

# Hidden Markov Model

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Hidden states  $Z = \{z_1, z_2, \dots, z_T\}$

Observations  $X = \{x_1, x_2, \dots, x_T\}$

$\alpha(z_t) = P(x_1, \dots, x_t, z_t)$  ← Joint proba of observations 1- $t$  and hidden state  $z_t$

①  $\beta(z_t) = P(x_{t+1}, \dots, x_T | z_t)$  ← Proba of all future observations given hidden state  $z_t$

② We will show

Corrected mistake in the given formula

$$\xi(z_{t-1}, z_t) = P(z_{t-1}, z_t | X) = \frac{\alpha(z_{t-1}) P(x_t | z_t) P(z_t | z_{t-1}) \beta(z_t)}{P(X)}$$

Bayes Rule:  $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

Diagram illustrating the derivation of the forward algorithm formula:

Left side (Numerator):  $P(x_1, \dots, x_{t-1} | z_{t-1}) P(x_t | z_t) P(x_{t+1}, \dots, x_T | z_t) P(z_t | z_{t-1}) P(z_{t-1})$

Right side (Denominator):  $P(X)$

Diagram shows the separation of the joint probability into conditional probabilities and the use of Bayes' Rule to simplify the expression.

$$\frac{\alpha(z_{t-1}) \cdot P(x_t | z_t) \cdot P(z_t | z_{t-1}) \cdot \beta(z_t)}{P(X)}$$

③ Compute  $P(X)$

$$P(X) = \sum_{t=0}^T \alpha(z_t) \quad \text{i.e. Sum over all } \alpha$$