Mathematical Methods

Differentiation and Anti-Differentiation Practice Test

Non calc	/ 24
Calc	/ 36
Total	/ 60
Percentage	

Name ________________________

SECTION ONE: RESOURCE FREE

TOTAL: 24 marks

EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet

WORKING TIME: 24 minutes

Show all of your working clearly. *Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.*

Question 1 **[1,1,2,2 = 6 Marks]**

Find the gradient function $\frac{dy}{dx}$ for each of the following:

a. $y = 3 - \frac{x}{5}$

$\frac{dy}{dx} = -\frac{1}{5}$ (1)

b. $y = 4x^a - 1$

$\frac{dy}{dx} = 4ax^{a-1}$ (1)

c. $y = 20x^4 - \frac{5}{x} - \frac{10}{x^2}$

$\frac{dy}{dx} = 80x^3 + \frac{5}{x^2} + \frac{20}{x^3}$ (2)

d. $y = (x+4)(x-7)$

$y = x^2 - 3x - 28$ (1)

$\frac{dy}{dx} = 2x - 3$ (1)

Question 2 **[4 Marks]**

Differentiate from first principles the following function:

$$y = 4x^2 + 7$$

$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{4(x+h)^2 + 7 - (4x^2 + 7)}{h}$ (1)

$= \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 + 7 - 4x^2 - 7}{h}$

$= \lim_{h \to 0} \frac{8xh + 4h^2}{h}$ (1)

$= \lim_{h \to 0} 8x + h$ (1)

$= 8x$ (1)

Question 3 **[2,2 = 4 Marks]**

Find the anti-derivative of the following:

a. $\frac{dy}{dx} = 2x^2 + 4$

$y = \frac{2x^3}{3} + 4x + c$ (2)

b. $\frac{dy}{dx} = x^3 - \frac{7}{x^n}$

$y = \frac{x^4}{4} + \frac{7}{(n+1)x^{n+1}}$ (2)

Question 4 **[4 Marks]**

Sketch a function that has the following properties.

$f(0) = 0$

$for\ x > 0, f(x) < 0$

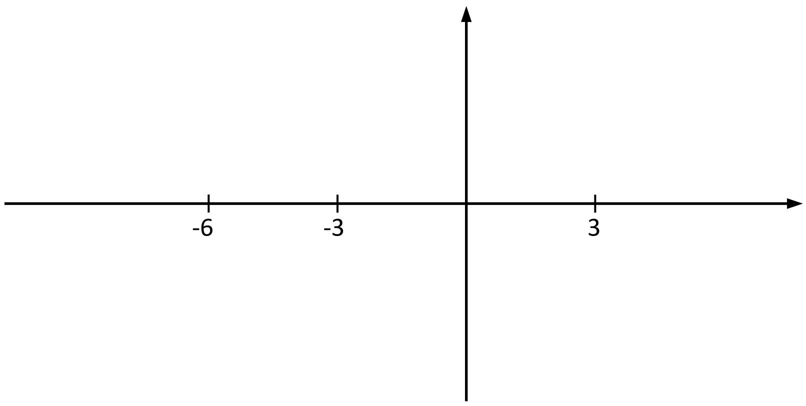
$for\ x > 3, f'(x) > 0$

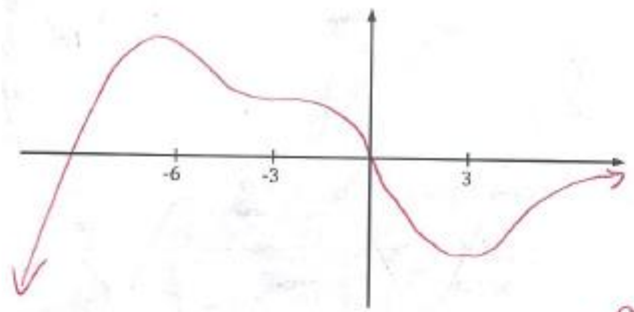
$f'(-3) = f'(3) = 0$

$for - 6 < x < 3, f'(x) \leq 0$

$as\ x \to \infty, f(x) \to 0$

$as\ x \to -\infty, f(x) \to -\infty$





1 as $x \to \pm\infty$

1 contains $(0,0)$

1 t.p at $x=3$

1 HPoI at $x=-3$

Question 5 **[4 Marks]**

Find y as a function of *x* given $\frac{dy}{dx} = 3x^5 + 4x^3 - 8x$ and $y = 1$ when $x = -1$.

$y = \int 3x^5 + 4x^3 - 8x \ dx$

$y = \frac{x^6}{2} + x^4 - 4x^2 + c$ (1)

$1 = \frac{1}{2} + 1 - 4 + c$ (1)

$c = \frac{7}{2}$ (1)

$\boldsymbol{y} = \frac{x^6}{2} + x^4 - 4x^2 + \frac{7}{2}$ (1)

END OF SECTION 1

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SECTION TWO: RESOURCE ALLOWED

TOTAL:	36 marks
EQUIPMENT:	pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet, scientific &/ or CAS calculator, 1 A4 page of notes
WORKING TIME:	36 minutes

Show all of your working clearly. *Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.*

Question 1 **[1,3 = 4 Marks]**

A particle travels such that its displacement *s* metres from O, the origin at time *t* seconds (where $t \geq 0$) is given by $s = t^3 + 2t^2 - 14t + 9$. Where rounding is appropriate, give answers correct to 2 decimal places.

a. Find the initial position of the particle.

$s(0) = 0 + 2(0) - 14(0) + 9 = 9$ (1)

b. Find when the particle is instantaneously at rest

$\frac{ds}{dt} = 0 = 3t^2 + 4t - 14$ (2)

$t = -2.9274 \text{ and } 1.5941\ s$

t must be positive

$t = 1.5941\ s$ (1)

Question 2 **[1,2,2,4,3 = 12 Marks]**

For the function $y = 2x^3 - 6x^2$ determine

a. the coordinates of points where the graph cuts the y-axis

$y = 2(0)^3 - 6(0)^2 = 0$

$(0,0)$ (1)

b. the coordinates of points where the graph cuts or touches the x-axis

$0 = 2x^2(x - 3)$

$x = 0, 3$

$(0,0) and\ (3,0)$ (2)

c. the behaviour of the function as $x \to \pm\infty$

$\lim_{x \to -\infty} 2x^3 - 6x^2 = -\infty$ (1)

$\lim_{x \to \infty} 2x^3 - 6x^2 = \infty$ (1)

d. the nature and location of any stationary points

$\frac{dy}{dx} = 0 = 6x^2 - 12x$

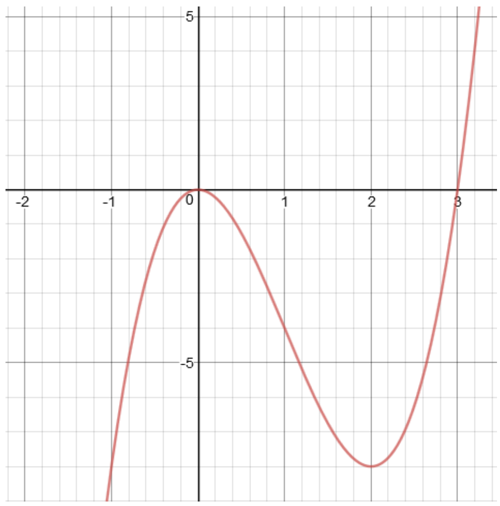
$0 = 6x(x - 2)$ (1)

$x = 0, 2$ (1)

$y(0) = 0$, (0,0) is a maximum (1)

$y(2) = 2(8) - 6(4) = -8$ (2,-8) is a minimum (1)

e. sketch of the graph of the function.



Question 3 **[1,1,1,4 = 7 Marks]**

Organisers of the 2007 *Slam–it Festival* know that if they sell tickets to their two day festival at $150 each they will sell 5000 tickets. For every 50c drop in ticket price, the number of tickets sold will increase by 50. It costs the organisers $250 000 per day to run the festival.

a. Write an expression that represents the price of the tickets, if the number of tickets sold is given by (5000 + 50*x*), where *x* is the number of 50c decreases.

$C(x) = 150 - 0.5x$ (1)

b. Hence, or otherwise, show that $R(x) = 750000 + 5000x - 25x^2$

$R(x) = C(x) \times N(x)$

$R(x) = (150 - 0.5x) \times (5000 + 50x)$ (1)

$R(x) = 750000 + 5000x - 25x^2$

c. Show that the total profit per day from the concert in terms of *x* is given as:

$$P(x) = 250000 + 5000x - 25x^2$$

$P(x) = R(x) - Cost(x)$

$P(x) = 750000 + 5000x - 25x^2 - 500000$ (1)

$P(x) = 250000 + 5000x - 25x^2$

d. What is the maximum profit the organisers can expect, and at what price should the tickets be sold to achieve this profit?

$\frac{dP}{dx} = 5000 - 50x$

$0 = 5000 - 50x$ (1)

$x = 100$ (1)

$C(100) = 150 - 0.5(100) = \$100$ (1)

$P(100) = 250000 + 5000(100) - 25(100)^2$

$P(100) = 250000 + 500000 - 250000$

$P(100) = \$500000$ (1)

Question 4 **[7 Marks]**

Ben is designing an open rectangular toy box (i.e. no top) that is to have a volume of 562500cm^3. The length of the wooden box is to be double the height.

Using calculus methods, determine the dimensions of the box that meet the volume requirement **and** minimises the amount of wood used to construct it.

$Area = lb + 2lh + 2bh$

$A = 2hb + 4h^2 + 2bh$

$A = 4bh + 4h^2$ (1)

$V = 2h \times b \times h = 562500$

$b = \frac{5625000}{2h^2}$ (1)

$A = 4h\left(\frac{562500}{2h^2}\right) + 4h^2$

$A = \frac{2(562500)}{h} + 4h^2$ (1)

$\frac{dA}{dh} = -\frac{2(562500)}{h^2} + 8h = 0$ (1)

$8h = \frac{2(562500)}{h^2}$

$h^3 = \frac{562500}{4} = 140625$

$h = 52cm$ (1)

$b = 1040.1cm$ (1)

$l = 104cm$ (1)

Question 5 **[2,4 = 6 Marks]**

A parabola is given by the equation: $y = 20 - 5x^2$

The parabola is shown below. A rectangle is to be placed between the x-axis and the parabola.

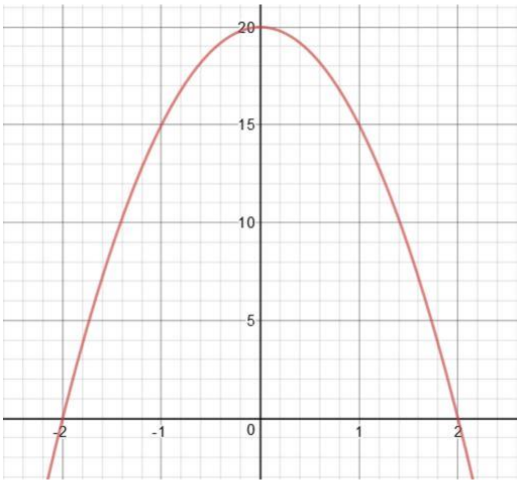
a. Find an equation for the area of the rectangle in terms of x.

Let x be the distance between the x axis and the point on the parabola.

$A = length \times height$

$A = 2xy$ (1)

$A = 2x(20 - 5x^2)$ (1)



b. Use a calculus method to find the dimensions of the rectangle that will maximise its area and state this maximum area.

$\frac{dA}{dx} = \frac{d}{dx}(40x - 10x^3)$

$\frac{dA}{dx} = 40 - 30x^2$ (1)

$0 = 40 - 30x^2$

$x = \sqrt{\frac{4}{3}} = 1.155$ (1)

$y = 20 - 5\left(\frac{4}{3}\right) = \frac{40}{3} = 13.33$ (1)

$A = 30.79$ (1)

END OF SECTION 2