}$$

\uparrow
M

\uparrow
N

\uparrow
P

$$M = NP$$

$$P = N^{-1} M$$

We require 16 neighbours because there are 16 unknowns
∴ We use 16 equations