Chapter 13 – Trigonometric ratios and applications

Solutions to Exercise 13A

1 **a** A and C are congruent.
The two side lengths and the angle included are equal (SAS)

b A, B and C are all congruent. The angles are all the same and the triangles have identical side lengths. (AAS and SSS)

c A and B are congruent. The side lengths are all the same (SSS).

2 **a**
$$\begin{aligned}\frac{x}{5} &= \cos 35^\circ\\ x &= 5 \times 0.8191\\ &= 4.10 \text{ cm}\end{aligned}$$

b
$$\begin{aligned}\frac{x}{10} &= \sin 45^\circ\\ x &= 10 \times 0.0871\\ &= 0.87 \text{ cm}\end{aligned}$$

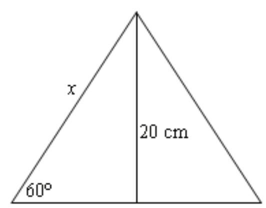
c
$$\begin{aligned}\frac{x}{8} &= \tan 20.16^\circ\\ x &= 8 \times 0.3671\\ &= 2.94 \text{ cm}\end{aligned}$$

d
$$\begin{aligned}\frac{x}{7} &= \tan 30^\circ 15'\\ x &= 7 \times 0.9661\\ &= 4.08 \text{ cm}\end{aligned}$$

e
$$\begin{aligned}\tan x^\circ &= \frac{10}{15}\\ &= 0.666\\ x &= 33.69^\circ\end{aligned}$$

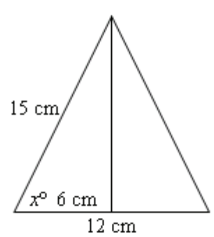
f
$$\begin{aligned}\frac{10}{x} &= \tan 40^\circ\\ 10 &= x \times 0.8390\\ x &= \frac{10}{0.8390}\\ &= 11.92 \text{ cm}\end{aligned}$$

3



$$\begin{aligned}\frac{20}{x} &= \sin 60^\circ\\ 20 &= x \times \frac{\sqrt{3}}{2}\\ x &= \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} \text{ cm}\end{aligned}$$

4



$$\begin{aligned}\cos x^\circ &= \frac{6}{15} = 0.4\\ x^\circ &= 66.42^\circ\end{aligned}$$

The third angle $= 180^\circ - 2 \times 66.42^\circ$
$= 47.16^\circ$

5 $\dfrac{h}{20} = \tan 49^\circ$

$x = 20 \times 1.1503$

≈ 23 m

6 a $\sin \angle ACB = \dfrac{1}{6}$

$\angle ACB = 9.59^\circ$

b $BC^2 = 6^2 - 1^2 = 35$

$BC = \sqrt{35}$ m

$= 5.92$ m

7 a $\cos\theta = \dfrac{10}{20} = 0.5$

$\theta = 60^\circ$

b $\dfrac{PQ}{20} = \sin 60^\circ$

$PQ = 20 \times 0.866$

$= 17.32$ m

8 a $\dfrac{3}{L} = \sin 26^\circ$

where L m is the length if the ladder

$3 = L \times 0.4383$

$L = \dfrac{3}{0.4383}$

$= 6.84$ m

b $\dfrac{3}{h} = \tan 26^\circ$

where h m is the height above the ground.

$3 = h \times 0.4877$

$h = \dfrac{3}{0.4877}$

$= 6.15$ m

9 $\sin\theta = \dfrac{13}{60} = 0.21666\ldots$

$\theta = 12.51^\circ$

10 $\dfrac{h}{200} = \sin 66^\circ$

$x = 200 \times 0.9135$

$= 182.7$ m

11 $\dfrac{400}{d} = \sin 16^\circ$

$400 = d \times 0.2756$

$d = \dfrac{400}{0.2756}$

$= 1451$ m

12 Since the diagonals are equal in length, the rhombus must be a square.

a $AC^2 = BC^2 + BA^2 = 2BC^2$

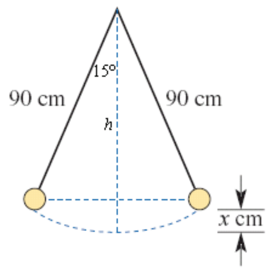
$100 = 2BC^2$

$BC^2 = 50$

$BC = \sqrt{50} = 5\sqrt{2}$ cm

b As the rhombus is a square, $\angle ABC = 90^\circ$.

13 Find the vertical height, h cm.



$$\frac{h}{90} = \cos 15^\circ$$
$$h = 90 \times 0.9659$$
$$h = 86.93 \text{ cm}$$
$$x = 90 - 86.93 = 3.07 \text{ cm}$$

14 $$\frac{15}{\left(\frac{L}{2}\right)} = \sin 52.5^\circ$$
$$15 = \frac{L}{2} \times 0.7933$$
$$L = \frac{30}{0.7933}$$
$$= 37.8 \text{ cm}$$

15 $$\frac{w}{50} = \tan 32^\circ$$
$$w = 50 \times 0.6248$$
$$= 31.24 \text{ cm}$$

16 $$h^2 + 1.7^2 = 4.7^2$$
$$h^2 = 4.7^2 - 1.7^2$$
$$= 19.2$$
$$h = 4.38 \text{ m}$$

17 $$\frac{50}{d} = \sin 60^\circ$$
$$50 = d \times 0.866$$
$$d = \frac{50}{0.866}$$
$$= 57.74 \text{ m}$$

18 Let length of the flagpole be l
$$\sin 60^\circ = \frac{l}{l+2}$$
$$\frac{\sqrt{3}}{2} = \frac{l}{l+2}$$
$$(l+2)\frac{\sqrt{3}}{2} = l$$
$$\left(\frac{\sqrt{3}}{2} - 1\right)l = -\sqrt{3}$$
$$l = \frac{\sqrt{3}}{\frac{-\sqrt{3}}{2} - 1}$$
$$l = \frac{2\sqrt{3}}{2 - \sqrt{3}}$$

19 Perimeter $=10 \Rightarrow x + h + opp = 10$
$$\cos 30^\circ = \frac{x}{h}$$
$$h = \frac{x}{\cos 30^\circ} = \frac{x}{\frac{\sqrt{3}}{2}} = \frac{2x}{\sqrt{3}}$$
$$\tan 30^\circ = \frac{opp}{x}$$
$$opp = x \tan 30^\circ = \frac{1}{\sqrt{3}}x$$
$$x + \frac{x}{\cos 30^\circ} + x \tan 30^\circ = 10$$
$$x + \frac{2x}{\sqrt{3}} + \frac{1}{\sqrt{3}}x = 10$$
$$(\sqrt{3} + 1)x = 10$$
$$x = \frac{10}{\sqrt{3} + 1} = 5(\sqrt{3} - 1)$$
$$x \approx 3.66$$

Solutions to Exercise 13B

1 a
$$\frac{x}{\sin 50^\circ} = \frac{10}{\sin 70^\circ}$$
$$x = \frac{10 \times \sin 50^\circ}{\sin 70^\circ}$$
$$= 8.15 \text{ cm}$$

b
$$\frac{y}{\sin 37^\circ} = \frac{6}{\sin 65^\circ}$$
$$x = \frac{6 \times \sin 37^\circ}{\sin 65^\circ}$$
$$= 3.98 \text{ cm}$$

c
$$\frac{x}{\sin 100^\circ} = \frac{5.6}{\sin 28^\circ}$$
$$x = \frac{5.6 \times \sin 100^\circ}{\sin 28^\circ}$$
$$= 11.75 \text{ cm}$$

d
$$x = 180^\circ - 38^\circ - 90^\circ$$
$$= 52^\circ$$
$$\frac{x}{\sin 52^\circ} = \frac{12}{\sin 90^\circ}$$
$$x = \frac{12 \times \sin 52^\circ}{\sin 90^\circ}$$
$$= 9.46 \text{ cm}$$

2 a
$$\frac{\sin\theta}{7} = \frac{\sin 72^\circ}{8}$$
$$\sin\theta = \frac{7 \times \sin 72^\circ}{8}$$
$$= 0.8321$$
$$\theta = 56.32^\circ$$
In this case θ cannot be obtuse. Since it is opposite a smaller side.

b
$$\frac{\sin\theta}{8.3} = \frac{\sin 42^\circ}{9.4}$$
$$\sin\theta = \frac{8.3 \times \sin 42^\circ}{9.4}$$
$$= 0.5908$$
$$\theta = 36.22^\circ$$
In this case θ cannot be obtuse. Since it is opposite a smaller side.

c
$$\frac{\sin\theta}{8} = \frac{\sin 108^\circ}{10}$$
$$\sin\theta = \frac{8 \times \sin 108^\circ}{10}$$
$$= 0.7608$$
$$\theta = 49.54^\circ$$
In this case θ cannot be obtuse. Since the given angle is obtuse.

d
$$\frac{\sin\theta}{9} = \frac{\sin 38^\circ}{8}$$
$$\sin\theta = \frac{9 \times \sin 38^\circ}{8}$$
$$= 0.6929$$
$$\theta = 43.84^\circ \text{ or } 180 - 43.84$$
$$= 131.16^\circ$$
$$\theta = 180 - 43.84 - 38 = 98.16^\circ$$
$$\text{or } 180 - 136.16 - 38 = 5.84^\circ$$

3 a
$$\begin{aligned} A &= 180^\circ - 59^\circ - 73^\circ \\ &= 48^\circ \\ \frac{b}{\sin 59^\circ} &= \frac{12}{\sin 48^\circ} \\ b &= \frac{12 \times \sin 59^\circ}{\sin 48^\circ} \\ &= 13.84 \text{ cm} \\ \frac{c}{\sin 73^\circ} &= \frac{12}{\sin 48^\circ} \\ c &= \frac{12 \times \sin 73^\circ}{\sin 48^\circ} \\ &= 15.44 \text{ cm} \end{aligned}$$

b
$$\begin{aligned} C &= 180^\circ - 75.3^\circ - 48.25^\circ \\ &= 56.45^\circ \\ \frac{a}{\sin 75.3^\circ} &= \frac{5.6}{\sin 48.25^\circ} \\ a &= \frac{5.6 \times \sin 75.3^\circ}{\sin 48.25^\circ} \\ &= 7.26 \text{ cm} \\ \frac{c}{\sin 56.45^\circ} &= \frac{5.6}{\sin 48.25^\circ} \\ c &= \frac{5.6 \times \sin 56.45^\circ}{\sin 48.25^\circ} \\ &= 6.26 \text{ cm} \end{aligned}$$

c
$$\begin{aligned} B &= 180^\circ - 123.2^\circ - 37^\circ \\ &= 19.8^\circ \\ \frac{b}{\sin 19.8^\circ} &= \frac{11.5}{\sin 123.2^\circ} \\ b &= \frac{11.5 \times \sin 19.8^\circ}{\sin 123.2^\circ} \\ &= 4.66 \text{ cm} \\ \frac{c}{\sin 37^\circ} &= \frac{11.5}{\sin 123.2^\circ} \\ c &= \frac{11.5 \times \sin 37^\circ}{\sin 123.2^\circ} \\ &= 8.27 \text{ cm} \end{aligned}$$

d
$$\begin{aligned} C &= 180^\circ - 23^\circ - 40^\circ \\ &= 117^\circ \\ \frac{b}{\sin 40^\circ} &= \frac{15}{\sin 23^\circ} \\ b &= \frac{15 \times \sin 40^\circ}{\sin 23^\circ} \\ &= 24.68 \text{ cm} \\ \frac{c}{\sin 117^\circ} &= \frac{15}{\sin 23^\circ} \\ c &= \frac{15 \times \sin 117^\circ}{\sin 23^\circ} \\ &= 34.21 \text{ cm} \end{aligned}$$

e
$$\begin{aligned} C &= 180^\circ - 10^\circ - 140^\circ \\ &= 30^\circ \\ \frac{a}{\sin 10^\circ} &= \frac{20}{\sin 140^\circ} \\ a &= \frac{20 \times \sin 10^\circ}{\sin 140^\circ} \\ &= 5.40 \text{ cm} \\ \frac{c}{\sin 30^\circ} &= \frac{20}{\sin 140^\circ} \\ c &= \frac{20 \times \sin 30^\circ}{\sin 140^\circ} \\ &= 15.56 \text{ cm} \end{aligned}$$

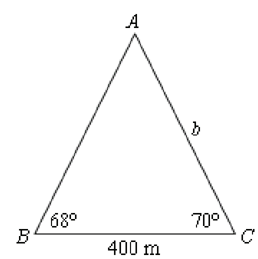
4 a
$$\begin{aligned} \frac{\sin B}{17.6} &= \frac{\sin 48.25^\circ}{15.3} \\ \sin B &= \frac{17.6 \times \sin 48.25^\circ}{15.3} \\ &= 0.8582 \\ B &= 59.12^\circ \text{ or } 180^\circ - 59.12^\circ \\ &= 120.88^\circ \end{aligned}$$

$$A = 180^\circ - 48.25^\circ - 59.12^\circ$$
$$= 72.63^\circ$$
$$\text{or } 180 - 48.25^\circ - 120.88^\circ$$
$$= 10.87^\circ$$
$$\frac{15.3}{\sin 48.25^\circ} = \frac{a}{\sin 72.63^\circ} \text{ or } \frac{a}{\sin 10.87^\circ}$$
$$a = \frac{15.3 \times \sin 72.63^\circ}{\sin 48.25^\circ}$$
$$\text{or } \frac{15.3 \times \sin 10.87^\circ}{\sin 48.25^\circ}$$
$$= 19.57 \text{ cm or } 3.87 \text{ cm}$$

b
$$\frac{\sin C}{4.56} = \frac{\sin 129^\circ}{7.89}$$
$$\sin C = \frac{4.56 \times \sin 129^\circ}{7.89}$$
$$= 0.4991$$
$$C = 26.69^\circ$$
$$A = 180^\circ - 129^\circ - 26.69^\circ$$
$$= 24.31^\circ$$
$$\frac{a}{\sin 24.31^\circ} = \frac{7.89}{\sin 129^\circ}$$
$$a = \frac{7.89 \times \sin 24.31^\circ}{\sin 129^\circ}$$
$$= 4.18 \text{ cm}$$

c
$$\frac{\sin B}{14.8} = \frac{\sin 28.35^\circ}{8.5}$$
$$\sin B = \frac{14.8 \times \sin 28.35^\circ}{85}$$
$$= 0.8268$$
$$B = 55.77^\circ \text{ or } 180 - 55.77 = 124.23^\circ$$
$$C = 180^\circ - 55.77^\circ - 28.35^\circ = 95.88^\circ$$
$$\text{or } 180^\circ - 124.23^\circ - 28.35^\circ$$
$$= 27.42^\circ$$
$$\frac{8.5}{\sin 28.35^\circ} = \frac{c}{\sin 95.88^\circ} \text{ or } \frac{c}{\sin 27.42^\circ}$$
$$c = \frac{8.5 \times \sin 95.88^\circ}{\sin 28.35^\circ}$$
$$\text{or } \frac{8.5 \times \sin 27.42^\circ}{\sin 28.35^\circ}$$
$$= 17.81 \text{ cm or } 8.24 \text{ cm}$$

5

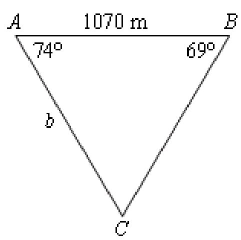


$$A = 180^\circ - 68^\circ - 70^\circ$$
$$= 42^\circ$$
$$\frac{b}{\sin 68^\circ} = \frac{400}{\sin 42^\circ}$$
$$b = \frac{400 \times \sin 68^\circ}{\sin 42^\circ}$$
$$= 554.26 \text{ m}$$

6

$$\angle APB = 46.2^\circ - 27.6^\circ$$
$$= 18.6^\circ \text{ (exterior angle property)}$$
$$\frac{a}{\sin 27.6^\circ} = \frac{34}{\sin 18.6^\circ}$$
$$PB = a = \frac{34 \times \sin 27.6^\circ}{\sin 18.6^\circ}$$
$$= 49.385 \text{ m}$$
$$\frac{h}{PB} = \sin 46.2^\circ$$
$$h = 49.385 \times 0.7217$$
$$= 35.64 \text{ m}$$

7



$$C = 180^\circ - 69^\circ - 74^\circ$$
$$= 37^\circ$$
$$\frac{b}{\sin 69^\circ} = \frac{1070}{\sin 37^\circ}$$
$$b = \frac{1070 \times \sin 69^\circ}{\sin 37^\circ}$$
$$= 1659.86 \text{ m}$$

8 a

$$X = 180^\circ - 120^\circ - 20^\circ$$
$$= 40^\circ$$
$$\frac{AX}{\sin 20^\circ} = \frac{50}{\sin 40^\circ}$$
$$= \frac{50 \times \sin 20^\circ}{\sin 40^\circ}$$
$$= 26.60 \text{ m}$$

b

$$Y = 180^\circ - 109^\circ - 32^\circ$$
$$= 39^\circ$$
$$\frac{AY}{\sin 109^\circ} = \frac{50}{\sin 39^\circ}$$
$$AY = \frac{50 \times \sin 109^\circ}{\sin 39^\circ}$$
$$= 75.12 \text{ m}$$

Solutions to Exercise 13C

1 $BC^2 = a^2$

$= b^2 + c^2 - 2bc\cos A$

$= 15^2 + 10^2 - 2 \times 15 \times 10 \times \cos 15°$

$= 325 - 300 \times \cos 15°$

$= 35.222$

$BC = 5.93$ cm

2 $\angle ABC = \angle B$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{5^2 + 8^2 - 10^2}{2 \times 5 \times 8} = -0.1375$$

$\therefore\ \angle ABC \approx 97.90°$

$\angle ACB = \angle C$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ac} = \frac{5^2 + 10^2 - 8^2}{2 \times 5 \times 10} = 0.61$$

$\therefore\ \angle ACB \approx 52.41°$

3 **a** $a^2 = b^2 + c^2 - 2bc\cos a$

$= 16^2 + 30^2 - 2 \times 16 \times 30 \times \cos 60°$

$= 1156 - 960 \times \cos 60°$

$= 676$

$a = 26$

b $b^2 = a^2 + c^2 - 2ac\cos B$

$= 14^2 + 12^2 - 2 \times 14 \times 12 \times \cos 53°$

$= 340 - 336 \times \cos 53°$

$= 137.7901$

$a \approx 11.74$

c $\angle ABC = \angle B$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{27^2 + 46^2 - 35^2}{2 \times 27 \times 46} = 0.6521$$

$\therefore\ \angle ABC \approx 49.29°$

d $b^2 = a^2 + c^2 - 2ac\cos B$

$= 17^2 + 63^2 - 2 \times 17 \times 63 \times \cos 120°$

$= 4258 - 2142 \times \cos 120°$

$= 5329$

$b = 73$

e $c^2 = a^2 + b^2 - 2ab\cos C$

$= 31^2 + 42^2 - 2 \times 31 \times 42 \times \cos 140°$

$= 2642 - 2604 \times \cos 140°$

$= 4719.77$

$c \approx 68.70$

f $\angle BCA = \angle C$

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{10^2+12^2-9^2}{2\times 10\times 12} = 0.6791$$

$\therefore\ \angle BCA \approx 47.22^\circ$

g $c^2 = a^2+b^2-2ab\cos C$

$= 11^2+9^2-2\times 11\times 9 \times \cos 43.2^\circ$

$= 202-198\times\cos 43.2^\circ$

$= 57.6642$

$c \approx 7.59$

h $\angle CBA = \angle B$

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{8^2+15^2-10^2}{2\times 8\times 15} = 0.7875$$

$\therefore\ \angle ABC \approx 38.05^\circ$

4 $c^2 = a^2+b^2-2ab\cos C$

$= 4^2+6^2-2\times 4\times 6\times\cos 20^\circ$

$= 52-48\times\cos 20^\circ$

$= 6.8947$

$c \approx 2.626$ km

5 $AB^2 = a^2+b^2-2ab\cos O$

$= 4^2+6^2-2\times 4\times 6\times\cos 30^\circ$

$= 52-48\times\cos 30^\circ$

$= 10.4307$

$AB \approx 3.23$ km

6 Label the points suitably: A and B are the hooks, and C is the 70° angle.

$c^2 = a^2+b^2-2ab\cos C$

$BD^2 = 42^2+54^2-2\times 42\times 54\times\cos 70^\circ$

$= 4680-4536\times\cos 70^\circ$

$= 3128.5966$

$BD \approx 55.93$ cm

7 a $\angle B = 180^\circ - 48^\circ = 132^\circ$

$AC^2 = a^2+c^2-2ac\cos B$

$= 5^2+4^2-2\times 5\times 4\times\cos 132^\circ$

$= 41-40\times\cos 132^\circ$

$= 67.7652$

$AC \approx 8.23$ cm

b $BD^2 = b^2+d^2-2bd\cos A$

$= 5^2+4^2-2\times 5\times 4\times\cos 48^\circ$

$= 41-40\times\cos 48^\circ$

$= 14.2347$

$BD \approx 3.77$ cm

8 a Use $\triangle ABD$.

$BD^2 = b^2+d^2-2bd\cos A$

$= 6^2+4^2-2\times 6\times 4\times\cos 92^\circ$

$= 52-48\times\cos 92^\circ$

$= 53.6751$

$BD \approx 7.326$ cm

b
$$\angle D = \angle BDC$$
$$\frac{\sin D}{5} = \frac{\sin 88^\circ}{7.3263}$$
$$\sin D = \frac{5 \times \sin 88^\circ}{7.3263}$$
$$= 0.6820$$
$$D = 43.0045^\circ$$
$$B = 180^\circ - 88^\circ$$
$$- 43.0045^\circ$$
$$= 48.9954^\circ$$
$$\frac{b}{\sin 48.9954^\circ} = \frac{7.3263}{\sin 88^\circ}$$
$$b = \frac{7.3263 \times \sin 48.9956^\circ}{\sin 88^\circ}$$
$$\approx 5.53 \text{ cm}$$

9 **a** Treat AB as c.
$$c^2 = a^2 + b^2 - 2ab\cos O$$
$$AB^2 = 70^2 + 90^2 - 2 \times 70$$
$$\times 90 \times \cos 65^\circ$$
$$= 13\,000 - 12\,600 \times \cos 65^\circ$$
$$= 7675.0099$$
$$AB \approx 87.61 \text{ m}$$

b
$$\cos \angle B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{70^2 + 87.6071^2 - 90^2}{2 \times 70 \times 87.6071}$$
$$= 0.3648$$
$$\angle AOB \approx 68.6010^\circ$$
Now use $\triangle OCB$.
Let $CB = a,\ OB = b,\ OC = c$.
$$CB = \frac{AB}{2} = 43.80$$
$$c^2 = a^2 + b^2 - 2ab\cos O$$
$$OC^2 = 43.8035^2 + 70^2 - 2 \times 43.8035$$
$$\times 70 \times 0.3648$$
$$= 4581.24$$
$$OC \approx 67.7 \text{ m}$$

Solutions to Exercise 13D

1 a
$$\begin{aligned}\text{Area} &= \frac{1}{2}ab\sin C\\ &= \frac{1}{2}\times 6\times 4\times \sin 70^\circ\\ &= 11.28\text{ cm}^2\end{aligned}$$

b
$$\begin{aligned}\text{Area} &= \frac{1}{2}yz\sin X\\ &= \frac{1}{2}\times 5.1\times 6.2\times \sin 72.8^\circ\\ &= 15.10\text{ cm}^2\end{aligned}$$

c
$$\begin{aligned}\text{Area} &= \frac{1}{2}nl\sin M\\ &= \frac{1}{2}\times 3.5\times 8.2\times \sin 130^\circ\\ &= 10.99\text{ cm}^2\end{aligned}$$

d
$$\angle C = 180 - 25 - 25 = 130^\circ$$
$$\begin{aligned}\text{Area} &= \frac{1}{2}ab\sin C\\ &= \frac{1}{2}\times 5\times 5\times \sin 130^\circ\\ &= 9.58\text{ cm}^2\end{aligned}$$

2 a Use the cosine rule to find $\angle B$. (Any angle will do.)
$$\begin{aligned}\cos\angle B &= \frac{a^2+c^2-b^2}{2ac}\\ &= \frac{3.2^2+4.1^2-5.9^2}{2\times 3.1\times 4.1}\\ &= -0.2957\\ \angle B &= 107.201^\circ\end{aligned}$$
$$\begin{aligned}\text{Area} &= \frac{1}{2}ac\sin B\\ &= \frac{1}{2}\times 3.2\times 4.1\\ &\quad\times \sin 107.201^\circ\\ &\approx 6.267\text{ cm}^2\end{aligned}$$

b Use the sine rule to fmd $\angle C$.
$$\begin{aligned}\frac{\sin C}{7} &= \frac{\sin 100^\circ}{9}\\ \sin C &= \frac{7\times\sin 100^\circ}{9}\\ &= 0.7659\\ C &= 49.992^\circ\\ \angle A &= 180^\circ - 100^\circ - 49.992^\circ\\ &= 30.007^\circ\end{aligned}$$
$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin A\\ &= \frac{1}{2}\times 9\times 7\times \sin 30.007^\circ\\ &\approx 15.754\text{ cm}^2\end{aligned}$$

c
$$\begin{aligned}E &= 180^\circ - 65^\circ - 66^\circ\\ &= 60^\circ\end{aligned}$$
$$\begin{aligned}\frac{e}{\sin 60^\circ} &= \frac{6.3}{\sin 55^\circ}\\ e &= \frac{6.3\times\sin 60^\circ}{\sin 55^\circ}\\ &= 6.6604\text{ cm}\end{aligned}$$
$$\begin{aligned}\text{Area} &= \frac{1}{2}ef\sin D\\ &= \frac{1}{2}\times 6.6604\times 6.3\times \sin 65^\circ\\ &\approx 19.015\text{ cm}^2\end{aligned}$$

d Use the cosine rule to find $\angle D$.

$$\cos\angle D = \frac{e^2+f^2-d^2}{2ef}$$

$$= \frac{5.1^2+5.7^2-5.9^2}{2\times 5.1\times 5.7}$$

$$= -0.4074$$

$$\angle D = 65.95^\circ$$

$$\text{Area} = \frac{1}{2}ef\sin D$$

$$= \frac{1}{2}\times 5.1\times 5.7\times \sin 65.95^\circ$$

$$\approx 13.274\text{ cm}^2$$

e $$\frac{\sin I}{12} = \frac{\sin 24^\circ}{5}$$

$$\sin I = \frac{12\times \sin 24^\circ}{5}$$

$$= 0.9671$$

$$I = 77.466^\circ \text{ or } 180^\circ - 74.466^\circ$$

$$= 102.533^\circ$$

$$G = 180^\circ - 24^\circ - 108.533^\circ$$

$$\text{or } 180^\circ - 24^\circ - 77.466^\circ$$

$$= 53.466^\circ \text{ or } 78.534^\circ$$

$$\text{Area} = \frac{1}{2}hi\sin G$$

$$= \frac{1}{2}\times 5\times 12\times \sin 53.466^\circ$$

$$\text{or } \frac{1}{2}\times 5\times 12\times \sin 78.534^\circ$$

$$\approx 24.105\text{ cm}^2 \text{ or } 29.401\text{ cm}^2$$

Note that although the diagram is drawn as if I is obtuse, you should not make this assumption. Diagrams are not neccessarily drawn to scale.

f $$I = 180 - 10 - 19$$

$$= 151^\circ$$

$$\frac{i}{\sin 151^\circ} = \frac{4}{\sin 19^\circ}$$

$$i = \frac{4\times \sin 151^\circ}{\sin 19^\circ}$$

$$= 5.9564$$

$$\text{Area} = \frac{1}{2}ih\sin G$$

$$= \frac{1}{2}\times 5.9564\times 4\times \sin 10^\circ$$

$$\approx 2.069\text{ cm}^2$$

Solutions to Exercise 13E

1 $l = \frac{105}{360} \times 2\pi r$

$= \frac{105}{360} \times 2 \times \pi \times 25$

≈ 45.81 cm

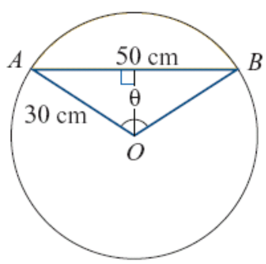
2 a $\theta = \frac{50}{30} = \frac{5}{3}$ radians

$= \frac{5}{3} \times \frac{180}{\pi}$ degrees

$= 95.4929°$

$= 95°30'$

b



$\sin\frac{\theta}{2} = \frac{25}{30} = 0.8333$

$\frac{\theta}{2} = 56.4426°$

$\theta = 112.885°$

$= 112°53'$

3 a Set your calculator to radian mode.

$\sin\frac{\theta}{2} = \frac{3}{7} = 0.4285$

$\frac{\theta}{2} = 0.4429$

$\theta = 0.8858$

$l = r\theta$

$= 7 \times 0.8858$

$= 6.20$ cm

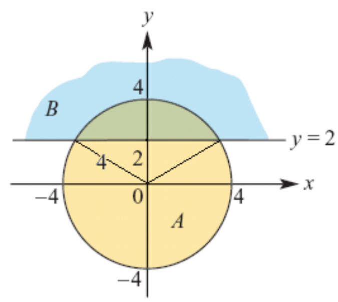
b This represents the minor segment area.

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 7^2 \times (0.8858 - \sin\ 0.8858)$

$= 2.73\text{ cm}^2$

4 A represents the interior of a circle of radius 4, centre the origin.



$\cos\frac{\theta}{2} = \frac{2}{4} = \frac{1}{2}$

$\frac{\theta}{2} = \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}$

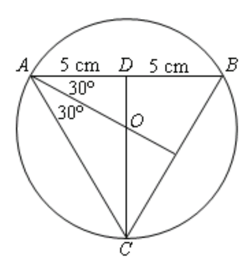
$A \cap B$ is a segment where $r = 4,\ \theta = \frac{2\pi}{3}$

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 4^2 \times \left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= 9.83\text{ cm}^2$

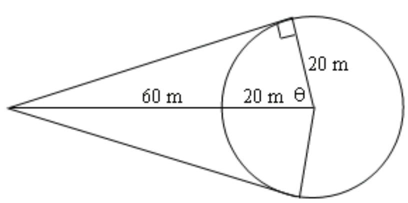
5



$$\text{Altitude } CD = 5\tan 60^\circ$$
$$= 5\sqrt{3} \text{ cm}$$
$$OD = 5\tan 30^\circ$$
$$= \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \text{ cm}$$
$$\text{Radius} = 5\sqrt{3} - \frac{5\sqrt{3}}{3}$$
$$= \frac{15\sqrt{3} - 5\sqrt{3}}{3}$$
$$= \frac{10\sqrt{3}}{3} \text{ cm}$$
$$\angle AOD = 60^\circ$$
$$\therefore\ \angle AOB = 120^\circ = \frac{2\pi}{3} \text{ radians}$$
$$\text{Area} = 3 \times \text{segment area}$$
$$= \frac{3}{2} \times r^2 \times (\theta - \sin\theta)$$
$$= \frac{3}{2} \times \frac{300}{9} \times \left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$$
$$= 50\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$$
$$= 61.42 \text{ cm}^2$$

6 a $C = 2\pi r$
$$= 2 \times \pi \times 20$$
$$= 40\pi \approx 125.66 \text{ m}$$

b

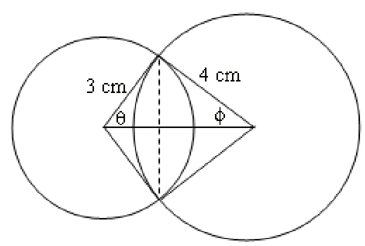


$$\cos\theta = \frac{20}{20 + 60} = 0.25$$
$$\theta = 1.3181 \text{ radians}$$
$$2\theta = 2.6362$$
$$\text{Proportion visible} = \frac{2.6362}{2\pi}$$
$$= 0.41956$$
$$\approx 41.96\%$$

7 a Use fractions of an hour (minutes).
$$l = \frac{25}{60} \times 2\pi r$$
$$= \frac{25}{60} \times 2 \times \pi \times 4$$
$$= \frac{10\pi}{3} \approx 10.47 \text{ m}$$

b Angle $= \frac{25}{60} \times 2\pi = \frac{5\pi}{6}$
$$\text{Area} = -\frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 4^2 \times \frac{5\pi}{6}$$
$$= \frac{20\pi}{3} \approx 20.94 \text{ m}^2$$

8

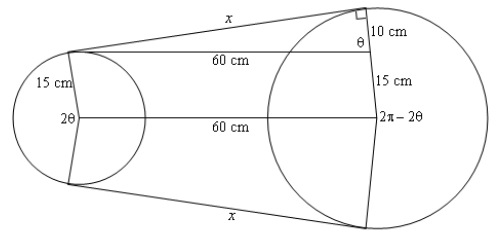


The required area is the sum of two segments.
Let the left area be A_1 and the right area A_2.
$$\tan\theta = \frac{4}{3}$$
$$\theta = 0.9272$$
$$2\theta = 1.8545$$
$$A_1 = \frac{1}{2} \times 3^2 \times (1.8545 - \sin\ 1.8545)$$
$$= 4.0256$$

$$\tan\phi = \frac{3}{4}$$
$$\phi = 0.6435$$
$$2\phi = 1.2870$$
$$A_2 = \frac{1}{2} \times 4^2 \times (1.2870 - \sin\ 1.2870)$$
$$= 2.6160$$
$$\text{Total area} = 4.0256 + 2.6160$$
$$= 6.64 \text{ cm}^2$$

9
$$A = \frac{1}{2}r^2\theta = 63$$
$$r^2\theta = 126$$
$$\theta = \frac{126}{r^2}$$
$$P = r + r + r\theta = 32$$
$$2r + r \times \frac{126}{r^2} = 32$$
$$2r + \frac{126}{r} = 32$$
$$2r^2 + 126 = 32r$$
$$2r^2 - 32r + 126 = 0$$
$$r^2 - 16r + 63 = 0$$
$$(r-7)(r-9) = 0$$
$$r = 7 \text{ or } 9 \text{ cm}$$
$$\theta = \frac{126}{r^2}$$
$$\text{When} r = 7,\ \theta = \frac{126}{7^2} = \left(\frac{18}{7}\right)^c$$
$$\text{When} r = 9,\ \theta = \frac{126}{9^2} = \left(\frac{14}{9}\right)^c$$

10 The following diagram can be deduced from the data:



$$x^2 = 60^2 - 10^2 = 3500$$
$$x = 10\sqrt{35}$$
$$\cos\theta = \frac{10}{60} = \frac{1}{6}$$
$$\theta = 1.4033$$
$$2\theta = 2.8066$$
$$2\pi - 2\theta = 3.4764$$

Length of belt on left wheel:

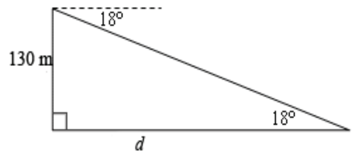
$$l = r\theta$$
$$= 15 \times 2.8066 = 42.1004$$

Length of belt on right wheel:

$$l = r\theta$$
$$= 25 \times 3.4764 = 86.9122$$
$$\text{Total} = 12 \times 10\sqrt{25} + 42.1004$$
$$+ 86.9112$$
$$\approx 247.33 \text{ cm}$$

Solutions to Exercise 13F

1

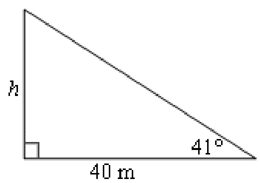


$$\frac{130}{d} = \tan 18^\circ$$

$$d = \frac{130}{\tan 18^\circ}$$

$$= 400.10 \text{ m}$$

2

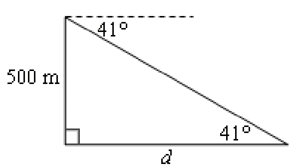


$$\frac{h}{40} = \tan 41^\circ$$

$$h = 40 \times 0.869$$

$$= 34.77 \text{ m}$$

3

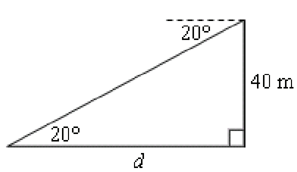


$$\frac{500}{d} = \tan 41^\circ$$

$$d = \frac{500}{\tan 41^\circ}$$

$$= 575.18 \text{ m}$$

4

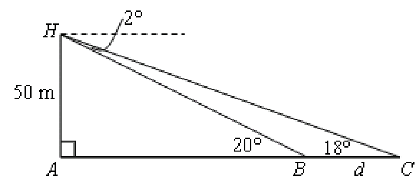


$$\frac{40}{d} = \tan 20^\circ$$

$$d = \frac{40}{\tan 20^\circ}$$

$$= 109.90 \text{ m}$$

5



$$\frac{50}{AB} = \tan 20^\circ$$

$$AB = \frac{50}{\tan 20^\circ}$$

$$= 137.373 \text{ m}$$

$$\frac{50}{AC} = \tan 18^\circ$$

$$AC = \frac{50}{\tan 18^\circ}$$

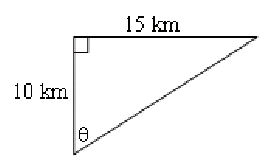
$$= 153.884 \text{ m}$$

$$d = AC - AB$$

$$= 153.884 - 137.373$$

$$\approx 16.51 \text{ m}$$

6

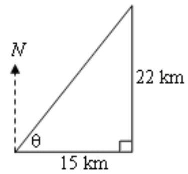


$$\tan\theta = \frac{15}{10} = 1.5$$

$$\theta \approx 56^\circ$$

The bearing is 056°.

7 a



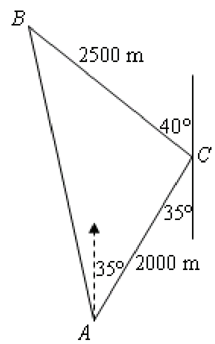
$\tan\theta = \dfrac{22}{15} = 1.466$

$\theta = 55.713°$

The bearing is $90° - \theta \approx 034°$.

b $180° + 34° = 214°$

8



a Use the cosine rule, where

$\angle C = 180 - 40 - 35 = 105°$

$$\begin{aligned} AB^2 &= c^2 \\ &= a^2 + b^2 - 2ab\cos C \\ &= 2500^2 + 2000^2 \\ &\quad - 2 \times 2500 \times 2000 \times \cos 105° \\ &= 12\,838\,190.4510 \end{aligned}$$

$AB = 3583.04$ m

b $\dfrac{2500}{\sin A} = \dfrac{3583.04}{\sin 105°}$

$A = 42.38°$

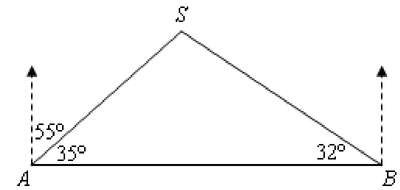
$\therefore$ bearing of B from A

$= (360 - 42.38 + 35)°$

$\approx 353°$

9 $207° - 180° = 027°$

10

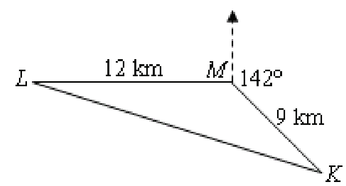


$\angle SAB = 90° - 55° = 35°$

$\angle SBA = 302° - 270° = 32°$

$\angle ASB = 180° - 35° - 32° = 113°$

11



$$\begin{aligned} \angle LMK &= 360° - 90° - 142° \\ &= 128° \end{aligned}$$

First, use the cosine rule to find LK.

$$\begin{aligned} LK^2 &= m^2 \\ &= k^2 + l^2 - 2kl\cos M \\ &= 12^2 + 9^2 - 2 \times 12 \times 9 \times \cos 128° \\ &= 357.9829 \end{aligned}$$

$LK = 18.920$

It is easier to use the sine rule to find $\angle MLK$.

$$\frac{\sin L}{9} = \frac{\sin 128°}{18.920}$$

$$\begin{aligned} \sin L &= \frac{\sin 128° \times 9}{18.920} \\ &= 0.3748 \end{aligned}$$

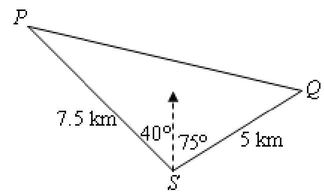
$$\begin{aligned} \angle MLK &= \angle L \\ &\approx 22.01° \end{aligned}$$

12 a $\angle BAN = 360° - 346° = 14°$

$\angle BAC = 14° + 35° = 49°$

b Use the cosine rule:

$$\begin{aligned} BC^2 &= a^2 \\ &= b^2 + c^2 = 2bc\cos A \\ &= 340^2 + 160^2 - 2 \times 340 \\ &\quad \times 160 \times \cos 49^\circ \\ &= 69\,820.7776 \\ BC &= 264.24 \text{ km} \end{aligned}$$

13 

Use the cosine rule:

$$\begin{aligned} \angle PSQ &= 115^\circ \\ PQ^2 &= s^2 \\ &= p^2 + q^2 - 2pq\cos A \\ &= 5^2 + 7.5^2 - 2 \times 5 \\ &\quad \times 7.5 \times \cos 115^\circ \\ &= 112.9464 \\ PQ &= 10.63 \text{ km} \end{aligned}$$

Solutions to Exercise 13G

1 a $FH^2 = 12^2 + 5^2$

$= 169$

$FH = 13$ cm

b $BH^2 = 13^2 + 8^2$

$= 233$

$BH = \sqrt{233} \approx 15.26$ cm

c $\tan \angle BHF = \dfrac{8}{13}$

$= 0.615$

$\angle BHF = 31.61°$

d $\angle BGH = 90°$ and $BH = \sqrt{233}$

$\cos \angle BGH = \dfrac{12}{\sqrt{233}}$

$= 0.786$

$\angle BGH = 38.17°$

2 a $AB = 2EF$

$EF = 4$ cm

b $\tan \angle VEF = \dfrac{VE}{EF}$

$= \dfrac{12}{4} = 3$

$\angle VEF = 71.57°$

c $VE^2 = 4^2 + 12^2$

$= 160$

$VE = \sqrt{160}$

$= 4\sqrt{10} \approx 12.65$ cm

d All sloping sides are equal in length. Choose VA.

$VA^2 = VE^2 + EA^2$

$= 160 + 4^2 = 176$

$VA = \sqrt{176}$

$= 4\sqrt{11} \approx 13.27$ cm

e $\angle VAD = \angle VAE$

$\tan \angle VAE = \dfrac{VE}{EA}$

$= \dfrac{4\sqrt{10}}{4}$

$= \sqrt{10} \approx 3.162$

$\angle VAE = 72.45°$

f Area of a triangular face

$= \dfrac{1}{2} \times AD \times VE$

$= \dfrac{1}{2} \times 8 \times 4\sqrt{10}$

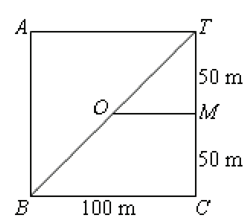
$= 16\sqrt{10}$ cm^2

Area of base $= 8 \times 8 = 64$ cm^2

Surface area $= 4 \times 16\sqrt{10} + 64$

≈ 266.39 cm^2

3 First, sketch the square base, and find the height h of the tree. Mark M as the mid-point of TC and O as the centre of the square.



$OM = TM = 50$ m

$$OT^2 = 50^2 + 50^2 = 5000$$
$$OT = \sqrt{5000}\text{ m}$$
$$\frac{h}{\sqrt{5000}} = \tan 20^\circ$$
$$h = \sqrt{5000} \times \tan 20^\circ$$
$$= 25.7365$$

At A and C,

$$\tan\theta = \frac{25.7365}{100} = 0.2573$$
$$\theta = 14.43^\circ$$

At B, $TB = 2 \times OT = 2\sqrt{5000}$ m

$$\tan\theta = \frac{25.7365}{\sqrt{5000}} = 0.1819$$
$$\theta = 10.31^\circ$$

4 **a** $\angle ABC = 180^\circ - 90^\circ - 45^\circ$

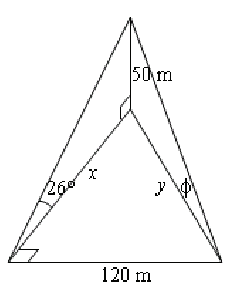
$$= 45^\circ$$

ABC is isosceles, and
$CB = AC = 85$ m.

b $\dfrac{XB}{BC} = \sin 32^\circ$

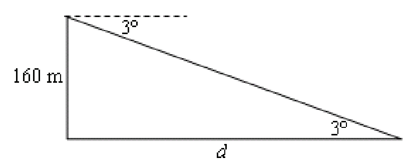
$$\frac{XB}{85} = \sin 32^\circ$$
$$XB = 85 \times \sin 32^\circ$$
$$= 45.04\text{ m}$$

5



$$\frac{50}{x} = \tan 26^\circ$$
$$x = \frac{50}{\tan 26^\circ}$$
$$= 102.515\text{ m}$$
$$y^2 = x^2 + 120^2$$
$$= 24\,909.364$$
$$y = \sqrt{24\,909.364}$$
$$= 157.827\text{ m}$$
$$\tan\phi = \frac{50}{y} = 0.316$$
$$\phi = 17.58^\circ$$

6 From the top of the cliff:



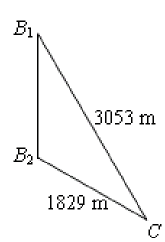
For the first buoy:

$$\frac{160}{d} = \tan 3^\circ$$
$$d = \frac{160}{\tan 3^\circ}$$
$$= 3052.981\text{ m}$$

For the second buoy

$$\frac{160}{d} = \tan 5^\circ$$
$$d = \frac{160}{\tan 5^\circ}$$
$$= 1828.808\text{ m}$$

From the cliff:



$\angle C = 337 - 308 = 29^\circ$

Use the cosine rule.

$$\begin{aligned} c^2 &= 3052.981^2 + 1828.808^2 \\ &\quad - 2 \times 3052.981 \times 1828.808 \\ &\quad \times \cos 29^\circ \\ &= 2\,898\,675.1436 \\ c &= 1702.55 \text{ m} \end{aligned}$$

7 a

$$\begin{aligned} AC^2 &= 12^2 + 5^2 = 169 \\ AC &= 13 \text{ cm} \\ \tan \angle ACE &= \frac{6}{13} \\ &= 0.4615 \\ \angle ACE &= 24.78^\circ \end{aligned}$$

b Triangle HDF is identical (congruent) to triangle AEC.

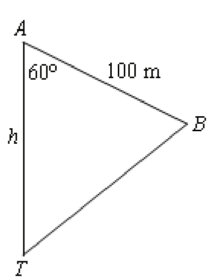
$\therefore \angle HFD = \angle ACE$

$$\begin{aligned} \angle HDF &= 90^\circ - 24.28^\circ \\ &= 65.22^\circ \end{aligned}$$

c

$$\begin{aligned} CH^2 &= 12^2 + 6^2 = 180 \\ CH &= \sqrt{180} \\ \tan \angle ECH &= \frac{EH}{CH} \\ &= \frac{5}{\sqrt{180}} = 0.3726 \\ \angle ECH &= 20.44^\circ \end{aligned}$$

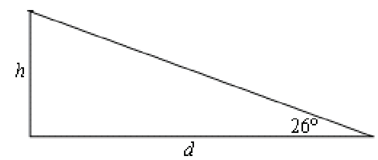
8 Looking from above, the following diagram applies.



Because the angle of elevation is 45°, AT will equal the height of the tower, h m. Use the cosine rule.

$$\begin{aligned} BT^2 &= h^2 + 100^2 - 2 \times h \times 100 \times \cos 60^\circ \\ &= h^2 + 100^2 - 200h \times \frac{1}{2} \\ &= h^2 - 100h + 100^2 \end{aligned}$$

From point B:



$$\begin{aligned} \frac{h}{d} &= \tan 26^\circ \\ d &= \frac{h}{\tan 26^\circ} \\ &= 2.050h \end{aligned}$$

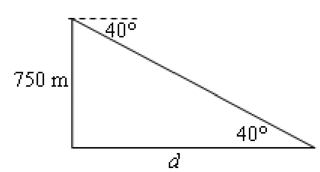
$$\begin{aligned} \therefore \quad 2.050^2h^2 &= h^2 - 100h + 100^2 \\ 4.2037h^2 &= h^2 - 100h + 10\,000 \end{aligned}$$

$$3.2037h^2 + 100h = 10\,000$$

Using the quadratic formula:

$h \approx 42.40$ m

9 Find the horizontal distance of A from the balloon.



$$\begin{aligned} \frac{750}{d} &= \tan 40^\circ \\ d &= \frac{750}{\tan 40^\circ} \\ &= 893.815 \text{ m} \end{aligned}$$

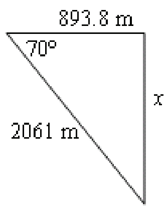
The distance of B from the balloon may be calculated in the same way:

$$\frac{750}{d} = \tan 20^\circ$$

$$d = \frac{750}{\tan 20^\circ}$$

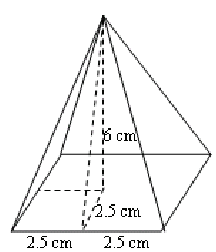
$$= 2060.608 \text{ m}$$

Draw the view from above and use the cosine rule.



$$x^2 = 893.8152 + 2060.6082$$
$$- 2 \times 893.815 \times 2060.608$$
$$\times \cos 70^\circ$$
$$= 3\,785\,143.5836$$
$$x = 1945.54 \text{ m}$$

10 a Find the length of an altitude:



$$a^2 = 2.5^2 + 6^2 = 42.45$$
$$a \approx 6.5 \text{ cm}$$

The sloping edges are also the hypotenuse of a right-angled triangle.

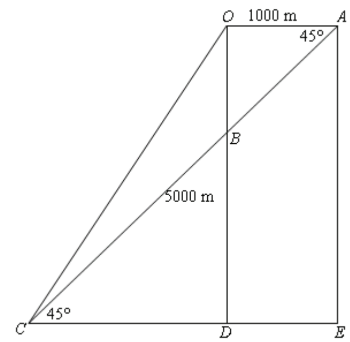
$$s^2 = 2.5^2 + 6.5^2 = 48.5$$
$$s \approx 6.96 \text{ cm}$$

b $\text{Area} = \frac{1}{2} \times 5 \times 6.5$

$$= 16.25 \text{ cm}^2$$

11 a $\text{Distance} = 300 \times \frac{1}{60} = 5 \text{ km}$

b Looking from above:



$$AE = 5000 \times \sin 45^\circ$$
$$= \frac{5000}{\sqrt{2}} \approx 3535.433$$
$$CE = 5000 \times \sin 45^\circ$$
$$= \frac{5000}{\sqrt{2}} \approx 3535.433$$
$$CD = CE - DE$$
$$= 3535.533 - 1000$$
$$= 2535.533$$
$$\tan \angle COD = \frac{2535.533}{3535.533}$$
$$= 0.7171$$
$$\angle COD = 35.65^\circ$$
$$\text{Bearing} = 180^\circ + 35.65^\circ$$
$$= 215.65^\circ$$

c Let the angle of elevation be θ.

$$OC^2 = 3535.533^2 + 2535.533^2$$
$$= 18\,928\,932$$
$$OC = 4350.739$$
$$\tan \theta = \frac{500}{4350.739}$$
$$= 0.1149$$
$$\theta = 6.56^\circ = 6^\circ 33'$$

Solutions to Review: Short-answer questions

1 The side that is opposite $\angle BAC$ is BC so apply the cosine rule to get:

$$BC^2 = AB^2 + CA^2 - 2 \times AB \times CA \times \cos(\angle BAC)$$

$$\cos(\angle BAC) = \frac{BC^2 - AB^2 - CA^2}{-2 \times AB \times CA} = \frac{6^2 - 4^2 - 5^2}{-2^2 4^2 5} = \frac{1}{8}$$

2 Apply the sine rule to get:

$$\frac{\sin(\angle ACB)}{8} = \frac{\sin(30°)}{10}$$

$$\sin(\angle ACB) = \frac{8\sin(30°)}{10} = \frac{2}{5}$$

3 **a** $l_{BC} = r\theta_{BOC}$

$2.4 = r \times 1.2$

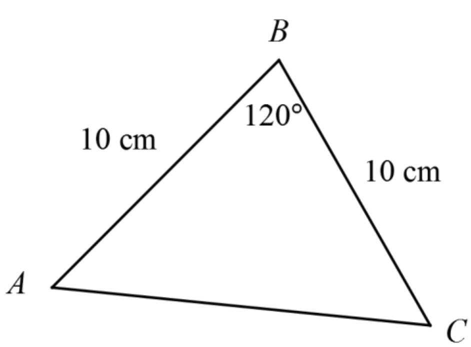
$r = 2\,\text{cm}$

b $l_{AB} = r\theta_{AOB}$

$1.4 = 2 \times \theta_{AOB}$

$\theta_{AOB} = 0.7$

4

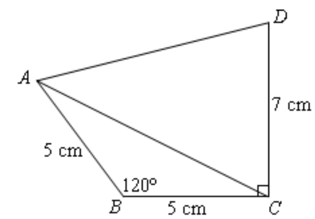


Use cosine rule to find AC:

$$AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos(B) = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos(120°) = 300$$

$AC = 10\sqrt{3}\,\text{cm}$

5



a Use the cosine rule.

$$AC^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 120° = 25 + 25 + 25 = 75$$

$AC = \sqrt{75} = 5\sqrt{3}\,\text{cm}$

b Area $= \frac{1}{2} \times 5 \times 5 \times \sin 120° = \frac{25\sqrt{3}}{4}\,\text{cm}^2$

c In isosceles triangle ABC,

$$\angle ACB = \angle BAC = \frac{1}{2}(180° - 120°) = 30°$$

$\angle ACD = 90° - 30° = 60°$

$$\text{Area of } ADC = \frac{1}{2} \times 7 \times AC \times \sin 60° = \frac{1}{2} \times 7 \times 5\sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{105}{4}\,\text{cm}^2$$

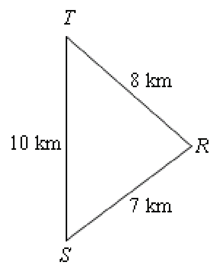
d Total area $= \dfrac{25\sqrt{3}}{4} + \dfrac{105}{4}$

$= \dfrac{25\sqrt{3} + 105}{4}$

$= \dfrac{5(5\sqrt{3} + 21)}{4}$ cm^2

6 $x = 180° - 37° = 143°$

7



$\cos S = \dfrac{10^2 + 7^2 - 8^2}{2 \times 10 \times 7}$

$= \dfrac{85}{140} = \dfrac{17}{28}$

8 First note that $AB = c = 5$ cm $\angle BAC = A = 60°$ and $AC = b = 6$ cm, so the angle is included. So start by finding $a = BC$ by the cosine rule.

$a^2 = b^2 + c^2 2bc \cos A$

$= 36 + 25 - 60 \cos 60°$

$= 36 + 25 - 30$

$= 31$

$a = \sqrt{31}$

Now use the sine rule.

$\dfrac{\sin B}{6} = \dfrac{\sin 60°}{\sqrt{31}}$

$\sin \angle ABC = \dfrac{6 \sin 60°}{\sqrt{31}}$

$= \dfrac{3\sqrt{3}}{\sqrt{31}} = \dfrac{3\sqrt{93}}{31}$

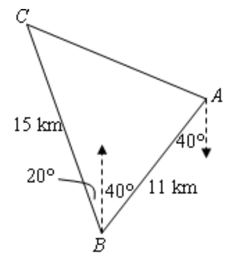
9 $A = \dfrac{1}{2} r^2 \theta$

$33 = \dfrac{1}{2} \times 6^2 \times \theta$

$= 18\theta$

$\theta = \dfrac{33}{18} = \dfrac{11}{6}$ (radians)

10



Use the cosine rule.

$AC^2 = b^2$

$= a^2 + c^2 - 2ac \cos B$

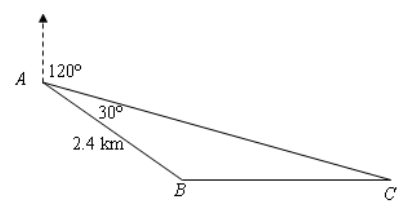
$= 11^2 + 15^2 - 2 \times 11 \times 15 \cos 60°$

$= 121 + 225 - 165$

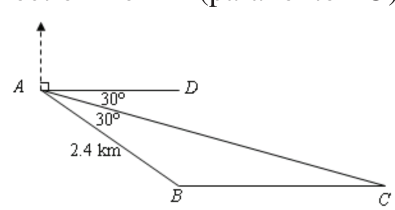
$= 181$

$AC = \sqrt{181}$ km

11 a



Draw a line AD in an easterly direction from A (parallel to BC).



$\angle DAC = 30°$

$\angle ACB = \angle DAC = 30°$

$\angle ABC = 180° - 30 - 30$

$= 120°$

$\therefore \quad BC = 2.4$ km

Use the cosine rule to find AC.

$AC^2 = b^2$

$= a^2 + c^2 - 2ac\cos B$

$= 2.4^2 + 2.4^2 - 2 \times 2.4$

$\times 2.4 \times \cos 120°$

$= 5.76 + 5.76 + 5.76 = 17.28$

$AC = \sqrt{1728}$

$= \sqrt{5.76 \times 3}$

$= 2.4\sqrt{3}$ or $\dfrac{12\sqrt{3}}{5}$ km

12 $l = r\theta$

$30 = 12\theta$

$\theta = \dfrac{30}{12} = \left(\dfrac{5}{2}\right)^c$

$A = \dfrac{1}{2} \times 12^2 \times \dfrac{5}{2}$

$= 180$ cm^2

13 a Find $\angle AOB$ first using cosine rule:

$3^2 = \sqrt{3}^2 + \sqrt{3}^2$

$- 2\sqrt{3}\sqrt{3}\cos(\angle AOB)$

$\cos(\angle AOB) = \dfrac{3^2 - \sqrt{3}^2 - \sqrt{3}^2}{-2\sqrt{3}\sqrt{3}}$

$\angle AOB = \cos^{-1}\left(\dfrac{3^2 - \sqrt{3}^2 - \sqrt{3}^2}{-2\sqrt{3}\sqrt{3}}\right)$

$= \dfrac{2\pi}{3}$

Hence, the length of arc ACB is:

$ACB = r\theta$

$= \sqrt{3} \times \dfrac{2\pi}{3}$

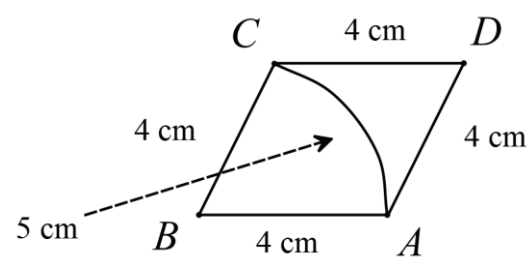
$= \dfrac{2\sqrt{3}\pi}{3}$ cm

b $A = \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2} \times \sqrt{3}^2 \times \left(2\pi - \dfrac{2\pi}{3}\right)$

$= 2\pi$ cm^2

14



The area of the unshaded region is equal to the area of the rhombus with the area of sector CBA subtracted from it.

$A_{\text{Rhombus}} = \text{base} \times \text{height}$

$= 4 \times (4\sin B)$

To find B:

$l = r\theta$

$5 = 4 \times \theta$

$\theta = \dfrac{5}{4}$

Thus

$A_{\text{Rhombus}} = 4 \times \left(4\sin\dfrac{5}{4}\right)$

$= 16\sin\dfrac{5}{4}$

To find the area of the sector CBA:

$A_{CBA} = \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2} \times 4^2 \times \dfrac{5}{4}$

$= 10$

To find the shaded area:

$A = \left[16\sin\left(\dfrac{5}{4}\right) - 10\right]$ cm^2

Solutions to Review: Multiple-choice questions

1 D Use the sine rule.

$$\frac{\sin Y}{y} = \frac{\sin X}{x}$$

$$\frac{\sin Y}{18} = \frac{\sin 62°}{21}$$

$$\sin Y = 18 \times \frac{\sin 62°}{21}$$

$$= 0.7568$$

$$Y = 49.2°$$

2 C Use the cosine rule.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$= 30^2 + 21^2 - 2 \times 30 \times 21 \times \frac{51}{53}$$

$$= 128.547$$

$$c \approx 11$$

3 C Use the cosine rule.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{5.2^2 + 6.8^2 - 7.3^2}{2 \times 5.2 \times 6.8}$$

$$= 0.2826$$

$$C \approx 74°$$

4 B Area $= \frac{1}{2}bc\sin A$

$$= \frac{1}{2} \times 5 \times 3 \times \sin 30°$$

$$= 3.75 \text{ cm}^2$$

5 A The other angles in the (isosceles) triangle are both

$$\frac{180° - 130°}{2} = 25°.$$

Use the sine rule.

$$\frac{10}{\sin 130°} = \frac{r}{\sin 25°}$$

$$r = \frac{10 \times \sin 25°}{\sin 130°}$$

$$\approx 5.52 \text{ cm}$$

6 A First find the angle at the centre using the cosine rule.

$$\cos C = \frac{6^2 + 6^2 - 5^2}{2 \times 6 \times 6}$$

$$= 0.6527$$

$$C = 49.248° = 0.8595^c$$

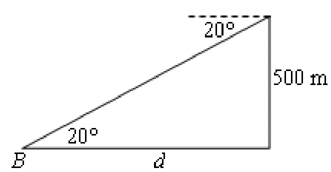
Segment area

$$= \frac{1}{2}r^2(\theta - \sin\theta)$$

$$= \frac{1}{2} \times 6^2 \times (0.8595 - \sin 0.8595)$$

$$\approx 1.8 \text{ cm}^2$$

7 D



$$\frac{500}{d} = \tan 20°$$

$$d = \frac{500}{\tan 20°}$$

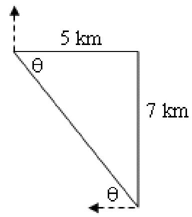
$$\approx 1374 \text{ m}$$

8 B $\tan\theta = \frac{80}{1300}$

$$= 0.0615$$

$$\theta = 3.521° \approx 4°$$

9 C



$$\tan\theta = \frac{7}{5} = 1.4$$

$$\theta = 54°$$

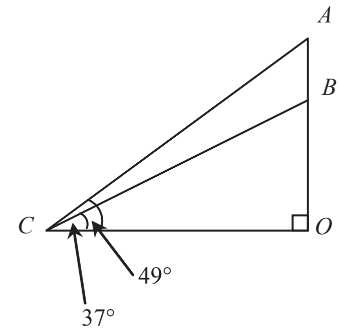
Bearing = 270° + 54° = 324°

10 A 215° − 180° = 035°

Solutions to Review: Extended-response questions

1 a $\angle ACB = 12°,\ \angle CBO = 53°,\ \angle CBA = 127°$

b $\angle CAB = 41°$



The sine rule applied to triangle ABC gives

$$\frac{CB}{\sin 41°} = \frac{60}{\sin 12°}$$

$$\therefore \quad CB = \frac{60 \sin 41°}{\sin 12°}$$

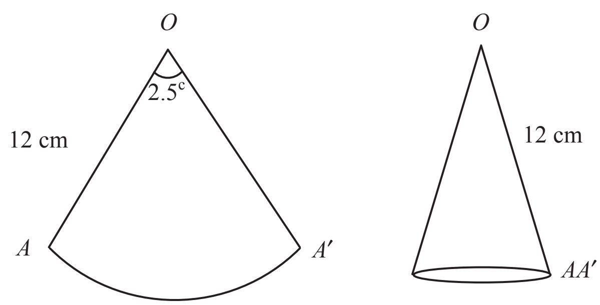
$$= 189.33, \text{ correct to two decimal places}$$

c $$\frac{OB}{CB} = \sin 37°$$

$$\therefore \quad OB = CB \sin 37°$$

$$= 113.94 \text{ m}$$

2 a



The circumference of the circular base $= 2.5 \times 12$

$= 30$ cm

Therefore $2\pi r = 30$

Solve for r, the radius of the base.

$$r = \frac{30}{2\pi}$$

$$= 4.77 \text{ cm, correct to two decimal places}$$

b Curved surface area of the cone = area of the sector

$$= \frac{1}{2} \times 144 \times 2.5$$

$$= 180 \text{ cm}^2$$

c The diameter length is required.

$$\begin{aligned}\text{Diameter} &= 2r\\ &= \frac{30}{\pi}\\ &= 9.55 \text{ cm}\end{aligned}$$

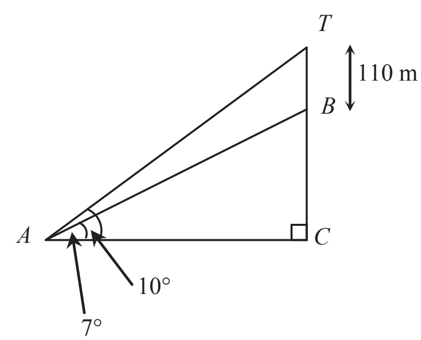
3 a $\angle TAB = 3^\circ,\ \angle ABT = 97^\circ$

$$\begin{aligned}\angle ATB &= (83 - 3)^\circ\\ &= 80^\circ\end{aligned}$$

b The sine rule applied to triangle ATB gives

$$\frac{AB}{\sin 80^\circ} = \frac{110}{\sin 3^\circ}$$

$$\begin{aligned}\therefore \quad CB &= \frac{110 \sin 80^\circ}{\sin 3^\circ}\\ &= 2069.87\end{aligned}$$

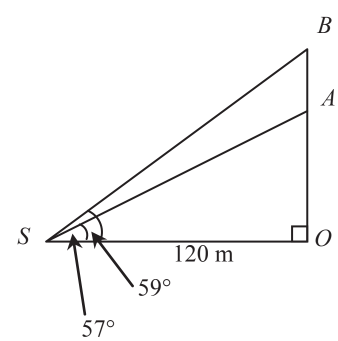


c
$$\begin{aligned}CB &= AB \sin 7^\circ\\ &= 252.25 \text{ m}\end{aligned}$$

4 a In right-angled triangle AOS

$$\frac{OA}{120} = \tan 57^\circ$$

$$\begin{aligned}\therefore \quad OA &= 120 \tan 57^\circ\\ &= 184.78 \text{ m, correct to two decimal places}\end{aligned}$$



b In right-angled triangle SOB

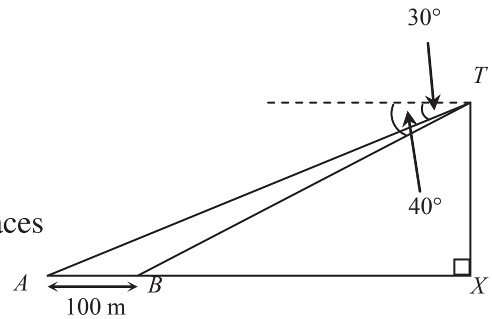
$$\frac{OB}{120} = \tan 59^\circ$$

$$\begin{aligned}\therefore \quad OB &= 120 \tan 59^\circ\\ &= 199.71 \text{ m, correct to two decimal places}\end{aligned}$$

c The distance $AB = OB - OA = 14.93$ m, correct to two decimal places.

5 a $\angle ATB = 10^\circ$

In triangle ATB the sine rule gives



$$\frac{100}{\sin 10^\circ} = \frac{AT}{\sin 140^\circ}$$

$$\therefore \quad AT = \frac{100 \sin 140^\circ}{\sin 10^\circ}$$

$$= 370.17 \text{ m, correct to two decimal places}$$

b Applying the sine rule again gives

$$\frac{BT}{\sin 30^\circ} = \frac{100}{\sin 10^\circ}$$

$$\therefore \quad BT = 287.94 \text{ m, correct to two decimal places}$$

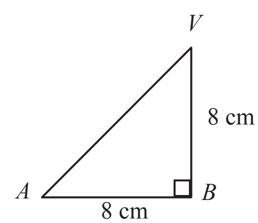
c In right-angled-triangle TBX

$$\frac{XT}{BT} = \sin 40^\circ$$

$$\therefore \quad XT = BT \sin 40^\circ$$

$$= 185.08 \text{ m, correct to two decimal places}$$

6 a Applying Pythagoras' theorem in triangle VBA



$$VA^2 = 8^2 + 8^2$$

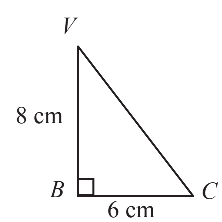
$$= 64 + 64$$

$$= 128$$

$$\therefore \quad VA = 8\sqrt{2}$$

The distance VA is $8\sqrt{2}$ cm.

b Applying Pythagoras' theorem in triangle VBC



$$VC^2 = 8^2 + 6^2$$

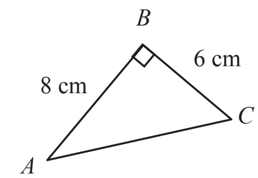
$$= 64 + 36$$

$$= 100$$

$$\therefore \quad VC = 10$$

The distance VC is 10 cm.

c Applying Pythagoras' theorem in triangle ABC



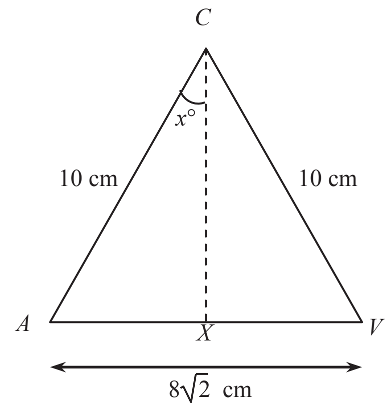
$$\begin{aligned} AC^2 &= 8^2 + 6^2 \\ &= 64 + 36 \\ &= 100 \end{aligned}$$

$\therefore \quad AC = 10$

The distance AC is 10 cm.

d Triangle VCA is isosceles with $VC = AC$

In right-angled triangle CXA



$$\begin{aligned} \sin x^\circ &= \frac{4\sqrt{2}}{10} \\ &= \frac{2\sqrt{2}}{5} \end{aligned}$$

$$\begin{aligned} \text{Therefore } x^\circ &= 34.4490\ldots^\circ \\ \text{and } \angle ACV &= 68.899\ldots^\circ \\ &= 68.9^\circ, \text{correct to one decimal place} \end{aligned}$$