Question 3 HW 2

Clean image = $I(x,y)$

Noise = $z(x,y) = \dfrac{e^{-(x^2+y^2)/2\sigma^2}}{\sigma\sqrt{2\pi}}$

Noisy image $N(x,y) = I(x,y) + \dfrac{e^{-(x^2+y^2)/2\sigma^2}}{\sigma\sqrt{2\pi}}$

(since the noise is additive)

We can assume image $I(x,y)$ has a constant variance α^2

$Var(I(x,y)) = \alpha^2$

$\therefore Var(N(x,y)) = \alpha^2 + \sigma^2$

$Mean(N(x,y)) = Mean(I(x,y)) + 0$

PDF of a Gaussian noise is given by $p(z) = \dfrac{1}{\sqrt{2\pi}\,\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$

$PDF(N(x,y)) = PDF(I(x,y)) * PDF(z(x,y))$ (convolution).

$\because$ The ~~ ~~ general formula for the sum of $Z = X + Y$ of 2 independent integer random variables is

$$P(Z=z) = \sum_{k=-\infty}^{\infty} P(X=k)\,P(Y=z-k)$$

~~we know 2x rep~~

$$\cancel{P_N(N) = \int_{-\infty}^{\infty} P_I(i) \cdot P_z(k-i)\,di}$$

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$P_N(N)$ ↓ PDF of $N(x,y)$; $P_I(i)$ ↓ PDF of $I(x,y)$; $P_z(k-i)$ ↓ PDF of $z(x,y)$

$$P_N(N) = \int_{-\infty}^{\infty} P_I(i) \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(k-i)^2}{2\sigma^2}}\,di$$

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