Master.

SHENTON
COLLEGE

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section One: Calculator-free

Your name: SOLUTION

Teacher name (circle one): Ai Friday Smith

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	8	8	50	51
Section Two: Calculator-assumed	13	13	100	96

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	6	
2	8	
3	8	
4	5	
5	4	
6	8	
7	5	
8	7	
S1 Total	51	

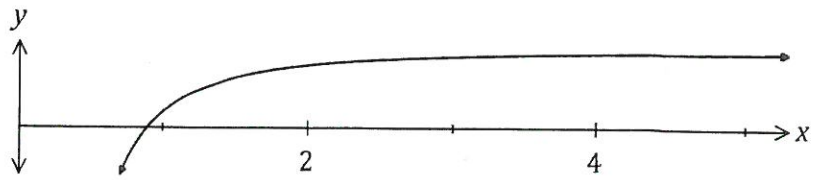
Section One: Calculator-free **(51 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

The curve shown below passes through the point $(1, 2)$ and is such that $\frac{dy}{dx} = \frac{16}{x^3}$.



(a) Determine the equation of the curve. (3 marks)

Remember +c

$\frac{dy}{dx} = \frac{16}{x^3}$

$y = \int \frac{16}{x^3}\,dx$ ✓ integrates derivative

$y = -\frac{8}{x^2} + c$

at $(1,2)$ $2 = -8 + c$ ✓ determines constant

$c = 10$

$y = -\frac{8}{x^2} + 10$ ✓ states equation

(b) Determine the area of the region enclosed by the curve, the x-axis, the line $x = 1$ and the line $x = 4$. (3 marks)

Area under curve $\int f(x)\,dx$.

3 mark Qn requires process

$A = \int_1^4 -\frac{8}{x^2} + 10\,dx$ ✓ Writes integral with limits

$= \left[\frac{8}{x} + 10x\right]_1^4$ ✓ integrates

$= (2 + 40) - (8 + 10)$

$= 42 - 18$

$= 24$ square units. ✓ evaluates and obtains area

Question 2 **(8 marks)**

A calculator program will generate a single random integer n, where $3 \leq n \leq 12$. The program is run once, and the discrete random variable X is the number of fours or fives obtained.

(a) Explain why X is a Bernoulli random variable. (2 mark)

Two mutually exclusive and exhaustive outcomes from a single trial.

2 possible values $X = \begin{cases} 1 & \text{if outcome 4 or 5} \\ 0 & \text{if outcome 3, 6, 7, 8, 9, 10, 11, 12} \end{cases}$

✓ Mentions M.E.E. exhaustive

✓ 1 0 concept

2 marks awarded for VERY extensive answer

(b) Determine $P(X = 1)$. (1 mark)

$$P(X=1) = \frac{2}{10} = \frac{1}{5}$$

✓ correct probability

(c) Determine the mean and standard deviation of X. (2 marks)

$$\bar{x} = \frac{2}{10} = \frac{1}{5}$$

$$\sigma_x = \sqrt{\frac{1}{5} \cdot \frac{4}{5}}$$

$$= \frac{2}{5}$$

✓ $\bar{x}$

✓ σ_x

Standard deviation ✓

The random variable Y is the number of fours or fives obtained in three consecutive runs of the program.

(d) Determine $P(Y \leq 1)$. (3 marks)

$$P(Y \leq 1) = P(Y=0) + P(Y=1)$$

$$= \left(\frac{4}{5}\right)^3 + \left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) \times 3$$

$$= \frac{64}{125} + \frac{48}{125}$$

$$= \frac{112}{125}$$

✓ $P(Y=0)$

✓ $P(Y=1)$

✓ correct prob.

Clear reasoning for 3 marks.

Process needs to be clear

Fraction work poor

Question 3

(a) Determine **(8 marks)**

(i) $\frac{d}{dx}\frac{\cos(4x)}{x^3}$ (2 marks)

$= \frac{x^3.(-4\sin(4x)) - \cos(4x).3x^2}{x^6}$

Well done by many. Quotient Rule on formula sheet

✓ applies quotient rule

✓ differentiates numerator correctly

(ii) $\frac{d}{dx}\int_x^5 (2\theta+5)^4\, d\theta$. (2 marks)

$= -\frac{d}{dx}\int_5^x (2\theta+5)^4\, d\theta$

$= -(2x+5)^4$

$\frac{d}{dx}$ Use of x not θ

✓ swaps limits and negates expression

✓ simplifies using 'x'

(b) Find the function A(t) given that $A(t) = \int_{\frac{1}{2}}^{t} \frac{d}{dx}\, x^2\sqrt{1-x^2}\, dx$. (2 marks)

$A(t) = \left[t^2\sqrt{1-t^2}\right]_{\frac{1}{2}}^{t}$

$= t^2\sqrt{1-t^2} - \frac{1}{4}\sqrt{1-\frac{1}{4}}$

$= t^2\sqrt{1-t^2} - \frac{\sqrt{3}}{8}$

Must have understanding of relationship between $\int_{\frac{1}{2}}^{t} \frac{d}{dx}(f(x)\, dx)$

✓ correct use of definite integral

✓ evaluates definite integral

(c) Determine $\int (8x+11)^3\, dx$. (2 marks)

$= \frac{(8x+11)^4}{32} + c$

Well done Some forgot to $\div 8$ by $\frac{d}{dx}(8x+11)$

✓ $(ax+b)^n$ antidifferentiated correctly. $\frac{(ax+b)^{n+1}}{n+1}$

✓ differentiates $8x+11$ to obtain correct integral + c

Question 4 **(5 marks)**

Let $f(x) = 5x + \frac{k}{2x}$, $x < 0$ and k is a constant. The graph of $y = f(x)$ has a stationary point when $x = -3$.

(a) Determine the value of k. (2 marks)

Stationary point $\therefore f'(-3) = 0$

$f'(x) = 5 - \frac{k}{2x^2}$ ✓ $f'(x)$

$f'(-3) = 5 - \frac{k}{18} = 0$

$5 = \frac{k}{18}$

$k = 90$ ✓ value of k

No follow through from incorrect derivative

$f(x) = 5x + \frac{k}{2}x$ ↑ constant

(b) Use the second derivative test to determine the nature of the stationary point. (3 marks)

$f'(x) = 5 - \frac{45}{x^2}$

$f''(x) = \frac{90}{x^3}$ ✓ $f''(x)$

$f''(-3) < 0$ ✓ uses < 0 test

$\therefore$ the stationary point is a Maximum. ✓ Gives correct nature.

Algebra is poor

In 3 mark Q state clearly WHY stationary point is a Max using 2nd derivative test.

Question 5 **(4 marks)**

A random variable X has a binomial probability distribution with a mean of 12 and variance of 3

(a) Determine the value of n and the value of p for this distribution. (2 marks)

$np = 12$

$np(1-p) = 3$

ie $12(1-p) = 3$

$1-p = \frac{1}{4}$

$p = \frac{3}{4}$

$\frac{3}{4}n = 12$

$n = 16$

✓ value of p

✓ value of n

(b) Determine the mean and variance of the distribution Y, where $Y = 5X + 3.3$. (2 marks)

$\bar{y} = 5(\bar{x}) + 3.3$

$= 5(12) + 3.3$

$= 63.3$

✓ $\bar{y}$

$\sigma_y^2 = 5^2 \times 3$

$= 75$

✓ var

σ^2 for variance not well known

Question 6 **(8 marks)**

A vehicle travelling in a straight line has a velocity of 10 ms^{-1} as it leaves point Q. The acceleration of the vehicle is given by $3 - 2t$ ms^{-2}, where t is the time in seconds since the vehicle left Q.

(a) Determine the velocity of the vehicle when $t = 4$. (3 marks)

3 marks Show steps

$v(t) = \int 3 - 2t \, dt$

$= 3t - t^2 + c$ ✓ integrates

$t = 0 \quad c = 10$

$v = 10 ms^{-1}$

$v(t) = 3t - t^2 + 10$ ✓ solves for c

$v(4) = 6 ms^{-1}$ ✓ $v(4) =$

(b) Determine how far from Q the vehicle first comes to rest for $t > 0$. (5 marks)

$v(t) = 0$ ✓ identifies $v = 0$

$0 = -t^2 + 3t + 10$

$0 = -(t - 5)(t + 2)$

$t = 5$ or $t = -2$ ✓ factorises and solves

$t > 0 \therefore t = 5$

Factorisation poor

Give both options and reject - 5 mark Q requires clear steps

Use of definite integral to be recommended

$x(t) = \int_0^5 -t^2 + 3t + 10 \, dt$ ✓ integral for $x(t)$

$= \left[\frac{-t^3}{3} + \frac{3t^2}{2} + 10t \right]_0^5$

$= \left(\frac{-125}{3} + \frac{75}{2} + 50 \right) - 0$ ✓ integrates and substitutes

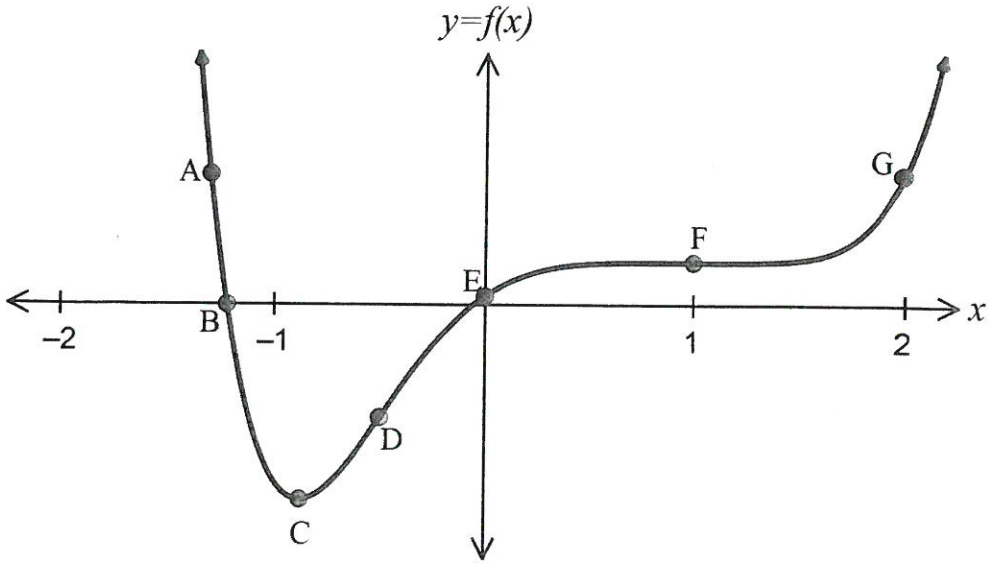
$= -\frac{250}{6} + \frac{225}{6} + \frac{300}{6}$

$= \frac{275}{6}$ m ✓ distance

Vehicle is $\frac{275}{6}$ m from Q

Question 7 **(5 marks)**

Consider the function $y = f(x)$ shown below. The points, A, B, C, D, E, F and G each lie on the graph.



Understanding in this question needs attention

(a) Which point/s labelled on the graph above satisfy the following,

(i) a point of inflection occurs? (1 mark)

D, F

states correct point/points ✓

(ii) $f'(x) = 0$ and $f''(x) \neq 0$? (1 mark)

C ✓

(iii) f is increasing and $f''(x) < 0$? (1 mark)

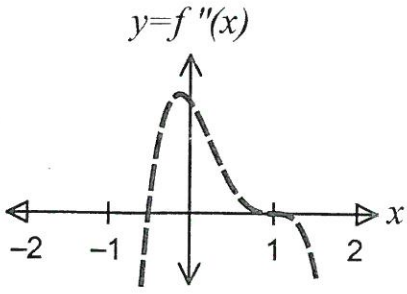
E ✓

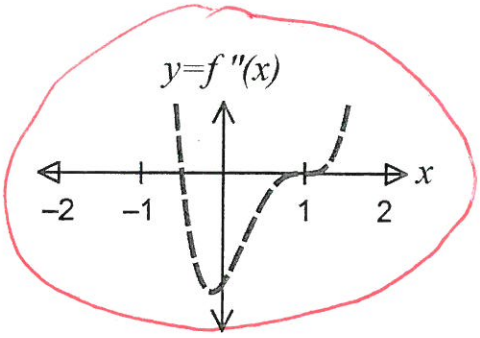
(iv) $f(x) > 0$ and the function is concave up? (1 mark)

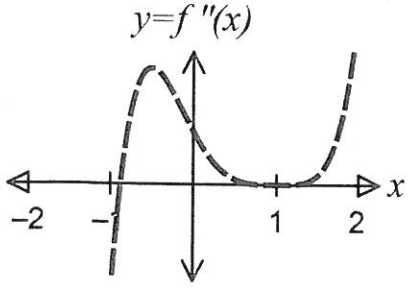
A, G ✓

(b) Circle the graph below that represents $f''(x)$. (1 mark)

circles correct graph ✓







Question 8 **(7 marks)**

(a) Determine $\frac{d}{dx}(2x\sqrt{4+x})$. (2 marks)

Well done use of Product Rule

$$\frac{d}{dx}\ 2x\sqrt{4+x}$$

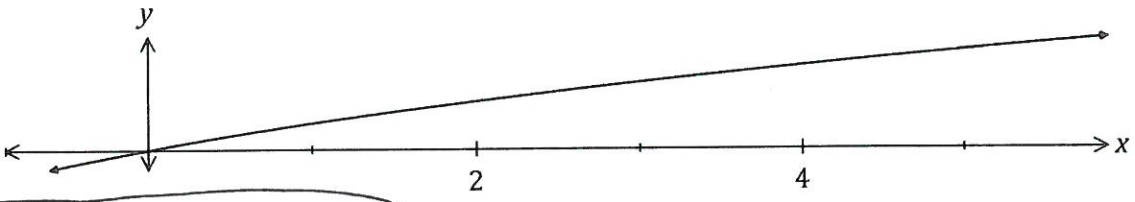
$$= 2x \cdot \tfrac{1}{2}(4+x)^{-\frac{1}{2}}(1) + \sqrt{4+x}\,.(2)$$

$$= \frac{x}{\sqrt{4+x}} + 2\sqrt{4+x}$$

✓ applies product rule

✓ applies chain rule

(b) Part of the graph of $y = \frac{x}{\sqrt{4+x}}$ is shown below.



Ability to see relationship ately

Using your answer from part (a) or otherwise, determine $\int_0^5 \frac{x}{\sqrt{4+x}}\,dx$. (5 marks)

$$\int_0^5 \frac{x}{\sqrt{4+x}}\,dx = \int_0^5 \frac{d}{dx}\ 2x\sqrt{4+x}\,dx - \int_0^5 2\sqrt{4+x}\,dx$$

$$= \left[2x\sqrt{4+x}\right]_0^5 - \left[\frac{2(4+x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^5$$

$$= \left[2x\sqrt{4+x}\right]_0^5 - \left[\tfrac{4}{3}(4+x)^{\frac{3}{2}}\right]_0^5$$

Need to lay out clear steps for 5 marks

$$= \left(10\sqrt{9} - 0\right) - \tfrac{4}{3}\left[9^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$$

$$= 30 - \tfrac{4}{3}(27 - 8)$$

$$= 30 - \tfrac{4}{3}(19)$$

Number work not strong

$$= \frac{90}{3} - \frac{76}{3}$$

$$= \frac{14}{3} \text{ sq units.}$$

✓ writes equation using answer from a

✓ uses $\int f'(x)\,dx = f(x)$

✓ $\int 2\sqrt{4+x}\,dx$

✓ substitutes bounds

✓ correct area

End of Section 1 Questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____