Rossmoyne Senior High School

MATHEMATICS 2C/2D
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words   _MARKING KEY_

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2013*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

Section One: Calculator-free　　　　　　　　　　　　　　　　**(50 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.
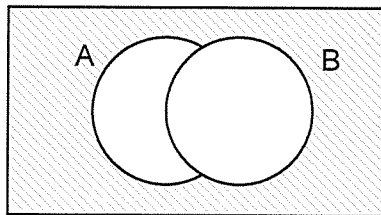
Working time for this section is 50 minutes.

Question 1　　　　　　　　　　　　　　　　　　　　　　　　**(7 marks)**

(a)　　Carefully shade the region $\overline{A \cup B}$ in the Venn diagram below.　　　　(1 mark)

(b)　　If $P(A) = \frac{2}{7}$, determine $P(\overline{A})$.　　　　　　　　　　　　　　　(1 mark)

$$\frac{5}{7}$$

(c)　　If $P(A) = 0.3$, $P(B) = \frac{1}{3}$ and $P(C) = 5\%$, state which event, out of A, B or C, is the most likely to occur. Justify your answer.　　　　　　　　　(2 marks)

> B
>
> because $\frac{1}{3}$ (33.3%) is larger than 0.3 (30%) and 5%.

(d)　　A student designed a driving test simulation by throwing a fair six-sided die, with a score of five or six representing a fail and any other score representing a pass. The student recorded the following scores from throwing the die 12 times:

6, 1, 5, 4, 5, 3, 5, 6, 4, 2, 3, 2.

(i)　　How many students passed?　　　　　　　　　　　　(1 mark)

7

(ii)　　Is this more or less than would be expected from the simulation design? Justify your answer.　　　　　　　　　　　　　　　　　(2 marks)

> Less than expected.
>
> Would expect $\frac{4}{6} \times 12 = 8$ to pass.

See next page

Question 2 (8 marks)

(a) Write the number 38 500 000 000 000 using scientific notation. (1 mark)

$$3.85 \times 10^{13}$$ ✓

(b) Factorise $x^2 + 14x + 40$. (1 mark)

$$(x+4)(x+10)$$ ✓

(c) Are the lines with equations $y = \frac{1}{3}x + 3$ and $y = 3x - \frac{1}{3}$ parallel, perpendicular or neither? (1 mark)

Neither ✓

(d) Find T_3 if $T_{n+1} = T_n - 7$ and $T_1 = 30$. (1 mark)

16 ✓

(e) Determine the coordinates of the y-intercept of the line with equation $3x + 2y = 12$. (2 marks)

$$y = -\frac{3}{2}x + 6$$ ✓

$$(0, \ 6)$$ ✓

(f) Determine n, if $\dfrac{7^2 \times 7^3}{\left(7^3\right)^2} = 7^n$. (2 marks)

$$\frac{7^5}{7^6} = 7^{-1}$$ ✓
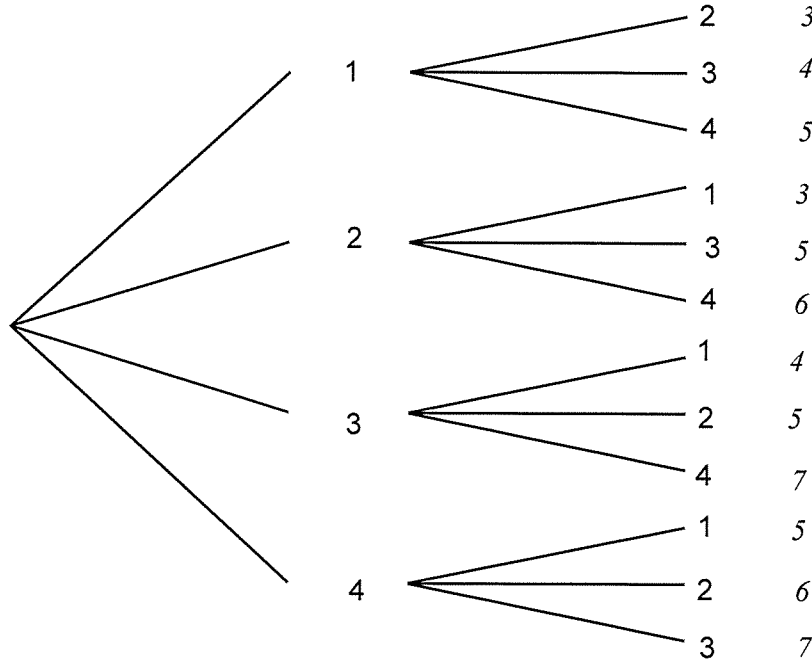
$$n = -1$$ ✓

See next page

Question 3 **(6 marks)**

A bag contains four balls marked with the numbers 1, 2, 3 and 4 respectively. A ball is randomly selected from the bag and its number noted. The ball is then placed on a table and a second ball is selected and its number noted.

(a) Complete this tree diagram to show the different pairs of numbers that could be selected.

(2 marks)

```
                              2    3
              1               3    4
                              4    5

                              1    3
              2               3    5
                              4    6

                              1    4
              3               2    5
                              4    7

                              1    5
              4               2    6
                              3    7
```

✓
✓ –1
✓ each
error/omission

(b) The numbers on the two balls selected are added together. Determine the probability that

(i) the total is seven. (1 mark)

$\boxed{\dfrac{2}{12}}$ ✓ f/t

(ii) the total is a multiple of three. (1 mark)

$\boxed{\dfrac{4}{12}}$ ✓

(iii) the total is odd, given that neither of the balls selected is marked with a four. (2 marks)
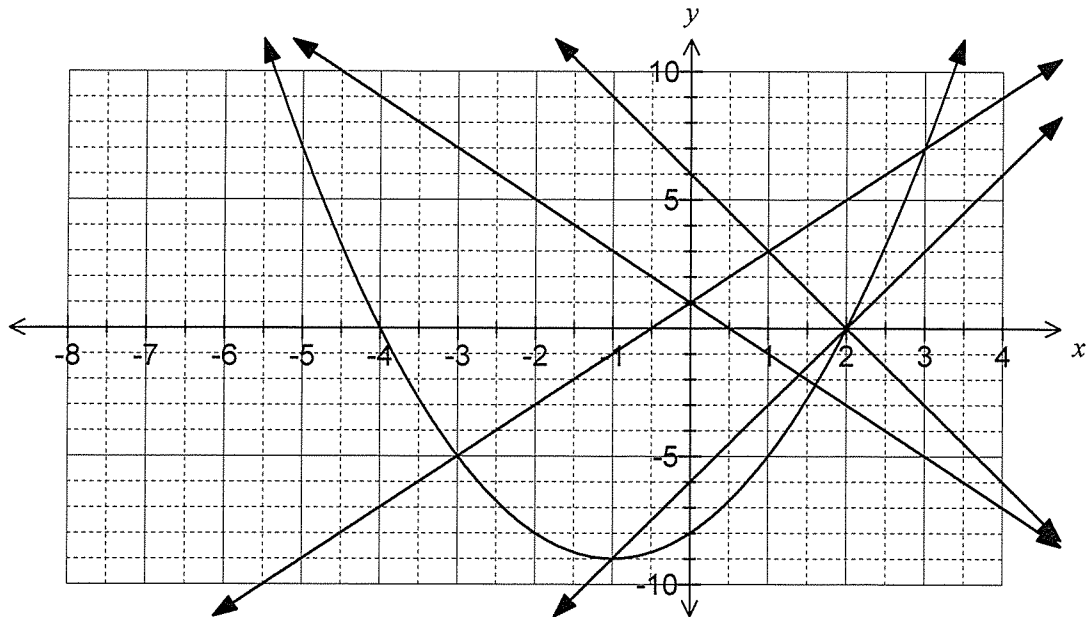
$\boxed{\dfrac{4}{6}}$ ✓ ✓ f/t

Question 4 (7 marks)

The graphs of $y = 2x + 1$, $y = 3x - 6$, $y = 1 - 2x$, $y = 6 - 3x$ and $y = x^2 + 2x - 8$ are shown.

Using the graph, or otherwise:

(a) Solve the equations

(i) $x^2 + 2x - 8 = 0$. (2 marks)

$$x = -4, \ x = 2$$

(ii) $x^2 + 2x - 8 = 3x - 6$. (2 marks)

$$x = -1, \ x = 2$$

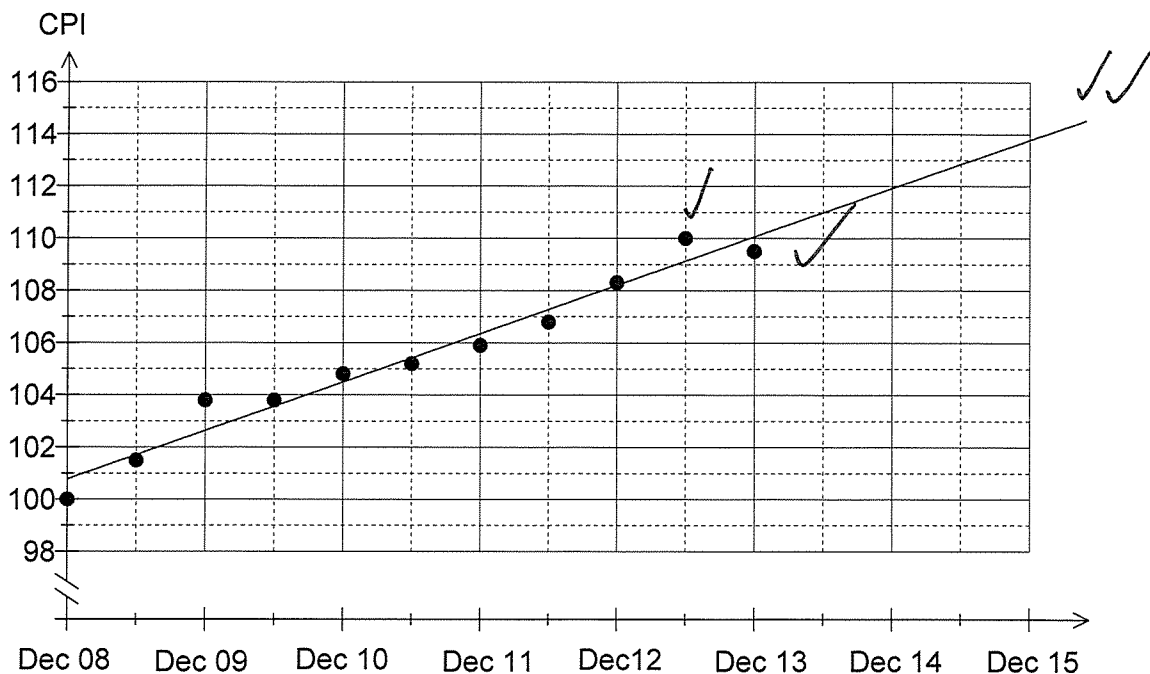(b) Solve the simultaneous equations $y - 2x = 1$ and $3x + y = 6$. (3 marks)

Find intersection of $y = 2x + 1$ and $y = 6 - 3x$.

$x = 1, \ y = 3$ ⇒ mention of using graphs OR method given

See next page

Question 5 **(8 marks)**

The graph below shows the consumer price index (CPI) at six monthly intervals from December 2008 until December 2012.

(a) The consumer price index values for June and December 2013 were 110.2 and 109.5 respectively. Add these two points to the graph above. (2 marks)

See above

(b) Fit a trend line to your graph. (2 marks)

(c) Estimate the consumer price index for

(i) June 2014. (1 mark)

≈ 111

(ii) December 2015. (1 mark)

≈ 114

(d) Which of the two estimates in (c) is the most reliable? Justify your answer. (2 marks)

June 2104. ✓

This prediction involves the least amount of extrapolation and so is more reliable than the prediction for December 2014.

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Question 6 (8 marks)

The graphs of $y = x^3$, $y = (x-1)(x-2)(x-3)$, $y = a(x-1)(x+1)^2$ and $y = -2(x+2)^3$ are shown below.
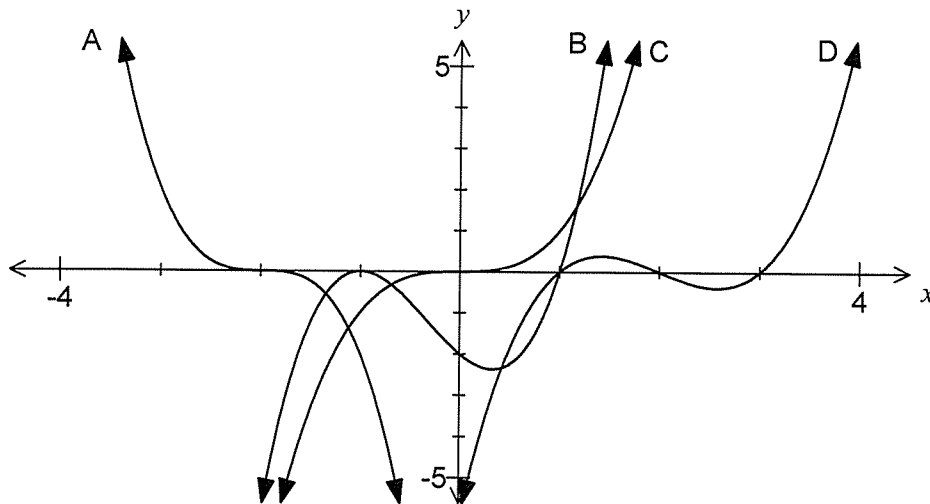
(a) Match each function with its graph (A, B, C or D) in the table below. (4 marks)

Function	$y = x^3$	$y = (x-1)(x-2)(x-3)$	$y = a(x-1)(x+1)^2$	$y = -2(x+2)^3$
Graph	C ✓	D ✓	B ✓	A ✓

(b) Determine the coordinates of the y-intercept of $y = -2(x+2)^3$. (2 marks)

$y = -2(0+2)^3$
$= -2 \times 8$
$= -16$

At (0, -16) ✓ ✓

(c) Use the graph of $y = a(x-1)(x+1)^2$ to determine the value of a. (2 marks)

Use y-intercept at (0, -2): ✓

$-2 = a(0-1)(0+1)^2$
$-2 = -a$
$a = 2$ ✓

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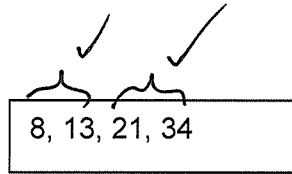
Question 7 (6 marks)

Terms of the Fibonacci sequence are given by the recursive rule $T_{n+2} = T_{n+1} + T_n$, $T_1 = 1$, $T_2 = 1$.

The first five terms of the Fibonacci sequence are 1, 1, 2, 3 and 5.

(a)   Determine the next four terms of the Fibonacci sequence (T_6 to T_9).   (2 marks)

> 8, 13, 21, 34

Consider the conjecture that in a run of any three consecutive Fibonacci numbers, just one of the numbers will be even.

(b)   Test the above conjecture with three examples.   (2 marks)

> 1, 1, **2** - one even number.
>
> 1, **2**, 3 - one even number.
>
> **2**, 3, 5 - one even number.
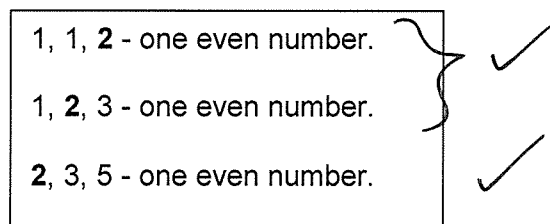
(c)   What conclusion can be drawn about the conjecture from your examples in (b)?   (2 marks)

> Based on the examples tested, the conjecture is true.
>
> However, an example may still exist that disproves the conjecture.

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____