

Perth College

Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNITS 1 AND 2

Section One:
Calculator-free

If required by your examination administrator, please
place your student identification label in this box

Student number: In figures

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In words

Your name

Solutions.

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	54	35
Section Two: Calculator-assumed	13	13	100	96	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (54 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(7 marks)

(a) The first four terms of a sequence are -1 , 3 , -9 and 27 .

(i) Write a recursive rule for the sequence above.

(1 mark)

$$T_{n+1} = -3T_n \quad T_1 = -1$$

(ii) Write a rule for the n^{th} term.

(1 mark)

$$T_n = -1(-3)^{n-1}$$

(iii) Determine the value of T_5 .

(1 mark)

$$-81 \checkmark$$

(b) The first three terms, in order, of an arithmetic sequence are $4x + 3$, $2x - 1$ and $x - 8$. Determine the fourth term of the sequence.

(4 marks)

$$2x - 1 - (4x + 3) = x - 8 - (2x - 1) \checkmark$$

$$2x - 1 - 4x - 3 = x - 8 - 2x + 1$$

$$-2x - 4 = -x - 7 \checkmark$$

$$-x = -3$$

$$\boxed{x = 3} \checkmark$$

$$\therefore 15, 5, -5, -15$$

$$\begin{array}{ccc} \curvearrowright & \curvearrowright & \curvearrowright \\ -10 & -10 & -10 \end{array}$$

$$\boxed{T_4 = -15} \checkmark$$

Question 2

(6 marks)

(a) State the 4th term of $(1 - 2x)^{10}$.

(3 marks)

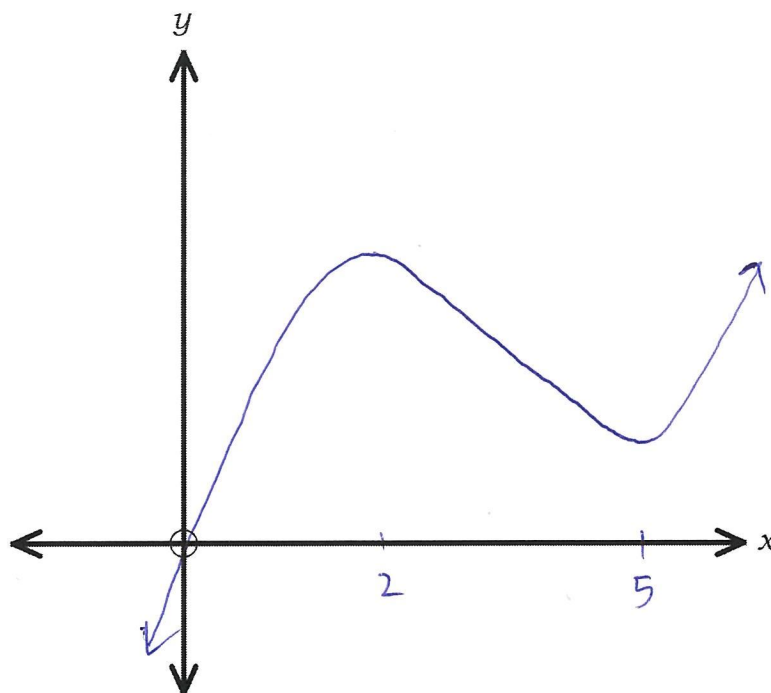
$$\binom{10}{3} (1)^7 (-2x)^3$$
$$= -960x^3$$

(b) Sketch the graph of a function that satisfies all conditions below:

(3 marks)

- has one root, which is the origin,
- $\frac{dy}{dx} = 0$ when $x = 2$ and $x = 5$,
- $\frac{dy}{dx} < 0$ when $2 < x < 5$
- $\frac{dy}{dx} > 0$ otherwise

-1 / condition not met.



Question 3

(8 marks)

Solve each equation below for x .

(a) $\frac{4x}{x-7} = \frac{3}{2}$

(2 marks)

$$8x = 3(x-7) \quad \checkmark$$

$$8x = 3x - 21$$

$$5x = -21$$

$$x = -\frac{21}{5} \quad \checkmark$$

(b) $(x+2)(x-2) = 3x$

(3 marks)

$$x^2 - 4 - 3x = 0 \quad \checkmark$$

$$(x-4)(x+1) = 0 \quad \checkmark$$

$$x = 4, x = -1 \quad \checkmark$$

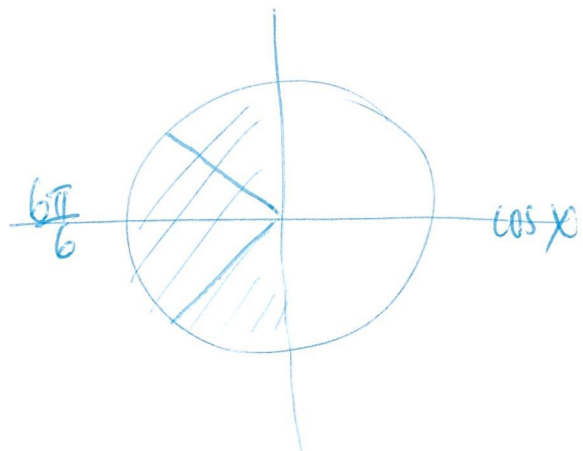
(c) $2 \cos x + \sqrt{3} = 0, 0 \leq x \leq 2\pi$

(3 marks)

$$\cos x = -\frac{\sqrt{3}}{2} \quad \checkmark$$

$$\text{ref } \theta = \frac{\pi}{6} \quad \checkmark$$

$$\therefore x = \frac{5\pi}{6}, \frac{7\pi}{6} \quad \frac{1}{2} \quad \frac{1}{2}$$



Question 4

(7 marks)

(a) Simplify

(i) $\frac{d}{dx}(2x^5 + 6x - 7).$ (1 mark)

$$10x^4 + 6$$



(ii) $\lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}.$ (1 mark)

$$4x^3$$



(b) Determine the gradient of the tangent to the curve $y = x^3 - 4x + 3$ when $x = -2$. (2 marks)

$$\frac{dy}{dx} = 3x^2 - 4$$



$$= 3(-2)^2 - 4$$

$$= 8$$



(c) Determine $f(x)$ given $f'(x) = 4x - 5$ and $f(2) = -3$. (3 marks)

$$f(x) = 2x^2 - 5x + C$$



$$-3 = 2(2)^2 - 5(2) + C$$

$$\boxed{C = -1}$$



$$\therefore f(x) = 2x^2 - 5x - 1$$



Question 5

(6 marks)

The derivative of a cubic polynomial is given by $\frac{dy}{dx} = 3x^2 - 2x - 24$.

The cubic passes through the point $(-1, -14)$.

(a) Determine the equation of the cubic.

(2 marks)

$$y = x^3 - x^2 - 24x + c \quad \checkmark$$

$$-14 = -1 - 1 + 24 + c$$

$$\boxed{c = -36}$$

$$\boxed{y = x^3 - x^2 - 24x - 36} \quad \checkmark$$

(b) Show that the cubic has a root when $x = -2$.

(1 mark)

$$y = (-2)^3 - (-2)^2 - 24(-2) - 36 \quad \checkmark$$

$$= 0$$

(c) Determine the other two roots of the cubic.

(3 marks)

$$\begin{array}{r} x^2 - 3x - 18 \\ (x+2) \overline{) x^3 - x^2 - 24x - 36} \\ \underline{-(x^3 + 2x^2)} \\ -3x^2 - 24x \\ \underline{-(-3x^2 - 6x)} \\ -18x - 36 \end{array}$$

$$x^2 - 3x - 18 \quad \checkmark$$

$$= (x-6)(x+3) \quad \checkmark$$

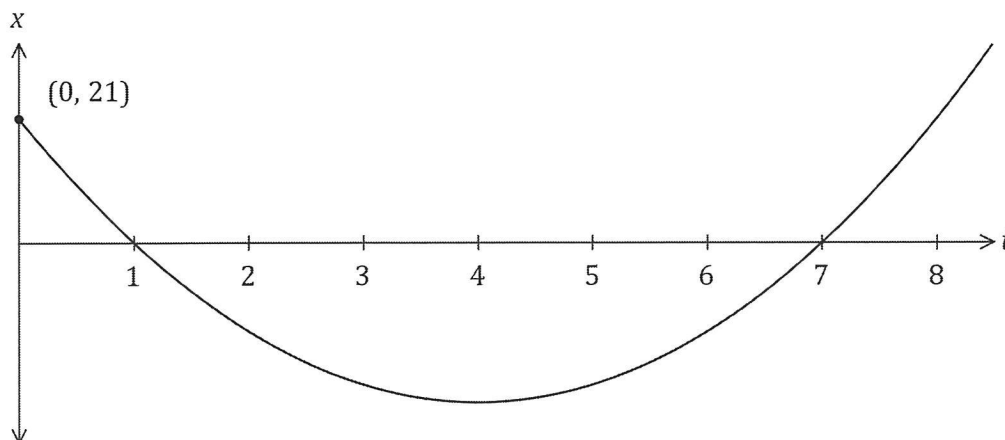
\therefore other roots are
 $x = 6, x = -3$
 $\checkmark_2 \quad \checkmark_2$

Question 6

(6 marks)

A small body moves in a straight line so that its displacement x from a fixed point O after t seconds is given by $x = at^2 + bt + c$ metres.

The position-time graph of the body is shown below.



- (a) Determine the values of the constants a , b and c .

(3 marks)

$$c = 21 \quad \checkmark$$

$$a = 3 \quad \checkmark$$

$$b = -24 \quad \checkmark$$

$$x = 3(t-1)(t-7)$$

$$= 3(t^2 - 8t + 7)$$

$$= 3t^2 - 24t + 21$$

- (b) Determine the displacement of the body when its velocity is 24 ms^{-1} .

(3 marks)

$$\frac{dx}{dt} = 6t - 24 \quad \checkmark$$

$$24 = 6t - 24$$

$$\boxed{t = 8} \quad \checkmark$$

$$x = 3(8)^2 - 24(8) + 21$$

$$= 192 - 192 + 21$$

$$\boxed{x = 21 \text{ m}} \quad \checkmark$$

Question 7

(6 marks)

- (a) Evaluate $\frac{m^{0.5}}{n^2}$ when $m = 4 \times 10^6$ and $n = 5 \times 10^2$, writing your answer in simplest form without the use of scientific notation. (3 marks)

$$\frac{(4 \times 10^6)^{0.5}}{(5 \times 10^2)^2} = \frac{2 \times 10^3}{25 \times 10^4}$$

$$= \frac{2}{250}$$

$$= \frac{1}{125}$$

✓ correctly expands.

✓ correctly simplifies power of 10.

✓ simplifies.

- (b) Determine the value of x when $4^x = 32\sqrt{2}$. (3 marks)

$$(2^2)^x = 2^5 \cdot 2^{\frac{1}{2}}$$

$$2x = 5\frac{1}{2}$$

$$2x = \frac{11}{2}$$

$$x = \frac{11}{4}$$

✓ recognises powers of 2

✓ correct use of index laws

✓ solves equation

Question 8

(8 marks)

Let $f(x) = x^2 + x$.

- (a) Determine the average rate of change when
- $x = 3$
- and
- $x = 5$
- .

(3 marks)

$$f(5) = 25 + 5 \\ = 30 \quad \checkmark$$

$$\text{av rate} = \frac{30 - 12}{2}$$

$$f(3) = 9 + 3 \\ = 12 \quad \checkmark$$

$$= 9 \quad \checkmark$$

- (b) Use first principles to determine the rate of change function
- $f'(x)$

(5 marks)

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h}$$

✓ correct substitution
✓ bracket.

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h}$$

✓ expands

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(2x + h + 1)}{\cancel{h}}$$

✓ factorises.

$$f'(x) = 2x + 1$$

✓ correctly states
 $f'(x)$
(must be $2x + 1$)

Supplementary page

Question number: _____

Markers use only		
Question	Maximum	Mark
1	7	
2	6	
3	8	
4	7	
5	6	
6	6	
7	6	
8	8	
S1 Total	54	
S1 Weighted	35%	
S2 Weighted	65%	
Total	100%	