

2.) Numerical iterations

Notation: before each update I show m, b , predictions \hat{y} , errors e_i , gradients, then updated m, b and new MSE.

Iteration 1

Start with: $m_0 = -1, b_0 = 1$

Predictions:

- For $x_1 = 1 : \hat{y}_1 = m_0 \times 1 + b_0 = -1 \times 1 + 1 = 0 \Rightarrow \hat{y}_1 = 0$
- For $x_2 = 3 : \hat{y}_2 = -1 \times 3 + 1 = -2 \Rightarrow \hat{y}_2 = -2$

Errors: $e_i = y_i - \hat{y}_i$

- $e_1 = 3 - 0 = 3 \Rightarrow e_1 = 3$
- $e_2 = 6 - (-2) = 6 + 2 = 8 \Rightarrow e_2 = 8$

Compute gradients

$$* \frac{d\bar{\delta}}{dm} = -1 \sum x_i e_i$$

$$\frac{d\bar{\delta}}{dm} = -1 (1 \times 3 + 3 \times 8) = -1 (3 + 24)$$

$$\Rightarrow \frac{d\bar{\delta}}{dm} = -27$$

$$* \frac{d\bar{\delta}}{db} = -1 \sum e_i$$

$$= -1 (3 + 8)$$

$$\Rightarrow \frac{d\bar{\delta}}{db} = -11$$

update Parameters:

$$\cdot m_1 = m_0 - \alpha \frac{d\delta}{dm}$$

$$m_1 = -1 - 0.1 \times (-27)$$

$$= -1 + 2.7$$

$$\underline{\underline{m_1 = 1.7}}$$

$$\cdot b_1 = b_0 - \alpha \frac{d\delta}{db}$$

$$b_1 = 1 - 0.1 (-11)$$

$$= 1 + 1.1$$

$$\underline{\underline{b_1 = 2.1}}$$

Check predictions and MSE after update.

$$\cdot \hat{y}_1^{(1)} \neq 1.0$$

$$\cdot \hat{y}_1^{(1)} = m_1 \times x_1 + b_1$$

$$= 1.7 \times 1 + 2.1$$

$$\hat{y}_1^{(1)} = 1.7 + 2.1$$

$$\hat{y}_1^{(1)} = 3.8$$

• Error

$$e_1 = y_1 - \hat{y}_1$$

$$e_1 = 3 - 3.8$$

$$\underline{e_1 = -0.8}$$

$$\hat{y}_2^{(1)} = m_1 x_2 + b_1$$

$$\hat{y}_2^{(1)} = 1.7 \times 3 + 2.1$$

$$= 5.1 + 2.1$$

$$\underline{\hat{y}_2^{(1)} = 7.2}$$

Error e_2

$$e_2 = y_2 - \hat{y}_2$$

$$e_2 = 6 - 7.2$$

$$\underline{e_2 = -1.2}$$

MSE after iteration 1:

$$MSE_1 = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$MSE_1 = \frac{1}{2} \sum (e_i)^2 \quad \text{for } i = 1 \text{ and } y_i - \hat{y}_i = e_i$$

$$MSE_1 = \frac{(e_1)^2 + (e_2)^2}{2} = \frac{(-0.8)^2 + (-1.2)^2}{2} = \frac{0.64 + 1.44}{2} = \frac{2.08}{2}$$

$$\underline{MSE_1 = 1.04}$$