CS663 Assigment 1 Question 3

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The probability mass function of a discrete random variable X assigns probabilities to possible values of a random variable. In the context of image processing,a probability mass function (PMF) refers to the probability distribution that describes the likelihood of occurrence of different pixel intensity values in a digital image. Mathematically, the PMF of an image is often denoted as $P(r)$, where 'r' represents the intensity value. The PMF is defined as the ratio of the number of pixels with a particular intensity value to the total number of pixels in the image. It can be calculated using the following formula:

$$P(r) = N(r)/N_{total}$$

Given that the PMF functions of images I and J are $p_I(i)$ and $p_J(j)$, we further deduce that the image I+J refers to element-wise addition of the corresponding pixel values from the two images. In other words, each pixel in the resulting image contains the sum of the corresponding pixels from images I and J. Suppose X and Y are two independent discrete random variables with distribution functions $m_1(x)$ and $m_2(x)$, and $Z = X + Y$, We would like to determine the distribution function $m_3(x)$ of Z.To do this, it is enough to determine the probability that Z takes on the value z, where z is an arbitrary integer. Suppose that $X = k$, where k is some integer. Then $Z = z$ if and only if $Y = z - k$. So the event $Z = z$ is the union of the pairwise disjoint events:

$$P(Z = z) = \sum_{1}^{\infty} P(X = k).P(Y = z - k)$$

This is also referred to as convolutions. The probability mass function (PMF) of the sum of two random variables is given by the convolution of their individual PMFs.

$$p_K(k) = \sum [p_I(i) * p_J(k - i)]$$

, where the sum is taken over all possible values of i that lead to k. Here, k represents the possible values of the sum I + J. For each k, you need to consider all the possible pairs of values (i, j) that satisfy i + j = k. Then, you multiply the probabilities of $p_I(i)$ and $p_J(j)$ and sum them up for all these pairs. This is similar to the convolution topic being covered in class.