Quantum Series

ENGINEERING

QUANTUM Series

Semester - 5 Mechanical Engineering

Heat and Mass Transfer

- Topic-wise coverage of entire syllabus in Question-Answer form.
- Short Questions (2 Marks)

Includes solution of following AKTU Question Papers
2015-16 • 2016-17 • 2017-18 • 2018-19 • 2019-20

CONTENTS

KME-501 : HEAT & MASS TRANSFER

1

UNIT

Introduction to Heat Transfer

PART-1

Thermodynamics and Heat Transfer, Modes of Heat Transfer : Conduction, Convection and Radiation, Effect of Temperature on Thermal Conductivity of Materials, Introduction to Combined Heat Transfer Mechanism, General Differential Heat Conduction Equation in Rectangular, Cylindrical and Spherical Coordinate Systems.

CONCEPT OUTLINE : PART-1

Thermodynamics : It is the science which deals with the relations among heat, work and properties of system which are in equilibrium. It describes state and changes in state of physical systems.

Heat Transfer : It is defined as the transmission of energy from one region to another as a result of temperature gradient.

Modes of Heat Transfer :

1. Conduction,
2. Convection, and
3. Radiation.

Thermal Conductivity : The amount of energy conducted through a body of unit area and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unity.

Combined Heat Transfer : This is the case which involves the use of two or more than two modes of heat transfer.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **What are the mechanisms of heat transfer ? How are they distinguished from each other ?** **AKTU 2014-15, Marks 10**

Answer

A. **Mechanisms of Heat Transfer :** Following are the three modes of heat transfer :

a. **Mechanism of Heat Transfer through Conduction :**

1. Thermal conduction is a mechanism of heat propagation from region of higher temperature to a region of lower temperature within a medium or between different mediums in direct physical contact.

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2. Conduction does not involves any movement of macroscopic portion of matter relative to one-another.
3. The conduction is done by :
   i. Due to random molecular motion *i.e.*, vibration of molecules about its equilibrium position and the concept is termed as micro form of heat transfer and is usually referred as diffusion of energy.
   ii. The thermal energy may be transferred by means of electrons which are free to move through the lattice structure of the material.
4. Heat transfer by conduction is prescribed by Fourier law :

$$Q = -kA\frac{dt}{dx}$$

Where, k = Conduction heat transfer coefficient,
A = Cross-sectional area perpendicular to direction of heat flow, and
$\frac{dt}{dx}$ = temperature gradient.

b. Mechanism of Heat Transfer by Convection :

1. Thermal convection is a process of energy transport affected by the circulation or mixing of a fluid medium.
2. Convection is possible only in a fluid medium and is directly linked with the transport of medium itself.
3. The effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid.
4. There are basically two types of convection :
   i. Natural or free convection, and
   ii. Forced convection.
5. Convective heat transfer is prescribed by Newton's law of cooling *i.e.*,

$$Q = hA(t_s - t_f)$$

Where, h = Convection heat transfer coefficient,
t_s = Surface temperature,
t_f = Fluid temperature, and
A = Area exposed to heat transfer.

c. Mechanism of Heat Transfer by Radiation :

1. The mechanism of heat transfer by radiation consists of three distinct phase :
   i. Conversion of thermal energy of the hot source into electromagnetic wave.
   ii. Passage of wave motion through intervening space.
   iii. Transformation of wave into heat.
2. Radiation heat transfer is governed by Stefan-Boltzmann law :

$$E_b = \sigma_b A T^4 \quad \text{(for black body)}$$

Where,
E_b = Energy radiated per unit time,
σ_b = Stefan-Boltzmann constant
$= 5.67 \times 10^{-8}$ W/m^{2}K^4, and
T = Absolute temperature.

B. Difference between Conduction, Convection and Radiation :

S. No.	Conduction	Convection	Radiation
1.	Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place.	Each particle absorbing heat is mobile.	Heat flows without any intervening medium in the form of electromagnetic waves.
2.	Medium is necessary for conduction.	Medium is necessary for convection.	Medium is not necessary for radiation.
3.	It is a slow process.	It is also a slow process.	It is a very fast process.
4.	Path of heat flow may be zig-zag.	Path may be zig-zag or curved.	Path is a straight line.
5.	Conduction takes place in solids.	Convection takes place in fluids.	Radiation takes place in gaseous and transparent media.
6.	The temperature of the medium increases through which heat flows.	In this process also the temperature of medium increases.	There is no change in the temperature of the medium.

Que 1.2. **Define thermal conductivity. Discuss the effect of temperature on thermal conductivity.**

Answer

A. Thermal Conductivity :

1. The amount of heat conducted through a body of unit area, and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unity.
2. The unit of thermal conductivity is W/mK or W/m °C.

B. Effect of Temperature on Thermal Conductivity :

1. Thermal conductivity of most of the metals decreases with increase in temperature.
2. Due to increase in temperature the density of solid decreases and consequently thermal conductivity also decreases.
3. In most of the liquids, the values of thermal conductivity tends to decrease with temperature due to decrease in density and with increase in temperature.
4. But in case of gases thermal conductivity increases with temperature because the thermal conductivity is directly proportional to the mean free path of the molecules.
5. So, gases with higher molecular weights have smaller thermal conductivity than those having lower molecular weight.
6. The dependence of thermal conductivity on temperature is as given below,

$$k = k_0 (1 + \beta t)$$

Where, k_0 = Thermal conductivity at 0 °C.

β = Temperature co-efficient of thermal conductivity which is positive for non-metal and insulating material and negative for metallic conductor.

7. Thermal conductivity of porous material depends upon the type of gas or liquid present in the voids.

Que 1.3. **Briefly explain the combined heat transfer mechanism.**

Answer

1. In actual conditions, more than one mode of heat transfer is involved.
2. For example, consider a plate of emissivity ϵ maintained at temperature T_s. The heat transfer from this plate will take place through both convection and radiation.
3. The heat loss from this plate is

$$q = q_{conv} + q_{rad}$$

$$q = hA(T_s - T_\infty) + \sigma A \in (T_s^4 - T_\infty^4)$$

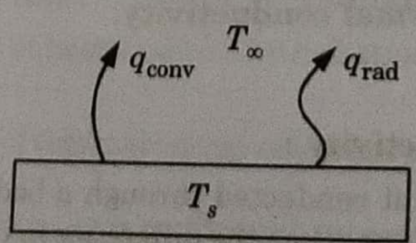


Fig. 1.3.1.

Que 1.4. **A brick (k = 1.2 W/m-K) wall 0.15 m thick separates hot combustion gases of a furnace from the outside ambient air which is at 25 °C. The outer surface temperature of the brick wall is found to be 100 °C. If the natural convection heat transfer coefficient on the outside of the brick wall is 20 W/m²-K and its emissivity is 0.8, calculate the inner surface temperature of the brick wall.**

AKTU 2013-14, Marks 10

Answer

Given : k = 1.2 W/m-K, L = 0.15 m, t_∞ = 25 °C = 25 + 273 = 298 K
t_2 = 100 °C = 273 + 100 = 373 K, h = 20 W/m²-K, ε = 0.8
To Find : Inner surface temperature of brick wall.

1. As we know that,

$$Q_{cond.} = Q_{conv.} + Q_{rad.}$$

$$k\left(\frac{t_1 - t_2}{L}\right) = h\,(t_2 - t_\infty) + \sigma\,\varepsilon\,(t_2^4 - t_\infty^4)$$

$$1.2 \times \left(\frac{t_1 - 373}{0.15}\right) = 20\,(373 - 298) + 5.67 \times 10^{-8} \times 0.8 \times [(373)^4 - (298)^4]$$

$$= 2020.3$$

$$t_1 = \frac{2020.3 \times 0.15}{1.2} + 373 = 625.5 \text{ K} = 352.5\ °\text{C}$$

Que 1.5. **A carbon steel plate (k = 45 W/m-K) 600 mm × 900 mm × 25 mm is maintained at 310 °C. Air at 15 °C blows over the hot plate. If convection heat transfer coefficient is 22 W/m²-°C and 250 W is lost from the plate surface by radiation. Calculate the inside plate temperature.**

AKTU 2017-18, Marks 10

Answer

Given : A = 600 mm × 900 mm = 0.6 × 0.9 = 0.54 m²
L = 25 mm = 0.025 m, t_s = 310 °C, t_f = 15 °C, h = 22 W/m²-°C
$Q_{rad.}$ = 250 W, k = 45 W/m-°C
To Find : Inside plate temperature.

1. In this case the heat conducted through the plate is removed from the plate surface by the combination of convection and radiation.
Heat conducted through the plate = Convection heat losses + Radiation heat losses

$$Q_{cond.} = Q_{conv.} + Q_{rad.}$$

$$-kA\,\frac{dt}{dx} = hA\,(t_s - t_f) + 250$$

$$-45 \times 0.54 \times \frac{(t_s - t_i)}{L} = 22 \times 0.54 \times (310 - 15) + 250$$

$$-45 \times 0.54 \times \frac{(310 - t_i)}{0.025} = 22 \times 0.54 \times 295 + 250$$

$$972\,(t_i - 310) = 3754.6$$

$$t_i = \frac{3754.6}{972} + 310 = 313.86\ °C$$

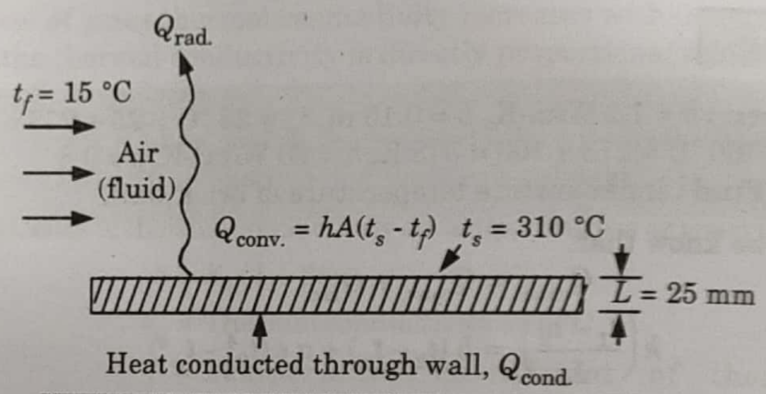


Fig. 1.5.1. Combination of conduction, convection and radiation heat transfer.

Que 1.6. **Derive the general heat conduction equation in rectangular or cartesian coordinates.**

Answer

1. Consider an infinitesimal rectangular parallelopiped (volume element) of sides dx, dy, and dz parallel, respectively, to the three axes (X, Y, Z) in a medium in which temperature is varying with location and time as shown in Fig. 1.6.1.
2. Let,

t = Temperature at the left face $ABCD$; this temperature may be assumed uniform over the entire surface, since the area of this face can be made arbitrarily small, and

$\frac{dt}{dx}$ = Rate of change of temperature along X-direction.

Then, $\left(\frac{\partial t}{\partial x}\right)dx$ = Change of temperature through distance dx, and

$t + \left(\frac{\partial t}{\partial x}\right)dx$ = Temperature on the right face $EFGH$ (at a distance dx from the left face $ABCD$).

Further, let, k_x, k_y, k_z = Thermal conductivities along X, Y and Z axes.

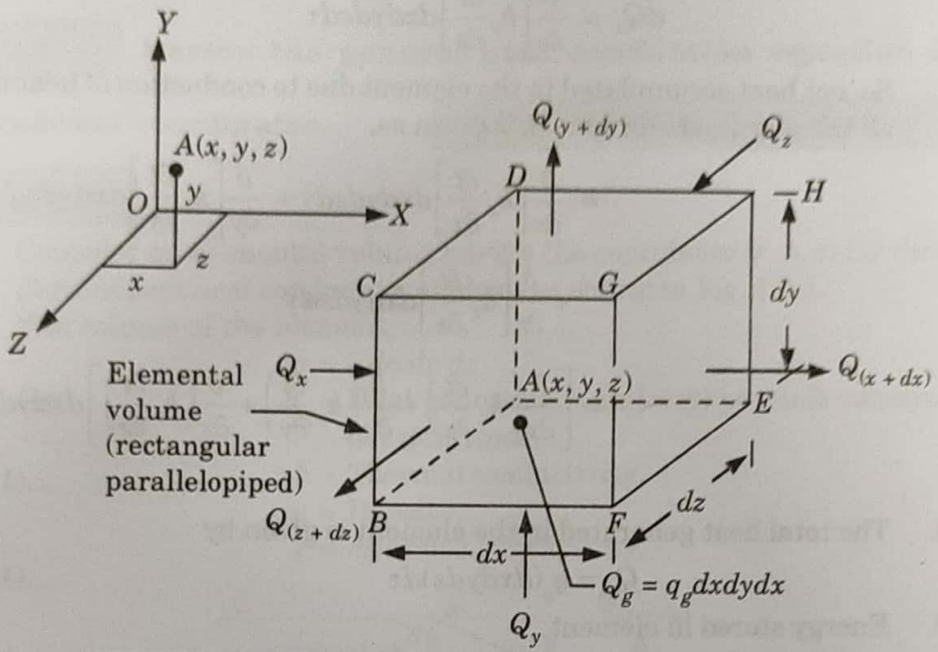


Fig. 1.6.1. Elemental volume for three-dimensional heat conduction analysis - Cartesian coordinates.

3. Quantity of heat flowing into element from the left face *ABCD* during the time interval $d\tau$ in *X*-direction is given by

Heat influx, $$Q_x = -k_x\,(dydz)\frac{\partial t}{\partial x}\,d\tau \qquad \text{...(1.6.1)}$$

4. During the same time interval $d\tau$ the heat flowing out of the right face of control volume (*EFGH*) will be

Heat efflux, $$Q_{(x+dx)} = Q_x + \frac{\partial}{\partial x}(Q_x)dx \qquad \text{...(1.6.2)}$$

5. Heat accumulation in the element due to heat flow in *X*-direction, subtracting eq. (1.6.2) from eq. (1.6.1), we get

$$dQ_x = Q_x - \left[Q_x + \frac{\partial}{\partial x}(Q_x)dx\right]$$

$$= -\frac{\partial}{\partial x}(Q_x)dx$$

$$= -\frac{\partial}{\partial x}\left[-k_x(dydz)\frac{\partial t}{\partial x}d\tau\right]dx$$

$$= \frac{\partial}{\partial x}\left[k_x\frac{\partial t}{\partial x}\right]dxdydzd\tau$$

6. Similarly the heat accumulated due to heat flow by conduction along *Y* and *Z*-directions in time $d\tau$ will be :

$$dQ_y = \frac{\partial}{\partial y}\left[k_y\frac{\partial t}{\partial y}\right]dxdydzd\tau$$

$$dQ_z = \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dxdydzd\tau$$

7. So, net heat accumulated in the element due to conduction of heat from all the coordinate directions is given as,

$$= \frac{\partial}{\partial x}\left[k_x \frac{\partial t}{\partial x}\right] dxdydzd\tau + \frac{\partial}{\partial y}\left[k_y \frac{\partial t}{\partial y}\right] dxdydzd\tau$$

$$+ \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dxdydzd\tau$$

$$= \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dxdydzd\tau \quad \text{...(1.6.3)}$$

8. The total heat generated in the element is given by

$$Q_g = q_g(dxdydz)d\tau \quad \text{...(1.6.4)}$$

9. Energy stored in element,

$$= \rho(dxdydz)c \frac{\partial t}{\partial \tau} d\tau \quad \text{...(1.6.5)}$$

10. Now using energy balance for the element, we have

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dxdydzd\tau + q_g(dxdydz)d\tau$$

$$= \rho(dxdydz)c \frac{\partial t}{\partial \tau} d\tau$$

11. Dividing both side by $dxdydzd\tau$, we have

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right) + q_g = \rho c \frac{\partial t}{\partial \tau}$$

This is known as the general heat conduction equation for 'non-homogeneous material', 'self heat generating' and 'unsteady three-dimensional heat flow'.

12. In case of homogeneous and isotropic material ($k_x = k_y = k_z = k$),

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial z^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho c}{k} \frac{\partial t}{\partial \tau}$$

or
$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial t}{\partial \tau}$$

where, $\alpha = \dfrac{k}{\rho . c}$ = Thermal diffusivity

Que 1.7. **Derive the general heat conduction equation in cylindrical coordinates.** **AKTU 2015-16, 2017-18; Marks 10**

Answer

1. Consider an elemental volume having the coordinate (r, ϕ, z) for three dimensional heat conduction analysis as shown in Fig. 1.7.1.
2. The volume of the element,

$$dv = r\, d\phi\, dr\, dz.$$

Let
q_g = Heat generation (uniform) per unit volume per unit time.
k = Thermal conductivity.
ρ = Density.
c = Specific heat capacity.

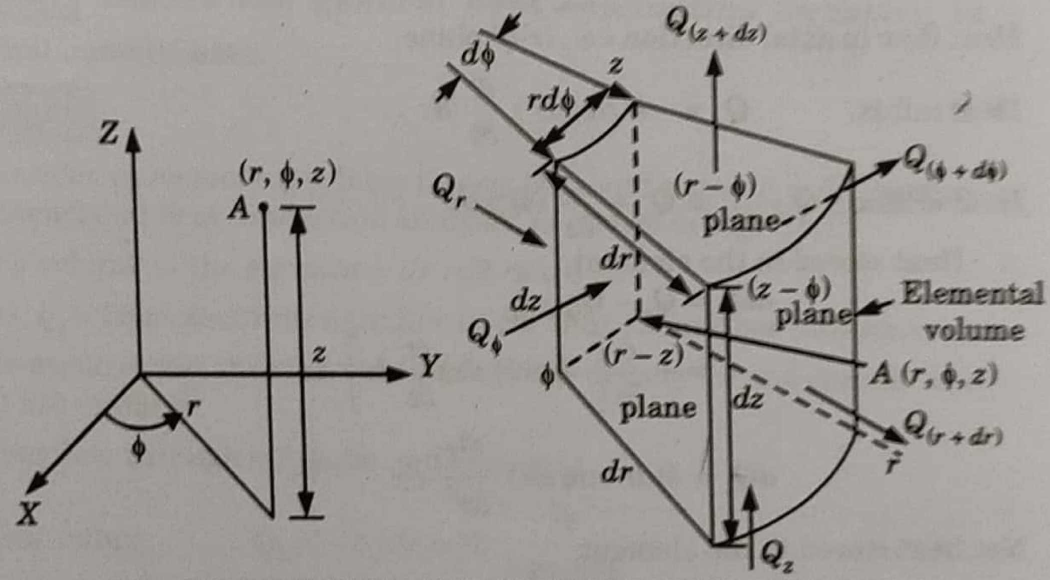


Fig. 1.7.1. Elemental volume for three-dimensional heat conduction analysis-cylindrical coordinates.

3. The heat flowing in the control volume in radial direction *i.e.*, $(z - \phi)$ plane :

Heat influx, $Q_r = -k(r d\phi\, dz)\,\dfrac{\partial t}{\partial r}\, d\tau$

Heat efflux, $Q_{(r+dr)} = Q_r + \dfrac{\partial}{\partial r}(Q_r)\, dr$

$\therefore$ Heat stored in the element,

$$dQ_r = Q_r - Q_{(r+dr)}$$

$$= -\frac{\partial}{\partial r}(Q_r)\, dr = -\frac{\partial}{\partial r}\left[-k(r\, d\phi\, dz)\frac{\partial t}{\partial r}\, d\tau\right] dr$$

$$= k(dr\, d\phi\, dz)\,\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) d\tau$$

$$dQ_r = k(dr\, rd\phi\, dz)\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right) d\tau$$

4. Heat flow in tangential direction *i.e.*, (r-z) plane

Heat influx, $\quad Q_\phi = -k(dr\, dz)\dfrac{\partial t}{r\partial\phi}\, d\tau$

Heat efflux, $\quad Q_{(\phi + d\phi)} = Q_\phi + \dfrac{\partial}{r\partial\phi}(Q_\phi)\, rd\phi$

$\therefore$ Heat stored in the element,

$$dQ_\phi = Q_\phi - Q_{(\phi + d\phi)}$$

$$= -\frac{\partial}{r\partial\phi}(Q_\phi)\, rd\phi = -\frac{\partial}{r\partial\phi}\left[-k(dr\, dz)\frac{\partial t}{r\partial\phi} d\tau\right] rd\phi$$

$$dQ_\phi = k(dr\, rd\phi\, dz)\frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2}\, d\tau$$

5. Heat flow in axial direction *i.e.*, (r-ϕ) plane

Heat influx, $\quad Q_z = -k(rd\phi\, dr)\dfrac{\partial t}{\partial z}\, d\tau$

Heat efflux, $\quad Q_{(z + dz)} = Q_z + \dfrac{\partial}{\partial z}(Q_z)\, dz$

$\therefore$ Heat stored in the element,

$$dQ_z = Q_z - Q_{(z + dz)}$$

$$= -\frac{\partial}{\partial z}\left[-k(rd\phi\, dr)\frac{\partial t}{\partial z} d\tau\right] dz$$

$$dQ_z = k(dr\, rd\phi\, dz)\frac{\partial^2 t}{\partial z^2}\, d\tau$$

6. Net heat stored in the element

$$= k.\, dr\, rd\phi\, dz\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}\right) d\tau$$

7. Heat generated within the control volume,

$$Q_g = q_g\, dr\, rd\phi\, dz\, d\tau$$

8. The increase in thermal energy in the element equal to

$$= \rho\, dr\, rd\phi\, dz\, c\frac{\partial t}{\partial\tau}\, d\tau$$

9. From the principle of conservation of energy :

$$kdr\, rd\phi\, dz\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}\right] d\tau + q_g\, dr\, rd\phi\, dzd\tau$$

$$= \rho dr\, rd\phi\, dzc\frac{\partial t}{\partial\tau}\, d\tau$$

or $$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2}+\frac{\partial^2 t}{\partial z^2}+\frac{q_g}{k}=\frac{\rho c}{k}\frac{\partial t}{\partial \tau}=\frac{1}{\alpha}\frac{\partial t}{\partial \tau} \quad ...(1.7.1)$$

This is the general heat conduction equation in cylindrical coordinates.

1. For steady-state, unidirectional heat flow in the radial direction, and with no internal heat generation, eq. (1.7.1) reduces to

$$\left(\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}\right)=0$$

or $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dt}{dr}\right)=0$$

Since $$\frac{1}{r}\neq 0$$

$$\frac{d}{dr}\left(r\frac{dt}{dr}\right)=0 \text{ or } r\frac{dt}{dr}=\text{constant}$$

Que 1.8. **Derive the general heat conduction equation in spherical coordinates.**

Answer

1. Consider an elemental volume having the coordinates (r, ϕ, θ), for three dimensional heat conduction analysis, as shown in Fig. 1.8.1.
2. The volume of the element = $dr\ rd\theta\ r\sin\theta\ d\phi$
3. Let, q_g = Heat generation (uniform) per unit volume per unit time. We assume that thermal conductivity (k), density (ρ) and specific heat (c) are uniform.
4. Heat flow through r-θ plane (ϕ-direction) :

Heat influx, $$Q_\phi=-k\,(dr\ rd\theta)\frac{\partial t}{r\sin\theta\partial\phi}d\tau \quad ...(1.8.1)$$

Heat efflux, $$Q_{(\phi+d\phi)}=Q_\phi+\frac{\partial}{r\sin\theta\partial\phi}(Q_\phi)\,r\sin\theta\,d\phi \quad ...(1.8.2)$$

∴ Heat accumulated in the element due to heat flow in the ϕ-direction,

$$dQ_\phi=Q_\phi-Q_{(\phi+d\phi)}$$
$$=-\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(Q_\phi)\,r\sin\theta\,d\phi$$
$$=-\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left[-k(dr\ rd\theta)\frac{1}{r\sin\theta}\frac{\partial t}{\partial\phi}d\tau\right]r\sin\theta d\phi$$
$$=k\,(dr\ rd\theta\ r\sin\theta\ d\phi)\frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}d\tau$$

5. Heat flow in r-ϕ (plane θ-direction) :

Heat influx, $$Q_\theta=-k\,(dr\ r\sin\theta\ d\phi)\frac{\partial t}{r\partial\theta}d\tau \quad ...(1.8.3)$$

Heat efflux, $Q_{(\theta + d\theta)} = Q_\theta + \dfrac{\partial}{r\partial\theta}(Q_\theta)\, r d\theta$ ...(1.8.4)

$\therefore$ Heat accumulated in the element due to heat flow in the θ-direction,

$$dQ_\theta = Q_\theta - Q_{(\theta+d\theta)} \quad \text{[Subtracting eq. (1.8.4) and (1.8.3)]}$$

$$= -\frac{\partial}{r\partial\theta}(Q_\theta)\, r d\theta$$

$$= -\frac{\partial}{r\partial\theta}\left[-k(dr\, r \sin\theta\, d\phi)\frac{\partial t}{r\,\partial\theta}\, d\tau\right] r\, d\theta$$

$$= \frac{k\, dr\, r d\phi\, r d\theta}{r \qquad r}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial t}{\partial\theta}\right] d\tau$$

$$= k\,(dr\, r d\theta\, r\sin\theta\, d\phi)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial t}{\partial\theta}\right] d\tau$$

6. Heat flow in θ-ϕ plane (r-direction) :

Heat influx, $Q_r = -k\,(r d\theta\, r\sin\theta\, d\phi)\dfrac{\partial t}{\partial r}\,\partial\tau$ ...(1.8.5)

Heat efflux, $Q_{(r+dr)} = Q_r + \dfrac{\partial}{\partial r}(Q_r) dr$ ...(1.8.6)

$\therefore$ Heat accumulation in the element due to heat flow in the r-direction,

$$dQ_r = Q_r - Q_{(r+dr)} = -\frac{\partial}{\partial r}(Q_r) dr$$

$$= -\frac{\partial}{\partial r}\left[-k\,(r d\theta\, r\sin\theta\, d\phi)\frac{\partial t}{\partial r} d\tau\right] dr$$

$$= k\, d\theta \sin\theta\, d\phi\, dr\frac{\partial}{\partial r}\left[r^2\frac{\partial t}{\partial r}\right] d\tau$$

$$= k\,(dr\, r d\theta\, r\sin\theta\, d\phi)\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial t}{\partial r}\right] d\tau$$

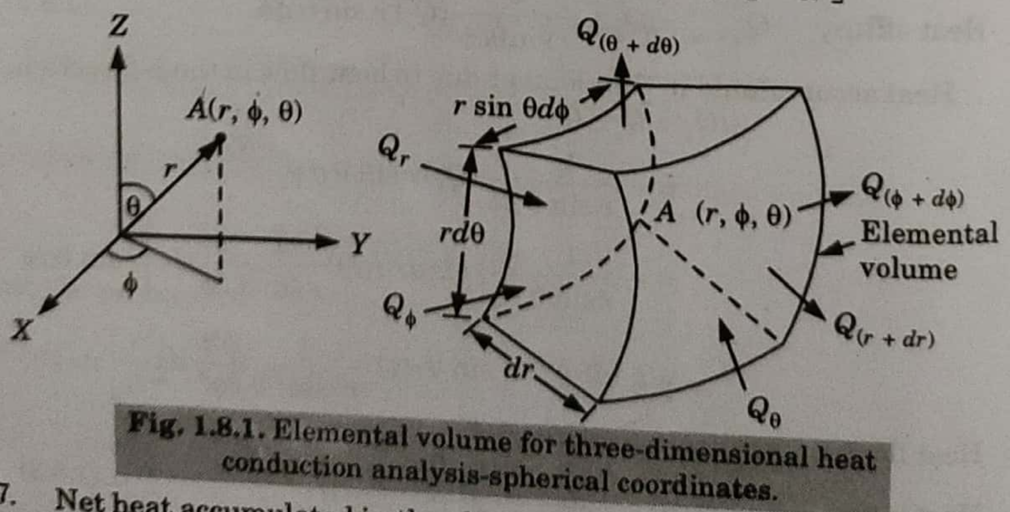


Fig. 1.8.1. Elemental volume for three-dimensional heat conduction analysis-spherical coordinates.

7. Net heat accumulated in the element

$$= k\, dr\, rd\theta\, r \sin\theta\, d\phi \left[\frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \phi^2} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial t}{\partial r} \right) \right] d\tau$$

8. The total heat generated within the element is given by,

$$Q'_g = q_g\, (dr\, rd\theta\, r \sin\theta\, d\phi)\, d\tau$$

9. The increase in thermal energy in the element

$$= \rho\, (dr\, rd\theta\, r \sin\theta\, d\phi)\, c \frac{\partial t}{\partial \tau} d\tau$$

10. From the principle of conservation of energy,

$$k\, dr\, rd\theta\, r \sin\theta\, d\phi \left[\frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \phi^2} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial t}{\partial r} \right) \right] d\tau$$

$$+ q_g\, (dr\, rd\theta\, r \sin\theta\, d\phi)\, d\tau = \rho\, (dr\, rd\theta\, r \sin\theta\, d\phi)\, c \frac{\partial t}{\partial \tau} d\tau$$

11. Dividing both sides by $k(dr\, rd\theta\, r \sin\theta\, d\phi)\, d\tau$, we get

$$\left[\frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \phi^2} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial t}{\partial r} \right) \right] + \frac{q_g}{k} = \frac{\rho c}{k} \frac{\partial t}{\partial \tau}$$

or
$$\left[\frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \phi^2} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial t}{\partial r} \right) \right] + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial t}{d\tau} \quad ...(1.8.7)$$

This equation is known as general heat conduction equation in spherical coordinates.

12. In case there are no heat sources present and the heat flow is steady and one-dimensional, then eq. (1.8.7) reduces to

$$\frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{dt}{dr} \right) = 0$$

PART-2

Initial and Boundary Conditions, Simple and Composite Systems in Rectangular, Cylindrical and Spherical Coordinates With and Without Energy Generation.

CONCEPT OUTLINE : PART-2

Initial and Boundary Conditions : To solve the differential equation, initial conditions are required to be in unsteady state heat conduction at (t = 0). Boundary conditions are referring to physical conditions existing at the boundary of the medium.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.9. **Define initial conditions and boundary conditions. Explain the different types of boundary conditions applied to heat conduction problem.**

Answer

A. Initial Conditions :

1. The initial conditions describe the temperature distribution in a medium at the initial moment of time, and these are needed only for the transient (time-dependent) problems.
2. The initial conditions can be expressed as

At $$\tau = 0;\ t = t\,(x, y, z) \qquad \text{...(1.9.1)}$$

3. For a uniform initial temperature distribution, a simple but typical form of the above identity can be recast as

At $$\tau = 0;\ t = t_0\,(x, y, z) \qquad \text{...(1.9.2)}$$

B. Boundary Conditions :

1. The boundary conditions refer to physical conditions existing at the boundaries of the medium, and specify the temperature or the heat flow at the surface of the body.

C. Types of Boundary Conditions :

The different types of boundary conditions applied to heat conduction problems are as follows :

a. Boundary Condition of First Kind (Prescribed Surface Temperature) :

1. The temperature distribution, t_s is given at a boundary surface for each moment of time.

$$t_s = t(x, y, z, \tau)$$

2. A typical example of boundary condition of the first kind for a slab is shown in Fig. 1.9.1.

$$t(x, y, \tau) = 0, \text{ at } x = 0$$
$$t(x, y, \tau) = 0, \text{ at } y = 0$$
$$t(x, y, \tau) = f_1(y), \text{ at } x = a$$
$$t(x, y, \tau) = f_2(x), \text{ at } y = b$$

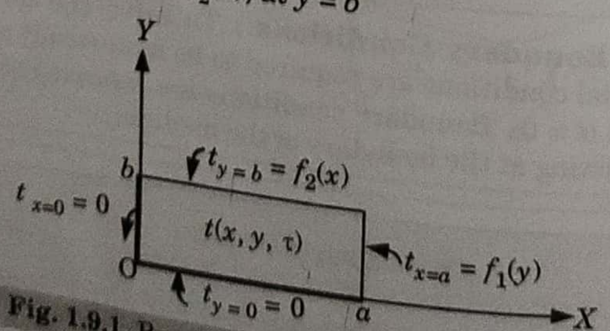


Fig. 1.9.1. Boundary condition of the first kind.

b. Boundary Condition of the Second Kind (Prescribed Heat Flux) :

1. In this case, the heat flux at a boundary is prescribed.

$$\frac{-k\partial t(x,\tau)}{\partial x} = q_0, \text{ at } x = 0$$

or
$$\frac{\partial t}{\partial x} = \frac{-q_0}{k} = F_0, \text{ at } x = 0$$

$$\frac{\partial t}{\partial x} = 0, \text{ at } x = 0$$

2. Here, $\frac{\partial t}{\partial x} = 0$ at $x = 0$ describes an insulated or adiabatic boundary such a condition can also exist at the plane of symmetry.

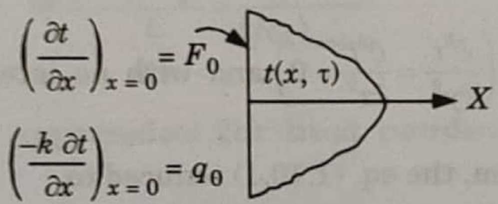


Fig. 1.9.2. Boundary condition of the second kind.

c. Boundary Condition of Third Kind (Convective Condition) :

1. This condition is encountered at a solid boundary when there is equality between heat transfer to the surface by conduction and that leaving the surface by convection.
2. At $x = 0$,

$$h_1(t_1 - t_{x=0}) = -k\left(\frac{\partial t}{\partial x}\right)_{x=0}$$

or
$$\left(k\frac{\partial t}{\partial x} + h_1 t\right)_{x=0} = h_1 t_1 = F_1$$

3. Similarly at $x = L$,

$$-k\left(\frac{\partial t}{\partial x}\right)_{x=L} = h_2(t_{x=L} - t_2)$$

or
$$\left(k\frac{\partial t}{\partial x} + h_2 t\right)_{x=L} = h_2 t_2 = F_2$$

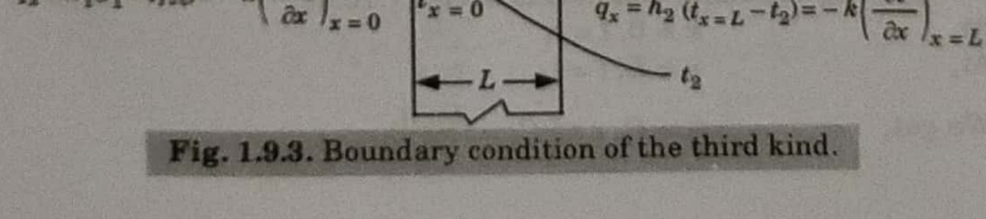


Fig. 1.9.3. Boundary condition of the third kind.

Que 1.10. **Derive the expression for heat conduction through a plane wall with uniform thermal conductivity for one-dimensional steady state, homogeneous and isotropic material.**

Answer

1. Consider a plane wall of homogeneous and isotropic material through which heat is flowing only in X-direction. The general heat conduction equation is,

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau} \qquad ...(1.10.1)$$

2. If the heat conduction takes place under steady state $\left(\frac{\partial t}{\partial \tau} = 0\right)$, one-dimensional $\left(\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0\right)$ and with no internal heat generation $\left(\frac{q_g}{k} = 0\right)$. Then, the eq. (1.10.1) reduced to,

$$\frac{\partial^2 t}{\partial x^2} = 0 \qquad ...(1.10.2)$$

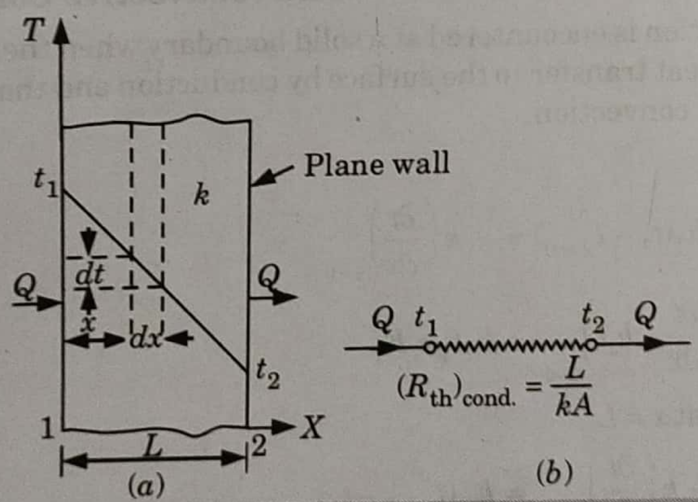


Fig. 1.10.1. Heat conduction through a plane wall.

3. By integrating the eq. (1.10.2) twice, we get

$$\frac{dt}{dx} = C_1 \text{ and } t = C_1 x + C_2 \qquad ...(1.10.3)$$

Where C_1 and C_2 are arbitrary constants.

4. By applying boundary conditions in eq. (1.10.3),

At, $x = 0,\ t = t_1$

At, $x = L,\ t = t_2$

We get, $t_1 = C_2$ and $C_1 = \frac{t_2 - t_1}{L}$

5. By putting the value of C_1 and C_2, in eq. (1.10.3),

$$t = \left(\frac{t_2 - t_1}{L}\right) x + t_1$$

It indicates that temperature distribution across a wall is linear and is independent of thermal conductivity.

6. Now, by using Fourier's equation,

$$Q = -kA\frac{dt}{dx} = -kA\frac{d}{dx}\left[\left(\frac{t_2 - t_1}{L}\right)x + t_1\right]$$

$$= -kA\frac{(t_2 - t_1)}{L} = kA\frac{(t_1 - t_2)}{L}$$

or

$$Q = \frac{(t_1 - t_2)}{\frac{L}{kA}} = \frac{t_1 - t_2}{(R_{th})_{cond.}}$$

Que 1.11. **Derive expression for heat conduction through a composite wall.**

Answer

1. Consider the transmission of heat through a composite wall consisting of a number of slabs as shown in Fig. 1.11.1.
2. Let L_A, L_B, L_C = Thicknesses of slabs A, B and C respectively (also called path length),
   k_A, k_B, k_C = Thermal conductivities of the slabs A, B, and C respectively,
   $t_1, t_4\ (t_1 > t_4)$ = Temperatures at the wall surfaces 1 and 4 respectively, and
   t_2, t_3 = Temperatures at the interfaces 2 and 3 respectively.
3. Since the quantity of heat transmitted per unit time through each slab/layer is same, so we have

$$Q = \frac{k_A A(t_1 - t_2)}{L_A} = \frac{k_B A(t_2 - t_3)}{L_B} = \frac{k_C A(t_3 - t_4)}{L_C}$$

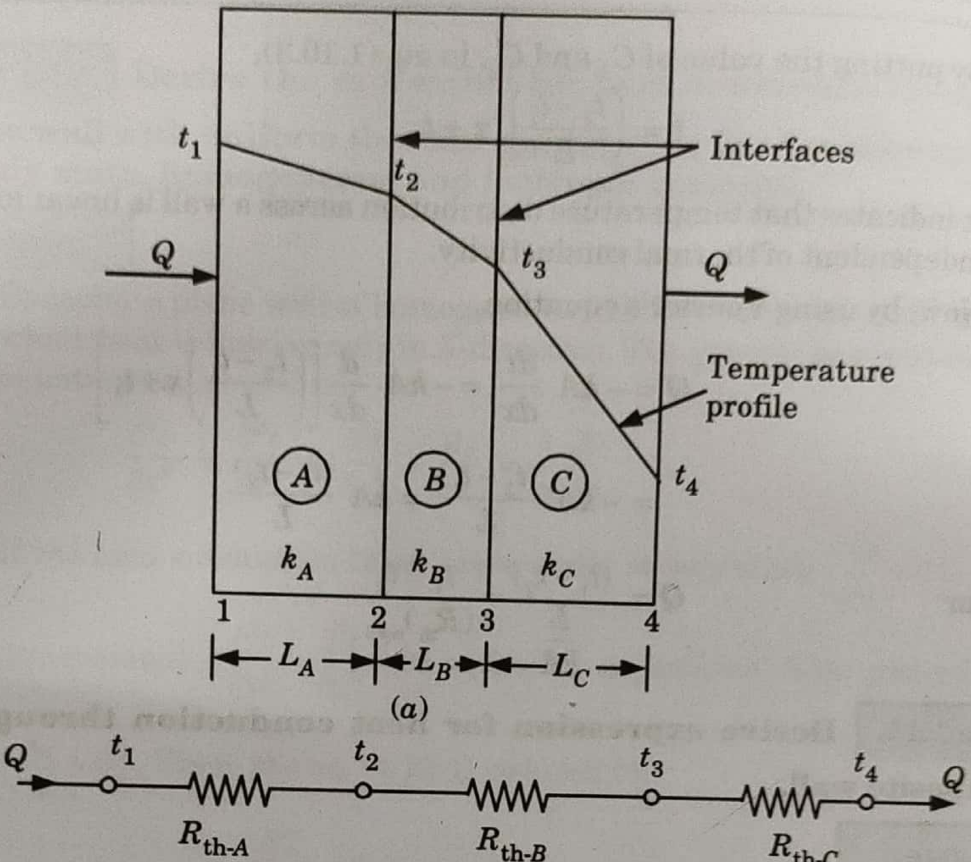
(Assuming that there is a perfect contact between the layers and no temperature drop occurs across the interface between the materials).

4. Rearranging the above expression, we get

$$t_1 - t_2 = \frac{QL_A}{k_A A} \quad ...(1.11.1)$$

$$t_2 - t_3 = \frac{QL_B}{k_B A} \quad ...(1.11.2)$$

$$t_3 - t_4 = \frac{QL_C}{k_C A} \quad ...(1.11.3)$$



$$R_{th-A} = \frac{L_A}{k_A A}, \quad R_{th-B} = \frac{L_B}{k_B A}, \quad R_{th-C} = \frac{L_C}{k_C A}$$

(b)

Fig. 1.11.1. Steady state conduction through a composite wall.

5. Adding eq. (1.11.1), eq. (1.11.2) and eq. (1.11.3), we have

$$(t_1 - t_4) = Q\left[\frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A}\right]$$

or, $$Q = \frac{A(t_1 - t_4)}{\left[\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_c}\right]} = \frac{(t_1 - t_4)}{\left[\frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A}\right]} = \frac{(t_1 - t_4)}{[R_{th-A} + R_{th-B} + R_{th-C}]}$$

6. If the composite wall consists of n slab/layers, then

$$Q = \frac{[t_1 - t_{(n+1)}]}{\sum_{1}^{n} \frac{L}{kA}}$$

Que 1.12. **Derive the expression for steady state heat conduction through a hollow cylinder with uniform thermal conductivity, whose inner surface is exposed to hot fluid and outer surface is exposed to a cold fluid.**

Answer

1. Consider a hollow cylinder made of material having constant thermal conductivity and insulated at both ends.
2. The general heat conduction equation in cylindrical co-ordinate is,

$$\frac{\partial^2 t}{dr^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{h} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau} \quad ...(1.12.1)$$

3. For steady state $\left(\frac{\partial t}{\partial \tau} = 0\right)$, unidirectional $(t \neq f[\phi, x])$ heat flow with no internal heat generation $(q_g = 0)$, then the eq. (1.12.1) becomes

$$\frac{d^2 t}{dr^2} + \frac{1}{r}\frac{dt}{dr} = 0$$

or
$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dt}{dr}\right] = 0$$

Since
$$\frac{1}{r} \neq 0,$$

$\therefore$
$$\frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0 \quad ...(1.12.2)$$

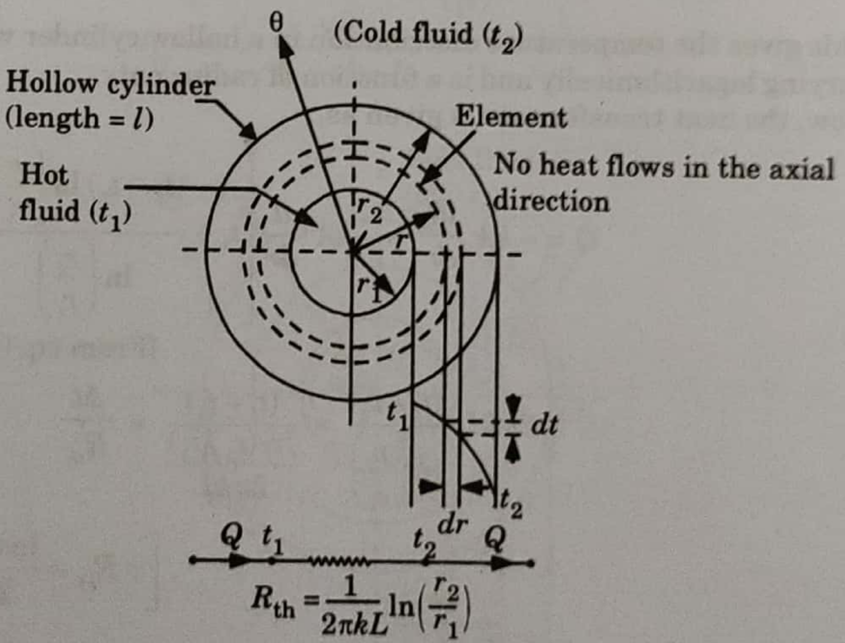


Fig. 1.12.1.

4. On integrating the eq. (1.12.2) two times, we get

$$r\frac{dt}{dr} = C \text{ or } \int dt = C\int \frac{dr}{r}$$

or
$$t = C \ln r + C_1 \quad ...(1.12.3)$$

5. Applying boundary condition :

At, $r = r_1, t = t_1$

At, $r = r_2, t = t_2$

We get, $t_1 = C \ln r_1 + C_1$ ...(1.12.4)

and $t_2 = C \ln r_2 + C_1$ ...(1.12.5)

6. From eq. (1.12.4) and eq. (1.12.5), we get

$$C = -\frac{t_1 - t_2}{\ln\left(\frac{r_2}{r_1}\right)}$$

and $$C_1 = t_1 + \frac{t_1 - t_2}{\ln(r_2/r_1)} \ln r_1$$

7. Substituting the value of C and C_1 in eq. (1.12.3) we get

$$t = t_1 + \frac{(t_1 - t_2)}{\ln\left(\frac{r_2}{r_1}\right)} \ln(r_1) - \frac{(t_1 - t_2)}{\ln\left(\frac{r_2}{r_1}\right)} \ln(r)$$

or $$\frac{t - t_1}{t_2 - t_1} = \frac{\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$ ...(1.12.6)

This gives the temperature distribution in a hollow cylinder which is varying logarithmically and is a function of radius only.

8. Now, the heat transfer rate is given as,

$$Q = -kA\frac{dt}{dr} = -kA\frac{d}{dr}\left[t_1 + \frac{(t_2 - t_1)\ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)}\right]$$

[From eq. (1.12.6)]

$$= 2\pi kL\frac{(t_1 - t_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{(t_1 - t_2)}{\frac{\ln(r_2/r_1)}{2\pi kL}} = \frac{\Delta t}{R_{th}}$$

$$\left[\because R_{th} = \frac{\ln(r_2/r_1)}{2\pi kL}\right]$$

So, $$Q = \frac{t_1 - t_2}{\frac{\ln(r_2/r_1)}{2\pi kL}}$$

This is the expression for rate of heat transfer through hollow cylinder which is insulated at both ends.

Que 1.13. **Derive the expression for heat conduction through a composite cylinder.**

Answer

1. Consider heat flows radially through a composite cylinder as shown in Fig. 1.13.1.
2. Let, t_{hf} and t_{cf} = The temperature of hot fluid and cold fluid respectively,
   h_{hf} and h_{cf} = Convective heat transfer coefficient of hot fluid and cold fluid respectively,
   k_A and k_B = Thermal conductivity of inside layer A and of outside layer B, and
   t_1, t_2, t_3 = Temperatures at point 1, 2 and 3.
3. The rate of heat transfer,

$$Q = h_{h_f} 2\pi r_1 L\,(t_{h_f} - t_1) = \frac{k_A(2\pi L)(t_1 - t_2)}{\ln\left(\dfrac{r_2}{r_1}\right)}$$

$$= \frac{k_B 2\pi L(t_2 - t_3)}{\ln\left(\dfrac{r_3}{r_2}\right)} = h_{cf} 2\pi r_3 L\,(t_3 - t_{cf})$$

4. By rearranging, we get

$$t_{h_f} - t_1 = \frac{Q}{h_{h_f} r_1 2\pi L} \qquad \text{...(1.13.1)}$$

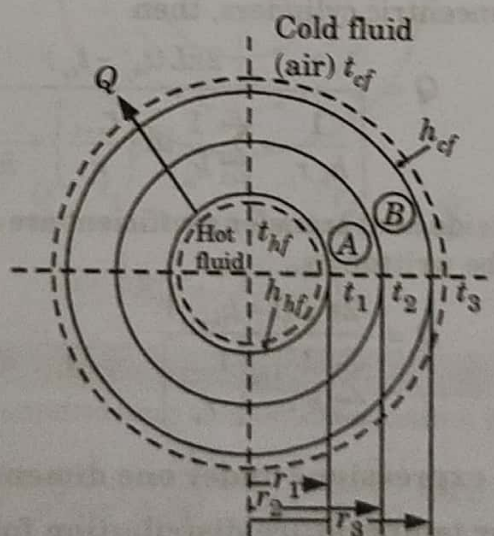


Fig. 1.13.1.

$$t_1 - t_2 = \frac{Q}{\dfrac{k_A 2\pi L}{\ln\left(\dfrac{r_2}{r_1}\right)}} \qquad \text{...(1.13.2)}$$

$$t_2 - t_3 = \frac{Q}{\dfrac{k_B 2\pi L}{\ln\left(\dfrac{r_3}{r_2}\right)}} \quad ...(1.13.3)$$

$$t_3 - t_{c_f} = \frac{Q}{h_{c_f} r_3 2\pi L} \quad ...(1.13.4)$$

5. Adding eq. (1.13.1), eq. (1.13.2), eq. (1.13.3) and eq. (1.13.4), we get

$$\frac{Q}{2\pi L}\left[\frac{1}{h_{h_f} r_1} + \frac{1}{\dfrac{k_A}{\ln\left(\dfrac{r_2}{r_1}\right)}} + \frac{1}{\dfrac{k_B}{\ln\left(\dfrac{r_3}{r_2}\right)}} + \frac{1}{h_{c_f} r_3}\right] = t_{h_f} - t_{c_f}$$

or
$$Q = \frac{2\pi L (t_{h_f} - t_{c_f})}{\left[\dfrac{1}{h_{h_f} r_1} + \dfrac{1}{\dfrac{k_A}{\ln\left(\dfrac{r_2}{r_1}\right)}} + \dfrac{1}{\dfrac{k_B}{\ln\left(\dfrac{r_3}{r_2}\right)}} + \dfrac{1}{h_{c_f} r_3}\right]}$$

or
$$Q = \frac{2\pi L (t_{h_f} - t_{c_f})}{\left[\dfrac{1}{h_{h_f} r_1} + \dfrac{\ln\left(\dfrac{r_2}{r_1}\right)}{k_A} + \dfrac{\ln\left(\dfrac{r_3}{r_2}\right)}{k_B} + \dfrac{1}{h_{c_f} r_3}\right]}$$

6. If there are n concentric cylinders, then

$$Q = \frac{2\pi L (t_{h_f} - t_{c_f})}{\left[\dfrac{1}{h_{h_f} r_1} + \displaystyle\sum_{n=1}^{n} \dfrac{1}{k_n} \ln\left\{\dfrac{r_{n+1}}{r_n}\right\} + \dfrac{1}{h_{c_f} r_{n+1}}\right]} \quad ...(1.13.5)$$

7. If inside and outside heat transfer coefficient are not considered, then eq. (1.13.5) can be written as,

$$Q = \frac{2\pi L (t - t_{(n+1)})}{\displaystyle\sum_{n=1}^{n} \frac{1}{k_n} \ln\left[\frac{r_{n+1}}{r_n}\right]}$$

Que 1.14. **Derive expressions under one dimensional steady state heat conduction for temperature distribution for the sphere.**

OR

Starting with an energy balance on a spherical shell volume element, derive the one dimensional transient heat conduction equation for a sphere with constant thermal conductivity and no heat generation [Fig. (1.14.1)].

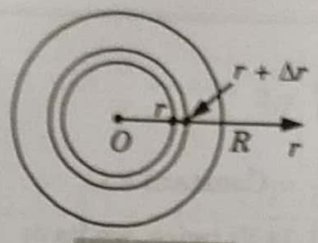


Fig. 1.14.1.

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Answer

1. Let thermal conductivity of a hollow sphere is uniform.
2. Let r_1 and r_2 are inner and outer radius and t_1 and t_2 are temperatures of inner surface and outer surface and k be thermal conductivity of the material with the given temperature range.

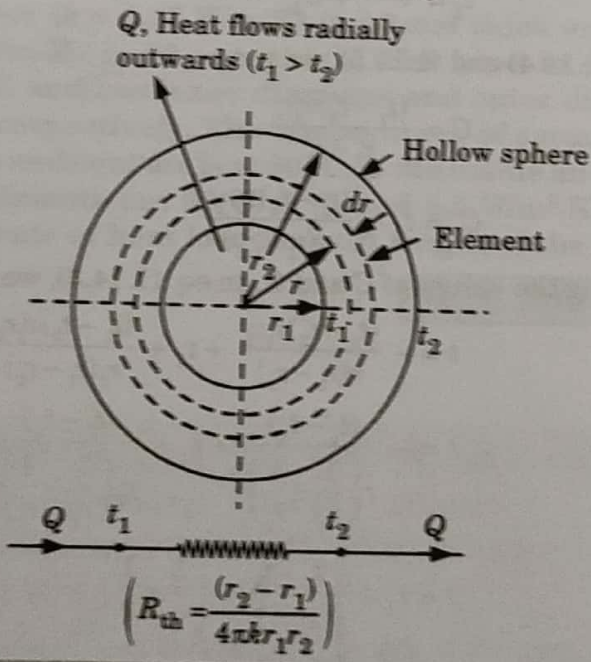

Fig. 1.14.2. Steady state conduction through a hollow sphere.

3. Applying the general heat conduction equation in spherical co-ordinates,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right)+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial t}{\partial \tau} \quad ...(1.14.1)$$

4. For steady state $\left(\frac{\partial t}{\partial \tau}=0\right)$, unidirectional heat flow in the radial direction and with no heat generation, eq. (1.14.1) reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{dt}{dr}\right)=0$$

$$\frac{d}{dr}\left(r^2 \frac{dt}{dr}\right) = 0 \qquad \left(\because \frac{1}{r^2} \neq 0\right)$$

or
$$r^2 \frac{dt}{dr} = \text{Constant} = C \qquad ...(1.14.2)$$

5. On integrating eq. (1.14.2) twice, we have

$$t = \frac{-C}{r} + C_1 \qquad ...(1.14.3)$$

6. Applying boundary condition,
   At $r = r_1, t = t_1$
   At $r = r_2, t = t_2$

   We get,
$$t_1 = \frac{-C}{r_1} + C_1 \qquad ...(1.14.4)$$

   and
$$t_2 = \frac{-C}{r_2} + C_1 \qquad ...(1.14.5)$$

7. From eq. (1.14.4) and (1.14.5), we get

$$C = \frac{(t_1 - t_2) r_1 r_2}{r_1 - r_2}$$

   and
$$C_1 = t_1 + \frac{(t_1 - t_2) r_1 r_2}{r_1 (r_1 - r_2)}$$

8. Substituting the values of C and C_1 in eq. (1.14.3), we have

$$t = -\frac{(t_1 - t_2) r_1 r_2}{r(r_1 - r_2)} + t_1 + \frac{(t_1 - t_2) r_1 r_2}{r_1 (r_1 - r_2)}$$

$$t = -\frac{(t_1 - t_2)}{r\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} + t_1 + \frac{(t_1 - t_2)}{r_1\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$$t = t_1 + \frac{t_1 - t_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \left[\frac{1}{r_1} - \frac{1}{r}\right] \qquad ...(1.14.6)$$

or
$$\frac{t - t_1}{t_2 - t_1} = \frac{\frac{1}{r} - \frac{1}{r_1}}{\frac{1}{r_2} - \frac{1}{r_1}}$$

or
$$\frac{t - t_1}{t_2 - t_1} = \frac{r_2}{r}\left[\frac{r - r_1}{r_2 - r_1}\right] \qquad ...(1.14.7)$$

Eq. (1.14.7) shows that temperature distribution associated with radial conduction through a sphere is represented by a hyperbola.

9. Now, from Fourier's equation,

$$Q = -kA\frac{dt}{dr} = -k4\pi r^2 \frac{d}{dr}\left[t_1 + \frac{t_1 - t_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}\left(\frac{1}{r_1} - \frac{1}{r}\right)\right]$$

$$= -k4\pi r^2 \frac{t_1 - t_2}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \times -\left(-\frac{1}{r^2}\right) = -k4\pi r^2 \frac{t_1 - t_2}{\left(\frac{r_1 - r_2}{r_1 r_2}\right)} \times \frac{1}{r^2} = -4\pi k \frac{(t_1 - t_2) r_1 r_2}{(r_1 - r_2)}$$

$$\therefore \quad Q = \frac{t_1 - t_2}{\frac{(r_2 - r_1)}{4\pi k r_1 r_2}} = \frac{t_1 - t_2}{R_{th}}$$

Where, $$R_{th} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

Que 1.15. **A steam pipe is covered with two layers of insulation. The inner layer (k = 0.17 W/m-K) is 30 mm thick and outer layer (k = 0.023 W/m-K) is 50 mm thick. The pipe is made of steel (k = 58 W/m-K) and has inner diameter and outer diameter of 160 and 170 mm, respectively. The temperature of saturated steam is 300 °C and the ambient air is at 50 °C. If the inside and outside heat transfer coefficients are 30 W/m²-K and 5.8 W/m²-K respectively, calculate the rate of heat loss per unit length of the pipe.**

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Answer

Given : $r_1 = 80$ mm , $t_i = t_1 = 300$ °C, $r_2 = 85$ mm, $t_4 = 50$ °C

$r_3 = 85 + 30 = 115$ mm, $r_4 = 115 + 50 = 165$ mm

$k_1 = 58$ W/m-K, $k_2 = 0.17$ W/m-K, $k_3 = 0.023$ W/m-K

$h_i = 30$ W/m²-K, $h_o = 5.8$ W/m²-K

To Find : Rate of heat loss per unit length of the pipe.

1. There are three concentric cylinders, hence $n = 3$, thus

$$\Sigma R = \frac{1}{2\pi L}\left[\frac{1}{r_1 h_i} + \frac{1}{h_o r_4} + \frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{k_3}\ln\left(\frac{r_4}{r_3}\right)\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{0.08 \times 30} + \frac{1}{0.165 \times 5.8}\right.$$

$$\left. + \frac{1}{58}\ln\left(\frac{85}{80}\right) + \frac{1}{0.17}\ln\left(\frac{115}{85}\right) + \frac{1}{0.023}\ln\left(\frac{165}{115}\right)\right]$$

$$= 3.014 \text{ K-m/W}$$

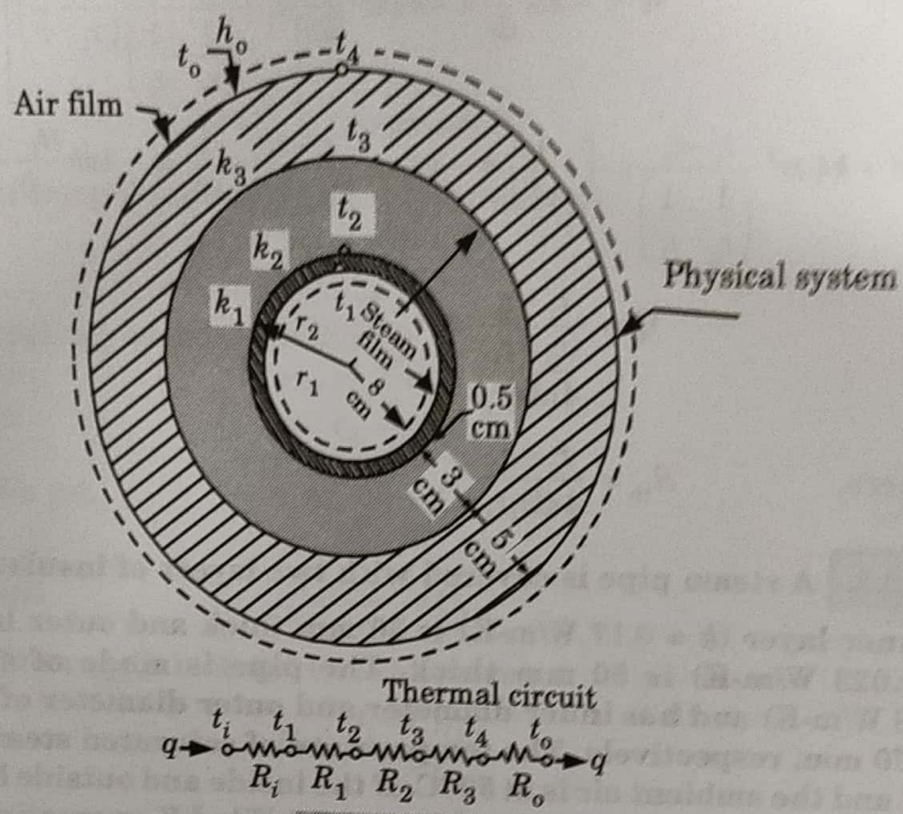

Fig. 1.15.1.

2. Heat loss per unit length of the pipe, $q = \dfrac{t_i - t_4}{\Sigma R} = \dfrac{300 - 50}{3.014}$

$$q = 82.95 \text{ W/m}$$

Que 1.16. **Consider a 0.8 m high and 1.5 m wide double pane window consisting of two 4 mm thick layers of glass ($k = 0.78$ W/m-°C) separated by a 10 mm wide stagnant air space ($k = 0.026$ W/m-°C). Determine the steady rate of heat transfer through this double pane window and the temperature of its inner surface for a day during which the room is maintained at 20 °C while the temperature of the outdoors is – 10 °C. Take the convection heat transfer coefficients on the inner and outer surface of the window to be $h_1 = 10$ W/m²-°C and $h_2 = 40$ W/m²-°C, which includes the effects of radiation.**

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Answer

Given : $w = 1.5$ m, $H = 0.8$ m, $x_1 = x_3 = 4$ mm $= 0.004$ m
$x_2 = 10$ mm $= 0.01$ m, $t_a = 20$ °C, $h_a = 10$ W/m²-°C,
$k_1 = k_3 = 0.78$ W/m-°C, $t_\infty = -10$ °C, $h_\infty = 40$ W/m²-°C, $k_2 = 0.026$ W/m-°C.
To Find : i. Heat transfer.
ii. Temperature of inner surface.

1. Under steady state,

$$Q = \frac{t_a - t_\infty}{\dfrac{1}{h_a A} + \dfrac{x_1}{k_1 A} + \dfrac{x_2}{k_2 A} + \dfrac{x_3}{k_3 A} + \dfrac{1}{h_\infty A}}$$

$$= \frac{20-(-10)}{\dfrac{1}{10\times1.2} + \dfrac{0.004}{0.78\times1.2} + \dfrac{0.01}{0.026\times1.2} + \dfrac{0.004}{0.78\times1.2} + \dfrac{1}{40\times1.2}}$$

$$= 69.24 \text{ W}$$

2. Also, $Q = h_i A_s (t_i - t_1)$

$$69.24 = 10 \times 1.2 \times (20 - t_1)$$

$$t_1 = 14.23\ °\text{C}$$

Que 1.17. **The insulation board for air conditioning purposes comprises three layers. A 12 cm thick layer of grass (k = 0.22 W/m-K) is sandwiched between 3 cm thick layer of plywood (k = 0.15 W/m-K) on each side. The bonding is achieved with glue which does not offer any resistance of heat flow. If the side surfaces of board are maintained at 40 °C and 20 °C temperature, determine the heat flux. How would the heat flux be affected if instead of glue the three pieces are fastened by steel bolts (k = 40 W/m-K) of 1.2 cm diameter at corners.**

AKTU 2015-16, Marks 10

Answer

Given : k_1 = 0.22 W/m-K, k_2 = 0.15 W/m-K, k_b = 40 W/m-K, D_b = 1.2 cm, A = 1 m²

To Find : i. Heat flux.

ii. Change in heat flux when three pieces are fastened by bolts instead of glue.

i. **When Pieces are Fastened by Glue :**

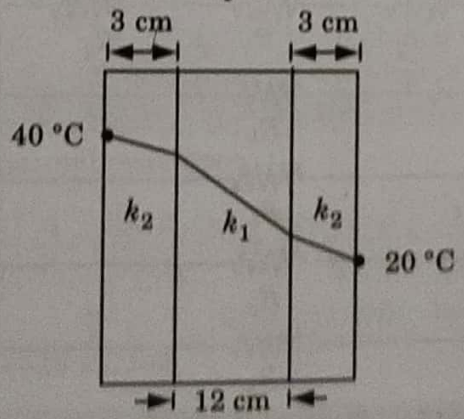

Fig. 1.17.1.

1. Thermal circuit for the given situation is shown below :

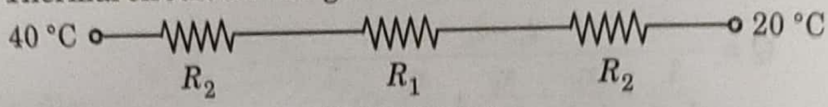

Fig. 1.17.2.

2. Now, $R_2 = \dfrac{L_2}{k_2 A} = \dfrac{0.03}{0.15 \times 1} = 0.2$

$$R_1 = \frac{L_1}{k_1 A} = \frac{0.12}{0.22 \times 1} = 0.545$$

$$R_{\text{eff.}} = 2R_2 + R_1 = 2 \times 0.2 + 0.545$$
$$= 0.4 + 0.545$$
$$= 0.945$$

3. Heat flux, $q_1 = \dfrac{\Delta t}{R_{\text{eff}}} = \dfrac{40 - 20}{0.945} = 21.164 \text{ W/m}^2$

ii. When they are Fastened by Bolts :

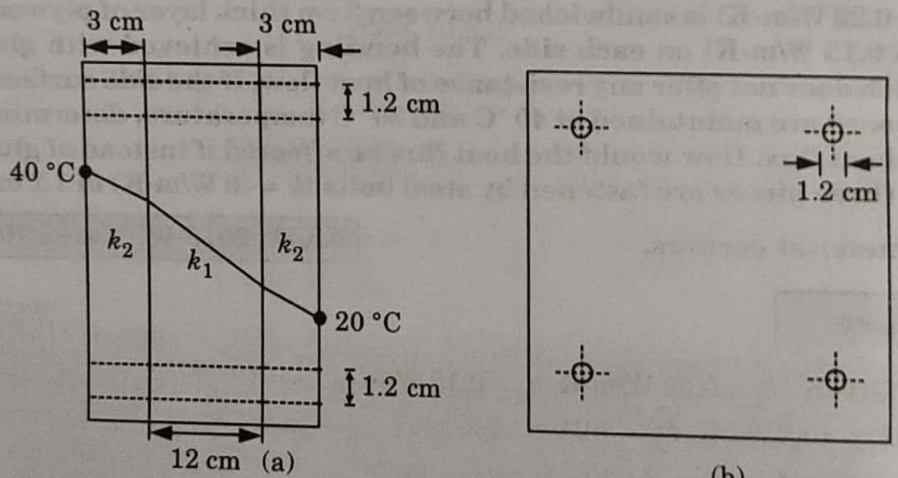

Fig. 1.17.3.

1. Thermal circuit for the given situation is shown below :

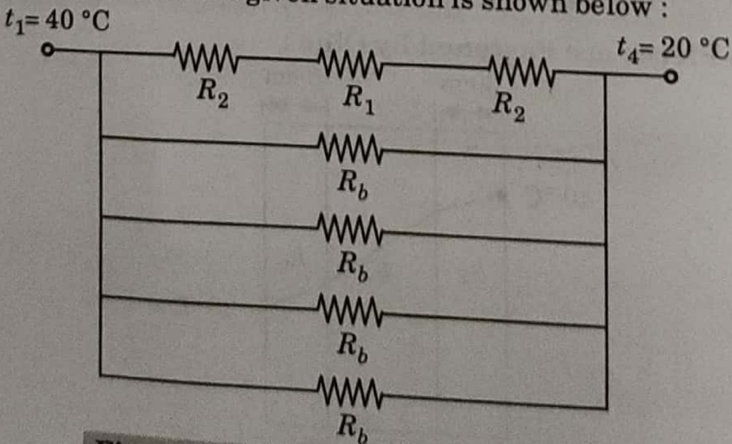

Fig. 1.17.4. Thermal circuit for the system.

2. Now, $R_b = \dfrac{L_b}{k_b A_b} = \dfrac{0.18}{40 \times \dfrac{\pi}{4}\left(\dfrac{1.2}{100}\right)^2} = 39.78$

$$\frac{1}{R_{\text{effective}}} = \frac{2}{R_b} + \frac{1}{R_s} + \frac{2}{R_b} \qquad \{R_s = R_1 + R_2 + R_2\}$$

$$= \frac{4}{R_b} + \frac{1}{R_s} = \frac{4R_s + R_b}{R_b R_s} = \frac{4 \times 0.945 + 39.78}{0.945 \times 39.78} = 1.16$$

So, $R_{eff} = 1/1.16 = 0.86$

3. Heat flux, $q_2 = \dfrac{\Delta t}{R_{eff}} = \dfrac{40 - 20}{0.86} = 23.2 \text{ W/m}^2$

4. Change in flux $= q_2 - q_1 = 23.2 - 21.164 = 2.036 \text{ W/m}^2$

Que 1.18. **A steam pipe of 5 cm inside diameter and 6.5 cm outside diameter is covered with a 2.75 cm radial thickness of high temperature insulation (k = 1.1 W/m-K). The surface heat transfer coefficient for inside and outside surfaces are 4650 W/m²-K and 11.5 W/m²-K. The thermal conductivity of the pipe material is 45 W/m-K. If the steam temperature is 200 °C and ambient air temperature is 25 °C, determine :**

a. Heat transfer per meter length of pipe.

b. Temperature at the interface.

c. Overall heat transfer coefficient. **AKTU 2016-17, Marks 10**

Answer

Given : $d_1 = 5$ cm or $r_1 = 2.5$ cm $= 0.025$ m

$d_2 = 6.5$ cm or $r_2 = 3.25$ cm $= 0.0325$ m, $x = 2.75$ cm,

$k_2 = 1.1$ W/m-K, $h_o = 11.5$ W/m²-K, $h_i = 4650$ W/m²-K,

$k_1 = 45$ W/m-K, $t_i = 200$ °C, $t_o = 25$ °C.

To Find :
i. Heat transfer per meter length of pipe.
ii. Temperature at the interface.
iii. Overall heat transfer coefficient.

1. Equivalent thermal resistance,

$$R_{th} = \frac{1}{2\pi L}\left[\frac{1}{h_i r_i} + \frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_o r_o}\right]$$

$$= \frac{1}{2\pi L}\left[\frac{1}{4650 \times 0.025} + \frac{\ln(3.25/2.5)}{45} + \frac{\ln(6/3.25)}{1.1} + \frac{1}{11.5 \times 0.06}\right]$$

$$= \frac{0.3216}{L}$$

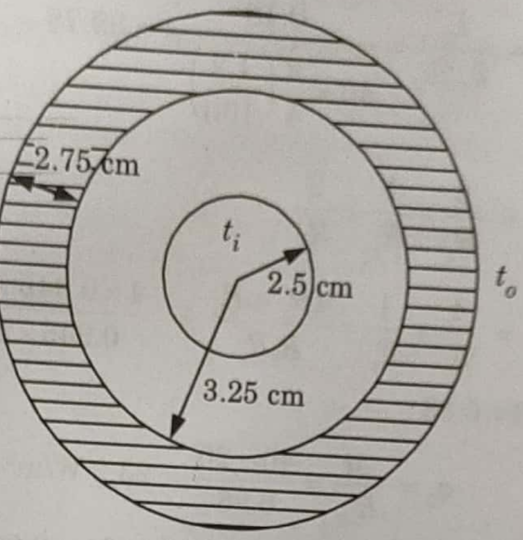

Fig. 1.18.1.

3. We know that, $Q = \dfrac{\Delta t}{R_{th}} = \dfrac{200 - 25}{\dfrac{0.3216}{L}}$

$$\frac{Q}{L} = 544.154 \text{ W/m} \quad \text{...(1.18.1)}$$

4. Let temperature at interface is t,

$$Q = \frac{\Delta t}{R_{th}}$$

$$Q = \frac{200 - t}{\dfrac{1}{(h_i r_i)(2\pi L)} + \dfrac{\ln(r_2 / r_1)}{2\pi k_1 L}}$$

$$= \frac{200 - t}{\dfrac{1.369 \times 10^{-3}}{L} + \dfrac{0.9279 \times 10^{-3}}{L}}$$

$$\frac{Q}{L} = \frac{200 - t}{2.2969 \times 10^{-3}} \quad \text{...(1.18.2)}$$

5. Equating eq. (1.18.1) and eq. (1.18.2), we get

$$544.154 = \frac{200 - t}{2.2969 \times 10^{-3}}$$

$$t = 198.75 \text{ °C.}$$

6. Overall heat transfer coefficient,

$$Q = UA\Delta t$$

$$U = \frac{Q}{A\Delta t} = \frac{Q}{2\pi r_3 L \Delta t} = \frac{544.154 \times L}{2\pi \times \dfrac{6}{100} \times L \times 175}$$

$$= 8.248 \text{ W/m}^2\text{-K}$$

Que 1.19. **A furnace wall is composed of 220 mm of fire brick, 150 mm of common brick, 50 mm of 85 % magnesia and 3 mm of steel plate on the outside. If the inside surface temperature is 1500 °C and outside surface temperature is 90 °C, estimate the temperature between layers and calculate the heat loss in kJ/hm². Assume, k (for fire brick) = 4 kJ/mh- °C, k (for common brick) = 2.8 kJ/mh-°C, k (for 85 % magnesia) = 2.4 kJ/mh-°C, k (steel) = 240 kJ/mh-°C.**

AKTU 2017-18, Marks 10

Answer

Given : L_A = 220 mm = 0.22 m, L_B = 150 mm = 0.15 m,
L_C = 50 mm = 0.05 m, L_D = 3 mm = 0.003 m, t_1 = 1500 °C, t_5 = 90 °C,
k_A = 4 kJ/mh-°C, k_B = 2.8 kJ/mh-°C, k_C = 2.4 kJ/mh-°C,
k_D = 240 kJ/mh-°C.

To Find :
i. Heat loss.
ii. Temperature between layers.

1. The equivalent thermal resistances of various layers are as follows :

$$R_{th\text{-}A} = \frac{L_A}{k_A} = \frac{0.22}{4} = 0.055 \text{ m}^2\text{h-°C/kJ}$$

$$R_{th\text{-}B} = \frac{L_B}{k_B} = \frac{0.15}{2.8} = 0.05357 \text{ m}^2\text{h-°C/kJ}$$

$$R_{th\text{-}C} = \frac{L_C}{k_C} = \frac{0.05}{2.4} = 0.02083 \text{ m}^2\text{h-°C/kJ}$$

$$R_{th\text{-}D} = \frac{L_D}{k_D} = \frac{0.003}{240} = 1.25 \times 10^{-5} \text{ m}^2\text{h-°C/kJ}$$

2. Total thermal resistance,

$$(R_{th})_{total} = 0.055 + 0.05357 + 0.02083 + 1.25 \times 10^{-5}$$
$$= 0.1294 \text{ m}^2\text{h-°C/kJ}$$

3. Heat loss,

$$q = \frac{(t_1 - t_5)}{(R_{th})_{total}} = \frac{(1500 - 90)}{0.1294} = 10896.445 \text{ kJ/hm}^2$$

4. Also,

$$q = \frac{t_4 - t_5}{R_{th\text{-}D}}$$

or

$$t_4 = t_5 + q\, R_{th\text{-}D} = 90 + 10896.445 \times 1.25 \times 10^{-5}$$
$$= 90.136 \text{ °C}$$

5. Similarly,

$$t_3 = t_4 + q\, R_{th\text{-}C} = 90.136 + 10896.445 \times 0.02083$$
$$= 317.109 \text{ °C}$$

and

$$t_2 = t_3 + q\, R_{th\text{-}B} = 317.109 + 10896.445 \times 0.05357$$
$$= 900.83 \text{ °C}$$

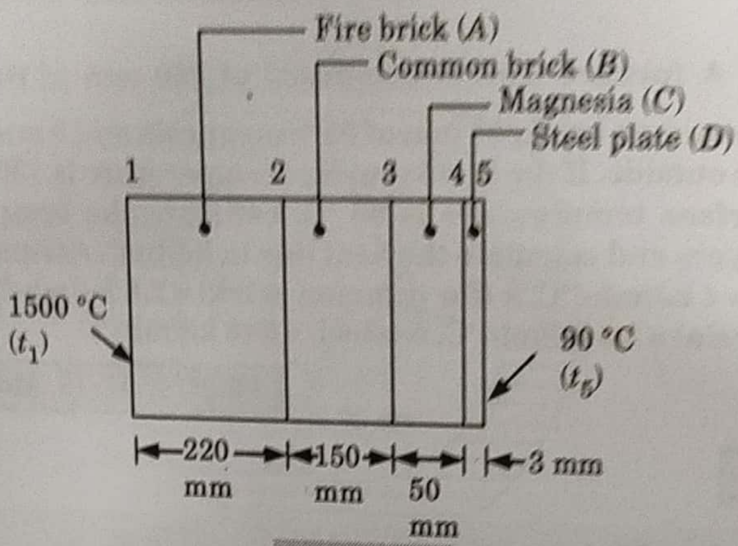

Fig. 1.19.1.

Que 1.20. **Derive the expression for maximum temperature within a plane wall with uniform heat generation when both the surfaces of the wall having same temperature.**

Answer

1. Consider a plane wall of thickness L, of uniform thermal conductivity k and in which heat sources are uniformly distributed in the whole volume.
2. Let the wall surface are maintained at temperature t_1 and t_2.
3. Assume that heat flow is one-dimensional, under steady state conditions and there is a uniform volumetric heat generation within the wall.
4. Consider an element of thickness dx at a distance x from the left face of the wall.
5. Heat conducted in at a distance x,

$$Q_x = -kA\frac{dt}{dx}$$

Heat generated in the element,

$$Q_g = q_g A dx$$

Heat conducted out at a distance $(x + dx)$,

$$Q_{(x+dx)} = Q_x + \frac{d}{dx}(Q_x)\,dx$$

6. From energy balance, we get,

$$Q_x + Q_g = Q_{x+dx}$$

$$Q_x + Q_g = Q_x + \frac{d}{dx}(Q_x)\,dx$$

$$Q_g = \frac{d}{dx}(Q_x)\,dx$$

$$q_g A\,dx = \frac{d}{dx}\left[-kA\frac{dt}{dx}\right]dx = -kA\,\frac{d^2t}{dx^2}\,dx$$

or $$\frac{d^2t}{dx^2} + \frac{q_g}{k} = 0 \quad ...(1.20.1)$$

7. Integrating eq. (1.20.1) we get

$$\frac{dt}{dx} = -\frac{q_g}{k}x + C_1 \quad ...(1.20.2)$$

8. On integrating eq. (1.20.2), we get.

$$t = -\frac{q_g}{2k}x^2 + C_1 x + C_2 \quad ...(1.20.3)$$

9. Now, if both the surfaces are at same temperature then,

at, $x = 0$, $t = t_1 = t_w$

and at, $x = L$, $t = t_2 = t_w$

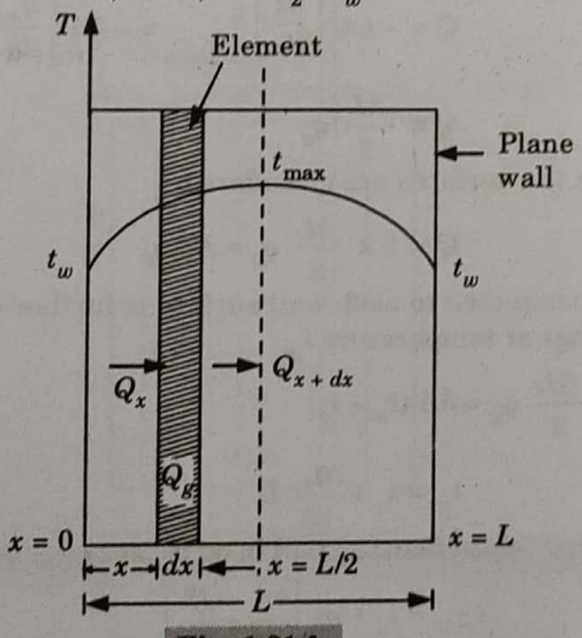

Fig. 1.21.1.

10. By applying above boundary conditions we in eq. (1.20.3), get,

$$C_2 = t_w$$

and $$C_1 = \frac{q_g}{2k}L$$

11. Substituting the value of C_1 and C_2 in eq. (1.20.3) :

$$t = -\frac{q_g}{2k}x^2 + \frac{q_g}{2k}L\,x + t_w$$

12. For the location of maximum temperature :

$$\frac{dt}{dx} = 0 \quad \text{or} \quad \frac{dt}{dx} = \frac{q_g}{2k}(L - 2x) = 0$$

Since, $\frac{q_g}{2k} \neq 0$, therefore, $(L - 2x) = 0$ or $x = \frac{L}{2}$

13. The maximum temperature occurs at $x = \frac{L}{2}$ *i.e.*, mid plane of plane wall and its value equals

$$t_{max} = \left[\frac{q_g}{2k}(L-x)\,x\right]_{x=\frac{L}{2}} + t_w$$

$$t_{max} = \left[\frac{q_g}{2k}\left(L-\frac{L}{2}\right)\frac{L}{2}\right] + t_w$$

$$t_{max} = \frac{q_g}{8k}L^2 + t_w \qquad \text{...(1.20.4)}$$

14. Heat transfer takes place towards both the surfaces and for each surface it is given by :

$$Q = -kA\left(\frac{dt}{dx}\right)_{x=0 \text{ or } x=L} = -kA\left[\frac{q_g}{2k}(L-2x)\right]_{x=0 \text{ or } x=L}$$

i.e.,
$$Q = \frac{AL}{2}q_g$$

15. When both the surfaces are considered

$$Q = 2 \times \frac{AL}{2}q_g = AL\,q_g$$

16. Also heat conducted to each wall surface is further dissipated to the surroundings at temperature t_a,

Thus, $\frac{AL}{2}q_g = hA\,(t_w - t_a)$

or
$$t_w = t_a + \frac{q_g}{2h}L \qquad \text{...(1.20.5)}$$

17. Substituting value of eq. (1.20.5) in eq (1.20.4), we get

$$t_{max} = t_a + q_g\left[\frac{L}{2h} + \frac{L^2}{8k}\right]$$

Que 1.21. **Derive the expression for maximum temperature within a cylinder with uniform heat generation in one-dimensional under steady state condition.**

Answer

1. Consider a cylindrical rod in which one-dimensional radial conduction is taking place under steady state condition.
2. Heat conducted in at radius r,

$$Q_r = -k\,2\pi rL\,\frac{dt}{dr}$$

3. Heat generated in the element,

$$Q_g = q_g \, 2\pi r \, dr \, L$$

Heat conducted out at radius $(r + dr)$,

$$Q_{r+dr} = Q_r + \frac{d}{dr}(Q_r)\, dr$$

4. Under steady state condition,

$$Q_r + Q_g = Q_{r+dr} = Q_r + \frac{d}{dr}(Q_r)\, dr$$

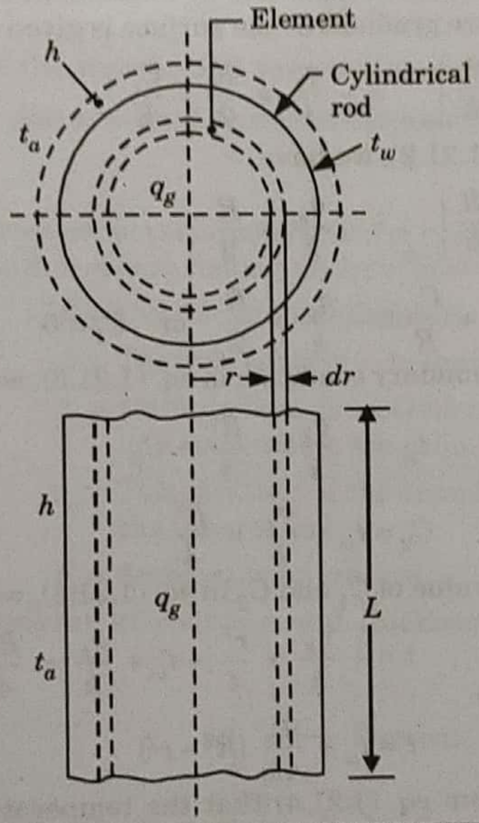

Fig. 1.21.1. Heat conduction in a solid cylinder with heat generation.

$$\therefore \quad Q_g = \frac{d}{dr}(Q_r)\, dr$$

$$q_g \, 2\pi r \, dr \, L = \frac{d}{dr}\left[-k \, 2\pi r L \frac{dt}{dr}\right] dr$$

$$\frac{d}{dr}\left[r \frac{dt}{dr}\right] = -\frac{q_g}{k}\, r \qquad ...(1.21.1)$$

5. Integrating the eq. (1.21.1), we get

$$r\frac{dt}{dr} = -\frac{q_g}{k} \times \frac{r^2}{2} + C_1$$

$$\frac{dt}{dr} = -\frac{q_g}{k} \times \frac{r}{2} + \frac{C_1}{r} \qquad ...(1.21.2)$$

6. Again integrating the eq. (1.21.2), we get,

$$t = -\frac{q_g}{k} \times \frac{r^2}{4} + C_1 \log r + C_2 \quad ...(1.21.3)$$

7. To get the value of C_1 and C_2, apply boundary condition :
   i. At, $r = R, \quad t = t_w$
   ii. At, $r = 0, \quad \frac{dt}{dr} = 0,$
   iii. Heat generated = Heat loss by conduction at the rod surface.
8. The temperature gradient at the surface is given by,

$$\left[\frac{dt}{dr}\right]_{r=R} = -\frac{q_g}{k} \times \frac{R}{2} + \frac{C_1}{R}$$

Also from eq. (1.21.2), we have

$$\left[\frac{dt}{dr}\right]_{r=R} = -\frac{q_g}{k} \times \frac{R}{2}$$

$$\therefore \quad -\frac{q_g}{k} \times \frac{R}{2} + \frac{C_1}{R} = -\frac{q_g}{k} \times \frac{R}{2} \quad \text{or} \quad C_1 = 0$$

9. Applying the boundary condition in eq. (1.21.3), we get,

$$t_w = -\frac{q_g}{k} \times \frac{R^2}{4} + C_2$$

or

$$C_2 = t_w + \frac{q_g}{k} \times \frac{R^2}{4}$$

10. Substitute the value of C_1 and C_2 in eq. (1.21.3), we get

$$t = -\frac{q_g}{k} \times \frac{r^2}{4} + t_w + \frac{q_g}{k} \times \frac{R^2}{4}$$

$$t = t_w + \frac{q_g}{4k}[R^2 - r^2] \quad ...(1.21.4)$$

11. It is evident from eq. (1.21.4) that the temperature distribution is parabolic and the location of maximum temperature is at $r = 0$ *i.e.*, centre of the cylinder.

So,

$$t_{max} = t_w + \frac{q_g}{4k} R^2 \quad ...(1.21.5)$$

12. By combining eq. (1.21.4) and eq. (1.21.5), we arrive at the following dimensionless form of temperature distribution,

$$\frac{t - t_w}{t_{max} - t_w} = \frac{\frac{q_g}{4k}[R^2 - r^2]}{\frac{q_g}{4k}.R^2} = \frac{(R^2 - r^2)}{R^2} = 1 - \left(\frac{r}{R}\right)^2$$

i.e.,

$$\frac{t - t_w}{t_{max} - t_w} = 1 - \left(\frac{r}{R}\right)^2$$

13. Also,

Energy generated within the rod = Energy dissipated by convection at the rod boundary

i.e., $q_g (\pi R^2 L) = h\, 2\pi RL (t_w - t_a)$

or $$t_w = t_a + \frac{q_g}{2h} \times R \qquad ...(1.21.6)$$

14. By putting the value of eq. (1.21.6) in eq. (1.21.5), we get

$$t_{max} = t_a + \frac{q_g}{2h} \times R + \frac{q_g}{4k} \times R^2$$

Que 1.22. **Derive the expression for maximum temperature with in a sphere in one dimensional under steady state condition.**

Answer

1. Consider one dimensional radial conduction of heat, under steady state conditions through a sphere having uniform heat generation.
2. Let, R = Outside radius of sphere,
   k = Thermal conductivity (uniform),
   q_g = Uniform heat generation per unit volume, per unit time within the solid,
   t_w = Temperature of the outside surface (wall) of the sphere, and
   t_a = Ambient temperature.
3. Consider an element at radius r and thickness, dr as shown in Fig. 1.22.1.

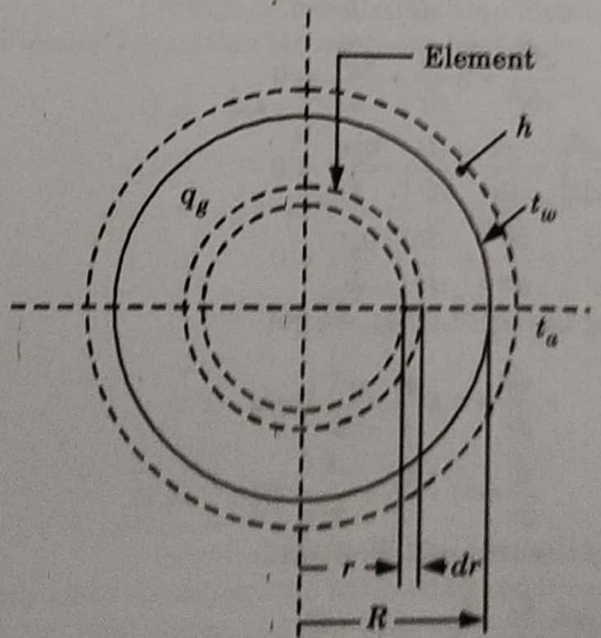

Fig. 1.22.1. Sphere with uniform heat generation.

4. Heat conducted in at radius r,

$$Q_r = -kA\frac{dt}{dr} = -k \times 4\pi r^2 \times \frac{dt}{dr}$$

Heat generated in the element,

$$Q_g = q_g \times A \times dr = q_g \times 4\pi r^2 \times dr$$

Heat conducted out at radius $(r + dr)$

$$Q_{(r+dr)} = Q_r + \frac{d}{dr}(Q_r)\, dr$$

5. Under steady state conditions, we have

$$Q_r + Q_g = Q_{(r+dr)}$$

$$= Q_r + \frac{d}{dr}(Q_r)\, dr$$

$$Q_g = \frac{d}{dr}(Q_r)\, dr$$

$$q_g \times 4\pi r^2 \times dr = \frac{d}{dr}\left[-4\pi k\, r^2 \times \frac{dt}{dr}\right] dr$$

$$q_g \times 4\pi r^2 \times dr = -4\pi k\, \frac{d}{dr}\left[r^2 \times \frac{dt}{dr}\right] dr$$

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 \times \frac{dt}{dr}\right] + \frac{q_g}{k} = 0$$

$$\frac{1}{r^2}\left[r^2 \times \frac{d^2t}{dr^2} + 2r \times \frac{dt}{dr}\right] + \frac{q_g}{k} = 0$$

$$\frac{d^2t}{dr^2} + \frac{2}{r} \times \frac{dt}{dr} + \frac{q_g}{k} = 0$$

$$r\frac{d^2t}{dr^2} + 2\frac{dt}{dr} + \frac{q_g r}{k} = 0$$

or
$$r\frac{d^2t}{dr^2} + \frac{dt}{dr} + \frac{dt}{dr} + \frac{q_g r}{k} = 0$$

or
$$\frac{dt}{dr}\left(r\frac{dt}{dr}\right) + \frac{dt}{dr} + \frac{q_g r}{k} = 0 \qquad ...(1.22.1)$$

6. Integrating the eq. (1.22.1), we get

$$r\frac{dt}{dr} + t + \frac{q_g}{k} \times \frac{r^2}{2} = C_1$$

or
$$\frac{d}{dr}(rt) + \frac{q_g}{k} \times \frac{r^2}{2} = C_1 \qquad ...(1.22.2)$$

7. Integrating the eq. (1.22.2), we get

$$rt + \frac{q_g}{k} \times \frac{r^3}{6} = C_1 r + C_2 \qquad ...(1.22.3)$$

8. At the centre of sphere, $r = 0$

$\therefore$
$$C_2 = 0$$

9. Applying boundary condition, at $r = R, t = t_w$ to eq. (1.22.3), we get

$$Rt_w + \frac{q_g}{k} \times \frac{R^3}{6} = C_1 R \qquad (\because \; C_2 = 0)$$

or
$$C_1 = t_w + \frac{q_g}{6k} R^2$$

10. By substituting the values of C_1 and C_2 in eq. (1.22.3), we have the temperature distribution as

$$rt + \frac{q_g}{k} \times \frac{r^3}{6} = \left[t_w + \frac{q_g}{6k} R^2\right] r$$

or
$$t + \frac{q_g}{6k} \times r^2 = t_w + \frac{q_g}{6k} R^2$$

or
$$t = t_w + \frac{q_g}{6k}(R^2 - r^2) \qquad ...(1.22.4)$$

11. From eq. (1.22.4) it is evident that the temperature distribution is parabolic; the maximum temperature occurs at the centre ($r = 0$) and its value is given by

$$t_{max} = t_w + \frac{q_g}{6k} R^2 \qquad ...(1.22.5)$$

12. From eq. (1.22.4) and eq. (1.22.5), we have

$$\frac{t - t_w}{t_{max} - t_w} = \frac{R^2 - r^2}{R^2} = 1 - \left(\frac{r}{R}\right)^2$$

i.e.,
$$\frac{t - t_w}{t_{max} - t_w} = 1 - \left(\frac{r}{R}\right)^2$$

(Temperature distribution in dimensionless form)

13. Invoking Fourier's equation (to evaluate heat flow), we have

$$Q = -kA\left(\frac{dt}{dr}\right)_{r=R}$$

$$= -k \times 4\pi R^2 \times \frac{d}{dr}\left[t_w + \frac{q_g}{6k}(R^2 - r^2)\right]_{r=R}$$

$$= -k \times 4\pi R^2 \left[\frac{q_g}{6k}(-2r)\right]_{r=R}$$

$$= k \times 4\pi R^2 \times \frac{q_g}{3k} \times R$$

$$Q = \frac{4}{3}\pi R^3 \times q_g$$

(= volume of sphere × heat generation capacity)

14. Under steady state conditions the heat conducted (or generated) should be equal to the heat convected from the outer surface of the sphere.

i.e.,
$$q_g \times \frac{4}{3}\pi R^3 = h \times 4\pi R^2 (t_w - t_a)$$

or $$t_w = t_a + \frac{q_g R}{3h} \quad ...(1.22.6)$$

15. Putting the value of eq. (1.22.6) in eq. (1.22.5), we get
The maximum temperature,

$$t_{max} = t_a + \frac{q_g}{3h} \times R + \frac{q_g}{6k} \times R^2$$

PART-3

Concept of Thermal Resistance, Analogy between Heat and Electricity Flow, Thermal Contact Resistance, Overall Heat Transfer Coefficient, Critical Radius of Insulation.

CONCEPT OUTLINE : PART-3

Thermal Resistance : It is the obstruction to the flow of heat transfer. It is analogous to the electric resistance in electrical system.

Analogy between Thermal and Electrical System :

Thermal System	Electrical System
Heat transfer, Q	Current, I
Change in temperature, dt	Change in voltage, dV
Thermal resistance, R_{th}	Electrical resistance, R

Overall Heat Transfer Coefficient : It is defined as the quantity which gives the heat transmitted per unit area per unit time per unit degree temperature difference between the bulk fluids on each side of the substance.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.23. **What do you mean by analogous system ? Explain the thermal resistance in case of conduction, convection and radiation.**

Answer

A. **Analogous System :**

1. When two physical systems are described by similar equations and have similar boundary conditions, then the system are said to be analogous.

2. The heat transfer process may be compared by analogy with the flow of electricity in an electrical system.

B. Thermal Resistance :

i. For Conduction :

As per Ohm's law :

Current, $$I = \frac{\text{Potential difference}}{\text{Electrical resistance}} = \frac{dV}{R}$$

By analogy, for conduction

$$Q = \frac{\text{Temperature difference } (dt)}{dx / kA}$$

$$R_{th} = \frac{dx}{kA}$$

R_{th} = Thermal conduction resistance.

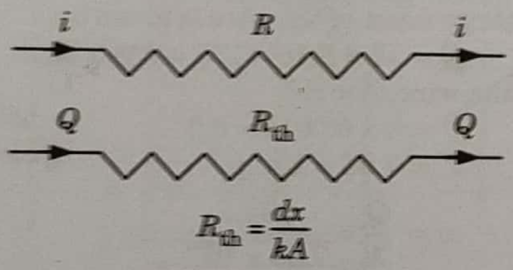

Fig. 1.23.1.

ii. For Convection : Convection heat transfer is given by Newton's law of cooling *i.e.*,

$$Q_{conv.} = h\,A(t_s - t_f)$$

$$Q_{conv.} = \frac{(t_s - t_f)}{1/hA}$$

$$R_{th_{conv.}} = \frac{1}{hA}$$

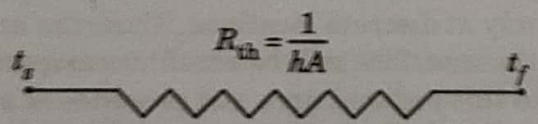

Fig. 1.23.2.

iii. For Radiation : Radiation heat transfer is governed by Stefan Boltzmann law *i.e.*,

$$Q = \sigma A T^4$$

or $$Q = \epsilon \sigma A (T_1^4 - T_2^4) \text{ (For real surface)}$$

or $$Q = \frac{(T_1 - T_2)}{\dfrac{1}{\epsilon \sigma A (T_1 + T_2)(T_1^2 + T_2^2)}}$$

So, $$(R_{th})_{Rad.} = \frac{1}{\epsilon \sigma A (T_1 + T_2)(T_1^2 + T_2^2)}$$

Que 1.24. **A wire of 1.2 mm diameter and 200 mm length is submerged horizontally in water at 7 bar. The wire carries a current of 135 A with an applied voltage of 2.18 V. If the surface of the wire is maintained at 200 °C, calculate the heat flux.**

AKTU 2014-15, Marks 05

Answer

Given : $d = 1.2$ mm $= 0.0012$ m, $l = 200$ mm $= 0.2$ m, $I = 135$ A
$V = 2.18$ V, $t_s = 200$ °C
To Find : Heat flux.

1. The electrical energy input of the wire is given by,
$$Q = VI = 2.18 \times 135 = 294.3 \text{ W}$$
2. Surface area of the wire, $A = \pi dl$
$$= \pi \times 0.0012 \times 0.2$$
$$= 7.54 \times 10^{-4} \text{ m}^2$$
3. Heat flux,
$$q = \frac{Q}{A} = \frac{294.3}{7.54 \times 10^{-4}}$$
$$= 0.39 \times 10^6 \text{ W/m}^2$$
$$= 0.39 \text{ MW/m}^2$$

Que 1.25. **Explain thermal contact resistance.**

AKTU 2013-14, Marks 05

Answer

1. In real systems, due to surface roughness and void spaces, the contact surfaces touch only at discrete locations. Thus, the area available for flow of heat at the interface will be small as compared to geometric face area. Due to this reduced area and presence of air voids, a large resistance to heat flow at the interface occurs. This resistance is known as thermal contact resistance.
Mathematically,
$$R_c = \frac{\Delta t}{Q} = \frac{\Delta t}{qA}$$
2. It causes temperature drop between two materials at the interface.

Que 1.26. **Derive an expression for overall heat transfer coefficient through a plane wall.**

Answer

1. Heat transfer between hot fluid and surface-1 of the wall take place due to convection. The amount of heat transfer is given as,

$$Q = h_{hf} A (t_{hf} - t_1) \quad ...(1.26.1)$$

2. Heat transfer within the wall takes place due to conduction. The amount of heat transfer is given as

$$Q = \frac{KA(t_1 - t_2)}{L} \quad ...(1.26.2)$$

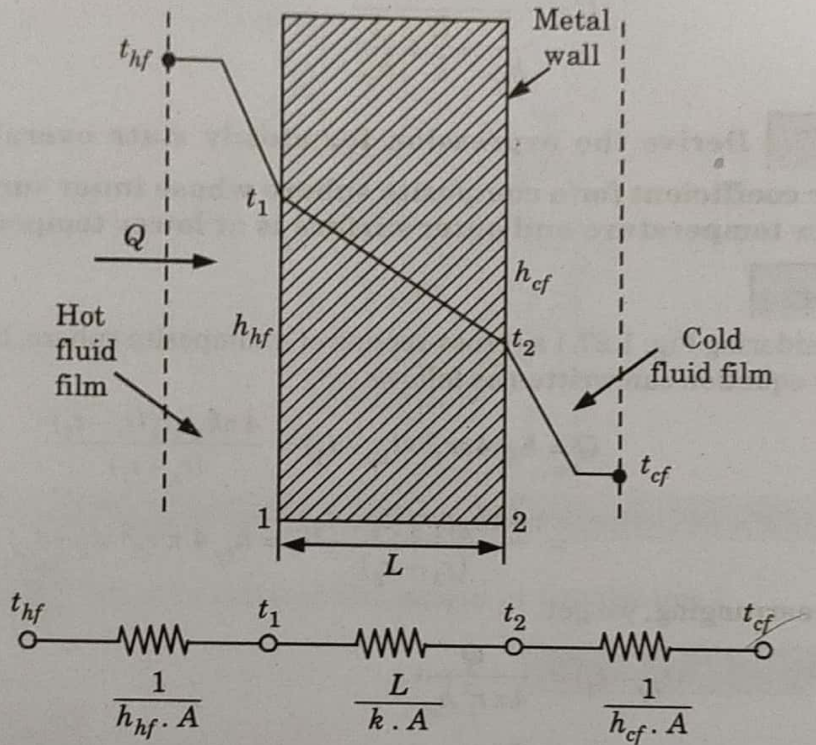

Fig. 1.26.1. The overall heat transfer through a plane wall.

3. Heat transfer between surface-2 of the wall and cold fluid takes place due to convection. The amount of heat transfer is given as

$$Q = h_{cf} A (t_2 - t_{cf}) \quad ...(1.26.3)$$

4. By rearranging the eq. (1.26.1), eq. (1.26.2) and eq. (1.26.3), we get

$$(t_{hf} - t_1) = \frac{Q}{h_{hf} A} \quad ...(1.26.4)$$

$$(t_1 - t_2) = \frac{QL}{kA} \quad ...(1.26.5)$$

$$(t_2 - t_{cf}) = \frac{Q}{h_{cf} A} \quad ...(1.26.6)$$

5. By adding eq. (1.26.4), eq. (1.26.5) and eq. (1.26.6), we get

$$(t_{hf} - t_{cf}) = Q \left[\frac{1}{h_{hf} A} + \frac{L}{kA} + \frac{1}{h_{cf} A} \right]$$

or

$$Q = \frac{A(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

6. If U is the overall coefficient of heat transfer, then

$$Q = U\,A(t_{h_f} - t_{c_f}) = \frac{A(t_{h_f} - t_{c_f})}{\dfrac{1}{h_{hf}} + \dfrac{L}{k} + \dfrac{1}{h_{cf}}}$$

or

$$U = \frac{1}{\dfrac{1}{h_{hf}} + \dfrac{L}{k} + \dfrac{1}{h_{cf}}}$$

Que 1.27. **Derive the expression for steady state overall heat transfer coefficient for a composite sphere whose inner surface is at higher temperature and outer surface is at lower temperature.**

Answer

1. Considering Fig. 1.27.1 as cross-section of a composite sphere, the heat flow equation can written as follows

$$Q = h_{h_f} 4\pi r_1^2 (t_{h_f} - t_1) = \frac{4\pi k_A r_1 r_2 (t_1 - t_2)}{(r_2 - r_1)}$$

$$= \frac{4\pi k_B r_2 r_3 (t_2 - t_3)}{(r_3 - r_2)} = h_{c_f} 4\pi r_3^2 (t_3 - t_{c_f})$$

2. By rearranging, we get

$$(t_{h_f} - t_1) = \frac{Q}{4\pi r_1^2 h_{h_f}} \quad \text{...(1.27.1)}$$

$$(t_1 - t_2) = \frac{Q(r_2 - r_1)}{4\pi k_A r_1 r_2} \quad \text{...(1.27.2)}$$

$$(t_2 - t_3) = \frac{Q(r_3 - r_2)}{4\pi k_B r_2 r_3} \quad \text{...(1.27.3)}$$

$$(t_3 - t_{c_f}) = \frac{Q}{4\pi r_3^2 h_{c_f}} \quad \text{...(1.27.4)}$$

3. Adding eq. (1.27.1), (1.27.2), (1.27.3) and (1.27.4), we get

$$(t_{h_f} - t_{c_f}) = \frac{Q}{4\pi}\left[\frac{1}{h_{h_f} r_1^2} + \frac{(r_2 - r_1)}{k_A r_1 r_2} + \frac{(r_3 - r_2)}{k_B r_2 r_3} + \frac{1}{h_{c_f} r_3^2}\right]$$

So,

$$Q = \frac{(t_{h_f} - t_{c_f})}{\dfrac{1}{4\pi}\left[\dfrac{1}{h_{h_f} r_1^2} + \dfrac{(r_2 - r_1)}{k_A r_1 r_2} + \dfrac{(r_3 - r_2)}{k_B r_2 r_3} + \dfrac{1}{h_{c_f} r_3^2}\right]} \quad \text{...(1.27.5)}$$

6. If U is the overall heat transfer coefficient. Then, heat transfer is given as,

$$Q = U\,A(t_{h_f} - t_{c_f}) \quad \text{...(1.27.6)}$$

7. Comparing the eq. (1.27.5) and (1.27.6), we get

$$U = \frac{4\pi}{A}\left[\frac{1}{\left\{\frac{1}{h_{h_f} r_1^2} + \frac{(r_2 - r_1)}{k_A r_1 r_2} + \frac{r_3 - r_2}{k_B r_2 r_3} + \frac{1}{h_{c_f} r_3^2}\right\}}\right]$$

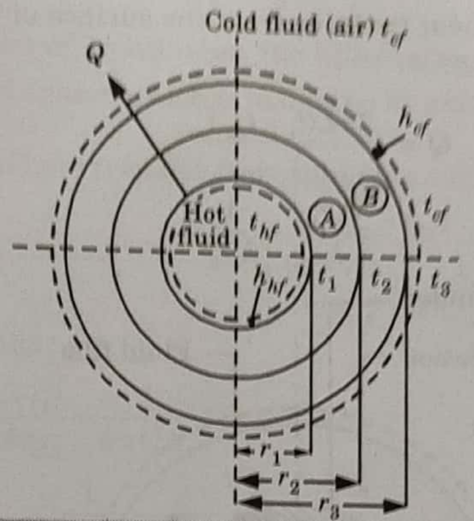

Fig. 1.27.1. Steady state conduction through a composite sphere.

Que 1.28. **What is critical thickness of insulation ?**

AKTU 2013-14, Marks 05

Answer

1. The thickness upto which heat flow rate increases and after which heat rate flow decreases is termed as critical thickness of insulation.
2. In case of cylinders and spheres it is called critical radius (r_c).
   i. For cylinder, $r_c = k / h$
   ii. For sphere, $r_c = 2k / h$
3. The addition of insulation always increases the conductive thermal resistance.
4. But when the total thermal resistance is made of conductive thermal resistance $[(R_{th})_{cond.}]$ and convective thermal resistance $[(R_{th})_{conv.}]$, the addition of insulation in some cases may reduce the convective thermal resistance due to increase in surface area, as in the case of a cylinder and sphere, and the total thermal resistance may actually decrease resulting in increased heat flow.

Que 1.29. **Prove that the critical radius of insulation for cylinder is, $r_c = \frac{k}{h_o}$ and for sphere is, $r_c = \frac{2k}{h_o}$.**

Answer

A. Critical Radius of Insulation for Cylinder :

1. Consider a solid cylinder of radius r_1 insulated with an insulation of thickness $(r_2 - r_1)$.
2. Then the rate of heat transfer from the surface of the solid cylinder to the surroundings is,

$$Q = \frac{2\pi L(t_1 - t_{air})}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k} + \frac{1}{h_o r_2}} \quad ...(1.29.1)$$

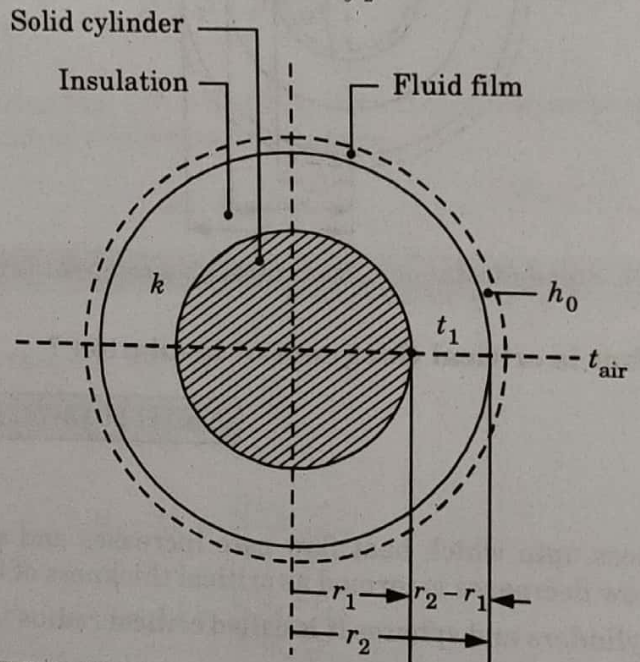

Fig. 1.29.1. Critical radius of insulation for cylinder.

3. From eq. (1.29.1) it is evident that as r_2 increases, the factor $\frac{\ln(r_2/r_1)}{k}$ increases but the factor $\frac{1}{h_o r_2}$ decreases. Then, the required condition is,

$$\frac{d}{dr_2}\left[\frac{\ln\left(\frac{r_2}{r_1}\right)}{k} + \frac{1}{h_o r_2}\right] = 0$$

$$\frac{1}{k}\frac{1}{r_2} + \frac{1}{h_o}\left(-\frac{1}{r_2^2}\right) = 0$$

$$\frac{1}{k} - \frac{1}{h_o r_2} = 0$$

$$h_o r_2 = k$$

$$r_2 = (r_c) = \frac{k}{h_o}$$

B. Critical Radius of Insulation for Sphere :

1. Consider a solid sphere which is insulated by a thickness of insulation $(r_2 - r_1)$.
2. Then the rate of heat transfer from sphere to surrounding is given by,

$$Q = \frac{(t_1 - t_{air})}{\left[\frac{(r_2 - r_1)}{4\pi k r_1 r_2} + \frac{1}{4\pi r_2^2 h_o}\right]}$$

3. For critical radius :

$$\frac{d}{dr_2}\left[\frac{(r_2 - r_1)}{4\pi k r_1 r_2} + \frac{1}{4\pi r_2^2 h_o}\right] = 0$$

$$\frac{1}{kr_2^2} - \frac{2}{r_2^3 h_o} = 0$$

$$2kr_2^2 = r_2^3 h_o$$

$$r_2 = r_c = \frac{2k}{h_o}$$

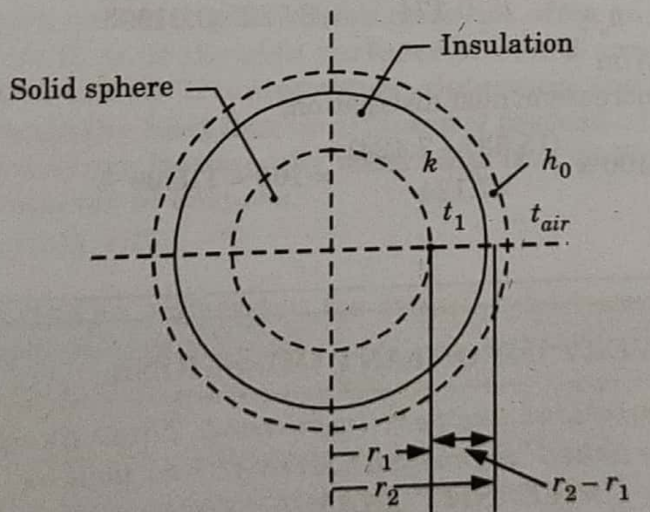

Fig. 1.29.2. Critical radius of insulation for sphere.

Que 1.30. **A wire of 6.5 mm diameter at a temperature of 60 °C is to be insulated by a material having k = 0.174 W/m-°C. Convective heat transfer coefficient (h_o) = 8.722 W/m²-°C. The ambient temperature is 20 °C for maximum heat loss. What is the minimum thickness of insulation and heat loss per meter length ? Also find the percentage increase in the heat dissipation.**

Answer

Given : $r_1 = \frac{6.5}{2} = 3.25 \text{ mm} = 0.00325 \text{ m}$

$k = 0.174 \text{ W/m-°C}$, $h_o = 8.722 \text{ W/m}^2\text{-°C}$, $t_{air} = 20 \text{ °C}$, $t_1 = 60 \text{ °C}$.

To Find :
i. Minimum thickness of insulation.
ii. Heat loss per meter length.
iii. Percentage increase in the heat dissipation.

1. We know that, the critical radius of insulation of cylinder

$$r_c = \frac{k}{h_o} = \frac{0.174}{8.722} = 0.01995 \text{ m} = 19.95 \text{ mm}$$

2. Minimum thickness of insulation $= r_c - r_1 = 19.95 - 3.25 = 16.7 \text{ mm}$
3. Heat loss without insulation,

$$Q_1 = \frac{2\pi L(t_1 - t_{air})}{\dfrac{1}{h_o r_1}} = \frac{2\pi \times 1 \times (60-20)}{\dfrac{1}{8.722 \times 0.00325}} = 7.124 \text{ W/m}$$

4. Heat loss with insulation when critical thickness is used,

$$Q_2 = \frac{2\pi L(t_1 - t_{air})}{\dfrac{\ln\left(\dfrac{r_c}{r_1}\right)}{k} + \dfrac{1}{h_o r_c}} = \frac{2\pi \times 1 \times (60-20)}{\dfrac{\ln\left(\dfrac{0.01995}{0.00325}\right)}{0.174} + \dfrac{1}{8.722 \times 0.01995}}$$

$= 15.537 \text{ W/m}$

5. Percentage increase in heat dissipation,

$$= \frac{Q_2 - Q_1}{Q_1} \times 100 = \frac{(15.537 - 7.124)}{7.124} \times 100 = 118.09 \%$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What are the mechanism of heat transfer ? How are they distinguished from each other ?

Ans. Refer Q. 1.1.

Q. 2. A carbon steel plate (k = 45 W/m-K) 600 mm × 900 mm × 25 mm is maintained at 310 °C. Air at 15 °C blows over the hot plate. If convection heat transfer coefficient is 22 W/m²-°C and 250 W is lost from the plate surface by radiation. Calculate the inside plate temperature.flow heat exchanger by using NTU method.

Ans. Refer Q. 1.5.

Q. 3. Derive the general heat conduction equation in rectangular or cartesian coordinates.

Ans. Refer Q. 1.6.

Q. 4. Derive the general heat conduction equation in cylindrical coordinates.

Ans. Refer Q. 1.7.

Q. 5. Derive expressions under one dimensional steady state heat conduction for temperature distribution for the sphere.

Ans. Refer Q. 1.14.

Q. 6. The insulation board for air conditioning purposes comprises three layers. A 12 cm thick layer of grass (k = 0.22 W/m-K) is sandwiched between 3 cm thick layer of plywood (k = 0.15 W/m-K) on each side. The bonding is achieved with glue which does not offer any resistance of heat flow. If the side surfaces of board are maintained at 40 °C and 20 °C temperature, determine the heat flux. How would the heat flux be affected if instead of glue the three pieces are fastened by steel bolts (k = 40 W/m-K) of 1.2 cm diameter at corners.

Ans. Refer Q. 1.17.

Q. 7. Derive an expression for overall heat transfer coefficient through a plane wall.

Ans. Refer Q. 1.26.

Q. 8. What is critical thickness of insulation ?

Ans. Refer Q. 1.28.

Q. 9. Prove that the critical radius of insulation for cylinder is,

$$r_c = \frac{k}{h_o} \text{ and for sphere is, } r_c = \frac{2k}{h_o}.$$

Ans. Refer Q. 1.29.

☺☺☺

2 UNIT

Fins & Transient Conduction

PART-1

Heat Transfer from Extended Surfaces, Fins of Uniform Cross-sectional Area, Errors of Measurement of Temperature in Thermometer Wells.

CONCEPT OUTLINE : PART-1

Fins : These are the extended surfaces used for increasing the heat transfer rate from the surfaces.

Efficiency of Fin (η_{fin}) : It is defined as the ratio of the actual heat transferred by the fin to the maximum heat transferable by fin, if entire fin area were at base temperature.

Effectiveness of Fin (ϵ_{fin}) : It is the ratio of the fin heat transfer rate to the heat transfer rate that would exist without a fin.

$$\epsilon_{fin} = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **What do you understand by fins and also explain types of fins ?**

Answer

A. Fins :

1. These are extended surfaces used for increasing the heat transfer rate from the surfaces whenever the available surface is found inadequate to transfer the required quantity of heat with the available temperature drop and convective heat transfer coefficient.
2. Heat transfer through fin is because of conduction and convection between its boundary and surrounding.

B. Types of Fins : Fins are of two types :

1. Fins of uniform cross-sectional area :
   a. Fins of rectangular profile, and
   b. Fins of circular profile.
2. Fins of non-uniform cross-sectional area :
   a. Longitudinal fin of trapezoidal profile,

b. Longitudinal fin of parabolic profile, and
c. Truncated conical spline.

Que 2.2. **What is the reason for the widespread use of fins on surfaces ?**

AKTU 2014-15, Marks 05

Answer

1. Generally fins are used to provide extra surface area to increase rate of heat transfer from existing surface.
2. In most of practical situation the heat transfer fluids involves liquid and gas.
3. In such a situation of heat transfer between liquid and gas, as gas is having less value of heat transfer coefficient it offers greater resistance for the heat transfer.
4. So to increase the rate of heat transfer on the gas side, extra surface area is provided by using fins.
5. Use of fins on surfaces is found in case of air-cooled cylinder, condenser tubes in refrigerator.

Que 2.3. **Derive an expression for temperature distribution and heat transfer rate in a straight fin of rectangular profile, when :**
i. Fin tip is insulated.
ii. Heat dissipation from an infinite long fin.
iii. The fin is of finite length and losses heat by convection.
Also write the advantages and application of fins.

OR

What are the applications of fins ? Establish an expression for temperature distribution in a straight fin of rectangular profile, when fin tip is insulated.

AKTU 2015-16, Marks 10

OR

Derive an expression of rectangular fin in case of heat dissipation from an infinite long fin. What are advantages and application of fins ?

AKTU 2017-18, Marks 10

Answer

A. Temperature Distribution and Heat Transfer Rate for Rectangular Fins :

1. For the proper design of fins, the knowledge of temperature distribution along the fin is necessary.
2. For analysis of heat flow through fin, following assumptions are made :
   i. Steady state heat conduction.

ii. No heat generation within the fin.

iii. One dimensional heat conduction.

iv. Homogeneous and isotropic fin material (*i.e.*, thermal conductivity of material is constant).

v. Uniform heat transfer coefficient (h) over the entire surface of the fin.

vi. Negligible thermal contact resistance.

vii. Negligible radiation.

3. Let, l = Length of the fin,
   b = Width of fin,
   y = Thickness of fin,
   t_0 = Temperature at the base of fin, and
   t_a = Temperature of the ambient / surrounding fluid.
4. Perimeter of fin, $P = 2(b + y)$.
5. Cross-sectional area of fin, $A_{CS} = by$.
6. Consider heat flows to and from an element of thickness dx at a distance x from the base.
7. Heat conducted into the element at plane x,

$$Q_x = -kA_{CS}\left[\frac{dt}{dx}\right]_x$$

8. Heat conducted out from element at plane $(x + dx)$,

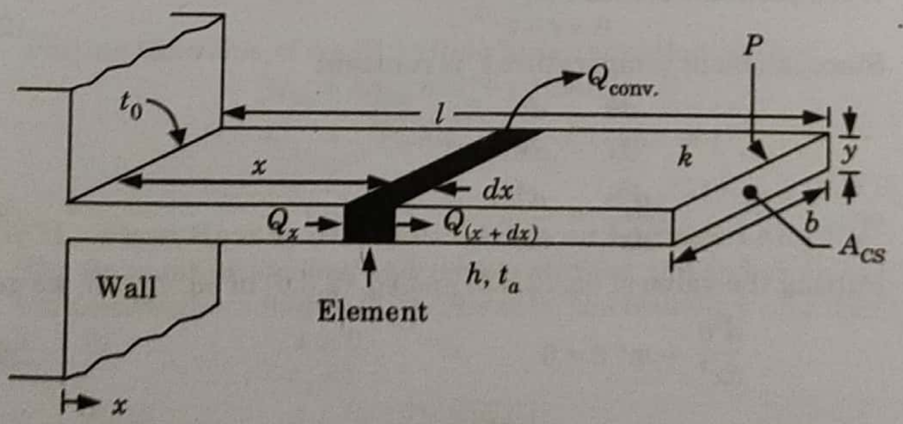

Fig. 2.3.1. Rectangular fin of uniform cross-section.

$$Q_{x+dx} = -kA_{CS}\left[\frac{dt}{dx}\right]_{x+dx}$$

9. Heat convected out of the element between the plane x and $(x + dx)$,

$$Q_{conv.} = h(P\,dx)(t - t_a) \quad ...(2.3.1)$$

10. Applying the energy balance on the element, we have

$$Q_x = Q_{x+dx} + Q_{conv.}$$

$$-kA_{CS}\left[\frac{dt}{dx}\right]_x = -kA_{CS}\left[\frac{dt}{dx}\right]_{x+dx} + h\,(P\,dx)\,(t - t_a) \quad ...(2.3.2)$$

11. By Taylor's expansion,

$$\left[\frac{dt}{dx}\right]_{x+dx} = \left(\frac{dt}{dx}\right)_x + \frac{d}{dx}\left(\frac{dt}{dx}\right)_x dx + \frac{d^2}{dx^2}\left(\frac{dt}{dx}\right)_x \frac{(dx)^2}{\underline{|2}} + ... \quad ...(2.3.3)$$

Substituting this in eq. (2.3.2), we have

$$-kA_{CS}\left[\frac{dt}{dx}\right]_x = -kA_{CS}\left[\frac{dt}{dx}\right]_x - kA_{CS}\frac{d}{dx}\left(\frac{dt}{dx}\right)_x dx$$

$$-kA_{CS}\frac{d^2}{dx^2}\left(\frac{dt}{dx}\right)_x \frac{(dx)^2}{\underline{|2}} + ... + h(P\,dx)(t - t_a)$$

Neglecting higher term as $dx \to 0$,

$$\therefore -kA_{CS}\left[\frac{dt}{dx}\right]_x = -kA_{CS}\left[\frac{dt}{dx}\right]_x - kA_{CS}\left(\frac{d^2t}{dx^2}\right) dx + h\,(P\,dx)\,(t - t_a)$$

$$kA_{CS}\left[\frac{d^2t}{dx^2}\right] dx - h\,(P\,dx)\,(t - t_a) = 0$$

or

$$\frac{d^2t}{dx^2} - \frac{hP}{kA_{CS}}(t - t_a) = 0 \quad ...(2.3.4)$$

12. If temperature excess θ is,

$$\theta = t - t_a \quad ...(2.3.5)$$

Since, ambient temperature t_a is constant

$$\therefore \quad \frac{d\theta}{dx} = \frac{dt}{dx}$$

or

$$\frac{d^2\theta}{dx^2} = \frac{d^2t}{dx^2} \quad ...(2.3.6)$$

13. Putting the value of eq. (2.3.5) and eq. (2.3.6) in eq. (2.3.4), we get

$$\frac{d^2\theta}{dx^2} - m^2\,\theta = 0 \quad ...(2.3.7)$$

Where, $m = \sqrt{\dfrac{hP}{kA_{CS}}}$

14. Eq. (2.3.4) and eq. (2.3.7) shows the general differential heat conduction equation for fins of uniform cross-sectional area.
15. The general solution of this linear and homogeneous second order differential equation is,

$$\theta = C_1\,e^{mx} + C_2\,e^{-mx}$$

or

$$t - t_a = C_1\,e^{mx} + C_2\,e^{-mx} \quad ...(2.3.8)$$

Where, C_1 and C_2 are the constants. These are determined by using proper boundary conditions.

Case I : When Fin Tip is Insulated :

1. When the end of fin is insulated, the boundary conditions are :
   i. At $x = 0, \quad \theta = \theta_0$
   ii. At $x = l, \quad \frac{dt}{dx} = 0$
2. Applying these boundary conditions in eq. (2.3.8), we get expression for temperature distribution,

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = \frac{\cosh\{m(l-x)\}}{\cosh(ml)} \quad ...(2.3.9)$$

Where, $\cosh\{m(l-x)\} = \frac{e^{m(l-x)} + e^{[-m(l-x)]}}{2}$

and, $\cosh(ml) = \frac{e^{ml} + e^{-ml}}{2}$

3. Differentiating eq. (2.3.9), we get

$$\frac{dt}{dx} = (t_0 - t_a)\left[\frac{\sinh\{m(l-x)\}}{\cosh(ml)}\right](-m)$$

$$\left[\frac{dt}{dx}\right]_{x=0} = -m(t_0 - t_a)\tanh(ml) \quad ...(2.3.10)$$

4. Rate of heat flow from the fin is given by

$$Q_{fin} = -kA_{CS}\left[\frac{dt}{dx}\right]_{x=0}$$

Putting the value of eq. (2.3.10) in above equation, we get

$$Q_{fin} = kA_{CS}\, m(t_0 - t_a)\tanh(ml)$$

$$Q_{fin} = \sqrt{PhkA_{CS}}\,(t_0 - t_a)\tanh(ml)$$

$$[\because m = \sqrt{hP/kA_{CS}}]$$

Case II : When Heat is Dissipated from an Infinite Long Fin :

1. The fin is infinitely long and temperature at the end of fin is that of ambient/surrounding fluid. In this case, the boundary conditions are :
   i. At $x = 0, \quad t = t_0$
   $t - t_a = t_0 - t_a$
   or, at $x = 0, \quad \theta = \theta_0$
   ii. At $x = \infty, \quad t = t_a$
   or $x = \infty, \quad \theta = 0$
2. Substituting these boundary condition in eq. (2.3.8), we get

$$\theta = \theta_0\, e^{-mx}$$

or $$(t - t_a) = (t_0 - t_a)\, e^{-mx} \quad ...(2.3.11)$$

3. Differentiating eq. (2.3.11) w.r.t. x, we get,

$$\left[\frac{dt}{dx}\right]_{x=0} = [-m(t_0 - t_a)\, e^{-mx}]_{x=0} = -m(t_0 - t_a) \quad ...(2.3.12)$$

4. Rate of heat flow across the base of fin is given by,

$$Q_{fin} = -kA_{CS}\left[\frac{dt}{dx}\right]_{x=0}$$

Putting the value of $\left[\frac{dt}{dx}\right]_{x=0}$ from eq. (2.3.12) in above equation, we get

$$Q_{fin} = \sqrt{PhkA_{CS}}\ (t_0 - t_a)$$

Case III : The Fin is of Finite Length and Losses Heat by Convection :

1. The boundary conditions are :
   i. At $x = 0, \theta = \theta_0$
   ii. Heat conducted to the fin at $(x = l)$ = Heat convected from the end to surrounding.

   i.e., $$-kA_{CS}\left[\frac{dt}{dx}\right]_{x=l} = hA_{SA}(t - t_a)$$

   Where, A_{SA} is surface area from which the convective heat transfer takes place and at the tip of fin, $A_{CS} = A_{SA}$.

   Thus, $$\frac{dt}{dx} = -\frac{h\theta}{k} \quad \text{at } x = l$$

2. Applying these boundary condition in eq. (2.3.8), we get

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh[m(l-x)] + \frac{h}{km}[\sinh\{m(l-x)\}]}{\cosh(ml) + \frac{h}{km}[\sinh(ml)]} \quad ...(2.3.13)$$

3. Differentiating eq. (2.3.13) w.r.t. x, we get

$$\left[\frac{dt}{dx}\right]_{x=0} = -m(t_o - t_a)\left[\frac{\sinh\{ml\} + \left[\frac{h}{km}\right]\cosh\{ml\}}{\cosh\{ml\} + \left[\frac{h}{km}\right]\sinh\{ml\}}\right] \quad ...(2.3.14)$$

4. Rate of heat flow through the fin is given by :

$$Q_{fin} = -kA_{CS}\left[\frac{dt}{dx}\right]_{x=0}$$

   Putting the value of $\left[\frac{dt}{dx}\right]_{x=0}$ from eq. (2.3.14) in above equation, we get

$$Q_{fin} = \sqrt{PhkA_{CS}}\ (t_o - t_a)\left[\frac{\tanh(ml) + \frac{h}{km}}{1 + \frac{h}{km}\tanh(ml)}\right]$$

B. Advantages :

1. By using the fin, heat transfer rate can be increased without any preventive maintenance.
2. It is the cheapest way for increasing the heat transfer rate from the hot bodies.

C. Applications :

1. Economisers for steam power plants,
2. Convectors for steam and hot-water heating systems,
3. Radiators for automobiles, and
4. Air cooled engine cylinder heads.

Que 2.4. **Explain effectiveness and efficiency of fin.**

Answer

A. Efficiency of Fin :

1. It is defined as the ratio of the actual heat transferred by the fin to the maximum heat transferable by fin, if entire fin area were at base temperature.

$$\eta_{fin} = \frac{\text{Actual heat transfer by the fin}(Q_{fin})}{\text{Maximum heat that would be transferred if whole surface of the fin is maintained at the base temperature }(Q_{max})}$$

2. For a fin which is infinitely long,

$$\eta_{fin} = \frac{\sqrt{PhkA_{CS}}\,(t_o - t_a)}{Phl(t_o - t_a)} = \sqrt{\frac{kA_{CS}}{hPl^2}} = \frac{1}{ml}$$

3. For a fin which is insulated at the tip,

$$\eta_{fin} = \frac{\sqrt{PhkA_{CS}}\,(t_o - t_a)\tanh(ml)}{Phl(t_o - t_a)} = \frac{\tanh(ml)}{ml}$$

B. Effectiveness of Fin :

1. It is the ratio of the fin heat transfer rate to the heat transfer rate that would exist without a fin.

Mathematically, $\varepsilon_{fin} = \dfrac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$

2. In case of infinitely long fin,

$$\varepsilon_{fin} = \sqrt{\frac{Pk}{hA_{CS}}}$$

3. For a straight rectangular fin of thickness '*y*' and width '*b*',

$$\varepsilon_{fin} = \sqrt{\frac{2k}{hy}}$$

Que 2.5. **Two pin fins are identical, except that the diameter of one of them is twice the diameter of the other. For which fin will the (*a*) effectiveness, and (*b*) fin efficiency be higher ? Explain.**

AKTU 2014-15, Marks 05

Answer

Let the diameter of fin I be d_1 and diameter of fin II be d_2.

Given : $d_1 = 2d_2$

a. **Effectiveness :**

1. The effectiveness of a fin is given as follows,

$$\varepsilon = \frac{kAm\,[t_s - t_\infty]\left(\dfrac{h \cosh mL + mk \sinh mL}{mk \cosh mL + h \sinh mL}\right)}{hA\,(t_s - t_\infty)}$$

2. Now,

$$\frac{\varepsilon_{\text{fin } I}}{\varepsilon_{\text{fin } II}} = \frac{\dfrac{kA_1 m(t_s - t_\infty)}{hA(t_s - t_\infty)}\left[\dfrac{h \cosh mL + mk \sinh mL}{mk \cosh mL + h \sinh mL}\right]}{\dfrac{kA_2 m(t_s - t_\infty)}{hA(t_s - t_\infty)}\left[\dfrac{h \cosh mL + mk \sinh mL}{mk \cosh mL + h \sinh mL}\right]}$$

$$\frac{\varepsilon_{\text{fin } I}}{\varepsilon_{\text{fin } II}} = \frac{A_1}{A_2} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2}$$

$$\frac{\varepsilon_{\text{fin } I}}{\varepsilon_{\text{fin } II}} = \frac{\frac{\pi}{4} (2d_2)^2}{\frac{\pi}{4} d_2^2}$$

$$\varepsilon_{\text{fin } I} = 4\,\varepsilon_{\text{fin } II}$$

3. So, effectiveness of fin I is higher than effectiveness of fin II.

b. **Efficiency :**

1. The efficiency of a fin is given as follows,

$$\eta_{\text{fin}} = \frac{1}{A\,[t_s - t_\infty]} \int_0^A (t - t_\infty)\, dA$$

2. Now,

$$\frac{\eta_{\text{fin } I}}{\eta_{\text{fin } II}} = \frac{\dfrac{1}{A_1\,[t_s - t_\infty]} \displaystyle\int_0^A [t - t_\infty]\, dA}{\dfrac{1}{A_2\,[t_s - t_\infty]} \displaystyle\int_0^A [t - t_\infty]\, dA} = \frac{A_2}{A_1}$$

$$\frac{\eta_{\text{fin } I}}{\eta_{\text{fin } II}} = \frac{\frac{\pi}{4} d_2^2}{\frac{\pi}{4} d_1^2} = \frac{\frac{\pi}{4} d_2^2}{\frac{\pi}{4} (2d_2)^2}$$

$$\frac{\eta_{fin\ I}}{\eta_{fin\ II}} = \frac{1}{4} \Rightarrow \eta_{fin\ I} = \frac{1}{4}\eta_{fin\ II} \qquad [\eta_{fin\ I} = 25\ \%\ \text{of}\ \eta_{fin\ II}]$$

3. So, efficiency of fin II is higher than efficiency of fin I.

Que 2.6. **A stainless steel fin (k = 20 W/m-K) having a diameter of 20 mm and a length of 0.1 m is attached to a wall at 300 °C. The ambient temperature is 50 °C and the heat transfer coefficient is 10 W/m²-K. The fin tip is insulated.**

Determine :

i. **The rate of heat dissipation from the fin.**

ii. **The temperature at the fin tip.** **AKTU 2013-14, Marks 10**

Answer

Given : d = 20 mm = 0.02 m, l = 0.1 m, t_o = 300 °C, t_a = 50 °C, k = 20 W/m-K, h = 10 W/m²-K

To Find : i. The rate of heat dissipation from the fin.
ii. The temperature at the fin tip.

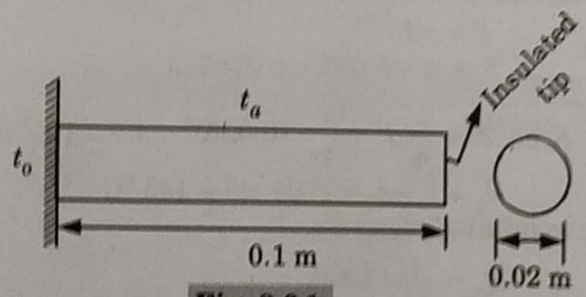

Fig. 2.6.1.

1. We know that, $$m = \sqrt{\frac{hP}{kA_{CS}}} = \sqrt{\frac{10 \times \pi \times d}{20 \times \frac{\pi}{4} \times d^2}} = \sqrt{\frac{10 \times 4}{20 \times 0.02}}$$

$$m = 10$$

2. Rate of heat transfer,

$$Q_{fin} = \sqrt{PhkA_{CS}}\ (t_o - t_a) \tanh (ml)$$

$$Q_{fin} = \sqrt{\pi \times 0.02 \times 10 \times 20 \times \frac{\pi}{4} \times (0.02)^2} \times (300 - 50) \times \tanh (10 \times 0.1)$$

$$= 11.96 \text{ W}$$

3. We know that,

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh \{m (l - x)\}}{\cosh (ml)}$$

Here, $x = l$

$$\therefore \quad \frac{t - t_a}{t_o - t_a} = \frac{1}{\cosh (ml)}$$

$$\frac{t - 50}{300 - 50} = \frac{1}{\cosh (10 \times 0.1)}$$

Temperature at fin tip, $t = 212\,°C$.

Que 2.7. **Aluminium fins 1.5 cm wide and 1.0 mm thick are placed on 2.5 cm diameter tube to dissipate the heat. The tube surface temperature is 170 °C and the ambient fluid temperature is 25 °C. Calculate the heat loss per fin for $h = 130\ W/m^2\text{-}°C$ for aluminium.**

AKTU 2014-15, Marks 10

Answer

Given : $b = 1.5$ cm $= 0.015$ m, $t = 1$ mm $= 0.001$ m, $d = 2.5$ cm $= 0.025$ m, $t_o = 170$ °C, $t_a = 25$ °C, $h = 130\ W/m^2\text{-}°C$, $k = 200$ W/m-°C
To Find : Heat loss per fin.

1. Perimeter, $P = \pi d$

$$= \pi \times 0.025 = 0.0785\ m$$

$$\text{Area, } A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.025)^2 = 4.9 \times 10^{-4}\ m^2$$

$$\theta_o = t_o - t_a = 170 - 25 = 145\ °C$$

2. Heat loss due to infinitely long fin,

$$Q_{fin} = \sqrt{hPkA}\ \theta_o$$

$$Q_{fin} = \sqrt{130 \times 0.0785 \times 200 \times 4.9 \times 10^{-4}} \times 145$$

$$Q_{fin} = 145.17\ W$$

Que 2.8. **What do you understand by thermometer well ? Estimate the error in temperature measurement in a thermometer well.**

Answer

A. **Thermometer Well :** It is defined as a small tube welded radially into a pipeline through which a fluid whose temperature is to be measured is flowing.

B. **Estimation of the Error in Temperature Measurement in a Thermometer Well :**

1. Let,

l = Length of the well/tube,

d = Internal diameter of the well/tube,

δ = Thickness of well/tube,

t_f = Temperature of the fluid flowing through the pipe (which is to be measured), and

t_o = Temperature of the pipe wall.

2. When the temperature of the fluid flowing through the pipeline is higher than the ambient temperature, the heat flows from the fluid towards the tube walls along the well. Consequently the temperature at the bottom of well becomes colder than the fluid flowing around, obviously the temperature shown by the thermometer will not be the true temperature of the fluid.

3. This error may be calculated by assuming the well to be a spine protruding from the wall of a pipe in which fluid is flowing.

4. It may be assumed, for simplicity, that there is no flow of heat from the tip of the well (*i.e.*, the tip of the well is insulated).

5. The temperature distribution at any distance x measured from pipe wall along the temperature well is given by

$$\frac{\theta_x}{\theta_o} = \frac{t_x - t_f}{t_o - t_f} = \frac{\cosh[m(l-x)]}{\cosh(ml)}$$

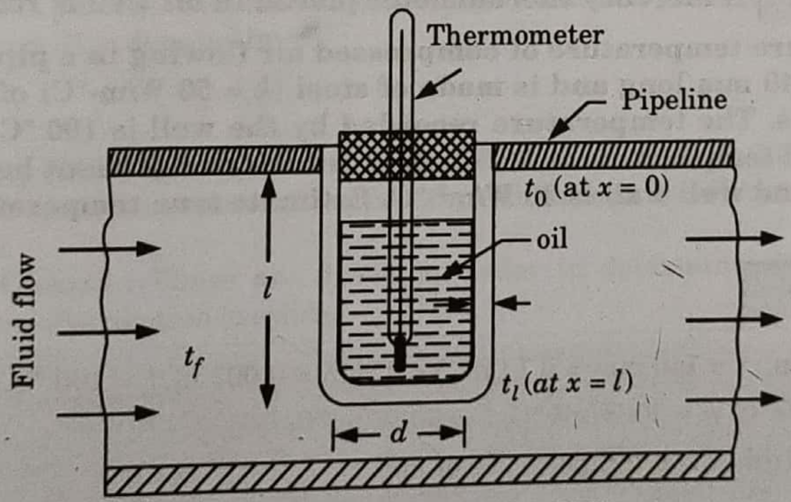

Fig. 2.8.1. Thermometer well.

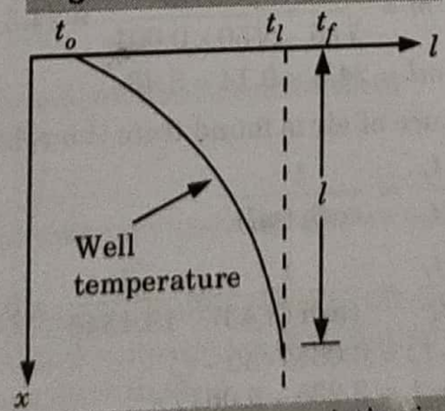

Fig. 2.8.2. Temperature variation in well.

6. At $x = l$, we have

$$\frac{t_l - t_f}{t_o - t_f} = \frac{\cosh[m(l - l)]}{\cosh(ml)} = \frac{1}{\cosh(ml)} \quad \text{[Thermometric error]}$$

Where, t_l = Temperature recorded by the thermometer at the bottom of the well.

7. Now, perimeter of the well,

$$P = \pi(d + 2\delta) \simeq \pi d$$

and cross-sectional area,

$$A_{cs} = \pi d\delta$$

$$\therefore \quad \frac{P}{A_{cs}} = \frac{\pi d}{\pi d\delta} = \frac{1}{\delta}$$

8. Then,

$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h}{k\delta}}$$

Thus, the temperature measured by the thermometer is not affected by the diameter of the well.

Que 2.9. **A mercury thermometer placed in oil well is required to measure temperature of compressed air flowing in a pipe. The well is 140 mm long and is made of steel (k = 50 W/m-°C) of 1 mm thickness. The temperature recorded by the well is 100 °C while pipe wall temperature is 50 °C. Heat transfer coefficient between the air and well wall is 30 W/m²-°C. Estimate true temperature of air.**

Answer

Given : l = 140 mm = 0.14 m, δ = 1 mm = 0.001 m, t_l = 100 °C, t_o = 50 °C, h = 30 W/m² °C, k = 50 W/m °C.
To Find : True temperature of air.

1. We know that,

$$m = \sqrt{\frac{h}{k\delta}} = \sqrt{\frac{30}{50 \times 0.001}} \simeq 24.5$$

2. Now,

$$ml = 24.5 \times 0.14 = 3.43$$

3. The true temperature of air is found from the relation,

$$\frac{t_l - t_f}{t_o - t_f} = \frac{1}{\cosh(ml)}$$

$$\frac{100 - t_f}{50 - t_f} = \frac{1}{\cosh(3.43)} = \frac{1}{15.4545} = 0.0647$$

$$(100 - t_f) = 0.0647\,(50 - t_f)$$

$$100 - t_f = 3.235 - 0.0647\, t_f$$

$$t_f = 103.46\ °C.$$

PART-2

Transient Heat Conduction, Lumped Capacitance Method, Time Constant, Unsteady State Heat Conduction in One Dimension only, Heisler Charts.

CONCEPT OUTLINE : PART-2

Transient Heat Conduction : Conduction of heat in unsteady state refers to the transient conditions wherein the heat flow and the temperature distribution at any point of the system vary continuously with time.

Newtonian Heating or Cooling : The process in which the internal resistance is assumed negligible in comparison with its surface resistance is called the Newtonian heating or cooling process.

Response of Thermocouple : It is defined as the time required for the thermocouple to attain the source temperature.

Time Constant : It is the time required for the temperature change to reach 36.8 percent of its final value in response to a step change in temperature. It is denoted by τ^*.

$$\tau^* = \frac{\rho VC}{hA_s}$$

Sensitivity : The time required by a thermocouple to reach its 63.2 percent of the initial value of temperature difference is called its sensitivity.

Heisler Charts : These are drawn in order to determine the temperature distribution in solids.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.10. **Explain transient heat conduction.**

Answer

1. The term transient or unsteady state designates a phenomenon which is time dependent.
2. Conduction of heat in unsteady state refers to the transient conditions wherein the heat flow and temperature distribution at any point of the system vary continuously with time.

3. Transient conditions occur in :
   i. Cooling of IC engine,
   ii. Automobile engines,
   iii. Heating and cooling of metal billets,
   iv. Heat treatment of metals by quenching, and
   v. Brick burning.
4. The temperature field in any transient problem is given by,

$$t = f(x, y, z, \tau)$$

5. The two types of variation in temperature during an unsteady state are as follows :

a. **Non-Periodic Variation :**

1. In a non-periodic transient state, the temperature at any point within the system varies non-linearly with time.
2. Examples :
   i. Heating of an ingot in a furnace, and
   ii. Cooling of bars, blanks and metal billets in steel works, etc.

b. **Periodic Variation :**

1. In a periodic transient state, temperatures undergo periodic changes which are either regular or irregular but definitely 'cyclic'.
2. A regular periodic variation is characterised by a harmonic sinusoidal or non-sinusoidal function, and irregular periodic variation is characterised by any function which is cyclic but not necessarily harmonic.
3. Examples : The temperature variations in :
   i. Cylinder of an IC engine,
   ii. Building during a period of 24 hours, and
   iii. Surface of earth during a period of 24 hours.

Que 2.11. **What do you mean by lump system analysis ? Derive the following expression for transient heat conduction**

$$\frac{t - t_\infty}{t_i - t_\infty} = \exp(-B_i F_0)$$

Where symbols have their usual meaning. Discuss the physical significance of Biot No. and Fourier no. **AKTU 2016-17, Marks 10**

OR

Prove that for a body whose thermal resistance is zero, the temperature required for cooling or heating can be obtained from the relation $(t - t_a)/(t_i - t_a) = \exp[-B_i F_o]$, where the symbols have their usual meanings.

AKTU 2017-18, Marks 10

Answer

A. Lump System Analysis :

1. The process in which the internal resistance is assumed negligible in comparison with its surface resistance is called the Newtonian heating or cooling process. The temperature in this process is considered to be uniform at a given time. Such an analysis is called lumped parameter analysis because the whole solid, whose energy at any time is a function of its temperature and total heat capacity is treated as one lump.

B. Expression :

1. Let us consider a body whose initial temperature is t_i throughout and which is placed suddenly in ambient air or any liquid at a constant temperature t_a.
2. The transient response of the body can be determined by relating its rate of change of internal energy with convective exchange at the surface *i.e.,*

$$Q = -\rho VC \frac{dt}{d\tau} = h A_S (t - t_a) \quad ...(2.11.1)$$

Where,
ρ = Density of solid, kg/m³,
t = Temperature of the body at any time, °C,
V = Volume of the body, m³,
A_S = Surface area of the body, m²,
C = Specific heat of body, J/kg-°C,
t_a = Ambient temperature, °C,
h = Unit surface conductance, W/m² °C, and
τ = Time, s.

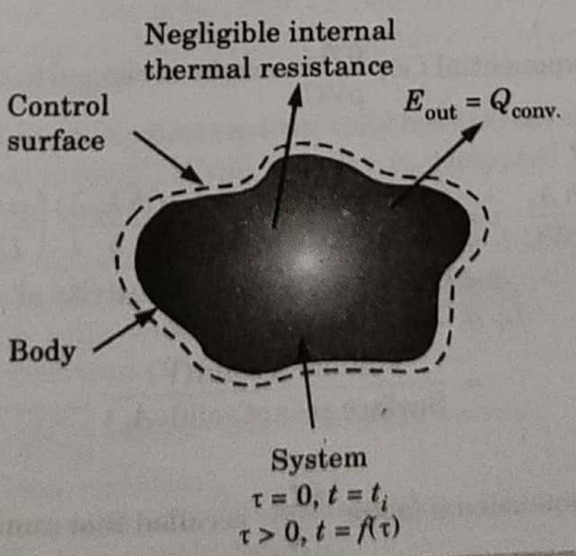

Fig. 2.11.1. General system for unsteady heat conduction.

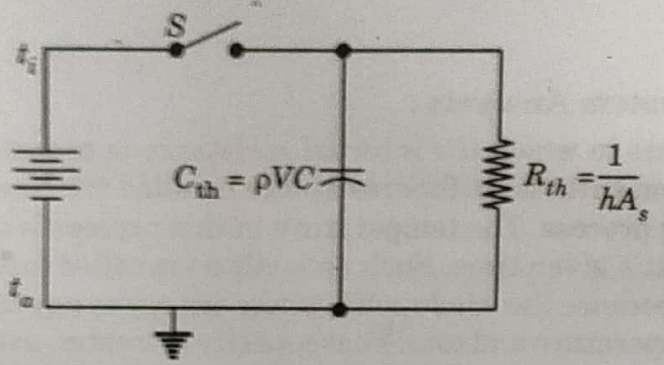

Fig. 2.11.2. Equivalent thermal circuit for lumped capacitance solid.

3. On rearranging the eq. (2.11.1), we get

$$\frac{dt}{t-t_a} = -\frac{hA_S}{\rho VC}d\tau$$

On integrating both sides, we get

$$\int\frac{dt}{t-t_a} = \frac{-hA_S}{\rho VC}\int d\tau$$

$$\ln(t-t_a) = \frac{-hA_S}{\rho VC}\tau + C_1 \quad ...(2.11.2)$$

The boundary conditions are :

At $\tau = 0$, $t = t_i$ (Initial surface temperature)

$\therefore$ $C_1 = \ln(t_i - t_a)$

Hence, $\ln(t-t_a) = \frac{-hA_S}{\rho VC}\tau + \ln(t_i - t_a)$

or

$$\frac{t-t_a}{t_i-t_a} = \frac{\theta}{\theta_i} = \exp\left[\frac{-hA_S}{\rho VC}\tau\right] \quad ...(2.11.3)$$

4. The power on exponential *i.e.*, $\frac{hA_S}{\rho VC}\tau$ can be arranged in dimensionless form as follows,

$$\frac{hA_S}{\rho VC}\tau = \left(\frac{hV}{kA_S}\right)\left(\frac{A_S^2 k}{\rho V^2 C}\tau\right) = \left(\frac{hL_c}{k}\right)\left(\frac{\alpha\tau}{L_c^2}\right) \quad ...(2.11.4)$$

Where, $\alpha = [k/\rho C]$ = Thermal diffusivity of solid.

L_c = Characteristic length.

$$= \frac{\text{Volume of solid }(V)}{\text{Surface area of solid }(A_S)}$$

5. In eq. (2.11.4),

i. The non-dimensional factor $\frac{hL_c}{k}$ is called Biot number B_i, and

ii. The non-dimensional factor $\frac{\alpha\tau}{L_c^2}$ is called the Fourier number F_o.

6. Using the non-dimensional term, eq. (2.11.3) becomes

$$\frac{\theta}{\theta_i} = \frac{t - t_a}{t_i - t_a} = e^{-B_i F_o}$$

C. Significance of Fourier Number and Biot Number :

1. Fourier number signifies the degree of penetration of heating or cooling effect through a solid.
2. Biot number gives an indication of the ratio of internal resistance (conduction) to surface resistance (convection).

Que 2.12. **Consider heat transfer between two identical hot solid bodies and the air surrounding them. The first solid is being cooled by a fan while the second one is allowed to cool naturally. For which solid is the lumped system analysis more likely to be applicable ? Why ?**

AKTU 2014-15, Marks 10

Answer

1. Lumped system analysis is applicable to the system when the Biot number is less than 0.1.

   Biot number, $B_i = \frac{hL_C}{k}$

   Where, h = Convective heat transfer coefficient,
   L_C = Characteristics length of a given geometry, and
   k = Thermal conductivity of a given material.

2. Now, in the given case, as both the systems having same material and geometry, Biot number depends only upon the value of heat transfer coefficient.
3. Now, in the first case the solid body is cooled by blowing a air with fan so the value of 'h' will be more as compared to second case in which the solid body is cooled by natural means.
4. So, value of Biot number is greater than 0.1 in first case than second case, hence the lumped system analysis will be applicable more likely to the solid which is being cooled by natural convection than forced convection.

Que 2.13. **Define the following terms :**

i. Time constant.
ii. Response of a thermocouple.
ii. Characteristic length.

Answer

i. **Time Constant :**

1. Time constant is the time required for the temperature change to reach 36.8 % of its final value in response to a step change in temperature. Mathematically,

$$\tau^* = \frac{\rho VC}{hA_s}$$

ii. **Response of Thermocouple :**

1. It is defined as the time required for the thermocouple to reach the source temperature when it is exposed to it.

iii. **Characteristic Length :**

1. It is the ratio of volume of the solid to the surface area of the solid. Mathematically,

$$L_C = \frac{V}{A_s}$$

Que 2.14. **For a plane wall subjected to uniform volumetric heat generation and exposed to a fluid at same temperature on both the sides, derive an expression for temperature distribution within the wall. Assume one dimensional case.**

Answer

1. Consider a plane wall of thickness L of uniform thermal conductivity k and in which heat sources are uniformly distributed in the whole volume.

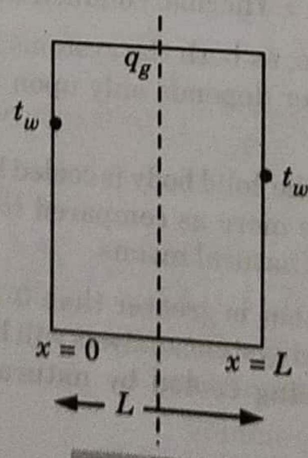

Fig. 2.14.1.

2. Let the wall temperature be t_w, since heat flow is one dimensional under steady state conditions, and there is a uniform volumetric heat generation within the wall, so, equation is :

$$\frac{d^2t}{dx^2} + \frac{q_g}{k} = 0 \qquad ...(2.14.1)$$

3. On integrating eq. (2.14.1), we get

$$\frac{dt}{dx} = \frac{-q_g}{k}x + C_1 \quad \text{...(2.14.2)}$$

4. Integrating eq. (2.14.2), we get

$$t = \frac{-q_g}{2k}x^2 + C_1 x + C_2 \quad \text{...(2.14.3)}$$

5. On applying boundary conditions, we get
At $x = 0, \quad t = t_w$
At $x = L, \quad t = t_w$
Using these boundary conditions in eq. (2.14.3), we have

$$\therefore \quad t_w = C_2$$

$$C_1 = \frac{q_g}{2k}L$$

6. Substituting the values of C_1 and C_2 in eq. (2.14.3), we get

$$t = \frac{-q_g}{2k}x^2 + \frac{q_g}{2k}Lx + t_w$$

$$t = \frac{q_g}{2k}(L-x)x + t_w \quad \text{...(2.14.4)}$$

7. For location of maximum temperature, differentiating eq. (2.14.4) w.r.t. x and equating the derivative to zero, we have

$$\frac{dt}{dx} = \frac{q_g}{2k}(L-2x) = 0$$

$$\because \quad \frac{q_g}{2k} \neq 0,$$

$$\therefore \quad L - 2x = 0$$

$$x = \frac{L}{2}$$

8. Thus the temperature distribution given by eq. (2.14.4) is parabolic and symmetrical about the midplane. The maximum temperature occurs at $x = \frac{L}{2}$.

$$\therefore \quad t_{max} = \left[\frac{q_g}{2k}(L-x)x\right]_{x=\frac{L}{2}} + t_w = \left[\frac{q_g}{2k}\left(L-\frac{L}{2}\right)\frac{L}{2}\right] + t_w$$

$$t_{max} = \frac{q_g}{8k}L^2 + t_w \quad \text{...(2.14.5)}$$

9. Heat transfer then takes place towards both the surfaces and for each surface, it is given by,

$$Q = -kA\left(\frac{dt}{dx}\right)_{x=0 \text{ or } x=L} = -kA\left[\frac{q_g}{2k}(L-2x)\right]_{x=0 \text{ or } x=L}$$

$$Q = \frac{AL}{2}q_g \quad \text{...(2.14.6)}$$

10. When both the surfaces are considered,

$$Q = 2 \times \frac{AL}{2} q_g = AL\, q_g \quad ...(2.14.7)$$

This heat dissipates to surrounding atmosphere at temperature t_a.

Thus,
$$\frac{AL}{2} q_g = hA\,(t_w - t_a)$$

or
$$t_w = t_a + \frac{q_g}{2h} L \quad ...(2.14.8)$$

11. Substituting value of t_w from eq. (2.14.8) in eq. (2.14.4), we get

$$t = t_a + \frac{q_g}{2h} L + \frac{q_g}{2k} (L - x)\, x$$

At $x = \frac{L}{2}$ *i.e.*, at the midplane

$$t = t_{max} = t_a + \frac{q_g}{2h} L + \frac{q_g}{8k} L^2$$

$$t_{max} = t_a + q_g \left[\frac{L}{2h} + \frac{L^2}{8k}\right]$$

Que 2.15. **A thermocouple junction which can be approximated as a 1 mm diameter sphere is used to measure a gas stream of t_a = 200 °C. Junction is at initially 25 °C. Determine how long it will take for the thermocouple to read 199 °C. Properties of the thermocouple junction are ρ = 8500 kg/m³, C = 320 J/kg-K, and k = 35 W/m-K. The heat transfer coefficient between the junction and the gas is 210 W/m²-K.**

AKTU 2013-14, Marks 10

Answer

Given : h = 210 W/m²-K, k = 35 W/m-K, C = 320 J/kg-K, ρ = 8500 kg/m³, D = 1 mm, R = 0.5 mm = 0.5 × 10⁻³ m, t_i = 25 °C, t_a = 200 °C, t = 199 °C

To Find : Time required to read 199 °C on thermocouple.

1. We know that,
$$L_c = \frac{\text{Volume}}{\text{Surface area}} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$$

2. Biot number,
$$B_i = \frac{hL_c}{k} = \frac{h\left(\frac{R}{3}\right)}{k} = \frac{210 \times 0.5 \times 10^{-3}}{35 \times 3} = 0.001$$

As $B_i < 0.1$, lumped capacitance method can be used.

2. Now,
$$\frac{t-t_a}{t_i-t_a} = \exp\left(-\frac{hA_s}{\rho VC}\tau\right)$$

$\because$
$$\frac{hA_s}{\rho VC} = \frac{h \times 4\pi R^2}{\rho \times \frac{4}{3}\pi R^3 C} = \frac{3h}{\rho RC}$$

$$= \frac{3 \times 210}{8500 \times 0.5 \times 10^{-3} \times 320} = 0.463$$

$\therefore$
$$\frac{199-200}{25-200} = e^{-0.463\,\tau}$$
$$0.00571 = e^{-0.463\,\tau}$$
$$-5.1655 = -0.463\tau$$
$$\tau = 11.17 \text{ s.}$$

Que 2.16. **During heat treatment, cylindrical pieces of 25 mm diameter, 30 mm height and at 30 °C are placed in furnace at 750 °C with convective heat transfer coefficient 80 W/m²-°C. Calculate the time required to heat the pieces to 600 °C. What will be shortfall in temperature if pieces are taken out from the furnace after 280 seconds ? Assume the following properties : k = 40 W/m-°C, C = 480 J/kg-K and ρ = 7850 kg/m³.** **AKTU 2015-16, Marks 10**

Answer

Given : d = 25 mm, r = 12.5 mm = 0.0125 m, H = 30 mm, t_i = 30 °C, t_a = 750 °C, h = 80 W/m²-°C, t = 600 °C, k = 40 W/m-°C, C = 480 J/kg-K, ρ = 7850 kg/m³

To Find : i. Time required to heat the pieces.
ii. Shortfall in temperature.

1. For a cylindrical piece, the characteristic length is,
$$L_C = \frac{\text{Volume}}{\text{Surface area}} = \frac{\pi r^2 H}{2\pi r(r+H)} = \frac{rH}{2(r+H)}$$
$$= \frac{0.0125 \times 0.03}{2(0.0125 + 0.03)} = 4.41 \times 10^{-3} \text{ m}$$

2. Biot number,
$$B_i = \frac{hL_C}{k} = \frac{80 \times 4.41 \times 10^{-3}}{40} = 0.00882 < 0.1$$

Since $B_i < 0.1$, the lumped parameter model can be adopted.

$\therefore$
$$\frac{t-t_a}{t_i-t_a} = \exp\left(-\frac{hA}{\rho VC}\tau\right)$$
$$\frac{hA}{\rho VC} = \frac{h}{\rho C}\left(\frac{A}{V}\right) = \frac{h}{\rho C L_C} = \frac{80}{7850 \times 480 \times 4.41 \times 10^{-3}}$$

$$= 0.004814$$

$$\therefore \quad \frac{600-750}{30-750} = \exp[-0.004814\,\tau]$$

$$\frac{30-750}{600-750} = \exp[0.004814\,\tau]$$

or $$0.004814\,\tau = \ln\left(\frac{30-750}{600-750}\right) = 1.5686$$

$\therefore$ Time required for heating,

$$\tau = \frac{1.5686}{0.004814} = 326 \text{ sec}$$

3. Let, t' = Temperature attained when the pieces are taken out from the furnace after 280 seconds.

Then, $$\frac{t'-750}{30-750} = \exp[-0.004814 \times 280] = \exp[-1.348] = \frac{1}{3.849}$$

or $$t' = 750 + \frac{30-750}{3.849} = 563\ °\text{C}$$

4. Shortfall in temperature $= t - t' = 600 - 563 = 37\ °\text{C}$.

Que 2.17. **What is time constant ? The steel ball bearings of 40 mm diameter and initially at uniform temperature of 600 °C are quenched in an oil bath maintained at 50 °C temperature. The heat transfer coefficient between the ball bearing and oil is 325 W/m²-K and the thermodynamics properties of the bearing can be taken as : k = 45 W/m-K and thermal diffusivity $\alpha = 1.25 \times 10^{-5}$ m²/s. Determine :**

a. The time duration for which bearing must remain in oil to attain 225 °C temperature.

b. The instantaneous heat transfer rate from the bearings when they are first immersed in oil and when they reach 225 °C.

AKTU 2016-17, Marks 7.5

Answer

A. Time Constant : Refer Q. 2.13, Page 2–18B, Unit-2.

B. Numerical :

Given : $d = 40$ mm, $t_i = 600$ °C, $t_a = 50$ °C, $h = 325$ W/m²-K, $k = 45$ W/m-K, $\alpha = 1.25 \times 10^{-5}$ m²/s.

To Find : a. The time duration for which bearing must remain in oil to attain 225 °C temperature.

b. The instantaneous heat transfer rate from the bearings when they are first immersed in oil and when they reach 225 °C.

1. We know that, $L_c = \dfrac{\text{Volume}}{\text{Surface area}} = \dfrac{\frac{4}{3}\pi r^3}{4\pi r^2} = \dfrac{r}{3} = \dfrac{0.04/2}{3}$

$= 6.67 \times 10^{-3}$ m

2. Biot number, $B_i = \dfrac{hL_C}{k} = \dfrac{325 \times 6.67 \times 10^{-3}}{45} = 0.0482$

Since $B_i < 0.1$, so lumped parameter analysis is applicable here.

3. Now, $\dfrac{hA}{\rho VC} = \dfrac{h}{\rho C}\left(\dfrac{A}{V}\right) = \dfrac{\alpha}{k} h\left(\dfrac{3}{r}\right)$ $\quad \left(\because \alpha = \dfrac{k}{\rho C}\right)$

$= \dfrac{1.25 \times 10^{-5}}{45} \times 325 \times \dfrac{3}{0.04/2} = 0.01354$

4. From lumped parameter analysis,

$$\frac{t - t_a}{t_i - t_a} = \exp\left(-\frac{hA}{\rho VC}\tau\right)$$

$$\therefore \quad \frac{225 - 50}{600 - 50} = \exp(-0.01354\,\tau)$$

$$0.01354\,\tau = \log_e 3.143 = 1.1452$$

$$\tau = \frac{1.1452}{0.01354} = 84.58 \text{ s}$$

5. The total energy transferred is,

$$Q_t = \rho VC\,(t_i - t_a)\left[\exp\left(-\frac{hA}{\rho VC}\tau\right) - 1\right]$$

Now, $\rho C = \dfrac{k}{\alpha} = \dfrac{45}{1.25 \times 10^{-5}} = 36 \times 10^5$

$$\exp\left(-\frac{hA}{\rho VC}\tau\right) = \exp(-0.01354 \times 84.58) = 0.3182$$

$$\therefore \quad Q_t = 36 \times 10^5 \times \frac{4}{3} \times \pi \times \left(\frac{0.04}{2}\right)^3 \times (600 - 50) \times (0.3182 - 1)$$

$= -45281$ J/s or 45.28 kW (Negative sign shows heat loss)

5. Instantaneous heat flow rate is given as,

$$Q_i = -hA\,(t_i - t_a)\exp\left(-\frac{hA}{\rho VC}\tau\right)$$

$$= -325 \times 4\pi \times \left(\frac{0.04}{2}\right)^2 \times (600 - 50) \times (0.3182) = 285.9 \text{ J/s}$$

$= 285.9$ W

Que 2.18. **A person is found dead at 5 p.m. in a room where temperature is 20 °C. The temperature of the body is measured to be 25 °C when found, and the heat transfer coefficient is estimated to be 8 W/m²-K. Modelling the human body into a 30 cm diameter,**

1.70 m long cylinder, calculate actual time of death of the person. Take thermo physical properties of the body :
$k = 6.08$ W/m-K, $\rho = 900$ kg/m³, $C = 4000$ J/kg-K

AKTU 2016-17, Marks 10

Answer

Given : $t_a = 20$ °C, $t = 25$ °C, $h = 8$ W/m²-K, $d = 30$ cm $= 0.3$ m
$r = 0.15$ m, $l = 1.70$ m, $k = 6.08$ W/m-K, $\rho = 900$ kg/m³, $C = 4000$ J/kg-K.
To Find : Actual time of death of the person. Temperature of live person is 37 °C.

1. From lumped parameter analysis,

$$\frac{t - t_a}{t_0 - t_a} = \exp\left[-\frac{hA_s}{\rho VC}\tau\right] = \exp\left[-\frac{8 \times 2\pi r L}{\rho \pi r^2 LC}\tau\right]$$

$$\frac{25 - 20}{37 - 20} = \exp\left[-\frac{8 \times 2}{900 \times 0.15 \times 4000}\tau\right]$$

$$-1.224 = \frac{-8 \times 2}{900 \times 0.15 \times 4000}\tau$$

$$\tau = 41310 \text{ sec} = 11.47 \text{ hr}$$

Hence, actual time of death of the person is 4.37 a.m.

Que 2.19. **An egg with mean diameter of 40 mm and initially at 20 °C is placed in a boiling water pan for 4 minutes and found to be boiled to the consumer's taste. For how long should a similar egg for same consumer be boiled when taken from a refrigerator at 5 °C. Take the following properties for egg :**
$k = 10$ W/m-°C, $\rho = 1200$ kg/m³, $C = 2$ kJ/kg-°C and $h = 100$ W/m²-°C.

AKTU 2016-17, Marks 7.5

Answer

Given : $R = 40/2 = 20$ mm $= 0.02$ m, $t_i = 20$ °C, $\tau = 4 \times 60 = 240$ sec, $k = 10$ W/m-°C, $\rho = 1200$ kg/m³, $C = 2$ kJ/kg-°C, $h = 100$ W/m²-°C, $t_a = 100$ °C.
To Find : Time taken to boiled the egg when it is taken from a refrigerator at 5 °C.

1. We know that,

Biot number, $$B_i = \frac{hL_C}{k}$$

$$B_i = \frac{h}{k} \times \frac{R}{3} = \frac{100 \times 0.02}{10 \times 3}$$

$$= 0.067 \qquad \left[\because L_C = \frac{4/3\,\pi R^3}{4\pi R^2} = \frac{R}{3}\right]$$

As $B_i < 0.1$, we can use lump theory.

2. From lump parameter analysis,

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA}{\rho VC}\tau\right] \quad ...(2.19.1)$$

$$\because \quad \frac{hA}{\rho VC} = \left(\frac{h}{\rho C}\right)\left(\frac{A}{V}\right) = \left(\frac{100}{1200 \times 2000}\right)\left(\frac{3}{R}\right)$$

$$= \left(\frac{100}{1200 \times 2000}\right)\left(\frac{3}{0.02}\right) = 0.00625$$

3. Substituting the above value in eq. (2.19.1), we get

$$\frac{t - 100}{20 - 100} = e^{-0.00625 \times 240} = e^{-1.50} = 0.223$$

or,

$$t = 100 + (20 - 100) \times 0.223 = 82.16\ °C \approx 82\ °C$$

4. Again using eq. (2.19.1), we get

$$\frac{82 - 100}{5 - 100} = e^{-0.00625\,\tau} = \frac{1}{e^{0.00625\tau}}$$

$$0.1895 = \frac{1}{e^{0.00625\tau}}$$

$$0.00625\,\tau = 1.6633$$

$$\tau = \frac{1.6633}{0.00625} = 266.13\ \text{s} = 4.435\ \text{minutes}$$

Que 2.20. **What are the Heisler charts ? How these charts are used to obtain temperature distribution when both conduction and convection resistances are almost of equal importance ?**

OR

Explain the utility of Heisler chart in transient heat conduction problem.

AKTU 2013-14, Marks 10

Answer

A. Heisler Charts :

1. The chart plotted with temperature $\left[\frac{t_0 - t_a}{t_i - t_a}\right]$ versus F_o (Fourier number) for various values of $\frac{1}{B_i}$ for solids of different geometrical shapes such as plates, cylinders and spheres are known as Heisler charts.
2. These charts provide the temperature history of solid at its mid planes ($x = 0$), temperatures at other locations are worked out by multiplying the mid plane temperature by correction factor read from charts.
3. From Heisler chart, the value of B_i (Biot number) and F_o (Fourier number) are calculated on the basis of characteristic parameter S which is semi-thickness in case of plates and surface radius in case of cylinder and spheres.

4. Heisler charts are extensively used to determine the temperature distribution.

B. Use of Heisler Charts to Obtain Temperature Distribution when both Conduction and Convection Resistances are almost of Equal Importance :

1. Consider the heating and cooling of a plane wall having a thickness of $2L$ and extending to infinity in y and z directions.
2. Let us assume that the wall initially is at uniform temperature t_i and both surfaces ($x = \pm L$) are suddenly exposed to and maintained at the ambient (surroundings) temperature t_a.
3. The governing differential equation is :

$$\frac{d^2t}{dx^2} = \frac{1}{\alpha}\frac{dt}{d\tau} \qquad ...(2.20.1)$$

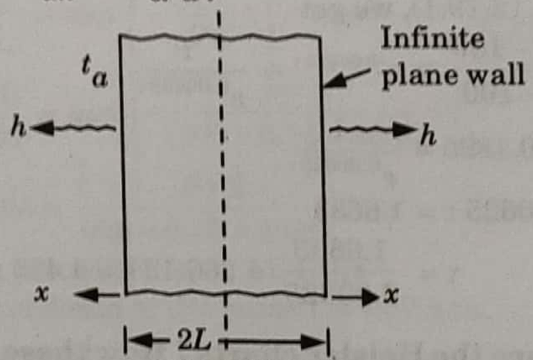

Fig. 2.20.1.

4. The boundary conditions are :
   i. At $\tau = 0, \quad t = t_i$
   ii. At $x = 0, \quad \frac{dt}{dx} = 0$
   iii. At $x = \pm L, kA\left(\frac{dt}{dx}\right) = hA\,(t - t_a)$
5. The solutions obtained after mathematical analysis indicate that

$$\frac{t - t_a}{t_i - t_a} = f\left(\frac{x}{l}, \frac{hl}{k}, \frac{\alpha\tau}{l^2}\right) \qquad ...(2.20.2)$$

6. From eq. (2.20.2), it is evident that when conduction resistance is not negligible, the temperature history becomes a function of Biot number $\left(\frac{hl}{k}\right)$, Fourier number $\left(\frac{\alpha\tau}{l^2}\right)$ and dimensionless parameter $\left(\frac{x}{l}\right)$ which indicates the location of point within the plate where temperature is to be obtained.

Que 2.21. **What is the significance of Heisler chart ? Describe various types of Heisler chart. What is characteristic length ?**

AKTU 2016-17, Marks 7.5

Answer

A. **Significance of Heisler Charts :** Refer Q. 2.20, Page 2–26B, Unit-2.

B. **Types of Heisler Charts :**

a. **Type I :**

1. These are plotted between dimensionless temperature ratio $(t_0 - t_a)/(t_i - t_a)$ at the centre and Fourier number F_0 with $1/B_i$ as a parameters.
2. These are used to determine the centre temperature from knowledge of B_i and F_0.

b. **Types II :**

1. These are plotted between dimensionless temperature ratio $(t_0 - t_a)/(t_i - t_a)$ and $1/B_i$ with x/l as a parameter.
2. These plots along with plots of type I can be used to determine the temperature at any location from a knowledge of B_i, F_0 and x/l.
3. The temperature at any location can be calculated as,

$$\left(\frac{t-t_a}{t_i-t_a}\right) = \left(\frac{t_0-t_a}{t_i-t_a}\right)\left(\frac{t-t_a}{t_0-t_a}\right)$$

c. **Type III :**

1. These are plotted between dimensionless heat transfer Q/Q_i and $F_0 B_i^2$ with B_i as a parameter.
2. These plots can be used to estimate the energy transferred during the time interval $\tau = 0$ to $\tau = \tau$ from a knowledge of B_i and F_0.

C. **Characteristic Length :** Refer Q. 2.13, Page 2–18B, Unit-2.

Que 2.22. **The nose section of a missile is formed of a 6 mm thick stainless plate and is held initially at uniform temperature of 88 °C. The missile enters the denser layers of the atmosphere at very high velocity. The effective temperature of air surrounding the nose region attains the value 2200 °C and the surface convective coefficient is estimated at 3405 W/m²-K. Make calculations for the maximum permissible time in these surroundings if the maximum metal temperature is not to exceed 1095 °C. Also workout the inside surface temperature under these conditions. The properties for steel are : ρ = 7800 kg/m³, k = 54 W/m-°C, C_p = 465 J/kg-K.**

AKTU 2015-16, Marks 10

Answer

Given : $L = 6$ mm $= 0.006$ m, $\rho = 7800$ kg/m³, $C_p = 465$ J/kg-K
$k = 54$ W/m-°C, $h = 3405$ W/m²-K, $t_i = 88$ °C, $t_a = 2200$ °C
$t = 1095$ °C

To Find : i. Maximum permissible time.
ii. Inside surface temperature.

1. Characteristic length, $L_C = \frac{L}{2} = \frac{0.006}{2} = 0.003$ m

2. Biot number, $B_i = \frac{hL_C}{k} = \frac{3405 \times 0.003}{54} = 0.189$

As $B_i > 0.1$, therefore lumped analysis cannot be applied in this case. Hence, Heisler charts are used to solve this problem.

3. Corresponding to $\frac{1}{B_i} = \frac{1}{0.189} = 5.29$ and $\frac{x}{L_C} = 1$ (outside surface from nose section), from Heisler charts,

$$\frac{t - t_a}{t_0 - t_a} = 0.9$$

4. Also,

$$\frac{t - t_a}{t_i - t_a} = \left[\frac{t_0 - t_a}{t_i - t_a}\right] \times \left[\frac{t - t_a}{t_0 - t_a}\right]$$

$$\frac{1095 - 2200}{88 - 2200} = \frac{t_0 - t_a}{t_i - t_a} \times 0.9$$

$$\frac{t_0 - t_a}{t_i - t_a} = \frac{1}{0.9} \times \frac{(-1105)}{(-2112)} = 0.5813$$

5. Now, again using the Heisler chart for $\frac{t_0 - t_a}{t_i - t_a} = 0.5813$ and $\frac{1}{B_i} = 5.29$.

We get, Fourier number, $F_0 = 4.39$

$$\frac{\alpha \tau}{L_C^2} = 4.39$$

$$\frac{k}{\rho C_p} \times \frac{\tau}{L_C^2} = 4.39 \qquad \left(\because \alpha = \frac{k}{\rho C_p}\right)$$

$$\frac{54}{7800 \times 465} \times \frac{\tau}{(0.003)^2} = 4.39$$

$$\tau = \frac{4.39 \times (0.003)^2 \times 7800 \times 465}{54}$$

$$\tau = 2.6537 \text{ sec}$$

6. The temperature at the inside surface ($x = 0$) is given by

$$\frac{t_0 - t_a}{t_i - t_a} = 0.5813$$

$$\frac{t_0 - 2200}{88 - 2200} = 0.5813$$

$$t_0 = 2200 + 0.5813\,(88 - 2200)$$

$$t_0 = 972.2944\ ^\circ\text{C}$$

Que 2.23. **A large metal plate of thickness 5 cm is initially at 460 °C. It is suddenly exposed to fluid at 100 °C with a convection coefficient of 142.5 W/m²-K. Find the time needed for its mid plane to reach a temperature of 316 °C and surface temperature at the same instant of time. Take k = 21.25 W/m-K and $\alpha = 1.2 \times 10^{-5}$ m²/sec.**

AKTU 2017-18, Marks 10

Answer

Given : L = 5 cm, t_i = 460 °C, t_a = 100 °C, h = 142.5 W/m²-K, t_0 = 316 °C, k = 21.25 W/m-K, $\alpha = 1.2 \times 10^{-5}$ m²/sec

To Find : i. Time.
ii. Surface temperature.

1. Characteristic length, $L_C = \frac{L}{2} = \frac{5}{2} = 2.5 \times 10^{-2}$ m

2. Biot number, $$B_i = \frac{hL_C}{k} = \frac{142.5 \times 2.5 \times 10^{-2}}{21.25} = 0.168$$

As $B_i > 0.1$, therefore lumped analysis cannot be applied in this case, further as $B_i < 100$, Heisler charts can be used to obtain the solution of the problem.

4. Now, $\frac{t_o - t_a}{t_i - t_a} = \frac{316 - 100}{460 - 100} = 0.6$ and $\frac{1}{B_i} = \frac{1}{0.168} = 5.95 \approx 6$

Corresponding to the above parametric values, from Heisler charts,

$$F_o = 3.4$$

$$\frac{\alpha\tau}{L_c^2} = 3.4$$

$$\frac{1.2 \times 10^{-5} \times \tau}{(2.5 \times 10^{-2})^2} = 3.4$$

$$\tau = 177 \text{ sec}$$

5. At surface, $\frac{x}{L} = 1$. Now corresponding to $x/L = 1$ and $1/B_i = 6$, from Heisler charts,

$$\frac{t - t_a}{t_o - t_a} = 0.92$$

$$\frac{t - 100}{316 - 100} = 0.92$$

$$t = 298.72 \text{ °C.}$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What is the reason for the widespread use of fins on surfaces ?

Ans. Refer Q. 2.2.

Q. 2. Derive an expression of rectangular fin in case of heat dissipation from an infinite long fin. What are advantages and application of fins ?

Ans. Refer Q. 2.3.

Q. 3. Explain effectiveness and efficiency of fin.

Ans. Refer Q. 2.4.

Q. 4. Consider heat transfer between two identical hot solid bodies and the air surrounding them. The first solid is being cooled by a fan while the second one is allowed to cool naturally. For which solid is the lumped system analysis more likely to be applicable ? Why ?

Ans. Refer Q. 2.12.

Q. 5. Define the following terms :

i. Time constant.

ii. Response of a thermocouple.

ii. Characteristic length.

Ans. Refer Q. 2.13.

Q. 6. What is the significance of Heisler chart ? Describe various types of Heisler chart. What is characteristic length ?

Ans. Refer Q. 2.21.

Q. 7. A large metal plate of thickness 5 cm is initially at 460 °C. It is suddenly exposed to fluid at 100 °C with a convection coefficient of 142.5 W/m²-K. Find the time needed for its mid plane to reach a temperature of 316 °C and surface temperature at the same instant of time. Take k = 21.25 W/m-K and $\alpha = 1.2 \times 10^{-5}$ m²/sec.

Ans. Refer Q. 2.23.

☺☺☺

3 UNIT

Forced & Natural Convection

PART-1

Hydrodynamic Boundary Layer, Thermal Boundary Layer, Approximate Integral Boundary Layer Analysis, Analogy between Momentum and Heat Transfer in Turbulent Flow over a Flat Surface, Mixed Boundary Layer.

CONCEPT OUTLINE : PART-1

Boundary Layer : The layer adjacent to the boundary is known as boundary layer. It is formed whenever there is relative motion between the boundary and the fluid.

Boundary Layer Thickness : It is defined as the distance from the boundary in which the velocity reaches 99 % of the velocity of the free stream.

Thermal Boundary Layer : It may be defined as that region where temperature gradient are present in the flow. These temperature gradients would result from a heat exchange process between the fluid and the wall.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **Explain the concept of boundary layer.**

Answer

1. The layer adjacent to the boundary is known as boundary layer. Boundary layer is formed whenever there is relative motion between the boundary and the fluid.
2. When a real fluid (viscous fluid) flows past a stationary solid boundary, a layer of fluid which comes in contact with the boundary surface adheres to it and condition of no slip occurs.
3. Thus the layer of fluid which cannot slip away from the boundary surface undergoes retardation, this retarded layer further causes retardation for the adjacent layers of the fluid, thereby developing a small region in the immediate vicinity of the boundary surface in which the velocity of the flowing fluid increases rapidly from zero at the boundary surface and approaches the velocity of main stream.
4. According to boundary layer theory, the extensive fluid medium around bodies moving in fluids can be divided into following two regions :

i. A thin layer adjoining the boundary called the boundary layer where the viscous shear takes place.

ii. A region outside the boundary layer where the flow behaviour is quite like that of an ideal fluid and the potential flow theory is applicable.

Que 3.2. **Derive the equation for boundary layer thickness.**

AKTU 2017-18, Marks 10

Answer

The equation for boundary layer thickness can be obtained in terms of displacement, momentum and energy that are discussed as below :

a. **Displacement Thickness :**

1. It can be defined as the distance, measured perpendicular to the boundary by which the main/free stream is displaced on account of formation of boundary layer. It is denoted by δ^*.

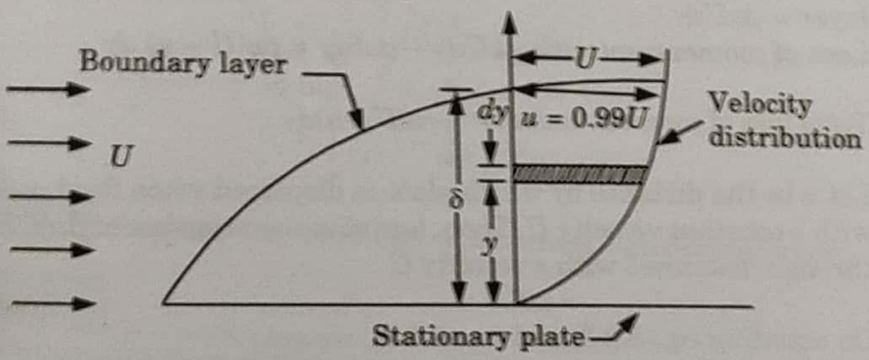

Fig. 3.2.1. Displacement thickness.

2. Let fluid of density ρ flow past a stationary plate with velocity U as shown in Fig. 3.2.1.
3. Consider an elementary strip of thickness dy at a distance y from the plate.
4. Mass flow per second through the elementary strip = $\rho u dy$
5. Mass flow per second through elementary strip, if the plate was not there = $\rho U dy$
6. Reduction of mass flow rate through elementary strip
$$= \rho(U - u)\, dy$$
7. Total reduction of mass flow rate due to introduction of plate
$$= \int_0^\delta \rho(U - u)dy \qquad ...(3.2.1)$$
8. Let the plate is displaced by a distance δ^* and velocity of flow for the distance δ^* is equal to the main/free stream velocity. Then, loss of mass of fluid/sec flowing through the distance δ^*
$$= \rho U \delta^* \qquad ...(3.2.2)$$

9. On equating eq. (3.2.1) and eq. (3.2.2), we get

$$\rho U\delta^* = \int_0^\delta \rho(U-u)dy$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy$$

b. Momentum Thickness :

1. It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid on account of boundary layer formation. It is denoted by θ.
2. Mass of flow per second through elementary strip = $\rho u dy$
3. Momentum/sec of this fluid inside the boundary layer
$$= \rho u dy \times u = \rho u^2 dy$$
4. Momentum/sec of the same mass of fluid before entering the boundary layer = $\rho u U dy$
5. Loss of momentum/sec = $\rho u U dy - \rho u^2 dy = \rho u(U-u)\,dy$
6. Total loss of momentum/sec = $\int_0^\delta \rho u(U-u)dy$ ...(3.2.3)
7. Let θ be the distance by which plate is displaced when fluid is flowing with a constant velocity U. Then, loss of momentum/sec of fluid flowing through distance θ with a velocity U
$$= \rho\theta U^2$$ ...(3.2.4)
8. On equating eq. (3.2.3) and eq. (3.2.4), we get

$$\rho\theta U^2 = \int_0^\delta \rho u(U-u)dy$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

c. Energy Thickness :

1. It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation. It is denoted by δ_e.
2. Mass of flow per second through elementary strip = $\rho u dy$
3. KE of this fluid inside the boundary layer
$$= \frac{1}{2}mu^2 = \frac{1}{2}(\rho u dy)u^2$$
4. KE of same mass of fluid before entering the boundary layer
$$= \frac{1}{2}(\rho u dy)U^2$$

5. Loss of KE through elementary strip

$$= \frac{1}{2}(\rho u dy)\,U^2 - \frac{1}{2}(\rho u dy)u^2 = \frac{1}{2}\rho u\,(U^2 - u^2)dy$$

6. So, total loss of KE of fluid $= \int_0^{\delta} \frac{1}{2}\rho u(U^2 - u^2)dy$ ...(3.2.5)

7. Let δ_e be the distance by which plate is displaced to compensate for reduction in KE. Then, loss of KE through δ_e of fluid flowing with velocity U

$$= \frac{1}{2}(\rho U \delta_e)\,U^2 \quad ...(3.2.6)$$

8. On equating eq. (3.2.5) and (3.2.6), we have

$$\frac{1}{2}(\rho U\delta_e)\,U^2 = \int_0^{\delta} \frac{1}{2}\rho u(U^2 - u^2)dy$$

$$\delta_e = \frac{1}{U^3}\int_0^{\delta} u(U^2 - u^2)dy$$

$$\delta_e = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right)dy$$

Que 3.3. **Explain the concept of thermal boundary layer.**

Answer

1. Whenever a flow of fluid takes place past a heated or cold surface, a temperature field is set up in the field next to the surface. If the surface of the plate is hotter than fluid, the temperature distribution will be as shown in the Fig. 3.3.1. The zone or the layer wherein the temperature field exists is called the thermal boundary layer.

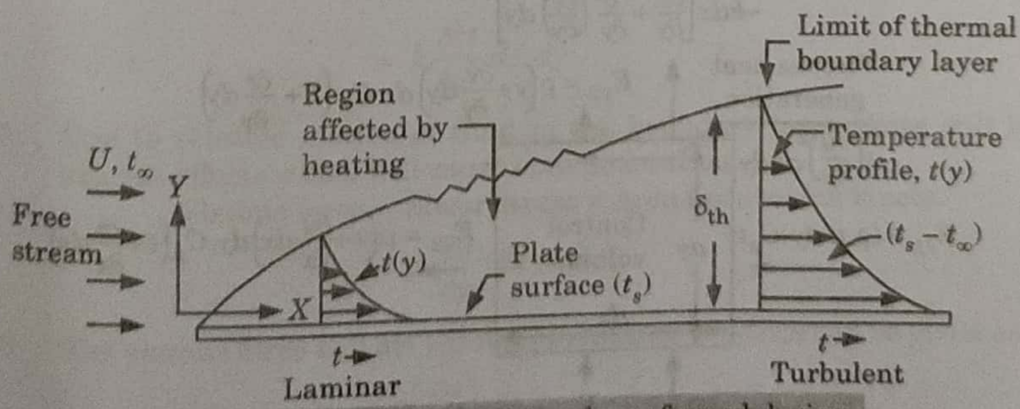

Fig. 3.3.1. Thermal boundary layer formed during flow of cool fluid over a warm plate.

2. Due to the exchange of heat between the plate and the fluid, temperature gradient occurs.

3. The thermal boundary layer thickness (δ_{th}) is arbitrarily defined as the distance y from the plate surface at which

$$\frac{t_s - t}{t_s - t_\infty} = 0.99$$

4. The temperature profile of the thermal boundary layer depends upon the viscosity, velocity of flow, specific heat and thermal conductivity of the fluid.

Que 3.4. **Derive the differential energy equation for flow past a flat plate. Also write down assumptions of it.**

OR

Derive an expression for energy equation of thermal boundary layer over flat plate.

AKTU 2016-17, Marks 7.5

Answer

A. Assumptions :

1. The flow is steady and incompressible.
2. Fluid's properties evaluated at the film temperature, $t_f = \dfrac{t_\infty - t_s}{2}$ are constant.
3. The body forces, viscous heating and conduction in the flow direction are negligible.

B. Derivation :

1. For an element of dimensions ($dx \times dy \times$ Unit depth) in the boundary layer the quantities of energy entering and leaving are shown in Fig. 3.4.1.

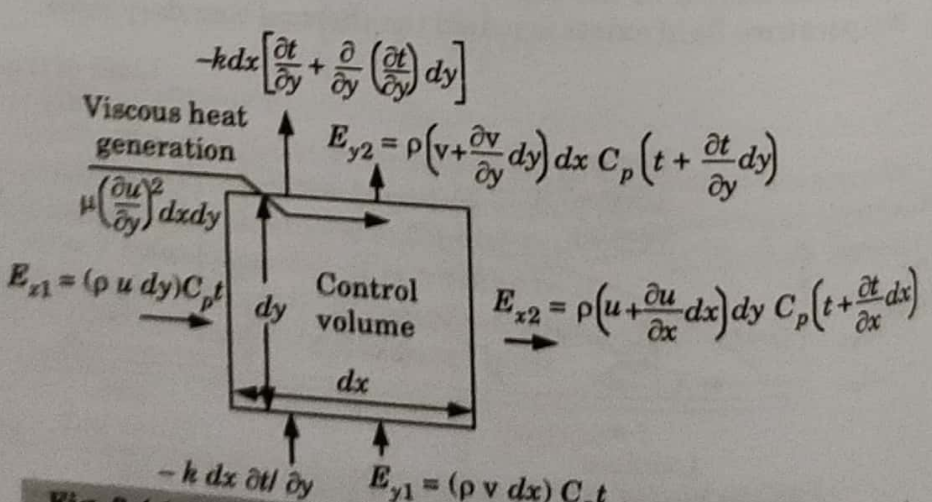

Fig. 3.4.1. Differential control volume for energy conservation in thermal boundary layer.

2. The rate of temperature change in the X-direction is being presumed small and as such conduction is to be considered only in the Y-direction.
3. The convective terms in the X and Y-directions have been written in terms of mass, temperature and specific heat which is assumed constant.

a. **In X-direction :**

i. Energy influx,

$$E_{x1} = \text{Mass} \times \text{Specific heat} \times \text{Temperature}$$
$$= (\rho u dy)\ C_p t$$

ii. Energy efflux,

$$E_{x2} = \rho\left(u + \frac{\partial u}{\partial x}dx\right) dy\ C_p \left(t + \frac{\partial t}{\partial x}\ dx\right)$$

Neglecting the product of small quantities,

$$E_{x2} = \rho C_p\ dy\left[ut + u\frac{\partial t}{\partial x}dx + t\frac{\partial u}{\partial x}dx\right]$$

iii. Net energy convection $= E_{x1} - E_{x2}$

$$= -\ \rho C_p\left[u\frac{\partial t}{\partial x} + t\frac{\partial u}{\partial x}\right]dx\ dy$$

b. **In Y-direction :**

i. Similarly, net energy convection in Y-direction,

$$= E_{y1} - E_{y2}$$

$$= (\rho\ v\ dx)\ C_p t - \rho C_p dx\left[vt + v\ dy\frac{\partial t}{\partial y} + t\frac{\partial v}{\partial y}\ dy\right]$$

$$= -\rho C_p\left[v\frac{\partial t}{\partial y} + t\frac{\partial v}{\partial y}\right]dx\ dy$$

4. Conduction heat rate in Y-direction,

$$= -\ kdx\frac{\partial t}{\partial y} - \left[-kdx\left\{\frac{\partial t}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial t}{\partial y}\right)\right\}dy\right]$$

$$= k\frac{\partial^2 t}{\partial y^2}\ dx\ dy$$

5. Due to relative motion of fluid in the boundary layer there will be viscous effects which will cause heat generation.

Viscous force = Shear stress × Area upon which it acts

$$= \mu\ \frac{\partial u}{\partial y}\ (dx \times 1)$$

6. The viscous force will act through a distance 's' which can be given as

$$s = \left(\frac{\partial u}{\partial y}\right) dy$$

$$\therefore\ \text{Viscous heat generation} = \mu\frac{\partial u}{\partial y}dx \times \frac{\partial u}{\partial y}dy = \mu\left(\frac{\partial u}{\partial y}\right)^2 dxdy$$

7. Now, for steady state condition, the algebraic sum of heat due to convection, conduction and viscous effect equals to zero. Thus

$$-\rho C_p\left(u\frac{\partial t}{\partial x}+t\frac{\partial u}{\partial x}+v\frac{\partial t}{\partial y}+t\frac{\partial v}{\partial y}\right)dxdy + k\frac{\partial^2 t}{\partial y^2}dxdy + \mu\left(\frac{\partial u}{\partial y}\right)^2 dxdy = 0$$

or $$-\rho C_p\left[u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}+t\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]+k\frac{\partial^2 t}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 = 0 \quad ...(3.4.1)$$

8. For a two-dimensional boundary layer flow, $\left(\frac{\partial u}{\partial x}\right)+\left(\frac{\partial v}{\partial y}\right) = 0$ and therefore eq. (3.4.1) can be written as,

$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 t}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2$$

which represents the differential energy equation for flow past a flat plate.

9. If the heat generation due to viscous effects is negligible, then the energy equation can be re-written as

$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 t}{\partial y^2} = \alpha\frac{\partial^2 t}{\partial y^2}$$

Where, $\alpha = \frac{k}{\rho C_p}$ = Thermal diffusivity

Que 3.5. **Derive the Von-Karman integral energy equation for the flow past a flat plate.**

AKTU 2015-16, Marks 10

Answer

1. Consider a control volume as shown in Fig. 3.5.1.

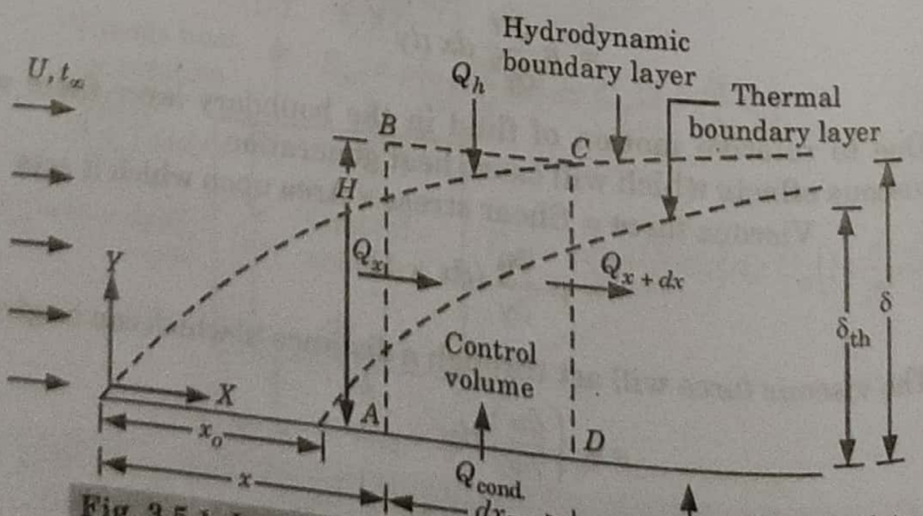

Fig. 3.5.1. Integral energy equation – control volume.

2. Following assumptions are made to derive the integral energy equation :
   i. ρ, C_p and k (thermo-plastic properties) of fluid remain constant within the operating range of the temperature
   ii. The heating of the plate commences at a distance x_0 from the leading edge of the plate.
3. For unit width of the plate we have,
   i. Mass of fluid entering through face $AB = \int_0^H \rho u \, dy$
   ii. Mass of fluid leaving through face $CD = \int_0^H \rho u \, dy + \frac{\partial}{\partial x}\left[\int_0^H \rho u \, dy\right] dx$
   iii. Mass of fluid entering the control volume through face BC

$$= \left[\int_0^H \rho u \, dy + \frac{\partial}{\partial x}\left\{\int_0^H \rho u \, dy\right\} dx\right] - \int_0^H \rho u \, dy$$

$$= \frac{\partial}{\partial x}\left[\int_0^H \rho u \, dy\right] dx$$

4. Heat influx through the face AB,

$$Q_x = \text{Mass} \times \text{Specific heat} \times \text{Temperature}$$

or,

$$Q_x = \left(\int_0^H \rho u \, dy\right) C_p t = \rho C_p \int_0^H ut \, dy$$

5. Heat efflux through the face CD,

$$Q_{x+dx} = \int_0^H ut \, dy + \frac{\partial}{\partial x}\left[\rho C_p \int_0^H ut \, dy\right] dx$$

6. Heat (energy) influx through the face BC (which is outside thermal boundary layer and there the temperature is constant at t_∞),

$$Q_h = \frac{\partial}{\partial x}\left[\int_0^H \rho u \, dy\right] dx \, C_p \, t_\infty$$

7. Heat conducted into the control volume through face AD,

$$Q_{\text{cond.}} = -kA\left[\frac{\partial t}{\partial y}\right]_{y=0} = -k \, dx\left(\frac{\partial t}{\partial y}\right)_{y=0}$$

8. The energy balance for the element is given by

$$\rho C_p \int_0^H ut \, dy + \frac{\partial}{\partial x}\left[\rho C_p \, t_\infty \int_0^H u \, dy\right] dx + \left[-k \, dx\left(\frac{\partial t}{\partial y}\right)_{y=0}\right]$$

$$= \rho C_p \int_0^H ut \, dy + \frac{\partial}{\partial x}\left[\rho C_p \int_0^H ut \, dy\right] dx$$

9. After simplification and rearrangement, we have

$$\frac{d}{dx}\int_0^H (t_\infty - t) u \, dy = \frac{k}{\rho C_p}\left(\frac{\partial t}{\partial y}\right)_{y=0} = \alpha\left(\frac{\partial t}{\partial y}\right)_{y=0} \quad \text{...(3.5.1)}$$

Eq. (3.5.1) is the integral equation for the boundary layer for constant properties and constant free stream temperature t_∞.

Que 3.6. **Explain the analogy between momentum and heat transfer in turbulent flow over a flat surface.**

AKTU 2015-16, Marks 10

OR

Derive an expression for Nusselt number for turbulent flow over flat plate using Colburn analogy.

AKTU 2016-17, Marks 10

Answer

1. As we know that, the inter-relationship between fluid friction and Newton's law of viscosity is given as,

$$\tau_0 = \mu \frac{du}{dy} \quad ...(3.6.1)$$

2. Heat flow along Y-direction follows the Fourier equation

$$\therefore \quad Q = -kA \frac{dt}{dy} \quad ...(3.6.2)$$

3. When Pr is unity, temperature and velocity profile are identical (for most of the gases, $0.6 < Pr < 1.0$)

$$i.e., \quad \frac{\mu C_p}{k} = 1 \text{ or } \frac{k}{\mu} = C_p \quad ...(3.6.3)$$

4. By combining eq. (3.6.1), eq. (3.6.2) and eq. (3.6.3), we get

$$Q = -C_p A \tau_0 \frac{dt}{du}$$

5. Separating the variables and integrating within the limits :
At the plate surface : $u = 0$ and $t = t_s$
At the outer edge of boundary layer : $u = U$ and $t = t_\infty$

$$\frac{Q}{C_p A \tau_0} \int_0^U du = -\int_{t_s}^{t_\infty} dt$$

$$\frac{Q}{C_p A \tau_0} U = (t_s - t_\infty)$$

$$\frac{Q}{A(t_s - t_\infty)} = \frac{\tau_0 C_p}{U} \quad ...(3.6.4)$$

6. But,

$$\frac{Q}{A(t_s - t_\infty)} = h_x \quad ...(3.6.5)$$

and,

$$\tau_0 = C_{fx} \times \frac{1}{2} \rho U^2 \quad ...(3.6.6)$$

7. Substituting eq. (3.6.5) and eq. (3.6.6) in eq. (3.6.4), we get

$$h_x = C_{fx} \times \frac{1}{2}\rho U^2 \times \frac{C_p}{U} = \frac{C_{fx}}{2}(\rho C_p U)$$

$$\frac{h_x}{\rho C_p U} = \frac{C_{fx}}{2}$$

$\frac{h_x}{\rho C_p U}$ is called the Stanton number St_x.

8. It represents the Nusselt number divided by the product of the Reynolds and Prandtl number, *i.e.*,

$$\frac{Nu_x}{Re_x\, Pr} = St_x = \frac{C_{fx}}{2} \quad ...(3.6.7)$$

Eq. (3.6.7) is called the Reynolds analogy.

9. Further in case of laminar boundary layer on a flat plate, we have

$$Nu_x = \frac{h_x\, x}{k} = 0.332\,(Re_x)^{1/2}\,(Pr)^{1/3} \quad ...(3.6.8)$$

10. Dividing both sides of the eq. (3.6.8) by $Re_x\,(Pr)^{1/3}$, we get

$$\frac{Nu_x}{Re_x\,(Pr)^{1/3}} = \frac{0.332}{(Re_x)^{1/2}} = \frac{C_{fx}}{2} \quad ...(3.6.9)$$

11. The LHS of the eq. (3.6.9) can be rewritten as

$$\frac{Nu_x}{Re_x\,(Pr)^{1/3}} = \frac{Nu_x}{Re_x\,Pr}(Pr)^{2/3} = St_x\,(Pr)^{2/3}$$

$$\therefore \quad St_x\,(Pr)^{2/3} = \frac{C_{fx}}{2} \quad ...(3.6.10)$$

Eq. (3.6.10) shows the inter-relationship between heat and momentum transfer and designated as Colburn analogy.

12. For $Pr = 1$, the Reynolds and Colburn analogies are same.

Que 3.7. **What do you understand by mixed boundary layer ?**

Answer

1. Fig. 3.7.1 shows flow over a plate.

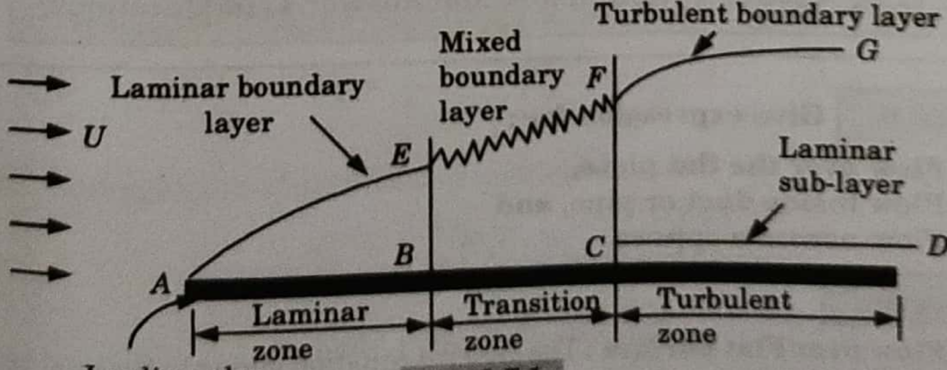

Fig. 3.7.1.

2. If the length of the plate is more than the distance x, calculated from equation, $Ux/\nu = 5 \times 10^5$, the thickness of boundary layer will go on increasing in the downstream direction.
3. Then the laminar boundary layer becomes unstable and motion of fluid within it is disturbed and irregular which leads to a transition from laminar to turbulent boundary layer.
4. This short length over which the boundary layer flow changes from laminar to turbulent is called transition zone or mixed zone, which is shown by distance BC.
5. Further downstream the transition zone, the boundary layer is turbulent and continues to grow in thickness.
6. This layer of boundary is called turbulent and continues to grow in thickness which is shown by the portion FG.

PART-2

Flow over a Flat Plate, Flow across a Single Cylinder and a Sphere, Flow Inside Ducts, Thermal Entrance Region, Empirical Heat Transfer Relations, Relation between Fluid Friction and Heat Transfer, Liquid Metal Heat Transfer.

CONCEPT OUTLINE : PART-2

Empirical Correlations for Forced Convection :

For Flow over Flat Surfaces :

$$Nu_x = 0.0292\,(Re_x)^{0.8}\,(Pr)^{0.33}$$

For Flow inside Duct or Pipe :

$$Nu_x = 0.023\,(Re)^{0.8}\,(Pr)^n$$

For Flow across a Sphere :

$$Nu_x = [0.97 + 0.68(Re)^{0.5}]\,(Pr)^{0.3}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.8. **Give expression for :**

a. **Flow over the flat plate,**
b. **Flow inside duct or pipe, and**
c. **Flow across a sphere.**

Answer

a. **Flow over Flat Surface :** The general equation giving to the local heat transfer coefficient for turbulent flow over flat plate is given by,

$$Nu_x = 0.0292\,(Re_x)^{0.8}\,(Pr)^{0.33}$$

b. **Flow inside Duct or Pipe :** The general equation giving to the local heat transfer coefficient for turbulent flow for duct or pipe is given by,

$$Nu_x = 0.023\,(Re_x)^{0.8}\,(P_r)^n$$

c. **Flow across a Sphere :** The general equation giving to the local heat transfer coefficient for turbulent flow for sphere is given by,

$$Nu_x = [0.97 + 0.68\,(Re)^{0.5}]\,(Pr)^{0.3}$$

Que 3.9. **Give the empirical correlations for forced convection.**

Answer

1. The following dimensionless numbers are used in forced convection :
   i. Nusselt number, $Nu = \dfrac{hL}{k}$
   ii. Reynold's number, $Re = \dfrac{\rho L \text{v}}{\mu}$
   iii. Prandtl number, $Pr = \dfrac{\mu C_p}{k}$
   iv. Stanton number, $St = \dfrac{h}{\rho C_p \text{v}}$
2. In order to determine the value of convection coefficient h, the following equations are used :

$$Nu = f_1\,(Re, Pr) = C_1\,(Re)^m\,(Pr)^n$$
$$St = f_2\,(Re, Pr) = C_2\,(Re)^a\,(Pr)^b$$

Que 3.10. **Describe the relation between fluid friction and heat transfer. How is the average friction and heat transfer coefficients determined in flow over a flat plate ?** **AKTU 2014-15, Marks 10**

Answer

A. **Relation between Fluid Friction and Heat Transfer :** Refer Q. 3.6, Page 3–10B, Unit-3.

B. **Expression for Average Friction :**

1. The local skin friction coefficient is given as

$$C_{fx} = \frac{\tau_0}{\frac{1}{2}\rho U^2} = \frac{0.646}{\sqrt{Re_x}}$$

2. Average value of skin friction coefficient,

$$\bar{C}_f = \frac{1}{L}\int_0^L C_{fx}\,dx = \frac{1}{L}\int_0^L \frac{0.646}{\sqrt{\rho U/\mu}}\frac{dx}{\sqrt{x}}$$

or,

$$\bar{C}_f = 1.292\sqrt{\frac{\mu}{L\rho U}} = \frac{1.292}{\sqrt{Re_L}}$$

Where, $Re_L = \dfrac{L\rho U}{\mu}$ is Reynolds number based on total length L of the flat plate.

C. **Expression for Heat Transfer Coefficient :**

1. We know that, $\dfrac{Q}{A} = h_x(t_s - t_\infty) = -k\left(\dfrac{dt}{dy}\right)_{y=0}$

or,
$$h_x = \frac{-k(dt/dy)_{y=0}}{t_s - t_\infty}$$

2. But,
$$\left(\frac{dt}{dy}\right)_{y=0} = \frac{3}{2}\left(\frac{t_s - t_\infty}{\delta_{th}}\right)$$

$$h_x = \frac{-k \times \dfrac{3}{2}\left(\dfrac{t_s - t_\infty}{\delta_{th}}\right)}{(t_s - t_\infty)} = \frac{3k}{2\delta_{th}} = \frac{3k}{2} \times \frac{1}{r\delta} \quad ...(3.10.1)$$

3. Substituting $r = \dfrac{0.975}{(Pr)^{1/3}}\left[1 - \left(\dfrac{x_0}{x}\right)^{3/4}\right]^{1/3}$ and $\delta = \dfrac{4.64x}{\sqrt{Re_x}}$ in eq. (3.10.1), we get

$$h_x = \frac{3k}{2} \times \frac{(Pr)^{1/3}}{0.975\left[1 - \left(\dfrac{x_0}{x}\right)^{3/4}\right]^{1/3}} \times \frac{\sqrt{Re_x}}{4.64x}$$

$$h_x = 0.332\,\frac{k}{x}(Pr)^{1/3}(Re)^{1/2} \times \frac{1}{\left[1 - \left(\dfrac{x_0}{x}\right)^{3/4}\right]^{1/3}}$$

4. When the plate is heated over the whole length *i.e.*, $x_0 = 0$, we have

$$h_x = 0.332\,\frac{k}{x}(Pr)^{1/3}(Re_x)^{1/2}$$

Que 3.11. **Explain drag force and drag coefficient.**

Answer

A. **Drag Force (F_D) :**

1. The drag force on elemental area
= Force due to pressure in the direction of fluid motion
+ Force due to shear stress in the direction of fluid motion

$= pdA \cos\theta + \tau_0 dA \cos(90° - \theta) = pdA \cos\theta + \tau_0 dA \sin\theta$

$\therefore$ Total drag, $F_D = \int p \cos\theta \, dA + \int \tau_0 \sin\theta \, dA$

2. The term $\int p \cos\theta \, dA$ is called the pressure drag or form drag while the term $\int \tau_0 \sin\theta \, dA$ is called the friction drag or skin drag or shear drag.

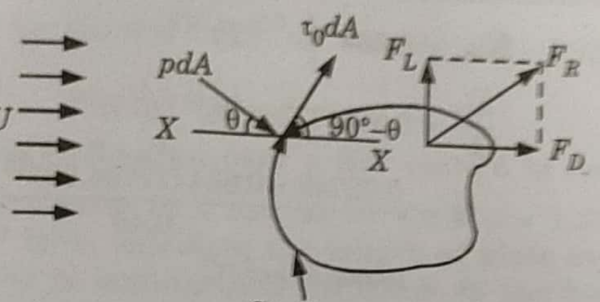

Fig. 3.11.1.

B. **Local Coefficient of Drag :** It is defined as the ratio of the shear stress τ_0 to the quantity $\frac{1}{2}\rho U^2$. It is denoted by C^*_D.

Hence, $$C^*_D = \frac{\tau_0}{\frac{1}{2}\rho U^2}$$

C. **Average Coefficient of Drag :** It is defined as the ratio of the total drag force to the quantity $1/2\,\rho A U^2$. It is also called coefficient of drag and it is denoted by C_D.

Hence, $$C_D = \frac{F_D}{1/2\,\rho A U^2}$$

Where, A = Area of the surface (or plate),
U = Free stream velocity, and
ρ = Mass density of fluid.

Que 3.12. **Air at 27 °C and 1 atm flows over a flat plate at a velocity 3 m/s. The plate is heated over its entire length to a temperature of 70 °C. Calculate the heat transferred if the plate length is 45 cm and width is 1 m. Properties of air, ν = 17.36 × 10^{-6} m²/s, k = 0.02749 W/m-K, C_p = 1.006 kJ/kg-K, Pr = 0.7. Take, $\bar{Nu}_L = 0.664\, Re_L^{0.5} Pr^{1/3}$.**

AKTU 2013-14, Marks 10

Answer

Given : t_∞ = 27 °C, t_s = 70 °C, U = 3 m/s, L = 45 cm = 0.45 m, w = 1 m, ν = 17.36 × 10^{-6} m²/s, k = 0.02749 W/m-K, C_p = 1.006 kJ/kg-K, Pr = 0.7

To Find : Heat transferred.

1. Reynolds number, $Re = \frac{UL}{\nu} = \frac{3 \times 0.45}{17.36 \times 10^{-6}} = 77764.98$

$\because$ $Re < 5 \times 10^5$, hence flow is laminar.

2. Average Nusselt number,

$$\bar{Nu}_L = 0.664\ Re^{0.5}\ Pr^{1/3}$$

$$\frac{\bar{h}L}{k} = 0.664\,(77764.98)^{0.5}\,(0.7)^{1/3}$$

$$\bar{h} = \frac{0.02749 \times 0.664\,(77764.98)^{0.5} \times (0.7)^{1/3}}{0.45}$$

$$\bar{h} = 10.044\ \text{W/m}^2\text{-°C}$$

3. Heat transfer, $Q = \bar{h} A_s (t_s - t_\infty)$

$$= 10.044 \times 0.45 \times 1 \times (70 - 27) = 194.351\ \text{W}$$

Que 3.13. **Air at 2 atm and 200 °C is heated as it flows at a velocity of 12 m/s through a tube with a diameter of 3 cm. A constant heat flux condition is maintained at the wall and the wall temperature is 20 °C above the air temperature all along the length of the tube. Calculate :**

i. the heat transfer per unit length of the tube, and

ii. the increase in bulk temperature of air over a 4 m length of the tube. Properties of air, $Pr = 0.681$, $\mu = 2.57 \times 10^{-5}$ kg/ms, $k = 0.0386$ W/m-K, $C_p = 1.025$ kJ/kg-K, $\rho = 1.493$ kg/m³.

Use : $\bar{Nu} = 0.023\ Re^{0.8}\ Pr^{0.4}$.

AKTU 2013-14, Marks 10

Answer

Given : $t_\infty = 200$ °C, $t_w = 220$ °C, $U = 12$ m/s, $d = 3$ cm $= 0.03$ m
$Pr = 0.681$, $\mu = 2.57 \times 10^{-5}$ kg/ms, $k = 0.0386$ W/m-K,
$C_p = 1.025$ kJ/kg-K, $\rho = 1.493$ kg/m³

To Find : i. Heat transfer per unit length of tube.
ii. Increase in bulk temperature of air.

1. Reynolds number,

$$Re = \frac{\rho U d}{\mu} = \frac{1.493 \times 12 \times 0.03}{2.57 \times 10^{-5}} = 0.2091 \times 10^5$$

$\because$ $R_e < 5 \times 10^5$, hence flow is laminar.

2. Nusselt number,

$$\bar{Nu} = 0.023\ Re^{0.8}\ Pr^{0.4}$$

$$\frac{hd}{k} = 0.023\ Re^{0.8}\ Pr^{0.4}$$

$$h = \frac{0.0386 \times 0.023 \times (0.2091 \times 10^5)^{0.8} \times (0.681)^{0.4}}{0.03}$$

$$h = 72.57 \text{ W/m}^2\text{-K}$$

3. Heat transferred per unit length of the tube,

$$Q = hA_s(t_w - t_\infty) = 72.57 \times \pi d \times 1 \times (220 - 200)$$
$$= 72.57 \times \pi \times 0.03 \times 20 = 136.79 \text{ W/m}$$

4. Bulk temperature increase of air over a 4 m length of the tube

$$Q = mC_p\,(\Delta t) = (\rho AU)\,C_p\,(\Delta t)$$

$$4 \times 136.79 = 1.493 \times \frac{\pi}{4} \times (0.03)^2 \times 12 \times 1.025 \times 10^3 \times (\Delta t)$$

$$\Delta t = 42.15\,°\text{C}$$

Que 3.14. **Air is flowing over a flat plate 5 m long and 2.5 m wide with a velocity of 4 m/s at 15 °C. If ρ = 1.208 kg/m³ and $\nu = 1.47 \times 10^{-5}$ m²/s, calculate the length of plate over which the boundary layer is laminar and thickness of the boundary layer (laminar), shear stress at the location where boundary layer ceases to be laminar and the total drag force on the both sides of that portion of the plate where boundary layer is laminar.**

AKTU 2014-15, Marks 10

Answer

Given : $L = 5$ m, $w = 2.5$ m, $U = 4$ m/s, $\rho = 1.208$ kg/m³, $\nu = 1.47 \times 10^{-5}$ m²/s

To Find :
i. Length of plate over which the boundary layer is laminar, and thickness of the boundary layer (laminar).
ii. Shear stress at the location where boundary layer ceases to be laminar.
iii. Total drag force on the both sides of that portion of plate where boundary layer is laminar.

1. Reynold number, $Re = \dfrac{UL}{\nu} = \dfrac{4 \times 5}{1.47 \times 10^{-5}} = 1.361 \times 10^6.$

Hence on the front portion, boundary layer is laminar and on the rear, it is turbulent.

$$Re = \frac{Ux}{\nu} = 5 \times 10^5$$

$$\therefore \quad \frac{4 \times x}{1.47 \times 10^{-5}} = 5 \times 10^5$$

$$x = \frac{5 \times 10^5 \times 1.47 \times 10^{-5}}{4} = 1.837 \text{ m}$$

Hence the boundary layer is laminar on 1.837 m length of the plate.

2. Thickness of the boundary layer (laminar), $\delta = \dfrac{5x}{\sqrt{Re}}$

$$\delta = \frac{5 \times 1.837}{\sqrt{5 \times 10^5}} = 0.01299 \text{ m or } 12.99 \text{ mm}$$

3. Local coefficient of drag, $C_{fx} = \frac{0.664}{\sqrt{Re}} = \frac{0.664}{\sqrt{5 \times 10^5}} = 0.000939$

Shear stress, $\tau_0 = C_{fx} \times \frac{1}{2} \rho U^2$

$$= 0.000939 \times \frac{1}{2} \times 1.208 \times 4^2 = 0.00907 \text{ N/m}^2$$

4. Total drag force on both sides of plate,

$$F_D = 2\bar{C}_f \times \frac{1}{2} \rho A U^2$$

Where, $\bar{C}_f$ = Average coefficient of drag (or skin friction)

$$= \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{5 \times 10^5}} = 1.878 \times 10^{-3}$$

and area of the plate, $A = xL = 1.837 \times 2.5 = 4.59 \text{ m}^2$

$$\therefore \quad F_D = 2 \times 1.878 \times 10^{-3} \times \frac{1}{2} \times 1.208 \times 4.59 \times 4^2$$

$$= 0.167 \text{ N}$$

Que 3.15. **A flat plate is 2 m long, 0.8 m wide and 3 mm thick. Its density and specific heat is 3000 kg/m³ and 700 J/kg-K respectively. The plate is having initial temperature of 80 °C. A stream of air at 20 °C blown over both surfaces of the plate along its width, at a velocity of 2 m/s. Calculate rate of heat dissipation from the plate and initial rate of cooling of the plate. The properties of air are $\rho = 1.09$ kg/m³, $k = 0.028$ W/m-°C, $C = 1007$ J/kg-K, $\mu = 2.03 \times 10^{-5}$ kg/m-s, $Pr = 0.698$**

$Nu_u = 0.664 (Re_x)^{0.5} (Pr)^{0.33}$ for Laminar Flow

$Nu_u = 0.0336$ [illegible] $836 (Pr)^{0.33}$ for Turbulent Flow

AKTU 2015-16, Marks 7.5

Answer

Given : $L = 2$ m, $w = 0.8$ m, $t = 3$ mm $= 0.003$ m
$\rho_p = 3000$ kg/m³, $C_p = 700$ J/kg-K, $(t_p)_i = 80$ °C, $\rho_a = 1.09$ kg/m³,
$k = 0.028$ W/m-°C, $C = 1007$ J/kg-K, $\mu = 2.03 \times 10^{-5}$ kg/m-s,
$Pr = 0.698$, For air, $t_\infty = 20$ °C, $U = 2$ m/s

To Find : i. Heat dissipation rate.
ii. Initial rate of cooling of plate.

1. Air blows over both surfaces of plate along width.
Hence, characteristic length, $L_c = w = 0.8$ m

2. Now, Reynolds number,

$$Re = \frac{\rho_a U L_c}{\mu}$$

$$= \frac{1.09 \times 2 \times 0.8}{2.03 \times 10^{-5}} = 8.59 \times 10^4$$

$\because$ $Re < 5 \times 10^5$, hence flow is laminar.

So $Nu_a = 0.664\,(Re)^{0.5}\,(Pr)^{0.33}$

$= 0.664\,(8.59 \times 10^4)^{0.5}\,(0.698)^{0.33}$

$= 172.837$

3. Since,

$$Nu_a = \frac{hL_c}{k}$$

$$Nu_a = \frac{h \times 0.8}{0.028}$$

$$\Rightarrow \quad \frac{h \times 0.8}{0.028} = 172.837$$

$$h = 6.0493 \text{ W/m}^2\text{-}^\circ\text{C.}$$

4. Now, heat transfer rate for one surface,

$$Q = hA\,\Delta t$$

$$Q = 6.0493 \times 2 \times 0.8 \times (80 - 20)$$

$$Q = 580.7328 \text{ W}$$

5. For both surfaces,

$$Q_T = 2 \times 580.7328 = 1161.4656 \text{ W}$$

6. Now, initial cooling rate of plate for one surface,

$$Q_P = \dot{m} C_p \,\Delta t$$

$$= \rho_P\, AUC_p \Delta t$$

$$= 3000 \times 2 \times 0.8 \times 2 \times 700 \times (80 - 20)$$

$$= 403.2 \text{ MW}$$

7. For both surface,

$$(Q_P)_T = 2 \times 403.2 = 806.4 \text{ MW}$$

Que 3.16. **Castor oil at 25 °C flows at a velocity of 0.1 m/s past a flat plate in a certain process. If the plate is 4.5 m long and is maintained at a uniform temperature of 95 °C. Calculate the following using exact solution :**

i. The hydrodynamic and thermal boundary layer thickness on one side of the plate,

ii. The total drag force per unit width on one side of the plate,

iii. The local heat transfer coefficient at the trailing edge, and the heat transfer rate.

AKTU 2017-18, Marks 10

Answer

Given : $t_\infty = 25\ °C$, $t_s = 95\ °C$, $L = 4.5$ m, $U = 0.1$ m/s.

To Find :
i. The hydrodynamic and thermal boundary layer thickness on one side of the plate.
ii. The total drag force per unit width on one side of the plate.
iii. The local heat transfer coefficient at the trailing edge, and the heat transfer rate.

Data Assumed : $\nu = 0.65 \times 10^{-4}\ m^2/s$, $\alpha = 7.2 \times 10^{-8}\ m^2/s$, $k = 0.213\ W/m\text{-}°C$, $\rho = 956.8\ kg/m^3$

1. Reynolds number at the end of the plate,

$$Re = \frac{UL}{\nu} = \frac{0.1 \times 4.5}{0.65 \times 10^{-4}} = 6923$$

$\because$ $Re < 5 \times 10^5$, hence flow is laminar.

2. The hydrodynamic boundary layer thickness,

$$\delta = \frac{5L}{\sqrt{Re}} = \frac{5 \times 4.5}{\sqrt{6923}}$$

$$= 0.2704 \text{ m} = 270.4 \text{ mm}$$

3. The thermal boundary layer thickness,

$$\delta_{th} = \frac{\delta}{(Pr)^{1/3}}$$

Where, $$Pr = \frac{\nu}{\alpha} = \frac{0.65 \times 10^{-4}}{7.2 \times 10^{-8}} = 902.77$$

$\therefore$ $$\delta_{th} = \frac{0.2704}{(902.77)^{1/3}} = 0.02798 \text{ m} = 27.98 \text{ mm}$$

4. The average skin friction coefficient is given by,

$$\bar{C}_f = \frac{1.328}{\sqrt{Re}}$$

$$\bar{C}_f = \frac{1.328}{\sqrt{6923}} = 0.01596$$

5. The total drag force,

$$F_D = \bar{C}_f \times \frac{1}{2} \rho U^2 \times \text{Area of plate (for one side)}$$

$$= 0.01596 \times \frac{1}{2} \times 956.8 \times (0.1)^2 \times (4.5 \times 1)$$

$$= 0.3436 \text{ N/m}$$

6. $$Nu_x = \frac{h_x x}{k} = 0.332\ (Re_x)^{1/2}\ (Pr)^{1/3}$$

$$= 0.332 \times (6923)^{1/2} (902.77)^{1/3} = 266.98$$

or, $$h_x = \frac{266.98 \times k}{x} = \frac{266.98 \times 0.213}{4.5} = 12.64 \text{ W/m}^2\text{-°C}$$

7. Local heat transfer coefficient at the trailing edge,

$$\bar{h} = 2h_x = 2 \times 12.64 = 25.28 \text{ W/m}^2\text{-°C}$$

8. Heat transfer rate, $Q = \bar{h} A_s (t_s - t_\infty)$

$$= 25.28 \times (4.5 \times 1) \times (95 - 25) = 7963.2 \text{ W}$$

Que 3.17. **Give the relation for liquid metal heat transfer.**

Answer

1. The example of liquid metal are sodium, potassium, lead, etc. and in the liquid metal, Prandtl number is very less than 1 *i.e.*, $Pr << 1$.
2. For constant temperature,

$$Nu = 4.8 + 0.0156\, Pe^{0.85}\, Pr^{0.08}$$

Where, $Pe = Re\, Pr$

3. For constant heat flux condition,

$$Nu = 4.82 + 0.0185\, Pe^{0.827}$$

PART-3

Physical Mechanism of Natural Convection, Buoyant Force Empirical Heat Transfer Relations for Natural Convection over Vertical Plates and Cylinders, Horizontal Plates and Cylinders and Sphere, Combined Free and Forced Convection.

CONCEPT OUTLINE : PART-3

Natural Convection : It occurs when the fluid circulates by virtue of the natural differences in densities of the hot and cold fluids.

Buoyant force or Buoyancy Force : The difference in the density of the cold fluid and hot fluid causes the fluid to flow in the upward direction. The force accounting this flow is known as buoyancy force.

Empirical Heat Transfer Relations for Free Convection :

For Vertical Plates :

i. In case of laminar flow, $Nu = 0.59\, (Gr\, Pr)^{1/4}$
ii. In case of turbulent flow, $Nu = 0.10\, (Gr\, Pr)^{1/3}$

For Cylinders :

i. In case of laminar flow, $Nu = 0.53\, (Gr\, Pr)^{1/4}$
ii. In case of turbulent flow, $Nu = 0.13\, (Gr\, Pr)^{1/3}$

For Spheres :

$$Nu = 2 + 0.43 (Gr\, Pr)^{1/4}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.18. **What do you understand by buoyant force and centre of buoyancy ?**

Answer

A. **Buoyant Force :** Whenever a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the buoyant force or simply buoyancy.

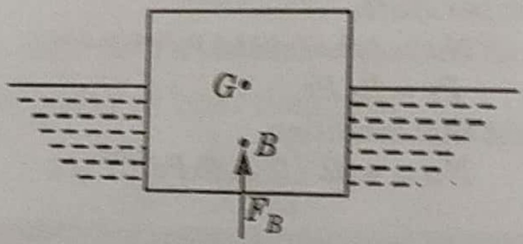

Fig. 3.18.1.

B. **Centre of Buoyancy :**

1. It is defined as a point about which the force of buoyancy is supposed to act.
2. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.
3. In Fig. 3.18.1, F_B is the force of buoyancy and point B is the centre of buoyancy.

Que 3.19. **For the case of free convection boundary layer on a heated vertical plate, show that :**

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(t - t_\infty) + v\frac{\partial^2 u}{\partial y^2}$$

Answer

1. In free convection, the additional force is the body force ρg and hence the momentum equation gets modified to

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \rho g - \frac{\partial p}{\partial x} \quad ...(3.19.1)$$

2. In free convection, outside the boundary layer, $(y \to \infty)$ as ρ tends to ρ_∞ whereas u and v approach zero. So, eq. (3.19.1) becomes,

$$0 = -\frac{\partial p}{\partial x} - \rho_\infty g$$

or

$$\frac{\partial p}{\partial x} = -\rho_\infty g \quad ...(3.19.2)$$

3. From eq. (3.19.1) and eq. (3.19.2), we get

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + g(\rho_\infty - \rho) \quad ...(3.19.3)$$

4. Further the density difference $(\rho_\infty - \rho)$ may be expressed in terms of the volume coefficient of expansion β which is defined as,

$$\beta = -\frac{1}{\rho}\left(\frac{\partial p}{\partial t}\right) = \frac{1}{\rho}\left(\frac{\rho_\infty - \rho}{t - t_\infty}\right)$$

5. So, eq. (3.19.3) becomes,

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \rho g\,\beta(t - t_\infty)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + g\beta(t - t_\infty) \quad \left\{\because\ v = \frac{\mu}{\rho}\right\}$$

Que 3.20. **Which dimensionless numbers are used to develop the empirical correlations for free convection ? Give the empirical heat transfer relations for natural convection over vertical plates, horizontal plates, cylinders and spheres.**

Answer

A. **Different Dimensionless Numbers used in Free Convection :** The dimensionless numbers used for empirical relations for heat transfer in natural convection are :

1. Nusselt number, $(Nu) = \frac{hL}{k}$
2. Grashoff number, $(Gr) = \frac{L^3 \beta g \Delta t}{v^2}$
3. Prandtl number, $(Pr) = \frac{\mu C_p}{k}$

B. **Empirical Heat Transfer Relations for Free Convection :**

a. **Vertical Plates :** The commonly used relation are :

i. **For Laminar Flow :**

$\overline{Nu}_L = 0.59\,(Gr\,Pr)^{1/4}$ for $(10^4 < Gr\,Pr < 10^9)$

ii. **For Turbulent Flow :**

$\overline{Nu}_L = 0.10\,(Gr\,Pr)^{1/3}$ for $(10^9 < Gr\,Pr < 10^{12})$

b. **Horizontal Plates :**

Case I : Upper surface heated or the lower surface cooled :

i. **For Laminar Flow :**

$$\bar{N}u_L = 0.54\,(Gr\,Pr)^{1/4} \text{ for } (10^5 < Gr\,Pr \le 2 \times 10^7)$$

ii. **For Turbulent Flow :**

$$\bar{N}u_L = 0.14\,(Gr\,Pr)^{1/3} \text{ for } (2 \times 10^7 < Gr\,Pr \le 3 \times 10^{10})$$

Case II. Lower surface heated or upper surface cooled :

i. **For Laminar Flow :**

$$\bar{N}u_L = 0.27\,(Gr\,Pr)^{1/4} \text{ for } (3 \times 10^5 < Gr\,Pr \le 3 \times 10^{10})$$

ii. **For Turbulent Flow :**

$$\bar{N}u_L = 0.107\,(Gr\,Pr)^{1/3} \text{ for } (7 \times 10^6 < Gr\,Pr \le 11 \times 10^{10})$$

c. **Cylinders :** The relation for cylinders is given by,

i. **For Laminar Flow :**

$$\bar{N}u_L = 0.53\,(Gr\,Pr)^{1/4} \text{ for } (10^4 < Gr\,Pr < 10^9)$$

ii. **For Turbulent Flow :**

$$\bar{N}u_L = 0.13\,(Gr\,Pr)^{1/3} \text{ for } (10^9 < Gr\,Pr < 10^{12})$$

d. **Spheres :** The relation for sphere is given by,

$$\bar{N}u_L = 2 + 0.43\,(Gr\,Pr)^{1/4} \text{ for } (1 < Gr\,Pr < 10^5)$$

Que 3.21. **Show that in natural convection heat transfer :**

$$\bar{N}u = f(Gr, Pr)$$

AKTU 2013-14, Marks 10

Answer

1. The heat transfer coefficient in case of natural or free convection, like forced convection depends upon the variables v, ρ, k, μ, C_p and L or D.
2. Since the fluid circulation in free convection is owing to difference in density between the various fluid layers due to temperature gradient and not by external agency, therefore, velocity v is no longer an independent variable but depends upon the following factors :
   i. Δt *i.e.*, the difference of temperatures between the heated surface and the undisturbed fluid.
   ii. β *i.e.*, coefficient of volume expansion of the fluid.
   iii. g *i.e.*, acceleration due to gravity.

   ($g\beta\Delta t$ is considered as one physical factor.)
3. Thus heat transfer coefficient 'h' may be expressed as follows :

$$h = f\,(\rho, L, \mu, C_p, k, g\beta\Delta t)$$

or

$$f_1\,(\rho, L, \mu, k, h, C_p, g\beta\Delta t) = 0 \qquad \ldots(3.21.1)$$

The parameter ($g\beta\Delta t$) represents the buoyant force and has the dimensions of $[LT^{-2}]$.

4. Total number of variables, $n = 7$
Fundamental dimensions in the problem are M, L, T, θ and hence, $m = 4$.
Number of dimensionless π-terms $= n - m = 7 - 4 = 3$
5. Now the eq. (3.21.1) can be written as,

$$f_1(\pi_1, \pi_2, \pi_3) = 0 \quad ...(3.21.2)$$

6. We choose ρ, L, μ and k as the repeating variables with unknown exponents.

$$\therefore \quad \pi_1 = \rho^{a1} L^{b1} \mu^{c1} k^{d1} h$$
$$\pi_2 = \rho^{a2} L^{b2} \mu^{c2} k^{d2} C_p$$
$$\pi_1 = \rho^{a3} L^{b3} \mu^{c3} k^{d3} g\beta\Delta t$$

a. **π_1-Term :**

$$M^0 L^0 T^0 \theta^0 = (ML^{-3})^{a1} (L)^{b1} (ML^{-1} T^{-1})^{c1} (MLT^{-3} \theta^{-1})^{d1} (MT^{-3} \theta^{-1})$$

Equating the exponents of M, L, T and θ respectively, we get

For M : $\quad 0 = a_1 + c_1 + d_1 + 1$

For L : $\quad 0 = -3a_1 + b_1 - c_1 + d_1$

For T : $\quad 0 = -c_1 - 3d_1 - 3$

For θ : $\quad 0 = -d_1 - 1$

Solving the above equations, we get

$a_1 = 0, b_1 = 1, c_1 = 0, d_1 = -1$

$$\therefore \quad \pi_1 = Lk^{-1} h \quad \text{or} \quad \pi_1 = \frac{hL}{k} = \bar{N}u$$

b. **π_2-Term :**

$$M^0 L^0 T^0 \theta^0 = (ML^{-3})^{a2} (L)^{b2} (ML^{-1} T^{-1})^{c2} (MLT^{-3} \theta^{-1})^{d2} (L^2 T^{-2} \theta^{-1})$$

Equating the exponents of M, L, T, θ respectively, we get

For M : $\quad 0 = a_2 + c_2 + d_2$

For L : $\quad 0 = -3a_2 + b_2 - c_2 + d_2 + 2$

For T : $\quad 0 = -c_2 - 3d_2 - 2$

For θ : $\quad 0 = -d_2 - 1$

Solving the above equations, we get

$$a_2 = 0, b_2 = 0, c_2 = 1, d_2 = -1$$

$$\therefore \quad \pi_2 = \mu k^{-1} C_p \quad \text{or} \quad \pi_2 = \frac{\mu C_p}{k} = Pr$$

c. **π_3-Term :**

$$M^0 L^0 T^0 \theta^0 = (ML^{-3})^{a3} (L)^{b3} (ML^{-1} T^{-1})^{c3} (MLT^{-3} \theta^{-1})^{d3} (LT^{-2})$$

Equating the exponents of M, L, T, θ respectively, we get

For M : $\quad 0 = a_3 + c_3 + d_3$

For L : $\quad 0 = -3a_3 + b_3 - c_3 + d_3 + 1$

For T : $\quad 0 = -c_3 - 3d_3 - 2$

For θ : $\quad 0 = -d_3$

Solving the above equations, we get

$$a_3 = 2, b_3 = 3, c_3 = -2, d_3 = 0$$

$$\therefore \quad \pi_3 = \rho^2 L^3 \mu^{-2} (g\beta\Delta t)$$

or $$\pi_3 = \frac{(g\beta\Delta t)\,\rho^2 L^3}{\mu^2} = \frac{(g\beta\Delta t)\,L^3}{\nu^2} = Gr$$

7. Putting the values of π_1, π_2 and π_3 in eq. (3.21.2), we have
$$\bar{N}u = f(Gr, Pr)$$

Que 3.22. **A horizontal pipe 1 ft (0.3048 m) in diameter is maintained at a temperature of 250 °C in a room where the ambient air is at 15 °C. Calculate the free convection heat loss per meter of length.**

AKTU 2014-15, Marks 10

Answer

Given : $D = 0.3048$ m, $t_o = 250$ °C, $t_\infty = 15$ °C
To Find : Free convection heat loss per meter of length.

1. For the horizontal cylinder, with laminar flow relation of convection heat transfer coefficient,
$$\bar{h} = 1.32\left(\frac{\Delta t}{D}\right)^{1/4} = 1.32\left(\frac{250-15}{0.3048}\right)^{1/4}$$
$$= 6.955 \text{ W/m}^2\text{-°C}$$
2. We know that, $Q_{conv.}$ per unit length,
$$Q_{conv.} = \bar{h}A\,(t_o - t_\infty)$$
$$= 6.955 \times \pi \times 0.3048 \times 1 \times (250 - 15)$$
$$= 1565.055 \text{ W} = 1.565 \text{ kW}$$

Que 3.23. **A 350 mm long glass plate is hung vertically in the air at 24 °C while its temperature is maintained at 80 °C. Calculate the boundary layer thickness at the trailing edge of the plate. If a similar plate is placed in a wind tunnel and air is blown over it at a velocity of 5 m/s, find the boundary layer thickness at its trailing edge. Also determine the average heat transfer coefficient, for natural and forced convection for the above mentioned data.**

AKTU 2017-18, Marks 10

Answer

Given : $L = 350$ mm $= 0.35$ m, $t_\infty = 24$ °C, $t_s = 80$ °C, $U = 5$ m/s
To Find : i. Boundary layer thickness at trailing edge for natural and forced convection.
ii. Average heat transfer coefficient for natural and forced convection.

1. Film temperature, $t_f = \dfrac{t_s + t_\infty}{2} = \dfrac{80 + 24}{2} = 52$ °C

2. The properties of air at 52 °C are : $k = 28.15 \times 10^{-3}$ W/m-°C,

$$\nu = 18.41 \times 10^{-6} \text{ m}^2\text{/s}, Pr = 0.7$$

$$\beta = \left(\frac{1}{52 + 273}\right) = 3.07 \times 10^{-3} \text{ K}^{-1}$$

3. In case of free convection the boundary layer thickness is calculated as below :

i. Grashof number,

$$Gr = \frac{L^3 g \beta (t_s - t_\infty)}{\nu^2}$$

$$= \frac{(0.35)^3 \times 9.81 \times 3.07 \times 10^{-3} \times (80 - 24)}{(18.41 \times 10^{-6})^2}$$

$$= 2.133 \times 10^8$$

ii. Rayleigh number,

$$Ra = Gr\, Pr = 2.133 \times 10^8 \times 0.7 = 1.493 \times 10^8$$

iii. As we know that,

$$\frac{\delta}{x} = 3.93\,(0.952 + Pr)^{1/4}\,(Gr)^{-1/4}\,(Pr)^{-1/2}$$

or,

$$\delta = 0.35\,[3.93\,(0.952 + 0.7)^{1/4}\,(2.133 \times 10^8)^{-1/4}\,(0.7)^{-1/2}]$$

$$= 0.0154 \text{ m}$$

4. In case of forced convection the boundary layer thickness is calculated as below :

i. Reynolds number,

$$Re = \frac{UL}{\nu} = \frac{5 \times 0.35}{18.41 \times 10^{-6}} = 9.505 \times 10^4$$

Since $Re < 5 \times 10^5$, hence the boundary layer is laminar.

ii. Now, boundary layer thickness is,

$$\delta = \frac{5L}{\sqrt{Re}} = \frac{5 \times 0.35}{\sqrt{9.505 \times 10^4}} = 0.00567 \text{ m} = 5.67 \text{ mm}$$

5. In case of free convection average heat transfer coefficient is calculated as below :

i. Nusselt number,

$$Nu_L = \frac{\bar{h}L}{k} = 0.677\,(Pr)^{1/2}\,(0.952 + Pr)^{-1/4}\,(Gr)^{1/4}$$

$$= 0.677\,(0.7)^{1/2}\,(0.952 + 0.7)^{-1/4}\,(2.133 \times 10^8)^{1/4}$$

$$= 60.378$$

ii. Heat transfer coefficient,

$$\bar{h} = \frac{k}{L} \times 60.378 = \frac{28.15 \times 10^{-3}}{0.35} \times 60.378$$

$$= 4.856 \text{ W/m}^2\text{-°C}$$

6. In case of forced convection average heat transfer coefficient is calculated as below :

i. Nusselt number,

$$\bar{Nu}_L = \frac{\bar{h}L}{k} = 0.664\,(Re)^{1/2}\,(Pr)^{1/3}$$

$$= 0.664\,(9.505 \times 10^4)^{1/2}\,(0.7)^{1/3} = 181.78$$

ii. Heat transfer coefficient,

$$\bar{h} = \frac{k}{L} \times 181.78 = \frac{28.15 \times 10^{-3}}{0.35} \times 181.78$$

$$= 14.62 \text{ W/m}^2\text{-°C}$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Derive the equation for boundary layer thickness.

Ans. Refer Q. 3.2.

Q. 2. Derive an expression for energy equation of thermal boundary layer over flat plate.

Ans. Refer Q. 3.4.

Q. 3. Derive the Von-Karman integral energy equation for the flow past a flat plate.

Ans. Refer Q. 3.5.

Q. 4. Derive an expression for Nusselt number for turbulent flow over flat plate using Colburn analogy.

Ans. Refer Q. 3.6.

Q. 5. Describe the relation between fluid friction and heat transfer. How is the average friction and heat transfer coefficients determined in flow over a flat plate ?

Ans. Refer Q. 3.10.

Q. 6. Castor oil at 25 °C flows at a velocity of 0.1 m/s past a flat plate in a certain process. If the plate is 4.5 m long and is maintained at a uniform temperature of 95 °C. Calculate the following using exact solution :

i. The hydrodynamic and thermal boundary layer thickness on one side of the plate,

ii. The total drag force per unit width on one side of the plate, and

iii. The local heat transfer coefficient at the trailing edge, and the heat transfer rate.

Ans. Refer Q. 3.16.

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4 UNIT

Thermal Radiation

PART-1

Basic Radiation Concept, Radiation Properties of Surface, Black Body Radiation, Planck's Law, Wein's Displacement Law, Stefan Boltzmann Law, Kirchoff 's Law, Gray Body, Shape Factor, Black Body Radiation.

CONCEPT OUTLINE : PART-1

Radiation Heat Transfer : It is defined as the transfer of energy from a system in the form of electromagnetic mechanism which is caused by the temperature difference.

Black Body : The body that absorbs all the radiant energy reaching its surface is known as black body.

Gray Body : If the radiative properties, α, ρ, τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called gray body.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **What do you understand by radiation heat transfer ? Also write the surface emission properties.**

Answer

A. Radiation Heat Transfer :

1. It is defined as the transfer of energy from a system in the form of electromagnetic mechanism which is caused by the temperature difference.
2. Factors on which rate of emission of radiation by a body depends are as follows :
   i. Temperature of the surface,
   ii. The nature of the surface, and
   iii. Wavelength of radiation.

B. Surface Emission Properties :

a. Total Emissive Power (*E*) :

1. It is defined as the total amount of radiation emitted by a body per unit area and time.
2. According to Stefan Boltzmann "the total emissive power of a black body is proportional to the fourth power of its absolute temperature."

$$E_b = \sigma t^4 \text{ W/m}^2$$

$$E_b = \sigma A t^4 \text{ W}$$

Where, σ = Stefan-Boltzmann constant.
$= 5.67 \times 10^{-8}$ W/m^2-K^4

b. **Monochromatic Emissive Power (E_λ) :** It is defined as the rate of energy radiated per unit area of the surface per unit wavelength.
Total emissive power is :

$$E = \int_0^\infty E_\lambda \, d\lambda \text{ W/m}^2$$

c. **Emission from real surface-emissivity :** The emissive power from a real surface is given by

$$E = \varepsilon \sigma A t^4 \text{ W}$$

Where, E = Emissivity of the material.

d. Intensity of radiation.

e. Radiation density and pressure.

f. **Radiosity (J) :** It refers to all of the radiant energy leaving a surface.

Que 4.2. **What do you mean by black body ? Also write the properties of black body.**

Answer

A. **Black Body :**

1. The body that absorbs all the radiant energy reaching its surface is known as black body.
2. For a black body : $\alpha = 1, \rho = 0, \tau = 0$

B. **Properties of Black Body :**

1. It absorbs all the incident radiation falling on it and does not transmit or reflect regardless of wavelength and direction.
2. It emits maximum amount of thermal radiations at all wavelengths at any specified temperature.
3. It is a diffuse emitter (*i.e.*, the radiation emitted by a black body is independent of direction).

Que 4.3. **Explain the following terms :**

a. **White body,**

b. **Gray body, and**

c. **Opaque body.**

Answer

a. **White Body :**

1. A body which reflects all incident radiation falling on it is known as white body.
2. For a white body : $\rho = 1, \alpha = 0$ and $\tau = 0$.

b. **Gray Body :**

1. If the radiative properties α, ρ, τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called gray body.

2. It is also defined as one whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation.

c. **Opaque Body :**

1. A body which does not transmit any part of incident radiation is known as opaque body.
2. For a opaque body. $\tau = 0$. So, $\alpha + \rho = 1$.

Que 4.4. **Define the properties :**

i. Emissivity.
ii. Absorptivity.
iii. Reflectivity.
iv. Transmissivity.

AKTU 2014-15, Marks 10

Answer

i. **Emissivity (ε) :** It is defined as the ability of a body to radiate heat. It is also defined as the ratio of the emissive power of a body to the emissive power of a black body of equal temperature.

ii. **Absorptivity (α) :** It is the ratio of amount of radiation absorbed to the total incident radiation.

$$\alpha = G_a / G$$

iii. **Reflectivity (ρ) :** It is the ratio of amount of radiation reflected to the total incident radiation.

$$\rho = G_r / G$$

iv. **Transmissivity (τ) :** It is the ratio of amount of radiation transmitted to the total incident radiation.

$$\tau = G_t / G$$

Que 4.5. **A flat plate 5 m² receives normally radiant energy with an intensity of 660 W/m². The absorptivity of the plate is 2 times its transmissivity and 3 times its reflectivity. Find the energy absorbed, transmitted and reflected in watts.**

Answer

Given : $\alpha = 2\tau$ or $\tau = \dfrac{\alpha}{2}$, $Q = 660$ W/m², $\alpha = 3\rho$ or $\rho = \dfrac{\alpha}{3}$, $A = 5$ m²

To Find : Energy absorbed, transmitted and reflected.

1. We know that, $\alpha + \rho + \tau = 1$,

Then,
$$\alpha + \frac{\alpha}{3} + \frac{\alpha}{2} = 1$$
$$\frac{6\alpha + 3\alpha + 2\alpha}{6} = 1$$
$$11\alpha = 6$$

$$\alpha = \frac{6}{11}$$

So, $$\tau = \frac{6}{22} \text{ or } \frac{3}{11}$$

and $$\rho = \frac{6}{33} \text{ or } \frac{2}{11}$$

2. Energy absorbed per unit area,

$$= 660 \times \alpha = 660 \times \frac{6}{11} = 360 \text{ W/m}^2$$

Total energy absorbed = 360 × 5 = 1800 W

3. Energy reflected per unit area,

$$= 660 \times \rho = 660 \times \frac{2}{11} = 120 \text{ W/m}^2$$

Total energy reflected = 120 × 5 = 600 W

4. Energy transmitted per unit area,

$$= 660 \times \tau = 660 \times \frac{3}{11} = 180 \text{ W/m}^2$$

Total energy transmitted = 180 × 5 = 900 W

Que 4.6. **Explain the following terms :**

a. **Kirchhoff's law,**
b. **Planck's law,**
c. **Stefan-Boltzmann law, and**
d. **Wein's displacement law.**

Answer

a. **Kirchhoff's Law :**

1. Kirchhoff's law states that the ratio of total emissive power (E) to absorptivity (α) is constant for all bodies which are in thermal equilibrium with their environment.

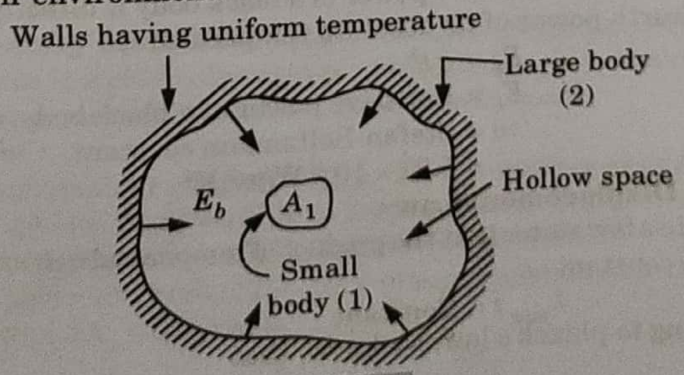


Fig. 4.6.1.

2. At equilibrium, the energy absorbed by body is equal to energy emitted by body. ...(4.6.1)

For body 1 : $A_1E_1 = \alpha_1 A_1 E_b$

For body 2 : $A_2E_2 = \alpha_2 A_2 E_b$ ...(4.6.2)

3. From eq. (4.6.1) and eq. (4.6.2), we have

$$E_b = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha} \quad ...(4.6.3)$$

4. According to the emissivity relation,

$$\varepsilon = \frac{E}{E_b}$$

or

$$E_b = \frac{E}{\varepsilon} \quad ...(4.6.4)$$

5. Comparing eq. (4.6.3) and eq. (4.6.4), we get

$$\varepsilon = \alpha$$

This shows that the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.

b. Planck's Law :

1. The laws governing the distribution of radiant energy over wavelength for a black body at a fixed temperature were formulated by Planck.
2. According to Planck, spectral emissive power $(E_\lambda)_b$ at any temperature and wavelength is given as,

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda t}\right) - 1}$$

Where

$C_1 = 2\pi c^2 h = 3.742 \times 10^8$ Wμm^4/m^2,
$C_2 = ch / k = 1.4388 \times 10^4$ μ mK,
t = Absolute temperature, k
λ = Wavelength, in μm
c = Speed of light,
h = Planck's constant, and
k = Boltzmann constant = 1.3805×10^{-23} J/K

c. Stefan-Boltzmann Law :

1. It states that the emissive power of a black body is directly proportional to the fourth power of its absolute temperature.

$$E_b = \sigma t^4$$

Where,
E_b = Emissive power of a black body, and
σ = Stefan-Boltzmann constant.
= 5.67×10^{-8} W/m^2 K^4

d. Wein's Displacement Law :

1. The Wein's law states that the product of temperature (t) and wavelength (λ_{max}) is constant *i.e.*,

$$\lambda_{max} t = \text{Constant}$$

2. According to planck's law, we know, that

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda t}\right) - 1}$$

3. $(E_\lambda)_b$ becomes maximum (if t remains constant) when

$$\frac{d\,(E_\lambda)_b}{d\lambda} = 0$$

or
$$\frac{d\,(E_\lambda)_b}{d\lambda} = \frac{d}{d\lambda}\left[\frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda t}\right)-1}\right] = 0$$

or
$$\frac{\left[\exp\left(\frac{C_2}{\lambda t}\right)-1\right](-5\,C_1\lambda^{-6}) - C_1\lambda^{-5}\left[\exp\left(\frac{C_2}{\lambda t}\right)\frac{C_2}{t}\left(\frac{-1}{\lambda^2}\right)\right]}{\left[\exp\left(\frac{C_2}{\lambda t}\right)-1\right]^2} = 0$$

or $$-5\,C_1\lambda^{-6}\exp\left(\frac{C_2}{\lambda t}\right) + 5\,C_1\lambda^{-6} + C_1C_2\lambda^{-5}\,\frac{1}{\lambda^2 t}\,\exp\left(\frac{C_2}{\lambda t}\right) = 0$$

4. Dividing both sides by $5\,C_1\lambda^{-6}$, we get

$$-\exp\left(\frac{C_2}{\lambda t}\right) + 1 + \frac{1}{5}\,C_2\,\frac{1}{\lambda t}\exp\left(\frac{C_2}{\lambda t}\right) = 0 \quad ...(4.6.5)$$

5. Solving eq. (4.6.5) by trial and error method, we get

$$\frac{C_2}{\lambda t} = \frac{C_2}{\lambda_{max}\,t} = 4.965$$

$$\lambda_{max}\,t = \frac{C_2}{4.965} = \frac{1.439\times10^4}{4.965} = 2898\ \mu\,\text{mK}$$

$$\lambda_{max}\,t = 2898\ \mu\,\text{mK}$$

Que 4.7. **Define intensity of radiation. Prove that the intensity of radiation is 1/π times of the total emissive power.**

AKTU 2015-16, Marks 7.5

Answer

A. **Intensity of Radiation :** It is defined as the rate of energy leaving a surface in a given direction per unit solid angle per unit area of emitting surface normal to the mean direction in space.

B. **Proof :**

1. Let us consider radiation from the elementary area dA_1 at the centre of a sphere. Suppose this radiation is absorbed by a second elemental area dA_2, a portion of the hemispherical surface.
2. The projected area of dA_1 on a plane perpendicular to the line joining dA_1 and $dA_2 = dA_1 \cos\theta$.
3. The solid angle subtended by $dA_2 = \dfrac{dA_2}{r^2}$

$\therefore$ The intensity of radiation, $$I = \frac{dQ_{12}}{dA_1\cos\theta \times \dfrac{dA_2}{r^2}} \quad ...(4.7.1)$$

Where, dQ_{12} is the rate of radiation heat transfer from dA_1 to dA_2.

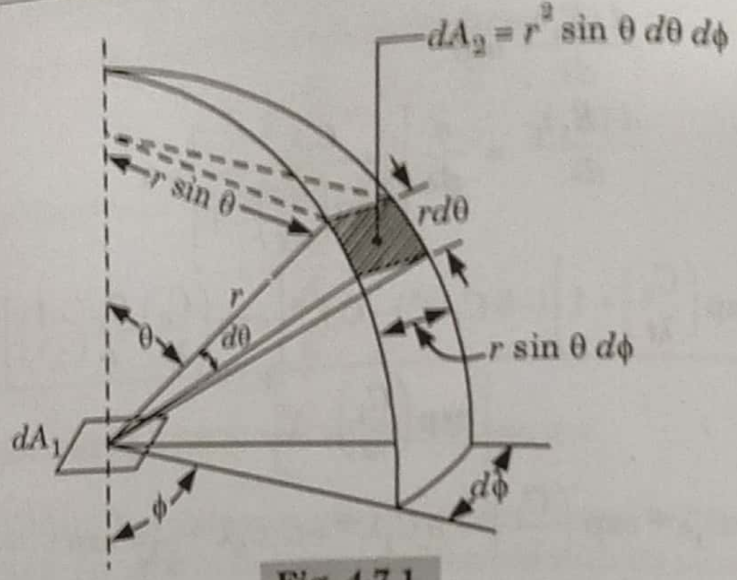


Fig. 4.7.1.

4. Since, $dA_2 = rd\theta\,(r \sin\theta\, d\phi)$

or $dA_2 = r^2 \sin\theta\, d\theta\, d\phi$ ...(4.7.2)

5. From eq. (4.7.1) and eq. (4.7.2), we get

$$dQ_{12} = I\, dA_1 \sin\theta \cos\theta\, d\theta\, d\phi$$

6. The total radiation through the hemisphere is given by

$$Q = IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{\phi=0}^{\phi=2\pi} \sin\theta \cos\theta\, d\theta\, d\phi$$

$$= 2\pi IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin\theta \cos\theta\, d\theta$$

$$= \pi IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin 2\theta\, d\theta$$

or, $Q = \pi I\, dA_1$

7. Since, $Q = E\, dA_1$

$\therefore$ $EdA_1 = \pi I\, dA_1$

or, $E = \pi I$

$$I = \frac{E}{\pi}$$

The above equation shows that intensity of radiation is $1/\pi$ times of the total emissive power.

PART-2

Radiation Exchange between Diffuse Non-Black Bodies in an Enclosure, Radiation Shield, Radiation Combined with Conduction and Convection, Absorption and Emission in Gaseous Medium, Solar Radiation, Green House Effect.

CONCEPT OUTLINE : PART-2

Shape Factor : It is defined as the fraction of radiation energy leaving one surface and falling on the other surface.

Radiation Shield : The object that reduces the radiation heat transfer by effectively increasing the surface resistances without actually removing any heat from the system is called radiation shield.

Irradiation : It is defined as the total radiation incident upon a surface per unit time per unit area.

Radiosity : It indicates the total radiation leaving a surface per unit time per unit area.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.8. **A small convex object of area A_1, temperature t_1 and emissivity ε_1 is enclosed within a large enclosure at temperature t_2 and emissivity ε_2. Derive an expression for the net heat exchange between the two objects.**

AKTU 2017-18, Marks 10

OR

What are the different cases of radiation heat exchange between non-black surfaces ? Explain any one of them.

Answer

A. Different Cases of Radiation Heat Exchange between Non-Black Bodies :

1. Infinite parallel plates.
2. Infinite long concentric cylinders.
3. Small gray bodies.
4. Small body in a large enclosure.

B. Small Body in a Large Enclosure :

1. Consider a small body placed in a large enclosure.
2. The large gray enclosure acts like a black body, absorbing practically all the radiation incident upon it and reflecting negligibly small energy back to the small gray body.
3. In this case $F_{1-2} = 1$, since all the radiations emitted by the small body would be intercepted by the outer large enclosure.
4. Thus, energy emitted by the enclosure $1 = A_1\varepsilon_1 \sigma t_1^4$

   Energy emitted by the enclosure $2 = A_2\varepsilon_2 \sigma t_2^4$

   Energy incident upon the small body $1 = F_{2-1} A_2\varepsilon_2 \sigma t_2^4$

Energy absorbed by the small body $1 = \alpha_1 F_{2-1} A_2 \varepsilon_2 \sigma t_2^4$
$$= \varepsilon_1 \varepsilon_2 A_2 F_{2-1} \sigma t_2^4$$

5. The net radiant heat exchange between the small body 1 and outer large enclosure 2,
$$Q_{12} = \varepsilon_1 A_1 \sigma t_1^4 - \varepsilon_1 \varepsilon_2 A_2 F_{2-1} \sigma t_2^4$$
6. If $t_1 = t_2$ and $Q_{12} = 0$, we have
$$A_1 = A_2 \varepsilon_2 F_{2-1}$$
and
$$Q_{12} = \varepsilon_1 A_1 \sigma (t_1^4 - t_2^4)$$
$$= f_{1-2} A_1 \sigma (t_1^4 - t_2^4)$$
Where, f_{1-2} represents the equivalent emissivity or interchange factor.

Que 4.9. What do you mean by shape factor ? Write its salient features.

AKTU 2015-16, Marks 7.5

OR

Derive reciprocity theorem and write down the salient features of shape factor.

AKTU 2016-17, Marks 10

Answer

A. **Shape Factor :** It is defined as the fraction of radiative energy that is diffused from one surface and strikes the other surface directly with no intervening reflections.

B. **Salient Features of Shape Factor :**

1. When heat transfer takes place between two bodies, the shape factor relation is given by,
$$A_1 F_{1-2} = A_2 F_{2-1}$$
This is called reciprocity theorem.
2. For flat and convex surfaces, the shape factor with respect to itself is zero because the energy leaving the surface never returns on the same surface
i.e.,
$$F_{1-1} = F_{2-2} = F_{3-3} = \ldots F_{ii} = 0.$$
3. For concave surfaces, the shape factor with respect to itself is never zero.
4. The shape factor is purely a function of geometrical parameters.

C. **Proof of Reciprocity Theorem :**

1. Considering heat exchange between elementary areas dA_1 and dA_2 of two black radiating bodies, separated by a non absorbing medium and having areas A_1 and A_2 and temperatures t_1 and t_2 respectively.
2. Let $d\omega_1$ be angle subtended at dA_1 by dA_2 and $d\omega_2$ angle subtended at dA_2 by dA_1. Then
$$d\omega_1 = \frac{dA_2 \cos \theta_2}{r^2}$$
and
$$d\omega_2 = \frac{dA_1 \cos \theta_1}{r^2}$$

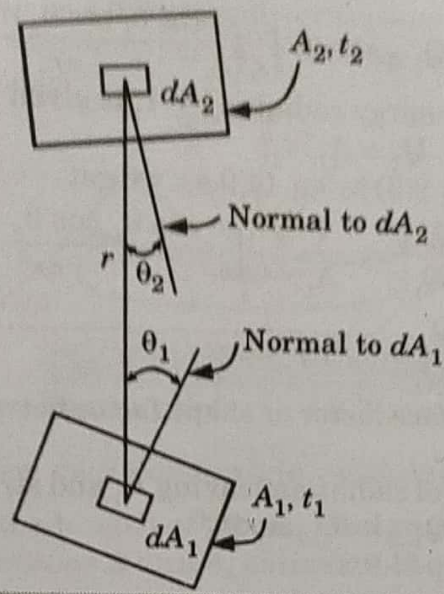


Fig. 4.9.1. Radiation heat exchange between two black surfaces.

3. The energy leaving dA_1 in the direction given by the angle per unit solid angle

$$= I_{b1}\, dA_1 \cos\theta_1$$

Where, I_b = Black body intensity, and

$dA_1 \cos\theta_1$ = Projection of dA_1 on the line between the centres.

4. The rate of radiant energy leaving dA_1 and striking on dA_2 is given by,

$$dQ_{1-2} = I_{b1}\, dA_1 \cos\theta_1\, d\omega_1 = \frac{I_{b1} \cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{r^2} \quad ...(4.9.1)$$

5. Now, the quantity of energy radiated by dA_2 and absorbed by dA_1 is given by,

$$dQ_{2-1} = \frac{I_{b2} \cos\theta_2 \cos\theta_1\, dA_2\, dA_1}{r^2} \quad ...(4.9.2)$$

6. The net rate of transfer of energy between dA_1 and dA_2 is

$$dQ_{12} = dQ_{1-2} - dQ_{2-1} = \frac{dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{r^2}(I_{b1} - I_{b2})$$

But, $\quad I_{b1} = \dfrac{E_{b1}}{\pi}$ and $I_{b2} = \dfrac{E_{b2}}{\pi}$

$$\therefore \quad dQ_{12} = \frac{dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}(E_{b1} - E_{b2})$$

or $$dQ_{12} = \frac{\sigma\, dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}(t_1^4 - t_2^4) \quad (\because E_b = \sigma t^4)$$

7. The rate of total heat transfer for the total areas A_1 and A_2 is given by

$$Q_{12} = \int dQ_{12} = \sigma(t_1^4 - t_2^4) \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1 dA_2}{\pi r^2}$$

8. Now, from eq. (4.9.1), we get

$$Q_{1-2} = I_{b1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1 dA_2}{r^2}$$

$$Q_{1-2} = \sigma t_1^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{\pi r^2} \quad ...(4.9.3)$$

9. The rate of total energy radiated by A_1 is given by

$$Q_1 = A_1 \sigma t_1^4 \quad ...(4.9.4)$$

10. On dividing eq. (4.9.3) by eq. (4.9.4), we get

$$\frac{Q_{1-2}}{Q_1} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{\pi r^2} \quad ...(4.9.5)$$

or

$$\frac{Q_{1-2}}{Q_1} = F_{1-2}$$

Where F_{1-2} is surface factor or shape factor between the two radiating surfaces.

11. Thus the amount of radiation leaving A_1 and striking A_2 is,

$$Q_{1-2} = F_{1-2} A_1 \sigma t_1^4 \quad ...(4.9.6)$$

12. Similarly, from eq. (4.9.2)

$$Q_{2-1} = \sigma t_2^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{\pi r^2} \quad ...(4.9.7)$$

13. The rate of total energy radiated by A_2 is given by

$$Q_2 = A_2 \sigma t_2^4 \quad ...(4.9.8)$$

14. On dividing eq. (4.9.7) by eq. (4.9.8), we get

$$\frac{Q_{2-1}}{Q_2} = \frac{1}{A_2}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 \, dA_2}{\pi r^2}$$

or

$$\frac{Q_{2-1}}{Q_2} = F_{2-1}$$

where F_{2-1} is the shape factor of A_2 with respect to A_1.

15. The amount of radiation leaving A_2 and arriving A_1 is,

$$Q_{2-1} = F_{2-1} A_2 \sigma t_2^4 \quad ...(4.9.9)$$

16. Thus, the net rate of heat transfer between two black surfaces A_1 and A_2 is given by

$$Q_{12} = A_1 F_{1-2} \sigma t_1^4 = A_2 F_{2-1} \sigma t_2^4$$

17. When the surface are maintained at same temperature, $t_1 = t_2$, there can be no heat exchange

$\therefore$

$$0 = (A_1 F_{1-2} - A_2 F_{2-1}) \sigma t^2 \quad [\because t_1 = t_2 = t]$$

Since, σ and t are both non zero quantities,

$\therefore$ $A_1 F_{1-2} - A_2 F_{2-1} = 0$

or

$$A_1 F_{1-2} = A_2 F_{2-1}$$

This is known as reciprocity theorem.

Que 4.10. **Explain diffuse emitter and radiation shape factor.**

AKTU 2017-18, Marks 10

Answer

A. **Diffuse Emitter :**

1. A surface whose emittance is same for all directions is called a diffuse emitter.

2. No real surface can be a diffuse emitter because electromagnetic wave theory predicts a zero emittance at $\theta = \pi/2$ for all materials.
3. Example : Black body.

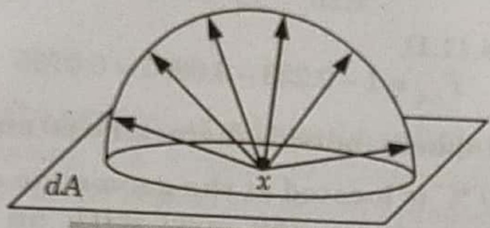


Fig. 4.10.1. Diffuse emitter.

B. Radiation Shape Factor : Refer Q. 4.9, Page 4–10B, Unit-4.

Que 4.11. **Consider two concentric cylinders having diameters 10 cm and 20 cm and a length of 20 cm. Designating the open ends of the cylinders as surfaces 3 and 4, estimate the shape factor, F_{3-4}.**

AKTU 2014-15, Marks 10

Answer

Given : $H = 20$ cm, $D_1 = 10$ cm $\Rightarrow R_1 = 5$ cm,
$D_2 = 20$ cm $\Rightarrow R_2 = 10$ cm.
To Find : Shape factor, F_{3-4}.

1. Using graph for radiation shape factor for two concentric cylinders of finite length, we get
$$F_{2-1} = 0.4126 \text{ and } F_{2-2} = 0.3286$$
2. Using the reciprocity relation, we have
$$A_1F_{1-2} = A_2F_{2-1}$$
$$F_{1-2} = (d_2/d_1)F_{2-1} = (20/10)(0.4126) = 0.8253$$
3. For surface 2, we have
$$F_{2-1} + F_{2-2} + F_{2-3} + F_{2-4} = 1.0$$
4. From symmetry $F_{2-3} = F_{2-4}$, so that
$$F_{2-3} = F_{2-4} = \left(\frac{1}{2}\right)(1 - 0.4126 - 0.3286) = 0.1294$$
5. Using reciprocity again, $A_2F_{2-3} = A_3F_{3-2}$

and
$$F_{3-2} = \frac{\pi(20)(20)}{\pi(20^2 - 10^2)/4}\, 0.1294 = 0.6901$$
6. We observe that $F_{1-1} = F_{3-3} = F_{4-4} = 0$ and for surface 3
$$F_{3-1} + F_{3-2} + F_{3-4} = 1.0 \quad \text{...(4.11.1)}$$
7. Now, for surface 1
$$F_{1-2} + F_{1-3} + F_{1-4} = 1.0$$
and from symmetry $F_{1-3} = F_{1-4}$, so that
$$F_{1-3} = \left(\frac{1}{2}\right)(1 - 0.8253) = 0.0874$$
8. Using reciprocity rule, we get

$$A_1F_{1\text{-}3} = A_3F_{3\text{-}1}$$

$$F_{3\text{-}1} = \frac{\pi(10)(20)}{\pi(20^2 - 10^2)/4}\,0.0874 = 0.233$$

9. Then from eq. (4.11.1),

$$F_{3\text{-}4} = 1 - 0.233 - 0.6901 = 0.0769$$

Que 4.12. **A small sphere (outside diameter = 60 mm) with a surface temperature of 300 °C is located at the geometric centre of a large sphere (inside diameter = 360 mm) with an inner surface temperature of 15 °C. Calculate how much of heat emitted from the large sphere inner surface is incident upon the outer surface of the small sphere, assuming that both surfaces approach black body behaviour. What is the net exchange of heat between the two spheres ?**

AKTU 2013-14, Marks 10

Answer

Given : r_1 (small sphere) = 60/2 = 30 mm = 0.03 m,
r_2 (large sphere) = 360/2 = 180 mm = 0.18 m,
t_1 = 300 °C = 573 K, t_2 = 15 °C = 288 K

To Find : i. Amount of heat emitted from the large sphere to small sphere.
ii. Net exchange of heat between two spheres.

1. Since all the radiation being emitted by the small sphere is incident upon and absorbed by the inner surface of the large sphere, therefore, configuration factor between 1 and 2 is $F_{1-2} = 1$.
2. Now,

$$A_1F_{1-2} = A_2F_{2-1} \quad \text{(Reciprocity theorem)}$$

$$4\pi r_1^2 \times 1 = 4\pi r_2^2 \times F_{2-1}$$

$$F_{2-1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{0.03}{0.18}\right)^2 = 0.0278$$

Thus 2.78 % of the heat emitted from the inner surface of the large sphere is incident upon the small sphere and absorbed by it.

3. Also from energy balance for the large sphere,

$$F_{2-1} + F_{2-2} = 1$$

$$F_{2-2} = 1 - F_{2-1} = 1 - 0.0278 = 0.9722$$

Thus, 97.22 % of emission from the large sphere is absorbed by the inner surface of the sphere itself.

4. Net interchange of heat between the two spheres is,

$$Q_{net} = F_{1-2}A_1\sigma(t_1^4 - t_2^4)$$

$$= 1 \times 4\pi \times (0.03)^2 \times 5.67 \times 10^{-8} \times [(573)^4 - (288)^4]$$

$$Q_{net} = 64.72 \text{ W}$$

Que 4.13. **What do you understand by radiation shields and derive the relation for radiation network for two parallel infinite planes separated by one shield.**

Answer

A. **Radiation Shield :** The shield that reduces the radiation heat transfer by effectively increasing the surface resistances without actually removing any heat from the system are known as radiation shield.

B. **Derivation :**

1. Let us consider two parallel plates, 1 and 2, each of area $A(A_1 = A_2 = A)$ at temperature t_1 and t_2 respectively with a radiation shield placed between them as shown in Fig. 4.13.1.
2. With no radiation shield, the net heat exchange between the parallel plates is given by,

$$(Q_{12})_{net} = \frac{A\,\sigma\,(t_1^4 - t_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \quad ...(4.13.1)$$

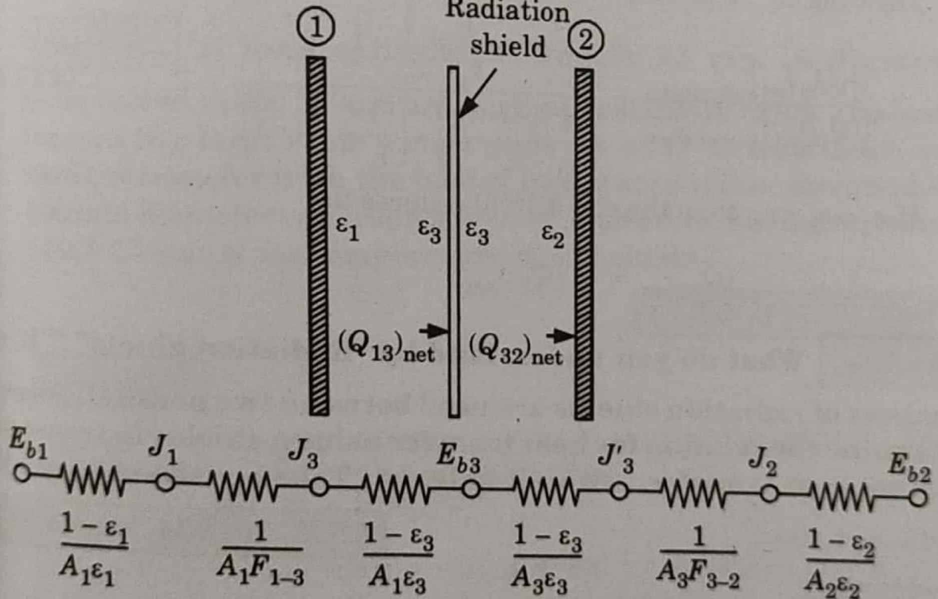


Fig. 4.13.1. Radiation network for two parallel infinite planes separated by one shield.

3. Let the emissivity of the radiation shield is ε_3.
4. Now, the heat exchange between surfaces 1, 3 and 3, 2 is,

$$(Q_{13})_{net} = \frac{A\,\sigma\,(t_1^4 - t_3^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1}$$

and

$$(Q_{32})_{net} = \frac{A\,\sigma\,(t_3^4 - t_2^4)}{\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1}$$

5. Since the radiation shield does not deliver or remove heat from the system,

$$\therefore \quad (Q_{13})_{net} = (Q_{32})_{net}$$

$$\frac{A\,\sigma\,(t_1^4 - t_3^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1} = \frac{A\,\sigma\,(t_3^4 - t_2^4)}{\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1} \quad ...(4.13.2)$$

6. On simplifying eq. (4.13.2), we get

$$t_3{}^4 = \frac{t_1^4\left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1\right) + t_2^4\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right)}{\left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right)} \quad ...(4.13.3)$$

7. On putting value of t_3 in the left side of eq. (4.13.2), we have

$$(Q_{12})_{net} = \frac{A\,\sigma\,(t_1^4 - t_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1\right)} \quad ...(4.13.4)$$

8. Dividing eq. (4.13.4) by eq. (4.13.1), we get

$$\frac{[(Q_{12})_{net}]_{\text{with shield}}}{[(Q_{12})_{net}]_{\text{without shield}}} = \frac{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1\right)} \quad ...(4.13.5)$$

9. If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$, then the eq. (4.13.5) reduces to

$$(Q_{32})_{net} = \frac{1}{2}\,(Q_{12})_{net}$$

Que 4.14. **What do you understand by "Radiation Shield" ? If n numbers of radiation shields are used between two parallel plates, determine the relation for heat transfer using n shields in terms of original heat transfer (without shields). Take emissivities of all surfaces to be ε.**

AKTU 2013-14, Marks 10

Answer

A. Radiation Shield : Refer Q. 4.13, Page 4–15B, Unit-4.

B. Derivation :

1. In the general case where there are n shields, all the surface resistances would be the same, since the emissivities are equal.
2. There will be two surface resistances for each shield and one for each heat transfer surface.
3. There will also be $(n + 1)$ 'space resistances' but the configuration factor is unity for each infinite parallel plane.

$$\text{Total resistance } (R)_{n\text{-shields}} = \left[(2n + 2)\left(\frac{1-\varepsilon}{\varepsilon}\right) + (n + 1)\right] \Big/ A$$

$$= \left[(n+1)\left(\frac{2}{\varepsilon}-1\right)\right] \Big/ A \quad ...(4.14.1)$$

4. The radiant heat transfer rate between two infinitely large parallel plates separated by n-shields is,

$$(Q)_{n\text{-shields}} = \frac{1}{(n+1)\left(\dfrac{2}{\varepsilon}-1\right)} A\sigma (t_1^4 - t_2^4) \quad ...(4.14.2)$$

5. For no shields ($n = 0$), the resistance is given by,

$$(R)_{\text{without shield}} = \left[\frac{2}{\varepsilon}-1\right] \Big/ A \quad ...(4.14.3)$$

and thus, $$(Q)_{\text{without shields}} = \frac{A\sigma(t_1^4 - t_2^4)}{(2/\varepsilon - 1)} \quad ...(4.14.4)$$

6. From eq. (4.14.1) to eq. (4.14.4), we get

$$\frac{(Q)_{n\text{-shields}}}{(Q)_{\text{without shields}}} = \frac{(R)_{\text{without shields}}}{(R)_{n\text{-shields}}} = \frac{1}{n+1}$$

Que 4.15. **A long cylindrical heater 25 mm in diameter is maintained at 660 °C and has surface resistivity of 0.8. The heater is located in a large room whose walls are at 27 °C. How much will the radiant transfer from the heater be reduced if it is surrounded by a 300 mm diameter radiation shield of aluminium having an emissivity of 0.2 ? What is the temperature of the shield ?**

AKTU 2014-15, Marks 10

Answer

Given : $r_1 = \frac{25}{2} = 12.5 \text{ mm} = 0.0125 \text{ m}$,

$r_3 = \frac{300}{2} = 150 \text{ mm} = 0.15 \text{ m}$, $t_1 = 660 + 273 = 933 \text{ K}$,

$t_2 = 27 + 273 = 300 \text{ K}$, $\varepsilon_1 = 0.8$, $\varepsilon_3 \text{ (shield)} = 0.2$.

To Find : i. Reduction in heat transfer.

ii. Temperature of shield, t_3.

1. Considering L is the length of the heater, the heat lost by the heater to the room is given by, ...(4.15.1)

$$Q = A_1 \varepsilon_1 \sigma[t_1^4 - t_2^4]$$

$$\therefore \quad Q = 2\pi r_1 L \times 0.8 \times 5.67 \times 10^{-8} \times [(933)^4 - (300)^4]$$

$$= 2\pi \times 0.0125 \times L \times 0.8 \times 5.67 \times (933^4 - 300^4) \times 10^{-8}$$

$$= 0.356 \times L \times (7577.5 - 81)$$

or, $$q = \frac{Q}{L} = 0.356 \times (7577.5 - 81) = 2668.7 \text{ W} = 2.67 \text{ kW/m}$$

2. When the cylinder is enclosed in a radiation shield then the heat flow is given by

$$Q' = \frac{A_1\sigma(t_1^4 - t_3^4)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_3} - 1\right)\frac{r_1}{r_3}} = A_3\varepsilon_3\sigma(t_3^4 - t_2^4)$$

3. As heat lost by heater to shield is further lost by shield to the room, where suffix '3' belongs to shield.

$$q' = \frac{Q'}{L} = \frac{2\pi r_1\sigma(t_1^4 - t_3^4)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_3} - 1\right)\frac{r_1}{r_3}} = \frac{2\pi r_3\sigma(t_3^4 - t_2^4)}{\frac{1}{\varepsilon_3}} \quad ...(4.15.2)$$

or $$\frac{r_1(t_1^4 - t_3^4)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_3} - 1\right)\frac{r_1}{r_3}} = \frac{r_3(t_3^4 - t_2^4)}{\frac{1}{\varepsilon_3}} \quad ...(4.15.3)$$

4. Substituting the given values in eq. (4.15.3), we get

$$\frac{0.0125\,(933^4 - t_3^4)}{\frac{1}{0.8} + \left(\frac{1}{0.2} - 1\right)\times\frac{0.0125}{0.15}} = \frac{0.15\,(t_3^4 - 300^4)}{\frac{1}{0.2}}$$

$$\frac{0.0125\,(933^4 - t_3^4)}{1.58} = 0.03\,(t_3^{\,4} - 300^4)$$

$$933^4 - t_3^{\,4} = 3.792\,(t_3^{\,4} - 300^4) = 3.792\,t_3^{\,4} - 3.792 \times 300^4$$

$$4.792\,t_3^{\,4} = 933^4 + 3.792 \times 300^4 = 7.885 \times 10^{11}$$

$$t_3^{\,4} = 1.64 \times 10^{11} \text{ or } t_3 = 636.4 \text{ K or } 363.4\ ^\circ\text{C}$$

5. Substituting the value of t_3 in eq. (4.15.2), we get

$$q' = \frac{2\pi \times 0.15 \times 5.67 \times 10^{-8} \times [(636.4)^4 - (300)^4]}{\frac{1}{0.2}}$$

$$= 1.0688 \times (1640.3 - 81) = 1666.6 \text{ W/m} = 1.67 \text{ kW/m}$$

6. ∴ Percentage reduction in heat flow

$$\Delta q = \frac{q - q'}{q} \times 100 = \frac{2.67 - 1.67}{2.67} \times 100 = 37.45\ \%$$

Que 4.16. **Consider radiative heat transfer between two large parallel planes of surface emissivity 0.8. How many thin radiation shields of emissivity 0.05 be placed between the surfaces to reduce the radiation heat transfer by factor 75 ?** **AKTU 2015-16, Marks 10**

Answer

Given : $\varepsilon_p = 0.8$, $\varepsilon = 0.05$

To Find : Number of shields.

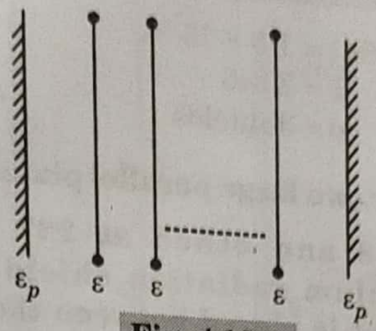


Fig. 4.16.1.

1. Let, there are n shields between two parallel planes.
2. Then, increase in surface resistance = $2n$ and increase in space resistance = $n + 1$.
3. Total resistance with n shields,

$$(R_{th})_n = \frac{1-\varepsilon_P}{\varepsilon_P} + 2n\left(\frac{1-\varepsilon}{\varepsilon}\right) + (n+1) \times \frac{1}{F} + \frac{1-\varepsilon_P}{\varepsilon_P}$$

$$(R_{th})_n = 2\left(\frac{1-\varepsilon_P}{\varepsilon_P}\right) + 2n\left(\frac{1-\varepsilon}{\varepsilon}\right) + \frac{n+1}{F}$$

4. On putting the given values,

$$(R_{th})_n = 2 \times \left(\frac{1-0.8}{0.8}\right) + 2n \times \left(\frac{1-0.05}{0.05}\right) + \frac{n+1}{1}$$

($\because$ For large parallel planes; $F = 1$)

$$= 0.5 + 38n + n + 1$$

$$(R_{th})_n = 1.5 + 39n$$

5. Heat exchange per unit area with shields between two surfaces,

$$(q)_n = \frac{\sigma(t_1^4 - t_2^4)}{(R_{th})_n}$$

$$(q)_n = \frac{\sigma(t_1^4 - t_2^4)}{1.5 + 39n} \quad ...(4.16.1)$$

6. When, there is no shield between surfaces, heat transfer per unit area,

$$q = \frac{\sigma(t_1^4 - t_2^4)}{2 \times \left(\frac{1-\varepsilon_P}{\varepsilon_P}\right) + \frac{1}{F}}$$

$$q = \frac{\sigma(t_1^4 - t_2^4)}{2 \times \left(\frac{1-0.8}{0.8}\right) + 1} = \frac{\sigma(t_1^4 - t_2^4)}{1.5}$$

7. Since, it is given that

$$(q)_n = \frac{1}{75} \times q$$

$$\frac{\sigma(t_1^4 - t_2^4)}{1.5 + 3.9n} = \frac{1}{75} \times \frac{\sigma(t_1^4 - t_2^4)}{1.5}$$

$$(1.5 + 39n) = 1.5 \times 75$$
$$n = 2.846$$
$$n \approx 3 \text{ shields}$$

Que 4.17. **Consider two large parallel plates one at $t_1 = 727$ °C with emissivity $\varepsilon_1 = 0.8$ and other at 227 °C with emissivity $\varepsilon_2 = 0.4$. An aluminium radiation shield with an emissivity, $\varepsilon_s = 0.05$ on both sides is placed between the plates. Calculate the percentage reduction in heat transfer rate between the two plates as a result of shield. Use $\sigma = 5.67 \times 10^{-8}$ W/m²K⁴.**

AKTU 2017-18, Marks 10

Answer

Given : $t_1 = 727$ °C $= 727 + 273 = 1000$ K, $\varepsilon_1 = 0.8$,
$t_2 = 227$ °C $= 227 + 273 = 500$ K, $\varepsilon_2 = 0.4$,
$\varepsilon_s = \varepsilon_3 = 0.05$, $\sigma = 5.67 \times 10^{-8}$ W/m² K⁴
To Find : Percentage reduction in heat transfer.

1. Without shield,
Q_{12} (per unit area)

$$= \frac{\sigma(t_1^4 - t_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{5.67\left[\left(\dfrac{1000}{100}\right)^4 - \left(\dfrac{500}{100}\right)^4\right]}{\dfrac{1}{0.8} + \dfrac{1}{0.4} - 1} = \frac{53156}{2.75} = 19329 \text{ W}$$

2. With shield,

$$(Q_{13})_{net} = (Q_{32})_{net}$$

$$\frac{A\sigma(t_1^4 - t_3^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1} = \frac{A\sigma(t_3^4 - t_2^4)}{\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_2} - 1}$$

$$\frac{\left(\dfrac{1000}{100}\right)^4 - \left(\dfrac{t_3}{100}\right)^4}{\dfrac{1}{0.8} + \dfrac{1}{0.05} - 1} = \frac{\left(\dfrac{t_3}{100}\right)^4 - \left(\dfrac{500}{100}\right)^4}{\dfrac{1}{0.05} + \dfrac{1}{0.4} - 1}$$

$$\frac{10000 - x^4}{1.25 + 20 - 1} = \frac{x^4 - 625}{20 + 2.5 - 1} \qquad \left(\text{Assuming, } \frac{t_3}{100} = x\right)$$

$$10000 - x^4 = \frac{20.25}{21.5}(x^4 - 625)$$
$$10000 - x^4 = 0.942\,(x^4 - 625)$$
$$= 0.942\,x^4 - 588.75$$
$$1.942\,x^4 = 10588.75 \text{ or } x = 8.59$$
$$T_3 = 100 \times 8.59 = 859 \text{ K}$$

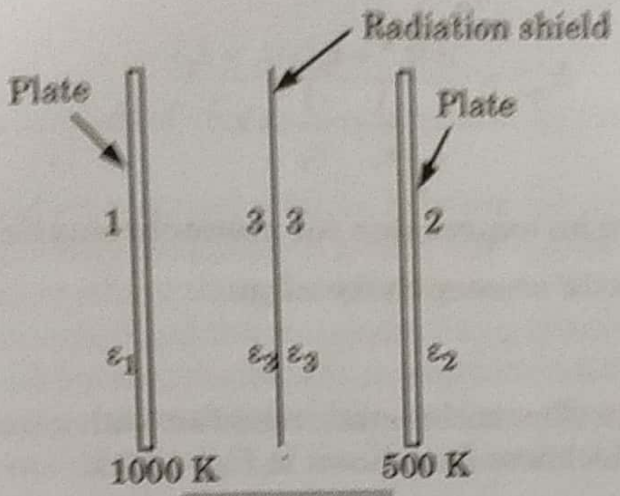


Fig. 4.17.1.

3. $$(Q_{12})_{net} \text{ (per unit area)} = \frac{5.67\left[\left(\frac{1000}{100}\right)^4 - \left(\frac{859}{100}\right)^4\right]}{\frac{1}{0.8} + \frac{1}{0.05} - 1}$$

$$= \frac{4555.3}{20.25} \times 5.67 = 1273.2 \text{ W}$$

4. Reduction in heat flow due to shield,

$$(Q_{12})_{net} - (Q_{12})_{net} = 19329 - 1273.2 = 18055.8 \text{ W}$$

5. Percentage reduction in heat transfer $= \frac{18055.8}{19329} = 93.41\ \%$

Que 4.18. **Derive the expression for radiation combined with convection.**

Answer

1. Total heat transfer by both radiation and convection is given by,

$$q = q_{conv.} + q_{rad.}$$
$$= h_{conv.}(t_g - t_w) + h_{rad.}(t_g - t_w)$$
$$= (h_{conv.} + h_{rad.})(t_g - t_w)$$

Where, t_g = Gas temperature,
t_w = Wall temperature,
$h_{rad.}$ = Radiation heat transfer coefficient, and
$h_{conv.}$ = Convection heat transfer coefficient.

2. The radiation heat transfer coefficient (h rad) is defined as,

$$(t_g - t_w)\, h_{rad.} = \varepsilon\sigma\,[t_g^4 - t_w^4]$$

or
$$h_{rad.} = \frac{\varepsilon\sigma(t_g^4 - t_w^4)}{(t_g - t_w)} = \varepsilon\sigma\,(t_g^2 + t_w^2)(t_g + t_w)$$

3. Now, the $h_{rad.}$ value for the case of two large parallel plates is given by

$$\frac{Q}{A} = \frac{\sigma(t_1^4 - t_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = h_{rad.}(t_1 - t_2)$$

$$h_{rad.} = \frac{\sigma(t_1^2 + t_2^2)(t_1 + t_2)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Que 4.19. **Derive an expression for monochromatic transmittivity and monochromatic absorptivity of gas.**

Answer

1. Consider a beam of monochromatic radiation with intensity $I_{\lambda 0}$ entering a gas layer of thickness L as shown in Fig. 4.19.1.

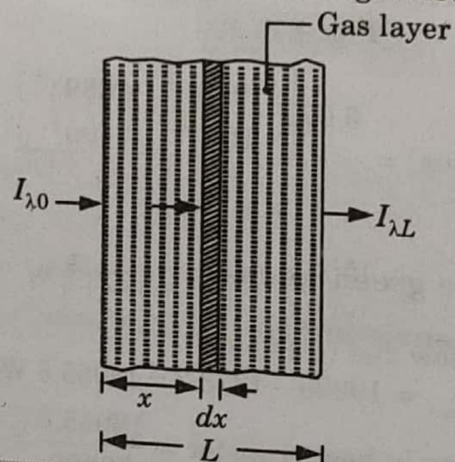


Fig. 4.19.1. Monochromatic radiation passing through an absorbing gas.

2. As the beam passes through the gas layer, its intensity gets reduced and the decrease is given by

$$dI_{\lambda x} = -k_\lambda I_{\lambda x} d_x \quad \text{...(4.19.1)}$$

Where, $I_{\lambda x}$ = Monochromatic intensity at a distance x, and
k_λ = Monochromatic absorption coefficient,

3. Integrating eq. (4.19.1) between the limits $x = 0$ and $x = L$, we get

$$\int_{x=0}^{x=L} \frac{dI_{\lambda x}}{I_{\lambda x}} = \int_{x=0}^{x=L} -k_\lambda dx$$

$$\ln \frac{I_{\lambda L}}{I_{\lambda o}} = -k_\lambda L$$

$$I_{\lambda L} = I_{\lambda o} e^{-k_\lambda L} \quad \text{...(4.19.2)}$$

Where, $I_{\lambda L}$ = Radiation intensity at $x = L$.

4. The ratio $I_{\lambda L}/I_{\lambda o}$ is the monochromatic transmittivity τ_λ of the gas.
5. In general, the gases do not reflect radiant energy *i.e.*, their reflectivity is zero, therefore,

$$\alpha_\lambda + \tau_\lambda = 1 \quad \text{or,} \quad \alpha_\lambda = 1 - \tau_\lambda = 1 - e^{-k_\lambda L}$$

The quantity $(1 - e^{-k_\lambda L})$ represents the monochromatic absorptivity of the gas.

Que 4.20. **Write a short note on solar radiation.**

Answer

1. Solar radiation consists of energy that is emitted by various layers of the sun.
2. It is a function of geometrical relation between the surface and the sun which is continuously changing on a daily and annual basis.
3. Solar radiation reaching the earth surface is partly reflected, some of the energy is absorbed and the remainder being shared thermally to the land masses and oceans, chemically in vegetation through the process of photosynthesis and mechanically in the form of wind.
4. The approximate distribution of the flow of sun's energy to the earth's surface is :

i. 9 % is scattered.
ii. 15 % is absorbed in the atmosphere and out of it 4 % reaches the earth's surface by convection.
iii. 43 % is transmitted to the earth directly and by diffuse radiation.
iv. 33 % is reflected back to space.

Que 4.21. **What is the green house effect ? Why is it a matter of great concern among environmental scientists ?**

Answer

A. **Green House Effect :**

1. The green house effect is a process by which radiative energy leaving a planetary surface is absorbed by atmospheric gases like carbon dioxide, methane, water vapours, etc., called green house gases.
2. They transfer this energy to other components of the atmosphere and it is re-radiated in all directions.
3. This transfer of energy to the surface increases the temperature of environment and leads to global warming.

B. **Reason to Consider it as a Matter of Great Concern :** It is matter of great concern among environmental scientists because of the following reasons :

1. If the emission of green house gases continues, the global temperature will increase.
2. It results in floods.
3. Global warming will lead to changes in the rainfall pattern.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What do you understand by radiation heat transfer ? Also write the surface emission properties.

Ans. Refer Q. 4.1.

Q. 2. What do you mean by black body ? Also write the properties of black body.

Ans. Refer Q. 4.2.

Q. 3. Define the properties :

i. Emissivity. **ii. Absorptivity.**

iii. Reflectivity. **iv. Transmissivity.**

Ans. Refer Q. 4.4.

Q. 4. Explain the following terms :

a. Kirchhoff's law,

b. Planck's law,

c. Stefan-Boltzmann law, and

d. Wein's displacement law.

Ans. Refer Q. 4.6.

Q. 5. Define intensity of radiation. Prove that the intensity of radiation is $1/\pi$ times of the total emissive power.

Ans. Refer Q. 4.7.

Q. 6. What are the different cases of radiation heat exchange between non-black surfaces ? Explain any one of them.

Ans. Refer Q. 4.8.

Q. 7. Derive reciprocity theorem and write down the salient features of shape factor.

Ans. Refer Q. 4.9.

Q. 8. Explain diffuse emitter and radiation shape factor.

Ans. Refer Q. 4.10.

Q. 9. What do you understand by "Radiation Shield" ? If n numbers of radiation shields are used between two parallel plates, determine the relation for heat transfer using n shields in terms of original heat transfer (without shields). Take emissivities of all surfaces to be ε.

Ans. Refer Q. 4.14.

Q. 10. Consider radiative heat transfer between two large parallel planes of surface emissivity 0.8. How many thin radiation shields of emissivity 0.05 be placed between the surfaces to reduce the radiation heat transfer by factor 75 ?

Ans. Refer Q. 4.16.

☺☺☺

5

UNIT

Heat Exchanger

PART-1

Types of Heat Exchangers, Fouling Factors, Overall Heat Transfer Coefficient, Logarithmic Mean Temperature Difference (LMTD) Method, Effectiveness-NTU Method, Compact Heat Exchangers.

CONCEPT OUTLINE : PART-1

Heat Exchanger : It is a device in which heat transfer takes place between two fluids at different temperatures *i.e.*, hot fluid and cold fluid.

Logarithmic Mean Temperature Difference (LMTD) : It is defined as that temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under various conditions of temperature difference.

Fouling or Scaling : The phenomenon of rust formation and deposition of fluid impurities is called fouling. The reciprocal of scale heat transfer coefficient, h_s is called the fouling factor.

$$R_f = \frac{1}{h_s} = \frac{1}{U_{\text{dirty}}} - \frac{1}{U_{\text{clean}}}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **What is heat exchanger ? Give the general classification of heat exchanger.**

OR

Discuss the general arrangement of parallel flow, counter flow and cross flow heat exchangers.

AKTU 2014-15, Marks 05

Answer

A. Heat Exchanger :

1. It is a device in which heat transfer takes place between two fluids at different temperature *i.e.*, hot fluid and cold fluid.
2. Example : Boilers, condensers, intercoolers, preheaters, regenerators, automobile radiators, etc.

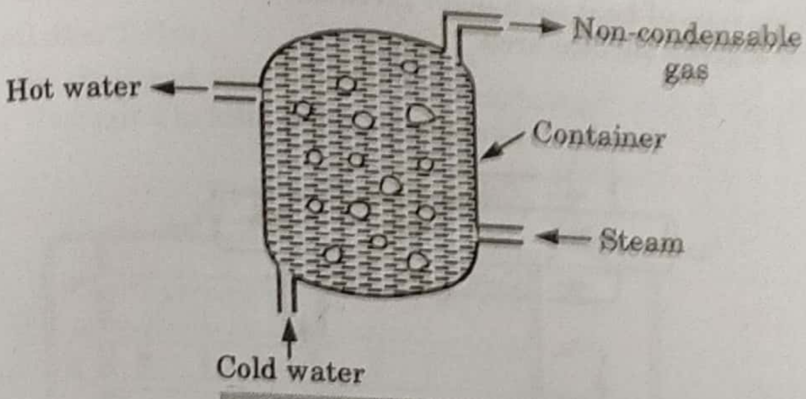


Fig. 5.1.1. Heat exchanger.

B. **Classification of Heat Exchanger :** The classification of heat exchanger is as follows :

a. **Classification according to Nature of Heat Exchange :**

i. **Direct Contact Heat Exchangers :**

1. In direct contact heat exchanger, the heat exchange takes place by direct mixing of hot and cold fluids.
2. Example : Cooling tower, jet condenser, etc.

ii. **Indirect Contact Heat Exchangers :**

1. In indirect contact heat exchanger, the heat exchange takes place by indirect contact of hot and cold fluids *i.e.*, there is a wall between two fluids.
2. Example : Gas turbine, IC engines, etc.

b. **Classification according to Relative Direction of Fluid Motion :**

i. **Parallel Flow Heat Exchangers :**

1. In a parallel flow heat exchanger, the hot and cold fluid flow in the same directions.
2. Example : Water heaters, oil coolers, etc.

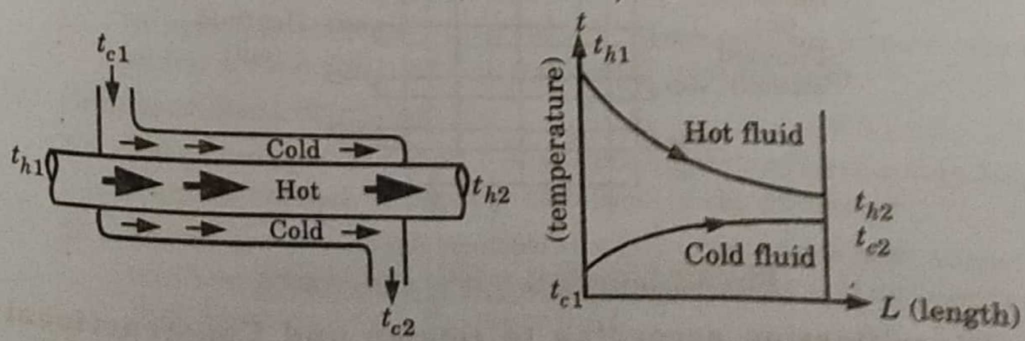


Fig. 5.1.2. Parallel flow heat exchanger.

ii. **Counter Flow Heat Exchangers :**

1. In a counter flow heat exchanger, the hot and cold fluids flow in the opposite directions.

2. This type of heat exchanger gives maximum rate of heat transfer for a given surface area.

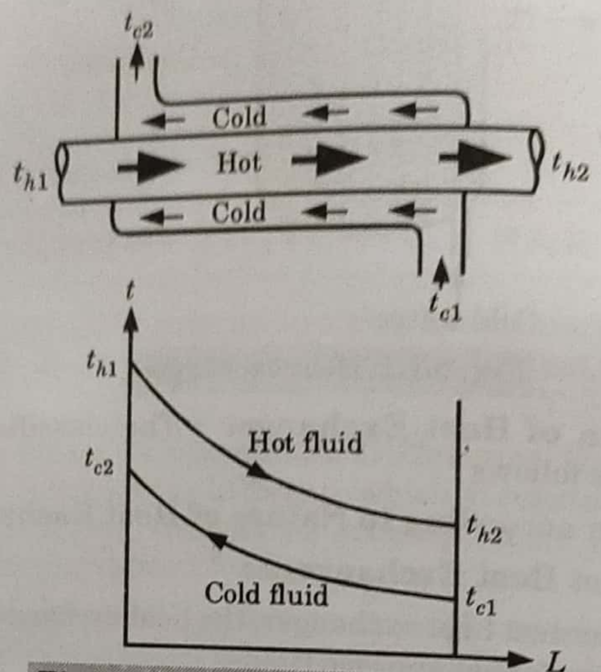


Fig. 5.1.3. Counter flow heat exchanger.

iii. Cross Flow Heat Exchangers :

1. In a cross flow heat exchanger, the two fluids cross each other at right angle.
2. Example : Automobile radiators, cooling unit of refrigeration system, etc.

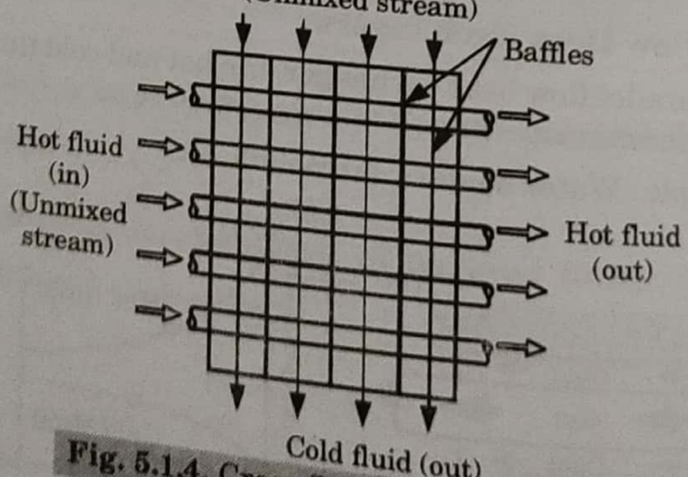


Fig. 5.1.4. Cross flow heat exchanger.

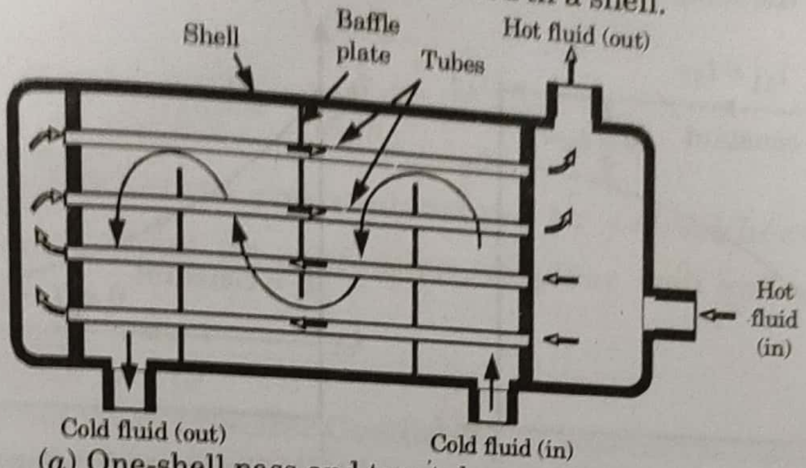
c. Classification according to Design and Constructional Features :

i. Concentric Tubes :

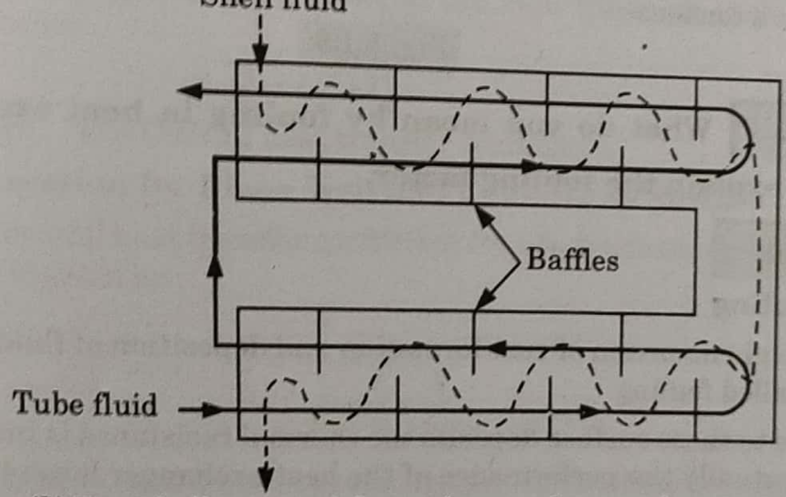
1. In concentric tubes, there are two tubes carrying hot fluid and cold fluid and the direction of flow may be parallel or counter.

ii. **Shell and Tube :**

1. In shell and tube type of heat exchanger, one of the fluids flows through a bundle of tubes enclosed in a shell.



(*a*) One-shell pass and two-tube pass heat exchanger.



(*b*) Two-shell pass and four-tube pass heat exchanger.

Fig. 5.1.5. Shell and tube heat exchangers.

iii. **Multiple Shell and Tube Passes :**

1. In multiple shell and tube passes, there are two or more shells which are used to increase the overall heat transfer.

iv. **Compact Heat Exchangers :**

1. These are special purpose heat exchangers and have a very large transfer surface area per unit volume of the exchanger.
2. These are generally employed when convective heat transfer coefficient associated with one of the fluids is much smaller than that associated with the other fluid.
3. Example : Plate fin, flattened fin tube exchangers, etc.

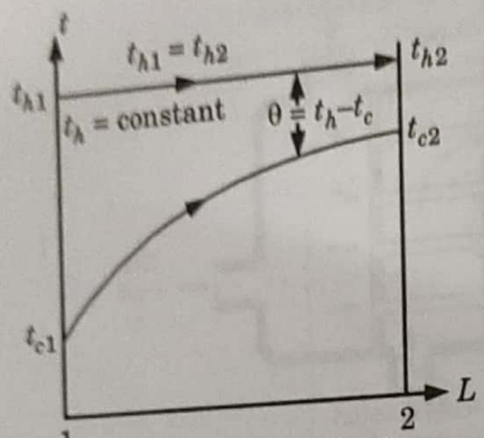
d. **Classification according to Physical State of Fluids :**

i. **Evaporator :**

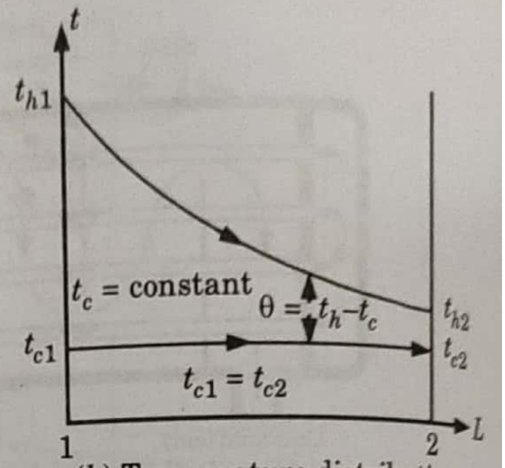
1. In evaporators, the cold fluid remains at constant temperature while the temperature of hot fluid decreases from inlet to outlet.

ii. **Condenser :**

1. In condenser, the hot fluid remains at constant temperature while the temperature of cold fluid increases from inlet to outlet.



(a) Temperature distribution in a condenser.



(b) Temperature distribution in an evaporator.

Fig. 5.1.6.

Que 5.2. **What do you mean by fouling in heat exchangers ? Briefly explain the fouling factor.**

Answer

A. **Fouling :**

1. The phenomenon of rust formation and deposition of fluid impurities is called fouling.
2. Due to these surface deposits the thermal resistance is increased and eventually the performance of the heat exchanger lowers.

B. **Fouling Factor :**

1. Since it is difficult to ascertain the thickness and thermal conductivity of the scale deposits, the effect of scale on heat flow is considered by specifying an equivalent scale heat transfer coefficient h_s.
2. The reciprocal of scale heat transfer coefficient, h_s is called the fouling factor, R_f.

Thus
$$R_f = \frac{1}{h_s}$$

3. Fouling factors are determined experimentally by testing the heat exchanger in both the clean and dirty conditions. The fouling factor, R_f is thus defined as :

$$R_f = \left(\frac{1}{h_s}\right) = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}}$$

4. If h_{si} and h_{so} be the heat transfer coefficients for the scale deposited on the inside and outside surfaces respectively, then the thermal resistance

to scale formation on the inside surface (R_{si}) and outside surface (R_{so}) are given by

$$R_{si} = \frac{1}{A_i h_{si}}$$

$$R_{so} = \frac{1}{A_0 h_{so}}$$

Que 5.3. **What do you understand by overall heat transfer coefficient ? Write its expression for plane wall and cylindrical surfaces ?**

Answer

A. **Overall Heat Transfer Coefficient :** It is defined as a quantity which gives the heat transmitted per unit area per unit time per degree temperature difference between the bulk fluids on each side of the substance.

$$Q = UA\Delta T$$

Where, U = Overall heat transfer coefficient.

B. **Expression for Plane Wall and Cylindrical Surface :**

1. The overall heat transfer coefficient for two fluids separated by a plane wall is given by :

$$U = \frac{1}{\frac{1}{h_i} + \frac{l}{k} + \frac{1}{h_0}}$$

2. The overall heat transfer coefficient for fluids which are separated by a tube wall is given by :

i. For inner surface :

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{k} \ln\left(\frac{r_0}{r_i}\right) + \left(\frac{r_i}{r_0}\right) \times \frac{1}{h_0}}$$

ii. For outer surface :

$$U_0 = \frac{1}{\left(\frac{r_0}{r_i}\right)\frac{1}{h_c} + \frac{r_0}{k} \ln\left(\frac{r_0}{r_i}\right) + \frac{1}{h_0}}$$

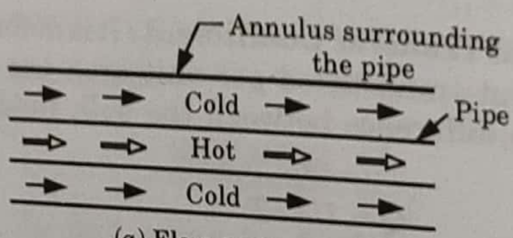
Que 5.4. **What are assumptions made to derive expression for LMTD ? Derive expression for logarithmic mean temperature difference (LMTD) for parallel flow heat exchanger.**

Answer

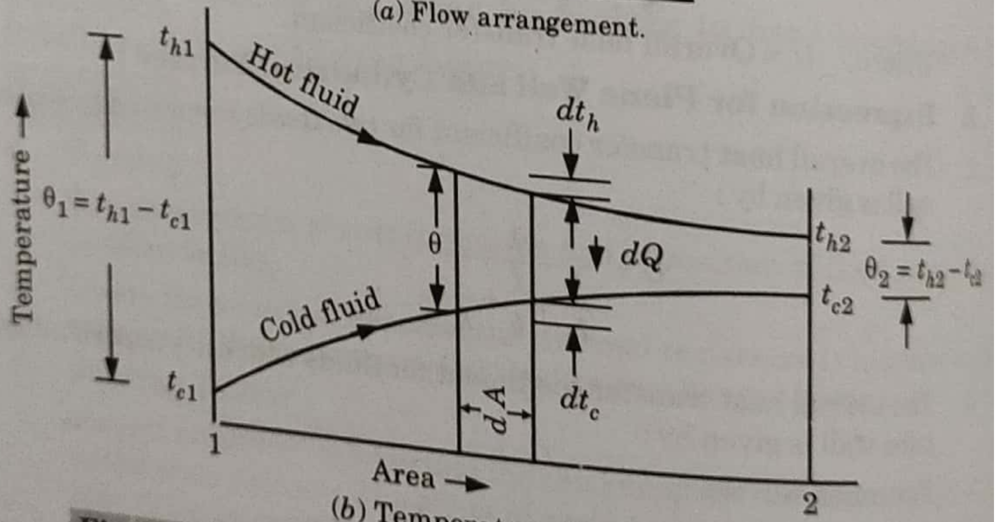
A. Assumptions for LMTD Expression :

1. The overall heat transfer coefficient (U) is constant.
2. Steady flow condition.
3. Constant specific heat and mass flow rates of both fluids.
4. Heat exchanger should be perfectly insulated.
5. Change in potential and kinetic energies are negligible.
6. Axial conduction along the tubes of the heat exchanger is negligible.

B. Expression for LMTD for Parallel Flow Heat Exchanger :



(a) Flow arrangement.



(b) Temperature distribution

Fig. 5.4.1. Calculation of LMTD for a parallel flow heat exchanger.

1. Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through the elementary area is given by,

$$dQ = UdA\,(t_h - t_c) = UdA\Delta t$$

2. The energy balance over a differential area dA is given by,

$$dQ = -\dot{m}_h C_{ph} dt_h = \dot{m}_c C_{pc} dt_c$$

$$= UdA\,(t_h - t_c) \quad \text{...(5.4.1)}$$

or

$$dt_h = \frac{-dQ}{\dot{m}_h C_{ph}} = -\frac{dQ}{C_h}$$

and,

$$dt_c = \frac{dQ}{\dot{m}_c C_{pc}} = \frac{dQ}{C_c}$$

$$\therefore \quad dt_h - dt_c = -dQ\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

$$d\theta = -dQ\left[\frac{1}{C_h}+\frac{1}{C_c}\right] \quad ...(5.4.2)$$

Where, C_h = Heat capacity of hot fluid.

C_c = Heat capacity of cold fluid.

$\dot{m}_h$ and $\dot{m}_c$ = Mass flow rates of hot and cold fluid, and

C_{ph} and C_{pc} = Specific heat of hot and cold fluid.

3. Substituting the value of dQ from eq. (5.4.1) in eq. (5.4.2), we get

$$d\theta = -UdA\,(t_h - t_c)\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

or
$$d\theta = -UdA\,\theta\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

or
$$\frac{d\theta}{\theta} = -UdA\left[\frac{1}{C_h}+\frac{1}{C_c}\right] \quad ...(5.4.3)$$

4. Integrating the eq. (5.4.3) between inlet and outlet conditions, we get

$$\int_1^2 \frac{d\theta}{\theta} = -\left[\frac{1}{C_h}+\frac{1}{C_c}\right]\int_{A=0}^{A=A} UdA$$

or
$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left[\frac{1}{C_h}+\frac{1}{C_c}\right] \quad ...(5.4.4)$$

5. Now, the total heat transfer rate between the two fluids is given by,

$$Q = C_h\,(t_{h1} - t_{h2}) = C_c\,(t_{c2} - t_{c1})$$

or
$$\frac{1}{C_h} = \frac{t_{h1} - t_{h2}}{Q} \quad ...(5.4.5)$$

$$\frac{1}{C_c} = \frac{t_{c2} - t_{c1}}{Q} \quad ...(5.4.6)$$

6. Substituting the value of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ from eq. (5.4.5) and eq. (5.4.6) in eq. (5.4.4), we get

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left[\frac{t_{h1} - t_{h2}}{Q}+\frac{t_{c2} - t_{c1}}{Q}\right]$$

$$= \frac{UA}{Q}\left[(t_{h2} - t_{c2}) - (t_{h1} - t_{c1})\right] = \frac{UA}{Q}(\theta_2 - \theta_1)$$

$$Q = \frac{UA(\theta_2 - \theta_1)}{\ln\left(\frac{\theta_2}{\theta_1}\right)}$$

or $Q = UA\,\theta_m$

Where, θ_m = Logarithmic mean temperature difference (LMTD).

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln\left(\frac{\theta_2}{\theta_1}\right)} = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)}$$

Que 5.5. Derive expression for LMTD for counter flow heat exchanger.

Answer

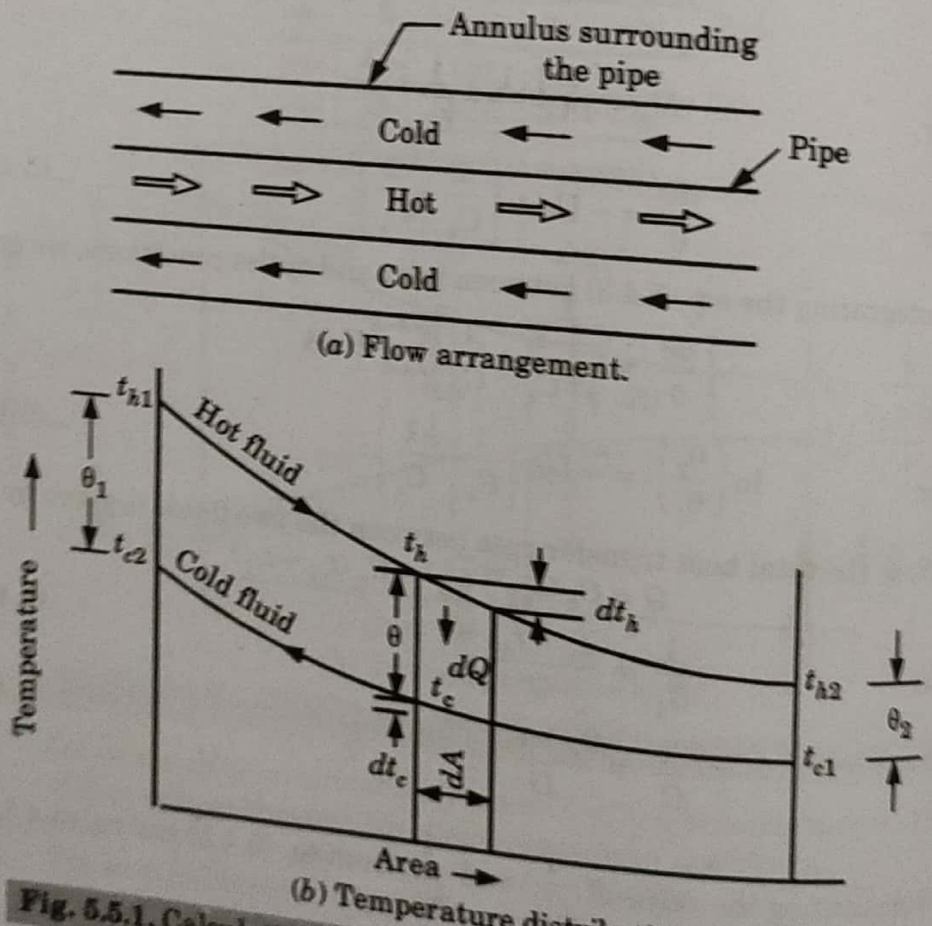


Fig. 5.5.1. Calculation of LMTD for a counter flow heat exchanger.

1. Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area dA is given by,

$$dQ = UdA\,(t_h - t_c) = UdA\Delta t \quad \text{...(5.5.1)}$$

2. The energy balance over dA is,

$$dQ = -\dot{m}_h C_{ph} dt_h = -\dot{m}_c C_{pc} dt_c \quad \text{...(5.5.2)}$$

3. Temperature of both fluids is :

$$dt_h = \frac{-dQ}{\dot{m}_h C_{ph}} = \frac{-dQ}{C_h}$$

and,

$$dt_c = \frac{-dQ}{\dot{m}_c C_{pc}} = \frac{-dQ}{C_c}$$

$$dt_h - dt_c = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

or

$$d\theta = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \quad ...(5.5.3)$$

4. Putting value of dQ from eq. (5.5.1) in eq. (5.5.3),

$$d\theta = -UdA\,(t_h - t_c)\left[\frac{1}{C_h} - \frac{1}{C_c}\right] = -UdA\theta\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

or,

$$\frac{d\theta}{\theta} = -UdA\left[\frac{1}{C_h} - \frac{1}{C_c}\right]. \quad ...(5.5.4)$$

5. On integrating eq. (5.5.4) from $A = 0$ to $A = A$, we have

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \quad ...(5.5.5)$$

6. Now, the total heat transfer rate between the two fluids is given by,

$$Q = C_h\,(t_{h1} - t_{h2}) = C_c\,(t_{c2} - t_{c1})$$

or

$$\frac{1}{C_h} = \frac{t_{h1} - t_{h2}}{Q}$$

or

$$\frac{1}{C_c} = \frac{t_{c2} - t_{c1}}{Q}$$

7. Substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ in eq. (5.5.5), we get

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left[\frac{t_{h1} - t_{h2}}{Q} - \frac{t_{c2} - t_{c1}}{Q}\right]$$

$$= \frac{-UA}{Q}\,[(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})]$$

$$= \frac{-UA}{Q}\,(\theta_1 - \theta_2) = \frac{UA}{Q}\,(\theta_2 - \theta_1)$$

or

$$Q = \frac{UA\,(\theta_2 - \theta_1)}{\ln\left(\dfrac{\theta_2}{\theta_1}\right)}$$

or

$$Q = UA\,\theta_m$$

Where, θ_m = Logarithmic mean temperature difference (LMTD).

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln\left(\frac{\theta_2}{\theta_1}\right)} = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)}$$

Que 5.6. **Under what conditions is the effectiveness – NTU method definitely preferred over the LMTD method in heat exchanger analysis ?**

AKTU 2014-15, Marks 05

Answer

1. In the thermal analysis of various types of heat exchangers LMTD method has been used.
2. This method is pretty simple and can be used in the design of heat exchangers when all the terminal temperatures are known or easily determined.
3. The difficulty arises if the temperatures of the fluids entering or leaving the heat exchanger are not known.
4. This type of situation is encountered in the selection of a heat exchanger or when the exchanger is to be run at off design conditions.
5. In such cases, it is preferable to utilise an altogether different method known as the effectiveness-NTU method.

Que 5.7. **Write down the significance of NTU method in heat exchanger. Derive an expression of effectiveness for counter flow heat exchanger by using NTU method.** **AKTU 2016-17, Marks 10**

Answer

A. Significance of NTU Method :

1. A heat exchanger can be designed by the LMTD, when inlet and outlet conditions are specified.
2. When the problem is to determine the inlet or exit temperature for a particular heat exchanger, the analysis is performed easily by using a method based on effectiveness of the heat exchanger (concept first proposed by Nusselt) and number of transfer units (NTU).

B. Effectiveness for Counter Flow Heat Exchanger :

1. The heat exchange dQ through an area dA of the heat exchanger is given by

$$dQ = UdA\,(t_h - t_c) \quad \text{...(5.7.1)}$$

$$= -\dot{m}C_{ph}dt_h = -\dot{m}C_{pc}dt_c$$

$$= -C_h\,dt_h = -C_c\,dt_c \quad \text{...(5.7.2)}$$

2. From eq. (5.7.2), we have

$$dt_h = -\frac{dQ}{C_h} \text{ and } dt_c = -\frac{dQ}{C_c}$$

$$\therefore \quad d(t_h - t_c) = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] = dQ\left[\frac{1}{C_c} - \frac{1}{C_h}\right] \quad ...(5.7.3)$$

3. Substituting the value of dQ from eq. (5.7.1) in eq. (5.7.3), we get

$$\frac{d(t_h - t_c)}{t_h - t_c} = U\,dA\left[\frac{1}{C_c} - \frac{1}{C_h}\right] \quad ...(5.7.4)$$

4. On integrating the eq. (5.7.4), we get

$$\ln\left[\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}}\right] = UA\left[\frac{1}{C_c} - \frac{1}{C_h}\right]$$

$$\ln\left[\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}}\right] = \frac{UA}{C_c}\left[1 - \frac{C_c}{C_h}\right]$$

$$\frac{t_{h2} - t_{c1}}{t_{h1} - t_{c2}} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}] \quad ...(5.7.5)$$

5. The expression for effectiveness is :

$$\varepsilon = \frac{C_h\,(t_{h1} - t_{h2})}{C_{\min}\,(t_{h1} - t_{c1})} = \frac{C_c\,(t_{c2} - t_{c1})}{C_{\min}\,(t_{h1} - t_{c1})}$$

Hence,

$$t_{h2} = t_{h1} - \frac{\varepsilon\,C_{\min}\,(t_{h1} - t_{c1})}{C_h} \quad ...(5.7.6)$$

$$t_{c2} = t_{c1} + \frac{\varepsilon\,C_{\min}\,(t_{h1} - t_{c1})}{C_c} \quad ...(5.7.7)$$

6. Substituting values from eq. (5.7.6) and eq. (5.7.7) in eq. (5.7.5) we get,

$$\frac{\left[t_{h1} - \dfrac{\varepsilon\,C_{\min}\,(t_{h1} - t_{c1})}{C_h}\right] - t_{c1}}{t_{h1} - \left[t_{c1} + \dfrac{\varepsilon\,C_{\min}\,(t_{h1} - t_{c1})}{C_c}\right]} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}]$$

$$\frac{(t_{h1} - t_{c1})\left[1 - \dfrac{\varepsilon\,C_{\min}}{C_h}\right]}{(t_{h1} - t_{c1})\left[1 - \dfrac{\varepsilon\,C_{\min}}{C_c}\right]} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}]$$

$$\frac{1 - \dfrac{\varepsilon\,C_{\min}}{C_h}}{1 - \dfrac{\varepsilon\,C_{\min}}{C_c}} = \exp\,[(UA/C_c)\,\{1 - (C_c/C_h)\}] \quad ...(5.7.8)$$

7. Assume $C_c < C_h$, $C_c = C_{min}$ and $C_h = C_{max}$. Substituting these values in eq. (5.7.8), we get,

$$\frac{1 - \frac{\varepsilon C_{min}}{C_{max}}}{1 - \frac{\varepsilon C_{min}}{C_{min}}} = \exp\left[(UA/C_{min})\{1 - (C_{min}/C_{max})\}\right]$$

$$\frac{1 - \frac{\varepsilon C_{min}}{C_{max}}}{1 - \varepsilon} = \exp\left[(UA/C_{min})\{1 - (C_{min}/C_{max})\}\right]$$

$$1 - \frac{\varepsilon C_{min}}{C_{max}} = \exp\left[(UA/C_{min})\{1 - (C_{min}/C_{max})\}\right] - \exp\left[(UA/C_{min})\{1 - (C_{min}/C_{max})\}\right]\varepsilon$$

$$1 - \exp\left[(UA/C_{min})\{1 - (C_{min}/C_{max})\}\right] = \varepsilon\left[\frac{C_{min}}{C_{max}} - \exp\{(UA/C_{min})(1 - C_{min}/C_{max})\}\right]$$

or

$$\varepsilon = \frac{1 - \exp[(UA/C_{min})\{1 - (C_{min}/C_{max})\}]}{\frac{C_{min}}{C_{max}} - \exp[(UA/C_{min})\{1 - (C_{min}/C_{max})\}]}$$

$$= \frac{\exp[(UA/C_{min})\{1 - (C_{min}/C_{max})\}] - 1}{\exp[(UA/C_{min})\{1 - (C_{min}/C_{max})\}] - \frac{C_{min}}{C_{max}}}$$

$$\varepsilon = \frac{1 - \exp[(-UA/C_{min})\{1 - (C_{min}/C_{max})\}]}{1 - \frac{C_{min}}{C_{max}}\exp[(-UA/C_{min})\{1 - (C_{min}/C_{max})\}]}$$

Since $C_{min}/C_{max} = R$ and $UA/C_{min} = \text{NTU}$ ...(5.7.9)

Hence,

$$\varepsilon = \frac{1 - \exp[-\text{NTU}(1 - R)]}{1 - R\exp[-\text{NTU}(1 - R)]}$$

Que 5.8. **Derive an expression of effectiveness for parallel flow heat exchanger by using NTU method.** **AKTU 2016-17, Marks 7.5**

OR

Derive an expression for effectiveness by NTU method for parallel flow.

Answer **AKTU 2017-18, Marks 10**

1. The heat exchange dQ through an area dA is given by,

$$dQ = UdA\,(t_h - t_c) \qquad \text{...(5.8.1)}$$

$$= -\dot{m}\,C_{ph}\,dt_h = \dot{m}\,C_{pc}\,dt_c$$

$$= -C_h\,dt_h = C_c dt_c \quad \text{...(5.8.2)}$$

2. From eq. (5.8.2), we get

$$dt_h = \frac{-dQ}{C_h} \text{ and } dt_c = \frac{dQ}{C_c} \quad \text{...(5.8.3)}$$

$$d(t_h - t_c) = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \quad \text{...(5.8.4)}$$

3. Putting the value of dQ from eq. (5.8.1) in eq. (5.8.4), we have

$$\frac{d(t_h - t_c)}{(t_h - t_c)} = -UdA\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \quad \text{...(5.8.5)}$$

4. On integrating the eq. (5.8.5), we get

$$\ln\left[\frac{(t_{h2} - t_{c2})}{(t_{h1} - t_{c1})}\right] = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln\left[\frac{(t_{h2} - t_{c2})}{(t_{h1} - t_{c1})}\right] = \frac{-UA}{C_h}\left[1 + \frac{C_h}{C_c}\right]$$

$$\left[\frac{(t_{h2} - t_{c2})}{(t_{h1} - t_{c1})}\right] = \exp\left[-\frac{UA}{C_h}\left[1 + \left(\frac{C_h}{C_c}\right)\right]\right] \quad \text{...(5.8.6)}$$

5. The expression of effectiveness is :

$$\varepsilon = \frac{C_h(t_{h1} - t_{h2})}{C_{min}(t_{h1} - t_{c1})} = \frac{C_c(t_{c2} - t_{c1})}{C_{min}(t_{h1} - t_{c1})}$$

So,

$$t_{h2} = t_{h1} - \frac{\varepsilon C_{min}(t_{h1} - t_{c1})}{C_h} \quad \text{...(5.8.7)}$$

$$t_{c2} = t_{c1} + \frac{\varepsilon C_{min}(t_{h1} - t_{c1})}{C_c} \quad \text{...(5.8.8)}$$

6. On putting value of t_{h2} and t_{c2} from eq. (5.8.7) and (5.8.8) in eq. (5.8.6), we get

$$\frac{1}{(t_{h1} - t_{c1})}\left[(t_{h1} - t_{c1}) - \varepsilon C_{min}(t_{h1} - t_{c1})\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\right] = \exp\left[-\left(\frac{UA}{C_h}\right)\left(1 + \frac{C_h}{C_c}\right)\right]$$

or

$$1 - \varepsilon\,C_{min}\left[\frac{1}{C_h} + \frac{1}{C_c}\right] = \exp\left[-\left(\frac{UA}{C_h}\right)\left(1 + \frac{C_h}{C_c}\right)\right]$$

or

$$\varepsilon = \frac{1 - \exp\left[-\left(\frac{UA}{C_h}\right)\left\{1 + \frac{C_h}{C_c}\right\}\right]}{C_{min}\left[\frac{1}{C_h} + \frac{1}{C_c}\right]} \quad \text{...(5.8.9)}$$

7. If $C_c > C_h$, then $C_{min} = C_h$ and $C_{max} = C_c$, hence eq. (5.8.9) will be,

$$\varepsilon = \frac{1 - \exp\left[-\left(\frac{UA}{C_{min}}\right)\left\{1 + \frac{C_{min}}{C_{max}}\right\}\right]}{1 + \left(\frac{C_{min}}{C_{max}}\right)} \quad ...(5.8.10)$$

8. If $C_c < C_h$, then $C_{min} = C_c$ and $C_{max} = C_h$, hence eq. (5.8.9) will be,

$$\varepsilon = \frac{1 - \exp\left[-\left(\frac{UA}{C_{max}}\right)\left\{1 + \frac{C_{max}}{C_{min}}\right\}\right]}{1 + \left(\frac{C_{min}}{C_{max}}\right)} \quad ...(5.8.11)$$

9. On rearranging eq. (5.8.10) and eq. (5.8.11), we have

$$\varepsilon = \frac{1 - \exp\left[-\left(\frac{UA}{C_{min}}\right)\left\{1 + \frac{C_{min}}{C_{max}}\right\}\right]}{1 + \left(\frac{C_{min}}{C_{max}}\right)} \quad ...(5.8.12)$$

Since, $R = \frac{C_{min}}{C_{max}}$ and $\frac{UA}{C_{min}} = \text{NTU}$

$\therefore$ $$\varepsilon = \frac{1 - \exp[-(\text{NTU})(1 + R)]}{1 + R}$$

Que 5.9. **After a long time in service, a counter flow oil cooler is checked to ascertain if its performance has deteriorated due to fouling. In the test a standard oil flowing at 2.0 kg/s is cooled from 420 K to 380 K by a water supply of 1.0 kg/s at 300 K at inlet. If the heat transfer surface is 3.33 m² and the design value of the overall heat transfer coefficient is 930 W/m²-K, how much has it been reduced by fouling ? Take C_p of oil as 2330 J/kg-K, C_p of water 4174 J/kg-K.**

AKTU 2013-14, Marks 10

Answer

Given : $\dot{m}_h = 2$ kg/s, $t_{h1} = 420$ K, $t_{h2} = 380$ K, $A = 3.33$ m², $\dot{m}_c = 1$ kg/s, $t_{c2} = 300$ K, $U = 930$ W/m²-K, $(C_p)_h = 2330$ J/kg-K, $(C_p)_c = 4174$ J/kg-K

To Find : Reduction in overall heat transfer coefficient due to fouling.

1. Heat balance over the exchanger is given as,

Heat lost by hot fluid = Heat gain by cold fluid

$$(\dot{m}\,C_p)_h\,(t_{h1} - t_{h2}) = (\dot{m}\,C_p)_c\,(t_{c1} - t_{c2})$$

$$2 \times 2330 \times (420 - 380) = 1 \times 4174 \times (t_{c1} - 300)$$

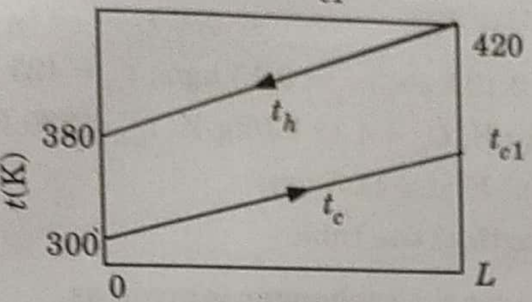


Fig. 5.9.1.

$\therefore \qquad t_{c1} = 344.7 \text{ K}$

2. We know that,

$$\theta_m = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln\left(\dfrac{t_{h1} - t_{c1}}{t_{h2} - t_{c2}}\right)}$$

$$= \frac{(420 - 344.7) - (380 - 300)}{\ln[(420 - 344.7)/(380 - 300)]} = \frac{75.3 - 80}{\ln\left(\dfrac{75.3}{80}\right)} = 77.6$$

3. Heat exchange due to fouling, $Q = U'A\,\theta_m$

$$U' = \frac{Q}{A\theta_m} = \frac{(\dot{m}C_p)_h\,(t_{h1} - t_{h2})}{A\theta_m}$$

$$= \frac{2 \times 2330 \times (420 - 380)}{3.33 \times 77.6} = \frac{186400}{3.33 \times 77.6} = 721 \text{ W/m}^2\text{-K}$$

4. Reduction in U due to fouling $= \dfrac{U - U'}{U} = \dfrac{930 - 721}{930} \times 100 = 22.5\,\%$

Que 5.10. **A counter flow heat exchanger is used to cool lubricating oil of a large industrial gas turbine engine. The oil flows through the tube at 0.19 kg/s (C_p = 2.18 kJ/kg-K) and the coolant water flows in the annulus in the opposite direction at the rate of 0.15 kg/s (C_p = 4.18 kJ/kg-K). The oil enters at 425 K and leaves at 345 K while the coolant enters at 285 K. How long must the tube be made to perform this duty if the heat transfer coefficient from oil to tube surface is 2250 W/m²-K and from tube to water is 5650 W/m²-K ? The tube has a mean diameter of 12.5 mm and its wall presents negligible resistance to heat transfer.**

AKTU 2015-16, Marks 10

Answer

Given : $m_o = 0.19$ kg/s, $m_w = 0.15$ kg/s, $t_{o1} = 425$ K, $t_{o2} = 345$ K, $C_o = 2.18$ kJ/kg-K, $C_w = 4.18$ kJ/kg-K, $t_{w1} = 285$ K, $h_o = 2250$ W/m²-K, $h_w = 5650$ W/m²-K, $d = 12.5$ mm

To Find : Length of the tube.

1. Heat balance over the exchanger is given as,

Heat lost by oil = Heat gained by water

$$m_o C_o (t_{o1} - t_{o2}) = m_w C_w (t_{w2} - t_{w1})$$

$$0.19 \times 2.18 \times (425 - 345) = 0.15 \times 4.18 \times (t_{w2} - 285)$$

∴ Outlet temperature of water,

$$t_{w2} = 285 + \frac{0.19 \times 2.18 \times 80}{0.15 \times 4.18} = 337.85 \text{ K}$$

2. By applying energy balance on the oil, the heat transfer is given by

$$Q = m_o C_o (t_{o1} - t_{o2}) = 0.19 \times 2.18 \times (425 - 345)$$
$$= 33.136 \text{ kJ/s} = 33.136 \times 10^3 \text{ J/s}$$

3. For counter flow heat exchanger,

$$\theta_1 = t_{o1} - t_{w2} = 425 - 337.85 = 87.15 \text{ K}$$
$$\theta_2 = t_{o2} - t_{w1} = 345 - 285 = 60 \text{ K}$$

∴ Log-mean temperature difference is,

$$\theta_m = \frac{\theta_1 - \theta_2}{\log_e (\theta_1 / \theta_2)} = \frac{87.15 - 60}{\log_e (87.15 / 60)} = 72.73 \text{ K}$$

4. Let, U = Overall heat transfer coefficient between oil and water.

We know that, $$\frac{1}{U} = \frac{1}{h_0} + \frac{1}{h_w}$$

$$U = \frac{h_0 h_w}{h_0 + h_w} = \frac{2250 \times 5650}{2250 + 5650} = 1609.18 \text{ W/m}^2\text{-K}$$

5. Also we know that, $Q = U A \theta_m$

∴ $$A = \frac{Q}{U\theta_m} = \frac{33.136 \times 10^3}{1609.18 \times 72.79} = 0.2829 \text{ m}^2$$

6. Required length of the tube is given by,

$$l = \frac{A}{\pi d} = \frac{0.2829}{\pi \times 12.5 \times 10^{-3}} = 7.21 \text{ m}$$

Que 5.11. Engine oil (C_p = 2100 J/kg-°C) is to be heated from 20 °C to 60 °C at a rate of 0.3 kg/s in a 2 cm diameter thin walled copper tube by condensing steam outside at a temperature of 130 °C (h_{fg} = 2174 kJ/kg) for an overall heat transfer coefficient of

650 W/m²-°C. Determine the rate of heat transfer and the length of the tube required to achieve it. **AKTU 2015-16, Marks 7.5**

Answer

Given : $C_{Po} = 2100$ J/kg-°C, $t_{oi} = 20$ °C, $t_{oe} = 60$ °C, $\dot{m}_o = 0.3$ kg/s
$d = 2$ cm $= 0.02$ m, $t_{wi} = 130$ °C, $U = 650$ W/m²-°C, $h_{fg} = 2174$ kJ/kg
To Find : i. Rate of heat transfer.
ii. Length of the tube.

1. We know that, heat transfer rate

$$Q = \dot{m}_o C_{Po} (t_{oe} - t_{oi})$$
$$= 0.3 \times 2100 \times (60 - 20) = 25200 \text{ W}$$
$$Q = 25.2 \text{ kW}$$

2. LMTD is given as, $\theta_m = \dfrac{\theta_1 - \theta_2}{\ln\left(\dfrac{\theta_1}{\theta_2}\right)}$

Where, $\theta_1 = t_{wi} - t_{oi} = 130 - 20 = 110$ °C
and $\theta_2 = t_{wi} - t_{oe} = 130 - 60 = 70$ °C

$$\therefore \quad \theta_m = \frac{110 - 70}{\ln\left(\dfrac{110}{70}\right)} = 88.5 \text{ °C}$$

3. We know that, $Q = UA\theta_m$

$$A = \frac{Q}{U\theta_m} = \frac{25200}{650 \times 88.5}$$
$$A = 0.438 \text{ m}^2$$

4. Also, $A = \pi dl$

$$l = \frac{A}{\pi d} = \frac{0.438}{\pi \times 2.0 \times 10^{-2}}$$
$$l = 6.97 \text{ m}$$
$$l \approx 7.0 \text{ m}$$

Que 5.12. **A steam condenser is transferring 250 kW of thermal energy at a condensing temperature of 65 °C. The cooling water enters the condenser at 20 °C with a flow rate of 7500 kg/hr. Calculate the LMTD. If overall heat transfer coefficient for the condenser surface is 1250 W/m² -°C. What surface area is required to handle this load ? What error would be introduced if the arithmetic mean temperature difference is used rather than the log mean temperature difference ?** **AKTU 2016-17, Marks 10**

Answer

Given : $Q = 250$ kW, $t_{h1} = t_{h2} = 65$ °C, $t_{c1} = 20$ °C, $\dot{m}_c = 7500$ kg/hr, $U = 1250$ W/m²-°C.

To Find : i. LMTD.
ii. Surface area.
iii. Error.

1. For the condensing steam, the temperature remains constant throughout the flow passage, *i.e.*,

$$t_{h1} = t_{h2} = 65 \text{ °C}$$

2. We know that energy balance on the cooling water gives,

$$Q = \dot{m}_c C_c (t_{c2} - t_{c1})$$

$$250 = \frac{7500}{3600} \times 4.186 \times (t_{c2} - 20)$$

Exit temperature of cooling water,

$$t_{c2} = 20 + \frac{250 \times 3600}{7500 \times 4.186} = 48.67 \text{ °C}$$

3. Log mean temperature difference,

$$\theta_m = \frac{\theta_1 - \theta_2}{\log_e(\theta_1 / \theta_2)}$$

Where,

$$\theta_1 = t_{h1} - t_{c1} = 65 - 20 = 45 \text{ °C}$$
$$\theta_2 = t_{h2} - t_{c2} = 65 - 48.67 = 16.33 \text{ °C}$$

$\therefore$

$$\theta_m = \frac{45 - 16.33}{\log_e(45/16.33)} = 28.28 \text{ °C}$$

4. Now, we know that $Q = UA\theta_m$

$\therefore$ Area required to handle the load is,

$$A = \frac{Q}{U\theta_m} = \frac{250 \times 10^3}{1250 \times 28.28} = 7.07 \text{ m}^2$$

5. The arithmetic mean temperature difference is

$$\bar{\theta} = \frac{\theta_1 + \theta_2}{2} = \frac{45 + 16.33}{2} = 30.66 \text{ °C}$$

$\therefore$

$$\bar{A} = \frac{Q}{U\bar{\theta}} = \frac{250 \times 10^3}{1250 \times 30.66} = 6.52 \text{ m}^2$$

6.

$$\text{Error} = \frac{A - \bar{A}}{A} \times 100 = \frac{7.07 - 6.52}{7.07} \times 100 = 7.77\,\%$$

Que 5.13. **A chemical having specific heat of 3.3 kJ/kg-K flowing at the rate of 20000 kg/hr enters a parallel flow heat exchanger at 120 °C. The flow rate of cooling water is 50000 kg/hr with an inlet temperature of 20 °C. The heat transfer area is 10 m² and the overall heat transfer coefficient is 1050 W/m²-°C. Determine :**

a. **The effectiveness of the heat exchanger.**
b. **The output temperature of water and chemical.**
Specific heat of the water is 4.186 kJ/kg.

AKTU 2016-17, Marks 10

Answer

Given : $C_{ph} = 3.3$ kJ/kg-K, $\dot{m}_h = \dfrac{20000}{3600} = 5.56$ kg/s, $t_{h1} = 120$ °C,

$\dot{m}_c = \dfrac{50000}{3600} = 13.89$ kg/s, $C_{pc} = 4.186$ kJ/kg-K, $t_{c1} = 20$ °C, $A = 10$ m²,

$U = 1050$ W/m²-C°

To Find : a. The effectiveness of the heat exchanger.
b. The output temperature of water and chemical.

1. Hot fluid capacity rate, $C_h = \dot{m}_h C_{ph} = 5.56 \times 3.3 = 18.35$

   Cold fluid capacity rate, $C_c = \dot{m}_c C_{pc} = 13.89 \times 4.186 = 58.14$

   We find that, $C_h < C_c$

2. Heat lost by hot fluid = Heat gained by cold fluid

$$5.56 \times 3.3 \times (120 - t_{h2}) = 13.89 \times 4.186 \times (t_{c2} - 20)$$

or,
$$(120 - t_{h2}) = 3.17\,(t_{c2} - 20) \qquad \text{...(5.13.1)}$$

3. Now,
$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{1050 \times 10}{18.35 \times 1000} = 0.572$$

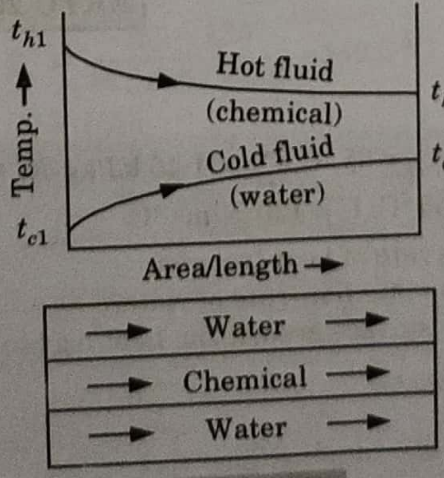


Fig. 5.13.1.

4. Effectiveness,
$$\varepsilon = \frac{1-\exp[-NTU(1+R)]}{1+R}$$

$\therefore$
$$\varepsilon = \frac{1-\exp[-0.572(1+0.316)]}{(1+0.316)}$$
$$\left(\because R = \frac{C_{min}}{C_{max}} = \frac{18.35}{58.14} = 0.316\right)$$
$$= \frac{1-0.471}{1.316} = 0.402$$

5. Also,
$$\varepsilon = \frac{C_h(t_{h1}-t_{h2})}{C_{min}(t_{h1}-t_{c1})}$$

or,
$$0.402 = \frac{(120-t_{h2})}{(120-20)} \quad (\because C_h = C_{min})$$

or,
$$t_{h2} = 120 - 0.402\,(120-20) = 79.8\ °C$$

6. Substituting the value of t_{h2} in eq. (5.13.1), we get
$$(120-79.8) = 3.17\,(t_{c2}-20)$$

$\therefore$
$$t_{c2} = \frac{(120-79.8)}{3.17} + 20 = 32.7\ °C$$

Que 5.14. **In a counter flow double pipe heat exchanger, water is heated from 25 °C to 65 °C by an oil with a specific heat of 1.45 kJ/kg-K and mass flow rate of 0.9 kg/s. The oil is cooled from 230 °C to 160 °C. If the overall heat transfer coefficient is 420 W/m²-°C. Calculate the following :**

i. the rate of heat transfer,

ii. the mass flow rate of water, and

iii. the surface area of the heat exchanger.

AKTU 2017-18, Marks 10

Answer

Given : $t_{c1} = 25$ °C, $t_{c2} = 65$ °C, $C_{ph} = 1.45$ kJ/kg-K, $\dot{m}_h = 0.9$ kg/s, $t_{h1} = 230$ °C, $t_{h2} = 160$ °C, $U = 420$ W/m²-°C

To Find : i. The rate of heat transfer.
ii. The mass flow rate of water.
iii. The surface area of the heat exchanger.

1. The rate of heat transfer,
$$Q = \dot{m}_h C_{ph}\,(t_{h1} - t_{h2})$$
$$Q = 0.9 \times 1.45 \times (230 - 160) = 91.35 \text{ kJ/s}$$

2. Heat lost by oil (hot fluid) = Heat gained by water (cold fluid)

$$\dot{m}_h C_{ph} (t_{h1} - t_{h2}) = \dot{m}_h C_{pc} (t_{c2} - t_{c1})$$

$$91.35 = \dot{m}_c \times 4.187 \times (65 - 25)$$

Mass flow rate of water, $\dot{m}_c = \dfrac{91.35}{4.187 \times (65 - 25)} = 0.545$ kg/s

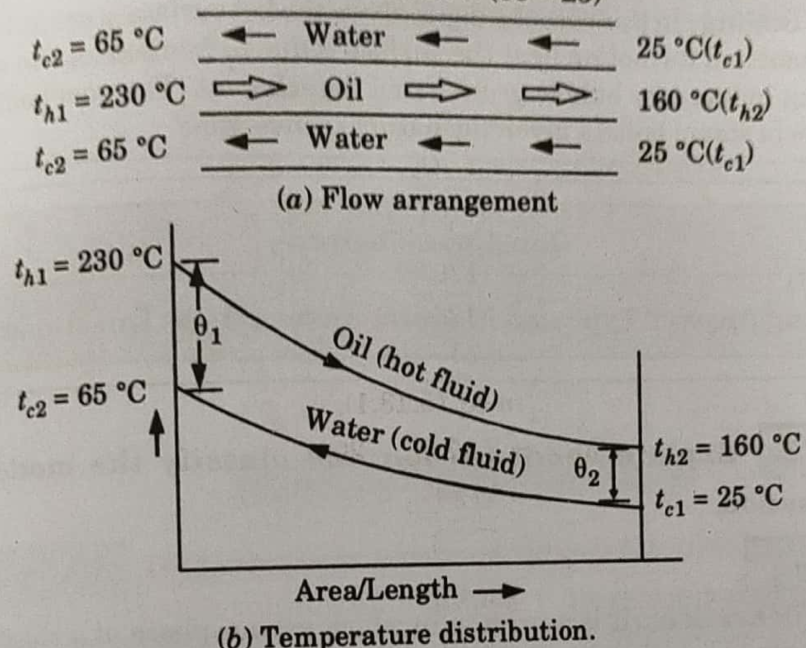


Fig. 5.14.1. Counter-flow heat exchanger.

3. Logarithmic mean temperature difference (LMTD) is given by,

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(t_{h1} - t_{c2}) - (t_{h2} - t_{c1})}{\ln[(t_{h1} - t_{c2})/(t_{h2} - t_{c1})]}$$

$$= \frac{(230 - 65) - (160 - 25)}{\ln[(230 - 65)/(160 - 25)]}$$

$$\theta_m = \frac{165 - 135}{\ln[(165/135)]} = 149.5\ °\mathrm{C}$$

4. We know that, $Q = UA\theta_m$

$$\therefore \quad A = \frac{Q}{U\theta_m} = \frac{91.35 \times 10^3}{420 \times 149.5} = 1.45\ \mathrm{m}^2$$

PART-2

Introduction to Condensation Phenomena, Heat Transfer Relations for Laminar Film Condensation on Vertical Surface and on Outside & Inside of a Horizontal Tube, Effect of Non-Condensable Gases, Dropwise Condensation, Boiling Modes, Pool Boiling, Hysteresis in Boiling Curve, Forced Convection Boiling, Heat Pipes.

CONCEPT OUTLINE : PART-2

Condensation : It is a in which vapour phase of a substance changes to liquid phase by releasing its latent heat of vapourisation.
Boiling : It is defined as the convective heat transfer process that involves a phase change from liquid to vapour state.
Pool Boiling: In this case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment. The pool boiling occurs in steam boilers involving natural convection.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.15. **Explain condensation and classify the modes of condensation.**

Answer

A. **Condensation :** It is a process in which vapour phase of a substance changes to liquid phase by releasing its latent heat of vaporisation.

B. **Modes of Condensation :** Following are the modes of condensation :

a. **Filmwise Condensation :**
   1. In film condensation, the condensation liquid wet the surface and forms a liquid film.

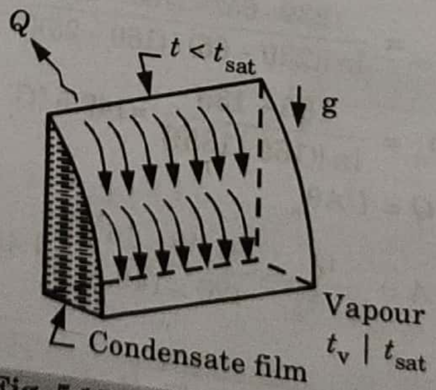


Fig. 5.15.1. Filmwise condensation.

   2. This liquid film gradually deposits on the surface which acts as a resistance to heat flow from the surface and hence it prevents the heat transfer between the condensing vapour and surface.

b. **Dropwise Condensation :**

1. In dropwise condensation, the vapour condenses into small liquid droplets of various sizes which fall down the surface randomly.
2. In this condensation process, a large portion of the area of solid surface is directly exposed to vapour without an insulating film of condensate liquid, consequently higher heat transfer rate are achieved.

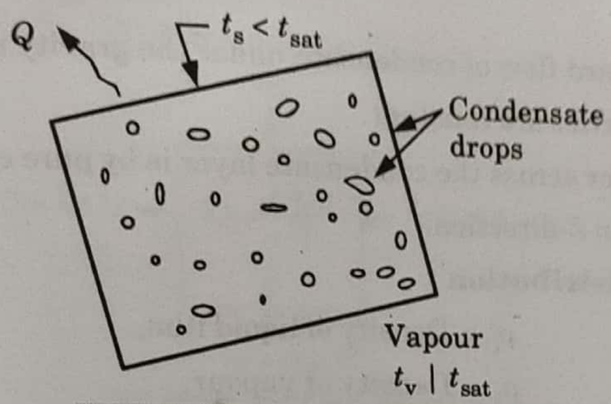


Fig. 5.15.2. Dropwise condensation.

Que 5.16. **Differentiate between dropwise condensation and filmwise condensation.**

AKTU 2015-16, Marks 7.5

Answer

S. No.	Filmwise Condensation	Dropwise Condensation
1.	Liquid condensate wets the surface.	It does not wet the surface.
2.	Liquid condensate spreads out and does not collect in droplets.	The liquid condensate collects in droplets.
3.	It forms continuous films over the entire surface.	It does not form any film over the surface.
4.	Continuous film offers resistance and restricts further transfer of heat between vapour and surface.	There is no film barrier to heat flow, therefore high heat transfer rate are experienced between vapour and surface.

Que 5.17. **Derive the expression of velocity distribution, mass flow rate, heat flux and film heat transfer coefficient for laminar film condensation on a vertical plate.**

Answer

A. Assumptions of Nusselt's Analysis :

1. Plate is maintained at constant temperature t_s, that is less than saturation temperature of vapour t_{sat}.
2. Vapour has low velocity and it does not exert viscous shear force at liquid.
3. The downward flow of condensate under the gravity is laminar.
4. Fluid properties are constant.
5. Heat transfer across the condensate layer is by pure conduction.
6. Unit depth in Z-direction.

B. Velocity Distribution :

1. Let,

ρ_l = Density of liquid film,

ρ_v = Density of vapour,

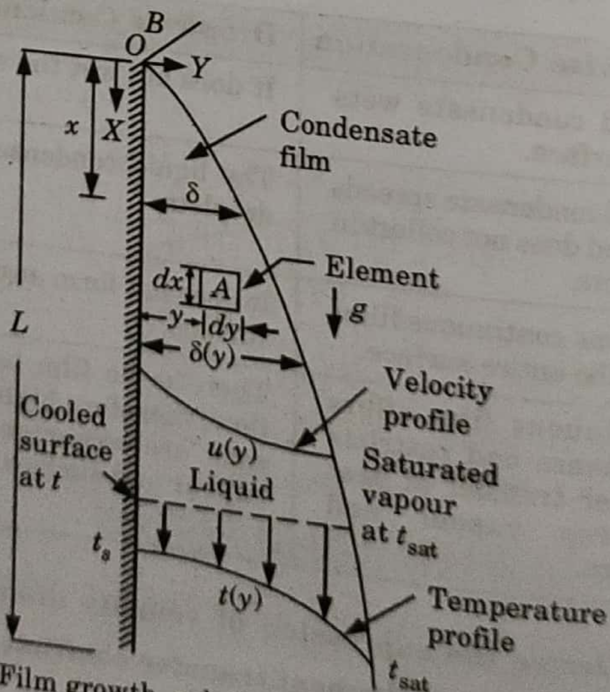
h_{fg} = Latent heat of condensation,

k = Conductivity of liquid film,

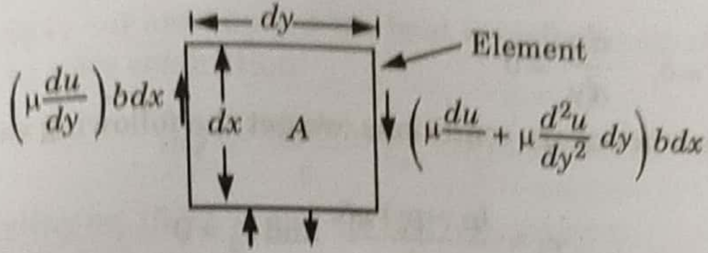
μ = Absolute viscosity of liquid film,

t_s = Surface temperature, and

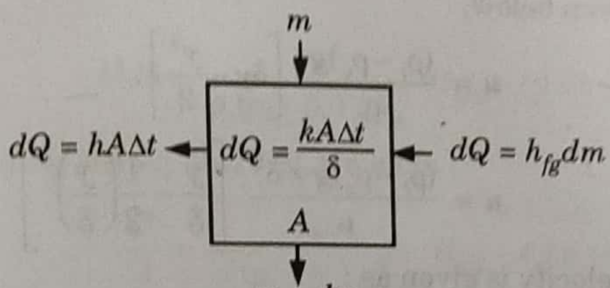
t_{sat} = Saturation temperature of vapour at prevailing pressure.



(a) Film growth, velocity and temperature profiles



(b) Force balance



(c) Mass and heat balance

Fig. 5.17.1.

2. To find an expression for velocity distribution u as a function of y from the wall surface. Let us consider the equilibrium between gravity and viscous forces on an elementary volume of the liquid film.
3. Gravitational force on the element

$$= \rho_l g(bdxdy) - \rho_v g(bdxdy) \quad ...(5.17.1)$$

4. Viscous shear force on the element

$$= \mu\frac{du}{dy}(bdx) - \left[\mu\frac{du}{dy} + \mu\frac{d^2u}{dy^2}dy\right](bdx) \quad ...(5.17.2)$$

5. On equating eq. (5.17.1) and eq. (5.17.2), we get

$$\rho_l g(bdxdy) - \rho_v g(bdxdy) = \mu\frac{du}{dy}(bdx) - \left[\mu\frac{du}{dy} + \mu\frac{d^2u}{dy^2}dy\right]bdx$$

$$\frac{d^2u}{dy^2} = \frac{-(\rho_l - \rho_v)g}{\mu} \quad ...(5.17.3)$$

6. On integrating the eq. (5.17.3), we have

$$\frac{du}{dy} = \frac{-(\rho_l - \rho_v)g}{\mu}y + c_1 \quad ...(5.17.4)$$

7. On integrating the eq. (5.17.4), we get

$$u = \frac{-(\rho_l - \rho_v)(y^2/2)g}{\mu} + c_1y + c_2 \quad ...(5.17.5)$$

8. The relevant boundary conditions are :

At $y = 0$, $u = 0$

At $y = \delta$, $\dfrac{du}{dy} = 0$

Using these boundary conditions, we get the following value of c_1 and c_2 :

$$c_1 = \frac{(\rho_l - \rho_v)\, g\delta}{\mu} \text{ and } c_2 = 0$$

9. Substituting the values of c_1 and c_2 in eq. (5.17.5), we get the velocity profile as given below,

$$u = \frac{(\rho_l - \rho_v)g}{\mu}\left[\delta y - \frac{y^2}{2}\right]$$

$$u = \frac{(\rho_l - \rho_v)g \times \delta^2}{\mu}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right]$$

10. Mean flow velocity is given as :

$$u_m = \frac{1}{\delta}\int_0^\delta u\,dy$$

$$= \frac{1}{\delta}\int_0^\delta \frac{(\rho_l - \rho_v)\, g \times \delta^2}{\mu}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] dy$$

$$u_m = \frac{(\rho_l - \rho_v)\, g \times \delta^2}{3\mu}$$

C. Mass Flow Rate :

1. Mass flow rate (m) = Mean flow velocity (u_m) × Flow area × Density

$$m = \frac{(\rho_l - \rho_v)g \times \delta^2}{3\mu} \times b\delta \times \rho_l$$

$$= \frac{\rho_l(\rho_l - \rho_v)\, gb\delta^3}{3\mu} \qquad ...(5.17.6)$$

2. On differentiating eq. (5.17.6) w.r.t. x (or δ), we have

$$dm = \frac{d}{dx}\left[\frac{\rho_l(\rho_l - \rho_v)\, gb\delta^3}{3\mu}\right] dx$$

$$= \frac{d}{d\delta}\left[\frac{\rho_l(\rho_l - \rho_v)\, gb\delta^3}{3\mu}\right]\frac{d\delta}{dx}\, dx$$

$$dm = \left[\frac{\rho_l(\rho_l - \rho_v)\, gb\delta^2}{\mu}\right] d\delta$$

D. Heat Flux :

1. Heat flow rate = Rate of energy release due to condensation at the surface

$$dQ = h_{fg} \times dm$$

$$dQ = h_{fg}\left[\frac{\rho_l(\rho_l-\rho_v)\,gb\delta^2}{\mu}\right]d\delta \quad ...(5.17.7)$$

2. According to our assumption the heat transfer across the condensate layer is by pure conduction.

Hence, $$dQ = \frac{k(bdx)}{\delta}\,(t_{sat}-t_s) \quad ...(5.17.8)$$

3. On equating eq. (5.17.7) and eq. (5.17.8), we have

$$\frac{h_{fg}\rho_l(\rho_l-\rho_v)\,gb\delta^2}{\mu}\,d\delta = \frac{k(bdx)}{\delta}\,(t_{sat}-t_s)$$

$$\delta^3 d\delta = \frac{k\mu}{\rho_l(\rho_l-\rho_v)\,gh_{fg}}\,(t_{sat}-t_s)\,dx \quad ...(5.17.9)$$

4. On integrating the eq. (5.17.9), we have

$$\frac{\delta^4}{4} = \frac{k\mu}{\rho_l(\rho_l-\rho_v)\,gh_{fg}}\,(t_{sat}-t_s)\,x + c_1$$

5. Now, putting the boundary condition :

$$\delta = 0 \text{ at } x = 0 \text{ give } c_1 = 0.$$

Hence, $$\delta = \left[\frac{4k\mu(t_{sat}-t_s)x}{\rho_l(\rho_l-\rho_v)g\,h_{fg}}\right]^{\frac{1}{4}} \quad ...(5.17.10)$$

E. Film Heat Transfer Coefficient :

1. According to the assumption,

$$dQ = \frac{k(bdx)}{\delta}\,(t_{sat}-t_s) \quad ...(5.17.11)$$

2. The heat flow rate can also be expressed as,

$$dQ = h_x\,(bdx)\,(t_{sat}-t_s) \quad ...(5.17.12)$$

Where, h_x is the local heat transfer coefficient.

3. On equating eq. (5.17.11) and eq. (5.17.12), we get

$$\frac{k(bdx)}{\delta}\,(t_{sat}-t_s) = h_x\,(bdx)\,(t_{sat}-t_s)$$

$$h_x = \frac{k}{\delta} \quad ...(5.17.13)$$

4. On substituting value of δ from eq. (5.17.10) in eq. (5.17.13), we get

$$h_x = \left[\frac{\rho_l(\rho_l-\rho_v)k^3 g\,h_{fg}}{4\,\mu x(t_{sat}-t_s)}\right]^{\frac{1}{4}}$$

Que 5.18. **Explain boiling and discuss the various types of boiling.**

Answer

A. **Boiling :**

1. It is a convective heat transfer process that involves a phase change from liquid to vapour state.
2. It is also defined as evaporation at a solid-liquid surface. This is possible only when the temperature of the surface (t_s) exceeds the saturation temperature corresponding to the liquid pressure (t_{sat}).
3. Heat is transferred from the solid surface to the liquid according to the law

$$Q = hA_s(t_s - t_{sat}) = hA_s\Delta t_e$$

Where, Δt_e = Excess temperature $= (t_s - t_{sat})$

B. **Types of Boiling :** There are mainly four types of boiling :

a. **Saturated Boiling :** In saturated boiling, the temperature of liquid exceeds the saturation temperature and bubbles are begin to form at the heating surface and goes to the upper surface of the liquid and finally leaves the surface.

b. **Pool Boiling :** This is the case when the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment. It occurs in steam boilers involving natural convection.

c. **Forced Convection Boiling :** In forced convection boiling, the motion of fluid through the heated surface is induced by external means *i.e.*, fans, pump, etc.

d. **Sub-cooled Boiling :** In sub-cooled boiling, the temperature of the liquid is below the saturation temperature and boiling take place near the heated surface and the bubbles are formed at the heated surface which get vanished on travelling a short distance.

Que 5.19. **Draw the boiling curve and identify the different boiling regimes. Also, explain the characteristics of each regime.**

AKTU 2014-15, Marks 05

OR

Define pool boiling and also explain regimes of pool boiling with the help of diagram.

AKTU 2017-18, Marks 10

Answer

A. **Pool Boiling :** Refer Q. 5.18, Page 5–30B, Unit-5.

B. **Regimes of Boiling :** Different boiling regimes are given below :

a. **Free Convection Boiling :**
1. Free convection boiling exists when excess temperature (Δt_e) is less than 5 °C.
2. In this region, the liquid near the surface is superheated slightly and convection currents circulate the liquid and evaporation takes place at the liquid surface.

b. **Nucleate Boiling :**
1. Nucleate boiling exists when temperature Δt_e is between 5 °C to 50 °C ($5 \le \Delta t_e \le 50$ °C).
2. This type of boiling exists in region II and III.
3. With the increase in Δt_e (excess temperature) the formation of bubbles on the surface of the wire at certain localised spots commences. The bubbles condense in the liquid without reaching the liquid surface. In fact, it is the region II where nucleate boiling starts.
4. With further increase in Δt_e the bubbles are formed more rapidly and rise to the surface of the liquid resulting in rapid evaporation, as indicated in the region III.
5. The nucleate boiling is thus characterised by formation of bubbles at the nucleation sites and the resulting liquid agitation.

c. **Film Boiling :**
1. Film boiling is also called unstable boiling or transition boiling which exists between 50 °C to 150 °C (50 °C $\le \Delta t_e \le$ 150 °C).
2. In this region bubble formation is very high and vapour film form only on the fraction of surface.
3. The film acts as a thermal resistance in heat flow path, hence heat transfer rate decreases.

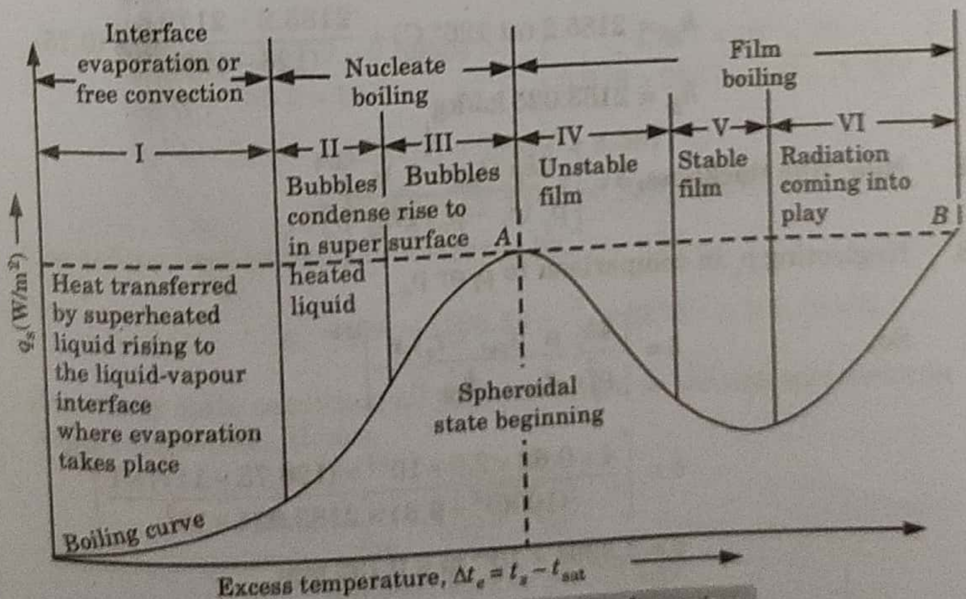


Fig. 5.19.1. The boiling curve for water.

Que 5.20. **Discuss about effect of non-condensable gases.**

Answer

1. The non-condensable gases near to the surface acts as a thermal resistance to the condensation process.
2. The rate of condensation decreases when the vapour is contaminated with non-condensable gases.
3. Even with a few percent by volume of air in a steam in condensation can reduce 50 % heat transfer coefficient.

Que 5.21. **Dry saturated steam at a pressure of 2.45 bar condenses on the surface of a vertical tube of height 1 m. The tube surface temperature is kept at 117 °C. Estimate the thickness of the condensate film.**

AKTU 2013-14, Marks 05

Answer

Given : p = 2.45 bar, ρ = 1000 kg/m^3 (dry saturated), t_s = 117 °C
k = 0.68 W/m-°C, x = 1 m, $\mu = 2.5 \times 10^{-4}$ Ns/m^2
To Find : Thickness of the condensate film.

1.
$$t_{sat} = 126.1 \text{ (at 2.4 bar)} + \frac{(127.4 - 126.1)}{(2.5 - 2.4)} \times 0.05$$

$$t_{sat} = 126.75° \text{ C}$$

$$h_{fg} = 2185.2 \text{ (at 126° C)} + \frac{2185.2 - 2179.4}{(126 - 128)} \times (0.75)$$

$$h_{fg} = 2183.025 \text{ kJ/kg}$$

2. Now, film thickness, $\delta = \left[\dfrac{4k\mu\,(t_{sat} - t_s)x}{\rho_l\,(\rho_l - \rho_v)\,gh_{fg}}\right]^{1/4}$

3. Neglecting ρ_v in comparison to ρ_l or ρ_ω

So,
$$\delta = \left[\frac{4k}{\rho_l^2}\,\frac{\mu}{g}\,\frac{(t_{sat} - t_s)\,x}{h_{fg}}\right]^{1/4}$$

$$\delta = \left[\frac{4 \times 0.68 \times 2.5 \times 10^{-4} \times (126.75 - 117) \times 1}{(1000)^2 \times 9.81 \times 2183.025 \times 10^3}\right]^{1/4}$$

$$\delta = 1.3264 \times 10^{-4} \text{ m} = 0.132 \text{ mm}$$

Que 5.22. **Estimate the power required to boil water in a copper pan, 0.35 m in diameter. The pan is maintained at 120 °C by an electric heater. What is the evaporation rate? Properties: Saturated water at 100 °C, $\rho_l = 957.9\ kg/m^3$, $\rho_v = 0.5955\ kg/m^3$, $C_{pl} = 4.217\ kJ/kg\text{-}K$, $\mu_l = 279 \times 10^{-6}\ Ns/m^2$, $Pr_l = 1.76$, $h_{fg} = 2257\ kJ/kg$ and $\sigma = 58.9 \times 10^{-3}\ N/m$. Use**

$$q'' = \mu_l * h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left[\frac{C_{p_l}\,\Delta t_e}{C_{s_f}\,h_{f_g}\,Pr_l^n}\right]^3$$

$C_{sf} = 0.013$, $n = 1.0$ **AKTU 2013-14, Marks 10**

Answer

Given : $d = 0.35$ m, $t_s = 120$ °C, $t_{sat} = 100$ °C
To Find : i. Power required to boil water.
ii. Evaporation rate.

1. The excess temperature, $\Delta t_e = t_s - t_{sat} = 120 - 100 = 20$ °C
2. As per boiling curve, for $\Delta t_e = 20$ °C, nucleate pool boiling will occur and for this, the following correlation holds good :

$$q'' = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{\frac{1}{2}}\left[\frac{C_{pl}\Delta t_e}{C_{sf} h_{fg} Pr_l^n}\right]^3$$

$$q'' = 279 \times 10^{-6} \times 2257 \times 10^3 \times$$

$$\left[\frac{9.81 \times (957.9 - 0.5955)}{58.9 \times 10^{-3}}\right]^{\frac{1}{2}} \times \left[\frac{4.217 \times 10^3 \times 20}{0.013 \times 2257 \times 10^3 \times (1.76)^1}\right]^3$$

$$= 629.703 \times 399.30 \times 4.357$$

$$= 1095525.86\ W/m^2 \text{ or } 1095.53\ kW/m^2$$

3. The boiling heat transfer rate is given by,

$$q_s = q''A = 1095.53 \times \frac{\pi}{4} \times (0.35)^2 = 105.40\ kW$$

4. For steady state condition, all the heat added to the pan will result in water evaporation. Hence,

$$\dot{m}_w = \frac{q_s}{h_{fg}} = \frac{105.40 \times 10^3}{2257 \times 10^3}$$

$$= 0.0467\ kg/s \text{ or } 168.12\ kg/h$$

PART-3

Introduction, Fick's Law of Diffusion, Steady State Equimolar Counter Diffusion, Steady State Diffusion through a Stagnant Gas Film.

CONCEPT OUTLINE : PART-3

Mass Transfer : The process of transfer of mass as a result of species concentration difference in a system/mixture is called mass transfer.

Fick's Law of Diffusion : It states that the mass flux of species A per unit area is proportional to the concentration gradient.

Mass Concentration : The mass concentration or density ρ_A of species A in a multi-component mixture is defined as the mass of A per unit volume of the mixture. It is expressed in kg/m^3 unit.

Molar Concentration : The molar concentration C_A of species A is defined as the number of moles of species per unit volume of the mixture. It is expressed as kg mole/m^3 unit.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.23. **Explain Fick's law of mass diffusion.**

AKTU 2013-14, Marks 05

OR

Discuss briefly the Fick's law of diffusion.

AKTU 2014-15, Marks 05

Answer

1. Fick's law of diffusion states that the mass flux of species A per unit area is proportional to the concentration gradient.
2. So, diffusion rate of species A is given by,

$$N_A = \frac{m_A}{A} = -D_{AB}\left(\frac{dC_A}{dx}\right) \quad ...(5.23.1)$$

Where,

m_A = Mass flow rate of species A by diffusion,

A = Area through which mass is flowing,

N_A = Mass flux of species A,

D_{AB} = Diffusion coefficient for binary mixture of species A and B,

C_A = Concentration of species A, and

$\frac{dC_A}{dx}$ = Concentration gradient for species A.

3. Similarly, diffusion rate for species B is given by,

$$N_B = \frac{m_B}{A} = -D_{BA}\left(\frac{dC_B}{dx}\right)$$

4. Fick's law can be expressed in terms of partial pressure of species by using perfect gas equation.

For species A :

$$p_A = \rho_A R_A T = \rho_A \frac{GT}{M_A}$$

$$\rho_A = C_A = \frac{p_A M_A}{GT}$$

Where, p_A = Partial pressure of species A,

ρ_A = Density of species A,

G = Universal gas constant, and

M_A = Molecular weight of species A.

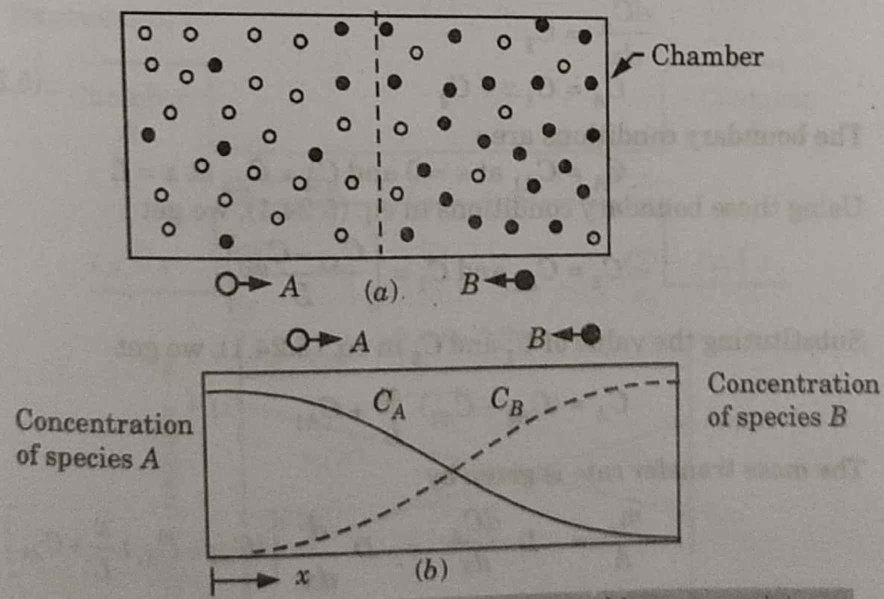


Fig. 5.23.1. Mass transfer by diffusion in a binary gas mixture.

5. Substituting the value of ρ_A in eq. (5.23.1), we get

$$N_A = \frac{m_A}{A} = -D_{AB}\frac{d}{dx}\left[\frac{p_A M_A}{GT}\right]$$

or
$$N_A = \frac{m_A}{A} = -D_{AB}\frac{M_A}{GT}\left(\frac{dp_A}{dx}\right)$$

6. Similarly, for species B:
$$N_B = \frac{m_B}{A} = -D_{AB}\frac{M_B}{GT}\left(\frac{dp_B}{dx}\right)$$

Que 5.24. **Derive expression for steady state diffusion of gases and liquids through plain membrane (solids).**

Answer

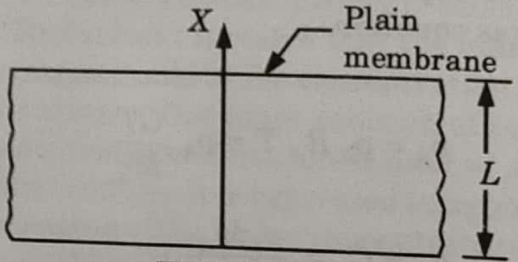


Fig. 5.24.1.

1. Considering the diffusion along X-axis, then the controlling equation is,
$$\frac{d^2C_A}{dx^2} = 0$$
2. On integration, we have
$$\frac{dC_A}{dx} = C_1$$
$$C_A = C_1 x + C_2 \quad ...(5.24.1)$$
3. The boundary conditions are :
$$C_A = C_{A1} \text{ at } x = 0 \text{ and } C_A = C_{A2} \text{ at } x = L$$
Using these boundary conditions in eq. (5.24.1), we get
$$C_2 = C_{A1} \text{ and } C_1 = \left[\frac{C_{A2} - C_{A1}}{L}\right]$$
4. Substituting the value of C_1 and C_2 in eq. (5.24.1), we get
$$C_A = (C_{A2} - C_{A1})\frac{x}{L} + C_{A1}$$
5. The mass transfer rate is given by
$$\frac{m_A}{A} = -D\frac{dC_A}{dx} = -D\frac{d}{dx}\left[(C_{A2} - C_{A1})\frac{x}{L} + C_{A1}\right]$$
$$= -D\left(\frac{C_{A2} - C_{A1}}{L}\right)$$
or
$$\frac{m_A}{A} = \frac{D}{L}(C_{A1} - C_{A2})$$

$$\frac{m_A}{A} = \frac{(C_{A1} - C_{A2})}{\left(\frac{L}{D}\right)}$$

Where, (L / D) is known as diffusion resistance.

6. The diffusion rate in the radial direction of a cylindrical system of inner radius r_1 and outer radius r_2 and length L is given by,

$$m_A = \frac{D(C_{A1} - C_{A2})}{\Delta x} A_m$$

Where, $\Delta x = (r_2 - r_1)$ and $A_m = \dfrac{2\pi L(r_2 - r_1)}{\ln(r_2 / r_1)}$

7. In case of sphere, $\Delta x = (r_2 - r_1)$ and $A_m = 4\pi r_1 r_2$

Que 5.25. **Define equimolar counter diffusion. Derive expression for steady state equimolar counter diffusion.**

Answer

A. **Equimolar Counter Diffusion :** Equimolar counter diffusion between species A and B of a binary gas mixture is defined as an isothermal diffusion process in which each molecule of component A is replaced by each molecule of component B and vice-versa.

B. **Expression :**

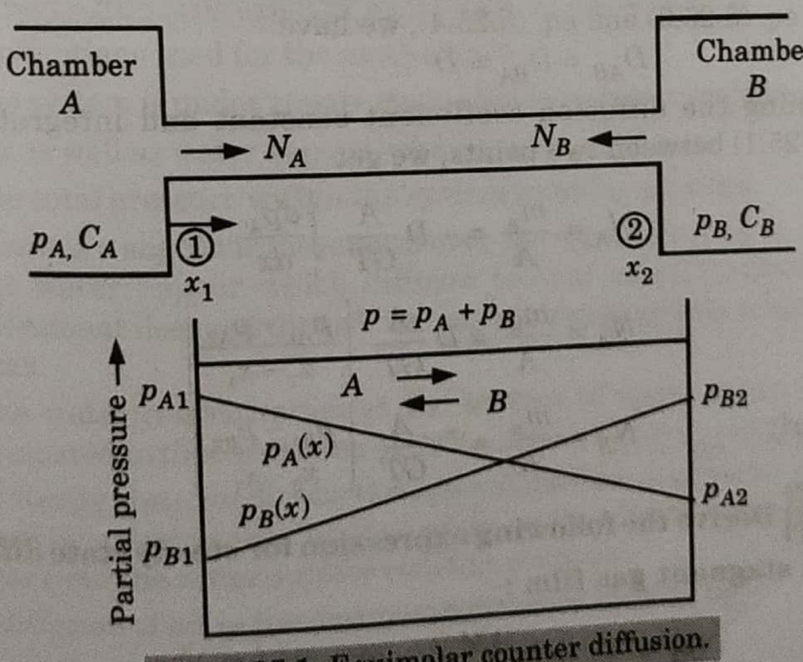


Fig. 5.25.1. Equimolar counter diffusion.

1. According to Fick's law, the molar diffusion rates of species A and B are given by,

$$N_A = \frac{m_A}{A} = -D_{AB} \frac{A}{GT}\left(\frac{dp_A}{dx}\right) \qquad \text{...(5.25.1)}$$

and, $$N_B = \frac{m_B}{A} = -D_{BA}\frac{A}{GT}\left(\frac{dp_B}{dx}\right) \quad ...(5.25.2)$$

Where, p_A and p_B = Partial pressures, and
N_A and N_B = Molar diffusion rates of the species A and B respectively.

2. According to Dalton's law of partial pressures,

$$p = p_A + p_B$$

On differentiating w.r.t. x, we get

$$\frac{dp}{dx} = \frac{dp_A}{dx} + \frac{dp_B}{dx}$$

3. Since, the total pressure of the system remains constant under steady conditions,

$$\therefore \quad \frac{dp}{dx} = \frac{dp_A}{dx} + \frac{dp_B}{dx} = 0$$

or, $$\frac{dp_A}{dx} = \frac{-dp_B}{dx} \quad ...(5.25.3)$$

4. According to steady state condition, the total molar flux relative to stationary coordinate must be zero.

So, $N_A + N_B = 0$ or $N_A = -N_B$

or $$-D_{AB}\frac{A}{GT} \times \frac{dp_A}{dx} = D_{BA}\frac{A}{GT} \times \frac{dp_B}{dx} \quad ...(5.25.4)$$

5. From eq. (5.25.3) and eq. (5.25.4), we have

$$D_{AB} = D_{BA} = D$$

6. Assuming the diffusion coefficient constant and integrating the eq. (5.25.1) between two points, we get

$$N_A = \frac{m_A}{A} = -D\frac{A}{GT}\int_1^2 \frac{dp_A}{dx}$$

or $$N_A = \frac{m_A}{A} = D\frac{A}{GT}\left[\frac{p_{A1} - p_{A2}}{x_2 - x_1}\right]$$

7. Similarly, $$N_B = \frac{m_B}{A} = D\frac{A}{GT}\left[\frac{p_{B1} - p_{B2}}{x_2 - x_1}\right]$$

Que 5.26. **Derive the following expression for steady state diffusion through a stagnant gas film :**

$$(m_w)_{total} = D\frac{AM_w}{GT}\frac{p}{(x_2 - x_1)}\ln\left(\frac{p_{a2}}{p_{a1}}\right)$$

Answer

1. Let us consider isothermal evaporation of water from a surface and its subsequent diffusion through the stagnant layer of gas over it.

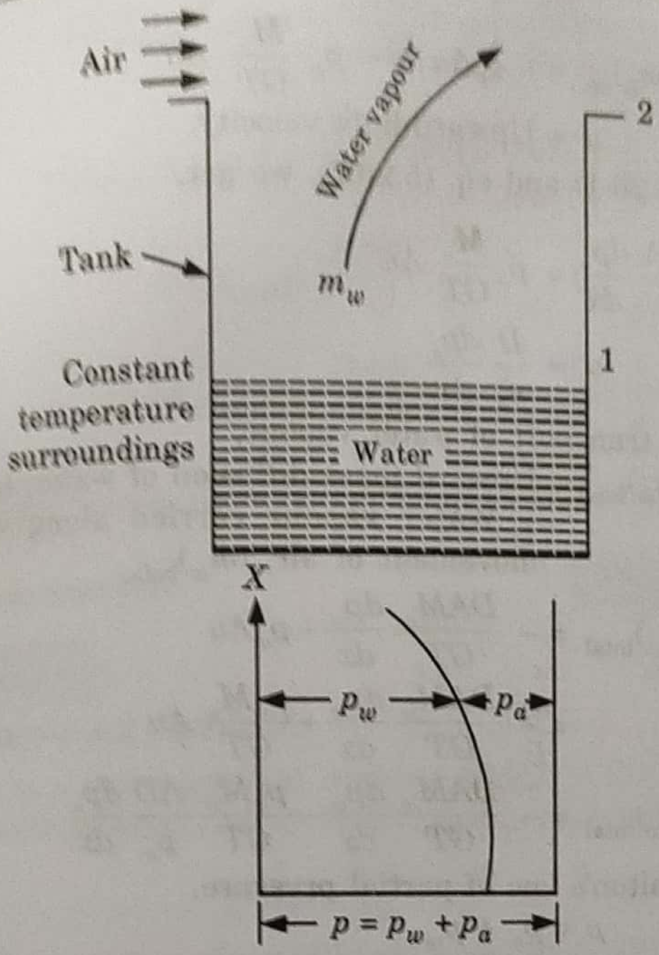


Fig. 5.26.1. Diffusion of water vapour through air.

2. The assumptions used for the analysis are :
   i. The system is under steady state and isothermal conditions.
   ii. Air as well as water vapour behaves as an ideal gas.
   iii. The total pressure within the system remains constant.
   iv. There is a slight air movement over the top of the tank to remove the water vapour which diffuses to that point; however, this movement does not disturb the concentration profile of air in the tank.
   v. The water concentration at the surface of water is much more compared to that at the top of the tank (*i.e.*, $C_{w1} > C_{w2}$ or $C_{a2} > C_{a1}$).
3. Under steady state conditions the upward movement of water must be balanced by a downward diffusion of air so that concentration at any distance from the water surface remains constant.

Mass diffusion of air in the downward direction is given by,

$$(m_a)_{down} = -D\frac{AM_a}{GT}\frac{dp_a}{dx} \quad ...(5.26.1)$$

Where,

A = Cross-sectional area of the tank, and

$\frac{dp_a}{dx}$ = Partial pressure gradient of air.

4. Bulk mass transfer of air upward,

$$(m_a)_{up} = -\rho_a A u = -p_a \frac{M_a}{GT} Au \quad ...(5.26.2)$$

Where, u = Upward bulk velocity.

5. Equating eq. (5.26.1) and eq. (5.26.2), we get,

$$\frac{DM_a A}{GT}\frac{dp_a}{dx} = p_a \frac{M_a}{GT} Au$$

or $$u = \frac{D}{p_a}\frac{dp_a}{dx}$$

6. The total mass transport of water vapour,

$(m_w)_{total}$ = Upward mass diffusion of water, $(m_w)_{diffusion}$ + Water vapour carried along with bulk movement of air, $(m_w)_{bulk}$

or $$(m_w)_{total} = -\frac{DAM_w}{GT}\frac{dp_w}{dx} + p_w Au$$

$$= -\frac{DAM_w}{GT}\frac{dp_w}{dx} + \frac{p_w M_w}{GT} Au$$

or $$(m_w)_{total} = -\frac{DAM_w}{GT}\frac{dp_w}{dx} - \frac{p_w M_w}{GT}\frac{AD}{p_a}\frac{dp_a}{dx} \quad ...(5.26.3)$$

7. According to Dalton's law of partial pressure,

$$p = p_a + p_w$$

On differentiating, we get

$$\frac{dp}{dx} = \frac{dp_a}{dx} + \frac{dp_w}{dx}$$

Since, the total pressure in the tank remains constant,

hence, $$\frac{dp}{dx} = 0$$

Thus, $$\frac{dp_a}{dx} = -\frac{dp_w}{dx} \quad ...(5.26.4)$$

8. Substituting eq. (5.26.4) in eq. (5.26.3), we get

$$(m_w)_{total} = -\frac{DAM_w}{GT}\frac{dp_w}{dx} - \frac{p_w M_w}{GT}\frac{AD}{p_a}\frac{dp_w}{dx}$$

$$= -\frac{DAM_w}{GT}\frac{dp_w}{dx}\left[1 + \frac{p_w}{p_a}\right]$$

$$= -\frac{DAM_w}{GT}\frac{dp_w}{dx}\left[\frac{p_a + p_w}{p_a}\right]$$

or $$(m_w)_{total} = -\frac{DAM_w}{GT}\frac{dp_w}{dx}\left[\frac{p}{p - p_w}\right] \quad ...(5.26.5)$$

9. Eq. (5.26.5) is known as Stefan's law for diffusion of an ideal gaseous component through a practically stagnant and ideal constituent of the binary system.

10. Integrating eq. (5.26.5) between x_1 and x_2, we get

$$(m_w)_{\text{total}} \int_{x_1}^{x_2} dx = -\frac{DAM_w}{GT} p \int_{p_{w_1}}^{p_{w_2}} \frac{dp_w}{p - p_w}$$

$$(m_w)_{\text{total}} (x_2 - x_1) = \frac{DAM_w}{GT} p \ln\left[\frac{p_{w2} - p}{p_{w1} - p}\right]$$

$$(m_w)_{\text{total}} = \frac{DAM_w}{GT} \frac{p}{(x_2 - x_1)} \ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right]$$

or,
$$(m_w)_{\text{total}} = \frac{DAM_w}{GT} \frac{p}{(x_2 - x_1)} \ln\left(\frac{p_{a2}}{p_{a1}}\right)$$

Que 5.27. **Discuss physical significance of Sherwood number and Schmidt number.**

AKTU 2013-14, Marks 05

Answer

A. **Sherwood Number :**

1. It is a dimensionless number which is the ratio of mass transfer coefficient multiply by length to the concentration profile.

$$Sh = \frac{h_m l}{D}$$

2. The mass transfer coefficient under flow condition can be worked out by the method of dimensional analysis.

B. **Schmidt Number :**

1. It is also a dimensionless number which is the ratio of velocity and concentration profiles.

$$Sc = v/D$$

2. It forms the connecting link between velocity and concentration profiles, these profiles show identical behaviour when $Sc = 1$.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Discuss the general arrangement of parallel flow, counter flow and cross flow heat exchangers.

Ans. Refer Q. 5.1.

Q. 2. Write down the significance of NTU method in heat exchanger. Derive an expression of effectiveness for counter flow heat exchanger by using NTU method.

Ans. Refer Q. 5.7.

Q. 3. Derive an expression for effectiveness by NTU method for parallel flow.

Ans. Refer Q. 5.8.

Q. 4. In a counter flow double pipe heat exchanger, water is heated from 25 °C to 65 °C by an oil with a specific heat of 1.45 kJ/kg-K and mass flow rate of 0.9 kg/s. The oil is cooled from 230 °C to 160 °C. If the overall heat transfer coefficient is 420 W/m²-°C. Calculate the following :

i. the rate of heat transfer,

ii. the mass flow rate of water, and

iii. the surface area of the heat exchanger.

Ans. Refer Q. 5.14.

Q. 5. Differentiate between dropwise condensation and filmwise condensation.

Ans. Refer Q. 5.16.

Q. 6. Define pool boiling and also explain regimes of pool boiling with the help of diagram.

Ans. Refer Q. 5.19.

Q. 7. Discuss briefly the Fick's law of diffusion.

Ans. Refer Q. 5.23.

Q. 8. Discuss physical significance of Sherwood number and Schmidt number.

Ans. Refer Q. 5.27.

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