Question 5 HW 2

Task: To express f_k which is the image obtained after successive convolution of f with mean filter of size $(2a+1)\times(2a+1)$ k times.

In a mean filter all the coefficients are equal & sum to 1.
This gives us the central pixel value to be replaced by arithmetic mean of all neighbouring pixels.

$$g(x,y) = \frac{1}{(2a+1)^2} \sum_{i=-a}^{a} \sum_{j=-a}^{a} f(x+j, y+i)$$

$f_1 = f * G$

$f_2 = f_1 * G = (f * G) * G$

Since convolution is associative we can also say $f_2 = f * (G * G)$

$\vdots$

$f_k = f_{k-1} * G$

$= ((f * G) * G) * G \dots * G) * G$

$= f * \underbrace{(G * G * G \dots * G)}_{k \text{ times.}}$

We repeatedly convolve the mean filter with itself K times

$f_k = f * (G^k)$

On multiple passes of ~~gaussian~~ mean kernel, we observe due to the central limit theorem that the resultant ~~gauss~~ kernel is a Gaussian kernel. Hence we have G^k as a Gaussian kernel

(Central Limit Theorem: The central limit theorem states if you have a population with mean μ, and take sufficiently large samples from the population with replacement, then the distribution of sample of means will be approx. normal distribution).

~~Since k is a large number & randomness~~