Test 2

Area Under Curve, F.T.O.C. Exponential Functions
Semester One 2018
Year 12 Mathematics Methods
Calculator Assumed

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Teacher:

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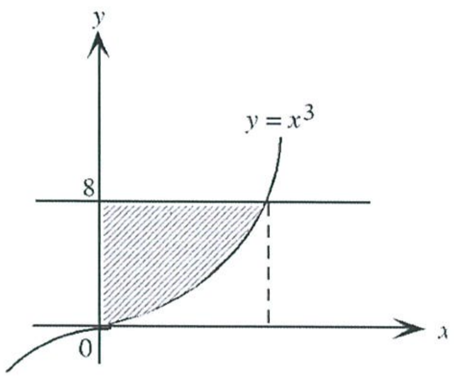
Date: Friday 16th March 7.45am

You may have a formula sheet for this section of the test.

Total________/40 **45 minutes +5 minutes READING**

Question 1 **(2 marks)**

The graphs with equations $y = x^3$ and $y = 8$ are shown. Write an expression that shows what the area of the shaded region is equal to:



$x^3 = 8$

$x = 2$

$A = \int_0^2 |8 - x^3| \, dx$ ✓ ✓

Question 2 **(5 marks)**

(a) Calculate $f'(0)$ when $f(x) = e^{2x}(1+5x)^3$. (3 marks)

$f'(x) = 2e^{2x}(1+5x)^3 + e^{2x} \times 3(1+5x)^2 \times 5$ ✓

$f'(0) = 2e^0(1+0)^3 + e^0 \times 15 \times (1+0)^2$ ✓ (substitute)

$= 2 \times 1 \times 1 + 1 \times 15 \times 1$

$= 2 + 15$

$= 17$ ✓

(b) Determine $\frac{d}{dx}\int_x^5 \sqrt{t^2+1}\, dt$. (2 marks)

$= -\frac{d}{dx}\int_5^x \sqrt{t^2+1}\, dt$

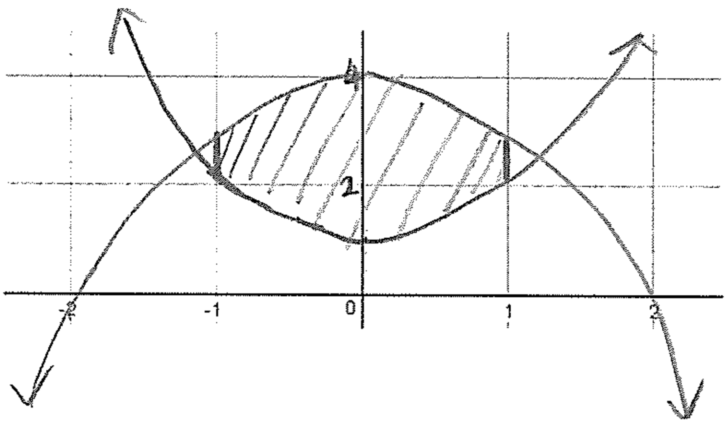
$= -\sqrt{x^2+1}$

✓ ✓

Question 3 **(4 marks)**

Show how to calculate the area of the region enclosed by the curves with equations $y = x^2 + 1$ and $y = 4 - x^2$ and the lines $x = -1$ and $x = 1$.

Draw a sketch to help show your solution. Show your working.



$x^2+1 = 4 - x^2$

$2x^2 = 3$

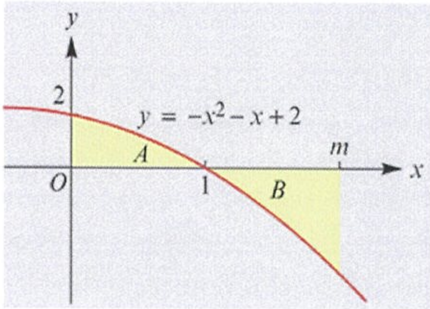
$x = \pm\sqrt{\frac{3}{2}}$

$$\int_{-1}^{1} \left|(4-x^2) - (x^2+1)\right| dx = \frac{14}{3}$$

Question 4 **(4 marks)**

The graph of $y = -x^2 - x + 2$ shown

Find the value of m such that A and B have the same area.



$$\text{Area of } A = \int_0^1 \left| -x^2 - x + 2 \right| dx = \frac{7}{6}$$

$$B = \int_1^m (-x^2 - x + 2)\, dx = -\frac{7}{6}$$

$$\Rightarrow \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x + c \right]_1^m = -\frac{7}{6}$$

$$\Rightarrow \left(-\frac{m^3}{3} - \frac{m^2}{2} + 2m + c \right) - \left(-\frac{1}{3} - \frac{1}{2} + 2 + c \right) = -\frac{7}{6}$$

$\therefore m = 1.81 \quad (m > 1)$

Question 5 **(4 marks)**

Given $\frac{dy}{dx} = ae^{-x} + 2$ and that when $x = 0$, $\frac{dy}{dx} = 5$ and $y = 1$,

Find the value of y when $x = 2$.

$x = 0 \quad \frac{dy}{dx} = ae^0 + 2 = a + 2 = 5 \quad \therefore a = 3$

$\frac{dy}{dx} = 3e^{-x} + 2$

$y = -3e^{-x} + 2x + c$

$x = 0, \quad y = -3e^0 + 0 + c = 1 \quad \therefore c = 4$

$x = 2, \quad y = -3e^{-2} + 2 \times 2 + 4$

$= -3e^{-2} + 8$

≈ 7.59

Question 6 **(8 marks)**

A group of biologists has decided that colonies of a native Australian animal are in danger if their populations are less than 1000. One such colony had a population of 2300at the start of 2011. The population was growing continuously such that $P = P_0 e^{0.065t}$ where P is the number of animals in the colony t years after the start of 2011.

(a) Determine, to the nearest 10 animals, the population of the colony at the start of 2014. (2 marks)

$P = 2300 \times e^{0.065 \times 3}$

≈ 2795.2 ✓

≈ 2800 ✓

(b) Determine the rate of change of the colony's population when $t = 2.5$ years. (2 marks)

$P' = 0.065 \times P_0 e^{0.065 \times 2.5}$ ✓

$= 0.065 \times 2300 \times e^{0.065 \times 2.5}$

≈ 175.879 (ok).

≈ 176 ✓

(c) At the beginning of 2017, a disease caused the colony's population to decrease continuously at the rate of 8.25% of the population per year. If this rate continues, when will the colony become "in danger"? Give your answer to the nearest month. (4 marks)

$P(6) = 2300 \times e^{0.065 \times 6}$

≈ 3397 ✓

From 2017:

$P(t) = 3397 e^{-0.0825t} = 1000$ ✓

$t = 14.8$ ✓ $(0.8 \times 12 = 9.6 \Rightarrow$ 10th month$)$

During October 2031. ✓

Question 7 **(9 marks)**

(a) What is the sign of $f(x) = x^3 - 6x^2 + 12x - 8$ from $x = 0$ to $x = 2$? (1 mark)

Negative ✓

(b) What is the sign of $f(x) = x^3 - 6x^2 + 12x - 8$ from $x = 2$ to $x = 4$? (1 mark)

positive ✓

(c) Find $\int_0^4 (x^3 - 6x^2 + 12x - 8)dx$. (2 mark)

$= 0$ ✓

(d) Find $\int_0^2 (x^3 - 6x^2 + 12x - 8)dx$. (2 mark)

$= -4$ ✓

(e) What is the area between $f(x) = x^3 - 6x^2 + 12x - 8$ and the $x-$axis from $x = 0$ to $x = 4$? (2 marks)

$\int_2^4 f(x)\,dx = \int_0^4 f(x)\,dx - \int_0^2 f(x)\,dx = 4$

$\therefore$ Area $= |-4| + 4 = 8$ ✓✓

(f) Explain why the answers to (c) and (e) are different. (1 marks)

Because the part from 0 to 2 is below the axis and part from 2 to 4 is above ✓

Question 8 **(6 ~~5~~ marks)**

The population of mice in a closed habitat is known to increase according to the function:

$P'(t) = \frac{t}{3} + 6$, where $P'(t)$ is measured in hundreds of mice per month and t is measured in months. The measurement of the population commences at $t = 0$,

(a) What is the total change in the population in the first 3 months after measuring commenced? (2 marks)

$$\int_0^3 \frac{t}{3} + 6 \, dt = 19.5$$ ✓

$$= 1950$$ ✓

(b) How long will it take for the increase in the population of mice to reach 4200? (2 marks)

$$\int_0^x \frac{t}{3} + 6 \, dt = 42$$ ✓

$$\left.\frac{t^2}{6} + 6t \right]_0^x = 42$$

$$\frac{x^2}{6} + 6x = 42$$

$$x = 6$$

After 6 months. ✓