



MATHEMATICS METHODS Year 12

Section One: Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that **you do not use pencil**, except in diagrams.

Question 1

(9 marks)

Evaluate the following.

(a) $\int_{-1}^2 \frac{d}{dx} x^2 e^{x^3} dx$ (2 marks)

(b) $\frac{d}{dx} (\sin(2x) + e^{x^3})$ (2 marks)

(c) $\int -3xe^{2x^2} dx$ (2 marks)

Question 1 continued

(d) $\int_0^{\frac{\pi}{2}} 2 + \cos \frac{x}{2} dx$, giving your answer as an exact value.

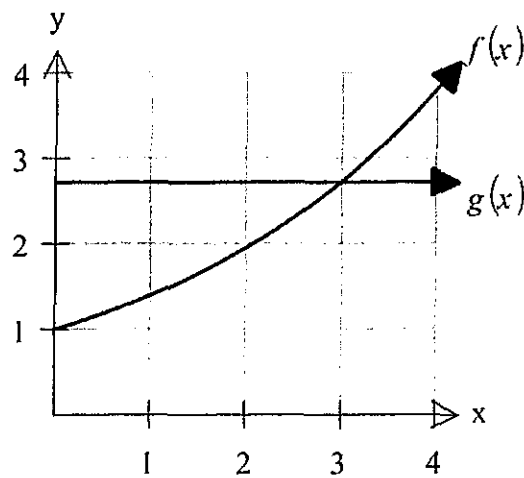
(3 marks)

See next page

Question 2

(3 marks)

The functions $f(x) = e^{\frac{x}{3}}$ and $g(x) = e$ are graphed below, intersecting at $(3, e)$.



Determine the area bound by the two curves and the y-axis.

Question 3**(3 marks)**

Determine the x coordinates of all stationary points on the function

$$f(x) = \int_0^{x^2} e^{t^2} - e \, dt$$

End of questions



Christ Church
Grammar School

2017
UNIT TEST 3

MATHEMATICS METHODS Year 12

Section Two:

Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes

Working time for this section: 30 minutes

Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

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Question 4**(6 marks)**

$X \sim B[n, p]$, with $\text{Var}(X) = 1.5$ and $E(X) = 2$.

(a) Determine the value of n and p .

(4 marks)

(b) Determine an expression for $P(X \geq 1)$. Do not simplify.

(2 marks)

Question 5**(5 marks)**

A population grows continuously such that $\frac{dP}{dt} = 0.09P$, where P is the size of the population t years after observation commenced. When observation commenced, the size of the population was 350 000.

(a) Determine an expression for P in terms of t .

(1 mark)

(b) How long will it take for the population to reach 1 000 000?

(2 marks)

(c) A second population is increasing at the same rate as that from part (a), however the initial population is unknown. How long will it take for this population to double in size?

(2 marks)

Question 6**(5 marks)**

A particle Q , with an initial displacement of 3m moves along a straight line. Its velocity $v \text{ ms}^{-1}$ after t seconds is given by $v(t) = 2 - 4e^{-0.5t}$.

(a) Determine:

(i) The displacement of the particle in terms of t . (2 marks)

(ii) The acceleration of the particle when it is at rest. (2 marks)

(b) Describe the speed of Q for large values of t . (1 mark)

Question 7

(8 marks)

Consider the discrete probability distribution shown below

x	-1	0	1	2	3
$P(X=x)$	0.25	0.3	0.1	p	q

(a) Determine:

i) $P(X > -1)$ (1 mark)

ii) $P(X = -1 | X \leq 1)$ (1 mark)

(b) Given that $E(X) = 0.85$ determine:

i) the values of p and q . (3 marks)

ii) $E(2X - 1)$ (1 mark)

(c) Given that $SD(X) = 1.6$, determine:

i) $SD(3X)$ (1 mark)

ii) $Var(3X)$ (1 mark)

Question 8

(6 marks)

It is known that 3% of cars manufactured in a particular assembly line will have some kind of defect.

(a) If a random sample of 7 cars is selected for testing, find that probability that:

i) no cars have a defect. (1 mark)

ii) exactly 3 cars have a defect. (1 mark)

iii) no cars have a defect, given that less than 5 cars have a defect. (2 marks)

(b) What is the largest number of cars that can be selected in a random sample such that the probability of there being at least 1 defective car is less than 20%.

(2 marks)

End of questions

Additional working space

Question number: _____

See next page



MATHEMATICS METHODS Year 12

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Teacher name _____

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Question 1

(9 marks)

Evaluate the following.

(a) $\int_{-1}^2 \frac{d}{dx} x^2 e^{x^3} dx$

(2 marks)

$$= \left[x^2 \cdot e^{x^3} \right]_{-1}^2 \quad \checkmark$$

$$= (2)^2 \cdot e^8 - ((-1)^2 \cdot e^{-1})$$

$$= 4e^8 - \frac{1}{e} \quad \checkmark$$

(b) $\frac{d}{dx} (\sin(2x) + e^{x^3})$

(2 marks)

$$2 \cos 2x + 3x^2 e^{x^3} \quad \checkmark \quad \checkmark$$

(c) $\int -3xe^{2x^2} dx$

(2 marks)

$$= -\frac{3}{4} \int 4x \cdot e^{2x^2} dx$$

$$= -\frac{3}{4} e^{2x^2} + C \quad \checkmark \quad \checkmark$$

(d) $\int_0^{\frac{\pi}{2}} 2 + \cos \frac{x}{2} dx$, giving your answer as an exact value.

(3 marks)

$$= \left[2x + 2\sin \frac{x}{2} \right]_0^{\frac{\pi}{2}} \quad \checkmark$$

$$= 2 \frac{\pi}{2} + 2\sin \frac{\pi}{2} - \left(2(0) + 2\sin \frac{0}{2} \right)$$

$$= \pi + 2\sin \frac{\pi}{2} - 0 \quad \checkmark$$

$$= \pi + 2 \times \frac{\sqrt{2}}{2}$$

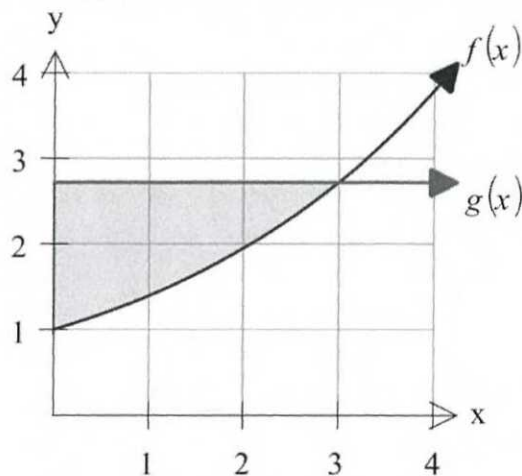
$$= \pi + \sqrt{2} \quad \checkmark$$

3

Question 2

(3 marks)

The functions $f(x) = e^{\frac{x}{3}}$ and $g(x) = e$ are graphed below, intersecting at $(3, e)$.



Determine the area bound by the two curves and the y-axis.

$$= \int_0^3 e - e^{\frac{x}{3}} dx \quad \checkmark$$

$$= \left[ex - 3e^{\frac{x}{3}} \right]_0^3 \quad \checkmark$$

$$= 3e - 3e^1 - (ex(0) - 3e^0)$$

$$= 3e - 3e + 3e^0$$

$$= \underline{\underline{3}} \text{ units}^2 \quad \checkmark$$

3

Question 3

(3 marks)

Determine the x coordinates of all stationary points on the function

$$f(x) = \int_0^{x^2} e^{t^2} - e \, dt$$

$$f'(x) = [e^{(x^2)^2} - e][2x]$$

$$\therefore f'(x) = \underline{2x(e^{x^4} - e)} \quad \checkmark$$

$$\text{Stat pt } f'(x) = 0$$

$$\therefore \underline{2x(e^{x^4} - e) = 0} \quad \checkmark$$

$$\therefore 2x = 0 \quad \text{OR} \quad e^{x^4} - e = 0$$

$$e^{x^4} = e$$

$$x^4 = 1$$

$$x = \pm 1$$

$$\therefore \text{Soln}$$

$$x = 0, \pm 1 \quad \checkmark$$

3

Question 5

(5 marks)

A population grows continuously such that $\frac{dP}{dt} = 0.09P$, where P is the size of the population t years after observation commenced. When observation commenced, the size of the population was 350 000.

(a) Determine an expression for P in terms of t .

(1 mark)

$$P_t = 350\,000 e^{0.09t} \quad \checkmark$$

(b) How long will it take for the population to reach 1 000 000?

(2 marks)

$$1\,000\,000 = 350\,000 e^{0.09t}$$

$$t = \frac{11.66^{469}}{\text{yrs}} \quad \checkmark$$

$$t > 11.665$$

(c) A second population is increasing at the same rate as that from part (a), however the initial population is unknown. How long will it take for this population to double in size?

(2 marks)

$$2P_0 = P_0 e^{0.09t} \quad \checkmark$$

$$2 = e^{0.09t}$$

$$t = 7.70^{163} \text{ yrs} \quad \checkmark$$

$$\therefore t > 7.70163534$$

5

Question 4

(6 marks)

 $X \sim B[n, p]$, with $\text{Var}(X) = 1.5$ and $E(X) = 2$.(a) Determine the value of n and p .

(4 marks)

$$\text{Var}(X) : np(1-p) = 1.5, \quad E(X) = np = 2$$

$$\therefore 2(1-p) = 1.5$$

$$1-p = 0.75$$

$$p = \underline{0.25}$$

$$0.25n = 2$$

$$\therefore \underline{n = 8}$$

(b) Determine an expression for $P(X \geq 1)$. Do not simplify.

(2 marks)

$$1 - 0.75^8$$

$$\text{OR} \quad \sum_{x=1}^8 {}^8C_x (0.25)^x (0.75)^{8-x}$$

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Calculator-assumed

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Question 6

5
(4 marks)

A particle Q , with an initial displacement of 3m moves along a straight line. Its velocity $v \text{ ms}^{-1}$ after t seconds is given by $v(t) = 2 - 4e^{-0.5t}$.

(a) Determine:

- (i) The displacement of the particle in terms of t . (1 mark)

$$x(t) = 2t + 8e^{-0.5t} + c \quad \checkmark \quad \text{but } x(0) = 3$$

$$3 = 8e^0 + c$$

$$c = -5$$

$$\therefore x(t) = 2t + 8e^{-0.5t} - 5 \quad \checkmark$$

- (ii) The acceleration of the particle when it is at rest. (2 marks)

$$\text{At rest } v(t) = 0$$

$$2 - 4e^{-0.5t} = 0$$

$$t = 1.39 \quad \checkmark$$

$$a(t) = 2e^{-0.5t}$$

$$a(1.39) = 0.9981 \text{ ms}^{-2} \quad \checkmark$$

- (b) Describe the speed of Q for large values of t .

(1 mark)

tends to 2 ms^{-1} \checkmark

Question 7

(8 marks)

Consider the discrete probability distribution shown below

x	-1	0	1	2	3
$P(X=x)$	0.25	0.3	0.1	p	q

(a) Determine:

i) $P(X > -1)$

(1 mark)

$$0.75 \quad \checkmark$$

ii) $P(X = -1 | X \leq 1)$

(1 mark)

$$\frac{0.25}{0.65} = 0.3846 \quad \checkmark \quad \left(\frac{5}{13}\right)$$

(b) Given that $E(X) = 0.85$ determine:i) the values of p and q .

(3 marks)

$$\sum P(X=x) = 1 \quad \therefore p+q = 0.35 \quad \text{--- (1)} \quad \checkmark$$

$$-0.25 + 0 + 0.1 + 2p + 3q = 0.85$$

$$2p + 3q = 1 \quad \text{--- (2)} \quad \checkmark$$

Solve (1) & (2) on CP.

$$\therefore p = \underline{0.05}, \quad q = \underline{0.3} \quad \checkmark$$

ii) $E(2X - 1)$

(1 mark)

$$= 2(0.85) - 1$$

$$= \underline{0.7} \quad \checkmark$$

See next page

(c) Given that $SD(X) = 1.6$, determine:

i) $SD(3X)$

(1 mark)

$$3 \times 1.6 = \underline{4.8} \quad \checkmark$$

ii) $Var(3X)$

(1 mark)

$$= 4.8^2 \\ = \underline{23.04} \quad \checkmark$$

Question 8

(6 marks)

It is known that 3% of cars manufactured in a particular assembly line will have some kind of defect.

(a) If a random sample of 7 cars is selected for testing, find that probability that:

i) no cars have a defect.

(1 mark)

$$x \sim B(7, 0.03) \quad \underline{0.8080} \quad \checkmark$$

ii) exactly 3 cars have a defect.

(1 mark)

$$\underline{0.0008366} \quad \checkmark$$

iii) no cars have a defect, given that less than 5 cars have a defect.

(2 marks)

$$\frac{0.8080}{1} \quad \checkmark = \underline{0.8080} \quad \checkmark$$

(b) What is the largest number of cars that can be selected in a random sample such that the probability of there being at least 1 defective car is less than 20%.

(2 marks)

$$\therefore \text{None Defective} > 0.8$$

$$0.97^n > 0.8 \quad \checkmark$$

$$n < 7.33$$

$$\therefore n = 7 \quad \checkmark$$

End of questions

8

Question number: _____

See next page
