TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 1 2018

Section 1 Calculator Free

Differentiation, Applications of Differentiation, Anti Differentiation

STUDENT'S NAME MARKING KEY

DATE: Thursday 1st March **TIME:** 30 minutes **MARKS**: 28

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Determine $\frac{dy}{dx}$ for the following. Do not simplify your answers.

(a) $y = \frac{x}{3x^3 - 2x + 5}$ [2]

$$\frac{dy}{dx} = \frac{3x^3 - 2x + 5 - x(9x^2 - 2)}{(3x^3 - 2x + 5)^2}$$

(b) $y = \sqrt[3]{(2x^3 + 7)^5}(2 - x)$ [3]

$$y = (2x^3 + 7)^{5/3}(2 - x)$$

$$\frac{dy}{dx} = \frac{5(2x^3 + 7)^{2/3}}{3}(6x^2)(2 - x) - \sqrt[3]{(2x^3 + 7)^5}$$

2. (3 marks)

Given $y = \frac{u^3}{3} + 3u$ and $x = \frac{u+1}{2}$, determine $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$u = 2x - 1$$

$$= u^2 + 3 \quad \times 2$$

$$= 2(u^2 + 3)$$

$$\frac{dy}{dx} = 2\left[(2x-1)^2 + 3\right]$$

$$= 2(2x-1)^2 + 6$$

3. (5 marks)

Determine the value(s) of a under which the curve $y = x^3 + ax^2 + 3x + 2$ will have exactly one stationary point.

$$\frac{dy}{dx} = 3x^2 + 2ax + 3$$

one solution when $b^2 - 4ac = 0$

$$(2a)^2 - 4(3)(3) = 0$$

$$4a^2 - 36 = 0$$

$$4a^2 = 36$$

$$a^2 = 9$$

$$a = \pm 3$$

4. (9 marks)

Determine each of the following.

(a) $\int 3x^2 - \frac{1}{\sqrt{x}} + e \; dx$ [3]

$= \frac{3x^3}{3} - 2x^{1/2} + ex + c$

$= x^3 - 2\sqrt{x} + ex + c$

(b) $\int \frac{2x^3 - 4x^2}{5x^2} dx$ [3]

$= \int \frac{2x^3}{5x^2} - \frac{4x^2}{5x^2} \, dx$

$= \int \frac{2x}{5} - \frac{4}{5} \, dx$

$= \frac{2x^2}{10} - \frac{4x}{5} + c$

(c) $\int \frac{-3}{\sqrt{7x+9}} dx$ [3]

$f(x) = 7x + 9$

$f'(x) = 7$

$= \int -3(7x+9)^{-1/2} dx$

$= \frac{1}{7} \int (7) -3(7x+9)^{-\frac{1}{2}} dx$

$= \frac{1}{7} \left[\frac{-3(7x+9)^{\frac{1}{2}}}{\frac{1}{2}} \right] + c$

$= \frac{-6(7x+9)^{\frac{1}{2}}}{7} + c$

5. (6 marks)

Using calculus techniques;

(a) Determine all stationary points of the function $y = \dfrac{x^3}{3} + 2x^2 + 3x - 2$ [4]

$$\frac{dy}{dx} = x^2 + 4x + 3$$

$$x^2 + 4x + 3 = 0$$

$$(x+3)(x+1) = 0$$

$$x = -3 \ ; \ x = -1$$

$$y_{|x=-3} = \frac{-27}{3} + 2(9) + 3(-3) - 2$$

$$= -2$$

$$y_{|x=-1} = \frac{(-1)^3}{3} + 2(-1)^2 + 3(-1) - 2$$

$$= -\frac{10}{3}$$

Stationary pts are $(-3, -2)$ and $(-1, -\frac{10}{3})$

(b) Showing full algebraic reasoning state the nature of each of these stationary points. 2 [$\cancel{3}$]

$$\frac{d^2y}{dx^2} = 2x + 4$$

$x = -3$ $\quad \dfrac{d^2y}{dx^2} < 0 \quad \therefore$ Maximum

$x = -1$ $\quad \dfrac{d^2y}{dx^2} > 0 \quad \therefore$ Minimum.

TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 1 2018

Section 2 Calculator Assumed

Differentiation, Applications of Differentiation, Anti Differentiation

STUDENT'S NAME MARKING KEY

DATE: Thursday 1st March **TIME:** 20 mins **MARKS**: 24

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser.

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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6. (3 marks)

A small metal sphere with a radius of 0.58 cm is dipped in gold. The coating of the gold is 0.02 cm thick. Use the derivative to approximate the increase in volume of the sphere.

$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} \qquad V = \frac{4}{3}\pi r^3$$

$$\Delta V = \frac{dV}{dr} \times \Delta r$$

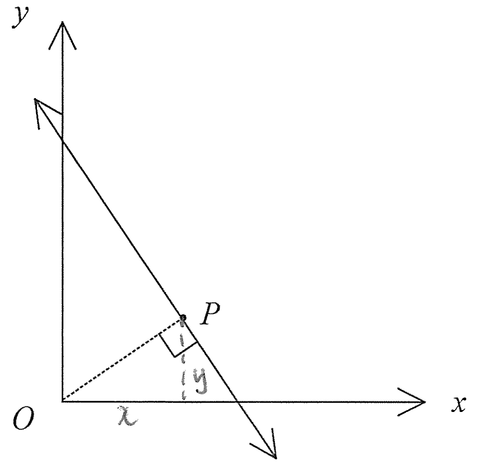
$$= 4\pi r^2 \times 0.02$$

$$= 0.08\pi(0.58)^2$$

$= 0.0845 \text{ cm}^3$ increase in sphere volume.

7. (6 marks)

An ant crawls along the line $y = -10x + 38$ drawn on the axes below.



Not drawn to scale

(a) Given the minimum distance occurs at P, show that the length of OP is $\sqrt{x^2 + (-10x + 38)^2}$. [2]

$OP^2 = x^2 + y^2$

$OP = \sqrt{x^2 + y^2}$

$= \sqrt{x^2 + (-10x + 38)^2}$

(b) Using calculus techniques, determine the minimum distance between the ant and the origin and the location this occurs. [4]

$\frac{dOP}{dx} = \frac{101x - 380}{\sqrt{101x^2 - 760x + 1444}}$

$0 = 101x - 380$

$x = 3.7624 \quad , \quad y = 0.376$

$\frac{d^2OP}{dx^2} > 0$ at $x = 3.7624$ $\therefore$ min

$OP = \sqrt{3.7624^2 + 0.376^2}$

$= 3.78$ cm

9. (7 marks)

The cost of a listed share in C cents, is modelled by $C = 75\sqrt{1+0.8t}$ for $t \geq 0$, where t is the number of years after 2000.

(a) Determine the cost per share in 2000. [1]

$C(0) = 75$ cents

(b) Determine the average rate of cost rise between 2000 and 2010. [2]

$$\text{Average rate} = \frac{C(10) - C(10)}{10}$$

$$= 15$$

(c) Determine the instantaneous rate of cost rise in 2005. [2]

$$C'(t) = \frac{30\sqrt{5}}{\sqrt{4t+5}}$$

$$C'(5) = 6\sqrt{5}$$

(d) Determine when the instantaneous rate of cost rise is 10 cents per year. [2]

$$C'(t) = 10$$

$$\frac{30\sqrt{5}}{\sqrt{4t+5}} = 10$$

$$t = 10$$

10. (8 marks)

A particle M moves in rectilinear motion such that its acceleration, a, in m/s^2 at any time, t, seconds(s) is given by:

$$a = 6t - 3 \text{ where } t \geq 0.$$

After 2 seconds, the particle's displacement is $-23m$ and it is travelling at a velocity of $-30ms^{-1}$

(a) By first determining the expression of velocity in terms of *t*, calculate the velocity of the particle after 1 second from its origin. [4]

$V = \int a$

$= \int 6t - 3 \, dt$

$= 3t^2 - 3t + c$ ✓

$-30 = 3(2)^2 - 3(2) + c$

$c = -36$ ✓

$\therefore V(t) = 3t^2 - 3t - 36$ ✓

$V(1) = 3(1)^2 - 3(1) - 36$

$= -36 ms^{-1}$ ✓

(b) Determine the distance travelled by particle M from *t*= 2 to *t*= 5. [4]

$x(2) = -23$

$x(t) = \int 3t^2 - 3t - 36$

$= t^3 - \frac{3t^2}{2} - 36t + c$ ✓

$-23 = (2)^3 - \frac{3(2)^2}{2} - 36(2) + c$

$-23 = 8 - 6 - 72 + c$

$c = 47$

$\therefore x(t) = t^3 - \frac{3t^2}{2} - 36t + 47$ ✓

Solve $V(t) = 0$

$V(t) = 0$ when $t = 4$. ✓

$x(4) = -57$

$x(5) = -45.5$

$\therefore$ distance travelled is $45.5m$ ✓