Chapter 2 – Coordinate geometry and linear relations

Solutions to Exercise 2A

1 **a** $A(2, 12), B(8, 4)$

$x = \frac{1}{2}(2 + 8) = 5$

$y = \frac{1}{2}(12 + 4) = 8$

M is at (5, 8).

b $A(-3, 5), B(4, -4)$

$x = \frac{1}{2}(-3 + 4) = 0.5$

$y = \frac{1}{2}(5 + -4) = 0.5$

M is at (0.5, 0.5).

c $A(-1.6, 3.4), B(4.8, -2)$

$x = \frac{1}{2}(-1.6 + 4.8) = 1.6$

$y = \frac{1}{2}(3.4 + -2) = 0.7$

M is at (1.6, 0.7).

d $A(3.6, -2.8), B(-5, 4.5)$

$x = \frac{1}{2}(3.6 + -5) = -0.7$

$y = \frac{1}{2}(-2.8 + 4.5) = 0.85$

M is at $(-0.7, 0.85)$

2 A is (1,1), B is (5,5) and C is (11, 2).

AB: $x = y = \frac{1}{2}(5 - 1) = 3$

Midpoint is at (3, 3).

BC: $x = \frac{1}{2}(5 + 11) = 8$

$y = \frac{1}{2}(5 + 2) = 3.5$

Midpoint is at (8, 3.5).

AC: $x = \frac{1}{2}(1 + 11) = 6$

$y = \frac{1}{2}(1 + 2) = 1.5$

Midpoint is at (6, 1.5).

3 A (3.1, 7.1), B (8.9,10.5)

$x = \frac{1}{2}(3.1 + 8.9) = 6$

$y = \frac{1}{2}(7.1 + 10.5) = 8.8$

C is at (6, 8.8).

4 **a** $X(-4, 2), M(0, 3)$

For midpt x: $0 = \frac{1}{2}(-4 + x)$

$\therefore \quad x = 4$

For midpt y: $3 = \frac{1}{2}(2 + y)$

$1 + \frac{y}{2} = 3$

$\frac{y}{2} = 2, \therefore \ y = 4$

Point Y is at (4,4).

b $X(-1, -3), M(0.5, -1.6)$

For midpt x: $0.5 = \frac{1}{2}(-1 + x)$

$1 = -1 + x, \therefore x = 2$

For midpt y: $-1.6 = \frac{1}{2}(-3 + y)$

$-3.2 = -3 + 2, \therefore y = -0.2$

Point Y is at (2, −0.2).

c $X(6, 3), M(2, 1)$

For midpt x: $2 = \frac{1}{2}(6 + x)$

$4 = 6 + x, \therefore \ x = -2$

For midpt y: $1 = \frac{1}{2}(-3 + y)$

$2 = -3. + y, \therefore y = 5$

Point Y is at (−2, 5).

d $X(4, -3), M(0, -3)$

For midpt x: $0 = \frac{1}{2}(4 + x)$

$\therefore x = 4$

For midpt y: does not change so $y = -3$ Point Y is at $(-4, -3)$

5 At midpoint: $x = \frac{1}{2}(1 + a); y = \frac{1}{2}(4 + b)$

$x = \frac{1}{2}(1 + a) = 5$

$1 + a = 10, \therefore a = 9$

$y = \frac{1}{2}(4 + b) = -1$

$4 + b = -2, \therefore b = -6$

6 **a** Distance between $(3, 6)$ and $(-4, 5)$

$= \sqrt{(6 - 5)^2 + (3 - (-4))^2}$

$= \sqrt{1^2 + 7^2}$

$= \sqrt{50} = 5\sqrt{2} \approx 7.07$

b Distance between $(4, 1)$ and $(5, -3)$

$= \sqrt{(4 - 5)^2 + (1 - (-3))^2}$

$= \sqrt{(-1)^2 + 4^2}$

$= \sqrt{17} \approx 4.12$

c Distance between $(-2, -3)$ and $(-5, -8)$

$= \sqrt{(-2 - (-5))^2 + (-3 - (-8))^2}$

$= \sqrt{3^2 + 5^2}$

$= \sqrt{34} \approx 5.83$

d Distance between (6,4) and $(-7, 4)$

$= \sqrt{(6 - (-7))^2 + (4 - 4)^2}$

$= \sqrt{13^2 + 0^2}$

$= 13.00$

7 $A = (-3, -4), B = (1, 5), C = (7, -2)$

$$
\begin{aligned}
AB &= \sqrt{(1 - (-3))^2 + (5 - (-4))^2} \\
&= \sqrt{4^2 + 9^2} \\
&= \sqrt{97} \\
BC &= \sqrt{(7 - 1)^2 + (-2 - 5)^2} \\
&= \sqrt{6^2 + (-7)^2} \\
&= \sqrt{85} \\
AC &= \sqrt{(7 - (-3))^2 + (-2 - (-4))^2} \\
&= \sqrt{10^2 + 2^2} \\
&= \sqrt{104}
\end{aligned}
$$

$P = \sqrt{97} + \sqrt{85} + \sqrt{104} \approx 29.27$

8 $A(6, 6), B(10, 2), C(-1, 5), D(-7, 1)$

For P: $x = \frac{1}{2}(6 + 10) = 8$

$y = \frac{1}{2}(6 + 2) = 4$

P is at $(8, 4)$.

For M: $x = \frac{1}{2}(-1 + -7) = -4$

$y = \frac{1}{2}(5 + 1) = 3$

M is at $(-4, 3)$.

$$
\begin{aligned}
\therefore PM &= \sqrt{(-4 - 8)^2 + (3 - 4)^2} \\
&= \sqrt{(-12)^2 + (-1)^2} \\
&= \sqrt{145} \approx 12.04
\end{aligned}
$$

9
$$
\begin{aligned}
DM &= \sqrt{(-6 - 0)^2 + (1 - 6)^2} \\
&= \sqrt{(-6)^2 + (-5)^2} \\
&= \sqrt{61} \\
DN &= \sqrt{(3 - 0)^2 + (-1 - 6)^2} \\
&= \sqrt{3^2 + 7^2} \\
&= \sqrt{58}
\end{aligned}
$$

DN is shorter.

Solutions to Exercise 2B

1 **a** $m = \dfrac{4-0}{0-(-1)} = 4$

b $m = \dfrac{6-0}{3-0} = 2$

c $m = \dfrac{1-0}{4-0} = \dfrac{1}{4}$

d $m = \dfrac{4-0}{0-1} = -4$

e $m = \dfrac{3-0}{3-0} = 1$

f $m = \dfrac{3-0}{-3-0} = -1$

g $m = \dfrac{10-0}{6-(-2)} = \dfrac{5}{4}$

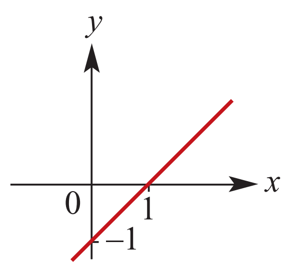
h $m = \dfrac{8-2}{0-3} = \dfrac{6}{-3} = -2$

i $m = \dfrac{5-0}{0-4} = \dfrac{5}{-4} = -\dfrac{5}{4}$

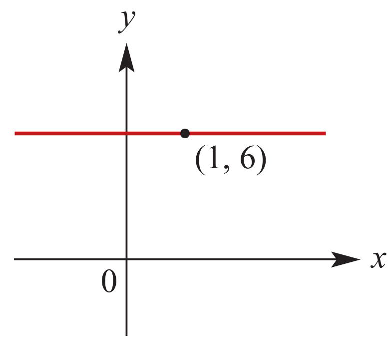
j $m = \dfrac{4-0}{0-(-3)} = \dfrac{4}{3}$

k Rise = zero so $m = 0$

2 Line with gradient 1:



3 $m = 0$ so $y = mx + c$ means $y = c$.
Here $c = 6$:



4 **a** $m = \dfrac{4-3}{2-6} = -\dfrac{1}{4}$

b $m = \dfrac{-6-4}{1-(-3)} = -\dfrac{5}{2}$

c $m = \dfrac{-3-7}{11-6} = -2$

d $m = \dfrac{0-8}{6-5} = -8$

e Rise = zero so $m = 0$

f $m = \dfrac{0-(-6)}{-6-0} = -1$

g $m = \dfrac{16-9}{4-3} = 7$

h $m = \dfrac{36-25}{6-5} = 11$

i $m = \dfrac{64-25}{-8-(-5)} = \dfrac{39}{-3} = -13$

j $m = \dfrac{100-1}{10-1} = \dfrac{99}{9} = 11$

k $m = \dfrac{1000-1}{10-1} = \dfrac{999}{9} = 111$

l $m = \dfrac{64-125}{4-5} = \dfrac{-61}{-1} = 61$

5 **a** $m = \dfrac{6a - 2a}{3a - 5a}$

$= \dfrac{4a}{-2a} = -2$

b $m = \dfrac{2b - 2a}{5b - 5a}$

$= \dfrac{2}{5}\left(\dfrac{b - a}{b - a}\right) = \dfrac{2}{5}$

6 **a** $m = \dfrac{a - 6}{7 - (-1)}$

$= \dfrac{a - 6}{8} = 6$

$a - 6 = 48, \therefore a = 54$

b $m = \dfrac{7 - 6}{b - 1}$

$= \dfrac{1}{b - 1} = -6$

$1 = 6(1 - b) = 6 - 6b$

$6b = 5, \therefore b = \dfrac{5}{6}$

7 We only need positive angles, so negative ones have 180° added.

a $(0, 3), (-3, 0); m = \dfrac{0 - (-3)}{3 - 0} = 1$
Angle $= \tan^{-1}(1) = 45°$

b $(0, -4), (4, 0); m = \dfrac{0 - (-4)}{4 - 0} = 1$
Angle $= \tan^{-1}(1) = 45°$

c $(0, 2), (-4, 0); m = \dfrac{0 - 2}{-4 - 0} = \dfrac{1}{2}$
Angle $= \tan^{-1}\left(\dfrac{1}{2}\right) = 26.57°$

d $(0, -5), (-5, 0); m = \dfrac{0 - (-5)}{-5 - 0} = -1$
Angle $= \tan^{-1}(-1) + 180° = 135°$

8 **a** $(-4, -2), (6,8); m = \dfrac{8 - (-2)}{6 - (-4)} = 1$
Angle $= \tan^{-1}(1) = 45°$

b $(2,6), (-2, 4); m = \dfrac{4 - 6}{-2 - 2} = \dfrac{1}{2}$
Angle $= \tan^{-1}\left(\dfrac{1}{2}\right) = 26.57°$

c $(-3, 4), (6, 1); m = \dfrac{1 - 4}{6 - (-3)} = -\dfrac{1}{3}$
Angle $= \tan^{-1}\left(-\dfrac{1}{3}\right) = 161.57°$

d $(-4, -3), (2, 4); m = \dfrac{4 - (-3)}{2 - (-4)} = \dfrac{7}{6}$
Angle $= \tan^{-1}\left(\dfrac{7}{6}\right) = 49.4°$

e $(3b, a), (3a, b); m = \dfrac{b - a}{3a - 3b}$
$= (b - a)\left(\dfrac{1}{-3}\right) = -\dfrac{1}{3}$
Angle $= \tan^{-1}\left(-\dfrac{1}{3}\right) = 161.57°$

f $(c, b), (b, c); m = \dfrac{c - b}{b - c} = -1$
Angle $= \tan^{-1}(-1) + 180° = 135°$

9 **a** $\tan 45° = 1$

b $\tan 135° = -1$

c $\tan 60° = \sqrt{3}$

d $\tan 120° = -\sqrt{3}$

Solutions to Exercise 2C

1 a $m = 3, c = 6$

b $m = -6, c = 7$

c $m = 3, c = -6$

d $m = -1, c = -4$

2 a $y = mx + c; m = 3, c = 5$
so $y = 3x + 5$

b $y = mx + c; m = -4, c = 6$
so $y = -4x + 6$

c $y = mx + c; m = 3, c = -4$
so $y = 3x - 4$

3 a $y = 3x - 6$; Gradient $= 3$; y-axis intercept $= -6$

b $y = 2x - 4$; Gradient $= 2$; y-axis intercept $= -4$

c $y = \frac{1}{2}x - 2$; Gradient $= \dfrac{1}{2}$; y-axis intercept $= -2$

d $y = \dfrac{1}{3}x - \dfrac{5}{3}$; Gradient $= \dfrac{1}{3}$; y-axis intercept $= -\dfrac{5}{3}$

4 a $2x - y = 9$

$$-y = -2x + 9$$
$$y = 2x - 9, \therefore m = 2, c = -9$$

b $3x + 4y = 10$

$$4y = -3x + 10$$
$$y = -\frac{3}{4}x + \frac{5}{2}, \therefore m = -\frac{3}{4}, c = \frac{5}{2}$$

c

$-x - 3y = 6$

$$-3y = x + 6$$
$$y = -\frac{1}{3}x - 2, \therefore m = -\frac{1}{3}, c = -2$$

d $5x - 2y = 4$

$$-2y = -5x + 4$$
$$y = \frac{5}{2}x - 2, \therefore m = \frac{5}{2}, c = -2$$

5 a The equation is of the form
$y = 3x + c$;
When $x = 6, y = 7$
$\therefore 7 = 3 \times 6 + c$
$\therefore c = -11$
The equation is $y = 3x - 11$

b The equation is of the form
$y = -2x + c$;
When $x = 1, y = 7$
$\therefore 7 = -2 \times 1 + c$
$\therefore c = 9$
The equation is $y = -2x + 9$

6 a $(-1, 4), (2, 3)$

$$m = \frac{3 - 4}{2 - (-1)} = -\frac{1}{3}$$

Using $(2, 3)$: $y = -\dfrac{2}{3} + c = 3$

$$c = \frac{11}{3}$$

$$\therefore y = -\frac{1}{3}x + \frac{11}{3}$$
$$3y = -x + 11$$

$\therefore x + 3y = 11$

b $(0, 4), (5, -3)$

$m = \dfrac{-3-4}{5-0} = -\dfrac{7}{5}$

Using (0,4): $y = c = 4$

$\therefore y = -\dfrac{7}{5}x + 4$

$5y = -7x + 20$

$\therefore 7x + 5y = 20$

c $(3, -2), (4, -4)$

$\therefore m = \dfrac{-4-(-2)}{4-3} = -2$

Using $(3, -2)$: $y = -2 \times 3 + c = -2$

$c = 4$

$\therefore y = -2x + 4$

$\therefore 2x + y = 4$

d $(5, -2), (8, 9)$

$\therefore m = \dfrac{9-(-2)}{8-5} = \dfrac{11}{3}$

Using $(5, -2)$: $y = \dfrac{11}{3} \times 5 + c = -2$

$c + \dfrac{55}{3} = -2$

$c = -\dfrac{61}{3}$

$\therefore\ y = \dfrac{11}{3}x - \dfrac{61}{3}$

$3y = 11x - 61$

$\therefore\ -11x + 3y = -61$

7 a The line passes through the point (0, 6) and (1, 8).

Therefore gradient $= \dfrac{8-6}{1-0} = 2$

b The equation is $y = 2x + 6$

8 a The equation is of the form

$y = 2x + c$;

When $x = 1, y = 6$

$\therefore 6 = 2 \times 1 + c$

$\therefore c = 4$

The equation is $y = 2x + 4$

b The equation is of the form

$y = -2x + c$;

When $x = 1, y = 6$

$\therefore 6 = -2 \times 1 + c$

$\therefore c = 8$

The equation is $y = -2x + 8$

9 a The equation is of the form

$y = 2x + c$;

When $x = -1, y = 4$

$\therefore 4 = 2 \times (-1) + c$

$\therefore c = 6$

The equation is $y = 2x + 6$

b The equation is of the form

$y = -2x + c$;

When $x = 0, y = 4$

$\therefore c = 4$

The equation is $y = -2x + 4$

c The equation is of the form

$y = -5x + c$;

When $x = 3, y = 0$

$\therefore 0 = -5 \times 3 + c$

$\therefore c = 15$

The equation is $y = -5x + 15$

10 a $y = mx + c$; $m = \dfrac{0-4}{6-0} = -\dfrac{2}{3}$

Using (0,4), $c = 4$

$y = -\dfrac{2x}{3} + 4$

b $y = mx + c$; $m = \dfrac{-6-0}{0-(-3)} = -\dfrac{6}{3} = -2$

Using $(0, -6)$, $c = -6$

$y = -2x - 6$

c $y = mx + c$; $m = \dfrac{0-4}{4-0} = -\dfrac{4}{4} = -1$
Using (0,4), $c = 4$
$y = -x + 4$

d $y = mx + c$; $m = \dfrac{3-0}{0-2} = -\dfrac{3}{2}$
Using (0,3):
$y = -\dfrac{3}{2}x + 3$

11 a Gradient $= \dfrac{6-4}{3-0} = \dfrac{2}{3}$
Passes through $(0,4)$, $\therefore c = 4$
Therefore equation is $y = \dfrac{2}{3}x + 4$

b Gradient $= \dfrac{2-0}{4-1} = \dfrac{2}{3}$
When $x = 1, y = 0$
$\therefore 0 = \dfrac{2}{3} \times 1 + c$
$\therefore c = -\dfrac{2}{3}$
Therefore equation is $y = \dfrac{2}{3}x - \dfrac{2}{3}$

c Gradient $= \dfrac{3-0)}{3-(-3)} = \dfrac{1}{2}$
When $x = -3, y = 0$
$\therefore 0 = \dfrac{1}{2} \times (-3) + c$
$\therefore c = \dfrac{3}{2}$
Therefore equation is $y = \dfrac{1}{2}x + \dfrac{3}{2}$

d Gradient $= \dfrac{0-3}{4-(-2)} = -\dfrac{1}{2}$
When $x = 4, y = 0$
$\therefore 0 = -\dfrac{1}{2} \times 4 + c$
$\therefore c = 2$
Therefore equation is $y = -\dfrac{1}{2}x + 2$

e Gradient $= \dfrac{8-2}{4.5-(-1.5)} = 1$
When $x = -1.5, y = 2$
$\therefore 2 = 1 \times (-1.5) + c$
$\therefore c = 3.5$
Therefore equation is $y = x + 3.5$

f Gradient $= \dfrac{-2-1.75}{4.5-(-3)} = -0.5$
When $x = -3, y = 1.75$
$\therefore 1.75 = -0.5 \times (-3) + 0.25$
$\therefore c = 0.25$
Therefore equation is
$y = -0.5x + 0.25$

12 a Axis intercepts: (0,4) and $(-1, 0)$
$m = \dfrac{4-0}{0-(-1)} = 4$,
$c = 4$ so $y = 4x + 4$

b Specified points: $(-3, 2)$ and (0,0)
$m = \dfrac{2-0}{-3-0} = -\dfrac{2}{3}$
$c = 0$ so $y = -\dfrac{2x}{3}$

c Axis intercepts: $(-2, 0)$ and $(0, -2)$
$m = \dfrac{0-(-2)}{-2-0} = 1$
$c = -2$ so $y = -x - 2$

d Axis intercepts: (2,0) and $(0, -1)$
$m = \dfrac{0-(-1)}{2-0} = \dfrac{1}{2}$,
$c = -1$ so $y = \dfrac{x}{2} - 1$

e $m = 0$, $c = 3.5$ so $y = 3.5$

f m undefined. Vertical line is $x = k$ so $x = -2$

13 P and Q are on the line $y = mx + c$;
$m = \dfrac{1-(-3)}{2-1} = 4$
Using Q at (2,1):
$y = 4 \times 2 + c = 1$ so $c = -7$
Line PQ has equation $y = 4x - 7$
Q and R are on the line $y = ax + b$:

$a = \dfrac{3-1}{2.5-2} = \dfrac{2}{0.5} = 4$
Using Q at (2, 1):
$y = 4 \times 2 + b = 1$ so $b = -7$
Line QR also has equation $y = 4x - 7$
P, Q and R are collinear.

14 **a** $y + x = 1$
Does not pass through (0, 0) because $y = 1 - x$ has $c = 1$

b $y + 2x = 2(x + 1)$
Does not pass through (0,0): this equation simplifies to $y = 2$, so y is never 0.

c $x + y = 0$
Passes through (0,0) because $c = 0$

d $x - y = 1$
Does not pass through (0,0) because $y = x + 1$ has $c = 1$

15 **a** $x = 4$

b $y = 11$

c $x = 11$

d $y = -1$

Solutions to Exercise 2D

1 **a** $x + y = 4$

If $x = 0, y = 4$; if $y = 0, x = 4$

Axis intercepts are at (0,4) and (4,0)

b $x - y = 4$

If $x = 0, y = -4$; if $y = 0, x = 4$

Axis intercepts are at $(0, -4)$ and (4,0)

c $-x - y = 6$

If $x = 0, y = -6$; if $y = 0, x = -6$

Axis intercepts are at $(0, -6)$ and $(-6, 0)$

d $y - x = 8$

If $x = 0, y = 8$; if $y = 0, x = -8$

Axis intercepts are at (0,8) and $(-8, 0)$

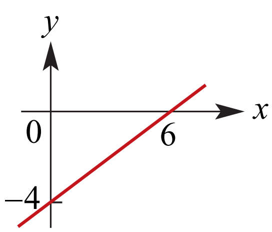
2 **a** $2x - 3y = 12$

If $x = 0, -3y = 12$

$\therefore y = \dfrac{12}{-3} = -4$

If $y = 0, 2x = 12$

$\therefore x = \dfrac{12}{2} = 6$

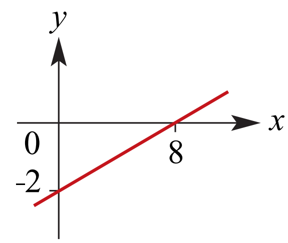


b $x - 4y = 8$:

If $x = 0, -4y = 8$

$\therefore y = \dfrac{8}{-4} = -2$

If $y = 0, x = 8$



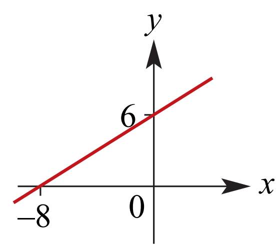
c $-3x + 4y = 24$

If $x = 0, 4y = 24$

$\therefore y = \dfrac{24}{4} = 6$

If $y = 0, -3x = 24$

$\therefore x = \dfrac{24}{-3} = -8$



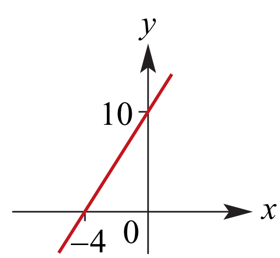
d $-5x + 2y = 20$

If $x = 0, 2y = 20$

$\therefore y = \dfrac{20}{2} = 10$

If $y = 0, -5x = 20$

$\therefore x = \dfrac{20}{-5} = -4$



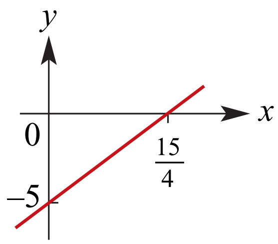
e $4x - 3y = 15$

If $x = 0, -3y = 15$

$\therefore y = \dfrac{15}{-3} = -5$

If $y = 0, 4x = 15$

$\therefore x = \dfrac{15}{4} = 3.75$



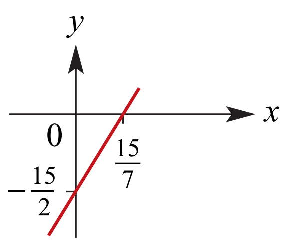
f $7x - 2y = 15$

If $x = 0, -2y = 15$

$\therefore y = \dfrac{15}{-2} = -7.5$

If $y = 0, 7x = 15$

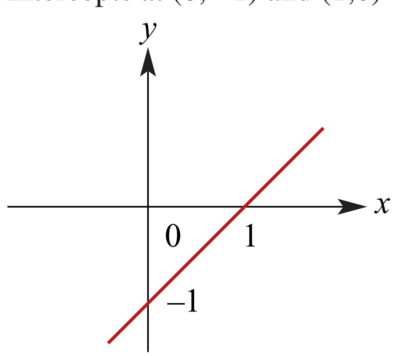
$\therefore x = \dfrac{15}{7}$



3 **a** $y = x - 1$

If $x = 0, y = -1$; if $y = 0, x = 1$

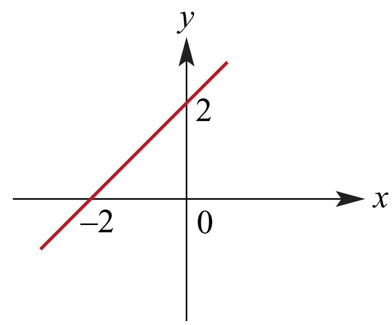
Intercepts at $(0, -1)$ and $(1,0)$



b $y = x + 2$

If $x = 0, y = 2$; if $y = 0, x = -2$

Intercepts at $(0, 2)$ and $(-2, 0)$

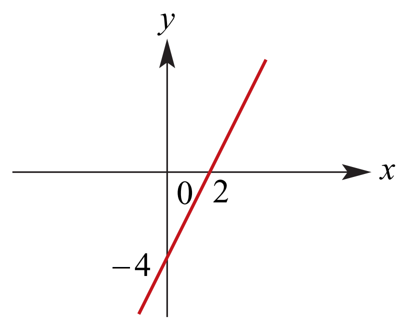


c $y = 2x - 4$

If $x = 0, y = -4$;

if $y = 0$, $2x - 4 = 0$, so $x = 2$

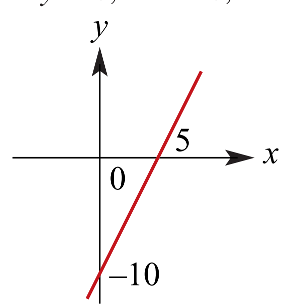
Intercepts at $(0, -4)$ and $(2,0)$



4 **a** $y = 2x - 10$

If $x = 0, y = -10$ so $(0, -10)$

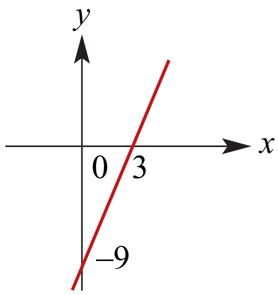
If $y = 0, 2x = 10, x = 5$ so $(5,0)$



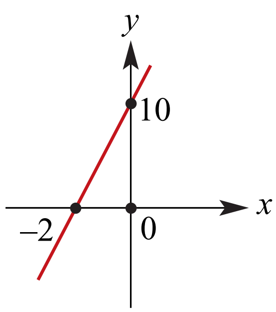
b $y = 3x - 9$

If $x = 0, y = -9$ so $(0, -9)$

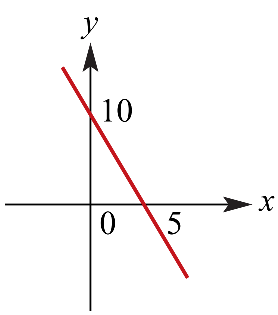
If $y = 0, 3x = 9, x = 3$ so $(3,0)$



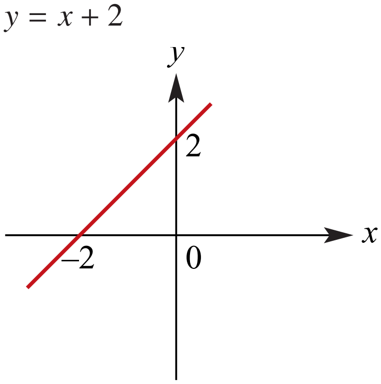
c $y = 5x + 10$
If $x = 0, y = 10$ so (0,10)
If $y = 0, 5x + 10 = 0,$
so $5x = -10$ and $x = -2$ so $(-2, 0)$



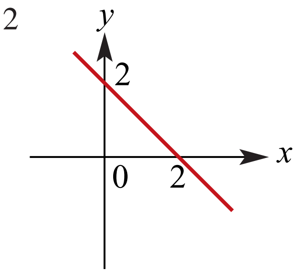
d $y = -2x + 10$
If $x = 0, y = 10$ so (0,10)
If $y = 0, -2x + 10 = 0,$
so $2x = 10$ and $x = 5$ so (5,0)



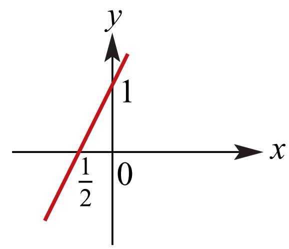
5 a $y = x + 2$



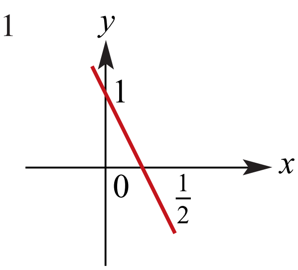
b $y = -x + 2$



c $y = 2x + 1$

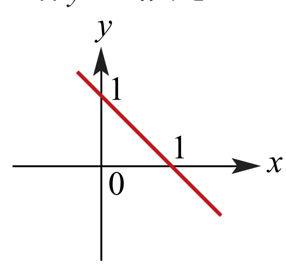


d $y = -2x + 1$



6 a $x + y = 1$

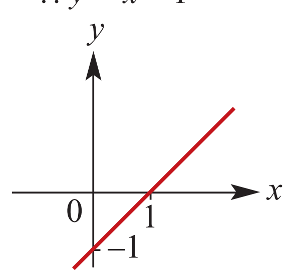
$\therefore y = -x + 1$



b $x - y = 1$

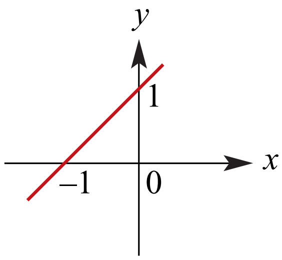
$x - 1 = y$

$\therefore y = x - 1$



c $y - x = 1$

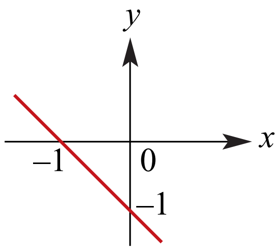
$\therefore y = x + 1$



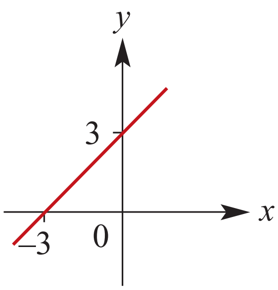
d $-x - y = 1$

$-y = x + 1$

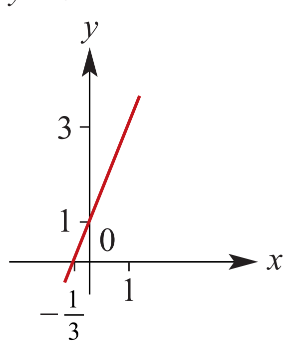
$\therefore y = -x - 1$



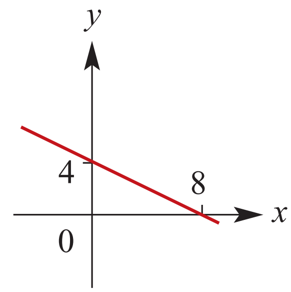
7 **a** $y = x + 3$



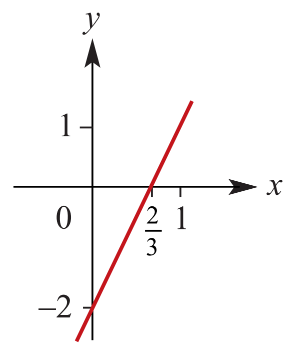
b $y = 3x + 1$



c $y = 4 - \frac{1}{2}x$



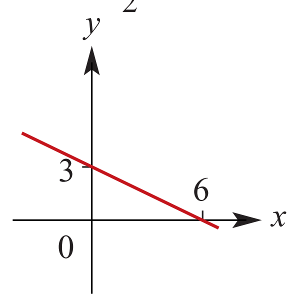
d $y = 3x - 2$



e $4y + 2x = 12$

$4y = 12 - 2x$

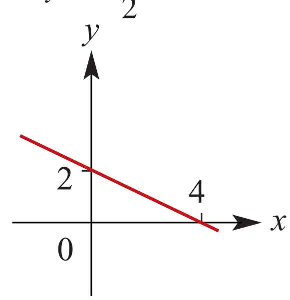
$\therefore y = -\frac{x}{2} + 3$



f $3x + 6y = 12$

$6y = 12 - 3x$

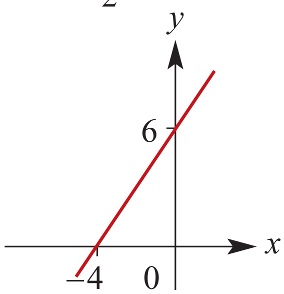
$\therefore y = -\frac{x}{2} + 2$



g $4y - 6x = 24$

$4y = 24 + 6x$

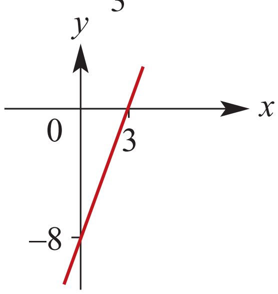
$\therefore y = \dfrac{3x}{2} + 6$



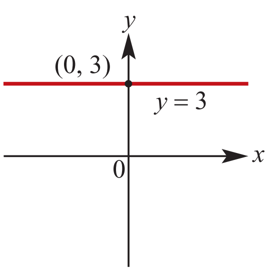
h $8x - 3y = 24$

$-3y = 24 - 8x$

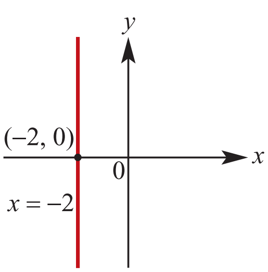
$\therefore y = \dfrac{8x}{3} - 8$



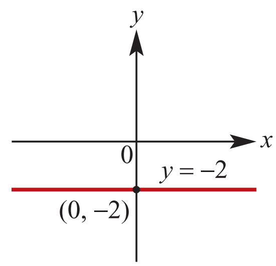
8 a



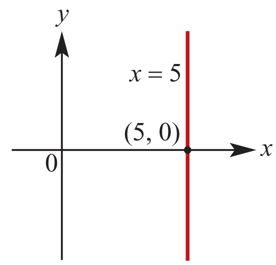
b



c



d



9 a $y = x$ so $m = 1; 45°$

b $y = -x$ so $m = -1; 135°$

c $y = x + 1$ so $m = 1; 45°$

d $x + y = 1$
$y = -x + 1$ so $m = -1; 135°$

e $y = 2x$ so $m = 2$;
$\tan^{-1}(2) = 63.43°$

f $y = -2x; m = -2$;
$\tan^{-1}(-2) + 180° = 116.57°$

10 a $y = 3x + 2; m = 3$
$\tan^{-1}(3) = 71.57°$

b $2y = -2x + 1$
$\therefore y = -x + \dfrac{1}{2}; m = -1$
$\tan^{-1}(-1) = 135°$

c $2y - 2x = 6$
$y - x = 3$
$\therefore y = x + 3; m = 1$
$\tan^{-1}(1) = 45°$

d $3y + x = 7$

$3y = -x + 7$

$\therefore y = -\frac{x}{3} + \frac{7}{3}; m = -\frac{1}{3}$

$\tan^{-1}\left(-\frac{1}{3}\right) + 180° = 161.57°$

11 A straight line has equation $y = 3x - 4$

$(0, a): a = -4$

$(b, 0): 0 = 3b - 4$

$3b = 4, \therefore b = \frac{4}{3}$

$(1, d): d = 3 - 4 = -1$

$(e, 10): 10 = 3e - 4$

$3e = 14, \therefore e = \frac{14}{3}$

Solutions to Exercise 2E

1 a Gradient = 2

Equation is of the form $y = 2x + c$

When $x = 4, y = -2$

$\therefore -2 = 2 \times 4 + c$

$\therefore c = -10$

The equation is $y = 2x - 10$

b Gradient $= -\frac{1}{2}$

Equation is of the form $y = -\frac{1}{2}x + c$

When $x = 4, y = -2$

$\therefore -2 = -\frac{1}{2} \times 4 + c$

$\therefore c = 0$

The equation is $y = -\frac{1}{2}x$

c Gradient $= -\frac{1}{2}$

Equation is of the form $y = -2x + c$

When $x = 4, y = -2$

$\therefore -2 = -2 \times 4 + c$

$\therefore c = 6$

The equation is $y = -2x + 6$

d Gradient $= -\frac{1}{2}$

Equation is of the form $y = \frac{1}{2}x + c$

When $x = 4, y = -2$

$\therefore -2 = \frac{1}{2} \times 4 + c$

$\therefore c = -4$

The equation is $y = \frac{1}{2}x - 4$

e $2x - 3y = 4$

$$-3y = -2x + 4$$

$$\therefore \quad y = \frac{2}{3}x - \frac{4}{3}$$

So the gradient we want is $\frac{2}{3}$.

Using the point $(4, -2)$:

$$y - (-2) = \frac{2}{3}(x - 4)$$

$$y = \frac{2}{3}(x - 4) - 2$$

$$y = \frac{2}{3}x - \frac{8}{3} - 2$$

$$y = \frac{2}{3}x - \frac{14}{3}$$

$$3y = 2x - 14$$

$$\therefore 2x - 3y = 14$$

f $2x - 3y = 4$

$$\therefore 3y = 2x - 4$$

$$\therefore y = \frac{2}{3}x - \frac{4}{3}$$

Gradient $= -\frac{3}{2}$

Equation is of the form $y = -\frac{3}{2}x + c$

When $x = 4, y = -2$

$\therefore -2 = -\frac{3}{2} \times 4 + c$

$\therefore c = 4$

The equation is $y = -\frac{3}{2}x + 4$

g $x + 3y = 5$

$$\therefore 3y = -x + 5$$

$$\therefore y = -\frac{1}{3}x + \frac{5}{3}$$

Gradient $= -\frac{1}{3}$

Equation is of the form $y = -\frac{1}{3}x + c$

When $x = 4, y = -2$

$\therefore -2 = -\frac{1}{3} \times 4 + c$

$\therefore c = -\frac{2}{3}$

The equation is $y = -\frac{1}{3}x - \frac{2}{3}$

h $x + 3y = -4$

$\therefore 3y = -x - 4$

$\therefore y = -\frac{1}{3}x - \frac{4}{3}$

Gradient = 3

Equation is of the form $y = 3x + c$

When $x = 4, y = -2$

$\therefore -2 = 3 \times 4 + c$

$\therefore c = -14$

The equation is $y = 3x - 14$

2 a $2y = 6x + 4; y = 3x + 4$

Parallel: $m = 3$ for both

b $x = 4 - y; 2x + 2y = 6$

Parallel: $m = -1$ for both

c $3y - 2x = 12; y + \frac{1}{3} = \frac{2}{3}x$

Parallel: $m = \frac{2}{3}$ for both

d $4y - 3x = 4; 3y = 4x - 3$

Not parallel:

$4y - 3x = 4$

$4y = 3x + 4$

$\therefore \quad y = \frac{3x}{4} + 1$

$3y = 4x - 3$

$\therefore y = \frac{4x}{3} - 1$

3 a $y = 4$ (The y-coordinate)

b $x = 2$ (The x-coordinate)

c $y = 4$ (The y-coordinate)

d $x = 3$ (The x-coordinate)

4 Gradient of $y = -\frac{1}{2}x + 6$ is $-\frac{1}{2}$.

So perpendicular gradient is

$-1 \div -\frac{1}{2} = 2$

Using the point (1,4):

$y - 4 = 2(x - 1)$

$y = 2(x - 1) + 4$

$\therefore \; y = 2x + 2$

5 $A(1, 5)$ and $B(-3, 7)$

Midpoint

$M(\frac{1 + (-3)}{2}, \frac{7 + 5}{2}) = M(-1, 6)$

Gradient $AB = \frac{7 - 5}{-3 - 1} = -\frac{1}{2}$

$\therefore$ gradient of line perpendicular to $AB = 2$. The equation of the line is of the form $y = 2x + c$

When $x = -1, y = 6$

$\therefore 6 = 2 \times (-1) + c$

$\therefore c = 8$

Equation of line is $y = 2x + 8$

6 Gradient of $AB = \frac{-3 - 2}{2 - 5} = \frac{5}{3}$

Gradient of $BC = \frac{3 - (-3)}{-8 - 2} = -\frac{3}{5}$

Product of these gradients

$= -\frac{3}{5} \times \frac{5}{3} = -1$

AB and BC are perpendicular, so ABC is a right–angled triangle.

7 $A(3, 7), B(6, 1), C(-8, 3)$

Gradient $AB = \frac{7 - 1}{3 - 6} = -2$

Gradient $BC = \frac{8 - 1}{20 - 6} = \frac{1}{2}$

$\therefore AB \perp BC$

8 Gradient of $RS = \frac{4 - 6}{6 - 2} = -\frac{1}{2}$

Gradient of $ST = \frac{-4 - 4}{2 - 6} = 2$

Product of these gradients = -1, so RS

and ST are perpendicular.

Gradient of $TU = \dfrac{-2-(-4)}{-2-2} = -\dfrac{1}{2}$

Gradient of $UR = \dfrac{6-(-2)}{2-(-2)} = 2$

Similarly, TU and UR are perpendicular, as are ST and TU, and RS and UR.
So $RSTU$ must be a rectangle.

9 $4x - 3y = 10$

$-3y = 10 - 4x$

$3y = 4x - 10$

$\therefore \quad y = \dfrac{4}{3}x - \dfrac{10}{3}$

Gradient $= \dfrac{4}{3}$

$4x - ly = m$

$-ly = m - 4x$

$ly = 4x - m$

$\therefore \quad y = \dfrac{4}{l}x - \dfrac{m}{l}$

Gradient $= \dfrac{4}{l}$

These lines are perpendicular, so their gradients multiplied equal -1:

$\dfrac{4}{3} \times \dfrac{4}{l} = -1$

$\dfrac{16}{3} = -l$

$\therefore \quad l = -\dfrac{16}{3}$

At intersection (4, 2) the y and x values are equal. From $4x - ly = m$:

$m = 16 - 2\left(-\dfrac{16}{3}\right)$

$= 16 + \dfrac{32}{3} = \dfrac{80}{3}$

10 **a** The line perpendicular to AB through B has gradient $-\dfrac{1}{2}$ and passes through $(-1, 6)$.
The equation of this line is
$y = -\dfrac{1}{2}x + \dfrac{11}{2}$.

b Intersects AB when
$2x + 3 = -\dfrac{1}{2}x + \dfrac{11}{2}$.
$\therefore x = 1, y = 5$ are the coordinates of point B.

c The coordinates of A and B are $(0, 3)$ and $(1, 5)$ respectively.
$\therefore$ the coordinates of C are $(2, 7)$.

Solutions to Exercise 2F

1 The point $(2, 7)$ is on the line $y = mx - 3$.
Hence $7 = 2m - 3$
That is, $m = 5$

2 The point $(3, 11)$ is on the line $y = 2x + c$.
Hence $11 = 2 \times 3 + c$ That is, $c = 5$

3 **a** Gradient of line perpendicular to the line $y = mx + 3$ is $-\frac{1}{m}$ The y-intercept is 3.
The equation of the second line is $y = -\frac{x}{m} + 3$.

b If $(1, -4)$ is on the line, $-4 = -\frac{1}{m} + 3$.
Hence $-\frac{1}{m} = -7$.
That is, $m = \frac{1}{7}$

4 $8 = m \times 3 + 2$
$m = 2$

5 $f: R \to R, f(x) = mx - 3, m \in R/\{0\}$

a x-axis intercept: $mx - 3 = 0, \therefore x = \frac{3}{m}$

b $6 = 5m - 3$
$5m = 9$
$m = \frac{9}{5}$

c x-axis intercept ≤ 1 for $\frac{3}{m} \leq 1$,
$\therefore m \geq 3$

d $y = f(x)$ has gradient $= m$, so a perpendicular line has gradient $= -\frac{1}{m}$.
Using the straight line formula for the point $(0, -3)$:
$y - (-3) = -\frac{1}{m}(x - 0)$
$\therefore y = -\frac{1}{m}x - 3$
OR $my + x = -3m$

6 $f: R \to R, f(x) = 2x + c$, where $c \in R$

a x-axis intercept: $2x + c = 0, \therefore x = -\frac{c}{2}$

b $6 = 5 \times 2 + c$
$c = -4$

c $-\frac{c}{2} \leq 1$
$c \geq -2$

d $y = f(x)$ has gradient $= 2$, so a perpendicular line has gradient $= -\frac{1}{2}$.
Using the straight line formula for the point $(0, c)$:
$y - c = -\frac{1}{2}(x - 0)$
$\therefore y = -\frac{1}{2}x + c$

7 $\frac{x}{a} - \frac{y}{12} = 4$

a When $y = 0, \frac{x}{a} = 4, \therefore x = 4a$
The coordinates of the x-axis intercept are $(4a, 0)$.

b Rearranging to make y the subject.
$y = \frac{12x}{a} - 48$
The gradient of the line is $\frac{12}{a}$

c **i** When the gradient is $2, \frac{12}{a} = 2, \therefore a = 6$

ii When the gradient is $-2, \frac{12}{a} = -2, \therefore a = -6$

iii $OM = \sqrt{\left(\frac{c}{4}\right)^2 + \left(\frac{c}{2}\right)^2}$

$= \sqrt{\frac{5c^2}{16}}$

If $OM = 2\sqrt{5}$ then $\sqrt{\frac{5c^2}{16}} = 2\sqrt{5}$

$\therefore c = 8$

8 **a** When $y = 0, x = \frac{c}{2}$

b $y = -2x + c$. When $x = 1, y = 7$

$\therefore 7 = -2 + c$

$\therefore c = 9.$

c $\frac{c}{2} \le 1 \Leftrightarrow c \le 2$

d Line perpendicular to $y = -2x + c$ has gradient $\frac{1}{2}$

Therefore $y = \frac{1}{2}x + c$

e $A(\frac{c}{2}, 0)$ and $B(0, c)$

i The midpoint of line segment AB has coordinates $\left(\frac{c}{4}, \frac{c}{2}\right)$

If $\left(\frac{c}{4}, \frac{c}{2}\right) = (3, 6)$ then $c = 12$

ii The area of the triangle $AOB = \frac{1}{2} \times c \times \frac{c}{2} = \frac{c^2}{4}$

If the area is 4, $\frac{c^2}{4} = 4$ which implies $c^2 = 16$. Therefore $c = 4$ since $c > 0$

9 $3x + by = 12$

a $3x + by = 12$

$by = -3x + 12$

$y = -\frac{3}{b}x + \frac{12}{b}$

$\therefore$ y-axis intercept is $\frac{12}{b}$.

b $\therefore$ gradient $= -\frac{3}{b}$

c **i** $-\frac{3}{b} = 1$

$b = -3$

ii $-\frac{3}{b} = -2$

$b = \frac{3}{2}$

d Gradient of perpendicular line is $\frac{b}{3}$

The line is of the form $y = \frac{b}{3}x + c$

When $x = 4, y = 0$

$0 = \frac{b}{3} \times 4 + c$

$c = -\frac{4b}{3}$

$\therefore y = \frac{b}{3}x - \frac{4b}{3}$ or $3y = bx - 4b$

Solutions to Exercise 2G

1 At $n = 0, w = \$350,$ paid at \$20 per n

$\therefore w = 20n + 350; n \in N \cup \{0\}$

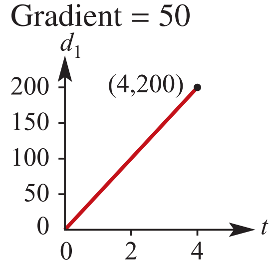
2 **a** At $t = 0, d_1 = 0$ and $v = 50$ km/h

$\therefore d_1 = vt = 50\,t$

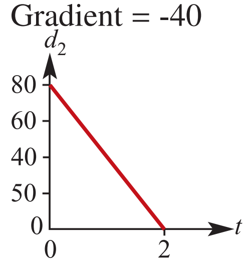
b At $t = 0, d_2 = 80$ and $v = -40$ km/h

$\therefore d_2 = 80 - 40\,t$

c Gradient = 50



Gradient = -40



3 **a** At $t = 0, V = 0,$ fills at 5 L/min

$\therefore V = 5t$

b At $t = 0, V = 10,$ fills at 5L/min

$\therefore V = 5t + 10$

4 **a** At $t = 0, v = 500,$ empties at 2.5 L/min

$\therefore v = -2.5t + 500$

b Since the bag is emptying, $v \leq 500$
The bag cannot contain a negative volume so $v \geq 0$

$\therefore 0 \leq v \leq 500$
The bag does not go back in time so $t \geq 0$
The bag empties when

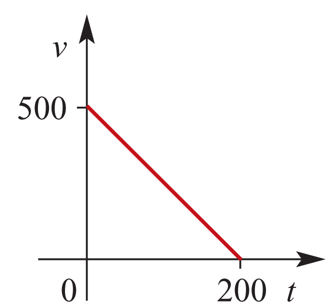
$$-2.5t + 500 = 0$$

$$2.5t = 500$$

$$t = 200$$

After that the function no longer holds true,
$\therefore 0 \leq t \leq 200$

c



5 At $n = 0, C = 2.6, C$ per km = 1.5
$\therefore C = 1.5n + 2.6$

6 **a** At $x = 0, C = 85, C$ per km = 0.24
$\therefore C = 0.24x + 85$

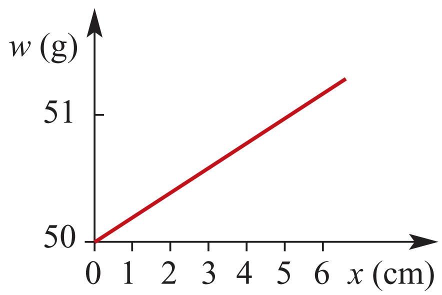
b When $x = 250,$

$C = 0.24\,(250) + 85$

$= 60 + 85 = \$145$

7 At $t = 0, d = 200$ km,
$v = -5$ km/h from B
$\therefore d = -5t + 200$

8 **a**



b $w = 50 + 0.2x$

c If $w = 52.5$ g,

$x = 5 \times (52.5 - 50)$

$= 5 \times 2.5 = 12.5$ cm

9 **a** $C = an + b$

If $n = 800, C = 47$; if

$n = 600, C = 35$

$$\begin{array}{rcl} 800a + b &=& 47 \\ 600a + b &=& 35 \\ \hline 200a &=& 12 \end{array}$$

$\therefore a = \frac{12}{200} = \frac{3}{50} = 0.06$

Substitute into 2nd equation:

$600 \times \frac{3}{50} + b = 35$

$36 + b = 35$

$b = -1$

$\therefore C = 0.06n - 1$

b If $n = 1000$,

$c = 0.06(1000) - 1$

$= 60 - 1 = \$59$

10 **a** $C = an + b$

If $n = 160, C = 975$; if

$n = 120, C = 775$

$$\begin{array}{rcl} 160a + b &=& 975 \\ 120a + b &=& 775 \\ \hline 40a &=& 200 \end{array}$$

$\therefore a = \frac{200}{40} = 5$

Substitute into 2nd equation:

$600 + b = 775$

$b = 175$

$\therefore C = 5n + 175$

b Yes, because $b \neq 0$

c When $n = 0$, $C = \$\ 175$

Solutions to Exercise 2H

1 The lines $x + y = 6$ and $2x + 2y = 13$ both have gradient -1 but different y-intercepts.

2 Let $x = \lambda$. Then solution is $\{(\lambda, 6 - \lambda) : \lambda \in R\}$

3 **a** $m = 4$. The line $y = 4x + 6$ is parallel to the line $y = 4x - 5$

b $m \neq 4$

c $15 = 5m + 6$
$\therefore m = \dfrac{9}{5}$
Check: (5,15) lies on the line $y = 4x - 5$

4 $6 = 4 + k$ and $6 = 2m - 4$
$\therefore k = 2$ and $m = 5$

5
$$2(m - 2) + 8 = 4 \ldots (1)$$
$$2m + 24 = k \ldots (2)$$
From (1) $2m - 4 + 8 = 4$
$$m = 0$$
From (2) $k = 24$

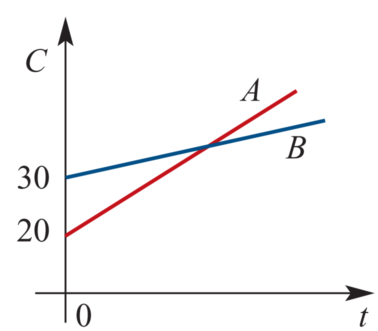
6 The simultaneous equations have no solution when the cooresponding lines have the same gradient and no point in common.
Gradient of $mx - y = 5$ is m.
Gradient of $3x + y = 6$ is -3.
$\therefore$ lines are parallel when $m = -3$

7
Gradient of $3x + my = 5$ is $-\dfrac{3}{m}$.
Gradient of $(m + 2)x + 5y = m$ is $-\dfrac{m+2}{5}$.
If the gradients are equal
$$-\frac{3}{m} = -\frac{m+2}{5}$$
$15 = m^2 + 2m$
$m^2 + 2m - 15 = 0$
$(m + 5)(m - 3) = 0$
$$m = -5 \text{ or } m = 3$$

a When $m = -5$ the equations become
$3x - 5y = 5$
$-3x + 5y = -5$
They are equations of the same line.
There are infinitely many solutions.

b When $m = 3$ the equations become
$3x + 3y = 5$
$5x + 5y = 3$
They are the equations of parallel lines with no common point.
No solutions

8 A: $C = 10t + 20$
B: $C = 8t + 30$

a


b Costs are equal when

$$10t + 20 = 8t + 30$$
$$2t = 10, \therefore t = 5$$

9 Day 1:

John: $v = \frac{1}{a}$ m/s

Michael: $v = \frac{1}{b}$ m/s

$d = vt = 50$ m, so Michael's time is:
$t = 50\,\frac{1}{v} = 50\,b$
Similarly, John's time is:
$t = 50\,\frac{1}{v} = 50\,a$
Michael wins by 1 second
$\therefore 50\,a = 50\,b + 1$
Day 2:
John runs only 47 m:
$t = 47\,a$
Michael runs the same time:
$t = 50\,b$
Michael wins by 0.1 seconds
$\therefore 47\,a = 50\,b + 0.1$
From day 1 : $50\,b = 50\,a - 1$
$\therefore 47\,a = 50\,a - 1 + 0.1$

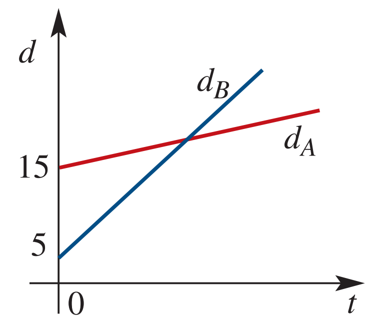
$$3\,a = 0.9, \therefore a = 0.3$$
$$50\,b = 50 \times 0.3 - 1 = 14$$
$$\therefore b = 0.28$$

Michael's speed:
$v = \frac{1}{b} = \frac{1}{0.28} = \frac{25}{7}$ m/s

10 $d_A = 10t + 15$

$d_B = 20t + 5$
t is the time in hours after 1.00 p.m.

a

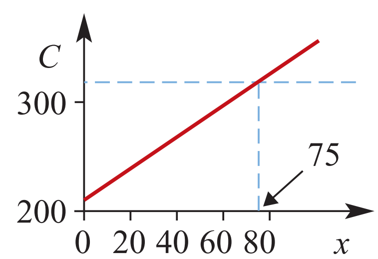


b $d_A = d_B$ when
$20t + 5 = 10t + 15$

$$10t = 10, \therefore t = 1$$

11 **a** A: $C = 1.6x + 210$
B: $C = 330$

b

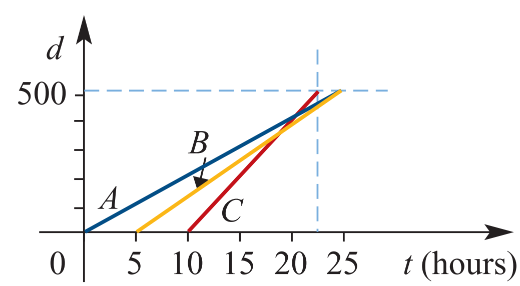


c Costs are equa1 when
$l.6x + 210 = 330$

$$1.6x = 120$$
$$x = \frac{120}{1.6} = 75$$

When $x < 75$, method A is cheaper;
when $x > 75$, method B is cheaper.
So the fixed charge method is cheaper
when $x > 75$.

12 **a**



b C wins the race

c $A(t) = 20t$

$B(t) = 25(t - 5)$

$C(t) = 40(t - 10)$

After 25 hours,

$A(25) = 500$

$B(25) = 25(25 - 5) = 500$

$C(25) = 40(25 - 10) = 600$

C completes the course in t hours where:

$40(t - 10) = 500$

$$t - 10 = \frac{500}{40} = 12.5$$

$$t = 12.5 + 10 = 22.5$$

d C, leaving 5 hours after B, overtakes B $13\frac{1}{2}$ hours after B had started and then overtakes A 20 hours after A had started. C wins the race with a total handicap time of $22\frac{1}{2}$ hours ($12\frac{1}{2}$ hours for journey +10 hours handicap) with A and B deadheating for 2nd, each with a total handicap time of 25 hours.

13 $y = -\frac{3}{4}x$ meets $y = \frac{3}{2}x - 12$ when

$$-\frac{3x}{4} = \frac{3x}{2} - 12$$

$$-3x = 6x - 48$$

$$9x = 48$$

$$x = \frac{48}{9} = \frac{16}{3} = 5\tfrac{1}{3}$$

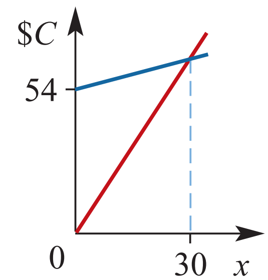
$$\therefore y = -\frac{3}{4} \times \frac{16}{3} = -4$$

The paths cross at $\left(5\frac{1}{3}, -4\right)$

14 a Public transport: $\$C = 2.8x$

Bus: $\$C = x + 54$

b



c Costs are equal when $2.8x = x + 54$

$$1.8x = 54$$

$$x = 30$$

It is more economical if there are more than 30 students.

15 a Anne: when $t = 0, d = 0$;

$v = 20$ km/h $= \frac{1}{3}$km/min

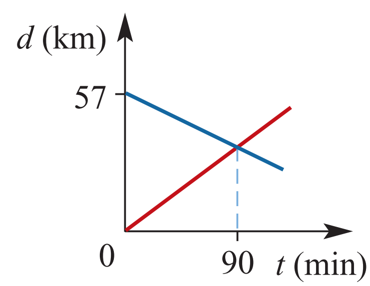
$\therefore d = \frac{t}{3}$

Maureen: when $t = 0, d = 57$;

$v = -18$ km/h $= -\frac{3}{10}$km/min

$\therefore d = 57 - \frac{3}{10}t$

b



c They meet when

$$\frac{t}{3} = 57 - \frac{3t}{10}$$
$$\frac{t}{3} + \frac{3t}{10} = 37$$
$$10t + 9t = 1710$$
$$19t = 1710$$
$$t = \frac{1710}{19} = 90$$

They meet after 90 min, i.e. 10.30 a.m.

d Substitute into either equation, but Anne's is easier:
$d = \frac{t}{3} = \frac{90}{3} = 30$
Anne has traveled 30 km, so Maureen must have traveled $57 - 30 = 27$ km.

Solutions to Review: Short-answer questions

1 a $A(1,2)$ and $B(5,2)$: y does not change.
Length $AB = 5 - 1 = 4$
Midpoint $x = \dfrac{1+5}{2} = 3$
$\therefore$ midpoint is at (3, 2).

b $A(-4,-2)$ and $B(3,-7)$
Length $AB = \sqrt{(-4-3)^2 + (-2-(-7))^2}$
$= \sqrt{(-7)^2 + 5^2} = \sqrt{74}$
Midpoint $x = \dfrac{-4+3}{2} = -\dfrac{1}{2}$
$y = \dfrac{-2+-7}{2} = -\dfrac{9}{2}$
$\therefore$ midpoint is at $\left(-\dfrac{1}{2}, -\dfrac{9}{2}\right)$.

c $A(3,4)$ and $B(7,1)$
Length $AB = \sqrt{(7-3)^2 + (1-4)^2}$
$= \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$
Midpoint $x = \dfrac{3+7}{2} = 5$
$y = \dfrac{4+1}{2} = \dfrac{5}{2}$
$\therefore$ midpoint is at $\left(5, \dfrac{5}{2}\right)$.

2 a $m = \dfrac{12-3}{8-4} = \dfrac{9}{4}$

b $m = \dfrac{-6-4}{8-(-3)} = -\dfrac{10}{11}$

c x does not change so gradient is undefined.

d $m = \dfrac{0-a}{a-0} = -1$

e $m = \dfrac{b-0}{a-0} = \dfrac{b}{a}$

f $m = \dfrac{0-b}{a-0} = -\dfrac{b}{a}$

3 If $m = 4$ then $y = 4x + c$

a Passing though $(0,0)$; $y = 4x$

b Passing though $(0,5)$; $y = 4x + 5$

c Passing though (1, 6);
$y = 4 + c = 6, \therefore c = 2$
$y = 4x + 2$

d Passing though (3, 7);
$y = 12 + c = 7, \therefore c = -5$
$y = 4x - 5$

4 a $y = 3x - 5$
Using $(1, a)$, $a = 3 - 5 = -2$

b $y = 3x - 5$
Using $(b, 15)$, $3b - 5 = 15$
$3b = 20, \therefore b = \dfrac{20}{3}$

5 $y = mx + c$; $m = \dfrac{-4-2}{3-(-5)} = -\dfrac{3}{4}$
Using $(3,-4)$:
$-4 = \left(-\dfrac{3}{4}\right)3 + c$
$\dfrac{9}{4} - 4 = c = -\dfrac{7}{4}$
$y = -\dfrac{3}{4}x - \dfrac{7}{4}$
$4y = -3x - 7$
$\therefore 3x + 4y = -7$

6 $y = mx + c : m = -\dfrac{2}{3}$
Using $(-4, 1)$:
$y = -\dfrac{2}{3}(-4) + c = 1$

$\frac{8}{3} + c = 1 \therefore c = -\frac{5}{3}$

$y = -\frac{2}{3x} - \frac{5}{3}$

$3y = -2x - 5$

$\therefore 2x + 3y = -5$

7 **a** Lines parallel to the x-axis are $y = c$.
Using (5, 11), $y = 11$

b Parallel to $y = 6x + 3$ so gradient
$m = 6$
When $x = 0, y = -10$, so $c = -10$
$y = 6x - 10$

c
$$3x - 2y + 5 = 0$$
$$-2y = -3x - 5$$
$$\therefore \quad y = \frac{3}{2}x + \frac{5}{2}$$
$m = \frac{3}{2}$ so perpendicular gradient
$= -\frac{2}{3}$
Using (0,1), $c = 1$
$$y = -\frac{2}{3}x + 1$$
$$3y = -2x - 3$$
$$\therefore \quad 2x + 3y = -3$$

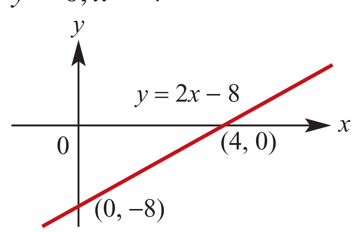
8 $y = mx + c$
Line at 45° to x-axis
$m = \tan(45°) = 1$
Using (2, 3): $3 = 1 \times 2 + c$
So $c = 1$ and $y = x + 1$
$\therefore y = x + 1$

9 $y = mx + c$:
Line at 135° to x-axis,
$m = \tan 135° = -1$
Using $(-2, 3)$: $3 = -1 \times -2 + c$
So $c = 1$ and $y = -x + 1$
$\therefore x + y = 1$

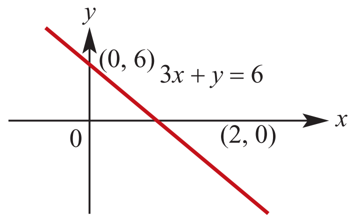
10 Gradient of a line perpendicular to
$y = -3x + 2$ is $\frac{1}{3}$.
Therefore required line is of the form
$y = \frac{1}{3}x + c$.
When $x = 4, y = 8$
$\therefore 8 = \frac{1}{3} \times 4 + c$
Hence $c = 8 - \frac{4}{3} = \frac{20}{3}$
$y = \frac{1}{3}x + \frac{20}{3}$

11 $y = 2x + 1$
When $x = 0, y = 1. \therefore a = 1$
When $y = 0, 2x + 1 = 0. \therefore b = -\frac{1}{2}$
When $x = 2, y = 5. \therefore d = 5$
When $y = 7, 2x + 1 = 7. \therefore e = 3$

12 **a** $y = 2x - 8$
When $x = 0, y = -8$ and when
$y = 0, x = 4$

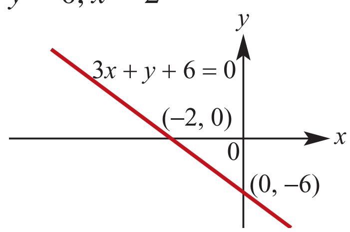


b $3x + y = 6$
When $x = 0, y = 6$ and when
$y = 0, x = 2$



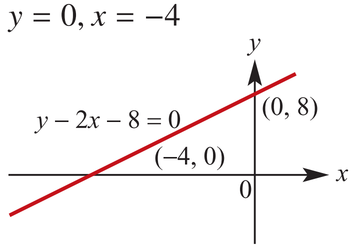
c $3x + y + 6 = 0$

When $x = 0, y = -6$ and when $y = 0, x = 2$



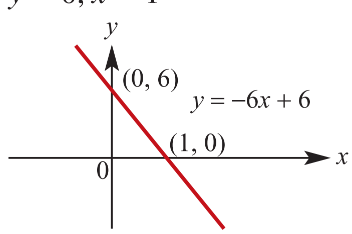
d $y - 2x - 8 = 0$

When $x = 0, y = 8$ and when $y = 0, x = -4$



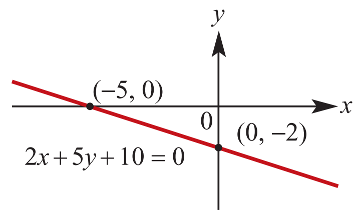
e $y = -6x + 6$

When $x = 0, y = 6$ and when $y = 0, x = 1$



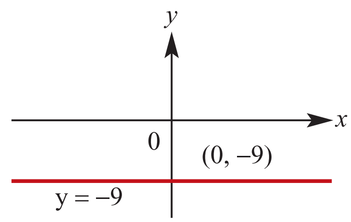
f $2x + 5y + 10 = 0$

When $x = 0, y = -2$ and when $y = 0, x = -5$



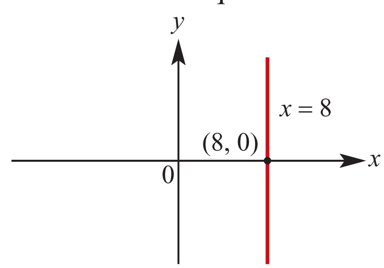
13 a Line is of the form $y = c$.

$\therefore y = -9$ is the equation

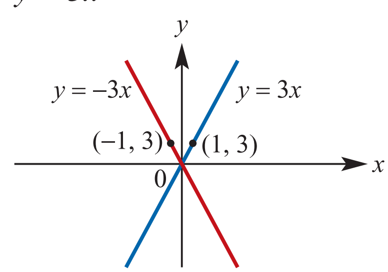


b Line is of the form $x = a$.

$\therefore x = 8$ is the equation.



c **i** $y = 3x$



ii $y = -3x$

14 a $d = 60t$

b gradient is 60

15 $S = 800 + 500n$

16 **a** $y = ax + 2$

When $x = 2, y = 6$

$\therefore 6 = 2a + 2$

$\therefore a = 2$

The equation is $y = 2x + 2$

b **i** When $y = 0, ax + 2 = 0$

$\therefore x = \frac{-2}{a}$

ii $\frac{-2}{a} > 1$

$-2 < a$

(Since $a < 0$)

$\therefore 0 > a > -2$

Equivalently $-2 < a < 0$

c $x + 3 = ax + 2$ Substitute in

$x - ax = 2 - 3$

$x(1 - a) = -1$

$x = \frac{1}{a - 1}$

$y = x + 3$ to find the y-coordinate.

$y = \frac{1}{a - 1} + 3$

$y = \frac{3a - 2}{a - 1}$

.

The coordinates of the point of intersection are $\left(\frac{1}{a - 1}, \frac{3a - 2}{a - 1}\right)$

Solutions to Review: Multiple-choice questions

1 D Midpoint $x = \dfrac{4+6}{2} = 5$

Midpoint $y = \dfrac{12+2}{2} = 7$

Midpt is at (5, 7)

2 E Midpoint x-coordinate

$6 = \dfrac{-4+x}{2}, \therefore x = 16$

Midpoint y-coordinate

$3 = \dfrac{-6+y}{2}, \therefore y = 12$

$\therefore x + y = 28$

3 A Gradient $= \dfrac{-10-(-8)}{6-5} = -2$

4 E Gradient $= \dfrac{2a-(-3a)}{4a-9a} = -1$

5 C $y = mx + c; m = 3$

Using $(1, 9)$:

$9 = 3 + c$ so $c = 6$

$y = 3x + 6$

6 D $y = mx + c; m = \dfrac{-14-(-6)}{-2-2} = 2$

Using $(2, -6)$:

$y = 4 + c = -6; c = -10$

So $y = 2x - 10$

7 B $y = 2x - 6$

Using $(a, 2)$:

$y = 2a - 6 = 2, \therefore a = 4$

8 E Axis intercepts at $(1, 0)$ and $(0, -3)$:

$y = mx + c : c = -3$

Using $(1, 0)$: $0 = m - 3$ so $m = 3$

$y = 3x - 3$

9 C $5x - y + 7 = 0$

$$-y = -5x - 7$$

$$\therefore \quad y = 5x + 7$$

Gradient = 5

$$ax + 2y - 11 = 0$$

$$2y = -ax + 11$$

$$\therefore \quad y = -\frac{a}{2}x + \frac{11}{2}$$

Parallel lines mean gradients are equal:

$-\dfrac{a}{2} = 5, \therefore a = -10$

10 E

$C = 2.5x + 65 = 750$

$2.5x = 685, \therefore x = 274$

11 C

$$2ax + 2by = 3 \ldots (1)$$

$$3ax - 2by = 7 \ldots (2)$$

Add (1) and (2)

$$5ax = 10$$

$$\therefore x = \frac{2}{a}$$

Substitute in (1)

$$y = -\frac{1}{2b}$$

Solutions to Review: Extended-response questions

1 **a** $C = 100n + 27.5n + 50 + 62.5n = 550 + 190n$

b

$$\begin{aligned} C \le 3000 \quad \therefore\ 550 + 190n &\le 3000 \\ \therefore\quad 190n &\le 2450 \\ \therefore\quad n &\le \frac{2450}{190} \\ \therefore\quad n &< 12.9 \end{aligned}$$

The cruiser can be hired for up to and including 12 days by someone wanting to spend no more than \$3000.

c $300n < 550 + 190n$

$110n < 550$

$n < 5$

It's cheaper to hire from the rival company for cruises less than 5 days.

2 **a** It is the cost of the plug.

b It is the cost per metre of the cable.

c 1.8

d

$$\begin{aligned} 24.5 &= 4.5 + 1.8x \\ \therefore\quad 20 &= 1.8x \\ \therefore\quad x &= \frac{20}{1.8} \\ &= \frac{100}{9} \\ &= 11\frac{1}{9} \end{aligned}$$

$11\frac{1}{9}$ metres of cable would give a total cost of \$ 24.50.

3 **a** It is the maximum profit when the bus has no empty seats, i.e. $x = 0$.

b $$\begin{aligned} P &< 0 \\ 1020 - 24x &< 0 \\ -24x &< -1020 \\ x &> \frac{-1020}{-24} \\ x &> 42.5 \end{aligned}$$

43 empty seats is the least number to cause a loss on a single journey.

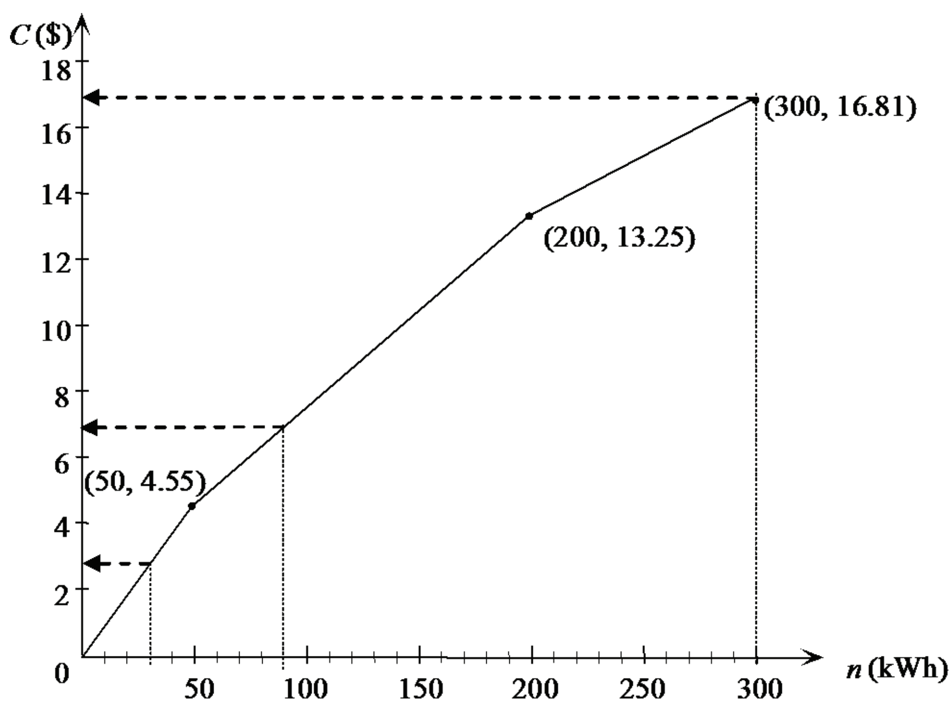
c The profit reduces by \$24 for each empty seat.

4 a i $C = 0.091n, \qquad 0 < n \le 50$

ii $$\begin{aligned} C &= 0.058(n - 50) + 0.091 \times 50 \\ &= 0.058n - 2.9 + 4.55 \\ &= 0.058n + 1.65, \qquad 50 < n \le 200 \end{aligned}$$

iii $$\begin{aligned} C &= 0.0356(n - 200) + 0.058 \times 200 + 1.65 \\ &= 0.0356n - 7.12 + 11.6 + 1.65 \\ &= 0.0356n + 6.13, \qquad n > 200 \end{aligned}$$

b



i When $n = 30$ kWh,

from the graph $C \approx \$3$

from the formula $C = 0.091 \times 30$

$= \$2.73$

ii When $n = 90$ kWh,

from the graph $C \approx \$7$

from the formula $C = 0.058 \times 90 + 1.65$

$= \$6.87$

iii When $n = 300$ kWh,

from the graph $C \approx 17$

from the formula $C = 0.0356 \times 300 + 6.13$

$= \$16.81$

c When $C = 20$,

$$20 = 0.0356n + 6.13$$

$$\therefore \quad 13.87 = 0.0356n$$

$$\therefore \quad n = 389.60\ldots$$

Approximately 390 kWh of electricity could be used for \$20.

5 a Let $(x_1, y_1) = (2, 10)$ and $(x_2, y_2) = (8, -4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 10}{8 - 2}$$

$$= \frac{-14}{6}$$

$$= \frac{-7}{3}$$

$$y - y_1 = m(x - x_1)$$

$$\therefore \quad y - 10 = \frac{-7}{3}(x - 2)$$

$$\therefore \quad y - 10 = \frac{-7}{3}x + \frac{14}{3}$$

$$\therefore \quad y + \frac{7}{3}x = 10 + \frac{14}{3}$$

$$\therefore \quad y + \frac{7}{3}x = \frac{44}{3}$$

$$\therefore \quad 3y + 7x = 44$$

$$\therefore y = -\frac{7}{3}x + 14\frac{2}{3}$$

The equation describing the aircraft's flight path is $7x + 3y = 44$.

b When $x = 15$, $7 \times 15 + 3y = 44$

$$\therefore \quad 105 + 3y = 44$$

$$\therefore \quad 3y = -61$$

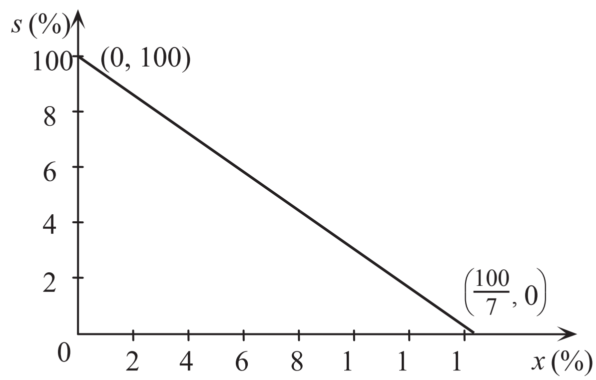
$$\therefore \quad y = \frac{-61}{3}$$

$$= -20\frac{1}{3}$$

When $x = 15$, the aircraft is $20\frac{1}{3}$ km south of O.

6 a $s = 100 - 7x$

b



c $100 - 7x \geq 95$

$$\therefore \quad -7x \geq -5$$

$$\therefore \quad x \leq \frac{5}{7}$$

i.e. $\frac{5}{7}$% air can be left in the concrete for at least 95% strength.

d $100 - 7x = 0$

$$\therefore \quad -7x = -100$$

$$\therefore \quad x = \frac{100}{7}$$

$$= 14\frac{2}{7}$$

i.e. the concrete contains $14\frac{2}{7}$% air when at 0% strength.

e Concrete at 0% strength would not be useful, therefore not a sensible model.

f $\{x : 0 \leq x \leq 14\frac{2}{7}\}$

7 a For line AB, let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (4, 6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{4 - 0} = 1$$

(Alternatively, $y = mx + c$, where $m = 1$ as shown and $c = 2, \therefore\ y = x + 2$.)

Now $\quad y - y_1 = m(x - x_1)$

$\therefore \quad y - 2 = 1(x - 0)$

$\therefore \quad y = x + 2$ is the equation of line AB.

For line CD, let $(x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (5, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{5 - 3} = 2$$

Now $\quad y - y_1 = m(x - x_1)$

$\therefore \quad y - 0 = 2(x - 3)$

$\therefore \quad y = 2x - 6$

The equation of line CD is $y = 2x - 6$.

b The lines intersect where $\quad x + 2 = 2x - 6$

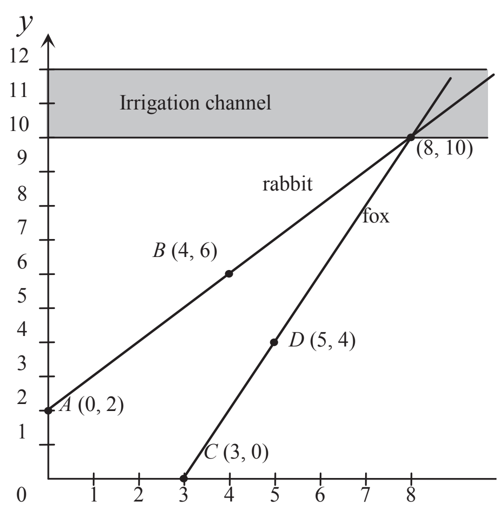
$\therefore \quad 2 = x - 6$

$\therefore \quad x = 8$

When $x = 8$, $\quad y = 2 \times 8 - 6$

$\therefore \quad y = 10$

i.e. the coordinates of the point of intersection are (8, 10), and the paths of the rabbit and the fox meet on the near bank of the irrigation channel.



8 a For the equation of line AB, let $(x_1, y_1) = (-4.5, 2)$ and $(x_2, y_2) = (0.25, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7 - 2}{0.25 - (-4.5)}$$
$$= \frac{5}{4.75}$$
$$= \frac{20}{19}$$

Now $y - y_1 = m(x - x_1)$

$\therefore$ $y - 2 = \frac{20}{19}(x - (-4.5))$

$\therefore$ $y = \frac{20}{19}x + \frac{90}{19} + 2$

$\therefore$ $y = \frac{20}{19}x + \frac{128}{19}$

The equation of line AB is $y = \frac{20}{19}x + \frac{128}{19}$.

$\therefore$ the y-coordinate of the point V is the y-axis intercept of $\frac{128}{19}$.

b For the equation of line VC, let $(x_1, y_1) = (0, \frac{128}{19})$ and $(x_2, y_2) = (5, 1.5)$.

$C = \frac{128}{19}$ and $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{1.5 - \frac{128}{19}}{5 - 0} -$$

$$= \frac{\frac{57}{38} - \frac{256}{38}}{5} = \frac{1}{5} \times -\frac{199}{38} = -\frac{199}{190}$$

Now $y = mx + c$

$y = -\frac{199}{190}x + \frac{128}{19}$ is the equation of the line VC.

c Cuts AB and VC are not equally inclined to the vertical axis because the gradients of AB and VC (although opposite in sign) are not the same size, (gradient $AB \approx 1.053$, gradient $VC \approx -1.047$).

9 a For the equation of line PQ, let $(x_1, y_1) = (4, -75)$ and $(x_2, y_2) = (36, -4)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4 - (-75)}{36 - 4} \\ &= \frac{71}{32} \end{aligned}$$

Now $\quad y - y_1 = m(x - x_1)$

$\therefore \quad y - (-75) = \frac{71}{32}(x - 4)$

$\therefore \quad y + 75 = \frac{71}{32}x - \frac{71}{8}$

$\therefore \quad y = \frac{71}{32}x - \frac{671}{8}$ is the equation of line PQ.

When $7x = 20$,

$$\begin{aligned} y &= \frac{71}{32} \times 20 - \frac{671}{8} \\ &= \frac{355}{8} - \frac{671}{8} \\ &= \frac{-316}{8} \\ &= -39\frac{1}{2} \end{aligned}$$

i.e. line PQ does not pass directly over a hospital located at $H(20, -36)$.

b When $y = -36$, $\quad -36 = \frac{71}{32}x - \frac{671}{8}$

$\therefore \quad \frac{383}{8} = \frac{71}{32}x$

$\therefore \quad x = \frac{383}{8} \times \frac{32}{71} = 21\frac{41}{71}$

i.e. when $y = -36$, the aircraft is $1\frac{41}{71}$ km east of H.

10 a 5 km due south of E is $D(68, 30)$.

For the equation of line AD, let $(x_1, y_1) = (48, 10)$ and $(x_2, y_2) = (68, 30)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30 - 10}{68 - 48} = 1$$

Now $\quad y - y_1 = m(x - x_1)$

$\therefore \quad y - 10 = 1(x - 48)$

$\therefore \quad y = x - 38$ is the equation ofthe new runway.

b $A(48, 10)$

$$\therefore \quad B((48+8), y) = (56, y)$$

When $x = 56$,

$$\begin{aligned} y &= x - 38 \\ &= 56 - 38 \\ &= 18 \end{aligned}$$

$\therefore$ the coordinates of B are (56, 18).

c Consider the line connecting $C(88, -10)$ and $D(68, 30)$.
Let $(x_1, y_1) = (88, -10)$ and $(x_2, y_2) = (68, 30)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{30 - (-10)}{68 - 88} \\ &= \frac{40}{-20} = -2 \end{aligned}$$

Now $\quad y - y_1 = m(x - x_1)$

$$\begin{aligned} \therefore \quad y - (-10) &= -2(x - 88) \\ \therefore \quad y + 10 &= -2x + 176 \\ \therefore \quad y &= -2x + 166 \text{ is the equation defining the line } CD. \end{aligned}$$

d $y = 10$ for an auxiliary beacon due east of A.

$$\begin{aligned} \therefore \quad 2x + 10 &= 166 \\ \therefore \quad 2x &= 156 \\ \therefore \quad x &= 78 \end{aligned}$$

i.e. the coordinates of the auxiliary beacon are (78, 10).

11 **a** Gradient of $AB = \dfrac{6 - 8}{8 - 2}$

$$\begin{aligned} &= -\frac{2}{6} \\ &= -\frac{1}{3} \end{aligned}$$

As $AD \perp AB$, gradient of $AD = 3$.
Equation of the line through A and D is

$$\begin{aligned} y - 8 &= 3(x - 2) \\ &= 3x - 6 \\ \therefore \quad y &= 3x + 2 \end{aligned}$$

b D lies on the line with equation $y = 3x + 2$, and has an x-coordinate of 0.
When $x = 0, y = 2 \therefore D$ is the point $(0, 2)$.

c Let M be the midpoint of AB, with coordinates (x, y).

$$x = \frac{2+8}{2} = 5$$

and $$y = \frac{8+6}{2} = 7$$

$\therefore$ M is the point (5, 7).

The point M lies on the perpendicular bisector of AB that has a gradient 3.

The equation of the perpendicular bisector of AB is

$$y - 7 = 3(x - 5)$$
$$= 3x - 15$$
$$\therefore \quad y = 3x - 8$$

d Let C be the point (a, b).

C lies on the lines with equation $y = 3x - 8$ and $3y = 4x - 14$

$$\therefore \quad b = 3a - 8 \qquad (1)$$

and $$3b = 4a - 14 \qquad (2)$$

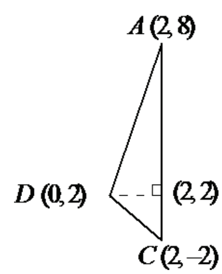
Substituting (1) into (2) yields

$$3(3a - 8) = 4a - 14$$
$$\therefore \quad 9a - 24 = 4a - 14$$
$$\therefore \quad 5a = 10$$
$$\therefore \quad a = 2$$
$$\therefore \quad b = 3(2) - 8 = -2$$

$\therefore$ C is the point (2, −2).

e Area of triangle ADC

Using $\frac{1}{2}$ (base)(height), area ΔADC is simply $\frac{1}{2}(2)(10) = 10$ square units [since AC = base length = 10 units]



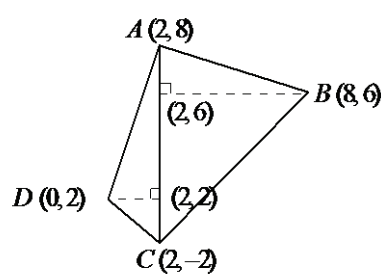
f Area ΔABC is simply $\frac{1}{2}(6)(10) = 30$ square units

Area of quadrilateral $ABCD$

$= \text{area of } \Delta ADC + \text{ area of } \Delta ABC$

$= 10 + 30$

$= 40$ square units



12 **a** $C = 40x + 30\,000$

b When $x = 6000$,

$$\begin{aligned} C &= 40 \times 6000 + 30\,000 \\ &= 270\,000 \\ \text{Cost per wheelbarrow} &= \frac{270\,000}{6000} \\ &= 45 \end{aligned}$$

i.e. overall cost per wheelbarrow is \$45.

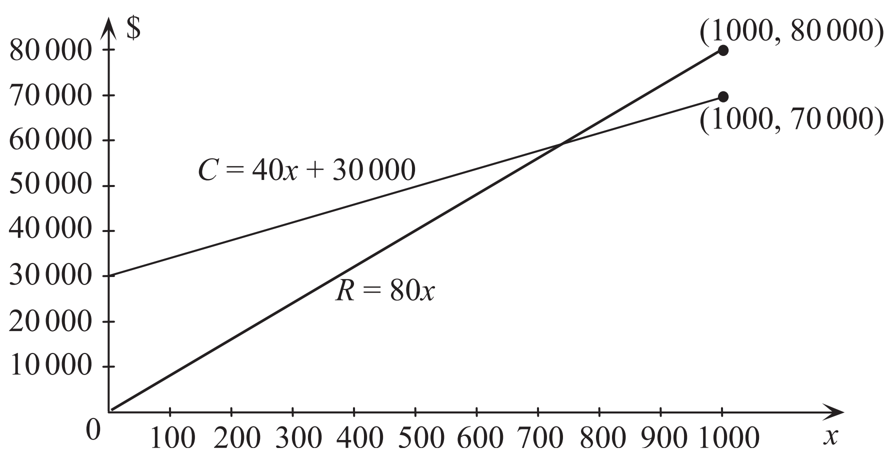
c Cost per wheelbarrow = \$46

$$\begin{aligned} \therefore \quad \frac{40x + 30\,000}{x} &= 46 \\ \therefore \quad 40x + 30\,000 &= 46x \\ \therefore \quad 30\,000 &= 6x \\ \therefore \quad x &= 5000 \end{aligned}$$

i.e. 5000 wheelbarrows must be made for an overall cost of \$46 each.

d $R = 80x$

e



f $R > C$, $\quad \therefore 80x > 40x + 30\,000$

$$\begin{aligned} \therefore \quad 40x &> 30\,000 \\ \therefore \quad x &> 750 \end{aligned}$$

i.e. minimum number of wheelbarrows to make a profit is 751.

g $P = R - C$

$$\begin{aligned} &= 80x - (40x + 30\,000) \\ &= 40x - 30\,000 \end{aligned}$$

13 **a** Cost of Method 1 $= 100 + 0.08125 \times 1560$

$= 226.75$

Cost of Method 2 $= 4 \times 27.5 + 0.075 \times 1560$

$= 227$

i.e. Method 1 is cheaper for 1560 units.

b

	Number of units of electricity			
	0	1000	2000	3000
Cost ($) by Method 1	100	181.25	262.50	343.75
Cost ($) by Method 2	110	185	260	335

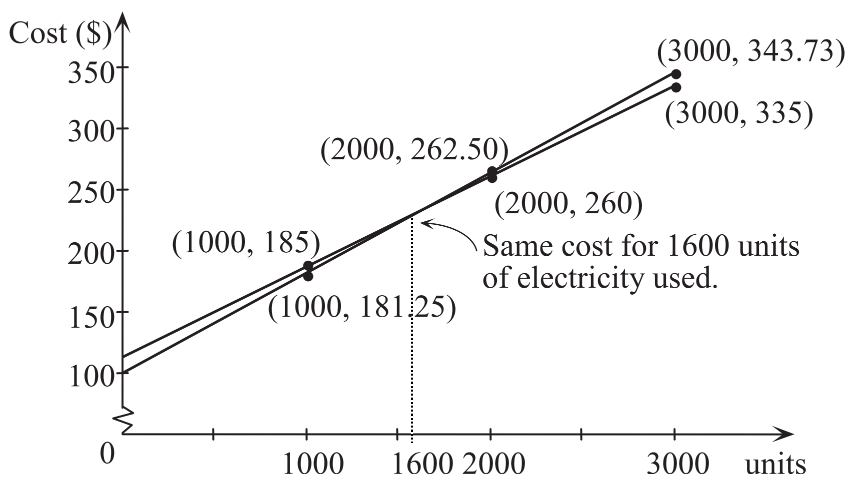
Calculations for Method 1:

$$100 + 0.08125 \times 0 = 100$$
$$100 + 0.08125 \times 1000 = 181.25$$
$$100 + 0.08125 \times 2000 = 262.50$$
$$100 + 0.08125 \times 3000 = 343.75$$

Calculations for Method 2:

$$4 \times 27.5 + 0.075 \times 0 = 110$$
$$4 \times 27.5 + 0.075 \times 1000 = 185$$
$$4 \times 27.5 + 0.075 \times 2000 = 260$$
$$4 \times 27.5 + 0.075 \times 3000 = 335$$

c



d $C_1 = 100 + 0.08125x$

$C_2 = 4 \times 27.5 + 0.075 \times x$

$= 110 + 0.075x$

When $C_1 = C_2$,
$$\begin{aligned} 100 + 0.08125x &= 110 + 0.075x \\ 0.00625x &= 10 \\ x &= 1600 \end{aligned}$$

14 a M is the point directly below the intersection of lines AC and BD.
Find the equations of the lines AC and BD.
M is the midpoint of both AC and BD.
Use the midpoint formula to find the coordinates of M.
Using line AC:
$$\begin{aligned} M &= \left(\frac{10+24}{2}, \frac{16+8}{2}\right) \\ &= (17, 12) \end{aligned}$$
i.e. the point to which the hook must be moved has coordinates (17, 12).

b (17, 12) is a point on the line, and m = gradient of AB
where $(x_1, y_1) = (10, 16)$
and $(x_2, y_2) = (16, 20)$
$$\therefore \quad m = \frac{20-16}{16-10} = \frac{2}{3}$$
$\therefore$ equation of line parallel to AB is
$$y - 12 = \frac{2}{3}(x - 17)$$
$$\therefore \quad y = \frac{2}{3}x - \frac{34}{3} + 12$$
$$\therefore \quad 3y = 2x + 2$$

15 a Find equations of lines PA, AB, BC, CD and DP using the formula $y - y_1 = m(x - x_1)$, where $m = -\dfrac{y_2 - y_1}{x_2 - x_1}$.

For line PA $$m = \frac{60 - 120}{100 - 0}$$
$$= -\frac{3}{5}$$
$$y - 120 = -\frac{3}{5}(x - 0)$$
$$\therefore \quad y = -\frac{3}{5}x + 120$$

For line AB $$m = \frac{100 - 60}{200 - 100}$$
$$= \frac{2}{5}$$
$$y - 60 = \frac{2}{5}(x - 100)$$
$$\therefore \quad y = \frac{2}{5}x + 20$$

For line BC $$m = \frac{200 - 100}{160 - 200}$$
$$= -\frac{5}{2}$$
$$y - 100 = -\frac{5}{2}(x - 200)$$
$$y = -\frac{5}{2}x + 600$$

For line CD $$m = \frac{160 - 200}{60 - 160}$$
$$= \frac{2}{5}$$
$$y - 200 = \frac{2}{5}(x - 160)$$
$$\therefore \quad y = \frac{2}{5}x + 136$$

For line DP $$m = \frac{120 - 160}{0 - 60}$$
$$= \frac{2}{3}$$
$$y - 120 = \frac{2}{3}(x - 0)$$
$$\therefore \quad y = \frac{2}{3}x + 120$$

b

$$m_{PA} = -\frac{3}{5}$$

$$m_{AB} = \frac{2}{5}$$

$$m_{BC} = -\frac{5}{2}$$

$$m_{CD} = \frac{2}{5}$$

and $$m_{DP} = \frac{2}{3}$$

Now $$m_{AB} = m_{CD} = \frac{2}{5}$$

and $$m_{AB} \times m_{BC} = \frac{2}{5} \times -\frac{5}{2} = -1$$

$\therefore$ line BC is perpendicular to lines AB and CD, which are parallel.
Hence $\angle ABC$ and $\angle BCD$ are right angles.