SHENTON COLLEGE

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Examination Semester Two 2017
Question/Answer Booklet

MATHEMATICS METHODS UNITS 3 and 4
Section Two (Calculator-assumed)

Teacher (Circle One) Mrs Friday Mr Smith

Your name _____

Time allowed for this section
Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section
To be provided by the supervisor
Question/answer booklet for Section Two.
Formula sheet. (retained from section one)

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on up to two unfolded sheets of A4 paper, and up to three calculators, CAS, graphic or scientific approved for use in the WACE examinations.

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

	Number of questions	Working time (minutes)	Marks available
Section One Calculator Free	8	50	52
Section 2 Calculator Assumed	**13**	**100**	**96**
		Total	148

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Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the Year 12 *Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
     Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil, except in diagrams**.

STRUCTURE OF THIS PAPER

QUESTION	MARKS AVAILABLE	MARKS AWARDED
9	7	
10	9	
11	8	
12	7	
13	8	
14	6	
15	7	
16	8	
17	8	
18	7	
19	8	
20	7	
21	6	
TOTAL	96	

Section Two: Calculator-assumed **65% (96 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

The capacity, X mL, of glass bottles made in a factory can be modelled by a normal distribution with mean μ and standard deviation 1.5 mL.

(a) If $\mu = 364$, determine

(i) $P(X \geq 362)$. (1 mark)

(ii) $P(X < 362 \mid X < 363)$. (2 marks)

(iii) the value of x, if $P(X \geq x) = \frac{3}{11}$. (1 mark)

(b) Given that $P(X < k) = 0.119$,

(i) determine the value of μ in terms of k. (2 marks)

(ii) determine μ if $k = 95$. (1 mark)

Question 10 **(9 marks)**

A fair die has one face numbered 1, three faces numbered 2 and two faces numbered 3.

(a) Determine the probability that the second even number occurs on the fourth throw of the dice. (3 marks)

(b) The die is thrown twice and X is the sum of the two scores.

(i) Complete the table below to show the probability distribution of X. (2 marks)

x	2	3	4	5	6
$P(X = x)$		$\frac{1}{6}$	$\frac{13}{36}$	$\frac{1}{3}$	

(ii) Determine $P(X = 5 \mid X \geq 5)$. (2 marks)

(iii) Calculate $\mathrm{E}(X)$. (2 marks)

Question 11 **(8 marks)**

From a random survey of 524 users of a free music streaming service, it was found that 386 would stop using it if they had to pay.

(a) Based on this survey, calculate the percentage of users who would stop using the service. (1 mark)

(b) Calculate the approximate margin of error for a 90% confidence interval estimate of the proportion of users who would stop using the service. (3 marks)

(c) Determine a 90% confidence interval for the proportion of users who would stop using the service. (2 marks)

(d) If 50 identical surveys were carried out and a 90% confidence interval for the proportion was calculated from each survey, determine the probability that exactly 48 of the intervals will contain the true value of the proportion. (2 marks)

See next page

Question 12 **(7 marks)**

The lifetime, T hours, of an electronic component is a continuous random variable with probability density function given by

$$f(t) = 0.005e^{-0.005t}, \qquad 0 \leq t < \infty.$$

(a) Determine the probability that a randomly chosen component has a lifetime of less than 450 hours. (2 marks)

(b) An engineer buys 12 of the components. If they operate independently of each other, determine the probability that at least 11 of them will not last 450 hours. (2 marks)

(c) A component has already been operating for exactly 440 hours. Determine the probability that it will fail within the next 36 hours. (3 marks)

See next page

Question 13 **(8 marks)**

160 black and 840 white spherical beads, identical except for their colour, are placed in a container and thoroughly mixed.

In experiment A, a bead is randomly selected, its colour noted and then replaced until a total of 20 beads have been selected.

(a) The random variable X is the number of black beads selected in experiment A.
Determine $P(X > 5)$. (2 marks)

(b) Experiment A is repeated 10 times. Determine the probability that at least one black bead is selected in each of these experiments. (2 marks)

In experiment B, a bead is randomly selected, its colour noted and then replaced until a total of 65 beads have been selected.

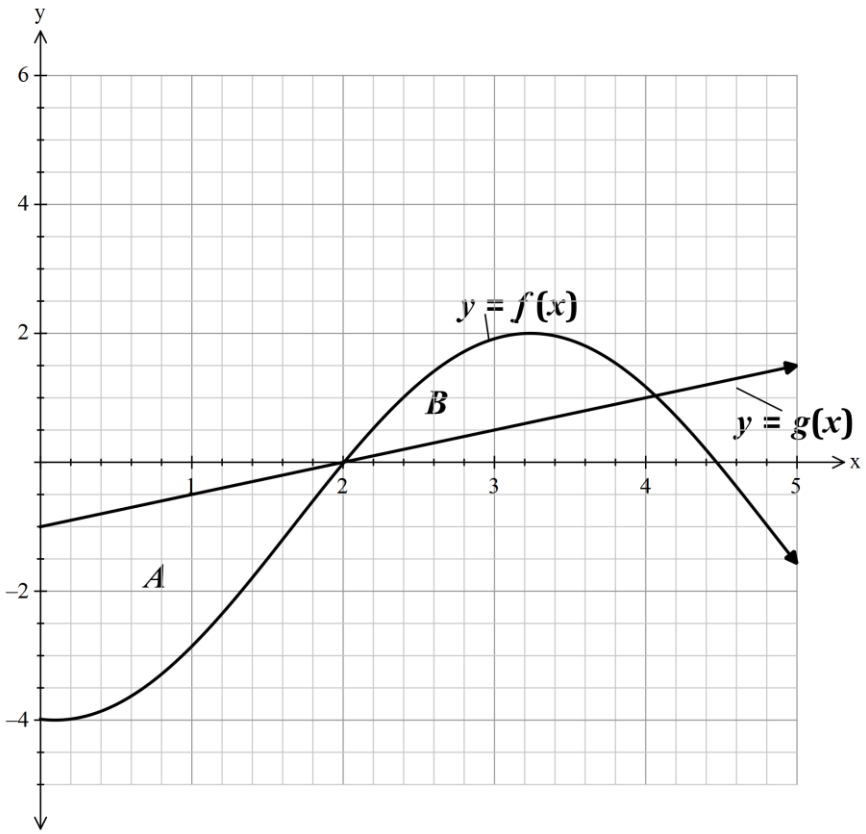
Experiments A and B are repeated a large number of times, with the proportions of black beads in each experiment, $\hat{p}_A$ and $\hat{p}_B$ respectively, recorded.

(c) The distribution of which proportion, $\hat{p}_A$ or $\hat{p}_B$, is most likely to approximate normality? Explain your answer and state the mean and standard deviation of the normal distribution for the proportion you have chosen. (4 marks)

See next page

Question 14 **(6 marks)**

Region A, on the graph below, is defined as the area enclosed between the $y-$ axis, $g(x)$ and $f(x)$ while **Region B**, is defined as the area enclosed between $g(x)$ and $f(x)$ and $2 \le x \le 4$.



The following information is known:

$$\int_0^2 f(x)dx = -5.1$$

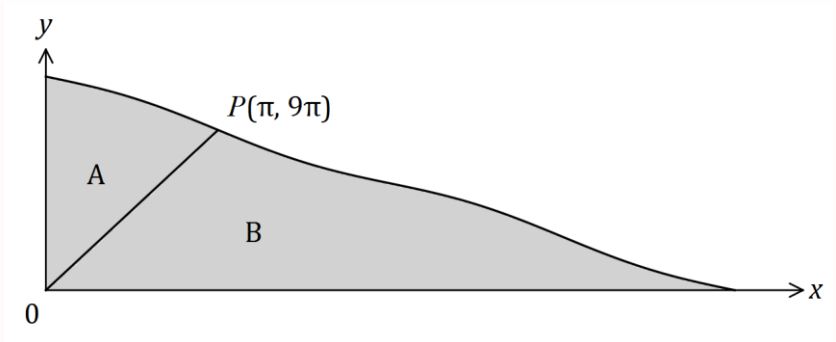
$$\int_0^4 f(x)dx = -2.18$$

(i) Determine the area of Region A. (2 marks)

(ii) Show that the area of Region B = 1.92 (4 marks)

Question 15 **(7 marks)**

The curve $y = 12\pi - 3x + \sin x$ is shown below passing through $P(\pi, 9\pi)$.



A straight line joins the origin to P, dividing the shaded area into two regions, A and B.

(a) Show that when $x = 4\pi$, $y = 0$. (1 mark)

(b) Determine the exact value of $\int_0^{\pi} (12\pi - 3x + \sin x)\, dx$. (2 marks)

(c) Determine the ratio of the area of region A to the area of region B in the form $1 : k$. (4 marks)

See next page

Question 16 **(8 marks)**

A researcher wants to estimate the proportion of Western Australian school-aged students who participate in organised sport during school holidays. The researcher plans to collect sample data by visiting schools and asking students.

(a) Discuss two different sources of bias that may occur when the researcher collects their sample data and suggest a procedure to avoid bias. (4 marks)

(b) Determine, to the nearest 10, the sample size the researcher should use to ensure that the margin of error of a 90% confidence interval is no more than 6%. (3 marks)

(c) Comment on how your answer to (b) would change if the researcher had a reliable estimate that the population proportion was close to 20%. (1 mark)

See next page

Question 17 **(8 marks)**

The mass, X g, of wasted metal when a cast is made is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{x}{2a^2} & 0 \leq x \leq 2a, \\ 0 & \text{elsewhere,} \end{cases}$$

where a is a positive constant.

(a) Determine $\mathrm{E}(X)$ in terms of a. (2 marks)

(b) The total mass of wasted metal from a random sample of 25 casts was 500 g.
Estimate the value of a. (2 marks)

(c) If $a = 9$, determine

(i) $P(X \geq 12)$. (1 mark)

(ii) $\mathrm{Var}(X)$. (3 marks)

Question 18 **(7 marks)**

A polynomial function $f(x)$ is such that $\int_2^6 4f(x)\,dx = 12$.

(a) Evaluate $\int_6^2 f(x)\,dx$ (2 marks)

(b) Determine the value of $\int_2^3 (f(x) + 3x^2)\,dx + \int_3^6 (1 + f(x))\,dx$. (5 marks)

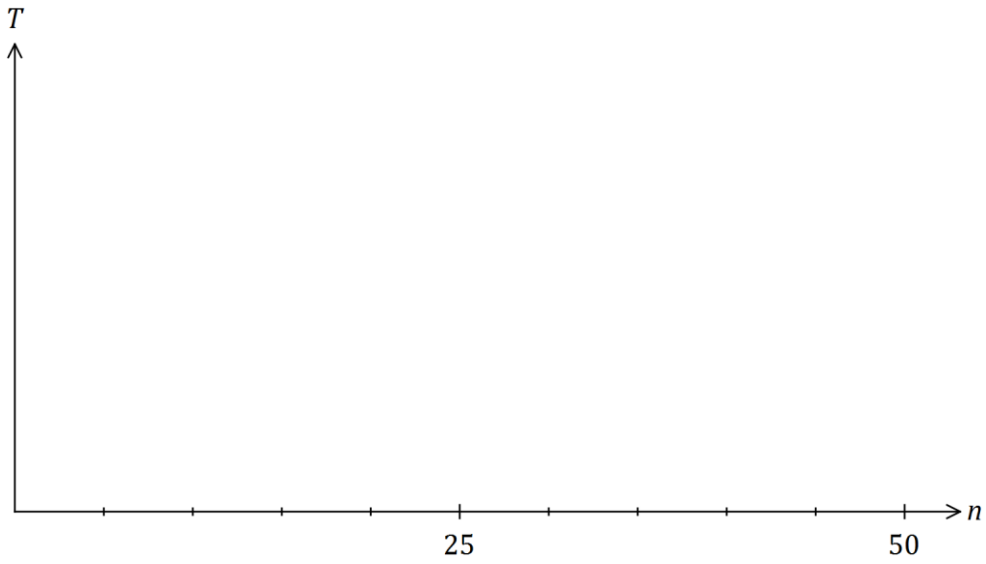
See next page

Question 19 **(8 marks)**

Hick's law, shown below, models the average time, T seconds, for a person to make a selection when presented with n equally probable choices.

$$T = a + b\log_2(n+1), \text{ where } a \text{ and } b \text{ are positive constants.}$$

(a) Draw the graph of T vs n on the axes below when $a = 5$ and $b = 10$. (3 marks)



(b) When a pizzeria had 12 choices of pizza, the average time for patrons to make a choice was 40 seconds. After increasing the number of choices by 15, the average time to make their choice increased by 20%.

Modelling the relationship with Hick's law, predict the average time to make a choice if patrons were offered a choice of 16 pizzas. (5 marks)

Question 20 **(7 marks)**

The acceleration of a particle is given by $a = 3\sin(2t)$ where distance is measure in metres and time, t, in seconds. Initially, the velocity of the particle is 4 m/s and its distance from the equilibrium position is 2 metres.

(a) Determine an equation for the velocity of the particle. (3 marks)

(b) Determine the displacement of the particle when t = 2 (accurate to 2 decimal places). (4 marks)

Question 21 **(6 marks)**

A popcorn container of capacity 660 mL is made from paper and has the shape of an open inverted cone of radius r and height h.

Determine the least area of paper required to make the container.

End of Questions

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Additional working space

Question number: _____

See next page

Additional working space

Question number: _____

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Additional working space

Question number: _____

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