

**Mathematics Methods Year 12**  
**Test 2      2016**

Section 1    Calculator Free  
**Exponential Function, Fundamental Theorem**

**STUDENT'S NAME** \_\_\_\_\_

*SOLUTIONS*

**DATE:** Friday 1<sup>st</sup> April

**TIME:** 33 minutes

**MARKS:** 33

**INSTRUCTIONS:**

Standard Items:      Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.      (7 marks)

Determine  $\frac{dy}{dx}$  for each of the following. Do not simplify.

(a)       $y = e^{\sqrt{x}}$

$$y = e^{x^{\frac{1}{2}}}$$
$$y' = \frac{1}{2} x^{-\frac{1}{2}} e^{x^{\frac{1}{2}}}$$

[2]

(b)       $y = \sin(e^{2x})$

$$y' = \cos e^{2x} \cdot 2e^{2x}$$

[2]

(c)       $y = (\cos x) e^{\cos x}$

$$y' = -\sin x e^{\cos x} + \cos x (-\sin x) e^{\cos x}$$

[3]

2. (10 marks)

(a) Evaluate exactly  $\int_0^2 x e^{4-x^2} dx$

[4]

$$\begin{aligned} &= -\frac{1}{2} \int_0^2 -2x e^{4-x^2} dx \\ &= -\frac{1}{2} \left[ e^{4-x^2} \right]_0^2 \\ &= -\frac{1}{2} \left[ e^0 - e^4 \right] \\ &= -\frac{1}{2} (1 - e^4) \end{aligned}$$

$$\text{or } \frac{e^4 - 1}{2}$$

(b) Determine  $\int \frac{4e^{2x} + 4x}{(e^{2x} + x^2)^3} dx$

[3]

$$\begin{aligned} &= \int 2(2e^{2x} + 2x)(e^{2x} + x^2)^{-3} dx \\ &= 2 \underbrace{(e^{2x} - x^2)^{-2}}_{-2} + C \\ &= - (e^{-2x} - x^2)^{-2} + C \end{aligned}$$

(c) Determine  $\int_{\pi}^{x^2} \left( \frac{d}{dt} e^{e^{-t}} \right) dt$

[3]

$$\begin{aligned} &= \int_{\pi}^{x^2} (-e^{-t} e^{e^{-t}}) dt \\ &= \left[ e e^{-t} \right]_{\pi}^{x^2} \\ &= e e^{-x^2} - e e^{-\pi} \end{aligned}$$

3. (4 marks)

Given  $y = \frac{e^x}{3+e^x}$

(a) determine  $\frac{dy}{dx}$

$$y' = \frac{e^x(3+e^x) - e^x \cdot e^x}{(3+e^x)^2}$$
$$= \frac{3e^x + e^{2x} - e^{2x}}{(3+e^x)^2}$$

[2]

(b) explain why  $\frac{dy}{dx} \neq 0$

$$3e^x > 0$$
$$(3+e^x)^2 > 0$$
$$\therefore \frac{dy}{dx} \neq 0$$

[2]

4. (5 marks)

(a) Determine  $\frac{dy}{dx}$  given  $y = xe^x$

$$y = e^x + xe^x$$

[2]

(b) Hence determine  $\int xe^x dx$

[3]

$$\int (e^x + xe^x) dx = xe^x$$

$$\int e^x dx + \int xe^x dx = xe^x$$

$$\int xe^x dx = xe^x - \int e^x dx$$
$$= xe^x - e^x + c$$

5. (4 marks)

Given  $y = \int_{-3}^x \frac{t^2 - 2}{\sqrt{t}} dt$ , use the incremental formula  $\delta y \approx \frac{dy}{dx} \times \delta x$  to determine the change in  $y$  if  $x$  changes from 4 to 4.02.

$$y' = \frac{d}{dx} \int_{-3}^x \frac{t^2 - 2}{\sqrt{t}} dt$$
$$= \frac{x^2 - 2}{\sqrt{x}}$$

$$\delta y \approx \frac{x^2 - 2}{\sqrt{x}} \times \delta x$$

$$\delta x = 0.02$$

$$x = 4$$

$$\approx \frac{16 - 2}{\sqrt{4}} \times 0.02$$

$$= 7 \times 0.02$$

$$= 0.14$$

**Mathematics Methods Year 12**  
**Test 2      2016**

Section 2    Calculator Assumed  
**Exponential Function, Fundamental Theorem**

**STUDENT'S NAME** \_\_\_\_\_

**DATE:** Friday 1<sup>st</sup> April

**TIME:** 20 minutes

**MARKS:** 21

**INSTRUCTIONS:**

Standard Items:      Pens, pencils, drawing templates, eraser

Special Items:      Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

6.      (4 marks)

Determine the value of  $x$  for which  $\int_x^{-1} (1-t^2) dt$  has a relative minimum. Justify it is a minimum value. [4]

$$\begin{aligned}
 & \frac{d}{dx} \int_x^{-1} (1-t^2) dt \\
 &= - \frac{d}{dx} \int_{-1}^x (1-t^2) dt \\
 &= - (1-x^2)
 \end{aligned}$$

$$\begin{aligned}
 - (1-x^2) &= 0 \\
 1-x^2 &= 0 \\
 x &= \pm 1
 \end{aligned}$$

$$\frac{d^2}{dx^2} = 2x$$

$$x=1 \quad \frac{d^2}{dx^2} = 2 \quad \therefore \text{MIN}$$

$$x=-1 \quad \frac{d^2}{dx^2} = -2 \quad \therefore \text{MAX}$$

7. (4 marks)

Sugar is being dissolved in a solution at a rate given by  $\frac{dS}{dt} = -20e^{-0.1t}$  where  $S$  is the amount, in grams, of undissolved sugar after  $t$  seconds.

(a) how much sugar is initially in the solution

[2]

$$\frac{dS}{dt} = -0.1(200)e^{-0.1t}$$

$$\therefore S_0 = 200$$

(b) how long does it take for half the sugar to dissolve.

[2]

$$100 = 200e^{-0.1t}$$

$$t = 6.9 \text{ secs}$$

8. (5 marks)

A particular rock is dropped into a swimming pool and it sinks vertically to the bottom. Due to water resistance, the rock does not have a constant velocity on the way to the bottom. Its velocity,  $v$  centimetres per second,  $t$  seconds after it hits the surface of the water is given by

$$v = 8(2 - e^{-0.8t}) \quad \text{for } 0 \leq t \leq 7$$

- (a) What is the initial velocity of the rock in the water? [1]

$$\begin{aligned} v_0 &= 8(2 - e^0) \\ &= 8 \end{aligned}$$

- (b) What is the acceleration of the rock after 4 seconds? [2]

$$\begin{aligned} v &= 16 - 8e^{-0.8t} \\ a &= 6.4e^{-0.8t} \end{aligned}$$

$$(t=4) \quad = 0.26 \text{ m/s}$$

- (c) Terminal velocity is an expression used to describe the velocity that is approached but never exceeded. Determine the terminal velocity reached by the rock in the water. [2]

$$16 \text{ m/sec.}$$

9. (8 marks)

The amount  $A$  of a drug (in milligrams) in the bloodstream will decline at a rate proportional to the current amount. That is  $\frac{dA}{dt} = -\left(\frac{1}{k}\right)A$ .

where  $k$  hours is a constant called the elimination time and time  $t$  is measured in hours.

- (a) Write down the formula for  $A(t)$ , the amount of the drug in the bloodstream after  $t$  hours, in terms of  $t$ ,  $k$  and the initial amount  $A_0$ . [2]

$$A = A_0 e^{-\frac{t}{k}}$$

- (b) What proportion of the drug remains in the bloodstream after  $k$  hours? [3]

$$\begin{aligned} A &= A_0 e^{-\frac{k}{k}} \\ &= A_0 e^{-1} \\ &= \frac{A_0}{e} \end{aligned}$$

$$\text{PROPORTION LEFT IS } \frac{1}{e} \quad \text{OR } 0.368$$

The drug sodium pentobarbitol can be used to tranquilize animals. A dog is tranquilized if its bloodstream contains at least 45 milligrams of the drug for each kilogram of the dog's weight. The elimination time for the drug is 6 hours.

- (c) What single dose of this drug should be given in order to tranquilize a 12 kilogram dog for 1 hour? [3]

$$540 = A_0 e^{-\frac{1}{6}}$$

$$A_0 = 638 \text{ milligrams}$$