Hidden Markov Model

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Hidden states $Z = \{z_1, z_2 \dots, z_T\}$

Observations $X = \{x_1, x_2 \dots, x_T\}$

$\alpha(z_t) = P(x_1, \dots x_t, z_t)$ ← Joint proba of observations $1 \to t$ and hidden state z_t

(1) $\beta(z_t) = P(x_{t+1} \dots x_T | z_t)$ ← Proba of all future observations given hidden state z_t

We will show

$$\xi(z_{t-1}, z_t) = P(z_{t-1}, z_t | X) = \frac{\alpha(z_{t-1}) P(x_t|z_t) P(z_t|z_{t-1}) \beta(z_t)}{P(X)}$$

Corrected Mistake in the given formula

$$\Rightarrow \frac{P(X | z_{t-1}, z_t) \cdot P(z_{t-1}, z_t)}{P(X)}$$

Bayes Rule: $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

d-seperation

$$\frac{P(x_1, \dots, x_{t-1} | z_{t-1}) P(x_t|z_t) P(x_{t+1}, \dots, x_T | z_t) P(z_t | z_{t-1}) P(z_{t-1})}{P(X)}$$

$$\frac{\alpha(z_{t-1}) \cdot P(x_t|z_t) \cdot P(z_t|z_{t-1}) \cdot \beta(z_t)}{P(X)}$$

(2) Compute $P(X)$

$$P(X) = \sum_{t=0}^{T} \alpha(z_t) \quad \text{i.e. Sum over all } \alpha$$