

Name: _____

PRACTICE EXAM (MID-YEAR)**Year 12 Mathematics Methods Exam 1**

- Time allowed: 15 minutes reading time
60 minutes writing time
- CAS calculators may be used.
- Part 1: Ten short-answer questions (15 marks).
- Part 2: Six extended-answer questions (25 marks).

Part 1**Short-answer questions**

Ten short-answer questions

15 marks

Show your working where appropriate.

Question 1

Given the acceleration $a = 2 \text{ cm/s}^2$, and the initial velocity, v , is 1 cm/s and initial displacement, x , is 4 cm , find the velocity and displacement at $t = 2 \text{ s}$.

[2 marks]**Question 2**

Evaluate $\int_0^1 \sqrt{x} dx$.

[1 mark]

Question 3

Forty balls numbered 1 to 40 are used in Lotto and selected randomly. To win the jackpot, you have to correctly guess six numbers.

- a Write down an expression for the probability of winning the jackpot if you buy a Lotto ticket.

[1 mark]

- b Write down an expression for the probability that you choose at least two of the six winning numbers.
(Leave in terms of nC_r .)

[1 mark]**Question 4**

Given $y = \frac{x^3}{3}$, find the change in y , δy , if x increases from 2 to 2.01.

[1 mark]**Question 5**

Find the expression that can be used to determine the area between the curves $y = x^2(x^2 - 4)$ and $y = 4 - x^2$.
Show all working.

[2 marks]

Question 6

Find the equation of the tangent to the function $y = \tan(x)$ at $x = 0$.

[1 mark]**Question 7**

Determine the turning point on the equation $y = 3 \sin(x - \frac{\pi}{3})$ for $0 < x < \pi$.

[2 marks]**Question 8**

Given a uniform probability density function defined for $2 \leq x \leq 6$, determine $P(3 \leq x \leq 4)$.

[1 mark]

Question 9

Sketch a function that has the following properties for $a > 0$.

x	-4	$-a$	0	a	4
f	0		0		0
f'	$+$	0	$-$	0	$+$
f''	$-$	$-$	0	$+$	$+$

[2 marks]

Question 10

Find where $f'(x) < 0$ given $f(x) = e^{-3x^2}$.

[1 mark]

Part 2

Extended-answer questions

Six extended-answer questions

25 marks

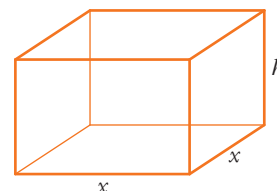
Show your working where appropriate.

Question 1

A square-based box with base $x \times x$ metres and height h metres is to have a volume of 4 m^3 .

To use as little material as possible, the surface area needs to be minimised.

- a** Prove that the surface area can be expressed as $\frac{16}{x} + 2x^2$
i.e. $\text{SA} = \frac{16}{x} + 2x^2$



- b** Find the values of both x and h for the minimum surface area.

[4 marks]

Question 2

Given the function $f(x) = x(x - 1)(x + 1) = x^3 - x$

- a** sketch the graph of the function

b determine $\int_{-1}^1 (x^3 - x) dx$ and interpret the result

c determine the area between the function f and the x -axis.

[4 marks]

Question 3

Two spinners, each with the four faces painted 1, 2, 3, 4, were spun together and the totals of the two spinners noted.

A game costs \$1 to play. A payment of \$2 is paid for a total of 2, 5 or 8.

a Find the expected payout.

b How could the payouts be changed to make the game fair?

[5 marks]

Question 4

a Find $\int \frac{\sqrt{x} - 1}{\sqrt{x}} dx$.

b Hence, evaluate $\int_1^4 \frac{\sqrt{x} - 1}{\sqrt{x}} dx$.

[4 marks]

Question 5

Given $y = \frac{x^2}{\sqrt{x} + 1}$, find the equation of the tangent at $x = 0$.

[4 marks]

Question 6

The manager of the local supermarket has determined the maximum profit, P , is related to the number of checkout workers, x , available to serve customers according to the formula $P(x) = -25x^2 + 300x$. If customers have to wait too long, they don't return to the shop.

a How many checkout workers should the manager employ to maximise profits?

b What is the maximum profit?

[4 marks]