Construct the hypo-cycloid, following the procedure under Construction: Fig. 5.36.

It may be noted that when the diameter of the generating circle is equal to the radius of the directing circle, the hypo-cycloid traced is a straight line.

5.6 INVOLUTE

An involute is a curve traced by a point, on a perfectly flexible thread, while unwinding from a circle or a polygon; the thread being kept tight.

Problem 29 *Draw the involute of an equilateral triangle of side 20 and draw a normal and a tangent at a distance 60 from the centre of the triangle.* (May/June 2010,JNTU)

Construction (Fig. 5.38)

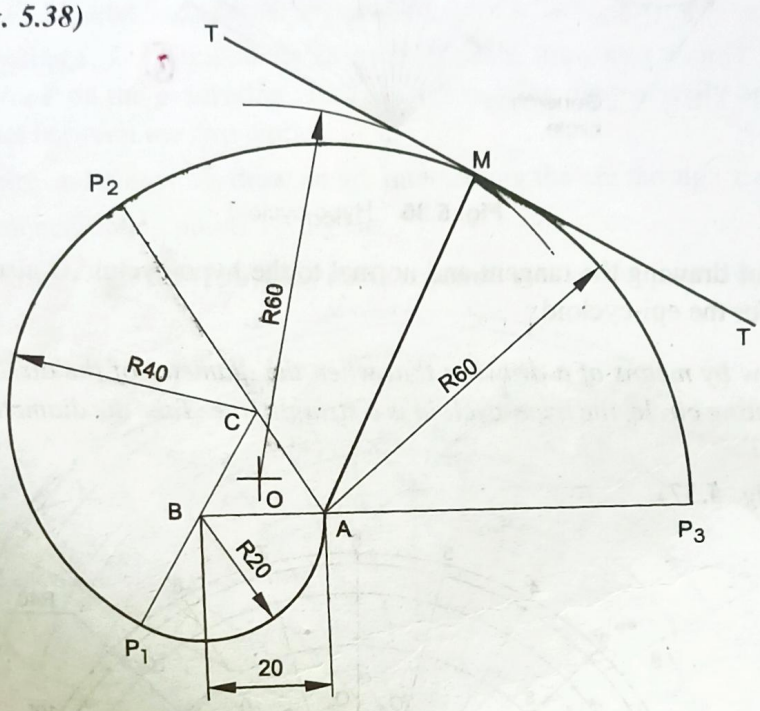

Fig. 5.38 Involute of a triangle

(i) Draw the given triangle ABC of side 20 and locate its centre O.

(ii) Assuming A as the starting point; with B as centre and radius BA (=20), draw an arc intersecting the line CB extended at P_1.

(iii) With centre C and radius CP_1 (= 2× 20), draw an arc intersecting the line AC extended at P_2.

(iv) With centre A and radius AP_2 (=3×20), draw an arc intersecting the line BA produced at P_3.

The curve through A, P_1, P_2, P_3 is the required involute.

To draw a tangent and a normal to the curve

(i) With O as centre (centre of the triangle) and radius 60, draw an arc intersecting the involute at M. The point M lies on that part of the arc, for which A is centre.

(ii) Join A, M; forming the normal to the curve.

(iii) A line T-T, perpendicular to AM at M is the required tangent.

Problem 30 *Draw the involute of a regular hexagon of side 20. Draw a tangent and a normal to the curve at a distance 100 from the centre of the hexagon.*

Construction (Fig. 5.39)

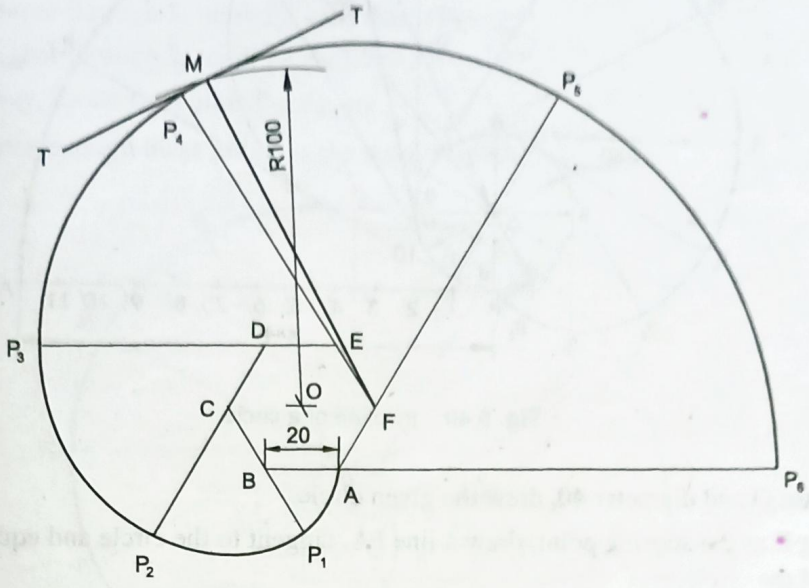

Fig. 5.39 Involute of a hexagon

(i) Draw the hexagon ABCDEF of side 20 and locate its centre O.

Assuming that the thread is unwound from A in the clock-wise direction, the starting point for the involute is A,

(ii) With centre B and radius BA (=20), draw an arc intersecting the line CB extended at P_1.

(iii) With centre C and radius CP_1 (=2 ×20), draw an arc intersecting the line DC extended at P_2.

(iv) In a similar way, obtain the other points P_3, P_4, etc.

A smooth curve through the above points is the required involute.

To draw a tangent and a normal to the curve

(i) With O as centre and radius 100, draw an arc intersecting the involute at M. The point M lies on that part of the arc, for which F is the centre.

(ii) Join F, M; forming normal to the curve.

(iii) A line T-T, perpendicular to FM at M is the required tangent.

Problem 31 *Draw the involute of a circle of 40 diameter. Also, draw a tangent and a normal to the curve at a point 95 from the centre of the circle.* (May 2012, JNTU)

Construction (Fig. 5.40)

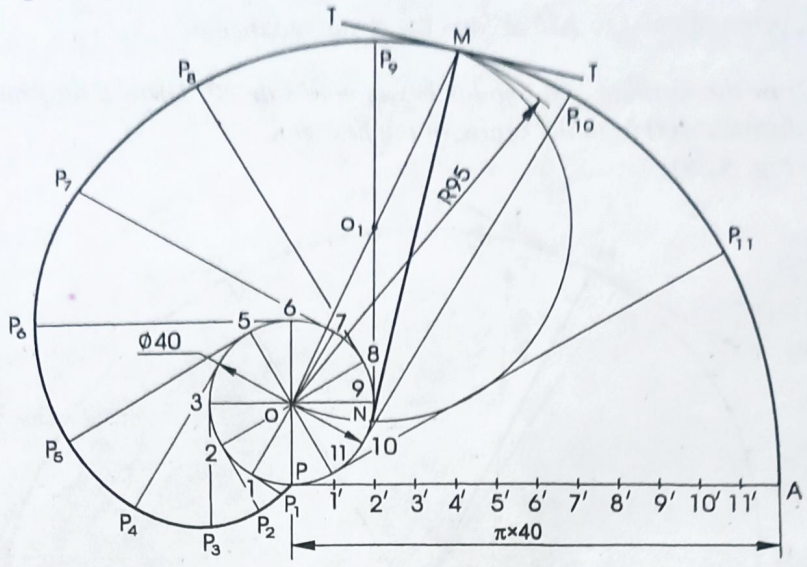

Fig. 5.40 Involute of a circle

(i) With centre O and diameter 40, draw the given circle.

(ii) Assuming P as the starting point, draw a line PA, tangent to the circle and equal to the circumference of the circle.

(iii) Divide the circle and the line PA into the same number of equal parts and number them as shown.

(iv) Draw a tangent to the circle at the point 1 and locate P_1 on it such that, $1P_1 = P1'$.

(v) Draw a tangent to the circle at point 2 and locate P_2 on it such that, $2P_2 = P2'$.

(vi) Locate other points P_3, P_4, etc., in a similar way.

A smooth curve through these points is the required involute.

From the construction, it is obvious that a tangent to the circle is normal to the involute. So, to draw the tangent and normal,

(i) Locate the point M on the curve, which is at 95 from the centre of the circle.

(ii) Join M, O and locate its mid-point O_1.

(iii) With centre O_1 and radius O_1 M, draw a semi-circle intersecting the given circle at N.

(iv) Join N, M; forming the normal to the curve and a line T-T, perpendicular to NM at M is the tangent to the curve

Problem 32 *A thread of length 165 is wound round a circle of 40 diameter. Trace the path of end point of the thread.* (May/June 2011,2012, JNTU)

Construction (Fig. 5.41)

(i) With centre O and radius 20, draw the given circle.

(ii) From point A on the circle, draw a line AP, tangential to the circle and equal to 165, the length of the thread.

(iii) Divide the circle into 12 equal parts and mark the chord lengths along the line AP.

(iv) Draw tangents to the circle at points 1, 2, etc.

(v) Along the tangent through 1, mark P_1 such that, $1P_1 = P1'$.

(vi) Along the tangent through 2, mark P_2 such that, $2P_2 = P2'$.

(vii) In a similar way, locate the points P_3, P_4, etc.

A smooth curve through these points is the required path.

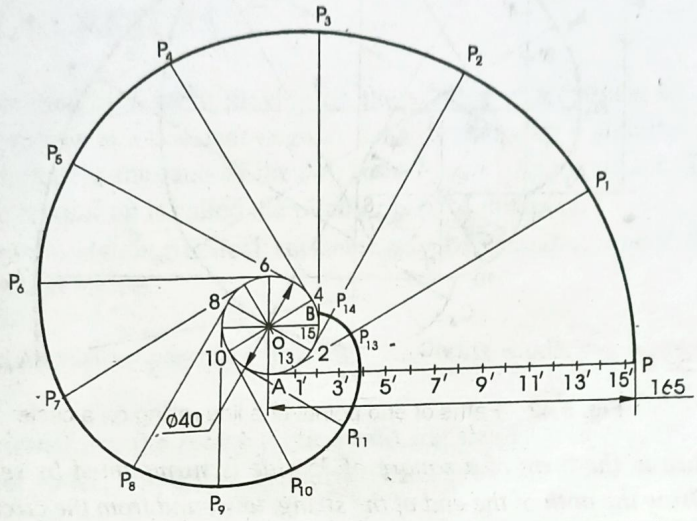

Fig. 5.41 Path of end point of thread, wound round a circle

NOTE (i) The points P_{13}, P_{14} and P_{15} are located along the tangents through 1, 2 and 3.

(ii) The point B on the circle is located such that, the chord length 15-B = 15′ -P.

Problem 33 *A line AC of 150 long, is tangential to a circle of diameter 60. Trace the paths of A and C, when the line AC rolls on the circle without slipping.*

Construction (Fig. 5.42)

(i) Draw a circle of diameter 60.

(ii) Draw the tangent AC to the circle at A, of length 150.

(iii) Divide the circle into a number of equal parts, say 12 and number them as shown.

(iv) Mark 1′ , 2′ , 3′ , etc., on AC such that, A-1′ = 1′ -2′ = 2′ -3′ , etc., = $1/12^{th}$ circumference of the circle.

(v) When the line AC rolls on the circle and 1′ coincides with 1, locate the positions of A and C; such that $1\text{-}A_1 = A\text{-}1'$ and $1\text{-}C_1 = 1'\text{-}C$.

(vi) Similarly, locate the end points for different positions of the line, as it rolls on the circle. Join A, A_1, A_2, A_3, etc., and C, C_1, C_2, C_3, etc., representing the paths of A and C.

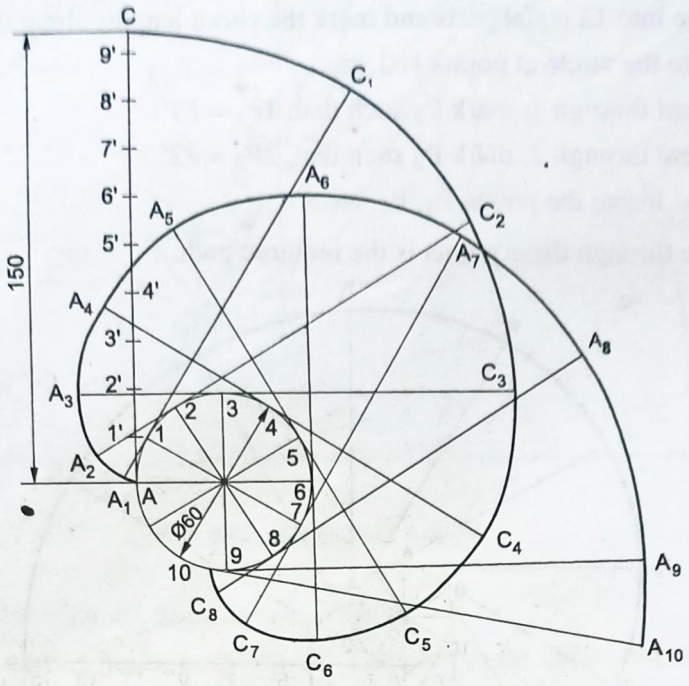

Fig. 5.42 Paths of end points of a line rolling on a circle

Problem 34 *A disc in the form of a square of 35 side is surmounted by semi-circles on the two opposite sides. Draw the path of the end of the string, unwound from the circumference of the disc.*

Construction (Fig. 5.43)

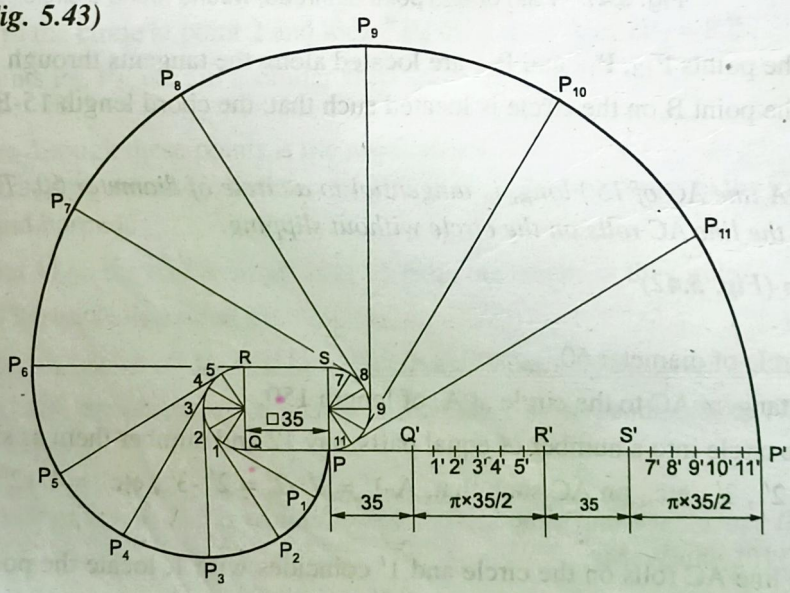

Fig. 5.43 Path of end point of a string unwound from a disc

(i) Draw the square PQRS of side 35, surmounted by two semi-circles on the two opposite sides.

(ii) Divide the semi-circles into six equal parts and number them as shown.

(iii) Extend QP to P′ such that, $PP' = 2 \times 35 + \pi \times 35$ and mark the division points as shown.

(iv) Draw tangents to the semi-circles at 1, 2, – – –– R, 7, 8, – – –– 11.

(v) Assuming that the string is unwound from P, locate P_1 along the tangent at 1 such that, $1P_1 = P1'$.

(vi) Locate P_2 along the tangent at 2 such that, $2P_2 = P2'$ and so on.

(vii) Join the points, P, P_1, P_2, etc., by a smooth curve forming the path of the end of the string.

5.7 HELICAL SURFACES

Helix is a curve, generated by a point moving on the surface of a cylinder or cone in a circular direction. The point moves at a constant angular velocity and with a simultaneous uniform rate of advance in axial direction; the ratio of the two movements being constant. The amount of the axial advance for one revolution is called the pitch or lead of the helix.

Threaded elements containing helical surfaces may be classified under the following two groups, depending upon their use.

(i) Fasteners, for holding various parts in a structure, viz., bolts, studs, cap screws, machine screws and set-screws.

(ii) Elements, which transform the rotary motion into translatory motion, viz., screw conveyor, lifting screw, wheel puller, lead screw, cylindrical cam, propeller blade, helical chute, etc.

5.8 SINGLE HELIX

Single helix is a curve, generated by a single moving point on the surface of a cylinder or cone in a circumferential direction.

Problem 35 *Draw a helix of pitch equal to 45, upon a cylinder of 40 diameter and develop the surface of the cylinder along with the helix. Assume the starting point P to be on the left extreme horizontal centre line in the top view.*

HINT Assume the generating point P to move upwards and in anti-clockwise direction.

Construction (Fig. 5.44)

(i) Draw the projections of the cylinder.

(ii) Divide the base and pitch of the helix into the same number of equal parts, say 12.

(iii) Draw the generators in the front view, corresponding to the division points in the top view of the cylinder.