CS-663 Assignment 3 Question 6

To prove : $\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = f(t)$
where $\mathcal{F}$ is a continuous fourier operator

Proof :

$$\mathcal{F}(f(t)) = F(u) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi ut} dt$$

and $$\mathcal{F}^{-1}(F(u)) = f(t) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ut} du \quad -(i)$$

also, considering $f(\cdot)$ and $F(\cdot)$ as functions of some arguments,

$$\mathcal{F}(F(u))(\omega) = \int_{-\infty}^{\infty} F(u) e^{-j2\pi \omega u} du$$

replace ω with t on both sides

$$\mathcal{F}(F(u))(t) = \int_{-\infty}^{\infty} F(u) e^{-j2\pi ut} du$$

$$= f(-t) \quad \text{(from eq(i))}$$

$$\therefore \boxed{\mathcal{F}(\mathcal{F}(f(t))) = f(-t)} \quad -(ii)$$

$$\therefore \quad \mathcal{F}(\mathcal{F}(\mathcal{F}(f(t)))) = \mathcal{F}(f(-t))$$

$$\therefore \quad \mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = \mathcal{F}(\mathcal{F}(f(-t)))$$

$$= f(-(-t))$$

(from eq (ii))

$$= f(t)$$

Hence proved.