TRINITY COLLEGE

Mathematics Methods Year 12
Test 2 2016

Section 1 Calculator Free

Exponential Function, Fundamental Theorem

STUDENT'S NAME _____

DATE: Friday 1st April **TIME:** 33 minutes **MARKS**: 33

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

Determine $\frac{dy}{dx}$ for each of the following. Do not simplify.

(a) $y = e^{\sqrt{x}}$ [2]

(b) $y = \sin\left(e^{2x}\right)$ [2]

(c) $y = (\cos x)\, e^{\cos x}$ [3]

2. (10 marks)

(a) Evaluate exactly $\int_0^2 x e^{4-x^2} dx$ [4]

(b) Determine $\int \frac{4e^{2x} + 4x}{(e^{2x} + x^2)^3} dx$ [3]

(c) Determine $\int_{\pi}^{x^2} \left(\frac{d}{dt} e^{e^{-t}} \right) dt$ [3]

3. (4 marks)

Given $y = \dfrac{e^x}{3 + e^x}$

(a) determine $\dfrac{dy}{dx}$ [2]

(b) explain why $\dfrac{dy}{dx} \neq 0$ [2]

4. (5 marks)

(a) Determine $\dfrac{dy}{dx}$ given $y = x e^x$ [2]

(b) Hence determine $\int x e^x dx$ [3]

5. (4 marks)

Given $y = \int_{-3}^{x} \frac{t^2 - 2}{\sqrt{t}} dt$, use the incremental formula $\delta y \approx \frac{dy}{dx} \times \delta x$ to determine the change in y if x changes from 4 to 4.02.

TRINITY COLLEGE

Mathematics Methods Year 12
Test 2 2016

Section 2 Calculator Assumed

Exponential Function, Fundamental Theorem

STUDENT'S NAME _____

DATE: Friday 1st April **TIME:** 20 minutes **MARKS**: 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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6. (4 marks)

Determine the value of x for which $\int_x^{-1}(1-t^2)\,dt$ has a relative minimum. Justify it is a minimum value. [4]

7. (4 marks)

Sugar is being dissolved in a solution at a rate given by $\frac{dS}{dt} = -20e^{-0.1t}$ where S is the amount, in grams, of undissolved sugar after t seconds.

(a) how much sugar is initially in the solution [2]

(b) how long does it take for half the sugar to dissolve. [2]

8. (5 marks)

A particular rock is dropped into a swimming pool and it sinks vertically to the bottom. Due to water resistance, the rock does not have a constant velocity on the way to the bottom. Its velocity, v centimetres per second, t seconds after it hits the surface of the water is given by

$$v = 8(2 - e^{-0.8t}) \quad \text{for} \quad 0 \le t \le 7$$

(a) What is the initial velocity of the rock in the water? [1]

(b) What is the acceleration of the rock after 4 seconds? [2]

(c) Terminal velocity is an expression used to describe the velocity that is approached but never exceeded. Determine the terminal velocity reached by the rock in the water. [2]

9. (8 marks)

The amount A of a drug (in milligrams) in the bloodstream will decline at a rate proportional to the current amount . That is $\frac{dA}{dt} = -\left(\frac{1}{k}\right)A$.

where *k* hours is a constant called the elimination time and time t is measured in hours.

(a) Write down the formula for $A(t)$, the amount of the drug in the bloodstream after *t* hours , in terms of *t*, *k* and the initial amount A_0 . [2]

(b) What proportion of the drug remains in the bloodstream after *k* hours ? [3]

The drug sodium pentobarbitol can be used to tranquilize animals. A dog is tranquilized if its bloodstream contains at least 45 milligrams of the drug for each kilogram of the dog's weight. The elimination time for the drug is 6 hours.

(c) What single dose of this drug should be given in order to tranquilize a 12 kilogram dog for 1 hour? [3]