Name:_____

Section 1:   Calculator Free   39 marks   40 minutes

1.   [1, 1, 1, 2 marks]

Suppose that two variables x and y are related by $y = 6^x$.

a)   Use the *definition of a logarithm* to express x in terms of y.

b)   Given that $\log_6 2 = q$, write the following in terms of q:

  i) $\log_6 24$

  ii) $\log_6 0.5$

  iii) $\log_6 3$

2. [2, 2, 2 marks]

Solve the following, giving your answers in exact form involving logarithms where necessary.
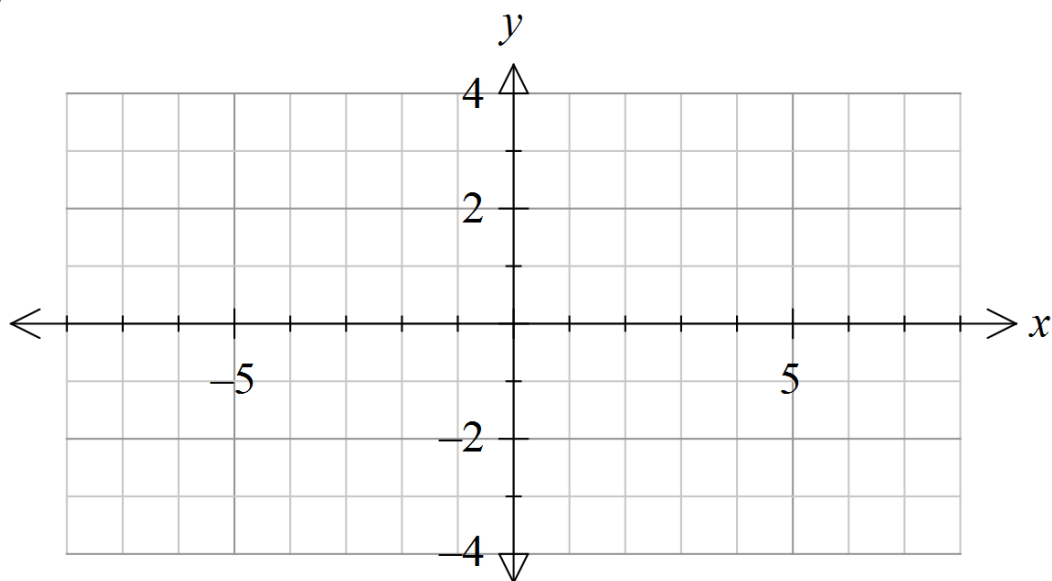
a) $3^{x-4} = 14$

b) $\log(x+4) - \log(x-5) = 1$
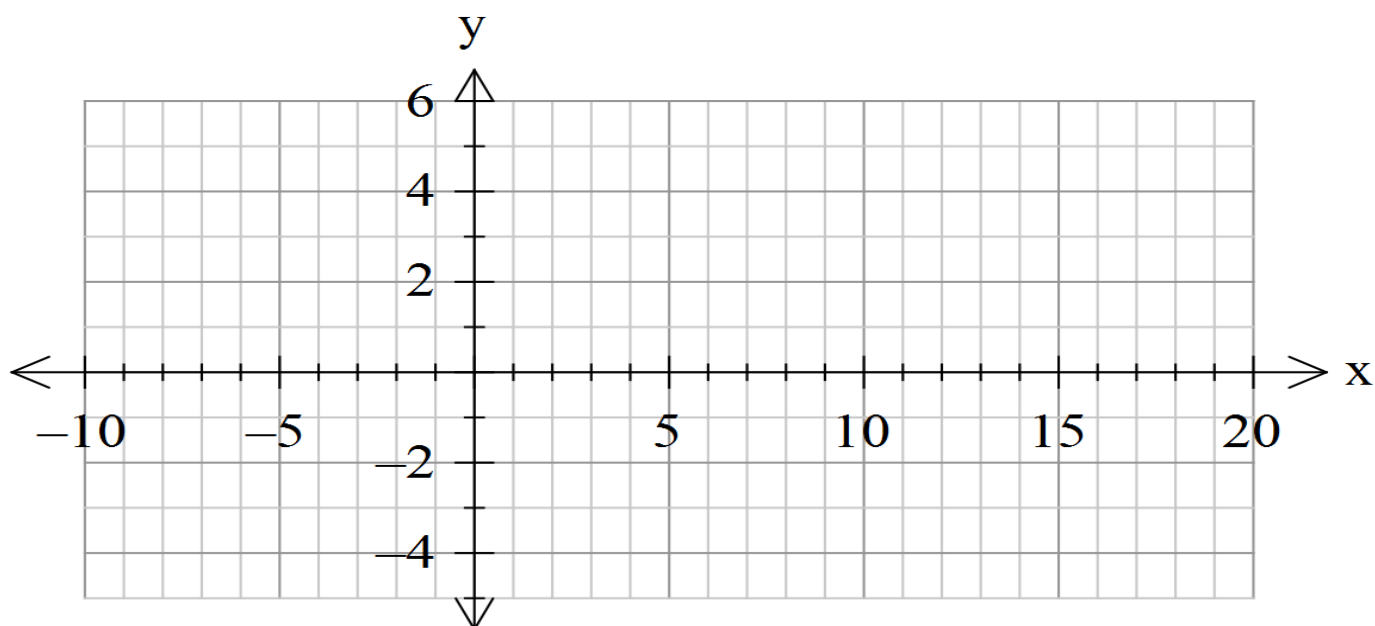
c) $11\left(3^x\right) = 5 + 3^{x+2}$

3. **[3, 3, 2 marks]**

On the sets of axes below, sketch the functions:

a) $y = \log_3(x - 2)$

b) $y = \log_{0.5}x + 4$

c) Use the graph to solve $\log_{0.5}x = 0.5$

4. **[3, 2 marks]**

Let $g(x) = \dfrac{\ln x}{x^2}$, for $x > 0$.

(a)   Use the quotient rule to show that $g'(x) = \dfrac{1 - 2 \ln x}{x^3}$.

(b)   The graph of g has a maximum point at A. Find the x-coordinate of A.

5.   **[2 marks]**

Find the derivative with respect to x of $y = \ln(x^3 + x^2)$

6. **[2, 1, 2, 2, 2 marks]**

a) Given the function $g(x) = x \ln x - x + 1$, determine $g'(x)$

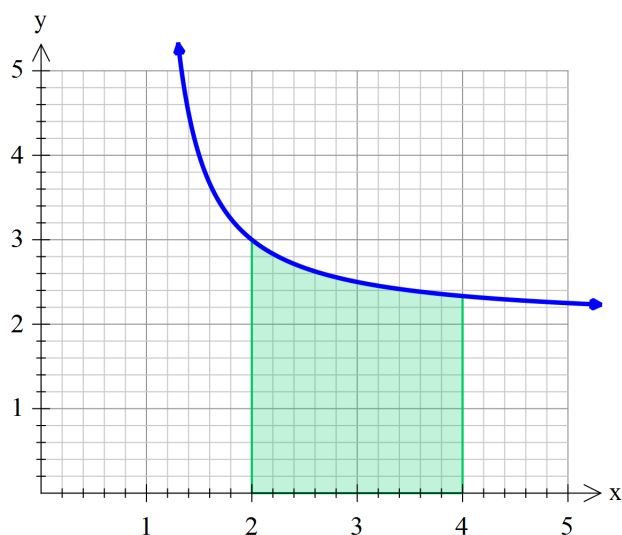b) Hence determine an expression for $\int \ln(x)\,dx$

c) Evaluate $\displaystyle\int_{1}^{2} \ln(x)\,dx$

d) Evaluate $\displaystyle\int_{1}^{2} \ln \sqrt{x}\,dx$

e) Determine an expression for $\displaystyle\int_{a}^{b} \ln(x)\,dx$ in terms of a and b, such that $b > a > 0$

7.   [2, 2 marks]

Consider the function $f(x) = 2 + \dfrac{1}{x-1} \; ; x > 1$

The region enclosed by the graph of $f(x)$, the x-axis and the lines $x = 2$ and $x = 4$, is shaded below.

(a)   Find $\displaystyle \int f(x)\,dx$.

(a)   Find $\displaystyle \int f(x)\,dx$.

(b)   Find a simplified expression for the exact area of A.

Name:_____

Section 2:   Calculator & Notes Allowed   13 marks   15 minutes

8.   [1, 1, 3 marks]

The faintest sound that can be heard by the human ear has intensity

$$I_0 = 10^{-16}$$ watts per square centimetre.

Noise levels, β, are measured in decibels and are related to intensity:

$$\beta = 10 \log \frac{I}{I_0}$$ decibels

Where I is the intensity of sound in watts per square centimetre.

a)   The maximum intensity which a human ear can tolerate is 10^{-4} watts per square centimetre. Determine the corresponding value of β.

b)   Busy motor traffic has a noise level of 70 decibels. Determine the corresponding intensity.

c)   The graph (without scales) of $\log I$ against β is sketched below; it is linear. By expressing $\log I$ in terms of β, determine the gradient and the intercept on the vertical axis.
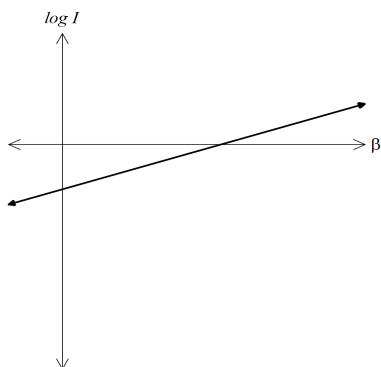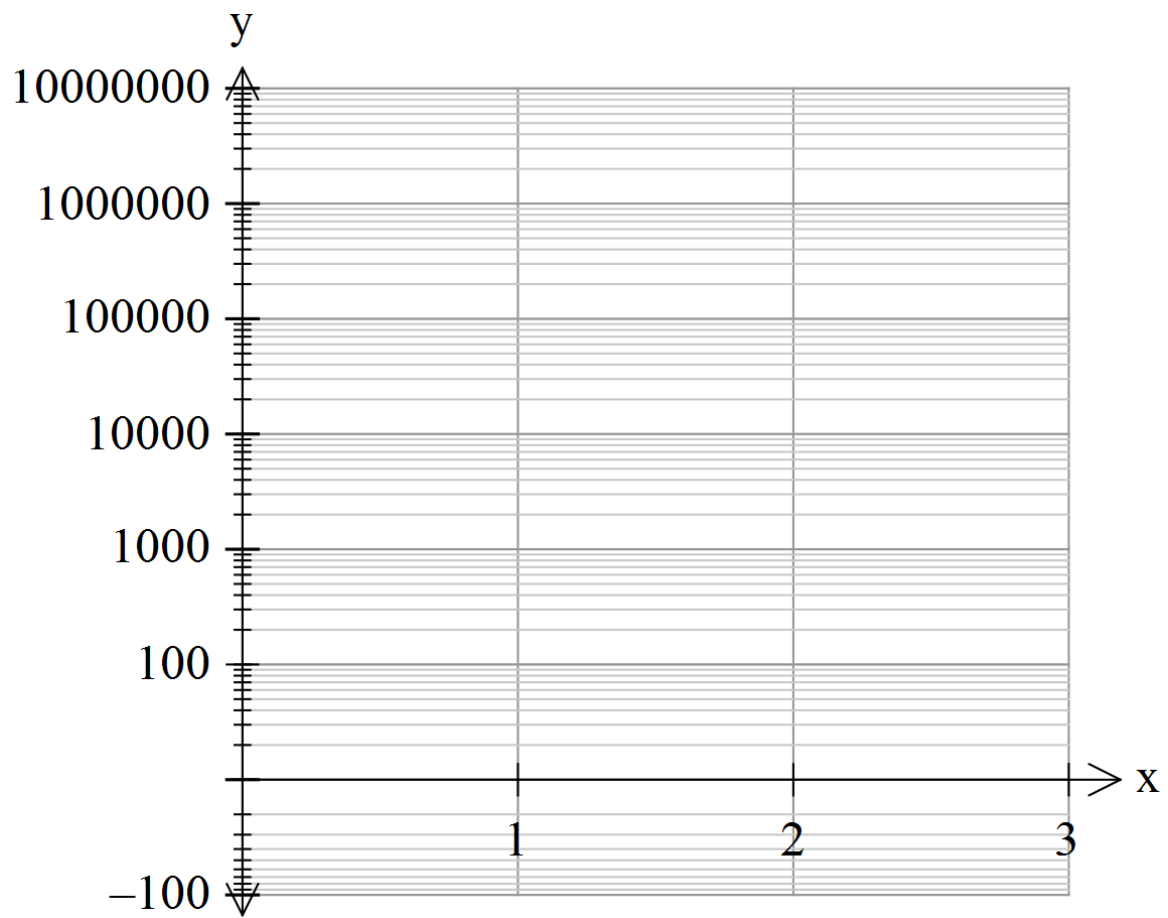
9. **[3, 2 marks]**

A particle P moves along a straight line. Its velocity, v ms^{-1} at time t seconds, is given by
$$v = 10\ln(t + 3) + 2 \ \ for \ \ t \geq 0$$

(a) Find the initial velocity and acceleration

(b) Find the acceleration of P when its velocity is 20 ms^{-1}

10. **[3 marks]**

Draw the graph of $y = 2 \times 10^{2x}$,for $x \geq 0$; using semi-log grid.