DLVC QUESTIONS

T-1

. Solve$(2x + e^y)dx + xe^y dy$?

. Find the orthogonal trajectories of the family of curve$x^2 + y^2 = a^2$?

. State newton's law of cooling?

. A. Solve$(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$?

B. Solve$\frac{dy}{dx} + \cot x \,.\, y = \cos x$?

. A) Solve $x\frac{dy}{dx} + y = x^3y^6$?

B) A body is originally 80^0C and cools downs to 60^0C in 20 minutes if the temperature of the air is 40^0C, find the temperature of the body 40 min ?

5. A) The number N of bacteria in culture grew at a rate proposinonal to N. The value of N was initially 100 and increases to 332 in one hour. What was value of N after 1 ½ hours?

B) If a voltage of $20 \cos 5t$ is applied to a series circuit consisting of 10 ohm resistor and 2 henry inductor determine the current at any time t?

TI –II

1. Define Wronskian?

(2) Solve $(2x+e^y)\,dx + (xe^y)\,dy = 0$

Sol:- $M\,dx + N\,dy = 0$

$M = 2x+e^y$ $\qquad$ $N = xe^y$

$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2x+e^y)$ $\qquad$ $\frac{\partial N}{\partial x} = e^y(1)$

$\frac{\partial N}{\partial x} = e^y$

$= 0+e^y$

$\frac{\partial M}{\partial y} = e^y$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

The given equation is an exact differential equation.

General solution,

$\Rightarrow \int M\,dx + \int N\,dy = c$

y is constant $\quad$ x is not involving

$\Rightarrow \int (2x+e^y)\,dx + \int (xe^y)\,dy = c$

$\Rightarrow \int 2x\,dx + \int e^y\,dx = c$

$\Rightarrow \cancel{2}\frac{x^2}{\cancel{2}} + e^y x = c$

$\Rightarrow x^2 + xe^y = c$

$\Rightarrow \boxed{x(x+e^y) = c}$

(2) Find the orthogonal trajectories of the family of the curves $x^2+y^2=a^2$.

Sol:- Given,

$$x^2+y^2=a^2$$

Differentiating with respect to x on both sides,

$$\Rightarrow \frac{d}{dx}(x^2+y^2)=\frac{d}{dx}(a^2)$$

$$\Rightarrow 2x+2y\frac{dy}{dx}=0$$

$$\Rightarrow 2\left(x+y\frac{dy}{dx}\right)=0$$

$$\Rightarrow x+y\frac{dy}{dx}=0$$

Replace, $\frac{dy}{dx}$ with $-\frac{dx}{dy}$

$$x+y\left(\frac{-dx}{dy}\right)=0$$

$$x=y\frac{dx}{dy}$$

$$\frac{1}{y}dy=\frac{1}{x}dx$$

$$\int\frac{1}{y}dy=\int\frac{1}{x}dx$$

$$\log y=\log x+\log c$$

$$\cancel{\log} y=\cancel{\log}(xc)$$

$$\boxed{y=xc}$$

(3) State Newton's Law of Cooling

Sol:- Newton's Law of Cooling:

The rate of change of the temperature of a body is proportional to the difference of the temperature of the body and that of the surrounding medium (usually air).

Let 'θ' be the temperature of the body at time 't' and 'θ_0' be the temperature of the surrounding medium (usually air).

By the Newton's law of cooling we have,

$$\frac{d\theta}{dt} \propto (\theta - \theta_0)$$

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

where, k is the positive Constant.

4.

(A) Solve $(xy^3+y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$.

Sol:- Given,

$$(xy^3+y)dx + 2(x^2y^2 + x + y^4)\,dy = 0 \quad -①$$

$$M\,dx + N\,dy = 0$$

$M = xy^3 + y \qquad N = 2(x^2y^2 + x + y^4)$

$\frac{\partial M}{\partial y} = 3xy^2 + 1 \qquad \frac{\partial N}{\partial x} = 2(2xy^2 + 1 + 0)$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\because \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

the given eq① is not an Exact Differential Equation.

$$\Rightarrow \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{xy^3+y}\left[(4xy^2+2) - (3xy^2+1)\right]$$

$$= \frac{1}{y(xy^2+1)}\left[4xy^2 + 2 - 3xy^2 - 1\right]$$

$$= \frac{1}{y(\cancel{xy^2+1})}(\cancel{xy^2+1}) = \frac{1}{y} = g(y)$$

Integrating Factor,

$$I.F = e^{\int g(y)dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

Multiplying I.F with eq ①,

$$\Rightarrow y\left[(xy^3+y)\,dx + 2(x^2y^2+x+y^4)\,dy\right] = 0$$

$$\Rightarrow (xy^4+y^2)\,dx + 2(x^2y^3+xy+y^5)\,dy = 0 \quad \text{②}$$

$$M_1\,dx + N_1\,dy = 0$$

$$\Rightarrow M_1 = xy^4+y^2 \qquad N_1 = 2(x^2y^3+xy+y^5)$$

$$\frac{\partial M_1}{\partial y} = 4xy^3+2y \qquad \frac{\partial N_1}{\partial x} = 2(2xy^3+y+0)$$

$$\frac{\partial N_1}{\partial x} = 4xy^3+2y$$

$$\because \frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$$

$\therefore$ Eq ② is an Exact Differential Equation,

then,

General Solution,

$$\Rightarrow \int M_1\,dx + \int N_1\,dy = c$$

y is constant $\qquad$ x is not involving

$$\Rightarrow \int (xy^4+y^2)\,dx + \int 2(\cancel{x^2y^3} + \cancel{xy} + y^5)\,dy = c$$

$$\Rightarrow y^4\int x\,dx + y^2\int dx + 2\int y^5\,dy = c$$

$$\Rightarrow \frac{x^2y^4}{2} + xy^2 + 2\left(\frac{y^6}{6}\right) = c$$

$$\Rightarrow \frac{3x^2y^4+6xy^2+2y^6}{6} = c$$

$$\Rightarrow \boxed{3x^2y^4+6xy^2+2y^6 = c}$$

4.

(B) Solve $\frac{dy}{dx} + \cot x \, y = \cos x$

Sol:- Given. $\frac{dy}{dx} + \cot x y = \cos x$

Compare with, $\frac{dy}{dx} + P.y = Q$

$\Rightarrow P = \cot x \quad Q = \cos x$

$\int p \, dx = \int \cot x \, dx = \log(\sin x)$

$I.F = e^{\int p dx} = e^{\log(\sin x)} = \sin x$

General solution,

$\Rightarrow y.(I.F) = \int Q \, (I.F) \, dx + C$

$\Rightarrow y.\sin x = \int \cos x . \sin x \, dx + c$

$\Rightarrow y.\sin x = \frac{1}{2} \int 2 \sin x . \cos x \, dx + c$

$\Rightarrow y.\sin x = \frac{1}{2} \int \sin 2x \, dx + C$

$\Rightarrow y.\sin x = \frac{1}{2}\left(\frac{-\cos 2x}{2}\right) + c$

$\Rightarrow \boxed{y.\sin x = \frac{-\cos 2x}{4} + c}$

5.

(A) Solve $x\frac{dy}{dx} + y = x^3 y^6$

Sol:- Given,

$x\frac{dy}{dx} + y = x^3 y^6$

$\frac{dy}{dx} + \frac{y}{x} = x^2 y^6$

$\frac{1}{y^6}\frac{dy}{dx} + \frac{y}{x} \cdot \frac{1}{y^{6\,5}} = x^2$

$\Rightarrow \frac{1}{y^6}\frac{dy}{dx} + \frac{1}{x}\left(\frac{1}{y^5}\right) = x^2 \quad -①$

$$\frac{1}{y^5} = u$$

$$\left.\begin{aligned} &\frac{d}{dy}\left(\frac{1}{y^5}\right)\frac{dy}{dx} = \frac{du}{dx} \\ &\frac{-1}{y^6}\frac{dy}{dx} = \frac{du}{dx} \\ &\frac{1}{y^6}\frac{dy}{dx} = \frac{-du}{dx} \end{aligned}\right\} \text{Substitute in eq①}$$

$$\Rightarrow \frac{-du}{dx} + \frac{1}{x}(u) = x^2$$

$$\Rightarrow \frac{du}{dx} - \frac{1}{x}\cdot u = -x^2$$

$$\Rightarrow \frac{du}{dx} + \left(\frac{-1}{x}\right)\cdot u = -x^2 \Rightarrow P = \frac{-1}{x}, \quad Q = -x^2$$

$$\int P\,dx = -\int \frac{1}{x}\,dx = -\log x = \log x^{-1}$$

$$I.F = e^{\int P\,dx} = e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$$

General Solution,

$$u(I.F) = \int Q(I.F)\,dx + c$$

$$u\cdot\frac{1}{x} = \int (-x^2)\frac{1}{x}\,dx + c$$

$$\frac{1}{y^5}\cdot\frac{1}{x} = -\int x\,dx + c$$

$$\frac{1}{xy^5} = -\frac{x^2}{2} + c$$

$$2 = -x^3y^5 + xy^5c$$

$$\Rightarrow x^3y^5 - xy^5c = -2$$

5.

(B) A body is originally $80^\circ c$ and cools down to $60^\circ c$ in 20 min, If the temperature of the air is $40^\circ c$. Then, find the temperature of the body after 40 min.

Sol. Let 'θ' be the temperature of the body at time 't'.

By Newton's law of cooling, we have

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

where, θ_0 is the temperature of the air

$\Rightarrow \theta_0 = 40^\circ$

$$\frac{d\theta}{dt} = -k(\theta - 40)$$

$$\Rightarrow \frac{d\theta}{(\theta - 40)} = -k\,dt$$

$$\int \frac{d\theta}{(\theta - 40)} = \int -k\,dt$$

$$\log(\theta - 40) = -kt + \log c$$

$$\log(\theta - 40) - \log c = -kt$$

$$\log\left(\frac{\theta - 40}{c}\right) = -kt$$

$$\frac{(\theta - 40)}{c} = e^{-kt}$$

$$\boxed{\theta - 40 = ce^{-kt}} \text{ — (1)}$$

where, $t = 0$, $\theta = 80^\circ c$

$$80 - 40 = ce^{-k(0)}$$

$$40 = ce^{0} \quad (e^0 = 1)$$

$$\boxed{c = 40}$$

when, $t = 20$, $\theta = 60^\circ C$

$$60 - 40 = ce^{-k(20)}$$

$$\cancel{20}^{1} = \cancel{40}_{2} e^{-20k}$$

$$1 = 2e^{-20k}$$

$$\boxed{e^{-20k} = \frac{1}{2}}$$

$$\cancel{20k = \log \frac{1}{2}}$$

$$\Rightarrow \boxed{k = \frac{1}{20} \log 2}$$

$$\frac{1}{2} = e^{-20k}$$

$$\log \frac{1}{2} = \log e^{-20k}$$

$$\log \frac{1}{2} = -20k$$

$$\log 1 - \log 2 = -20k$$

$$0 - \log 2 = -20k$$

$$\cancel{-}\log 2 = \cancel{-}20k$$

$$k = \frac{1}{20} \log 2$$

Substitute c & k values in eq ①

$$\theta - 40 = 40 e^{-\left(\frac{1}{20} \log 2\right) t} \quad -③$$

when, $t = 40$

then eq ③ becomes,

$$\theta - 40 = 40 e^{-\left(\frac{1}{\cancel{20}} \log 2\right) \cancel{40}^{2}}$$

$$\theta - 40 = 40 e^{-\log 4}$$

$$\theta - 40 = 40 \cancel{e}^{\cancel{\log} 4^{-1}}$$

$$\theta - 40 = \cancel{40}^{10}\left(\frac{1}{\cancel{4}}\right)$$

$$\theta = 10 + 40$$

$$\boxed{\theta = 50^\circ C}$$

6(A) The number N of bacterial in culture grew at a rate proportional to N. The value of N was initially 100 and increases to 332 in one hour. What was value of N after 1 ½ hours?

Sol:- We know that,

$$N = ce^{kt} \quad -(1)$$

where, c = initial number

t = time

k = constant

$c = 100$

$$N = 100\, e^{kt} \quad -(2)$$

$N = 332$, $t = 1$ hr in eq (2)

$$332 = 100\, e^{k(1)}$$

$$\Rightarrow e^{k} = \frac{332}{100}$$

$N = ?$, $t = 1 + \frac{1}{2} = \frac{3}{2}$ hr

$$N = 100\, e^{k \cdot 3/2}$$

$$= 100 \left(\frac{332}{100}\right)^{3/2}$$

$$\boxed{N = 605}$$ ~~approx~~ (ans)

3) If a voltage of $20 \cos 5t$ is applied to a series circuit consisting of 10Ω resistor and $2H$ inductor. Determine the current at any time t.

Let i be the current flowing in the circuit containing resistance R

$E = 20 \cos 5t$, $R = 10\,V$, $L = 2H$.

By voltage law, we have

$$\frac{di}{dt} + \frac{R}{L} i = E$$

$$\frac{di}{dt} + \frac{10}{2} i = 20 \cos 5t$$

ie $$\frac{di}{dt} + 5i = 20 \cos 5t$$

$$P = 5 , \; Q = 20 \cos 5t$$

I.F $\quad e^{\int P\,dt} = e^{\int 5\,dt} = e^{5t}$

G.S $\quad i\,(I.F) = \int Q\,(I.F)\,dt + c$

$$i\,(e^{5t}) = \int 20 \cos 5t \,(e^{5t})\,dt + c$$

$$\int e^{ax} \cos bx \,dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$$

$$i\,e^{5t} = 20 \int e^{5t} \cos 5t \,dt + c$$

$$i.\,e^{5t} = 20 \left[\frac{e^{5t}}{5^2+5^2}(5 \cos 5t + 5 \sin 5t)\right] + c$$

$$i.\,e^{5t} = 20 \left[\frac{e^{5t}}{50} 5 (\cos 5t + \sin 5t)\right] + c$$

$$i.\,e^{5t} = 2 e^{5t} (\cos 5t + \sin 5t) + c \quad \text{—} ①$$

sub $i = 0$, $t = 0$

$$0 = 2 e^{5(0)} (\cos 5(0) + \sin 5(0)) + c$$

$$0 = 2(1)(1+0) + c$$

$$0 = 2 + c \Rightarrow c = -2$$

from ① $\Rightarrow$ $\quad i\,e^{5t} = 2 e^{5t} (\cos 5t + \sin 5t) - 2.$

$$\boxed{i = 2 (\cos 5t + \sin 5t) - 2 e^{-5t}}$$

) Find the charge and current in RC circuit. If $R = 20\Omega$, $C = 0.01F$ and $v(t) = 20 \sin 2t$ with $q(0) = 0$

we are given,

$R = 20\Omega$, $C = 0.01F$, $E = 20 \sin 2t$

diff eqn of given circuit is