Christ Church Grammar School

Mathematics Department
Year 11 Mathematics Methods

Semester 1 Examination, 2020

Question/Answer Booklet

MATHEMATICS METHODS

UNIT 1

Section Two:
Calculator Assumed

Solutions

Student Name: ____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	14	14	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the Christ Church Grammar School reporting and assessment policy. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (97 Marks)**

This section has **fourteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) In city Z, the fare for a 8.5 km taxi ride is $\$16.70$ and the fares for all taxi rides are directly proportional to the distance travelled.

Determine

(i) the fare for a 12 km taxi ride. (2 marks)

Solution
$\frac{12}{8.5} \times 16.70 = \$23.58$
Specific behaviours
✓ indicates use of direct proportion ✓ correct fare, to 2 decimal places

Alternative Solution
$F \propto d$ $F = kd$ $k = \frac{16.70}{8.5} = 1.965\ (3dp)$ $F = 1.965 \times 12 = \$23.58$
Specific behaviours
✓ indicates use of direct proportion and calculates k ✓ correct fare, to 2 decimal places

(ii) the distance a fare of $\$26.72$ would cover. (2 marks)

Solution
$\frac{26.72}{16.70} \times 8.5 = 13.6\text{ km}$
Specific behaviours
✓ indicates use of direct proportion ✓ correct distance

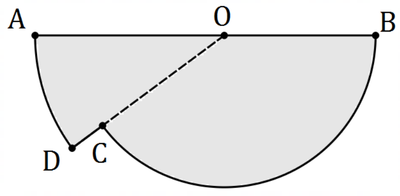
Alternative Solution
$F = 1.965d$ $d = \frac{26.72}{1.965} = 13\text{ km}$
Specific behaviours
✓ indicates use of direct proportion ✓ correct distance

(b) A straight line makes an angle of $30°$ with the positive x-axis and passes through the point with coordinates $(0, 1)$. Determine the exact equation of the line. (2 marks)

Solution
$m = \tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $y = \frac{\sqrt{3}}{3}x + 1$
Specific behaviours
✓ gradient ✓ equation in exact form

Question 10 **(5 marks)**

Shape $AOBCDA$ below consists of sector BOC of circle centre O joined to sector DOA of a different circle, also centre O. AB is a straight line of length 65 cm, arc AD is 12 cm long and $\angle AOD = 0.32$ radians.



(a) Determine the length OA. (2 marks)

Solution
Let $OA = R$ so that $0.32R = 12$ $R = 37.5 \text{ cm}$
Specific behaviours
✓ correct use of arc length   ✓ correct length

(b) Determine the area of the shape. (3 marks)

Solution
$A_{DOA} = \frac{1}{2} \times 37.5^2 \times 0.32$ $= 225$ Let $OB = r$ $r = 65 - 37.5$ $= 27.5$ $A_{BOC} = \frac{1}{2} \times 27.5^2(\pi - 0.32)$ $= 1067$ $\text{Area} = 225 + 1067$ $= 1\,292 \text{ cm}^2$
Specific behaviours
✓ area of sector DOA ✓ radius and angle of sector BOC ✓ area of shape

Question 11 **(8 marks)**

The height h metres of a particle above level ground is defined as a function of time t seconds as follows:

$$h(t) = 68.75 + 15t - 5t^2, \qquad 0 \le t \le 5.5\,.$$

(a) Determine the height of the particle when

(i) $t = 0$. (1 mark)

(ii) $t = 4.5$. (1 mark)

Solution
$h(0) = 68.75$ m $h(4.5) = 35$ m
Specific behaviours
✓ (i) correct ✓ (ii) correct

(b) Determine the maximum height reached by the particle and the time it reached this height. (2 marks)

Solution
From graph of $h(t)$: Maximum height: $h = 80$ m when $t = 1.5$ s.
Specific behaviours
✓ correct height ✓ correct time

(c) Determine the time(s) that the particle was at a height of 75 m. (2 marks)

Solution
From graph of $h(t)$: $h = 75$ when $t = 0.5$ s, 2.5 s
Specific behaviours
✓ one time ✓ both times

(d) State the range of the function $h(t)$ for the given domain. (2 marks)

Solution
Range of h: $0 \le h \le 80$ Or Accept $\{h \in \mathbb{R}: 0 \le h \le 80\}$
Specific behaviours
✓ upper limit ✓ lower limit, correct inequality Note: no penalty for just writing inequality in this context.

Question 12 **(6 marks)**

The graph $y = f(x)$, where $f(x) = x^2 + bx + c$ has a turning point at $(-2, -1)$.

(a) State the equation of the line of symmetry for the graph of $y = f(x)$. (1 mark)

Solution
$x = -2$
Specific behaviours
✓ correct equation

(b) Determine the value of the constant b and the value of the constant c. (3 marks)

Solution
$f(x) = (x+2)^2 - 1$ $= x^2 + 4x + 4 - 1$ $b = 4$ $c = 3$
Specific behaviours
✓ writes $f(x)$ in squared form ✓ value of b ✓ value of c

(c) The graph of $y = f(x)$ is translated 3 units to the right and 5 units upwards. Determine the equation of the resulting curve. (2 marks)

Solution
New turning point at $(-2+3, -1+5) = (1, 4)$. Equation is $y = (x-1)^2 + 4 = x^2 - 2x + 5$
Specific behaviours
✓ identifies new turning point ✓ correct equation (either form)

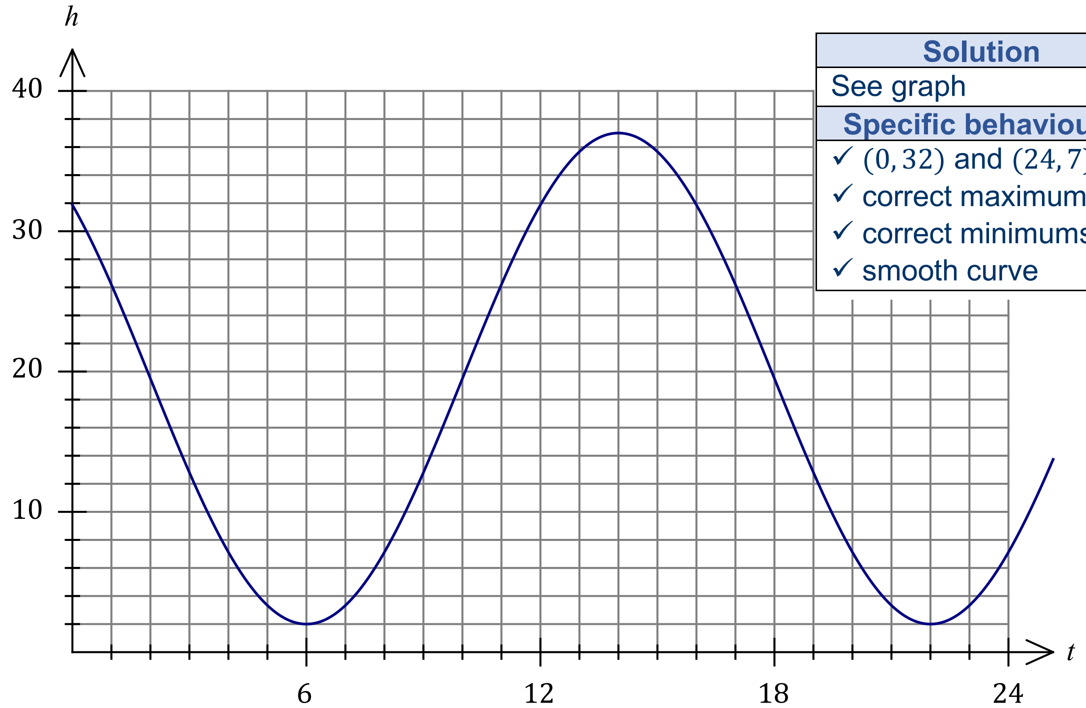
Question 13 **(8 marks)**

The height above ground level, h m, of a seat on a steadily rotating Ferris wheel t minutes after the wheel begins to move is given by $h = 19.5 + 17.5\cos\left(\frac{\pi t}{8} + \frac{\pi}{4}\right)$.

(a) Determine the initial height of the seat. (1 mark)

Solution
$h(0) = 31.87$ m
Specific behaviours
✓ correct height (accept 1 or 2 decimal places, but do not penalise if more decimal places)

(b) Graph the height of the seat against time on the axes below. (4 marks)



Solution
See graph
Specific behaviours
✓ $(0, 32)$ and $(24, 7)$
✓ correct maximum
✓ correct minimums
✓ smooth curve

(c) Determine

(i) the maximum height above ground reached by the seat. (1 mark)

Solution
$h_{MAX} = 37$ m
Specific behaviours
✓ correct height

(ii) the time taken, to the nearest second, for the seat to first reach a height of 5 m above ground level. (2 marks)

Solution
$h = 5 \Rightarrow t = 4.49$
$0.49 \times 60 = 29$
$t = 4$ m 29 s $(269$ s$)$
Specific behaviours
✓ time as decimal
✓ time to nearest second

Question 14 **(10 marks)**

(a) Express

(i) $35°$ in radians. (1 mark)

Solution
$35° \times \dfrac{\pi}{180°} = \dfrac{7\pi}{36}$ Or $0.6109^{\,r}$ or $(radians)$
Specific behaviours
✓ correct measure

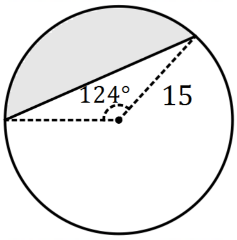
(ii) $\dfrac{11\pi}{15}$ in degrees. (1 mark)

Solution
$\dfrac{11\pi}{15} \times \dfrac{180°}{\pi} = 132°$
Specific behaviours
✓ correct measure

(b) A minor segment subtends an angle of $124°$ in a circle of radius 15 cm.

(i) Sketch a diagram to show the circle and minor segment. (1 mark)

Solution



Specific behaviours

✓ shows minor segment with given information

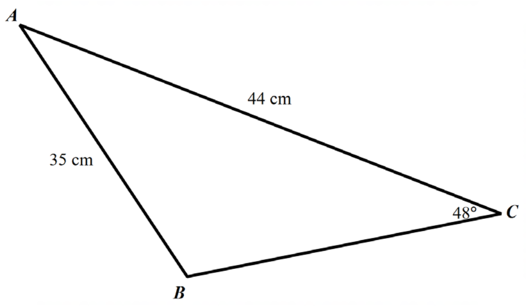
(ii) Determine the area of the minor segment. (3 marks)

Solution
$124° \times \dfrac{\pi}{180°} = \dfrac{31\pi}{45}\ (2.164^r)$
$A = \dfrac{1}{2}(15)^2\left(\dfrac{31\pi}{45} - \sin\dfrac{31\pi}{45}\right)$
$A = 150\ \text{cm}^2$
Specific behaviours
✓ converts degrees to radian measure and uses radian measure (exact or decimal value) ✓ substitutes correctly (exact or decimal value) ✓ correct area, any rounding (units not required, but preferred) **Note:** if degree is used in radian formula, then no 2nd mark

Alternative Solution
$Area\ sector = \dfrac{124}{360} \times \pi \times 15^2 = 243.473$
$Area\ triangle = \dfrac{1}{2} \times 15 \times 15 \times \sin 124 = 93.2667$
$Area\ segment = Area\ sector - area\ triangle$ $= 150\ \text{cm}^2\ (150.2\ \text{cm}^2)$
Specific behaviours
✓ calculates area of sector correctly ✓ calculates area of triangle correctly ✓ correct area, any rounding (units not required, but preferred)

(c) In triangle ABC, $AC = 44$ cm, $AB = 35$ cm and $\angle ACB = 48°$. Determine the smallest possible area of the triangle. (4 marks)

Solution



$$\frac{44}{\sin B} = \frac{35}{\sin 48°}$$

$$B = 69.1° \text{ or } 110.9°$$

For smallest area need $\angle A$ to be small as possible:

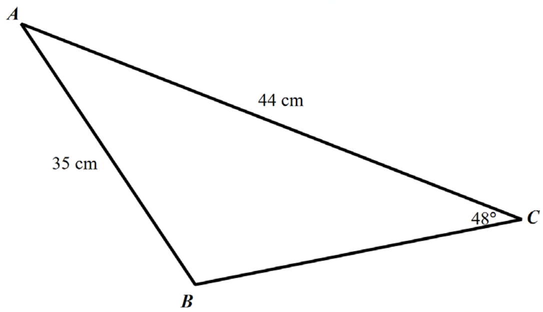
$$A = 180° - 48° - 110.9° = 21.1°$$

$$\text{Area} = \frac{1}{2}(44)(35)\sin 21.1°$$
$$= 277 \text{ cm}^2$$

Specific behaviours

✓ size of angle B
✓ size of angle A
✓ identifies correct angle for smallest size area
✓ correct area (accept rounding to 1 or 2 decimal places.)

Alternative Solution



$$35^2 = x^2 + 44^2 - 2x(44)\cos 48°$$
$$x = 16.9591 \; or \; x = 41.9243$$

For smallest area $x = 16.9591$

$$\text{Area} = \frac{1}{2}(44)(16.9591)\sin 48°$$
$$= 277.267 \text{ cm}^2$$
$$= 277.267 \text{ cm}^2$$

Note: If you around $x = 16.9591$ to $x = 17$, then
$A = 277.936 \text{ cm}^2$
$A = 278 \text{ cm}^2$, so penalise one mark for rounding too early.

Specific behaviours

✓ used the cosine rule to find third side
✓ solves for x, showing both solutions.
✓ identifies correct value for smallest size area
✓ correct area (accept rounding to 1 or 2 decimal places.)

Question 15 **(7 marks)**

Let $f(x) = \sqrt{12 - 2x} - 2$ and $g(x) = 16 + x$.

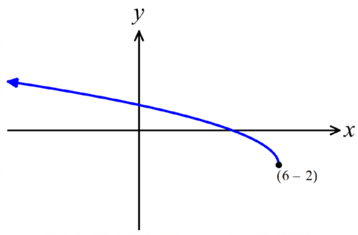
(a) Evaluate $f(-2) - g(-2)$. (2 marks)

Solution
$f(-2) - g(-2) = \sqrt{16} - 2 - (16 - 2)$ $= 2 - 14$ $= -12$
Specific behaviours
✓ evaluates f correctly ✓ correct value

(b) State the domain of $f(x)$. (2 marks)

Solution
$12 - 2x \geq 0$ $-2x \geq -12$ $x \leq 6$ Domain: $\{x \in \mathbb{R}: x \leq 6\}$
Specific behaviours
✓ correct inequality ✓ correct notation Note: penalise for incorrect notation and make a note so not penalised in question 19 as well.

Alternative Solution
Draws sketch/graph $x \leq 6$ Domain: $\{x \in \mathbb{R}: x \leq 6\}$
Specific behaviours
✓ correct inequality ✓ correct notation Note: penalise for incorrect notation and make a note so not penalised in question 19 as well.



(c) State the range of $g(x)$. (1 mark)

Solution
$y \in \mathbb{R}$
Specific behaviours
✓ correct range (symbols or words)

(d) Determine the coordinates of the point(s) of intersection of $y = f(x)$ and $y = g(x)$. (2 marks)

Solution
Using graph/CAS: $(-12, 4)$
Specific behaviours
✓ x-coordinate ✓ y-coordinate

Question 16 **(8 marks)**

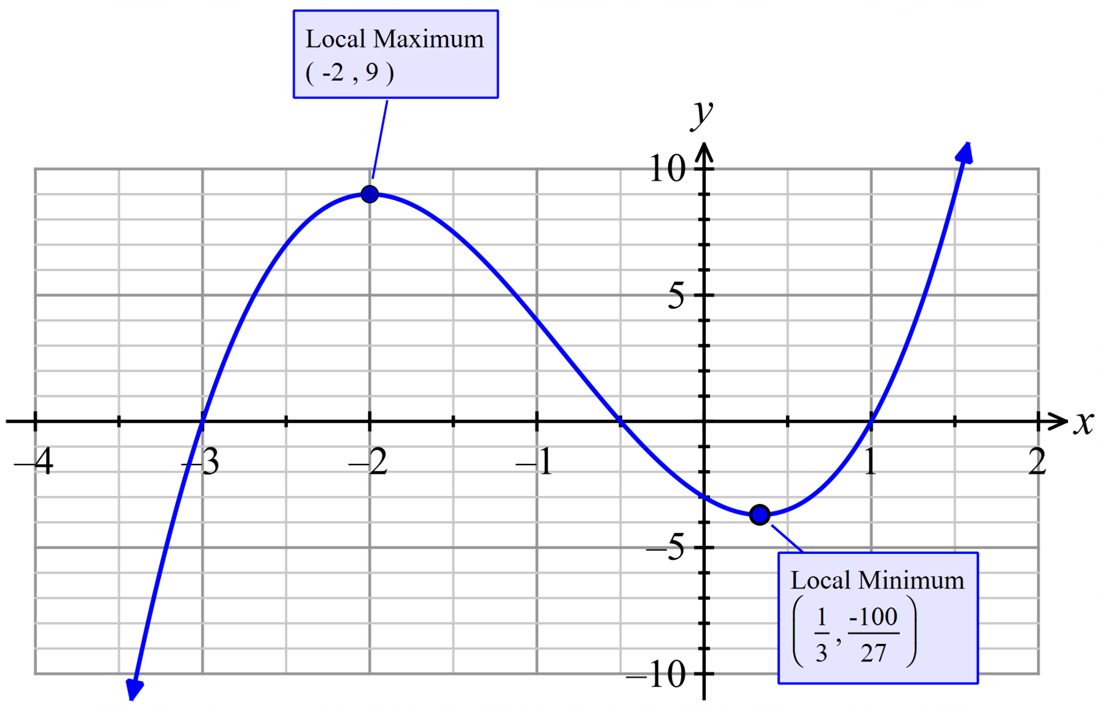
A polynomial of degree 3 passes through the points with coordinates $(0, -3), (1, 0), (-3, 0)$ and $(-0.5, 0)$.

(a) Determine the equation of the polynomial in expanded form. (4 marks)

Solution
Using roots: $y = a(x-1)(x+3)(x+0.5)$ Use 4th point: $x = 0 \Rightarrow -3 = a(-1)(3)(0.5)$ $a = 2$ Expand: $y = 2(x-1)(x+3)(x+0.5)$ $= 2x^3 + 5x^2 - 4x - 3$
Specific behaviours
✓ factored form using roots ✓ substitutes fourth point ✓ correct value of a ✓ correct expanded form

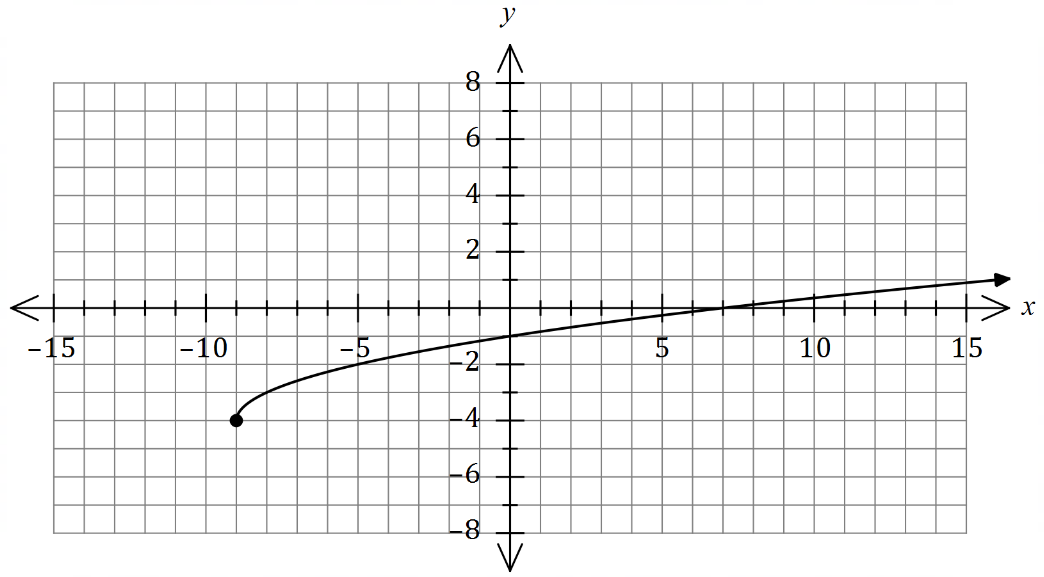
(b) Draw the graph of the polynomial on the axes below, indicating the coordinates of all turning points. (4 marks)

Solution
See graph
Specific behaviours
✓ all roots and y-intercept $(-3, 0), (0{,}5, 0), (1, 0)$ *and* $(0, -3)$ ✓ labelled maximum $(-2{,}9)$ ✓ labelled minimum $\left(\frac{1}{3}, \frac{-100}{27}\right) = (3.3, -3.7)$ ✓ smooth curve



Question 17 **(8 marks)**

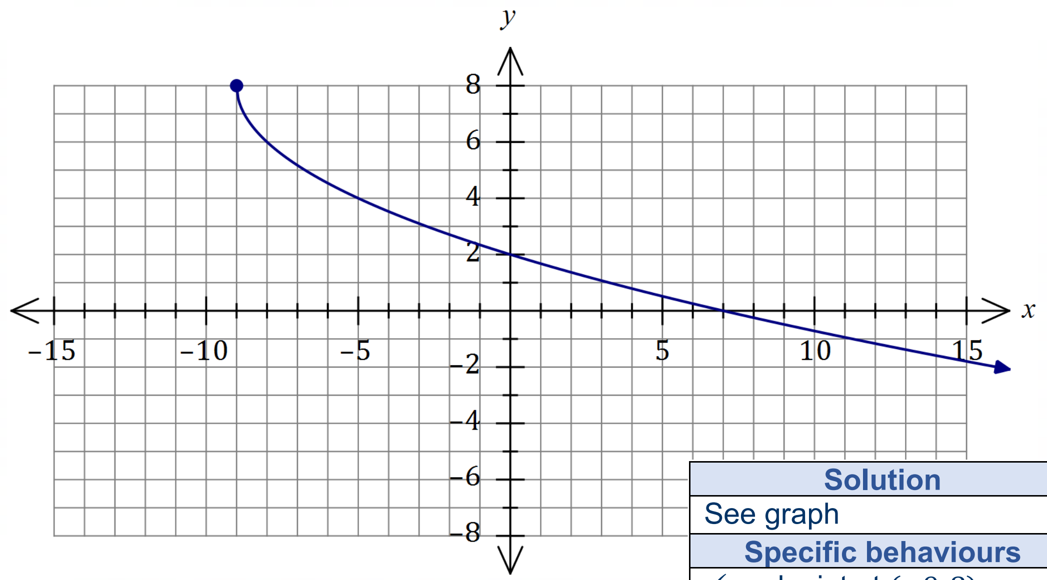
The graph of $y = f(x)$ is drawn below, where $f(x) = \sqrt{x+a} + b$.



(a) Determine the value of the constant a and the value of the constant b. (2 marks)

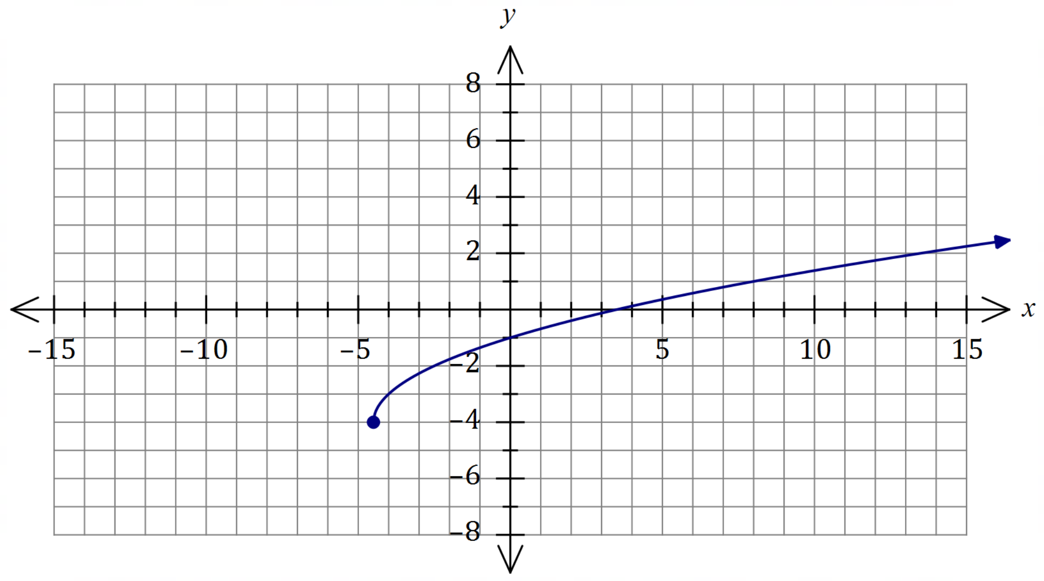
Solution
$a = 9, \qquad b = -4$
Specific behaviours
✓ value of a
✓ value of b

(b) Draw the graph of $y = -2f(x)$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ endpoint at $(-9, 8)$
✓ thru' $(0,2)$ and $(7, 0)$
✓ smooth curve

(c) Draw the graph of $y = f(2x)$ on the axes below. (3 marks)

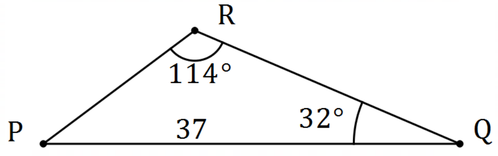


Solution
See graph
Specific behaviours
✓ endpoint at $(-4.5, -4)$ ✓ thru' $(0, -1)$ and $(3.5, 0)$ ✓ smooth curve

Question 18 **(8 marks)**

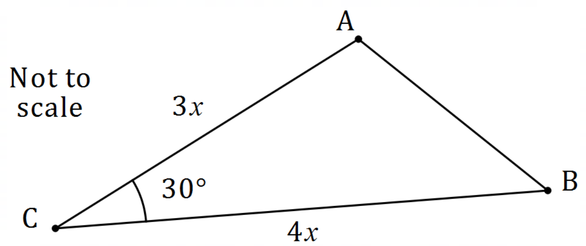
(a) Determine the area of triangle PQR when $\angle PQR = 32°$, $\angle PRQ = 114°$ and $PQ = 37$ cm.

(4 marks)

Solution

$$\frac{q}{\sin 32°} = \frac{37}{\sin 114°} \Rightarrow q = 21.46$$
$$\begin{aligned}\text{Area} &= \frac{1}{2}(37)(21.46)\sin(34°)\\ &= 222\text{ cm}^2\end{aligned}$$
Specific behaviours
✓ sketch of triangle or evidence of correct values used in rule ✓ correct use of sine rule ✓ length of second side ✓ correct area

(b) The area of triangle ABC is 75 cm^2, $\angle ACB = 30°$ and $3BC = 4AC$ as shown in the diagram. Determine the length of AB.

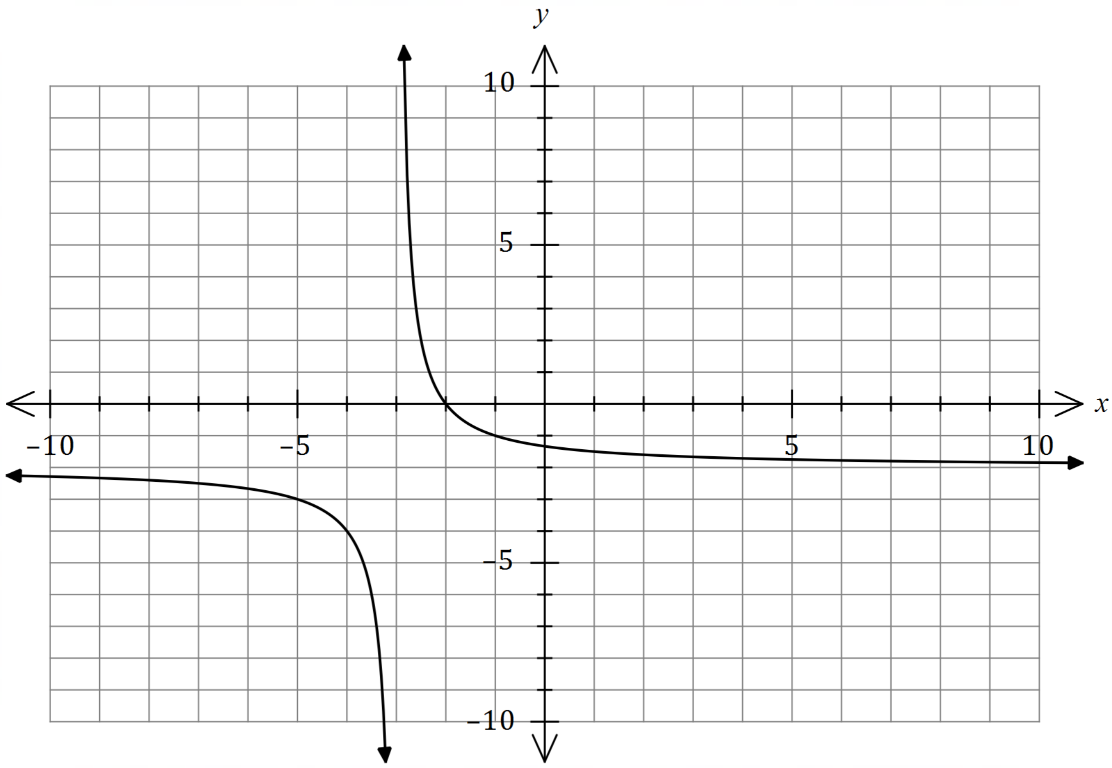
(4 marks)



Solution
$$\frac{1}{2}(4x)(3x)\sin(30°) = 75$$ $$x = 5$$
$$AB^2 = 15^2 + 20^2 - 2(15)(20)\cos(30°)$$ $$AB = 10.27\text{ cm}$$
Specific behaviours
✓ area equation ✓ value of x ✓ cosine rule ✓ length of AB

Question 19 **(5 marks)**

The graph of $y = f(x)$ is shown, where $f(x) = \dfrac{a}{x+b} + c$ and a, b and c are constants.



(a) Determine the value of a, the value of b and the value of c. (3 marks)

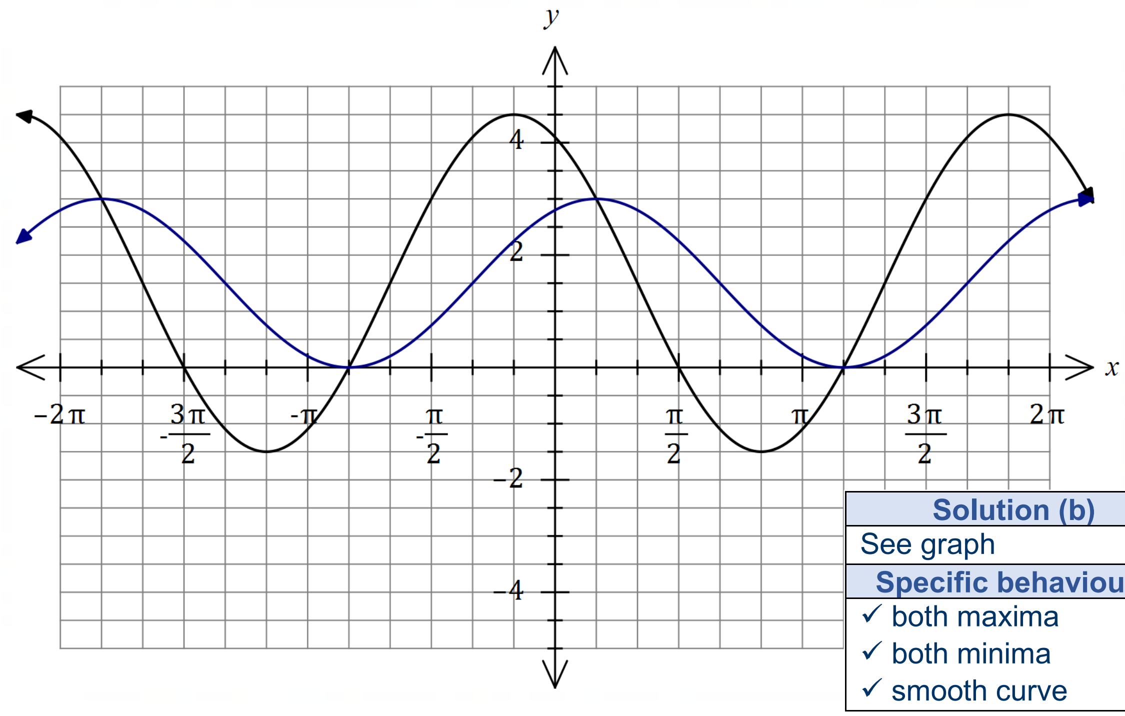
Solution
$b = 3, \quad c = -2$ $(-2, 0) \Rightarrow 0 = \dfrac{a}{-2+3} - 2$ $a = 2$
Specific behaviours
✓ value of a ✓ value of b ✓ value of c

(b) State the domain and range of $f(x)$. (2 marks)

Solution
Domain: $\{x \in \mathbb{R}: x \neq -3\}$ Range: $\{y \in \mathbb{R}: y \neq -2\}$
Specific behaviours
✓ correct domain ✓ correct range Note: must have curly brackets and should have $x \in \mathbb{R}$ and $y \in \mathbb{R}$ and only penalise once.

Question 20 **(8 marks)**

The graph of $y = a + b\cos(x + c)$ is drawn below, where a, b and c are positive constants.



Solution (b)
See graph
Specific behaviours
✓ both maxima ✓ both minima ✓ smooth curve

(a) Determine the value of a, the value of b and the value of c, where $c < \pi$. (3 marks)

Solution
$a = 1.5, \quad b = 3, \quad c = \dfrac{\pi}{6}$
Specific behaviours
✓ value of a ✓ value of b ✓ value of c

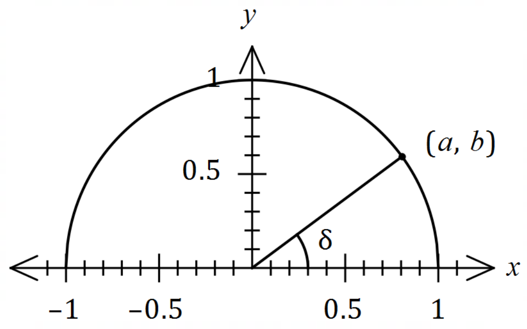
(b) On the same axes, draw the graph of $y = a + \dfrac{b}{2}\cos(x - c)$. (3 marks)

(c) Solve $b\cos(x + c) = \dfrac{b}{2}\cos(x - c)$ for $-\pi \le x \le \pi$. (2 marks)

Solution
Using intersection of graphs: $x = -\dfrac{5\pi}{6}, \quad x = \dfrac{\pi}{6}$ OR $x = -2.6180, \quad x = 0.5236$
Specific behaviours
✓ a correct solution, anywhere ✓ two solutions as given (as exact values or decimals; decimals to at least 2 decimal places)

Question 21 **(5 marks)**

Consider part of the unit circle shown below.



Determine, in terms of a and / or b, an expression for each of the following:

(a) $\cos\delta°$. (1 mark)

Solution
$\cos\delta° = a$
Specific behaviours
✓ correct expression

(b) $\sin(180° - \delta°)$. (1 mark)

Solution
$\sin(180° - \delta°) = \sin\delta°$ $= b$
Specific behaviours
✓ correct expression

(c) $\cos(\delta° - 90°)$. (1 mark)

Solution
$\cos(\delta° - 90°) = \sin\gamma$ $= b$
Specific behaviours
✓ correct expression

(d) $\sin(2\delta°)$. (2 marks)

Solution
$\sin(2\gamma°) = \sin(\delta° + \delta°)$ $= \sin\delta°\cos\delta° + \cos\delta°\sin\delta°$ $= (b)(a) + (a)(b)$ $= 2ab$
Specific behaviours
✓ halves angle and uses sum identity ✓ correct expression

Question 22 **(5 marks)**

(a) A box of chocolates contains nine different chocolates. Determine the number of different selections that can be made when

(i) two chocolates are chosen from the box. (1 mark)

Solution
$\binom{9}{2} = 36$
Specific behaviours
✓ correct value

(ii) one, two or three chocolates are chosen from the box. (2 marks)

Solution
$\binom{9}{1} + \binom{9}{2} + \binom{9}{3} = 9 + 36 + 84$ $= 129$
Specific behaviours
✓ correct selects 1, 2 and 3 chocolates ✓ correct total value

(b) (i) Determine the value of r if $\binom{9}{3} = \binom{9}{r}$ (1 mark)

Solution
$\binom{9}{3} = \binom{9}{r}$ then $r = 6$
Specific behaviours
✓ correct value

(ii) Explain how Pascal's triangle can be used to find the solution to part (i). (1 mark)

Solution
The values in Pascal's triangle are symmetrical so $\binom{9}{6}$ is the same value as $\binom{9}{3}$, just starting from the other side of Pascal's triangle.
Specific behaviours
✓ valid explanation

End of questions

Supplementary page

Question number: _________