Chapter 9 – Probability

Solutions to Exercise 9A

1 Toss of a coin: sample space = $\{H, T\}$

2 Die rolled: sample space $= \{1, 2, 3, 4, 5, 6\}$

3 **a** 52 cards

b 4 suits

c Spades, hearts, diamonds, clubs

d Hearts, diamonds = red; spades, clubs = black

e 13 cards in each suit.

f 'Picture cards' are Jack, Queen, King and Ace

g 4 aces

h 16 'picture cards'

4 **a** $\{0, 1, 2, 3, 4, 5\}$

b $\{0, 1, 2, 3, 4, 5, 6\}$

c $\{0, 1, 2, 3\}$

5 **a** $\{0, 1, 2, 3, 4, 5 \ldots\}$

b $\{0, 1, 2, 3, 4, 5 \ldots 41\}$

c $\{1, 2, 3, 4, 5 \ldots\}$

6 **a** 'An even number' in die roll $= \{2, 4, 6\}$

b 'More than two female students' $= \{FFF\}$

c 'More than four aces' = $\{\}$ or $\varnothing$

7 $\varepsilon = \{1, 2, 3, \ldots, 20\}$, $n(\varepsilon) = 20$

a Let A be the event the number is divisible by 2.
$A = \{2, 4, \ldots, 20\}$, $n(A) = 10$,
$$\Pr(A) = \frac{n(A)}{n(\varepsilon)} = \frac{10}{20} = \frac{1}{2}$$

b Let B be the event the number is divisible by 3.
$B = \{3, 6, \ldots, 18\}$, $n(A) = 6$,
$$\Pr(B) = \frac{n(B)}{n(\varepsilon)} = \frac{6}{20} = \frac{3}{10}$$

c Let C be the event the number is divisible by both 2 and 3.
$C = \{6, 12, 18\}$, $n(C) = 3$,
$$\Pr(C) = \frac{n(C)}{n(\varepsilon)} = \frac{3}{20}$$

8 $\varepsilon = \{1, 2, 3, \ldots, 15\}$, $n(\varepsilon) = 15$

a Let A be the event the number is less than 5.
$A = \{1, 2, 3, 4\}$, $n(A) = 4$,
$$\Pr(A) = \frac{n(A)}{n(\varepsilon)} = \frac{4}{15}$$

b Let B be the event the number is greater than or equal to 6.
$B = \{6, 7, \ldots, 15\}$, $n(B) = 10$,
$$\Pr(B) = \frac{n(B)}{n(\varepsilon)} = \frac{10}{15} = \frac{2}{3}$$

c Let C be the event the number is a

number from 5 to 8 inclusive.
$C = \{5, 6, 7, 8\}$, $n(C) = 4$,
$\Pr(C) = \dfrac{n(C)}{n(\varepsilon)} = \dfrac{4}{15}$

9 a 13 clubs: $\Pr(\clubsuit) = \dfrac{13}{52} = \dfrac{1}{4}$

b 26 red cards: $\Pr(\text{red}) = \dfrac{26}{52} = \dfrac{1}{2}$

c 16 picture cards: $\Pr(\text{picture}) = \dfrac{16}{52} = \dfrac{4}{13}$

d a red picture card Pr(red picture) = $\dfrac{8}{52} = \dfrac{2}{13}$

10 a 36 cards < 10: $\Pr(< 10) = \dfrac{36}{52} = \dfrac{9}{13}$

b 40 cards ≤ 10: $\Pr(\le 10) = \dfrac{40}{52} = \dfrac{10}{13}$

c Even number = $\{2, 4, 6, 8, 10\}$ so 20 evens: $\Pr(\text{even}) = \dfrac{20}{52} = \dfrac{5}{13}$

d 4 aces: $\Pr(\text{ace}) = \dfrac{4}{52} = \dfrac{1}{13}$

11 a $\Pr(\text{29 November}) = \dfrac{1}{365}$

b $\Pr(\text{November}) = \dfrac{30}{365} = \dfrac{6}{73}$

c 30 days between 15 January and 15 February, not including either day:
$\therefore \Pr = \dfrac{6}{73}$

d 90 (non-leap) days in the first three months of the year: $\therefore \Pr = \dfrac{90}{365} = \dfrac{18}{73}$

12 $\varepsilon = \{A_1, U, S, T, R, A_2, L, I, A_3\}$, $n(\varepsilon) = 9$

a $\Pr(\{T\}) = \dfrac{1}{9}$

b Pr(an A is drawn) =
$\Pr(\{A_1, A_2, A_3\}) = \dfrac{3}{9} = \dfrac{1}{3}$

c Let V be the event a vowel is drawn
$V = \{A_1, A_2, A_3, U, I\}$, $n(V) = 5$
$\Pr(V) = \dfrac{5}{9}$

d Let C be the event a consonant is drawn
$C = \{S, T, R, L\}$, $n(C) = 4$
$\Pr(C) = \dfrac{4}{9}$

13 $\Pr(1) + \Pr(2) + \Pr(3) + \Pr(5) + \Pr(6) + \Pr(4) = 1$

$$\therefore \frac{1}{12} + \frac{1}{6} + \frac{1}{8} + \frac{1}{6} + \frac{1}{8} + \Pr(4) = 1$$

$$\frac{2 + 4 + 3 + 4 + 3}{24} + \Pr(4) = 1$$

$$\therefore \Pr(4) = 1 - \frac{16}{24} = 1 - \frac{2}{3} = \frac{1}{3}$$

14 $\Pr(1) = 0.2, \Pr(3) = 0.1, \Pr(4) = 0.3$
$\Pr(1) + \Pr(3) + \Pr(4) = 0.6$
$\therefore \Pr(2) = 1 - 0.6 = 0.4$

15 a $\Pr(1) = \dfrac{1}{3}$

b $\Pr(1) = \dfrac{1}{8}$

c $\Pr(1) = \dfrac{1}{4}$

16 $\varepsilon = \{M, T, W, Th, F, Sa, Su\}$ $n(\varepsilon) = 7$

a $\Pr(\text{Born on Wednesday}) = \dfrac{1}{7}$

b Pr(Born on a weekend) =
Pr({Sa,Su}) = $\frac{2}{7}$
Pr(Not born on a weekend)= $1 - \frac{2}{7} = \frac{5}{7}$

17 $n(\varepsilon) = 52$

a $n(\text{Club}) = 13$
$\Pr(\text{Club}) = \frac{13}{52} = \frac{1}{4}$
$\Pr(\text{Not Club}) = 1 - \frac{1}{4} = \frac{3}{4}$

b $n(\text{Red}) = 26$
$\Pr(\text{Red}) = \frac{26}{52} = \frac{1}{2}$
$\Pr(\text{Not Red}) = 1 - \frac{1}{2} = \frac{1}{2}$

c Picture cards are Kings, Queens and Jacks
$n(\text{Picture Card}) = 12$
$\Pr(\text{Picture Card}) = \frac{12}{52} = \frac{3}{13}$
$\Pr(\text{Not Red}) = 1 - \frac{3}{13} = \frac{10}{13}$

d $n(\text{Red Picture}) = 6$
$\Pr(\text{Red Picture}) = \frac{6}{52} = \frac{3}{26}$
$\Pr(\text{Not Red}) = 1 - \frac{3}{26} = \frac{23}{26}$

18 $\varepsilon = \{1, 2, 3, 4\}$
$\Pr(1) = \Pr(2) = \Pr(3) = x$ and
$\Pr(4) = 2x$.
$\therefore x + x + x + 2x = 1$
$\therefore x = \frac{1}{5}$
$\therefore \Pr(1) = \Pr(2) = \Pr(3) = \frac{1}{5}$ and
$\Pr(4) = \frac{2}{5}$

19 $\varepsilon = \{1, 2, 3, 4, 5, 6\}$

a $\Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = x$,
$\Pr(6) = 2x$ and $\Pr(1) = \frac{x}{2}$.
$\therefore x + x + x + x + 2x + \frac{x}{2} = 1$
$\therefore \frac{13x}{2} = 1 \therefore x = \frac{2}{13}$
$\therefore \Pr(2) = \Pr(3) = \Pr(4) = \Pr(5) = \frac{2}{13}$
$\Pr(6) = \frac{4}{13}$ and $\Pr(1) = \frac{1}{13}$

b $\frac{9}{13}$

Solutions to Exercise 9B

1 **a** Pr(head) $= \frac{34}{100} = \frac{17}{50} = 0.34$

b Pr(ten) $= \frac{20}{200} = \frac{1}{10} = 0.10$

c Pr(two heads) $= \frac{40}{150} = \frac{4}{15}$

d Pr(three sixes) $= \frac{1}{200}$ or 0.005

2 **a** 20 trials is far too few to obtain reliable data.

b Pr(two heads) $= \frac{1}{4}$, Pr(one head) $= \frac{1}{2}$, Pr(no heads) $= \frac{1}{4}$

c Results may resemble **b**, but could be anything with such a small sample.

d 100 trials is certainly better. For example, with 95% confidence limits, the number of (H, H) results over 20 trials would be between 1 and 9. Over 100 trials we would expect between 16 and 34.

e To find the probabilities exactly would require an infinite number of trials.

3 Die 1 shows Pr(6) $= \frac{78}{500} = 0.156$

Die 2 shows Pr(6) $= \frac{102}{700} = 0.146$

Die 1 has a higher observed probability of throwing a 6.

4 Total number of balls = 400; 340 red and 60 black.

Proportion of red $= \frac{340}{400} = \frac{17}{20} = 0.85$

b Proportion of red in sample =
$= \frac{48}{60} = \frac{4}{5} = 0.8$

c Proportion of red in sample =
$= \frac{54}{60} = \frac{9}{10} = 0.9$

d Expected number of red balls$= 0.85 \times 60 = 51$

5 Estimate of probability
$= \frac{890}{2000} = \frac{89}{200} = 0.445$

6 **a** Area of blue section
$= \frac{\pi(1)^2}{4} = \frac{\pi}{4} \approx 0.7855$
Area of square$= 1 \times 1 = 1$.

Proportion of square that is blue$= \frac{\pi}{4} \approx 0.7855$

b Probability of hitting the blue region$= \frac{\pi}{4} \approx 0.7855$

7 Area of board$= \pi(14)^2 = 196\pi$
Area of shaded region $= \pi(14)^2 - \pi(7)^2$

$= 196\pi - 49\pi$

$= 147\pi$

Probabilty that the dart will hit the shaded area$= \frac{147}{196} = \frac{3}{4}$

8 **a** Pr(Red section) $= \frac{120}{360} = \frac{1}{3}$

b Pr(Yellow section) $= \frac{60}{360} = \frac{1}{6}$

c Pr(Not Yellow section) $= 1 - \frac{1}{6} = \frac{5}{6}$

9 Area of square = 1 m^2.
Area of circle $= \pi \times 0.4^2 = 0.16\pi$

a Probability of hitting the shaded part $= 0.16\pi$

b Probability of hitting the unshaded part $= 1 - 0.16\pi \approx 0.4973$

10 **a** **i** Area of square= x^2

ii Area of larger circle
$= \pi(\frac{x}{2})^2 = \frac{1}{4}\pi x^2$

iii Area of smaller circle
$= \pi(\frac{x}{4})^2 = \frac{1}{16}\pi x^2$

b **i** Probability of landing inside the smaller circle= $\dfrac{\frac{1}{16}\pi x^2}{x^2} = \dfrac{\pi}{16}$

ii Probability of landing inside the smaller circle= $\dfrac{(\frac{1}{4} - \frac{1}{16})\pi x^2}{x^2} = \dfrac{3\pi}{16}$

iii Probability of landing in the outer shaded region= $\dfrac{x^2 - \frac{1}{4}\pi x^2}{x^2} = 1 - \dfrac{\pi}{4}$

Solutions to Exercise 9C

1 $\varepsilon = \{HH, HT, TH, TT\}$

a Pr(No heads) = Pr($\{TT\}$) = $\frac{1}{4}$.

b Pr(More than one tail) = Pr($\{TT\}$) = $\frac{1}{4}$.

2 **a** Pr(First toss is a head) = $\frac{1}{2}$

b Pr(Second toss is a head) = $\frac{1}{2}$

c Pr(Both tosses are heads) = $\frac{1}{4}$

3 Sample space =
$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
There is only 1 way of getting 2 or 12, 2 ways of getting 3 or 11, 3 ways of getting 4 or 10 etc.

a
$$\begin{aligned}\Pr(\text{even}) &= \Pr(2) + \Pr(4) + \Pr(6) \\ &\quad + \Pr(8) + \Pr(10) + \Pr(12) \\ &= \frac{1+3+5+5+3+1}{36} \\ &= \frac{1}{2}\end{aligned}$$

b $\Pr(3) = \frac{2}{36} = \frac{1}{18}$

c
$$\begin{aligned}\Pr(< 6) &= \Pr(2) + \Pr(3) \\ &\quad + \Pr(4) + \Pr(5) \\ &= \frac{1+2+3+4}{36} \\ &= \frac{10}{36} = \frac{5}{18}\end{aligned}$$

4 Sample space =
$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

a $\Pr(10) = \frac{3}{36} = \frac{1}{12}$

b
$$\begin{aligned}\Pr(\text{odd}) &= \Pr(3) + \Pr(5) + \Pr(7) \\ &\quad + \Pr(9) + \Pr(11) \\ &= \frac{2+4+6+4+2}{36} \\ &= \frac{1}{2}\end{aligned}$$

c
$$\begin{aligned}\Pr(\le 7) &= \frac{1+2+3+4+5+6}{36} \\ &= \frac{21}{36} = \frac{7}{12}\end{aligned}$$

5 $\varepsilon =$
$\{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

a Pr(exactly one tail) =
Pr($\{HHT, HTH, THH\}$) = $\frac{3}{8}$

b Pr(exactly two tails) =
Pr($\{HTT, TTH, THT\}$) = $\frac{3}{8}$

c Pr(exactly three tails) = Pr($\{TTT\}$) = $\frac{1}{8}$

d Pr(no tails) = Pr($\{HHH\}$) = $\frac{1}{8}$

6 $\varepsilon =$
$\{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

a Pr(the third toss is a head) =
Pr($\{HHH, HTH, THH, TTH\}$) = $\frac{1}{2}$

b
Pr(second and third tosses are heads) =

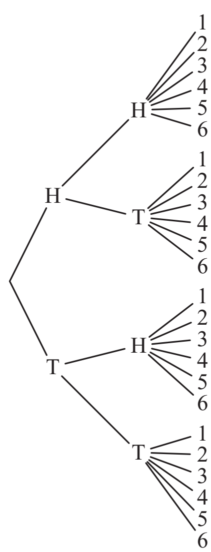
$\Pr(\{HHH, THH\}) = \frac{1}{4}$

c Pr(at least one head and one tail) = $\Pr(\{HHT, HTH, THH, TTH, THT, HTT, \}) = \frac{3}{4}$

7 12 equally likely outcomes:

$$\Pr(\text{even}, H) = \Pr(2, H) + \Pr(4, H) + \Pr(6, H) = \frac{3}{12} = \frac{1}{4}$$

8 a



b i

$$\Pr(\text{2 heads and a 6}) = \Pr(\{(H, H, 6)\} = \frac{1}{24}$$

ii

Pr(1 head, 1 tail and an even number)

$= \Pr(\{(H, T, 6), (H, T, 4), (H, T, 2)$

$, (T, H, 6), (T, H, 4), (T, H, 2)\}$

$= \frac{6}{24}$

$= \frac{1}{4}$

iii Pr(2 tails and an odd number)

$= \Pr(\{(T, T, 1), (T, T, 3), (T, T, 5)\}$

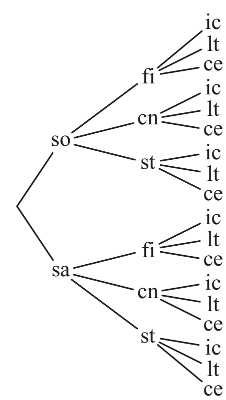
$= \frac{3}{24}$

$= \frac{1}{8}$

iv Pr(an odd number on the die)

$= \frac{1}{2}$

9 a



b i

$$\Pr(\text{soup, fish and lemon tart}) = \Pr(\{(so, fi, it)\} = \frac{1}{18}$$

ii Pr(fish)

$= \frac{1}{3}$

iii

Pr(salad and chicken)

$= \Pr(\{(sa, c, lt), (sa, c, ic)), (sa, c, ce))\}$

$= \frac{3}{18}$

$= \frac{1}{6}$

iv Pr(no lemon tart)

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

c This increases the number of choices for the entree to 3 and the dessert 4. There are $3 \times 3 \times 4 = 36$ choices.

i Pr(soup, fish and lemon tart)

$= \Pr(\{(so, fi, it)\}$

$= \frac{1}{36}$

ii Pr(all courses)

$= \frac{1}{2}$

iii Pr(only two courses)

$= \frac{15}{36}$

$= \frac{5}{12}$

iv Pr(only the main courses)

$= \frac{3}{36}$

$= \frac{1}{12}$

10 a (1, 1)(2, 1)(3, 1)(4, 1)(5, 1)
(1, 2)(2, 2)(3, 2)(4, 2)(5, 2)
(1, 3)(2, 3)(3, 3)(4, 3)(5, 3)
(1, 4)(2, 4)(3, 4)(4, 4)(5, 4)
(1, 5)(2, 5)(3, 5)(4, 5)(5, 5)

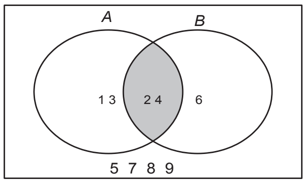
b i $\Pr(5) = \frac{4}{25}$

ii Pr(different) = 1 − Pr(same) = $1 - \frac{1}{5} = \frac{4}{5}$

iii Pr(second number two more than first number) = $\frac{3}{25}$

Solutions to Exercise 9D

1 $\in \ = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$A = \{1, 2, 3, 4\},\ B = \{2, 4, 6\}$.



a $A \cup B = \{1, 2, 3, 4, 6\}$

b $A \cap B = \{2, 4\}$

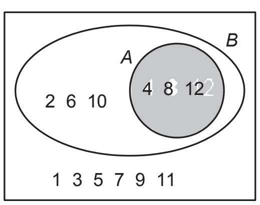
c $A' = \{5, 6, 7, 8, 9, 10\}$

d $A \cap B' = \{1, 3\}$

e $(A \cap B)' = \{1, 3, 5, 6, 7, 8, 9, 10\}$

f $(A \cup B)' = \{5, 7, 8, 9, 10\}$

2 $\in \ = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{\text{multiples of four}\}$
$B = \{\text{even numbers}\}$



a $A' = \{1, 2, 3, 5, 6, 7, 9, 10, 11\}$

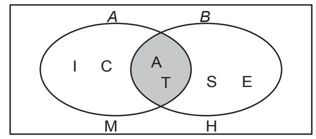
b $B' = \{1, 3, 5, 7, 9, 11\}$

c $A \cup B = \{2, 4, 6, 8, 10, 12\}$

d $(A \cup B)' = B' = \{1, 3, 5, 7, 9, 11\}$

e $A' \cap B' = B' = \{1, 3, 5, 7, 9, 11\}$

3 $\in \ = \{\text{MATHEICS}\}, A = \{\text{ATIC}\}, B = \{\text{TASE}\}$



a $A' = \{\text{E, H, M, S,}\}$

b $B' = \{\text{C, H, I, M}\}$

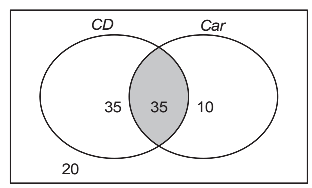
c $A \cup B = \{\text{A, C, E, I, S, T}\}$

d $(A \cup B)' = \{\text{H, M}\}$

e $A' \cup B' = \{\text{C, E, H, I, M, S}\}$

f $A' \cap B' = \{\text{H, M}\}$

4



$\varepsilon = 100$ students

a 20 students own neither a car nor smart phone.

b 45 students own either but not both.

5 $\varepsilon = \{1, 2, 3, 4, 5, 6\}$;
$A = \{2, 4, 6\},\ B = \{3\}$

a $(A \cup B) = \{2, 3, 4, 6\}$
$\therefore \Pr(A \cup B) = \dfrac{2}{3}$

b $(A \cap B) = \{\}$
$\therefore \Pr(A \cap B) = 0$

c $A' = \{1, 3, 5\}$
$\therefore \Pr(A') = \dfrac{1}{2}$

d $B' = \{1, 2, 4, 5, 6\} \therefore \Pr(B') = \dfrac{5}{6}$

6 $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$;
$A = \{2, 4, 6, 8, 10, 12\}$, $B = \{3, 6, 9, 12\}$

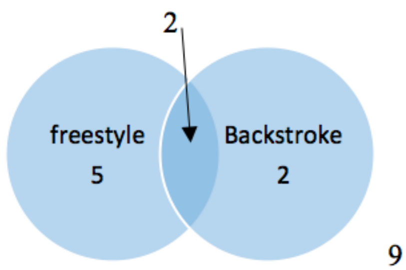
a $\Pr(A) = \dfrac{6}{12} = \dfrac{1}{2}$

b $\Pr(B) = \dfrac{4}{12} = \dfrac{1}{3}$

c $\{A \cap B\} = \{6, 12\}, \therefore \Pr(A \cap B) = \dfrac{1}{6}$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$= \frac{2}{3}$$

7



a $\Pr(\text{Swims freestyle}) = \dfrac{7}{18}$

b $\Pr(\text{Swims backstroke}) = \dfrac{4}{18} = \dfrac{2}{9}$

c $\Pr(\text{Swims freestyle and backstroke}) = \dfrac{2}{18} = \dfrac{1}{9}$

d $\Pr(\text{is on the swimming team}) = \dfrac{9}{18} = \dfrac{1}{2}$

8 $A = \{1, 2, 3, 4, 6, 12\}$ and $B = \{2, 3, 5, 7\}$

a $\Pr(A) = \dfrac{6}{20} = \dfrac{3}{10}$

b $\Pr(B) = \dfrac{4}{20} = \dfrac{1}{5}$

c $\Pr(A \cap B) = \dfrac{2}{20} = \dfrac{1}{10}$

d $\Pr(A \cup B) = \dfrac{8}{20} = \dfrac{2}{5}$

9 $\Pr(A) = 0.5, \Pr(B) = 0.4$, and
$\Pr(A \cap B) = 0.2$.
$\Pr(A \cup B) = 0.5 + 0.4 - 0.2 = 0.7$

10 $\Pr(A) = 0.35, \Pr(B) = 0.24$, and
$\Pr(A \cap B) = 0.12$.
$\Pr(A \cup B) = 0.35 + 0.24 - 0.12 = 0.47$

11 $\Pr(A) = 0.28, \Pr(B) = 0.45$, and $A \subset B$

a $\Pr(A \cap B) = \Pr(B) = 0.28$

b

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$= 0.28 + 0.45 - 0.28$$
$$= 0.45$$

12 $\Pr(A) = 0.58, \Pr(B) = 0.45$, and $B \subset A$

a $\Pr(A \cap B) = \Pr(B) = 0.45$

b

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$= 0.45 + 0.58 - 0.45$$
$$= 0.58$$

13 $\Pr(A) = 0.3, \Pr(B) = 0.4$, and $A \cap B = \varnothing$

a $\Pr(A \cap B) = 0$

b

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ &= 0.3 + 0.4 - 0\\ &= 0.7\end{aligned}$$

14 $\Pr(A) = 0.08, \Pr(B) = 0.15$, and $A \cap B = \varnothing$

a $\Pr(A \cap B) = 0$

b

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ &= 0.08 + 0.15 - 0\\ &= 0.23\end{aligned}$$

15 $\Pr(A) = 0.3, \Pr(B) = 0.4$, and $A \cup B = 0.5$

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ 0.5 &= 0.3 + 0.4 - \Pr(A \cap B)\end{aligned}$$

$\therefore \Pr(A \cap B) = 0.2$

16 $\Pr(A) = 0.24, \Pr(B) = 0.44$, and $A \cup B = 0.63$

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ 0.63 &= 0.24 + 0.44 - \Pr(A \cap B)\end{aligned}$$

$\therefore \Pr(A \cap B) = 0.05$

17 $\Pr(A) = 0.3, \Pr(B) = 0.4$, and $A \cap B' = 0.2$

$$\begin{aligned}\Pr(A \cup B') &= \Pr(A) + \Pr(B') - \Pr(A \cap B')\\ &= 0.3 + 0.6 - 0.2\\ &= 0.7\end{aligned}$$

18 Pr(Soccer) = 0.18, Pr(Tennis) = 0.25 and Pr(Soccer and Tennis) = 0.11

Pr(Soccer **or** Tennis) = 0.18 + 0.25 − 0.11
= 0.32

19 Pr(Chinese) = 0.22, Pr(French) = 0.35 and Pr(Chinese and French) = 0.14

a

Pr(Chinese **or** French) = 0.22 + 0.35 − 0.14
= 0.43

b Probability of exactly oneof these languages
$= \Pr(C \cup F) - \Pr(C \cap F) = 0.29$

Solutions to Exercise 9E

1 $\Pr(A) = 0.6,\ \Pr(A \cap B) = 0.4,$
$\Pr(A' \cap B) = 0.1$

	B	B'	
A	$\Pr(A \cap B) = 0.4$	$\Pr(A \cap B') = 0.2$	$\Pr(A) = 0.6$
A'	$\Pr(A' \cap B) = 0.1$	$\Pr(A' \cap B') = 0.3$	$\Pr(A) = 0.4$
	$\Pr(B) = 0.5$	$\Pr(B') = 0.5$	1

a $\Pr(A \cap B') = 0.2$

b $\Pr(B) = 0.5$

c $\Pr(A' \cap B') = 0.3$

d $\Pr(A \cup B) = 1 - 0.3 = 0.7$

2 $\Pr(A') = 0.25,\ \Pr(A' \cap B) =$
$0.12, \Pr(B) = 0.52:$

	B	B'	
A	$\Pr(A \cap B) = 0.4$	$\Pr(A \cap B') = 0.35$	$\Pr(A) = 0.75$
A'	$\Pr(A' \cap B) = 0.12$	$\Pr(A' \cap B') = 0.13$	$\Pr(A') = 0.25$
	$\Pr(B) = 0.52$	$\Pr(B') = 0.48$	1

a $\Pr(A) = 0.75$

b $\Pr(A \cap B) = 0.4$

c $\Pr(A \cup B) = 1 - 0.13 = 0.87$

d $\Pr(B') = 0.48$

3 $\Pr(C \cup D) = 0.85$
$\therefore \Pr(C' \cap D') = 0.15,\ \Pr(C) = 0.45$
and $\Pr(D') = 0.37$:

	D	D'	
C	$\Pr(C \cap D) = 0.23$	$\Pr(C \cap D') = 0.22$	$\Pr(\mathrm{C}) = 0.45$
C'	$\Pr(C' \cap D) = 0.4$	$\Pr(C' \cap D') = 0.15$	$\Pr(C') = 0.55$
	$\Pr(D) = 0.63$	$\Pr(D') = 0.37$	1

a $\Pr(D) = 0.63$

b $\Pr(C \cap D) = 0.23$

c $\Pr(C \cap D') = 0.22$

d $\Pr(C' \cup D') = 1 - 0.23 = 0.77$

4 $\Pr(E \cup F) = 0.7$
$\therefore \Pr(E' \cap F') = 0.3$
$\Pr(E \cap F) = 0.15, \Pr(E') = 0.55:$

	F	F'	
E	$\Pr(E \cap F) = 0.15$	$\Pr(E \cap F') = 0.3$	$\Pr(E) = 0.45$
E'	$\Pr(E' \cap F) = 0.25$	$\Pr(E' \cap F') = 0.3$	$\Pr(E') = 0.55$
	$\Pr(F) = 0.4$	$\Pr(F') = 0.6$	1

a $\Pr(E) = 0.45$

b $\Pr(F) = 0.4$

c $\Pr(E' \cap F) = 0.25$

d $\Pr(E' \cup F) = 1 - 0.3 = 0.7$

5 $\Pr(A) = 0.8,\ \Pr(B) = 0.7,$
$\Pr(A' \cap B') = 0.1$:

	B	B'	
A	$\Pr(A \cap B) = \boldsymbol{0.6}$	$\Pr(A \cap B') = \boldsymbol{0.2}$	$\Pr(A) = 0.8$
A'	$\Pr(A' \cap B) = \boldsymbol{0.1}$	$\Pr(A' \cap B') = 0.1$	$\Pr(A') = \boldsymbol{0.2}$
	$\Pr(B) = 0.7$	$\Pr(B') = \boldsymbol{0.3}$	1

a $\Pr(A \cup B) = 1 - 0.1 = 0.9$

b $\Pr(A \cap B) = 0.6$

c $\Pr(A' \cap B) = 0.1$

d $\Pr(A \cup B') = 1 - 0.1 = 0.9$

6 $\Pr(G) = 0.85,\ \Pr(L) = 0.6,$
$\Pr(L \cup G) = 0.5$:

	L	L'	
G	$\Pr(G \cap L) = 0.5$	$\Pr(G \cap L') = \boldsymbol{0.35}$	$\Pr(G) = 0.85$
G'	$\Pr(G' \cap L) = \boldsymbol{0.1}$	$\Pr(G' \cap L') = \boldsymbol{0.05}$	$\Pr(G') = \boldsymbol{0.15}$
	$\Pr(L) = 0.6$	$\Pr(L') = \boldsymbol{0.4}$	1

a $\Pr(G \cup L) = 1 - 0.05 = 0.95$, so 95% favoured at least one proposition.

b $\Pr(G' \cap L') = 0.05$, so 5% favoured neither proposition

7 a

	C	C'	
A	$\Pr(A \cap C) = \frac{4}{52}$	$\Pr(A \cap C') = \frac{12}{52}$	$\Pr(A) = \frac{16}{52}$
A'	$\Pr(A' \cap C) = \frac{9}{52}$	$\Pr(A' \cap C') = \frac{27}{52}$	$\Pr(A') = \frac{36}{52}$
	$\Pr(C) = \frac{13}{52}$	$\Pr(C') = \frac{39}{52}$	1

b **i** $\Pr(A) = \frac{16}{52} = \frac{4}{13}$ (all picture cards)

ii $\Pr(C) = \frac{13}{52} = \frac{1}{4}$ (all hearts)

iii $\Pr(A \cap C) = \frac{4}{52} = \frac{1}{13}$ (picture hearts)

iv $\Pr(A \cup C) = \frac{25}{52}$ (all hearts or pictures)

v $\Pr(A \cup C') = \frac{43}{52}$ (all club, diamond and spades or pictures)

8 $\Pr(M \cap F) = \frac{1}{6}$ or $\frac{10}{60}$
$\Pr(M) = \frac{3}{10} = \frac{18}{60}$
$\Pr(F') = \frac{7}{15} = \frac{28}{60}$

	F	F'	
M	$\Pr(M \cap F) = \frac{10}{60}$	$\Pr(M \cap F') = \frac{8}{60}$	$\Pr(M) = \frac{18}{60}$
M'	$\Pr(M \cap F) = \frac{22}{60}$	$\Pr(M \cap F') = \frac{20}{60}$	$\Pr(M) = \frac{42}{60}$
	$\Pr(F) = \frac{32}{60}$	$\Pr(F') = \frac{28}{60}$	60

a $\Pr(F) = \frac{32}{60} = \frac{8}{15}$

b $\Pr(M') = \frac{42}{60} = \frac{7}{10}$

c $\Pr(M \cap F') = \frac{8}{60}$ or $\frac{2}{15}$

d $\Pr(M' \cap F') = \frac{20}{60}$ or $\frac{1}{3}$

9 $\Pr(F) = 0.65$
$\Pr(W) = 0.72$
$\Pr(W' \cap F') = 0.2$

	F	F'	
W	$\Pr(W \cap F)$ $= \boldsymbol{0.57}$	$\Pr(W \cap F')$ $= \boldsymbol{0.15}$	$\Pr(W)$ $= 0.72$
W'	$\Pr(W' \cap F)$ $= \boldsymbol{0.08}$	$\Pr(W' \cap F')$ $= 0.2$	$\Pr(W')$ $= \boldsymbol{0.28}$
	$\Pr(F) = 0.65$	$\Pr(F') = \mathbf{0.35}$	1

a $\Pr(W \cup F) = 1 - 0.2 = 0.8$

b $\Pr(W \cap F) = 0.57$

c $\Pr(W') = 0.28$

d $\Pr(W' \cap F) = 0.08$

10 $\Pr(H \cap N') = 0.05$
$\Pr(H' \cap N) = 0.12$
$\Pr(N') = 0.19$

	N	N'	
H	$\Pr(H \cap N)$ $= \boldsymbol{0.69}$	$\Pr(H \cap N')$ $= 0.05$	$\Pr(H)$ $= \boldsymbol{0.74}$
H'	$\Pr(H' \cap N)$ $= 0.12$	$\Pr(H' \cap N')$ $= \boldsymbol{0.14}$	$\Pr(H')$ $= \boldsymbol{0.26}$
	$\Pr(N) = \mathbf{0.81}$	$\Pr(N') = 0.19$	1

a $\Pr(N) = 0.81$

b $\Pr(H \cap N) = 0.69$

c $\Pr(N) = 0.74$

d $\Pr(H \cup N) = 1 - 0.14 = 0.86$

11 $\Pr(B) = \frac{40}{60} = \frac{2}{3}$
$\Pr(S) = \frac{32}{60} = \frac{8}{15}$
$\Pr(B' \cap S') = 0$

	B	B'	
S	$\Pr(S \cap B)$ $= \frac{12}{60}$	$\Pr(S \cap B')$ $= \frac{20}{60}$	$\Pr(S)$ $= \frac{32}{60}$
S'	$\Pr(S' \cap B)$ $= \frac{28}{60}$	$\Pr(S' \cap B')$ $= 0$	$\Pr(S')$ $= \frac{28}{60}$
	$\Pr(B) = \frac{40}{60}$	$\Pr(B') = \frac{20}{60}$	60

a $\Pr(B' \cap S') = 0$

b $\Pr(B \cup S) = 1$

c $\Pr(B \cap S) = \frac{12}{60} = \frac{1}{5} = 0.2$

d $\Pr(B' \cap S) = \frac{20}{60} = \frac{1}{3}$

12 $\Pr(H) = \frac{35}{50} = 0.7$
$\Pr(S) = \frac{38}{50} = 0.76$
$\Pr(H' \cap S') = \frac{6}{50} = 0.12$

	H	H'	
S	$\Pr(S \cap H)$ $= \boldsymbol{0.58}$	$\Pr(S \cap H')$ $= \boldsymbol{0.18}$	$\Pr(S)$ $= 0.76$
S'	$\Pr(S' \cap H)$ $= \boldsymbol{0.12}$	$\Pr(S' \cap H')$ $= 0.12$	$\Pr(S')$ $= \boldsymbol{0.24}$
	$\Pr(H)$ $= 0.7$	$\Pr(H')$ $= \boldsymbol{0.3}$	1

a $\Pr(H \cup S) = 1 - 0.12 = 0.88$

b $\Pr(H \cap S) = 0.58$

c $\Pr(H' \cap S) + \Pr(H \cap S')$

$= 0.12 + 0.18$

$= 0.3$

d $\Pr(H \cap S') = 0.12$

Solutions to Exercise 9F

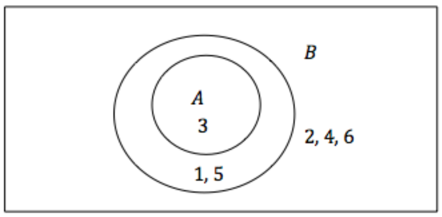
1 $A = \{6\}, \; B = \{3, 4, 5, 6\}$

$\therefore A \cap B = \{6\}$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

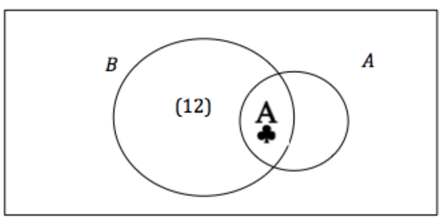
$$= \frac{1}{6} \div \frac{4}{6} = \frac{1}{4}$$

2



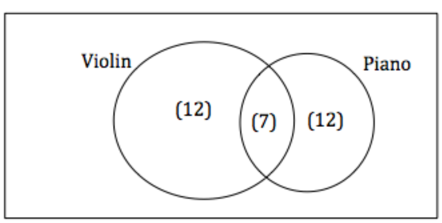
$\Pr(A|B) = \frac{1}{3}$

3



$\Pr(A|B) = \frac{1}{13}$

4



$\Pr(\text{Violin}|\text{Piano}) = \frac{7}{19}$

5 $\Pr(\text{Double six}|\text{ A double}) = \frac{1}{6}$

6 **a** $\Pr(\text{iPad}|\text{ iPhone}) = \frac{4}{17}$

b $\Pr(\text{iPhone}|\text{ iPad}) = \frac{4}{7}$

7 $\Pr(\text{Think yes}|\text{ Male}) = \frac{35}{60} = \frac{7}{12}$

8 **a** $\Pr(\text{Prefers sport}) = \frac{375}{500} = \frac{3}{4}$

b $\Pr(\text{Prefers sport}|\text{Male}) = \frac{225}{300} = \frac{3}{4}$

9

$\cap$	S	A	R	O	T
F	42	61	22	12	137
NF	88	185	98	60	431
Tot	130	246	120	72	568

a $\Pr(S) = \frac{130}{568} = \frac{65}{284}$

b $\Pr(F) = \frac{137}{568}$

c $$\Pr(F|S) = \frac{\Pr(F \cap S)}{\Pr(S)}$$

$$= \frac{42}{568} \div \frac{130}{568}$$

$$= \frac{42}{130} = \frac{21}{65}$$

d $$\Pr(F|A) = \frac{\Pr(F \cap A)}{\Pr(A)})$$

$$= \frac{61}{568} \div \frac{246}{568} = \frac{61}{246}$$

10 $\Pr(A) = 0.6, \Pr(B) = 0.3, \Pr(B|A) = 0.1$

a $\Pr(A \cap B) = \Pr(B|A) \times \Pr(A) = 0.06$

b $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

$= \dfrac{0.06}{0.3} = 0.2$

11 **a** $\Pr(B|A) = \dfrac{\Pr(A \cap B)}{\Pr(A)}$

$= \dfrac{04}{0.7} = \dfrac{4}{7}$

b $\Pr(A \cap B) = \Pr(A|B) \times \Pr(B)$

$= 0.6(0.5) = 0.3$

c $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

$\therefore \Pr(B) = \dfrac{\Pr(A \cap B)}{\Pr(A|B)}$

$= \dfrac{03}{0.44} = \dfrac{15}{22}$

12 $\Pr(A) = 0.5,\ \Pr(B) = 0.4,\ \Pr(A \cup B) = 0.7$
$\Pr(A \cap B) + \Pr(A \cup B) = \Pr(A) + \Pr(B)$

$\cap$	B	B'		
A	0.2	0.3	0.5	$\Pr(A)$
A'	0.2	0.3	0.5	$\Pr(A')$
	0.4	0.6	1	
	$\Pr(B)$	$\Pr(B')$		

a $\Pr(A \cap B) = 0.5 + 0.4 - 0.7 = 0.2$

b $\Pr(A|B) = \dfrac{0.2}{0.4} = 0.5$

c $\Pr(B|A) = \dfrac{0.2}{0.5} = 0.4$

13 $\Pr(A) = 0.6,\ \Pr(B) = 0.54,$
$\Pr(A \cap B') = 0.4$

$\cap$	B	B'		
A	0.2	0.4	0.6	$= \Pr(A)$
A'	0.34	0.06	0.4	$= \Pr(A')$
	0.54	0.46	1	
	$= \Pr(B)$	$= \Pr(B')$		

a $\Pr(A \cap B) = 0.2$

b $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

$= \dfrac{0.2}{0.54} = \dfrac{10}{27}$

c $\Pr(B|A) = \dfrac{\Pr(A \cap B)}{\Pr(A)}$

$= \dfrac{0.2}{0.6} = \dfrac{1}{3}$

14 $\Pr(A) = 0.4, \Pr(B) = 0.5, \Pr(A|B) = 0.6$

a $\Pr(A \cap B) = \Pr(A|B) \times \Pr(B) = 0.3$

b $\Pr(B|A) = \dfrac{0.3}{0.4}$

$= \dfrac{3}{4} = 0.75$

15 $\Pr(H) = 0.6,\ \Pr(W|H) = 0.8$
$\therefore\ \Pr(H \cap W) = \Pr(W|H) \times \Pr(H)$

$= 0.8(0.6) = 0.48$

$\Pr(W|H') = 0.4$
$\therefore\ \Pr(H' \cap W) = \Pr(W|H') \times \Pr(H')$

$= 0.4^2 = 0.16$

$\cap$	W	W'		
H	0.48	0.12	0.6	$\Pr(H)$
H'	0.16	0.24	0.4	$\Pr(H')$
	0.64	0.36	1	
	$\Pr(W)$	$\Pr(W')$		

$\Pr(H' \cap W) = 0.16 = 16\%$

16 $\Pr(C) = 0.15,\ \Pr(F) = 0.08,$
$\Pr(C \cap F) = 0.03$

$$\Pr(F|C) = \frac{\Pr(C \cap F)}{\Pr(C)} = \frac{0.03}{0.15} = \frac{1}{5}$$

17 (with replacement)

a $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

b $\Pr(A,\ A) = \left(\frac{1}{13}\right)^2 = \frac{1}{169}$

c $\Pr(R,\ B) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

d If the picture cards are Knight, King, Queen, Ace then
$\Pr(P,\ P) = \left(\frac{4}{13}\right)^2 = \frac{16}{169}$
If only Knight, King, Queen
$\Pr(P,\ P) = \left(\frac{3}{13}\right)^2 = \frac{9}{169}$

18 (without replacement)

a $\left(\frac{13}{52}\right)\left(\frac{12}{51}\right) = \frac{1}{17}$

b $\Pr(A,\ A) = \left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = \frac{1}{221}$

c $\Pr(R,\ B) = \left(\frac{26}{52}\right)\left(\frac{26}{51}\right) = \frac{13}{51}$

d $\Pr(P,\ P) = \left(\frac{16}{52}\right)\left(\frac{15}{51}\right) = \frac{20}{221}$

19 $\Pr(W) = 0.652,\ \Pr(A|W) = 0.354$

$$\Pr(A \cap W) = \Pr(A|W) \times \Pr(W) = 0.231$$

20 $\varepsilon = 28,\ G = 15,\ B = 14 = (6G + 8G')$

$\therefore B' = (9G + 5G')$

a $\Pr(G) = \frac{15}{28}$

b $\Pr(B) = \frac{14}{28} = \frac{1}{2}$

c $\Pr(B') = 1 - \frac{1}{2} = \frac{1}{2}$

d $\Pr(B|G) = \frac{\Pr(G \cap B)}{\Pr(G)} = \frac{6}{28} \div \frac{15}{28} = \frac{2}{5}$

e $\Pr(G|B) = \frac{\Pr(G \cap B)}{\Pr(B)} = \frac{6}{28} \div \frac{14}{28} = \frac{3}{7}$

f $\Pr(B|G') = \frac{\Pr(G' \cap B)}{\Pr(G')} = \frac{8}{28} \div \frac{13}{28} = \frac{8}{13}$

g $\Pr(B' \cap G') = \frac{5}{28}$

h $\Pr(B \cap G) = \frac{6}{28} = \frac{3}{14}$

21 **a** $\Pr(R) = 0.85$

b $\Pr(L|R) = 0.60$

c $\Pr(L \cap R) = \Pr(L|R) \times \Pr(R) = 0.51$

d $\Pr(L) = 0.51$ since L is a subset of R.

22 U = 'students who prefer not to wear a uniform'
E = 'students in Yr 11'
E' = 'students in Yr 12'
$\Pr(U|E) = 0.25 = \frac{1}{4}$

$\Pr(U|E') = 0.40 = \dfrac{2}{5}$

$\Pr(E) = 320/600 = \dfrac{8}{15}$

$$\begin{aligned}\Pr(U \cap E) &= \Pr(U|E) \times \Pr(E)\\ &= \left(\frac{8}{15}\right)\frac{1}{4} = \frac{2}{15}\end{aligned}$$

$$\begin{aligned}\Pr(U \cap E') &= \Pr(U|E')\Pr(E')\\ &= \left(\frac{7}{15}\right)\frac{2}{5} = \frac{14}{75}\end{aligned}$$

$$\begin{aligned}\therefore \Pr(U) &= \Pr(U \cap E') + \Pr(U \cap E)\\ &= \frac{2}{15} + \frac{14}{75} = \frac{24}{75} = 32\%\end{aligned}$$

However, these are students who prefer *not* to wear uniform.
Students in favour are therefore 68%.

23 $\Pr(B \cap G) = 0.4\left(\dfrac{4}{9}\right) = 0.178$

$\Pr(B \cap G') = 0.35\left(\dfrac{5}{9}\right) = 0.194$

$\Pr(B' \cap G) = 0.6\left(\dfrac{4}{9}\right) = 0.267$

$\Pr(B' \cap G') = 0.65\left(\dfrac{5}{9}\right) = 0.361$

$\cap$	B	B'	
G	0.178	0.267	0.444
G'	0.194	0.361	0.556
	0.372	0.628	1

a **i** $\Pr(G) = \dfrac{400}{900} = 0.444$

ii $\Pr(B|G) = 0.40$ (40%)

iii $\Pr(B|G') = 0.35$ (35%)

iv $$\begin{aligned}\Pr(B \cap G) &= \Pr(B|G) \times \Pr(G)\\ &= 0.4(0.444) = 0.178\end{aligned}$$

v $$\begin{aligned}\Pr(B \cap G') &= \Pr(B|G') \times \Pr(G')\\ &= 0.35\left(\frac{500}{900}\right) \cong 0.194\end{aligned}$$

b $\Pr(B) = \dfrac{335}{900} \cong 0.372$

c **i** $$\begin{aligned}\Pr(G|B) &= \frac{\Pr(B \cap G)}{\Pr(B)}\\ &= \frac{0.178}{0.372} \cong 0.478\end{aligned}$$

ii $$\begin{aligned}\Pr(G|B') &= \frac{\Pr(B' \cap G)}{\Pr(B')}\\ &= \frac{0.267}{0.628} = 0.425\end{aligned}$$

24 N': 12% of $480 = D$;
N: 5% of $620 = D$

a **i** $$\begin{aligned}\Pr(N) &= \frac{620}{620 + 480}\\ &\approx 0.564\end{aligned}$$

ii $\Pr(D|N) = 0.05$ (5%)

iii $\Pr(D|N') = 0.12$ (= 12%)

iv $$\begin{aligned}\Pr(D \cap N) &= \Pr(D|N) \times \Pr(N)\\ &= 0.05(0.563)\\ &= 0.0282\end{aligned}$$

v
$$\begin{aligned}\Pr(D \cap N') &= \Pr(D|N') \times \Pr(N')\\ &= 0.12(0.437) \approx 0.052\end{aligned}$$

b $12\%(480) + 5\%(620) = \dfrac{89}{1100} = 0.081$

c $$\begin{aligned}\Pr(N|D) &= \frac{\Pr(D \cap N)}{\Pr(D)}\\ &= \frac{0.028}{0.081} \approx 0.35\end{aligned}$$

25 $B1 = 3M,\ 3M'$; $B2 = 3M,\ 2M'$;
$B3 = 2M,\ 1M'$

a $\Pr(M \cap B1) = \frac{1}{3}\left(\frac{1}{2}\right) = \frac{1}{6}$

b $$\begin{aligned}\Pr(M) &= \Pr(M \cap B1) + \Pr(M \cap B2) \\ &\quad + \Pr(M \cap B3) \\ &= \frac{1}{3}\left(\frac{1}{2}\right) + \frac{1}{3}\left(\frac{3}{5}\right) + \frac{1}{3}\left(\frac{2}{3}\right) \\ &= \frac{1}{6} + \frac{1}{5} + \frac{2}{9} = \frac{53}{90}\end{aligned}$$

c $$\begin{aligned}\Pr(B1|M) &= \frac{\Pr(M \cap B1)}{\Pr(M)} \\ &= \frac{1}{6} \div \frac{53}{90} = \frac{15}{53}\end{aligned}$$

26 $A,\ B \neq \varnothing$

a $\Pr(A|B) = 1$
$\therefore \Pr(A \cap B) = \Pr(B)$
$\therefore B$ is a subset of A, i.e. $B \subseteq A$

b $\Pr(A|B) = 0$
$\therefore A$ and B are mutually exclusive or $A \cap B = \varnothing$

c $\Pr(A|B) = \frac{\Pr(A)}{\Pr(B)}$
$\therefore \Pr(A \cap B) = \Pr(A)$
$\therefore A$ is a subset of B, i.e. $A \subseteq B$

Solutions to Exercise 9G

1 Do you think private individuals should be allowed to carry guns?

	Male	Female	
Yes	35	30	65
No	25	10	35
Total	60	40	100

Pr(male and support guns) $= 0.35$;
Pr(male) $\times$ Pr(support guns) $= 0.39 \neq 0.35$;
therefore not independent

2

	Male	Female	Total
Sport	225	150	375
Music	75	50	125
Total	300	200	500

Pr(male and prefer sport) $= 0.45$;
Pr(male) $\times$ Pr(prefer sport) $= 0.45$;
therefore independent

3

Type of accident	Speeding Yes	No	Total
Serious	42	61	103
Minor	88	185	273
Total	130	246	376

Pr(speeding and serious) ≈ 0.112;
Pr(speeding) $\times$ Pr(serious) $= 0.095 \neq 0.112$;
therefore not independent

4 $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{1, 2, 3, 4, 5, 6\}$,
$B = \{1, 3, 5, 7, 9, 11\}$,
$C = \{4, 6, 8, 9\}$
$\therefore \Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{2}$, $\Pr(C) = \frac{1}{3}$

a $A \cap B = \{1, 3, 5\}$
$\therefore \Pr(A \cap B) = \frac{1}{4}$
$\Pr(A)\Pr(B) = \frac{1}{4}$ so A and B are independent.

b $A \cap C = \{4, 6\}$
$\therefore \Pr(A \cap C) = \frac{1}{6}$
$\Pr(A)\Pr(C) = \frac{1}{6}$ so A and C are independent.

c $B \cap C = \{9\}$
$\therefore \Pr(B \cap C) = \frac{1}{12}$
$\Pr(B)\Pr(C) = \frac{1}{6}$ so B and C are not independent.

5 $\Pr(A \cap B)$
= Pr(even number and square number)
$= \Pr(\{4\}) = \frac{1}{6}$
$\Pr(A) = \frac{3}{6} = \frac{1}{2}$
and $\Pr(B) = \Pr(\{1, 4\}) = \frac{2}{6} = \frac{1}{3}$
$\therefore \Pr(A \cap B) = \Pr(A) \times \Pr(B)$

6 $\Pr(A) = 0.3$, $\Pr(B) = 0.1$,
$\Pr(A \cap B) = 0.1$
$\Pr(A)\Pr(B) = 0.03 \neq 0.1$, so A and B are not independent.

7 $\Pr(A) = 0.6$, $\Pr(B) = 0.7$, and A and B are independent

a $\Pr(A|B) = \Pr(A) = 0.6$

b $\Pr(A \cap B) = \Pr(A)\Pr(B)$
$= 0.6(0.7) = 0.42$

c $\Pr(A \cap B) = \Pr(A) + \Pr(B)$
$- \Pr(A \cap B)$
$\Pr(A \cup B) = 0.6 + 0.7 - 0.42 = 0.88$

8 $\Pr(A \cap B) = \Pr(A)\Pr(B)$
$= 0.5(0.2) = 0.1$
$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$= 0.5 + 0.2 - 0.1 = 0.6$

9

Blood group	0	A	B	AB
Pr	0.5	0.35	0.1	0.05

a $\Pr(A) = 0.35$

b $\Pr(A,\ B) = 0.35(0.1) = 0.035$

c $\Pr(A,\ A) = 0.35^2 = 0.1225$

d $\Pr(O,\ AB) = 0.05(0.5) = 0.025$

10 $N = 165$:

	H	N	L
M	88	22	10
F	11	22	12

a $\Pr(N) = \frac{44}{165} = \frac{4}{15}$

b $\Pr(F \cap H) = \frac{11}{165} = \frac{1}{15}$

c $\Pr(F \cup H) = \Pr(F) + \Pr(H)$
$- \Pr(F \cap H)$
$= \frac{45 + 99 - 11}{165} = \frac{133}{165}$

d $\Pr(F|L) = \frac{\Pr(F \cap L)}{\Pr(L)}$
$= \frac{12}{165} \div \frac{22}{165} = \frac{6}{11}$

e $\Pr(L|F) = \frac{\Pr(F \cap L)}{\Pr(F)}$
$= \frac{12}{165} \div \frac{45}{165} = \frac{4}{15}$
F and L are not independent. If they were, then
$\Pr(L|F) = \Pr(L)\Pr(L)$
$= \frac{45}{165} = \frac{3}{11} \neq \frac{4}{15}$

11 $\Pr(A) = \frac{20}{36} = \frac{5}{9}$
$\Pr(B) = \frac{9}{36} = \frac{1}{4}$
$\Pr(A \cap B) = \frac{5}{36} = \Pr(A)\Pr(B)$
$\therefore$ A and B are independent.

12 $\Pr(W) = 0.4,\ \Pr(M) = 0.5$
$\Pr(W|M) = 0.7 = \frac{\Pr(W \cap M)}{\Pr(M)}$

a $\Pr(W \cap M) = \Pr(W|M) \times \Pr(M)$
$= 0.7(0.5) = 0.35$

b $\Pr(M|W) = \frac{\Pr(W \cap M)}{\Pr(W)}$
$= \frac{035}{0.4} = \frac{7}{8}$ or 0.875

13 $N = 65$:

	T	F	S
L	13	4	1
M	8	10	3
H	2	16	8

a $\Pr(L) = \frac{18}{65}$

b $\Pr(S) = \frac{12}{65}$

c $\Pr(T) = \frac{23}{65}$

d $\Pr(M) = \dfrac{21}{65}$

e $\Pr(L \cap F) = \dfrac{4}{65}$

f $\Pr(T \cap M) = \dfrac{8}{65}$

g $\Pr(L|F) = \dfrac{4}{30} = \dfrac{2}{15}$

h $\Pr(I|M) = \dfrac{8}{21}$

Income is not independent of age, e.g.:

$\Pr(L \cap F) = \dfrac{4}{65} = 0.0615$, but

$\Pr(L)\Pr(F) = \left(\dfrac{18}{65}\right)\left(\dfrac{30}{65}\right) = 0.128$

You would not expect middle managers' income to be independent of age.

14 $N = 150$:

	G	G'
F	48	16
F'	24	62

a **i** $\Pr(G|F) = \dfrac{48}{64} = \dfrac{3}{4} = 0.75$

ii $\Pr(G \cap F) = \dfrac{48}{150} = 0.32$

iii $\Pr(G \cup F) = \dfrac{88}{150} = \dfrac{44}{75} = 0.587$

b $\Pr(G)\Pr(F) = \left(\dfrac{48+24}{150}\right)\left(\dfrac{48+16}{150}\right)$

$= \left(\dfrac{72}{150}\right)\left(\dfrac{64}{150}\right) = 0.2048$

$\Pr(G)\Pr(F) \neq \Pr(G \cap F)$

$\therefore$ G and F are not independent.

c G and F not mutually exclusive:

$\Pr(G \cap F) \neq 0$

Solutions to Exercise 9H

1 We know the answer is $\frac{1}{8}$. Binomial with $p = \frac{1}{2}$ and $n = 3$.
Simulate with random integers 0 and 1 with your calculator.

2 Binomial, $n = 5$ and $p = \frac{1}{2}$ It can be simulated with using random integers 0 and 1 with your calculator in a .
$\Pr(X \geq 3)$:

X	3	4	5
$\Pr(X = x)$	0.3125	0.15625	0.03125

One in every two simulations would be expected to give this result.

3 Binomial, $n = 10$ and $p = 0.2$
$\Pr(X \geq 5) = 0.032793$:
Simulate with random integers1-5.
Choose one value to be correct for each question.

4 There are many possibilities here, but simplest would be to use a random number table, where each souvenir is given a number from 0 to 9.
(The average number of purchases needed is exactly given by:
$1 + \frac{10}{9} + \frac{10}{8} + \frac{10}{7} + \ldots + \frac{10}{2} + \frac{10}{1} \approx 29.3)$
This is known as the 'Collector's Problem'

Solutions to Review: Short-answer questions

1 **a** Six ways of getting 7

$\therefore \Pr(7) = \frac{6}{36} = \frac{1}{6}$

b $\Pr(7') = 1 - \frac{1}{6} = \frac{5}{6}$

2 $\Pr(O) = 0.993$

$\therefore \Pr(O') = 1 - 0.993 = 0.007$

3 **a** $\Pr(\text{divisible by 3}) = \frac{100}{300} = \frac{1}{3}$

b $\Pr(\text{divisible by 4}) = \frac{75}{300} = \frac{1}{4}$

c $\Pr(\text{divisible by 3 or by 4})$

$= \frac{1}{3} + \frac{1}{4} - \Pr(\text{divisible by 12})$

$= \frac{7}{12} - \frac{25}{300} = \frac{1}{2}$

4 $30\ R$, $20\ B$

$\therefore\ \Pr(R) = 0.6$

a $\Pr(R,\ R) = 0.6^2 = 0.36$

b No replacement:

$\Pr(R, R) = \left(\frac{3}{5}\right)\left(\frac{29}{49}\right) = \frac{87}{245}$

5 $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 4, 9\}$

If $A + B = C$,

$C = \{2, 5, 10, 4, 7, 12, 6, 9, 14, 8, 11,$

$16, 10, 13, 18\}$

Of these, only $\{6, 9, 12, 18\}$ are divisible by 3.

$\Pr(\text{sum divisible by 3}) = \frac{4}{15}$

6 **a** $\epsilon = \{156, 165, 516, 561, 615, 651\}$

b $\Pr(> 400) = \frac{4}{6} = \frac{2}{3}$

c $\Pr(\text{even}) = \frac{2}{6} = \frac{1}{3}$

7 STATISTICIAN has 5 vowels and 7 consonants.

a $\Pr(\text{vowel}) = \frac{5}{12}$

b $\Pr(\text{T}) = \frac{3}{12} = \frac{1}{4}$

8 $\Pr(I) = 0.6,\ \Pr(J) = 0.1,\ \Pr(D) = 0.3$

a $\Pr(I, J, I) = 0.6\,(0.1)\,0.6$

$= 0.036$

b $\Pr(D, D, D) = 0.3^3 = 0.027$

c $\Pr(I,\ D,\ D) + \Pr(J,\ D,\ D) +$

$\Pr(D,\ I,\ D) + \Pr(D,\ J,\ D) +$

$\Pr(D,\ D,\ I) + \Pr(D,\ D,\ J)$

$= 3\,(0.6\ + 0.1)\left(0.3^2\right)$

$= 0.189$

d $\Pr(J') = 0.9$

$\therefore \Pr(J', J', J') = 0.9^3 = 0.729$

9 $\Pr(R) = \frac{1}{3}$, $\Pr(B) = \frac{2}{3}$

a $\Pr(R,\ R,\ R) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

b $\Pr(B,\ R,\ B) = \frac{2}{3}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{27}$

c $\Pr(R,\ B,\ B) + \Pr(B,\ R,\ B) +$

$\Pr(B,\ B,\ R)$
$= 3\left(\frac{4}{27}\right) = \frac{4}{9}$

d $\Pr(\geq 2B) = \Pr(B,\ B,\ B) + \Pr(2B)$

$$= \left(\frac{2}{3}\right)^3 + \frac{4}{9} = \frac{20}{27}$$

10 $\Pr(A) = 0.6,\ \Pr(B) = 0.5$
If A and B are mutually exclusive,
$\Pr(A \cap B) = 0$
By definition,

$$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$= 1.1 > 0$$

This is impossible, so they cannot be mutually exclusive.

11

$\cap$	B	B'	
A	0.1	0.5	0.6
A'	0.4	0	0.4
	$\Pr(B) = 0.5$	$\Pr(B') = 0.5$	1

a $\Pr(A \cap B') = 0.5$

b $\Pr(A' \cap B') = 0$

c $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B$
$= 0.6 + 0.5 - 0.1$
$= 1$

12 **a** $\frac{7}{18}$

b $\frac{1}{2}$

13 $\Pr(B) = \frac{1}{3}\ \therefore\ \Pr(B') = \frac{2}{3}$

a $\Pr(A|B') = \dfrac{\Pr(A \cap B')}{\Pr(B')} = \frac{3}{7}$

$\therefore \Pr(A \cap B') = \frac{3}{7}\left(\frac{2}{3}\right) = \frac{2}{7}$

$\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \frac{2}{3}$

$\therefore \Pr(A \cap B) = \frac{2}{3}\left(\frac{1}{3}\right) = \frac{2}{9}$

b $\Pr(A) = \frac{2}{9} + \frac{2}{7} = \frac{32}{63}$

c $\Pr(B'|A) = \dfrac{\Pr(A \cap B')}{\Pr(A)}$

$$= \frac{2}{7} \div \frac{32}{63} = \frac{9}{16}$$

14

Pr	O	N	U	Tot
H	0.1	0.08	0.02	0.2
H'	0.15	0.45	0.2	0.8
Tot	0.25	0.53	0.22	1

a $\Pr(H) = 0.2$

b $\Pr(H|O) = \dfrac{\Pr(H \cap O)}{\Pr(O)}$

$$= \frac{0.1}{0.25} = 0.4$$

15 $\Pr(A) = 0.3, \Pr(B) = 0.6, \Pr(A \cap B) = 0.2$

a $\Pr(A \cup B) = \Pr(A) + \Pr(B)$
$- \Pr(A \cap B) = 0.7$

$\cap$	B	B'	
A	0.2	0.1	0.3
A'	0.4	0.3	0.7
	0.6	0.4	1

b $\Pr(A' \cap B') = 0.3$

c $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

$$= \frac{0.2}{0.6} = \frac{1}{3}$$

d $\Pr(B|A) = \dfrac{\Pr(A \cap B)}{\Pr(A)}$

$= \dfrac{0.2}{0.3} = \dfrac{2}{3}$

16 a $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

If $\Pr(A|B) = 1$, then $\dfrac{\Pr(A \cap B)}{\Pr(B)} = 1$

$\therefore \Pr(A \cap B) = \Pr(B)$
$\therefore$ B is a subset of A.

b $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

If $\Pr(A|B) = 0$, then $\dfrac{\Pr(A \cap B)}{\Pr(B)} = 0$

$\therefore \Pr(A \cap B) = 0$
$\therefore$ A and B are mutually exclusive or disjoint.

c $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

If $\Pr(A|B) = \Pr(A)$, then

$\dfrac{\Pr(A \cap B)}{\Pr(B)} = \Pr(A)$

$\therefore \Pr(A \cap B) = \Pr(A)\Pr(B)$
$\therefore$ A and B are independent.

Solutions to Review: Multiple-choice questions

1 B $\Pr(< 50) = 1 - \Pr(\geq 50)$

$= 1 - 0.7 = 0.3$

2 C $\Pr(G) = 1 - \Pr(G')$

$= 1 - 0.7 = 0.3$

3 A 4 Ts in 10

$\therefore \Pr(\text{T}) = \dfrac{2}{5}$

4 C $\Pr(C) = 1 - \Pr(C')$

$= 1 - \dfrac{18}{25} = \dfrac{7}{25}$

5 D $\Pr(J \cup \spadesuit) = \dfrac{16}{52} = \dfrac{4}{13}$

6 A Area outside circle $= 16 - \pi(1.5)^2\ \text{m}^2$

$\therefore\ \Pr = 1 - \dfrac{2.25\pi}{16} \cong 0.442$

7 D $\Pr(\text{Head and a six}) = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$

8 E $\Pr(A) = 0.35,\ \Pr(A \cap B) = 0.18,$

$\Pr(B) = 0.38$

$\Pr(A \cup B) = 0.35 + 0.38 - 0.18$

$= 0.55$

9 A $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$= 0.47 + 0.28 - 0.28 = 0.47$

10 B $\Pr(B) = 0.32$

$\Pr(F) = 0.57$

$\Pr(F \cap B) = 0.11$

	B	B'	
F	0.11	0.46	0.57
F'	0.21	0.22	0.43
	0.32	0.68	1

11 B $\Pr(G) = \dfrac{3}{10} = \dfrac{9}{30},$

$\Pr(M) = \dfrac{2}{3} = \dfrac{20}{30},$

$\Pr(G' \cap M) = \dfrac{7}{15} = \dfrac{14}{30},$

	M	M'	
G	$\Pr(G \cap M) = \dfrac{6}{30}$	$\Pr(G \cap M) = \dfrac{3}{30}$	$= \dfrac{9}{30}$
G'	$\Pr(G' \cap M) = \dfrac{14}{30}$	$\Pr(G' \cap M) = \dfrac{7}{30}$	$= \dfrac{21}{30}$
	$\Pr(M) = \dfrac{20}{30}$	$\Pr(M') = \dfrac{10}{30}$	1

$\Pr(G' \cap M') = \dfrac{7}{30}$

12 B

13 A

14 E $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$

$= \dfrac{8}{21} \div \dfrac{4}{7} = \dfrac{2}{3}$

15 C $\Pr(G,\ G) = 0.6(0.7) = 0.42$

16 A $\Pr(G, G) + \Pr(G, G')$

$= 0.42 + (0.4)^2$

$= 0.58$

17 B $\Pr(A \cap B) = \Pr(A)\Pr(B)$

$= 0.35(0.46) = 0.161$

$\Pr(A \cup B) = \Pr(A) + \Pr(B)$

$- \Pr(A \cap B)$

$= 0.35 + 0.46 - 0.161$

$= 0.649$

18 D The reliability

$= 0.85 + 0.95 - 0.85 \times 0.95$

$= 0.9925$

Solutions to Review: Extended-response questions

1 Let A = number of days it takes to build scenery.
Let B = number of days it takes to paint scenery.
Let C = number of days it takes to print programs.

a Pr(building and painting scenery together taking exactly 15 days)

$$= \Pr(A = 7) \times \Pr(B = 8) + \Pr(A = 8) \times \Pr(B = 7)$$
$$= \frac{3}{10} \times \frac{1}{10} + \frac{4}{10} \times \frac{3}{10}$$
$$= \frac{3 + 12}{100}$$
$$= 0.15$$

b Pr(all 3 tasks taking exactly 22 days)

$$= \Pr(A = 6) \times \Pr(B = 8) \times \Pr(C = 8) + \Pr(A = 7) \times \Pr(B = 7) \times \Pr(C = 8) +$$
$$\Pr(A = 7) \times \Pr(B = 8) \times \Pr(C = 7) + \Pr(A = 8) \times \Pr(B = 6) \times \Pr(C = 8) +$$
$$\Pr(A = 8) \times \Pr(B = 7) \times \Pr(C = 7) + \Pr(A = 8) \times \Pr(B = 8) \times \Pr(C = 6)$$
$$= \frac{3 \times 1 \times 2 + 3 \times 3 \times 2 + 3 \times 1 \times 4 + 4 \times 6 \times 2 + 4 \times 3 \times 4 + 4 \times 1 \times 4}{1000}$$
$$= \frac{6 + 18 + 12 + 48 + 48 + 16}{1000}$$
$$= \frac{148}{1000}$$
$$= 0.148$$

2 **a** For bowl A, $\Pr(2 \text{ apples}) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$

For bowl B, $\Pr(2 \text{ apples}) = \frac{7}{8} \times \frac{6}{7} = \frac{3}{4}$

b For bowl A, $\Pr(2 \text{ apples with replacement}) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

For bowl B, $\Pr(2 \text{ apples with replacement}) = \frac{7}{8} \times \frac{7}{8} = \frac{49}{64}$

c Let A be the event that bowl A is chosen.

Then $$\Pr(A|2\text{ apples}) = \frac{\Pr(A \cap 2\text{ apples without replacement})}{\Pr(2\text{ apples without replacement})}$$
$$= \frac{\frac{1}{2} \times \frac{3}{28}}{\frac{1}{2}\left(\frac{3}{28} + \frac{21}{28}\right)} = \frac{\frac{3}{28}}{\frac{3+21}{28}}$$
$$= \frac{3}{24} = \frac{1}{8} = 0.125$$

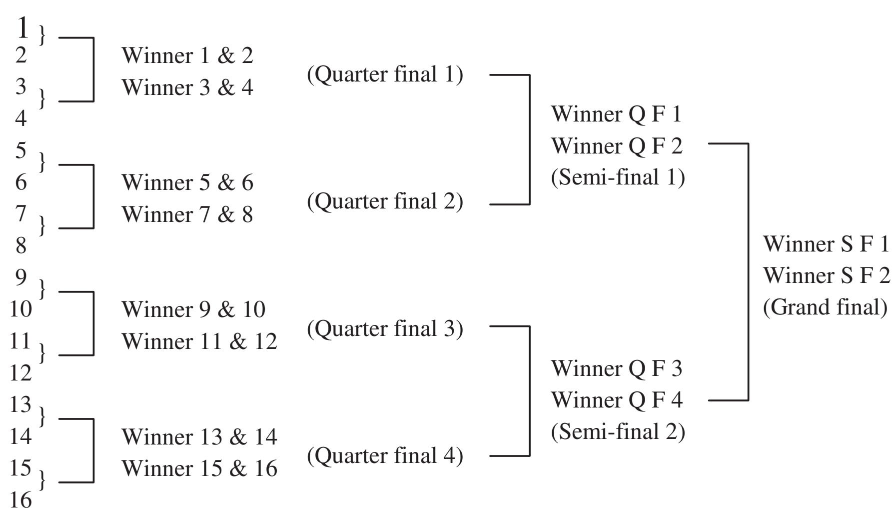
d $$\Pr(A|2\text{ apples}) = \frac{\Pr(A \cap 2\text{ apples with replacement})}{\Pr(2\text{ apples with replacement})}$$
$$= \frac{\frac{1}{2} \times \frac{9}{64}}{\frac{1}{2}\left(\frac{9}{64} + \frac{49}{64}\right)}$$
$$= \frac{9}{58}$$
$$\cong 0.125$$

3 a $\frac{4}{5}$

b Pr(running the day after) $= \frac{4}{5} \times \frac{4}{5} + \frac{1}{5} \times \frac{1}{4} = 0.69$

c Pr(running exactly twice in the next three days) $= \frac{4}{5} \times \frac{4}{5} \times \frac{1}{5} + \frac{4}{5} \times \frac{1}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{4} \times \frac{4}{5} = 0.208$

4 a The following structure is assumed.



Pr(Player winning 1 match) = 0.5

Pr(Player winning 2 matches) $= 0.5 \times 0.5$

Pr(Player winning 3 matches) $= 0.5 \times 0.5 \times 0.5$

Pr(Player winning 4 matches) $= 0.5 \times 0.5 \times 0.5 \times 0.5$

$\therefore$ expected number of matches $= 0.5 \times 1 + 0.5^2 \times 2 + 0.5^3 \times 3 + 0.5^4 \times 4$

$$= \frac{13}{8}$$

b If probability of winning is 0.7, expected number of matches

$$= 0.7 \times 1 + 0.7^2 \times 2 + 0.7^3 \times 3 + 0.7^4 \times 4$$

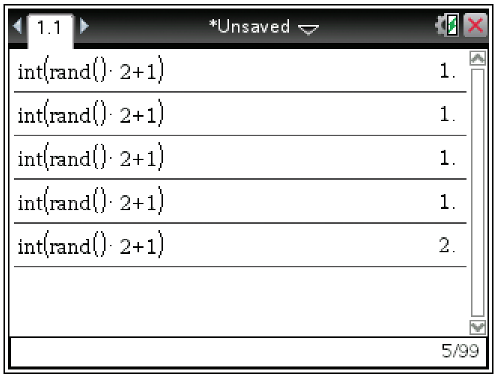
$$= \frac{18\,347}{5000}$$

$$\approx 3.7$$

Simulation

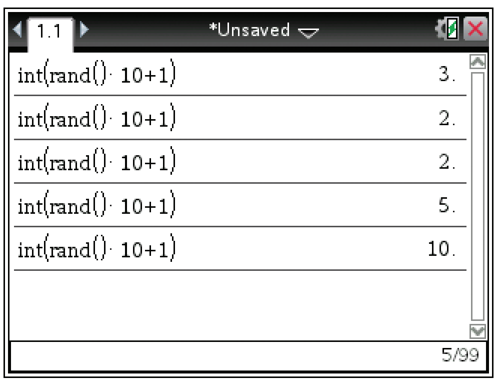
a Use **int(rand()*2 + 1)**

If 1 occurs, a win is recorded. If 2 occurs, a loss is recorded. Sequence stops as soon as 2 is obtained. In the example to the right, the player plays 2 matches.



b Use **int(rand*10 + 1)**

If a digit 1 – 7 inclusive is obtained, a win is recorded. If 8 or 9 or 10 is obtained, a loss is recorded. In the example to the right, the player plays 5 matches.



5 a Theoretical answer

For teams A and B,

$$\begin{aligned}\text{probability of winning} &= \frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\ &= \frac{1}{4}+\frac{1}{8}\\ &= \frac{3}{8} \text{ or } 0.73\end{aligned}$$

For teams C and D,

$$\text{probability of winning} = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8} \text{ or } 0.125$$

Simulation

TI: The following program simulates the final series and assigns equal probability of winning to each of the two teams in any game. It displays the winner of each game, and lastly the winner of the series.

```
prog5
Define LibPub prog5()=
Prgm
Local w,x,y,z
Disp "Game 1"
randInt(1,2)→x
If x=1 Then
Disp "A wins"
EndIf
If x=2 Then
Disp "B wins"
EndIf
Disp "Press ENTER to continue"
Disp "Game 2"
randInt(1,2)→y
If y=1 Then
Disp "C wins"
EndIf
If y=2 Then
Disp "D wins"
EndIf
Disp "Press ENTER to continue"
Disp "Game 3"
0/99
```

```
prog5
randInt(1,2)→z
If x=1 and z=1 Then
Disp "B wins"
EndIf
If x=2 and z=1 Then
Disp "A wins"
EndIf
If y=1 and z=2 Then
Disp "C wins"
EndIf
If y=2 and z=2 Then
Disp "D wins"
EndIf
Disp "Press ENTER to continue"
Disp "Game 4"
randInt(1,2)→w
If x=1 and w=1 or x=2 and z=1 and w=2 Then
Disp "A wins"
EndIf
If x=2 and w=1 or x=1 and z=1 and w=2 Then
Disp "B wins"
EndIf
0/99
```

```
If y=1 and z=2 and w=2 Then
Disp "C wins"
EndIf
If y=2 and z=2 and w=2 Then
Disp "D wins"
EndIf
EndPrgm
```

b Simulation

TI: The following program uses a simulation of 100 final series to estimate the probability of each team winning a final series. The estimated probabilities are displayed.

```
prog6
Define LibPub prog6()=
Prgm
Local a,b,c,d,n,w,x,y,z
0→a
0→b
0→c
0→d
For n,1,100
randInt(1,2)→x
randInt(1,2)→y
randInt(1,2)→z
randInt(1,2)→w
If x=1 and w=1 or x=2 and z=1 and w=2 Then
a+1→a
ElseIf x=2 and w=1 or x=1 and z=1 and w=2 Then
b+1→b
ElseIf y=1 and z=2 and w=2 Then
c+1→c
ElseIf y=2 and z=2 and w=2 Then
d+1→d
EndIf
EndFor
Disp "Pr(A wins) =",a/100
Disp "Pr(B wins) =",b/100
Disp "Pr(C wins) =",c/100
Disp "Pr(D wins) =",d/100
EndPrgm
```