CS-663 Assignment 3 Q3

Convolution theorem for 2D discrete fourier transforms :

$$\mathcal{F}(f * h)(u,v) = F(u,v)\,H(u,v)$$

if f is of size $W_1 \times W_2$ and h of size $K_1 \times K_2$:

→ Symmetrically zero-pad f and h so they acquire size $(W_1 + K_1 - 1) \times (W_2 + K_2 - 1)$

in general, 2D DFT $= \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}$

for images:

$$\rightarrow F(u,v) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{W_1} + \frac{vy}{W_2}\right)}$$

(2D Discrete fourier transform)

$$\rightarrow f(x,y) * h(x,y) = \sum_{j=0}^{W_2-1} \sum_{i=0}^{W_1-1} h(i,j)\, f\left(x - \frac{i}{W_1},\, y - \frac{j}{W_2}\right)$$

(2D convolution)

$$\therefore g(x,y) = \sum_{k=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} f(x-i,\, y-k)\, h(i,k)$$

$f(x,y) * h(x,y)$

Take fourier transform on both sides

$$G(u,v) = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} f(x-i, y-k)\, h(i,k)\, e^{-j2\pi(xu+yv)}$$

$$= \sum_{x,y} \sum_{i,k} f(x-i, y-k)\, e^{-j2\pi(xu+yv)}$$

$$= \sum_{x,y} \sum_{i,k} f(x-i, y-k)\, e^{-j2\pi((x-i)u+(y-k)v)}\, h(i,k)\, e^{-j2\pi(iu+kv)}$$

$$= \sum_{x,y} f(x-i, y-k)\, e^{-j2\pi((x-i)u+(y-k)v)} \sum_{i,k} h(i,k)\, e^{-j2\pi(iu+kv)}$$

Let $x-i = x'$

$y-k = y'$

$$= \sum_{x'y'} f(x', y')\, e^{-j2\pi(x'u+y'v)} \sum_{i,k} h(i,k)\, e^{-j2\pi(iu+kv)}$$

$$= FT(f(x', y')) \cdot FT(h(i,k))$$

$$= F(u,v) \cdot H(u,v)$$

$$\therefore\ G(u,v) = F(u,v) \cdot H(u,v)$$

Hence Proved