Question 4 HW 2

Task : To prove / disprove

(a) Laplacian mask with -4 in center is separable.

A separable filter can be expressed as outer product of 2 1D filters

$F(x, y)$ is separable if $F(x,y) = g(x) * h(y)$

Laplacian mask :
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

We need to find 2 1D filters $g(x)$ & $h(y)$ that produce this.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} d & e & f \end{bmatrix}$$

$ad = 0 \quad ae = 1 \quad af = 0 \Rightarrow d = 0, f = 0, a = e = 1$

$bd = 1 \quad be = -4 \quad bf = 1 \Rightarrow \quad ?$

$cd = 0 \quad ce = 1 \quad cf = 0 \Rightarrow d = 0, f = 0, c = e = 1$

We see that there is no possible 2 1D filters that can get this

$\therefore$ This is disproved as there are no 1D filters which when convolved in both directions produce -4 in center.

(b) To implement a 2D Laplacian filter using only 1D convolutions, ~~the filter must be separable~~. ~~Since the Laplacian mask is not separable this statement is also disproved~~.

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) - 2f(x) + f(x-1)$$

We first apply double derivative convolution in x direction and then in y direction

I_x = convolve(I, L_x) ; I_y = convolve(I, L_y)

$$L\_x = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \quad L\_y = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$

We convolve I with L_x & L_y and add I_x and I_y. This gives us the same answer as applying Laplacian mask

$\therefore$ This is proved.