Dominating sets using Boolean Algebra

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***Abstract*— Graphs are important tools for problem solving and have found applications in several disciplines such as traffic control, ecology, biology and computers storage allocation problems etc. A vertex v in a graph G is said to dominate both itself and its neighbours, that is v dominates every vertex in its closed neighbourhood N[v]. Therefore, v dominates degGv+1 vertices of G. In this paper we discuss dominating set of interval graphs using boolean algebra.**

I. INTRODUCTION

A Graph [1] ***G*** is defined as an ordered pair of a finite set [2] ***V*** of vertices and a finite set ***E*** of edges, where a vertex is a node and an edge is a connection between any two vertices [3]. A graph can be represented using adjacency matrix and incidence matrix.

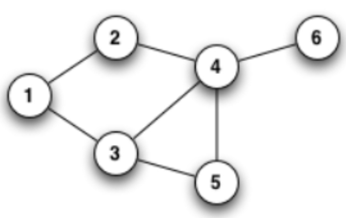

Fig. 1. A simple undirected graph

In graph theory, a dominating set for a graph G = (V, E) is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number is the number of vertices in a smallest dominating set for G.

A set S of vertices in G is called a dominating set [4] for G if every vertex of G dominated by some vertex in S. Alternatively, S is a dominating set of G if every vertex of G either belongs to S or is adjacent to some vertex in S. The domination number $Y(G)$ of a graph G is the minimum cardinality of a dominating set of G. A dominating set S is called an independent dominating set if no two vertices of S are adjacent. The independent domination number $Y_i(G)$ of G is the minimum cardinality taken over all independent dominating sets of G.

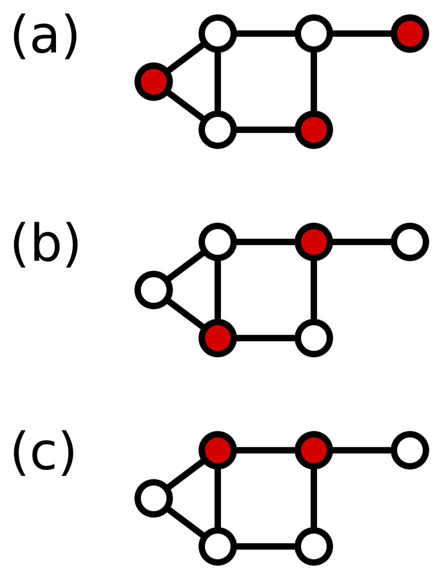

Fig. 2. Example

Figures (a)–(c) above show three examples of dominating sets for a graph. In each example, each white vertex is adjacent to at least one red vertex, and it is said that the white vertex is dominated by the red vertex. The domination number of this graph is 2: the examples (b) and (c) show that there is a dominating set with 2 vertices, and it can be checked that there is no dominating set with only 1 vertex for this graph.

A. *Minimal Dominating Set*

A dominating S set of a graph G is called a minimal dominating set if no proper subset of S is a dominating set. Every vertex of a graph is a minimal dominating set if and only if the graph is a complete graph. If a graph has an isolated vertex then any minimal dominating set of the graph contains all such graphs.

B. Interval Family and Interval Graph

Let I = I1, I2,. . . . . . .,In be an interval family where each I_i is an interval on the real line and I_i= [a_i, b_i] for i=1, 2,. . . .n. Where a_i, b_i are called the left and right end points of I_i respectively. Without loss of generality, we consider that all end points of the intervals in I are distinct numbers between 1 and 2n. Two intervals i and j are said to intersect each other if they have non empty intersection.

A graph G = (V, E) is an interval graph if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. That is if i = [a_i, b_i] and j = [a_j, b_j] then i and j intersect means either $a_j < b_i$ or $a_i < b_j$. For each interval i let nbd[i] denote the set of intervals that intersect i (including i). Let min(i)denote the smallest interval and max(i) the largest interval in nbd[i] . Let us now define the non intersecting interval NI(i) of the interval i as below: NI(i) = j, if $b_i < a_j$ and there do not exist an interval k such that $b_i < a_k < a_j$. If there is no such j, then define NI(i)=null.

II. METHODOLOGY

This paper deals with checking if dominating sets can be found out using boolean algebra [5] or not. But before looking for a solution using boolean algebra, let us see a general approach to solve the problem.

A. Method 1

- First we have to initialize a set 'S' as empty.
- Take any edge 'e' of the graph connecting the vertices (say A and B).
- Add one vertex between A and B (let say A) to our set S.
- Delete all the edges in the graph connected to A.
- Go back to step 2 and repeat, if some edge is still left in the graph.
- The final set S is a Dominant Set of the graph.

B. Method 2: Using boolean algebra

A method obtained for dominating sets in a graph will be developed by using an interval family. Let

$$I = I_1, I_2, \ldots\ldots, I_n$$

be an interval family and G is an interval graph of I. Now we will find the minimal dominating set for the following interval graph from an interval family I. Let

$$V = v_1, v_2, v_3, v_4, v_5, v_6, v_7$$

are the vertices of an interval graph G. To dominate a vertex I_i we must include I_i or any of the vertices adjacent to I_i. A minimum set satisfying this condition for every vertex I_i is a desired set. Therefore for every vertex I_i in G and let us form a Boolean product of sums

$$(x + y)x = x$$

and

$$xx = x$$

the interval

$$I = I_{i1}, I_{i2}, \ldots\ldots, I_{in}$$

where, $I_{i1}, I_{i2}, \ldots\ldots, I_{in}$ are the vertices adjacent to I_i and d is a degree of I_i.

$$S = \Pi\{I_{i1}, I_{i2}, \ldots\ldots, I_{in}\}$$

for all I_i in G. Here, S is expressed as a sum of products, in which each term will represent a minimal dominating set.

III. RESULTS

The objective of this paper was to prove whether or not the dominating set problem is solvable by boolean algebra or not. In this paper we have proposed the solution which proves that this problem of dominating sets can be solvable by the boolean algebra.

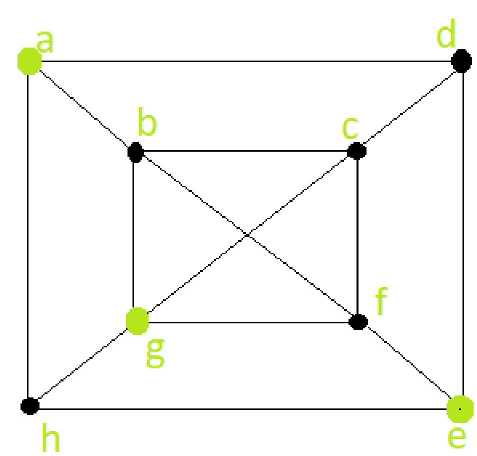

Fig. 3.

In the result graph of 8 vertices and 14 edges shown in Fig. 3, every black vertex is adjacent to at least one green vertex, which is then said that the black vertex is dominated by the green vertex. So, the dominant set of this graph is S = {a, g, e}.

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