WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS METHODS

SEMESTER ONE 2018

TEST 4: ANTIDIFFERENTIATION, APPLICATIONS OF CALCULUS AND FUNDAMENTAL THEOREM OF CALCULUS

Thursday 24 May | Name: Solutions.

Time: 45 minutes | **Part A:** $\frac{}{27}$ | **Part B:** $\frac{}{16}$ | **Total:** $\frac{}{43}$ | %

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet for both sections, and an A4 page of notes, plus up to 3 permitted calculators in the Calculator Allowed section.

Calculator Free

1. [6 marks]

Determine the anti-derivative of

a) $(4-3x)^2$

$$\int (4-3x)^2 dx = \frac{(4-3x)^3}{-3.3} + c = -\frac{1}{9}(4-3x)^3 + c.$$

[2]

b) $5x^4 - \frac{9}{\sqrt{x}}$

$$\int (5x^4 - 9x^{-1/2})\,dx = x^5 - \frac{9x^{1/2}}{1/2} + c = x^5 - 18\sqrt{x} + c.$$

[2]

c) $\frac{10x}{x^2+5}$

$$\int \frac{10x}{x^2+5}\,dx = 5\int \frac{2x}{x^2+5}\,dx = 5\ln(x^2+5) + c$$

$(x^2+5 > 0 \quad \forall x \in \mathbb{R})$

[2]

2. [4 marks]

Determine the following, simplyfying your answers:

a) $\int \frac{1-x^3}{x^2}\, dx$ $= \int (x^{-2} - x)\, dx$

$= \frac{x^{-1}}{-1} - \frac{x^2}{2} + c$

$= -\frac{1}{x} - \frac{x^2}{2} + c.$

[2]

b) $\frac{d}{dx}\left(\int_x^7 \frac{2t}{t^2-5}\, dt\right)$

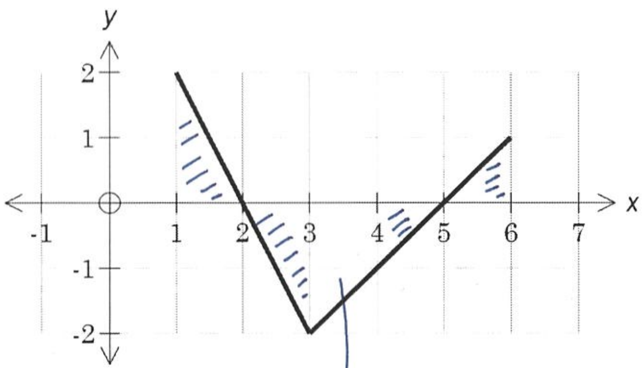
$= -\frac{2x}{x^2-5}$

[2]

3. [3 marks]

Let the graph of $f(x)$ between $x = 1$ and $x = 6$ be as shown.

Evaluate $\int_1^6 f(x)\, dx$.



$I = \frac{1}{2}(1\times 2) - \frac{1}{2}(3\times 2) + \frac{1}{2}(1\times 1)$

$= 1 - 3 + \frac{1}{2}$

$= -\frac{3}{2}$

OR.

$I = \left[\left(\frac{2+1}{2}\right)\times 1\right] \times -1$ below axis

$= -\frac{3}{2}.$

[3]

4. [7 marks]

A particle P moves in a straight horizontal line such that its acceleration at time t seconds is given by $a = k(2t - 5)$, where k is a positive constant.

Given that at time $t = 0$, P is at rest at the origin and that at time $t = 6$, its velocity is $1.5\ ms^{-1}$,

a) find the acceleration of P in terms of t.

$$v = \int k(2t-5)\,dt$$

$$= k.(t^2 - 5t) + c.$$

$v(0) = 0 \Rightarrow c = 0$

$k \neq 0$

$$v = k(t^2 - 5t).$$

$$v(6) = 1{\cdot}5 = k(36-30)$$

$$\Rightarrow k = \frac{1{\cdot}5}{6} = \frac{3}{12} = \frac{1}{4}$$

$$a = \frac{1}{4}(2t-5)$$

[4]

b) show that the displacement of the particle, x metres, from O at time t is given by

$$x = \frac{1}{24}t^2(2t - 15)$$

Since $v = \frac{1}{4}(t^2 - 5t)$

$$x = \int \frac{1}{4}(t^2 - 5t)\,dt$$

$$= \frac{1}{4}\left(\frac{t^3}{3} - \frac{5t^2}{2}\right) + c.$$

$x(0) = 0 \Rightarrow c = 0.$

$$x = \frac{1}{4}\left(\frac{t^3}{3} - \frac{5t^2}{2}\right)$$

$$= \frac{1}{4}\left[\frac{2t^3}{6} - \frac{15t^2}{6}\right]$$

$$= \frac{1}{24}t^2(2t - 15)$$ shown.

[3]

5. [4 marks]

Use $\int_{-2}^{4} f(x)\, dx = 8$ and $\int_{-2}^{1} f(x)\, dx = 1$ to evaluate the following:

a) $\int_{-2}^{4} -5f(x)\, dx$ $= -5\int_{-2}^{4} f(x)\, dx$

$= -5 \times 8 \quad = -40.$

[1]

b) $\int_{1}^{4} f(x)\, dx$ $= \int_{-2}^{4} f(x)\, dx - \int_{-2}^{1} f(x)\, dx = 8 - 1 = 7$

[1]

c) $\int_{-2}^{4} [f(x) - 2x]\, dx$ $= \int_{-2}^{4} f(x)\, dx - \int_{-2}^{4} 2x\, dx$

$= 8 - [x^2]_{-2}^{4}$

$= 8 - \{16 - 4\} \quad = -4.$

[2]

6. [3 marks]

The rate of flow of a liquid into a container is given by $\frac{dV}{dt} = e^{0.5t}$, where V is the volume in cubic centimetres and t is the time in seconds.

Find the volume of liquid in the container after 3 seconds if the container intially holds 10 cm^3.

$\frac{dV}{dt} = e^{0.5t}$

$V = \int e^{0.5t}\, dt$

$= 2e^{0.5t} + c.$ ✓

$V(0) = 10 = 2 \times 1 + c \Rightarrow c = 8.$ ✓

$V(t) = 2e^{0.5t} + 8$

$V(3) = (2e^{1.5} + 8)\ \text{cm}^3.$ ✓

[3]

WESLEY COLLEGE

By daring & by doing

NAME: ______________________________

(16 marks)

Resourced

7. [5 marks]

a) Find $\frac{dy}{dx}$ given $y = x.\sin x$

$$\frac{dy}{dx} = \sin x + x\cos x.$$

[2]

b) Use your answer to part (a) to find $\int (x.\cos x)dx$

$$\therefore \int \frac{dy}{dx}.dx = \int \sin x \, dx + \int (x\cos x)\,dx$$ ✓

$$y + c_1 = -\cos x + c_2 + \int (x\cos x)\,dx.$$ ✓

$$\therefore \int (x\cos x)\,dx = x\sin x + \cos x + C.$$ ✓

[3]

8. [6 marks]

The velocity of a body moving along a straight line is given by $v = -3t^2 - 2t + 5$ m/s where t is the time in seconds. The initial displacement of the body from a fixed point O is 3 metres.

a) Find the displacement of the body when $t = 5$.

$$x = \int(-3t^2 - 2t + 5)\,dt = -t^3 - t^2 + 5t + c. \quad x(0) = 3 \Rightarrow c = 3.$$

$$x(5) = -5^3 - 5^2 + 25 + 3 = -122m.$$

[2]

b) Find the instantaneous speed at $t = 5$ seconds

$$|v(5)| = 80 ms^{-1}$$

[1]

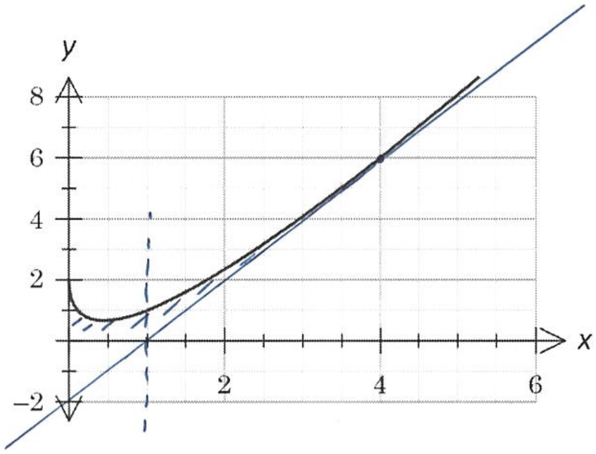
c) What is the average speed of the body over the first 5 seconds?

$$\text{Av. speed} = \frac{\int_0^5 |-3t^2 - 2t + 5|\,dt}{5} = \frac{131}{5} = 26.2 \text{ m/s}.$$ ✓ ✓ ✓

[3]

9. [5 marks]

A sketch of the curve C with equation $y = 3x - 4\sqrt{x} + 2$ has been given below.



a) Using the tanLine command, or otherwise, determine the equation of the tangent, which has x-coordinate 4.
Draw the tangent on the sketch.

$y = 2x - 2$.

[2]

b) Write down the integral(s) that will determine the area of the region captured by C, the tangent to C at A and the positive coordinate axes and state the area.

$$\text{Area} = \int_0^1 (3x - 4\sqrt{x} + 2)\,dx + \int_1^4 \left[(3x - 4\sqrt{x} + 2) - (2x - 2)\right] dx.$$

$$= \frac{5}{6} + \frac{5}{6}$$

[3]

$$= \frac{5}{3} \text{ sq. units.}$$