$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

Compute eigenvalues, eigenvectors and A^m.

$\rightarrow \lambda \vec{v} = A\vec{v}$

$A\vec{v} - \lambda\vec{v} = (A - \lambda I) \cdot \vec{v} = 0$

$\rightarrow \det(A - \lambda I) = 0$

$$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = -\lambda^3 + 5\lambda^2 + 4\lambda - 20 = -(\lambda+2)\cdot(\lambda-2)\cdot(\lambda-5) = 0$$

Eigenvalues: $\lambda_1 = -2$; $\lambda_2 = 2$; $\lambda_3 = 5$

$$1 \rightarrow \begin{bmatrix} 3 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \vec{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$2 \rightarrow \begin{bmatrix} -1 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \vec{v}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$3 \rightarrow \begin{bmatrix} -4 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^m = A_1 \cdot A_2 \cdot \ldots A_m \left\{ A^1 = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix} \right. \begin{matrix} (1 \cdot 5 - 2^1) \\ (1 \cdot 5 - 2^1) - 2^1 \end{matrix}$$

$$A^2 = \begin{bmatrix} 11 & 7 & 7 \\ 7 & 11 & 7 \\ 7 & 7 & 11 \end{bmatrix} \begin{matrix} (3 \cdot 5 - 2^2) - 2^2 \\ \underbrace{(3 \cdot 5 - 2^2)} \end{matrix}$$

$$A^3 = \begin{bmatrix} 39 & 39 & 47 \\ 39 & 47 & 39 \\ 47 & 39 & 39 \end{bmatrix} \begin{matrix} (11 \cdot 5 - 2^3) \\ (11 \cdot 5 - 2^3) - 2^3 \end{matrix}$$

$$A^4 = \begin{bmatrix} 219 & 203 & 203 \\ 203 & 219 & 203 \\ 203 & 203 & 219 \end{bmatrix} \begin{matrix} (47 \cdot 5 - 2^4) - 2^4 \\ (47 \cdot 5 - 2^4) \end{matrix}$$

$x_{max}^m = x_{max}^{m-1} \cdot 5 - 2^m$

$\Rightarrow x_{min}^m = (x_{max}^{m-1} \cdot 5 - 2^m) - 2^m$

$m=5$ $x_{max}^5 = 219 \cdot 5 - 2^5$

$\rightarrow x_{min} \{ (219 \cdot 5 - 2^5) - 2^5$

$$\Rightarrow A^5 = \begin{bmatrix} (219\cdot5-2^5)-2^5 & (219\cdot5-2^5)-2^5 & 219\cdot5-2^5 \\ (219\cdot5-2^5)-2^5 & 219\cdot5-2^5 & (219\cdot5-2^5)-2^5 \\ 219\cdot5-2^5 & (219\cdot5-2^5)-2^5 & (219\cdot5-2^5)-2^5 \end{bmatrix}$$

$\max \quad z = x_1 + 2x_2 + 3x_3$

s.t. $\quad x_1 + x_2 \leq 4 \qquad x_3 \leq 5 \qquad x_1, x_2, x_3 \geq 0$

Without Simplex:

$\rightarrow \underline{0} + 2 \cdot \underline{4} + 3 \cdot \underline{5} = 23$

With Simplex:

	z	x_1	x_2	x_3	s_1	s_2	b	
s_1	0	1	1	0	1	0	4	$4/0 = \infty$
s_2	0	0	0	1	0	1	5	$5/1 = 5 \leftarrow$
	1	-1	-2	-3	0	0	0	

	z	x_1	x_2	x_3	s_1	s_2	b	
s_1	0	1	1	0	1	0	4	$4/1 = 4 \leftarrow$
x_3	0	0	0	1	0	1	5	$5/0 = \infty$
	1	-1	-2	0	0	3	15	

	z	x_1	x_2	x_3	s_1	s_2	b
x_2	0	1	1	0	1	0	4
x_3	0	0	0	1	0	1	5
	1	1	0	0	2	3	23

$\Rightarrow x_1 = 0 \,;\, x_2 = 4 \,;\, x_3 = 5 \quad \Rightarrow z = 23$

Show that the binary bayes classifier is linear & give its function.

• The features are normally distributed and the prior probabilities are $p_1 = p_2 = \frac{1}{2}$

$$P(y=1|x) \geq P(y=0|x)$$

$$\frac{P(x|y=1)\cdot P(y=1)}{P(x|y=0)\cdot P(y=0)} \geq 1$$

by naive bayes assumptions $P(x|y) = \prod_{i=1}^{N} P(x_i|y)$

$$\frac{P(y=1)}{P(y=0)} \cdot \prod_{i=1}^{N} \frac{P(x_i|y=1)}{P(x_i|y=0)} \geq 1$$

Let $p = P(y=1)$; $P(x_i|y)$ is normally distributed:

$$\frac{p}{1-p} + \prod_{i=1}^{N} \frac{\cancel{\frac{1}{\sqrt{2}^{d}\cdot\sqrt{\det(\Sigma)}}}\cdot \exp\left(-\frac{1}{2}(x-\mu_{i1})\Sigma_1^{-1}(x-\mu_{i1})\right)}{\cancel{\sqrt{2}^{d}\cdot\sqrt{\det(\Sigma)}}\cdot \exp\left(-\frac{1}{2}(x-\mu_{i0})\Sigma_0^{-1}(x-\mu_{i0})\right)} \geq 0$$

Take log

$$\log\left(\frac{p}{1-p}\right) + \prod_{i=1}^{N} \left(-\frac{1}{2}(x-\mu_{i1})\cancel{\Sigma_1^{-1}}(x-\mu_{i1})\} - \left(-\frac{1}{2}(x-\mu_{i0})\cancel{\Sigma_0^{-1}}(x-\mu_{i0})\right)\right) \geq 0$$

$$\log\left(\frac{p}{1-p}\right) + \prod_{i=1}^{N} \left(\cancel{-\frac{1}{2}x^2} + x_i\mu_{i1} + \frac{1}{2}\mu_{i1}^2\right) - \left(\cancel{-\frac{1}{2}x_i^2} + x_i\mu_{i0} + \frac{1}{2}\mu_{i0}^2\right. \geq 0$$

$$\underbrace{\log\left(\frac{p}{1-p}\right)}_{0} + \prod_{i=1}^{N} \left(x_i \cdot (u_{i1} - u_{i0}) + \frac{1}{2}u_{i1}^2 - \frac{1}{2}u_{i0}^2\right) \geq 0$$

if $p = 0.5$

$$\rightarrow \prod_{i=1}^{N} \left(\underbrace{x_i}_{x} \cdot \underbrace{(u_{i1} - u_{i0})}_{w} + \underbrace{\frac{1}{2}u_{i1}^2 - \frac{1}{2}u_{i0}^2}_{b}\right) \geq 0$$

$x \cdot w + b \Rightarrow$ Linear