TRINITY COLLEGE

Mathematics Methods Units 3/4
Test 4   2017

Section 1   Calculator Free

Calculus involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME   _____

DATE: Thursday 20 July   **TIME:** 25 minutes   **MARKS**: 26

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
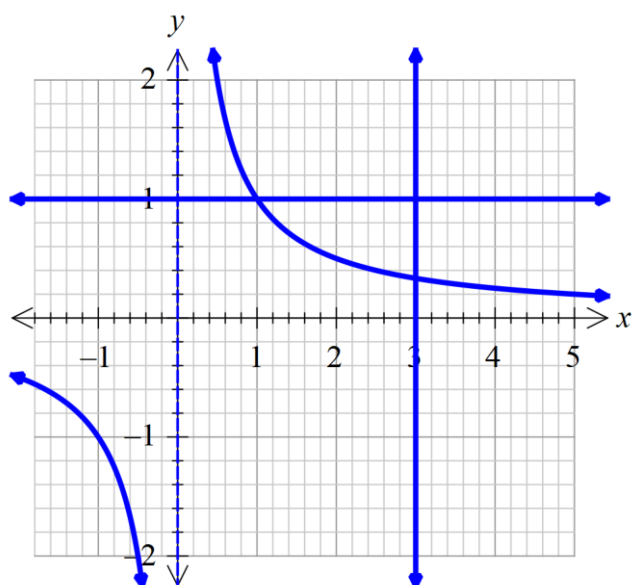
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.   (4 marks)

Determine the equation of the tangent to the curve $y = x^2 \ln x^2$ at the point $x = 1$

2. (7 marks)

(a) (i) Determine the coordinates of the point of intersection between the curve $y = \dfrac{1}{x}$ and the line $y = 1$ [1]

(ii) Hence or otherwise, determine the exact area of the region trapped between the curve $y = \dfrac{1}{x}$, the line $x = 3$, the *x-axis*, the *y-axis* and the line $y = 1$. [4]

(b) $\displaystyle\int \dfrac{5x}{x^2 - 1}\, dx$ [2]

3. (11 marks)

   (a) Differentiate each of the following with respect to x.

   (i) $y = \dfrac{\ln x}{x^3}$ [3]

   (ii) $y = (x + \ln \sin x)^4$ [3]

   (iii) $y = \ln \sqrt{\dfrac{e^{5x}}{x^2 - 1}}$ [3]

   (b) If $f(x) = \displaystyle\int_1^x \ln \sqrt{t} \; dt$, determine $f'(e^2)$ [2]

4. (4 marks)

A continuous random variable, X, has a probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{5}e^{-\frac{x}{5}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The median of X is m. Determine the exact value of m.

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Mathematics Methods Units 3/4
Test 4   2017

Section 2   Calculator Assumed
Calculus involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME _____

DATE: Thursday 20 July   **TIME:** 25 minutes   **MARKS**: 29

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this
                         assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5.   (6 marks)

Let $x = \log_n 5$ and $y = \log_n 4$.

(a)   Write $x - \dfrac{y}{2}$ as a single logarithmic term.   [2]

(b)   Express the following in terms of *x* and/or *y*.

(i) $\log_n 100$ [2]

(ii) $\log_5 4$ [2]

6. (15 marks)

The time (in minutes) that it takes a student to complete a puzzle is a random variable X with a probability density function given by:

$$f(x) = \begin{cases} \dfrac{20x - x^2}{1125} & 5 \le x \le 20 \\ 0 & elsewhere \end{cases}$$

(a) Determine the probability that it takes exactly 6 minutes to complete the puzzle. [1]

(b) Determine the probability that it takes less than 10 minutes to complete the puzzle. [2]

(c) Determine the probability that it takes between 8 and 10 minutes to complete the puzzle given that it takes less than 10 minutes. [2]

(d) Determine the expected time it takes to complete the puzzle. [2]

(e) Determine the standard deviation of the random variable X. [2]

(f)   Determine the mean and variance of $5 - 2X$. [2]

The time (in minutes) that it takes a student to complete a second more challenging puzzle is a random variable Y with a cumulative probability distribution function given by

$$F(y) = 1 - \frac{10}{y}$$

(g)   Determine the probability that it takes a student longer than 25 minutes to complete the second (more challenging) puzzle. [2]

(h)   Determine the quickest possible time for solving this second (more challenging) puzzle. [2]

7.   (8 marks)

The decibel scale for sound, measured in decibels (dB), is defined as $D = 20\log_{10}\left(\dfrac{P}{P_0}\right)$, where P is the pressure of the sound being measured and P_0 is a fixed reference pressure.

(a)   Complete the table below, giving values rounded to one decimal place.   [2]

P	$0.5P_0$	P_0	$2P_0$	$4P_0$	$8P_0$
D	-6.0				

(b)   Sketch the graph of $D = 20\log_{10}\left(\dfrac{P}{P_0}\right)$ on the axes below labelling all key features   [3]
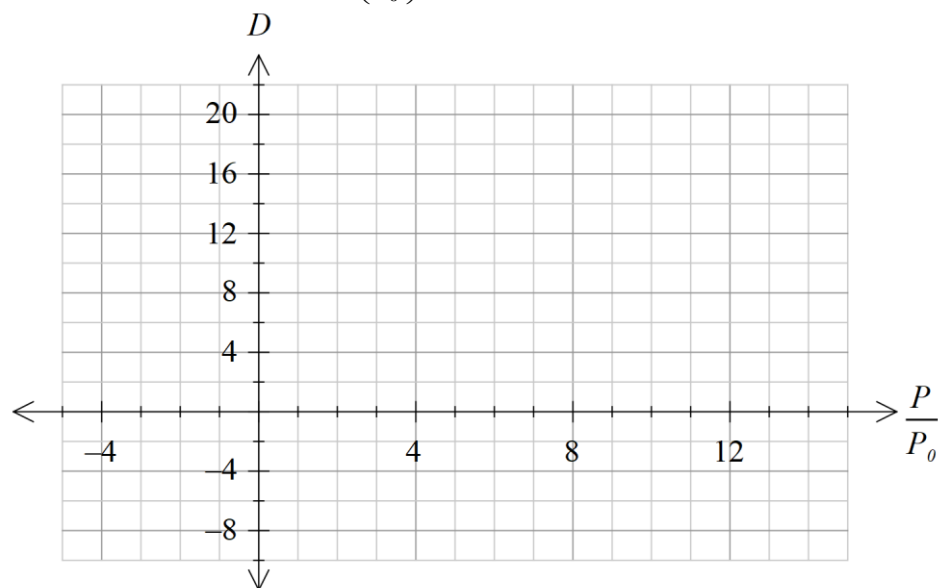
(c)   When measured at similar distances, the sound produced by a dishwashing machine measures 47 dB, while that produced by lawn mower measures 96 dB. How many times greater is the sound pressure of the mower to that of the dishwasher?   [3]