Rossmoyne Senior High School

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

See next page

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	54	35
Section Two: Calculator-assumed	13	13	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

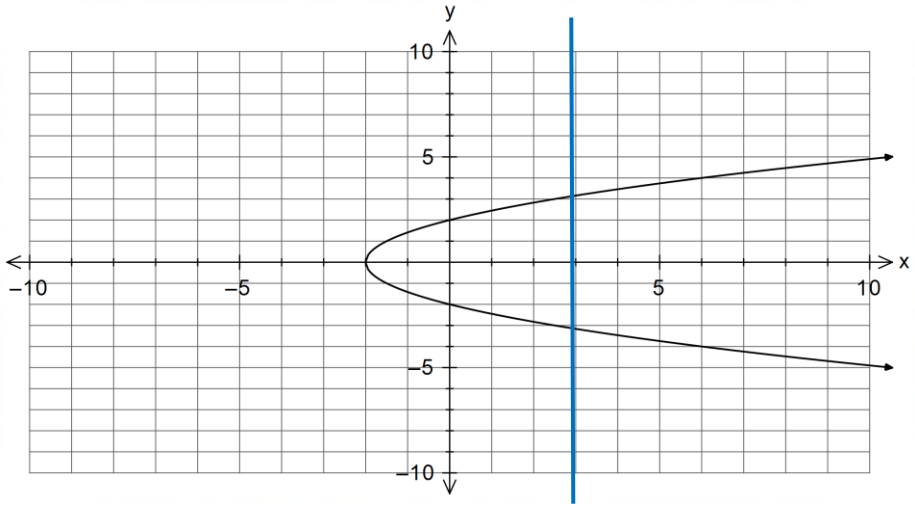
Section Two: Calculator-assumed **65% (97 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

The graph below is given in the form $y^2 = a(x-b)$.



(a) Find the values of a and b. (2 marks)

Solution
$a = 2$ $b = -2$
Specific behaviours
✓ correct value of a
✓ correct value of b

(b) State the equation of the axis of symmetry. (1 mark)

Solution
$y = 0$
Specific behaviours
✓ correct equation

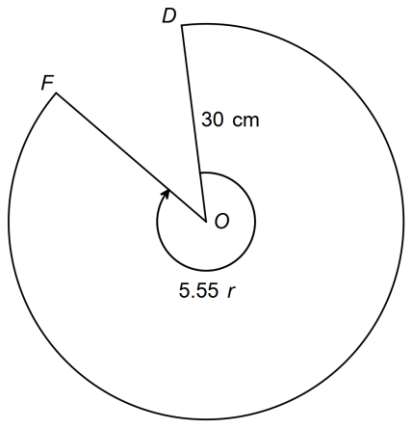
(c) Show the vertical line test on the graph above and explain how it is used to show whether this graph is a function or not. (2 marks)

Solution
See graph. The vertical line intersects the graph twice, indicating that it is a one-to-many relation and therefore is not a function.
Specific behaviours
✓ shows vertical line intersecting graph ✓ states that the vertical line intersects the graph twice

See next page

Question 10 **(5 marks)**

(a) The diagram below shows a sector of a circle with centre O. The radius of the circle is 30cm and $\angle DOF = 5.55$ radians. Calculate the length of the major arc DF. (2 marks)



Solution
$L = r\theta$ $= 30 \times 5.55$ $= 166.5cm$
Specific behaviours
✓ uses arc length formula with radians ✓ correct arc length NB: Answer only ok

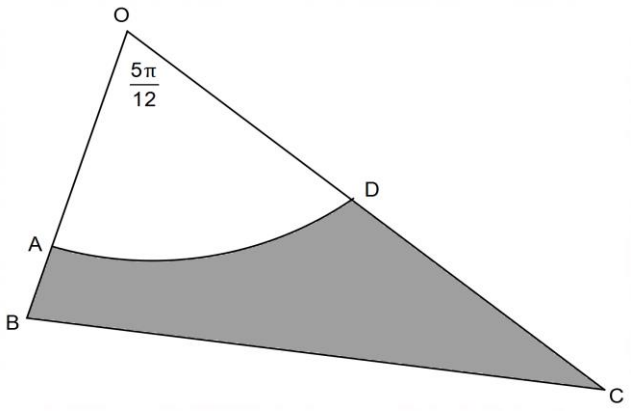
(b) In the diagram below AOD is a sector of the circle with centre O. BOC is a triangle.

In sector AOD, the radius is 30 cm and angle AOD is $\frac{5\pi}{12}$ radians.

In triangle OBC, OB = 38 cm and OC = 55 cm.

Calculate the shaded area of the shape with the vertices of ABCD rounded to 3 decimal places.

(3 marks)



Solution
$A_{\triangle} = 0.5ab\sin\theta$ $= (0.5)(38)(55)\sin\left(\frac{5\pi}{12}\right)$ $= 1009.3925cm^2$ $A_{sector} = 0.5r^2\theta$ $= (0.5)(30)^2\left(\frac{5\pi}{12}\right)$ $= 589.0486cm^2$ $A_{shaded} = 1009.3925 - 589.0486$ $= 420.344cm^2$
Specific behaviours
✓ area of triangle ✓ area of sector ✓ shaded area, correct to 3dp NB: No marks awarded if calculator is in degree mode as it will give a negative answer for the final area (area of triangle will be 23.87cm^2)

See next page

Question 11 **(8 marks)**

A study of the achievements of 360 students enrolled in a university course yielded the following information:

- 50% of all students achieved a distinction
- 60% of those who did not achieve a distinction studied part-time
- 45% of those who studied full-time did not achieve a distinction

(a) Use the above information to complete the following table. (4 marks)

	Full-time	Part-time	Totals
Distinction	**88**	**92**	**180**
No distinction	**72**	**108**	**180**
Totals	**160**	**200**	360

Solution
$360 \times 0.5 = 180, \quad 180 \times 0.6 = 108, \quad 180 - 108 = 72$ $\frac{72}{x} = 0.45 \Rightarrow x = 160, \quad 160 - 72 = 88, \quad 180 - 88 = 92$
Specific behaviours
✓ splits total to D/ND; ✓ splits no distinction to FT/PT ✓ calculates PT total; ✓ completes table

(b) Determine the probability that a randomly chosen student from the study

(i) achieved a distinction and studied full-time. (1 mark)

Solution
$P = \frac{88}{360} = \frac{11}{45} = 0.2\overline{4}$
Specific behaviours
✓ correct probability

(ii) achieved a distinction or studied full-time. (1 mark)

Solution
$P = \frac{360 - 108}{360} = \frac{252}{360} = \frac{7}{10}$
Specific behaviours
✓ correct probability

(c) Sets F and D are subsets of the students in the study. A student belongs to F if they studied full-time and to D if they achieved a distinction. Use set notation to describe the subset of part-time students who achieved a distinction. (2 marks)

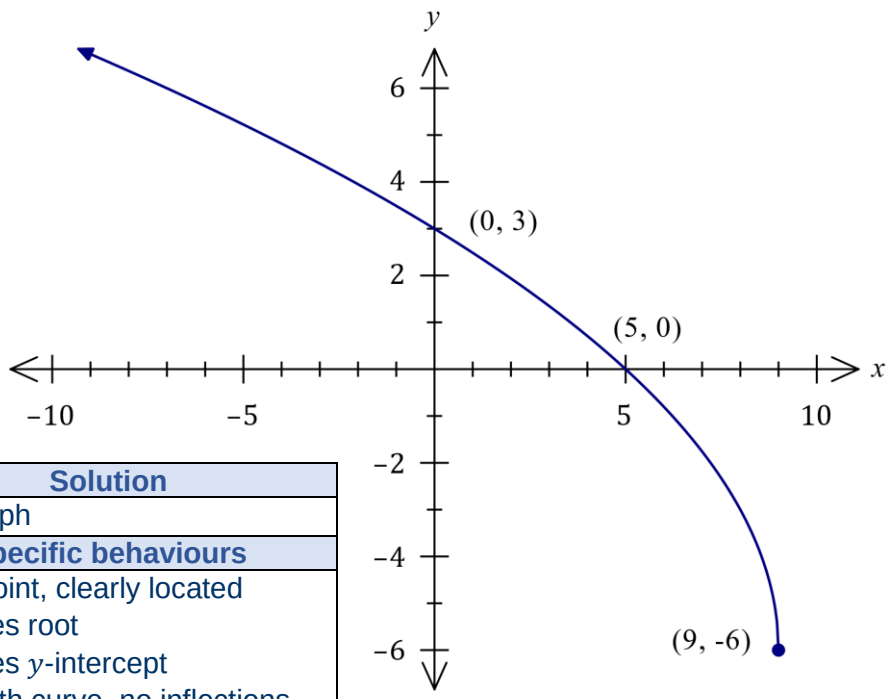
Solution
$\bar{F} \cap D$
Specific behaviours
✓ uses complement notation ✓ correct set notation

See next page

Question 12 **(8 marks)**

Let $f(x) = 3\sqrt{9-x} - 6$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (4 marks)



Solution
See graph
Specific behaviours
✓ endpoint, clearly located
✓ locates root
✓ locates y-intercept
✓ smooth curve, no inflections

(b) Describe the transformation(s) required to obtain the graphs of the following functions from the graph of $y = 3\sqrt{9-x} - 6$:

(i) $y = \sqrt{9-x} - 2$. (2 marks)

Solution
$y = \frac{1}{3}f(x)$. **Vertical dilation** OR **dilation parallel to the y-axis** OR **dilation from the x-axis** of scale factor $\frac{1}{3}$.
Specific behaviours
✓ both "dilation" and correct direction in description
✓ correct scale factor

(ii) $y = 3\sqrt{1-x} - 6$. (2 marks)

Solution
$y = f(x-8)$. Horizontal translation of 8 units to the left.
Specific behaviours
✓ states a translation
✓ correct distance and direction

See next page

Question 13 **(8 marks)**

(a) Triangle ABC is such that $b = 15$ cm, $c = 18$ cm and $\angle A = 125°$. Determine, with justification, the length of side a. (2 marks)

Solution
$a^2 = 15^2 + 18^2 - 2(15)(18)\cos 125°$ $a = 29.3$ cm
Specific behaviours
✓ clearly shows use of cosine rule ✓ correct length

(b) Triangle PQR is such that $p = 48.1$ cm, $q = 41.5$ cm and $\angle Q = 45°$. Determine all possible areas of this triangle. (6 marks)

Solution

$$\frac{\sin P}{48.1} = \frac{\sin 45°}{41.5}$$

First solution:

$$\angle P_1 = 55°$$
$$\angle R_1 = 180° - 45° - 55° = 80°$$
$$A_1 = \frac{1}{2}(48.1)(41.5)\sin 80°$$
$$A_1 = 983 \text{ cm}^2$$

Second solution:

$$\angle P_2 = 180° - 55° = 125°$$
$$\angle R_2 = 180° - 45° - 125° = 10°$$
$$A_2 = \frac{1}{2}(48.1)(41.5)\sin 10°$$
$$A_2 = 174 \text{ cm}^2$$

Areas are 174 cm^2 and 983 cm^2.

Specific behaviours

✓ shows use of sine rule
✓ calculates one value for $\angle P$
✓ derives $\angle R$ and shows use of area formula
✓ calculates one correct area
✓ calculates second set of values for $\angle P$ and $\angle R$
✓ calculates second area

See next page

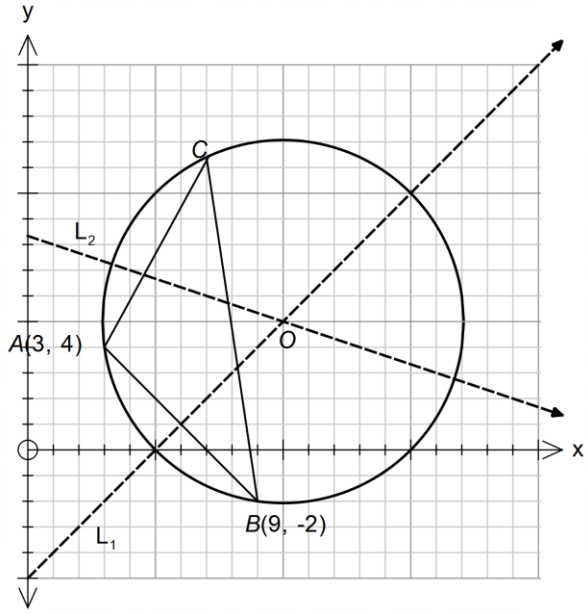
Question 14 **(5 marks)**

Triangle *ABC* is shown below where $A(3,4)$ and $B(9,-2)$.

The line L_1 is the perpendicular bisector of side *AB*.

The line L_2 intersects side *AC* and has the equation $3y+x=25$.

O is the centre of the circle $(x-10)^2+(y-5)^2=50$ which passes through the vertices of $\triangle ABC$.



Show algebraically that *O* is the intersection of L_1 and L_2.

Solution	Alternate Solution
$m_{AB}=\frac{-2-4}{9-3}$ $=\frac{-6}{6}$ $=-1$ $m_{L_1}=1$ $L_1: y=x-5$ $L_1=L_2$ $y+5=25-3y$ $4y=20$ $y=5$ $x=10$ $\therefore Int@\quad (10,5)$ From the circle equation, the centre is also at (10,5). $\therefore$ Lines intersect at O.	$m_{AB}=\frac{-2-4}{9-3}$ $=\frac{-6}{6}$ $=-1$ $m_{L_1}=1$ $L_1: y=x-5$ $L_2: y=-\frac{x}{3}+\frac{25}{3}$ $Centre \rightarrow (10,5)$ $5=-\frac{10}{3}+\frac{25}{3}$ $5=\frac{15}{3}$ $5=5$ From the circle equation, the centre is also at (10,5). $\therefore$ Lines intersect at O.
Specific behaviours	**Specific behaviours**
✓ gradient of L_1 ✓ equation of L_1 ✓ equates L_1 and L_2 ✓ solves to find the coordinates of the intersection ✓ states that the intersection is the same point as the centre of the circle, as can be determined from the circle equation	✓ gradient of L_1 ✓ equation of L_1 ✓ checks centre lies on L_1 ✓ checks centre lies on L_2 ✓ concludes that the lines intersect at the centre of the circle

See next page

Question 15 **(9 marks)**

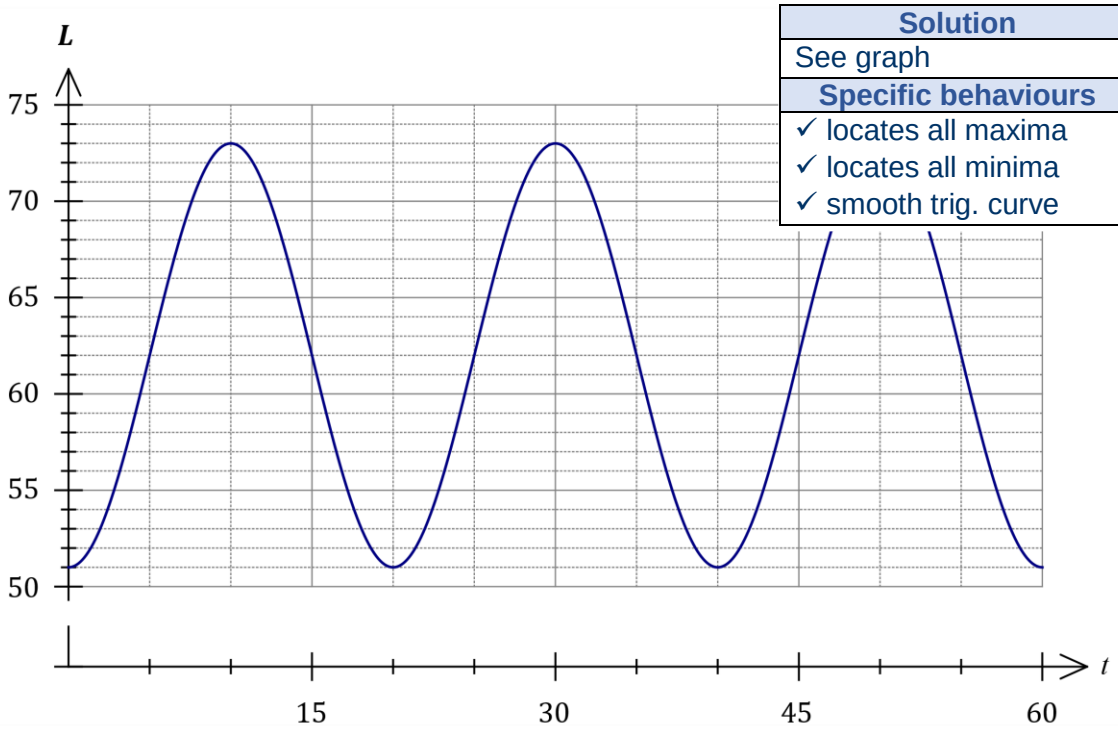
The loudness L of sound, in decibels, emitted by a machine t minutes after it is switched on can be modelled by

$$L = 62 - 11\cos\left(\frac{\pi t}{10}\right)$$

(a) Determine the initial loudness emitted by the machine. (1 mark)

Solution
$L(0) = 51$ dB
Specific behaviours
✓ correct value with units

(b) Draw the graph of L against t on the axes below for the first 60 minutes. (3 marks)



Solution
See graph
Specific behaviours
✓ locates all maxima
✓ locates all minima
✓ smooth trig. curve

(c) State the maximum loudness emitted by the machine and the time this maximum was first reached. (2 marks)

Solution
$L_{MAX} = 73$ dB when $t = 10\ mins.$
Specific behaviours
✓ correct maximum with units (penalise once in this question)
✓ correct time

(d) A health and safety inspector can deem a machine unserviceable if the loudness it emits exceeds 70 dB for more than 15 minutes in any hour that it is running. Determine, with justification, whether this machine could be deemed unserviceable. (3 marks)

Solution
Exceeds value for $7.59 < t < 12.41$ during first cycle. $3(12.41 - 7.59) = 14.5$ minutes per hour - and so machine could not be deemed unserviceable.
Specific behaviours
✓ identifies interval endpoints
✓ calculates minutes per hour (allow 14.4 mins)
✓ uses calculations to draw conclusion

See next page

Question 16 **(8 marks)**

A souvenir shop sells T-shirts in two colours and three sizes. Sales records for the past year are shown below.

	Small	Medium	Large
Blue	210	420	310
White	230	450	180

Assume that the shop holds a large stock and that sales continue in similar proportions. Where relevant, round your answers in this question to three decimal places.

(a) A customer randomly selects a T-shirt for purchase. Determine:

(i) the size and colour of the least likely T-shirt and the probability that this T-shirt is selected. (2 marks)

Solution

Least likely: White, large.

$$P = \frac{180}{1800} = 0.1$$

Specific behaviours

✓ type of T-shirt
✓ calculates probability

(ii) the probability that the T-shirt selected is not small. (2 marks)

Solution

$$P = \frac{870 + 490}{1600} = \frac{1360}{1800} = \frac{34}{45} \approx 0.756$$

Specific behaviours

✓ counts required sizes (1360)
✓ calculates probability
NB: Answer only ok, don't penalise rounding here

(b) A customer randomly selects two T-shirts for purchase. Determine the probability that the T-shirts are:

(i) both medium. (2 marks)

Solution

$$P(MM) = \left(\frac{870}{1800}\right)^2 = \frac{841}{3600} \approx 0.234$$

Specific behaviours

✓ probability of one medium
✓ calculates probability

(ii) of different colours. (2 marks)

Solution

$$P(BW) = \frac{940}{1800} \times \frac{860}{1800} = \frac{2021}{8100} \quad (\approx 0.2495)$$

$$\therefore P(WB \cup BW) = 2 \times \frac{2021}{8100} = \frac{2021}{4050} \approx 0.499$$

Specific behaviours

✓ probability of one then the other
✓ calculates probability

See next page

Question 17 **(8 marks)**

(a) Let $f(x) = x^2 + bx + c$, where b and c are constants. The graph of $y = f(x)$ has an axis of symmetry with equation $x = -3$ and an axis intercept at $(0, 5)$.

(i) State the value of the constant c. (1 mark)

Solution
c is the y-intercept: $c = 5$.
Specific behaviours
✓ correct value

(ii) Determine the value of the constant b. (2 marks)

Solution
Axis of symmetry has equation $x = -\frac{b}{2a}$: $$-3 = -\frac{b}{2} \Rightarrow b = 6$$
Specific behaviours
✓ indicates appropriate method ✓ calculates value

(b) Let $g(x) = 2(x - 2)^2 - 7$. Determine

(i) the coordinates of the turning point of the graph of $y = g(x)$. (1 mark)

Solution
Turning point is at $(2, -7)$.
Specific behaviours
✓ correct coordinates

(ii) the domain and range of $g(x)$. (2 marks)

Solution
Domain: $x \in \mathbb{R}$, and range: $y \geq -7$.
Specific behaviours
✓ states domain ✓ states range

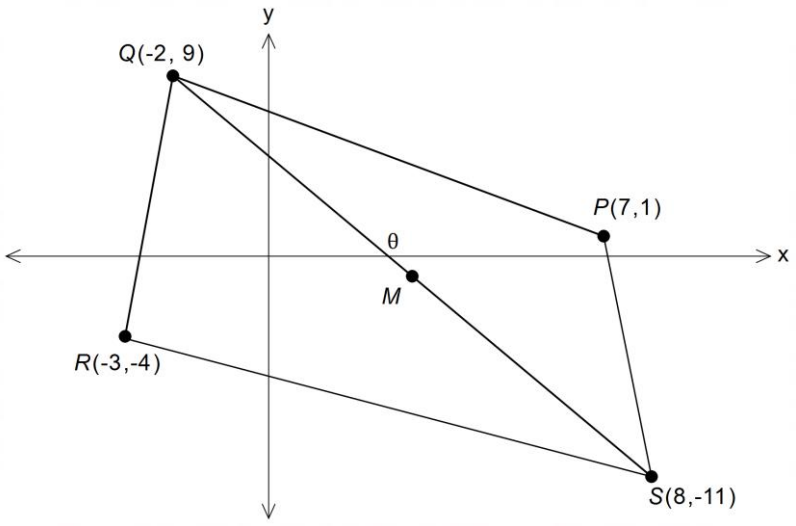
(iii) the coordinates of the turning point of the graph of $y = g(x - 3) + 2$. (2 marks)

Solution
Graph has been translated 3 units right and 2 units upwards and so new turning point at $(5, -5)$.
Specific behaviours
✓ indicates correct use of one translation ✓ correct coordinates

See next page

Question 18 **(9 marks)**

In the diagram *PQRS* is a quadrilateral having vertices *P*(7, 1), *Q*(−2, 9), *R*(−3, −4) and *S*(8, −11). *M* is the midpoint of *QS*.



(a) If a line is drawn from *P* to *R*, determine the equation of the line *PR.* (2 marks)

Solution
$m_{PR} = \frac{1-(-4)}{7-(-3)}$ $= \frac{5}{10}$ $= \frac{1}{2}$ $y = mx + c$ $= \frac{1}{2}x + c$ $1 = \frac{7}{2} + c$ $c = -\frac{5}{2}$ $\therefore y = \frac{x-5}{2}$
Specific behaviours
✓ correct gradient   ✓ states equation of line

(b) Determine whether *M* lies on the line PR. (3 marks)

Solution
$M @ \left(\frac{-2+8}{2}, \frac{9-11}{2}\right)$ $M @ \ (3,-1)$ $y = \frac{x-5}{2}$ $y_{\|x=3} = \frac{3-5}{2}$ $= -1$ $\therefore$ M lies on the line PR.
Specific behaviours
✓ correct coordinates for M   ✓ substitutes M into equation and makes correct conclusion

See next page

Question 18 (cont.)

(c) Show that QS is perpendicular to PR. (2 marks)

Solution
$m_{PR} = \frac{1}{2}$ $m_{QS} = \frac{-11-9}{8-(-2)}$ $= -2$ $m_{PR} \times m_{QS} = 0.5 \times -2$ $= -1$ $\therefore QS \perp PR$
Specific behaviours
✓ gradient of QS ✓ multiplies by gradient of PR to get -1 and concludes that the lines are perpendicular

d) Calculate θ, the angle of inclination of QS. (2 marks)

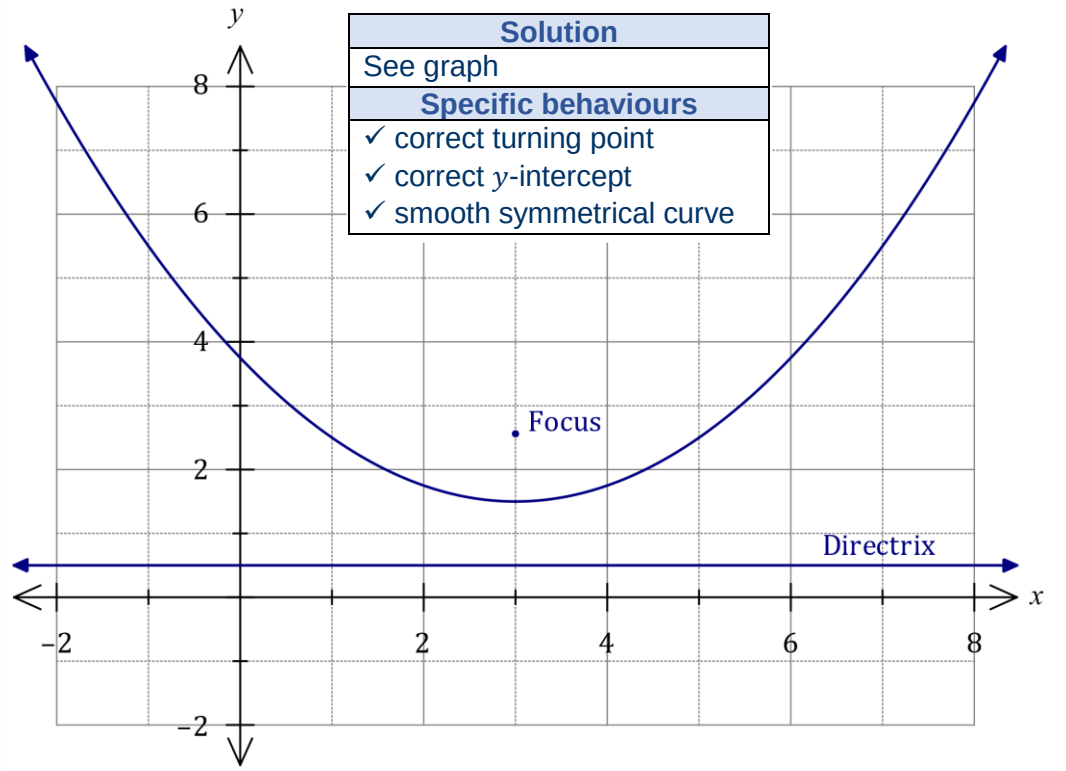
Solution
$\tan\theta = -2$ $\theta = 116.57°$
Specific behaviours
✓ equation for angle of inclination ✓ correct angle (accept any rounding) NB: answer only ok

See next page

Question 19 **(7 marks)**

The equation of a parabola is $y = \frac{1}{4}(x^2 - 6x + 15)$.

(a) Sketch the parabola on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ correct turning point
✓ correct y-intercept
✓ smooth symmetrical curve

All parabolas have a focal point and a directrix. For a parabola with equation $y = a(x - p)^2 + q$, the focal point is at $\left(p, q + \frac{1}{4a}\right)$ and the equation of the directrix is $y = q - \frac{1}{4a}$, where a, p and q are constants.

(b) Determine the focal point and directrix for this parabola and add them, with labels, to your sketch above. (4 marks)

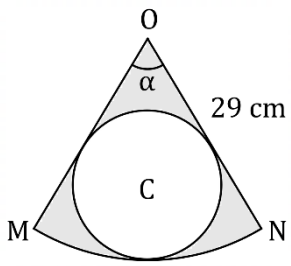
Solution
From graph, turning point at $(3, 1.5)$. Hence $a = \frac{1}{4}, \quad p = 3, \quad q = 1.5$ Focal point: $(3, 2.5)$ and directrix: $y = 0.5$.
Specific behaviours
✓ indicates turning point
✓ indicates values of all constants
✓ plots focus
✓ draws directrix

See next page

Question 20 **(7 marks)**

The diagram shows sector OMN of a circle centre O of radius 29 cm and $\alpha = 68°$.

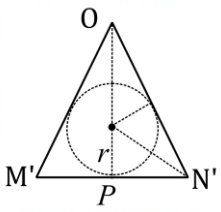
Circle C is inside the sector and just touches OM, ON and arc MN.

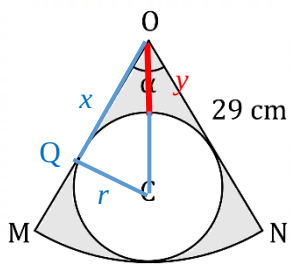


(a) Determine the area of sector OMN. (2 marks)

Solution
$A = \frac{68°}{360°} \times \pi(29)^2 = \frac{14297\pi}{90} \approx 499 \text{ cm}^2$
Specific behaviours
✓ indicates suitable method
✓ calculates area

(b) Show that the radius of circle C is 10.4 cm, correct to one decimal place. (3 marks)

Solution
Extend sector sides to form isosceles triangle:

$PN' = 29\tan\left(\frac{68°}{2}\right) = 19.56 \text{ cm}$
$r = 19.56\tan\left(\frac{56°}{2}\right) = 10.4 \text{ cm}$
Specific behaviours
✓ forms isosceles triangle, shows angles
✓ calculates PN'
✓ calculates radius

Alternate Solution
$2r + y = 29$
$(r+y)^2 = x^2 + r^2$
$\tan 34° = \frac{r}{x}$

CAS solve: $r = 10.4cm$
Specific behaviours
✓ forms triangle OCQ
✓ sets up three equations to solve simultaneously
✓ calculates radius

(c) Determine the area of the shaded region, inside sector OMN but outside circle C. (2 marks)

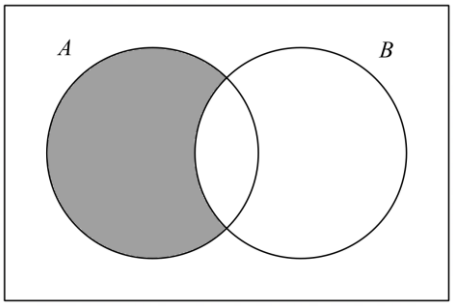
Solution
$A_C = \pi(10.4)^2 \approx 340$
Shaded area $= 499 - 340 = 159 \text{ cm}^2$
Specific behaviours
✓ calculates area of circle
✓ calculates shaded area, with units
NB: Answer only ok

See next page

Question 21 **(10 marks)**

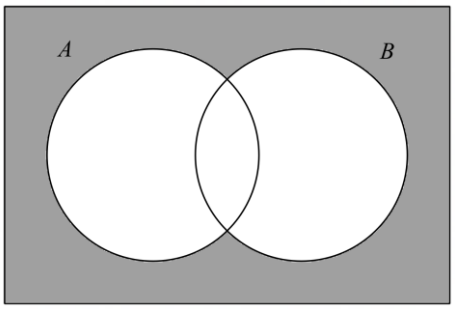
(a) Consider the two Venn diagrams below:

(i) Shade the region corresponding to $A \cap \overline{B}$. (1 mark)



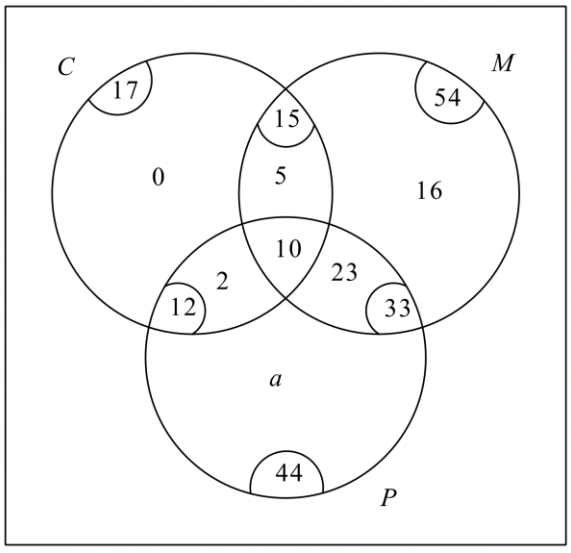
Solution
See diagram.
Specific behaviours
✓ correct shading

(ii) Use symbolic set notation to describe the shaded regions below. (1 mark)



Solution
$\overline{A \cup B}$ or $\overline{A} \cap \overline{B}$
Specific behaviours
✓ correct notation

(b) The following Venn diagram shows the number of students electing to study at least one of Chemistry (C), Maths (M) or Physics (P).



(i) Determine the value of a. (1 mark)

Solution
$a = 44 - 2 - 10 - 23 = 9$
Specific behaviours
✓ correct value of a

See next page

Question 21 (cont.)

(ii) Determine $n(C \cup M \cup P)$ (1 mark)

Solution
$n(C \cup M \cup P) = 54 + 2 + 9 = 65$
Specific behaviours
✓ correct value

(iii) If one student is selected at random from the group, determine the probability of the following scenarios, leaving your answers as unsimplified fractions:

(a) They elected to study Maths but not Physics. (2 marks)

Solution
$P(M \cap \overline{P}) = \frac{16+5}{65}$ $= \frac{21}{65}$
Specific behaviours
✓ correct numerator
✓ correct denominator

(b) They elected to study Maths and Physics, given that they did not study chemistry. (2 marks)

Solution
$P(M \cap P \mid \overline{C}) = \frac{23}{16+23+9}$ $= \frac{23}{48}$
Specific behaviours
✓ correct numerator
✓ correct denominator

(c) They elected to study two of the subjects, given that they did not elect to study all three subjects.

(2 marks)

Solution
$P(two \mid \overline{C \cap M \cap P}) = \frac{2+5+23}{55}$ $= \frac{30}{55}$
Specific behaviours
✓ correct numerator
✓ correct denominator

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Supplementary page

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