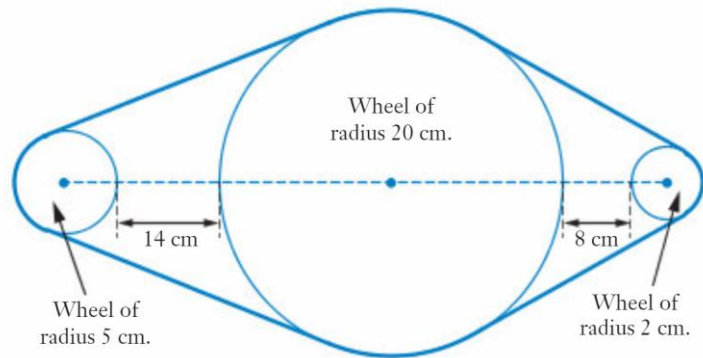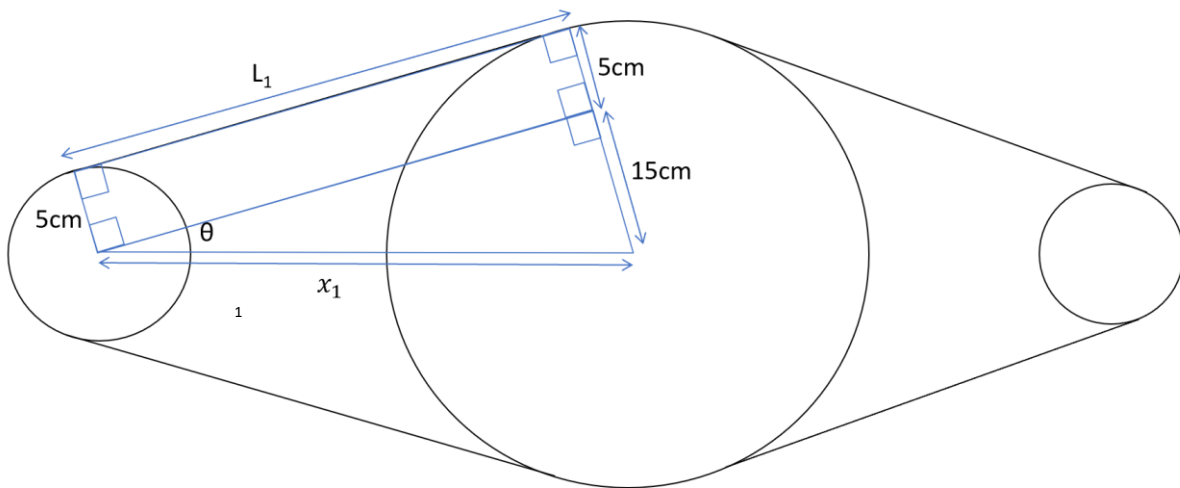## Question:

30 Find, to the nearest centimetre, the length of the continuous belt passing around the three wheels as shown in the diagram (not drawn to scale).

Wheel of radius 20 cm.

14 cm

8 cm

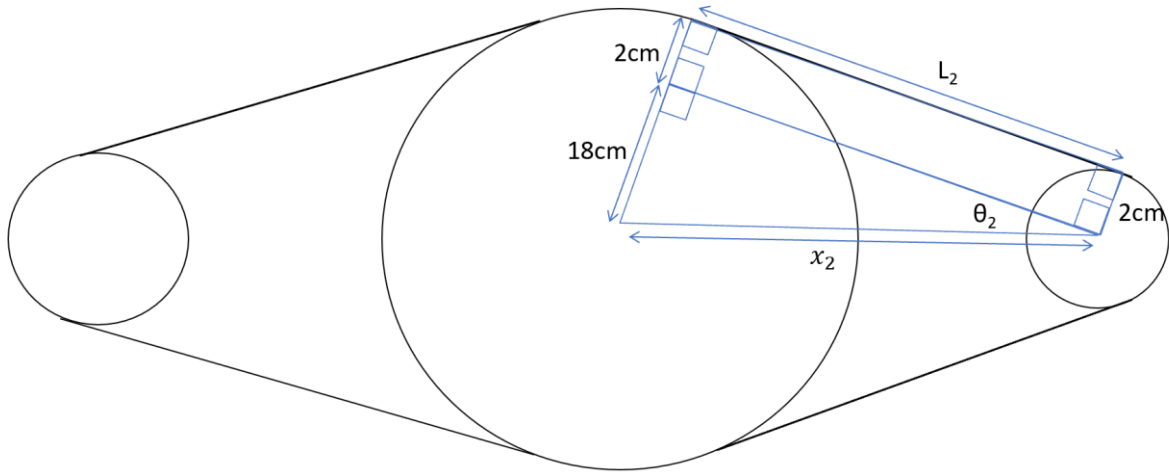Wheel of radius 5 cm.

Wheel of radius 2 cm.

Working:

$$x_1 = 20 + 14 + 5 = 39 \text{cm}$$

$$\theta_1 = \sin^{-1}\left(\frac{15}{39}\right) = 0.395 \text{ rad}$$

$$\tan 0.395 = \frac{15}{L_1} \rightarrow L_1 = \frac{15}{\tan 0.395} = 36 \text{cm}$$

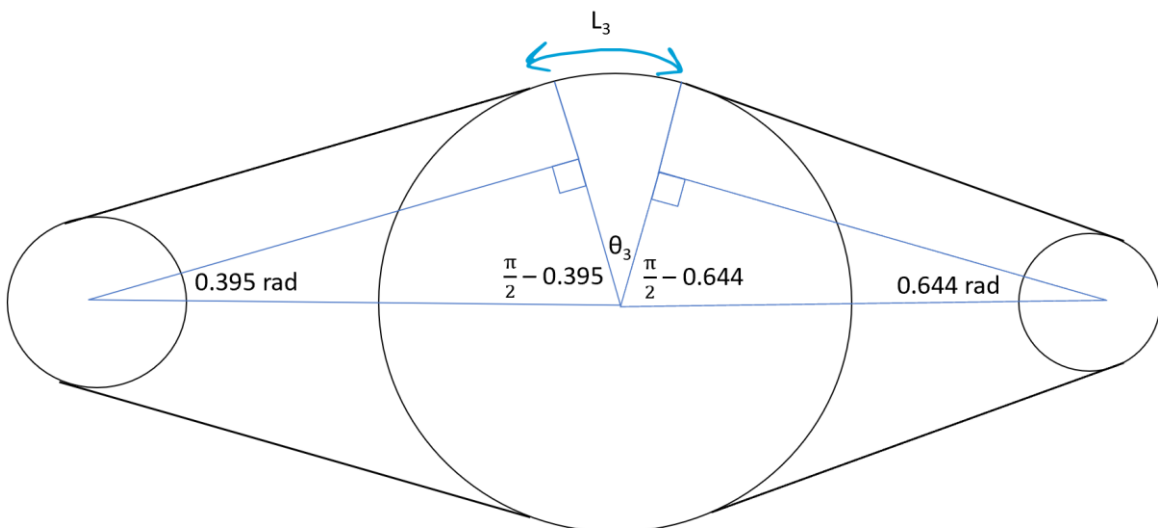$$L_{\text{left}} = 2 \times 36 = 72 \text{cm}$$

$$x_2 = 20 + 8 + 2 = 30\text{cm}$$

$$\theta_2 = \sin^{-1}\left(\frac{18}{30}\right) = 0.644 \text{ rad}$$

$$\tan 0.644 = \frac{18}{L_2} \rightarrow L_2 = \frac{18}{\tan 0.644} = 24$$

$$L_{\text{right}} = 2 \times 24 = 48\text{cm}$$

$$L_{\text{left}} + L_{\text{right}} = 72 + 48 = 120\text{cm}$$

All angles in each right triangle add up to π rad
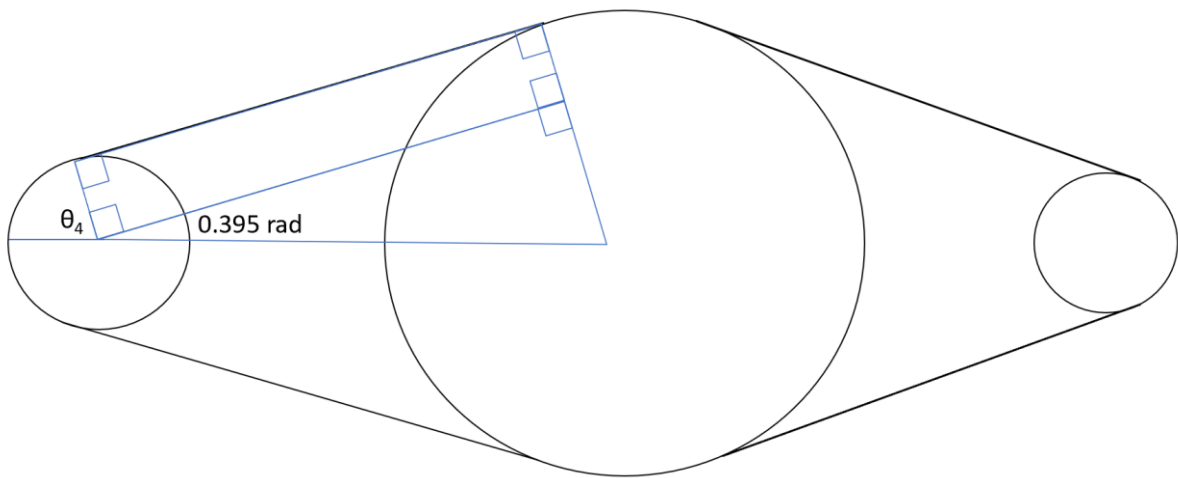
$$\text{Left} - \text{hand} - \text{side triangle: } 0.395 + \frac{\pi}{2} + \left(\frac{\pi}{2} - 0.395\right) = \pi \text{ rad}$$

$$\text{Right} - \text{hand} - \text{side triangle: } 0.644 + \frac{\pi}{2} + \left(\frac{\pi}{2} - 0.644\right) = \pi \text{ rad}$$

$$\theta_3 = \pi - \left(\frac{\pi}{2} - 0.395\right) - \left(\frac{\pi}{2} - 0.644\right) = 1.038 \text{ rad}$$
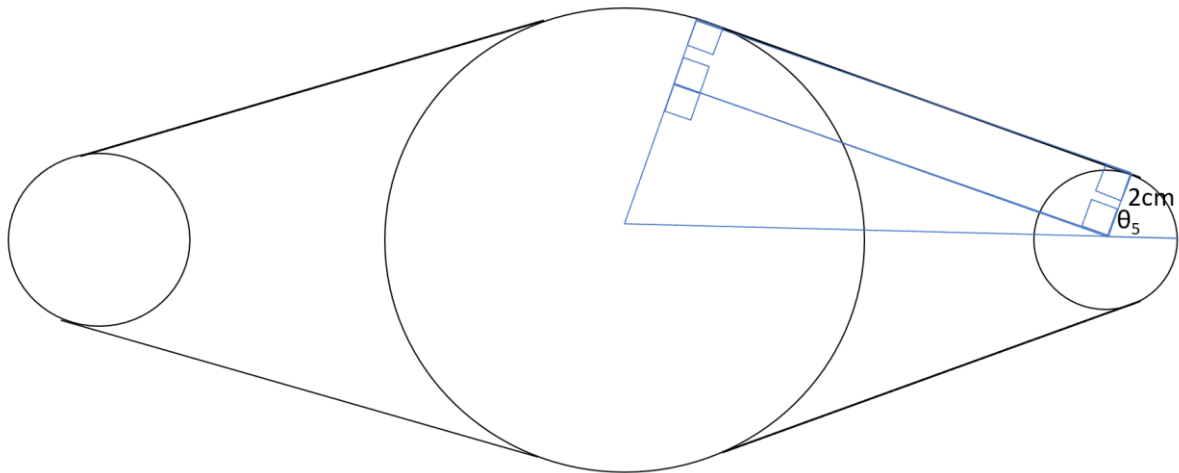
$$L_3 = r\theta = 20 \times 1.038 = 20.766\text{cm}$$

$$L_{\text{top+bottom}} = 2 \times 20.766 = 41.532\text{cm}$$

0.395 rad

θ_4

$$\theta_4 = \pi - 0.395 - \frac{\pi}{2} = 1.176 \text{ rad}$$

$$\theta_{\text{left}} = 2 \times 1.176 = 2.352 \text{ rad}$$

$$L_4 = r\theta = 5 \times 2.352 = 11.760\text{cm}$$

2cm

θ_5

$$\theta_5 = \pi - 0.644 - \frac{\pi}{2} = 0.927 \text{ rad}$$

$$\theta_{right} = 2 \times 0.927 = 1.855 \text{ rad}$$

$$L_5 = r\theta = 2 \times 1.855 = 3.709$$

$$L_{total} = L_{left} + L_{right} + L_{top+bottom} + L_4 + L_5$$

$$= 72 + 48 + 41.532 + 11.760 + 3.709$$

$$= 177.0009221 \text{cm} \approx 177 \text{cm}$$

Answer:

30 177 cm