CS-663 Assignment 3 Question 4

Given a 2D discrete signal $f(x,y)$ of size W_1 by W_2, its DFT is given as:

$$F_d(u,v) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y) \exp(-j 2\pi vy/W_2) \cdot \exp(-j 2\pi ux/W_1)$$

(we compute row wise FFT followed by column wise FFT).

We create an image with intensity 0 everywhere except the central row.

Analytical fourier transform ↓↓↓

P.T.O

Analytically taking fourier transform of this image:

$$F_d(u,v) = \frac{1}{\sqrt{201 \times 201}} \sum_{0}^{200} \sum_{0}^{200} f(x,y) \cdot \exp\left(-j2\pi vy/201\right) \cdot \exp\left(-j2\pi ux/201\right)$$

First we evaluate the column wise summation to get:

$$F_d(u,v) = \frac{255}{201} \sum_{x=0}^{200} \sum_{y=0}^{200} \delta(y-101) \exp\left(-j2\pi vy/201\right) \cdot \exp\left(-j\frac{2\pi ux}{201}\right)$$

$$= \frac{255}{201} \sum_{x=0}^{200} \exp\left(-j\frac{2\pi \times 101 \cdot v}{201}\right) \exp\left(-j\frac{2\pi ux}{201}\right)$$

Now row wise summation gives us:

$$F_d(u,v) = \frac{255}{201} \exp\left(-j \cdot \frac{202\pi}{201} v\right) \sum_{x=0}^{200} \exp\left(-j\frac{2\pi ux}{201}\right)$$

using formula for geometric progression:

$$F_d(u,v) = \frac{255}{201} e^{\frac{-j 202\pi u}{201}} \cdot \left(\frac{\left(e^{\frac{-j 2\pi u}{201}}\right)^{201} - 1}{e^{\frac{-j 2\pi u}{201}} - 1} \right)$$

$$= \frac{255}{201} e^{\frac{-j 202\pi v}{201}} \left(\frac{1 - e^{-j 2\pi u}}{1 - e^{\frac{-j 2\pi u}{201}}} \right)$$

Proof:

$e^{-j 2\pi u} = 1$ for any integer u

and $e^{-j 2\pi u/201} = 1$ for u as integral multiple of 201

$= 0$ everywhere except at $u = 0$ for $0 \le u \le 200$

$= \delta(u)$

$= \delta_{u0}$

Alternate proof:

$$F^{-1}(F(f(x))) = f(x)$$

and if $f(x) = 1$ then $F^{-1}(F(f(x))) = 1$

but $F(1) = \delta_{n0}$

$\therefore F^{-1}(\delta_{n0}) = 1$

$\Rightarrow \delta_{u0} = F(1)$

but $F(1) = \sum_{0}^{200} 1 \cdot e^{\frac{-j2\pi nu}{201}}$

$= \sum_{0}^{200} e^{\frac{-j2\pi nu}{201}}$

$\therefore \sum_{0}^{200} e^{\frac{-j2\pi nu}{201}} = \delta_{u0}$

$\therefore \boxed{F_a(u,v) = \frac{255}{201} e^{\frac{-j2\pi \cdot 101 v}{201}} \delta_{u0}}$

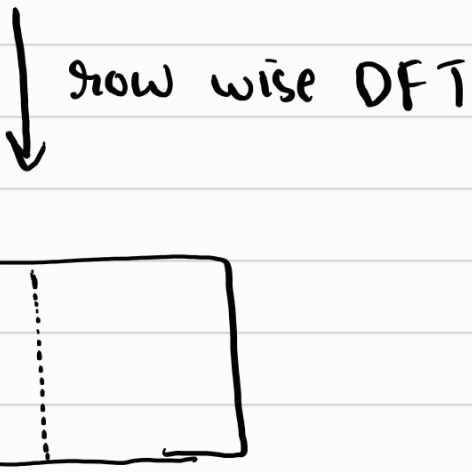
$e^{\frac{-j2\pi \cdot 101 v}{201}}$ → Sinusoid in vertical direction

δ_{u0} → delta in u direction

$\therefore$ A horizontal thin line gets converted to a dotted vertical line

Explanation : (switch white and black for handdrawn image)

Column wise DFT →



Images from matlab :

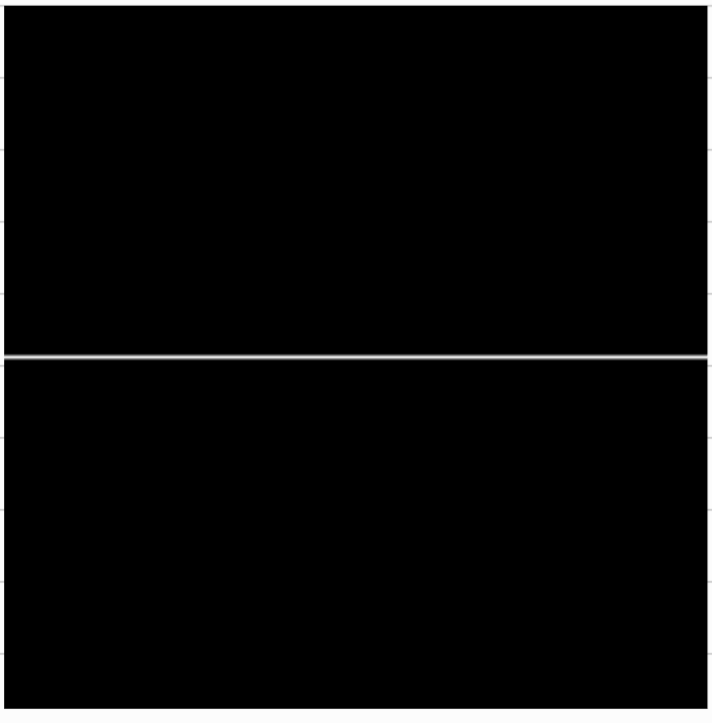
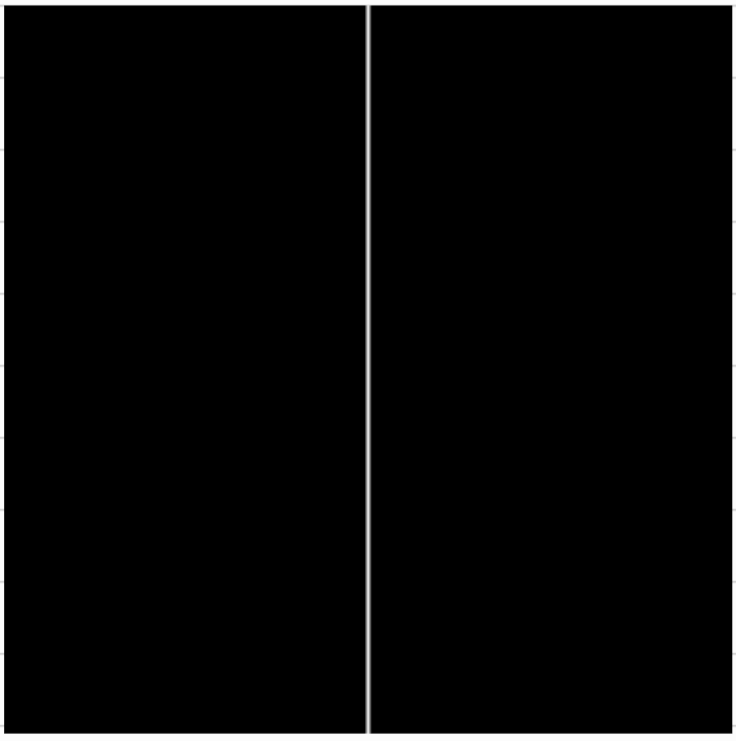
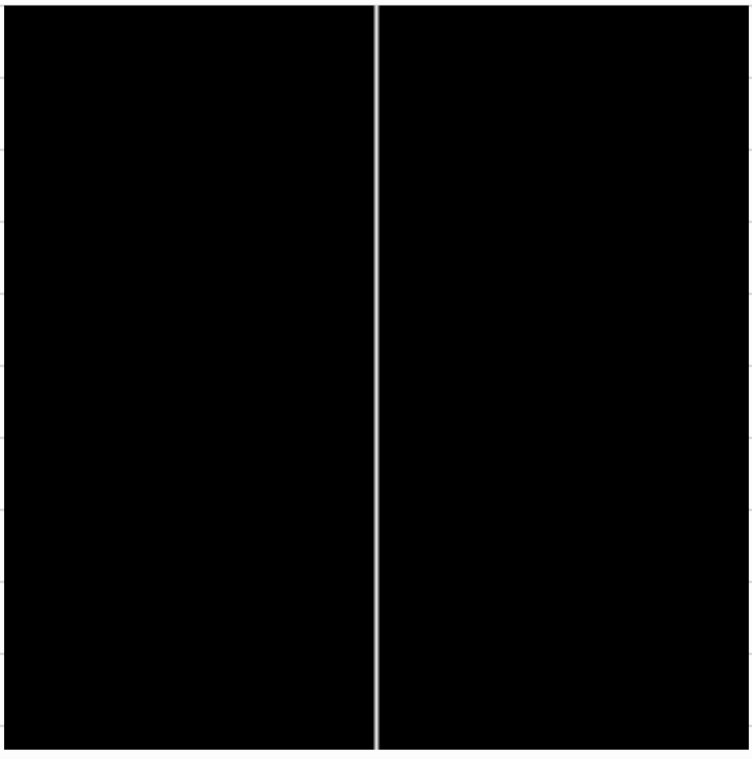


Image (original)



Fourier transform of image (real part)

Fourier transform (complex)



log scaled fourier transform

i.e. log (abs(FT)+1)

Matlab code :

```
z = zeros(201,201) ;
z(101 , : ) = 255 ;

figure(1) ; imshow(z);
FA = fftshift(fft2(z));
figure(2) ; imshow(FA);
figure(3) ; imshow(log(FA+1));
```