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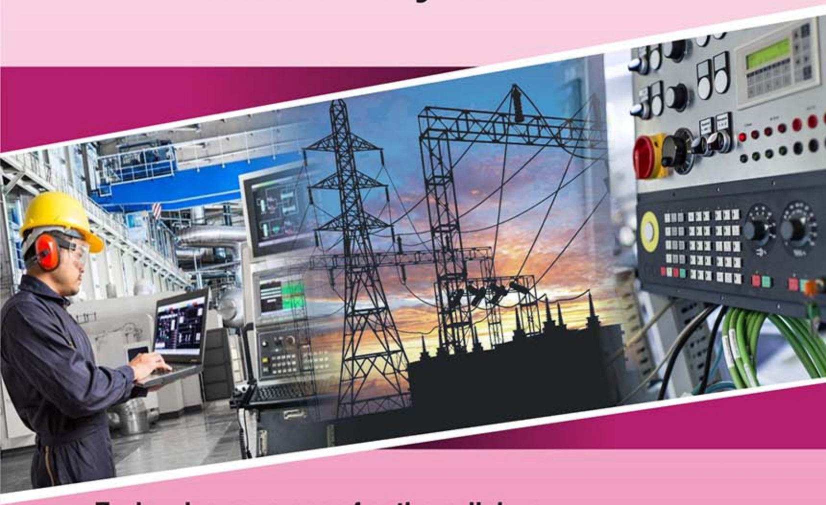

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CONTROL SYSTEM

By

Varun Gupta

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CONTROL SYSTEM

Pre-requisites of course: Basic signal systems

Course Outcome		**Knowledge Level, KL**
Upon the completion of the course, the student will be able to:		
CO 1	Obtain transfer functions to predict the correct operation of open loop and closed loop control systems and identify the basic elements, structures and the characteristics of feedback control systems.	**K3**
CO 2	Measure and evaluate the performance of basic control systems in time domain. Design specification for different control action.	**K4**
CO 3	Analyze the stability of linear time-invariant systems in time domain using Routh-Hurwitz criterion and root locus technique.	**K4**
CO 4	Determine the stability of linear time-invariant systems in frequency domain using Nyquist criterion and Bode plot.	**K4**
CO 5	Design different type of compensators to achieve the desired performance of control System by root locus and Bode plot method. Develop and analyze the intermediate states of the system using state space analysis.	**K5**

KL- Bloom's Knowledge Level (K1, K2, K3, K4, K5, K6)
K1 – Remember K2 – Understand K3 – Apply K4 – Analyze K5 – Evaluate K6 – Create

Detailed Syllabus:

Unit-I:
Control System Concepts: Elements of control systems, concept of open loop and closed loop systems, Examples and application of open loop and closed loop systems, Mathematical Modelling of Physical Systems (Electro Mechanical), Determination of transfer function by block diagram reduction techniques and signal flow method using Mason's gain formula, Basic Characteristics of negative feedback control systems.
Control System Components: Constructional and working concept of AC & DC servomotor, synchro's, stepper motor and tachometer.

Unit-II:
Time Response Analysis: Standard test signals, time response analysis of first and second order systems, time response specifications of second order system for unit step input, location of roots of characteristics equation and corresponding time response, steady state errors and error constants.
Basic modes of feedback control: Proportional, Derivative, Integral and PID controllers.

Unit-III:
Stability and Algebraic Criteria: Concept of stability and its necessary conditions, Routh-Hurwitz criteria and its limitations.
Root Locus Technique: Salient features of root locus plot, Procedure for plotting root locus, root contours.

Unit-IV:

Frequency Response Analysis: Frequency Response analysis from transfer function model, Construction of polar and inverse polar plots.

Stability in Frequency Domain: Nyquist stability criterion, Determination of gain and phase margin from Bode & Nyquist Plots, Correlation between time and Frequency Responses.

Unit-V

Introduction to Design: The design problems and preliminary considerations of lead, lag and lead-lag compensation networks, design of closed loop systems using compensation techniques in time and frequency domains.

State Space Technique: The concept of state & space, State-space model of physical system, conversion of state-space to transfer function model and vice-versa, State transition matrix, Concept of controllability and observability and their testing.

Text Book:

1. I. J. Nagrath & M. Gopal, “Control System Engineering”, 6th Ed. New Age International Publishers, 2018. 4th Edition
2. M. Gopla, “Control System Principles and Design”, McGraw Hill 4th Edition
3. Ogata, "Modern Control Engineering, 5th Edition", Pearson Education, 2015
4. B.C. Kuo & Farid Golnaraghi, “Automatic Control Systems”, 10th Edition, McGraw Hill
5. D. Roy Choudhary, “Modern Control Engineering”, Prentice Hall of India.
1. Salivahanan, "Control Systems Engineering", Pearson Education, 2015.
6. Ambikapathy, “Control Systems”, Khanna Publishers

Reference Books:

1. (Schaums Outlines Series) Joseph J. Distefano III, Allen R. Stubberud, Ivan J. Williams, “Control Systems”, 3rd Edition, McGraw Hill, Special Indian Edition, 2010.
2. Norman S. Mise, Control System Engineering, Wiley Publishing Co.
3. Ajit K Mandal, “Introduction to Control Engineering” New Age International.
4. R.T. Stefani, B.Shahian, C.J.Savant and G.H. Hostetter, “Design of Feedback Control Systems” Oxford University Press.
5. Samarjit Ghosh, “Control Systems theory and Applications”, Pearson Education.

1
UNIT

Control System Concepts

PART-1

Concept of Control System,
Physical Systems and their Mathematical Modeling.

CONCEPT OUTLINE : PART-1

- **Open loop control system :**
1. The control action is independent on the desired output.
2. In this system the output is not compared with the reference input.
- **Closed loop control system :** In a closed loop control system the control action is dependent on the output.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **Explain open loop and closed loop control system with the help of suitable examples.**

Answer

A. Open loop control system :

1. The open loop control system is also known as control system without feedback or non-feedback control system.
2. In open loop systems the control action is independent of the desired output.
3. In this system the output is not compared with the reference input.
4. The component of the open loop systems are controller and controlled process.

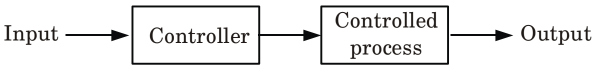

Fig. 1.1.1.

Examples :

1. Automatic washing machine
2. Immersion rod
3. A field control DC motor.

B. Closed loop control system :

1. Closed loop control system is also known as feedback control system.

2. In closed loop control system the control action is dependent on the desired output.
3. Any system having one or more feedback paths forms a closed loop system. In closed loop systems the output is compared with the reference input and error signal is produced.
4. The error signal is fed to the controller to reduce the error and desired output is obtained.

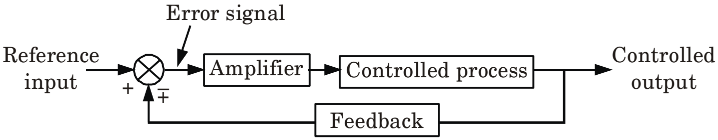

Fig. 1.1.2.

Examples :

1. Air conditioners
2. Autopilot aeroplane
3. Electric iron.

Que 1.2. **Compare the open loop control system and closed loop control system, also give few examples for each system.**

AKTU 2013-14, Marks 05

Answer

S. No.	Open loop	Closed loop
1.	Feedback is not present. So any change in output has no effect on the input.	Feedback is present. So changes in output effects input.
2.	It is not much accurate.	It is very accurate.
3.	It is very sensitive to errors and disturbances.	Less sensitive to errors and disturbances.
4.	It has small bandwidth.	It has large bandwidth.
5.	Simple in construction and is cheap.	Complicated in design and costly.
6.	Highly affected by non-linearity.	Less affected by non-linearity.
7.	**Examples :** Washing machine, traffic signal.	**Examples :** Electric iron, automatic gear.

Que 1.3. **Derive force-voltage analogy.**

Answer

Translation mechanical systems :

1. A translational spring-mass-damper system is shown in Fig. 1.3.1.

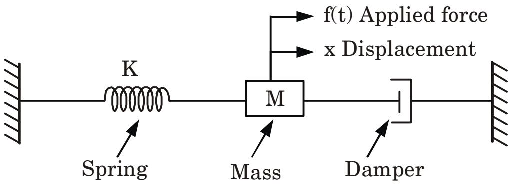

Fig. 1.3.1. Translational spring-mass-damper system.

2. Here, M = Mass of system

$f(t)$ = Applied force

D = Coefficient of damping

K = Spring deflection constant

3. The equation of motion for the system is obtained by applying D′ Alembert's principle,

$$M\frac{d^2x}{dt^2} = -D\frac{dx}{dt} - Kx + f(t) \quad ...(1.3.1)$$

4. Rearranging eq. (1.3.1), we get

$$M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx = f(t) \quad ...(1.3.2)$$

Force-voltage analogy :

1. The analogy of eq. (1.3.2) with the voltage equation of an electrical circuit can be established by considering electrical circuit shown in Fig. 1.3.2.

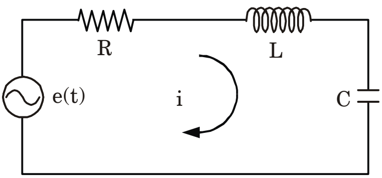

Fig. 1.3.2. Electrical circuit for force-voltage analogy.

2. The voltage equations for the circuit is as follows :

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = e(t) \quad ...(1.3.3)$$

3. As the current is the rate of flow of electric charge.

$$\therefore \quad i = \frac{dq}{dt} \quad ...(1.3.4)$$

4. Substituting eq. (1.3.4) in eq. (1.3.3),

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e(t) \quad ...(1.3.5)$$

5. Comparing eq. (1.3.5) and (1.3.2), we get

Table 1.3.1. Force-Voltage Analogy

Mechanical system	Electrical system
Force, f	Voltage, e
Velocity, v	Current, i
Displacement, x	Charge, q
Mass, M	Inductance, L
Damping coefficient, D	Resistance, R
Compliance, $1/K$ (stiffness, K)	Capacitance, C

Que 1.4. **Derive force-current analogy.**

Answer

Translational mechanical system : Refer Q. 1.3, Page 1–4C, Unit-1.

Equation of spring-mass-damper system

$$M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx = f(t) \quad ...(1.4.1)$$

Force-Current analogy :

1. The nodal equation as obtained for the circuit shown in Fig. 1.4.1 is analogous to spring-mass-damper system described by the eq. (1.4.1).

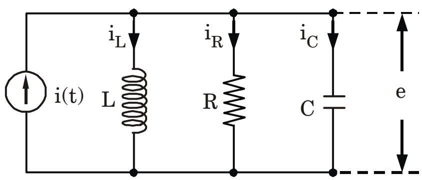

Fig. 1.4.1. Electrical circuit for force-current analogy.

2. According to nodal analysis

$$i_L + i_C + i_R = i(t) \quad ...(1.4.2)$$

3. The common voltage across the three parallel elements of the circuit is denoted as e. The following relations are obtained :

$$i_L = \frac{1}{L}\int e dt \quad ...(1.4.3)$$

$$i_R = \frac{1}{R}e \quad ...(1.4.4)$$

$$i_C = C\frac{de}{dt} \quad ...(1.4.5)$$

4. Substituting eq. (1.4.3), (1.4.4) and (1.4.5) in eq. (1.4.2),

$$\frac{1}{L}\int e dt + \frac{1}{R}e + C\frac{de}{dt} = i(t) \quad ...(1.4.6)$$

5. The voltage e is related to flux linkages associated with inductance L as follows :

$$e = \frac{d\phi}{dt} \quad ...(1.4.7)$$

6. Putting $e = \frac{d\phi}{dt}$ in eq. (1.4.6), we get

$$\frac{1}{L}\phi + \frac{1}{R}\frac{d\phi}{dt} + C\frac{d^2\phi}{dt^2} = i(t)$$

$$C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi = i(t) \quad ...(1.4.8)$$

7. Comparing eq. (1.4.1) and (1.4.8), we get

Table 1.4.1. Force-Current Analogy

Mechanical system	**Electrical system**
Force, f	Current, i
Velocity, v	Voltage, e
Displacement, x	Flux linkage, ϕ
Mass, M	Capacitance, C
Damping coefficient, D	Conductance, G
Stiffness, K (compliance, $1/K$)	Inductance, L

PART-2

Construction and Working of AC and DC Servomotor, Synchros, Stepper Motor and Tachometer.

CONCEPT OUTLINE : PART-2

- Servomotors are of two types :

1. AC servomotor
2. DC servomotor

- **Synchro :** A synchro is an electromagnetic transducer commonly used to convert angular position of a shaft into an electric signal.
- **Stepper motor :** The stepper motor is a special type of synchronous motor which is designed to rotate through a specific angle for each electrical pulse received from its control unit.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.5. Describe the AC servomotor for control application.

AKTU 2013-14, Marks 10

OR

Explain construction and working of AC servomotor.

AKTU 2014-15, Marks 10

Answer

Construction :

1. Fig. 1.5.1 shows the schematic diagram of a 2ϕ AC servomotor.
2. The stator has two distributed windings which are displaced from each other by 90 electrical degrees.
3. One winding is called the reference or fixed phase and other winding is called control phase.

Working principle :

1. Reference phase is supplied from a constant voltage source $V_r \angle 0°$. The other winding *i.e.*, control phase is supplied with a variable voltage of the same frequency as the reference phase but its phase is displaced by 90° (electrically).

2. The control phase is usually supplied from a servo amplifier.
3. The speed and torque of the rotor are controlled by the phase difference between the control voltage and the reference phase voltages.
4. The direction of rotation of the rotor can be reversed by reversing the phase difference from leading to lagging between the control phase voltage and the reference phase voltage.

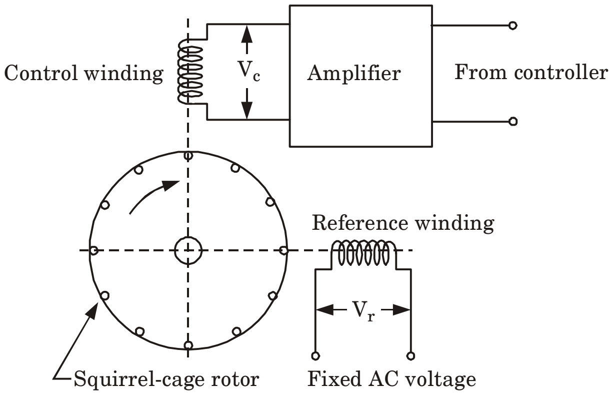

Fig. 1.5.1.

Applications :

1. In feedback control systems
2. In aircraft and spacecraft
3. In tracking and guidance systems
4. In robotics
5. In radar and machine tools
6. In process controller
7. In radio controlled cars and airplanes.

Que 1.6. **Write a short note on armature-controlled DC servo motor.**

Answer

1. Consider the armature controlled DC motor (DC servo motor) and assume that the demagnetizing effect of armature reaction is neglected, field voltage is constant and magnetic circuit is linear. Armature controlled DC servo motor is shown in Fig. 1.6.1.

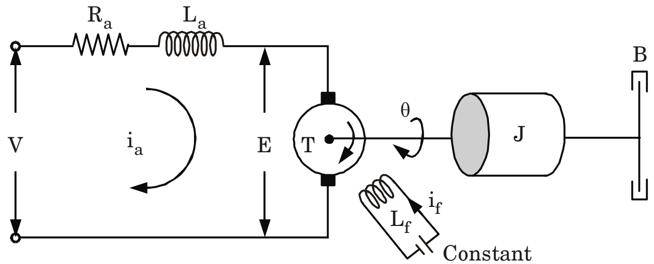

Fig. 1.6.1.

2. Let R_a = Armature resistance

L_a = Armature inductance

i_a = Armature current

E = Induced emf in armature

V = Applied armature voltage

θ = Angular displacement of the motor shaft

T = Torque developed by motor

J = Equivalent moment of inertia of motor shaft and load referred to the motor

B = Equivalent viscous friction coefficient.

3. Apply KVL in armature circuit

$$V = \frac{L_a di_a}{dt} + R_a i_a + E \qquad ...(1.6.1)$$

4. In the armature-controlled DC motor, the field current is held constant. For a constant field current, the flux becomes constant, and the torque becomes directly proportional to the armature current so that

$$T \propto \phi\, i_a$$

$$T = Ki_a \qquad ...(1.6.2)$$

5. When armature is rotating, an emf is induced

$$E = \frac{K_b d\theta}{dt} \qquad ...(1.6.3)$$

6. The armature current produces the torque which is applied to the inertial mass and friction hence the force balance equation is

$$\frac{Jd^2\theta}{dt^2} + \frac{Bd\theta}{dt} = T = KI_a \qquad ...(1.6.4)$$

7. Taking the Laplace transform on both sides of eq. (1.6.4) and (1.6.1)

$$(sL_a + R_a)\, I_a(s) + E(s) = V(s) \qquad \text{[Initial condition is zero]}$$

$$(Js^2 + Bs)\, \theta(s) = T(s) = KI_a(s) \qquad ...(1.6.5)$$

8. The transfer function of this system is obtained as

$$\frac{\theta(s)}{V(s)} = \frac{K}{s[L_a J s^2 + (BL_a + JR_a)s + R_a B + KK_b]} \quad ...(1.6.6)$$

9. Block diagram is shown in Fig. 1.6.2.

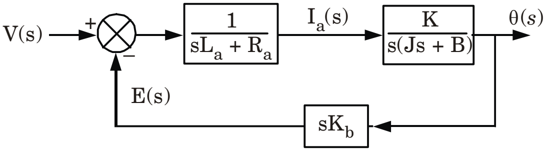

Fig. 1.6.2.

Que 1.7. **Write a short note on field controlled DC servomotor.**

Answer

1. A schematic diagram of a field controlled DC motor (DC servo motor) shown in Fig. 1.7.1.

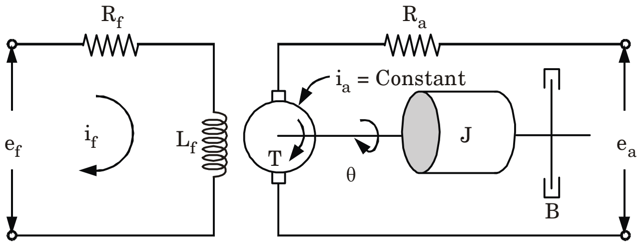

Fig. 1.7.1.

2. Here, R_f = Field winding resistance
L_f = Field winding inductance
I_f = Field winding current
R_a = Armature resistance
i_a = Armature current
θ = Angular displacement.

3. The torque T developed by the motor is proportional to product of the air-gap flux ϕ and armature current i_a so we get

$$T = K_1 \phi i_a \quad ...(1.7.1)$$

where K_1 is constant.

4. But the air gap flux ϕ and the field current i_f are proportional for the usual operating range of the motor and i_a is assumed to be constant, we can rewrite the above equation as

$$T = K_2\, i_f \qquad ...(1.7.2)$$

where K_2 is a constant.

5. The equations for this system are

$$L_f \frac{di_f}{dt} + R_f i_f = e_f \qquad ...(1.7.3)$$

and $$\frac{Jd^2\theta}{dt^2} + \frac{Bd\theta}{dt} = T = K_2 i_f \qquad ...(1.7.4)$$

6. By taking the Laplace transform on both sides of eq. (1.7.3) and (1.7.4) where all initial conditions are zero, we get

$$(L_f s + R_f)\, i_f(s) = E_f(s) \qquad ...(1.7.5)$$

$$(Js^2 + Bs)\, \theta(s) = K_2 i_f(s) \qquad ...(1.7.6)$$

7. From the above equations, the transfer function of this system is obtained as

$$\frac{\theta(s)}{E_f(s)} = \frac{K_2}{s(L_f s + R_f)(Js + B)} \qquad ...(1.7.7)$$

8. Block diagram is shown in Fig. 1.7.2.

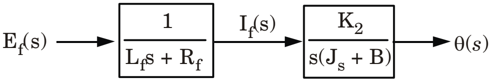

Fig. 1.7.2.

Que 1.8. Explain principle of synchros and how it works as an error detector.

Answer

A. Principle of synchros :

1. It is a rotary transducer that converts angular displacement into an AC voltage or an AC voltage into an angular displacement.
2. A synchros system consists of
   i. A control transmitter (CX) and
   ii. A control transformer (CT).
3. The control transmitter consists of a stator and a rotor. The rotor is a dumb-bell shaped magnetic structure. The supply is given to the rotor by means of slip rings, which are actually mounted on the stator housing.
4. The secondaries are in the skewed slot all along the periphery of the stator and are 120° apart because of their mechanical displacement.
5. The induced secondary voltage will depend upon the angle of the rotor shaft. For reference the zero degree position of the shaft is defined when the rotor is in alignment with the coil S_2.

6. In this position, the voltage in coil S_2 is maximum, and similarly the maximum voltage in coils S_1 and S_3 will result at 120° and 240° positions respectively. The voltage in S_2 is a function of θ and so is the voltage in S_1 and S_3. Thus,

$$E_{0s2} = A \cos \theta$$
$$E_{0s1} = A \cos (\theta - 120°)$$
$$E_{0s3} = A \cos (\theta - 240°)$$

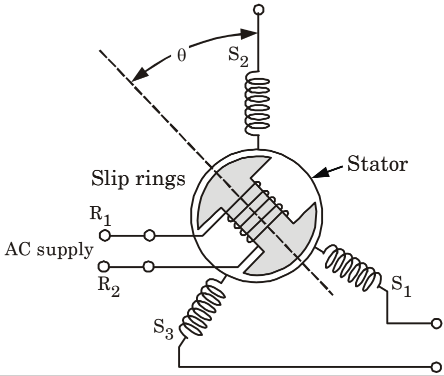

Fig. 1.8.1. Schematic diagram of synchro transmitter.

7. The connections of the synchro are made between the terminals and hence

$$\begin{aligned} E_{s1s2} &= E_{0s1} - E_{0s2} \\ &= A \cos (\theta - 120°) - A \cos \theta \\ &= A\left[-\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta - \cos\theta\right] \\ &= \sqrt{3}A\left[-\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right] \end{aligned}$$

8. Therefore,
$$E_{s2s1} = \sqrt{3}A\left[\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right]$$
$$= \sqrt{3}A\cos(\theta + 30°) \quad ...(1.8.1)$$

Similarly,
$$E_{s3s2} = \sqrt{3}A\cos(\theta + 150°) \quad ...(1.8.2)$$
$$E_{s1s3} = \sqrt{3}A\cos(\theta + 270°) \quad ...(1.8.3)$$

B. Synchros as an error detector :

1. This is the most commonly used error detector in AC system and is known by several names such as Selsyn, Telesyn Circutrol, Dichloyn, Teletorque and Autosyn, etc. It has high sensitivity and infinite resolution.
2. The schematic diagram of synchro-pair error detector is shown in Fig. 1.8.2.

3. The stator windings of the control transformer are connected electrically to the control transmitter and hence the magnetic field established in the control transformer depends upon the terminal voltages of the control transmitter, which are functions of the angular position of the transmitter rotor.
4. Since the voltage induced in the rotor of the control transformer depends upon the angle at which its turns are cut by the magnetic field of the stator, this induced voltage is determined both by the angular position of the transformer rotor and the angular direction of the stator magnetic field.
5. Numerically, the voltage induced in the transformer rotor is equal to the sine of the difference angle between θ_r and θ_L multiplied by the maximum voltage induced.
6. When the control transformer is connected as a component of the servo system, the correct zero position must be selected.
7. The voltages given by eq. (1.8.1), (1.8.2) and (1.8.3) are now impressed across the corresponding stator terminals of the control transformer.
8. When the rotor positions of the two synchros are in perfect alignment, the voltage generated across the terminals of the CT rotor windings is zero.
9. When the two rotor shafts are not in alignment, the rotor voltage of the CT is approximately a sine function of the difference between the two shaft angles.

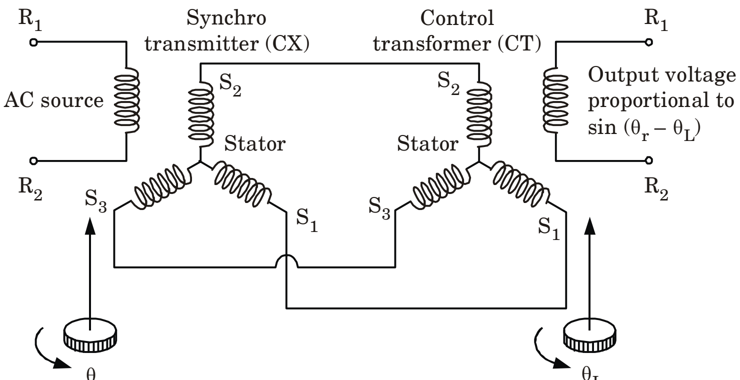

Fig. 1.8.2. Synchro error detector.

Que 1.9. Explain the working principle of stepper motor with neat diagram. Also give applications.

Answer

A. Stepper motor :

1. A stepper motor is a form of AC motor. The input given to this motor is in the form of electric pulses.
2. For every input pulse, the motor shaft turns through a specified number of degrees, called a step.
3. Shaft of stepper motors moves through one angular step for each input pulse. The range of step size may vary from 0.72° to 90°.

B. Principle of operation :

1. Stators of stepper motors have salient poles on which concentrated windings are wound. These windings may be appropriately connected so as to result in two, three or four phase windings, on the stator.
2. The rotors of stepper motors carries no winding. The rotors are made from either permanent-magnet or ferromagnetic material.
3. A stepper motor is fed through an external drive circuit. The function of this drive circuit is to receive input voltage pulses and to deliver appropriate currents to the stator windings of the stepper motor.
4. These currents set up air-gap field which moves through one angular step for each input pulse. The rotor follows the axis of air-gap magnetic field on account of the development of reluctance torque and/or permanent-magnet torque.
5. Stepper motors vary widely in their designs and configurations, these can be classified into three main types; variables-reluctance motors, permanent-magnet motors and hybrid motors.

C. Operation of variable reluctance stepper motor :

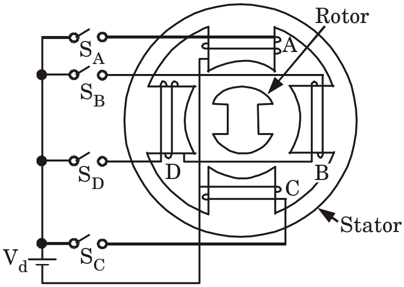

Fig. 1.9.1. 4ϕ, 4/2 pole variable reluctance stepper motor.

1. It is a 4ϕ, 4/2-pole (4 poles in stator and 2 in rotor), single-stack, variable reluctance stepper motor. Four phases A, B, C, and D are connected to DC source and are energized in the sequence A, B, C, D, A.
2. When winding A is excited, the rotor aligns with axis of phase A. The rotor is stable in this position and cannot move until phase A is de-energized.

3. Next phase B is excited, A is disconnected. The rotor moves through 90° in clockwise direction to align with the resultant air-gap field which now lies along the axis of phase B.
4. Further, phase C is excited and B is disconnected, the rotor moves further a step of 90° in the clockwise direction. Thus, as the phases are excited in the sequence A, B, C, D, A the rotor moves through a step of 90° at each transition in clockwise direction.
5. The rotor completes one revolution in four steps. The direction of rotation can be reversed by reversing the sequence of switching the windings, *i.e.*, A, D, C, B, A.
6. The magnitude of step angle for any variables reluctance or permanent magnet stepper motor is given by :

$$\beta = \frac{360°}{MN_r}$$

where, β = Step angle

M = Number of stator phases or stacks

N_r = Number of rotor teeth or rotor poles.

7. The step angle is also expressed as,

$$\beta = \frac{N_s - N_r}{N_s N_r} \times 360°$$

where, N_s = Number of stator teeth or stator poles.

By choosing different combinations of number of rotor teeth or stator exciting coils, any desired step angle can be obtained.

D. Applications :

1. Paper feed motors in typewriters and printer
2. Positioning of print heads
3. Pen in XY-plotters
4. Recording heads in computer disk drives.

Que 1.10. Write a short note on tachometer.

Answer

Tachometer :

1. Tachometer is electromechanical device that convert mechanical energy into electrical energy.
2. The device works as a generator with the output voltage proportional to the magnitude of the angular velocity.

Types of tachometer :

A. DC tachometer :

1. The most common type of DC tachometer contains an iron-core rotor.

2. The magnetic field is provided by a permanent magnet, and no external supply voltage is necessary.
3. The windings on the rotor (armature) are connected to the commutator segments and the output voltage is taken across a pair of brushes that ride on the commutator segments.
4. The DC tachometers provide visual speed readout of a rotating shaft. Such tachometers are directly connected to a voltmeter which is calibrated in rpm.

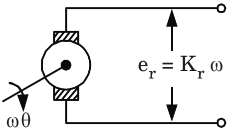

Fig. 1.10.1.

B. AC tachometer :

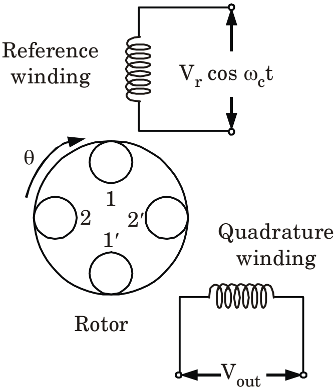

Fig. 1.10.2. AC tachometer.

1. It consists of two stator field coils or windings placed 90° electrical apart or mounted at right angles to each other. These windings are reference winding and quadrature winding.
2. A rotating element, *i.e.*, a rotor, is placed in the air gap between the magnetic structure. Generally the rotor is a thin aluminium cup to minimize the losses and low inertia.
3. A sinusoidal voltage of rated value is applied to the primary winding. A secondary winding is placed at a 90 degree angle mechanically with respect to the primary winding.
4. When the rotor shaft is rotated, the magnitude of the sinusoidal output voltage is proportional to the rotor speed.

Mathematical modeling of tachometers :

1. The dynamics of the tachometer can be represented by the equation

$$e_t(t) = K_t \frac{d\theta(t)}{dt} = k_t\omega(t) \quad ...(1.10.1)$$

where, $e_t(t)$ = Output voltage

$\theta(t)$ = Rotor displacement

$\omega(t)$ = Rotor velocity

K_t = Tachometer constant

2. The transfer function of the tachometer is obtained by taking the Laplace transform of eq. (1.10.1),

$$\therefore \quad \frac{E_t(s)}{\theta(s)} = K_t s$$

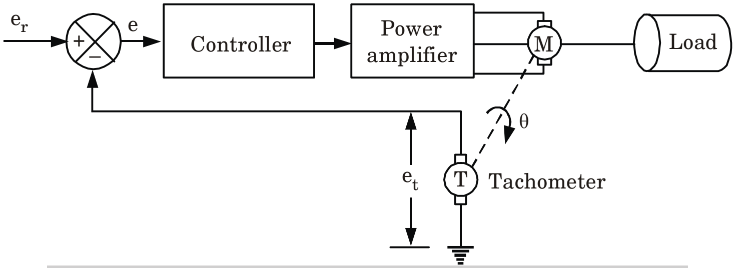

Fig. 1.10.3. Velocity-control system with tachometer feedback.

PART-3

Transfer Function Models, Block Diagram Algebra, Signal Flow Graph, Mason's Gain Formula.

CONCEPT OUTLINE : PART-3

- **Transfer function :** It is the ratio of output to input terms expressed in Laplace transform.

  $T(s) = C(s) / R(s)$
- **Signal flow graph :** A signal flow graph is pictorial representation of the simultaneous equations describing a system.
- **Mason's gain formula :**

$$T = \frac{1}{\Delta}\sum_{k=1}^{n} P_k \Delta_k .$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.11. **What is transfer function ? Write advantages and disadvantages of transfer function.**

Answer

A. Transfer function : Transfer function is defined as the Laplace transform of output to the Laplace transform of input when all initial conditions are set to zero.

$$T(s) = \frac{L[c(t)]}{L[r(t)]} = \left.\frac{C(s)}{R(s)}\right|_{\text{All initial condition are zero}}$$

B. Advantages :

1. Stability analysis of the system can easily be carried out.
2. The use of Laplace transform approach allows converting integral-differential time domain equations to simple algebraic equations.

C. Disadvantages :

1. It is applied only to linear and time invariant systems.
2. Initial conditions do affect the system performance but these are neglected while determining transfer function and hence lose their importance.

Que 1.12. **Find the transfer function of the electrical network shown in Fig. 1.12.1.**

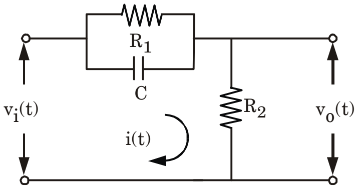

Fig. 1.12.1.

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Answer

1. Taking Laplace transform of circuit components,

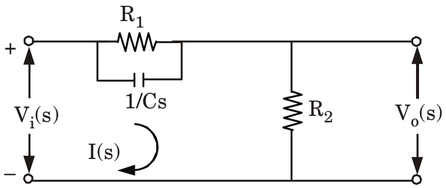


Fig. 1.12.2.

2. Apply KVL in the loop

$$-V_i(s) + [R_1 \| 1/Cs]\ I(s) + R_2\ I(s) = 0$$

$$V_i(s) = \left\{\left[\frac{R_1 \dfrac{1}{Cs}}{R_1 + \dfrac{1}{Cs}}\right] + R_2\right\} I(s)$$

3. And, $$V_0(s) = R_2 I(s)$$

$$V_0(s) = \frac{R_2 V_i(s)}{\left[\dfrac{R_1 \dfrac{1}{Cs}}{R_1 + \dfrac{1}{Cs}}\right] + R_2} = \frac{R_2 V_i(s)}{\left[\left(\dfrac{R_1}{R_1 Cs + 1}\right) + R_2\right]}$$

4. Transfer function, $$\frac{V_0(s)}{V_i(s)} = \frac{R_2 (R_1 Cs + 1)}{R_2 (R_1 Cs + 1) + R_1}$$

Que 1.13. **Explain basic rules of block diagram reduction. Define blocks, summing point and branch point. Explain the block diagram reduction rules.**

Answer

A. Branch point : A branch point or a take off point is a point from which the signal from a block goes concurrently to other blocks or running points.

B. Summing point : The output of control system is feedback to the summing point where it is compared with the reference input.

C. Blocks :

1. A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals.
2. A system is represented by using block diagrams. These are easy to construct even for the complicated system. It is easy to visualize the function of individual element in the block diagram of a system.

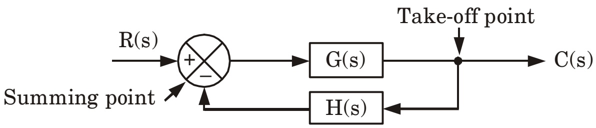
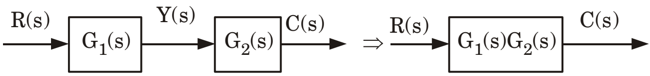


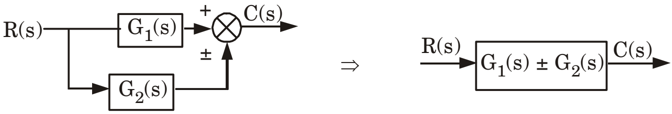
Fig. 1.13.1. Schematic diagram of closed loop system.

D. Block diagram reduction rules :

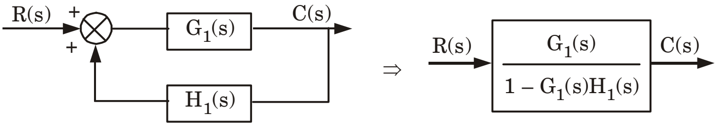
1. Series connection :



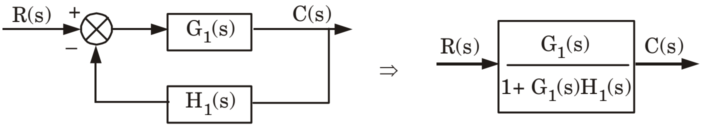
2. Parallel connection :



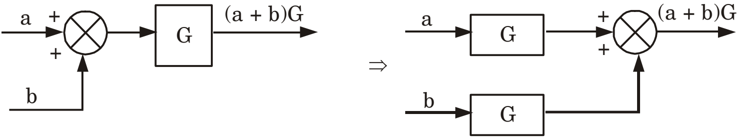
3. Positive feedback connection :



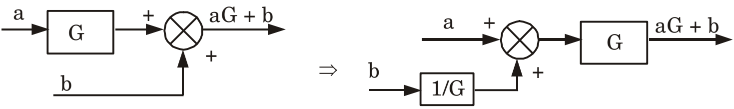
4. Negative feedback connection :



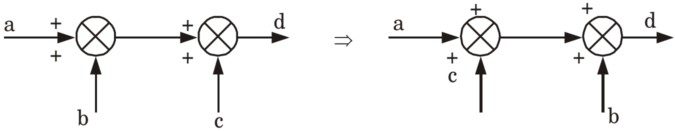
5. Moving the summing point ahead of the block :



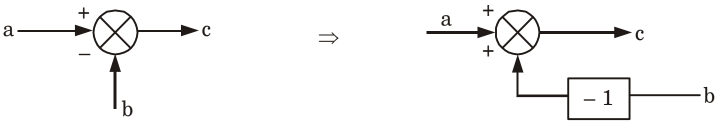
6. Moving the summing point before the block :



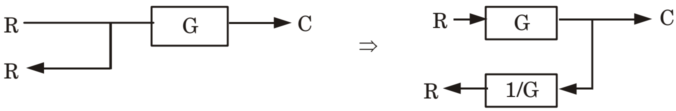
7. Interchanging input :



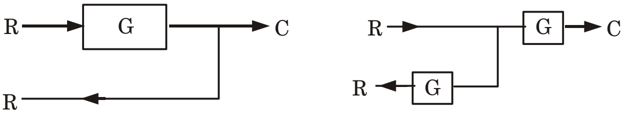
8. Multiplication by (–1) to the input :



9. Moving a take-off point ahead a block :



10. Moving a take-off point before a block :



Que 1.14. **Find the single block equivalent of Fig. 1.14.1.**

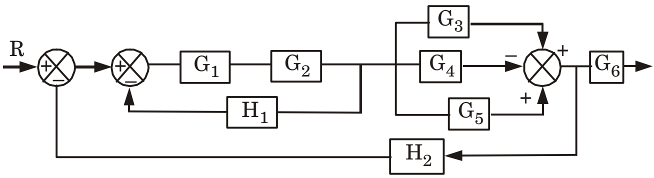


Fig. 1.14.1.

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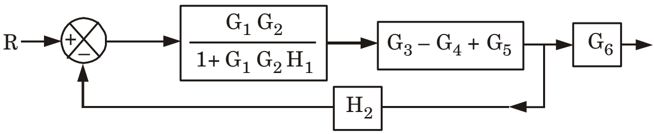
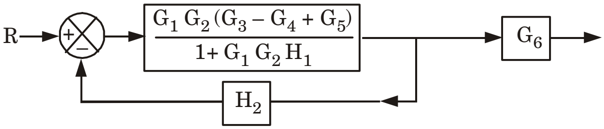
Answer

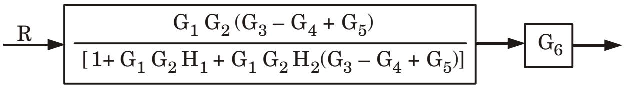
1.



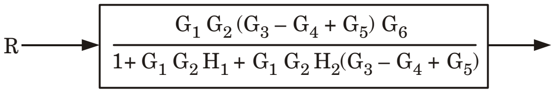
2.



3.



4. Single block equivalent



Que 1.15. **Reduce the block diagram shown in Fig. 1.15.1 to a single block representation.**

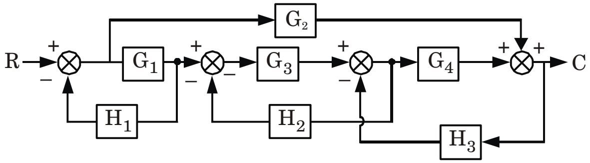


Fig. 1.15.1.

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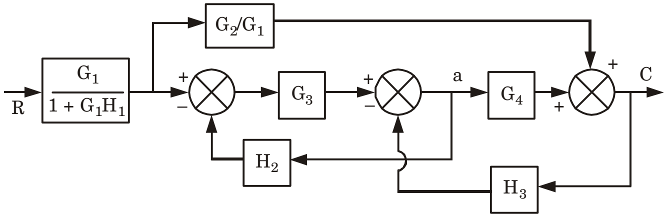
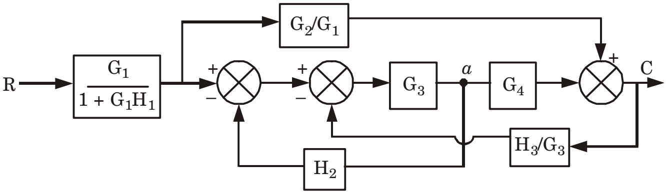
Answer

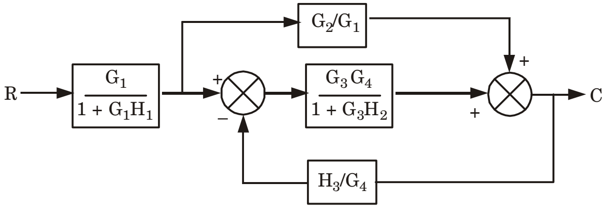
Step 1 :



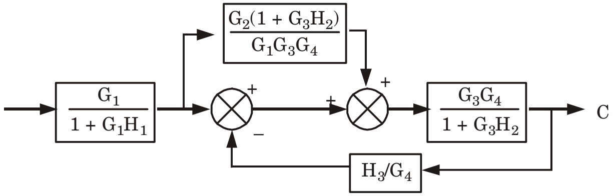
Step 2 : Shift summing point a before G_3.



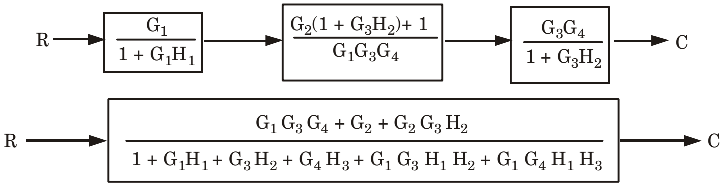
Step 3 : Interchange summing points.



Step 4 : Shift summing point before block.



Step 5 : Interchange summing points.



Que 1.16. **Determine the transfer function $C(s)/R(s)$ for the block diagram shown in Fig. 1.16.1.**

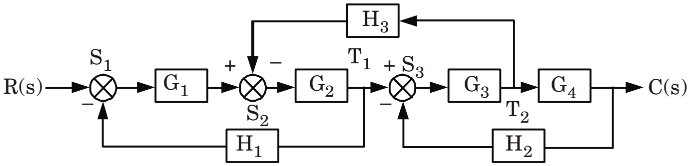


Fig. 1.16.1.

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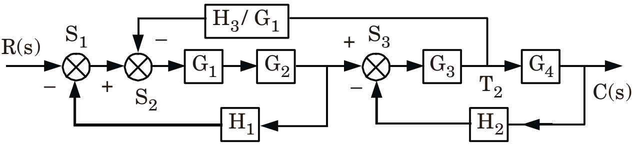
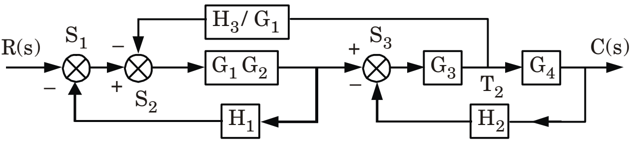
Answer

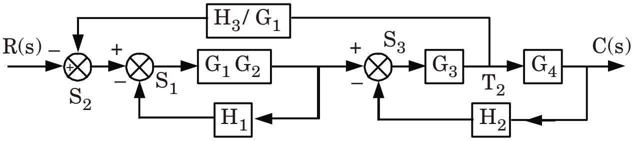
Step 1 : Shift S_2 before G_1, we get



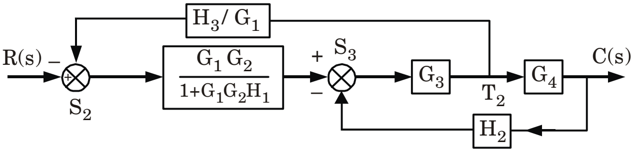
Step 2 : G_1 and G_2 are in cascade.



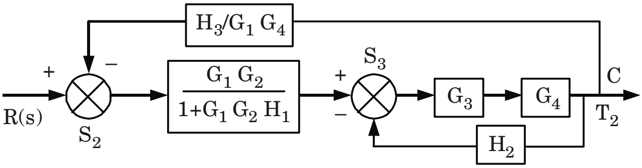
Step 3 : Interchanging summing points



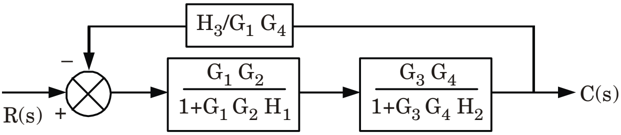
Step 4 : Solve the inner loop.



Step 5 : Shift T_2 after G_4.



Step 6 : Solve the inner loop.



Step 7 : Forward blocks are in cascade and final result will be

$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}{1+\dfrac{G_2G_3H_3}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)+G_2G_3H_3}$$

Que 1.17. **Using block diagram reduction techniques, find the closed loop transfer function of the system whose block diagram is given in Fig. 1.17.1.**

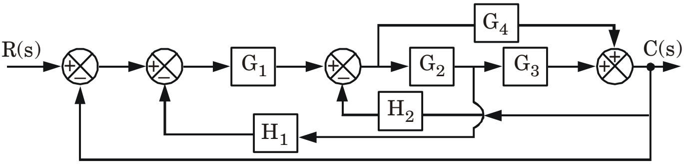


Fig. 1.17.1.

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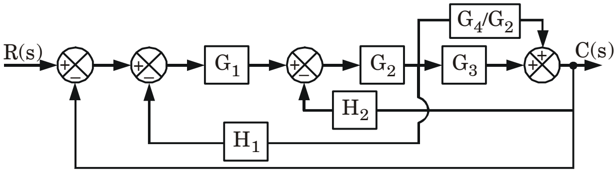
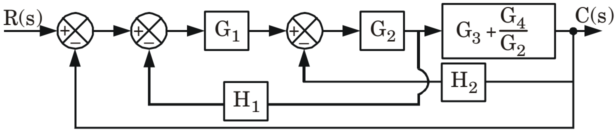
Answer

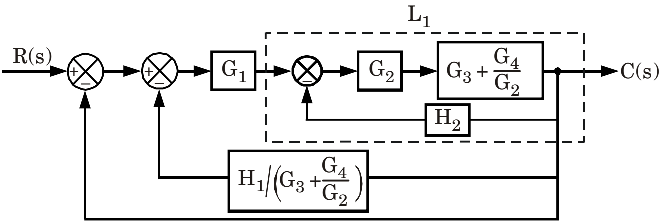
Step 1:



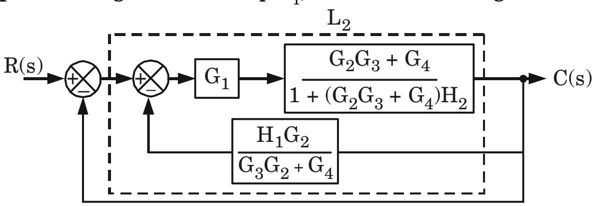
Step 2:



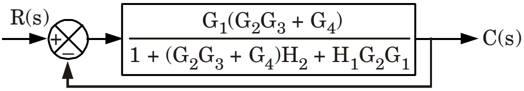
Step 3 :



Step 4 : Solving feedback loop L_1, reduced block diagram is



Step 5 : Solving feedback loop L_2, the reduced block diagram is



Step 6 : Solving feedback loop,

$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_2 + H_1G_2G_1}}{1 + \dfrac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_2 + H_1G_2G_1} \times 1}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + (G_2G_3 + G_4)H_2 + H_1G_2G_1 + (G_2G_3 + G_4)G_1}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_2 + G_4H_2 + H_1G_2G_1 + G_1G_2G_3 + G_1G_4}$$

Que 1.18. **Briefly explain Mason's gain formula.**

Answer

1. The overall gain can be determined by Mason's gain formula given below

$$T = \sum_{k=1}^{k} \frac{P_k \Delta_k}{\Delta}$$

where, P_k = Forward path gain of k_{th} path from a specified input node to an output node.

Δ = Determinant which involves closed-loop gain and mutual interactions between non-touching loops.

= 1 – [Sum of all individual loop gain]

+ [Sum of loop gain products of all possible pair of non-touching loops]

– [Sum of loop gain products of all possible triplets of non-touching loops]

+ [..........] – [...........]

Δ_k = Path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

2. The path factor Δ_k for the k_{th} path is equal to the value of the graph determinant of a signal flow graph which exists after erasing the k_{th} path from the graph.

Que 1.19. **Define various terminologies used in signal flow graph.**

Answer

1. **Source node :** The node having only outgoing branches is known as source or input node. *e.g.*, x_0 is source node.
2. **Sink node :** The node having only incoming branches is known as sink or output node. *e.g.*, x_5 is sink node.
3. **Chain node :** A node having incoming and outgoing branches is known as chain node. *e.g.* x_1, x_2, x_3 and x_4.

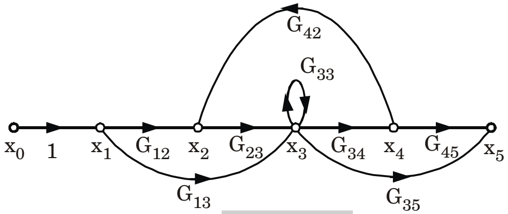


Fig. 1.19.1.

4. **Forward path :** A path from the input to output node is defined as forward path. *e.g.*

$x_0 - x_1 - x_2 - x_3 - x_4 - x_5$	First forward path.
$x_0 - x_1 - x_3 - x_4 - x_5$	Second forward path.
$x_0 - x_1 - x_3 - x_5$	Third forward path.
$x_0 - x_1 - x_2 - x_3 - x_5$	Fourth forward path.

5. **Feedback loop :** A path which originates from a particular node and terminating at the same node, travelling through at least one other node, without tracing any node twice is called feedback loop. For example, $x_2 - x_3 - x_4 - x_2$.
6. **Self loop :** A feedback loop consisting of only one node is called self loop. *i.e.*, G_{33} at x_3 is self loop.
7. **Path gain :** The product of branch gains while going through a forward path known as path gain. *i.e.* path gain for path $x_0 - x_1 - x_2 - x_3 - x_4 - x_5$ is, $1 \times G_{12} \times G_{23} \times G_{34} \times G_{45}$. This can be also called forward path gain.
8. **Dummy node :** If there exists incoming and outgoing branches both at first and last node representing input and output variables, then as per definition these cannot be called source and sink nodes. In such a case a separate input and output nodes can be created by adding branches with gain 1. Such nodes are called dummy nodes.

   e.g.

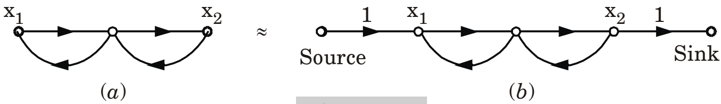


Fig. 1.19.2.

9. **Non-touching loops :** If there is no node common in between the two or more loops, such loops are said to be non-touching loops.

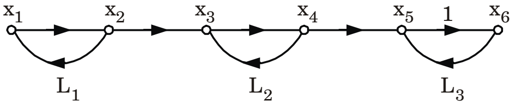


Fig. 1.19.3. Three non-touching loops.

10. **Loop gain :** The product of all the gains of the branches forming a loop is called loop gain. For a self loop, gain indicated along it is its gain. Generally such loop gain are denoted by 'L' *e.g.* L_1, L_2 etc.

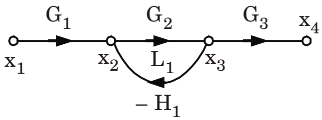


Fig. 1.19.4.

Loop gain, $L_1 = -G_2H_1$.

Que 1.20. **Find the transfer function of the signal flow graph shown in Fig. 1.20.1, using Mason's gain formula.**

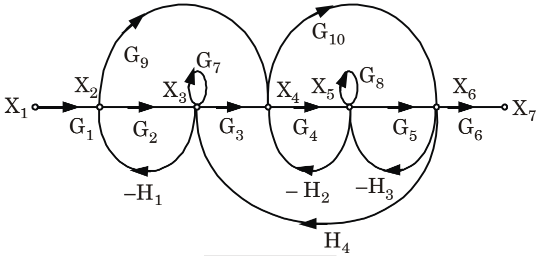


Fig. 1.20.1.

AKTU 2016-17, Marks 10

Answer

1. Forward path and gains are :

$$P_1 = G_1\, G_2\, G_3\, G_4\, G_5\, G_6, \qquad \Delta_1 = 1$$
$$P_2 = G_1\, G_9\, G_4\, G_5\, G_6, \qquad \Delta_2 = 1 - G_7$$
$$P_3 = G_1\, G_9\, G_{10}\, G_6, \qquad \Delta_3 = 1 - G_7 - G_8 + G_7\, G_8$$
$$P_4 = G_1\, G_2\, G_3\, G_{10}\, G_6, \qquad \Delta_4 = 1 - G_8$$

2. Loops and gains are :

$$L_1 = -G_2 H_1$$
$$L_2 = -G_4 H_2$$
$$L_3 = -G_5 H_3$$
$$L_4 = -G_3 G_4 G_5 H_4$$
$$L_5 = G_7$$
$$L_6 = G_8$$
$$L_7 = H_1 G_9 G_{10} H_4$$
$$L_8 = H_1 G_9 G_4 G_5 H_4$$
$$L_9 = -H_4 G_3 G_{10}$$
$$L_{10} = H_2 G_{10} H_3$$
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 \text{ --------- } + L_{10}) + (L_1 L_2 + L_1 L_3 + L_1 L_6 + L_1 L_{10} + L_7 L_6 + L_5 L_6 + L_5 L_2 + L_5 L_3 + L_5 L_{10} + L_9 L_5 + L_9 L_3)$$

3. Using Mason's gain formula, $T(s) = \dfrac{X_7}{X_1} = \dfrac{\sum_{k=1}^{4} P_k \Delta_k}{\Delta}$

$$= \frac{G_1G_2G_3G_4G_5G_6 + G_1G_9G_4G_5G_6(1-G_7) + G_1G_9G_{10}G_6(1-G_7-G_8+G_7G_8)}{\begin{array}{l}1+(G_2H_1+G_4H_2+G_5H_3+G_3G_4G_5H_4-G_7-G_8-H_1G_9G_{10}H_4 \\ -H_1G_9G_5H_4G_4+H_4G_3G_{10}-H_2G_{10}H_3)+(G_2H_1G_4H_2+G_2H_1G_5H_3 \\ -G_2H_1G_8-G_2H_1H_2G_{10}H_3+H_1G_9G_4G_5H_4G_8+G_7G_8-G_7G_4H_2 \\ -G_7G_5H_3+G_7H_2G_{10}H_3-H_4G_3G_{10}G_7+H_4G_3G_{10}G_5H_3)\end{array}}$$

Que 1.21. **Draw the signal flow graph and determine the overall transfer function of the block diagram shown in Fig. 1.21.1.**

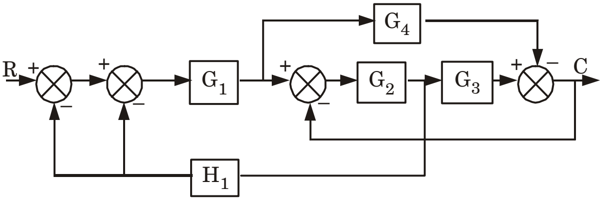


Fig. 1.21.1.

AKTU 2014-15, Marks 10

Answer

1. Signal flow graph of given block diagram :

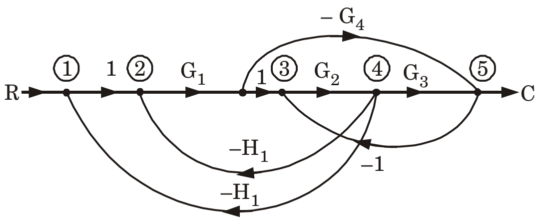


Fig. 1.21.2.

2. Forward paths : $P_1 = G_1G_2G_3$ $\therefore \Delta_1 = 1 - 0 = 1$

$P_2 = G_1\,(-\,G_4) = -\,G_1G_4$ $\therefore \Delta_2 = 1 - 0 = 1$

3. Loops :

$$L_1 = G_1G_2(-\,H_1) = -\,G_1G_2H_1$$
$$L_2 = G_1G_2(-H_1) = -\,G_1G_2H_1$$
$$L_3 = G_2G_3(-\,1) = -\,G_2G_3$$
$$L_4 = G_1(-\,G_4)(-1)G_2(-H_1) = G_1G_2G_4H_1$$
$$L_5 = G_1(-\,G_4)(-1)G_2(-H_1) = -\,G_1G_2G_4H_1$$
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$$
$$= 1-(-\,G_1G_2H_1 - G_1G_2H_1 - G_2G_3 - G_1G_2G_4H_1 - G_1G_2G_4H_1)$$

4. Then transfer function,

$$TF = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 - G_1G_4}{1 + 2G_1G_2H_1 + 2G_1G_2G_4H_1 + G_2G_3}$$

PART-4

Open Loop and Closed Loop System and their Sensitivity Analysis.

CONCEPT OUTLINE : PART-4

- Sensitivity is change in variable due to variation in parameters of control system.
  Mathematically,

$$S_K^A = \frac{\partial A / A}{\partial K / K}$$

The notation S_K^A denotes sensitivity of variable A with respect to parameter K.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.22. **Discuss the effect of feedback on :**

i. Overall gain

ii. Stability

iii. Noise and Disturbance.

AKTU 2013-14, Marks 05

Answer

i. Overall Gain :

For open loop :

$$T(s) = \frac{C(s)}{R(s)} = G(s)$$

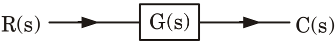


Fig. 1.22.1.

For closed loop :

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

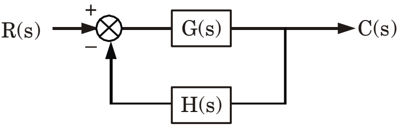


Fig. 1.22.2.

So, for a closed loop control system the gain $G(s)$ reduces by a factor $1/[1 + G(s)H(s)]$ as compared to open loop control system.

ii. Stability :

1. Consider an open loop system with overall transfer function as

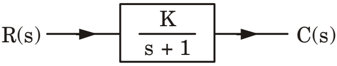


Fig. 1.22.3.

$$G(s) = \frac{K}{s + T}$$

The open loop pole is located at $s = -T$.

2. If unity negative feedback is introduced in the system, the overall transfer function of a closed loop system becomes

$$\frac{C(s)}{R(s)} = \frac{\dfrac{K}{s+T}}{1 + \dfrac{K}{s+T}} = \frac{K}{s + (K + T)}$$

3. Thus the closed loop pole is now located at $s = -(K + T)$

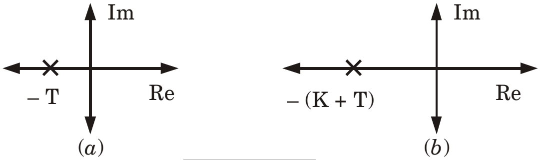


Fig. 1.22.4.

4. Thus, the feedback controls the time response *i.e.*, dynamics of the system by adjusting location of its poles. The stability of the system depends on the location of poles in s-plane. Thus it can be concluded that the feedback affects the stability of the system.

iii. Noise and Disturbances :

1. Every control system has some non-linearities present in it. The dominant non-linearities like friction, dead zone, saturation etc., affect the output of the system adversely.
2. Some external disturbance signals also make the system output inaccurate.
3. The examples of such external disturbances are high frequency noise in electronic applications, thermal noise in amplifier tubes, wind gusts on antenna of radar system etc.
4. The disturbance may be in the forward path, feedback path or output of a system.

a. Disturbance in the forward path :

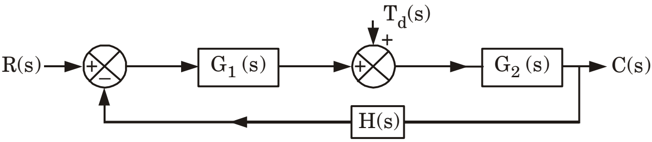


Fig. 1.22.5.

1.
$$G(s) = G_2(s)$$
$$H'(s) = -G_1(s)\,H(s)$$
$$\frac{C(s)}{T_d(s)} = \frac{G_2(s)}{1-[G_2(s)\,(-G_1(s)\,H(s))]}$$
$$\frac{C(s)}{T_d(s)} = \frac{G_2(s)}{1+G_1(s)\,G_2(s)\,H(s)}$$
$$C(s) = \frac{T_d(s)\,G_2(s)}{1+G_1(s)\,G_2(s)\,H(s)}$$

2. In the denominator assume that $1 << G_1(s)\,G_2(s)\,H(s)$, hence we get
$$C(s) = \frac{T_d(s)}{G_1(s)\,H(s)}$$
Thus to make the effect of disturbance on the output as small as possible the $G_1(s)$ must be selected as large as possible.

b. Disturbance in the feedback path :

1. These are produced due to the nonlinear behaviour of the feedback path element.

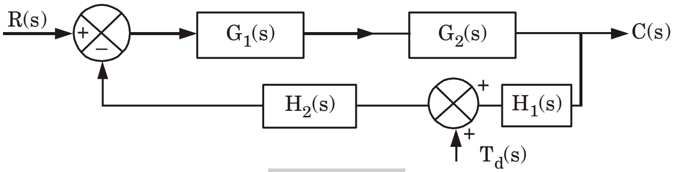


Fig. 1.22.6.

2. $$\frac{C(s)}{T_d(s)} = \frac{-G_1(s)\,G_2(s)\,H(s)}{1-\{-G_1(s)\,G_2(s)\,H_2(s)\,H_1(s)\}}$$

$$\frac{C(s)}{T_d(s)} = \frac{-G_1(s)\,G_2(s)\,H_2(s)}{1+G_1(s)\,G_2(s)\,H_2(s)\,H_1(s)}$$

3. For large value of G_1, G_2, H_1, H_2 in the denominator 1 can neglected [$\because$ $1 << G_1(s)G_2(s)H_1(s)H_2(s)$]

$$\frac{C(s)}{T_d(s)} = -\frac{1}{H_1\,(s)}$$

4. Thus by designing proper feedback element $H_1(s)$, the effect of disturbance in feedback path on output can be reduced.

c. Disturbance at the output :

1. Consider that there is disturbance $T_d(s)$ affecting the output directly with $R(s) = 0$, we get

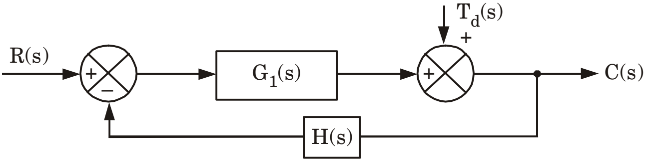


Fig. 1.22.7.

$$\frac{C(s)}{T_d(s)} = \frac{1}{1-[-G(s)\,H(s)]} = \frac{1}{1+G(s)\,H(s)}$$

2. For large value of $G(s)$ $H(s)$, 1 in denominator can be neglected [$\because$ $1 << G(s)\,H(s)$]

$$C(s) = \frac{T_d(s)}{G(s)\,H(s)}$$

3. Thus if disturbance is affecting the output directly then by changing the values of $G(s)$, $H(s)$ or both, the effect of disturbance can be minimized.
The feedback minimizes the effect of disturbance signals occurring in the control system.

Que 1.23. **What do you understand by the term sensitivity ? Consider the feedback control system shown in Fig. 1.23.1. The normal value of the process parameter '*K*' is 1. Determine the sensitivity of transfer function, $T(s) = \dfrac{C(s)}{R(s)}$ to variations in parameter '*K*', at ω = 5.**

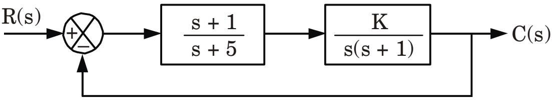


Fig. 1.23.1.

AKTU 2016-17, Marks 10

Answer

Sensitivity : Sensitivity is the change in variable due to variation in parameters of control system.

e.g.

$$S_G^T = \frac{\%\text{ Change in } T}{\%\text{ Change in } G} = \frac{\dfrac{\partial T}{T} \times 100}{\dfrac{\partial G}{G} \times 100}$$

Numerical :

Given : $G(s) = \dfrac{K}{s(s+5)}$, $H(s) = 1$

To Find : Sensitivity, S_G^T .

1. Forward gain, $G(s) = \dfrac{K}{s(s+5)}$

2. $$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{(s^2+5s+K)}$$

3. Sensitivity,
$$S_K^T = \frac{\dfrac{\partial T}{T} \times 100}{\dfrac{\partial K}{K} \times 100} = \frac{K}{T}\frac{\partial T}{\partial K}$$
$$= \frac{K(s^2+5s+K)}{K}\left[\frac{1}{(s^2+5s+K)} - \frac{K}{(s^2+5s+K)^2}\right]$$
$$= (s^2+5s+K)\left[\frac{s^2+5s+K-K}{(s^2+5s+K)^2}\right] = \frac{s(s+5)}{(s^2+5s+K)^2}$$

4. Putting $s = j\omega, K = 1$

$$= \frac{j\omega(j\omega+5)}{\left[(j\omega)^2\ 5\ j\omega+1\right]^2}$$

5. Again, putting $\omega = 5$

$$= \frac{j5\,(j5+5)}{[-25+j25+1]^2} = \frac{j25\,(j+1)}{[j25-24]^2}$$

$$= \frac{(-25+j25)}{(j25-24)^2}$$

$$|\,S_K^T\,| = \frac{\sqrt{25^2+25^2}}{\sqrt{25^2+24^2}} = 1.02$$

Que 1.24. **Discuss the effect of feedback on the following :**

i. Sensitivity

ii. Error

iii. Gain

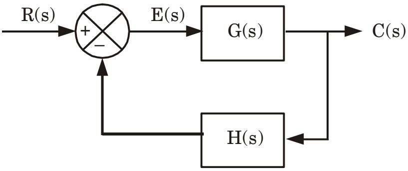


Fig. 1.24.1.

Answer

i. Sensitivity :

$$S_G^T = \frac{\%\text{ Change in } T}{\%\text{ Change in } G} = \frac{\dfrac{\partial T}{T}\times 100}{\dfrac{\partial G}{G}\times 100}$$

$$S_G^T = \frac{G}{T}\,\frac{dT}{dG}$$

We know $\qquad T = \dfrac{C}{R} = \dfrac{G}{1+GH}$

$$S_G^T = \frac{G}{\left(\dfrac{G}{1+GH}\right)}\left[\frac{(1+GH)-GH}{(1+GH)^2}\right]$$

$$S_G^T = (1+GH)\,\frac{1}{(1+GH)^2} = \frac{1}{1+GH}$$

So sensitivity of feedback system (with respect to G) is reduced by factor of $1 + GH$.

ii. Error : $\qquad E(s) = R(s) - H(s)\,C(s) \qquad$ ...(1.24.1)

But,
$$C(s) = \frac{G(s)}{1+G(s)\,H(s)}R(s)$$
$$E(s) = R(s) - \frac{H(s)\,G(s)}{1+G(s)\,H(s)}R(s)$$
$$= \left[\frac{1+G(s)\,H(s) - H(s)G(s)}{1+G(s)\,H(s)}\right]R(s)$$
$$E(s) = \frac{1}{1+G(s)H(s)}R(s) \qquad \text{...(1.24.2)}$$

Hence, error E(s) is reduced by $\frac{1}{1+G(s)H(s)}$

iii. Gain : $\text{Gain} = \frac{G}{1+GH}$

Hence, Gain reduces by factor of $1 + GH$.

Que 1.25. Calculate the sensitivity of the closed loop system shown in Fig. 1.25.1 with respect to the forward path transfer function at ω = 1.5 rad/sec.

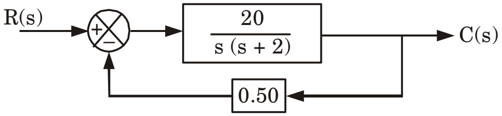


Fig. 1.25.1.

AKTU 2013-14, Marks 05

Answer

Given : $\omega = 1.5$ rad/sec, $G = 20/s(s+2)$, $H = 0.5$

To Find : Sensitivity, S_G^T.

1. Sensitivity with respect to forward path :

$$S_G^T = \frac{1}{1+GH}$$

$$\therefore \quad S_G^T = \frac{1}{1+\dfrac{20\times 0.5}{s(s+2)}} = \frac{s(s+2)}{s^2+2s+10} = \frac{s^2+2s}{s^2+2s+10}$$

Put $s = j\omega$,

$$S_G^T = \frac{(j\omega)^2 + 2j\omega}{(j\omega)^2 + 2(j\omega) + 10} = \frac{-2.25 + j3}{-2.25 + 10 + j3} = \frac{-2.25 + j3}{7.75 + j3}$$

$$= \frac{(-2.25 + j3) \times (-7.75 + j3)}{(j3 + 7.75)(j3 - 7.75)}$$

$$= \frac{17.43 - j30 - 9}{-9 - 60.06} = \frac{8.43 - j30}{-69}$$

$$|S_G^T| = \frac{\sqrt{(8.43)^2 + (30)^2}}{53.953} = \frac{31.16}{53.953} = 0.577$$

2. Feedback path :

$$S_G^T = \frac{-GH}{1+GH} = \frac{\dfrac{-20 \times 0.5}{s(s+2)}}{1 + \dfrac{20 \times 0.5}{s(s+2)}} = \frac{-10}{s^2 + 2s + 10}$$

Put $s = j\omega$

$$S_G^T = \frac{-10}{(j\omega)^2 + 2j\omega + 10} = \frac{-10}{(j1.5)^2 + j2 \times 1.5 + 10} = \frac{-10}{-2.25 + j3 + 10}$$

$$= \frac{-10}{7.75 + j3} \frac{(j3 - 7.75)}{(j3 - 7.75)} = \frac{-j30 + 77.5}{(j3)^2 - (7.75)^2}$$

$$= \frac{\sqrt{(-30)^2 + (77.5)^2}}{69} = \frac{83.10}{69} = 1.204$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Compare the open loop control system and closed loop control system, also give few examples for each system.

Ans. Refer Q. 1.2, Unit-1.

Q. 2. Describe the AC servomotor for control application.

Ans. Refer Q. 1.5, Unit-1.

Q. 3. Write a short note on armature-controlled DC servo motor.

Ans. Refer Q. 1.6, Unit-1.

Q. 4. Write a short note on tachometer.

Ans. Refer Q. 1.10, Unit-1.

Q. 5. Find the transfer function of the electrical network shown in Fig. 1.

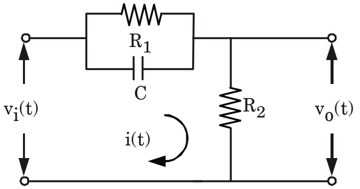


Fig. 1.

Ans. Refer Q. 1.12, Unit-1.

Q. 6. **Using block diagram reduction techniques, find the closed loop transfer function of the system whose block diagram is given in Fig. 2.**

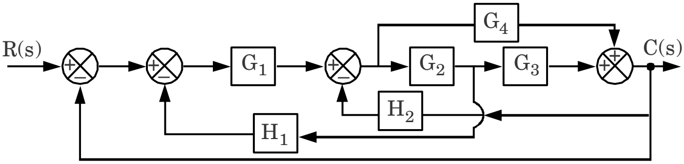


Fig. 2.

Ans. Refer Q. 1.17, Unit-1.

Q. 7. **Find the transfer function of the signal flow graph shown in Fig. 3, using Mason's gain formula.**

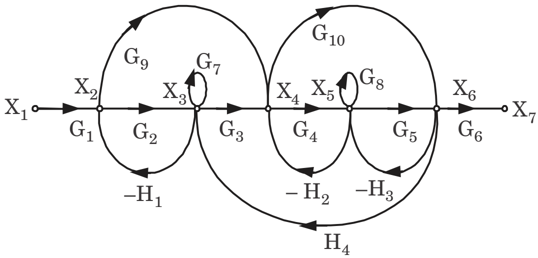


Fig. 3.

Ans. Refer Q. 1.20, Unit-1.

Q. 8. **Draw the signal flow graph and determine the overall transfer function of the block diagram shown in Fig. 4.**

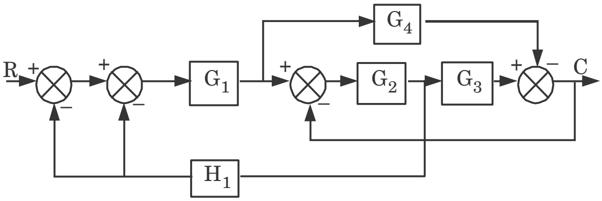


Fig. 4.

Ans. Refer Q. 1.21, Unit-1.

Q. 9. Discuss the effect of feedback on the following :
i. Sensitivity ii. Error iii. Gain

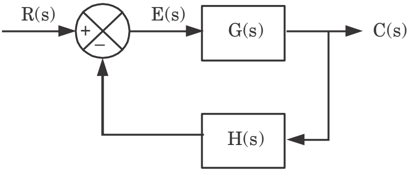


Fig. 5.

Ans. Refer Q. 1.24, Unit-1.

Q. 10. Calculate the sensitivity of the closed loop system shown in Fig. 6 with respect to the forward path transfer function at ω = 1.5 rad/sec.

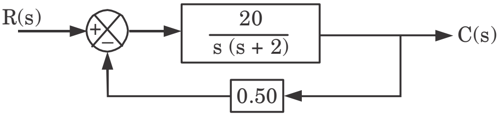


Fig. 6.

Ans. Refer Q. 1.25, Unit-1.

☺☺☺

2 UNIT

Time Response Analysis

PART-1

Standard Test Signals, Time Response of First and Second Order Systems, Time Response Specifications.

CONCEPT OUTLINE : PART-1

- **Standard test signals :**
  There are four basic standard test signals. These are :

i. Step signal

ii. Ramp signal

iii. Parabolic signal

iv. Impulse signal.

- **Time response system for second order :**

  Transfer function, $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n}$

  For unit step input, $c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **Explain various standard test signals, and also find relation between them.**

AKTU 2014-15, Marks 05

Answer

1. **Unit step :** Signals which start at time $t = 0$ and have magnitude of unity are called unit step signals.
   They are represented by a unit step function $u(t)$.

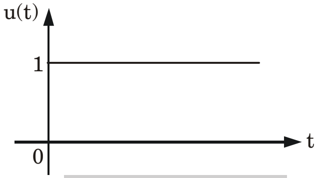


Fig. 2.1.1. Unit step.

They are defined mathematically as :

$$u(t) = \begin{cases} 1\,; & t \geq 0 \\ 0\,; & t < 0 \end{cases}$$

2. Unit ramp : Signals which start from zero and are linear in nature with a constant slope m are called unit ramp signals.

They are represented by a unit ramp function $r(t)$.

They are defined mathematically as :

$$r(t) = \begin{cases} mt\,; & t \geq 0 \\ 0\,; & t < 0 \end{cases}$$

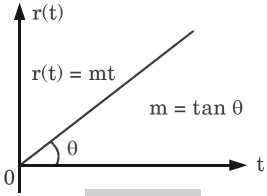


Fig. 2.1.2.

3. Unit impulse : Signals which act for very small time but have large amplitude are called unit impulse functions.

They are represented by $\delta(t)$.

They are defined mathematically as,

$$\delta(t) = \begin{cases} 0\,; & t \neq 0 \\ 1\,; & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

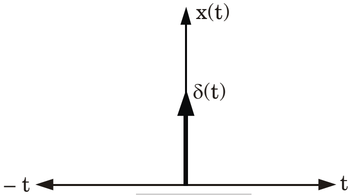


Fig. 2.1.3.

4. Unit Parabolic Signal : The continuous-time unit parabolic function $p(t)$, also called acceleration signal starts at $t = 0$, and is defined as :

$$p(t) = \begin{cases} \dfrac{t^2}{2}\,; & \text{for } t \geq 0 \\ 0\,; & \text{for } t < 0 \end{cases}$$

or $$p(t) = \frac{t^2}{2} u(t)$$

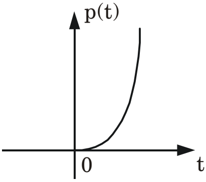


Fig. 2.1.4. Unit parabolic signal.

Relation :

1. Relation between impulse and step signal :

$$\delta(t) = \frac{d}{dt}u(t)$$

2. Relation between step and ramp signal :

$$u(t) = \frac{d}{dt}r(t)$$

3. Relation between ramp and parabolic signal :

$$r(t) = \frac{d}{dt}\left(\frac{t^2}{2}\right).$$

Que 2.2. Discuss the time response of first order system with unit step, unit impulse and unit ramp inputs.

Answer

1. Consider a first order system with unity feedback.

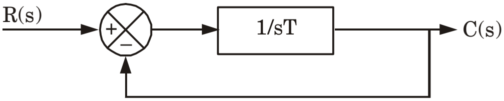


Fig. 2.2.1.

2.
$$\frac{C(s)}{R(s)} = \frac{\frac{1}{sT}}{\left(1+\frac{1}{sT}\right)} = \frac{1}{(1+sT)}$$

$$\therefore \quad C(s) = \frac{1}{(1+sT)}R(s) \quad ...(2.2.1)$$

3. **Response to unit step input :**

$$r(t) = u(t)$$

$$R(s) = \frac{1}{s}$$

Putting in eq. (2.2.1), we get

$$C(s) = \frac{1}{s}\frac{1}{(1+sT)}$$

$$= \frac{1}{s} - \frac{1}{\left(s + \frac{1}{T}\right)}$$

Taking inverse Laplace transform, we have

$$L^{-1}[C(s)] = c(t) = L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left[\frac{1}{\left(s + \frac{1}{T}\right)}\right]$$

$$c(t) = 1 - e^{-t/T}$$

4. Response to unit impulse input :

$$r(t) = \delta(t)$$

$$R(s) = 1$$

Putting in eq. (2.2.1),

$$C(s) = \frac{1}{(sT+1)}1 = \frac{1}{T}\frac{1}{\left(s + \frac{1}{T}\right)}$$

Taking inverse Laplace transform, we have

$$L^{-1}[C(s)] = \frac{1}{T}L^{-1}\left[\frac{1}{\left(s + \frac{1}{T}\right)}\right]$$

$$c(t) = \frac{1}{T}e^{-t/T}$$

5. Response to unit ramp input :

$$r(t) = t$$

$$R(s) = \frac{1}{s^2}$$

Putting in eq. (2.2.1), we get

$$C(s) = R(s)\ \frac{1}{(sT+1)} = \frac{1}{s^2(sT+1)}$$

$$= \frac{1}{s^2} - \frac{T}{s} + \frac{T}{\left(s + \frac{1}{T}\right)}$$

Taking inverse Laplace transform, we have

$$L^{-1}[C(s)] = L^{-1}\left(\frac{1}{s^2}\right) - TL^{-1}\left(\frac{1}{s}\right) + TL^{-1}\left[\frac{1}{\left(s + \frac{1}{T}\right)}\right]$$

$$c(t) = t - T + Te^{-t/T}$$

Que 2.3. **Give comparison between open loop and closed loop system. The impulse response of unity feedback closed loop system is, $c(t) = -te^{-t} + 2e^{-t}$, find the open loop transfer function.**

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Answer

Comparison between open loop and closed loop System : Refer Q. 1.2, Page 1 –3C, Unit-1.

Numerical :

Given : $r(t) = \delta(t)$, $c(t) = -te^{-t} + 2e^{-t}$
To Find : Transfer function.

1. Laplace transform gives

$$R(s) = 1$$
$$c(t) = -t\,e^{-t} + 2\,e^{-t}$$

2. Laplace transform of $c(t)$ gives

$$C(s) = \frac{-1}{(s+1)^2} + \frac{2}{(s+1)}$$

$$C(s) = \frac{-1+2(s+1)}{(s+1)^2} = \frac{(2s+1)}{(s+1)^2} \quad ...(2.3.1)$$

3. For unity feedback control system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$C(s) = \frac{G(s)}{1+G(s)} R(s) \quad ...(2.3.2)$$

4. Putting value of $C(s)$ and $R(s)$ in eq. (2.3.2)

$$\frac{(2\,s+1)}{(s+1)^2}(1+G(s)) = G(s)$$

$$\frac{(2\,s+1)}{(s+1)^2} = G(s)\left[1 - \frac{(2\,s\;+\;1)}{(s+1)^2}\right]$$

$$= G(s)\left[\frac{s^2+1+2s-2s-1}{(s+1)^2}\right]$$

5. Open loop transfer function is

$$\therefore \qquad G(s) = \frac{(2\,s+1)}{s^2}$$

Que 2.4. **Derive the expression for the time response of a second order control system subjected to a unit step input.**

OR

Derive the expression for step response of second order control system for underdamped response. **AKTU 2015-16, Marks 10**

Answer

1. Consider the second order system with unity feedback.

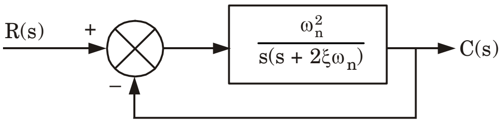

Fig. 2.4.1.

The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 / s(s+2\xi\omega_n)}{1+\omega_n^2 / s(s+2\xi\omega_n)}$$

where, ξ = Damping factor or Damping ratio

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

Then output, $$C(s) = R(s)\frac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n^2} \qquad ...(2.4.1)$$

2. For unit step input

$$r(t) = 1$$

$$R(s) = \frac{1}{s}$$

Then $$C(s) = \frac{1}{s}\,\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad ...(2.4.2)$$

3. In eq. (2.4.2) putting $[s^2 + 2\xi\omega_n s + \omega_n^2] = [(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)]$ and breaking it into partial fraction

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{[(s+\xi\omega_n)^2 + \omega_n^2(1-\xi^2)]} \qquad ...(2.4.3)$$

Put $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\,\omega_n)^2 + \omega_d^2} \qquad ...(2.4.4.)$$

4. Rewrite eq. (2.4.4)

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\,\omega_n)^2 + \omega_d^2} - \frac{\xi\,\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\xi\,\omega_n)^2 + \omega_d^2} \qquad ...(2.4.5)$$

5. Taking inverse Laplace transform of eq. (2.4.5),

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d}\,e^{-\xi\omega_n t}\sin\omega_d t$$

Put $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}[(\sqrt{1-\xi^2})\cos\omega_d t + \xi\sin\omega_d t] \quad ...(2.4.6)$$

6. Put $\sin\phi = \sqrt{1-\xi^2}$,

$\therefore$ $\cos\phi = \xi$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}(\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi) \quad ...(2.4.7)$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$

and $\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

7. Eq. (2.4.7) is rewritten as

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right] \quad ...(2.4.8)$$

8. The term ω_n is called natural frequency of oscillations. Term $\omega_d = \omega_n\sqrt{1-\xi^2}$ is called damped frequency of oscillations and the term ξ is called damping ratio or damping factor .

a. Underdamped case ($0 < \xi < 1$) : From eq. (2.4.8), time constant is $1/\xi\omega_n$ and the response having damped oscillations with overshoot and undershoot is known as underdamped response.

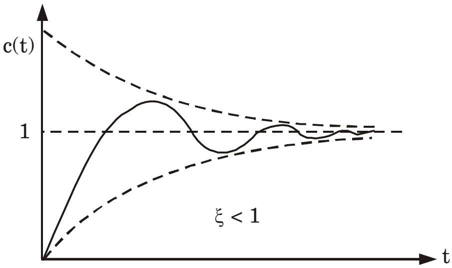

Fig. 2.4.2. Underdamped oscillations.

b. Undamped case ($\xi = 0$) :

Put $\xi = 0$

Eq. (2.4.8) becomes,

$$c(t) = 1 - \sin(\omega_n t + \pi/2)$$

$$c(t) = 1 - \cos\omega_n t$$

The system will oscillate at undamped frequency ω_n.

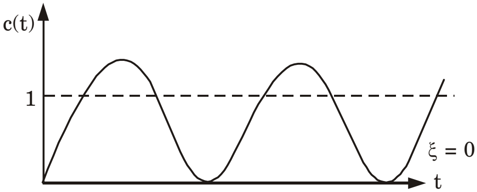

Fig. 2.4.3. Undamped oscillations.

c. Critically damped case (ξ = 1) :

At $\xi = 1$

$$c(t) = 1 + e^{-\xi\omega_n t}(1 + \omega_n t)$$

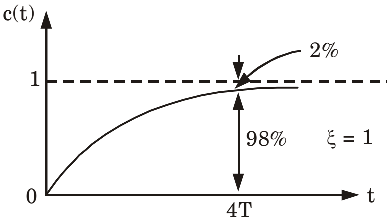

Fig. 2.4.4. Critically damped.

d. Overdamped case (ξ > 1) :

$$C(s) = \frac{\omega_n^2}{\left(s + \xi\omega_n + \omega_n\sqrt{\xi^2 - 1}\right)\left(s + \xi\omega_n - \omega_n\sqrt{\xi^2 - 1}\right)s}$$

Taking inverse Laplace both side,

At $\xi > 1$
$$c(t) = 1 + \underbrace{\frac{e^{-\left(\xi - \sqrt{\xi^2 - 1}\right)\omega_n t}}{2\sqrt{\xi^2 - 1}(\xi + \sqrt{\xi^2 - 1})}}_{\text{neglect}} - \frac{e^{-\left(\xi - \sqrt{\xi^2 - 1}\right)\omega_n t}}{2\sqrt{\xi^2 - 1}.(\xi - \sqrt{\xi^2 - 1})}$$

$$c(t) = 1 - \frac{e^{-\left(\xi - \sqrt{\xi^2 - 1}\right)\omega_n t}}{2\sqrt{\xi^2 - 1}(\xi - \sqrt{\xi^2 - 1})}$$

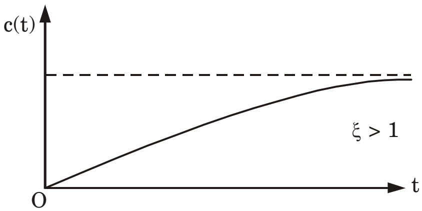

Fig. 2.4.5. Overdamped.

Que 2.5. **Prove that the servomechanism system is a second order system *i.e.*,**

$$T(s) = \frac{\omega_n^2}{(s^2 + 2\xi\,\omega_n s + \omega_n^2)}$$

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Answer

1.

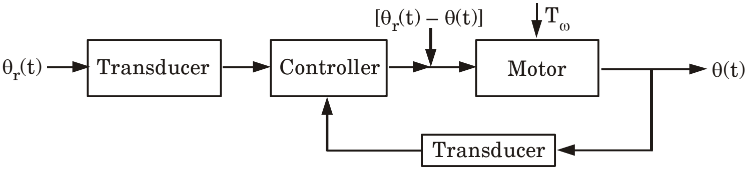

Fig. 2.5.1.

2. The rotation of motor can be described by the differential equation

$$J\ddot{\theta}(t) + B\dot{\theta}(t) = T_m + T_w \quad ...(2.5.1)$$

where, T_m = Torque developed by motor

T_w = Wind torque (disturbance)

B = Coefficient of viscous friction

J = Moment of inertia of motor and load

3. The torque developed by the motor is assumed to be proportional to $u(t)$, the input voltage to the motor, so that

$$T_m(t) = Ku(t) = K[\theta_r(t) - \theta(t)]$$

4. Putting $T_m(t)$ in eq. (2.5.1)

$$J\ddot{\theta}(t) + B\dot{\theta}(t) = K[\theta_r(t) - \theta(t)] \quad [T_w = 0]$$

$$\therefore J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = K\theta_r(t) \quad ...(2.5.2)$$

5. Taking Laplace transform on both sides

$$Js^2\theta(s) + Bs\,\theta(s) + K\theta(s) = K\theta_r(s)$$

$$[Js^2 + Bs + K]\,\theta(s) = K\theta_r(s)$$

$$\frac{\theta(s)}{\theta_r(s)} = \frac{K}{[Js^2 + Bs + K]} = \left[\frac{K/J}{s^2 + \frac{B}{J}s + \frac{K}{J}}\right]$$

6. Putting $K/J = \omega_n^2$

$$\frac{B}{J} = 2\xi\omega_n$$

$$\frac{\theta(s)}{\theta_r(s)} = \frac{C(s)}{R(s)} = \frac{\omega_n^{\,2}}{s^2 + 2\xi\,\omega_n\,s + \omega_n^{\,2}}$$

Thus, the servomechanism system is a second order system.

PART-2

Steady State Errors and Errors Constants.

CONCEPT OUTLINE : PART-2

- The difference between the steady-state response and desired reference give steady state error,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1 \pm G(s)H(s)}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.6. **Discuss steady state error.**

Answer

1. The difference between the steady-state response and desired reference give steady state error.
2. If the actual output of control system during steady state error deviates from the reference input, then system possess steady state error.
3. Steady error helps in determining the accuracy, so the steady state error should be minimum.
4. The steady state performance of a control system is assessed by the magnitude of the steady state error possessed by the system and the system input can be step or ramp or parabolic.
5. The magnitude of steady state error in a closed loop control system depends on its open loop control function. *i.e.*, $G(s)H(s)$ of the system.

Fig. 2.6.1. Canonical diagram.

6. $$E(s) = R(s) - B(s)$$

But $$B(s) = C(s)H(s)$$

$\therefore$ $$E(s) = R(s) - C(s)H(s)$$

Also $$C(s) = G(s)E(s)$$

7. $$E(s) = R(s) - G(s)E(s)H(s)$$

$$E(s)[1 + G(s)\, H(s)] = R(s)$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)} \quad \text{...(2.6.1)}$$

If positive feedback is used, the expression will be

$$E(s) = \frac{R(s)}{1-G(s)H(s)} \quad \text{...(2.6.2)}$$

8. Combining eq. (2.6.1), and (2.6.2)

$$E(s) = \frac{R(s)}{1 \pm G(s)H(s)} \quad \text{...(2.6.3)}$$

For a unity feedback system $H(s) = 1$

9. $\therefore$ $$E(s) = \frac{R(s)}{1 \pm G(s)} \quad \text{...(2.6.4)}$$

10. In time domain $e(t) = L^{-1}[E(s)]$. It is the expression of error valid for all time. Steady state error is defined as :

$$e_{ss} = \lim_{t\to\infty} e(t)$$

11. Using final value theorem

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1 \pm G(s)H(s)}$$

Que 2.7. **Derive an expression for K_p, K_v, and K_a for type-1 system.**

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OR

Derive an expression for K_p, K_v and K_a for type-1, type-2 and type-3 system.

Answer

i. **Unit step input :**

Here $$r(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0 \end{cases}$$

$$R(s) = \frac{1}{s}$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)} \qquad \text{...(2.7.1)}$$

$$E(s) = \frac{1}{s + sG(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{1}{s + sG(s)H(s)}$$

$$= \frac{1}{1 + \lim_{s \to 0} G(s)H(s)} = \frac{1}{1 + K_p}$$

where, $K_p = \lim_{s \to 0} G(s)H(s)$

K_p = Position error constant.

For step input, $e_{ss} = \dfrac{1}{1 + K_p}$

Case 1 : For type '0'

K_p = Constant

e_{ss} = Constant

Case 2 : For type '1'

$K_p = \infty$

$$e_{ss} = \frac{1}{1 + \infty} = 0$$

Case 3 : For type '2'

$K_p = \infty$

$$e_{ss} = \frac{1}{1 + \infty} = 0$$

ii. Ramp input :

Here $r(t) = \begin{cases} t & ; \quad t > 0 \\ 0 & ; \quad t < 0 \end{cases}$

$$R(s) = \frac{1}{s^2}$$

$$E(s) = \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$= \frac{1}{\lim_{s \to 0} sG(s)H(s)}$$

$$e_{ss} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

K_v = Velocity error constant.

Case 1 : For type '0'

$$K_v = \lim_{s\to 0} sG(s)H(s) = 0$$

$$e_{ss} = \frac{1}{K_v} = \infty$$

Case 2 : For type '1'

K_v = constant

$$e_{ss} = \frac{1}{K_v} = \text{constant}$$

Case 3 : For type '2'

$$K_v = \infty$$

$$e_{ss} = 0$$

iii. Parabolic input :

Here
$$r(t) = \begin{cases} (1/2)\,t^2 & ; \quad t > 0 \\ 0 & ; \quad t < 0 \end{cases}$$

$$R(s) = \frac{1}{s^3}$$

$$E(s) = \frac{1}{s^3 + s^3 G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$e_{ss} = \frac{1}{\lim_{s\to 0} s^2 G(s)H(s)} = \frac{1}{K_a}$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

K_a = Acceleration error constant.

Case 1 : For type '0'

$$K_a = 0$$

$$e_{ss} = \frac{1}{0} = \infty$$

Case 2 : For type '1'

$$K_a = 0$$

$$e_{ss} = \frac{1}{0} = \infty$$

Case 3 : For type '2'

K_a = Constant

e_{ss} = Constant

Que 2.8. **Find K_p, K_v, K_a for the system having :**

i. $G(s) = \dfrac{10}{s^2}$ **and** $H(s) = 0.7$

ii. $G(s) = \dfrac{5}{s^2 + 3s + 5}$, $H(s) = 0.6$

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Answer

i. **Given :** $G(s) = 10/s^2$ and $H(s) = 0.7$
To Find : K_p, K_v and K_a.

1.
$$K_p = \lim_{s \to 0} G(s)\, H(s) = \lim_{s \to 0} \frac{10}{s^2} \times 0.7$$
$$K_p = \infty$$

2.
$$K_v = \lim_{s \to 0} s\, G(s)\, H(s) = \lim_{s \to 0} s \times \frac{10}{s^2} \times 0.7$$
$$K_v = \lim_{s \to 0} \frac{7}{s}$$
$$K_v = \infty$$

3.
$$K_a = \lim_{s \to 0} s^2\, G(s)\, H(s) = \lim_{s \to 0} s^2 \times \frac{10}{s^2} \times 0.7$$
$$K_a = 7$$

ii. **Given :** $G(s) = \dfrac{5}{(s^2 + 3s + 5)}$, $H(s) = 0.6$

To Find : K_p, K_v and K_a.

1.
$$K_p = \lim_{s \to 0} \frac{5 \times 0.6}{(s^2 + 3s + 5)}$$
$$K_p = \frac{3}{5}$$

2.
$$K_v = \lim_{s \to 0} \frac{s \times 5}{(s^2 + 3s + 5)} \times 0.6 = \lim_{s \to 0} \frac{3s}{(s^2 + 3s + 5)}$$
$$K_v = 0$$

3.
$$K_a = \lim_{s \to 0} \frac{s^2 \times 5}{(s^2 + 3s + 5)} \times 0.6$$
$$K_a = 0$$

Que 2.9. **A unity feedback system has transfer function $G(s) = \dfrac{K}{s(s + 2)(s^2 + 2s + 5)}$, determine steady state error if input is $r(t) = 2 + 4t + \dfrac{t^2}{2}$.**

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Answer

Given : $G(s) = \dfrac{K}{s(s+2)(s^2+2s+5)}$, $H(s) = 1$

$$r(t) = 2 + 4t + \frac{t^2}{2}$$

To Find : Steady state error, e_{ss}.

1.
$$G(s)\,H(s) = \frac{K}{s(s+2)(s^2+2s+5)}$$

$$K_p = \lim_{s \to 0} G(s)\,H(s) = \infty$$

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s)$$

$$= \lim_{s \to 0} \frac{sK}{s(s+2)(s^2+2s+5)} = \frac{K}{10}$$

$$K_a = \lim_{s \to 0} s^2\,G(s)\,H(s) = 0$$

2. Now,
$$r(t) = 2 + 4t + \frac{t^2}{2}$$

$A_1 = 2$ (Constant)

$A_2 = 4$ (Coeff. of t)

$A_3 = 1$ (Coeff. of $t^2/2$)

$$e_{ss} = \frac{A_1}{1+K_p} + \frac{A_2}{K_v} + \frac{A_3}{K_a} = \frac{2}{\infty} + \frac{4}{\frac{K}{10}} + \frac{1}{0}$$

$$= 0 + 40/K + \infty = \infty$$

Que 2.10. **Discuss different type of test signal used for analysis of control system in time domain.**

The reference input to a unity feedback system is shown in Fig. 2.10.1. The open loop transfer function of the system is

$$\mathbf{G(s)} = \frac{\mathbf{400(s+1)}}{\mathbf{(s+2)(s+8)}}$$

Calculate the steady state error.

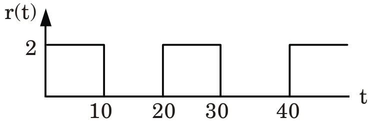


Fig. 2.10.1.

AKTU 2016-17, Marks 10

Answer

Types of test signal : Refer Q. 2.1, Page 2–2C, Unit-2.

Numerical :

Given : $G(s) = \dfrac{400(s+1)}{(s+2)(s+8)}$, $H(s) = 1$

To Find : Steady state error.

1. Laplace of periodic waveform using formula :

$$R(s) = \frac{R_1(s)}{1-e^{-TS}}$$

2. First we have to find Laplace of $r(t)$,

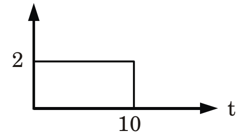


Fig. 2.10.2.

By integration method

$$R_1(s) = \int_0^{10} 2e^{-st}\, dt$$

$$= \frac{2}{(-s)}\left[e^{-st}\right]_0^{10}$$

$$R_1(s) = \frac{2}{s}(1-e^{-10s})$$

4. Here, $T = 20$, So $R(s) = \dfrac{R_1(s)}{1-e^{-TS}}$

$$R(s) = \frac{\frac{2}{s}(1-e^{-10s})}{(1-e^{-20s})}$$

$$R(s) = \frac{\frac{2}{s}(1-e^{-10s})}{(1-e^{-10s})(1+e^{-10s})}$$

$$R(s) = \frac{2}{s(1+e^{-10s})}$$

5. Using formula

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0}\frac{s\dfrac{2}{s(1+e^{-10s})}}{1+\dfrac{400(s+1)}{(s+2)(s+8)}}$$

$$= \lim_{s \to 0} \frac{\dfrac{2}{(1+e^{-10s})}}{\dfrac{(s+2)(s+8)+400(s+1)}{(s+2)(s+8)}}$$

$$= \lim_{s \to 0} \frac{2(s+2)(s+8)}{[(s+2)(s+8)+400(s+1)](1+e^{-10s})}$$

$$= \frac{2(0+2)(0+8)}{[(0+2)(0+8)+400(0+1)](1+e^{0})}$$

$$= \frac{2 \times 2 \times 8}{2 \times 8 + 400} = \frac{32}{416}$$

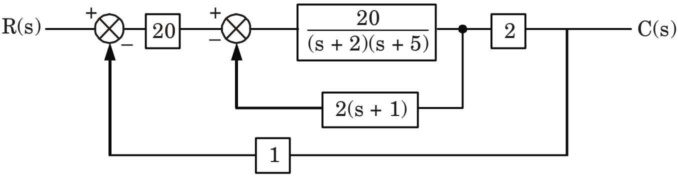
$$\therefore \qquad e_{ss} = 0.0769$$

Que 2.11. **For the system shown in figure, determine the type of system, error coefficient and the error for the following inputs :**

i. $r(t) = 6$

ii. $r(t) = 8t$

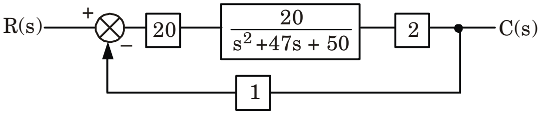
iii. $r(t) = 10 + 4t + 15t^2$



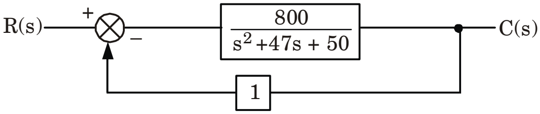
AKTU 2017-18, Marks 10

Answer

Step 1 :



Step 2 :



So, $$G(s) = \frac{800}{s^2 + 47s + 50} \text{ and } H(s) = 1$$

$$G(s)\,H(s) = \frac{800}{s^0(s^2 + 47s + 50)}$$

$\therefore$ Type of system is zero.

i. $r(t) = 6$

So, $R(s) = \dfrac{6}{s} \Rightarrow A = 6$, which is coefficient of $\dfrac{1}{s}$

Now, $K_p = \lim\limits_{s \to 0} G(s)\, H(s) = \dfrac{800}{50}$

Now, $e_{ss} = \dfrac{A}{1+K_p} = \dfrac{6}{1+\dfrac{800}{50}} = \dfrac{6 \times 50}{850} = 0.353$

ii. $r(t) = 8\, t$

So, $R(s) = \dfrac{8}{s^2} \Rightarrow A = 8$, which is coefficient of $\dfrac{1}{s^2}$

Now, $K_v = \lim\limits_{s \to 0} sG(s)\, H(s)$

$$= \lim_{s \to 0} \frac{s \times 800}{s^2 + 47s + 50} = 0$$

Now, $e_{ss} = \dfrac{A}{K_v} = \dfrac{8}{0} = \infty$

iii. $r(t) = 10 + 4t + 15t^2$

$$= 10 + 4t + 30\,\frac{t^2}{2}$$

Now, So, $A_1 = 10$, $A_2 = 4$, $A_3 = 30$

$K_p = \lim\limits_{s\to 0} G(s)H(s)$

$= \dfrac{800}{850} = 0.941$

$K_v = \lim\limits_{s\to 0} sG(s)H(s)$

$= 0$

$K_a = \lim\limits_{s\to 0} s^2\, G(s)H(s)$

$= 0$

Now, $e_{ss} = \dfrac{A_1}{1+K_p} + \dfrac{A_2}{K_v} + \dfrac{A_3}{K_a}$

$$= \frac{10}{1+0.941} + \frac{4}{0} + \frac{30}{0} = \infty$$

Que 2.12. **For a unity feedback system having**

$$\boldsymbol{G(s) = \frac{35(s+4)}{s(s+2)(s+5)}}$$

find (i) the type of the system, (ii) all error coefficients and (iii) errors for ramp input with magnitude 5. **AKTU 2017-18, Marks 10**

Answer

Given : $G(s) = \dfrac{35(s+4)}{s(s+2)(s+5)}$, $H(s) = 1$

To Find : Type of the system, All error coefficients and Errors for ramp input with magnitude 5.

i. **Type :** System is type 1 system. As we have s^1 in common in denominator.

ii. **Error coefficients :**

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$= \lim_{s \to 0} \frac{35(s+4)}{s(s+2)(s+5)} \times 1 = \infty$$

$$K_v = \lim_{s \to 0} s\, G(s)H(s)$$

$$= \lim_{s \to 0} \frac{s \times 35\,(s+4)}{s\,(s+2)\,(s+5)} \times 1$$

$$= \frac{35 \times 4}{10} = 14$$

$$K_a = \lim_{s \to 0} s^2\, G(s)H(s)$$

$$= \lim_{s \to 0} s^2 \frac{35(s+4)}{s(s+2)(s+5)} \times 1 = 0$$

iii. **Error :** Magnitude, $A = 5$ and $K_v = 14$

$$e_{ss} = \frac{A}{K_v} = \frac{5}{14}$$

$$e_{ss} = 0.357.$$

Que 2.13. **A unity feedback system has an OLTF $G(s) = \dfrac{K(s+2)}{s(s^3+7s^2+42s)}$. Find the static error constant and e_{ss} due to an input $r(t) = t^2u(t)$.**

AKTU 2017-18, Marks 10

Answer

Given : $G(s) = \dfrac{K(s+2)}{s(s^3+7s^2+42s)}$, $r(t) = t^2u(t)$

To Find : K_a and e_{ss}.

1. $$r(t) = t^2u(t) = 2\,\frac{t^2}{2}u(t)$$

$\therefore$ $A = 2.$ [Coeff. of $t^2/2$]

2. Now, $$G(s)H(s) = \frac{K(s+2)}{s(s^3+7s^2+42s)} = \frac{K(s+2)}{s^2(s^2+7s+42)}$$

3. Now, $$K_a = \lim_{s \to 0} s^2\, G(s)\, H(s) = \lim_{s \to 0} \frac{s^2\, K(s+2)}{s^2(s^2+7s+42)} = \frac{2K}{42} = \frac{K}{21}$$

4. $$e_{ss} = \frac{A}{K_a} = \frac{2}{\dfrac{K}{21}} = \frac{42}{K}$$

PART-3

Time Response Specifications, Design Specifications of Second Order Systems.

CONCEPT OUTLINE : PART-3

- **Delay time (t_d) :** It is the time required for response to reach 50 % of final value in first time.
- **Rise time (t_r) :** It is the time required for the response to rise from 10 % to 90 % of its final value for overdamped and 0 to 100 % for underdamped system.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.14. **Define the following terms of second order system :**

i. Delay time **ii. Rise time**

iii. Peak time **iv. Steady state error**

v. Settling time

AKTU 2013-14, Marks $6\frac{2}{3}$

OR

Draw time domain response curve of a second order system and indicate important specifications.

AKTU 2014-15, Marks 05

Answer

1. **Delay time (t_d) :** It is the time required for the response to reach 50 % of the final value in first time.
2. **Rise time (t_r) :** It is the time required for the response to rise from 10 % to 90 % of its final value for overdamped system and 0 to 100 % for underdamped systems.

$$t_r = \frac{\pi - \tan^{-1}\dfrac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

3. **Peak time (t_p) :** The peak time is the time required for the response to reach the first peak of the time response or first peak overshoot.

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

4. **Maximum overshoot (M_p) :** It is the normalized difference between the peak of the time response and steady output. The maximum percent overshoot is defined as

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$

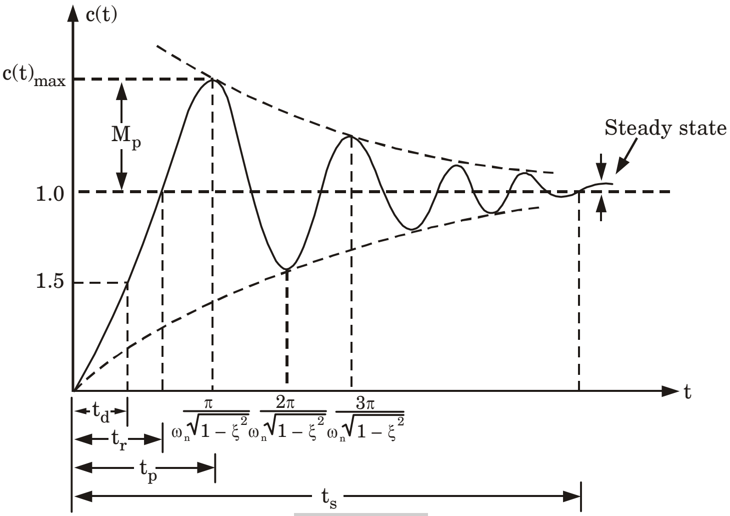


Fig. 2.14.1.

5. **Settling time (t_s) :** The setting time is the time required for the response to reach and stay within the specified range (2 % to 5 %) of its final value.

$$t_s = \frac{4}{\xi\omega_n}$$

6. **Steady state error (e_{ss}) :** It is the difference between actual output and desired output as time 't' tends to infinity.

$$e_{ss} = \lim_{t\to\infty}\left[r(t) - c(t)\right]$$

Que 2.15. **Derive the expression for :**

i. Rise time

ii. Peak overshoot time for the second order.

AKTU 2013-14, Marks $6\frac{2}{3}$

Answer

A. The rise time, t_r :

1. The time response of a second order control system is given by

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin[(\omega_n\sqrt{1-\xi^2})t + \phi]$$

2. Time t_r is the time at which response reaches 100 % *i.e.*, $c(t) = 1$.

$$1 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}}\sin[(\omega_n\sqrt{1-\xi^2})t_r + \phi]$$

$$\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}}\sin[(\omega_n\sqrt{1-\xi^2})t_r + \phi] = 0$$

$$\sin[(\omega_n\sqrt{1-\xi^2})t_r + \phi] = 0 \qquad [\sin n\pi = 0,\ n = 1]$$

$$[(\omega_n\sqrt{1-\xi^2})t_r + \phi] = \pi$$

3. $\therefore$

$$t_r = \frac{\pi - \phi}{\omega_n\sqrt{1-\xi^2}}$$

where,

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

B. Maximum overshoot, M_p :

1.

$$M_p = c(t)_{\max} - 1$$

$$\% M_p = \frac{c(t)_{\max} - 1}{1} \times 100$$

2.

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin[(\omega_n\sqrt{1-\xi^2})t + \phi]$$

$$\therefore \quad \frac{dc(t)}{dt} = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_n\sqrt{1-\xi^2}\cos[\omega_n\sqrt{1-\xi^2})t+\phi] - \frac{(-\xi\omega_n)e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin[(\omega_n\sqrt{1-\xi^2})t+\phi]$$

3. For maximum, Put $\frac{dc(t)}{dt} = 0$

$$\therefore \quad \frac{e^{-\xi w_n t}}{\sqrt{1-\xi^2}}\{-\omega_n\sqrt{1-\xi^2}\cos[(\omega_n\sqrt{1-\xi^2})t+\phi] + \xi\omega_n\sin[(\omega_n\sqrt{1-\xi^2})t+\phi]\} = 0$$

$$\therefore \quad \omega_n\sqrt{1-\xi^2}\cos[(\omega_n\sqrt{1-\xi^2})t+\phi] = \xi\omega_n\sin[(\omega_n\sqrt{1-\xi^2})t+\phi]$$

$$\therefore \quad \tan[(\omega_n\sqrt{1-\xi^2})t+\phi] = \frac{\sqrt{1-\xi^2}}{\xi}$$

4. Putting, $\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

$$\tan\left[(\omega_n\sqrt{1-\xi^2})t+\tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] = \frac{\sqrt{1-\xi^2}}{\xi} \qquad ...(2.15.1)$$

5. The general solution of eq. (2.15.1) is

$$(\omega_n\sqrt{1-\xi^2})t = n\pi \qquad ...(2.15.2)$$

where $n = 0, 1, 2...$

At $n = 1$, $\quad t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

6. Putting $t = t_p$ in $c(t)$,

$$c(t)_{\max} = 1-\frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\sin[(\omega_n\sqrt{1-\xi^2})t_p+\phi]$$

$$= 1-\frac{e^{-\xi\omega_n\frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}}\sin\left[(\omega_n\sqrt{1-\xi^2})\frac{\pi}{\omega_n\sqrt{1-\xi^2}}+\phi\right]$$

$$= 1-\frac{e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}}\sin(\pi+\phi) = 1-\frac{e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}}(-\sin\phi)$$

As, $\quad \phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$, $\therefore \sin\phi = \sqrt{1-\xi^2}$

Hence, $\quad c(t)_{\max} = 1+\frac{e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}}\sqrt{1-\xi^2}$

$$c(t)_{\max} = 1 + e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

7. $$M_p = c(t)_{\max} - 1$$

$$M_p = \left(1 + e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}\right) - 1$$

$$M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\% M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \times 100$$

Que 2.16. Derive expression for resonant frequency and resonant peak for second order control system. **AKTU 2015-16, Marks 10**

Answer

1. For second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n^2}$$

2. Putting $s = j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{-\omega^2 + 2\xi\,\omega_n j\omega + \omega_n^2}$$

$$M = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\left(\frac{\omega}{\omega_n}\right)}$$

Let $\frac{\omega}{\omega_n} = x$

$$M = \frac{1}{1 - x^2 + j2\xi x} = \frac{1}{\sqrt{(1-x^2)^2 + 4\xi^2 x^2}}$$

3. To maximize magnitude, put $\frac{dM}{dx} = 0$,

$$\frac{dM}{dx} = \frac{d}{dx}[(1-x^2)^2 + 4\xi^2 x^2]^{-1/2}$$

$$= -\frac{1}{2}[(1-x^2)^2 + 4\xi^2 x^2]^{-3/2}\frac{d}{dx}[(1-x^2)^2 + 4\xi^2 x^2]$$

$$= -\frac{1}{2}\frac{1}{[(1-x^2)^2 + 4\xi^2 x^2]^{3/2}}[2(1-x^2)(-2x) + 8\xi^2 x]$$

$$= 0$$

$$4x[x^2 + 2\xi^2 - 1] = 0$$

As, $x \neq 0$

$\therefore$ $x^2 = 1 - 2\xi^2$

$$x = \sqrt{1 - 2\xi^2}$$

$$\omega_r = \omega_n \sqrt{1 - 2\xi^2}$$

4. $$|M_r| = \frac{1}{\sqrt{(1 - x^2)^2 + 4\xi^2 x^2}}$$

$$|M_r| = \frac{1}{\sqrt{(1 - (\sqrt{1 - 2\xi^2}))^2 + 4\xi^2(\sqrt{1 - 2\xi^2})^2}}$$

$$|M_r| = \frac{1}{\sqrt{4\xi^4 + 4\xi^2(1 - 2\xi^2)}}$$

$$M_r = \frac{1}{2\xi\sqrt{1 - \xi^2}}$$

Que 2.17. **The open loop transfer function of a unity feedback system is :**

$$G(s) = \frac{\alpha}{s(1 + \beta s)}$$

For this system overshoot reduces from 0.6 to 0.2 due to change in α only. Show that :

$$\frac{(\beta\alpha_1 - 1)}{(\beta\alpha_2 - 1)} \cong 43.$$

where α_1 and α_2 are values of α for 0.6 and 0.2 overshoot respectively.

AKTU 2013-14, Marks $6\frac{2}{3}$

Answer

Given : $G(s) = \dfrac{\alpha}{s(1 + \beta s)}$, $H(s) = 1$, M_p = from 0.6 to 0.2

To Prove : $\dfrac{\beta\alpha_1 - 1}{\beta\alpha_1 - 2} \cong 43.$

1. M_p reduces from 0.6 to 0.2 due to change in α only.
   Transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} = \frac{\alpha / s\,(1+\beta s)}{1 + \dfrac{\alpha}{s(1+\beta s)}} = \frac{\alpha}{\beta s^2 + s + \alpha} = \left(\frac{\alpha/\beta}{s^2 + \dfrac{1}{\beta}s + \dfrac{\alpha}{\beta}}\right)$$

2. Compare denominator by $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$2\xi\omega_n = \frac{1}{\beta} \quad \therefore \quad \xi = \frac{1}{2\omega_n \beta}$$

$$\omega_n = \sqrt{\frac{\alpha}{\beta}}$$

$$\xi = \frac{\sqrt{\beta}}{2\beta\sqrt{\alpha}} = \frac{1}{2\sqrt{\alpha\beta}}$$

3. Maximum overshoot,

$$M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\log_e 0.6 = \log_e e^{\frac{-\xi_1\pi}{\sqrt{1-\xi_1^2}}}$$

$$\log_e 0.6 = -\frac{\xi_1 \pi}{\sqrt{1-\xi_1^2}}$$

$$-0.5 = -\frac{\xi_1 \pi}{\sqrt{1-\xi_1^2}}$$

$$(1-\xi_1{}^2)\, 0.25 = \xi_1{}^2 \pi^2$$

$$\frac{(1-\xi_1^2)}{\xi_1^2} = \frac{9.87}{0.25} = 39.48$$

$$39.48\, \xi_1{}^2 + \xi_1{}^2 - 1 = 0$$

$$40.48\, \xi_1{}^2 = 1$$

$$\xi_1 = 0.157 = \frac{1}{2\sqrt{\alpha_1\beta}}$$

$$\alpha_1\beta = 10.142$$

$$\therefore \quad \alpha_1\beta - 1 = 9.142 \qquad ...(2.17.1)$$

4. Similarly for ξ_2 at $\alpha_2 = 0.2$, we get

$$\xi_2 = 0.4538 = \frac{1}{2\sqrt{\alpha_2\,\beta}} \qquad ...(2.17.2)$$

$$\alpha_2\beta = 1.214$$

$$\therefore \quad \alpha_2\beta - 1 = 0.214$$

5. Dividing eq. (2.17.1) and (2.17.2), we get

$$\frac{\alpha_1\,\beta - 1}{\alpha_2\,\beta - 1} = 42.72$$

$$\cong 43$$

Que 2.18. **A unity feedback system has a forward path transfer function $G(s) = \frac{(s+2)}{s\,(s+1)}$. Determine rise time, peak time and settling time (2 % tolerance).**

AKTU 2014-15, Marks 05

Answer

Given : $G(s) = \dfrac{(s+2)}{s(s+1)}$, $H(s) = 1$

To Find : t_r, t_p and t_s.

1. The characteristic equation for unity feedback control system is given by

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{(s+2)}{s(s+1)} = 0$$

$$s^2 + s + s + 2 = 0$$

$$s^2 + 2s + 2 = 0$$

2. Comparing by standard second order characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \sqrt{2} \text{ rad/sec}$$

$$2\xi\omega_n = 2$$

$$\xi = \frac{1}{\sqrt{2}} = 0.707$$

3. Rise time, $t_r = \dfrac{\pi - \phi}{\omega_d}$

$$\phi = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right] = 45° = 0.7853 \text{ (in radian)}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{2}\sqrt{1-(0.707)^2} = 1$$

$$t_r = 2.3562 \text{ sec}$$

4. Peak time, $t_p = \dfrac{\pi}{\omega_d} = 3.1415$ sec

5. Setting time, t_s (for 2%) $= \dfrac{4}{\xi\omega_n} = \dfrac{4}{(0.707)(\sqrt{2})}$

$$t_s = 4 \text{ sec.}$$

Que 2.19. **Derive the expression for second order system response when subjected to unit impulse input for damping ratio (ξ) < 1. An unity feedback system is characterized by an open loop transfer function.**

$$G(s) = \frac{K}{s(s+10)}$$

Determine the gain ‘K’ so that the system will have a damping ratio of 0.5. For this value of ‘K’, determine the settling time, peak overshoot and time to peak overshoot for a unit step input.

AKTU 2016-17, Marks 10

Answer

Derivation for second order system response :

1. For unit impulse function, $R(s) = 1$ and the output of a second order system is given by

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad ...(2.19.1)$$

2. In eq. (2.19.1) rewriting the term
$(s^2 + 2\xi\omega_n s + \omega_n^2)$ as $[(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)]$

$$C(s) = \frac{\omega_n^2}{[(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)]}$$

$$C(s) = \frac{\omega_n^2}{\omega_n\sqrt{1 - \xi^2}} \frac{\omega_n\sqrt{1 - \xi^2}}{[(s + \xi\omega_n)^2 + (\omega_n\sqrt{1 - \xi^2})^2]} \quad ...(2.19.2)$$

3. Taking inverse Laplace transform on both sides of eq. (2.19.2),

$$c(t) = \frac{\omega_n}{\sqrt{1 - \xi^2}} e^{-\xi\omega_n t} \sin[(\omega_n\sqrt{1 - \xi^2})t] \quad ...(2.19.3)$$

4. For $\xi < 1$,

$$c(t) = \frac{\omega_n}{\sqrt{1 - \xi^2}} e^{-\xi\omega_n t} \sin[(\omega_n\sqrt{1 - \xi^2})t] \quad ...(2.19.4)$$

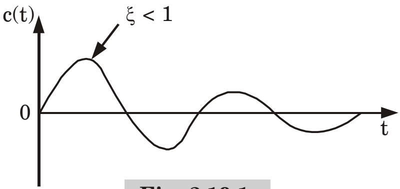


Fig. 2.19.1.

5. The time response for $\xi < 1$ is decaying exponential oscillations and the output at times goes negative also.

Numerical :

Given : $G(s) = \dfrac{K}{s(s+10)}$, $\xi = 0.5$, $H(s) = 1$

To Find : K, t_s, t_p and M_p.

1.
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{K}{s^2 + 10s + K} \quad ...(2.19.5)$$

2. For second order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad ...(2.19.6)$$

3. Comparing eq. (2.19.5) and (2.19.6), we get

$$\omega_n = \sqrt{K} \text{ rad/sec}$$

$$\because \quad 2\xi\omega_n = 10$$

$$2 \times 0.5 \times \sqrt{K} = 10$$

$$\sqrt{K} = 10$$

$$\therefore \quad K = 100$$

4. Peak overshoot, $M_p = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} = e^{-\left(\frac{0.5\pi}{\sqrt{1-0.5^2}}\right)} = 0.1630$

5. Settling time, $t_s = \dfrac{4}{\xi\omega_n} = \dfrac{4}{0.5 \times 10} = 0.8$ sec

6. Peak time, $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi}{10\sqrt{1-(0.5)^2}} = 0.36$ sec.

PART-4

Proportional, Derivative, Integral and PID Compensations, Design Considerations for Higher Order System and Performance Indices.

CONCEPT OUTLINE : PART-4

- **P controller (Proportional controller) :** It is a control system technology based on a response in proportion to the difference between what is set as a desired process variable (or set point) and the current value of the variable.
- **PI controller (Proportional integral controller) :** A controller in the forward path which changes the controller output corresponding to the proportional plus integral of error signal is called PI controller.
- **PID controller (Proportional, integral and derivative controller) :** It is a close loop system which has feedback control system and it compares the process variable (feedback variable) with set point and generates an error signal and according to that it adjusts the output of system.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.20. **Write a short note on proportional control.**

Answer

Proportional control :

1. In proportional control the actuating signal for the control action in a control system is proportional to the error signal.
2. The error signal being the difference between the reference input signal and the feedback signal obtained from the output.
3. For the system considered as shown in Fig. 2.20.1, the actuating signal is proportional to the error signal, therefore, the system is called proportional control system.
4. Consider a second order system where controller input is error itself and proportional constant is $K = 1$ as shown in Fig. 2.20.1.

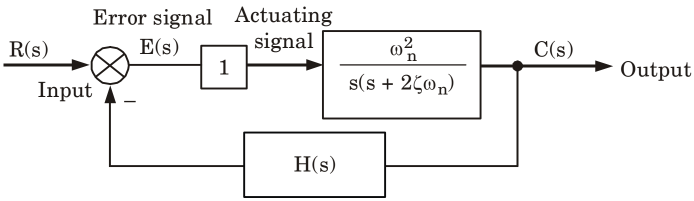


Fig. 2.20.1.

$$G(s)H(s) = \frac{\omega_n^2}{s(s + 2\xi\,\omega_n)}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n^2}$$

For this system damping ratio is ξ and natural frequency ω_n.

5. And for steady state error

$$K_p = \lim_{s \to 0} G(s)H(s) = \infty$$

and

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \frac{\omega_n}{2\xi}$$

Effects :

1. Steady state error is reduced.
2. Disturbance signal rejection occurs.
3. Relative stability is improved.

Que 2.21. **Write a short note on proportional derivative compensator stating its merits and demerits.**

AKTU 2014-15, Marks 05

OR

What is the effect of PD (Proportional Derivative) Controller on steady state error due to a unit ramp input in second order system ? Prove mathematically.

AKTU 2013-14, Marks 05

OR

Write a short note on PD controller and synchros.

AKTU 2015-16, Marks 10

Answer

1. A controller in the forward path, which changes the controller output corresponding to proportional plus derivative of error signal is called PD controller.

2. Output of controller = $K\, e(t) + T_d \dfrac{de(t)}{dt}$

   Taking Laplace transform

$$= K\,E(s) + sT_d\,E(s) = E(s)\,[K + sT_d]$$

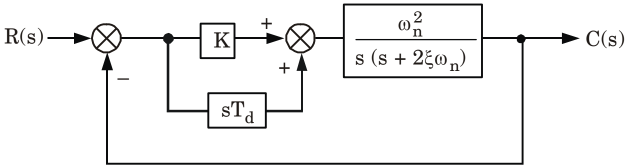


Fig. 2.21.1.

3. Assume, $K = 1$, we can write,

$$G(s) = \frac{(1 + sT_d)\,\omega_n^2}{s(s + 2\xi\,\omega_n)}$$

$$\frac{C(s)}{R(s)} = \frac{(1 + sT_d)\,\omega_n^2}{s^2 + s\,[2\xi\,\omega_n + \omega_n^2\,T_d] + \omega_n^2}$$

4. Comparing denominator with standard form,

$$2\xi'\,\omega_n = 2\xi\,\omega_n + \omega_n^2\,T_d$$

$$\therefore \quad \xi' = \xi + \frac{\omega_n\,T_d}{2}$$

5. Because of this controller, damping ratio increased by factor $\dfrac{\omega_n\,T_d}{2}$.

$$K_p = \lim_{s \to 0} G(s)H(s) = \infty$$

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \frac{\omega_n}{2\xi}$$

   without PD controller,

$$G(s)\,H(s) = \frac{\omega_n^2}{s(s+2\varepsilon\omega_n)} \qquad [K=1]$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = \frac{\omega_n}{2\varepsilon}$$

As there is no change in coefficients, error also will remain same.

6. **Effects :**

i. It increases damping ratio.

ii. 'TYPE' of the system remains unchanged.

iii. It reduces peak overshoot.

iv. It reduces settling time.

v. Steady state error remains unchanged.

7. **Demerits :**

1. It amplifies the noise signal.
2. It may cause saturation effect in the actuator.
3. It does not improve e_{ss} because of generation of a zero in the transfer function.

Synchros : Refer Q. 1.8, Page 1–11C, Unit-1.

Que 2.22. Discuss the effect on the performance of a second order control system of

i. Derivative control

ii. Integral control.

AKTU 2016-17, Marks 10

OR

Discuss the PI and PD controller with their applications. Also find the different error constant for P, I and D.

AKTU 2013-14, Marks 10

Answer

i. **Derivative controller :** Refer Q. 2.21, Page 2–31C, Unit-2.

Application of PD controller :

1. In an industrial plant, a PD controller is applied in applications where overshoot cannot be tolerated. Such as batch pH neutralization.
2. In some positioning applications.

ii. **Proportional integral controller (*PI*) :**

1. A controller in the forward path, which changes the controller output corresponding to the proportional plus integral of the error signal is called PI controller.
2. Output of controller $= K\,e(t) + K_i \int e(t)\,dt$

Taking Laplace $= K\ E(s) + \frac{K_i}{s} E(s) = E(s)\left[K + \frac{K_i}{s}\right]$

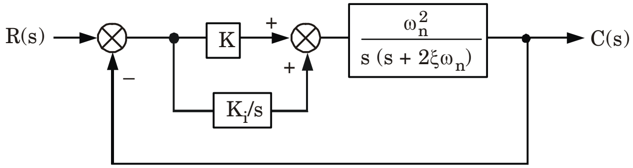


Fig. 2.22.1.

3. Assume, $K = 1$, we can write,

$$G(s) = \frac{\left[1 + \frac{K_i}{s}\right]\omega_n^2}{s(s + 2\xi\ \omega_n)} = \frac{(K_i + s)\ \omega_n^2}{s^2(s + 2\xi\ \omega_n)}$$

i.e., system becomes TYPE 2 in nature.

$$\frac{C(s)}{R(s)} = \frac{(K_i + s)\ \omega_n^2}{s^3 + 2\xi\ \omega_n\ s^2 + s\ \omega_n^2 + K_i\ \omega_n^2}$$

i.e, it becomes third order.

4. Now as order increases by one, system relatively becomes less stable as K_i must be designed in such a way that system will remain in stable condition. Second order system is always stable.

while $K_p = \lim_{s \to 0} G(s)H(s) = \infty$, $e_{ss} = 0$

$K_v = \lim_{s \to 0} s\ G(s)H(s) = \infty$, $e_{ss} = 0$

5. Hence as type is increased by one, error becomes zero for ramp type of input *i.e.*, steady state of system gets improved and system becomes more accurate in nature.
6. **Applications of PI controller :**
i. Pressure control
ii. Temperature control
7. **Effects :**
i. It increases order of the system.
ii. It increases TYPE of the system.
iii. Design of K_i must be proper to maintain stability of system.
vi. Steady state error reduces tremendously for same type of inputs.

Que 2.23. Explain PID controller.

Answer

PID controller (Proportional, integral and derivative controller) :

1. It is a close loop system which has feedback control system and it compares the process variable (feedback variable) with set point and generates an error signal and according to that it adjusts the output of system.

2. It is the combination of proportional, integral and derivative controller.
3. A PD (Proportional derivative) type of controller improves transient part without affecting the steady state part.
4. While PI (Proportional integral) type of controller improves steady state part without affecting the transient part.
5. In PID, both PI and PD effects are incorporated. Hence both transient as well as steady state part of the response can be improved.
6. This can be realized as shown in the Fig. 2.23.1.

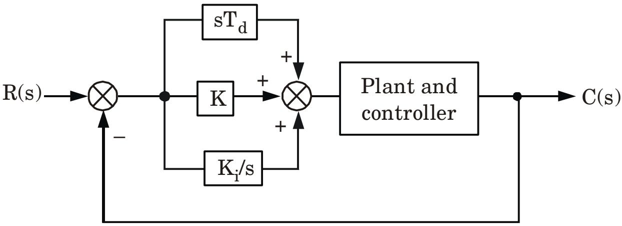


Fig. 2.23.1.

7. The output of PID controller is time domain is,

$$\text{Controller output} = Ke(t) + K_i \int e(t)\,dt + T_d \frac{de(t)}{dt}$$

8. Taking the Laplace transform, controller output in s-domain is,

$$\text{Controller output} = E(s)\left[K + \frac{K_i}{s} + s\,T_d\right]$$

Que 2.24. **Determine K_t so that ξ = 0.6. Find the corresponding time domain specification for system in Fig. 2.24.1.**

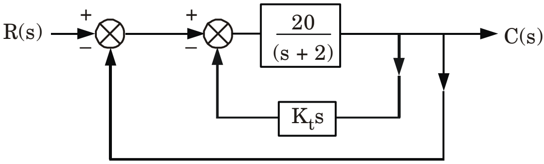


Fig. 2.24.1.

Answer

1. $$\text{Inner loop} = \frac{\dfrac{20}{s(s+2)}}{1 + \dfrac{20sK_t}{s(s+2)}} = \frac{20}{s\,(s + 2 + 20K_t)}$$

2. $\therefore$ $$\frac{C(s)}{R(s)} = \frac{\dfrac{20}{s(s+2+20K_t)}}{1 + \dfrac{20}{s(s+2+20K_t)}} = \frac{20}{s^2 + s\,(2 + 20K_t) + 20}$$

3. $\therefore\ \omega_n^2 = 20,$ $\omega_n = \sqrt{20}$

$$2\xi\omega_m = 2 + 20\,K_t$$

$$\therefore \quad \xi = \frac{2 + 20\,K_t}{2\sqrt{20}} = 0.6 \text{ (given)}$$

$$K_t = 0.1683$$

Que 2.25. **Define about performance index in brief.**

Answer

Performance Index : The goodness of the system performance can be based on the performance index and helps in designing a control system.

a. **Integral Square Error (ISE) :**

$$I = \int_0^{\infty} e^2(t)dt$$

It provides a good compromise between the rise time to limit the effect of a large initial error, reduction of peak overshoot and settling time. When $\xi = 0.5$, I is minimum. It is difficult to calculate for higher order system.

b. **Integral of Absolute Value of Error :**

$$I = \int_0^{\infty} |e(t)|\,dt$$

There are fewer penalties for large error and large penalty for small error. It requires use of computer and has poor sensitivity. A system based on this criteria has enough damping and transient performance.

c. **Integral of Time Multiplied Square Criteria :**

$$I = \int_0^{\infty} te^2(t)dt$$

Large initial error is weighted lightly, while errors occurring late are weighted heavily.

d. **Integral of Square Time Multiplied Square Criteria :**

$$I = \int_0^{\infty} t^2e^2(t)dt$$

Here, more weight is given to large initial errors.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Discuss the time response of first order system with unit step, unit impulse and unit ramp inputs.

Ans. Refer Q. 2.2, Unit-2.

Q. 2. Derive an expression for K_p, K_v, and K_a for type-1 system.

Ans. Refer Q. 2.7, Unit-2.

Q. 3. A unity feedback system has transfer function $G(s) = \dfrac{K}{s(s+2)(s^2+2s+5)}$, determine steady state error if input is $r(t) = 2 + 4t + \dfrac{t^2}{2}$.

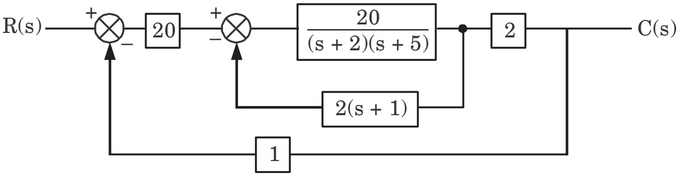
Ans. Refer Q. 2.9, Unit-2.

Q. 4. For the system shown in figure, determine the type of system, error coefficient and the error for the following inputs :

i. $r(t) = 6$,

ii. $r(t) = 8t$

iii. $r(t) = 10 + 4t + 15t^2$



Ans. Refer Q. 2.11, Unit-2.

Q. 5. Define the following terms of second order system :

i. Delay time

ii. Rise time

iii. Peak time

iv. Steady state error

v. Settling time

Ans. Refer Q. 2.14, Unit-2.

Q. 6. Derive expression for resonant frequency and resonant peak for second order control system.

Ans. Refer Q. 2.16, Unit-2.

Q. 7. A unity feedback system has a forward path transfer function $G(s) = \dfrac{(s+2)}{s(s+1)}$. Determine rise time, peak time and settling time (2% tolerance).

Ans. Refer Q. 2.18, Unit-2.

Q. 8. Write a short note on proportional derivative compensator stating its merits and demerits.

Ans. Refer Q. 2.21, Unit-2.

Q. 9. Discuss the effect on the performance of a second order control system of

i. Derivative control

ii. Integral control.

Ans. Refer Q. 2.22, Unit-2.

☺☺☺

3
UNIT

Stability and Algebraic Criteria

PART-1

Concept of Stability and its Necessary Conditions, Routh-Hurwitz Criteria and its Limitations.

CONCEPT OUTLINE : PART-1

- **Concept of stability :** A stable system is one that will remain at rest unless excited by external sources and will return to rest if all the excitations are removed.
- **Routh stability criterion :**

1. Routh stability criterion checks the system stability.
2. If the elements of the first column of the Routh array have :

i. No sign change, it means system is stable.

ii. Sign change, it means system is unstable.

iii. If one row becomes zero, it means system is marginally stable.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **Define stability. State the necessary conditions for system to be absolutely stable.** **AKTU 2017-18, Marks 10**

Answer

Stability :

1. A system is stable if its response (*i.e.*, the transfer function) approaches zero as time approaches infinity.
2. In other words, a system is stable if every bounded input produces a bounded output.

Types of stability :

1. **Absolutely stable :** If a system output is stable for all variations of its parameters, then the system is called absolutely stable system. It gives the information about whether system is stable or unstable.
2. **Relative stability :** The system is said to be relatively more stable or unstable on the basis of settling time. System is said to be relatively more stable if settling time for the system is less than that of the other system. Relative stability gives degree of stability or how close it to instability.

Necessary conditions for absolutely stable :

i. All the coefficients of characteristics equation must have same sign.

ii. There should be no missing term.

iii. All poles of transfer function should be in left half of s-plane.

iv. The degree of denominator polynomial of transfer function is greater or equal to that of numerator polynomial.

Que 3.2. **What is Routh-Hurwitz stability criterion ? Explain briefly.**

OR

What are the limitations of Routh-Hurwitz criterion ?

AKTU 2017-18, Marks 10

OR

Explain the method of forming Routh array.

Answer

The Routh-Hurwitz stability criterion is a mathematical test that is a necessary and sufficient condition for the stability of system.

Routh's criterion :

1. The necessary and sufficient condition for the system to be stable is that all the terms in the first column of Routh's array (*i.e.* a_0, a_1, b_1, c_1, d_1 ....a_n) must have same sign.
2. There should not be any sign change in the first column of Routh's array.
3. If there is any sign change then

i. System is unstable.

ii. Number of sign changes is equal to the number of poles lying in the right half of the s-plane.

Method of forming a Routh array :

$$\begin{array}{l|lllll}
s^n & a_0 & a_2 & a_4 & a_6 & \ldots\ldots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 & \\
s^{n-2} & b_1 & b_2 & b_3 & & \\
s^{n-3} & c_1 & c_2 & c_3 & & \\
\vdots & \vdots & \vdots & \vdots & & \\
s^0 & a_n & & & &
\end{array}$$

i. Coefficients for first two rows are written directly from characteristics equation,

$$b_1 = \frac{a_1\, a_2 - a_0\, a_3}{a_1}$$

$$b_2 = \frac{a_1\, a_4 - a_0\, a_5}{a_1}$$

$$b_3 = \frac{a_1\, a_6 - a_0\, a_7}{a_1}$$

ii. From 2nd and 3rd row, 4th row can be obtained as

$$c_1 = \frac{b_1\, a_3 - a_1\, b_2}{b_1}$$

$$c_2 = \frac{b_1\, a_5 - a_1\, b_3}{b_1}$$

iii. This process is to be continued till the coefficient for s_0 is obtained which will be a_n. From this array stability of a system can be predicated.

Special cases :

1. When the 1st term in a row is zero, but all other terms is non-zero then substitute a small positive number ε for zero & proceed to evaluate the rest of the elements. When the 1st column term is zero, it means that there is an imaginary root.

2. **All zero row :** In this case, write auxiliary equation from preceding row, differentiate this equation & substitute all zero row by the coefficient obtained by differentiating the auxiliary equation. This case occurs when the roots are in pairs. The system is said to be limitedly stable.

Application : Routh's criterion can be applied to determine range of certain parameters of a system to ensure stability. It is usually to find the range of the open loop gain K for closed loop stability.

Limitations :

1. It is valid only if the characteristics equation is algebraic.
2. If any coefficient of the characteristics equation is complex or contain power of e then this criterion cannot be applied.
3. It gives information about how many roots are lying in the RHS of the s-plane but values of the roots are not available. Also it cannot distinguish between real and complex roots.

Que 3.3. **The characteristics equation of a system is given $(s^4 + 20s^3 + 15s^2 + 2s + K) = 0$, determine the range of the K, for system to be stable.**

AKTU 2017-18, Marks 10

Answer

Given : $G(s) = s^4 + 20s^3 + 15s^2 + 2s + K = 0$
To Find : Range of K.

1. Routh array,

$$\begin{array}{c|ccc} s^4 & 1 & 15 & K \\ s^3 & 20 & 2 & 0 \\ s^2 & 149 & K & 0 \\ s^1 & \dfrac{298-20K}{149} & 0 & \\ s^0 & K & & \end{array}$$

2. For system to be stable

$$\frac{298-20K}{149} > 0 \text{ and } K > 0$$

3. Range of K is $0 < K < 14.9$

Que 3.4. **Using Routh's stability criterion, determine the range of *K* for open loop transfer function**

$$G(s)\,H(s) = \frac{K}{s(s+1)(1+2s)}$$

AKTU 2015-16, Marks 10

Answer

Given : $G(s)\,H(s) = \dfrac{K}{s(s+1)(1+2s)}$

To Find : Range of K.

1. Using characteristics equation

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{K}{s(s+1)(1+2s)} = 0$$

$$s(s+1)(1+2s) + K = 0$$

$$s(2s^2 + 3s + 1) + K = 0$$

$$2s^3 + 3s^2 + s + K = 0$$

2. Using Routh array :

$$\begin{array}{c|cc} s^3 & 2 & 1 \\ s^2 & 3 & K \\ s^1 & \dfrac{3-2K}{3} & 0 \\ s^0 & K & \end{array}$$

3. For system to be stable :

$$\frac{3-2K}{3} > 0 \text{ and } K > 0$$

$$\therefore \qquad 0 < K < \frac{3}{2}$$

Que 3.5. **Explain the working principle of stepper motor with neat diagram.**
The characteristics equation for feedback control is, $s^3 + 5s^2 + 12s + K = 0$. Find the range of K for all the roots to lie to the left of $s = 1$.

AKTU 2016-17, Marks 10

Answer

Stepper motor : Refer Q. 1.9, Page 1–13C, Unit-1.
Numerical :

Given : Characteristics equation is $s^3 + 5s^2 + 12s + K = 0$
To Find : Range of K.

1. Putting $s = s - 1$

$$(s - 1)^3 + 5(s - 1)^2 + 12(s - 1) + K = 0$$
$$s^3 - 1 + 3s - 3s^2 + 5s^2 + 5 - 10s + 12s - 12 + K = 0$$
$$s^3 + 2s^2 + 5s + (K - 8) = 0$$

2. **Routh array :**

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 2 & (K-8) \\ s^1 & \dfrac{10-(K-8)}{2} & 0 \\ s^0 & (K-8) & \end{array}$$

3. To lie all roots to the left side of $s = -1$, there should not be any sign change in first column of Routh array.

$$\frac{10-(K-8)}{2} > 0$$
$$10 - (K - 8) > 0$$
$$18 - K > 0$$

$\therefore$ $K < 18$ ...(3.5.1)

Also, $K - 8 > 0$

$K > 8$ ...(3.5.2)

From eq. (3.5.1) and (3.5.2), range of K is

$$8 < K < 18$$

Que 3.6. **For a system having characteristic equation $2s^4 + 4s^2 + 1 = 0$, find the following :**

i. The number of roots in the left half of s-plane.
ii. The number of roots in the right half of s-plane.
iii. The number of roots on the imaginary axis. Use the Routh Hurwitz Criterion.

AKTU 2013-14, Marks 10

Answer

Given : Characteristic equation is $2s^4 + 4s^2 + 1 = 0$
To Find : Number of roots in left, right and on imaginary axis.

1. **Routh array :**

$$\begin{array}{c|ccc} s^4 & 2 & 4 & 1 \\ s^3 & 0 & 0 & \\ s^2 & & & \\ s^1 & & & \\ s^0 & & & \end{array}$$

2. Auxiliary equation

$$2s^4 + 4s^2 + 1 = 0$$

3. Differentiating w.r.t s,

$$8s^3 + 8s = 0$$
$$s^3 + s = 0$$

4. Again forming Routh array,

$$\begin{array}{c|ccc} s^4 & 2 & 4 & 1 \\ s^3 & 1 & 1 & \\ s^2 & 2 & 1 & \\ s^1 & 1 & & \\ s^0 & 1 & & \end{array}$$

5. No sign change but one row (s^3 row) is zero then the system is marginally stable.

i. The number of roots in left half of s-plane is two.

ii. The number of roots in right half of s-plane is zero.

iii. The number of roots on the imaginary axis is two.

Que 3.7. **Determine the stability of a closed loop control system whose characteristic equation is $s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$**

AKTU 2014-15, Marks 10

Answer

Given : Characteristic equation is $s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$
To Find : Stability.

1. Routh array is :

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 1 & 2 & 10 \\ s^3 & 0 & 1 & \\ s^2 & & & \\ s^1 & & & \\ s^0 & & & \end{array}$$

2. Putting ε (very small positive value) in place of 0 in the first column Routh array

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 1 & 2 & 10 \\ s^3 & \varepsilon(+\text{ ve}) & 1 & 0 \\ s^2 & \dfrac{2\varepsilon-1}{\varepsilon}(-\text{ ve}) & 10 & \\ s^1 & \dfrac{\left(\dfrac{2\varepsilon-1}{\varepsilon}\right)-10\varepsilon}{\left(\dfrac{2\varepsilon-1}{\varepsilon}\right)}(+\text{ ve}) & 0 & \\ s^0 & 10 & & \end{array}$$

3. Since there is two sign change in the first column of Routh array, therefore system is unstable.

PART-2

Root Locus Techniques : Root Contour, Construction of Root Loci, Effect of Transportation Lag and Root Locus of Non Minimal Phase System and Effect of Pole-Zero Cancellation.

CONCEPT OUTLINE : PART-2

- **Root-locus :**

1. Root-locus is used for checking the stability of the system.
2. Angle of asymptotes, $\theta_q = \dfrac{180°(2q+1)}{(P-Z)}$

   where $q = 0, 1, 2, \ldots (P - Z - 1)$
3. Centroid, $\sigma = \dfrac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P-Z}$

   where, P = Number of poles
   Z = Number of zeros

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.8. **Explain step by step procedure for plotting root locus.**

OR

Write short note on :
i. Centroid
ii. Breakaway points
iii. Steady state error.

AKTU 2017-18, Marks 10

Answer

Root locus :

1. Root locus is the plot of loci of the root of the complementary equation when one or more parameter of open-loop transfer function is varied, mostly the only one variable available is the gain K. The negative K has no significance. Hence vary K from 0 to ∞, the plot obtained is called root locus.
2. It gives the complete dynamic response of system. It provides the measure of sensitivity of roots of variation in the system.

Rules for construction of root loci :

1. The root locus is symmetrical about the real axis.
2. Each branch of the root locus originates form an open-loop pole where $K = 0$ and terminates on an open loop zeros or on infinity where $K = \infty$.
3. The number of branches of the root locus terminating on infinity is equal to $P - Z$.
4. A point on the real axis lies on the locus, if number of open loop poles and zeros on the real axis to the right of this point is odd.
5. **Angles of asymptotes :** The $(P - Z)$ root locus branches that approaches to infinity are along straight line is called asymptote. Asymptotes making angles with the real axis are given by
$$\theta_q = \frac{(2q+1)180°}{P-Z}; \text{ where } q = 0, 1, 2, ..., (P - Z - 1)$$
6. **Centroid :** The point of intersection of the asymptotes with the real axis called centroid is at $s = \sigma$, where
$$\sigma = \frac{\Sigma \text{ Real part of poles } - \Sigma \text{ Real part of zeros}}{\text{Number of poles } - \text{ Number of zeros}}$$
7. **Break points :** The break points (breakaway and breakin points) of the root locus are determined from the roots of the equation $\dfrac{dK}{ds} = 0.$

The r branches of the root locus which meet at a point break away at an angle of $\pm \dfrac{180°}{r}$.

8. **Angle of departure :** The angle of departure from an open loop pole is given by

$$\theta_d = 180° + \phi;$$

where ϕ is the net angle contribution at the zero of all other open-loop poles and zeros.

9. **Angle of arrival :** The angle of arrival at open loop zero is given by

$$\theta_a = 180° - \phi,$$

where ϕ is the net angle contribution of all other open loop poles and zeros.

10. The point of intersection of the root locus branches with the imaginary axis and the critical value of K can be determined by use of the Routh's criterion.

11. **Gain :** The open-loop gain K in pole-zero from at any point s_0 on the root locus is given by

$$K = \frac{\text{Product of phasor lengths from } s_0 \text{ to open-loop poles}}{\text{Product of phasor lengths from } s_0 \text{ to open-loop zeros}}$$

Root contour :

1. The root locus technique for the study of stability of closed loop system from open loop transfer function can be extended by varying parameter other than K in system from 0 to ∞.
2. Consider a system shown in the Fig. 3.8.1.

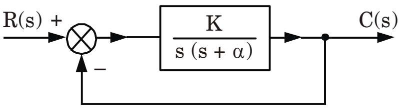


Fig. 3.8.1.

3. The open loop transfer function is $G(s)H(s) = \dfrac{K}{s(s+\alpha)}$
4. The locus of roots of characteristic equation obtained by varying parameter of the system other than K from 0 to ∞ is called root contour of the system.
5. The parameters like α, K are to be varied simultaneously, while sketching the root contours.

Steady state error : Refer Q. 2.6, Page 2–11C, Unit-2.

Que 3.9. **Sketch the root locus for the open loop transfer function of a unity feedback control system given below and determine value of K at $\xi = 0.5$, $G(s)\,H(s) = \dfrac{K}{s(s+1)(s+3)}$**

AKTU 2014-15, Marks 10

OR

Construct root loci for open loop transfer function :

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+3)}$$

AKTU 2015-16, Marks 10

Answer

Given : $G(s)\,H(s) = \dfrac{K}{s(s+1)(s+3)}$, $H(s) = 1$, $\xi = 0.5$

To Sketch : Root locus and value of K at $\xi = 0.5$.

A.

1. The open-loop poles are at $s = 0$, $s = -1$, $s = -3$.
2. There is no open loop zeros.
3. Number of poles, $P = 3$
   Number of zeros, $Z = 0$
   $P - Z = 3 - 0 = 3$ *i.e.*, three branches of root locus end at infinity.
4. **Angle of asymptotes :**

$$\theta_q = \frac{(2q+1)180°}{P-Z} \quad \text{where } q = 0, 1, 2, \ldots(P - Z - 1)$$

$$\theta_0 = \frac{(2\times 0+1)\times 180°}{(3-0)} = 60°$$

$$\theta_1 = \frac{(2\times 1+1)\times 180°}{(3-0)} = 180°$$

$$\theta_2 = \frac{(2\times 2+1)\times 180°}{(3-0)} = 300°$$

5. **Centroid of asymptotes :**

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P - Z}$$

$$= \frac{(0-1-3)-(0)}{3-0} = -1.33.$$

6. **Breakaway points :** Between open-loop poles $s = 0$ and $s = -1$, there exists a breakaway point.
   The characteristic equation is
   $s\,(s+1)(s+3) + K = 0$

$$K = -(s^3 + 4s^2 + 3s)$$

$$\therefore \quad \frac{dK}{ds} = -(3s^2 + 8s + 3) = 0$$

$$\therefore \quad 3s^2 + 8s + 3 = 0$$

$$s = -1.33 \pm 0.88 = -0.42 \text{ and } -2.21$$

As the breakaway point has to lie between $s = 0$ and $s = -1$, the valid breakaway point is $s = -0.42$.

7. Intersection with $j\omega$ axis :

Characteristic equation, $1 + G(s)\,H(s) = 0$

$$s\,(s + 1)\,(s + 3) + K = 0$$

$$s^3 + 4s^2 + 3s + K = 0$$

Routh array :

s^3	1	3
s^2	4	K
s^1	$(12 - K)/4$	
s^0	K	

The value of K at imaginary axis :

$$\frac{(12 - K)}{4} = 0 \quad \therefore \quad K = 12$$

Auxiliary equation,

$$4s^2 + 12 = 0$$

$$s = \pm\, j\,1.73$$

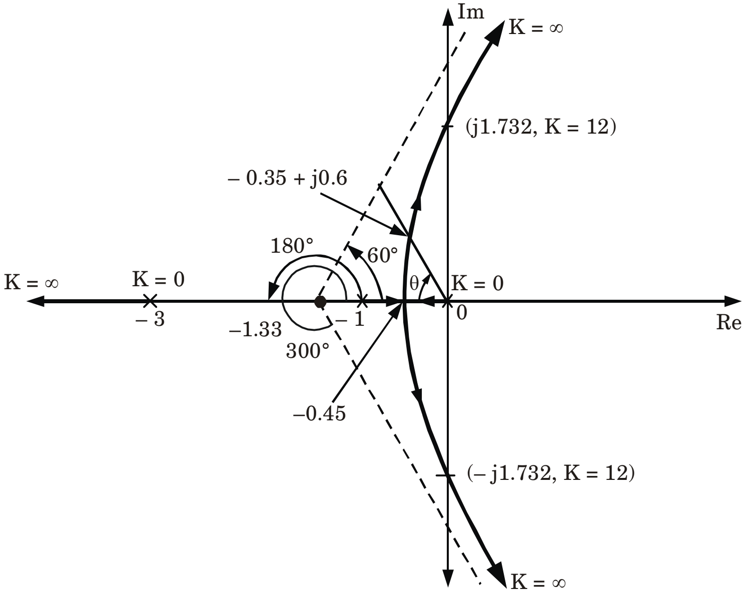


Fig. 3.9.1. Root locus for G(s) = K/s(s + 1)(s + 3).

B.

1. Since $\xi = 0.5$

 and $\cos\theta = \xi$

 $\cos\theta = 0.5$

 $\therefore$ $\theta = 60°$

2. From the origin a line at angle $\theta = 60°$ is drawn as shown in Fig. 3.9.1, which intersects the root locus plot at $s = (-0.35 + j0.6)$. As the point $s = (-0.35 + j0.6)$ lies on the root locus the following equation is satisfied

$$|G(s)\,H(s)| = 1 \quad \text{or} \quad \left|\frac{K}{s(s+1)(s+3)}\right| = 1$$

3. Put $s = (-0.35 + j0.6)$, we get

$$\left|\frac{K}{(-0.35+j0.6)[(-0.35+j0.6)+1][(-0.35+j0.6)+3]}\right| = 1$$

$$\therefore \qquad \frac{K}{1.66} = 1 \quad \text{or} \quad K = 1.66$$

Que 3.10. Sketch the root locus for a system having :

$$G(s) = \frac{K}{(s+1)} \text{ and } H(s) = \frac{(s+1)}{(s^2+4s+5)}$$

and comment on the result. **AKTU 2013-14, Marks 10**

Answer

1. The characteristics equation of the system is

$$1 + G(s)H(s) = 0$$

$$1 + \left(\frac{K}{s+1}\right)\left(\frac{s+1}{s^2+4s+5}\right) = 0$$

$$1 + \left(\frac{K}{s^2+4s+5}\right) = 0$$

$$s^2 + 4s + K + 5 = 0$$

2. Two roots of the above equation are

$$s_1, s_2 = -2 \pm j\sqrt{K+1}$$

For various value of K the roots are

K	0	3	8	∞
s_1	$-2+j1$	$-2+j2$	$-2+j3$	$-2+j\infty$
s_2	$-2-j1$	$-2-j2$	$-2-j3$	$-2-j\infty$

The roots determined above are plotted in s-plane as shown in Fig. 3.10.1.

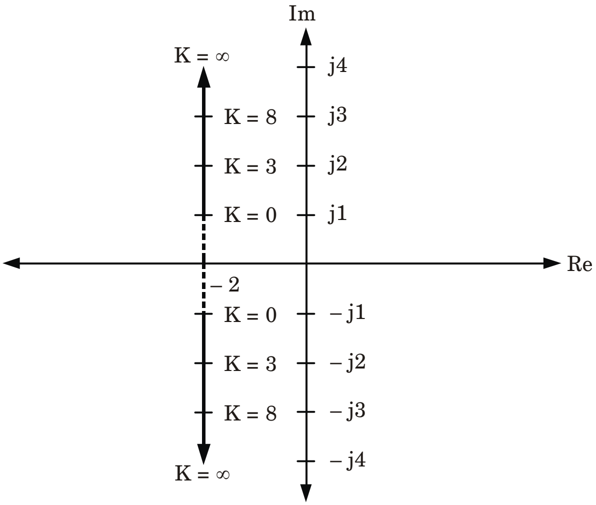


Fig. 3.10.1.

Conclusion :

1. For all values of K from 0 to ∞ the roots are complex with negative real plot indicating that the time response is underdamped.
2. Whatever may be positive values of K there is no chance for the real part of the roots to become positive, hence the system is inherently stable.

Que 3.11. **Sketch the root locus for the closed loop control system with** $G(s) = \dfrac{K}{s(s+1)(s^2+4s+5)}$. **AKTU 2016-17, Marks 10**

Answer

Given : $G(s) = \dfrac{K}{s(s+1)(s^2+4s+5)}$

To Draw : Root locus.

1. Poles *i.e.*, $s = 0$, $s = -1$, $s = -2 + j$ and $s = -2 - j$
2. There is no open loop zero.
3. Number of poles, $P = 4$

   Number of zeros, $Z = 0$

   $P - Z = 4 - 0 = 4$ *i.e.*, four branches of root locus terminates at infinity.
4. **Angle of asymptotes :**

$$\theta_q = \frac{(2q+1)}{P-Z} \times 180°$$

where, $q = 0, 1, 2, \ldots\ldots$ upto $(P - Z - 1)$

$$\theta_0 = \frac{2 \times 0 + 1}{4-0} \times 180° = 45°$$

$$\theta_1 = \frac{2 \times 1 + 1}{4-0} \times 180° = 135°$$

$$\theta_2 = \frac{2 \times 2 + 1}{4-0} \times 180° = 225°$$

$$\theta_3 = \frac{2 \times 3 + 1}{4-0} \times 180° = 315°$$

5. Centroid of asymptotes :

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P-Z}$$

$$= \frac{(0-1-2+j-2-j)-(0)}{4-0} = \frac{-5}{4} = -1.25$$

6. Breakaway points : The characteristic equation is $1 + G(s)\, H(s) = 0$

$$s(s+1)(s^2+4s+5) + K = 0$$

$$s^4 + 5s^3 + 9s^2 + 5s + K = 0$$

$$\therefore \quad dK/ds = 4s^3 + (5 \times 3s^2) + (9 \times 2s) + 5 \times 1 = 0$$

$$4s^3 + 15s^2 + 18s + 5 = 0$$

$$(s + 0.4)(4s^2 + 13.4s + 12.5) = 0$$

$$(s + 0.4)\{(s + 1.675 - j\,0.565)(s + 1.675 + j\,0.565)\} = 0$$

Therefore three breakaway points are obtained

$s = -0.4$ on real axis and

$s = -1.7 + j\,0.6$ and $s = -1.7 - j\,0.6$

7. Intersection points with imaginary axis :

The characteristic equation is $s^4 + 5s^3 + 9s^2 + 5s + K = 0$

The Routh array :

$$\begin{array}{c|ccc} s^4 & 1 & 9 & K \\ s^3 & 5 & 5 & 0 \\ s^2 & 8 & K & \\ s^1 & 5 - \dfrac{5K}{8} & 0 & \\ s^0 & K & & \end{array}$$

Value of K at imaginary axis :

$$5 - \frac{5K}{8} = 0; \quad K = 8$$

Solving auxiliary equation formed from the s^2 terms in Routh array, therefore

$$8s^2 + K = 0$$

$$8s^2 + 8 = 0$$
$$s^2 + 1 = 0$$
$$s = \pm\, j$$

8. Angle of departure from complex pole :

$$\phi_d = 180° - (\phi_P - \phi_Z)$$
$$\phi_{P1} = 180° - \tan^{-1}(1/2) = 153.43°$$
$$\phi_{P2} = 180° - \tan^{-1}(1/1) = 135°$$
$$\phi_{P3} = 90°$$
$$\phi_{(-2+j)} = 180° - (\phi_{P1} + \phi_{P2} + \phi_{P3})$$

$\therefore$ $\phi_{(-2+j)} = 180° - (153.43° + 135° + 90°) = -198.43°$

and $\phi_{(-2-j)} = +198.43°$

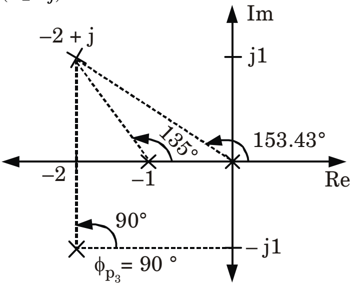


Fig. 3.11.1.

9. Root locus :

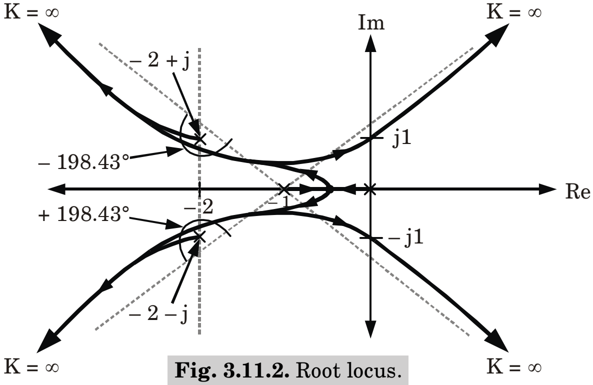


Fig. 3.11.2. Root locus.

Que 3.12. **Construct the RL (root locus) for a unity feedback system with OLTF $G(s) = \dfrac{K(s+1)}{s^2(s+9)}$.**

AKTU 2017-18, Marks 10

Answer

Given : $G(s) = \dfrac{K(s+1)}{s^2(s+9)}$, $H(s) = 1$

To Draw : The root locus.

1. The open loop poles *i.e.*, $s = 0$, $s = 0$, $s = -9$
2. Open loop zero are at $s = -1$
3. Number of poles, $P = 3$

   Number of zeros, $Z = 1$

   $P - Z = 3 - 1 = 2$ *i.e.*, two branches of root locus terminates at infinity.
4. **Angle of asymptotes :**

$$\theta_q = \frac{(2q+1)}{P-Z} \times 180°$$

where, $q = 0, 1, 2, \ldots\ldots$ upto $(P - Z - 1)$

$$\theta_0 = \frac{(2\times 0+1)}{3-1} \times 180° = 90°$$

$$\theta_1 = \frac{2\times 1+1}{3-1} \times 180° = 270°$$

5. **Centroid of asymptotes :**

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P-Z}$$

$$= \frac{0-9+1}{2} = \frac{-8}{2} = -4$$

6. **Breakaway points :**

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1)}{s^2(s+9)} \times 1 = 0$$

$$s^2(s+9) + K(s+1) = 0$$

$$K = \frac{-s^2(s+9)}{(s+1)}$$

$$\frac{dK}{ds} = -\frac{(s+1)(3s^2+18s) - s^2(s+9)(1)}{(s+1)^2} = \frac{-2s(s+3)^2}{(s+1)^2} = 0$$

$$-2s(s+3)^2 = 0$$

$$s = 0, \quad s = -3$$

7. **Intersection with imaginary axis :**

   $s^3 + 9s^2 + Ks + K = 0$

   Routh array :

$$\begin{array}{c|cc} s^3 & 1 & K \\ s^2 & 9 & K \\ s^1 & 8K/9 & \\ s^0 & K & \end{array}$$

Value of K at imaginary axis,

$$\frac{8K}{9} = 0$$

$$K = 0$$

Now, $$9s^2 + K = 0$$

$$9s^2 = 0$$

$$s = 0$$

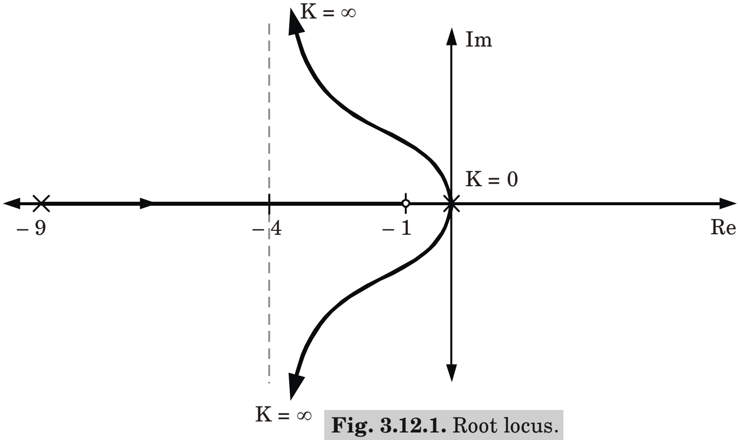


Fig. 3.12.1. Root locus.

Que 3.13. **Sketch the RL for a unity feedback system with OLTF**

$$\mathbf{G(s) = \frac{K(s^2 + 2s + 10)}{s^2 + 4s + 5}}.$$

AKTU 2017-18, Marks 10

Answer

Given : $G(s) = \dfrac{K(s^2 + 2s + 10)}{s^2 + 4s + 5}$, $H(s) = 1$

To Draw : Root locus.

1. The open loops poles are at $s = -2 - j$, $s = -2 + j$
2. The open loop zeros are at $s = -1 - j3$, $s = -1 + j3$
3. Number of poles = 2

   Number of zeros = 2

   $P - Z = 2 - 2 = 0$ *i.e.*, none of branches of root locus terminates at infinity.
4. Angle of departure and arrival can't be calculated because the centroid, $\sigma = \infty$.

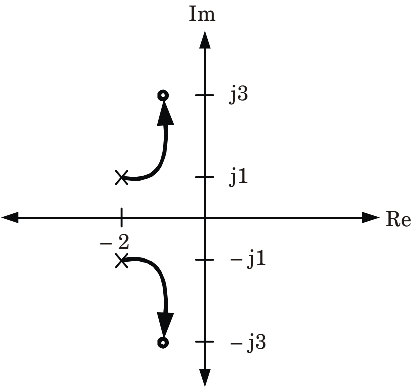


Fig. 3.13.1.

Que 3.14. **Consider a unity feedback system with a forward path transfer function $G(s) = \dfrac{K(s-4)}{(s+2)(s-1)}$**

Draw the root locus. **AKTU 2017-18, Marks 10**

Answer

Given : $G(s) = \dfrac{K(s-4)}{(s+2)(s-1)}$

To Draw : Root locus.

1. The open loop poles are at $s = -2$, $s = 1$
2. The open loop zeros at $s = 4$
3. Number of poles = 2
   Number of zeros = 1
   $P - Z = 1$ *i.e.*, one branch of root locus terminates at infinity.
4. **Centroid of asymptotes :**

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P - Z} = \frac{1-2-4}{1} = -5$$

5. **Breakaway point :**

$$1 + G(s)H(s) = 0$$

$$\frac{1 + K(s-4)}{(s+2)(s-1)} = 0$$

$$s^2 + s - 2 + K(s-4) = 0$$

$$K = \frac{2 - s - s^2}{(s-4)}$$

$$\frac{dK}{ds} = \frac{(s-4)(-1-2s)-(2-s-s^2)(1)}{(s-4)^2}$$

$$= \frac{-s^2+8s+2}{(s-4)^2} = 0$$

$$s = 8.24 \text{ (Breakin point)}$$

and $$s = -0.24 \text{ (Breakaway point)}$$

6. **Angle of asymptotes :**

$$\theta_q = \frac{(2q+1)}{P-Z} \times 180° = \frac{1}{2-1} \times 180° = 180°$$

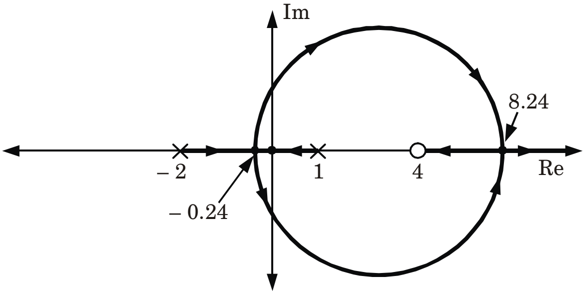


Fig. 3.14.1. Root locus.

Que 3.15. **Discuss the effects of pole-zero cancellation.**

Answer

Pole-zero cancellation :

1. Let us assume that some of the poles of G are at undesirable locations and that we want to place them more appropriately.
2. We can insert a controller in series before the system in order to cancel out the undesirable poles and keep only those which are suitable.
3. The undesirable poles of G are not removed by this technique, but that their effect is cancelled out by the coincident zeros of the controller.
4. This is known as pole-zero cancellation. A difficulty with pole zero cancellation is that it may not be exact.

Effects of pole-zero cancellation are :

1. If the specification on allowable steady state errors cannot be met, a low frequency pole can be cancelled and replaced with a lower frequency pole, yielding a large forward loop.
2. If poles with small damping ratios are present in the plant transfer function, they may be cancelled and replaced with poles which have larger damping rates.

Que 3.16. Discuss the effects of adding poles to root locus.

Answer

Effects of addition of open loop poles :

1. Root locus shifts towards imaginary axis.
2. System stability relatively decreases.
3. System becomes more oscillatory in nature.
4. Range of operating value of K for stability of the system decreases.

Example :

1. Consider, $G(s)H(s) = K / s(s + 2)$
   Corresponding root locus is shown in the Fig. 3.16.1.

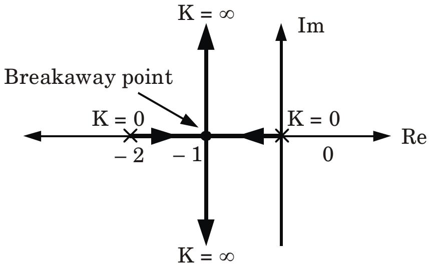


Fig. 3.16.1.

2. Now if pole at $s = -4$ added to $G(s)H(s)$ root locus becomes as shown in the Fig. 3.16.2.

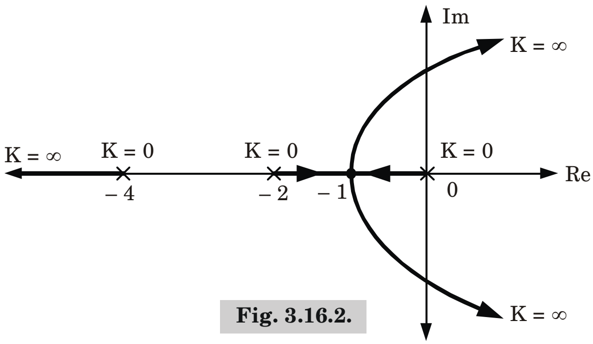


Fig. 3.16.2.

Que 3.17. Discuss the effects of adding zeros to root locus.

Answer

Effects of addition of zeros :

1. Root locus shifts to left, away from imaginary axis.

2. Relative stability of the system increases.
3. System becomes less oscillatory.
4. Range of operating values of K for system stability increases.

Example :

1. Root loci for $G(s)\,H(s) = \dfrac{K}{s(s+2)}$ is shown in the Fig. 3.17.1.

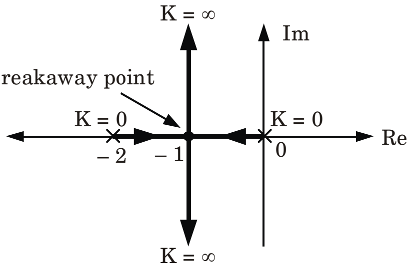


Fig. 3.17.1.

2. Now if zero at $s = -4$ added to $G(s)H(s)$ root locus becomes as shown in Fig. 3.17.2.

$$G(s)H(s) = \frac{K\,(s+4)}{s(s+2)}$$

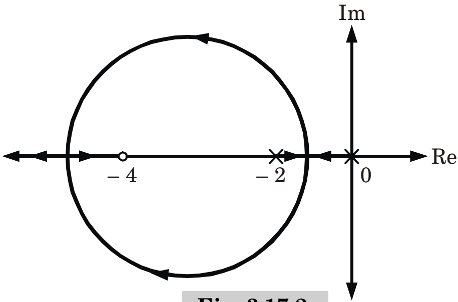


Fig. 3.17.2.

3. It can be seen that root locus shift towards left. So as roots move towards left half of s-plane, hence relative stability increases.

Que 3.18. Define transportation lag. Explain its effect with suitable example.

Answer

1. The transportation lag is the delay between the time an input signal is applied to a system and the time the system reacts to that input signal.

2. Transportation lags are common in industrial applications. They are often called dead time.
3. Consider a system having open-loop transfer function

$$G(s)H(s) = \frac{Ke^{-sT}}{s(s+2)}$$

where T is the transportation delay in seconds and is given as 1 s.
4. Let us draw the root loci for K varying in the range $0 < K < \infty$.
5. If the transportation delay is small, then we can assume

$$e^{-sT} = 1 - sT$$

6. Rewriting $G(s)H(s)$ as

$$G(s)H(s) = \frac{K(1-s)}{s(s+2)}; \; T = 1$$

$$G(s)H(s) = \frac{-K(s-1)}{s(s+2)}$$

7. The characteristic equation becomes

$$1 - G(s)H(s) = 0$$

The root loci are drawn as shown in Fig. 3.18.1.

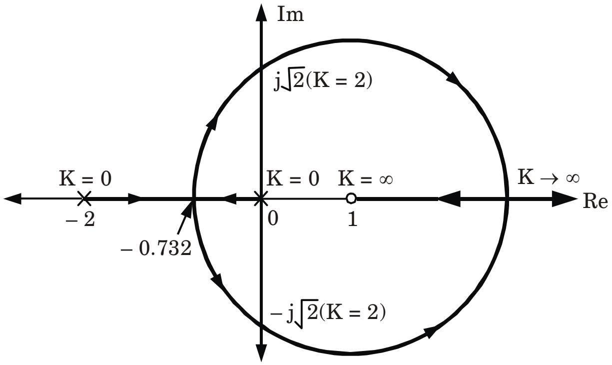


Fig. 3.18.1. Root-loci plot of $1 + [Ke^{-s}/s(s+2)] = 0$ with e^{-s} is approximated as $(1-s)$.

Effect : It reduces the stability of a system and limits the achievable response time of the system.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Define stability. State the necessary conditions for system to be absolutely stable.

Ans. Refer Q. 3.1, Unit-3.

Q. 2. The characteristics equation of a system is given $(s^4 + 20s^3 + 15s^2 + 2s + K) = 0$, determine the range of the K, for system to stable.

Ans. Refer Q. 3.3, Unit-3.

Q. 3. For a system having characteristic equation $2s^4 + 4s^2 + 1 = 0$, find the following :

i. The number of roots in the left half of s-plane.

ii. The number of roots in the right half of s-plane.

iii. The number of roots on the imaginary axis. Use the Routh Hurwitz Criterion.

Ans. Refer Q. 3.6, Unit-3.

Q. 4. Write short note on :

i. Centroid

ii. Breakaway points

iii. Steady state error.

Ans. Refer Q. 3.8, Unit-3.

Q. 5. Sketch the root locus for a system having :

$$G(s) = \frac{K}{(s+1)} \text{ and } H(s) = \frac{(s+1)}{(s^2 + 4s + 5)}$$

and comment on the result.

Ans. Refer Q. 3.10, Unit-3.

Q. 6. Construct the RL (root locus) for a unity feedback system with OLTF $G(s) = \dfrac{K(s+1)}{s^2(s+9)}$.

Ans. Refer Q. 3.12, Unit-3.

Q. 7. Sketch the RL for a unity feedback system with OLTF G(s) = $\dfrac{K(s^2+2s+10)}{s^2+4s+5}$.

Ans. Refer Q. 3.13, Unit-3.

Q. 8. Discuss the effects of pole-zero cancellation.

Ans. Refer Q. 3.15, Unit-3.

Q. 9. Discuss the effects of adding poles to root locus.

Ans. Refer Q. 3.16, Unit-3.

Q. 10. Discuss the effects of adding zeros to root locus.

Ans. Refer Q. 3.17, Unit-3.

☺☺☺

4 UNIT

Frequency Response Analysis

PART-1

Frequency Response Analysis from Transfer Function Model, Correlation between Time and Frequency Response.

CONCEPT OUTLINE : PART-1

- **Frequency response :**

i. The magnitude and phase relationship between the sinusoidal input and steady-state output of a system is called frequency response.
i.e., Sinusoidal input, $r(t) = A \sin \omega t$
Steady state output, $c(t) = B \sin (\omega t + \phi)$

ii. The frequency response is easily evaluated from sinusoidal transfer function by replacing s by $j\omega$.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. What do you understand by frequency response ? What are its various methods ? Discuss their relative advantages, disadvantages and limitations.

Answer

A. Frequency response :

1. The magnitude and phase relationship between the sinusoidal input and the steady state output of a system is termed as the frequency response.
2. Linear system with sinusoidal input
$$r(t) = A \sin \omega t$$
3. Steady state output of
$$c(t) = B \sin (\omega t + \phi)$$

B. Various methods of frequency response :

1. Polar plots
2. Inverse polar plots
3. Nyquist stability criterion
4. Bode plot

C. Advantages of frequency domain approach :

1. Without the knowledge of the transfer function, the frequency response of stable open loop system can be obtained experimentally.

2. These methods are easy to use for design of control and for finding absolute as well as relative stability of the system.
3. Even if the system has moderate non-linearity it can be depicted by an approximate transfer function.
4. There is a close relation between frequency response of a system and its step response.

D. Disadvantages of frequency response :

1. For systems with very large time constants, the frequency test is cumbersome to perform as the time required for the output to reach the steady-state for each frequency of the test signal is excessively long.
2. The frequency response test obviously cannot be performed on non-interruptible systems.

E. Limitations of frequency response methods :

1. These methods can be applied only to linear systems.
2. For an existing system, obtaining frequency response is possible only if the time constants are up to few minutes.
3. Obtaining frequency response practically is fairly time consuming.

Que 4.2. **Explain various frequency domain specifications.**

Answer

1. **Resonant peak (M_r) :** Resonance peak is the peak value of magnitude of closed loop frequency response.

   For second order, $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

2. **Resonance frequency :** The frequency at which resonant peak occurs is called resonant frequency.

   For second order, $\omega_r = \omega_n\sqrt{1-2\xi^2}$

3. **Bandwidth :** It is the range of frequency up to ω_c, which is cut off frequency. At ω_c the magnitude of closed loop transfer function is 3 dB down from its zero frequency level.

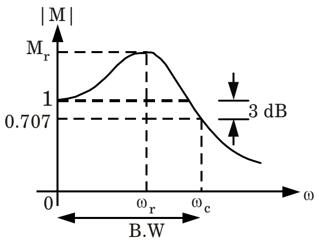


Fig. 4.2.1.

4. **Cut-off rate :** The cut-off rate is the frequency rate at which the magnitude ratio decreased beyond the cut-off frequency ω_c.

Que 4.3. What is closed loop frequency response ? Give an account of the correlation between time response and frequency response for a second order system with relevant expressions.

AKTU 2016-17, Marks 10

Answer

Closed loop frequency response :

1. Consider the transfer function for closed loop system,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

2. For unity feedback, $H(s) = 1$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} \quad \text{...(4.3.1)}$$

Put $s = j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)} \quad \text{...(4.3.2)}$$

3. The polar plot of eq. (4.3.2) is shown in Fig. 4.3.1.

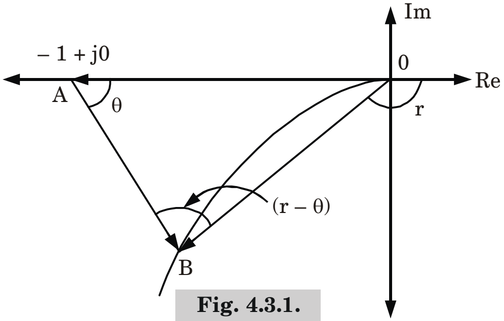


Fig. 4.3.1.

4. From Fig. 4.3.1, $\vec{OB} = G(j\omega)$

$$\vec{OA} = -1$$

$$\vec{AB} = \vec{OB} - \vec{OA} = G(j\omega) - (-1)$$

$$\vec{AB} = 1 + G(j\omega)$$

5. From eq. (4.3.2)

$$\left|\frac{C(j\omega)}{R(j\omega)}\right| = M(\omega) = \frac{\vec{OB}}{\vec{AB}}$$

$$\frac{\angle C(j\omega)}{\angle R(j\omega)} = \frac{\angle \vec{OB}}{\angle \vec{AB}} = \frac{\angle r}{\angle \theta} = \angle (r - \theta)$$

$$\therefore \qquad \frac{C(j\omega)}{R(j\omega)} = M(\omega)\, e^{j\,\phi(\omega)}$$

where $M(j\omega)$ is the magnitude and $\phi(\omega) = r - \theta$.

6. Frequency response consists of two parts :
i. Magnitude,
ii. Phase angle.
iii. Both can be plotted against different values of ω.
7. Hence frequency response of closed loop system is plot of magnitude and phase angle.

Correlation :

1. For 2^{nd} order system, the transfer function is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where, ξ = Damping factor
ω_n = Natural frequency of oscillations

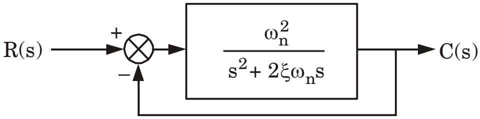


Fig. 4.3.2.

2. Closed loop frequency response is,

$$\frac{C(j\omega)}{R(j\omega)} = T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n(j\omega) + \omega_n^2} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2j\xi\left(\frac{\omega}{\omega_n}\right)}$$

$$= \frac{1}{(1 - u^2) + j2\xi u} \qquad \text{...(4.3.3)}$$

where $u = \omega/\omega_n$, normalized driving frequency.

$$\therefore \qquad |T(j\omega)| = M = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \qquad \text{...(4.3.4)}$$

and $$\angle T(j\omega) = \phi = -\tan^{-1}\frac{2\xi u}{1-u^2} \qquad \text{...(4.3.5)}$$

3. The steady state output is

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \sin\left(\omega t - \tan^{-1}\frac{2\xi u}{1-u^2}\right)$$

∴ From eq. (4.3.4) and (4.3.5) when

$$u = 0, M = 1 \text{ and } \phi = 0$$

$$u = 1, M = \frac{1}{2\xi} \text{ and } \phi = -\frac{\pi}{2}$$

$$u = \infty, M = 0 \text{ and } \phi = -\pi$$

4. The frequency where M has a peak value is called the resonant frequency. At this frequency the slope of magnitude curve is zero.

If ω_r = Resonant frequency.

$u_r = \omega_r/\omega_n$ is normalized resonant frequency.

$$\left.\frac{dM}{du}\right|_{u=u_r} = -\frac{1}{2}\frac{\left[-4(1-u_r^2)u_r + 8\xi^2 u_r\right]}{\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{3/2}} = 0$$

$$-4(1-u_r^2)\,u_r + 8\xi^2 u_r = 0$$

$$-4u_r(1-u_r^2-2\xi^2) = 0$$

∴ $$u_r = \sqrt{1-2\xi^2}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2} \quad \text{...(4.3.6)}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad \text{...(4.3.7)}$$

5. The phase angle ϕ of $T(j\omega)$ at resonant frequency is

$$\phi_r = -\tan^{-1}\left[\frac{\sqrt{1-2\xi^2}}{\xi}\right]$$

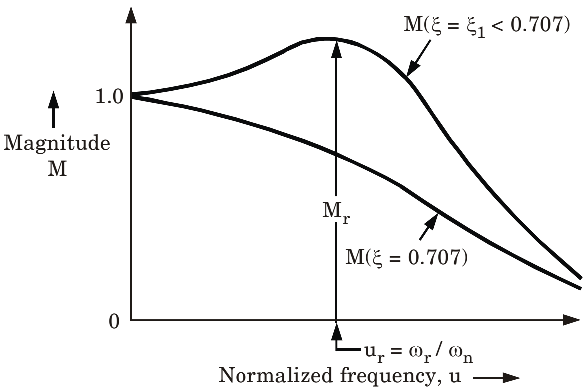


Fig. 4.3.3. Frequency response magnitude characteristics.

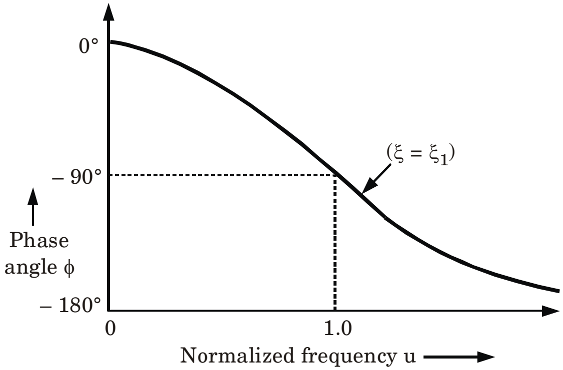


Fig. 4.3.4. Frequency response phase characteristic.

Bandwidth :

1. The frequency at which M has value of $\frac{1}{\sqrt{2}}$ and is called cut-off frequency ω_c. The signal frequencies above cut-off are attenuated.
2. The range of frequencies for which $M \geq \frac{1}{\sqrt{2}}$ is known as bandwidth ω_b.

   The low pass filters has bandwidth equal to cut-off. ω_b indicates the noise filtering characteristics of the system.

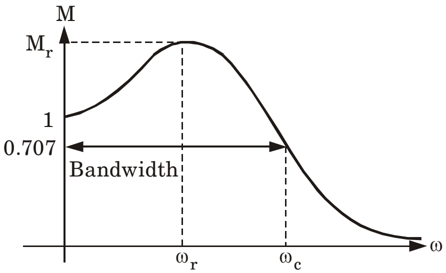


Fig. 4.3.5.

3. Normalized bandwidth, $u_b = \omega_b/\omega_n$

$$\therefore \qquad M = \frac{1}{\sqrt{(1-u_b^2)^2 + (2\xi u_b)^2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \qquad u_b = [1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}\,]^{\frac{1}{2}}$$

$$\omega_b = \omega\,[1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}\,]^{\frac{1}{2}}$$

4. The damped frequency of oscillations ω_d and peak overshoot M_p of the step response for $0 \leq \xi \leq 1$ are

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$M_p = \exp\left[\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right]$$

5. For $\xi > \frac{1}{\sqrt{2}}$, M_r does not exists so the correlation breaks down

$$\frac{\omega_r}{\omega_d} = \frac{\sqrt{1-2\xi^2}}{\sqrt{1-\xi^2}}$$

Que 4.4. **The steady state output of the system for a sinusoidal input of unit magnitude and variable frequency ω is given as**

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2+4z^2u^2}}\sin\left(\omega t - \tan^{-1}\frac{27u}{\sqrt{1-u^2}}\right)$$

Determine :

i. Resonant frequency **ii. Resonant peak**

iii. Bandwidth **iv. Phase angle.**

AKTU 2016-17, Marks 10

Answer

Given : $c(t) = \frac{1}{\sqrt{(1-u^2)^2+4z^2u^2}}\sin\left(\omega t - \tan^{-1}\frac{27u}{\sqrt{1-u^2}}\right)$

To Find : i. Resonant frequency
ii. Resonant peak
iii. Bandwidth.

1. $$M = \frac{1}{\sqrt{(1-u^2)^2+4z^2u^2}}$$

2. $$\phi = -\tan^{-1}\frac{27u}{\sqrt{1-u^2}}$$

3. The frequency where M has a peak value is known as resonant frequency. At this frequency, the slope of the magnitude curve is zero. Let ω_r be the resonant frequency and $u_r = \frac{\omega_r}{\omega_n}$

$$\left.\frac{dM}{du}\right|_{u=u_r} = -\frac{1}{2}\frac{[-4(1-u_r^2)u_r + 8z^2u_r]}{[(1-4_r^2)^2+4z^2u_r^2]^{3/2}} = 0$$

$$-4(1-u_r^2) = 8z^2$$

$$1-u_r^2 = 2z^2$$

$$u_r = \sqrt{1-2z^2}$$

$$\therefore \quad \omega_r = \omega_n\sqrt{1-2z^2}$$

4. Maximum value of magnitude is known as resonant peak is given by

$$M_r = \frac{1}{\sqrt{(1-u_r^2)^2 + 4z^2u_r^2}} = \frac{1}{\sqrt{(1-1+2z^2)^2 + 4z^2(1-2z^2)}}$$

$$= \frac{1}{\sqrt{4z^4 + 4z^2(1-2z^2)}} = \frac{1}{2z\sqrt{z^2+1-2z^2}} = \frac{1}{2z\sqrt{1-z^2}}$$

5. The range of frequencies over which M is equal to or greater than $\frac{1}{\sqrt{2}}$:

Putting $u_b = \frac{\omega_b}{\omega_n}$

$$M = \frac{1}{\sqrt{(1-u_b^2)^2 + 4z^2u_b^2}} = \frac{1}{2}$$

$$(1 - u_b^{\,2})^2 + 4z^2u_b^{\,2} = 2$$

$$1 + u_b^{\,4} - 2u_b^{\,2} + 4z^2u_b^{\,2} - 2 = 0$$

$$u_b = [1 - 2z^2 + \sqrt{2 - 4z^2 + 4z^4}\,]^{1/2}$$

$$\therefore \qquad \omega_b = \omega_n\,[1 - 2z^2 + \sqrt{1 - 4z^2 + 4z^4}\,]^{1/2}$$

PART-2

Construction of Polar and Inverse Polar Plots, Nyquist Stability Criterion.

CONCEPT OUTLINE : PART-2

- **Polar plot :** Polar plot of a transfer function $G(j\omega)$ is a plot of magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity.

$$M = |G(j\omega)\,H(j\omega)| = \text{Magnitude}$$

$$\phi = \angle G(j\omega)\,H(j\omega) = \text{Phase}$$

- **Nyquist stability criterion :**

i. There is no encirclement of $-1 + j0$ point. This implies that system is stable if there are no poles of $G(s)\,H(s)$ in right half of s-plane, otherwise unstable.

ii. There is a counter clockwise encirclement of $-1 + j0$ point. This implies that the system is stable if the number of counter clockwise encirclements is same as the number of poles of $G(s)$ $H(s)$ in the right-half of s-plane, otherwise system is unstable.

iii. There is a clockwise encirclement of $-1 + j0$ point. In this case the system is unstable.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.5. **Explain Polar plot. Discuss phase margin and gain margin on polar plot.**

OR

Define the following :

i. Resonance frequency

ii. Bandwidth

iii. Cut-off rate

iv. Phase margin

v. Gain margin

Also discuss the advantages of frequency domain analysis.

AKTU 2013-14, Marks 10

Answer

Resonance frequency : Refer Q. 4.2, Page 4–3C, Unit-4.

Bandwidth : Refer Q. 4.2, Page 4–3C, Unit-4.

Cut-off rate : Refer Q. 4.2, Page 4–3C, Unit-4.

A. Polar plot :

1. Polar plot of a transfer function $G(j\omega)$ is a plot of the magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar coordinates as ω is varied from zero to infinity. Therefore it is the locus of vectors $|G(j\omega)| \angle G(j\omega)$ as ω is varied from zero to infinity.
2. In frequency response we have

$$M = |G(j\omega)\, H(j\omega)| = \text{Magnitude}$$

$$\phi = \angle G(j\omega)\, H(j\omega) = \text{Phase}$$

Values of M and ϕ are obtained by varying the input frequency ω from 0 to ∞.

Table 4.5.1.

ω	$M = \|G(j\omega)\, H(j\omega)\|$	$\phi = \angle G(j\omega)\, H(j\omega)$
0	M_0	ϕ_0
ω_1	M_1	ϕ_1
ω_2	M_2	ϕ_2
⋮	⋮	⋮
∞	M_∞	ϕ_∞

Above table is used for polar plot.

Advantage :

It shows the frequency response characteristics of a system over the entire frequency range in a single plot.

Disadvantage : It does not indicate the contribution of each individual factor of the open loop transfer function.

B. Procedure to sketch the polar plot :

1. Determine $G(s)$.
2. Put $s = j\omega$ in $G(s)$.
3. Evaluate $|G(j\omega)|$ at $\omega = 0$ and $\omega = \infty$.
4. Evaluate phase angle at $\omega = 0$ and $\omega = \infty$.
5. Separate real and imaginary parts.
6. Put imaginary part = 0 and find ω. Using ω find $G(j\omega)$.
7. Put real part = 0 and find ω. Using ω find $G(j\omega)$.

C. Phase margin :

1. The phase margin is that amount of the additional phase lag at the gain cross over frequency required to bring the system to the verge of instability.
2. Gain cross over frequency is the frequency at which the magnitude of open loop transfer function ($|G(j\omega)|$) is unity.
3. Phase margin is equal to 180° plus the angle of $G(j\omega)$ at the gain cross over point.

$$\phi_m = 180° + \phi$$

D. Gain margin :

1. It is the reciprocal of magnitude $|G(j\omega)|$ at the frequency at which the phase angle is –180°.

   Gain Margin (GM),

$$K_g = \frac{1}{|G(j\omega_c)|}$$

2. When the plot passes through the point $(-1 + j0)$, the gain margin is zero it means that the loop gain can no longer be increased because the system is at verge of instability.
3. If gain margin is positive, it means that the system is stable and negative gain margin means the system is unstable.

E. Stability :

1. To determine the stability of the system first draw the polar plot for given transfer function.
2. If the critical point $(-1 + j0)$ is within the plot, it means the system is unstable.

3. If the polar plot passes through the point (–1 + j0) the system is marginally stable and if the critical point is outside the polar plot, the system is stable.

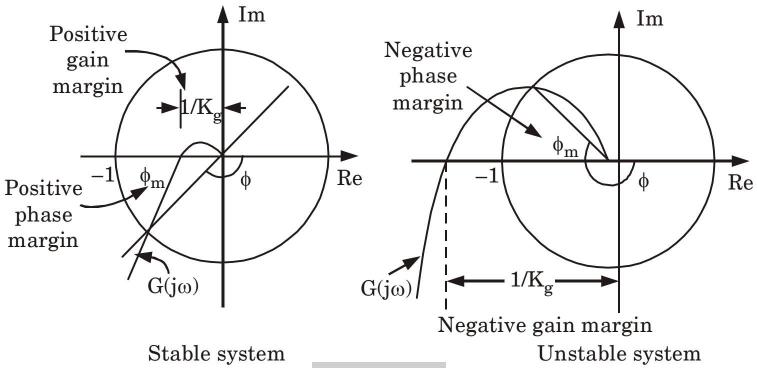


Fig. 4.5.1.

F. Advantages of frequency domain analysis :

1. In frequency domain the design and parameter adjustment of the open-loop transfer function of a system for closed loop performance is easy than time domain.
2. The effects of noise disturbance and parameter variation are easy to visualize and access.
3. The transient response of a system can be obtained from its frequency response through Fourier integral.
4. Nyquist criteria is a powerful frequency domain method of finding stability as well as relative stability of a system.
5. The frequency response is easily evaluated from sinusoidal transfer function by replacing s by $j\omega$.
6. The transfer function $T(j\omega)$ is a complex function of frequency and has both magnitude and a phase angle.

Que 4.6. **Sketch the polar plots for**

i. Type 0 system, $G(s) = \dfrac{3}{(s+1)(s+2)}$

ii. Type 1 system, $G(s) = \dfrac{12}{s(s+1)(s+2)}$

Answer

Given : i. $G(s) = \dfrac{3}{(s+1)(s+2)}$ ii. $G(s) = \dfrac{12}{s(s+1)(s+2)}$

To Sketch : Polar plot.

i.

1. $$G(s) = \frac{3}{(s+1)(s+2)}$$

2. Putting $s = j\omega$, $G(s) = \dfrac{3}{(1+j\omega)(2+j\omega)}$

3. $$M = \frac{3}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}} \quad \text{and} \quad \phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

For various value of ω, the value of M and ϕ are :

Table 4.6.1.

ω	M	ϕ
0	1.5	0°
5	0.109	– 146.89
10	0.029	– 178.28
⋮	⋮	⋮
∞	0	– 180°

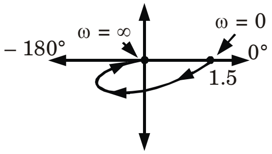


Fig. 4.6.1.

ii.

1. $$G(s) = \frac{12}{s(s+1)(s+2)}$$

2. Putting $s = j\omega$, $G(j\omega) = \dfrac{12}{j\omega(1+j\omega)(2+j\omega)}$

3. $$M = \frac{12}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}} \quad \text{and} \quad \phi = -90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

Table 4.6.2.

ω	M	ϕ
0	∞	– 90°
5	0.087	– 236.8°
10	0.012	– 268.2°
⋮	⋮	⋮
∞	0	– 270°

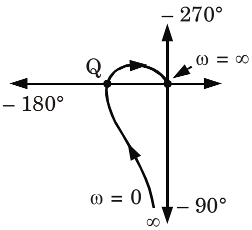


Fig. 4.6.2.

The co-ordinates of Q can be obtained by rationalizing $G(j\omega)$.

Que 4.7. **Sketch the inverse polar plot of**

$$G(s) = \frac{10}{(s+10)}$$

Answer

Given : $G(s) = \dfrac{10}{(s+10)}$

To Sketch : Inverse polar plot.

1. $$G(s) = \frac{10}{(s+10)}$$

2. Putting $$s = j\omega,\ G(j\omega) = \frac{10}{(j\omega+10)} = \frac{1}{1+j(0.1\omega)}$$

3. $\therefore$ $$\frac{1}{G(j\omega)} = 1 + j(0.1\omega) = M \angle \phi$$

where, $M = \sqrt{1+(0.1\omega)^2}$ and $\phi = \tan^{-1}(0.1\omega)$

Table 4.7.1.

ω	M	ϕ
0	1	0°
1	1.005	5.71°
10	1.414	45°
100	10.05	84.28°
⋮	⋮	⋮
∞	∞	+ 90°

The inverse polar plot is shown in the Fig. 4.7.1.

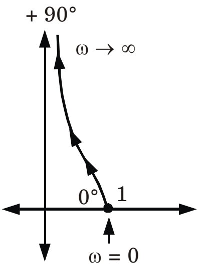


Fig. 4.7.1. Inverse polar plot.

Que 4.8. **Sketch the polar plot for**

i. $G(s) = \dfrac{10e^{-s}}{s+1}$

ii. $G(s) = \dfrac{32}{(s+4)(s^2+4s+8)}$

and find its points of intersection with the real and imaginary axes. **AKTU 2017-18, Marks 10**

Answer

i. Given : $G(s) = \dfrac{10e^{-s}}{s+1}$

To Sketch : Polar plot.

1. Putting $s = j\omega$

$$G(j\omega) = \frac{10e^{-j\omega}}{j\omega + 1}$$

2. Magnitude :

$$|G(j\omega)| = \frac{|10e^{-j\omega}|}{\sqrt{\omega^2+1}} = \frac{10}{\sqrt{\omega^2+1}}$$

3. Phase angle :

$$\angle G(j\omega) = -\omega - \tan^{-1}\omega$$

4. Here the magnitude decreases from unity monotonically and the phase angle also decreases monotonically and indefinitely, the polar plot of the given transfer function is spiral as shown in Fig. 4.8.1.

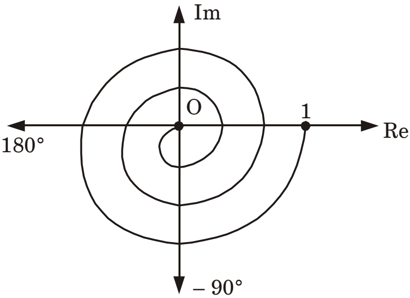


Fig. 4.8.1.

ii. Given : $G(s) = \dfrac{32}{(s+4)(s^2+4s+8)}$

To Sketch : Polar plot.

1. Putting $s = j\omega$

$$G(j\omega) = \frac{32}{(j\omega+4)[j\omega^2+4j\omega+8]}$$

$$= \frac{32}{(j\omega+4)[(8-\omega^2)+4j\omega]}$$

$$= \frac{32(4-j\omega)[(8-\omega^2)-4j\omega]}{(16+\omega^2)[(8-\omega^2)^2+16\omega^2]}$$

$$= \frac{32[4(8-\omega^2)-4\omega^2-j\{\omega(8-w^2)+16\omega\}]}{(16+\omega^2)[(8-\omega^2)^2+16\omega^2]}$$

2. Saparating into real and imaginary parts

$$G(j\omega) = \frac{32[4(8-\omega^2)-4\omega^2]}{(16+\omega^2)[(8-\omega^2)^2+16\omega^2]} - \frac{j32[\omega(8-\omega^2)+16\omega]}{(16+\omega^2)[(8-\omega^2)^2+16\omega^2]}$$

3. Intersection point with real axis,

$$\omega(8-\omega^2)+16\omega = 0$$
$$8\omega-\omega^3+16\omega = 0$$
$$\omega^3-24\omega = 0$$
$$\omega(\omega^2-24) = 0$$
$$\omega_1 = 0, -4.899 \text{ and } +4.899$$

4. Intersection points on imaginary axis,

$$4\omega^2-4(8-\omega^2) = 0$$
$$4\omega^2 = 4(8-\omega^2)$$
$$4\omega^2 = 32-4\omega^2$$
$$\omega^2 = 4$$
$$\omega = \pm 2$$

5. $$|G(j\omega)| = \frac{32}{\sqrt{\omega^2+16\sqrt{(8-\omega^2)+16\omega^2}}}$$

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{4\omega}{8-\omega^2}\right)$$

6. The value of $G(j\omega)$ at $\omega = 0$, and $\omega = \infty$

$$|G(j0)| = \frac{32}{4 \times 8} = 1$$
$$|G(j\infty)| = 0$$
$$\angle G(j0) = 0°$$
$$\angle G(j\infty) = -270,$$

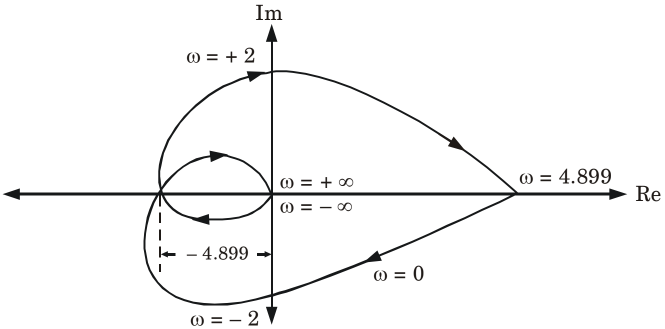


Fig. 4.8.2.

Que 4.9. **What is Nyquist stability criterion ? Explain phase margin and Gain Margin in polar plot.** **AKTU 2014-15, Marks 10**

Answer

1. Nyquist stability criterion calculates closed loop stability with the help of open loop transfer function without any complex calculations.
2. Complex calculations are done in polar plot.
3. If open loop transfer function is $G(s)H(s)$ then closed loop poles $[1 + G(s)H(s) = 0]$.

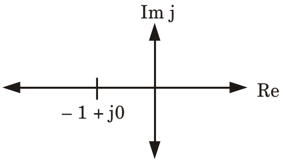


Fig. 4.9.1.

Encirclement :

1. If $-1 + j0$ point is encircled and in anti-clockwise direction.

$$N = P - Z$$
$$N = \text{–ve (for clockwise)}$$

P = number of poles of $G(s)H(s)$ right half of s-plane.

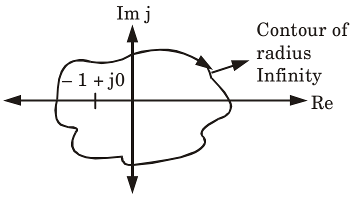


Fig. 4.9.2.

We check Z,
If $Z > 0$ the given system is unstable.
If $Z = 0$ the given system is stable.
Phase margin and gain margin in polar plot : Refer Q. 4.5, Page 4–10C, Unit-4.

Que 4.10. **For the $G(s) = 1/s(s-2)$, $H(s) = 1$. Sketch the Nyquist plot and determine the stability of the system.** **AKTU 2013-14, Marks 10**

Answer

Given : $G(s) = \dfrac{1}{s(s-2)}$, $H(s) = 1$

To Sketch : Nyquist plot.

1. $$G(s) = \frac{1}{s(s-2)}$$
$$G(s) = \frac{-1}{s(2-s)}$$
2. Put $s = j\omega$
$$G(j\omega) = \frac{-1}{j\omega(2-j\omega)}$$
$$|G(j\omega)| = \frac{1}{\omega\sqrt{4+\omega^2}}$$
3. $$\angle G(j\omega) = 180° - 90° + \tan^{-1}\frac{\omega}{2}$$
$$= 90° + \tan^{-1}\left(\frac{\omega}{2}\right)$$

Table 4.10.1.

ω	M	ϕ
0	∞	90°
1	0.4472	116.56°
10	9.8×10^{-3}	168.69°
100	9.99×10^{-5}	178.85°
∞	0 Amplitude decreases	180° Angle increases

4. This system is unstable because contour has infinite radius, so it encircle (– 1, 0) point.

$$N = P - Z;\ N = -1$$

$P = 1$ (number of open loop pole in right half of s-plane)

$$Z = 2$$

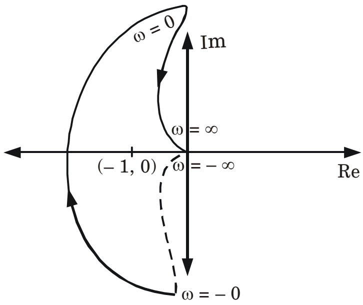


Fig. 4.10.1.

Que 4.11. **Draw the Nyquist plot for the open loop transfer function given below and comment on closed loop stability**

$$G(s)H(s) = \frac{1.5\,(s+4)}{s(s-2)}.$$

AKTU 2014-15, Marks 10

Answer

Given : $G(s)H(s) = \dfrac{1.5(s+4)}{s(s-2)}$

To Draw : Nyquist plot.

1. Put $s = j\omega$, $G(j\omega)H(j\omega) = \dfrac{1.5\,(j\omega+4)}{j\omega\,(j\omega-2)} = \dfrac{-1.5\,(j\omega+4)}{j\omega\,(2-j\omega)}$

2. $$M = |G(j\omega)\,H(j\omega)| = \frac{1.5\sqrt{16+\omega^2}}{\omega\sqrt{4+\omega^2}}$$

3. $$\angle\phi = \angle G(j\omega)\, H(j\omega) = 90° + \tan^{-1}\left(\frac{\omega}{4}\right) + \tan^{-1}\left(\frac{\omega}{2}\right)$$

Table 4.11.1.

ω	M	ϕ
0	∞	90°
0.1	29.97	94.29°
0.5	5.866	111.16°
10	0.1584	236.89°
100	0.015	266.57
∞	0	270°

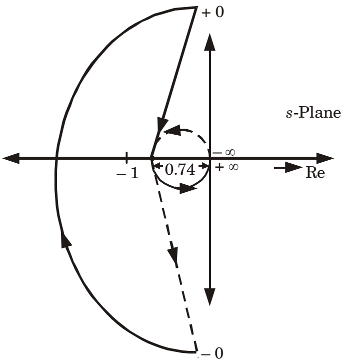


Fig. 4.11.1.

4. Because in type 1 system – 0 meets + 0 by angle π clockwise.
Because $-1 + j0$ $(-1, 0)$ is encircled and one time in clockwise direction
$N = -1$, $P = 1$

5. Number of poles of $G(s)H(s)$ on right half of s-plane

$$N = P - Z$$
$$Z = 2$$

So the given system is unstable.

Que 4.12. **Sketch the Nyquist plot for the system with open loop transfer function**

$$G(s)\, H(s) = \frac{60}{(s+1)(s+2)(s+5)}$$

and comment on stability. **AKTU 2015-16, Marks 10**

Answer

Given : $G(s)\,H(s) = \dfrac{60}{(s+1)(s+2)(s+5)}$

To Sketch : Nyquist plot.

1. Put $s = j\omega$, $G(j\omega)\,H(j\omega) = \dfrac{60}{(j\omega+1)(j\omega+2)(j\omega+5)}$

2. Magnitude, $M = |G(j\omega)\,H(j\omega)| = \dfrac{60}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})(\sqrt{\omega^2+25})}$

3. Phase $\phi = \angle G(j\omega)H(j\omega)$

$$= -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

Table 4.12.1.

ω	M	ϕ
0	6	0°
1.12	3.402	– 90.11°
4.12	0.4769	– 180°
10	0.0523	133.58
∞	0	90° (– 270°)

4. $N = P - Z$

$P = 0$ (No pole on RH plane)

$N = 0$ (No encirclement of $-1 + j0$ point)

$Z = 0$ (Number of roots of closed loop characteristic equation having positive real parts).

So the given system is stable system.

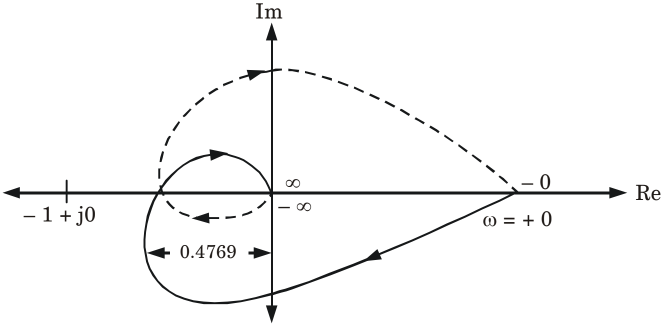


Fig. 4.12.1.

Que 4.13. Using Nyquist stability criterion, investigate the stability of a unity feedback system with open loop transfer function.

$$G(s) = \frac{(s - z_1)}{s(s + p_1)}, z_1, p_1 > 0$$

Answer

Given : $G(s) = \dfrac{(s - z_1)}{s(s + p_1)}$

To Check : Stability of system.

1.
$$G(j\omega)\,H(j\omega) = \frac{j\omega - z_1}{j\omega(j\omega + p_1)} = \frac{\sqrt{\omega^2 + z_1^2}}{\omega\sqrt{\omega^2 + p_1^2}}$$

$$\phi = \angle 90° - \tan^{-1}\left[\frac{\omega(p_1 + z_1)}{(p_1 z_1 - \omega^2)}\right]$$

2. Now,

$$\lim_{\omega \to 0} G(j\omega)\,H(j\omega) = \infty\angle + 90°$$

$$G(j\sqrt{p_1 z_1})\,H(j\sqrt{p_1 z_1}) = \frac{1}{p_1\sqrt{p_1 z_1}}\angle 0°$$

$$\lim_{\omega \to \infty} G(j\omega)\,H(j\omega) = 0\angle - 90°$$

3. Thus the locus comes down in the first quadrant, crosses the positive real axis into the fourth quadrant, and approaches the origin from an angle of – 90°.

4. Path $\overline{def}$ maps into the origin, and $\overline{ija}$ maps into on semicircle at infinity. The resulting plot is shown in Fig. 4.13.1.

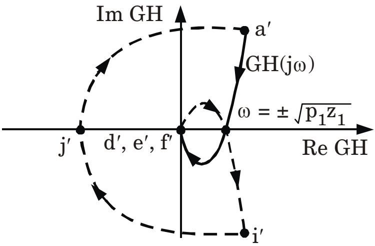


Fig. 4.13.1.

Que 4.14. For a closed loop system whose transfer function is $G(s)\,H(s) = \dfrac{Ke^{-sT}}{s(s+1)}$, determine the maximum value of the gain 'K' for stability.

AKTU 2016-17, Marks 10

Answer

Given : $G(s)\,H(s) = \dfrac{Ke^{-sT}}{s(s+1)}$

To Find : Maximum value of K.

1. Putting $$s = j\omega$$

$$\therefore \quad G(j\omega)\,H(j\omega) = \frac{Ke^{-j\omega T}}{j\omega(j\omega+1)}$$

2. $$|G(j\omega)\,H(j\omega)| = \frac{K}{\omega\sqrt{1+\omega^2}}$$

$$\angle G(j\omega)\,H(j\omega) = -\tan^{-1}\omega - 90° - \omega T \times \frac{180°}{\pi}$$

$$= -57.3\omega T - 90° - \tan^{-1}\omega$$

3. Intersection of Nyquist plot with negative axis of $G(s)\,H(s)$ plane is determined by using following relation,

$$\angle G(j\omega)\,H(j\omega) = -180°\ (2k+1)$$

where, $k = 0, 1, 2$ ....

4. It the first instant, the Nyquist plot intersects the negative real axis of $G(s)\,H(s)$ plane for $k = 0$, and the frequency at intersection point is ω_2.

5. Therefore, $$\angle G(j\omega_2)\,H(j\omega_2) = -180°$$

$$-57.3\omega_2 T - 90° - \tan^{-1}\omega_2 = -180°$$

$$-57.3\omega_2 T - \tan^{-1}\omega_2 = -90°$$

6. $$T = 0.5$$

$$-28.65\omega_2 - \tan^{-1}\omega_2 = 90°$$

7. Using trial and error, $$\omega_2 = 1.3075 \text{ rad/sec}$$

$$G(j\omega_2)\,H(j\omega_2) = \frac{K}{\omega_2\sqrt{1+\omega_2^2}} = \frac{K}{1.3075\sqrt{1+(1.3075)^2}} = 0.4646\,K$$

8. For stability the point $(-1 + j0)$ be placed outside the Nyquist plot

$$0.4646\,K < 1$$

$$\therefore \quad K < 2.152$$

PART-3

Determination of Gain and Phase Margin from Bode and Nyquist Plots, Nichol Charts.

CONCEPT OUTLINE : PART-3

- **Bode plot :** In a bode plot we have two plots.

  Magnitude plot : Which is drawn in between magnitude of transfer function in dB and $\log_{10} \omega$.

  Phase plot : Which is drawn in between phase angle of transfer function in degree and $\log_{10} \omega$.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.15. Write procedure to draw Bode plot from given open loop transfer function.

Answer

A. Bode plot : In this plot we have two plots namely Magnitude plot; which is drawn in between magnitude of transfer function in dB and $\log_{10} \omega$ (ω is frequency) and phase plot; which is drawn in between phase angle of transfer function in degree and $\log_{10}\omega$ generally.

B. Procedure to draw Bode plot :

Step 1 :

Replace $s = j\omega$ from open loop transfer function $G(s)$ to have $G(j\omega)$.

Step 2 :

Change all the factor of $G(j\omega)$ in standard form and make a table with remarks for both magnitude plot and phase plot.

Some standard results and remarks is summarized here :

Table 4.15.1.

S. No.	Factors of G(jω)	Corner Frequencies	Remarks for Magnitude Plot	Remarks for Phase Plot
1.	$\frac{K}{j\omega}$	None	Straight line of constant slope – 20 dB/ decade passing through 20 log K at $\omega = 1$	Constant – 90°
2.	$K.j\omega$	None	Slope = + 20 dB/ decade straight line of constant slope – 20 dB/ decade through 20 log K at $\omega = 1$	Constant +90°
3.	$\frac{1}{1 + j\omega T_1}$	$\omega_{c1} = \frac{1}{T_1}$	Straight line of slope – 20 dB/ decade for $\omega > \omega_{c1}$	Phase angle = $-\tan^{-1}(\omega T)$
4.	$(1 + j\omega T_1)$	$\omega_{c2} = \frac{1}{T_2}$	Straight line of slope + 20 dB/ decade for $\omega > \omega_{c2}$	Phase angle = $-\tan^{-1}(\omega T)$
5.	$\frac{1}{1 + 2j\xi\left(\frac{\omega}{\omega_n}\right) + \left(\frac{\omega}{\omega_n}\right)^2}$	$\omega_{c3} = \omega_n$	Straight line of slope – 40 dB/ decade for $\omega > \omega_{c3}$	Phase angle = $-\tan^{-1}\left[\frac{\xi(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right]$
6.	$1 + 2j\xi\left(\frac{\omega}{\omega_n}\right) + \left(\frac{\omega}{\omega_n}\right)^2$	$\omega_{c4} = \omega_n$	Straight line of slope + 40 dB/ decade for $\omega > \omega_{c4}$	Phase angle = $+\tan^{-1}\left[\frac{\xi(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right]$

Step 3 :
From the table, draw the magnitude plot for each of the factor by calculating resultant slope and starting with lowest corner frequency and ending with the highest one.

Step 4 :
Add all the expression of phase angle to have different values of ϕ (resultant phase).

Step 5 :
Draw ϕ with $\log_{10}\omega$ to have phase plot.

Que 4.16. **Explain gain margin and phase margin and how to determine them using Bode plot.**

Answer

A. Gain Margin (*GM*) :

1. It is the margin in gain which is allowed till the system reaches on the verge of instability.
2. Mathematically, gain margin is reciprocal of magnitude of $G(j\omega)H(j\omega)$ at phase crossover frequency.

$$GM = 1 / |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

B. Phase Margin (*PM*) : It is the amount of additional phase lag which can be introduced in the system till it reaches on the verge of instability.

Mathematically, $PM = 180° + \angle G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$

C. Calculation of *GM* and *PM* from Bode plot :

1. $$GM = -\,20\ \log|G(j\omega)\ H(j\omega)|_{\omega\,=\,\omega_{pc}}\ \text{dB}$$
$$= 0\ \text{dB} - \log|G(j\omega)\ H(j\omega)|_{\omega\,=\,\omega_{pc}}\ \text{dB}$$
$20\ \log|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$ can be directly known from magnitude plot.

2. Now, $$\text{PM} = 180° + \angle G(j\omega)H(j\omega)\big|_{\omega=\omega_{gc}}$$
$\angle G(j\omega)H(j\omega)\big|_{\omega=\omega_{gc}}$ can be directly known on phase angle plot.

3. **Gain crossover frequency (ω_{gc}) :**
Frequency at which magnitude of $G(j\omega)H(j\omega)$ is unity.

4. **Phase crossover frequency (ω_{pc}) :**
Frequency at which phase angle of $G(j\omega)H(j\omega)$ is –180°.

5. Stable system = *PM* and *GM* positive.
Unstable system = *PM* and *GM* negative
Marginally stable = *PM* = 0, *GM* = 0

Que 4.17. **Draw Bode plot (log magnitude plot) for the transfer function.**

$$G(s) = \frac{20s}{s^2 + 20s + (100)^2}$$

AKTU 2016-17, Marks 10

Answer

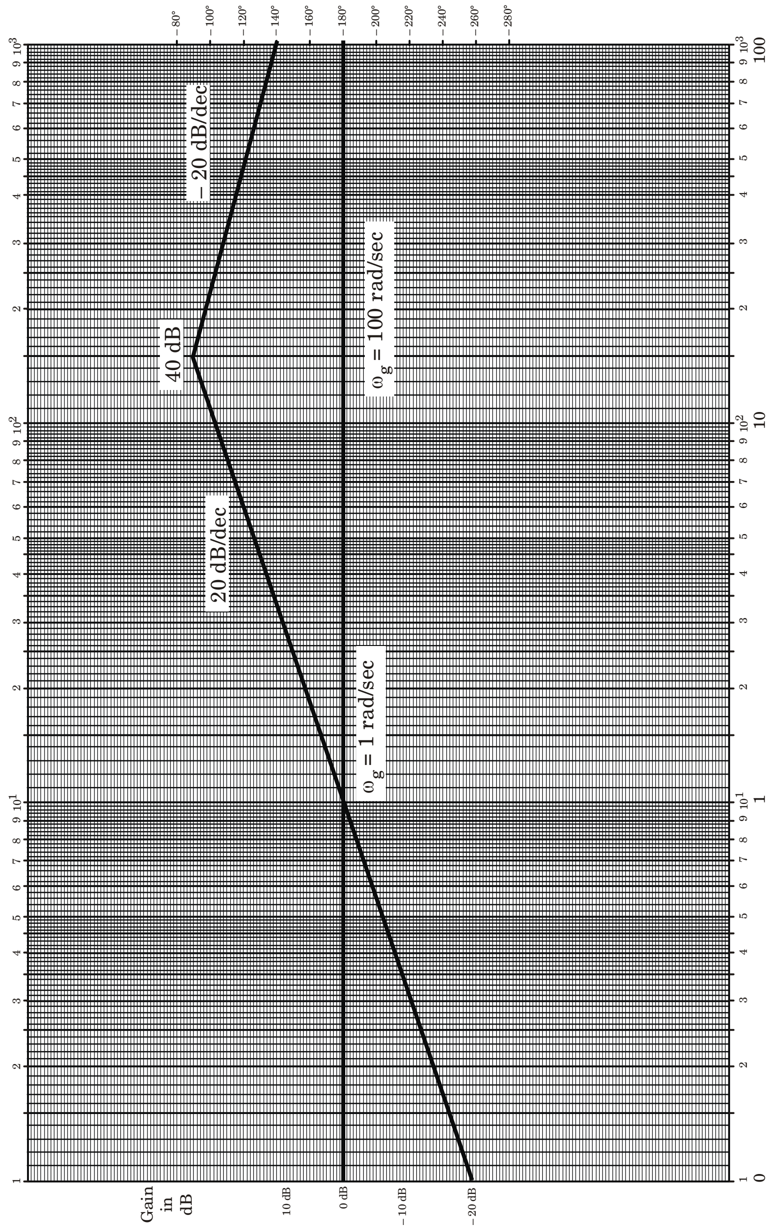
Given : $G(s) = 20s / [s^2 + 20s + (100)^2]$
To Draw : Bode plot.

1. $$G(s) = \frac{20s}{(100)^2\left[1 + \dfrac{20s}{(100)^2} + \left(\dfrac{s}{100}\right)^2\right]}$$

S. No.	Factor	Corner frequency	Asymptotic log-magnitude characteristic
1.	s	None	Straight line of constant slope (20 dB / dec) passing through $\omega = 1$
2.	$\dfrac{1}{\left[1 + \dfrac{20s}{(100)^2} + \left(\dfrac{s}{100}\right)^2\right]}$	$\omega_1 = 100$ rad/sec	Straight line of constant slope (– 40 dB/dec) originating from $\omega = 100$ rad/sec.

$$K = 20 / (100)^2 = 0.002$$

Bode plot :


– 80°
– 100°
– 120°
– 140°
– 160°
– 180°
– 200°
– 220°
– 240°
– 260°
– 280°
– 20 dB/dec
40 dB
20 dB/dec
ω_g = 100 rad/sec
ω_g = 1 rad/sec
Gain in dB
10 dB
0 dB
– 10 dB
– 20 dB
100
10
1
0

Que 4.18. **Draw the bode plot for the transfer function given as, apply correction to the magnitude plot for the quadratic term and comment on stability.**

$$G(s)\,H(s) = \frac{5}{s^2\,(1 + 0.1s)\,(s^2 + 0.4s + 1)}$$

AKTU 2014-15, Marks 10

Answer

Given : $G(s)\,H(s) = \dfrac{5}{s^2(1+0.1s)(s^2+0.4s+1)}$

To Sketch : Bode plot.

1. $G(s)H(s)$ is in time constant form.
2. Factors :

i. $K = 5$, $20 \log K = 13.98$ dB, straight line parallel to Log ω axis.

ii. $\dfrac{1}{s^2}$, 2 poles at origin, straight line of slope – 40 dB / dec passing through intersection point of $\omega = 1$ and 0 dB.

iii. Quadratic pole, $\dfrac{1}{1+0.4s+s^2}$, $\omega_n^2 = 1$ *i.e.*, $\omega_{c1} = \omega_n = 1$

Straight line of slope – 40 dB / dec for $\omega \geq 1$.

iv. Comparing middle term with $2\,\xi\,\omega_n = 0.4$, $\xi = 0.2$

$\therefore$ Correction = – 20 Log $2\,\xi$ = + 7.95 dB at $\omega = \omega_n = 1$.

Thus magnitude plot will shift upwards by + 7.95 dB at $\omega = \omega_n = 1$.

iv. Simple pole, $\dfrac{1}{1+0.1s}$, $T_2 = 0.1$, $\omega_{C2} = \dfrac{1}{T_2} = 10$, straight line of slope – 20 dB/dec for $\omega \geq 10$.

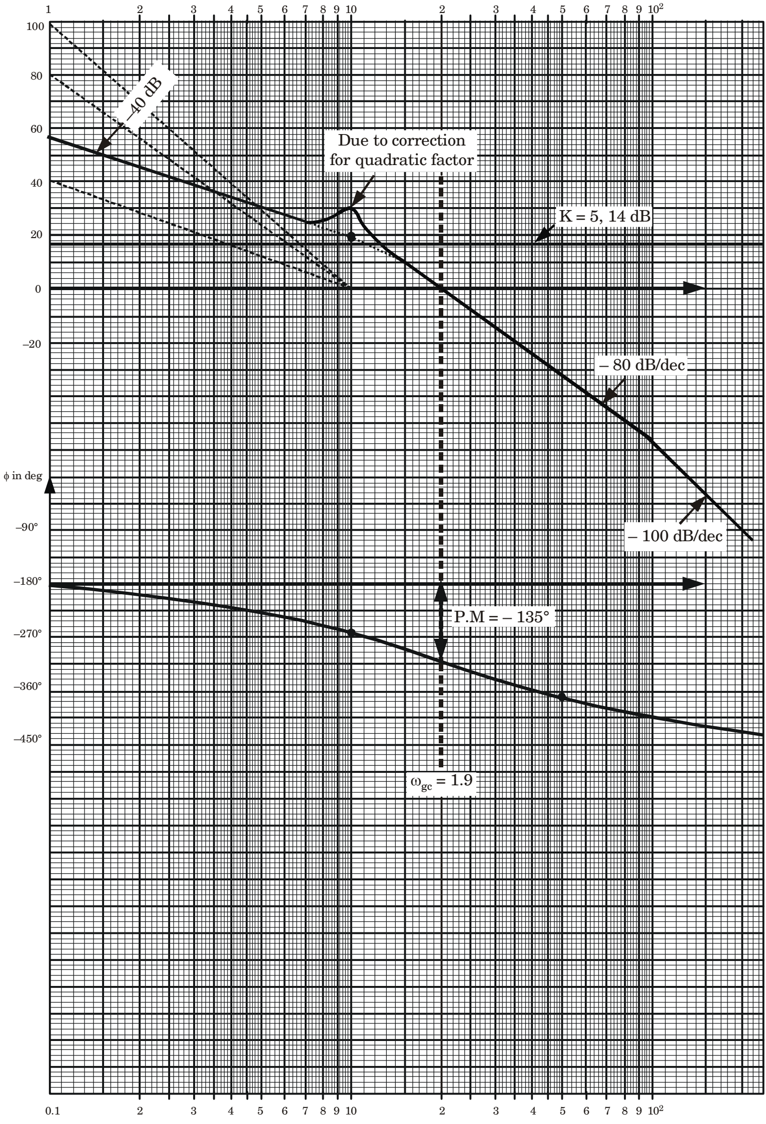
Table 4.18.1.

Range of ω	$0 < \omega < 1$	$1 \leq \omega < 10$	$10 \leq \omega < \infty$
Resultant slope in dB/dec	– 40	– 40 – 40 = – 80	– 80 – 20 = – 100

3. Phase angle :

$$G(j\omega)H(j\omega) = \frac{5}{(j\omega)^2(1+0.1j\omega)\,[(1-\omega^2)+0.4\;j\omega]}$$

Bode plot :



–40 dB
Due to correction for quadratic factor
K = 5, 14 dB
– 80 dB/dec
– 100 dB/dec
φ in deg
P.M = – 135°
ω_{gc} = 1.9

Table 4.18.2.

ω	$\dfrac{1}{(j\omega)^2}$	$-\tan^{-1}\left\{\dfrac{0.4\omega}{1-\omega^2}\right\}$	$-\tan^{-1} 0.1\omega$	ϕ
0.1	– 180°	– 2.31°	– 0.57°	– 182.88°
1	– 180°	– 90°	– 5.71°	– 275.71°
5	– 180°	+ 4.76° – 180° = –175.23°	– 26.56°	– 381.79°
∞	– 180°	– 180°	– 90°	– 450°

4. **Comment :** From the Bode plot $\omega_{pc} \to 0$ and phase angle plot does not cross – 180°. Thus system is unstable with G.M. = $-\infty$ dB and P.M. = – 135° at ω_{gc} = 1.9 rad/s.

Que 4.19. **A unity feedback control systems has :**

$$G(s) = \frac{40}{s(s+2)(s+5)}$$

Draw the Bode Plot. Find Gain Margin. **AKTU 2013-14, Marks 10**

Answer

Given : $G(s) = \dfrac{40}{s(s+2)(s+5)}$

To Draw : Bode plot.

1. **Magnitude plot :**

$$G(s) = \frac{40}{s(s+2)(s+5)} = \frac{40}{s \times 2\left(1+\dfrac{s}{2}\right) 5\left(1+\dfrac{s}{5}\right)} = \frac{4}{s\left(1+\dfrac{s}{2}\right)\left(1+\dfrac{s}{5}\right)}$$

First corner frequency (pole) (ω_1) = 2 rad/s
Second corner frequency (pole) (ω_2) = 5 rad/s

2. **Phase plot :**

$$G(s) = \frac{4}{s\left(1+\dfrac{s}{2}\right)\left(1+\dfrac{s}{5}\right)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{4}{j\omega(1+0.5j\omega)(1+0.2j\omega)}$$

$$\phi = -90^\circ - \tan^{-1} 0.5\omega - \tan^{-1} 0.2\omega$$

Magnitude plot :

Table 4.19.1.

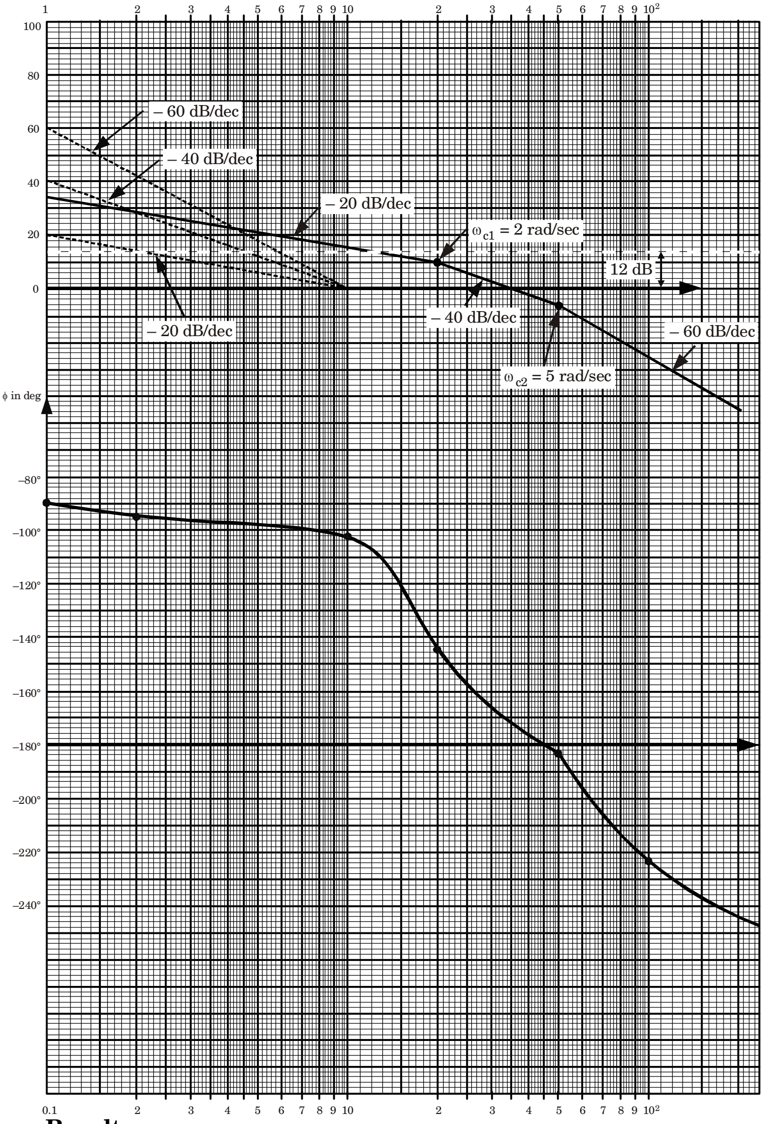
S. No.	Factor	Corner frequency characteristics	Asymptotic log magnitude
1.	$1/s$	None	Straight line of constant slope (– 20 dB/dec) passing through at $\omega = 1$
2.	$1/(1 + 0.5s)$	$\omega_1 = 2$	Straight line of constant slope (– 20 dB/dec) originating from $\omega_1 = 2$
3.	$1/(1 + 0.2s)$	$\omega_2 = 5$	Straight line of constant slope (– 20 dB/dec) originating from $\omega_2 = 5$
4.	4	None	Straight line of constant slope of 0 dB/dec starting from 20 log 4 = 12 dB point

Phase plot :

Table 4.19.2

S. No.	ω (rad/sec)	ϕ (degrees)
1.	0	– 90°
2.	0.2	– 98°
3.	1	– 127.874°
4.	2	– 156.8°
5.	5	– 203.198°
6.	10	– 232.125°
7.	15	– 243.97°

Bode plot :



Result :

i. Gain crossover frequency = 2.8 rad/s

ii. Phase cross over frequency = 3.3 rad/s

iii. Gain margin = 3 dB

iv. Phase margin = 8°.

Que 4.20. **For a unity feedback system, the open loop transfer function is**

$$G(s)\,H(s) = \frac{2(s+0.25)}{s^2\,(s+1)\,(s+0.5)}$$

Draw bode plot and determine gain margin, phase margin.

AKTU 2015-16, Marks 10

Answer

Given : $G(s)\,H(s) = \dfrac{2(s+0.25)}{s^2(s+1)\,(s+0.5)}$

To Draw : Bode plot.

1. $$G(s)\,H(s) = \frac{\dfrac{2\times 0.25}{0.5}\left(\dfrac{s}{0.25}+1\right)}{s^2(s+1)\left(\dfrac{s}{0.5}+1\right)} = \frac{(4s+1)}{s^2(s+1)\,(2s+1)}$$

2. $K = 1$, 20 Log $K = 0$ dB, No effect on Bode Plot.

3. $\dfrac{1}{s^2}$, 2 Poles at origin, straight line of slope – 40 dB/dec

4. $(1 + 4s)$, simples zero, $T_1 = 4$, $\omega_{C1} = \dfrac{1}{T_1} = 0.25$,

   straight line of slope + 20 dB/dec for $\omega \geq 0.25$

5. $\dfrac{1}{1+2s}$, simple zero, $T_2 = 2$, $\omega_{C2} = \dfrac{1}{T_2} = 0.5$,

   straight line of slope – 20 dB/sec for $\omega \geq 0.5$

6. $\dfrac{1}{1+s}$, simple zero, $T_3 = 1$, $\omega_{C3} = \dfrac{1}{T_3} = 1$,

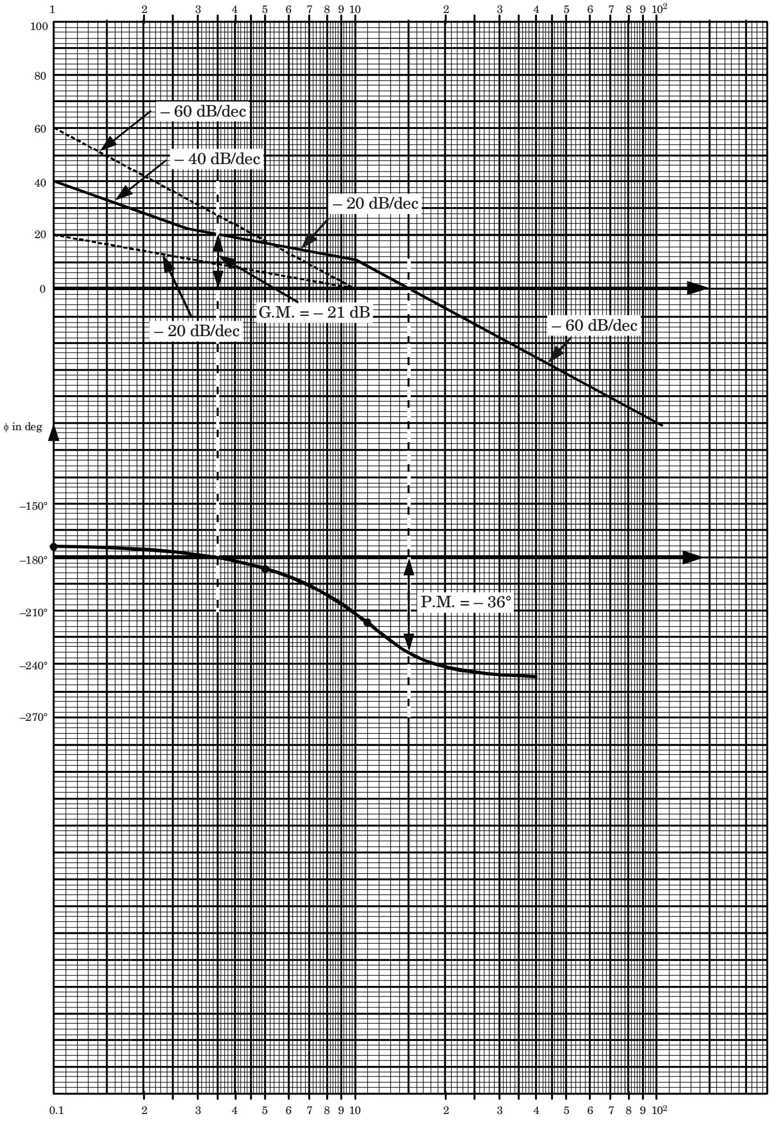
   straight line of slope – 20 dB/sec for $\omega \geq 1$

Table 4.20.1.

Range of ω	$0 < \omega < 0.25$	$0.25 \leq \omega < 0.5$	$0.5 \leq \omega < 1$	$1 \leq \omega < \infty$
Resultant slope in dB/dec	– 40	– 40 + 20 = – 20	– 20 – 20 = – 40	– 40 – 20 = – 60

7. **Phase angle table :** $G(j\omega)H(j\omega) = \dfrac{(1+4\,j\omega)}{(j\omega)^2\,(1+2\,j\omega)\,(1+\,j\omega)}$

Bode plot :



1
2
3
4
5
6
7
8
9
10
2
3
4
5
6
7
8
9
10²
100
80
60
40
20
0
– 60 dB/dec
– 40 dB/dec
– 20 dB/dec
– 20 dB/dec
G.M. = – 21 dB
– 60 dB/dec
ϕ in deg
–150°
–180°
–210°
–240°
–270°
P.M. = – 36°
0.1

Table 4.20.2.

ω	$\dfrac{1}{(j\omega)^2}$	$+\tan^{-1}4\omega$	$+\tan^{-1}2\omega$	$-\tan^{-1}\omega$	ϕ
0.1	– 180°	+ 21.8°	– 11.3°	– 5.71°	– 175.2°
0.5	– 180°	+ 63.43°	– 45°	– 26.56°	– 188.13°
1	– 180°	+ 75.96°	– 63.43°	– 45°	– 212.47°
∞	– 180°	+ 90°	– 90°	– 90°	– 270°

Result : PM = – 36°, GM = – 21 dB.

Que 4.21. Discuss constant magnitude loci.

OR

Using Nyquist stability criterion, investigate the stability of a unity feedback system with open loop transfer function.

$$G(s) = \frac{(s - z_1)}{s(s + p_1)}, \; z_1, p_1 > 0$$

Also discuss the significance of M-circle.

AKTU 2016-17, Marks 10

Answer

A. Numerical : Refer Q. 4.13, Page 4–22C, Unit-4.

B. M-circle :

1. To obtain the constant magnitude loci (M-circle), let us first note that $G(j\omega)$ is a complex quantity and can be written as follows :

$$G(j\omega) = X + jY$$

where X and Y are real quantities

2. Then M is given by

$$M = \frac{|X + jY|}{|1 + X + jY|}$$

and M^2 is

$$M^2 = \frac{X^2 + Y^2}{(1 + X)^2 + Y^2}$$

3. Hence, $X^2(1 - M^2) - 2M^2X - M^2 + (1 - M^2)Y^2 = 0$ ...(4.21.1)

If $M = 1$, then from above eq. (4.21.1) we obtain $X = -1/2$. This is the equation of a straight line parallel to the Y axis and passing through point $(-1/2, 0)$.

4. If $M \neq 1$, equation can be written as

$$X^2 + \frac{2M^2}{M^2-1}X + \frac{M^2}{M^2-1} + Y^2 = 0$$

5. If the term $M^2/(M^2-1)^2$ is added to both sides of this last equation we obtain

$$\left[X^2 + \frac{M^2}{M^2-1}\right]^2 + Y^2 = \frac{M^2}{(M^2-1)^2} \quad ...(4.21.2)$$

6. Eq. (4.21.2) is the equation of circle with centre $X = \frac{-M^2}{(M^2-1)}$, $Y = 0$ and with radius $|M/(M^2-1)|$.
7. The constant M loci on the $G(s)$ plane are a family of circle. The centre and radius of the circle for a given value of M can be easily calculated.

C. Significance : M-circle can be used to obtain closed loop frequency response from open loop frequency response.

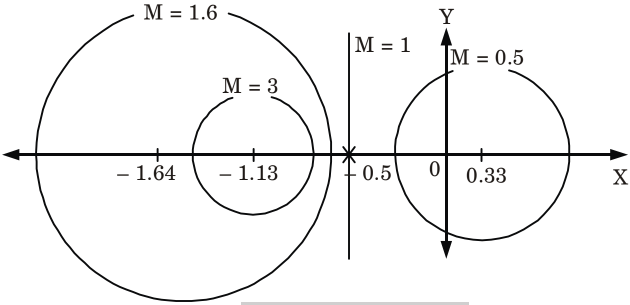


Fig. 4.21.1. M-circles.

Que 4.22. Discuss constant phase angle loci for frequency response system.

OR

Discuss N-circle construction.

Answer

1. N-circles having constant phase.

   $G(j\omega) = x + jy$ where x and y are real number.

2. Also $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{x+jy}{1+x+jy} \quad ...(4.22.1)$$

3. If α is the phase shift of the output with respect to input, we have

$$\alpha = \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x}$$

$$\tan \alpha = \tan\left[\tan^{-1}\frac{y}{x} - \tan^{-1}\frac{y}{1+x}\right] = \frac{\dfrac{y}{x} - \dfrac{y}{1+x}}{1 + \dfrac{y}{x}\left[\dfrac{y}{1+x}\right]}$$

$$\tan \alpha = \frac{y}{x^2 + x + y^2}$$

4. If $\tan \alpha = N$, we have

$$x^2 + x + y^2 - \frac{1}{N}y = 0 \quad ...(4.22.2)$$

This can be modified to

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2 \quad ...(4.22.3)$$

5. This is again an equation of circle having centre $\left[-\frac{1}{2}, \frac{1}{2N}\right]$ and radius $\sqrt{\frac{1}{4} + \frac{1}{4N^2}}$.

As $x = 0$, $y = 0$ and $x = -1$, $y = 0$ satisfy this eq. (4.22.3) irrespective value of N. Each circle passes through the origin and (–1, 0) point.

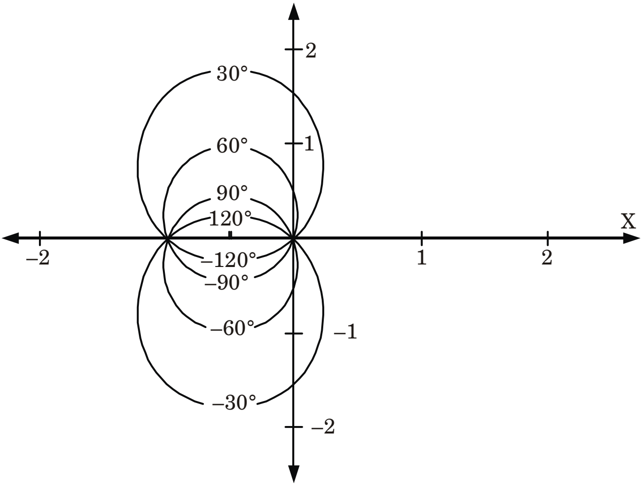


Fig. 4.22.1. N-circles.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain various frequency domain specifications.

Ans. Refer Q. 4.2, Unit-4.

Q. 2. What is closed loop frequency response ? Give an account of the correlation between time response and frequency response for a second order system with relevant expressions.

Ans. Refer Q. 4.3, Unit-4.

Q. 3. Sketch the polar plot for

i. $G(s) = \dfrac{10e^{-s}}{s+1}$

ii. $G(s) = \dfrac{32}{(s+4)(s^2+4s+8)}$

and find its points of intersection with the real and imaginary axes.

Ans. Refer Q. 4.8, Unit-4.

Q. 4. What is Nyquist stability criterion ? Explain phase margin and Gain Margin in polar plot.

Ans. Refer Q. 4.9, Unit-4.

Q. 5. Sketch the Nyquist plot for the system with open loop transfer function

$$G(s)\,H(s) = \frac{60}{(s+1)\,(s+2)\,(s+5)}$$

and comment on stability.

Ans. Refer Q. 4.12, Unit-4.

Q. 6. Using Nyquist stability criterion, investigate the stability of a unity feedback system with open loop transfer function.

$$G(s) = \frac{(s - z_1)}{s(s + p_1)},\ z_1, p_1 > 0$$

Ans. Refer Q. 4.13, Unit-4.

Q. 7. Explain gain margin and phase margin and how to determine them using Bode plot.

Ans. Refer Q. 4.16, Unit-4.

Q. 8. Draw Bode plot (log magnitude plot) for the transfer function.

$$G(s) = \frac{20s}{s^2 + 20s + (100)^2}.$$

Ans. Refer Q. 4.17, Unit-4.

Q. 9. Draw the bode plot for the transfer function given as, apply correction to the magnitude plot for the quadratic term and comment on stability.

$$G(s)\,H(s) = \frac{5}{s^2\,(1 + 0.1s)\,(s^2 + 0.4s + 1)}$$

Ans. Refer Q. 4.18, Unit-4.

Q. 10. Discuss constant magnitude loci.

Ans. Refer Q. 4.21, Unit-4.

☺☺☺

5 UNIT

Introduction to Design

PART-1

The Design Problems and Preliminary Consideration of Lead, Lag and Lead-Lag Compensation Networks, Design of Closed Loop System using Compensation Techniques in Time and Frequency Domain.

CONCEPT OUTLINE : PART-1

- **Compensation :** If a system is to be redesigned so as to meet the required specifications, it is necessary to alter the system by adding an external device to it. Such a redesign or alteration of system using an additional suitable device is called compensation of a control system.
- **Phase-lead compensation :**

$$\frac{E_o(s)}{E_i(s)} = \frac{\alpha(1+sT)}{(1+s\alpha T)}$$

- **Phase-lag compensation :**

$$\frac{E_o(s)}{E_i(s)} = \frac{1+sT}{1+s\beta T}$$

- **Phase lead-lag compensation :**

i. Lead-lag compensator is combination of lead network and lag network.

ii. Transfer function,

$$G_c(s) = K_c \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{\beta T_2}\right)}{\left(s+\frac{\beta}{T_1}\right)\left(s+\frac{1}{\beta T_2}\right)}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **Define Phase-lead compensation technique. How would you draw its pole zero configuration ?**

Answer

1. Fig. 5.1.1 shows a phase-lead network where in the phase of output voltage leads the phase of input voltage for sinusoidal input.

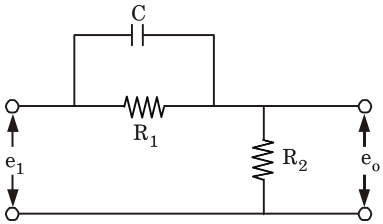


Fig. 5.1.1. Phase lead network.

2. The transfer function of a phase lead network,

$$\frac{E_o(s)}{E_i(s)} = \frac{\alpha(1+sT)}{(1+s\alpha T)} \quad ...(5.1.1)$$

$$\frac{E_o}{E_i} = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

where, $\alpha < 1$

$$\alpha = \frac{R_2}{R_1+R_2}$$

and $T = R_1 C$

3. The transfer function given by eq. (5.1.1) can be expressed in sinusoidal form as

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{\alpha(1+j\omega T)}{(1+j\omega\alpha T)} \quad ...(5.1.2)$$

4. The pole zero configuration of eq. (5.1.2) shown in Fig. 5.1.2.

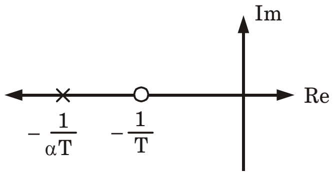


Fig. 5.1.2. Pole zero configuration.

Bode plot :

1. The two corner frequencies are

$$\omega = \frac{1}{T}\text{, lower corner frequency}$$

$$\omega = \frac{1}{\alpha T}\text{, upper corner frequency}$$

2. The maximum phase lead ϕ_m occurs at mid-frequency ω_m between upper and lower corner frequencies

$$\therefore \quad \log_{10}\omega_m = \frac{1}{2}\left[\log_{10}\left(\frac{1}{T}\right)+\log_{10}\left(\frac{1}{\alpha T}\right)\right]$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

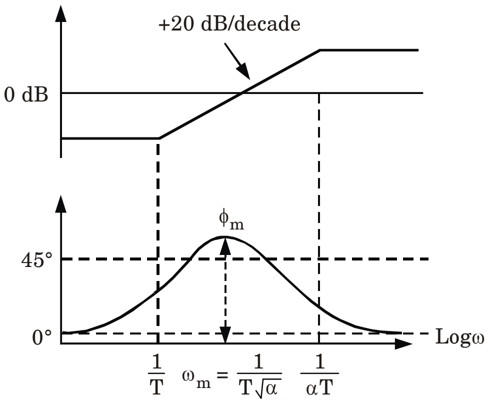


Fig. 5.1.3. Bode plot of lead compensator.

3. The phase angle $\angle E_o(j\omega)/E_i(j\omega)$ can be calculated as

$$\angle\frac{E_o(j\omega)}{E_i(j\omega)} = \tan^{-1}(\omega T) - \tan^{-1}(\omega\alpha T)$$

4. At $\omega = \omega_m = \dfrac{1}{\sqrt{\alpha T}}$,

The phase angle is

$$\phi_m = \tan^{-1}\left[\frac{1}{T\sqrt{\alpha}}T\right] - \tan^{-1}\left[\frac{1}{\alpha\sqrt{T}}\alpha T\right]$$

$$= \tan^{-1}\left[\frac{1}{\sqrt{\alpha}}\right] - \tan^{-1}\left[\sqrt{\alpha}\right]$$

$$\therefore \quad \tan\phi_m = \frac{\dfrac{1}{\sqrt{\alpha}} - \sqrt{\alpha}}{1 + \dfrac{1}{\sqrt{\alpha}}\sqrt{\alpha}}$$

$$\tan \phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

and $$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

Que 5.2. **Explain the Bode Plot Method of designing Lead compensator.**

OR

What are the effects and limitations of lead compensation ?

Answer

Step 1 : Consider equation

$$G_c(s) = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

Let $K_c\alpha = K$ = DC gain

$$G_c(s) = K\frac{Ts+1}{\alpha Ts+1}$$

This control system is shown in Fig. 5.2.1.

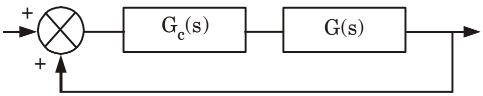


Fig. 5.2.1.

The open loop transfer function is :

$$G_c(s)\, G(s) = \frac{K(Ts+1)}{\alpha Ts+1} G(s)$$

$$= \frac{Ts+1}{\alpha Ts+1} KG(s) = \frac{Ts+1}{\alpha Ts+1} G_1(s)$$

where $G_1(s) = KG(s)$

Now, determine gain K to satisfy the requirement on the given static error coefficient.

Step 2 : Using above value of K, find phase margin from Bode plot of $G_1(j\omega)$ which is the gain adjusted but uncompensated system. Generally phase margin is specified in design problems.

Step 3 : Determine the necessary phase lead (ϕ_m) required to be added by the following equation

$$\phi_m = \phi_1 - \phi_2 + \phi'$$

where, ϕ_1 = Specified phase margin

ϕ_2 = Phase margin obtained in step 2

ϕ' = Safety margin.

Step 4 : Determine the value of the attenuation factor (α) using equation

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

Step 5 : Determine the frequency where the magnitude of uncompensated system $G_1(j\omega)$ is equal to $-20 \log (1/\sqrt{\alpha})$ dB. This is the new gain crossover frequency which corresponds to

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

The maximum phase shift (ϕ_m) occurs at this frequency.

Step 6 : Determine the corner frequencies of the lead compensator as :

Zero of lead compensator $\omega_{c1} = 1/T$

Pole of lead compensator $\omega_{c2} = 1/\alpha T$

Step 7 : Calculate constant K_c from value of K determined in step 1 and that of α determined in step 4, by using equation $K_c = K\alpha$.

Step 8 : Determine gain margin (G.M.). If it is not satisfactory and not according to design requirements repeat the design process by modifying the pole-zero location of the lead compensator until the satisfactory result is obtained.

Effects of lead compensation :

1. The lead compensator adds a dominant zero and a pole. This increases the damping of the closed loop system.
2. It improves the phase margin of the closed loop system.
3. The slope of the magnitude plot in Bode diagram of the forward path transfer function is reduced at the gain cross frequency. This improves gain and phase margins improving the relative stability.
4. It increases bandwidth of the closed loop system. More the bandwidth, faster is the response.

Limitations of lead compensation :

1. Lead compensation requires an additional increase in gain to offset the attenuation inherent in the lead network.
2. The compensated system may have a larger undershoot than overshoot. So tendency to over compensate system may lead to a conditionally stable system.
3. The maximum lead angle available from a single lead network is about 60°. Thus if lead of more than 70° to 90° is required a multistage lead compensators are required.

Que 5.3. **For the open loop transfer function, $G(s)\,H(s) = \dfrac{10}{s(1 + 0.2s)}$ design a suitable compensator such that the system will have a phase margin of at least 45°.**

AKTU 2014-15, Marks 10

Answer

Given : $G(s)H(s) = \dfrac{10}{s(1+0.2s)}$

To Design : Compensator.

1. Given transfer function is of type-1 system,

$$G(s)H(s) = \frac{10}{s\left(1+\dfrac{s}{5}\right)}$$

2. Starting point $\omega = K^{1/n}$ [Here $n = 1$]

$$\omega = 10 \text{ (cut 0 dB axis)}$$

3. Starting slope $= -20$ dB/decade

4. $$\text{P.M} = 180° + \angle\phi, \quad \text{at } \omega = \omega_g = 35.54°$$

Here, $\omega_p = 0$ rad/s

and $\text{G.M} = \infty$ dB

$$\phi_m = 45° - 35.54° + \text{tolerance}$$

$$\phi_m = 45° - 35.54° + 5° = 14.46°$$

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m} = \left(\frac{1-\sin 14.46°}{1+\sin 14.46°}\right)$$

$$\alpha = 0.6$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

Table 5.3.1.

ω	∠φ
1	–101.30°
3	–120.96°
10	–153.43°
100	–177.13°
∞	–180°

5. $$20\log_{10}|G_c(j\omega)| = -\,20\log_{10}\frac{1}{\sqrt{\alpha}}$$

$$= -\,20\log_{10}(\alpha)^{-1/2} = 10\log_{10}\alpha = -\,2.218 \text{ dB}$$

6. Lower corner frequency $= \dfrac{1}{T} = \sqrt{\alpha}\,\omega_m$

$$\omega_m = 8.2 \text{ rad/s} \qquad \text{(at } -2.218 \text{ dB)}$$
$$= \sqrt{0.6} \times 8.2 = 6.35 \text{ rad/s}$$

7. Upper corner frequency

$$= \frac{1}{\alpha T} = \frac{\sqrt{\alpha}\omega_m}{\alpha} = \frac{\omega_m}{\sqrt{\alpha}} = \frac{8.2}{\sqrt{0.6}} = 10.58 \text{ rad/s}$$

Phase lead compensation network

$$G_c(j\omega) = \frac{1 + j\omega T}{1 + j\omega\alpha T}$$

$$G_c(j\omega) = \frac{1 + j\omega \dfrac{1}{6.35}}{1 + j\omega \times 0.6 \times \dfrac{1}{6.35}} = \left(\frac{1 + j0.1574\omega}{1 + j0.09448\omega}\right)$$

8. So, compensated network

$$G'(s) = G_c(s)\ G(s)$$
$$G'(s) = \left(\frac{1 + 0.1574s}{1 + 0.09448s}\right)\left(\frac{1}{s(1 + 0.2s)}\right)$$

Que 5.4. Design a phase lead compensator for a negative unity feedback system with plant transfer function

$$G_p(s) = \frac{K}{s(s+10)(s+1000)}$$ **to satisfy the conditions :**

phase margin in atleast 45°, static error constant = 1000 s^{-1}.

AKTU 2016-17, Marks 10

Answer

Given : $G_p(s) = \dfrac{K}{s(s+10)(s+1000)}$, Phase margin = 45°

$K_v = 1000 \text{ sec}^{-1}$

To Design : Phase lead compensator.

1. $$K_v = \lim_{s \to 0} sG_p(s) = \lim_{s \to 0} s \frac{K}{s(s+10)(s+1000)}$$

$$K_v = \frac{K}{10 \times 1000}$$
$$K = 10 \times 1000\ K_v = 10^7$$

2. $$G_p(s) = \frac{10^7}{10000s\left(1 + \dfrac{s}{10}\right)\left(1 + \dfrac{s}{1000}\right)} = \frac{10^3}{s\left(1 + \dfrac{s}{10}\right)\left(1 + \dfrac{s}{1000}\right)}$$

3. Starting point at 0 dB axis at $\omega = K^{1/n}$

Here, $n = 1$ (Type of the system)

$$\omega = 1000 \text{ rad/sec}$$

4. First corner frequency, ω_{c1} = 10 rad/sec
Second corner frequency, ω_{c2} = 1000 rad/sec
Initial slope = – 20 dB/sec

$$\angle G_p(j\omega) = -90 - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{1000}\right)$$

5. We have got $\omega_p = \omega_g$ = 100 rad/sec.
So, gain margin = 0 dB and phase margin = 180° – 180° = 0°

$$\phi_m = 45° - 0° + \text{tolerance} = 45° - 0° + 5° = 50°$$

6. The value of α parameter of the phase lead network is given by

$$\alpha = \left(\frac{1 - \sin\phi_m}{1 + \sin\phi_m}\right) = 0.1324$$

In decibel,

At $\omega = \omega_m$, magnitude in dB $= -20 \log_{10} \dfrac{1}{\sqrt{\alpha}} = 10 \log_{10} \alpha = -8.78$ dB

Table 5.4.1.

ω	$\angle G_p(j\omega)$
1	– 95.76°
10	– 135.57°
40	– 168.25°
90	– 178.80°

7. At a gain of – 8.78 dB, the frequency is 165 rad/sec
The lower corner frequency

$$= \frac{1}{T} = \sqrt{\alpha}\,\omega_m = (\sqrt{0.1324}) \times 165 = 60 \text{ rad/sec}$$

8. The upper corner frequency

$$= \frac{1}{\alpha T} = \frac{1}{0.1324} \times 60 = 453.1722 \text{ rad/sec}$$

9. Lead compensation network, $G_c(s) = G_c(j\omega) = \left(\dfrac{1 + j\omega T}{1 + j\omega\alpha T}\right)$

$$= \frac{\left(1 + j\omega\dfrac{1}{60}\right)}{\left(1 + j\omega\dfrac{1}{453.1722}\right)} = \frac{(1 + j\omega\, 0.0166)}{(1 + j\omega\, 0.022066)}$$

10. So, the open loop transfer function of the compensated system is

$$G'(s) = G_p(s)\, G_c(s) = \frac{1000}{s(s+10)(s+1000)} \times \frac{(1 + j\omega\, 0.0166)}{(1 + j\omega\, 0.022066)}$$

$$= \frac{1000\,(1 + 0.0166s)}{s\,(s+10)(s+1000)(1 + 0.22066s)}$$

Bode plot :

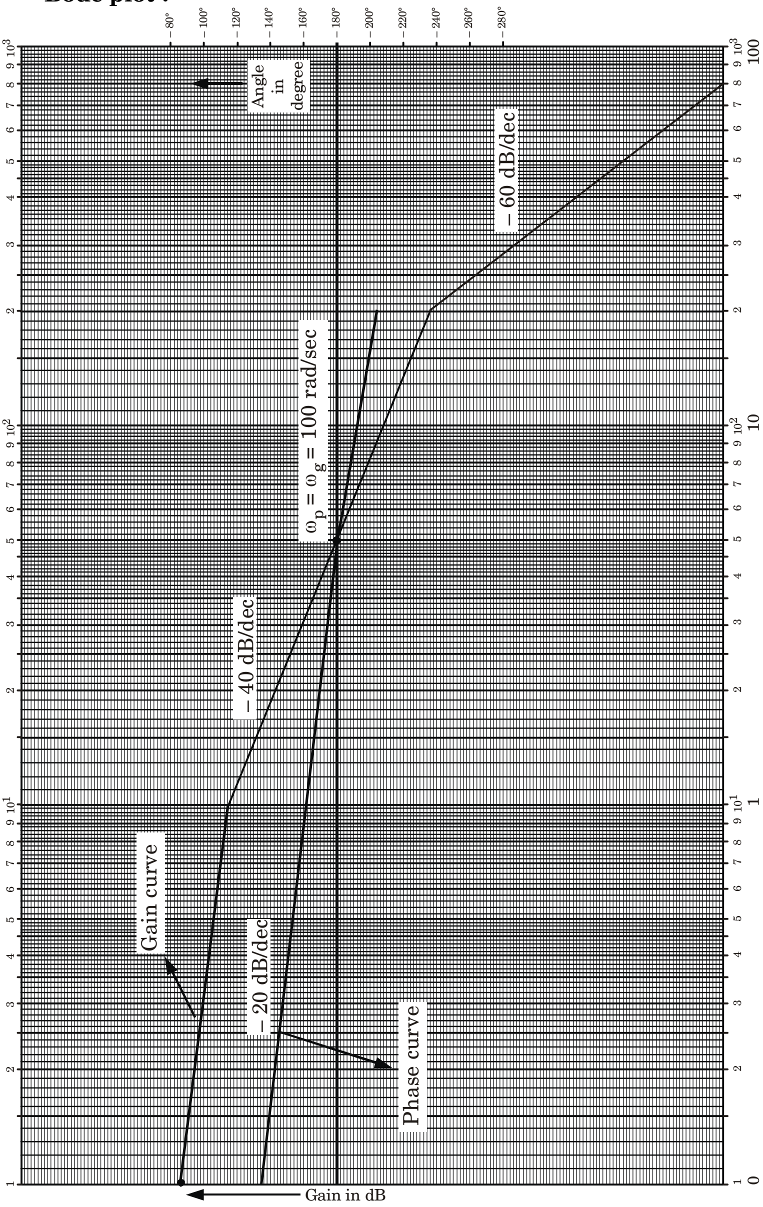


Fig. 5.4.1.

Que 5.5. **Define phase-lag compensation techniques with the help of *R*-*C* network. Also draw its pole zero configuration.**

OR

Write short notes of the following :

a. Lead compensator.

b. Lag compensator.

c. Gain margin and phase margin. **AKTU 2015-16, Marks 15**

Answer

A. Lead compensator : Refer Q. 5.1, Page 5–3C, Unit-5.

B. Lag compensator :

1.

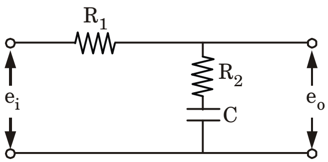


Fig. 5.5.1. Phase-lag network.

The transfer function of phase-lag network is shown in Fig. 5.5.1,

$$\frac{E_o(s)}{E_i(s)} = \frac{1+sT}{1+s\beta T} \quad ...(5.5.1)$$

where $\beta > 1$, $\beta = \dfrac{R_1 + R_2}{R_2}$

and $T = R_2C$

2. The transfer function given by eq. (5.5.1) can be expressed in sinusoidal form as

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{1+j\omega T}{1+j\omega\beta T} \quad ...(5.5.2)$$

3. Bode plot for transfer function of eq. (5.5.2) is shown in Fig. 5.5.2.

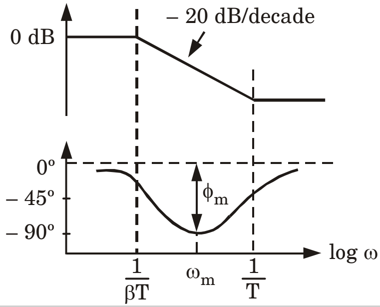


Fig. 5.5.2. Bode plot of lag compensator.

4. The two corner frequencies are $\omega = \frac{1}{T}$, upper corner frequency for zero at $s = -\frac{1}{T}$, $\omega = \frac{1}{\beta T}$, lower corner frequency for a pole at $s = -\frac{1}{\beta T}$

5. The maximum phase-lag, ϕ_m occurs at mid frequency ω_m between upper and lower corner frequencies.

$$\therefore \quad \log_{10} \omega_m = \frac{1}{2}\left[\log\left(\frac{1}{\beta T}\right) + \log_{10}\left(\frac{1}{T}\right)\right]$$

$$\therefore \quad \omega_m = \frac{1}{\sqrt{\beta T}}$$

6. The phase angle $\angle E_o(j\omega)/E_i(j\omega)$ calculated as

$$\angle \frac{E_o(j\omega)}{E_i(j\omega)} = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T)$$

At $\omega = \omega_m = \frac{1}{\sqrt{\beta T}}$, the phase angle is ϕ_m :

$$\tan \phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\sin \phi_m = \frac{1-\beta}{1+\beta}$$

Pole-zero configuration is shown in Fig. 5.5.3.

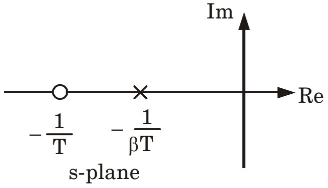


Fig. 5.5.3. Pole zero configuration.

C. **Phase margin and gain margin :** Refer Q. 4.16, Page 4–25C, Unit-4.

Que 5.6. **What are the effects and limitations of lag compensation ?**

OR

Explain Bode plot method to design a lag compensator. Also give its significances.

Answer

Basically, lag compensator is a low pass filter and its main function is to provide attenuation in the high frequency range to give sufficient phase margin.

Step 1 : Let the system transfer function of controller be

$$G_c(s) = K_c \frac{s + \frac{1}{\tau}}{s + \frac{1}{\beta\tau}} \qquad (\beta > 1)$$

$$G_c(s) = \beta K_c \frac{\tau s + 1}{\beta \tau s + 1}$$

Let $K_c\beta = K,$

The OLTF of the compensated system is

$$G_c(s)G(s) = K \frac{\tau s + 1}{\beta\tau s + 1} G(s) = \frac{\tau s + 1}{\beta\tau s + 1} KG(s)$$

$$= \frac{\tau s + 1}{\beta\tau s + 1} G_1(s) \qquad \text{where } G_1(s) = KG(s)$$

Now, determine gain K to satisfy the requirement on the given static velocity error constant.

Step 2 : Using the value of K determined in step 1, draw Bode plot of $G_1(j\omega)$. Find the phase margin (ϕ).

Step 3 : Using specified phase margin (ϕ_2), find the required phase margin ϕ_1

$$\phi_1 = \phi_2 + \phi', \text{ where } \phi' = 5° \text{ to } 12°$$

Step 4 : Find the frequency at which the phase angle of the open loop transfer function is equal to – 180° plus the required margin (ϕ_1). This is the new gain crossover frequency.

Step 5 : Determine attenuation necessary to bring the magnitude curve down to 0 dB at the new gain crossover frequency. This change is due to the factor (β) and the attenuation is – 20 log β. For this shift find the value of β.

Step 6 : Determine the other corner frequency (corresponding to pole of lag compensator) from $\omega_{c1} = 1/(\beta\tau)$.

Step 7 : Using the value of K determined in step 1 and β determined in step 5, calculate constant K_c as

$$K_c = \frac{K}{\beta}$$

Step 8 : Using the transfer function of lag compensator, draw bode plot and verify specifications.

Effects (significance) of lag compensation :

1. Lag compensator allows high gain at low frequencies thus it is basically a low pass filter. Hence it improves the steady state performance.

Limitations of lag compensation :

1. In lag compensation, the attenuation characteristics is used for the compensation. The phase lag characteristic is of no use in the compensation.
2. The attenuation due to lag compensator shifts the gain crossover frequency to a low frequency point. Thus the bandwidth of the system gets reduced.
3. Reduced bandwidth means slower response. Thus, rise time and settling time are usually longer. The transient response lasts for longer time.

Que 5.7. **Explain lead-lag compensator. What are the effects of lead-lag compensation ?**

Answer

1.

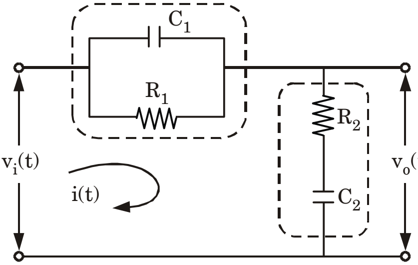


Fig. 5.7.1. Lead-lag compensator in time-domain.

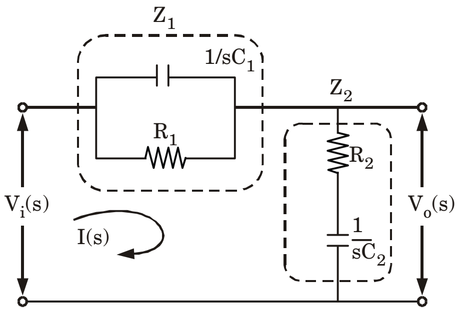


Fig. 5.7.2. In s-domain.

Lead-lag compensator is combination of lead network and lag network. From Fig. 5.7.1, we have

$$Z_1 = R_1 \,||\, \frac{1}{sC_1} = \frac{R_1 \times \dfrac{1}{sC_1}}{R_1 + \dfrac{1}{sC_1}} = \frac{R_1}{sR_1C_1 + 1} \qquad \text{...(5.7.1)}$$

and $$Z_2 = R_2 + \frac{1}{sC_2} = \frac{(sR_2C_2 + 1)}{sC_2} \quad ...(5.7.2)$$

2. Applying Kirchhoff's voltage law to the shown Fig. 5.7.1 network

$$V_i(s) = (Z_1 + Z_2)\, I(s) \quad ...(5.7.3)$$

$$V_0(s) = Z_2 I(s) \quad ...(5.7.4)$$

Hence the transfer function of the system

$$G_c(s) = \frac{V_o(s)}{V_i(s)} \quad ...(5.7.5)$$

3. Putting value of $V_0(s)$ and $V_i(s)$ in eq. (5.7.5), we have

$$G_c(s) = \frac{Z_2 I(s)}{(Z_1 + Z_2) I(s)}$$

$$G_c(s) = \frac{Z_2}{Z_1 + Z_2} \quad ...(5.7.6)$$

4. Putting value of Z_1 and Z_2 in eq. (5.7.6), we have

$$G_c(s) = \frac{\dfrac{(sR_2C_2 + 1)}{sC_2}}{\dfrac{R_1}{sR_1C_1 + 1} + R_2 + \dfrac{1}{sC_2}}$$

$$G_c(s) = \frac{(sR_2C_2 + 1)(sR_1C_1 + 1)}{s^2R_1R_2C_1C_2 + sR_2C_2 + sR_1C_1 + sR_1C_2 + 1}$$

$$= \frac{\left(s + \dfrac{1}{R_2C_2}\right)\left(s + \dfrac{1}{R_1C_1}\right)}{s^2 + \left[\dfrac{1}{R_1C_1} + \dfrac{1}{R_2C_2} + \dfrac{1}{R_2C_1}\right]s + \dfrac{1}{R_1C_1R_2C_2}}$$

$$G_c(s) = \frac{\left(s + \dfrac{1}{T_1}\right)\left(s + \dfrac{1}{T_2}\right)}{\left(s + \dfrac{\beta}{T_1}\right)\left(s + \dfrac{1}{\beta T_2}\right)}$$

where, $$T_1 = R_1C_1,\ T_2 = R_2C_2$$

$$\frac{\beta}{T_1} + \frac{1}{\beta T_2} = \frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}$$

and $$\alpha\beta T_1T_2 = R_1R_2C_1C_2$$

and $$\alpha\beta = 1$$

5. Poles are at $$s = -\frac{\beta}{T_1}, -\frac{1}{\beta T_2}$$

Zeros are at $s = -\dfrac{1}{T_1}, -\dfrac{1}{T_2}$

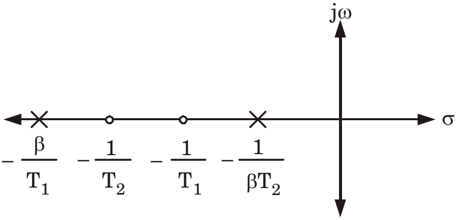


Fig. 5.7.3. Pole-zero configurations.

6. The various corner frequencies of the lag-lead compensator are,

$$\omega_{C1} = \frac{\beta}{T_1}, \text{ for a simple pole}$$

$$\omega_{C2} = \frac{1}{T_1}, \text{ for a simple zero}$$

$$\omega_{C3} = -\frac{1}{T_1}, \text{ for a simple zero}$$

$$\omega_{C4} = -\frac{1}{\beta T_2}, \text{ for a simple pole}$$

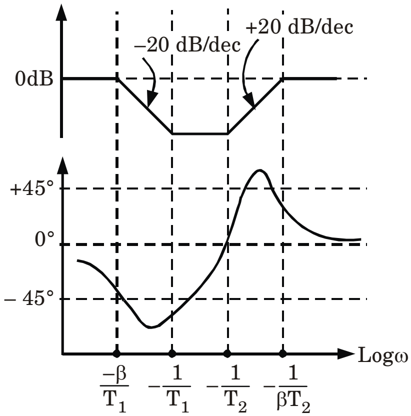


Fig. 5.7.4. Bode plot of lag-lead compensator.

Effects :

1. Lag-lead compensator increases the low frequency gain which improves the steady state.

2. It increases bandwidth of the system which makes the system response very fast.
3. In general, the phase lead position of this compensator is used to achieve large bandwidth and hence shorter rise time and settling time. While the phase lag portion provides the major damping of the system.

PART-2

The Concept of State and Space, State-Space Model of Physical System, Conversion of State-Space to Transfer Function Model and Vice-Versa.

CONCEPT OUTLINE : PART-2

- An n^{th} order differential equation is not generally suitable for computer solution; it is the best to obtain a set of n first-order differential equation, using a set of auxiliary variables called state variables and this approach is called State Variable Analysis.
- **Transfer Matrix :**

$$\text{Transfer matrix} = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B$$

$$\text{Transfer function} = C[sI - A]^{-1} B = \frac{C \,\text{adj}[(sI - A)]}{|sI - A|}$$

- **Diagonalization :** Transformation of a matrix into a diagonal matrix so that the diagonal elements are represented by eigen values is called diagonalization.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.8. **Define state and state variable ? What are the advantages of state space techniques ?** **AKTU 2017-18, Marks 10**

OR

Define the following terms :

i. State

ii. State variables

iii. State vector

iv. State space

v. State equation

Answer

i. **State :** The state of a dynamic system is the smallest set of variables such that the knowledge of these variables at $t = t_0$ with the knowledge of the input for $t \geq t_0$ completely determines the behaviour of the system for any time $t \geq t_0$.

ii. **State variables :** The variables involved in determining the state of dynamics system are called state variables.

iii. **State vector :** If we need n variable to completely describe the behaviour of a given system, then these n state variables may be considered as n component of a vector x. Such a vector is called state vector.

iv. **State space :** The n-dimensional space whose coordinate axes consists of the x_1 axis, x_2 axis ..., x_n axis is called state space. Any state can be represented by a point in the state space.

v. **State equation :** A state space representation is a mathematical model of the physical system as a set of output, input and state variables related by a differential equation is known as state equation.

Advantages :

1. The method takes into account the effect of all initial conditions.
2. It can be applied to non-linear as well as time varying conditions.
3. It can be conveniently applied to multiple input multiple output systems.
4. The system can be designed for the optimal conditions precisely by using this modern method.
5. Any type of the input can be considered for designing the system.
6. As the method involves matrix algebra, can be conveniently adopted for the digital computers.

Que 5.9. **Discuss state space representation.**

Answer

1. In a state variable system, state variables are represented by $x_1(t)$, $x_2(t)$ ...
2. Input variables by $u_1(t)$, $u_2(t)$, ...
3. Output variables by $y_1(t)$, $y_2(t)$ ...

   where,

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix} ; u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ \vdots \\ u_n(t) \end{bmatrix} ; y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ \vdots \\ y_n(t) \end{bmatrix}$$

4. So, dynamics of a system can be represented by n^{th} order differential equation

$$\frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + a_2 \frac{d^{n-2} y(t)}{dt^{n-2}} \dots + a_{n-1} \frac{dy(t)}{dt} + a_n y(t) = u(t) \quad \text{...(5.9.1)}$$

5. The knowledge of initial conditions $y(0), \frac{dy(0)}{dt}, \dots, \frac{d^{n-1}y(0)}{dt^{n-1}}$, along with the input $u(t)$ for $t \geq 0$, completely determines the future behavior of the system.

6. We take $y(t), \frac{dy(t)}{dt}, \dots, \frac{d^{n-1}y(t)}{dt^{n-1}}$ as a set of n-state variables.

7. Let us define

$$x_1 = y$$
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\cdots$$
$$\dot{x}_{n-1} = x_n$$

8. Now from eq. (5.9.1)

$$\dot{x}_n = -a_n x_1 - a_{n-1} x_2 \dots - a_1 x_n + u$$

9. We can write this set of equations in matrix form as,

$$\dot{x} = Ax + Bu$$

Output : $y = Cx$

where, $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$;

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}; \; B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

10. For n^{th} order single-input-single-output system :

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

A = System matrix of order $(n \times n)$

B = Input coupling matrix of order $(n \times 1)$

C = Output coupling matrix of order $(1 \times n)$

x = State vector of order $(n \times 1)$

u = Scalar input of order (1×1)

y = Scalar output of order (1×1)

11. For multivariable system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}; \; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}; \; u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & & & \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{bmatrix}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & & & \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix}$$

Que 5.10. Describe different representation of state model in details.

Answer

Basic ways to represent state models are :

a. State space representation using differential equation :

1. The state model is represented using these state variables called as phase variables.
2. The phase variable is defined as those particular state variables which are obtained from one of the system variables and its derivatives.
3. Transfer function can be obtained by differential equation

$$\frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_{n-1} \frac{dy(t)}{dt} + a_n y(t) = b \; u(t)$$

$$\dot{y}^{(n)} + a_1 \dot{y}^{(n-1)} + \dots + a_{n-1} \dot{y}^{(1)} + a_n = b \; u(t)$$

Put

$$x_1 = y$$
$$x_2 = \dot{y} = \dot{x}_1$$
$$x_3 = \ddot{y} = \dot{x}_2$$
$$x_n = \dot{y}^{(n-1)} = \dot{x}_{n-1}$$
$$x_1 = y$$

$$\dot{x}_1 = x_2 = \dot{y}$$
$$\dot{x}_2 = x_3 = \ddot{y}$$
$$\vdots \quad \vdots \quad \vdots$$
$$\dot{x}_n = \dot{y}^{(n)} = bu - a_n - ....a_2\dot{y}^{(n-2)} - a_1\dot{y}^{(n-1)}$$
$$\dot{x}_n = bu - a_n x_3 - a_{n-1}x_2 - .... - a_2 x_{n-1} - a_1 x_n$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ b \end{bmatrix} u$$

$$\dot{x} = Ax + Bu$$

$$y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + [0]u$$

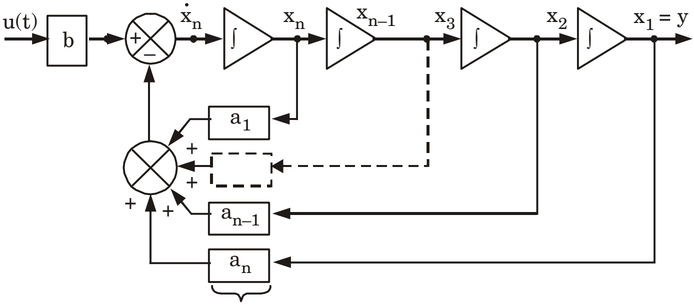


Fig. 5.10.1. The block diagram.

b. Direct decomposition : In direct decomposition we represent the denominator polynomial as

$$= \frac{1}{(s[s+a]+b)s+c}$$

We can do it for higher order also.

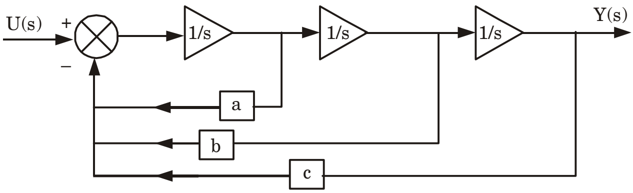


Fig. 5.10.2.

c. **Cascaded decomposition :** Here transfer function is written in terms of product of two or more small transfer functions.

$$\frac{Y(s)}{U(s)} = K\frac{(s+b_1)}{(s+a_1)} \times \frac{(s+b_2)}{(s+a_2)}$$

Each group is then decomposed by direct decomposition and the state diagram of the transfer functions is cascaded.

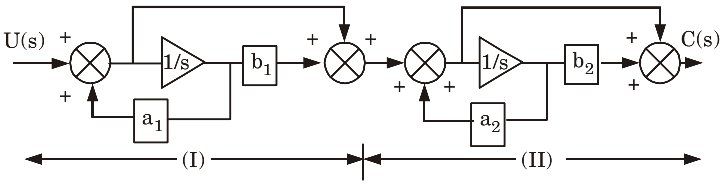


Fig. 5.10.3.

d. **Parallel decomposition :** This method is also called canonical form.

The given transfer function

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \ldots. + b_n}{a_0 s^n + a_1 s^{n-1} + \ldots. + a_n}$$

Then factorized $\quad T(s) = \dfrac{c_1}{s+a_1} + \dfrac{c_2}{s+a_2} + \ldots. + \dfrac{c_n}{s+a_n}$

Simulate all terms using direct decomposition and add in parallel to have complete representation.

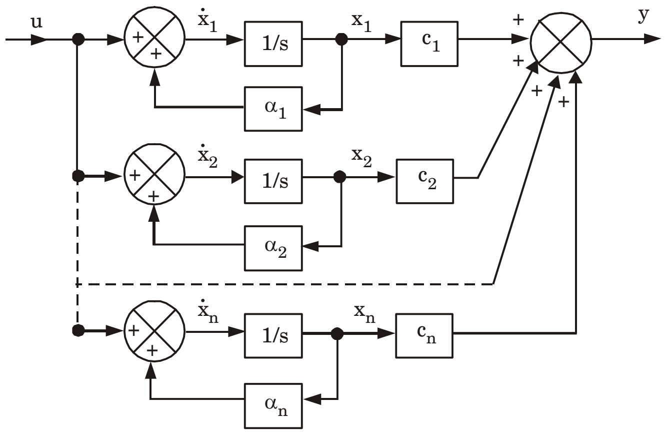


Fig. 5.10.4.

Que 5.11. **A system is described by the following differential equation. Represent the system in the state space.**

$$\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = u_1(t) + 3u_2(t) + 4u_3(t) \text{ and}$$

output are $y_1 = 4\frac{dx}{dt} + 3u_1,\ y_2 = \frac{d^2x}{dt^2} + 4u_2 + u_3$.

AKTU 2017-18, Marks 10

Answer

1. Select the state variables as

$$x_1 = x$$

$$\dot{x}_1 = \dot{x} = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -3x_3 - 4x_2 - 4x_1 + u_1(t) + 3u_2(t) + 4u_3(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -4 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

2. Output :

$$y_1 = 4x_2 + 3u_1$$
$$y_2 = x_3 + 4u_2 + u_3$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Que 5.12. **What is Transfer Function ? Also derive the expression for transfer function of a state model.**

Answer

A. **Transfer Function :** Transfer function is defined as the ratio of Laplace transform of output of the system to the Laplace transform of input, under the assumptions that all initial conditions are zero.

$$T(s) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} = \frac{C(s)}{R(s)}$$

B. **Derivation :**

1. Let us consider a vector matrix differential equation

$$\dot{x} = Ax + Bu$$

and output, $y = Cx$

2. Now, taking Laplace transform with zero initial conditions

$$sX(s) = AX(s) + BU(s)$$

$$X(s) = [sI - A]^{-1} BU(s)$$

and $Y(s) = CX(s)$

$$Y(s) = C\ [sI - A]^{-1} BU(s)$$

3. For a single-input-single-output system, Y and U are scalars.
4. Now transfer matrix can be given as

$$\text{Transfer matrix} = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B$$

5. Transfer function $= C[sI - A]^{-1} B = \dfrac{C\ \text{adj}([sI - A])B}{\det\ [sI - A]}$
6. Denominator part *i.e.* $|sI - A|$ is called the characteristic equation.

$$|sI - A| = 0$$

7. n^{th} degree characteristic equation $|sI - A| = 0$ has n roots or eigen values.

Que 5.13. Write the state variable formulation of the parallel RLC network.

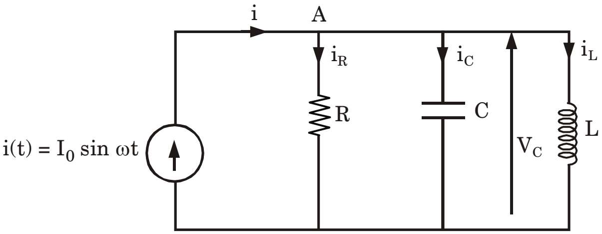


Fig. 5.13.1.

Answer

1. KCL at node A,

$$i_R + i_C + i_L = I_0 \sin \omega t$$

$$\frac{V_C(t)}{R} + \frac{1}{L}\int V_C(t)\, dt + \frac{C dV_C(t)}{dt} = I_0 \sin \omega t$$

2. Differentiating *w.r.t.* t and dividing by C,

$$\frac{1}{RC}\frac{dV_C(t)}{dt}+\frac{1}{LC}V_C(t)+\frac{d^2V_C(t)}{dt^2}=\frac{I_0\omega}{C}\cos\omega t$$

$$\frac{d^2V_C(t)}{dt^2}+\frac{1}{RC}\frac{dV_C(t)}{dt}+\frac{1}{LC}V_C(t)=\frac{I_0\omega}{C}\cos\omega t \quad ...(5.13.1)$$

3. Let, $V_C(t) = x_1(t)$

$$\dot{x}_1(t) = x_2(t)$$

Eq. (5.13.1) becomes,

$$\dot{x}_2(t) = -\frac{1}{RC}x_2(t)-\frac{1}{LC}x_1(t)+\frac{I_0\omega}{C}\cos\omega t$$

4. State space representation

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ \dfrac{-1}{LC} & \dfrac{-1}{RC}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ \dfrac{I_0\omega}{C}\cos\omega t\end{bmatrix}$$

5. Output

$$V_C = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Que 5.14. Define the following terms :

i. State **ii. State variables**

iii. State vector **iv. State space**

v. State equation

Also write the properties of state transition matrix.

AKTU 2013-14, Marks 10

OR

What is the State Transition Matrix ? Derive its expression. Enlist its properties with proofs.

Answer

i. **State variables :** Refer Q. 5.8, Page 5–17C, Unit-5.

ii. **State :** Refer Q. 5.8, Page 5–17C, Unit-5.

iii. **State vector :** Refer Q. 5.8, Page 5–17C, Unit-5.

iv. **State space :** Refer Q. 5.8, Page 5–17C, Unit-5.

v. **State equation :** Refer Q. 5.8, Page 5–17C, Unit-5.

State transition matrix : The matrix $\phi(t)$ = exp (At) is an $n \times n$ matrix and it helps in transition from initial state $X(0)$ to any other state $x(t)$ for $t > 0$, hence $\phi(t)$ is called state transition matrix.

Derivation :

1. Consider homogeneous equation, with $u(t) = 0$

$$\dot{x} = Ax$$

Laplace transform,

$$sX(s) - x(0) = A\,X(s)$$
$$(sI - A)\,X(s) = x(0)$$
$$I = \text{Identity matrix}$$
$$s = \text{Scalar Laplace operator.}$$

2. Multiplying both sides by $[sI - A]^{-1}$

$$X(s) = [sI - A]^{-1}\,x(0)$$
$$X(s) = \phi(s)\,x(0)$$

where,

$$\phi(s) = [sI - A]^{-1}$$
$$\phi(s) = \frac{1}{s}\left[I - \frac{A}{s}\right]^{-1} = \frac{I}{s} + \frac{A}{s^2} + \frac{A^2}{s^3} + \ldots$$

3. Taking inverse Laplace transform of $X(s)$,

$$x(t) = L^{-1}\,[X(s)] = L^{-1}\,[sI - A]^{-1}\,x(0)$$
$$= L^{-1}\left[\frac{I}{s} + \frac{A}{s^2} + \frac{A^2}{s^3} + \ldots\right]x(0)$$
$$= \left[I + At + \frac{A^2t^2}{\lfloor 2} + \ldots\right]x(0)$$
$$x(t) = e^{At}\,x(0) = \phi(t)\,x(0)$$

4. State transition matrix

$$\phi(t) = L^{-1}\,[sI - A]^{-1} = L^{-1}\,[\phi(s)]$$

5. So, state transition matrix (STM) can be given as,

$$\phi(t) = e^{At} = I + At + \frac{1}{\lfloor 2}A^2t^2 + \frac{1}{\lfloor 3}A^3t^3 + \ldots$$

Properties of STM :

1. $$\phi(0) = e^{A0} = I$$

Proof : $$\phi(0) = e^{A \times 0} = I$$

2. $$\phi^{-1}(t) = \phi(-t)$$

Proof : $$\phi^{-1}(t) = \frac{1}{\phi(t)} = \frac{1}{e^{At}} = e^{-At} = \phi(-t)$$

3. $\phi(t_2 - t_1)\,\phi(t_1 - t_0) = \phi(t_2 - t_0)$

Proof :

$\phi(t_2 - t_1)\phi(t_1 - t_0) = e^{A(t_2 - t_1)}\,e^{A(t_1 - t_0)} = e^{A(t_2 - t_1 + t_1 - t_0)} = e^{A(t_2 - t_0)}$
$= \phi(t_2 - t_0)$

4. $[\phi(t)]^k = \phi(kt)$

Proof : $\phi(t)^k = \phi(t).\phi(t)$ ... k times $= e^{At}\,e^{At}$ ... k times $= e^{Akt} = \phi(kt)$

5. $\phi(t_1 + t_2) = \phi(t_1)\,\phi\,(t_2) = \phi(t_2)\,\phi(t_1)$

Proof :

$\phi(t_1 + t_2) = e^{A(t_1 + t_2)} = e^{At_1}\,e^{At_2} = e^{At_2}\,e^{At_1} = \phi(t_1)\,\phi(t_2) = \phi(t_2)\,\phi(t_1)$

Que 5.15.

i. **Derive the transfer function from state model.**
ii. **Obtain the complete solution of non-homogeneous state equation using time domain method.**
iii. **Discuss the significance of lag network. Also draw its *s*-plane representation and Bode plot.**

AKTU 2016-17, Marks 10

OR

What are homogeneous and non-homogeneous systems ? Derive the solution of the two systems in terms of the state variables.

Answer

i. **Derivation of the transfer function from state model :** Refer Q. 5.12, Page 5–23C, Unit-5.

ii.

A. **Homogeneous system :** If in a state model of a system, the matrix A is a constant matrix and input control forces zero, then the equation takes the form, $\dot{x}(t) = A\,X(f)$, such an equation is called homogeneous equation and the system is called homogeneous system.

Solution of homogeneous state equation :

1. $$\dot{x} = Ax \qquad \{u(t) = 0 \text{ for homogeneous equation}\}$$
2. Taking Laplace transform
$$sX(s) - x\,(0) = A\,X(s)$$
Hence, $[sI - A]\,X(s) = x\,(0)$
where I is identity matrix and s is the scalar Laplace operator.
3. Premultiplying both side by $[sI - A]^{-1}$
$$X(s) = [sI - A]^{-1}\,x(0) = \phi(s)\,x(0)$$
4. Taking the inverse Laplace transform of $X(s)$
$$x(t) = L^{-1}\left[\frac{1}{s} + \frac{A}{s^2} + \frac{A_2}{s^3} + \ldots\right]x(0)$$
$$x(t) = \phi(t)\,x(0)$$
$\phi(t) = e^{At}$ is $(n \times n)$ matrix and is called State Transition Matrix.
$\phi(t)$ is unique solution of
$$\frac{d\phi(t)}{dt} = A\,\phi(t)\,,\ \phi(0) = I$$

B. **Non-homogeneous system :** If in a state model of a system, if A is a constant matrix and input control forces are applied to the system, then the system is called non-homogeneous system.

Solution of non-homogeneous equation :

1. Consider state equation
$$\dot{x}\,(t) = Ax(t) + Bu(t)$$

$$U(s) = L[u(t)]$$
$$X(s) = L[x(t)]$$

2. Taking Laplace transform,
$$sX(s) - x(0) = AX(s) + BU(s)$$
$$[sI - A]\,X(s) = x(0) + BU(s)$$

3. Premultiplying by $[sI - A]^{-1}$
$$X(s) = [sI - A]^{-1}\,x(0) + [sI - A]^{-1}\,BU(s)$$

4. Taking inverse Laplace transform
$$x(t) = L^{-1}\{X(s)\}$$
$$= L^{-1}[sI - A]^{-1}\,x(0) + L^{-1}[sI - A]^{-1}\,BU(s)$$
$$x(t) = \phi(t)x(0) + \int_0^t \phi(t-\tau)\,Bu(\tau)\,d\tau$$
$$= e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)\,d\tau$$

5. If the initial time is other than zero, say t_0 then
$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^t e^{A(t-\tau)}Bu(\tau)\,d\tau$$
$$= \phi(t-t_0)\,x(t_0) + \int_{t_0}^t \phi(t-\tau)\,Bu(\tau)\,d\tau$$

6. $$x(t) = x_c(t) + x_p(t)$$
where $x_c(t) = \phi(t)\,x(0)$
is the complementary solution of the state vector

and $$x_p(t) = \int_{t_0}^t \phi(t-\tau)\,Bu(\tau)\,d\tau$$
is a particular solution of vector.

iii. **Significance of lag network :** Refer Q. 5.6, Page 5–12C, Unit-5.
***s*-plane representation and Bode plot of lag network :** Refer Q. 5.5, Page 5–11C, Unit-5.

Que 5.16. **Obtain state equation of a given transfer function**

a. $$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

b. $$\frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s+4)}$$

AKTU 2015-16, Marks 10

Answer

Given : $$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

To Find : State representation.

1. $Y(s)\ [s^3 + 2s^2 + 3s + 1] = U(s)$
   Taking inverse Laplace transform

$$\frac{d^3y}{dt^3} + \frac{2d^2y}{dt^2} + \frac{3dy}{dt} + y(t) = u(t)$$

2. Choosing state variable

$$y(t) = y_1(t)$$

$$\frac{dy(t)}{dt} = \dot{y}_1(t) = y_2(t)$$

$$\frac{d^2y(t)}{dt} = \dot{y}_2(t) = y_3(t)$$

$$\frac{d^3y(t)}{dt} = \dot{y}_3(t) = y_4(t)$$

3. $y_4 + 2y_3 + 3y_2 + y_1(t) = u(t)$

$$\dot{y}_3 = -2y_3 - 3y_2 - y_1(t) + u(t)$$

$$\begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \\ \dot{y}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

Given : $\dfrac{Y(s)}{U(s)} = \dfrac{1}{(s+1)(s+4)}$

To Find : State representation.

1. $Y(s)\ [(s^2 + 5s + 4)] = U(s)$
   Taking inverse Laplace

$$\frac{d^2y}{dt^2} + \frac{5dy}{dt} + 4y(t) = u(t)$$

2. $$\frac{d^2y}{dt^2} = \dot{y}_2(t) = y_3(t)$$

$$\frac{dy}{dt}(t) = y_2(t) = \dot{y}_1(t)$$

$$y(t) = y_1(t)$$

3. $\dot{y}_2(t) + 5y_2(t) + 4y_1(t) = u(t)$

$$\dot{y}_2(t) = -5y_2(t) - 4y_1(t) + u(t)$$

$$\begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Que 5.17. **A feedback system has a closed loop transfer function**

$$\frac{C(s)}{R(s)} = \frac{10(s+4)}{s(s+1)(s+3)}$$

Construct the phase variable state model.

AKTU 2013-14, Marks 10

Answer

Given : $\dfrac{C(s)}{R(s)} = \dfrac{10\,(s+4)}{s(s+1)(s+3)}$

To Find : Phase variable state model.

1. $$\frac{C(s)}{R(s)} = \frac{10\,(s+4)}{s(s^2+4s+3)}$$

$$\frac{C(s)}{R(s)} = \frac{10\,(s+4)}{s^3+4s^2+3s}$$

2. $$\frac{C(s)}{R(s)} = \frac{C(s)}{X(s)}\,\frac{X(s)}{R(s)}$$

$$\frac{X(s)}{R(s)} = \frac{1}{s^3+4s^2+3s}$$

3. $$\frac{d^3x(t)}{dt^3} + 4\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} = r(t)$$

Let $$x(t) = x_1(t)$$

$$\frac{dx(t)}{dt} = x_2(t) = \dot{x}_1(t)$$

$$\frac{d^2x(t)}{dt^2} = x_3\,(t) = \dot{x}_2(t)$$

$$\dot{x}_3(t) = \frac{d^3x(t)}{dt^3} = r(t) - 3x_2(t) - 4x_3(t)$$

4. $$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1\,(t) \\ x_2\,(t) \\ x_3\,(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

5. $$\frac{C(s)}{X(s)} = 10\,s + 40$$

$$C(s) = 10\,s\,X(s) + 40\,X(s)$$

$$c(t) = 10\,\frac{dx(t)}{dt} + 40x\,(t)$$

$$c(t) = 10\, x_2\,(t) + 40\, x_1\,(t)$$

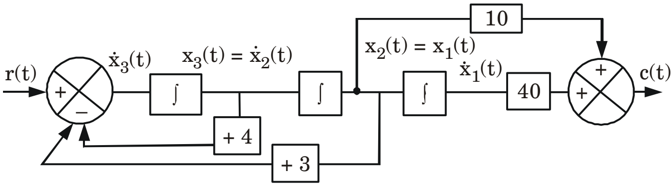


Fig. 5.17.1. Phase variable state model.

Que 5.18. Determine the state model from transfer function of a system given as

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 3s + 2}{s^3 + 9s^2 + 26s + 24}$$

AKTU 2014-15, Marks 10

Answer

1. $$\frac{Y(s)}{X(s)} = (s^2 + 3s + 2)$$

   Taking inverse Laplace both sides

   $$\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t) = y(t)$$

2. $$\frac{X(s)}{U(s)} = \frac{1}{s^3 + 9s^2 + 26s + 24}$$

   Taking inverse Laplace both sides

   $$\frac{d^3x(t)}{dt^3} + 9\frac{d^2x(t)}{dt^2} + 26\frac{dx(t)}{dt} + 24x(t) = u(t)$$

3. Choose state variables

   $$x(t) = x_1(t)$$
   $$\frac{dx(t)}{dt} = \dot{x}_1(t) = x_2(t)$$
   $$\frac{d^2x(t)}{dt^2} = \dot{x}_2(t) = x_3(t)$$

   So, $$\dot{x}_3(t) = 4(t) - 9x_3(t) - 26x_2(t) - 24x_1(t)$$
   $$y(t) = 2x_1(t) + 3x_2(t) + x_3(t)$$

4. In state space representation form

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [2 \quad 3 \quad 1] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

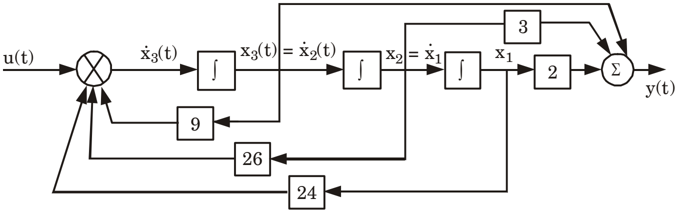


Fig. 5.18.1. Block diagram.

PART-3

Similarity Transformation of the control System, Concept of Controllability and Observability and their Testing.

CONCEPT OUTLINE : PART-3

- **Controllability :** A system is said to be controllable if the state of the system can be transferred to another desired state over a given time period by using input.
- **Observability :** A system is said to be observable if all the state of the system can be determined on the basis of knowledge of the output of the system at given time.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.19. **Discuss similarity transformation in control system.**

Answer

1. The choice of state is not unique for a given system. Suppose that there exists a set of state variables

$$x = [x_1 \quad x_2 \quad x_3 \quad .... \quad x_n]^T$$

so that a linear or similarity transformation

$$x = Mz; \quad |M| \neq 0, \quad M^{-1} \text{ exists}$$

i.e.,

$$z = M^{-1} x \qquad ...(5.19.1)$$

transforms to another set of state variables

$$z = [z_1\ z_2\ z_3 \ldots z_n]^T$$

2. Differentiating eq. (5.19.1), we get

$$\dot{z} = M^{-1}\dot{x} = M^{-1}[Ax + Bu] = M^{-1}Ax + M^{-1}Bu$$
$$= M^{-1}A[Mz] + M^{-1}Bu$$
$$= [M^{-1}AP]z + [M^{-1}B]u$$
$$= \tilde{A}z + \tilde{B}u$$

i.e., $\dot{z} = \tilde{A}z + \tilde{B}u$

and output $y = C\,M\,z = \tilde{C}\,z$

i.e., $y = \tilde{C}\,z$

where $\tilde{A} = M^{-1}A\,M$

$\tilde{B} = M^{-1}B$

$\tilde{C} = C\,M$

where M is a non-singular transformation matrix.

3. Hence by similarity transformation, the transformed system can be represented by the vector-matrix differential equation as

$$\dot{z} = \tilde{A}z + \tilde{B}u$$

and output $y = \tilde{C}\,z$

where $\tilde{A} = M^{-1}A\,M$

$\tilde{B} = M^{-1}B$

$\tilde{C} = C\,M$

Que 5.20. **State and explain controllability and observability in view of Kalman and Gilbert test.**

The state equation for a system is, $\dot{x} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$.

AKTU 2016-17, Marks 10

Answer

1. **Controllability :** A system is said to be controllable if the state of the system can be transferred to another desired state over a given time period by using input.
2. **Observability :** A system is said to be observable if all the state of the system can be determined on the basis of knowledge of the output of the system at given time.

Kalman's test for controllability :

1. Consider n^{th} order multiple input linear time invariant system represented by its state equation as,

$$\dot{x} = Ax + Bu$$

2. The necessary and sufficient condition for the system to be completely state controllable is that the rank of the composite matrix Q_c is n.
3. The composite matrix Q_c is given by,

$$Q_c = [B : AB : A^2B : \ldots A^{n-1}B]$$

Gilbert's test for controllability :

1. For the Gilbert's test it is necessary that the matrix A must be in canonical form. Hence the given state model is required to be transformed to the canonical form first, to apply the Gilbert's test.
2. Consider single input linear time invariant system represented by,

$$\dot{z} = A\,x + B\,u$$

where A is not in the canonical form. Then it can be transformed to the canonical form by the transformation,

$$x = M\,z$$

where M = Model matrix

3. The transformed state model,

$$\dot{z} = \tilde{A}\,z + \tilde{B}\,u$$

where
$$\tilde{A} = M^{-1}AM$$
$$\tilde{B} = M^{-1}B$$

4. In such a case the necessary and sufficient condition for the complete state controllability is that the vector matrix $\tilde{B}$ should not have any zero elements. If it has zero elements then the corresponding state variables are not controllable.
5. If the eigen values are repeated then matrix A cannot be transformed to Jordan canonical form. If A has eigen values $\lambda_1, \lambda_1, \lambda_2, \lambda_2, \lambda_3, \lambda_4, \ldots \lambda_n$ then the transformation results Jordan canonical form shown in matrix below

$$J = \begin{bmatrix} \lambda_1 & 1 & 0 & 0 & 0 & 0 \ldots\ldots\ldots 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 \ldots\ldots\ldots 0 \\ 0 & 0 & \lambda_2 & 1 & 0 & 0 \ldots\ldots\ldots 0 \\ 0 & 0 & 0 & \lambda_2 & 1 & 0 \ldots\ldots\ldots 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 \ldots\ldots\ldots 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 \ldots\ldots\ldots 0 \\ \vdots & \vdots & & & & \\ 0 & 0 \ldots & \ldots & \ldots & \ldots & \ldots\ldots\ldots \lambda_n \end{bmatrix}$$

Jordan block

6. In such a case, the condition for the complete state controllability is that the elements of any row of $\tilde{B}$ that corresponds to the last row of each Jordan block are not all zero.

Kalman's test for observability :

1. State equation

$$\dot{x} = A\,x + B\,u$$

and $$y = C\,x$$

2. The system is completely observable if and only if the rank of the composite matrix Q_o is n.

Composite matrix Q_o is given by,

$$Q_o = [C^T : A^T\, C^T : .... \,(AT)^{n-T}CT]$$

where C^T = Transpose of matrix C

A^T = Transpose of matrix A

Gilbert's test for observability :

1. For Gilbert's test, the state model must be expressed in the canonical form. Consider the state model of linear time invariant system as,

$$\dot{x} = A\,x + B\,u$$

and $$y = C\,x$$

2. Use the transformation $x = Mz$ where M is the model matrix.

$$\therefore \quad y = CM\,z = \tilde{C}\,z$$

where $$\tilde{C} = CM$$

For a single input single output system,

$$y = \tilde{C}z = \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \dots & \tilde{C}_{1n} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$$

$$= \tilde{C}_{11} z_1 + \tilde{C}_{12} z_1 + \dots + \tilde{C}_{1n} z_n$$

3. For the system to be observable, each term corresponding to each state must be observed in the output. Hence none of the coefficient of $\tilde{C}$ must be zero.

Numerical :

Given : $\dot{x} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$

To Find : Controllability.

1. $$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, n = 2$$

2. $$Q_c = [B\ AB]$$

3. $$AB = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

4. $$Q_c = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$$

5. $$|Q_c| = \begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix} = 2 - 2 = 0$$

Rank of Q_c is $r \neq n$

Hence, system is not completely controllable.

Que 5.21. **A system characterised by the transfer function $\dfrac{Y(s)}{U(s)} = \dfrac{2}{s^3 + 6s^2 + 11s + 6}$. Find the state and output equation in matrix form and also test the controllability and observability of the given system.**

AKTU 2015-16, Marks 15

Answer

Given : $$\frac{Y(s)}{U(s)} = \frac{2}{s^3 + 6s^2 + 11s + 6}$$

To Find : State matrix; Controllability, Q_c; Observability, Q_o.

1. $Y(s)\ [(s^3 + 6s^2 + 11s + 6)] = 2U(s)$

2. $$\dot{x}_1 = x_2,\ \dot{x}_2 = x_3,\ \dot{x}_3 = -6x_1 - 11x_2 - 6x_3 + u$$

3. $$y = 2x_1$$

$$= \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4. **Controllability :**

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$$

$$Q_c = [B : AB : A^2B]$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -6 & -11 & -6 \\ 36 & 60 & 25 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & 1 \\ -6 & -11 & -6 \\ 36 & 60 & 25 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 25 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 25 \end{bmatrix} = 0\begin{vmatrix} 1 & -6 \\ -6 & 25 \end{vmatrix} - 0\begin{vmatrix} 0 & -6 \\ 1 & 25 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & -6 \end{vmatrix}$$

$$Q_c = 0 - 0 + (-1)$$

$$Q_c = -1$$

$$|Q_c| \neq 0$$

It is of rank 3 and controllable.

5. **Observability :**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, C^T = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$Q_0 = [C^T : A^TC^T : (A^T)^2 C^T]$$

$$A^TC^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -22 \\ 0 & 1 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$(A^T)^2 = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & 11 \\ 0 & 1 & -6 \end{bmatrix}\begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & 11 \\ 0 & 1 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -6 & 36 \\ 0 & -11 & 60 \\ 1 & -6 & 25 \end{bmatrix}$$

$$(A^T)^2C^T = \begin{bmatrix} 0 & -6 & 36 \\ 0 & -11 & 60 \\ 1 & -6 & 25 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= 2\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & 2 \\ 0 & 0 \end{vmatrix} = 2[4 - 0] - 0 + 0$$

$$|Q_o| = 8$$

Therefore, its rank is 3 and the system is observable.

Que 5.22. **A linear time invariant system is characterized by the state variable model. Examine the controllability and observability of the system**

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & -4 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 40 \\ 10 \\ 0 \end{bmatrix}; \mathbf{C} = [0 \quad 0 \quad 1]$$

AKTU 2017-18, Marks 10

Answer

Given : $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -3 \\ 0 & 0 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 40 \\ 10 \\ 0 \end{bmatrix}$, $C = [0 \quad 0 \quad 1]$

To Test : Controllability and observability.

1. **Controllability test :**

i. $$AB = \begin{bmatrix} 0 \\ 40 \\ 0 \end{bmatrix}, \quad A^2B = \begin{bmatrix} 0 \\ -30 \\ 0 \end{bmatrix}$$

ii. Controllability test matrix is given by,

$$Q_c = [B : AB : A^2B] = \begin{bmatrix} 40 & 0 & 0 \\ 10 & 40 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$|Q_c| = 0$$

iii. Thus the rank of Q_c is 2. Hence the system is not controllable.

2. **Obervability test :**

i. $$A^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -3 & -4 \end{bmatrix}, \quad C^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

ii. $$A^T C^T = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix}, \quad (A^T)^2 C^T = \begin{bmatrix} 0 \\ 0 \\ 16 \end{bmatrix}$$

iii. The observability test matrix is given by

$$Q_o = [C^T : A^T C^T : (A^T)^2 C^T]$$

$$Q_o = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -4 & 16 \end{bmatrix}$$

iv. Its rank is 1. Hence the system is not observable.

Que 5.23. **Check the controllability and observability of a system having following coefficient matrices.**

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{C} = [1 \ \ 0 \ \ 0]$$

AKTU 2014-15, Marks 10

Answer

Given : $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $C = [1 \quad 0 \quad 0]$

To Test : Controllability and Observability.

A. Controllability test :

1. $$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -3 & -3 \end{bmatrix}$$

$$A^2 B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -3 \\ 11 & 11 \end{bmatrix}$$

2. Controllability test matrix is given by

$$Q_c = [B : AB : A^2 B]$$

$$Q_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -3 & -3 \\ 1 & 1 & -3 & -3 & 11 & 11 \end{bmatrix}$$

3. Consider $\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 1 \neq 0 \,;\ |Q_c| \neq 0$

Hence rank of Q_c is equal to its order *i.e.*, 3. Therefore the system is controllable.

B. Observability test :

1.
$$C = [1 \;\; 0 \;\; 0] \,;\ C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \,;\ A^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(A^T)^2 C^T = (A^T)(A^T C^T) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2. Observability test matrix is given by

$$Q_o = [C^T : A^T C^T : (A^T)^2 C^T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|Q_o| = 1 \neq 0$$

Hence its rank is 3 equal to its order *i.e.*, 3, therefore system is completely observable.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Explain the Bode Plot Method of designing Lead compensator.

Ans. Refer Q. 5.2, Unit-5.

Q. 2. For the open loop transfer function, $G(s)\,H(s) = \dfrac{10}{s(1+0.2s)}$ design a suitable compensator such that the system will have a phase margin of at least 45°.

Ans. Refer Q. 5.3, Unit-5.

Q. 3. Design a phase lead compensator for a negative unity feedback system with plant transfer function.

$$G_p(s) = \frac{K}{s(s+10)(s+1000)}$$ **to satisfy the conditions :**

phase margin in atleast 45°, static error constant = 1000 s^{-1}.

Ans. Refer Q. 5.4, Unit-5.

Q. 4. Define the following terms :

i. State **ii. State variables**

iii. State vector **iv. State space**

v. State equation

Also write the properties of state transition matrix.

Ans. Refer Q. 5.8, Unit-5.

Q. 5. Obtain state equation of a given transfer function

a. $$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

b. $$\frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s+4)}$$

Ans. Refer Q. 5.16, Unit-5.

Q. 6. A linear time invariant system is characterized by the state variable model. Examine the controllablity and observability of the system

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & -4 \end{bmatrix}; B = \begin{bmatrix} 40 \\ 10 \\ 0 \end{bmatrix}; \quad C = [0 \quad 0 \quad 1]$$

Ans. Refer Q. 5.22, Unit-5.

Q. 7. Check the controllability and observability of a system having following coefficient matrices.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } C = [1 \quad 0 \quad 0]$$

Ans. Refer Q. 5.23, Unit-5.

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1 UNIT

Control System Concepts (2 Marks Questions)

1.1. Explain open loop and closed loop system with physical examples. **AKTU 2015-16, Marks 02**

OR

Discuss open loop and closed loop system giving suitable example. **AKTU 2016-17, Marks 02**

OR

Draw the block diagram and explain the open loop control system and closed loop control system.

AKTU 2017-18, Marks 02

OR

What are the major types of control systems ? Explain them in detail with examples. **AKTU 2017-18, Marks 02**

Ans. There are two types of control system :

1. Open loop control system :

Fig. 1.1.1

In open loop control systems the control action is independent of the desired output. In this system the output is not compared with the reference input.

Example : Washing machine, Immersion rod, Time operated traffic control, DC shunt motor.

2. Closed loop control system :

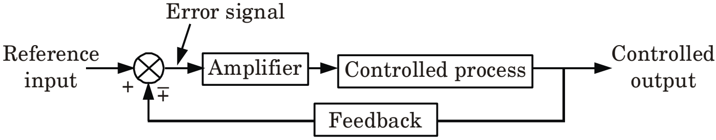


Fig. 1.1.2.

In a closed loop control system the output has an effect on control action through a feedback as shown in Fig. 1.1.2.

Example : Automatic steering control system, Driving system of an automobile, Home heating system, Ship stabilization system.

1.2. Discuss the effect of feedback on the time constant of a control system.

AKTU 2016-17, Marks 02

Ans.

1. Consider an open loop system with overall transfer function as,

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{1+sT}$$

2. System is subjected to unit step input,

$$R(s) = 1/s,$$

$$\therefore \quad c(t) = L^{-1}[C(s)] = K[1 - e^{-t/T}] \quad \text{...(1.2.1)}$$

Here time constant = T

3. Response of system for unit step when feedback having gain h is added. Then new response,

$$c'(t) = \frac{K}{1+Kh}\left[1 - e^{\frac{1}{(T/1+Kh)}}\right]$$

New time constant due to the feedback = $(T/1 + Kh)$.

4. For positive value of h and $K > 1$, the time constant $(T/1 + Kh)$ is less than T. Hence feedback reduces the time constant of a system.

1.3. Why is negative feedback invariably preferred in a closed loop system ?

AKTU 2017-18, Marks 02

Ans. The negative feedback results in better stability in steady state and rejects any disturbance signals. It has also low sensitivity to parameter variations. Hence negative feedback is preferred in closed loop systems.

1.4. Explain the working of AC servomotor with neat diagram.

AKTU 2016-17, Marks 02

Ans. **Working principle :**

1. Reference phase is supplied from a constant voltage source $V_r \angle 0°$. The other winding *i.e.*, control phase is supplied with a variable voltage of the same frequency as the reference phase but its phase is displaced by 90° (electrically).
2. The control phase is usually supplied from a servo amplifier.
3. The speed and torque of the rotor are controlled by the phase difference between the control voltage and the reference phase voltages.

4. The direction of rotation of the rotor can be reversed by reversing the phase difference from leading to lagging between the control phase voltage and the reference phase voltage.

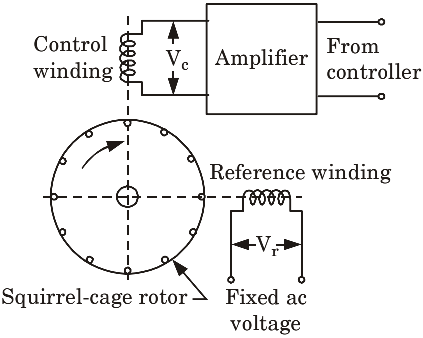
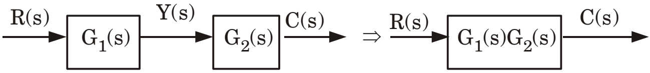


Fig. 1.4.1. Schematic diagram of 2ϕ servomotor.

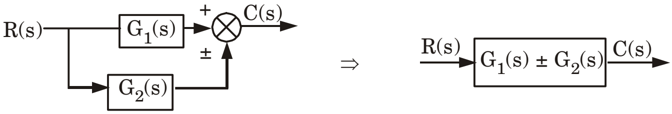
1.5. What is the basis for framing the rules of block diagram reduction technique ? **AKTU 2017-18, Marks 02**

Ans.

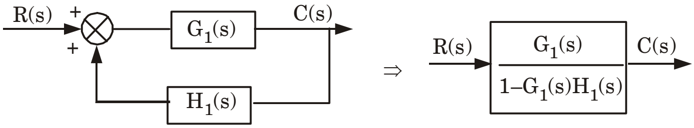
1. Series connection :



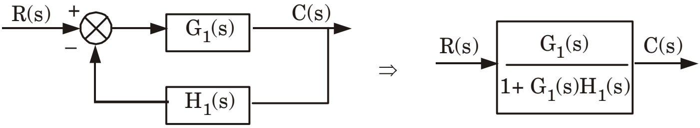
2. Parallel connection :



3. Positive feedback connection :



4. Negative feedback connection :



1.6. What is a signal flow graph ? **AKTU 2017-18, Marks 02**

Ans. A signal flow graph is a pictorial representation of the simultaneous equations describing the system. This is an alternate approach which does not require any reduction process because of availability of a flow graph gain formula which relates the input and output system variables.

1.7. Explain Mason's gain formula. **AKTU 2015-16, Marks 02**

Ans. The overall gain can be determined by Mason's gain formula given below :

$$T = \sum_{k=1}^{k} \frac{P_k \Delta_k}{\Delta}$$

where, P_k = The forward path gain

Δ = The graph determinant which involves closed-loop gain and mutual interactions between non-touching loops.

Δ_k = The path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

1.8. What are the advantages of negative feedback control system ?

Ans.

1. Better frequency response.
2. Less distortion.
3. Less gain or voltage drift.
4. Less temperature drift.

1.9. What are the disadvantages of positive feedback control system ?

Ans.

1. Poor frequency response.
2. More distortion.
3. More drift.

1.10. What is transfer function ?

Ans. It is defined as the Laplace transform of output to the Laplace transform of input when all initial conditions are set to zero.

$$T(s) = \frac{L[c(t)]}{L[r(t)]} = \frac{C(s)}{R(s)}$$

1.11. What is source and sink ?

Ans.

1. **Source :** Source is the input node in the signal flow graph and it has only outgoing branches.
2. **Sink :** Sink is an output node in the signal flow graph and it has only incoming branches.

1.12. What are the basic elements used for modeling mechanical translational system ?

Ans. The model of mechanical translational system can be obtained by using three basic elements mass, spring and damper.

1.13. Derive the transfer function $E_0(s)/E_1(s)$ of network shown

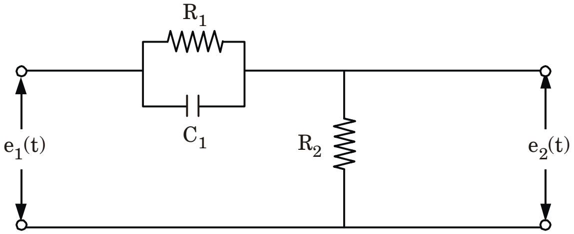


Fig. 1.13.1.

AKTU 2017-18, Marks 02

Ans.

1. Apply KVL in the loop,

$$-V_i(s) + \left[R_1 \parallel \frac{1}{C_1 s}\right] I(s) + R_2 \; I(s) = 0$$

$$V_i(s) = \left\{\left[\frac{R_1 \dfrac{1}{C_1 s}}{R_1 + \dfrac{1}{C_1 s}}\right] + R_2\right\} I(s)$$

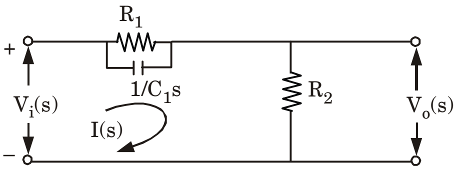


Fig. 1.13.2.

2. And, $V_0(s) = R_2 \, I(s)$

$$V_0(s) = \frac{R_2 \, V_i(s)}{\left[\dfrac{R_1 \dfrac{1}{C_1 s}}{R_1 + \dfrac{1}{C_1 s}}\right] + R_2} = \frac{R_2 \, V_i \, (s)}{\left[\left(\dfrac{R_1}{R_1 \, C_1 s + 1}\right) + R_2\right]}$$

$\therefore$ Transfer function, $\dfrac{V_0(s)}{V_i(s)} = \dfrac{R_2 \, (R_1 \, C_1 s + 1)}{R_2 \, (R_1 \, C_1 s + 1) + R_1}$.

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2
UNIT

Time Response Analysis (2 Marks Questions)

2.1. Mention the nature of transient response of second order control system for different types of damping.

Ans. For $\xi < 0$: Positive roots and the response diverges out.
For $\xi = 0$: Sustained oscillations.
For $0 < \xi < 1$: Underdamped.
For $\xi = 1$: Critically damped.
For $\xi > 1$: Overdamped.

2.2. Define rise time and delay time for second order control system. AKTU 2015-16, Marks 02

Ans.

1. **Rise Time (t_r) :** Rise time is defined as the time required for the response to rise from 10 % to 90 % (for overdamped) or from 0 % to 100 % (for underdamped) of its final value.

$$\text{Rise time } (t_r) = \frac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_d}$$

2. **Delay Time (t_d) :** Delay time is the time required for the response to reach the half of the final value in the first attempt.
The delay time in terms of ξ and ω_n is given by :

$$t_d = \frac{1+0.7\xi}{\omega_n}$$

2.3. What is peak time ?

Ans. Peak time is the time required for the response to reach the first peak of the overshoot.

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

2.4. What is settling time ?

Ans. Settling time is the time required for the response curve to reach and stay within a specified percentage (2 % to 5 %) of its final value.

For 2 % band, $t_s = \dfrac{4}{\xi\omega_n}$

and for 5 % band, $t_s = \dfrac{3}{\xi\omega_n}$

2.5. Define steady-state error.

Ans. It indicates the error between the actual output and desired output as t tends to infinity *i.e.*

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$$

It is denoted by e_{ss}.

2.6. Define the P, PI and PID controllers.

AKTU 2017-18, Marks 02

Ans. **P controller (Proportional controller) :** It is a control system technology based on a response in proportion to the difference between what is set as a desired process variable (or set point) and the current value of the variable.

PI controller (Proportional integral controller) : A controller in the forward path which changes the controller output corresponding to the proportional plus integral of error signal is called PI controller.

PID controller (Proportional, integral and derivative controller): It is a Close loop system which has feedback control system and it compares the Process variable (feedback variable) with set Point and generates an error signal and according to that it adjusts the output of system.

2.7. Give the comparison between PI and PID controller.

AKTU 2016-17, Marks 02

Ans.

S. No.	PI	PID
1.	It is a combination of proportional and integral control action.	It is a combination of proportional, integral and derivative controller.
2.	It helps in reducing steady state error.	It helps in reducing steady state error and also getting steady state conditions quickly.

2.8. What is transient and steady state response ?

Ans. The transient response is the response of the system when the input changes from one state to another. The response of the system as $t \to \infty$ is called steady state response.

2.9. List the time domain specifications.

Ans. The time domain specifications are :
1. Delay time
2. Rise time
3. Peak time
4. Maximum overshoot
5. Settling time.

2.10. Why derivative control is not employed in isolation ?

Ans. A derivative control mode in isolation produces no corrective efforts for any constant errors, because, it acts only on rate of change of error.

2.11. What is the effect on system performance when a proportional controller is introduced in a system ?

Ans. The proportional controller improves the steady-state tracking accuracy, disturbance signal rejection and relative stability of the system. It also increases the loop gain of the system which results in reducing the sensitivity of the system to parameter variations.

2.12. Why derivative controller is not used in control systems ?

Ans. The derivative controller produces a control action based on rate of change of error signal and it does not produce corrective measures for any constant error. Hence derivative controller is not used in control systems.

2.13. Discuss the significance of various time domain specifications. **AKTU 2016-17, Marks 02**

Ans.
1. **Delay time (T_d) :** It is the time required for the response to reach half the final value the very first time.
2. **Rise time (T_r) :** The rise time is the time required for the response to rise from
   a. 10 % to 90 % of its final value, (overdamped systems).
   b. 5 % to 95 % of its final value, (critical damped systems).
   c. 0 % to 100 % of its final value (underdamped systems).
3. **Peak time (T_p) :** The peak time is the time required for the response to reach the first (maximum) peak of the overshoot.
4. **Maximum overshoot (M_p) :** It is the maximum peak value of the response curve measured from unity.
5. **The settling time (T_s) :** It is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2 % or 5 %).

2.14. The OLTF of a unity feedback system is $G(s) = 4(s + a)/s(s + 1)(s + 4)$ find the expression for error $E(s)$ and hence find the value of a so that the e_{ss} due to a unit ramp is 0.125.

AKTU 2017-18, Marks 02

Ans.

Given : $G(s) = \dfrac{4(s+a)}{s(s+1)(s+4)}$; $r(t) = 0.125\, t$

To Find : e_{ss}.

1. $$r(t) = 0.125\, t$$

$$R(s) = \frac{0.125}{s^2}$$

2. $$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)\,H(s)} = \lim_{s \to 0} \frac{\dfrac{s \times 0.125}{s^2}}{1 + \dfrac{4\,(s+a)}{s(s+1)\,(s+4)} \times 1}$$

$$= \lim_{s \to 0} \frac{\dfrac{0.125}{s} \times s(s+1)\,(s+4)}{s(s+1)\,(s+4) + 4(s+a)} = \lim_{s \to 0} \frac{0.125 \times (s+1)\,(s+4)}{s(s+1)\,(s+4) + 4(s+a)}$$

$$= \frac{0.125 \times 1 \times 4}{4a}$$

$$\therefore \quad e_{ss} = \frac{1}{4a}$$

2.15. Consider the system as shown in Fig. 2.15.1. Determine the value of a such that the damping ratio is 0.5.

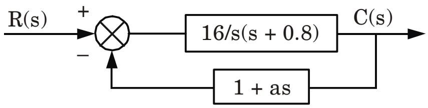

Fig. 2.15.1.

AKTU 2015-16, Marks 02

Ans.

1. $$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} = \frac{\dfrac{16}{s(s+0.8)}}{1 + \dfrac{16}{s(s+0.8)} \times (1 + as)}$$

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + 0.8s + 16 + 16as} \qquad \text{...(2.15.1)}$$

2. Comparing this eq. (2.15.1) with $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$

we get, $\omega_n^2 = 16$

$\omega_n = 4$

3. Damping factor is given as $\xi = 0.5$

$$2\xi\omega_n = (0.8 + 16a)$$
$$2 \times 0.5 \times 4 = 0.8 + 16a$$
$$4 = 0.8 + 16a$$
$$a = 0.2$$

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3 UNIT

Stability and Algebraic Criteria (2 Marks Questions)

3.1. State the necessary and sufficient condition of Routh-Hurwitz criterion. **AKTU 2015-16, Marks 02**

Ans.
1. All roots of characteristic equation should lie in the left half of *s*-plane, *i.e.*, all coefficients must be positive.
2. There should be no missing term.

3.2. Establish the relation between Routh and Hurwitz stability criterion. **AKTU 2016-17, Marks 02**

Ans.
1. In the case of the Routh criterion, $a_0 > 0$, $a_1 > 0$, $b_1 > 0$, $c_1 > 0$, $d > 0$, ...

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_3} = \frac{\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}}{a_3}$$

2. Again, in the case of the Hurwitz criterion

$$a_3 > 0, \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0$$

3. From the Routh criterion, we also observe that $b_1 > 0$ and $a_3 > 0$. Therefore,

$$b_1 = \frac{\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}}{a_3} > 0$$

since $$a_3 > 0, \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0$$

This is also true for c_1, d_1 for the Routh criterion.

4. Hence, the Hurwitz criterion and the Routh criterion are basically the same and draw the same conclusion.

3.3. What do you understand by relative stability ? Explain.

AKTU 2016-17, Marks 02

Ans. The system is said to be relatively more stable or unstable on the basis of settling time. System is said to be relatively more stable if settling time for that system is less than of the other system.

3.4. Define the term centroid and breakaway point.

AKTU 2015-16, Marks 02

Ans. **Centroid :** All asymptotes intersect the real axis at a common point known as centroid.

$$\sigma = \frac{\Sigma \text{ Real part of open loop poles} - \Sigma \text{ Real part of open loop zeros}}{P - Z}$$

where, P = Number of open loop poles
Z = Number of open loop zeros.

Breakaway point : Breakaway point is defined as the point at which root locus comes out of the real axis.

3.5. Differentiate between two-phase servomotor and normal induction motor.

Ans.

S. No.	Two-Phase Servomotor	Induction Motor
1.	The rotor of the servomotor is built with high resistance so that its X/R is small.	The rotor construction is usually squirrel cage or drag-cup type. The diameter of the rotor is kept small in order to reduce inertia.
2.	The voltage applied to the two stator windings is seldom balanced.	In this, the voltage applied to the two stator is not balanced.

3.6. What is the effect of addition of pole on root locus ?

Ans. The addition of a pole to the system open loop transfer function pushes the root locus to the right of original root locus, thereby reducing the system stability and making the system more oscillatory.

3.7. Define asymptote.

Ans. The line which touches the curve at infinity is known as asymptote. For $(P - Z)$ branches, angle of such asymptote is given by :

$$\theta_q = \frac{(2q + 1) \times 180°}{P - Z}$$

where $q = 0, 1, 2, ... (P - Z - 1)$.

3.8. What is the effect of addition of zero on root locus ?

Ans. The addition of a zero to the system pushes the root locus to the left by original root locus, thereby making the system more stable and less oscillatory.

3.9. What is root contour ?

Ans. The locus of root of characteristics equation obtained by varying parameter of the system other than K from 0 to ∞ is called root contour of the system.

3.10. Define BIBO stability.

Ans. A linear system is said to have BIBO stability if every bounded (finite) input results in a bounded (finite) output.

3.11. What will be the nature of impulse response when the roots of characteristic equation are lying on imaginary axis ?

Ans. If the roots of characteristic equation lie on imaginary axis the nature of impulse response is oscillatory.

3.12. What is dominant pole ?

Ans. The dominant pole is a pair of complex conjugate pole which decides transient response of the system. In higher order systems the dominant poles are very close to origin and all other poles of the system are widely separated and so they have less effect on transient response of the system.

3.13. How will you find the gain K at a point on root locus ?

Ans. The gain K at a point $s = s_a$ on root locus is given by,

$$K = \frac{\text{Product of length of vector from open loop poles to the point } s_a}{\text{Product of length of vector from open loop zeros to the point } s_a}$$

3.14. Determine the stability of the system whose characteristics equation is given by $2s^4 + 2s^3 + s^2 + 3s + 2 = 0$.

AKTU 2017-18, Marks 02

Ans.

1. System equation $2s^4 + 2s^3 + s^2 + 3s + 2 = 0$.
2. By the Routh array

$$\begin{array}{c|ccc} s^4 & 2 & 1 & 2 \\ s^3 & 2 & 3 & 0 \\ s^2 & -2 & 3 & \\ s^0 & 6 & & \end{array}$$

3. Since there is sign change in the first column of Routh table, therefore system is unstable.

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4 UNIT

Frequency Response Analysis (2 Marks Questions)

4.1. Explain in brief :
i. Gain margin
ii. Phase margin.

AKTU 2016-17, Marks 02

Ans.
i. Gain margin : It is the reciprocal of magnitude $|G(j\omega)|$ at the frequency at which the phase angle is – 180°.
Gain Margin (GM),

$$K_g = \frac{1}{|G(j\omega_c)|}$$

where, ω_c = Phase cross-over frequency.

ii. Phase margin : The phase margin is that amount of the additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.
Phase margin is equal to 180° plus the angle of $G(j\omega)$ at the gain crossover point.

$$\phi_m = 180° + \phi$$

4.2. Define gain crossover frequency.

Ans. Gain crossover frequency is the frequency at which magnitude of open loop transfer function is unity.

4.3. What is phase crossover frequency ?

Ans. Phase crossover frequency is the frequency at which phase of open loop transfer function is – 180°.

4.4. Write the condition to test the stability of Nyquist plot.

Ans. Condition to test stability, $N = Z - P$.
where, N = Number of encirclement of $(-1 + j0)$ point in clockwise direction.
Z = Number of closed loop poles on the right half of s-plane.
For system to be stable, Z should be zero *i.e.*, $N = -P$.

4.5. Explain the significances of constant M and N circles.

AKTU 2015-16, Marks 02

Ans. They are useful for determining the closed loop frequency response from the given open loop. This is accomplished by superimposing the log-magnitude versus phase angle plot of $G(j\omega)$.

4.6. Write the condition to test stability of Bode plot.

Ans.

1.	Stable system	$\omega_{gc} < \omega_{pc}$	GM = positive PM = positive
2.	Unstable system	$\omega_{gc} > \omega_{pc}$	GM = negative PM = negative
3.	Marginally stable system	$\omega_{gc} = \omega_{pc}$	GM = 0 PM = 0

4.7. What is Nichols plot ?

Ans. The Nichols plot is a frequency response plot of the open loop transfer function of a system. It is a graph between magnitude of $G(j\omega)$ in dB and the phase of $G(j\omega)$ in degree, plotted on a ordinary graph sheet.

4.8. What is All-Pass system ?

Ans. All Pass systems are systems with all pass transfer functions. In all pass transfer functions, the magnitude is unity at all frequencies and the transfer function will have anti-symmetric pole zero pattern (*i.e.*, for every pole in the left half *s*-plane, there is a zero in the mirror image position with respect to imaginary axis).

4.9. What is non-minimum phase transfer function ?

Ans. A transfer function which has one or more zeros in the right half *s*-plane is known as non-minimum phase transfer function.

4.10. Define corner frequency ?

Ans. The magnitude plot can be approximated by asymptotic straight lines. The frequencies corresponding to the meeting point of asymptotes are called corner frequency. The slope of the magnitude plot changes at every corner frequency.

4.11. Write the expression for resonant peak and resonant frequency.

Ans.

1. Resonant peak, $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$
2. Resonant frequency,

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

4.12. What are the advantages of Bode Plot ?

Ans.

1. The magnitudes are expressed in dB, and so, a simple procedure is available to add magnitude to each one by one.
2. The approximate bode plot can be quickly sketched, and the corrections can be made at corner frequency to get the exact plot.
3. The frequency domain specifications can be easily determined.

4.13. Draw the polar plot of open loop transfer function $\frac{1}{s^2}$.

AKTU 2015-16, Marks 02

Ans.

Given : $G(s)\,H(s) = 1 / s^2$
To Find : Polar plot.

1. Putting $s = j\omega$

$$G(j\omega)\,H(j\omega) = \frac{1}{(j\omega)^2} = \frac{-1}{\omega^2}$$

2. Magnitude, $M = |G(j\omega)\,H(j\omega)| = \frac{1}{\omega^2}$

and phase angle, $\phi = -180°$

ω	M
0	∞
2	1/4
4	1/16
⋮	⋮
∞	0

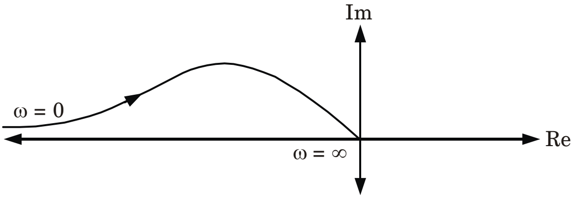

Fig. 4.13.1.

4.14. Show that the polar plot of $G(s) = K/(s + a)$ is a semicircle. Also find its centre and radius.

AKTU 2017-18, Marks 02

Ans.

Given : $G(s) = \dfrac{K}{s+a}$

To Find : Centre and Radius.

Solution :

1. Let $s = j\omega$

$$G(j\omega) = \frac{K}{j\omega + a} = \frac{K}{\sqrt{\omega^2 + a^2}} \angle \tan^{-1}\frac{\omega}{a}$$

2. At $\omega = 0$, $|G(j\,0)| = \dfrac{K}{a}$

$$\angle G(j\,0) = -0°$$

3. At $\omega = -\infty$, $|G(j\infty)| = 0$

$$\angle G(j\infty)| = -90°$$

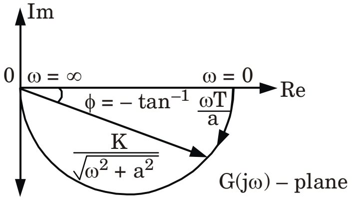

Fig. 4.14.1.

4. Diameter $= |G(j\omega)|_{\omega=0} - |G(j\omega)|_{\omega=\infty}$

$$= \frac{K}{a} - 0 = \frac{K}{a}$$

So, Radius $= K/2a$

5. Center $= \dfrac{|G(j\omega)|_{\omega=0} - |G(j\omega)|_{\omega=\infty}}{2} = K/2a.$

☺☺☺

5 UNIT

Introduction to Design (2 Marks Questions)

5.1. What is the need of compensation in control system ?

AKTU 2015-16, Marks 02

Ans. If a system is to be redesigned so as to meet the required specifications, it is necessary to alter the system by adding and external device to it. Such a redesign or alteration of system using an additional suitable device is called compensation of a control system.

5.2. Differentiate between lag and lead network in view of their Bode plot.

AKTU 2016-17, Marks 02

Ans.

S. No.	Lead	Lag
1.	The lead network is basically a high-pass filter.	The lag network is essentially a low-pass filter.
2.	The primary function of the lead compensator is to reshape the frequency-response curve to provide sufficient phase-lead angle.	The primary function of a lag compensator is to provide attenuation in the high-frequency range to give a system sufficient phase margin.

5.3. What are state and state variables ?

AKTU 2015-16, Marks 02

Ans. **State :** State of a system is the minimum amount of information needed along with initial conditions at $t = t_0$ and input excitation so that future response of system can be completely described at any time $t > t_0$.

State Variables : A set of at least n variables $x_1(t)$, $x_2(t)$ ... $x_n(t)$, are needed to completely describe how a system will behave in future, along with initial state and input excitation. These minimal set of variables which can determine the state of a system are known as state variables.

5.4. Discuss the advantages of state variable technique over transfer function approach. AKTU 2016-17, Marks 02

Ans.

1. This method can be applied to linear or non linear, time variant or time invariant system.
2. This method can be designed for optimal conditions.

5.5. Write the characteristics of lead networks.

Ans.

1. A lead network has zero at $s = -1/T$ and a pole at $s = -1/\alpha T$.
2. Since $0 < \alpha < 1$, we see that the zero is always located to the right of the pole in the complex s-plane.
3. For a small value of α the pole is located far to the left.

5.6. Write the characteristics of lag networks.

Ans.

1. In the complex plane, a lag network has a pole at $s = -1/\beta T$ and a zero at $s = -1/T$.
2. The pole is located to the right of the zero.
3. The lag network is essentially a low-pass filter.

5.7. Write the properties of state transition matrix.

Ans.

1. $\phi(0) = 1,$
2. $\phi^{-1}(t) = \phi(-t)$
3. $[\phi(t)]^k = \phi(kt)$

5.8. Define diagonalization.

Ans. Transformation of a matrix into a diagonal matrix such as the diagonal elements are represented by eigen values is called diagonalization.

5.9. Define Kalman's Test.

Ans. A *LTI* continuous system is completely controllable if and only if the rank of the controllability matrix, is defined as

$$Q_c = [B : AB : A^2B; \ldots, A^{n-1}B],$$

is equal to rank n.

5.10. What are the factors to be considered for choosing series or shunt/feedback compensation ?

Ans.

1. Nature of signals in the system.
2. Power levels at various points.
3. Components available.
4. Economic considerations.

5.11. What is compensation ?

Ans. The compensation is the design procedure in which the system behaviour is altered to meet the desired specifications, by introducing additional device called compensator.

5.12. What are the advantages in frequency domain design ?

Ans.

1. The effect of disturbances, sensor noise and plant uncertainties are easy to visualize and assess in frequency domain.
2. The experimental information can be used for design purposes.

5.13. Define state transition matrix.

Ans. State transition matrix $\phi(t) = e^{At}$

$$\text{where } e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \ldots + \sum_{i=0}^{n} \frac{A^i t^i}{t!}$$

5.14. Check whether given matrix is controllable or not :

$$Q_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & -5 & 11 \end{bmatrix}$$

Ans.

$$|Q_c| = 0\begin{vmatrix} 1 & -5 \\ -5 & 11 \end{vmatrix} - 0\begin{vmatrix} 0 & -5 \\ 1 & 11 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & -5 \end{vmatrix}$$

$$|Q_c| = [0 - 0] + [0 - 1] = -1$$

Thus the rank of Q_c is 3 and order is also 3. Hence, the system is completely controllable.

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B.Tech.
(SEM. V) ODD SEMESTER THEORY EXAMINATION, 2015-16
CONTROL SYSTEM

Time : 3 Hours **Max. Marks : 100**

SECTION–A

Note : Attempt **all** sections. **All** sections carry **equal** marks. Write answer of each part in short. **(2 × 10 = 20)**

1. a. Explain open loop and closed loop system with physical examples.

b. State the necessary and sufficient condition of Routh-Hurwitz criterion.

c. Explain the significances of constant M and N circles.

d. What is the need of compensation in control system ?

e. Draw the polar plot of open loop transfer function $\frac{1}{s^2}$.

f. What are state and state variables ?

g. Consider the system as shown in Fig. 1. Determine the value of *a* such that the damping ratio is 0.5.

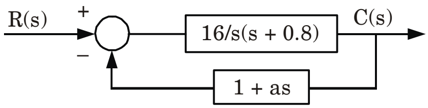

Fig. 1.

h. Define rise time and delay time for second order control system.

i. Explain Mason's gain formula.

j. Define the term centroid and breakaway point.

SECTION–B

Note : Attempt any **five** questions of the following. **(10 × 5 = 50)**

2. Determine the transfer function $C(s)/R(s)$ for the block diagram shown in Fig. 2.

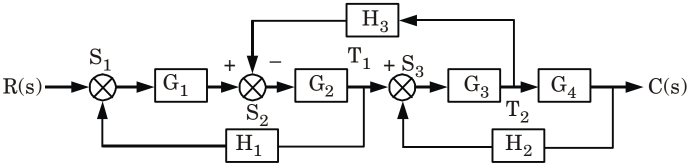

Fig. 2.

3. Derive the expression for step response of second order control system for underdamped response.

4. Using Routh's stability criterion, determine the range of K for open loop transfer function

$$G(s)\,H(s) = \frac{K}{s(s+1)(1+2s)}$$

5. Construct root loci for open loop transfer function :

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+3)}$$

6. Derive expression for resonant frequency and resonant peak for second order control system.

7. Sketch the Nyquist plot for the system with open loop transfer function

$$G(s)\,H(s) = \frac{60}{(s+1)(s+2)(s+5)}$$

and comment on stability.

8. Write short notes on PD controller and synchro.

9. Obtain state equation of a given transfer function

a. $$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

b. $$\frac{Y(s)}{U(s)} = \frac{1}{(s+1)(s+4)}$$

SECTION–C

Note : Attempt any **two** questions of the following. **(15 × 2 = 30)**

10. For a unity feedback system, the open loop transfer function is

$$G(s)\,H(s) = \frac{2(s + 0.25)}{s^2\,(s+1)\,(s+0.5)}$$

Draw bode plot and determine gain margin, phase margin.

11. A system characterised by the transfer function

$$\frac{Y(s)}{U(s)} = \frac{2}{s^3 + 6s^2 + 11s + 6}$$ **. Find the state and output equation in matrix form and also test the controllability and observability of the given system.**

12. Write short notes of the following :

a. Lead compensator.

b. Lag compensator.

c. Gain margin and phase margin.

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SOLUTION OF PAPER (2015-16)

SECTION–A

Note : Attempt **all** sections. **All** sections carry **equal** marks. Write answer of each part in short. **(2 × 10 = 20)**

1. a. Explain open loop and closed loop system with physical examples.

Ans. There are two types of control system :

1. Open loop control system :

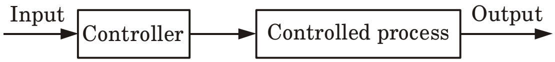

Fig. 1.

In open loop control systems the control action is independent of the desired output. In this system the output is not compared with the reference input.

Example : Washing machine, Immersion rod, Time operated traffic control, DC shunt motor.

2. Closed loop control system :

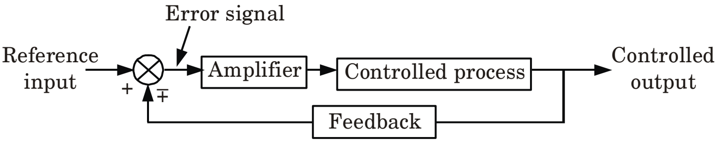

Fig. 2.

In a closed loop control system the output has an effect on control action through a feedback as shown in Fig. 2.

Example : Automatic steering control system, Driving system of an automobile, Home heating system, Ship stabilization system.

b. State the necessary and sufficient condition of Routh-Hurwitz criterion.

Ans.

1. All roots of characteristic equation should lie in the left half of s-plane, *i.e.*, all coefficients must be positive.
2. There should be no missing term.

c. Explain the significances of constant M and N circles.

Ans. They are useful for determining the closed loop frequency response from the given open loop. This is accomplished by superimposing the log-magnitude versus phase angle plot of $G(j\omega)$.

d. What is the need of compensation in control system ?

Ans. If a system is to be redesigned so as to meet the required specifications, it is necessary to alter the system by adding and external device to it. Such a redesign or alteration of system using an additional suitable device is called compensation of a control system.

e. Draw the polar plot of open loop transfer function $\frac{1}{s^2}$.

Ans.

Given : $G(s)\,H(s) = 1 / s^2$
To Find : Polar plot.

1. Putting $s = j\omega$

$$G(j\omega)\,H(j\omega) = \frac{1}{(j\omega)^2} = \frac{-1}{\omega^2}$$

2. Magnitude, $M = |G(j\omega)\,H(j\omega)| = \frac{1}{\omega^2}$

and phase angle, $\phi = -180°$

ω	M
0	∞
2	1/4
4	1/16
$\vdots$	$\vdots$
∞	0

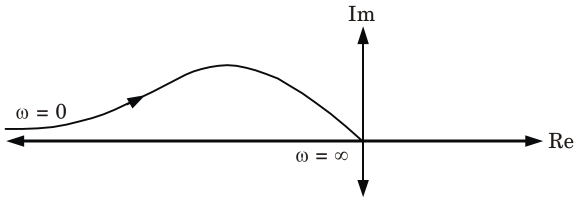

Fig. 3.

f. What are state and state variables ?

Ans. **State :** State of a system is the minimum amount of information needed along with initial conditions at $t = t_0$ and input excitation so that future response of system can be completely described at any time $t > t_0$.

State Variables : A set of at least n variables $x_1(t)$, $x_2(t)$ ... $x_n(t)$, are needed to completely describe how a system will behave in future, along with initial state and input excitation. These minimal set of variables which can determine the state of a system are known as state variables.

g. Consider the system as shown in Fig. 4. Determine the value of *a* such that the damping ratio is 0.5.

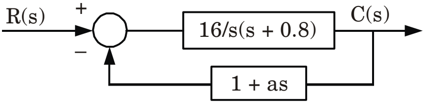

Fig. 4.

Ans.

1. $$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} = \frac{\dfrac{16}{s(s+0.8)}}{1 + \dfrac{16}{s(s+0.8)} \times (1 + as)}$$

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + 0.8s + 16 + 16as} \quad ...(1)$$

2. Comparing this eq. (1) with $\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$

we get, $\omega_n^2 = 16$

$\omega_n = 4$

3. Damping factor is given as $\xi = 0.5$

$$2\xi\omega_n = (0.8 + 16a)$$
$$2 \times 0.5 \times 4 = 0.8 + 16a$$
$$4 = 0.8 + 16a$$
$$a = 0.2$$

h. Define rise time and delay time for second order control system.

Ans.

1. **Rise Time (t_r) :** Rise time is defined as the time required for the response to rise from 10 % to 90 % (for overdamped) or from 0 % to 100 % (for underdamped) of its final value.

$$\text{Rise time } (t_r) = \frac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_d}$$

2. **Delay Time (t_d) :** Delay time is the time required for the response to reach the half of the final value in the first attempt.
The delay time in terms of ξ and ω_n is given by :

$$t_d = \frac{1 + 0.7\xi}{\omega_n}$$

i. Explain Mason's gain formula.

Ans. The overall gain can be determined by Mason's gain formula given below :

$$T = \sum_{k=1}^{k} \frac{P_k \Delta_k}{\Delta}$$

where, P_k = The forward path gain

Δ = The graph determinant which involves closed-loop gain and mutual interactions between non-touching loops.

Δ_k = The path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

j. Define the term centroid and breakaway point.

Ans. Centroid : All asymptotes intersect the real axis at a common point known as centroid.

$$\sigma = \frac{\Sigma \text{ Real part of open loop poles} - \Sigma \text{ Real part of open loop zeros}}{P - Z}$$

where, P = Number of open loop poles

Z = Number of open loop zeros.

Breakaway point : Breakaway point is defined as the point at which root locus comes out of the real axis.

SECTION–B

Note : Attempt any **five** questions of the following. **(10 × 5 = 50)**

2. Determine the transfer function $C(s)/R(s)$ for the block diagram shown in Fig. 5.

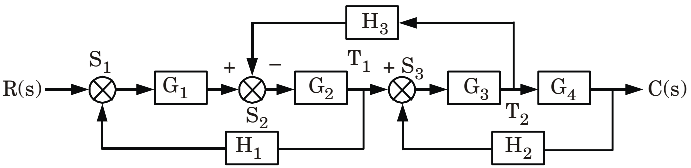

Fig. 5.

Ans. Step 1 : Shift S_2 before G_1, we get

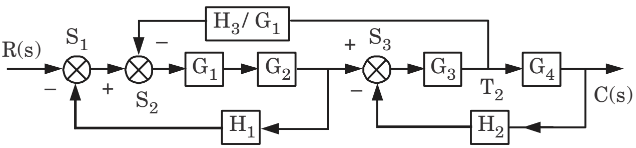

Fig. 6.

Step 2 : G_1 and G_2 are in cascade.

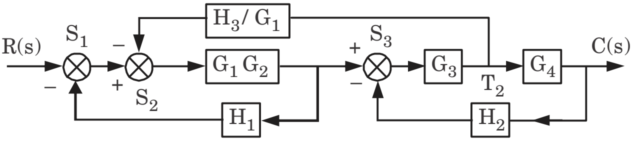

Fig. 7.

Step 3 : Interchanging summing points

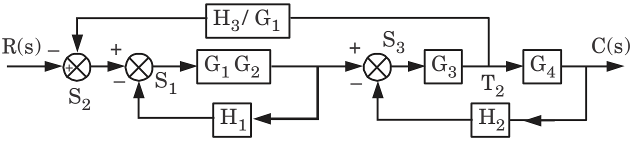

Fig. 8.

Step 4 : Solve the inner loop.

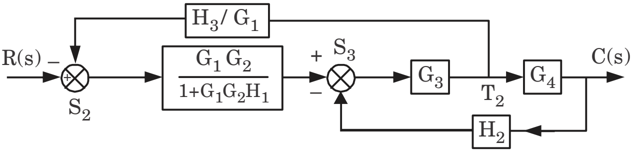

Fig. 9.

Step 5 : Shift T_2 after G_4.

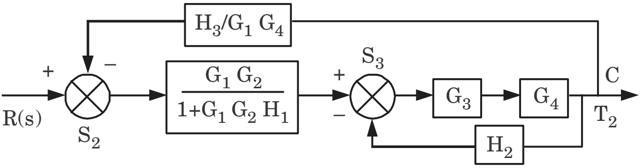

Fig. 10.

Step 6 : Solve the inner loop.

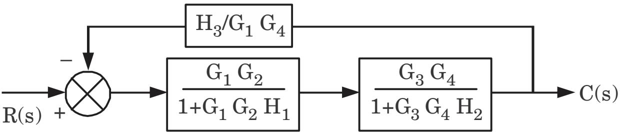

Fig. 11.

Step 7 : Forward blocks are in cascade and final result will be

$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}{1+\dfrac{G_2G_3H_3}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)+G_2G_3H_3}$$

3. Derive the expression for step response of second order control system for underdamped response.

Ans.

1. Consider the second order system with unity feedback.

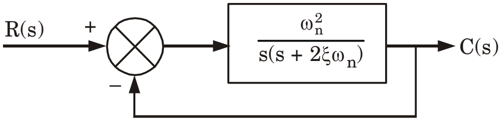

Fig. 12.

The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 / s(s+2\xi\omega_n)}{1+\omega_n^2 / s(s+2\xi\omega_n)}$$

where, ξ = Damping factor or Damping ratio

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s^2+2\xi\omega_n s+\omega_n^2)}$$

Then output,

$$C(s) = R(s)\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad ...(1)$$

2. For unit step input

$$r(t) = 1$$

$$R(s) = \frac{1}{s}$$

Then $$C(s) = \frac{1}{s}\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad ...(2)$$

3. In eq. (2) putting $[s^2+2\xi\omega_n s+\omega_n^2] = [(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)]$ and breaking it into partial fraction

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{[(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)]} \quad ...(3)$$

Put $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} \quad ...(4)$$

4. Rewrite eq. (4)

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2} \quad ...(5)$$

5. Taking inverse Laplace transform of eq. (5),

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d} e^{-\xi\omega_n t}\sin\omega_d t$$

Put $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}[(\sqrt{1-\xi^2})\cos\omega_d t + \xi\sin\omega_d t] \quad ...(6)$$

6. Put $\sin\phi = \sqrt{1-\xi^2}$,

$\therefore$ $\cos\phi = \xi$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}(\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi) \quad ...(7)$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$

and $\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

7. Eq. (7) is rewritten as

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right] \quad ...(8)$$

8. The term ω_n is called natural frequency of oscillations. Term $\omega_d = \omega_n\sqrt{1-\xi^2}$ is called damped frequency of oscillations and the term ξ is called damping ratio or damping factor .

a. Underdamped case ($0 < \xi < 1$) : From eq. (8), time constant is $1/\xi\omega_n$ and the response having damped oscillations with overshoot and undershoot is known as underdamped response.

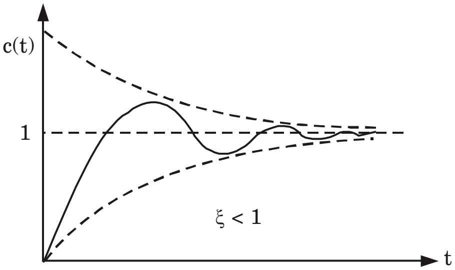

Fig. 13. Underdamped oscillations.

4. Using Routh's stability criterion, determine the range of K for open loop transfer function

$$G(s)\,H(s) = \frac{K}{s(s+1)(1+2s)}$$

Ans.

Given : $G(s)\,H(s) = \dfrac{K}{s(s+1)(1+2s)}$

To Find : Range of K.

1. Using characteristics equation

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{K}{s(s+1)(1+2s)} = 0$$

$$s(s+1)(1+2s) + K = 0$$

$$s(2s^2 + 3s + 1) + K = 0$$

$$2s^3 + 3s^2 + s + K = 0$$

2. Using Routh array :

$$\begin{array}{c|cc} s^3 & 2 & 1 \\ s^2 & 3 & K \\ s^1 & \dfrac{3-2K}{3} & 0 \\ s^0 & K & \end{array}$$

3. For system to be stable :

$$\frac{3-2K}{3} > 0 \text{ and } K > 0$$

$$\therefore \qquad 0 < K < \frac{3}{2}$$

5. Construct root loci for open loop transfer function :

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+3)}$$

Ans.

Given : $G(s)\,H(s) = \dfrac{K}{s(s+1)(s+3)}$, $H(s) = 1$, $\xi = 0.5$

To Sketch : Root locus and value of K at $\xi = 0.5$.

1. The open-loop poles are at $s = 0$, $s = -1$, $s = -3$.
2. There is no open loop zeros.
3. Number of poles, $P = 3$
   Number of zeros, $Z = 0$

$P - Z = 3 - 0 = 3$ *i.e.*, three branches of root locus end at infinity.

4. Angle of asymptotes :

$$\theta_q = \frac{(2q+1)180°}{P-Z} \qquad \text{where } q = 0, 1, 2, \ldots (P - Z - 1)$$

$$\theta_0 = \frac{(2 \times 0 + 1) \times 180°}{(3-0)} = 60°$$

$$\theta_1 = \frac{(2 \times 1 + 1) \times 180°}{(3-0)} = 180°$$

$$\theta_2 = \frac{(2 \times 2 + 1) \times 180°}{(3-0)} = 300°$$

5. Centroid of asymptotes :

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P-Z}$$

$$= \frac{(0-1-3)-(0)}{3-0} = -1.33.$$

6. Breakaway points : Between open-loop poles $s = 0$ and $s = -1$, there exists a breakaway point.

The characteristic equation is

$$s(s+1)(s+3) + K = 0$$

$$K = -(s^3 + 4s^2 + 3s)$$

$$\therefore \quad \frac{dK}{ds} = -(3s^2 + 8s + 3) = 0$$

$$\therefore \quad 3s^2 + 8s + 3 = 0$$

$$s = -1.33 \pm 0.88 = -0.42 \text{ and } -2.21$$

As the breakaway point has to lie between $s = 0$ and $s = -1$, the valid breakaway point is $s = -0.42$.

7. Intersection with $j\omega$ axis :

Characteristic equation, $1 + G(s)H(s) = 0$

$$s(s+1)(s+3) + K = 0$$

$$s^3 + 4s^2 + 3s + K = 0$$

Routh array :

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 4 & K \\ s^1 & (12-K)/4 & \\ s^0 & K & \end{array}$$

The value of K at imaginary axis :

$$\frac{(12-K)}{4} = 0 \quad \therefore \quad K = 12$$

Auxiliary equation,

$$4s^2 + 12 = 0$$

$$s = \pm j\,1.73$$

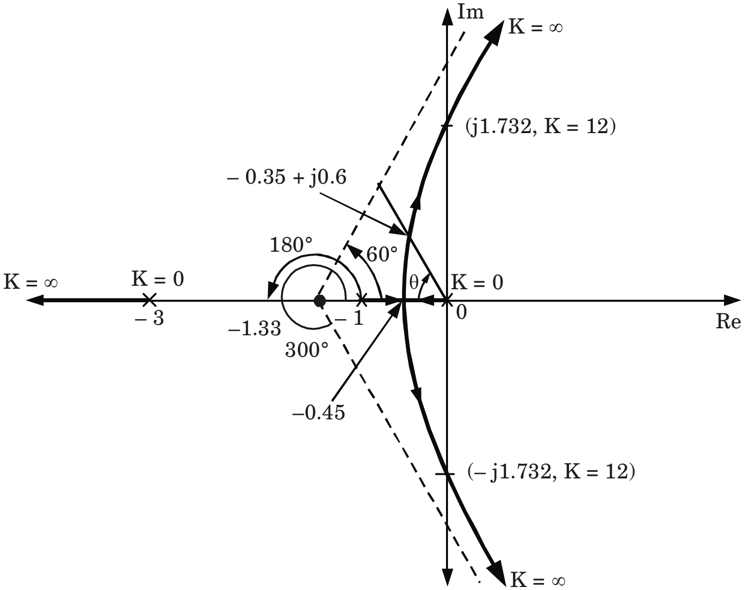

Fig. 14. Root locus for G(s) = K/s(s + 1)(s + 3).

6. Derive expression for resonant frequency and resonant peak for second order control system.

Ans.

1. For second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n^2}$$

2. Putting $s = j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{-\omega^2 + 2\xi\,\omega_n j\omega + \omega_n^2}$$

$$M = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\left(\frac{\omega}{\omega_n}\right)}$$

Let $\frac{\omega}{\omega_n} = x$

$$M = \frac{1}{1 - x^2 + j2\xi x} = \frac{1}{\sqrt{(1 - x^2)^2 + 4\xi^2 x^2}}$$

3. To maximize magnitude, put $\frac{dM}{dx} = 0$,

$$\frac{dM}{dx} = \frac{d}{dx}[(1-x^2)^2 + 4\xi^2 x^2]^{-1/2}$$

$$= -\frac{1}{2}[(1-x^2)^2 + 4\xi^2 x^2]^{-3/2}\frac{d}{dx}[(1-x^2)^2 + 4\xi^2 x^2]$$

$$= -\frac{1}{2}\frac{1}{[(1-x^2)^2 + 4\xi^2 x^2]^{3/2}}[2(1-x^2)(-2x)+8\xi^2 x]$$

$$= 0$$

$$4x[x^2 + 2\xi^2 - 1] = 0$$

As, $x \neq 0$

$\therefore$ $x^2 = 1 - 2\xi^2$

$$x = \sqrt{1-2\xi^2}$$

Resonant frequency,

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

4. $$|M_r| = \frac{1}{\sqrt{(1-x^2)^2 + 4\xi^2 x^2}}$$

$$|M_r| = \frac{1}{\sqrt{(1-(\sqrt{1-2\xi^2}))^2 + 4\xi^2(\sqrt{1-2\xi^2})^2}}$$

$$|M_r| = \frac{1}{\sqrt{4\xi^4 + 4\xi^2(1-2\xi^2)}}$$

Resonant peak,

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

7. Sketch the Nyquist plot for the system with open loop transfer function

$$G(s)\,H(s) = \frac{60}{(s+1)(s+2)(s+5)}$$

and comment on stability.

Ans.

Given : $G(s)\,H(s) = \dfrac{60}{(s+1)(s+2)(s+5)}$

To Sketch : Nyquist plot.

1. Put $s = j\omega$, $G(j\omega)\,H(j\omega) = \dfrac{60}{(j\omega+1)(j\omega+2)(j\omega+5)}$

2. Magnitude, $M = |G(j\omega)\,H(j\omega)| = \dfrac{60}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})(\sqrt{\omega^2+25})}$

3. Phase $\phi = \angle G(j\omega)H(j\omega)$

$$= -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

Table 1.

ω	*M*	φ
0	6	0°
1.12	3.402	– 90.11°
4.12	0.4769	– 180°
10	0.0523	133.58
∞	0	90° (– 270°)

4. $N = P - Z$
   $P = 0$ (No pole on RH plane)
   $N = 0$ (No encirclement of $-1 + j0$ point)
   $Z = 0$ (Number of roots of closed loop characteristic equation having positive real parts).

   So the given system is stable system.

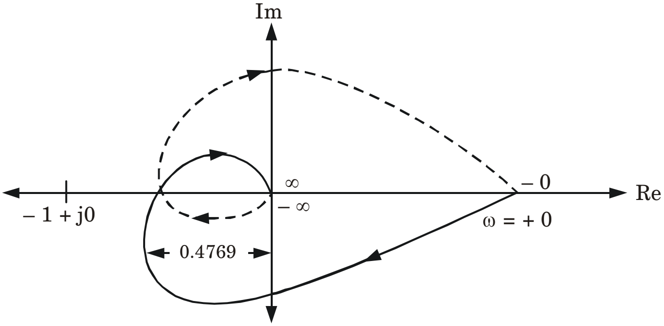

Fig. 15.

8. **Write short notes on PD controller and synchro.**

Ans.

A. PD controller :

1. A controller in the forward path, which changes the controller output corresponding to proportional plus derivative of error signal is called PD controller.

2. Output of controller = $K\, e(t) + T_d \dfrac{de(t)}{dt}$

   Taking Laplace transform

$$= K\,E(s) + sT_d\,E(s) = E(s)\,[K + sT_d]$$

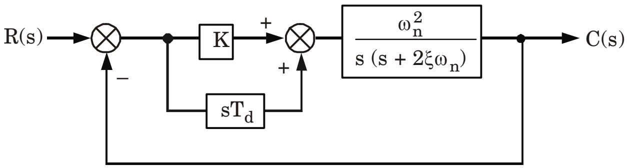

Fig. 16.

3. Assume, $K = 1$, we can write,

$$G(s) = \frac{(1 + sT_d)\,\omega_n^2}{s(s + 2\xi\,\omega_n)}$$

$$\frac{C(s)}{R(s)} = \frac{(1 + sT_d)\,\omega_n^2}{s^2 + s\,[2\xi\,\omega_n + \omega_n^2\,T_d] + \omega_n^2}$$

4. Comparing denominator with standard form,

$$2\xi'\,\omega_n = 2\xi\,\omega_n + \omega_n^2\,T_d$$

$$\therefore \qquad \xi' = \xi + \frac{\omega_n\,T_d}{2}$$

5. Because of this controller, damping ratio increased by factor $\frac{\omega_n\,T_d}{2}$.

$$K_p = \lim_{s \to 0} G(s)H(s) = \infty$$

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \frac{\omega_n}{2\xi}$$

without PD controller,

$$G(s)\,H(s) = \frac{\omega_n^2}{s(s + 2\varepsilon\omega_n)} \qquad [K = 1]$$

$$K_v = \lim_{s \to 0} sG(s)H(s) = \frac{\omega_n}{2\varepsilon}$$

As there is no change in coefficients, error also will remain same.

B. Synchros :

1. It is a rotary transducer that converts angular displacement into an AC voltage or an AC voltage into an angular displacement.
2. A synchros system consists of
 i. A control transmitter (CX) and
 ii. A control transformer (CT).
3. The control transmitter consists of a stator and a rotor. The rotor is a dumb-bell shaped magnetic structure. The supply is given to the rotor by means of slip rings, which are actually mounted on the stator housing.
4. The secondaries are in the skewed slot all along the periphery of the stator and are 120° apart because of their mechanical displacement.

5. The induced secondary voltage will depend upon the angle of the rotor shaft. For reference the zero degree position of the shaft is defined when the rotor is in alignment with the coil S_2.
6. In this position, the voltage in coil S_2 is maximum, and similarly the maximum voltage in coils S_1 and S_3 will result at 120° and 240° positions respectively. The voltage in S_2 is a function of θ and so is the voltage in S_1 and S_3. Thus,

$$E_{0s2} = A \cos \theta$$
$$E_{0s1} = A \cos (\theta - 120°)$$
$$E_{0s3} = A \cos (\theta - 240°)$$

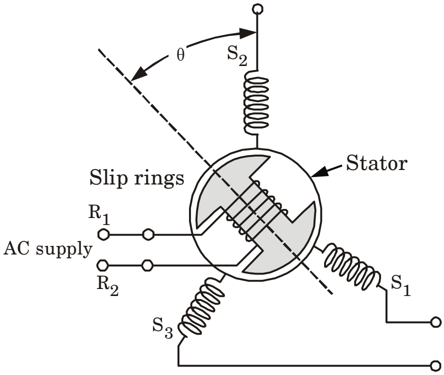

Fig. 17. Schematic diagram of synchro transmitter.

7. The connections of the synchro are made between the terminals and hence

$$\begin{aligned} E_{s1s2} &= E_{0s1} - E_{0s2} \\ &= A \cos (\theta - 120°) - A \cos \theta \\ &= A\left[-\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta - \cos\theta\right] \\ &= \sqrt{3}A\left[-\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right] \end{aligned}$$

8. Therefore, $E_{s2s1} = \sqrt{3}A\left[\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right]$

$$= \sqrt{3}A\cos(\theta + 30°) \quad ...(1)$$

Similarly, $E_{s3s2} = \sqrt{3}A\cos(\theta + 150°)$ ...(2)

$$E_{s1s3} = \sqrt{3}A\cos(\theta + 270°) \quad ...(3)$$

9. Obtain state equation of a given transfer function

a. $\dfrac{Y(s)}{U(s)} = \dfrac{1}{s^3 + 2s^2 + 3s + 1}$

b. $\dfrac{Y(s)}{U(s)} = \dfrac{1}{(s+1)(s+4)}$

Ans.

Given : $\dfrac{Y(s)}{U(s)} = \dfrac{1}{s^3 + 2s^2 + 3s + 1}$

To Find : State representation.

1. $Y(s)\ [s^3 + 2s^2 + 3s + 1] = U(s)$
   Taking inverse Laplace transform
   $$\frac{d^3y}{dt^3} + \frac{2d^2y}{dt^2} + \frac{3dy}{dt} + y(t) = u(t)$$
2. Choosing state variable
   $$y(t) = y_1(t)$$
   $$\frac{dy(t)}{dt} = \dot{y}_1(t) = y_2(t)$$
   $$\frac{d^2y(t)}{dt} = \dot{y}_2(t) = y_3(t)$$
   $$\frac{d^3y(t)}{dt} = \dot{y}_3(t) = y_4(t)$$
3. $y_4 + 2y_3 + 3y_2 + y_1(t) = u(t)$
   $$\dot{y}_3 = -2y_3 - 3y_2 - y_1(t) + u(t)$$
   $$\begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \\ \dot{y}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

Given : $\dfrac{Y(s)}{U(s)} = \dfrac{1}{(s+1)(s+4)}$

To Find : State representation.

1. $Y(s)\ [(s^2 + 5s + 4)] = U(s)$
   Taking inverse Laplace
   $$\frac{d^2y}{dt^2} + \frac{5dy}{dt} + 4y(t) = u(t)$$
2. $$\frac{d^2y}{dt^2} = \dot{y}_2(t) = y_3(t)$$
   $$\frac{dy}{dt}(t) = y_2(t) = \dot{y}_1(t)$$
   $$y(t) = y_1(t)$$
3. $\dot{y}_2(t) + 5y_2(t) + 4y_1(t) = u(t)$
   $$\dot{y}_2(t) = -5y_2(t) - 4y_1(t) + u(t)$$

$$\begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

SECTION–C

Note : Attempt any **two** questions of the following. **(15 × 2 = 30)**

10. For a unity feedback system, the open loop transfer function is

$$G(s)\,H(s) = \frac{2(s+0.25)}{s^2\,(s+1)\,(s+0.5)}$$

Draw bode plot and determine gain margin, phase margin.

Ans.

Given : $G(s)\,H(s) = \dfrac{2(s+0.25)}{s^2(s+1)\,(s+0.5)}$

To Draw : Bode plot.

1. $$G(s)\,H(s) = \frac{\dfrac{2\times 0.25}{0.5}\left(\dfrac{s}{0.25}+1\right)}{s^2(s+1)\left(\dfrac{s}{0.5}+1\right)} = \frac{(4s+1)}{s^2(s+1)\,(2s+1)}$$
2. $K = 1$, 20 Log K = 0 dB, No effect on Bode Plot.
3. $\dfrac{1}{s^2}$, 2 Poles at origin, straight line of slope – 40 dB/dec
4. $(1 + 4s)$, simples zero, $T_1 = 4$, $\omega_{C1} = \dfrac{1}{T_1} = 0.25$,

   straight line of slope + 20 dB/dec for $\omega \geq 0.25$
5. $\dfrac{1}{1+2s}$, simple zero, $T_2 = 2$, $\omega_{C2} = \dfrac{1}{T_2} = 0.5$,

   straight line of slope – 20 dB/sec for $\omega \geq 0.5$
6. $\dfrac{1}{1+s}$, simple zero, $T_3 = 1$, $\omega_{C3} = \dfrac{1}{T_3} = 1$,

   straight line of slope – 20 dB/sec for $\omega \geq 1$

Table 2.

Range of ω	$0 < \omega < 0.25$	$0.25 \leq \omega < 0.5$	$0.5 \leq \omega < 1$	$1 \leq \omega < \infty$
Resultant slope in dB/dec	– 40	– 40 + 20 = – 20	– 20 – 20 = – 40	– 40 – 20 = – 60

7. Phase angle table: $G(j\omega)H(j\omega) = \dfrac{(1+4j\omega)}{(j\omega)^2\,(1+2j\omega)\,(1+j\omega)}$

Bode plot :

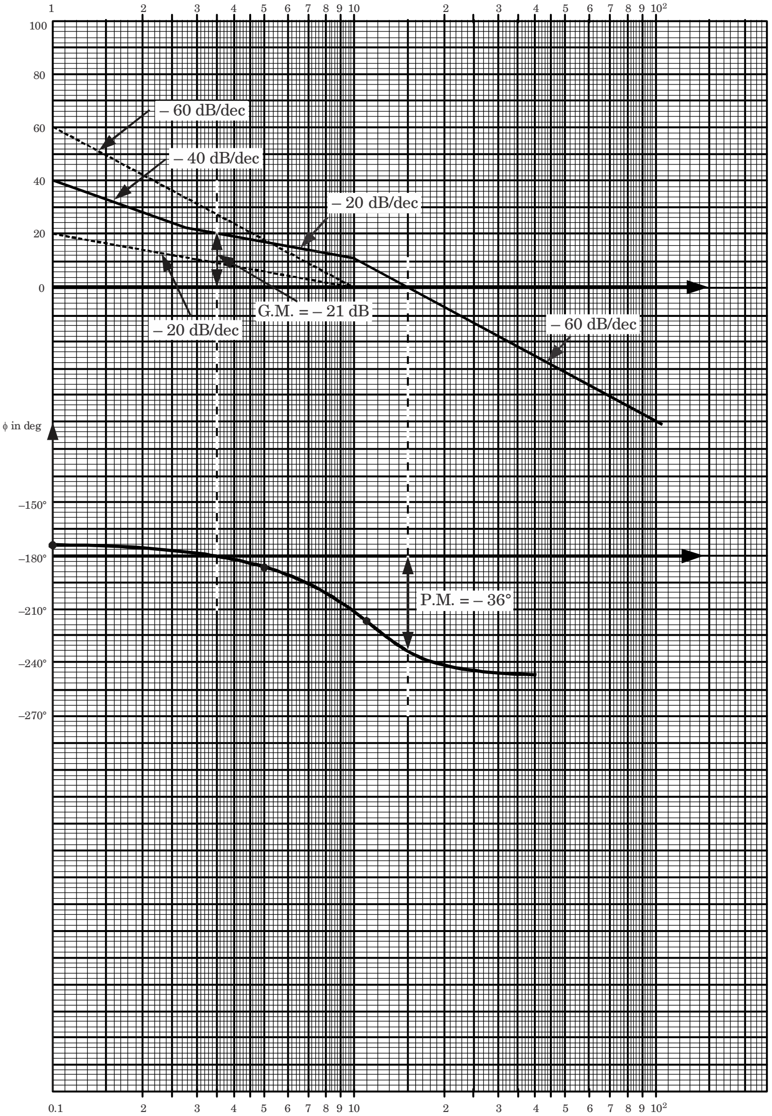

Table 3.

ω	$\frac{1}{(j\omega)^2}$	$+\tan^{-1}4\omega$	$+\tan^{-1}2\omega$	$-\tan^{-1}\omega$	ϕ
0.1	– 180°	+ 21.8°	– 11.3°	– 5.71°	– 175.2°
0.5	– 180°	+ 63.43°	– 45°	– 26.56°	– 188.13°
1	– 180°	+ 75.96°	– 63.43°	– 45°	– 212.47°
∞	– 180°	+ 90°	– 90°	– 90°	– 270°

Result : PM = – 36°, GM = – 21 dB.

11. A system characterised by the transfer function $\frac{Y(s)}{U(s)} = \frac{2}{s^3 + 6s^2 + 11s + 6}$. Find the state and output equation in matrix form and also test the controllability and observability of the given system.

Ans.

Given : $\frac{Y(s)}{U(s)} = \frac{2}{s^3 + 6s^2 + 11s + 6}$

To Find : State matrix; Controllability, Q_c; Observability, Q_o.

1. $Y(s)\,[(s^3 + 6s^2 + 11s + 6)] = 2U(s)$

2. $\dot{x}_1 = x_2,\ \dot{x}_2 = x_3,\ \dot{x}_3 = -6x_1 - 11x_2 - 6x_3 + u$
3. $y = 2x_1$

$$= \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4. **Controllability :**

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$$

$$Q_c = [B : AB : A^2B]$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -6 & -11 & -6 \\ 36 & 60 & 25 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & 1 \\ -6 & -11 & -6 \\ 36 & 60 & 25 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 25 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 25 \end{bmatrix} = 0\begin{vmatrix} 1 & -6 \\ -6 & 25 \end{vmatrix} - 0\begin{vmatrix} 0 & -6 \\ 1 & 25 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & -6 \end{vmatrix}$$

$$Q_c = 0 - 0 + (-1)$$

$$Q_c = -1$$

$$|Q_c| \neq 0$$

It is of rank 3 and controllable.

5. **Observability :**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, C^T = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$Q_0 = [C^T : A^TC^T : (A^T)^2\, C^T]$$

$$A^TC^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -22 \\ 0 & 1 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$(A^T)^2 = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & 11 \\ 0 & 1 & -6 \end{bmatrix}\begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & 11 \\ 0 & 1 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -6 & 36 \\ 0 & -11 & 60 \\ 1 & -6 & 25 \end{bmatrix}$$

$$(A^T)^2C^T = \begin{bmatrix} 0 & -6 & 36 \\ 0 & -11 & 60 \\ 1 & -6 & 25 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= 2\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & 2 \\ 0 & 0 \end{vmatrix} = 2[4 - 0] - 0 + 0$$

$$|Q_o| = 8$$

Therefore, its rank is 3 and the system is observable.

12. Write short notes of the following :
a. Lead compensator.
b. Lag compensator.
c. Gain margin and phase margin.

Ans.

A. Lead compensator :

1. Fig. 18 shows a phase-lead network where in the phase of output voltage leads the phase of input voltage for sinusoidal input.

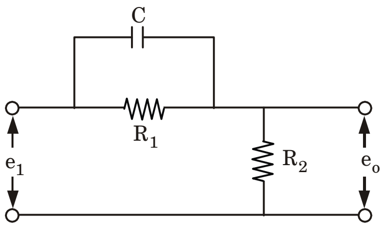

Fig. 18. Phase lead network.

2. The transfer function of a phase lead network,

$$\frac{E_o(s)}{E_i(s)} = \frac{\alpha(1+sT)}{(1+s\alpha T)} \quad ...(1)$$

$$\frac{E_o}{E_i} = \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

where, $\alpha < 1$

$$\alpha = \frac{R_2}{R_1 + R_2}$$

and $T = R_1 C$

3. The transfer function given by eq. (1) can be expressed in sinusoidal form as

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{\alpha(1+j\omega T)}{(1+j\omega\alpha T)} \quad ...(2)$$

4. The pole zero configuration of eq. (2) shown in Fig. 19.

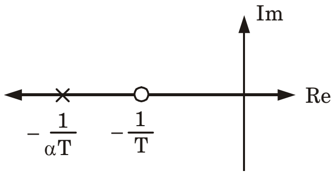

Fig. 19. Pole zero configuration.

Bode plot :

1. The two corner frequencies are

$$\omega = \frac{1}{T}, \text{ lower corner frequency}$$

$$\omega = \frac{1}{\alpha T}, \text{ upper corner frequency}$$

2. The maximum phase lead ϕ_m occurs at mid-frequency ω_m between upper and lower corner frequencies

$$\therefore \qquad \log_{10} \omega_m = \frac{1}{2}\left[\log_{10}\left(\frac{1}{T}\right) + \log_{10}\left(\frac{1}{\alpha T}\right)\right]$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}}$$

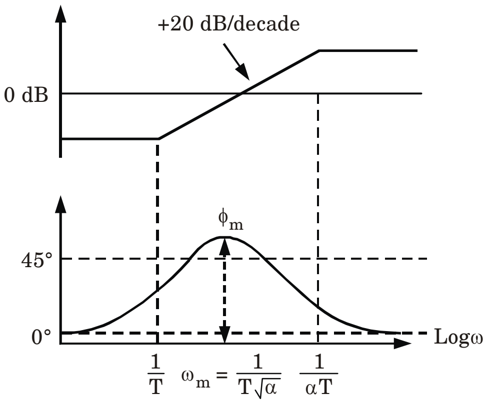

Fig. 20. Bode plot of lead compensator.

3. The phase angle $\angle E_o(j\omega)/E_i(j\omega)$ can be calculated as

$$\angle \frac{E_o(j\omega)}{E_i(j\omega)} = \tan^{-1}(\omega T) - \tan^{-1}(\omega\alpha T)$$

4. At $\qquad \omega = \omega_m = \frac{1}{\sqrt{\alpha} T},$

The phase angle is

$$\phi_m = \tan^{-1}\left[\frac{1}{T\sqrt{\alpha}} T\right] - \tan^{-1}\left[\frac{1}{\alpha\sqrt{T}} \alpha T\right]$$

$$= \tan^{-1}\left[\frac{1}{\sqrt{\alpha}}\right] - \tan^{-1}\left[\sqrt{\alpha}\right]$$

$\therefore$ $$\tan \phi_m = \frac{\frac{1}{\sqrt{\alpha}} - \sqrt{\alpha}}{1 + \frac{1}{\sqrt{\alpha}}\sqrt{\alpha}}$$

$$\tan \phi_m = \frac{1-\alpha}{2\sqrt{\alpha}}$$

and $$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

B. Lag compensator :

1.

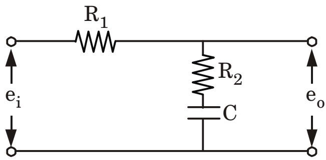

Fig. 21. Phase-lag network.

The transfer function of phase-lag network is shown in Fig. 21,

$$\frac{E_o(s)}{E_i(s)} = \frac{1+sT}{1+s\beta T} \quad ...(1)$$

where $\beta > 1$, $$\beta = \frac{R_1 + R_2}{R_2}$$

and $$T = R_2 C$$

2. The transfer function given by eq. (1) can be expressed in sinusoidal form as

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{1+j\omega T}{1+j\omega\beta T} \quad ...(2)$$

3. Bode plot for transfer function of eq. (2) is shown in Fig. 22.

4. The two corner frequencies are $\omega = \frac{1}{T}$, upper corner frequency for zero at $s = -\frac{1}{T}$, $\omega = \frac{1}{\beta T}$, lower corner frequency for a pole at $s = -\frac{1}{\beta T}$

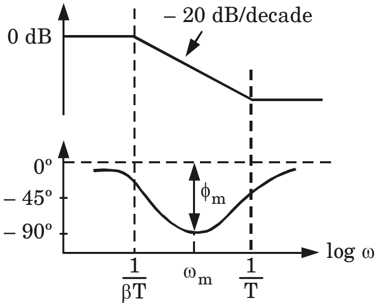

Fig. 22. Bode plot of lag compensator.

5. The maximum phase-lag, ϕ_m occurs at mid frequency ω_m between upper and lower corner frequencies.

$$\therefore \quad \log_{10} \omega_m = \frac{1}{2}\left[\log\left(\frac{1}{\beta T}\right) + \log_{10}\left(\frac{1}{T}\right)\right]$$

$$\therefore \quad \omega_m = \frac{1}{\sqrt{\beta T}}$$

6. The phase angle $\angle E_o(j\omega)/E_i(j\omega)$ calculated as

$$\angle \frac{E_o(j\omega)}{E_i(j\omega)} = \tan^{-1}(\omega T) - \tan^{-1}(\omega \beta T)$$

At $\omega = \omega_m = \dfrac{1}{\sqrt{\beta T}}$, the phase angle is ϕ_m :

$$\tan \phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\sin \phi_m = \frac{1-\beta}{1+\beta}$$

Pole-zero configuration is shown in Fig. 23.

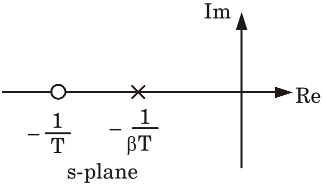

Fig. 23. Pole zero configuration.

C. Phase margin and gain margin :

a. Gain Margin (*GM*) :

1. It is the margin in gain which is allowed till the system reaches on the verge of instability.
2. Mathematically, gain margin is reciprocal of magnitude of $G(j\omega)H(j\omega)$ at phase crossover frequency.

$$GM = 1 / |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

b. Phase Margin (*PM*) : It is the amount of additional phase lag which can be introduced in the system till it reaches on the verge of instability.

Mathematically,

$$PM = 180° + \angle G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

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B.Tech.

(SEM. V) ODD SEMESTER THEORY EXAMINATION, 2016-17

CONTROL SYSTEM

Time : 3 Hours **Max Marks : 100**

SECTION-A

1. Attempt **all** parts. **(2 × 10 = 20)**

a. **Discuss open loop and closed loop system giving suitable example.**

b. **Discuss the effect of feedback on the time constant of a control system.**

c. **Explain the working of AC servomotor with neat diagram.**

d. **Give the comparison between PI and PID controller.**

e. **Discuss the significance of various time domain specifications.**

f. **Establish the relation between Routh and Hurwitz stability criterion.**

g. **Explain in brief :**

i. **Gain margin**

ii. **Phase margin.**

h. **What do you understand by relative stability ? Explain.**

i. **Differentiate between lag and lead network in view of their Bode plot.**

j. **Discuss the advantages of state variable technique over transfer function approach.**

SECTION-B

Attempt any **three** questions from this section. **(10 × 3 = 30)**

1. **Using block diagram reduction techniques, find the closed loop transfer function of the system whose block diagram is given in Fig. 1.**

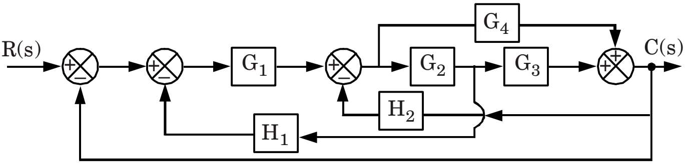


Fig. 1.

2. **Derive the expression for second order system response when subjected to unit impulse input for damping ratio (ξ) < 1.**
   An unity feedback system is characterized by an open loop transfer function.

$$G(s) = \frac{K}{s(s+10)}$$

   Determine the gain 'K' so that the system will have a damping ratio of 0.5. For this value of 'K', determine the settling time, peak overshoot and time to peak overshoot for a unit step input.

3. **For a closed loop system whose transfer function is**
   $G(s)\ H(s) = \dfrac{Ke^{-sT}}{s(s+1)}$, determine the maximum value of the gain 'K' for stability.

4. **What is closed loop frequency response ? Give an account of the correlation between time response and frequency response for a second order system with relevant expressions.**

5. i. **Derive the transfer function from state model.**
   ii. **Obtain the complete solution of non-homogeneous state equation using time domain method.**
   iii. **Discuss the significance of lag network. Also draw its *s*-plane representation and Bode plot.**

SECTION-C

Attempt **all** questions.

1. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. **Find the transfer function of the signal flow graph shown in Fig. 2, using Mason's gain formula.**

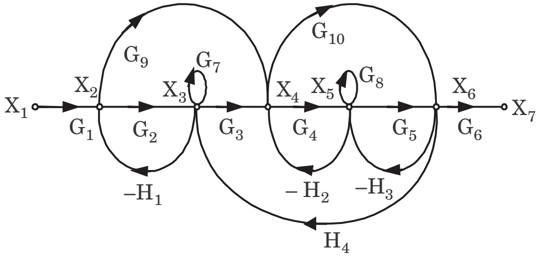


Fig. 2.

b. What do you understand by the term sensitivity ? Consider the feedback control system shown in Fig. 3. The normal value of the process parameter '*K*' is 1. Determine the sensitivity of transfer function $T(s) = \dfrac{C(s)}{R(s)}$ to variations in parameter '*K*', at ω = 5.

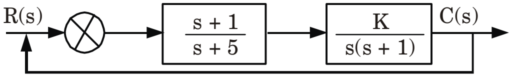


Fig. 3.

2. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. Discuss different type of test signal used for analysis of control system in time domain.
The reference input to a unity feedback system is shown in Fig. 4. The open loop transfer function of the system is

$$G(s) = \frac{400(s+1)}{(s+2)(s+8)}$$

Calculate the steady state error.

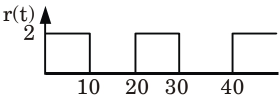


Fig. 4.

b. Discuss the effect on the performance of a second order control system of
i. Derivative control ii. Integral control.

3. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. Explain the working principle of stepper motor with neat diagram.

The characteristics equation for feedback control is, $s^3 + 5s^2 + 12s + K = 0$

Find the range of K for all the roots to lie to the left of $s = 1$

b. Sketch the root locus for the closed loop control system with $G(s) = \dfrac{K}{s(s+1)(s^2+4s+5)}$

4. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. The steady state output of the system for a sinusoidal input of unit magnitude and variable frequency ω is given as

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + 4z^2u^2}} \sin\left(\omega t - \tan^{-1}\frac{27u}{\sqrt{1-u^2}}\right)$$

Determine :

i. Resonant frequency **ii. Resonant peak**

iii. Bandwidth **iv. Phase angle.**

b. Draw Bode plot (log magnitude plot) for the transfer function.

$$G(s) = \frac{20s}{s^2 + 20s + (100)^2}$$

OR

Using Nyquist stability criterion, investigate the stability of a unity feedback system with open loop transfer function.

$$G(s) = \frac{(s - z_1)}{s(s+p_1)}, \; z_1, p_1 > 0$$

Also discuss the significance of M-circle.

5. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. State and explain controllability and observability in view of Kalman and Gilbert test.

The state equation for a system is, $\dot{x} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$

b. Design a phase lead compensator for a negative unity feedback system with plant transfer function.

$G_p(s) = \dfrac{K}{s(s+10)(s+1000)}$ **to satisfy the conditions :**

phase margin in atleast 45°, static error constant = 1000 s^{-1}

☺☺☺

SOLUTION OF PAPER (2016-17)

SECTION-A

1. Attempt **all** parts. **(2 × 10 = 20)**

a. Discuss open loop and closed loop system giving suitable example.

Ans. There are two types of control system :

1. Open loop control system :

Fig. 1.

In open loop control systems the control action is independent of the desired output. In this system the output is not compared with the reference input.

Example : Washing machine, Immersion rod, Time operated traffic control, DC shunt motor.

2. Closed loop control system :

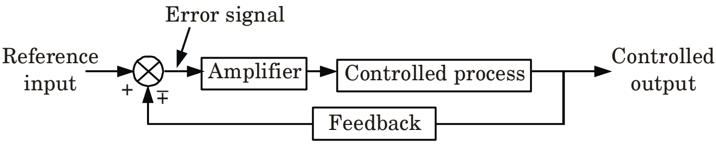


Fig. 2.

In a closed loop control system the output has an effect on control action through a feedback as shown in Fig. 2.

Example : Automatic steering control system, Driving system of an automobile, Home heating system, Ship stabilization system.

b. Discuss the effect of feedback on the time constant of a control system.

Ans.

1. Consider an open loop system with overall transfer function as,

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{1 + sT}$$

2. System is subjected to unit step input,

$$R(s) = 1/s,$$

$$\therefore \quad c(t) = L^{-1}[C(s)] = K[1 - e^{-t/T}] \quad \text{...(1)}$$

Here time constant = T

3. Response of system for unit step when feedback having gain h is added. Then new response,

$$c'(t) = \frac{K}{1 + Kh}\left[1 - e^{\frac{1}{(T/1+Kh)}}\right]$$

New time constant due to the feedback = $(T/1 + Kh)$.

4. For positive value of h and $K > 1$, the time constant $(T/1 + Kh)$ is less than T. Hence feedback reduces the time constant of a system.

c. Explain the working of AC servomotor with neat diagram.

Ans. Working principle :

1. Reference phase is supplied from a constant voltage source $V_r \angle 0°$. The other winding *i.e.*, control phase is supplied with a variable voltage of the same frequency as the reference phase but its phase is displaced by 90° (electrically).
2. The control phase is usually supplied from a servo amplifier.
3. The speed and torque of the rotor are controlled by the phase difference between the control voltage and the reference phase voltages.
4. The direction of rotation of the rotor can be reversed by reversing the phase difference from leading to lagging between the control phase voltage and the reference phase voltage.

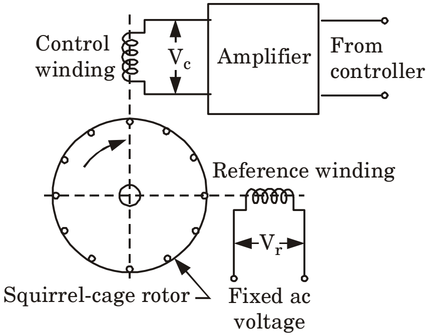


Fig. 3. Schematic diagram of 2ϕ servomotor.

d. Give the comparison between PI and PID controller.

Ans.

S. No.	PI	PID
1.	It is a combination of proportional and integral control action.	It is a combination of proportional, integral and derivative controller.
2.	It helps in reducing steady state error.	It helps in reducing steady state error and also getting steady state conditions quickly.

e. Discuss the significance of various time domain specifications.

Ans.

1. **Delay time (T_d) :** It is the time required for the response to reach half the final value the very first time.
2. **Rise time (T_r) :** The rise time is the time required for the response to rise from

a. 10 % to 90 % of its final value, (overdamped systems).

b. 5 % to 95 % of its final value, (critical damped systems).

c. 0 % to 100 % of its final value (underdamped systems).

3. **Peak time (T_p) :** The peak time is the time required for the response to reach the first (maximum) peak of the overshoot.
4. **Maximum overshoot (M_p) :** It is the maximum peak value of the response curve measured from unity.
5. **The settling time (T_s) :** It is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2 % or 5 %).

f. Establish the relation between Routh and Hurwitz stability criterion.

Ans.

1. In the case of the Routh criterion, $a_0 > 0$, $a_1 > 0$, $b_1 > 0$, $c_1 > 0$, $d > 0$, ...

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_3} = \frac{\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}}{a_3}$$

2. Again, in the case of the Hurwitz criterion

$$a_3 > 0, \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0$$

3. From the Routh criterion, we also observe that $b_1 > 0$ and $a_3 > 0$. Therefore,

$$b_1 = \frac{\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}}{a_3} > 0$$

since $$a_3 > 0, \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0$$

This is also true for c_1, d_1 for the Routh criterion.

4. Hence, the Hurwitz criterion and the Routh criterion are basically the same and draw the same conclusion.

g. Explain in brief :

i. Gain margin

ii. Phase margin.

Ans.

i. Gain margin : It is the reciprocal of magnitude $|G(j\omega)|$ at the frequency at which the phase angle is – 180°.

Gain Margin (GM),

$$K_g = \frac{1}{|G(j\omega_c)|}$$

where, ω_c = Phase cross-over frequency.

ii. Phase margin : The phase margin is that amount of the additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.

Phase margin is equal to 180° plus the angle of $G(j\omega)$ at the gain crossover point.

$$\phi_m = 180° + \phi$$

h. What do you understand by relative stability ? Explain.

Ans. The system is said to be relatively more stable or unstable on the basis of settling time. System is said to be relatively more stable if settling time for that system is less than of the other system.

i. Differentiate between lag and lead network in view of their Bode plot.

Ans.

S. No.	Lead	Lag
1.	The lead network is basically a high-pass filter.	The lag network is essentially a low-pass filter.
2.	The primary function of the lead compensator is to reshape the frequency-response curve to provide sufficient phase-lead angle.	The primary function of a lag compensator is to provide attenuation in the high-frequency range to give a system sufficient phase margin.

j. Discuss the advantages of state variable technique over transfer function approach.

Ans.

1. This method can be applied to linear or non linear, time variant or time invariant system.
2. This method can be designed for optimal conditions.

SECTION-B

Attempt any **three** questions from this section. **(10 × 3 = 30)**

1. Using block diagram reduction techniques, find the closed loop transfer function of the system whose block diagram is given in Fig. 4.

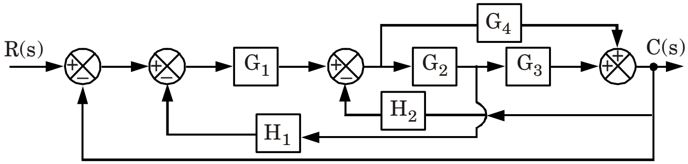
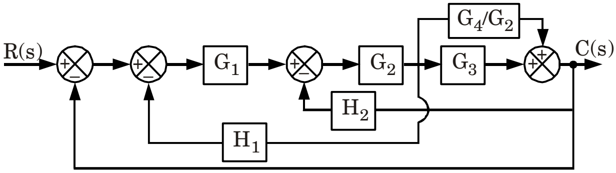


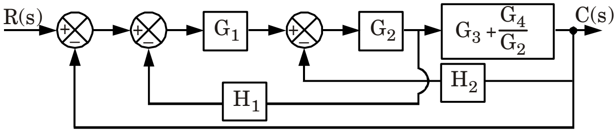
Fig. 4.

Ans.

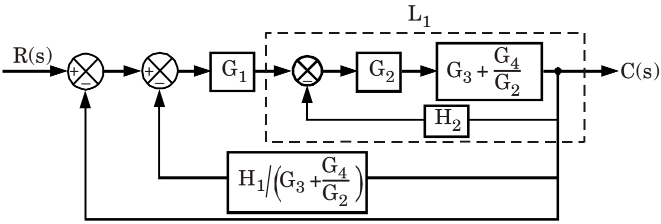
Step 1 :



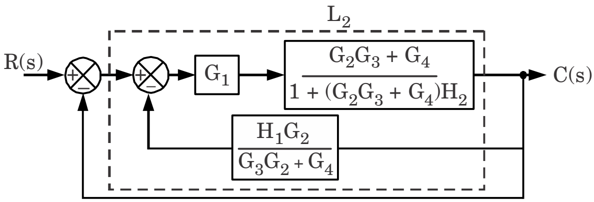
Step 2 :



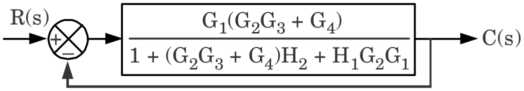
Step 3 :



Step 4 : Solving feedback loop L_1, reduced block diagram is



Step 5 : Solving feedback loop L_2, the reduced block diagram is



Step 6 : Solving feedback loop,

$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_2 + H_1G_2G_1}}{1 + \dfrac{G_1(G_2G_3 + G_4)}{1 + (G_2G_3 + G_4)H_2 + H_1G_2G_1} \times 1}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + (G_2G_3 + G_4)H_2 + H_1G_2G_1 + (G_2G_3 + G_4)G_1}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2H_2 + G_4H_2 + H_1G_2G_1 + G_1G_2G_3 + G_1G_4}$$

2. Derive the expression for second order system response when subjected to unit impulse input for damping ratio (ξ) < 1.
An unity feedback system is characterized by an open loop transfer function.

$$G(s) = \frac{K}{s(s+10)}$$

Determine the gain '*K*' so that the system will have a damping ratio of 0.5. For this value of '*K*', determine the settling time, peak overshoot and time to peak overshoot for a unit step input.

Ans. Derivation for second order system response :

1. For unit impulse function, $R(s) = 1$ and the output of a second order system is given by

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad ...(1)$$

2. In eq. (1) rewriting the term

$(s^2 + 2\xi\omega_n s + \omega_n^2)$ as $[(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)]$

$$C(s) = \frac{\omega_n^2}{[(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)]}$$

$$C(s) = \frac{\omega_n^2}{\omega_n\sqrt{1-\xi^2}} \; \frac{\omega_n\sqrt{1-\xi^2}}{[(s + \xi\omega_n)^2 + (\omega_n\sqrt{1-\xi^2})^2]} \quad ...(2)$$

3. Taking inverse Laplace transform on both sides of eq. (2),

$$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} \; e^{-\xi\omega_n t} \sin[(\omega_n\sqrt{1-\xi^2})t] \quad ...(3)$$

4. For $\xi < 1$,

$$c(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} \; e^{-\xi\omega_n t} \sin[(\omega_n\sqrt{1-\xi^2})t] \quad ...(4)$$

5. The time response for $\xi < 1$ is decaying exponential oscillations and the output at times goes negative also.

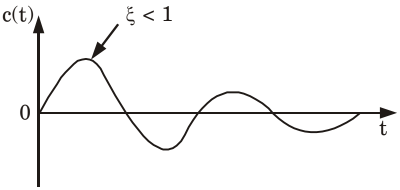


Fig. 5.

Numerical :

Given : $G(s) = \dfrac{K}{s(s+10)}$, $\xi = 0.5$, $H(s) = 1$

To Find : K, t_s, t_p and M_p.

1. $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$
$$= \frac{K}{s^2+10s+K} \qquad \text{...(5)}$$

2. For second order system,
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \qquad \text{...(6)}$$

3. Comparing eq. (5) and (6), we get
$$\omega_n = \sqrt{K} \text{ rad/sec}$$
$$\because \quad 2\xi\omega_n = 10$$
$$2 \times 0.5 \times \sqrt{K} = 10$$
$$\sqrt{K} = 10$$
$$\therefore \quad K = 100$$

4. Peak overshoot, $$M_p = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} = e^{-\left(\frac{0.5\pi}{\sqrt{1-0.5^2}}\right)} = 0.1630$$

5. Settling time, $$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5\times 10} = 0.8 \text{ sec}$$

6. Peak time, $$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{10\sqrt{1-(0.5)^2}} = 0.36 \text{ sec.}$$

3. For a closed loop system whose transfer function is $G(s)\,H(s) = \dfrac{Ke^{-sT}}{s(s+1)}$, determine the maximum value of the gain 'K' for stability.

Ans.

Given : $G(s)\,H(s) = \dfrac{Ke^{-sT}}{s(s+1)}$

To Find : Maximum value of K.

1. Putting $s = j\omega$

$$\therefore \quad G(j\omega)\,H(j\omega) = \frac{Ke^{-j\omega T}}{j\omega(j\omega+1)}$$

2. $$|G(j\omega)\,H(j\omega)| = \frac{K}{\omega\sqrt{1+\omega^2}}$$

$$\angle G(j\omega)\,H(j\omega) = -\tan^{-1}\omega - 90° - \omega T \times \frac{180°}{\pi}$$

$$= -57.3\omega T - 90° - \tan^{-1}\omega$$

3. Intersection of Nyquist plot with negative axis of $G(s)\,H(s)$ plane is determined by using following relation,

$$\angle G(j\omega)\,H(j\omega) = -180°\,(2k+1)$$

where, $k = 0, 1, 2 \ldots.$

4. It the first instant, the Nyquist plot intersects the negative real axis of $G(s)\,H(s)$ plane for $k = 0$, and the frequency at intersection point is ω_2.

5. Therefore, $\angle G(j\omega_2)\,H(j\omega_2) = -180°$

$$-57.3\omega_2 T - 90° - \tan^{-1}\omega_2 = -180°$$

$$-57.3\omega_2 T - \tan^{-1}\omega_2 = -90°$$

6. $$T = 0.5$$

$$-28.65\omega_2 - \tan^{-1}\omega_2 = 90°$$

7. Using trial and error, $\omega_2 = 1.3075$ rad/sec

$$G(j\omega_2)\,H(j\omega_2) = \frac{K}{\omega_2\sqrt{1+\omega_2^2}} = \frac{K}{1.3075\sqrt{1+(1.3075)^2}} = 0.4646\,K$$

8. For stability the point $(-1+j0)$ be placed outside the Nyquist plot

$$0.4646\,K < 1$$

$$\therefore \quad K < 2.152$$

4. What is closed loop frequency response ? Give an account of the correlation between time response and frequency response for a second order system with relevant expressions.

Ans. **Closed loop frequency response :**

1. Consider the transfer function for closed loop system,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

2. For unity feedback, $H(s) = 1$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} \quad ...(1)$$

Put $s = j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)} \quad ...(2)$$

3. The polar plot of eq. (2) is shown in Fig. 6.

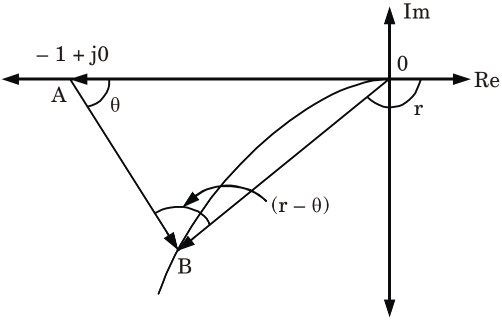


Fig. 6.

4. From Fig. 6, $\vec{OB} = G(j\omega)$

$$\vec{OA} = -1$$

$$\vec{AB} = \vec{OB} - \vec{OA} = G(j\omega) - (-1)$$

$$\vec{AB} = 1 + G(j\omega)$$

5. From eq. (2)

$$\left|\frac{C(j\omega)}{R(j\omega)}\right| = M(\omega) = \frac{\vec{OB}}{\vec{AB}}$$

$$\frac{\angle C(j\omega)}{\angle R(j\omega)} = \frac{\angle \vec{OB}}{\angle \vec{AB}} = \frac{\angle r}{\angle \theta} = \angle (r - \theta)$$

$$\therefore \quad \frac{C(j\omega)}{R(j\omega)} = M(\omega)\, e^{j\,\phi(\omega)}$$

where $M(j\omega)$ is the magnitude and $\phi(\omega) = r - \theta$.

6. Frequency response consists of two parts :
   i. Magnitude,
   ii. Phase angle.
   iii. Both can be plotted against different values of ω.

7. Hence frequency response of closed loop system is plot of magnitude and phase angle.

Correlation :

1. For 2nd order system, the transfer function is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where, ξ = Damping factor

ω_n = Natural frequency of oscillations

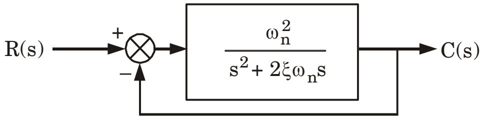


Fig. 7.

2. Closed loop frequency response is,

$$\frac{C(j\omega)}{R(j\omega)} = T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n(j\omega) + \omega_n^2} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2j\xi\left(\frac{\omega}{\omega_n}\right)}$$

$$= \frac{1}{(1-u^2) + j2\xi u} \quad ...(3)$$

where $u = \omega/\omega_n$, normalized driving frequency.

$$\therefore \quad |T(j\omega)| = M = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \quad ...(4)$$

and $$\angle T(j\omega) = \phi = -\tan^{-1}\frac{2\xi u}{1-u^2} \quad ...(5)$$

3. The steady state output is

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \sin\left(\omega t - \tan^{-1}\frac{2\xi u}{1-u^2}\right)$$

$\therefore$ From eq. (4) and (5) when

$u = 0$, $M = 1$ and $\phi = 0$

$u = 1$, $M = \frac{1}{2\xi}$ and $\phi = -\frac{\pi}{2}$

$u = \infty$, $M = 0$ and $\phi = -\pi$

4. The frequency where M has a peak value is called the resonant frequency. At this frequency the slope of magnitude curve is zero.

If ω_r = Resonant frequency.

$u_r = \omega_r/\omega_n$ is normalized resonant frequency.

$$\left.\frac{dM}{du}\right|_{u=u_r} = -\frac{1}{2}\frac{[-4(1-u_r^2)u_r + 8\xi^2 u_r]}{[(1-u_r^2)^2 + (2\xi u_r)^2]^{3/2}} = 0$$

$$-4(1-u_r^2)\,u_r + 8\xi^2 u_r = 0$$

$$-4u_r(1-u_r^2-2\xi^2) = 0$$

$$\therefore \quad u_r = \sqrt{1-2\xi^2}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2} \quad \text{...(6)}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad \text{...(7)}$$

5. The phase angle ϕ of $T(j\omega)$ at resonant frequency is

$$\phi_r = -\tan^{-1}\left[\frac{\sqrt{1-2\xi^2}}{\xi}\right]$$

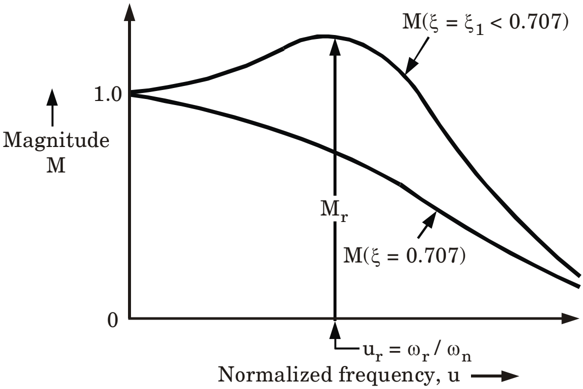


Fig. 8. Frequency response magnitude characteristics.

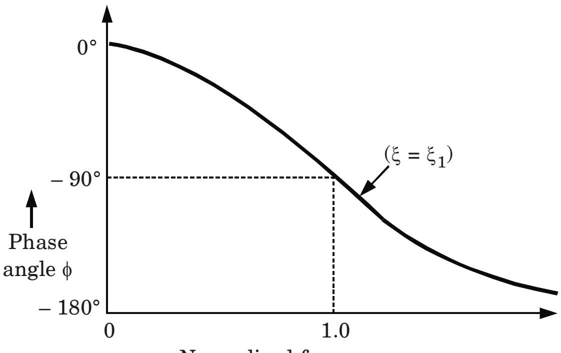


Fig. 9. Frequency response phase characteristic.

Bandwidth :

1. The frequency at which M has value of $\frac{1}{\sqrt{2}}$ and is called cut-off frequency ω_c. The signal frequencies above cut-off are attenuated.
2. The range of frequencies for which $M \geq \frac{1}{\sqrt{2}}$ is known as bandwidth ω_b. The low pass filters has bandwidth equal to cut-off. ω_b indicates the noise filtering characteristics of the system.

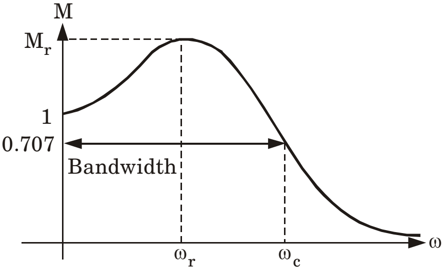


Fig. 10.

3. Normalized bandwidth, $u_b = \omega_b/\omega_n$

$$\therefore \quad M = \frac{1}{\sqrt{(1-u_b^2)^2 + (2\xi u_b)^2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad u_b = [1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}\,]^{\frac{1}{2}}$$

$$\omega_b = \omega\,[1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}\,]^{\frac{1}{2}}$$

4. The damped frequency of oscillations ω_d and peak overshoot M_p of the step response for $0 \leq \xi \leq 1$ are

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$M_p = \exp\left[\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right]$$

5. For $\xi > \frac{1}{\sqrt{2}}$, M_r does not exists so the correlation breaks down

$$\frac{\omega_r}{\omega_d} = \frac{\sqrt{1-2\xi^2}}{\sqrt{1-\xi^2}}$$

5. i. Derive the transfer function from state model.

ii. Obtain the complete solution of non-homogeneous state equation using time domain method.

iii. Discuss the significance of lag network. Also draw its *s*-plane representation and Bode plot.

Ans.

i. Derivation of the transfer function from state model :

1. Let us consider a vector matrix differential equation

$$\dot{x} = Ax + Bu$$

and output, $y = Cx$

2. Now, taking Laplace transform with zero initial conditions

$$sX(s) = AX(\mathrm{s}) + BU(s)$$

$$X(s) = [sI - A]^{-1} BU(s)$$

and $Y(s) = CX(s)$

$$Y(s) = C\ [sI - A]^{-1} BU(s)$$

3. For a single-input-single-output system, Y and U are scalars.

4. Now transfer matrix can be given as

$$\text{Transfer matrix} = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B$$

5. Transfer function $= C[sI - A]^{-1} B = \dfrac{C\ \text{adj}([sI - A])B}{\det\,[sI - A]}$

6. Denominator part *i.e.* $|sI - A|$ is called the characteristic equation.

$$|sI - A| = 0$$

7. n^{th} degree characteristic equation $|sI - A| = 0$ has n roots or eigen values.

ii. Solution of non-homogeneous equation :

1. Consider state equation

$$\dot{x}\ (t) = Ax(t) + Bu(t)$$

$$U(s) = L[u(t)]$$

$$X(s) = L[x(t)]$$

2. Taking Laplace transform,

$$sX(s) - x(0) = AX(s) + BU(s)$$

$$[sI - A]\ X(s) = x(0) + BU(s)$$

3. Premultiplying by $[sI - A]^{-1}$

$$X(s) = [sI - A]^{-1} x(0) + [sI - A]^{-1} BU(s)$$

4. Taking inverse Laplace transform

$$x(t) = L^{-1}\{X(s)\}$$

$$= L^{-1}[sI - A]^{-1} x(0) + L^{-1}[sI - A]^{-1} BU(s)$$

$$x(t) = \phi(t)x(0) + \int_0^t \phi(t - \tau) Bu(\tau)\, d\tau$$

$$= e^{At} x(0) + \int_0^t e^{A(t-\tau)} Bu(\tau)\, d\tau$$

5. If the initial time is other than zero, say t_0 then

$$x(t) = e^{A(t-t_0)}x(t_0)+\int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau$$

$$= \phi(t-t_0)\,x(t_0)+\int_{t_0}^{t}\phi(t-\tau)\,Bu(\tau)\,d\tau$$

6. $$x(t) = x_c(t) + x_p(t)$$

where $x_c(t)= \phi(t)\,x(0)$

is the complementary solution of the state vector

and $$x_p(t) = \int_{t_0}^{t}\phi(t-\tau)\,Bu(\tau)\,d\tau$$

is a particular solution of vector.

iii. Significance of lag network :

Basically, lag compensator is a low pass filter and its main function is to provide attenuation in the high frequency range to give sufficient phase margin.

Effects (significance) of lag compensation :

Lag compensator allows high gain at low frequencies thus it is basically a low pass filter. Hence it improves the steady state performance.

***s*-plane representation and Bode plot of lag network :**

1.

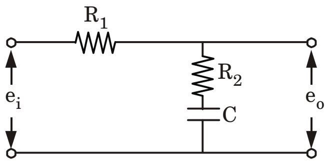


Fig. 11. Phase-lag network.

The transfer function of phase-lag network is shown in Fig. 11,

$$\frac{E_o(s)}{E_i(s)} = \frac{1+sT}{1+s\beta T} \quad ...(1)$$

where $\beta > 1$, $\beta = \dfrac{R_1 + R_2}{R_2}$

and $T = R_2C$

2. The transfer function given by eq. (1) can be expressed in sinusoidal form as

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{1+j\omega T}{1+j\omega\beta T} \quad ...(2)$$

3. Bode plot for transfer function of eq. (2) is shown in Fig. 12.

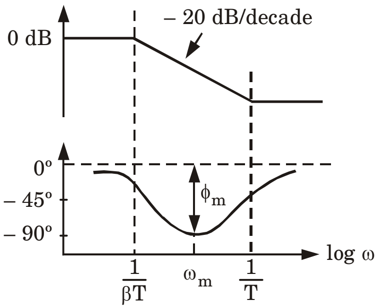


Fig. 12. Bode plot of lag compensator.

SECTION-C

Attempt **all** questions.

1. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. Find the transfer function of the signal flow graph shown in Fig. 13, using Mason's gain formula.

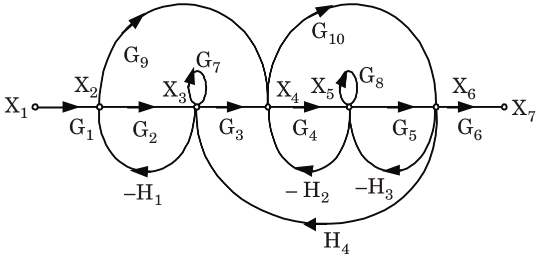


Fig. 13.

Ans.

1. Forward path and gains are :

$$P_1 = G_1 G_2 G_3 G_4 G_5 G_6, \quad \Delta_1 = 1$$
$$P_2 = G_1 G_9 G_4 G_5 G_6, \quad \Delta_2 = 1 - G_7$$
$$P_3 = G_1 G_9 G_{10} G_6, \quad \Delta_3 = 1 - G_7 - G_8 + G_7 G_8$$
$$P_4 = G_1 G_2 G_3 G_{10} G_6, \quad \Delta_4 = 1 - G_8$$

2. Loops and gains are :

$$L_1 = -G_2 H_1$$
$$L_2 = -G_4 H_2$$
$$L_3 = -G_5 H_3$$
$$L_4 = -G_3 G_4 G_5 H_4$$
$$L_5 = G_7$$
$$L_6 = G_8$$
$$L_7 = H_1 G_9 G_{10} H_4$$
$$L_8 = H_1 G_9 G_4 G_5 H_4$$

$$L_9 = -H_4\,G_3\,G_{10}$$
$$L_{10} = H_2\,G_{10}\,\mathrm{H}_3$$
$$\Delta = 1 - (L_1 + L_2 + L_3 + \mathrm{L}_4 \text{ --------- } + L_{10}) + (L_1\,L_2 + L_1\,L_3 + L_1\,L_6 + L_1\,L_{10} + L_7\,L_6 + L_5\,L_6 + L_5\,L_2 + L_5\,L_3 + L_5\,L_{10} + L_9\,L_5 + L_9\,L_3)$$

3. Using Mason's gain formula, $T(s) = \dfrac{X_7}{X_1} = \dfrac{\sum_{k=1}^{4} P_k \Delta_k}{\Delta}$

$$= \frac{G_1G_2G_3G_4G_5G_6 + G_1G_9G_4G_5G_6(1-G_7) + G_1G_9G_{10}G_6(1-G_7-G_8+G_7G_8)}{\begin{array}{l}1 + (G_2H_1 + G_4H_2 + G_5H_3 + G_3G_4G_5H_4 - G_7 - G_8 - H_1G_9G_{10}H_4 \\ -H_1G_9G_5H_4G_4 + H_4G_3G_{10} - H_2G_{10}H_3) + (G_2H_1G_4H_2 + G_2H_1G_5H_3 \\ -G_2H_1G_8 - G_2H_1H_2G_{10}H_3 + H_1G_9G_4G_5H_4G_8 + G_7G_8 - G_7G_4H_2 \\ -G_7G_5H_3 + G_7H_2G_{10}H_3 - H_4G_3G_{10}G_7 + H_4G_3G_{10}G_5H_3)\end{array}}$$

b. What do you understand by the term sensitivity ? Consider the feedback control system shown in Fig. 14. The normal value of the process parameter '*K*' is 1. Determine the sensitivity of transfer function $T(s) = \dfrac{C(s)}{R(s)}$ to variations in parameter '*K*', at ω = 5.

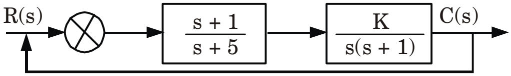


Fig. 14.

Ans. **Sensitivity :** Sensitivity is the change in variable due to variation in parameters of control system.

e.g.

$$S_G^T = \frac{\%\text{ Change in } T}{\%\text{ Change in } G} = \frac{\dfrac{\partial T}{T} \times 100}{\dfrac{\partial G}{G} \times 100}$$

Numerical :

Given : $G(s) = \dfrac{K}{s(s+5)}$, $H(s) = 1$

To Find : Sensitivity, S_G^T.

1. Forward gain, $G(s) = \dfrac{K}{s(s+5)}$

2. $$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{K}{(s^2 + 5s + K)}$$

3. Sensitivity, $$S_K^T = \frac{\frac{\partial T}{T} \times 100}{\frac{\partial K}{K} \times 100} = \frac{K}{T}\frac{\partial T}{\partial K}$$

$$= \frac{K(s^2+5s+K)}{K}\left[\frac{1}{(s^2+5s+K)} - \frac{K}{(s^2+5s+K)^2}\right]$$

$$= (s^2+5s+K)\left[\frac{s^2+5s+K-K}{(s^2+5s+K)^2}\right] = \frac{s(s+5)}{(s^2+5s+K)^2}$$

4. Putting $s = j\omega, K = 1$

$$= \frac{j\omega(j\omega+5)}{\left[(j\omega)^2\ 5\ j\omega+1\right]^2}$$

5. Again, putting $\omega = 5$

$$= \frac{j5\,(j5+5)}{[-25+j25+1]^2} = \frac{j25\,(j+1)}{[j25-24]^2}$$

$$= \frac{(-25+j25)}{(j25-24)^2}$$

$$|\,S_K^T\,| = \frac{\sqrt{25^2+25^2}}{\sqrt{25^2+24^2}} = 1.02$$

2. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. Discuss different type of test signal used for analysis of control system in time domain.

The reference input to a unity feedback system is shown in Fig. 15. The open loop transfer function of the system is

$$G(s) = \frac{400(s+1)}{(s+2)(s+8)}$$

Calculate the steady state error.

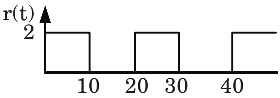


Fig. 15.

Ans. Types of test signal :

1. **Unit step :** Signals which start at time $t = 0$ and have magnitude of unity are called unit step signals.

They are represented by a unit step function $u(t)$.

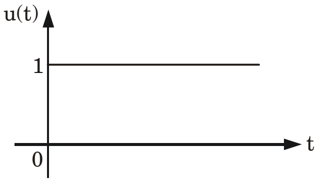


Fig. 16. Unit step.

They are defined mathematically as :

$$u(t) = \begin{cases} 1\,; & t \geq 0 \\ 0\,; & t < 0 \end{cases}$$

2. **Unit ramp :** Signals which start from zero and are linear in nature with a constant slope m are called unit ramp signals.
   They are represented by a unit ramp function $r(t)$.
   They are defined mathematically as :

$$r(t) = \begin{cases} mt\,; & t \geq 0 \\ 0\;\;; & t < 0 \end{cases}$$

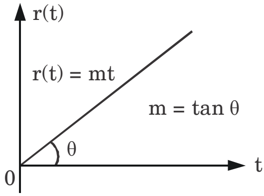


Fig. 17.

3. **Unit impulse :** Signals which act for very small time but have large amplitude are called unit impulse functions.
   They are represented by $\delta(t)$.
   They are defined mathematically as,

$$\delta(t) = \begin{cases} 0\,; & t \neq 0 \\ 1\;; & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

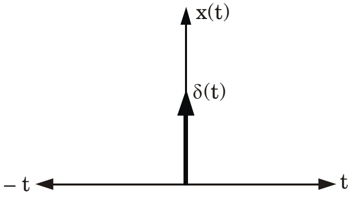


Fig. 18.

4. **Unit Parabolic Signal :** The continuous-time unit parabolic function $p(t)$, also called acceleration signal starts at $t = 0$, and is defined as :

$$p(t) = \begin{cases} \dfrac{t^2}{2} & ; \text{ for } \quad t \geq 0 \\ 0 & ; \text{ for } \quad t < 0 \end{cases}$$

or $$p(t) = \frac{t^2}{2} u(t)$$

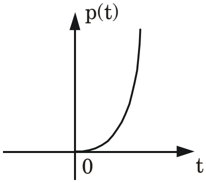


Fig. 19.

Relation :

1. Relation between impulse and step signal :

$$\delta(t) = \frac{d}{dt} u(t)$$

2. Relation between step and ramp signal :

$$u(t) = \frac{d}{dt} r(t)$$

3. Relation between ramp and parabolic signal :

$$r(t) = \frac{d}{dt}\left(\frac{t^2}{2}\right).$$

Numerical :

Given : $G(s) = \dfrac{400(s+1)}{(s+2)(s+8)}$, $H(s) = 1$

To Find : Steady state error.

1. Laplace of periodic waveform using formula :

$$R(s) = \frac{R_1(s)}{1 - e^{-TS}}$$

2. First we have to find Laplace of $r(t)$,

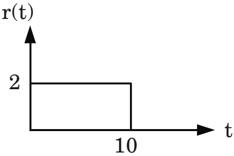


Fig. 20.

By integration method

$$R_1(s) = \int_0^{10} 2e^{-st}\, dt$$

$$= \frac{2}{(-s)}\left[e^{-st}\right]_0^{10}$$

$$R_1(s) = \frac{2}{s}(1 - e^{-10s})$$

4. Here, $T = 20$, So $R(s) = \dfrac{R_1(s)}{1 - e^{-TS}}$

$$R(s) = \frac{\frac{2}{s}(1 - e^{-10s})}{(1 - e^{-20s})}$$

$$R(s) = \frac{\frac{2}{s}(1 - e^{-10s})}{(1 - e^{-10s})(1 + e^{-10s})}$$

$$R(s) = \frac{2}{s(1 + e^{-10s})}$$

5. Using formula

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{s\,\dfrac{2}{s(1 + e^{-10s})}}{1 + \dfrac{400(s+1)}{(s+2)(s+8)}}$$

$$= \lim_{s \to 0} \frac{\dfrac{2}{(1 + e^{-10s})}}{\dfrac{(s+2)(s+8) + 400(s+1)}{(s+2)(s+8)}}$$

$$= \lim_{s \to 0} \frac{2(s+2)(s+8)}{[(s+2)(s+8) + 400(s+1)](1 + e^{-10s})}$$

$$= \frac{2(0+2)(0+8)}{[(0+2)(0+8) + 400(0+1)](1 + e^{0})}$$

$$= \frac{2 \times 2 \times 8}{2 \times 8 + 400} = \frac{32}{416}$$

$$\therefore \quad e_{ss} = 0.0769$$

b. Discuss the effect on the performance of a second order control system of
i. Derivative control ii. Integral control.

Ans.
i. Derivative controller :
Effects :
i. It increases damping ratio.

ii. 'TYPE' of the system remains unchanged.
iii. It reduces peak overshoot.
iv. It reduces settling time.
v. Steady state error remains unchanged.

ii. Proportional integral controller (*PI*) :

Effects :

i. It increases order of the system.
ii. It increases TYPE of the system.
iii. Design of K_i must be proper to maintain stability of system.
vi. Steady state error reduces tremendously for same type of inputs.

3. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. Explain the working principle of stepper motor with neat diagram.
The characteristics equation for feedback control is, $s^3 + 5s^2 + 12s + K = 0$
Find the range of K for all the roots to lie to the left of $s = 1$

Ans.

A. Stepper motor :

1. A stepper motor is a form of AC motor. The input given to this motor is in the form of electric pulses.
2. For every input pulse, the motor shaft turns through a specified number of degrees, called a step.
3. Shaft of stepper motors moves through one angular step for each input pulse. The range of step size may vary from 0.72° to 90°.

B. Principle of operation :

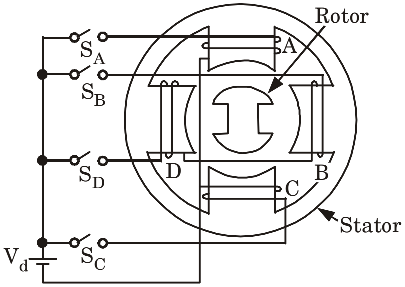


Fig. 21. 4ϕ, 4/2 pole variable reluctance stepper motor.

1. It is a 4ϕ, 4/2-pole (4 poles in stator and 2 in rotor), single-stack, variable reluctance stepper motor. Four phases *A*, *B*, *C*, and *D* are connected to DC source and are energized in the sequence *A*, *B*, *C*, *D*, *A*.
2. When winding *A* is excited, the rotor aligns with axis of phase *A*. The rotor is stable in this position and cannot move until phase *A* is de-energized.

3. Next phase B is excited, A is disconnected. The rotor moves through 90° in clockwise direction to align with the resultant air-gap field which now lies along the axis of phase B.
4. Further, phase C is excited and B is disconnected, the rotor moves further a step of 90° in the clockwise direction. Thus, as the phases are excited in the sequence A, B, C, D, A the rotor moves through a step of 90° at each transition in clockwise direction.
5. The rotor completes one revolution in four steps. The direction of rotation can be reversed by reversing the sequence of switching the windings, *i.e.*, A, D, C, B, A.
6. The magnitude of step angle for any variables reluctance or permanent magnet stepper motor is given by :

$$\beta = \frac{360°}{MN_r}$$

where, β = Step angle

M = Number of stator phases or stacks

N_r = Number of rotor teeth or rotor poles.

7. The step angle is also expressed as,

$$\beta = \frac{N_s - N_r}{N_s N_r} \times 360°$$

where, N_s = Number of stator teeth or stator poles.

By choosing different combinations of number of rotor teeth or stator exciting coils, any desired step angle can be obtained.

Numerical :

Given : Characteristics equation is $s^3 + 5s^2 + 12s + K = 0$
To Find : Range of K.

1. Putting $s = s - 1$

$(s - 1)^3 + 5\,(s - 1)^2 + 12\,(s - 1) + K = 0$

$s^3 - 1 + 3s - 3s^2 + 5s^2 + 5 - 10s + 12s - 12 + K = 0$

$s^3 + 2s^2 + 5s + (K - 8) = 0$

2. **Routh array :**

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 2 & (K-8) \\ s^1 & \dfrac{10-(K-8)}{2} & 0 \\ s^0 & (K-8) & \end{array}$$

3. To lie all roots to the left side of $s = -1$, there should not be any sign change in first column of Routh array.

$$\frac{10-(K-8)}{2} > 0$$

$$10 - (K - 8) > 0$$

$$18 - K > 0$$

$$\therefore \quad K < 18 \quad ...(1)$$

Also, $K - 8 > 0$

$$K > 8 \quad ...(2)$$

From eq. (1) and (2), range of K is

$$8 < K < 18$$

b. Sketch the root locus for the closed loop control system with $G(s) = \dfrac{K}{s(s+1)(s^2+4s+5)}$

Ans.

Given : $G(s) = \dfrac{K}{s(s+1)(s^2+4s+5)}$

To Draw : Root locus.

1. Poles *i.e.*, $s = 0$, $s = -1$, $s = -2 + j$ and $s = -2 - j$
2. There is no open loop zero.
3. Number of poles, $P = 4$

   Number of zeros, $Z = 0$

   $P - Z = 4 - 0 = 4$ *i.e.*, four branches of root locus terminates at infinity.
4. **Angle of asymptotes :**

$$\theta_q = \frac{(2q+1)}{P-Z} \times 180°$$

where, $q = 0, 1, 2, ...$ upto $(P - Z - 1)$

$$\theta_0 = \frac{2 \times 0 + 1}{4 - 0} \times 180° = 45°$$

$$\theta_1 = \frac{2 \times 1 + 1}{4 - 0} \times 180° = 135°$$

$$\theta_2 = \frac{2 \times 2 + 1}{4 - 0} \times 180° = 225°$$

$$\theta_3 = \frac{2 \times 3 + 1}{4 - 0} \times 180° = 315°$$

5. **Centroid of asymptotes :**

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P - Z}$$

$$= \frac{(0 - 1 - 2 + j - 2 - j) - (0)}{4 - 0} = \frac{-5}{4} = -1.25$$

6. **Breakaway points :** The characteristic equation is $1 + G(s)\, H(s) = 0$

$$s(s + 1)\,(s^2 + 4s + 5) + K = 0$$

$$s^4 + 5s^3 + 9s^2 + 5s + K = 0$$

$$\therefore \quad dK / ds = 4s^3 + (5 \times 3s^2) + (9 \times 2s) + 5 \times 1 = 0$$

$$4s^3 + 15s^2 + 18s + 5 = 0$$

$$(s + 0.4)\,(4s^2 + 13.4s + 12.5) = 0$$

$$(s + 0.4)\,\{(s + 1.675 - j\,0.565)\,(s + 1.675 + j\,0.565)\} = 0$$

Therefore three breakaway points are obtained

$s = -\,0.4$ on real axis and

$s = -\,1.7 + j\,0.6$ and $s = -\,1.7 - j\,0.6$

7. **Intersection points with imaginary axis :**

The characteristic equation is $s^4 + 5s^3 + 9s^2 + 5s + K = 0$

The Routh array :

$$\begin{array}{c|ccc} s^4 & 1 & 9 & K \\ s^3 & 5 & 5 & 0 \\ s^2 & 8 & K & \\ s^1 & 5 - \dfrac{5K}{8} & 0 & \\ s^0 & K & & \end{array}$$

Value of K at imaginary axis :

$$5 - \frac{5K}{8} = 0; \;\; K = 8$$

Solving auxiliary equation formed from the s^2 terms in Routh array, therefore

$$8s^2 + K = 0$$

$$8s^2 + 8 = 0$$

$$s^2 + 1 = 0$$

$$s = \pm\, j$$

8. **Angle of departure from complex pole :**

$$\phi_d = 180° - (\phi_P - \phi_Z)$$

$$\phi_{P1} = 180° - \tan^{-1}(1/2) = 153.43°$$

$$\phi_{P2} = 180° - \tan^{-1}(1/1) = 135°$$

$$\phi_{P3} = 90°$$

$$\phi_{(-2\,+j)} = 180° - (\phi_{P1} + \phi_{P2} + \phi_{P3})$$

$$\therefore \quad \phi_{(-2\,+j)} = 180° - (153.43° + 135° + 90°) = -\,198.43°$$

and $\phi_{(-2-j)} = +198.43°$

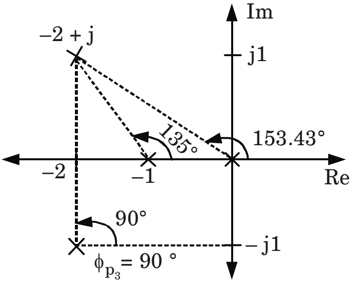


Fig. 22.

9. **Root locus :**

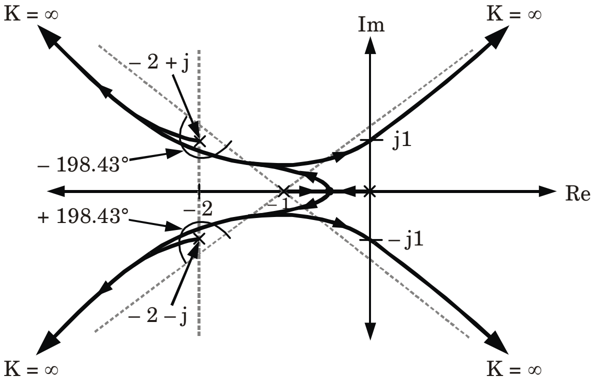


Fig. 23. Root locus.

4. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. **The steady state output of the system for a sinusoidal input of unit magnitude and variable frequency ω is given as**

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + 4z^2u^2}} \sin\left(\omega t - \tan^{-1}\frac{27u}{\sqrt{1-u^2}}\right)$$

Determine :

i. **Resonant frequency**

ii. **Resonant peak**

iii. **Bandwidth**

iv. **Phase angle.**

Ans.

Given : $c(t) = \dfrac{1}{\sqrt{(1-u^2)^2+4z^2u^2}} \sin\left(\omega t - \tan^{-1}\dfrac{27u}{\sqrt{1-u^2}}\right)$

To Find :
i. Resonant frequency
ii. Resonant peak
iii. Bandwidth.
iv. Phase angle

1. $$M = \frac{1}{\sqrt{(1-u^2)^2+4z^2u^2}}$$

2. $$\phi = -\tan^{-1}\frac{27u}{\sqrt{1-u^2}}$$

3. The frequency where M has a peak value is known as resonant frequency. At this frequency, the slope of the magnitude curve is zero. Let ω_r be the resonant frequency and $u_r = \dfrac{\omega_r}{\omega_n}$

$$\left.\frac{dM}{du}\right|_{u=u_r} = -\frac{1}{2}\frac{[-4(1-u_r^2)u_r + 8z^2u_r]}{[(1-4_r^2)^2+4z^2u_r^2]^{3/2}} = 0$$

$$-4(1-u_r^2) = 8z^2$$

$$1-u_r^2 = 2z^2$$

$$u_r = \sqrt{1-2z^2}$$

$$\therefore \quad \omega_r = \omega_n\sqrt{1-2z^2}$$

4. Maximum value of magnitude is known as resonant peak is given by

$$M_r = \frac{1}{\sqrt{(1-u_r^2)^2+4z^2u_r^2}} = \frac{1}{\sqrt{(1-1+2z^2)^2+4z^2(1-2z^2)}}$$

$$= \frac{1}{\sqrt{4z^4+4z^2(1-2z^2)}} = \frac{1}{2z\sqrt{z^2+1-2z^2}} = \frac{1}{2z\sqrt{1-z^2}}$$

5. The range of frequencies over which M is equal to or greater than $\dfrac{1}{\sqrt{2}}$: Putting $u_b = \dfrac{\omega_b}{\omega_n}$

$$M = \frac{1}{\sqrt{(1-u_b^2)^2+4z^2u_b^2}} = \frac{1}{2}$$

$$(1-u_b^2)^2 + 4z^2u_b^2 = 2$$

$$1 + u_b^{\,4} - 2u_b^{\,2} + 4z^2u_b^{\,2} - 2 = 0$$

$$u_b = [1 - 2z^2 + \sqrt{2 - 4z^2 + 4z^4}\,]^{1/2}$$

$$\therefore \qquad \omega_b = \omega_n [1 - 2z^2 + \sqrt{1 - 4z^2 + 4z^4}\,]^{1/2}$$

b. Draw Bode plot (log magnitude plot) for the transfer function.

$$G(s) = \frac{20s}{s^2 + 20s + (100)^2}$$

OR

Using Nyquist stability criterion, investigate the stability of a unity feedback system with open loop transfer function.

$$G(s) = \frac{(s - z_1)}{s(s + p_1)}, \; z_1, p_1 > 0$$

Also discuss the significance of M-circle.

Ans. **Bode plot :**

Given : $G(s) = 20s / [s^2 + 20s + (100)^2]$
To Draw : Bode plot.

1. $$G(s) = \frac{20s}{(100)^2 \left[1 + \dfrac{20s}{(100)^2} + \left(\dfrac{s}{100}\right)^2\right]}$$

S. No.	Factor	Corner frequency	Asymptotic log-magnitude characteristic
1.	s	None	Straight line of constant slope (20 dB / dec) passing through $\omega = 1$
2.	$\dfrac{1}{\left[1 + \dfrac{20s}{(100)^2} + \left(\dfrac{s}{100}\right)^2\right]}$	$\omega_1 = 100$ rad/sec	Straight line of constant slope (– 40 dB/ dec) originating from $\omega = 100$ rad/sec.

$$K = 20 / (100)^2 = 0.002$$

Bode plot :

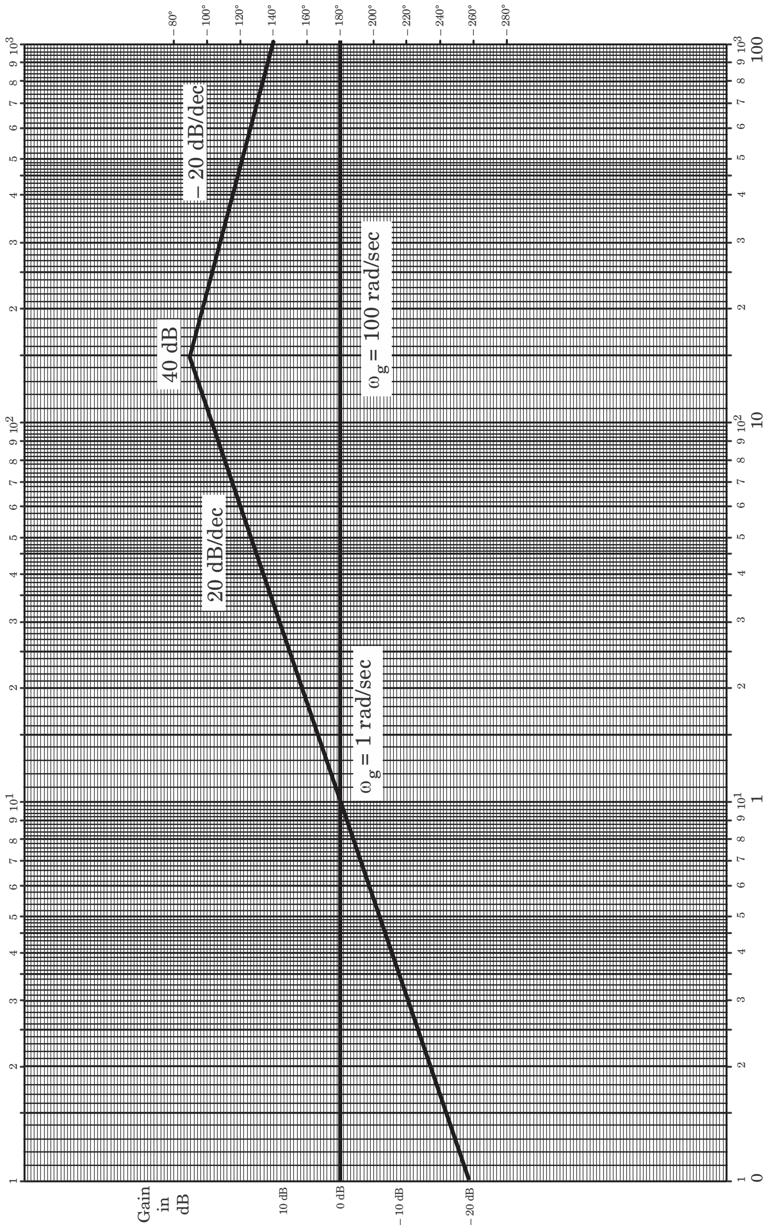


Fig. 24.

Nyquist stability :

Given : $G(s) = \dfrac{(s - z_1)}{s(s + p_1)}$

To Check : Stability of system.

1. $$G(j\omega)\, H(j\omega) = \frac{j\omega - z_1}{j\omega(j\omega + p_1)} = \frac{\sqrt{\omega^2 + z_1^2}}{\omega\sqrt{\omega^2 + p_1^2}}$$

$$\phi = \angle 90° - \tan^{-1}\left[\frac{\omega(p_1 + z_1)}{(p_1 z_1 - \omega^2)}\right]$$

2. Now,

$$\lim_{\omega \to 0} G(j\omega)\, H(j\omega) = \infty \angle + 90°$$

$$G(j\sqrt{p_1 z_1})\, H(j\sqrt{p_1 z_1}) = \frac{1}{p_1\sqrt{p_1 z_1}} \angle 0°$$

$$\lim_{\omega \to \infty} G(j\omega)\, H(j\omega) = 0 \angle -90°$$

3. Thus the locus comes down in the first quadrant, crosses the positive real axis into the fourth quadrant, and approaches the origin from an angle of – 90°.

4. Path $\overline{def}$ maps into the origin, and $\overline{ija}$ maps into on semicircle at infinity. The resulting plot is shown in Fig. 25.

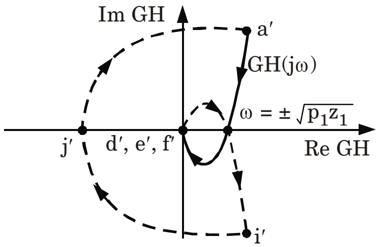


Fig. 25.

Significance of M-circle :

M-circle can be used to obtain closed loop frequency response from open loop frequency response.

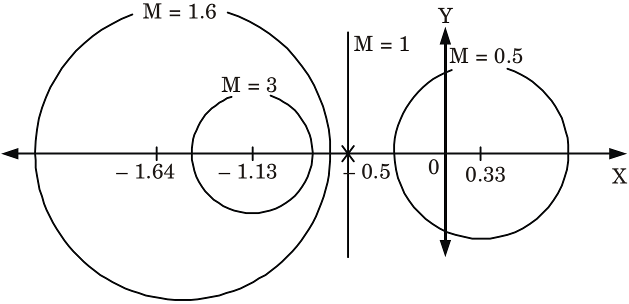


Fig. 26. M-circles.

5. Attempt any **one** part of the following. **(10 × 1 = 10)**

a. State and explain controllability and observability in view of Kalman and Gilbert test.

The state equation for a system is, $\dot{x} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$

Ans. Kalman's test for controllability :

1. Consider n^{th} order multiple input linear time invariant system represented by its state equation as,

$$\dot{x} = Ax + Bu$$

2. The necessary and sufficient condition for the system to be completely state controllable is that the rank of the composite matrix Q_c is n.
3. The composite matrix Q_c is given by,

$$Q_c = [B : AB : A^2B : \dots A^{n-1}B]$$

Gilbert's test for controllability :

1. For the Gilbert's test it is necessary that the matrix A must be in canonical form. Hence the given state model is required to be transformed to the canonical form first, to apply the Gilbert's test.
2. Consider single input linear time invariant system represented by,

$$\dot{z} = A\,x + B\,u$$

where A is not in the canonical form. Then it can be transformed to the canonical form by the transformation,

$$x = M\,z$$

where M = Model matrix

3. The transformed state model,

$$\dot{z} = \tilde{A}\,z + \tilde{B}\,u$$

where $\tilde{A} = M^{-1} AM$

$$\tilde{B} = M^{-1} B$$

4. In such a case the necessary and sufficient condition for the complete state controllability is that the vector matrix $\tilde{B}$ should not have any zero elements. If it has zero elements then the corresponding state variables are not controllable.
5. If the eigen values are repeated then matrix A cannot be transformed to Jordan canonical form. If A has eigen values $\lambda_1, \lambda_1, \lambda_2, \lambda_2, \lambda_3, \lambda_4, \ldots \lambda_n$ then the transformation results Jordan canonical form shown in matrix below

$$J = \begin{bmatrix} \lambda_1 & 1 & 0 & 0 & 0 & 0 \ldots\ldots\ldots 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 \ldots\ldots\ldots 0 \\ 0 & 0 & \lambda_2 & 1 & 0 & 0 \ldots\ldots\ldots 0 \\ 0 & 0 & 0 & \lambda_2 & 1 & 0 \ldots\ldots\ldots 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 \ldots\ldots\ldots 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 \ldots\ldots\ldots 0 \\ \vdots & \vdots & & & & \end{bmatrix}$$

Jordan block

6. In such a case, the condition for the complete state controllability is that the elements of any row of $\tilde{B}$ that corresponds to the last row of each Jordan block are not all zero.

Kalman's test for observability :

1. State equation

$$\dot{x} = A\,x + B\,u$$

and $$y = C\,x$$

2. The system is completely observable if and only if the rank of the composite matrix Q_o is n.

Composite matrix Q_o is given by,

$$Q_o = [C^T : A^T C^T : \ldots\ldots\ldots\ldots (AT)^{n-T} CT]$$

where C^T = Transpose of matrix C

A^T = Transpose of matrix A

Gilbert's test for observability :

1. For Gilbert's test, the state model must be expressed in the canonical form. Consider the state model of linear time invariant system as,

$$\dot{x} = A\,x + B\,u$$

and $$y = C\,x$$

2. Use the transformation $x = Mz$ where M is the model matrix.

$\therefore$ $$y = CM\,z = \tilde{C}\,z$$

where $$\tilde{C} = CM$$

For a single input single output system,

$$y = \tilde{C}z = \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \ldots. & \tilde{C}_{1n} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$$

$$= \tilde{C}_{11} z_1 + \tilde{C}_{12} z_1 + \ldots. + \tilde{C}_{1n} z_n$$

3. For the system to be observable, each term corresponding to each state must be observed in the output. Hence none of the coefficient of $\tilde{C}$ must be zero.

Numerical :

Given : $\dot{x} = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$

To Find : Controllability.

1. $$A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, n = 2$$

2. $$Q_c = [B\ AB]$$

3. $$AB = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

4. $$Q_c = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$$

5. $$|Q_c| = \begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix} = 2 - 2 = 0$$

Rank of Q_c is $r \neq n$

Hence, system is not completely controllable.

b. Design a phase lead compensator for a negative unity feedback system with plant transfer function.

$$G_p(s) = \frac{K}{s(s+10)(s+1000)}$$ **to satisfy the conditions :**

phase margin in atleast 45°, static error constant = 1000 s^{-1}

Ans.

Given : $G_P(s) = \dfrac{K}{s(s+10)(s+1000)}$, Phase margin = 45°

$K_v = 1000 \text{ sec}^{-1}$

To Design : Phase lead compensator.

1. $$K_v = \lim_{s \to 0} sG_p(s) = \lim_{s \to 0} s \frac{K}{s(s+10)(s+1000)}$$

$$K_v = \frac{K}{10 \times 1000}$$

$$K = 10 \times 1000\, K_v = 10^7$$

2. $$G_p(s) = \frac{10^7}{10000s\left(1+\dfrac{s}{10}\right)\left(1+\dfrac{s}{1000}\right)} = \frac{10^3}{s\left(1+\dfrac{s}{10}\right)\left(1+\dfrac{s}{1000}\right)}$$

3. Starting point at 0 dB axis at $\omega = K^{1/n}$

Here, $n = 1$ (Type of the system)

$\omega = 1000$ rad/sec

4. First corner frequency, $\omega_{c1} = 10$ rad/sec

Second corner frequency, $\omega_{c2} = 1000$ rad/sec

Initial slope = – 20 dB/sec

$$\angle G_p(j\omega) = -90 - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{1000}\right)$$

5. We have got $\omega_p = \omega_g = 100$ rad/sec.

So, gain margin = 0 dB and phase margin = 180° – 180° = 0°

$$\phi_m = 45° - 0° + \text{tolerance} = 45° - 0° + 5° = 50°$$

6. The value of α parameter of the phase lead network is given by

$$\alpha = \left(\frac{1-\sin\phi_m}{1+\sin\phi_m}\right) = 0.1324$$

In decibel,

At $\omega = \omega_m$, magnitude in dB $= -20 \log_{10} \dfrac{1}{\sqrt{\alpha}} = 10 \log_{10} \alpha = -8.78$ dB

Table 1.

ω	$\angle G_p(j\omega)$
1	– 95.76°
10	– 135.57°
40	– 168.25°
90	– 178.80°

7. At a gain of – 8.78 dB, the frequency is 165 rad/sec

The lower corner frequency

$$= \frac{1}{T} = \sqrt{\alpha}\, \omega_m = (\sqrt{0.1324}) \times 165 = 60 \text{ rad/sec}$$

8. The upper corner frequency

$$= \frac{1}{\alpha T} = \frac{1}{0.1324} \times 60 = 453.1722 \text{ rad/sec}$$

9. Lead compensation network, $G_c(s) = G_c(j\omega) = \left(\dfrac{1 + j\omega T}{1 + j\omega\alpha T}\right)$

$$= \frac{\left(1 + j\omega \dfrac{1}{60}\right)}{\left(1 + j\omega \dfrac{1}{453.1722}\right)} = \frac{(1 + j\omega\, 0.0166)}{(1 + j\omega\, 0.022066)}$$

10. So, the open loop transfer function of the compensated system is

$$G'(s) = G_p(s)\, G_c(s) = \frac{1000}{s(s + 10)(s + 1000)} \times \frac{(1 + j\omega\, 0.0166)}{(1 + j\omega\, 0.022066)}$$

$$= \frac{1000\,(1 + 0.0166s)}{s\,(s + 10)(s + 1000)(1 + 0.22066s)}$$

Bode plot :

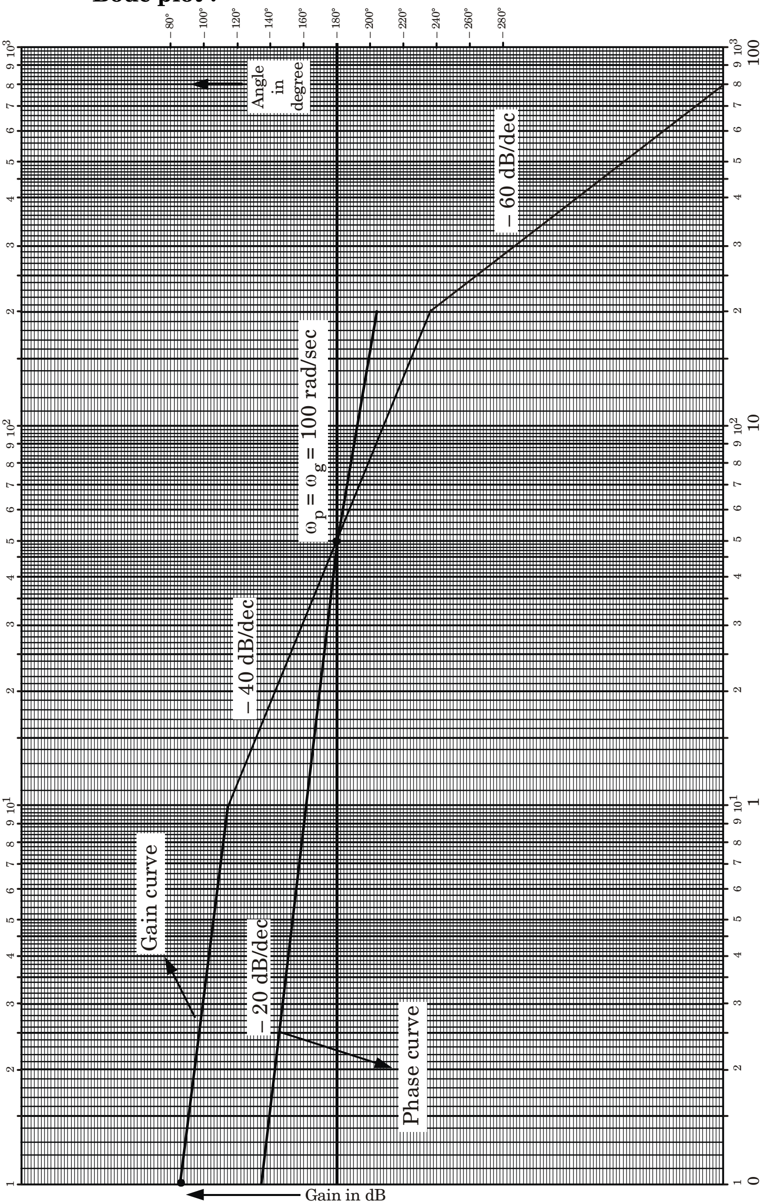


Fig. 27.

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B.Tech.

(SEM. V) ODD SEMESTER THEORY EXAMINATION, 2017-18

CONTROL SYSTEM

Time : 3 Hours **Max Marks : 100**

Note : Attempt all sections. If any missing data is required, then choose suitably.

SECTION-A

1. Attempt **all** questions in brief. **(2 × 10 = 20)**

a. What are the major types of control systems ? Explain them in detail with examples.

b. Define the P, PI and PID controllers.

c. Determine the stability of the system whose characteristics equation is given by $2s^4 + 2s^3 + s^2 + 3s + 2 = 0$.

d. Derive the transfer function $E_0(S)/E_1(S)$ of network shown in Fig. 1.

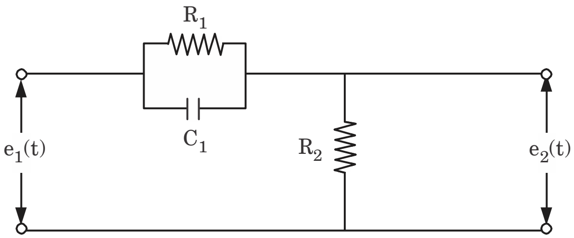

Fig. 1.

e. Show that the polar plot of $G(s) = K/(s + a)$ is a semicircle. Also find its centre and radius.

f. Draw the block diagram and explain the open loop control system and closed loop control system.

g. The OLTF of a unity feedback system is $G(s) = 4(s + a)/s(s + 1)(s + 4)$ find the expression for error $E(s)$ and hence find the value of a so that the e_{ss} due to a unit ramp is 0.125.

h. What is a signal flow graph ?

i. Why is negative feedback invariably preferred in a closed loop system ?

j. What is the basis for framing the rules of block diagram reduction technique ?

SECTION-B

2. Attempt any **three** of the following : **(10 × 3 = 30)**

a. For the system shown in figure, determine the type of system, error coefficient and the error for the following inputs :

i. $r(t) = 6$,

ii. $r(t) = 8t$

iii. $r(t) = 10 + 4t + 15t^2$

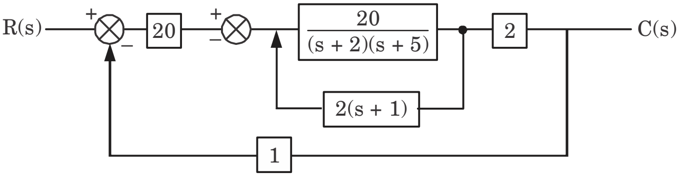

Fig. 2.

b. A linear time invariant system is characterized by the state variable model. Examine the controlablity and observability of the system

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & -4 \end{bmatrix}$$

$$B = \begin{bmatrix} 40 \\ 10 \\ 0 \end{bmatrix} \quad ; \quad C = [0 \quad 0 \quad 1]$$

c. Consider a unity feedback system with a forward path transfer function.

$$G(s) = \frac{K(s-4)}{(s+2)(s-1)}$$

Draw the root locus.

d. Write short note on :

i. Centroid

ii. **Breakaway points**

iii. **Steady state error.**

e. **For a unity feedback system having**

$$G(s) = \frac{35(s+4)}{s(s+2)(s+5)}$$

find (i) the type of the system, (ii) all error coefficients and (iii) errors for ramp input with magnitude 5.

SECTION-C

3. a. **A system is described by the following differential equation. Represent the system in the state space.**

$$\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = u_1(t) + 3u_2(t) + 4u_3(t) \text{ and output are}$$

$$y_1 = 4\frac{dx}{dt} + 3u_1, \quad y_2 = \frac{d^2x}{dt^2} + 4u_2 + u_3$$

b. **Define state and state variable ? What are the advantages of state space techniques ?**

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. **Define stability ? State the necessary conditions for system to be absolutely stable ?**

b. **What are the limitations of Routh Hurwitz criterion ?**

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. **The characteristics equation of a system is given ($s^4 + 20s^3 + 15s^2 + 2s + K = 0$), determine the range of the K, for system to stable.**

b. **Construct the RL (root locus) for a unity feedback system with OLTF** $G(s) = \dfrac{K(s+1)}{s^2(s+9)}$

6. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. **Sketch the RL (root loss) for a unity feedback system with OLTF** $G(s) = \dfrac{K(s^2+2s+10)}{s^2+4s+5}$

b. A unity feedback system shown in figure find the controller gain K_c and K_d so that the closed loop poles are placed at $s = 15 \pm j20$.

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A unity feedback system has an OLTF $G(s) = \dfrac{K(s+2)}{s(s^3 + 7s^2 + 42s)}$.

Find the static error constant and e_{ss} due to an input $r(t) = t^2u(t)$.

b Sketch the polar plot for

i. $G(s) = \dfrac{10e^{-s}}{s+1}$

ii. $G(s) = \dfrac{32}{(s+4)(s^2+4s+8)}$ **and find its points of intersection with the real and imaginary axes.**

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SOLUTION OF PAPER (2017-18)

Note : Attempt all sections. If any missing data is required, then choose suitably.

SECTION-A

1. Attempt **all** questions in brief. **(2 × 10 = 20)**

a. What are the major types of control systems ? Explain them in detail with examples.

Ans. There are two types of control system :

1. Open loop control system :

Fig. 1.

In open loop control systems the control action is independent of the desired output. In this system the output is not compared with the reference input.

Example : Washing machine, Immersion rod, Time operated traffic control, DC shunt motor.

2. Closed loop control system :

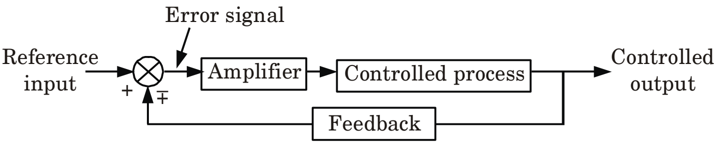


Fig. 2.

In a closed loop control system the output has an effect on control action through a feedback as shown in Fig. 2.

Example : Automatic steering control system, Driving system of an automobile, Home heating system, Ship stabilization system.

b. Define the P, PI and PID controllers.

Ans. **P controller (Proportional controller) :** It is a control system technology based on a response in proportion to the difference between what is set as a desired process variable (or set point) and the current value of the variable.

PI controller (Proportional integral controller) : A controller in the forward path which changes the controller output corresponding to the proportional plus integral of error signal is called PI controller.

PID controller (Proportional, integral and derivative controller): It is a Close loop system which has feedback control system and it compares the Process variable (feedback variable) with set Point and generates an error signal and according to that it adjusts the output of system.

c. Determine the stability of the system whose characteristics equation is given by $2s^4 + 2s^3 + s^2 + 3s + 2 = 0$.

Ans.

1. System equation $2s^4 + 2s^3 + s^2 + 3s + 2 = 0$.
2. By the Routh array

$$\begin{array}{c|ccc} s^4 & 2 & 1 & 2 \\ s^3 & 2 & 3 & 0 \\ s^2 & -2 & 3 & \\ s^0 & 6 & & \end{array}$$

3. Since there is sign change in the first column of Routh table, therefore system is unstable.

d. Derive the transfer function $E_0(S)/E_1(S)$ of network shown in Fig. 3.

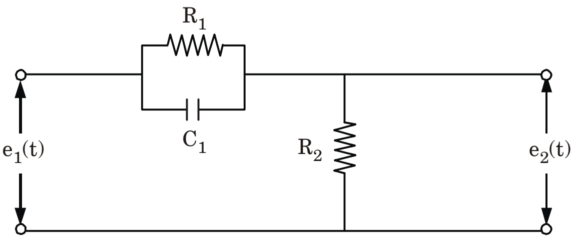


Fig. 3.

Ans.

1. Apply KVL in the loop,

$$-V_i(s) + \left[R_1 \parallel \frac{1}{C_1 s}\right] I(s) + R_2 \; I(s) = 0$$

$$V_i(s) = \left\{\left[\frac{R_1 \dfrac{1}{C_1 s}}{R_1 + \dfrac{1}{C_1 s}}\right] + R_2\right\} I(s)$$

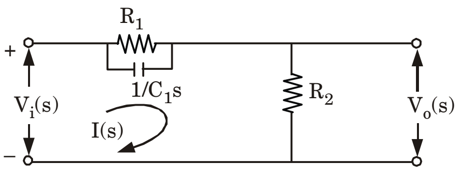


Fig. 4.

2. And, $V_0(s) = R_2 I(s)$

$$V_0(s) = \frac{R_2 V_i(s)}{\left[\dfrac{R_1 \dfrac{1}{C_1 s}}{R_1 + \dfrac{1}{C_1 s}}\right] + R_2} = \frac{R_2 V_i(s)}{\left[\left(\dfrac{R_1}{R_1 C_1 s + 1}\right) + R_2\right]}$$

$\therefore$ Transfer function, $\dfrac{V_0(s)}{V_i(s)} = \dfrac{R_2 (R_1 C_1 s + 1)}{R_2 (R_1 C_1 s + 1) + R_1}$.

e. Show that the polar plot of $G(s) = K/(s + a)$ is a semicircle. Also find its centre and radius.

Ans.

Given : $G(s) = \dfrac{K}{s+a}$

To Find : Centre and Radius.

Solution :

1. Let $s = j\omega$

$$G(j\omega) = \frac{K}{j\omega + a} = \frac{K}{\sqrt{\omega^2 + a^2}} \angle \tan^{-1}\frac{\omega}{a}$$

2. At $\omega = 0$, $|G(j\,0)| = \dfrac{K}{a}$

$\angle\, G(j\,0) = -0°$

3. At $\omega = -\infty$, $|G(j\infty)| = 0$

$\angle G(j\infty)| = -90°$

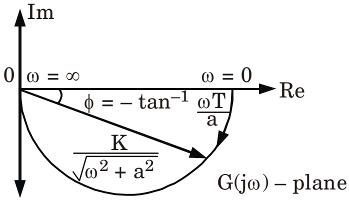


Fig. 5.

4. Diameter $= |G(j\omega)|_{\omega=0} - |G(j\omega)|_{\omega=\infty}$

$= \dfrac{K}{a} - 0 = \dfrac{K}{a}$

So, Radius $= K/2a$

5. Center $= \dfrac{|G(j\omega)|_{\omega=0} - |G(j\omega)|_{\omega=\infty}}{2} = K/2a$.

f. Draw the block diagram and explain the open loop control system and closed loop control system.

Ans. There are two types of control system :

1. **Open loop control system :**

Fig. 6.

In open loop control systems the control action is independent of the desired output. In this system the output is not compared with the reference input.

Example : Washing machine, Immersion rod, Time operated traffic control, DC shunt motor.

2. **Closed loop control system :**

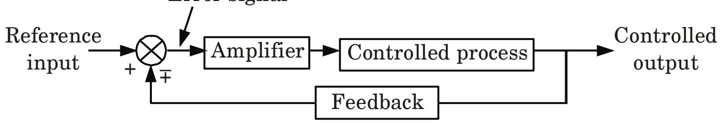


Fig. 7.

In a closed loop control system the output has an effect on control action through a feedback as shown in Fig. 7.

Example : Automatic steering control system, Driving system of an automobile, Home heating system, Ship stabilization system.

g. The OLTF of a unity feedback system is $G(s) = 4(s + a)/s(s + 1)(s + 4)$ find the expression for error $E(s)$ and hence find the value of a so that the e_{ss} due to a unit ramp is 0.125.

Ans.

Given : $G(s) = \dfrac{4(s+a)}{s(s+1)(s+4)}$; $r(t) = 0.125\, t$

To Find : e_{ss}.

1. $$r(t) = 0.125\, t$$

$$R(s) = \frac{0.125}{s^2}$$

2. $$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)\,H(s)} = \lim_{s\to 0} \frac{\dfrac{s \times 0.125}{s^2}}{1 + \dfrac{4\,(s+a)}{s(s+1)\,(s+4)} \times 1}$$

$$= \lim_{s\to 0} \frac{\dfrac{0.125}{s} \times s(s+1)\,(s+4)}{s(s+1)\,(s+4) + 4(s+a)} = \lim_{s\to 0} \frac{0.125 \times (s+1)\,(s+4)}{s(s+1)\,(s+4) + 4(s+a)}$$

$$= \frac{0.125 \times 1 \times 4}{4a}$$

$$\therefore \quad e_{ss} = \frac{1}{4a}$$

h. What is a signal flow graph ?

Ans. A signal flow graph is a pictorial representation of the simultaneous equations describing the system. This is an alternate approach which does not require any reduction process because of availability of a flow graph gain formula which relates the input and output system variables.

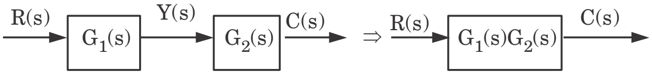
i. Why is negative feedback invariably preferred in a closed loop system ?

Ans. The negative feedback results in better stability in steady state and rejects any disturbance signals. It has also low sensitivity to parameter variations. Hence negative feedback is preferred in closed loop systems.

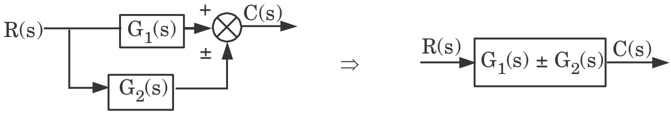
j. What is the basis for framing the rules of block diagram reduction technique ?

Ans.

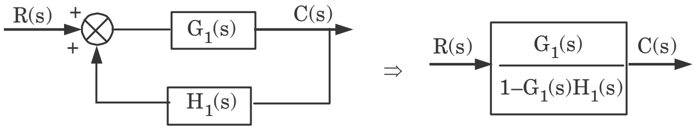
1. Series connection :



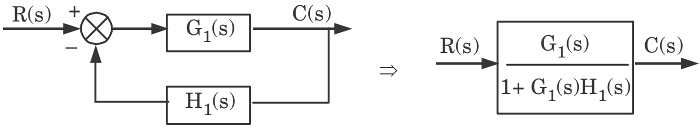
2. Parallel connection :



3. Positive feedback connection :



4. Negative feedback connection :



SECTION-B

2. Attempt any **three** of the following : **(10 × 3 = 30)**

a. For the system shown in figure, determine the type of system, error coefficient and the error for the following inputs :

i. $r(t) = 6$,
ii. $r(t) = 8t$
iii. $r(t) = 10 + 4t + 15t^2$

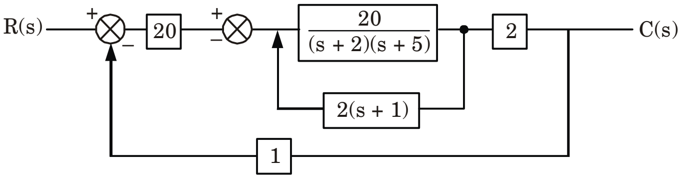
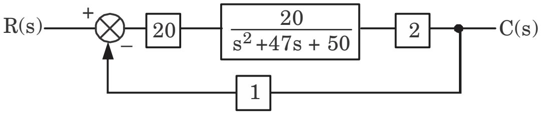
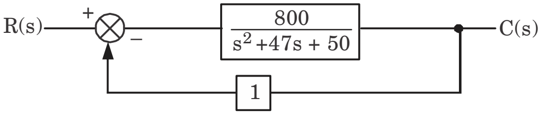


Fig. 8.

Ans. **Step 1 :**



Step 2 :



So, $$G(s) = \frac{800}{s^2 + 47s + 50} \text{ and } H(s) = 1$$

$$G(s)\,H(s) = \frac{800}{s^0(s^2 + 47s + 50)}$$

$\therefore$ Type of system is zero.

i. $r(t) = 6$

So, $R(s) = \dfrac{6}{s} \Rightarrow$ A = 6, which is coefficient of $\dfrac{1}{s}$

Now, $$K_p = \lim_{s \to 0} G(s)\,H(s) = \frac{800}{50}$$

Now, $$e_{ss} = \frac{A}{1 + K_p} = \frac{6}{1 + \dfrac{800}{50}} = \frac{6 \times 50}{850} = 0.353$$

ii. $r(t) = 8\,t$

So, $R(s) = \dfrac{8}{s^2} \Rightarrow$ A = 8, which is coefficient of $\dfrac{1}{s^2}$

Now, $$K_v = \lim_{s \to 0} sG(s)\,H(s)$$

$$= \lim_{s \to 0} \frac{s \times 800}{s^2 + 47s + 50} = 0$$

Now, $$e_{ss} = \frac{A}{K_v} = \frac{8}{0} = \infty$$

iii. $$r(t) = 10 + 4t + 15t^2$$

$$= 10 + 4t + 30\frac{t^2}{2}$$

Now, So, $A_1 = 10$, $A_2 = 4$, $A_3 = 30$

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$= \frac{800}{850} = 0.941$$

$$K_v = \lim_{s \to 0} sG(s)H(s)$$

$$= 0$$

$$K_a = \lim_{s \to 0} s^2\, G(s)H(s)$$

$$= 0$$

Now, $$e_{ss} = \frac{A_1}{1+K_p} + \frac{A_2}{K_v} + \frac{A_3}{K_a}$$

$$= \frac{10}{1+0.941} + \frac{4}{0} + \frac{30}{0} = \infty$$

b. A linear time invariant system is characterized by the state variable model. Examine the controlablity and observability of the system

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{-3} \\ \mathbf{0} & \mathbf{1} & \mathbf{-4} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{40} \\ \mathbf{10} \\ \mathbf{0} \end{bmatrix} \quad ; \quad \mathbf{C = [0 \quad 0 \quad 1]}$$

Ans.

Given : $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -3 \\ 0 & 0 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 40 \\ 10 \\ 0 \end{bmatrix}$, $C = [0 \quad 0 \quad 1]$

To Test : Controllability and observability.

1. Controllability test :

i. $$AB = \begin{bmatrix} 0 \\ 40 \\ 0 \end{bmatrix}, \quad A^2B = \begin{bmatrix} 0 \\ -30 \\ 0 \end{bmatrix}$$

ii. Controllability test matrix is given by,

$$Q_c = [B : AB : A^2B] = \begin{bmatrix} 40 & 0 & 0 \\ 10 & 40 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$|Q_c| = 0$$

iii. Thus the rank of Q_c is 2. Hence the system is not controllable.

2. Obervability test :

i. $$A^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -3 & -4 \end{bmatrix}, \quad C^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

ii. $$A^TC^T = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix}, \quad (A^T)^2\, C^T = \begin{bmatrix} 0 \\ 0 \\ 16 \end{bmatrix}$$

iii. The observability test matrix is given by

$$Q_o = [C^T : A^TC^T : (A^T)^2\, C^T]$$

$$Q_o = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -4 & 16 \end{bmatrix}$$

iv. Its rank is 1. Hence the system is not observable.

c. Consider a unity feedback system with a forward path transfer function.

$$G(s) = \frac{K(s-4)}{(s+2)(s-1)}$$

Draw the root locus.

Ans.

Given : $G(s) = \dfrac{K(s-4)}{(s+2)(s-1)}$

To Draw : Root locus.

1. The open loop poles are at $s = -2$, $s = 1$
2. The open loop zeros at $s = 4$
3. Number of poles = 2
   Number of zeros = 1
   $P - Z = 1$ *i.e.*, one branch of root locus terminates at infinity.
4. **Centroid of asymptotes :**

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P - Z} = \frac{1-2-4}{1} = -5$$

5. **Breakaway point :**

$$1 + G(s)H(s) = 0$$

$$\frac{1 + K(s-4)}{(s+2)(s-1)} = 0$$

$$s^2 + s - 2 + K(s - 4) = 0$$

$$K = \frac{2 - s - s^2}{(s - 4)}$$

$$\frac{dK}{ds} = \frac{(s-4)(-1-2s)-(2-s-s^2)(1)}{(s-4)^2}$$

$$= \frac{-s^2 + 8s + 2}{(s-4)^2} = 0$$

$s = 8.24$ (Breakin point)

and $s = -0.24$ (Breakaway point)

6. Angle of asymptotes :

$$\theta_q = \frac{(2q+1)}{P-Z} \times 180° = \frac{1}{2-1} \times 180° = 180°$$

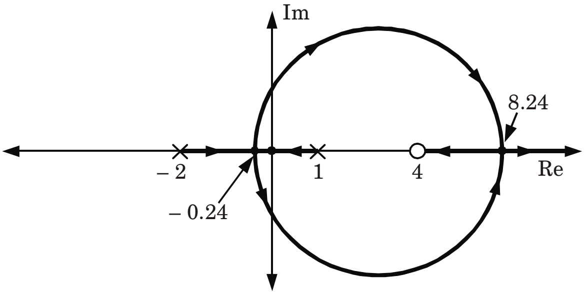


Fig. 9. Root locus.

d. Write short note on :
i. Centroid
ii. Breakaway points
iii. Steady state error.

Ans.

i. Centroid : The point of intersection of the asymptotes with the real axis called centroid is at $s = \sigma$, where

$$\sigma = \frac{\Sigma \text{ Real part of poles } - \Sigma \text{ Real part of zeros}}{\text{Number of poles } - \text{ Number of zeros}}$$

ii. Break points :

1. Breakaway point is a point on the root locus where multiple roots of the characteristic equation occurs, for a particular value of K.
2. Since breakaway point indicates values of multiple root, it is always on the root locus.
3. Steps to determine the co-ordinates of breakaway points are :

Step 1 : Construct the characteristic equation $1 + G(s)\,H(s) = 0$ of the system.

Step 2 : From this equation, separate the terms involving 'K' and terms involving 's'. Write the value of K in terms of s.

$$K = F(s)$$

Step 3 : Differentiate above equation w.r.t. 's', equate it to zero.

$$\frac{dK}{ds} = 0$$

Step 4 : Roots of the equation $\frac{dK}{ds} = 0$ gives us the breakaway points.

iii. Steady state error :

1. The difference between the steady-state response and desired reference give steady state error.
2. If the actual output of control system during steady state error deviates from the reference input, then system possess steady state error.
3. Steady error helps in determining the accuracy, so the steady state error should be minimum.
4. The steady state performance of a control system is assessed by the magnitude of the steady state error possessed by the system and the system input can be step or ramp or parabolic.
5. The magnitude of steady state error in a closed loop control system depends on its open loop control function. *i.e.*, $G(s)H(s)$ of the system.

e. For a unity feedback system having

$$G(s) = \frac{35(s+4)}{s(s+2)(s+5)}$$

find (i) the type of the system, (ii) all error coefficients and (iii) errors for ramp input with magnitude 5.

Ans.

Given : $G(s) = \dfrac{35(s+4)}{s(s+2)(s+5)}$, $H(s) = 1$

To Find : Type of the system, All error coefficients and Errors for ramp input with magnitude 5.

i. Type : System is type 1 system. As we have s^1 in common in denominator.

ii. Error coefficients :

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$= \lim_{s \to 0} \frac{35(s+4)}{s(s+2)(s+5)} \times 1 = \infty$$

$$K_v = \lim_{s \to 0} s\, G(s)H(s)$$

$$= \lim_{s \to 0} \frac{s \times 35\,(s+4)}{s\,(s+2)\,(s+5)} \times 1$$

$$= \frac{35 \times 4}{10} = 14$$

$$K_a = \lim_{s \to 0} s^2\, G(s)H(s)$$

$$= \lim_{s \to 0} s^2 \frac{35(s+4)}{s(s+2)(s+5)} \times 1 = 0$$

iii. Error : Magnitude, $A = 5$ and $K_v = 14$

$$e_{ss} = \frac{A}{K_v} = \frac{5}{14}$$

$$e_{ss} = 0.357.$$

SECTION-C

3. a. A system is described by the following differential equation. Represent the system in the state space.

$$\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = u_1(t) + 3u_2(t) + 4u_3(t) \text{ and output are}$$

$$y_1 = 4\frac{dx}{dt} + 3u_1, \quad y_2 = \frac{d^2x}{dt^2} + 4u_2 + u_3$$

Ans.

1. Select the state variables as

$$x_1 = x$$

$$\dot{x}_1 = \dot{x} = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -3x_3 - 4x_2 - 4x_1 + u_1(t) + 3u_2(t) + 4u_3(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

2. Output :

$$y_1 = 4x_2 + 3u_1$$

$$y_2 = x_3 + 4u_2 + u_3$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

b. Define state and state variable ? What are the advantages of state space techniques ?

Ans.

i. **State :** The state of a dynamic system is the smallest set of variables such that the knowledge of these variables at $t = t_0$ with the knowledge of the input for $t \geq t_0$ completely determines the behaviour of the system for any time $t \geq t_0$.

ii. **State variables :** The variables involved in determining the state of dynamics system are called state variables.

Advantages :

1. The method takes into account the effect of all initial conditions.
2. It can be applied to non-linear as well as time varying conditions.
3. It can be conveniently applied to multiple input multiple output systems.
4. The system can be designed for the optimal conditions precisely by using this modern method.
5. Any type of the input can be considered for designing the system.
6. As the method involves matrix algebra, can be conveniently adopted for the digital computers.

4. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Define stability ? State the necessary conditions for system to be absolutely stable ?

Ans. **Stability :**

1. A system is stable if its response (*i.e.*, the transfer function) approaches zero as time approaches infinity.
2. In other words, a system is stable if every bounded input produces a bounded output.

Necessary conditions for absolutely stable :

i. All the coefficients of characteristics equation must have same sign.
ii. There should be no missing term.
iii. All poles of transfer function should be in left half of s-plane.
iv. The degree of denominator polynomial of transfer function is greater or equal to that of numerator polynomial.

b. What are the limitations of Routh Hurwitz criterion ?

Ans. **Limitations :**

1. It is valid only if the characteristics equation is algebraic.
2. If any coefficient of the characteristics equation is complex or contain power of e then this criterion cannot be applied.
3. It gives information about how many roots are lying in the RHS of the s-plane but values of the roots are not available. Also it cannot distinguish between real and complex roots.

5. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. The characteristics equation of a system is given ($s^4 + 20s^3 + 15s^2 + 2s + K = 0$), determine the range of the K, for system to stable.

Ans.

Given : $G(s) = s^4 + 20s^3 + 15s^2 + 2s + K = 0$
To Find : Range of K.

1. Routh array,

$$\begin{array}{c|ccc} s^4 & 1 & 15 & K \\ s^3 & 20 & 2 & 0 \\ s^2 & 149 & K & 0 \\ s^1 & \dfrac{298-20K}{149} & 0 & \\ s^0 & K & & \end{array}$$

2. For system to be stable

$$\frac{298-20K}{149} > 0 \text{ and } K > 0$$

3. Range of K is $0 < K < 14.9$

b. Construct the RL (root locus) for a unity feedback system with OLTF $G(s) = \dfrac{K(s+1)}{s^2(s+9)}$

Ans.

Given : $G(s) = \dfrac{K(s+1)}{s^2(s+9)}$, $H(s) = 1$
To Draw : The root locus.

1. The open loop poles *i.e.*, $s = 0$, $s = 0$, $s = -9$
2. Open loop zero are at $s = -1$
3. Number of poles, $P = 3$
   Number of zeros, $Z = 1$
   $P - Z = 3 - 1 = 2$ *i.e.*, two branches of root locus terminates at infinity.
4. **Angle of asymptotes :**

$$\theta_q = \frac{(2q+1)}{P-Z} \times 180°$$

where, $q = 0, 1, 2, \ldots\ldots$ upto $(P - Z - 1)$

$$\theta_0 = \frac{(2\times 0+1)}{3-1} \times 180° = 90°$$

$$\theta_1 = \frac{2\times 1+1}{3-1} \times 180° = 270°$$

5. **Centroid of asymptotes :**

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P-Z}$$

$$= \frac{0-9+1}{2} = \frac{-8}{2} = -4$$

6. Breakaway points :

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1)}{s^2(s+9)} \times 1 = 0$$

$$s^2(s + 9) + K(s + 1) = 0$$

$$K = \frac{-s^2(s+9)}{(s+1)}$$

$$\frac{dK}{ds} = -\frac{(s+1)(3s^2+18s) - s^2(s+9)(1)}{(s+1)^2} = \frac{-2s(s+3)^2}{(s+1)^2} = 0$$

$$-2s(s + 3)^2 = 0$$

$$s = 0, \quad s = -3$$

7. Intersection with imaginary axis :

$$s^3 + 9s^2 + Ks + K = 0$$

Routh array :

$$\begin{array}{c|cc} s^3 & 1 & K \\ s^2 & 9 & K \\ s^1 & 8K/9 & \\ s^0 & K & \end{array}$$

Value of K at imaginary axis,

$$\frac{8K}{9} = 0$$

$$K = 0$$

Now, $\quad 9s^2 + K = 0$

$$9s^2 = 0$$

$$s = 0$$

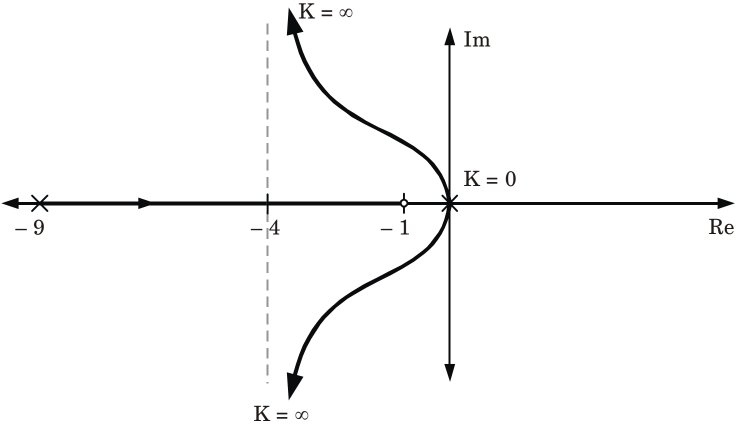


Fig. 10. Root locus.

6. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. Sketch the RL (root locus) for a unity feedback system with OLTF $G(s) = \dfrac{K(s^2 + 2s + 10)}{s^2 + 4s + 5}$

Ans.

Given : $G(s) = \dfrac{K(s^2 + 2s + 10)}{s^2 + 4s + 5}$, $H(s) = 1$

To Draw : Root locus.

1. The open loops poles are at $s = -2 - j$, $s = -2 + j$
2. The open loop zeros are at $s = -1 - j3$, $s = -1 + j3$
3. Number of poles = 2
   Number of zeros = 2
   $P - Z = 2 - 2 = 0$ *i.e.*, none of branches of root locus terminates at infinity.
4. Angle of departure and arrival can't be calculated because the centroid, $\sigma = \infty$.

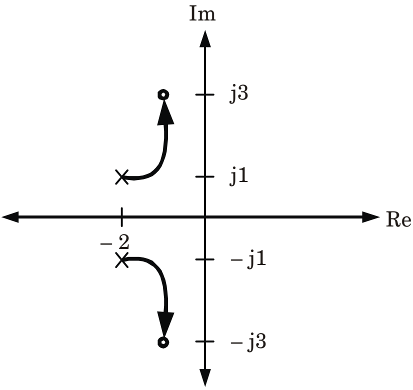


Fig. 11.

b. A unity feedback system shown in figure find the controller gain K_c and K_d so that the closed loop poles are placed at $s = 15 \pm j20$.

Ans. This question is incomplete as the figure mentioned is not given.

7. Attempt any **one** part of the following : **(10 × 1 = 10)**

a. A unity feedback system has an OLTF $G(s) = \dfrac{K(s+2)}{s(s^3 + 7s^2 + 42s)}$.

Find the static error constant and e_{ss} due to an input $r(t) = t^2u(t)$.

Ans.

Given : $G(s) = \dfrac{K(s+2)}{s(s^3+7s^2+42s)}$, $r(t) = t^2u(t)$

To Find : K_a and e_{ss}.

1. $$r(t) = t^2u(t) = 2\frac{t^2}{2}u(t)$$
   $\therefore$ $A = 2.$ [Coeff. of $t^2/2$]
2. Now, $$G(s)H(s) = \frac{K(s+2)}{s(s^3+7s^2+42s)} = \frac{K(s+2)}{s^2(s^2+7s+42)}$$
3. Now, $$K_a = \lim_{s\to 0} s^2\, G(s)\, H(s) = \lim_{s\to 0}\frac{s^2\, K(s+2)}{s^2(s^2+7s+42)} = \frac{2K}{42} = \frac{K}{21}$$
4. $$e_{ss} = \frac{A}{K_a} = \frac{2}{\frac{K}{21}} = \frac{42}{K}$$

b Sketch the polar plot for

i. $G(s) = \dfrac{10e^{-s}}{s+1}$

ii. $G(s) = \dfrac{32}{(s+4)(s^2+4s+8)}$ **and find its points of intersection with the real and imaginary axes.**

Ans.

i. Given : $G(s) = \dfrac{10e^{-s}}{s+1}$

To Sketch : Polar plot.

1. Putting $s = j\omega$
   $$G(j\omega) = \frac{10e^{-j\omega}}{j\omega+1}$$
2. Magnitude :
   $$|G(j\omega)| = \frac{|10e^{-j\omega}|}{\sqrt{\omega^2+1}} = \frac{10}{\sqrt{\omega^2+1}}$$

3. Phase angle :

$$\angle G(j\omega) = -\omega - \tan^{-1}\omega$$

4. Here the magnitude decreases from unity monotonically and the phase angle also decreases monotonically and indefinitely, the polar plot of the given transfer function is spiral as shown in Fig. 12.

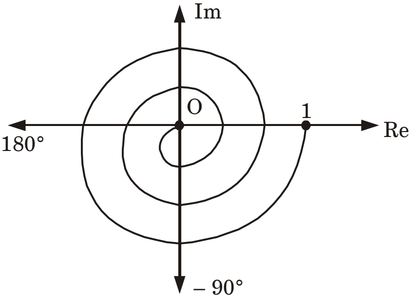


Fig. 12.

ii. Given : $G(s) = \dfrac{32}{(s+4)(s^2+4s+8)}$

To Sketch : Polar plot.

1. Putting $s = j\omega$

$$G(j\omega) = \frac{32}{(j\omega+4)[j\omega^2+4j\omega+8]}$$

$$= \frac{32}{(j\omega+4)[(8-\omega^2)+4j\omega]}$$

$$= \frac{32(4-j\omega)[(8-\omega^2)-4j\omega]}{(16+\omega^2)[(8-\omega^2)^2+16\omega^2]}$$

$$= \frac{32[4(8-\omega^2)-4\omega^2-j\{\omega(8-w^2)+16\omega\}]}{(16+\omega^2)[(8-\omega^2)^2+16\omega^2]}$$

2. Saparating into real and imaginary parts

$$G(j\omega) = \frac{32[4(8-\omega^2)-4\omega^2]}{(16+\omega^2)[(8-\omega^2)^2+16\omega^2]} - \frac{j32[\omega(8-\omega^2)+16\omega]}{(16+\omega^2)[(8-\omega^2)^2+16\omega^2]}$$

3. Intersection point with real axis,

$$\omega(8-\omega^2)+16\omega = 0$$
$$8\omega-\omega^3+16\omega = 0$$
$$\omega^3-24\omega = 0$$
$$\omega(\omega^2-24) = 0$$
$$\omega_1 = 0, -4.899 \text{ and } +4.899$$

4. Intersection points on imaginary axis,

$$4\omega^2-4(8-\omega^2) = 0$$
$$4\omega^2 = 4(8-\omega^2)$$
$$4\omega^2 = 32-4\omega^2$$

$$\omega^2 = 4$$
$$\omega = \pm 2$$

5. $$|G(j\omega)| = \frac{32}{\sqrt{\omega^2 + 16}\sqrt{(8-\omega^2) + 16\omega^2}}$$

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{4\omega}{8-\omega^2}\right)$$

6. The value of $G(j\omega)$ at $\omega = 0$, and $\omega = \infty$

$$|G(j0)| = \frac{32}{4 \times 8} = 1$$
$$|G(j\infty)| = 0$$
$$\angle G(j0) = 0°$$
$$\angle G(j\infty) = -270,$$

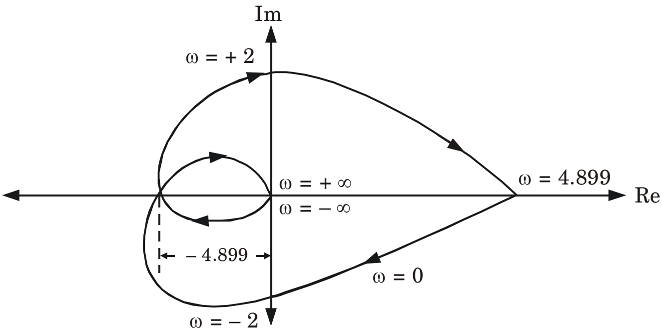


Fig. 13.

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B.Tech.
(SEM. V) ODD SEMESTER THEORY EXAMINATION, 2018-19
CONTROL SYSTEM

Time : 3 Hours **Max Marks : 70**

Note : Attempt all sections. If any missing data is required, then choose suitably.

SECTION-A

1. Attempt **all** parts of the following : **(2 × 7 = 14)**

a. **What is Mason's gain formula ?**

b. **What is an impulse response ?**

c. **What is steady state error ?**

d. **Define damping ratio.**

e. **Define gain crossover frequency and phase margin.**

f. **What is centroid in root locus ?**

g. **Define state variable and state space.**

SECTION-B

2. Attempt any **three** parts of the following : **(7 × 3 = 21)**

a. **For a unity feedback system the open loop transfer function is given by**

$$G(s) = K/s(s+2)(s^2+6s+25)$$

i. **Sketch the root locus.**

ii. **At what value of K the system become unstable.**

iii. **At this point of instability determine the frequency of oscillation of the system.**

b. **Explain the working of servomotor with suitable diagram and also derive the field controlled DC motor transfer function.**

c. **Draw the Nyquist plot for the unity feedback system whose open loop transfer function is**

$$G(s)\,H(s) = \frac{K}{s^2(1+sT)}$$

d. Determine the transfer function of the circuit given below :

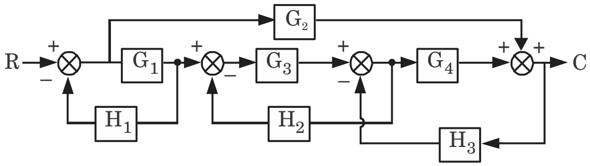


Fig. 1.

e. Establish the correlation between time response and frequency response analysis and suitably explain with diagrams.

SECTION-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Find the generalized error coefficients for a system whose $G(s)\,H(s) = 1/s(s+2)$ and also find the expression for steady state error for input $r(t) = 2 + 3t + 2t^3$.

b. Sketch the polar plot for the following transfer function

$$G(s) = (1 + 4s)/s^2\,(s+1)(2s+1)$$

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Explain P, PI, PID controllers and also give their advantages.

b. Derive the expressions for second order system for underdamped case and when the input is unit step.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Construct the state model of a system characterized by the differential equation. Give the block diagram representation of the state model.

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = u$$

b. A single input single output system is given as

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix},\ B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } C = [1 \ \ 0 \ \ 0]$$

Test for controllability and observability.

6. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. For the given transfer function $G(s)\,H(s) = 2/s(1 + 0.5s)\,(1 + 0.05s)$ Determine phase crossover frequency and gain margin.

b. The forward path transfer function of unity feedback control system is $G(s) = 100/s(s + 6.45)$. Find the resonance peak M_r, resonant frequency ω_r and bandwidth of the closed loop system.

7. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. What is the effect of adding pole to a system ? Discuss.

b. Explain the lag compensation.

☺☺☺

SOLUTION OF PAPER (2018-19)

Note : Attempt all sections. If any missing data is required, then choose suitably.

SECTION-A

1. Attempt **all** parts of the following : **(2 × 7 = 14)**

a. What is Mason's gain formula ?

Ans. The overall gain can be determined by Mason's gain formula given below :

$$T = \sum_{k=1}^{k} \frac{P_k \Delta_k}{\Delta}$$

where, P_k = The forward path gain

Δ = The graph determinant which involves closed-loop gain and mutual interactions between non-touching loops.

Δ_k = The path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

b. What is an impulse response ?

Ans. Impulse response is the response of system when we provide impulse signal as input.

c. What is steady state error ?

Ans. It indicates the error between the actual output and desired output as t tends to infinity *i.e.*

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$$

It is denoted by e_{ss}.

d. Define damping ratio.

Ans. Damping ratio indicates the amount of damping present in system and denoted by ξ.

e. Define gain crossover frequency and phase margin.

Ans.

1. **Gain crossover frequency :** Gain crossover frequency is the frequency at which magnitude of open loop transfer function is unity.
2. **Phase margin :** It is the reciprocal of magnitude $|G(j\omega)|$ at the frequency at which the phase angle is – 180°.

Gain Margin (GM),

$$K_g = \frac{1}{|G(j\omega_c)|}$$

where, ω_c = Phase cross-over frequency.

f. What is centroid in root locus ?

Ans. **Centroid :** All asymptotes intersect the real axis at a common point known as centroid.

$$\sigma = \frac{\Sigma \text{Real part of open loop poles} - \Sigma \text{Real part of open loop zeros}}{P - Z}$$

where, P = Number of open loop poles

Z = Number of open loop zeros.

Breakaway point : Breakaway point is defined as the point at which root locus comes out of the real axis.

g. Define state variable and state space.

Ans. **State Variables :** A set of at least n variables $x_1(t)$, $x_2(t)$ ... $x_n(t)$, are needed to completely describe how a system will behave in future, along with initial state and input excitation. These minimal set of variables which can determine the state of a system are known as state variables.

State space : The n-dimensional space whose coordinate axes consists of the x_1 axis, x_2 axis ..., x_n axis is called state space. Any state can be represented by a point in the state space.

SECTION-B

2. Attempt any **three** parts of the following : **(7 × 3 = 21)**

a. For a unity feedback system the open loop transfer function is given by

$$\boldsymbol{G(s) = K/s(s + 2)(s^2 + 6s + 25)}$$

i. Sketch the root locus.

ii. At what value of K the system become unstable.

iii. At this point of instability determine the frequency of oscillation of the system.

Ans.

Given : $G(s) = \dfrac{K}{s(s+2)(s^2+6s+25)}$

To Draw : Root locus.

1. Poles *i.e.*, $s = 0$, $s = -2$, $s = -3 + 4i$,

$$s = -3 - 4i$$

2. There is no open loop zero.
3. Number of poles, $P = 4$

   Number of zeros, $Z = 0$

$P - Z = 4 - 0 = 4$ *i.e.*, four branches of root locus terminates at infinity.

4. Angle of asymptotes :

$$\theta_q = \frac{(2q+1)}{P-Z} \times 180°$$

where, $q = 0, 1, 2, ...$ upto $(P - Z - 1)$

$$\theta_0 = \frac{2\times0+1}{4-0}\times180° = 45°$$

$$\theta_1 = \frac{2\times1+1}{4-0}\times180° = 135°$$

$$\theta_2 = \frac{2\times2+1}{4-0}\times180° = 225°$$

$$\theta_3 = \frac{2\times3+1}{4-0}\times180° = 315°$$

5. Centroid of asymptotes :

$$\sigma = \frac{\Sigma \text{ Real parts of poles} - \Sigma \text{ Real parts of zeros}}{P-Z}$$

$$= \frac{(0-2-3+4i-3-4i)-0}{4-0} = \frac{-8}{4} = -2$$

6. Breakaway points : The characteristic equation is

$$1 + G(s)\,H(s) = 0$$

$$s(s+2)(s^2+6s+25) + K = 0$$

$$s^4 + 8s^3 + 37s^2 + 50s + K = 0$$

$$\frac{dK}{ds} = 4s^3 + 24s^2 + 74s + 50$$

Therefore 3 breakaway points are obtained

$s = -0.9$ on real axis and

$$s = -2.55 \mp j\,2.72$$

7. Intersection points with imaginary axis :

The characteristic equation is

$s^4 + 8s^3 + 37s^2 + 50s + K = 0$

Routh array :

$$\begin{array}{c|ccc} s^4 & 1 & 37 & K \\ s^3 & 8 & 50 & 0 \\ s^2 & 30.75 & K & 0 \\ s^1 & 50 - \dfrac{8K}{30.75} & 0 & \\ s^0 & K & & \end{array}$$

Value of K at imaginary axis :

$$50 - \frac{8K}{30.75} = 0 \qquad \therefore \quad K = 192.2$$

8. Solving auxiliary equation formed from the s^2 terms in Routh array,

$$30.75\, s^2 + K = 0$$

$$30.75\, s^2 + 192.2 = 0$$

$$s = \mp j\, 2.5$$

So, $\omega = 2.5$ rad/sec

9. Angle of departure from complex pole :

$$\phi_d = 180° - (\phi_P - \phi_Z)$$

$$\phi_{P1} = 180° - \tan^{-1}\left(\frac{4}{3}\right) = 126.87°$$

$$\phi_{P2} = 180° - \tan^{-1}\left(\frac{1}{1}\right) = 135°$$

$$\phi_{P3} = 90°$$

$$\phi_{(-4+j3)} = 180° - (126.87° + 135° + 90°)$$

$$= -171.87$$

Root locus :

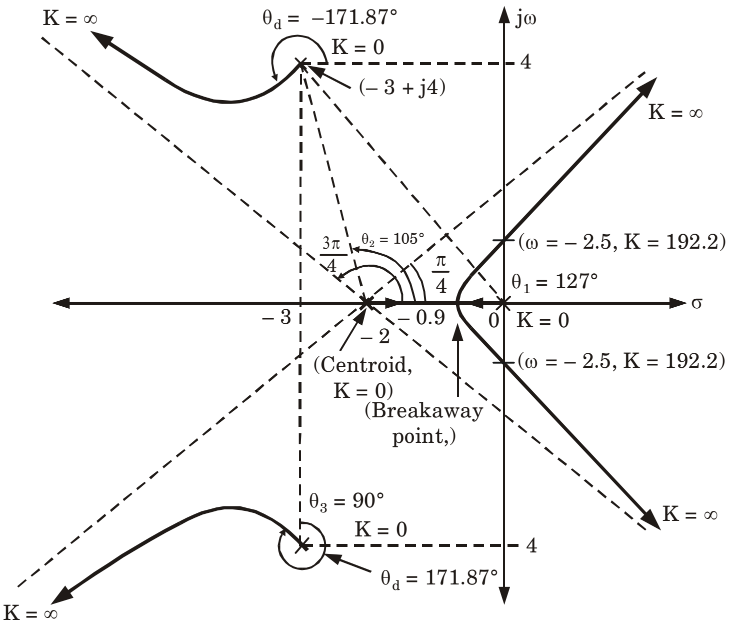


Fig. 1.

b. Explain the working of servomotor with suitable diagram and also derive the field controlled DC motor transfer function.

Ans.

1. **Servomotor :**
1. Fig. 2 shows the schematic diagram of a 2ϕ AC servomotor.
2. The stator has two distributed windings which are displaced from each other by 90 electrical degrees.
3. One winding is called the reference or fixed phase and other winding is called control phase.

Working principle :

1. Reference phase is supplied from a constant voltage source $V_r \angle 0°$. The other winding *i.e.*, control phase is supplied with a variable voltage of the same frequency as the reference phase but its phase is displaced by 90° (electrically).
2. The control phase is usually supplied from a servo amplifier.
3. The speed and torque of the rotor are controlled by the phase difference between the control voltage and the reference phase voltages.
4. The direction of rotation of the rotor can be reversed by reversing the phase difference from leading to lagging between the control phase voltage and the reference phase voltage.

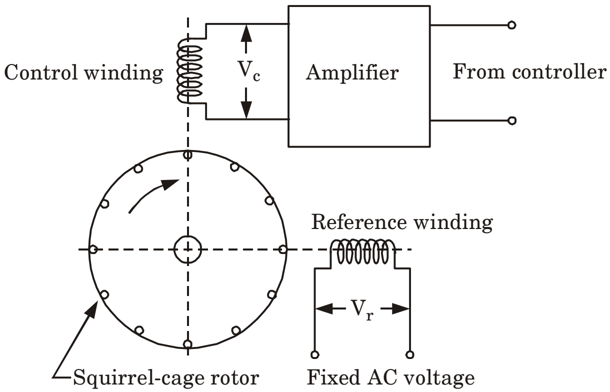


Fig. 2.

2. **Field controlled DC motor (DC Servomotor) transfer function :**
1. A schematic diagram of a field controlled DC motor (DC servo motor) shown in Fig. 3.

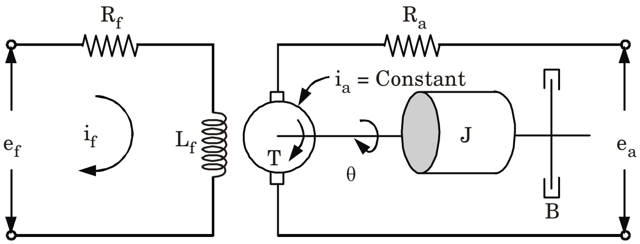


Fig. 3.

2. Here, R_f = Field winding resistance
L_f = Field winding inductance
I_f = Field winding current
R_a = Armature resistance
i_a = Armature current
θ = Angular displacement.

3. The torque T developed by the motor is proportional to product of the air-gap flux ϕ and armature current i_a so we get

$$T = K_1 \phi i_a \quad ...(1)$$

where K_1 is constant.

4. But the air gap flux ϕ and the field current i_f are proportional for the usual operating range of the motor and i_a is assumed to be constant, we can rewrite the above equation as

$$T = K_2 i_f \quad ...(2)$$

where K_2 is a constant.

5. The equations for this system are

$$L_f \frac{di_f}{dt} + R_f i_f = e_f \quad ...(3)$$

and $$\frac{Jd^2\theta}{dt^2} + \frac{Bd\theta}{dt} = T = K_2 i_f \quad ...(4)$$

6. By taking the Laplace transform on both sides of eq. (3) and (4) where all initial conditions are zero, we get

$$(L_f s + R_f)\, i_f(s) = E_f(s) \quad ...(5)$$
$$(Js^2 + Bs)\, \theta(s) = K_2 i_f(s) \quad ...(6)$$

7. From the above equations, the transfer function of this system is obtained as

$$\frac{\theta(s)}{E_f(s)} = \frac{K_2}{s(L_f s + R_f)(Js + B)} \quad ...(7)$$

8. Block diagram is shown in Fig. 4.

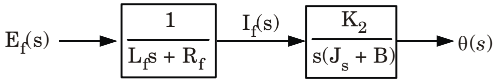


Fig. 4.

c. Draw the Nyquist plot for the unity feedback system whose open loop transfer function is

$$G(s)\,H(s) = \frac{K}{s^2(1+sT)}$$

Ans.

1. Putting $s = j\omega$

$$G(j\omega)\,H(j\omega) = \frac{K}{(j\omega)^2\,(1+j\omega T)}$$

2. $$|G(j\omega)\,H(j\omega)| = \frac{K}{\omega^2\sqrt{1+\omega^2T^2}}$$

3. $$\angle G(j\omega)\,H(j\omega) = -180° - \angle \tan^{-1}\omega T$$

4. At $\omega = 0$, $G(0)H(0) = \infty \angle -180°$

At $\omega = \infty$, $G(\infty)H(\infty) = 0 \angle -270° = 0 \angle 90°$

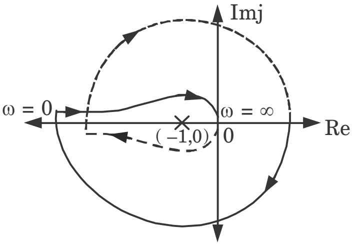


Fig. 5.

d. Determine the transfer function of the circuit given below :

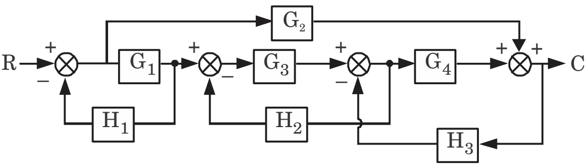
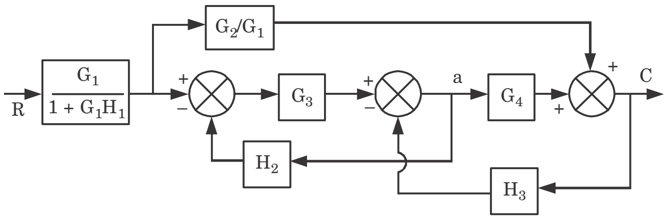
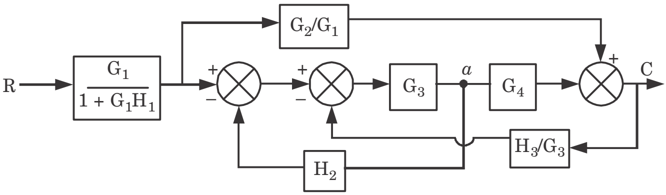


Fig. 6.

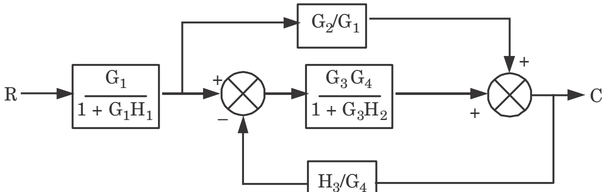
Ans. **Step 1 :**



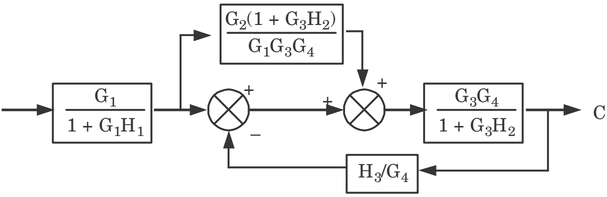
Step 2 : Shift summing point a before G_3.



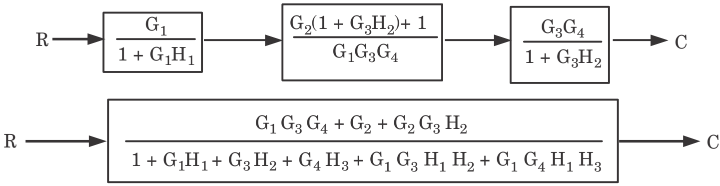
Step 3 : Interchange summing points.



Step 4 : Shift summing point before block.



Step 5 : Interchange summing points.



e. Establish the correlation between time response and frequency response analysis and suitably explain with diagrams.

Ans. **Correlation :**

1. For 2nd order system, the transfer function is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where, ξ = Damping factor

ω_n = Natural frequency of oscillations

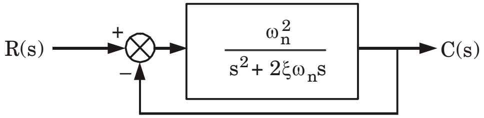


Fig. 7.

2. Closed loop frequency response is,

$$\frac{C(j\omega)}{R(j\omega)} = T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n(j\omega) + \omega_n^2} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2j\xi\left(\frac{\omega}{\omega_n}\right)}$$

$$= \frac{1}{(1-u^2) + j2\xi u} \quad ...(1)$$

where $u = \omega/\omega_n$, normalized driving frequency.

$$\therefore \quad |T(j\omega)| = M = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \quad ...(2)$$

and $$\angle T(j\omega) = \phi = -\tan^{-1}\frac{2\xi u}{1-u^2} \quad ...(3)$$

3. The steady state output is

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \sin\left(\omega t - \tan^{-1}\frac{2\xi u}{1-u^2}\right)$$

$\therefore$ From eq. (2) and (3) when

$$u = 0, M = 1 \text{ and } \phi = 0$$

$$u = 1, M = \frac{1}{2\xi} \text{ and } \phi = -\frac{\pi}{2}$$

$$u = \infty, M = 0 \text{ and } \phi = -\pi$$

4. The frequency where M has a peak value is called the resonant frequency. At this frequency the slope of magnitude curve is zero.

If ω_r = Resonant frequency.

$u_r = \omega_r/\omega_n$ is normalized resonant frequency.

$$\left.\frac{dM}{du}\right|_{u=u_r} = -\frac{1}{2}\frac{\left[-4(1-u_r^2)u_r + 8\xi^2 u_r\right]}{\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{3/2}} = 0$$

$$-4(1-u_r^2)\,u_r + 8\xi^2 u_r = 0$$

$$-4u_r(1-u_r^2-2\xi^2) = 0$$

$$\therefore \qquad u_r = \sqrt{1-2\xi^2}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2} \qquad \text{...(4)}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \qquad \text{...(5)}$$

5. The phase angle ϕ of $T(j\omega)$ at resonant frequency is

$$\phi_r = -\tan^{-1}\left[\frac{\sqrt{1-2\xi^2}}{\xi}\right]$$

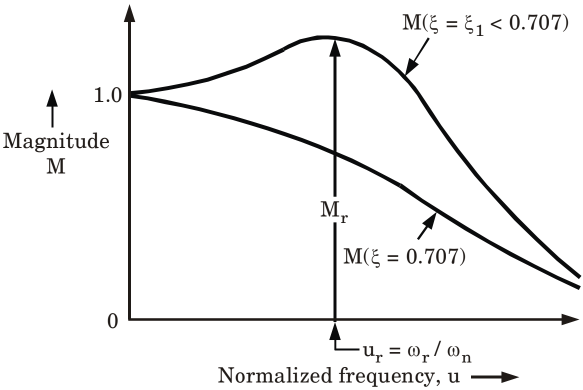


Fig. 8. Frequency response magnitude characteristics.

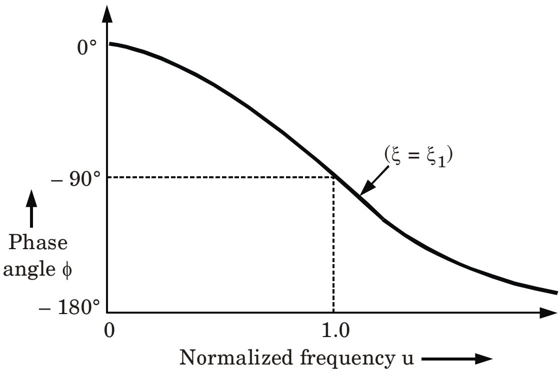


Fig. 9. Frequency response phase characteristic.

SECTION-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Find the generalized error coefficients for a system whose $G(s)\,H(s) = 1/s(s + 2)$ and also find the expression for steady state error for input $r(t) = 2 + 3t + 2t^3$.

Ans.

1. $r(t) = 2 + 3t + 2t^3;\ r'(t) = 3 + 6t^2\,;$

$r''(t) = 12t\,;\ r'''(t) = 12\,;\ r''''(t) = 0.$

The coefficients C_0, C_1, C_2 and C_3 need to be determined.

2. $$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} = \frac{1}{1+\dfrac{1}{s(s+2)}} = \frac{s^2+2s}{s^2+2s+1} = \frac{s^2+2s}{(s+1)^2}$$

3. $$E(s) = \frac{s^2+2s}{(s+1)^2}R(s) \quad \therefore F_e(s) = \frac{s^2+2s}{(s+1)^2}$$

4. $$C_0 = \lim_{s\to 0} F_e(s) = \lim_{s\to 0}\frac{s^2+2s}{(s+1)^2} = 0$$

5. $$C_1 = \lim_{s\to 0}\frac{dF_e(s)}{ds} = \lim_{s\to 0}\frac{d}{ds}\left\{\frac{s^2+2s}{(s+1)^2}\right\}$$

$$= \lim_{s\to 0}\frac{(s+1)^2(2s+2)-(s^2+2s)\,(2)\,(s+1)(1)}{(s+1)^4} = \lim_{s\to 0}\frac{2}{(s+1)^3} = 2$$

6. $$C_2 = \lim_{s\to 0}\frac{d^2F_e(s)}{ds^2} = \lim_{s\to 0}\frac{d}{ds}\left\{\frac{dF_e(s)}{ds}\right\} = \lim_{s\to 0}\frac{d}{ds}\left\{\frac{2}{(s+1)^3}\right\}$$

$$= \lim_{s\to 0}\frac{(s+1)^3(0)-2(3)\,(s+1)^2(1)}{(s+1)^6} = \lim_{s\to 0}-\frac{6}{(s+1)^4} = -6$$

7. $$C_3 = \lim_{s\to 0}\frac{d^3F_e(s)}{ds^3} = \lim_{s\to 0}\frac{d}{ds}\left\{\frac{d^2\,F_e(s)}{ds^2}\right\} = \lim_{s\to 0}\frac{d}{ds}\left\{-\frac{6}{(s+1)^4}\right\}$$

$$= \lim_{s\to 0}-\left\{\frac{(s+1)^4\,(0)-6\times 4(s+1)^3\,(1)}{(s+1)^8}\right\} = \lim_{s\to 0}\frac{24}{(s+1)^5} = 24$$

b. Sketch the polar plot for the following transfer function $G(s) = (1 + 4s)/s^2\,(s + 1)(2s + 1)$

Ans.

1. Putting $s = j\omega$, $$G(j\omega) = \frac{1+j4\omega}{(j\omega)^2\,(j\omega+1)\,(j2\,\omega+1)}$$

2. $$M = \frac{\sqrt{1+16\omega^2}}{\omega^2\sqrt{1+\omega^2}\,\sqrt{1+4\omega^2}}$$

3. $$\phi = \tan^{-1}4\omega - 180° - \tan^{-1}\omega - \tan^{-1}2\omega$$

4. The angle ϕ for different values of ω is tabulated in Table 1 :

Table 1.

ω	$\tan^{-1} 4\omega$	$-180°$	$-\tan^{-1}\omega$	$-\tan^{-1} 2\omega$	ϕ
0	0	$-180°$	0	0	$-180°$
∞	90°	$-180°$	$-90°$	$-90°$	$-270°$

5. The value of M, when $\omega = \infty$, $M = 0$, when $\omega = 0$, $M = \infty$
6. Polar plot is shown in Fig. 10.

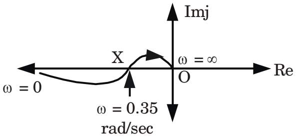


Fig. 10.

7. Polar plot makes an intercept on the negative real axis at point X. The intercept OX is calculate as given below :

$$G(j\omega)\,H(j\omega) = \frac{1 + 4j\omega}{(j\omega)^2(1 + j\omega)(1 + 2j\omega)}$$

Separating into real and imaginary parts

$$G(j\omega)\,H(j\omega) = -\left\{\frac{1 + 10\omega^2}{\omega^2[(1-2\omega^2)^2 + 9\omega^2]} + \frac{j(\omega - 8\omega^3)}{\omega^2[(1-2\omega^2)^2 + 9\omega^2]}\right\}$$

Equating imaginary part to zero

$$(\omega - 8\omega^3) = 0$$

$$8\omega^3 = 1$$

$$\omega = 0.35 \text{ rad/sec}$$

i.e., At $\omega = 0.35$ rad/sec,
plot crosses negative real axis.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Explain P, PI, PID controllers and also give their advantages.

Ans.

1. P controller :

1. In proportional control the actuating signal for the control action in a control system is proportional to the error signal.
2. The error signal being the difference between the reference input signal and the feedback signal obtained from the output.
3. For the system considered as shown in Fig. 11, the actuating signal is proportional to the error signal, therefore, the system is called proportional control system.
4. Consider a second order system where controller input is error itself and proportional constant is $K = 1$ as shown in Fig. 11.

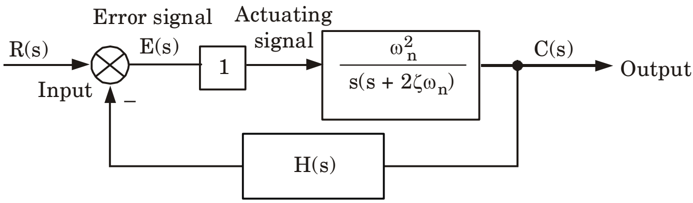


Fig. 11.

$$G(s)H(s) = \frac{\omega_n^2}{s(s+2\xi\,\omega_n)}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n^2}$$

For this system damping ratio is ξ and natural frequency ω_n.

5. And for steady state error

$$K_p = \lim_{s\to 0} G(s)H(s) = \infty$$

and $$K_v = \lim_{s\to 0} s\,G(s)H(s) = \frac{\omega_n}{2\xi}$$

Advantages :

1. Steady state error is reduced.
2. Disturbance signal rejection occurs.
3. Relative stability is improved.

2. Proportional integral controller (*PI*) :

1. A controller in the forward path, which changes the controller output corresponding to the proportional plus integral of the error signal is called PI controller.
2. Output of controller = $K\,e(t) + K_i \int e(t)\,dt$

Taking Laplace = $K\;E(s) + \frac{K_i}{s}E(s) = E(s)\left[K + \frac{K_i}{s}\right]$

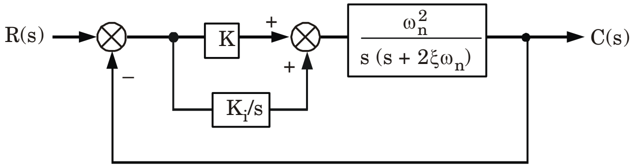


Fig. 12.

3. Assume, $K = 1$, we can write,

$$G(s) = \frac{\left[1+\frac{K_i}{s}\right]\omega_n^2}{s(s+2\xi\,\omega_n)} = \frac{(K_i + s)\,\omega_n^2}{s^2(s+2\xi\,\omega_n)}$$

i.e., system becomes TYPE 2 in nature.

$$\frac{C(s)}{R(s)} = \frac{(K_i + s)\,\omega_n^2}{s^3 + 2\xi\,\omega_n\,s^2 + s\,\omega_n^2 + K_i\,\omega_n^2}$$

i.e., it becomes third order.

4. Now as order increases by one, system relatively becomes less stable as K_i must be designed in such a way that system will remain in stable condition. Second order system is always stable.

while $$K_p = \lim_{s \to 0} G(s)H(s) = \infty \text{ , } e_{ss} = 0$$

$$K_v = \lim_{s \to 0} s\, G(s)H(s) = \infty, e_{ss} = 0$$

5. Hence as type is increased by one, error becomes zero for ramp type of input *i.e.*, steady state of system gets improved and system becomes more accurate in nature.

Advantages :

i. It increases order of the system.
ii. It increases TYPE of the system.
iii. Design of K_i must be proper to maintain stability of system.
vi. Steady state error reduces tremendously for same type of inputs.

3. PID controller :

1. It is a close loop system which has feedback control system and it compares the process variable (feedback variable) with set point and generates an error signal and according to that it adjusts the output of system.
2. It is the combination of proportional, integral and derivative controller.
3. A PD (Proportional derivative) type of controller improves transient part without affecting the steady state part.
4. While PI (Proportional integral) type of controller improves steady state part without affecting the transient part.
5. In PID, both PI and PD effects are incorporated. Hence both transient as well as steady state part of the response can be improved.
6. This can be realized as shown in the Fig. 13.

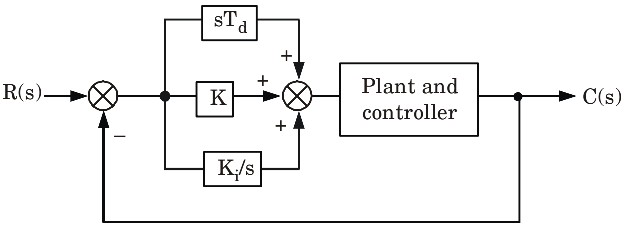


Fig. 13.

7. The output of PID controller is time domain is,

$$\text{Controller output} = Ke(t) + K_i \int e(t)\, dt + T_d \frac{de(t)}{dt}$$

8. Taking the Laplace transform, controller output in s-domain is,

$$\text{Controller output} = E(s)\left[K + \frac{K_i}{s} + s\,T_d\right]$$

Advantages :

1. Flexible and reliable.
2. Improves stability.

b. Derive the expressions for second order system for underdamped case and when the input is unit step.

Ans.

1. Consider the second order system with unity feedback.

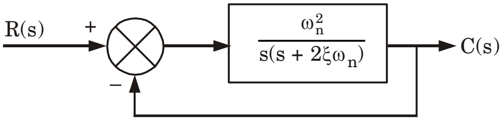


Fig. 14.

The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 / s(s+2\xi\omega_n)}{1+\omega_n^2 / s(s+2\xi\omega_n)}$$

where, ξ = Damping factor or Damping ratio

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s^2+2\xi\omega_n s+\omega_n^2)}$$

Then output, $C(s) = R(s)\dfrac{\omega_n^2}{s^2 + 2\xi\,\omega_n s + \omega_n^2}$ ...(1)

2. For unit step input

$$r(t) = 1$$

$$R(s) = \frac{1}{s}$$

Then $C(s) = \dfrac{1}{s}\,\dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ ...(2)

3. In eq. (2) putting $[s^2 + 2\xi\omega_n s + \omega_n^2] = [(s + \xi\omega_n)^2 + \omega_n^2(1-\xi^2)]$ and breaking it into partial fraction

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{[(s + \xi\omega_n)^2 + \omega_n^2(1-\xi^2)]} \quad ...(3)$$

Put $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s + \xi\,\omega_n)^2 + \omega_d^2} \quad ...(4)$$

4. Rewrite eq. (4)

$$C(s) = \frac{1}{s} - \frac{s + \xi\omega_n}{(s+\xi\,\omega_n)^2 + \omega_d^2} - \frac{\xi\,\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\xi\,\omega_n)^2+\omega_d^2} \quad ...(5)$$

5. Taking inverse Laplace transform of eq. (5),

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d}\,e^{-\xi\omega_n t}\sin\omega_d t$$

Put $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}[(\sqrt{1-\xi^2})\cos\omega_d t + \xi\sin\omega_d t] \quad ...(6)$$

6. Put $$\sin\phi = \sqrt{1-\xi^2},$$

$\therefore$ $$\cos\phi = \xi$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}(\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi) \quad ...(7)$$

where $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

and $$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

7. Eq. (7) is rewritten as

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[(\omega_n\sqrt{1-\xi^2})t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right] \quad ...(8)$$

8. The term ω_n is called natural frequency of oscillations. Term $\omega_d = \omega_n\sqrt{1-\xi^2}$ is called damped frequency of oscillations and the term ξ is called damping ratio or damping factor .

a. Underdamped case ($0 < \xi < 1$) : From eq. (8), time constant is $1/\xi\omega_n$ and the response having damped oscillations with overshoot and undershoot is known as underdamped response.

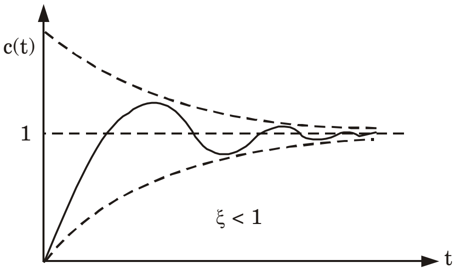


Fig. 15. Underdamped oscillations.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Construct the state model of a system characterized by the differential equation. Give the block diagram representation of the state model.

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = u$$

Ans.

1. $$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = u$$

2. Taking Laplace on both sides : $Y(s)(s^3 + 6s^2 + 11s + 6) = U(s)$
$$Y(s)/U(s) = 1/s^3 + 6s^2 + 11s + 6$$

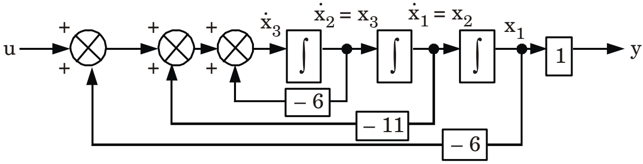


Fig. 16.

3. Choosing state variable : $\dot{x}_1 = x_2,\ \dot{x}_2 = x_3$
$$\dot{x}_3 = -6x_1 - 11x_2 - 6x_3 + u \text{ and } y = x_1$$

4. State model, $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = [1\ 0\ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

b. A single input single output system is given as
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } C = [1\ \ 0\ \ 0]$$
Test for controllability and observability.

Ans.

Given : $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $C = [1\ \ 0\ \ 0]$

To Test : Controllability and Observability.

A. Controllability test :

1. $$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -3 & -3 \end{bmatrix}$$
$$A^2B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -3 \\ 11 & 11 \end{bmatrix}$$

2. Controllability test matrix is given by

$$Q_c = [B : AB : A^2B]$$

$$Q_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -3 & -3 \\ 1 & 1 & -3 & -3 & 11 & 11 \end{bmatrix}$$

3. Consider $\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 1 \neq 0 \; ; \; |Q_c| \neq 0$

Hence rank of Q_c is equal to its order *i.e.*, 3. Therefore the system is controllable.

B. Observability test :

1. $$C = [1 \;\; 0 \;\; 0] \; ; \; C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \; ; \; A^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(A^T)^2 C^T = (A^T)(A^T C^T) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2. Observability test matrix is given by

$$Q_o = [C^T : A^T C^T : (A^T)^2 C^T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|Q_o| = 1 \neq 0$$

Hence its rank is 3 equal to its order *i.e.*, 3, therefore system is completely observable.

6. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. For the given transfer function $G(s) H(s) = 2/s(1 + 0.5s) (1 + 0.05s)$ Determine phase crossover frequency and gain margin.

Ans.

Given : $G(s) = \dfrac{2}{s(1 + 0.5s)(1 + 0.05s)}$

To Draw : Bode plot.

1. Magnitude plot :

$$G(s) = \frac{2}{s(1+0.5s)(1+0.05s)} = \frac{2}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{20}\right)}$$

First corner frequency (pole) $(\omega_1) = 2$ rad/s
Second corner frequency (pole) $(\omega_2) = 20$ rad/s

2. Phase plot :

$$G(s) = \frac{2}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{20}\right)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{2}{j\omega(1+0.5j\omega)(1+0.05j\omega)}$$

$$\phi = -90° - \tan^{-1} 0.5\omega - \tan^{-1} 0.05\omega$$

Magnitude plot :

Table 1.

S. No.	Factor	Corner frequency characteristics	Asymptotic log magnitude
1.	$1/s$	None	Straight line of constant slope (– 20 dB/dec) passing through at $\omega = 1$
2.	$1/(1 + 0.5s)$	$\omega_1 = 2$	Straight line of constant slope (– 20 dB/dec) originating from $\omega_1 = 2$
3.	$1/(1 + 0.05s)$	$\omega_2 = 20$	Straight line of constant slope (– 20 dB/dec) originating from $\omega_2 = 20$
4.	2	None	Straight line of constant slope of 0 dB/dec starting from 20 log 2 = 16 dB point

Phase plot :

Table 2.

S. No.	ω (rad/sec)	ϕ (degrees)
1.	0	– 90°
2.	0.2	– 96.28°
3.	1	– 119.42°
4.	2	– 140.71°
5.	5	– 172.23°
6.	10	– 195.25°
7.	15	– 209.27°

Bode plot :

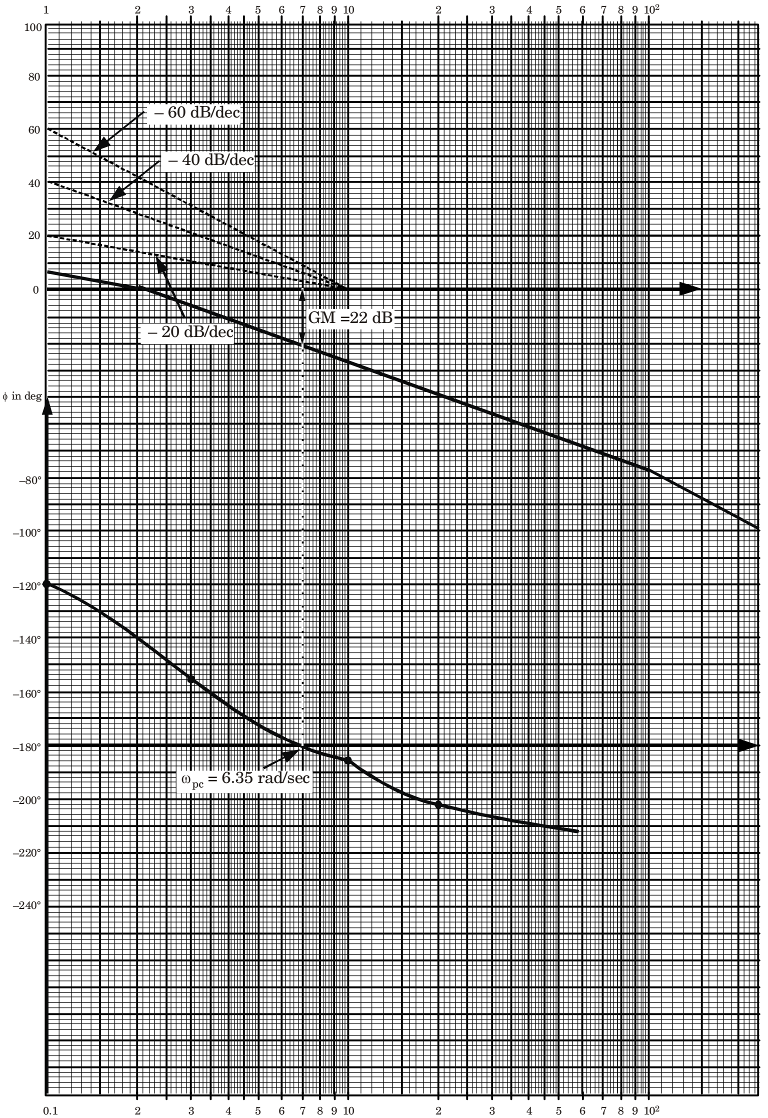


Fig. 17.

Results from bode plot :

Phase crossover frequency (ω_p) = 6.35 rad/sec

Gain margin = 22 dB

b. The forward path transfer function of unity feedback control system is $G(s) = 100/s(s + 6.45)$. Find the resonance peak M_r, resonant frequency ω_r and bandwidth of the closed loop system.

Ans.

1. $$G(s) = \frac{100}{s(s+6.45)} \text{ and } H(s) = 1$$

2. $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)} = \frac{100/(s(s+6.45))}{1+100/(s(s+6.45))}$$

$$= \frac{100}{s^2 + 6.45s + 100} \quad \text{...(1)}$$

3. Compare eq. (1) with $\dfrac{\omega_n{}^2}{s^2 + 2\xi\omega_n\, s + \omega_n{}^2}$

so, $\omega_n = \sqrt{100} = 10$ rad/sec

$2\xi\,\omega_n = 6.45 \quad \therefore \quad \xi = 0.32$

4. Resonant frequency, $\omega_r = \omega_n\sqrt{1-\xi^2} = 9.4$ rad/sec

5. Resonant peak, $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}} = \dfrac{1}{2\times 0.32\sqrt{1-0.32^2}} = 1.65$

6. Bandwidth, $\omega_b = \omega_n\sqrt{1 - 2\xi^2 + (2 - 4\xi^2 + 4\xi^4)^{1/2}} = 14.39$ rad/sec

7. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. What is the effect of adding pole to a system ? Discuss.

Ans.

1. Adding a pole at $s = -1/T$ to the forward-path transfer function of lead to

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)(1+Ts)}$$

2. We can obtain a qualitative indication on the bandwidth properties by referring to Fig. 19, which shows the plots of $|M(j\omega)|$ versus ω for $\omega_n = 1$, $\zeta = 0.707$, and various values of T. Since the system is now of the third order, it can be unstable for a certain set of system parameters.

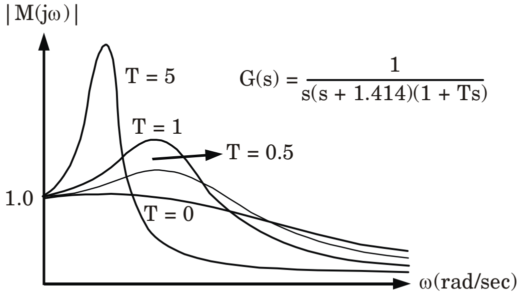


Fig. 18. Magnification curves for a third-order system with a forward-path transfer function G(s).

3. The effect of adding a pole to the forward-path transfer function is to make the closed-loop system less stable, while decreasing the bandwidth.
4. The unit-step responses of Fig. 19 show that for larger values of T, $T = 1$ and $T = 5$, the following relations are observed :
   i. The rise time increases with the decrease of the bandwidth,
   ii. The larger values of M_r also correspond to a larger maximum overshoot in the unit-step responses.

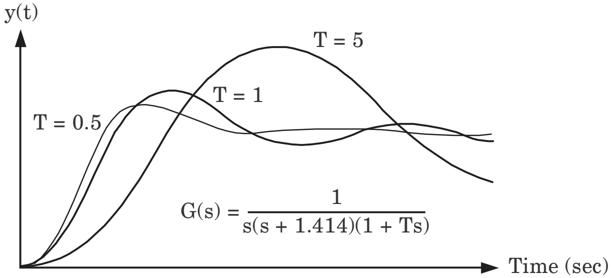


Fig. 19. Unit-step responses of a third-order system with a forward-path transfer function G(s).

b. Explain the lag compensation.

Ans.

A. Lag compensator :

1.

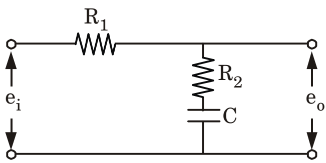


Fig. 20. Phase-lag network.

The transfer function of phase-lag network is shown in Fig. 21,

$$\frac{E_o(s)}{E_i(s)} = \frac{1+sT}{1+s\beta T} \quad ...(1)$$

where $\beta > 1$, $\beta = \dfrac{R_1 + R_2}{R_2}$

and $T = R_2C$

2. The transfer function given by eq. (1) can be expressed in sinusoidal form as

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{1+j\omega T}{1+j\omega\beta T} \quad ...(2)$$

3. Bode plot for transfer function of eq. (2) is shown in Fig. 21.

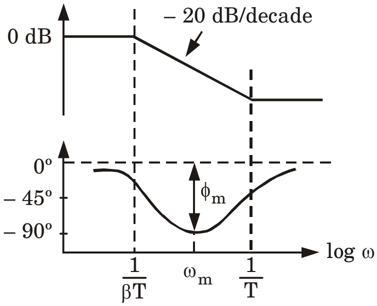


Fig. 21. Bode plot of lag compensator.

4. The two corner frequencies are $\omega = \dfrac{1}{T}$, upper corner frequency for zero at $s = -\dfrac{1}{T}$, $\omega = \dfrac{1}{\beta T}$, lower corner frequency for a pole at $s = -\dfrac{1}{\beta T}$

5. The maximum phase-lag, ϕ_m occurs at mid frequency ω_m between upper and lower corner frequencies.

$$\therefore \qquad \log_{10}\omega_m = \frac{1}{2}\left[\log\left(\frac{1}{\beta T}\right) + \log_{10}\left(\frac{1}{T}\right)\right]$$

$$\therefore \qquad \omega_m = \frac{1}{\sqrt{\beta T}}$$

6. The phase angle $\angle E_o(j\omega)/E_i(j\omega)$ calculated as

$$\angle\frac{E_o(j\omega)}{E_i(j\omega)} = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T)$$

At $\omega = \omega_m = \frac{1}{\sqrt{\beta T}}$, the phase angle is ϕ_m :

$$\tan \phi_m = \frac{1-\beta}{2\sqrt{\beta}}$$

$$\sin \phi_m = \frac{1-\beta}{1+\beta}$$

Pole-zero configuration is shown in Fig. 22.

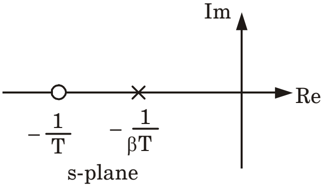


Fig. 22 Pole zero configuration.

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B.Tech.

(SEM. V) ODD SEMESTER THEORY EXAMINATION, 2019-20

CONTROL SYSTEM

Time : 3 Hours **Max Marks : 70**

Note : Attempt all sections. If any missing data is required, then choose suitably.

SECTION-A

1. Attempt **all** parts of the following : **(2 × 7 = 14)**

a. **The impulse response of unity feedback control system is $C(t) = te^{-t} + 2be^{-t}$, find transfer function.**

b. **Give example of type zero, type one and type two systems.**

c. **What is damping constant give its relation with time constant ?**

d. **What is centroid of asymptotes and how the centroid is calculated ?**

e. **Sketch polar plot of $G(s) = 1/(1 + \alpha S)$**

f. **Explain gain crossover frequency margin, phase crossover frequency.**

g. **Enlist the properties of state transition matrix.**

SECTION-B

2. Attempt any **three** parts of the following : **(7 × 3 = 21)**

a. **State and explain Masson's gain formula. For the system shown in the Fig. 1 find the overall transfer function of system using block diagram reduction.**

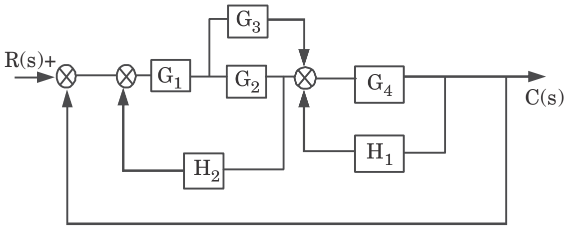

Fig. 1.

b. The close loop transfer function is given by
$T(s) = k\,(s + z)/ s^2 + 4s + 8$, where k, z is adjustable.

i. If $r(t) = t$ finds k and z so that steady error is zero.

ii. for the value of k, z obtain in part (i) find $e(\infty)$ for input $r(t) = t^2/2$

c. Explain stability on basis of location of poles and zeros. For a unity feedback system $G(s) = k/ s(s + 1)\,(1 + 2s)\,(1 + 3s)$. Determine range of k for stability, value of k for frequency of sustain oscillation.

d. Sketch Nyquist plot for $G(s)\,H(s) = 6 / s^2\,(s + 2)$ comment on stability.

e. Explain the term : State, State Space, State Vector. A SISO system has transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^3 + 7s^2 + 14s + 8}$$

write down the state equation and stage diagram.

SECTION-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Explain the effect of sensitivity on feedback system, Determine sensitivity of negative close loop system where $G(s) = 20/\ s(s + 4)$, $H(s) = 0.5$ wrt forward path transfer function, feedback path transfer function.

b. Derive block diagram of armature controlled and field controlled DC motor with proper labeling of circuit diagram.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Explain steady state error due to step input for type 0, 1 and 2 systems. Find steady stage error for – $G(s) = 10\,(1 + 4s)/s^2\,(1 + s)$, $H(s) = 1$, input $r(t) = 1 + t + t^2/2$.

b. i. Draw the root locus of characteristics equation for second order system as damping ratio varies from $-\infty$ to $+\infty$ keeping W_n constant.

ii. Explain the effect of adding poles and zero to transfer function.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A unity feedback system has an open loop transfer function.
Draw the root locus for the system.

$$G(s)\,H(s) = k\,(s+2)/(s+3)\,(s^2+2s+2)$$

b. What is the necessary condition for stability ? Explain limitation of Routh's stability method, construct Routh array and determine the stability of the system whose characteristics equation, $s^5 + 2s^4 + s^3 + 2s^2 + s + 4 = 0$

6. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Draw Bode Plot of unity feedback control system having

$$\text{OLTF } G(s) = \frac{10}{s(1+0.2s)\,(1+0.2s)}.$$

Determine GM, PM, gain cross over frequency, phase cross over frequency and discuss stability of closed loop system.

b. Explain the strengths of frequency response approach, establish correlation between frequency domain response and time domain response.

7. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. What is lag compensator ? What are the characteristics of lag compensation ? Explain the frequency response of lag compensator.

b. The state equations are given below, check controllability and observability of a system.

$$\dot{x}_1 = x_2 - x_3 + 3r$$
$$\dot{x}_2 = x_1 + x_2 + x_3 - 2r$$
$$\dot{x}_3 = x_1 + x_2 + r$$
$$y = x_1$$

☺☺☺

SOLUTION OF PAPER (2019-20)

Note : Attempt all sections. If any missing data is required, then choose suitably.

SECTION-A

1. Attempt **all** parts of the following : **(2 × 7 = 14)**

a. The impulse response of unity feedback control system is $C(t) = te^{-t} + 2be^{-t}$, find transfer function.

Ans. Numerical :

Given : $r(t) = \delta(t)$, $c(t) = te^{-t} + 2be^{-t}$
To Find : Transfer function.

1. Laplace transform gives

$$R(s) = 1$$

$$c(t) = t\,e^{-t} + 2b\,e^{-t}$$

2. Laplace transform of $c(t)$ gives

$$C(s) = \frac{1}{(s+1)^2} + \frac{2b}{(s+1)}$$

$$C(s) = \frac{1 + 2b\,(s+1)}{(s+1)^2} = \frac{(2bs + 2b + 1)}{(s+1)^2} \quad \text{...(1)}$$

3. For unity feedback control system

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$C(s) = \frac{G(s)}{1 + G(s)} R(s) \quad \text{...(2)}$$

4. Putting value of $C(s)$ and $R(s)$ in eq. (2)

$$\frac{(2\,bs + 2b + 1)}{(s+1)^2}\,(1 + G(s)) = G(s)$$

$$\frac{(2\,bs + 2b + 1)}{(s+1)^2} = G(s)\left[1 - \frac{(2\,bs + 2b + 1)}{(s+1)^2}\right]$$

$$= G(s)\left[\frac{s^2 + 1 + 2s - 2bs - 2b - 1}{(s+1)^2}\right]$$

5. Transfer function = $\dfrac{2bs + 2b + 1}{s^2 + 2s - 2bs - 2b}$

b. Give example of type zero, type one and type two systems.

Ans. Type zero system :

$$G(s)\,H(s) = \frac{k(s+z_1)(s+z_2)}{s^0(s+p_1)(s+p_2)}$$

Type one system :

$$G(s)\,H(s) = \frac{k(s+z_1)(s+z_2)}{s^1(s+p_1)(s+p_2)}$$

Type two system :

$$G(s)\,H(s) = \frac{k(s+z_1)(s+z_2)}{s^2(s+p_1)(s+p_2)}$$

c. What is damping constant give its relation with time constant ?

Ans. The damping constant is a measure of a system's settling time.

Relation with time constant : The damping constant is an inverse of time constant.

d. What is centroid of asymptotes and how the centroid is calculated ?

Ans. Centroid : All asymptotes intersect the real axis at a common point known as centroid.

$$\sigma = \frac{\Sigma \text{Real part of open loop poles} - \Sigma \text{Real part of open loop zeros}}{P - Z}$$

where, P = Number of open loop poles

Z = Number of open loop zeros.

e. Sketch polar plot of $G(s) = 1/(1 + \alpha S)$

Ans.

1. The frequency domain transfer function is,

$$G(j\omega)H(j\omega) = \frac{1}{1+\alpha j\omega} = \frac{1+j0}{1+j\omega\alpha}$$

$$|G(j\omega)H(j\omega)| = M = \frac{1}{\sqrt{1+\omega^2\alpha^2}}$$

$$\angle G(j\omega)\,H(j\omega) = \phi = \frac{\tan^{-1}\left(\frac{0}{1}\right)}{\tan^{-1}\left(\frac{\omega\alpha}{1}\right)} = \frac{0^\circ}{(\tan^{-1}\omega\alpha)} = -\tan^{-1}(\omega\alpha)$$

2. For various values of ω, the value of M and ϕ are :

ω	Magnitude (*M*)	Phase angle (φ)
0	1	0°
∞	0	– 90°

4. The corresponding polar plot is shown in Fig. 1.

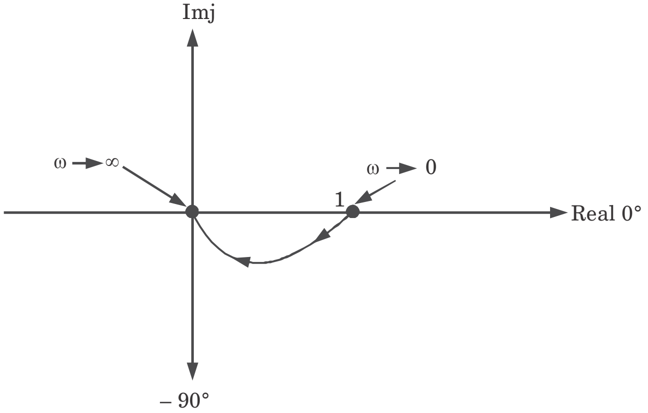

Fig. 1.

f. Explain gain crossover frequency margin, phase crossover frequency.

Ans.

a. Gain crossover frequency : It is the reciprocal of magnitude $|G(j\omega)|$ at the frequency at which the phase angle is – 180°.
Gain Margin (GM),

$$K_g = \frac{1}{|G(j\omega_c)|}$$

where, ω_c = Phase cross-over frequency.

b. Phase crossover frequency : The phase margin is that amount of the additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.
Phase margin is equal to 180° plus the angle of $G(j\omega)$ at the gain crossover point.

$$\phi_m = 180° + \phi$$

g. Enlist the properties of state transition matrix.

Ans.

1. $\phi(0) = 1$
2. $\phi^{-1}(t) = \phi(-t)$
3. $[\phi(t)]^k = \phi(kt)$

SECTION-B

2. Attempt any **three** parts of the following : **(7 × 3 = 21)**

a. State and explain Masson's gain formula. For the system shown in the Fig. 2 find the overall transfer function of system using block diagram reduction.

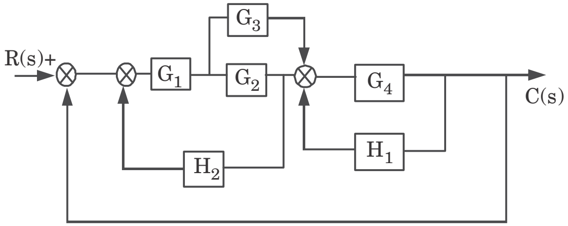

Fig. 2.

Ans.

i. Masson's gain formula :

1. The overall gain can be determined by Mason's gain formula given below

$$T = \sum_{k=1}^{k} \frac{P_k \Delta_k}{\Delta}$$

where, P_k = Forward path gain of k_{th} path from a specified input node to an output node.

Δ = Determinant which involves closed-loop gain and mutual interactions between non-touching loops.

= 1 – [Sum of all individual loop gain]

+ [Sum of loop gain products of all possible pair of non-touching loops]

– [Sum of loop gain products of all possible triplets of non-touching loops]

+ [..........] – [...........]

Δ_k = Path factor associated with the concerned path and involves all closed loops in the graph which are isolated from the forward path under consideration.

2. The path factor Δ_k for the k_{th} path is equal to the value of the graph determinant of a signal flow graph which exists after erasing the k_{th} path from the graph.

ii. Numerical :

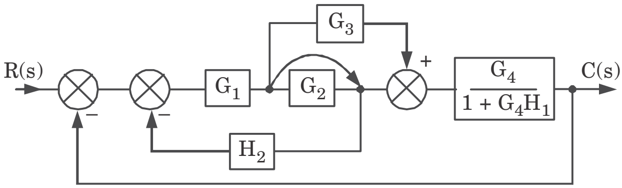

Fig. 3.

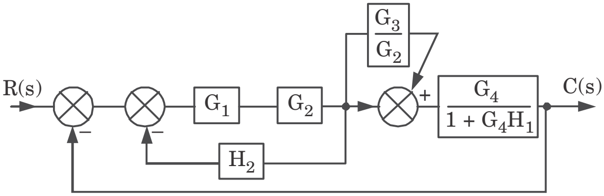

Fig. 4.

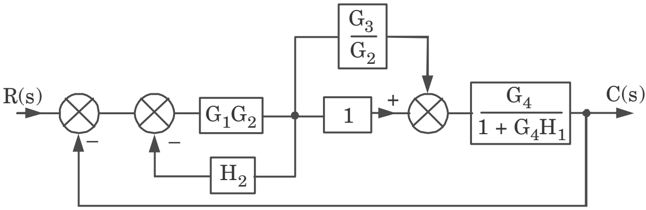

Fig. 5.

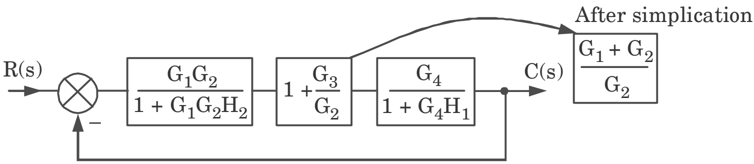

Fig. 6.

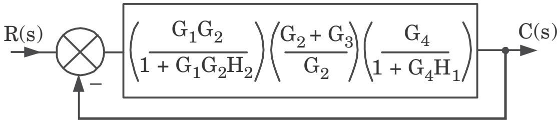

Fig. 7.

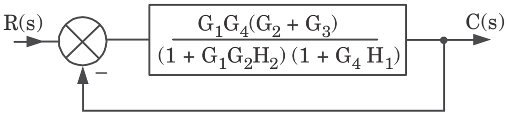

Fig. 8.

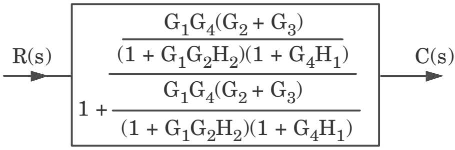

Fig. 9.

$$\frac{C(s)}{R(s)} = \frac{G_1G_4(G_2+G_3)}{1+G_1G_2H_2+G_4\,H_1+G_1G_2G_4H_1H_2+G_1\,G_4\,(G_2+G_3)}$$

b. The close loop transfer function is given by
$T(s) = k\,(s+z)/\,s^2+4s+8$, Where k, z is adjustable.
i. If $r(t) = t$ finds k and z so that steady error is zero.
ii. for the value of k, z obtain in part (i) find $e(\infty)$ for input $r(t) = t^2/2$

Ans.
i.

1. Given, $\dfrac{C(s)}{R(s)} = \dfrac{k(s+z)}{s^2+4s+8}$

   and, $e_{ss} = 0$

2. We know $e_{ss} = \lim\limits_{s\to 0} sE(s)$

   $0 = \lim\limits_{s\to 0} sE(s)$

   but $E(s) = R(s) - C(s)$

   then $0 = \lim\limits_{s\to 0} s\,[R(s) - C(s)]$

   or $R(s) = C(s)$

   $R(s) = \dfrac{k(s+z)}{s^2+4s+8}$

   $s^2 + 4s + 8 = k(s+z)$

3. Compare the coefficients of s in the both sides.

   s^1 coefficient $4 = k$

   s^0 coefficient $8 = kz$

   or, $z = 2$

ii.

1. Given, $r(t) = t^2/2$
2. Taking Laplace transform, $R(s) = -\dfrac{1}{s^3}$
3. We know $e_{ss} = \lim\limits_{s\to 0} sE(s)$

   $E(s) = R(s) - C(s) = R(s) - \dfrac{k(s+z)}{(s^2+4s+8)}\,R(s)$

4. Putting the value of k and z then we get,

   $E(s) = R(s) - \dfrac{4(s+2)}{(s^2+4s+8)}\,R(s)$

$$E(s) = \frac{(s^2 + 4s + 8 - 4s - 8)}{s^2 + 4s + 8} R(s)$$

$$e_{ss} = \left| \lim_{s \to 0} s \frac{s^2}{s^2 + 4s + 8} \times -\frac{1}{s^3} \right| = \frac{1}{8} = 0.125$$

c. Explain stability on basis of location of poles and zeros. For a unity feedback system $G(s) = k/ s(s + 1) (1 + 2s) (1 + 3s)$. Determine range of k for stability, value of k for frequency of sustain oscillation.

Ans.

A. Stability on basis of location :

1. The linear system is stable if the rightmost pole(s) is/are on the left-hand half plane (LHHP) on the s-plane.
2. The linear system is marginally stable if the rightmost pole(s) is/are simple order (first-order) on the $j\omega$ axis, including the origin on the s-plane.
3. The linear system is unstable if the rightmost pole(s) is/are on the right-hand half plane (RHHP) of the s-plane or if the rightmost pole(s) is/are multiple-order on the $j\omega$ axis on the s-plane.
4. Zeros do not affect system Stability.

B. Numerical :

$$G(s) = \frac{k}{s(s+1)(1+2s)(1+3s)}$$

$$H(s) = 1$$

1. Characteristics equation,

$$1 + G(s) H(s) = 0$$

$$1 + \frac{k}{s(s+1)(1+2s)(1+3s)} = 0$$

$$s(s + 1) (1 + 2s) (1 + 3s) + k = 0$$

$$6s^4 + 11s^3 + 6s^2 + s + k = 0$$

$$\begin{array}{c|ccc}
s^4 & 6 & 6 & k \\
s^3 & 11 & 1 & 0 \\
s^2 & \frac{60}{11} & k & 0 \\
s^1 & \dfrac{\frac{60}{11} - 11k}{\frac{60}{11}} & & 0 \\
s^0 & k & &
\end{array}$$

2. For system to be stable there should not be sign change in the first column.

$\therefore$ $k > 0$ for s^0

and $60 - 121\,k > 0$

$$60 > 121\,k$$

$$0.49 > k,\ k < 0.49$$

3. So the range of k is $0 < k < 0.49$

$$k_{mar} = 0.49$$

4. To find frequency, find out roots of auxiliary equation at marginal value of k

$$A(s) = \frac{60}{11}s^2 + k = 0$$

$$60\,s^2 + 11\,k = 0$$

$$60\,s^2 + 11 \times 0.49 = 0$$

$$60\,s^2 + 5.39 = 0$$

$$60\,s^2 = -5.39$$

$$s^2 = \frac{-5.39}{60}$$

$$s^2 = 0.089$$

$$s = \sqrt{-0.089} = \pm j0.298$$

5. Comparing with $s = \pm j\omega$

so ω = Frequency of oscillations = 0.298 rad/sec.

d. Sketch Nyquist plot for $G(s)\,H(s) = 6 / s^2\,(s + 2)$ comment on stability.

Ans.

1. In frequency domain the given transfer function will become

$$G(j\omega)\,H(j\omega) = \frac{6}{(j\omega)^2\,(j\omega + 2)}$$

2. Magnitude (M) :

$$|G(j\omega)H(j\omega)| = \frac{6}{\omega^2\sqrt{\omega^2 + 4}}$$

3. Phase angle (ϕ) :

$$\angle G(j\omega)H(j\omega) = \angle -180 - \tan^{-1}\frac{\omega}{2}$$

4. For various value of ω, the value of M and ϕ are :

ω	M	ϕ
0	∞	– 180°
∞	0	– 270°

5. The corresponding Nyquist plot is shown in Fig. 10.

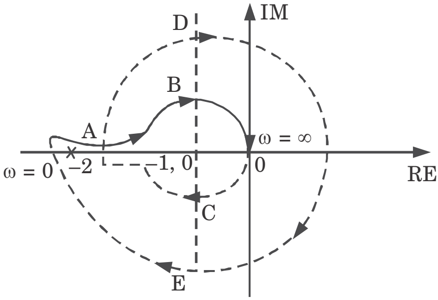

Fig. 10.

6. For stability

$$N = P - Z$$

where N = Number of encirclement

P = Number of poles enclosed by the contour

Z = Number of zeros enclosed by the contour

$$N = 2$$
$$P = 0$$
$$2 = 0 - Z$$
$$Z = 2$$

So the system is unstable because its value lies in the right half s-plane.

e. Explain the term : State, State Space, State Vector. A SISO system has transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^3 + 7s^2 + 14s + 8}$$

write down the state equation and stage diagram.

Ans.

A. i. State : The state of a dynamic system is the smallest set of variables such that the knowledge of these variables at $t = t_0$ with the knowledge of the input for $t \geq t_0$ completely determines the behaviour of the system for any time $t \geq t_0$.

ii. State space : The n-dimensional space whose coordinate axes consists of the x_1 axis, x_2 axis ..., x_n axis is called state space. Any state can be represented by a point in the state space.

iii. State vector : If we need n variable to completely describe the behaviour of a given system, then these n state variables may be considered as n component of a vector x. Such a vector is called state vector.

B. Numerical :

1. The given transfer function can be written as,

$$\frac{Y(s)}{X_1(s)} \cdot \frac{X_1(s)}{U(s)} = 1 \times \frac{1}{s^3 + 7s^2 + 14s + 8}$$

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3 + 7s^2 + 14s + 8}$$

2. The differential equation in time domain is written as,

$$\dddot{x}_1 + 7\ddot{x}_1 + 14\dot{x}_1 + 8x(t) = u(t)$$

Let

$$\dot{x}_1 = x_2 = 0.x_1 + 1.x_2 + 0.x_3 + 0.u$$
$$\dot{x}_2 = x_3 = 0.x_1 + 0.x_2 + 1.x_3 + 0.u$$
$$\dot{x}_3 = -8x_1 - 14x_2 - 7x_3 + 1.u$$

3. In matrix form,

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

As $\dfrac{Y(s)}{X_1(s)} = 1$

4. It can be written in time domain as

$$y(t) = x_1(t)$$
$$= 1.x_1 + 0.x_2 + 0.x_3$$

5. In matrix form

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

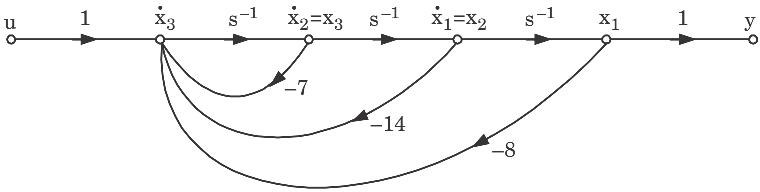

Fig. 11.

SECTION-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Explain the effect of sensitivity on feedback system, Determine sensitivity of negative close loop system where $G(s) = 20/ s(s + 4)$, $H(s) = 0.5$ wrt forward path transfer function, feedback path transfer function.

Ans.

i. Effect of sensitivity :

$$S_G^T = \frac{\%\text{ Change in } T}{\%\text{ Change in } G} = \frac{\dfrac{\partial T}{T} \times 100}{\dfrac{\partial G}{G} \times 100}$$

$$S_G^T = \frac{G}{T}\frac{dT}{dG}$$

We know $$T = \frac{C}{R} = \frac{G}{1+GH}$$

$$S_G^T = \frac{G}{\left(\dfrac{G}{1+GH}\right)}\left[\frac{(1+GH)-GH}{(1+GH)^2}\right]$$

$$S_G^T = (1+GH)\frac{1}{(1+GH)^2} = \frac{1}{1+GH}$$

So sensitivity of feedback system (with respect to G) is reduced by factor of $1 + GH$.

ii. Numerical :

Given : $20/s(s + 4)$, $H = 0.5$

To Find : Sensitivity, S_G^T

Assumption : $\omega = 1.5$ rad/sec

1. Sensitivity with respect to forward path :

$$S_G^T = \frac{1}{1+GH}$$

$$\therefore \quad S_G^T = \frac{1}{1+\dfrac{20\times 0.5}{s(s+4)}} = \frac{s(s+4)}{s^2+4s+10} = \frac{s^2+4s}{s^2+4s+10}$$

Put $s = j\omega$, $$S_G^T = \frac{(j\omega)^2+4j\omega}{(j\omega)^2+4(j\omega)+10} = \frac{(j1.5)^2+4(j1.5)}{(j1.5)^2+4(j1.5)+10}$$

$$= \frac{-2.25+j6}{-2.25+j6+10} = \frac{-2.25+j6}{7.75+j6}$$

$$= \frac{(-2.25+j6)\times(-7.75+j6)}{(j6+7.75)(j6-7.75)}$$

$$= \frac{17.43-j46.5-j13.5-36}{-36-60.06}$$

$$= \frac{-18.57-j60}{-96.06}$$

$$|S_G^T| = \frac{\sqrt{(-18.57)^2+(-60)^2}}{96.06} = \frac{62.80}{96.06} = 0.65$$

2. **Feedback path :**

$$S_G^T = \frac{-GH}{1+GH} = \frac{\dfrac{-20\times 0.5}{s(s+4)}}{1+\dfrac{20\times 0.5}{s(s+4)}} = \frac{-10}{s^2+4s+10}$$

Put $s = j\omega$

$$S_G^T = \frac{-10}{(j\omega)^2 + 4j\omega + 10} = \frac{-10}{(j1.5)^2 + 4(j1.5) + 10} = \frac{-10}{-2.25 + j6 + 10}$$

$$= \frac{-10}{7.75 + j6}\frac{(j6 - 7.75)}{(j6 - 7.75)} = \frac{-j60 + 77.5}{(j6)^2 - (7.75)^2} = \frac{-j60 + 77.5}{-36 - 60.06}$$

$$= \frac{-j60 + 77.5}{-96.06}$$

$$|S_G^T| = \frac{\sqrt{(-60)^2 + (77.5)^2}}{96.06} = \frac{98.01}{96.06} = 1.02$$

b. Derive block diagram of armature controlled and field controlled DC motor with proper labeling of circuit diagram.

Ans.

i. Armature controlled DC motor :

1. Consider the armature controlled DC motor (DC servo motor) and assume that the demagnetizing effect of armature reaction is neglected, field voltage is constant and magnetic circuit is linear. Armature controlled DC servo motor is shown in Fig. 12.

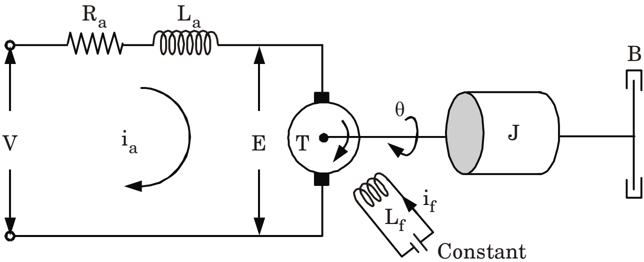

Fig. 12.

2. Let
   - R_a = Armature resistance
   - L_a = Armature inductance
   - i_a = Armature current
   - E = Induced emf in armature
   - V = Applied armature voltage
   - θ = Angular displacement of the motor shaft
   - T = Torque developed by motor
   - J = Equivalent moment of inertia of motor shaft and load referred to the motor
   - B = Equivalent viscous friction coefficient.
3. Apply KVL in armature circuit

$$V = \frac{L_a di_a}{dt} + R_a i_a + E \quad \text{...(1)}$$

4. In the armature-controlled DC motor, the field current is held constant. For a constant field current, the flux becomes constant, and the torque becomes directly proportional to the armature current so that

$$T \propto \phi\, i_a$$
$$T = Ki_a \quad \text{...(2)}$$

5. When armature is rotating, an emf is induced

$$E = \frac{K_b d\theta}{dt} \quad \text{...(3)}$$

6. The armature current produces the torque which is applied to the inertial mass and friction hence the force balance equation is

$$\frac{Jd^2\theta}{dt^2} + \frac{Bd\theta}{dt} = T = KI_a \quad \text{...(4)}$$

7. Taking the Laplace transform on both sides of eq. (4) and (1)

$$(sL_a + R_a)\, I_a(s) + E(s) = V(s) \quad \text{[Initial condition is zero]}$$
$$(Js^2 + Bs)\, \theta(s) = T(s) = KI_a(s) \quad \text{...(5)}$$

8. The transfer function of this system is obtained as

$$\frac{\theta(s)}{V(s)} = \frac{K}{s[L_a Js^2 + (BL_a + JR_a)s + R_a B + KK_b]} \quad \text{...(6)}$$

9. Block diagram is shown in Fig. 13.

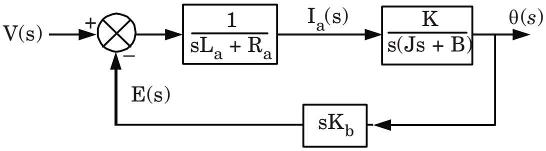

Fig. 13.

ii. Field controlled DC motor :

1. A schematic diagram of a field controlled DC motor (DC servo motor) shown in Fig. 14.

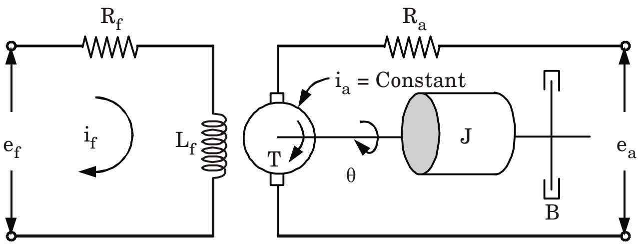

Fig. 14.

2. Here, R_f = Field winding resistance
L_f = Field winding inductance
I_f = Field winding current
R_a = Armature resistance
i_a = Armature current
θ = Angular displacement.

3. The torque T developed by the motor is proportional to product of the air-gap flux ϕ and armature current i_a so we get

$$T = K_1 \phi i_a \quad ...(1)$$

where K_1 is constant.

4. But the air gap flux ϕ and the field current i_f are proportional for the usual operating range of the motor and i_a is assumed to be constant, we can rewrite the above equation as

$$T = K_2 i_f \quad ...(2)$$

where K_2 is a constant.

5. The equations for this system are

$$L_f \frac{di_f}{dt} + R_f i_f = e_f \quad ...(3)$$

and $$\frac{Jd^2\theta}{dt^2} + \frac{Bd\theta}{dt} = T = K_2 i_f \quad ...(4)$$

6. By taking the Laplace transform on both sides of eq. (3) and (4) where all initial conditions are zero, we get

$$(L_f s + R_f)\, i_f(s) = E_f(s) \quad ...(5)$$

$$(Js^2 + Bs)\, \theta(s) = K_2 i_f(s) \quad ...(6)$$

7. From the above equations, the transfer function of this system is obtained as

$$\frac{\theta(s)}{E_f(s)} = \frac{K_2}{s(L_f s + R_f)(Js + B)} \quad ...(7)$$

8. Block diagram is shown in Fig. 15.

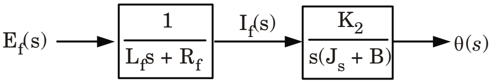

Fig. 15.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Explain steady state error due to step input for type 0, 1 and 2 systems. Find steady stage error for $-G(s) = 10(1 + 4s)/s^2(1 + s)$, $H(s) = 1$, input $r(t) = 1 + t + t^2/2$.

Ans.

A. Steady state error :

i. Unit step input :

Here $$r(t) = \begin{cases} 1; & t > 0 \\ 0; & t < 0 \end{cases}$$

$$R(s) = \frac{1}{s}$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)} \qquad ...(1)$$

$$E(s) = \frac{1}{s + sG(s)H(s)}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{1}{s + sG(s)H(s)}$$

$$= \frac{1}{1 + \lim_{s \to 0} G(s)H(s)} = \frac{1}{1 + K_p}$$

where, $K_p = \lim_{s \to 0} G(s)H(s)$

K_p = Position error constant.

For step input, $e_{ss} = \dfrac{1}{1 + K_p}$

Case 1 : For type '0'

K_p = Constant

e_{ss} = Constant

Case 2 : For type '1'

$K_p = \infty$

$$e_{ss} = \frac{1}{1 + \infty} = 0$$

Case 3 : For type '2'

$K_p = \infty$

$$e_{ss} = \frac{1}{1 + \infty} = 0$$

B. Numerical : $G(s) = \dfrac{10\,(1 + 4s)}{s^2\,(1 + s)}$

$H(s) = 1$

1. $$k_p = \lim_{s \to 0} G(s)\,H(s) = \lim_{s \to 0} \frac{10\,(1 + 4s)}{s^2(1 + s)} = \infty$$

2. $$k_v = \lim_{s \to 0} G(s) \cdot H(s) = \lim_{s \to 0} s \frac{10\,(1 + 4s)}{s^2(1 + s)} = \infty$$

3. $$k_a = \lim_{s \to 0} s^2\, G(s) \cdot H(s) = \lim_{s \to 0} \frac{10\,(1 + 4s)}{(1 + s)} = 10$$

$$r(t) = 1 + t + \frac{t^2}{2}$$

4. Steady state error is given by,

$$e_{ss} = \lim_{s \to 0} s \frac{R(s)}{1 + G(s)\, H(s)}$$

$$e_{ss} = \frac{R_1}{1 + k_p} + \frac{R_2}{k_v} + \frac{R_3}{k_a}$$

$$= \frac{1}{1 + \infty} + \frac{1}{\infty} + \frac{1/2}{10} = 0.05$$

b. i. Draw the root locus of characteristics equation for second order system as damping ratio varies from – ∞ to + ∞ keeping W_n constant.

ii. Explain the effect of adding poles and zero to transfer function.

Ans.

i. Root locus of characteristics equation for second order system :

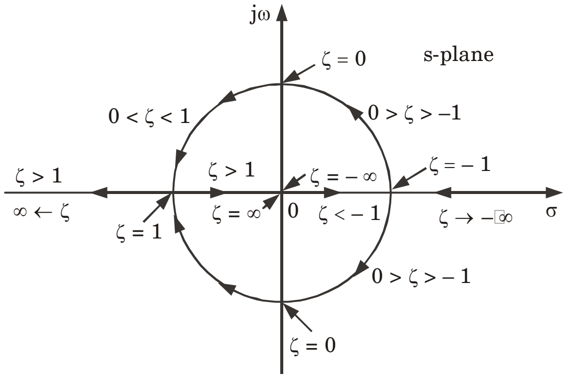

Fig. 16.

ii. A. Effect of adding poles to the forward path transfer function :

1. Adding a pole at $s = -1/T$ to the forward-path transfer function of lead to

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)(1 + Ts)}$$

2. We can obtain a qualitative indication on the bandwidth properties by referring to Fig. 17, which shows the plots of $|M(j\omega)|$ versus ω for $\omega_n = 1$, $\zeta = 0.707$, and various values of T. Since the system is now of the third order, it can be unstable for a certain set of system parameters.

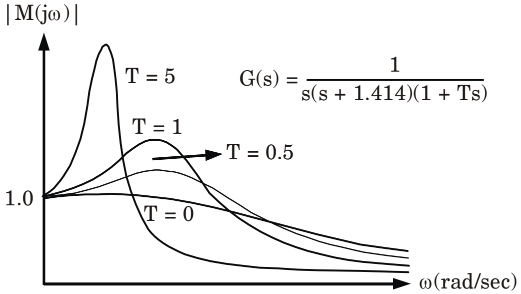

Fig. 17. Magnification curves for a third-order system with a forward-path transfer function G(s).

3. The effect of adding a pole to the forward-path transfer function is to make the closed-loop system less stable, while decreasing the bandwidth.
4. The unit-step responses of Fig. 18 show that for larger values of T, $T = 1$ and $T = 5$, the following relations are observed :
   i. The rise time increases with the decrease of the bandwidth,
   ii. The larger values of M_r also correspond to a larger maximum overshoot in the unit-step responses.

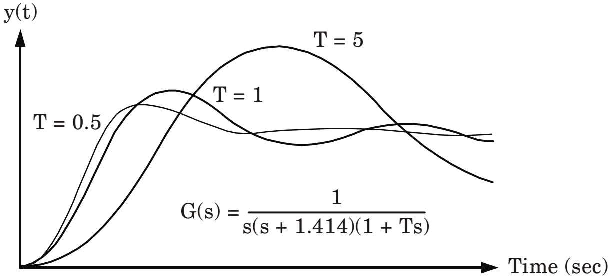

Fig. 18. Unit-step responses of a third-order system with a forward-path transfer function G(s).

B. Effect of adding a zero to the forward path transfer function :

1. Consider unity feedback system having forward path transfer function as,

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)} \quad \text{...(1)}$$

2. Let us add a zero at $s = -1/T$ to the transfer function so that eq. (1) becomes

$$G(s) = \frac{\omega_n^2(1 + Ts)}{s(s + 2\xi\omega_n)}$$

3. The closed-loop transfer function for unity feedback system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2(1+Ts)}{s^2+(2\xi\omega_n+T\omega_n^2)s+\omega_n{}^2}$$

$$\text{B.W.} = \sqrt{\left(-b+\frac{1}{2}\sqrt{b^2+4\omega_n^4}\right)}$$

where $\quad b = 4\xi^2\,\omega_n^2 + 4\xi\,\omega_n^3 + T - 2\omega_n^2 - \omega_n^4$

for $\quad \xi = 0.707$ and $\omega_n = 1$

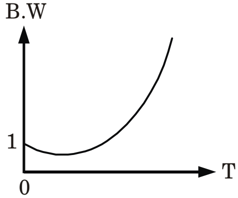

Fig. 19.

4. The general effect of adding a zero to the forward path transfer function is to increase the bandwidth of the closed-loop system.

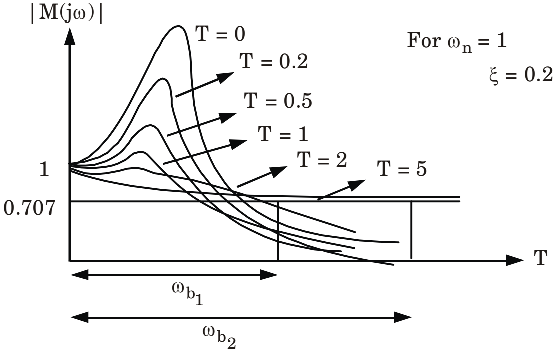

Fig. 20.

5. So bandwidth increases as T-increases

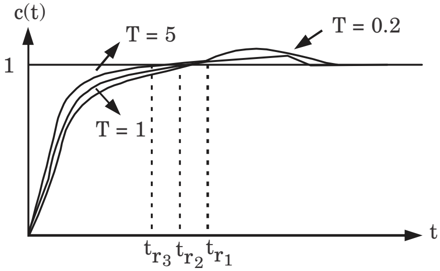

Fig. 21.

$t_{r3} < t_{r2} < t_{r1}$

So as T increases rise time decreases.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A unity feedback system has an open loop transfer function.
Draw the root locus for the system.

$$G(s)\,H(s) = k\,(s+2)/(s+3)\,(s^2+2s+2)$$

Ans.
$$G(s)\,H(s) = \frac{k\,(s+2)}{(s+3)\,(s^2+2s+2)}$$

Step-1 :

Poles, $P_1 = -3,\ P_2 = -1+j,\ P_3 = -1-j$

Zeros, $Z = -2$

No of poles = 3, No of zeros = 1

Step-2 : No of asymptotes $= P - Z = 3 - 1 = 2$

Angle of asymptotes

$$\theta_1 = \frac{(2k+1)\times 180°}{P-Z} = \frac{(2\times 0+1)\times 180°}{2} = 90°$$

$$\theta_2 = \frac{(2\times 1+1)\times 180°}{2} = 270°$$

Step-3 : Centroid

$$\sigma = \frac{\Sigma\ R.P \text{ of poles} - \Sigma R.P \text{ of zeros}}{P-Z}$$

$$= \frac{(-3-1-1)-(-2)}{2} = \frac{-5+2}{2} = -1.5$$

Step-4 : Breakaway point :

Characteristics equation,

$$1 + G(s)\,H(s) = 0$$

$$1 + \frac{k\,(s+2)}{(s+3)\,(s^2+2s+2)}$$

$(s+3)\,(s^2+2s+2) + k\,(s+2) = 0$

$s^3 + 5s^2 + 8s + 6 + k\,(s+2) = 0$

$$k = \frac{-s^3 - 5s^2 - 8s - 6}{s+2}$$

$$\therefore \qquad \frac{dk}{ds} = 0$$

$$\frac{d}{ds}\left[\frac{-s^3-5s^2-8s-6}{s+2}\right] = 0$$

$$(s+2)\,[-3s^2-10s-8]-[-s^3-5s^2-8s-6]\times 1 = 0$$

$$-2s^3-11s^2-20s-10 = 0$$

$$2s^3+11s^2+20s+10 = 0$$

$$s = -0.80,\ -2.34+0.84j - 2.34 - 0.84]$$

That breakaway point is not valid

Step-5 : Intersection point : Characteristics equation,

$$1 + G(s)\,H(s) = 0$$

$$1+\frac{k\,(s+2)}{(s+3)\,(s^2+2s+2)} = 0$$

$$s^3+5s^2+8s+6+ks+2k = 0$$

Apply Routh's criteria,

s^3	1	k + 8
s^2	5	6 + 2 k
s^1	$\frac{34+3k}{5}$	0
s^0	6 + 2k	0

$$\frac{34+3k}{5} = 0$$

$$34+3k = 0$$

$$3k = -34$$

$$k = -34/3 = -11.33$$

Auxiliary equation

$$5s^2+6+2k = 0$$

$$5s^2+6+2\times -11.33 = 0$$

$$s^2 = 3.33 = s = \pm 1.82$$

That intersection point is not valid.

Step-6 : Angle of departure :

$$\phi_d = 180° - [\phi_P - \phi_Z]$$

$$\phi_{P1} = 90°,\ \phi_{P2} = 26.56°$$

$$\phi_Z = 45°$$

$$\phi_d = 180° - [90° + 26.56° - 45°] = 108.44°$$

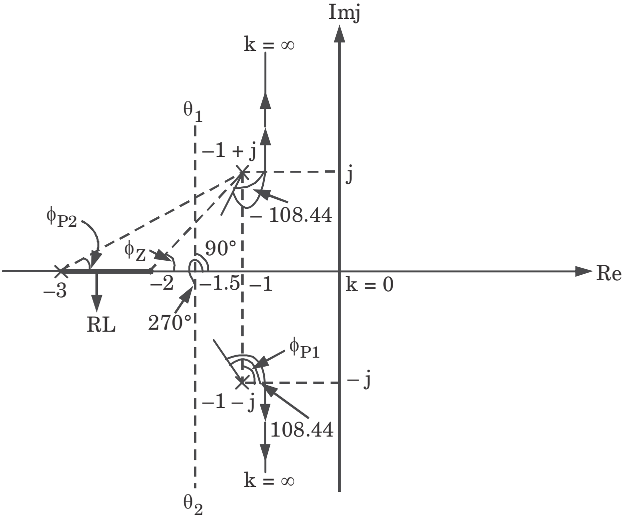

Fig. 22.

b. What is the necessary condition for stability ? Explain limitation of Routh's stability method, construct Routh array and determine the stability of the system whose characteristics equation, $s^5 + 2s^4 + s^3 + 2s^2 + s + 4 = 0$

Ans.

i. Necessary condition :

i. All the coefficients of characteristics equation must have same sign.

ii. There should be no missing term.

iii. All poles of transfer function should be in left half of s-plane.

iv. The degree of denominator polynomial of transfer function is greater or equal to that of numerator polynomial.

ii. Limitation :

1. It is valid only if the characteristics equation is algebraic.
2. If any coefficient of the characteristics equation is complex or contain power of e then this criterion cannot be applied.
3. It gives information about how many roots are lying in the RHS of the s-plane but values of the roots are not available. Also it cannot distinguish between real and complex roots.

iii. Numerical :

1. Routh array is :

$$\begin{array}{c|ccc} s^5 & 1 & 1 & 1 \\ s^4 & 2 & 2 & 4 \\ s^3 & 0 & -1 & \\ s^2 & & & \\ s^1 & & & \\ s^0 & & & \end{array}$$

2. Putting ε (very small positive value) in place of 0 in the first column Routh array

$$\begin{array}{c|ccc} s^5 & 1 & 1 & 1 \\ s^4 & 2 & 2 & 4 \\ s^3 & \varepsilon & -1 & 0 \\ s^2 & \dfrac{2\varepsilon+2}{\varepsilon}(+\text{ ve}) & 4 & \\ s^1 & \dfrac{-\left(\dfrac{2\varepsilon+2}{\varepsilon}\right)-4\varepsilon}{\left(\dfrac{2\varepsilon+1}{\varepsilon}\right)}(-\text{ ve}) & 0 & \\ s^0 & -4 & & \end{array}$$

3. Since there is sign change in the first column of Routh array, therefore system is unstable.

6. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Draw Bode Plot of unity feedback control system having

OLTF G(s) = $\dfrac{10}{s(1+0.2s)(1+0.2s)}$ **.**

Determine GM, PM, gain cross over frequency, phase cross over frequency and discuss stability of closed loop system.

Ans.

Given : $G(s) = \dfrac{10}{s(1+0.2s)(1+0.2s)}$

To Draw : Bode plot.

1. **Magnitude plot :**

$$G(s) = \frac{10}{s(1+0.2s)(1+0.2s)} = \frac{10}{5\left(1+\dfrac{s}{5}\right)\left(1+\dfrac{s}{5}\right)}$$

Corner frequency (ω_1 or ω_2) = 5 rad/s

2. Phase plot :

$$G(s) = \frac{10}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{5}\right)}$$

Put, $s = j\omega$

$$G(j\omega) = \frac{10}{(j\omega)(1+0.2j\omega)(1+0.2j\omega)}$$

$$\phi = -90° - \tan^{-1}(0.2\omega) - \tan^{-1}(0.2\omega)$$

Magnitude plot :

Table 1.

S. No.	Factor	Corner frequency	Asymptotic log magnitude
1.	$1/s$	None	Straight line of constant slope (– 20 dB/dec) passing through at $\omega = 1$
2.	$1/(1+0.2s)^2$	$\omega_1, \omega_2 = 5$	Straight line of constant slope (– 40 dB/dec) originating from $\omega_1, \omega_2 = 5$
3.	10	None	Straight line of constant slope 0 dB/dec starting from 20 log 10 = 20dB point.

Phase plot :

Table 2.

S. No.	ω (rad/sec)	ϕ (degrees)
1.	0	– 90°
2.	0.2	– 94.58°
3.	1	– 112.6°
4.	2	– 135.24°
5.	5	– 203.09°
6.	10	– 216.87°
7.	15	– 259.65°

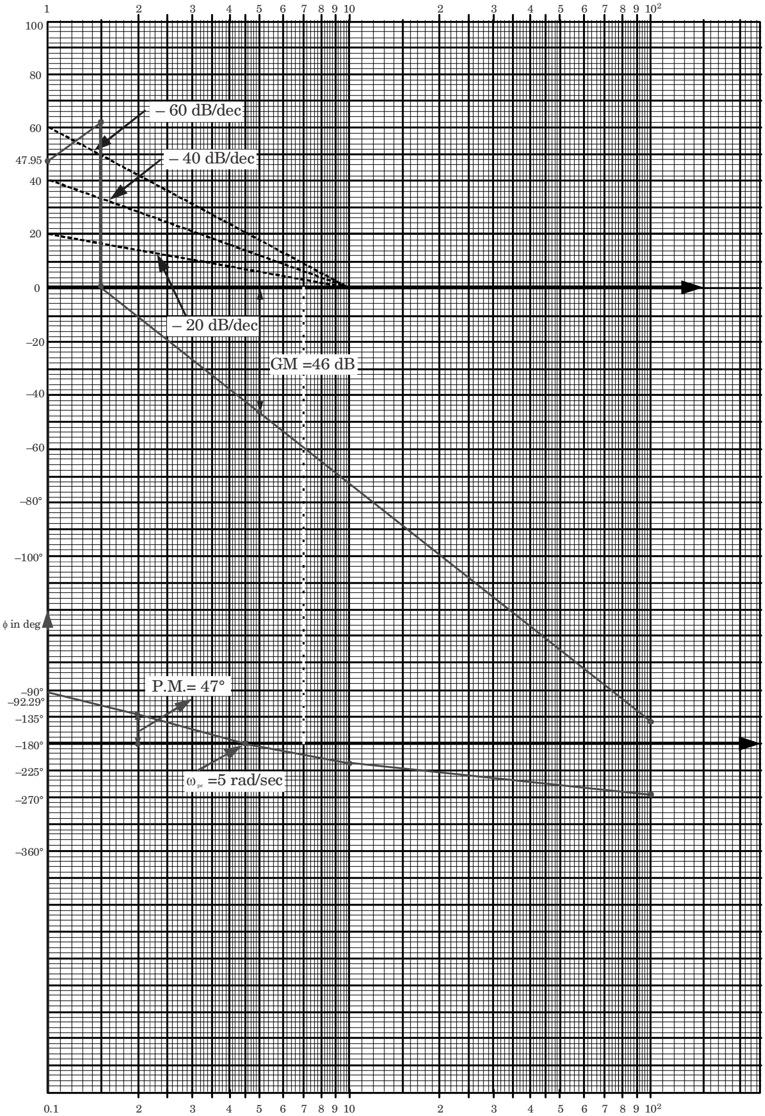

Fig. 23.

Results from bode plot :

A. Phase crossover frequency (ω_{pc}) = 5 rad/sec

B. Gain margin = 46 dB

C. Phase margin = 47°

D. Phase crossover frequency = 0.2 rad/sec

E. **Stability :** The gain margin as well as phase margin are both positive therefore, the system is stable.

b. Explain the strengths of frequency response approach, establish correlation between frequency domain response and time domain response.

Ans. Closed loop frequency response :

1. Consider the transfer function for closed loop system,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

2. For unity feedback, $H(s) = 1$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} \quad ...(1)$$

Put $s = j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)} \quad ...(2)$$

3. The polar plot of eq. (2) is shown in Fig. 25.

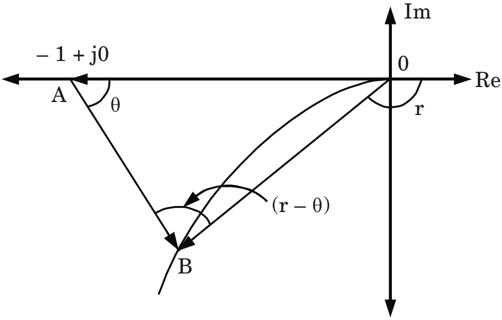

Fig. 25.

4. From Fig. 25, $\vec{OB} = G(j\omega)$

$$\vec{OA} = -1$$

$$\vec{AB} = \vec{OB} - \vec{OA} = G(j\omega) - (-1)$$

$$\vec{AB} = 1 + G(j\omega)$$

5. From eq. (2)

$$\left|\frac{C(j\omega)}{R(j\omega)}\right| = M(\omega) = \frac{\vec{OB}}{\vec{AB}}$$

$$\frac{\angle C(j\omega)}{\angle R(j\omega)} = \frac{\angle \vec{OB}}{\angle \vec{AB}} = \frac{\angle r}{\angle \theta} = \angle (r - \theta)$$

$$\therefore \qquad \frac{C(j\omega)}{R(j\omega)} = M(\omega)\, e^{j\,\phi(\omega)}$$

where $M(j\omega)$ is the magnitude and $\phi(\omega) = r - \theta$.

6. Frequency response consists of two parts :
   i. Magnitude,
   ii. Phase angle.
   iii. Both can be plotted against different values of ω.
7. Hence frequency response of closed loop system is plot of magnitude and phase angle.

Correlation :

1. For 2^{nd} order system , the transfer function is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where, ξ = Damping factor

ω_n = Natural frequency of oscillations

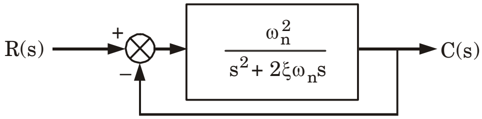

Fig. 26.

2. Closed loop frequency response is,

$$\frac{C(j\omega)}{R(j\omega)} = T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n(j\omega) + \omega_n^2} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2j\xi\left(\frac{\omega}{\omega_n}\right)}$$

$$= \frac{1}{(1 - u^2) + j2\xi u} \qquad ...(3)$$

where $u = \omega/\omega_n$, normalized driving frequency.

$$\therefore \qquad |T(j\omega)| = M = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \qquad ...(4)$$

$$\text{and} \qquad \angle T(j\omega) = \phi = -\tan^{-1}\frac{2\xi u}{1-u^2} \qquad ...(5)$$

3. The steady state output is

$$c(t) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \sin\left(\omega t - \tan^{-1}\frac{2\xi u}{1-u^2}\right)$$

$\therefore$ From eq. (4) and (5) when

$$u = 0, M = 1 \text{ and } \phi = 0$$

$$u = 1, M = \frac{1}{2\xi} \text{ and } \phi = -\frac{\pi}{2}$$

$$u = \infty, M = 0 \text{ and } \phi = -\pi$$

4. The frequency where M has a peak value is called the resonant frequency. At this frequency the slope of magnitude curve is zero.

If ω_r = Resonant frequency.

$u_r = \omega_r/\omega_n$ is normalized resonant frequency.

$$\left.\frac{dM}{du}\right|_{u=u_r} = -\frac{1}{2}\frac{\left[-4(1-u_r^2)u_r + 8\xi^2 u_r\right]}{\left[(1-u_r^2)^2 + (2\xi u_r)^2\right]^{3/2}} = 0$$

$$-4(1-u_r^2)\,u_r + 8\xi^2 u_r = 0$$

$$-4u_r(1-u_r^2 - 2\xi^2) = 0$$

$$\therefore \quad u_r = \sqrt{1-2\xi^2}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2} \quad \text{...(6)}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad \text{...(7)}$$

5. The phase angle ϕ of $T(j\omega)$ at resonant frequency is

$$\phi_r = -\tan^{-1}\left[\frac{\sqrt{1-2\xi^2}}{\xi}\right]$$

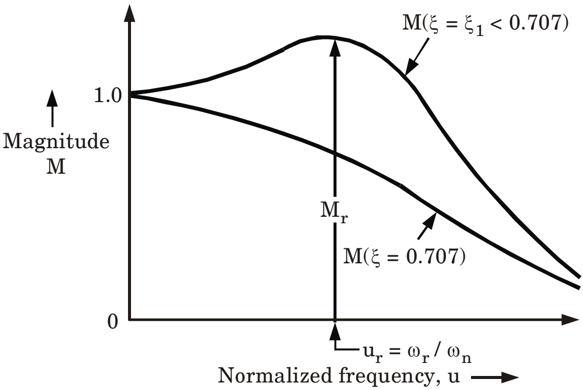

Fig. 27. Frequency response magnitude characteristics.

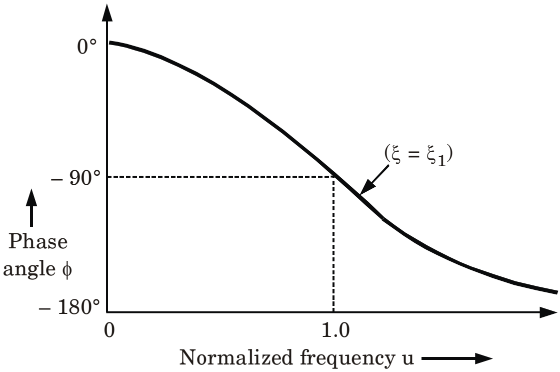

Fig. 28. Frequency response phase characteristic.

7. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. What is lag compensator ? What are the characteristics of lag compensation ? Explain the frequency response of lag compensator.

Ans.

A. Lag compensator : The lag compensator is an electrical network which produces a sinusoidal output having the phase lag when a sinusoidal input is applied.

B. Characteristics :

1. Lag compensator allows high gain at low frequencies thus it is basically a low pass filter. Hence it improves the steady state performance.
2. A lag compensator is to provide attenuation in the high frequency range to give a system sufficient phase margin.

C. Frequency response of lag compensator :

1. The transfer function of a lag compensator with unity zero-frequency gain is of the form,

$$D(s) = \frac{\tau s + 1}{\beta \tau s + 1} \quad \beta > 1,\ \tau > 0$$

2. The sinusoidal transfer function is of the form,

$$D(j\omega) = \frac{1 + j\omega\tau}{1 + j\omega\beta\tau}$$

3. The bode plot of $D(j\omega)$, shown in Fig. 29, has two corner frequencies at $\omega = 1/\beta\tau$ and $\omega = 1/\tau$.
4. The phase lag mainly occurs within and around the two corner frequencies.
5. It must be recognized here that any phase lag at the gain crossover frequency of the compensated system is undesirable.
6. To prevent detrimental effects of phase lag due to the lag compensator, the corner frequencies of the lag compensator must

be located substantially lower than the gain crossover frequency of the compensated system.

7. Fig. 29 also shows that in the high frequency range, the lag compensator has an attenuation of 20 log β dB, which is the property that is utilized to give a system sufficient phase margin.
8. The addition of a lag compensator results in an improvement in the ratio of control signal to noise signal in the loop.
9. The high frequency noise signals are attenuated by a factor $\beta > 1$, while the low frequency control signals undergo unit amplification (0 dB gain).
10. The choice of β is usually restricted because a very large β will appreciably reduce the gain crossover frequency and consequently speed of response of the system. A typical choice of β is 10.

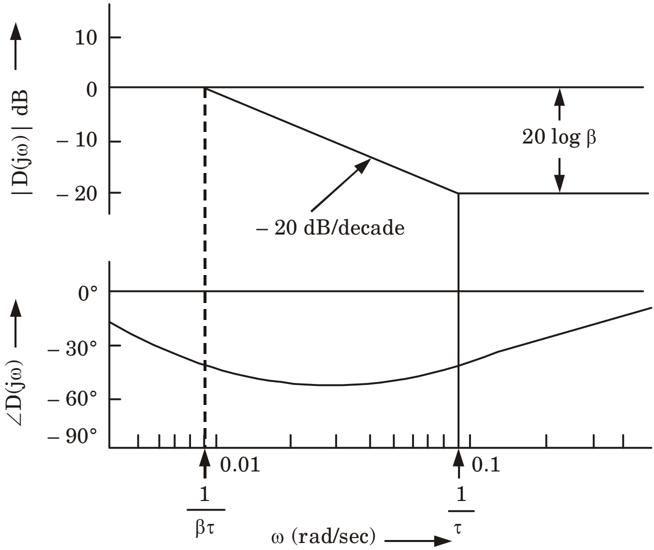

Fig. 29.

b. The state equations are given below, check controllability and observability of a system.

$$\dot{x}_1 = x_2 - x_3 + 3r$$
$$\dot{x}_2 = x_1 + x_2 + x_3 - 2r$$
$$\dot{x}_3 = x_1 + x_2 + r$$
$$y = x_1$$

Ans.

$$\dot{x}_1 = x_2 - x_3 + 3r$$
$$\dot{x}_2 = x_1 + x_2 + x_3 - 2r$$
$$\dot{x}_3 = x_1 + x_2 + r$$

$$y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} r$$

$$y = [1\;0\;0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

i. Controllability :

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \; C = [1\;0\;0]$$

$$AB = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 3 & 0 \\ 1 & 2 & 0 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 3 & 0 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$Q_C = [B : AB : A^2B]$$

$$= \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

$$Q_C = 0$$

Hence rank of Q_C is equal to its order *i.e.*, 3 Therefore the system is controllable.

ii. observability :

$$A^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

$$(A^T)^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

$$(A^T)^2 C^T = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Q_0 = [C^T : A^T C^T : (A^T)^2 C^T]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= 1$$

Hence rank of Q_0 is equal to its order *i.e.*, 3, so the system is observable.

☺☺☺