

KINGSWAY CHRISTIAN COLLEGE

MATHS DEPARTMENT5

Course:	Mathematics Methods Year 12
Assessment Task:	Test 5 – Discreet Random Variables and The Binomial Distribution
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Date:	10 th August 2017
Assessment Score :	/ 45
Year Score:	
Comments:	
Teacher signature:	
Parent/ Guardian sig	gnature:
Comments:	

METHODS YEAR 12

Test 5 2017

Discreet Random variables and Distributions

Calculator Allowed

Time: 45 mins

Marks:

/ 45

Calculators are allowed for this test, but no notes. Please show work out where needed.

Question 1

(3,4,3 = 10 marks)

The discrete random variable X can only take the values 0, 1, 2, 3, 4, 5. The probability distribution of X is given by the following

$$P(X = 0) = P(X = 1) = P(X = 2) = a$$

$$P(X=3) = P(X=4) = P(X=5) = b$$

where a and b are constants.

$$P(X \ge 2) = 3P(X < 2)$$

Determine the values of a and b.

Determine the values of
$$a$$
 and b .

 $a + 3b = 3(2a) \lor a + 3b = 1.$

Solve
$$\begin{cases} a+3b=6a \\ 3a+3b=1 \end{cases}$$
; $a,b \end{cases} -7 a=\frac{1}{8}$

(b) Show that the expectation of X is $\frac{23}{8}$ and determine the exact variance of X.

$$E(x) = (0 \times \frac{1}{8}) + (1 \times \frac{1}{8}) + (3 \times \frac{5}{24}) + (4 \times \frac{5}{24}) + (5 \times \frac{5}{24})$$

$$= \frac{3}{8} + \frac{60}{24} \quad \text{ony warking}.$$

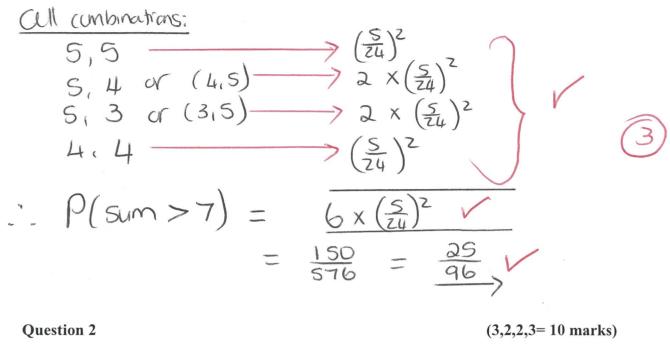
$$\frac{23}{8}$$

$$V(X) = (0^{2} \times \frac{1}{8}) + (1^{2} \times \frac{1}{8}) + (2^{2} \times \frac{1}{8}) + (3^{2} \times \frac{5}{24}) + (4^{2} \times \frac{5}{24})$$

$$-\left(\frac{23}{8}\right)^2 V$$
 ony working.

$$=\frac{533}{192}$$

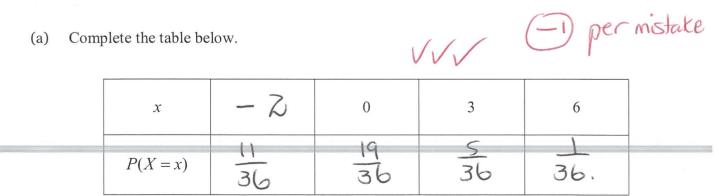
(c) Determine the exact probability that the sum of two independent observations from this distribution exceeds 7.



On a long train journey, a statistician is invited by a gambler to play a dice game. The game uses two ordinary dice which the statistician is to throw.

If the total score is 12, the statistician is paid \$6 by the gambler. If the total score is 8, the statistician is paid \$3 by the gambler. However, if both or either dice show a 1, the statistician pays the gambler \$2. Otherwise, no money changes hands.

Let \$X be the amount paid to the statistician by the gambler.



Explain why the table in part (a) describes a probability distribution for the discrete random variable X.

$$\sum_{i}^{\infty} p(x) = 1$$
 and $0 \le p \le 1$.

Show that, if the statistician played the game 100 times, his expected loss would be \$2.78, to the nearest cent.

$$E(x) = (-2 \times \frac{11}{36}) + (0 \times \frac{15}{36}) + (3 \times \frac{15}{36}) + (6 \times \frac{15}{36})$$

$$= -0.007. V$$

=
$$-0.007$$
. In 100 games he would lose 100x 0.027
= -2.7 which is a loss of $\frac{$2.78}{$}$ (2 dp).

Find the amount, a, that the \$6 would have to be changed to in order to make the game unbiased.

For the game to be unbrased:
$$E(x) = 0$$
.

For the game to be unbrased: $E(x) = 0$.

 $(-2 \times \frac{11}{36}) + (0 \times \frac{15}{36}) + (3 \times \frac{5}{36}) + (a \times \frac{1}{36}) = 0$.

:. Solve
$$\left(-\frac{22}{36} + \frac{15}{36} + \frac{a}{36} = 0, a\right)$$

$$\alpha = 7$$

(3 marks)

Given that $X \sim B(15, p)$ find the value of p such that P(X > 13) = 0.4

Show your working

· realize that it is 7/4 P(X > 13) = P(X > 14)= P(X = 14) + P(X = 15)p'' = (15) p'' + (15: solve (0,4= 15p14(1-p) + p15, p) : P=0,869698 P= 0,87. V · Correct aswer

Question 4

In a school of 480 students, 25% said they barracked for the Dockers.

State why "Supported the Dockers" is a Binomial random variable in this context.

Independent trials. Success | Failure. V (: Bernuli trials) 75% 25%

Determine μ and σ .

$$n = 480$$
 $p = 0.125$
 $= 480 \times 0.125$
 $M = 120$

$$0 = \sqrt{npq} / 1$$

$$= \sqrt{480 \times 0.75} \times 0.75$$

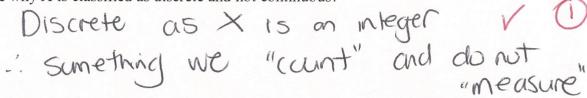
$$= \sqrt{90} / 149. / 190.$$

(2,4 = 6 marks)

A Study found that 80 per cent of people exhibiting common influenza symptoms recovered without taking any medication. A random sample of 30 people who had developed influenza symptoms was taken.

Let X denote the number of people in this sample who recovered without taking any medication.

(a) State why X is classified as discrete and not continuous?



(b) State the probability distribution of X and the mean and standard deviation of this distribution.

$$X \sim Bi (n; p)$$
 $M = 30 \times 0.8 = 24$
 $X \sim Bi (30; 0.8)$
 $G = \sqrt{npq}$

$$= \sqrt{30 \times 0.8 \times 0.2}$$

$$= 2.191$$

- (c) What is the probability, correct to three decimal places that
 - (i) Exactly 25 people recovered without any medication?

$$P(x=25) = 01172.$$

(ii) At least 24 but no more than 28 recovered without any medication?

$$P(24 \le x \le 28) = 0.596.$$
 (also accept 0.597).

A manufacturer of chocolate produces 3 times as many soft centred chocolates as hard centred ones. The chocolates are randomly packed in boxes of 20.

Let the Discreet Random Variable X = the number of hard centred chocolates per box.

- Find the probability that in a box there are
 - an equal number of soft centred and hard centred chocolates (i)

$$X \sim Bi (20i 0.025) V$$

 $P(X = 10) = 0,00992 V$

(ii) at least one hard centred chocolate.

$$P(x>1) = 1 - p(x=0)$$

= 0,003171.

fewer than 5 hard centred chocolates.

Pr than 5 hard centred chocolates.

$$P(X \le S) = P(X \le 4)$$

$$= 0, 41484$$

$$\approx 0, 4148$$

A random sample of 5 boxes is taken from the production line. Use your answer from (b) question (iii), to find the probability that exactly 3 of the boxes contain fewer than 5 hard centred chocolates.

Let the Discreet Random Variable Y= the number of boxes that contain fewer than 5 hard centred chocolates.

$$y \sim Bi(5; 0.41484)$$

 $p(y=3) = 0.24445$
 ≈ 0.2445