A hyperbola passing through any given point, located between the two asymptotes, making any angle other than 90°, may also be constructed (Fig. 5.29); following the method similar to construction (Fig. 5.28)

5.5 CYCLOIDAL CURVES

Cycloidal curves are generated by a point on the circumference of a circle, when it rolls without slipping along a straight or curved path. The rolling circle is called the generating circle and the fixed straight line / circle is called the directing line/ circle respectively.

5.5.1 Cycloid

A cycloid is a curve generated by a fixed point on the circumference of a circle, when it rolls along a straight line without slipping (Fig.5.30). Obviously, the size of the curve depends upon the diameter of the generating circle.

Problem 22 *Construct a cycloid, given the diameter of the generating circle as 40. Draw tangent to the curve at a point on it, 35 from the line.* (May/June 2008, 2010, May 2012, JNTU)

Construction (Fig. 5.30)

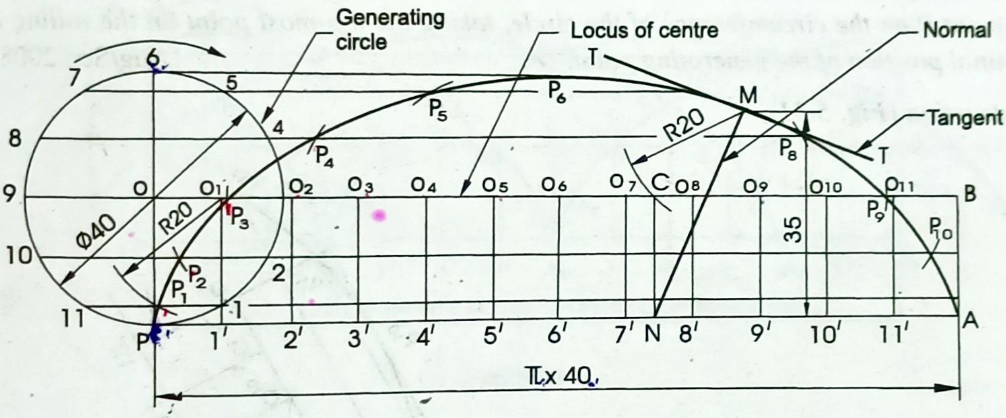

Fig. 5.30 Cycloid

(i) With centre O and radius 20, draw the generating circle.

(ii) Locate the initial position of the generating point P on the circumference of the circle.

(iii) Draw a line PA, tangential and equal to the circumference of the circle.

(iv) Divide the circle and the line PA into the same number of equal parts and number them as shown.

(v) Draw the line OB, parallel and equal to PA, which is the locus of the centre of the generating circle.

(vi) Erect perpendiculars at 1′, 2′, etc., to meet the line OB at O_1, O_2, etc.

(vii) Through the points 1, 2, 3, etc., draw lines parallel to PA.

(viii) With O_1 as centre and radius 20, draw an arc intersecting the line through 1 at P_1.

P_1 is the position of the point P, when the centre of the generating circle moves to O_1.

(ix) With O_2 as centre and radius 20, draw an arc intersecting the line through 2 at P_2.

(x) Similarly, locate the points P_3, P_4, etc.

A smooth curve passing through these points is the required cycloid.

To draw the tangent and normal

(i) Locate the point M on the curve, which is at 35 from the directing line.

(ii) With M as centre and radius 20, draw an arc intersecting the locus of the centre (OB) at C.

(iii) Through C, draw a line perpendicular to the directing line PA, meeting it at N (the point of contact of the generating circle, when its centre moves to C).

The line joining the points M and N is the required normal and a line T-T perpendicular to it and passing through M is the tangent to the cycloid.

Problem 23 *A circle of 50 diameter rolls on a horizontal line for half a revolution clock-wise and then on a line inclined at 60° to the horizontal for another half, clock-wise. Draw the curve traced by a point P on the circumference of the circle, taking the top-most point on the rolling circle as the initial position of the generating point.* (Aug/Sep 2008, JNTU)

Construction (Fig. 5.31)

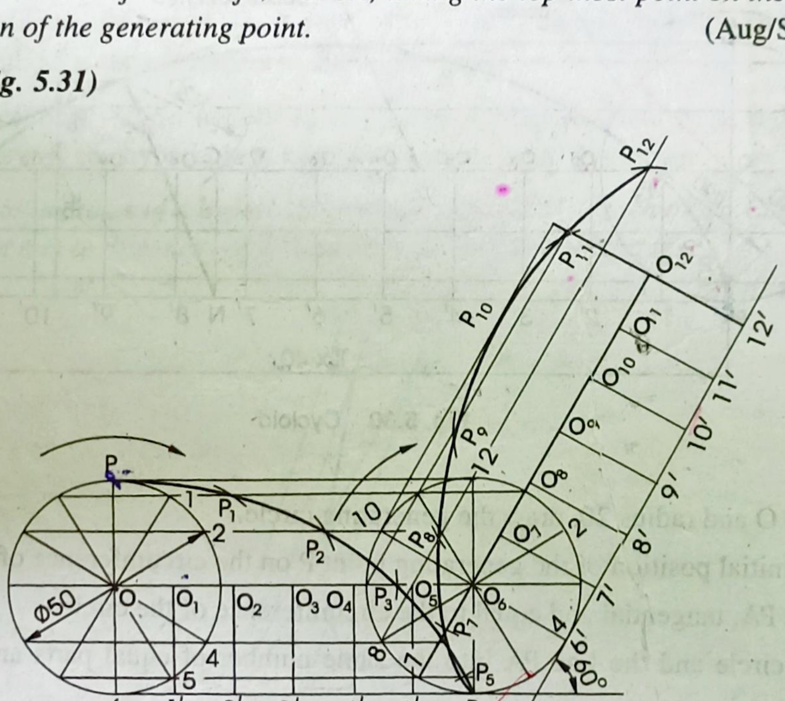

Fig. 5.31 Cycloid

(i) With centre O and radius 25, draw the given circle.

(ii) Divide the circle into a number of equal parts, say 12.

(iii) Mark the division points on the circle and draw a tangent (directing line) passing through 6.

(iv) Mark half of the circumference along the tangent and divide it into 6 equal parts.

(v) Locate the initial position of the generating point P.

(vi) Draw a smooth curve, representing the path of P, as the circle rolls along the directing line for half a revolution, following the Construction: Fig. 5.30.

(vii) With O_6 as centre, draw the circle, representing the position of the rolling circle after half revolution.

(viii) Draw a line tangential to the circle, making 60° with the directing line. This is the directing line for the next half revolution of the generating circle.

(ix) Trace the path of P for the next half revolution of the generating circle, as shown.

Problem 24 *ABC is an equilateral triangle of side 70. Trace the loci of vertices A, B and C, when the circle circumscribing ABC, rolls without slipping, along a fixed straight line, for one complete revolution.*

Construction (Fig. 5.32)

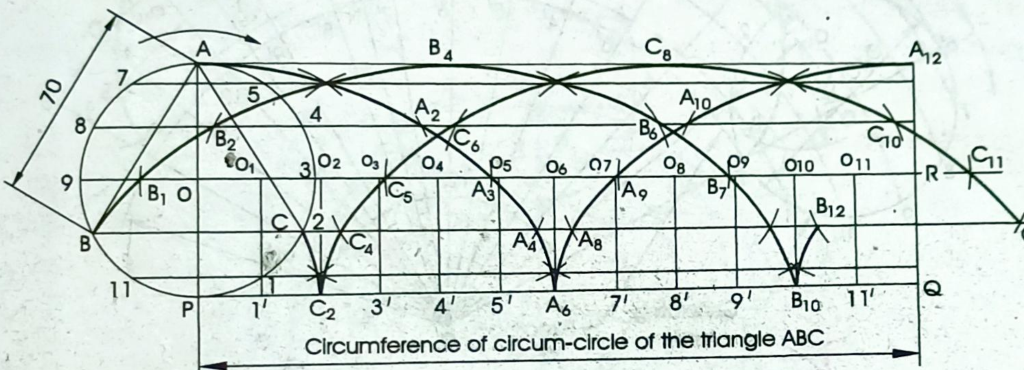

Fig. 5.32

(i) Draw the equilateral triangle ABC and draw its circum-circle.

(ii) Divide the circle into a number of equal parts such that the corners (vertices) of the triangle A, B and C coincide with the division points.

(iii) Draw the line PQ, tangential and equal to the circumference of the circle.

(iv) Divide the line PQ into the same number of equal parts, as that of the circle.

(v) Draw the line OR, parallel and equal to PQ, which is the locus of the centre of the (generating) circle.

(vi) Erect perpendiculars at 1′ , 2′ , etc., to meet the line OR at O_1, O_2, etc.

(vii) Follow the steps 7 to 10 of Construction: Fig. 5.30 suitably and obtain the loci of vertices A, B and C.

5.5.2 Epi-cycloid

An epi-cycloid is a curve traced by a point on the circumference of a circle, which rolls without slipping on another circle (directing circle) outside it.

Problem 25 *Draw an epi-cycloid of a circle of 40 diameter, which rolls on another circle of 120 diameter for one revolution clock-wise. Draw a tangent and a normal to it at a point 90 from the centre of the direcing circle.* **(Aug/Sep 2011, JNTU)**

Construction (Fig. 5.33)

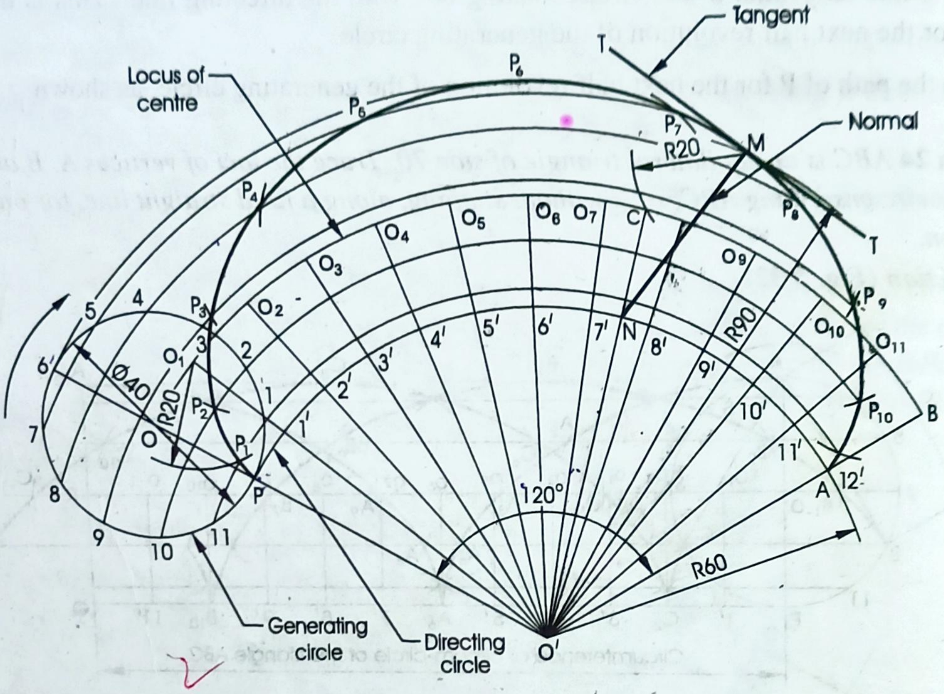

Fig. 5.33 Epi-cycloid

(i) Draw a part of the directing circle with O′ as centre and radius 60.

(ii) Draw any radial line O′ P and extend it.

(iii) Locate the point O on the above line such that, OP = 20, the radius of the generating circle.

(iv) With O as centre and radius 20, draw the generating circle.

(v) Locate the point A on the directing circle such that, the arc length PA is equal to the circumference of the generating circle.

The point A is obtained by setting $\angle PO'A = 360° \times 20/60 = 120°$.

(vi) With centre O′ and radius O′O, draw an arc intersecting the line O′ A produced at B.

The arc OB is the locus of the centre of the generating circle.

(vii) Divide the generating circle and the arc PA into the same number of equal parts and number them as shown.

(viii) Join O′ ,1′ ; O′ , 2′ ; etc., and extend, meeting the arc OB at O_1, O_2, etc.

(ix) Through the points 1, 2, 3, etc., on the generating circle, draw arcs with O′ as centre.

(x) With centre O_1 and radius 20, draw an arc intersecting the arc through 1 at P_1.

(xi) In a similar manner, obtain points P_2, P_3, etc.

A smooth curve through these points is the required epi-cycloid.

To draw the tangent and normal

(i) Locate the point M on the curve, which is at 90 from the centre of the directing circle.

(ii) With M as centre and radius 20, draw an arc intersecting the locus of the centre of the generating circle at C.

(iii) Join C to O′ , intersecting the directing circle at N. The line joining N to M is the required normal and a line T-T, perpendicular to it and passing through M is the required tangent.

NOTE When the diameters of the generating circle and directing circle are equal, the epi-cycloid traced is called a cardioid, as shown in Fig. 5.34.

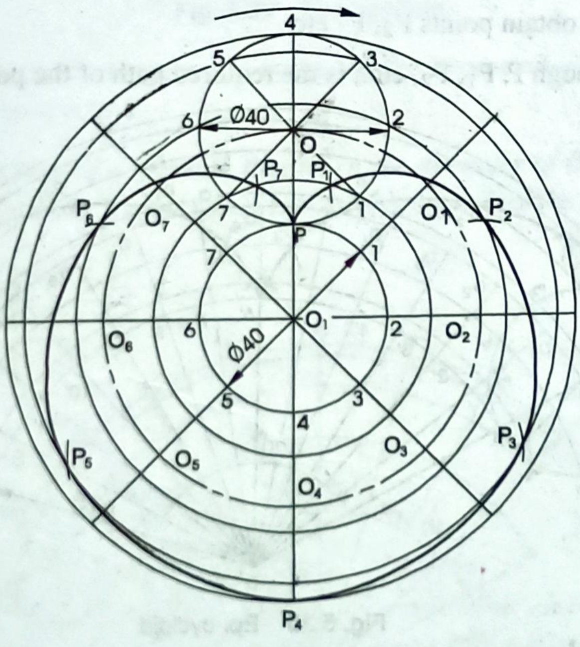

Fig. 5.34 Cardioid

Problem 26 *A circle of 50 diameter rolls without slipping on the outside of another circle of diameter 150. Show the path of a point on the periphery of the (generating) rolling circle, diametrically opposite to the initial point of contact between the circles.* (Aug/Sep 2011, JNTU)

Construction (Fig. 5.35)

(i) Draw a part of the directing circle with O′ as centre and radius 75.

(ii) Draw any radial line O′ A and extend it.

(iii) Locate the point O on the above line such that, OA = 25.

(iv) With O as centre and radius 25 (=OA), draw the generating circle.

(v) Locate the point B on the directing circle such that, the arc length AB is equal to the circumference of the generating circle.

The point B is obtained, by setting $\angle AO'B = 360^\circ \times \frac{25}{75} = 120^\circ$

(vi) With O′ as center and O′O as radius, draw an arc intersecting the line O′ B extended at C. The arc OC is the locus of the centre of the generating circle.

(vii) Divide the generating circle and the arc AB into the same number of equal parts and number them as shown.

(viii) Join O′ , 1′ ; O′ , 2′ ; etc; and extend; meeting the arc OC at O_1, O_2, etc.

(ix) Through the points 1, 2, 3, etc., on the generating circle, draw arcs with O′ as centre. Locate the point P on the generating circle, which is lying diametrically opposite to the initial point of contact between the two circles.

(x) With O as centre and radius 25, draw an arc intersecting the arc through 1 at P_1.

(xi) In a similar manner, obtain points P_2, P_3 etc.

A smooth curve through P, P_1, P_2, etc., is the required path of the point P, the epi-cycloid.

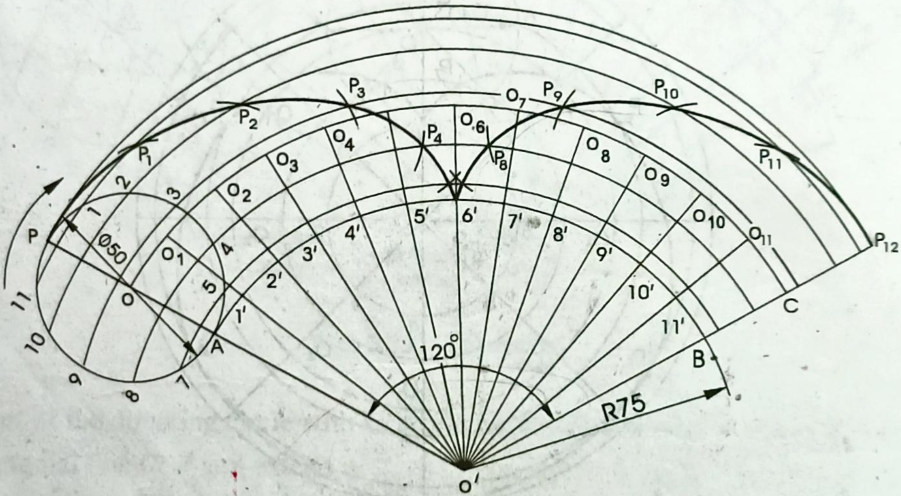

Fig. 5.35 Epi-cycloid

5.5.3 Hypo-cycloid

A hypo-cycloid is a curve traced by a point on the circumference of a generating circle, which rolls without slipping on another circle (directing circle), inside it.

Problem 27 *Draw, a hypo-cycloid of a circle of 40 diameter which rolls inside another circle of 160 diameter, for one revolution counter clock-wise. Draw a tangent and a normal to it at a point 65 from the centre of the directing circle.* (Aug/Sep 2008, JNTU)

A procedure similar to the above (Fig. 5.35), may be followed for constructing the hypo-cycloid (Fig. 5.36), keeping in view that the generating circle rolls inside the directing circle.

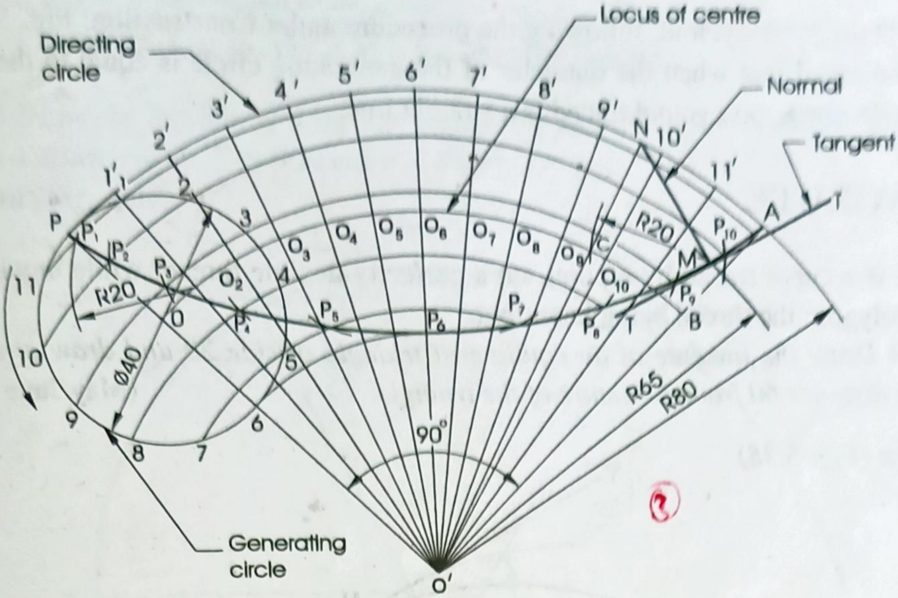

Fig. 5.36 Hypo-cycloid

The method of drawing the tangent and normal to the hypo-cycloid is also similar to the one that is followed for the epi-cycloid.

Problem 28 *Show by means of a drawing that when the diameter of the directing circle is twice that of the generating circle, the hypo-cycloid is a straight line. Take the diameter of the generating circle equal to 40.* (June 2008, JNTU)

Construction (Fig. 5.37)

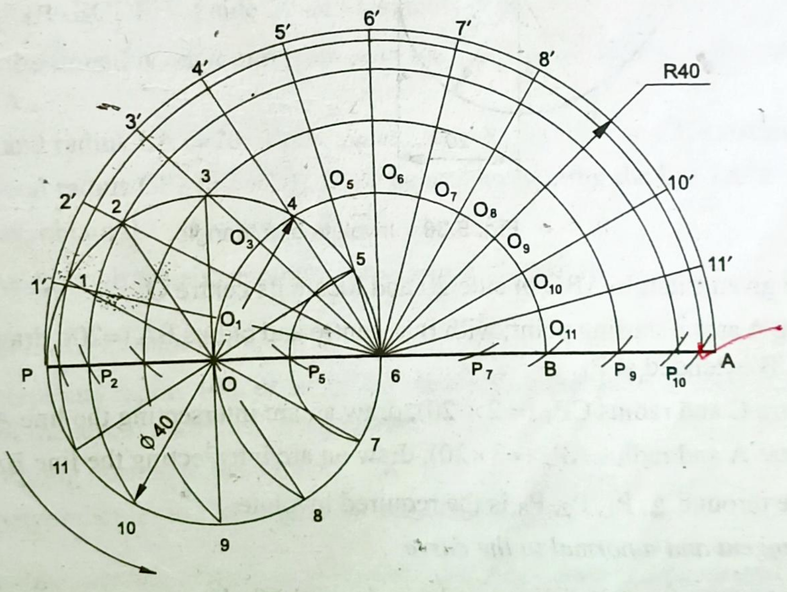

Fig. 5.37 Hypo-cycloid-straight line