

## Perth College

Semester Two Examination, 2017

Question/Answer booklet

### MATHEMATICS METHODS UNITS 1 AND 2

Section One:  
Calculator-free

If required by your examination administrator, please  
place your student identification label in this box

Student Number: In figures

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In words

Final

Your name

Answers

#### Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

#### Materials required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet

##### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

#### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					<b>100</b>

**Instructions to candidates**

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2. Write your answers in this Question/Answer booklet.
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4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
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Section One: Calculator-free

35% (52 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(7 marks)

- (a) The tenth term of an arithmetic sequence is 67 and the fourteenth term is 51.

Determine the sum of the first 20 terms of the sequence.

(4 marks)

$$\begin{aligned}
 67 &= a + 9d \\
 51 &= a + 13d \\
 16 &= -4d \\
 d &= -4 \checkmark \\
 a &= 103 \checkmark \\
 S_n &= \frac{20}{2} (206 + 19(-4)) \checkmark \\
 &= 1300 \checkmark
 \end{aligned}$$

- (b) The geometric series  $x, x^2, x^3, x^4 \dots$  has a sum to infinity of 24.

(3 marks)

- (i) State the first term.

$$x \checkmark$$

- (ii) State the common ratio.

$$x \checkmark$$

- (iii) Determine the value of  $x$ .  $24 = \frac{x}{1-x}$

$$24 - 24x = x$$

$$x = \frac{24}{25} \checkmark$$

## Question 2

(5 marks)

(a) Determine  $f'(x)$  if

(i)  $f(x) = 8x^5 - x + 1.$

(1 mark)

$$f'(x) = 40x^4 - 1 \quad \checkmark$$

(ii)  $f(x) = (3x + 5)^2.$

(2 marks)

$$f(x) = 9x^2 + 30x + 25 \quad \checkmark$$

$$f'(x) = 18x + 30$$

(b) The area of an oil spill, at time  $t$  hours, is given by  $A(t) = 0.25t^2 + 0.5t + 0.75 \text{ m}^2$ . Determine the instantaneous rate of change of area of the spill when  $t = 8$  hours.

(2 marks)

$$A'(t) = 0.5t + 0.5 \quad \checkmark$$

$$A'(8) = 0.5(8) + 0.5$$

$$= 4.5 \text{ m}^2/\text{h} \quad \checkmark$$

## Question 3

(8 marks)

The graph of  $y = ax^3 + bx + c$  has a stationary point at  $(2, 29)$  and a gradient of 18 when  $x = 1$ .

- (a) Determine the values of the constants  $a$ ,  $b$  and  $c$ .

(6 marks)

$$y' = 3ax^2 + b \quad \checkmark$$

$$18 = 3a + b \quad \checkmark$$

$$0 = 12a + b \quad \checkmark$$

$$18 = -9a$$

$$a = -2 \quad \checkmark$$

$$18 = 3(-2) + b$$

$$b = 24 \quad \checkmark$$

$$29 = -2(2)^3 + 24(2) + c$$

$$29 = -16 + 48 + c$$

$$c = -3 \quad \checkmark$$

- (b) Determine the coordinates of any other stationary points.

(2 marks)

$$y' = -6x^2 + 24$$

$$0 = -6x^2 + 24 \quad \checkmark$$

$$x = -2$$

$$\begin{pmatrix} -2 & -35 \end{pmatrix}$$

$\downarrow \quad \quad \downarrow$

Question 4

(7 marks)

- (a) Evaluate  $x^{2a} \div x^b$  when  $x = 16$ ,  $a = 1.5$  and  $b = 3.5$ .

(3 marks)

$$\frac{16^3}{16^{3.5}} \checkmark = \frac{1}{16^{0.5}} \checkmark = \frac{1}{4} \checkmark$$

- (b) The first two terms of a geometric sequence are  $1.5 \times 10^{-2}$  and  $3 \times 10^{-5}$  respectively. Calculate the fifth term of the sequence, giving your answer in scientific notation. (4 marks)

$$r = \frac{3 \times 10^{-5}}{1.5 \times 10^{-2}} = 2 \times 10^{-3} \checkmark$$

$$T_5 = 1.5 \times 10^{-2} (2 \times 10^{-3})^4 \checkmark$$

$$= 1.5 \times 10^{-2} \times 16 \times 10^{-12} \checkmark$$

$$= 24 \times 10^{-14} \downarrow \frac{1}{2}$$

$$= 2.4 \times 10^{-13} \downarrow \frac{1}{2}$$

## Question 5

(7 marks)

Solve the following equations for  $x$ :

(a)  $2 \cos x = 1, 0 \leq x \leq 360^\circ.$

(2 marks)

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ \quad \checkmark$$

$$x = 300^\circ \quad \checkmark$$

(b)  $\frac{x-3}{x+2} = \frac{2}{3}.$

(2 marks)

$$3(x-3) = 2(x+2) \quad \checkmark$$

$$3x - 9 = 2x + 4$$

$$x = 13 \quad \checkmark$$

(c)  $(2x-3)^2 - 36 = 0.$

(3 marks)

$$(2x-3)^2 = 36$$

$$2x-3 = \pm 6 \quad \checkmark$$

$$2x-3 = 6$$

$$2x-3 = -6$$

$$x = 4.5 \quad \checkmark$$

$$x = -1.5 \quad \checkmark$$

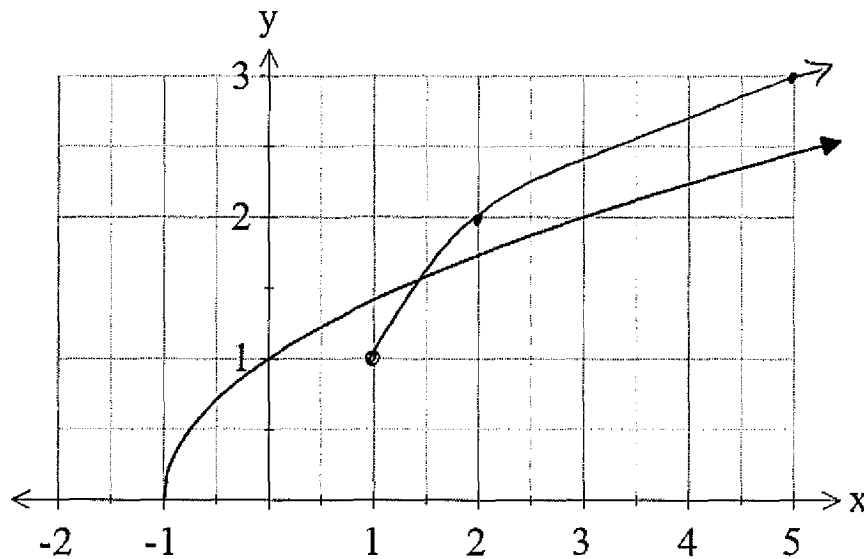
Question 6

(5 marks)

- (a) The graph of  $y = f(x)$  is shown below, where  $f(x) = \sqrt{x+1}$ .

Add the graph  $y = g(x)$ , where  $g(x) = \sqrt{x-1} + 1$ , to the axes.

(2 marks)



$(1, 1) \checkmark$   
 $(2, 2) \frac{1}{2}$   
 $(5, 3) \frac{1}{2}$

- (b) Using first principles, find the value of the gradient of the curve  $f(x) = x^3$  at the point where  $x = 5$ .

(3 marks)

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \checkmark$$

$$\lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \frac{1}{2}$$

$$\lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$\lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \frac{1}{2}$$

$$= 3(5)^2$$

$$= 75 \checkmark$$

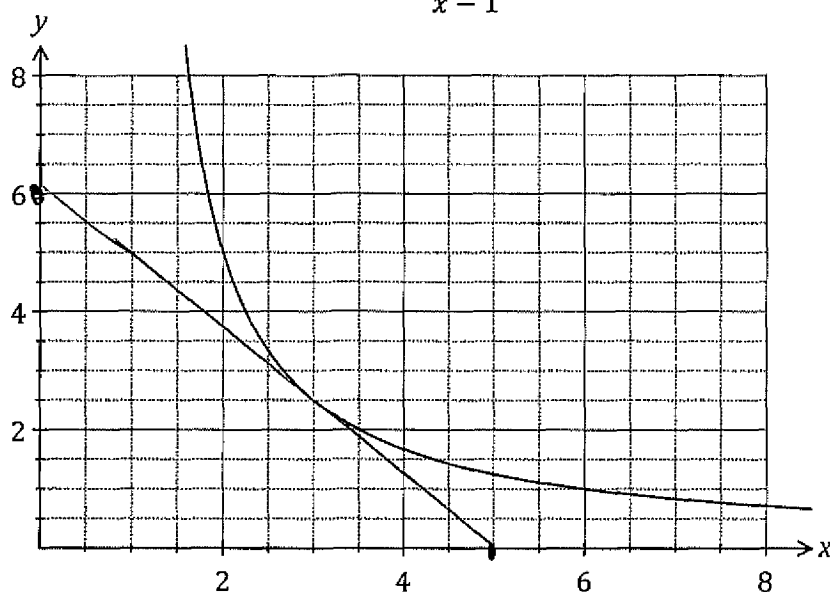


Question 7

(6 marks)

The graph of the function  $y = f(x)$  is shown below, where

$$f(x) = \frac{5}{x-1}$$



- (a) Draw the tangent to the graph at  $x = 3$  so that it cuts both axes, and use the tangent to estimate the value of  $f'(3)$ . (3 marks)

tangent ✓  
 tangent cuts y-axis between 5 and 7 ✓  
 tangent cuts x-axis between 4 and 6 ✓  
 rise for gradient ✓  
 run

- (b) Calculate the average rate of change of the function as  $x$  increases from 3 to 3.5. (3 marks)

$$f(3) = 2.5 \quad \frac{1}{2}$$

$$f(3.5) = 2 \quad \frac{1}{2}$$

$$\frac{2 - 2.5}{0.5} = -1 \quad \checkmark$$

## Question 8

(7 marks)

- (a) Determine the coefficient of the
- $a^3$
- term in the expansion of
- $(2a - 3)^4$
- .

(3 marks)

$$\binom{4}{1} (2a)^3 (-3) \checkmark$$

$$4 \times 2^3 \times -3 = -96 \checkmark$$

- (b) Consider the equation
- $x^3 - 4x^2 - 11x + 30 = 0$
- .

- (i) Show that
- $x = 2$
- is a solution of the equation.

(1 mark)

$$2^3 - 4(2)^2 - 11(2) + 30 = 0 \checkmark$$

- (ii) Determine all other solutions.

(3 marks)

$$\begin{array}{r} x-2 \overline{) x^3 - 4x^2 - 11x + 30} \\ - (x^3 - 2x^2) \end{array}$$

$$\begin{array}{r} -2x^2 - 11x \\ - (-2x^2 + 4x) \end{array} \checkmark$$

$$\begin{array}{r} -15x + 30 \\ - (-15x + 30) \\ 0 \end{array}$$

$$x^2 - 2x - 15 = 0 \checkmark$$

$$(x+3)(x-5) = 0$$

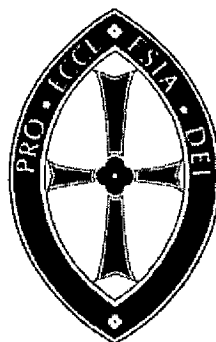
$$x = -3 \quad \downarrow \frac{1}{2}$$

$$x = 5 \quad \downarrow \frac{1}{2}$$

Additional working space

Question number: \_\_\_\_\_

Markers use only		
Question	Maximum	Mark
1	7	
2	5	
3	8	
4	7	
5	7	
6	5	
7	6	
8	7	
S1 Total	52	
S1 Wt ( $\times 0.6731$ )	35%	
S2 Wt	65%	
Total	100%	



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Question/Answer booklet

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Student Number: In figures

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In words

Final

Your name

Answers.

#### Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

#### Materials required/recommended for this section

##### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet (retained from Section One)

##### *To be provided by the candidate*

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Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

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Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(6 marks)

- (a) The tangent to the curve  $y = 10 + 2x - x^2$  at  $(2, 10)$  intersects the  $x$ -axis at  $(a, 0)$ . Determine the value of  $a$ .

(3 marks)

$$\begin{aligned}
 y' &= 2 - 2x \quad \checkmark \\
 \text{at } x &= 2 \\
 y' &= -2 \\
 y &= -2x + C \\
 10 &= -2(2) + C \\
 C &= 14 \\
 y &= -2x + 14 \quad \checkmark \\
 0 &= -2x + 14 \\
 x &= 7 \\
 a &= 7 \quad \checkmark
 \end{aligned}$$

- (b) If  $f'(x) = 1 + x - x^3$  and  $f(1) = 0$ , determine  $f(3)$ .

(3 marks)

$$\begin{aligned}
 f(x) &= x + \frac{x^2}{2} - \frac{x^4}{4} + C \quad \checkmark \\
 0 &= 1 + \frac{1^2}{2} - \frac{1^4}{4} + C \\
 C &= -\frac{5}{4} \quad \checkmark \quad -1.25 \\
 f(3) &= 3 + \frac{3^2}{2} - \frac{3^4}{4} - \frac{5}{4} \\
 &= -14 \quad \checkmark
 \end{aligned}$$

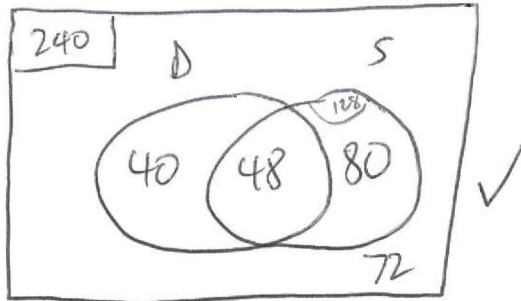
Question 10

(8 marks)

A group of 240 students were asked whether they had bought a drink or a snack from the school canteen. 72 had bought neither, 128 had bought a snack and 48 had bought both.

- (a) Determine the number of students who only bought a drink.

(2 marks)



$$n(D \cap \bar{S}) = 40 \checkmark$$

- (b) Determine the probability that a randomly chosen student from the group had bought

- (i) a snack or a drink.

(1 mark)

$$\frac{168}{240} \checkmark$$

- (ii) only a snack.

(1 mark)

$$\frac{80}{240} \checkmark$$

- (iii) a snack given that they had bought a drink.

(2 marks)

$$\frac{48}{88} \checkmark$$

- (c) For this group of students, are the events buying a snack and buying a drink independent? Justify your answer.

(2 marks)

$$P(S) = \frac{128}{240} = 0.53 \quad P(S|D) = \frac{48}{88} = 0.54 \checkmark$$

$$P(S) = \frac{128}{240}$$

$$P(D) = \frac{88}{240}$$

$$\frac{128}{240} \neq \frac{48}{88}$$

$\therefore$  Not independent  $\checkmark$   
only if calc.

$$P(S) \times P(D) = \frac{44}{225}$$

$$P(D \cap S) = \frac{48}{240}$$

$$\neq$$



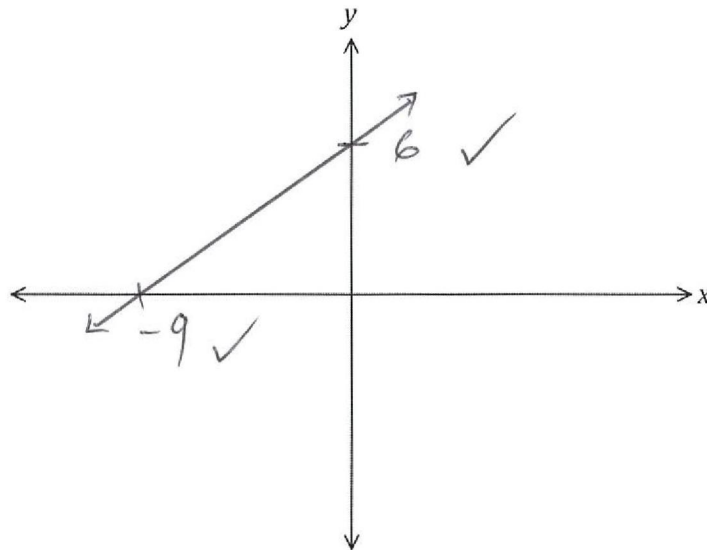
Question 11

(6 marks)

Line  $L_1$  has equation  $3y - 2x = 18$ .

- (a) Sketch the graph of  $L_1$ , showing all intercepts.

(2 marks)



- (b) Determine the equation of line  $L_2$  that is parallel to  $L_1$  and passes through the point with coordinates  $(-5, -6)$ . (2 marks)

$$3y - 2x = 18$$

$$y = \frac{2}{3}x + 6$$

$$y = \frac{2}{3}x + c$$

$$-6 = \frac{2}{3}(-5) + c$$

$$c = -\frac{8}{3}$$

$$y = \frac{2}{3}x - \frac{8}{3}$$

$$L = \left(-\frac{1}{2}\right)$$

must be exact.

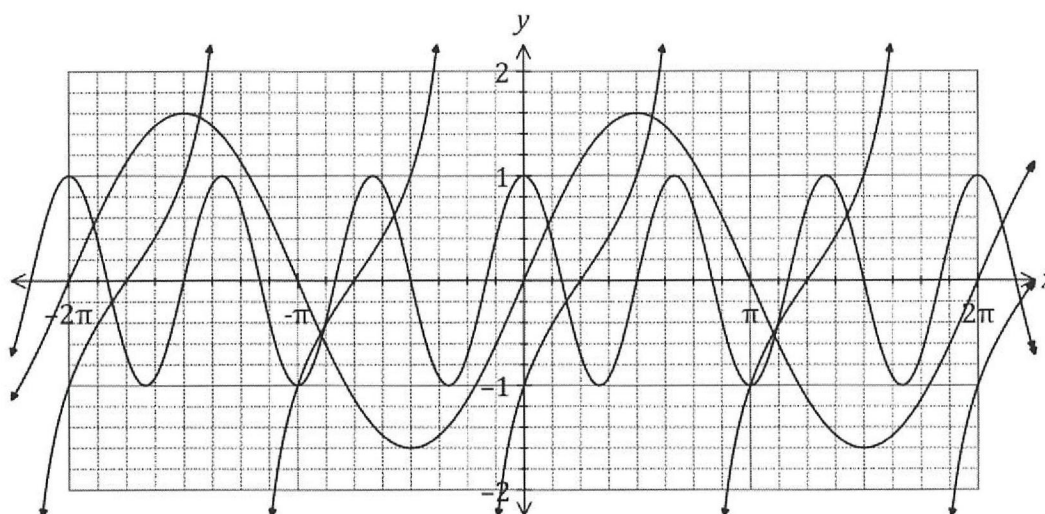
- (c) Determine the equation of line  $L_3$  that is perpendicular to  $L_1$  and has the same y-intercept as  $L_1$ . (2 marks)

$$y = -\frac{3}{2}x + 6$$

Question 12

(7 marks)

- (a) The graphs of  $y = a \sin x$ ,  $y = \cos(bx)$  and  $y = \tan(x + c)$  are shown below.



Determine the values of the constants  $a$ ,  $b$  and  $c$ .

(3 marks)

$$a = 1.6 \checkmark$$

$$b = 3 \checkmark$$

$$c = -\frac{\pi}{4} \text{ or } \frac{3\pi}{4} \checkmark$$

- (b) One day, the depth of water in a tidal basin was modelled (in radians) by  $d = 9.5 + 3.2 \cos(0.5t - 0.4)$ , where  $d$  was the depth in metres and  $t$  was the time, in hours, after midnight. For this day, determine

- (i) the depth of water at 4.30 am.

(2 marks)

$$t = 4.5 \text{ h } \checkmark$$

$$d = 9.5 + 3.2 \cos(0.5 \times 4.5 - 0.4) = 8.62 \text{ m } \checkmark$$

- (ii) the first time in the **afternoon** that the depth of water was 7 m.

(2 marks)

$$7 = 9.5 + 3.2 \cos(0.5t - 0.4) \checkmark \frac{1}{2}$$

$$t = 18.3 \text{ h } \checkmark$$

$$6:18 \text{ pm } \checkmark \quad 6:20 \text{ (12)}.$$

Question 13

(6 marks)

The quadratic function  $f(x) = ax^2 + bx + c$  passes through  $P(3, -10)$  and has roots at  $x = -5$  and  $x = 8$ .

- (a) Determine the values of the constants  $a$ ,  $b$  and  $c$ .

(3 marks)

$$f(x) = a(x+5)(x-8) \checkmark$$

$$f(x) = a(x^2 - 3x - 40)$$

$$-10 = a(3^2 - 3(3) - 40)$$

$$-10 = -40a$$

$$a = 0.25 \checkmark$$

$$b = -0.75 \frac{1}{2}$$

$$c = -10 \frac{1}{2}$$

- (b) State the y-intercept of the graph  $y = -5f(x)$ .

(1 mark)

$$y = -5[0.25x^2 - 0.75x - 10]$$

y int  $(-10)$ .  
 $-10 \times -5$ .

$$(0, 50) \text{ or } c = 50 \checkmark$$

- (c) State the roots of the graph  $y = f(2x)$ .

(2 marks)

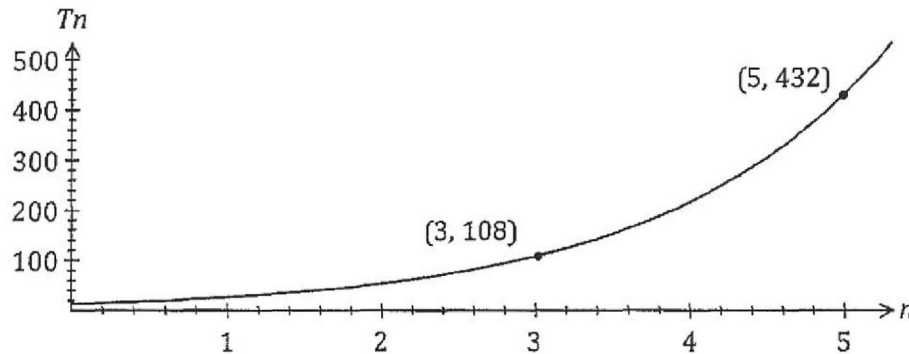
horizontal dilation factor  $\frac{1}{2}$

$$\text{roots } x = -2.5 \checkmark \quad x = 4 \checkmark$$

Question 14

(8 marks)

The number of followers of a social media influencer, counted at the start of five successive months, is shown in the exponential graph below.



The number of followers ( $T_n$ ) at the start of month  $n$  can be modelled by the recursive equation  $T_{n+1} = rT_n$ ,  $T_1 = a$ .

- (a) Use the graph to determine the values of  $r$  and  $a$ .

(3 marks)

$$108 \times r^2 = 432 \quad \checkmark$$

$$r = 2 \quad \checkmark$$

$$a = 27 \quad \checkmark$$

- (b) Assuming the growth rate continues,

- (i) how many followers are expected at the start of month 12?

(1 mark)

$$55296 \quad \checkmark$$

- (ii) at the start of which month will the number of followers first exceed 750 000?

(1 mark)

$$16^{\text{th}} \text{ month} \quad \checkmark$$

- (c) When the number of followers reached 1 million, the influencer fell out of favour and started to lose 25% of their followers each month. After how many months **from this time** will they have less than 2 000 followers? (3 marks)

$$2000 = 1000000 (0.75)^n \quad \checkmark$$

$$n = 21.6 \text{ months} \quad \checkmark$$

$$\text{accept } 22$$

$$\text{or use } a = 750000$$

$$2000 = 750000 (0.75)^{n-1}$$

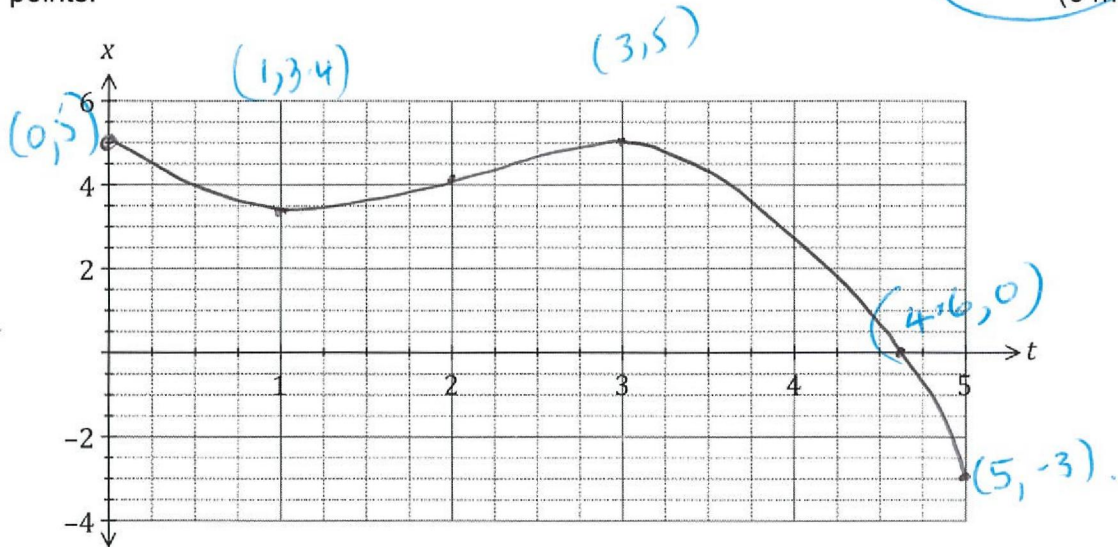
Question 15

(8 marks)

A particle is moving along a straight line so that its displacement,  $x$  metres, from a fixed point  $O$  after  $t$  seconds is given by

$$x = 5 - \frac{18t}{5} + \frac{12t^2}{5} - \frac{2t^3}{5}.$$

- (a) Sketch the displacement of the particle on the axes below for  $0 \leq t \leq 5$ , labelling key points. (3 marks)



- (b) Determine the velocity of the particle when  $t = 0.5$ .

(2 marks)

$$\frac{dx}{dt} = -\frac{18}{5} + \frac{24t}{5} - \frac{6t^2}{5}$$

at  $t = 0.5$   $\frac{dx}{dt} = -1.5 \text{ m/s}$  ✓

- (c) For how long during the first five seconds is the graph decreasing?

(1 mark)

$$0 < t < 1$$

$$3 < t < 5$$

3 seconds ✓

- (d) Determine the time(s) when the velocity of the particle is  $-3.6 \text{ m/s}$ .

(2 marks)

$$-3.6 = -\frac{18}{5} + \frac{24t}{5} - \frac{6t^2}{5}$$

$$t = 0 \text{ ✓}$$

$$t = 4 \text{ ✓}$$

Question 16

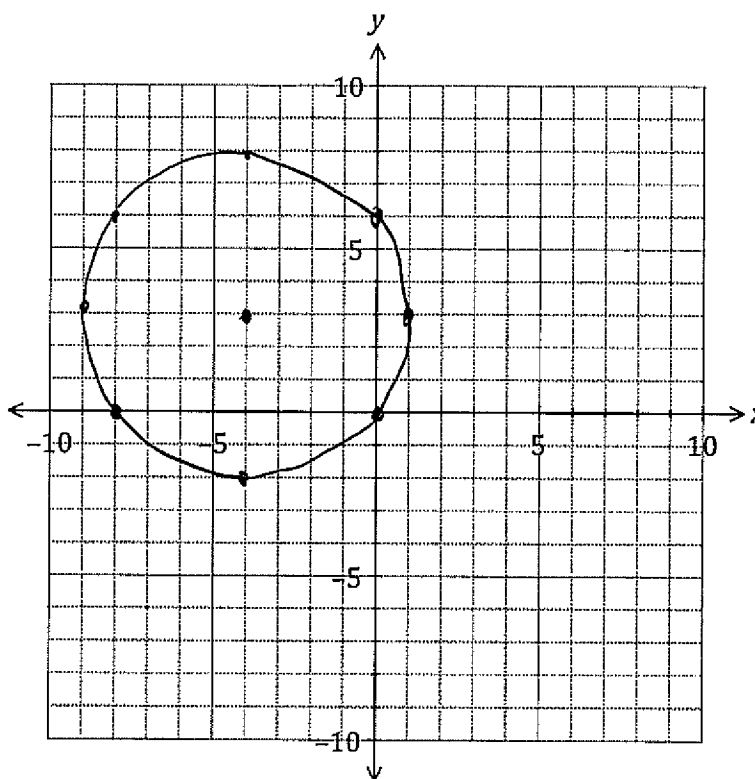
(6 marks)

(a) The variables  $x$  and  $y$  are related by  $(x + 4)^2 + (y - 3)^2 = 25$ .

(i) Sketch the graph of this relationship, showing all key features.

(3 marks)

Centre ✓  
radius ✓  
intercepts ✓  $-\frac{1}{2}$  per wrong



(ii) How does the vertical line test indicate that  $y$  is not a function of  $x$ ?

(1 mark)

cut the graph more than once ✓

(b) State the domain and range of the function  $f(x) = 4 - \sqrt{x + 3}$ .

(2 marks)

domain  $x \geq -3$  ✓

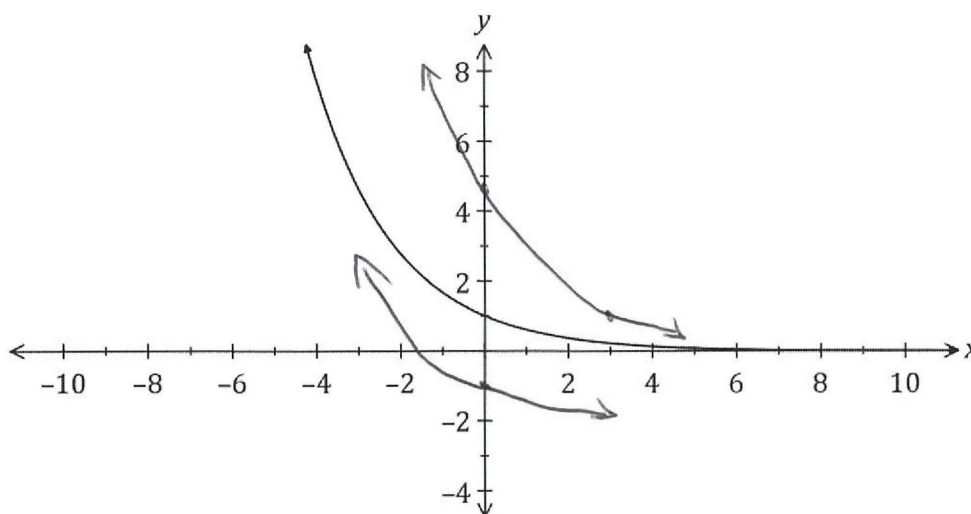
range  $y \leq 4$  ✓



Question 17

(8 marks)

The graph of  $y = a^x$  is shown below, where  $a$  is a positive constant.



(a) On the same axes, sketch and label the graphs of

(i)  $y = a^{x-3}$ . *goes through (3,1) ✓  
shape ✓* (2 marks)

(ii)  $y = a^x - 2$ . *y int (0,-1) ✓  
asymptote  $y = -2$  ✓* (2 marks)

(b) The graph of  $y = a^{x-3}$  intersects the graph of  $y = 1.2^x$  when  $x = 2.1$ .

Determine, giving your answers to 3 significant figures, *overall.*

(i) the y-coordinate of the point of intersection. (1 mark)

$$y = 1.2^{2.1} \approx 1.47 \quad \checkmark$$

*final (2.1, 1.47)*

(ii) the value of the constant  $a$ . (HINT: don't use any previously rounded solution) (3 marks)

$$1.2^{2.1} = a^{2.1-3} \quad \checkmark$$

$$a = 0.653 \quad \checkmark$$

*sig fig ✓*

*random guess (2)  
some sort of working (1)*

Question 18

(8 marks)

Five different letters are selected from the eleven in the word COMRADESHIP. The order in which the letters are selected is not important, so that the selection COMRA is the same as the selection RAMOC, and so on.

(a) Determine the number of different selections

(i) of five letters.

(2 marks)

$$\binom{11}{5} \checkmark = 462 \checkmark$$

(ii) of five letters that contain one vowel and four consonants.

(2 marks)

$$\binom{4}{1} \binom{7}{4} = 140 \checkmark$$

$\checkmark$

(b) Determine the probability that a random selection of five different letters

(i) includes the letters M and R.

(2 marks)

$$\frac{\binom{2}{2} \binom{9}{3}}{\binom{11}{5}} \checkmark = \frac{2}{11} \checkmark$$

$\left( \frac{84}{462} \right)$

(ii) includes at least one vowel.

(2 marks)

$$1 - \frac{\binom{4}{0} \binom{7}{5}}{\binom{11}{5}} \checkmark = \frac{21}{22}$$

$\left( \frac{441}{462} \right)$

$\frac{21}{462}$



Question 19

(8 marks)

Events  $A$  and  $B$  occur at random and it is known that  $P(B) = 0.6$  and  $P(A \cup B) = 0.72$ .

Determine  $P(A)$  when

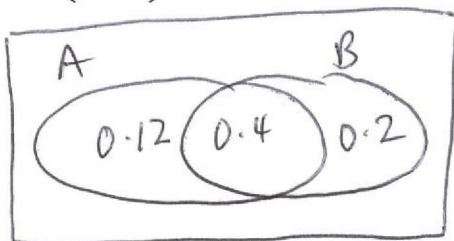
- (a)  $A$  and  $B$  are mutually exclusive.

(1 mark)

$$P(A) = 0.12 \checkmark$$

- (b)  $P(A \cap B) = 0.4$ .

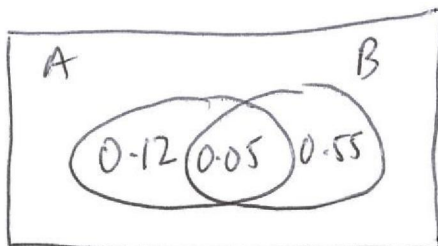
(1 mark)



$$P(A) = 0.52 \checkmark$$

- (c)  $P(\bar{A} \cap B) = 0.55$ .

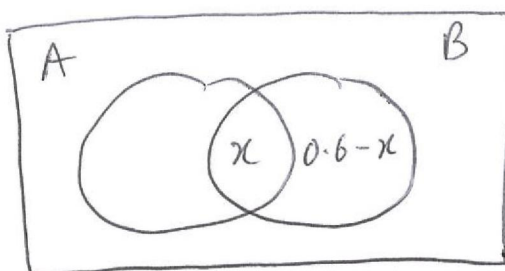
(1 mark)



$$P(A) = 0.17 \checkmark$$

- (d)  $P(A/B) = 0.3$ .

(2 marks)



$$\frac{x}{0.6} = \frac{1}{3}$$

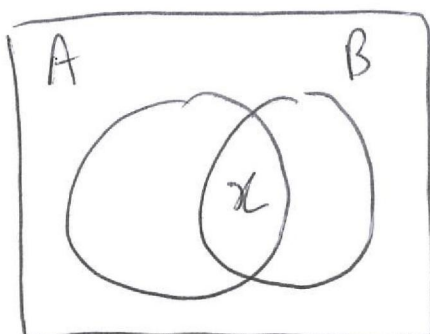
$$x = 0.2 \checkmark$$

$$P(A) = 0.32 \checkmark$$

$$P(A) \times 0.6 = P(A) + 0.6 - 0.72 \checkmark$$

- (e)  $A$  and  $B$  are independent.

(3 marks)



$$P(A) \times P(B) = P(A \cap B)$$

$$(0.2 + x) \times 0.6 = x \checkmark$$

$$x = 0.18 \checkmark$$

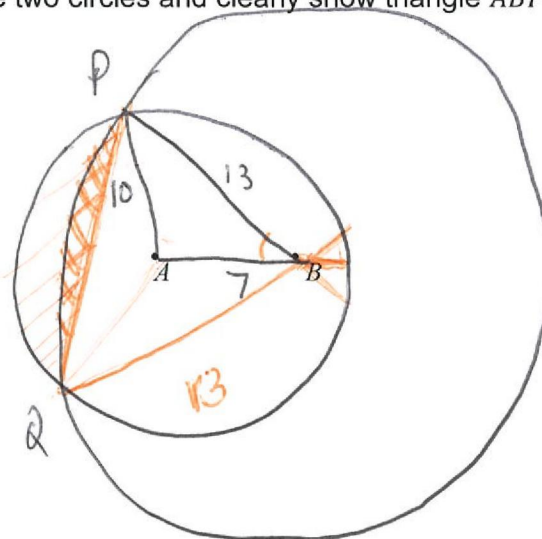
$$P(A) = 0.3 \checkmark$$

Question 20

(11 marks)

Two circles of radii 10 cm and 13 cm have centres at  $A$  and  $B$  respectively. The centres are 7 cm apart and the circles intersect at  $P$  and  $Q$ .

- (a) Sketch a diagram of the two circles and clearly show triangle  $ABP$ . (2 marks)



✓ circles  
✓ triangle

- (b) Show that  $\angle PBA = 49.6^\circ$ , when rounded to one decimal place. (2 marks)

$$\cos(B) = \frac{7^2 + 13^2 - 10^2}{2(7)(13)} \checkmark$$

$$\cos(B) = \left( \frac{59}{91} \right) \checkmark$$

$$10^2 = 13^2 + 7^2 - 2 \times 13 \times 7 \cos B$$

- (c) Determine the length of the chord  $PQ$  to the nearest millimetre. (2 marks)

$$PQ^2 = 13^2 + 13^2 - 2 \times 13 \times 13 \times \cos 99.2 \checkmark$$

$$PQ = 19.8 \text{ cm} \checkmark$$

$$PQ = 2r \sin \frac{1}{2} \theta$$

$$= 2 \left( \frac{13}{10} \right) \sin \frac{1}{2} (1.731)$$

- (d) (i) Find the obtuse  $\angle PBQ$ , in radians, correct to three decimal places. (1 mark)

$$49.6 \times 2 = 99.2^\circ \Rightarrow 1.731 \text{ R } \checkmark$$

- (ii) Find the reflex  $\angle PAQ$ , in radians, correct to three decimal places. (1 mark)

$$\cos(PAB) = \frac{10^2 + 7^2 - 13^2}{2(10)(7)}$$

$$14.8^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \cos A$$

$$49.205 = 1.714$$

$$196.41 \times 2 = 3.428 \checkmark$$

$$\frac{2\pi}{360} \times \cos A = \text{answer}$$

$$(\text{accept } 3.425)$$

Follow on from previous rounding  
146.22  $\Rightarrow$  3425

- (iii) Determine the area common to both circles. Answer to one decimal place. (3 marks)

$$\frac{1}{2}(13)^2(1.731 - \sin 1.731) + \frac{1}{2}(10)^2(3.428 - \sin 3.428)$$

$$= 248.4 \text{ cm}^2 \checkmark$$

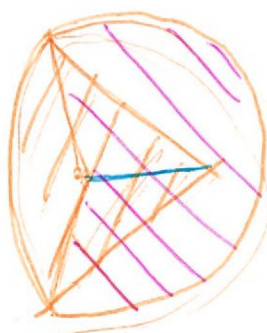
$$(\text{accept } 248.08 \text{ cm}^2)$$

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} 13^2 1.731$$

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} 10^2 3.464$$

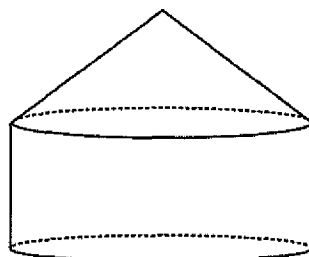


$$2 \times \frac{1}{2} (13)(17) \sin 49.6$$

## Question 21

(8 marks)

A composite solid is made from a cone and a cylinder, both of height  $h$  cm and radius  $r$  cm, as shown below.



The dimensions are such that the sum of  $h$  and  $9r$  is 45 cm.

- (a) Show that the volume of the solid is given by  $V = 60\pi r^2 - 12\pi r^3$ . (2 marks)

$$\begin{aligned}
 V &= \frac{1}{3}\pi r^2 h + \pi r^2 h \\
 &= \frac{1}{3}\pi r^2 (45 - 9r) + \pi r^2 (45 - 9r) \checkmark \\
 &= 15\pi r^2 - 3\pi r^3 + 45\pi r^2 - 9\pi r^3 \checkmark \\
 &= 60\pi r^2 - 12\pi r^3
 \end{aligned}$$

- (b) Use calculus techniques to determine the values of  $r$  and  $h$  that will maximise the volume of the solid, and state this maximum volume. (6 marks)

$$V' = 120\pi r - 36\pi r^2 \checkmark$$

$$0 = 120\pi r - 36\pi r^2 \checkmark$$

$$r = 0 \quad r = \frac{10}{3} \checkmark 3\bar{3}$$

$$h = 45 - 9\left(\frac{10}{3}\right) = 15 \text{ cm} \checkmark$$

$$V = 60(\pi)\left(\frac{10}{3}\right)^2 - 12(\pi)\left(\frac{10}{3}\right)^3$$

$$= 698.1 \text{ cm}^3 \checkmark$$

$$\begin{aligned}
 \text{at } r < \frac{10}{3} \quad V' &> 0 \\
 \text{at } r > \frac{10}{3} \quad V' &< 0 \checkmark \\
 \therefore \text{Max}
 \end{aligned}$$