Question 9 **(7 marks)**

The voltage between the plates of a discharging capacitor can be modelled by the function $V(t) = 14e^{kt}$, where V is the voltage in volts, t is the time in seconds and k is a constant.

It was observed that after three minutes the voltage between the plates had decreased to 0.6 volts.

(a) State the initial voltage between the plates. (1 mark)

(b) Determine the value of k. (2 marks)

(c) How long did it take for the initial voltage to halve? (2 marks)

(d) At what rate was the voltage decreasing at the instant it reached 8 volts? (2 marks)

Question 10 **(11 marks)**

The gradient function of f is given by $f'(x) = 12x^3 - 24x^2$.

(a) Show that the graph of $y = f(x)$ has two stationary points. (2 marks)

(b) Determine the interval(s) for which the graph of the function is concave upward. (3 marks)

(c) Given that the graph of $y = f(x)$ passes through $(1, 0)$, determine $f(x)$. (2 marks)

(d) Sketch the graph of $y = f(x)$, indicating all key features. (4 marks)

Question 11 **(8 marks)**

(a) Consider the area bounded by $y = x^3$, $y = \sqrt{x}$ and $x = 2$.

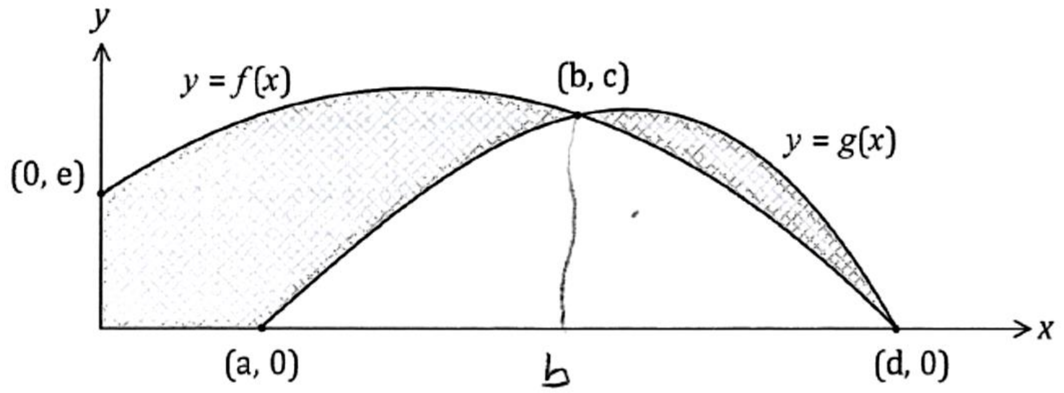
(i) Sketch the region described above on the axes provided. (2 marks)

(ii) Use calculus to find the exact area bounded by $y = x^3$, $y = \sqrt{x}$ and $x = 2$. (3 marks)

(b) The marginal cost function for producing x electronic components per day is
$M_c(x) = \dfrac{100}{\sqrt{x}} + 150$.
Determine the cost of increasing production from 100 components per day to 400 components per day. (3 marks)

Question 12 **(7 marks)**

The graphs of the functions f and g are shown below, intersecting at the points (b, c) and $(d, 0)$.

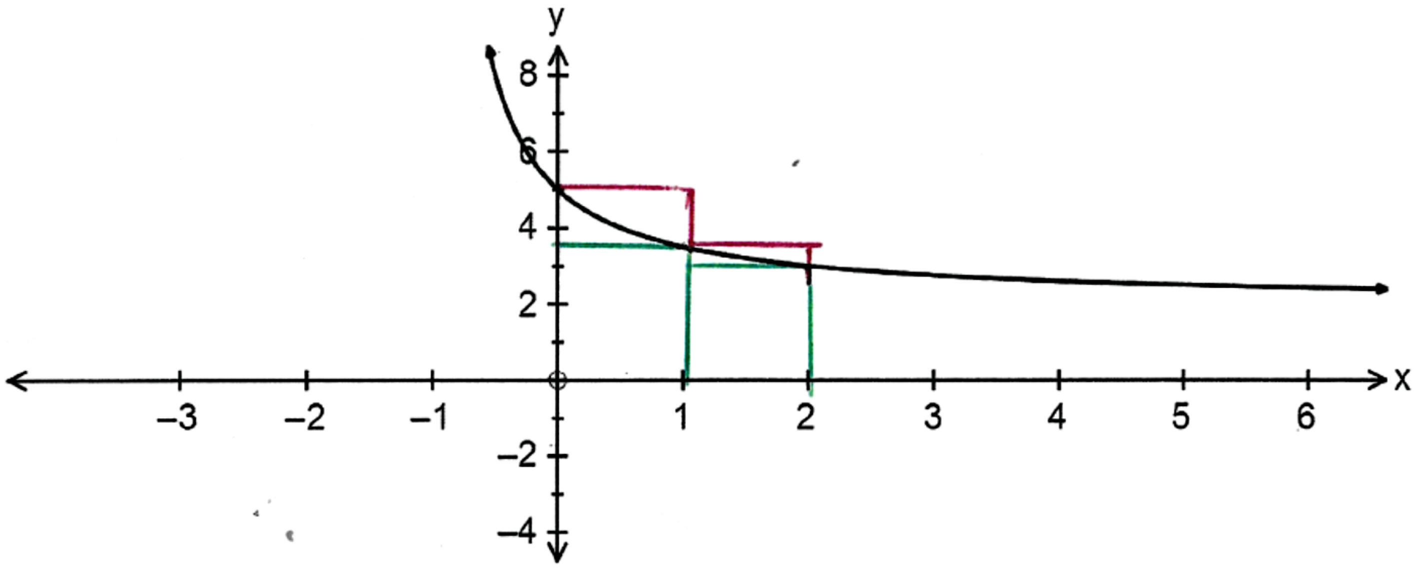


(a) Using definite integrals, write an expression for the area of the shaded region. (3 marks)

(b) Evaluate the area when $f(x) = 15 + 12x - 3x^2$ and $g(x) = -x^3 + 3x^2 + 13x - 15$.
(4 marks)

Question 14 **(7 marks)**

Consider the function $y = \dfrac{3}{x+1} + 2$ graphed below.



(a) Complete the table of values: (1 mark)

x	y
0	
1	
2	
3	
4	

(b) Use 4 upper rectangles and 4 lower rectangles to approximate the area under the curve from $0 \le x \le 4$. (5 marks)

(c) Suggest one change to the above procedure to improve the accuracy of the estimate. (1 mark)

Question 16 **(12 marks)**

Particle P leaves point A at time $t = 0$ seconds and moves in a straight line with acceleration given by

$$a = \frac{16}{(2t+1)^3}\ \mathrm{ms}^{-2}.$$

Particle P has an initial velocity of $-3\ \mathrm{ms}^{-1}$ and point A has a displacement of 4 metres from the origin.

(a) Calculate the initial acceleration of P. (1 mark)

(b) Is P ever stationary? If your answer is yes, determine the time(s) when this happens. If your answer is no, explain why. (3 marks)

(c) Calculate the displacement of P when $t = 12$ seconds. (2 marks)

(d) Calculate the change of displacement of P during the third second. (2 marks)

(e) Determine the maximum speed of P during the first three seconds and the time when this occurs. (2 marks)

(f) Calculate the total distance travelled by P during the first three seconds. (2 marks)

Question 18 **(8 marks)**

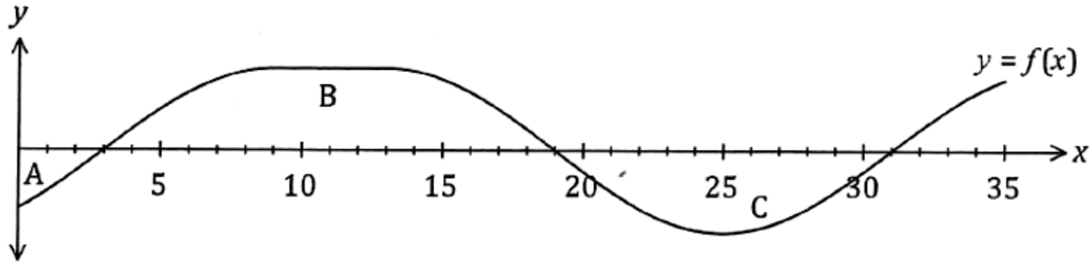
A storage container of volume 36π cm^3 is to be made in the form of a right circular cylinder with one end open. The material for the circular end costs 12c per square centimetre and for the curved side costs 9c per square centimetre.

(a) Show that the cost of materials for the container is $12\pi r^2 + \dfrac{648\pi}{r}$ cents, where r is the radius of the cylinder. (4 marks)

(b) Use calculus techniques to determine the dimensions of the container that minimise its material costs and state this minimum cost. (4 marks)

Question 19 **(9 marks)**

The graph of $y = f(x)$ is shown below. The areas between the curve and the $x-$axis for regions A, B and C are 3, 20 and 12 square units respectively.



(a) Evaluate

(i) $\int_0^{31} f(x)\,dx$. (1 mark)

(ii) $\int_{19}^{0} f(x)\,dx$. (2 marks)

(iii) $\int_3^{31} 2 - 3f(x)\,dx$. (3 marks)

It is also known that $A(31) = 0$, where $A(x) = \int_{10}^{x} f(t)\,dt$.

(b) Evaluate

(i) $A(19)$. (1 mark)

(ii) $A(0)$. (2 marks)