Test 1 : Thursday 25th February
Differentiation Techniques

This assessment contributes 4% towards the final year mark.

40 minutes are allocated for this task.

No notes of ANY nature are permitted.

Calculators are NOT permitted for this task.

Full marks may not be awarded to correct answers unless sufficient justification is given.

Name : SOLUTIONS + MARKING KEY

Score :

(out of 40)

Do NOT turn over this page until you are instructed to do so.

Q1. (11 marks)

Find the derivatives of the following functions, leaving answers with positive indices and simplifying answers where possible.

(a) $y = e^{4x} + \dfrac{3}{x^2}$ (2 marks)

$$\frac{dy}{dx} = 4e^{4x} - \frac{6}{x^3}$$

✓ $4e^{4x}$

✓ $-\dfrac{6}{x^3}$

(b) $y = \sin(3x) - 4\cos(2x)$ (3 marks)

$$\frac{dy}{dx} = 3\cos(3x) + 8\sin(2x)$$

✓ $\cos(3x)$ and $\sin(2x)$

✓ multipliers 3 and 8

✓ + sign

(c) $y = (1+5x)^4$ (2 marks)

$$\frac{dy}{dx} = 4(1+5x)^3 \times 5$$

$$= 20(1+5x)^3$$

✓ $4(1+5x)^3$

✓ coefficient of 20

(d) $y = x^2 e^{-2x}$ (leaving your answer in factorised form) (4 marks)

$$\frac{dy}{dx} = 2x e^{-2x} + x^2 \times -2e^{-2x}$$

$$= (2x - 2x^2)e^{-2x}$$

$$= 2x(1-x)e^{-2x}$$

✓ uses product rule correctly

✓ $2x e^{-2x}$ term correct

✓ $-2x^2 e^{-2x}$ term correct

✓ factorises fully

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Q2 **(8 marks)**

(a) Given that $f(x)=\dfrac{x^3-1}{x^3+1}$, find $f'(1)$ (4 marks)

$$f'(x) = \frac{3x^2.(x^3+1) - (x^3-1).3x^2}{(x^3+1)^2}$$

$$= \frac{3x^5 + 3x^2 - 3x^5 + 3x^2}{(x^3+1)^2}$$

$$= \frac{6x^2}{(x^3+1)^2}$$

$$f'(1) = \frac{6}{4} = \frac{3}{2} = 1.5$$

✓ correct quotient rule

✓ numerator correct

✓ denominator correct

✓ evaluates correctly at $x=1$

(b) Given that $g(x)=(1+\cos(3x))^2$ find $g'\left(\pi/6\right)$ (4 marks)

$$g'(x) = 2(1+\cos(3x)) \times -3\sin 3x$$

$$= -6\sin(3x)(1+\cos(3x))$$

$$g'(\pi/6) = -6\sin\frac{\pi}{2} \times \left(1+\cos\frac{\pi}{2}\right)$$

$$= -6 \times 1 \times 1$$

$$= -6$$

✓ $2(1+\cos 3x)$

✓ $-3\sin 3x$

✓ subs $x = \pi/6$

✓ evaluates correctly

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Q3. (8 marks)

Consider the graph of $y = \dfrac{x}{4+x^2}$.

(a) Find the values of x at which the gradient of the graph is equal to 0. (4 marks)

$$\frac{dy}{dx} = \frac{1.(4+x^2) - x.2x}{(4+x^2)^2}$$

✓ applies quotient rule

$$= \frac{4-x^2}{(4+x^2)^2}$$

✓ correct and simplified

✓ numerator $= 0$

$$\frac{dy}{dx} = 0 \Rightarrow 4 - x^2 = 0$$

$$\Rightarrow \underline{x = 2} \text{ or } \underline{x = -2}$$

✓ both answers

(b) Find the equation of the tangent to the curve at the point where x = 1. (4 marks)

$$x = 1 \quad \frac{dy}{dx} = \frac{4-1}{(4+1)^2} = \frac{3}{25}$$

✓ gradient at $x = 1$

and $y = \dfrac{1}{5}$

✓ y-value at $x = 1$

Tangent is $y = \dfrac{3}{25}x + c$

✓ eq^n of tangent with correct gradient

$$\left(1, \tfrac{1}{5}\right): \frac{1}{5} = \frac{3}{25} + c$$

$$\therefore c = \frac{2}{25}$$

$$\therefore y = \frac{3}{25}x + \frac{2}{25}$$

✓ evaluates c and writes eq^n.

Or $\dfrac{y - \frac{1}{5}}{x-1} = \dfrac{3}{25} \Rightarrow 25y - 5 = 3x - 3$

$$\Rightarrow 25y - 3x = 2$$

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Q4. (6 marks)

Consider the function given by $y = \dfrac{ax^2 + b}{cx + d}$.

Find the values of a, b , c and d given that $\dfrac{dy}{dx} = \dfrac{6x^2 + 18x + 5}{(2x+3)^2}$

$$\frac{dy}{dx} = \frac{2ax(cx+d) - (ax^2+b).c}{(cx+d)^2}$$

✓ numerator

✓ denominator

$$= \frac{acx^2 + 2adx - bc}{(cx+d)^2} = \frac{6x^2 + 18x + 5}{(2x+3)^2}$$

✓ c value and d value

$\therefore \quad c = 2, \quad d = 3$

$\therefore \quad 2a = 6$ and $6a = 18$ and $-bc = 5$

✓ compares numerators

$\therefore \quad a = 3, \quad b = -2.5$

✓ finds a

✓ finds b

$\underline{\underline{a = 3}}, \ \underline{\underline{b = -2.5}}, \ \underline{\underline{c = 2}}, \ \underline{\underline{d = 3}}$

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Q5. (7 marks)

Consider the function $y = e^{2x} \sin 3x$.

Show that $\dfrac{d^2y}{dx^2}$ is of the form $e^{2x}\left[a \sin 3x + b \cos 3x\right]$ and find the values of a and b.

$$y = e^{2x} \sin 3x$$

$$\frac{dy}{dx} = 2e^{2x} \cdot \sin 3x + e^{2x} \cdot 3\cos 3x$$

✓ 1st term $\frac{dy}{dx}$
✓ 2nd term $\frac{dy}{dx}$

$$= e^{2x}\left[2 \sin 3x + 3\cos 3x\right]$$

$$\frac{d^2y}{dx^2} = 2e^{2x}\left[2\sin 3x + 3\cos 3x\right]$$

✓ 1st term $\frac{d^2y}{dx^2}$

$$+ e^{2x}\left[6 \cos 3x - 9 \sin 3x\right]$$

✓ 2nd term $\frac{d^2y}{dx^2}$

$$= e^{2x}\left[4 \sin 3x + 6\cos 3x + 6 \cos 3x - 9 \sin 3x\right]$$

✓ takes out factor of e^{2x}

$$= e^{2x}\left[-5 \sin 3x + 12 \cos 3x\right]$$

✓ collects terms

$\therefore \quad a = -5$

$b = 12$

✓ states a and b

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