SADLER MATHEMATICS METHODS UNIT 1

WORKED SOLUTIONS

Chapter 1 Trigonometry

Exercise 1B

Question 1

a $\sin x = 0.4$
$$x = 24°$$

b $\sin x = 0.75$
$$x = 49°$$

c $\sin x = 0.8$
$$x = 53°$$

Question 2

a $\sin x = 0.2$
Ref angle $= 12°$
$$x = 180° - 12°$$
$$x = 168°$$

b $\sin x = 0.3$
Ref angle $= 17°$
$$x = 180° - 17°$$
$$= 163°$$

c $\sin x = 0.55$
Ref angle $= 33°$
$$x = 180° - 33°$$
$$= 147°$$

Question 3

a $\sin x = 0.5$

       $x = 30°, 150°$

b $\sin x = 0.15$

       $x = 9°, 171°$

c $\sin x = 0.72$

       $x = 46°, 134°$

Question 4

$$A = \frac{1}{2} \times a \times b \sin C$$
$$= \frac{1}{2} \times 5.4 \times 6.2 \times \sin 42°$$
$$= 11.2 \text{ cm}^2$$

Question 5

$$A = \frac{1}{2} \times 5.7 \times 7.3 \times \sin 68°$$
$$= 19.3 \text{ cm}^2$$

Question 6

$$A = \frac{1}{2} \times 5.8 \times 6.9 \times \sin 115°$$
$$= 18.1 \text{ cm}^2$$

Question 7

Missing angle 110°

$$A = \frac{1}{2} \times 6.7 \times 8.8 \times \sin 110°$$
$$= 27.7 \text{ cm}^2$$

Question 8

$$108 - (61 + 39) = 80°$$
$$A = \frac{1}{2} \times 5.2 \times 6.8 \times \sin 80$$
$$= 17.4 \text{ cm}^2$$

Question 9

$$A = \frac{1}{2} \times 16.1 \times 19.5 \times \sin 118°$$
$$= 138.6 \text{ cm}^2$$

Question 10

$$20.7 = \frac{1}{2} \times 6.2 \times x \times \sin 50°$$
$$x = \frac{20.7 \times 2}{6.2 \times \sin 50°} \text{ or use CP solve}$$
$$x = 8.7 \text{ cm}$$

Question 11

$$20.0 = \frac{1}{2} \times 7.2 \times x \times \sin 110°$$
$$x = 5.9 \text{ cm}$$

Question 12

$$15.0 = \frac{1}{2} \times 7.5 \times x \times \sin 30°$$
$$x = 8.0 \text{ cm}$$

Question 13

$$45.1 = \frac{1}{2} \times 14.9 \times x \times \sin 135°$$
$$x = 8.6 \text{ cm}$$

Question 14

$$25.5 = \frac{1}{2} \times 8.0 \times 6.4 \times \sin x$$
$$\sin x = \frac{25.5 \times 2}{8.0 \times 6.4}$$
$$x = 84.9°, 95.1°$$

Question 15

$$72.7 = \frac{1}{2} \times 9.8 \times 14.9 \times \sin x$$
$$\sin x = \frac{72.7 \times 2}{9.8 \times 14.9}$$
$$x = 84.7°, 95.3°$$

Exercise 1C

Question 1

$$\frac{x}{\sin 85°} = \frac{56}{\sin 74°}$$
$$x = 58.0$$

Question 2

$$180 - (74 + 34) = 72$$
$$\frac{x}{\sin 72°} = \frac{12.4}{\sin 74°}$$
$$x = 12.3$$

Question 3

$$\frac{\sin x°}{61} = \frac{\sin 48°}{56}$$
$$\sin x° = \frac{61 \times \sin 48°}{56}$$
$$x = 54°, 126° \ (48° + 54° < 180° \text{ and } 48° + 126° < 180°)$$

Question 4

$$\frac{\sin x°}{3.4} = \frac{\sin 123°}{11.8}$$
$$\sin x° = \frac{3.4 \times \sin 123°}{11.8}$$
$$x = 14°$$

Question 5

$$\frac{\sin x°}{16.3} = \frac{\sin 38°}{10.4}$$

$$\sin x° = \frac{16.3 \times \sin 38°}{10.4}$$

$$x = 75°, 105°$$

Question 6

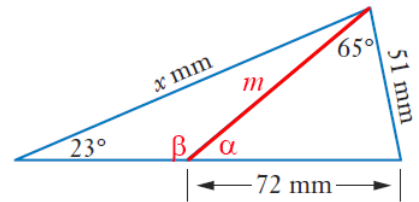$$\frac{\sin \alpha}{51} = \frac{\sin 65°}{72}$$

$$\alpha = 40°$$

$$\beta = 140°$$

$$\theta = 180 - 65 - 40 = 75°$$

$$\frac{m}{\sin 75°} = \frac{72}{\sin 65°}$$

$$m = 76.7 \text{ mm}$$

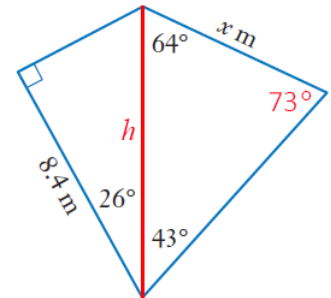$$\frac{x}{\sin 140°} = \frac{76.7}{\sin 23°}$$

$$x = 126 \text{ mm}$$

Question 7

$$180 - 64 - 43 = 73$$

$$\cos 26° = \frac{8.4}{h}$$

$$\therefore h = 9.346 \text{ m}$$

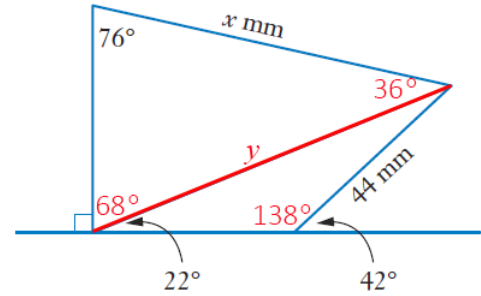$$\frac{x}{\sin 43°} = \frac{9.346}{\sin 73°}$$

$$x = 6.7 \text{ m}$$

Question 8

$$\frac{44}{\sin 22°} = \frac{y}{\sin 138°}$$
$$y = 78.59 \text{ mm}$$
$$\frac{78.59}{\sin 76°} = \frac{x}{\sin 68°}$$
$$x = 75 \text{ mm}$$

Question 9

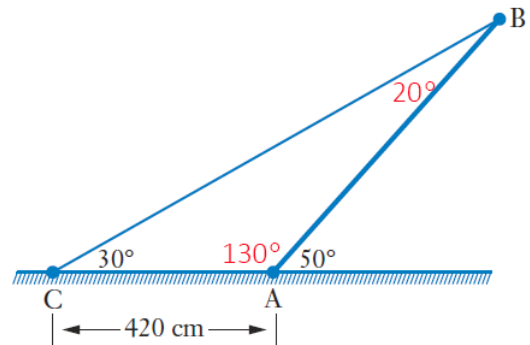$$\frac{\sin 20°}{420} = \frac{\sin 30°}{AB}$$
$$AB = \frac{420 \times \sin 30°}{\sin 20°}$$
$$= 614 \text{ cm}$$
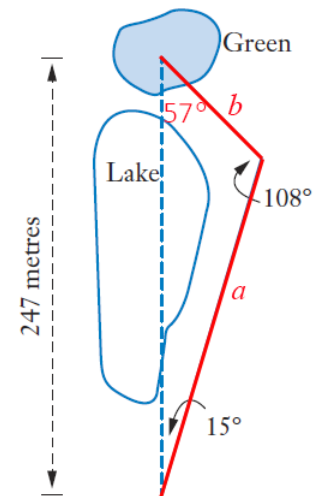

Question 10

$$\frac{a}{\sin 57°} = \frac{247}{\sin 108°}$$
$$a = 217.8$$
$$\frac{b}{\sin 15°} = \frac{247}{\sin 108°}$$
$$b = 67.2$$
$$\text{Difference} = 217.8 + 67.2 - 247$$
$$= 38 \text{ m}$$

Question 11

$$x^2 = 58^2 + 73^2 - 2(58)(73)\cos 52°$$
$$x = 59 \text{ mm}$$

Question 12

$$x^2 = 7.3^2 + 9.8^2 - 2(7.3)(9.8)\cos 114°$$
$$x = 14.4 \text{ cm}$$

Question 13

$$\cos x° = \frac{7.3^2 + 6.8^2 - 5.2^2}{2(7.3)(6.8)}$$
$$x = 43°$$

Question 14

$$\cos x = \frac{84^2 + 124^2 - 173^2}{2(84)(124)}$$
$$x = 11°$$

Question 15

$$x^2 = 23^2 + 31^2 - 2(23)(31)\cos 108°$$
$$x = 44 \text{ m}$$

Question 16

$$s^2 = 85^2 - 51^2$$
$$s = 68$$
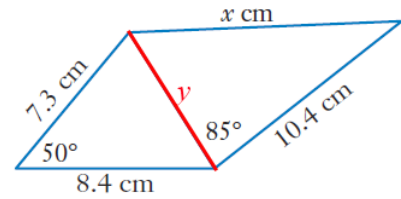$$\cos x = \frac{68^2 + 83^2 - 79^2}{2(68)(83)}$$
$$x = 62°$$

Question 17

$y^2 = 8.4^2 + 7.3^2 - 2(8.4)(7.3)\cos 50°$

$y = 6.71 \text{ cm}$

$x^2 = 6.71^2 + 10.4^2 - 2(6.71)(10.4)\cos 85°$

$x = 11.9 \text{ cm}$

Question 18

$\cos\alpha = \dfrac{135^2 + 128^2 - 123^2}{2(135)(128)}$

$\alpha = 55.7°$

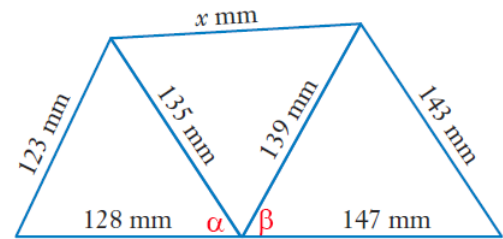$\cos\beta = \dfrac{139^2 + 147^2 - 143^2}{2(139)(147)}$

$\beta = 59.9°$
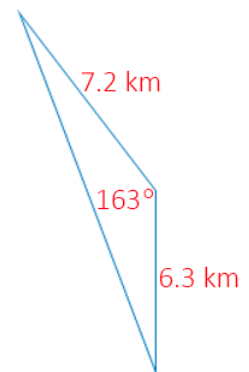
$180° - 55.7° - 59.9°$

$= 64.4°$

$x^2 = 135^2 + 139^2 - 2(135)(139)\cos 64.4°$

$x = 146 \text{ mm}$

Question 19

$SF^2 = 6.3^2 + 7.2^2 - 2(6.3)(7.2)\cos 163°$

$SF = 13.4 km$
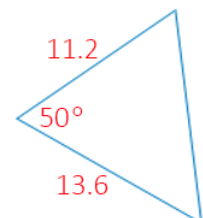
Question 20

Jim: $1.4 \times 8 = 11.2 \text{ m}$

Toni: $1.7 \times 8 = 13.6 \text{ m}$

$x^2 = 11.2^2 + 13.6^2 - 2(11.2)(13.6)\cos 50°$

$x = 10.7 \text{ m}$

Question 21

$$\frac{\sin x°}{19.8} = \frac{\sin 42°}{13.7}$$

$$\sin x° = \frac{19.8 \sin 42°}{13.7}$$

$$x = 75°, 105°$$

Question 22

$$\cos x = \frac{12.3^2 + 13.8^2 - 19.9^2}{2(12.3)(13.8)}$$

$$x = 99°$$

Question 23

$$x^2 = 481^2 + 563^2 - 2(481)(563)\cos 72°$$

$$x = 617 \text{ m}$$

Question 24

$$180° - 112° - 27° = 41°$$

$$\frac{x}{\sin 41°} = \frac{7.62}{\sin 112°}$$

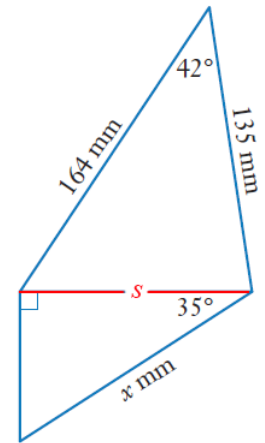$$x = \frac{7.62 \times \sin 41°}{\sin 112°}$$

$$= 5.39 \text{ km}$$

Question 25

$$s^2 = 164^2 + 135^2 - 2(164)(135)\cos 42°$$

$$s = 110.5 \text{ mm}$$

$$\cos 35° = \frac{110.5}{x}$$
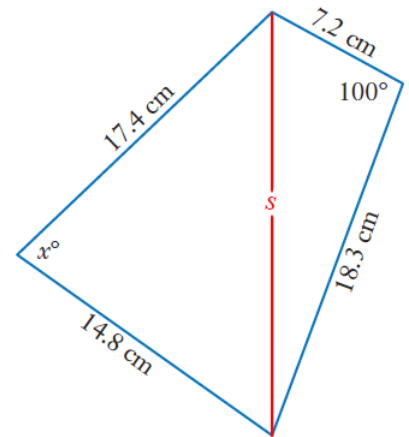
$$x = \frac{110.5}{\cos 35°}$$

$$= 135 \text{ mm}$$

Question 26

$$s^2 = 7.2^2 + 18.3^2 - 2(7.2)(18.3)\cos 100°$$

$$s = 20.8$$

$$\cos x = \frac{17.4^2 + 14.8^2 - 20.8^2}{2(17.4)(14.8)}$$

$$x = 80°$$

Question 27

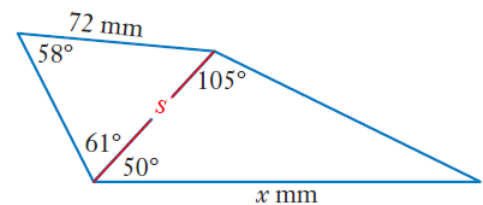$$\frac{s}{\sin 58°} = \frac{72}{\sin 61°}$$

$$s = \frac{72 \times \sin 58°}{\sin 61°}$$

$$= 69.8 \text{ mm}$$

Missing angle in Δ2 : 25°

$$\frac{x}{\sin 105°} = \frac{69.8}{\sin 25°}$$

$$x = \frac{69.8 \times \sin 105°}{\sin 25°}$$
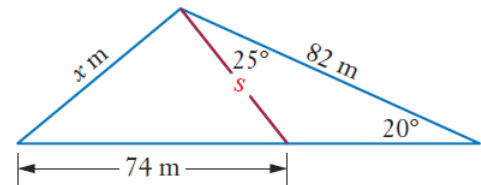
$$= 160 \text{ mm}$$

Question 28

Missing angle: 135°

$$\frac{s}{\sin 20°} = \frac{82}{\sin 135°}$$

$$s = \frac{82 \times \sin 20°}{\sin 135°}$$

$$= 39.7 \text{ m}$$

$$x^2 = 39.7^2 + 74^2 - 2(39.7)(74)\cos 45°$$
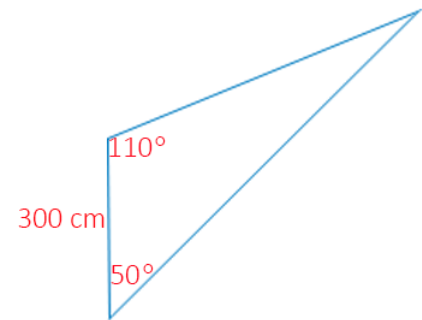
$$x = 54 \text{ m}$$

Question 29

Missing angle: 20°

$$\frac{\sin 20°}{300} = \frac{\sin 50°}{AC}$$

$$AC = \frac{300 \times \sin 50°}{\sin 20°}$$

$$= 672 \text{ cm}$$

$$\frac{\sin 20°}{300} = \frac{\sin 110°}{BC}$$

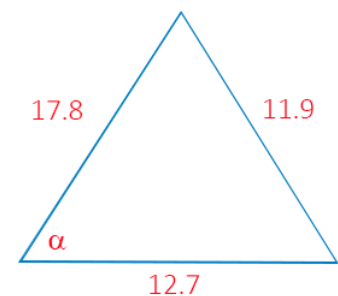$$BC = \frac{300 \times \sin 110°}{\sin 20°}$$

$$= 824 \text{ cm}$$

Question 30

Smallest angle opposite smallest side

$$\cos\alpha = \frac{17.8^2 + 12.7^2 - 11.9^2}{2(17.8)(12.7)}$$

$$\alpha = 42°$$

Question 31

$$\frac{\sin 72°}{9.1} = \frac{\sin B}{7.3}$$

$$\sin B = \frac{7.3 \times \sin 72°}{9.1}$$

$$B = 49.7°$$

$$\angle C = 180° - 72° - 49.7° = 58.3°$$

$$\frac{AB}{\sin 58.3°} = \frac{9.1}{\sin 72°}$$

$$AB = \frac{9.1 \times \sin 58.3°}{\sin 72°}$$

$$= 8.1 \text{ cm}$$
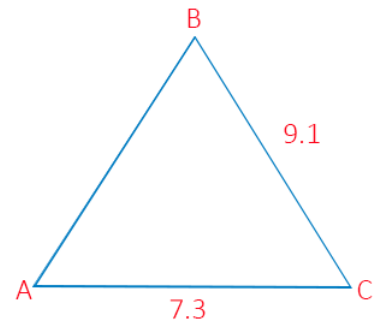
Question 32

$$BC^2 = 12.4^2 + 14.3^2 - 2(12.4)(14.3)\cos 43°$$

$$BC = 9.9 \text{ cm}$$

$$\cos C° = \frac{9.9^2 + 14.3^2 - 12.4^2}{2(9.9)(14.3)}$$

$$C = 58°$$

$$B = 180° - 43° - 58° = 79°$$

Question 33

$$AB^2 = 15.2^2 + 12.1^2 - 2(15.2)(12.1)\cos 70°$$

$$AB = 15.9 \text{ km}$$

Question 34

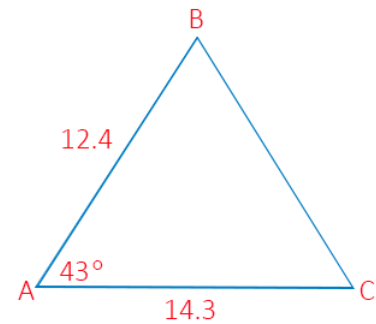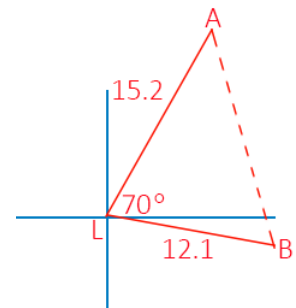$$\frac{7.3}{\sin 20°} = \frac{LQ}{\sin 100°}$$

$$LQ = \frac{7.3 \times \sin 100°}{\sin 20°}$$

$$= 21.0 \text{ km}$$
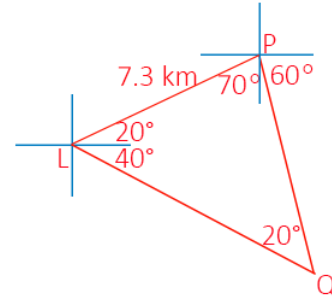
Question 35

$$\frac{h}{\sin 52°} = \frac{25}{\sin 68°}$$

$$h = \frac{25 \times \sin 52°}{\sin 68°}$$

$$= 21 \text{ m}$$
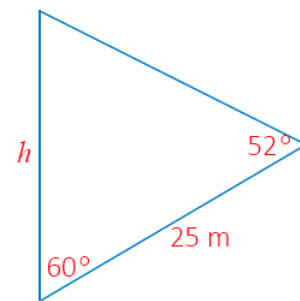
Question 36

$$BD^2 = 6.8^2 + 3.7^2 - 2(6.8)(3.7)\cos 48°$$

$$BD = 5.1 \text{ cm}$$

$$AC^2 = 3.7^2 + 6.8^2 - 2(3.7)(6.8)\cos 132°$$

$$AC = 9.7 \text{ cm}$$
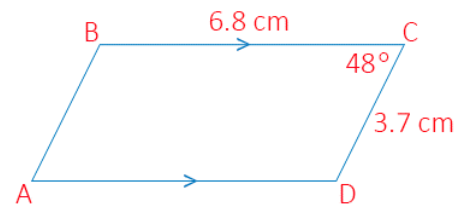
Question 37

$$AD^2 = 7.4^2 + 5.2^2 - 2(7.4)(5.2)\cos 63°$$

$$AD = 6.8 \text{ cm}$$

$$CD^2 = 7.4^2 + 5.2^2 - 2(7.4)(5.2)\cos 117°$$

$$CD = 10.8 \text{ cm}$$
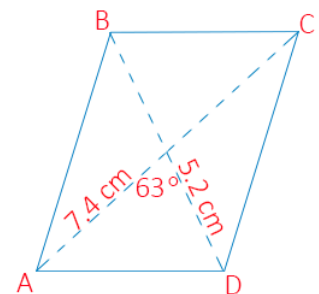
Question 38

a

$AC = 2.6$ m

$$\cos\theta = \frac{2.6^2 + 2^2 - 1^2}{2(2.6)(2)}$$

$$\theta = 20.2°$$
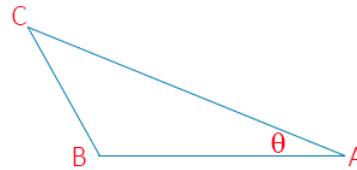
b

$$\cos\theta = \frac{2.1^2 + 2^2 - 1^2}{2(2.1)(2)}$$

$$\theta = 28.1°$$

Question 39

a ΔCHE is right angled with $HE = 450$cm

$$\frac{x}{\sin 20°} = \frac{900}{\sin 140°}$$

$$x = \frac{900 \times \sin 20°}{\sin 140°}$$

$$= 479 \text{ cm}$$

b $ED = \dfrac{1}{2} \times CE$

$\qquad = 239$ cm

c $DF^2 = 300^2 + 239.4^2 - 2(300)(239.4)\cos 20°$

$\quad DF = 111$ cm

d $CF^2 = 478.9^2 + 300^2 - 2(478.9)(300)\cos 20°$

$\quad CF = 222$ cm

Question 40

a $d^2 = 90^2 + 70^2 - 2(90)(70)\cos 150°$

    $d = 154.6$ mm

      $= 155$ mm (nearest mm)

b   At 5:10 the angle between the hands is 95°

    $d^2 = 90^2 + 70^2 - 2(90)(70)\cos 95°$

    $d = 118.7$ mm

      $= 119$ mm (nearest mm)

Question 41

a $\dfrac{SL}{\sin 40°} = \dfrac{2.5}{\sin 115°}$

    $SL = \dfrac{2.5 \times \sin 40°}{\sin 115°}$

      $= 1.77$ km

b $\dfrac{OS}{\sin 25°} = \dfrac{2.5}{\sin 115°}$

    $OS = \dfrac{2.5 \times \sin 25°}{\sin 115°}$

      $= 1.17$ km
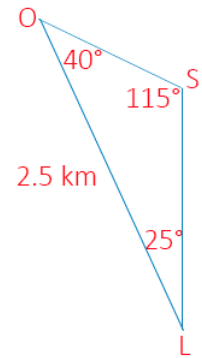
Question 42

a $AB^2 = 4^2 + 3^2$

$AB = 5$

$BC^2 = 8^2 + 8^2$

$BC = 8\sqrt{2}$

$AC^2 = 12^2 + 5^2$

$AC = 13$

$\cos B = \dfrac{5^2 + \left(8\sqrt{2}\right)^2 - 13^2}{2(5)(8\sqrt{2})}$

$B = 98°$

b $\tan \alpha = \dfrac{3}{4}$

$\alpha = 36.9°$

$\tan \beta = \dfrac{8}{8}$

$\beta = 45°$

$\therefore \angle ABC = 180° - 45° - 36.9°$

$= 98°$ (to nearest degree)
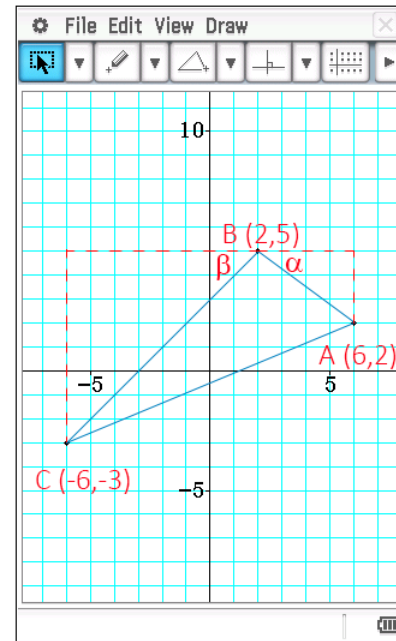
Question 43

$\dfrac{40}{\sin 15°} = \dfrac{BD}{\sin 20°}$

$BD = \dfrac{40 \times \sin 20°}{\sin 15°}$

$= 52.9$ m

$\sin 35° = \dfrac{h}{52.9}$

$h = 52.9 \times \sin 35°$

$= 30$ m (to nearest metre)
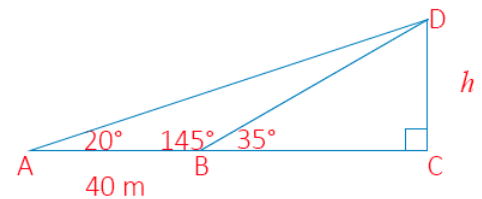
Question 44

$$\tan 17° = \frac{37}{AB}$$

$$AB = \frac{37}{\tan 17°}$$

$$= 121m$$

$$\tan 12° = \frac{37}{AC}$$

$$AC = \frac{37}{\tan 12°}$$

$$= 174m$$

$$BC^2 = 121^2 + 174^2 - 2(121)(174)\cos 30°$$

$$BC = 92m$$
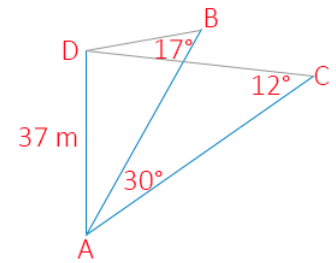
Question 45

From the information given, the ambiguous case exists.

We are given two sides but not the angle in between.

$$\frac{\sin\alpha}{19.4} = \frac{\sin 55°}{18.2}$$

$$\sin\alpha = \frac{19.4 \times \sin 55°}{18.2}$$

$$\alpha = 61° \text{ or } 119°$$

If $\angle I = 61°$

$$\therefore \angle H = 180° - 61° - 55°$$

$$= 64°$$
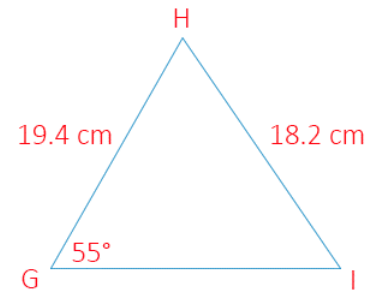
$$\frac{GI}{\sin 64°} = \frac{18.2}{\sin 55°}$$

$$GI = \frac{18.2 \times \sin 64°}{\sin 55°}$$

$$= 20 \text{ cm}$$

Missing measurements $61°, 64°, 20\text{ cm}$

If $\angle I = 119°$

$$\therefore \angle H = 180° - 119° - 55°$$

$$= 6°$$

$$\frac{GI}{\sin 6°} = \frac{18.2}{\sin 55°}$$

$$GI = \frac{18.2 \times \sin 6°}{\sin 55°}$$

$$= 2.3 \text{ cm}$$

Missing measurements $119°, 6°, 2.3\text{ cm}$

Question 46

$$\tan 20° = \frac{40}{AB}$$

$$AB = \frac{40}{\tan 20°}$$

$$= 110 \text{ m}$$

$$\tan 12° = \frac{40}{AC}$$

$$AC = \frac{40}{\tan 12°}$$

$$= 188 \text{ m}$$

$$BC^2 = 110^2 + 188^2 - 2(110)(188)\cos 48°$$

$$BC = 141 \text{ m}$$
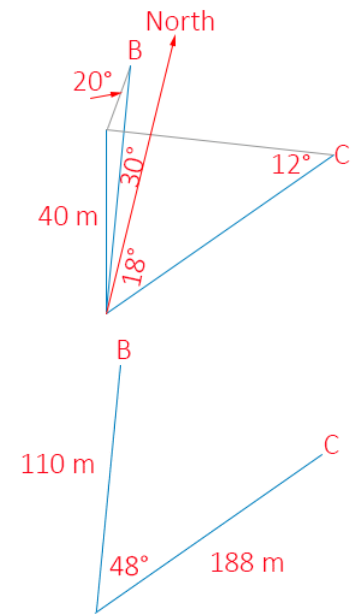
Question 47

a $\angle BCD = 80°$ (opposite angles in a cyclic quad are supplementary)

b

$$BD^2 = 7.2^2 + 6.1^2 - 2(7.2)(6.1)\cos 100°$$
$$BD = 10.2$$

$$\frac{\sin \angle ADB}{7.2} = \frac{\sin 100°}{10.2}$$

$$\sin \angle ADB = \frac{7.2 \times \sin 100°}{10.2}$$

$$\angle ADB = 44°$$

$$\frac{\sin \angle BDC}{8.2} = \frac{\sin 80°}{10.2}$$

$$\sin \angle BDC = \frac{8.2 \times \sin 80°}{10.2}$$

$$\angle BDC = 52°$$

$$\angle ADC = \angle ADB + \angle BDC$$
$$= 44° + 52°$$
$$= 96°$$

c

In $\triangle BDC, \angle DBC = 48°$

$$\frac{DC}{\sin 48°} = \frac{8.2}{\sin 52°}$$

$$DC = \frac{8.2 \times \sin 48°}{\sin 52°}$$

$$= 7.7 \text{ cm}$$

$\therefore$ Perimeter of $ABCD$

$$= 7.2 + 6.1 + 8.2 + 7.7$$
$$= 29.2 \text{ cm}$$
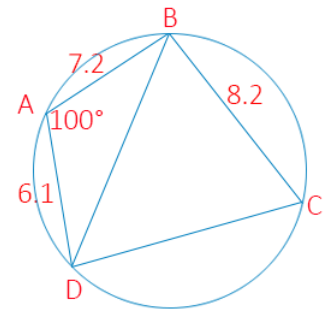
d

Area $\triangle ABC$

$= \dfrac{1}{2} \times 7.2 \times 6.1 \times \sin 100°$

$= 21.6$

Area $\triangle BDC$

$= \dfrac{1}{2} \times 8.2 \times 7.7 \times \sin 80°$

$= 31.1$

$\therefore$ Area $= 21.6 + 31.1 = 52.7$ cm^2

Question 48

a

$AC^2 = 10^2 + 12^2 - 2(10)(12)\cos\theta$

$\qquad = 244 - 240\cos\theta$

b

$x^2 = 14^2 + 9^2 - 2(14)(9)\cos\phi$

$\quad = 277 - 252\cos\phi$

c

θ and ϕ are opposite angles in a cyclic quadrilateral

$\therefore \phi = 180 - \theta$

$AC^2 = 244 - 240\cos\theta = 277 - 252\cos(180 - \theta)$

$\qquad -33 - 240\cos\theta = -252 \times (-\cos\theta)$

$\qquad\qquad -33 = 252\cos\theta + 240\cos\theta$

$\qquad\qquad -33 = 492\cos\theta$

$\qquad\qquad \cos\theta = \dfrac{-33}{492}$

$\qquad\qquad \theta = 94°$
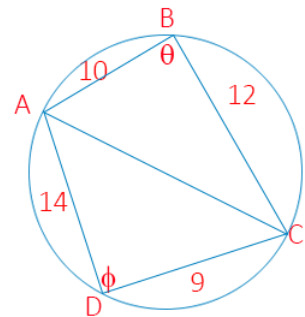
Question 49

$$\cos\theta = \frac{63^2 + 55^2 - 22^2}{2(63)(55)}$$

$$\theta = 20° \text{ (nearest degree)}$$

$$A = \frac{1}{2} \times ab\sin C$$

$$= \frac{1}{2} \times 63 \times 55 \times \sin 20°$$

$$= 592.5 \text{ m}^2$$

(exact use of θ gives 593.97m^2)
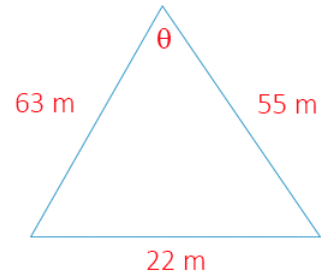
Heron's Rule

$$S = \frac{63 + 55 + 22}{2}$$

$$= 70$$

$$A = \sqrt{70(70-63)(70-55)(70-22)}$$

$$= 593.97 \text{ m}^2$$

Question 50

$$\frac{1}{2}(25+48+53) = 63$$

$$A_1 = \sqrt{63(63-25)(63-48)(63-53)}$$

$$= 599.2$$

$$\frac{1}{2}(33+38+45) = 58$$

$$A_2 = \sqrt{58(58-33)(58-38)(58-45)}$$

$$= 614$$

$$614 - 599 = 15m^2$$

Exercise 1D

Question 1

$\sin 0° = 1$

Question 2

$\sin 30° = \dfrac{1}{2}$

Question 3

$\tan 45° = 1$

Question 4

$\sin 60° = \dfrac{\sqrt{3}}{2}$

Question 5

$\cos 60° = \dfrac{1}{2}$

Question 6

$\cos 0° = 1$

Question 7

$\tan 90° = \text{ undefined}$

Question 8

$$\cos 45° = \frac{1}{\sqrt{2}}$$

Question 9

$$\cos 30° = \frac{\sqrt{3}}{2}$$

Question 10

$$\tan 60° = \sqrt{3}$$

Question 11

$$\sin 90° = 1$$

Question 12

$$\tan 0° = 0$$

Question 13

$$\cos 60° = \frac{1}{2}$$

Question 14

$$\sin 45° = \frac{1}{\sqrt{2}}$$

Question 15

$$\tan 30° = \frac{1}{\sqrt{3}}$$

Question 16

$$\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$$

Question 17

$$\cos 135° = -\cos 45° = -\frac{1}{\sqrt{2}}$$

Question 18

$$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$$

Question 19

$$\cos 120° = -\cos 60 = -\frac{1}{2}$$

Question 20

$$\cos 180° = -1$$

Question 21

$$\tan 135° = -\tan 45° = -1$$

Question 22

$\tan 120° = -\tan 60° = -\sqrt{3}$

Question 23

$\tan 150° = -\tan 30° = \dfrac{-1}{\sqrt{3}}$

Question 24

$\tan 180° = \tan 0° = 0$

Question 25

$\sin 180° = \sin 0° = 0$

Question 26

$\sin 150° = \sin 30° = \dfrac{1}{2}$

Question 27

$\sin 135° = \sin 45° = \dfrac{1}{\sqrt{2}}$

Question 28

$$\sin 60° = \frac{9}{x}$$

$$x = \frac{9}{\frac{\sqrt{3}}{2}}$$

$$= 9 \times \frac{2}{\sqrt{3}}$$

$$= \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{18\sqrt{3}}{3}$$

$$= 6\sqrt{3}$$

Question 29

$$x^2 + 9 = 49$$

$$x^2 = 40$$

$$x = \sqrt{40}$$

$$= 2\sqrt{10}$$

Question 30

$$\sin 45° = \frac{h}{10}$$

$$\frac{\sqrt{2}}{2} = \frac{h}{10}$$

$$h = \frac{10\sqrt{2}}{2}$$

$$= 5\sqrt{2}$$

$$\sin 60° = \frac{x}{5\sqrt{2}}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{5\sqrt{2}}$$

$$x = \frac{\sqrt{3}}{2} \times 5\sqrt{2}$$

$$= \frac{5\sqrt{6}}{2}$$

Question 31

$$x^2 = (2\sqrt{3})^2 + 4^2 - 2(2\sqrt{3})(4)\cos 150°$$

$$= 12 + 16 - 16\sqrt{3} \times (-\frac{\sqrt{3}}{2})$$

$$= 28 + \frac{16 \times 3}{2}$$

$$x^2 = 52$$

$$x = 2\sqrt{13}$$

Question 32

$$\frac{10}{\sin 45°} = \frac{m}{\sin 60°}$$

$$m = \frac{10 \times \sin 60°}{\sin 45°}$$

$$= 10 \times \frac{\sqrt{3}}{2} \times \sqrt{2}$$

$$= \frac{10\sqrt{6}}{2}$$

$$= 5\sqrt{6}$$

$$\tan 30° = \frac{x}{m}$$

$$x = m \times \tan 30°$$

$$= 5\sqrt{6} \times \frac{1}{\sqrt{30}}$$

$$= 5\sqrt{2}\, cm$$

Question 33

$$\cos 30° = \frac{x}{h_1}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{h_1}$$

$$\sqrt{3}h_1 = 2x$$

$$h_1 = \frac{2}{\sqrt{3}}x$$

$$\sin 60° = \frac{h_1}{h_2}$$

$$\frac{\sqrt{3}}{2} = \frac{2x}{\sqrt{3}h_2}$$

$$3h_2 = 4x$$

$$h_2 = \frac{4}{3}x$$

$$\frac{y}{\sin\theta} = \frac{h_2}{\sin 45°}$$

$$y = \frac{h_2 \times \sin\theta}{\sin 45°}$$

$$= \frac{4}{3}x \times \sin\theta \times \sqrt{2}$$

$$= \frac{4\sqrt{2}(x)\sin\theta}{3}$$

Question 34

$$h_1{}^2 = 4^2 + (2\sqrt{2})^2$$
$$= 16 + 8$$
$$h = \sqrt{24} = 2\sqrt{6}$$

$$\frac{S}{\sin\theta} = \frac{2\sqrt{6}}{\sin 60°}$$

$$S = \frac{2\sqrt{6}\sin\theta}{\dfrac{\sqrt{3}}{2}}$$

$$= 2\sqrt{6} \times \sin\theta \times \frac{2}{\sqrt{3}}$$

$$= 4\sqrt{2}\sin\theta$$

$$\frac{y}{\sin 45°} = \frac{4\sqrt{2}\sin\theta}{\sin\phi}$$

$$y = \frac{4\sqrt{2}\sin\theta}{\sin\phi} \times \frac{1}{2}$$

$$= \frac{4\sin\theta}{\sin\phi}$$

Exercise 1E

Question 1

a $30°$

b $\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

Question 2

a $45°$

b $\tan 45° = 1$

Question 3

a $60°$

b $\tan 60° = \sqrt{3}$

Question 4

a $120°$

b $\tan 120° = -\tan 60° = -\sqrt{3}$

Question 5

a $135°$

b $\tan 135° = -\tan 45° = -1$

Question 6

a $150°$

b $\tan 150° = -\tan 30° = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

Question 7

$\text{gradient} = \tan \theta$

Miscellaneous exercise one

Question 1

a

$$2(3x+4)+5(x-3)$$
$$=6x+8+5x-15$$
$$=11x-7$$

b

$$2(3x+4)-5(x-3)$$
$$=6x+8-5x+15$$
$$=x+23$$

c

$$5+2(5x-4)$$
$$=5+10x-8$$
$$=10x-3$$

d

$$5-2(5x-4)$$
$$=5-10x+8$$
$$=-10x+13$$

e

$$2(3-4x)+5(3x+1)$$
$$=6-8x+15x+5$$
$$=7x+11$$

f

$$2(3-4x)-5(3x+1)$$
$$=6-8x-15x-5$$
$$=-23x+1$$

g

$$(x+3)(x+5)$$
$$=x^2+8x+15$$

h

$$(x+3)(x-5)$$
$$= x^2 - 2x - 15$$

i

$$(2x+5)(x+3)$$
$$= 2x^2 + 6x + 5x + 15$$
$$= 2x^2 + 11x + 15$$

j

$$(2x-5)(x-3)$$
$$= 2x^2 - 5x - 6x + 15$$
$$= 2x^2 - 11x + 15$$

Question 2

a $2x + 8 = 2(x+4)$

b $6y + 9 = 3(2y + 3)$

c $16ab + 12ac + 8a^2 = 4a(4b + 3c + 2a)$

d $a^2 + a = a(a+1)$

e $x^2 + 7x - 8 = (x+8)(x-1)$

f $x^2 - 9x + 8 = (x-8)(x-1)$

g $x^2 + 5x - 14 = (x+7)(x-2)$

h $x^2 - 8x + 12 = (x-6)(x-2)$

i $x^2 - 16 = (x+4)(x-4)$

j $2a^2 - 18 = 2(a^2 - 9) = 2(a-3)(a+3)$

Question 3

a $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

b $\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

c $\sqrt{200} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

d $3\sqrt{5} \times 2\sqrt{5} = 6 \times 5 = 30$

e $\sqrt{15} \times \sqrt{3} = \sqrt{5} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{5}$

f $6\sqrt{3} \times \sqrt{6} = 6\sqrt{3} \times \sqrt{2} \times \sqrt{3} = 18\sqrt{2}$

g $3\sqrt{5} \times 7\sqrt{2} = 21\sqrt{10}$

h

$$(3\sqrt{2}+1)^2 = (3\sqrt{2}+1)(3\sqrt{2}+1)$$
$$= 18 + 3\sqrt{2} + 3\sqrt{2} + 1$$
$$= 19 + 6\sqrt{2}$$

Question 4

$$\cos 80° = \frac{a}{l} \qquad\qquad \cos 75° = \frac{a+20}{l}$$

$$l = \frac{a}{\cos 80°} \qquad\qquad l = \frac{a+20}{\cos 75°}$$

$$\frac{a+20}{\cos 75°} = \frac{a}{\cos 80°}$$

$$\cos 80°(a+20) = a\cos 75°$$

$$a\cos 80° + 20\cos 80° = a\cos 75°$$

$$a\cos 80° - a\cos 75° = -20\cos 80°$$

$$a(\cos 80° - \cos 75°) = -20\cos 80°$$
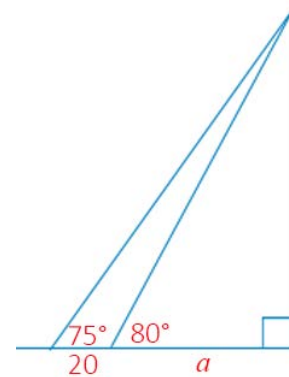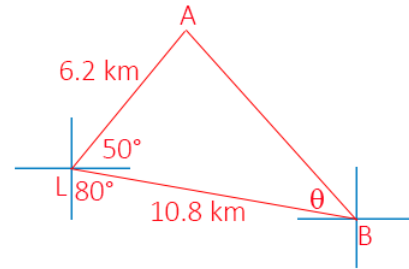
$$a = \frac{-20\cos 80°}{(\cos 80° - \cos 75°)}$$

$$= 40.78 cm$$

$$\therefore l = \frac{a}{\cos 80°}$$

$$= \frac{40.78}{\cos 80°}$$

$$= 235 \text{ cm (nearest cm)}$$

Question 5

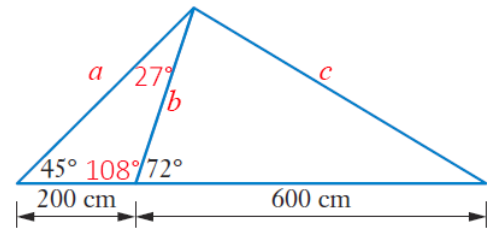$$AB^2 = 6.2^2 + 10.8^2 - 2(6.2)(10.8)\cos 60°$$

$$AB = 9.39$$

$$\frac{\sin\theta}{6.2} = \frac{\sin 60°}{9.39}$$

$$\sin\theta = \frac{6.2 \times \sin 60°}{9.39}$$

$$\theta = 35°$$

$\therefore$ Bearing of A from B is $280 + 35 = 315°$

Question 6

A circle with radius 'r' has a circumference of $2\pi r$ and an area of πr^2.

If the radius is multiplied by a factor of k, the circumference becomes $2 \times \pi \times k(r) = 2k\pi r = k(2\pi r)$.

The circumference increases by the same factor, k, and so is in direct proportion.

The relationship between radius and circumference is linear which is required for direct proportion.

When the radius is multiplied by k, the area of the circle becomes $\pi \times (kr)^2 = k^2 \pi r^2$.

The area is k^2 times bigger than the original. Area and radius are not in direct proportion.

The relationship between A and r is also not linear, $A = \pi r^2$, so they cannot be in direct proportion.

Question 7

$$\frac{200}{\sin 27°} = \frac{b}{\sin 45°}$$

$$b = \frac{200 \times \sin 45°}{\sin 27°}$$

$$= 311.5 \text{ cm}$$

$$\frac{a}{\sin 108°} = \frac{200}{\sin 27°}$$

$$a = \frac{200 \times \sin 108°}{\sin 27°}$$

$$= 419.0 \text{ cm}$$

$$c^2 = 800^2 + 419^2 - 2(800)(419)\cos 45°$$

$$c = 584.40 \text{ cm}$$

Length required:

$(800 + 584.40 + 419.0 + 311.5) \times 12 \div 100$ m

$= 253.788$ cm

$\therefore 260$ cm required