Chapter 3 – Quadratics

Solutions to Exercise 3A

1 a $2(x-4) = 2x-8$

b $-2(x-4) = -2x+8$

c $3(2x-4) = 6x-12$

d $-3(4-2x) = 6x-12$

e $x(x-1) = x^2-x$

f $2x(x-5) = 2x^2-10x$

2 a $(2x+4x)+1 = 6x+1$

b $(2x+x)-6 = 3x-6$

c $(3x-2x)+1 = x+1$

d $(-x+2x+4x)-3 = 5x-3$

3 a $8(2x-3)-2(x+4)$

$= 16x-24-2x-8$

$= 14x-32$

b $2x(x-4)-3x$

$= 2x^2-8x-3x$

$= 2x^2-11x$

c $4(2-3x)+4(6-x)$

$= 8-12x+24-4x$

$= 32-16x$

d $4-3(5-2x)$

$= 4-15+6x$

$= 6x-11$

4 a $2x(x-4)-3x$

$= 2x^2-8x-3x$

$= 2x^2-11x$

b $2x(x-5)+x(x-5)$

$= 2x^2-10x+x^2-5x$

$= 3x^2-15x$

c $2x(-10-3x)$

$= -20x-6x^2$

d $3x(2-3x+2x^2)$

$= 6x-9x^2+6x^3$

e $3x-2x(2-x)$

$= 3x-4x+2x^2$

$= 2x^2-x$

f $3(4x-2)-6x$

$= 12x-6-6x$

$= 6x-6$

5 a $(3x-7)(2x+4)$

$= 6x^2+12x-14x-28$

$= 6x^2-2x-28$

b $(x-10)(x-12)$

$= x^2-10x-12x+120$

$= x^2-22x+120$

c $(3x-1)(12x+4)$

$= 36x^2+12x-12x-4$

$= 36x^2-4$

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d $(4x-5)(2x-3)$

$= 8x^2 - 12x - 10x + 15$

$= 8x^2 - 22x + 15$

e $(x-\sqrt{3})(x-2)$

$= x^2 - 2x - \sqrt{3}x + 2\sqrt{3}$

$= x^2 - (2+\sqrt{3})x + 2\sqrt{3}$

f $(2x-\sqrt{5})(x+\sqrt{5})$

$= 2x^2 + 2\sqrt{5}x - \sqrt{5}x - 5$

$= 2x^2 + \sqrt{5}x - 5$

g $(3x-\sqrt{7})(x+\sqrt{7})$

$= 3x^2 + 3\sqrt{7}x - 2\sqrt{7}x - 14$

$= 3x^2 + \sqrt{7}x - 14$

h $(5x-3)(x+2\sqrt{2})$

$= 5x^2 + 10\sqrt{2}x - 3x - 6\sqrt{2}$

$= 5x^2 + (10\sqrt{2}-3)x - 6\sqrt{2}$

i $(\sqrt{5}x-3)(\sqrt{5}x-32\sqrt{2})$

$= 5x^2 - 32\sqrt{10}x - 3\sqrt{5}x + 96\sqrt{2}$

$= 5x^2 - (32\sqrt{10}+3\sqrt{5})x + 96\sqrt{2}$

6 a $(2x-3)(3x^2+2x-4)$

$= 6x^3 + 4x^2 - 8x - 9x^2 - 6x + 12$

$= 6x^3 - 5x^2 - 14x + 12$

b $(x-1)(x^2+x+1)$

$= x^3 + x^2 + x - x^2 - x - 1$

$= x^3 - 1$

c $(6-2x-3x^2)(4-2x)$

$= 24 - 12x - 8x + 4x^2 - 12x^2 + 6x^3$

$= 24 - 20x - 8x^2 + 6x^3$

d $(5x-3)(x+2) - (2x-3)(x+3)$

$= (5x^2 + 10x - 3x - 6)$

$\quad - (2x^2 + 6x - 3x - 9)$

$= (5x^2 + 7x - 6) - (2x^2 + 3x - 9)$

$= 3x^2 + 4x + 3$

e $(2x+3)(3x-2) - (4x+2)(4x-2)$

$= (6x^2 - 4x + 9x - 6)$

$\quad - (16x^2 - 8x + 8x - 4)$

$= (6x^2 + 5x - 6) - (16x^2 - 4)$

$= -10x^2 + 5x - 2$

7 a $(x-4)^2$

$= x^2 - 4x - 4x + 16$

$= x^2 - 8x + 16$

b $(2x-3)^2$

$= 4x^2 - 6x - 6x + 9$

$= 4x^2 - 12x + 9$

c $(6-2x)^2$

$= 36 - 12x - 12x + 4x^2$

$= 36 - 24x + 4x^2$

d $\left(x-\frac{1}{2}\right)^2$

$= x^2 - \frac{x}{2} - \frac{x}{2} + \frac{1}{4}$

$= x^2 - x + \frac{1}{4}$

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e $(x - \sqrt{5})^2$

$= x^2 - \sqrt{5}x - \sqrt{5}x + 5$

$= x^2 - 2\sqrt{5}x + 5$

f $(x - 2\sqrt{3})^2$

$= x^2 - 2\sqrt{3}x - 2\sqrt{3}x + 4(3)$

$= x^2 - 4\sqrt{3}x + 12$

8 **a** $(x - 3)(x + 3)$

$= x^2 - 3x + 3x - 92$

$= x^2 - 9$

b $(2x - 4)(2x + 4)$

$= 4x^2 + 8x - 8x - 16$

$= 4x^2 - 16$

c $(9x - 11)(9x + 11)$

$= 81x^2 + 99x - 99x + 121$

$= 81x^2 - 121$

d $(2x - 3)(2x + 3)$

$= 4x^2 - 9$

e $(2x + 5)(2x - 5)$

$= 4x^2 - 25$

f $(x - \sqrt{5})(x + \sqrt{5})$

$= x^2 - 5$

g $(2x + 3\sqrt{3})(2x + 3\sqrt{3})$

$= 4x^2 - 27$

h $(\sqrt{3} - \sqrt{7})(\sqrt{3} + \sqrt{7})$

$= 3x^2 - 7$

9 **a** $(x - y + z)(x - y - z)$

$= ((x - y) + z)((x - y) - z)$

$= (x - y)^2 - z^2$

$= x^2 - 2xy + y^2 - z^2$

b $(2a - b + c)(2a - b - c)$

$= ((2a - b) + c)((2a - b) - c)$

$= (2a - b)^2 - c^2$

$= 4a^2 - 4ab + b^2 - c^2$

c $(3w - 4z + u)(3w + 4z - u)$

$= (3w - (4z - u))((3w + (4z - u))$

$= (3w)^2 - (4z - u)^2$

$= 9w^2 - 16z^2 + 8zu - u^2$

d $(2a - \sqrt{5}b + c)(2a + \sqrt{5}b + c)$

$= (2a + c - \sqrt{5}b)(2a + c - \sqrt{5}b)$

$= (2a + c)^2 - 5b^2$

$= 4a^2 + 4ac + c^2 - 5b^2$

10 **a** **i** $A = x^2 + 2x + 1$

ii $A = (x + 1)^2$

b **i** $A = (x - 1)^2 + 2(x - 1) + 1$

ii $A = x^2$

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Solutions to Exercise 3B

1 a $2x + 4 = 2(x + 2)$

b $4a - 8 = 4(a - 2)$

c $6 - 3x = 3(2 - x)$

d $2x - 10 = 2(x - 5)$

e $18x + 12 = 6(3x + 2)$

f $24 - 16x = 8(3 - 2x)$

2 a $4x^2 - 2xy = 2x(2x - y)$

b $8ax + 32xy = 8x(a + 4y)$

c $6ab - 12b = 6b(a - 2)$

d $6xy + 14x^2y = 2xy(3 + 7x)$

e $x^2 + 2x = x(x + 2)$

f $5x^2 - 15x = 5x(x - 3)$

g $-4x^2 - 16x = -4x(x + 4)$

h $7x + 49x^2 = 7x(1 + 7x)$

i $2x - x^2 = x(2 - x)$

3 a $6x^3y^2 + 12y^2x2 = 6x^2y^2(x + 2)$

b $7x^2y - 6y^2x = xy(7x - 6y)$

c $8x^2y^2 + 6y^2x = 2xy^2(4x + 3)$

4 a $x^3 + 5x^2 + x + 5$

$= x^2(x + 5) + (x + 5)$

$= (x + 5)(x^2 + 1)$

b $xy + 2x + 3y + 6$

$= x(y + 2) + 3(y + 2)$

$= (x + 3)(y + 2)$

c $x^2y^2 - x^2 - y^2 + 1$

$= x^2(y^2 - 1) - (y^2 - 1)$

$= (x^2 - 1)(y^2 - 1)$

$= (x - 1)(x + 1)(y - 1)(y + 1)$

d $ax + ay + bx + by$

$= a(x + y) + b(x + y)$

$= (a + b)(x + y)$

e $a^3 - 3a^2 + a - 3$

$= a^2(a - 3) + (a - 3)$

$= (a^2 + 1)(a - 3)$

f $2ab - 12a - 5b + 30$

$= 2a(b - 6) - 5(b - 6)$

$= (b - 6)(2a - 5)$

g $2x^2 - 2x + 5x - 5$

$= 2x(x - 1) + 5(x - 1)$

$= (x - 1)(2x + 5)$

h $x^3 - 4x + 2x^2 - 8$

$= x(x^2 - 4) + 2(x^2 - 4)$

$= (x^2 - 4)(x + 2)$

$= (x - 2)(x + 2)(x + 2)$

i $x^3 - bx^2 - a^2x + a^2b$

$= x^2(x - b) - a^2(x - b)$

$= (x^2 - a^2)(x - b)$

$= (x - a)(x + a)(x - b)$

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5 **a** $x^2 - 36 = (x-6)(x+6)$

b $x^2 - 81 = (x-9)(x+9)$

c $x^2 - a^2 = (x-a)(x+a)$

d $4x^2 - 81 = (2x-9)(2x+9)$

e $9x^2 - 16 = (3x-4)(3x+4)$

f $25x^2 - y^2 = (5x-y)(5x+y)$

g $3x^2 - 48 = 3(x^2 - 16)$

$= 3(x-4)(x+4)$

h $2x^2 - 98 = 2(x^2 - 49)$

$= 2(x-7)(x+7)$

i $3ax^2 - 27a = 3a(x^2 - 9)$

$= 3a(x-3)(x+3)$

j $a^2 - 7 = (a - \sqrt{7})(a + \sqrt{7})$

k $2a^2 - 5 = (\sqrt{2}a - \sqrt{5})(\sqrt{2}a + \sqrt{5})$

l $x^2 - 12 = (x - \sqrt{12})(x + \sqrt{12}) =$
$(x - 2\sqrt{3})(x + 2\sqrt{3})$

6 **a** $(x-2)^2 - 16$

$= (x-2-4)(x-2+4)$

$= (x-6)(x+2)$

b $25 - (2+x)^2$

$= (5-(2+x))(5+(2+x))$

$= (3-x)(7+x)$

c

$3(x+1)^2 - 12 = 3((x+1)^2 - 4)$

$= 3(x+1-2)(x+1+2)$

$= 3(x-1)(x+3)$

d

$(x-2)^2 - (x+3)^2$

$= ((x-2)-(x+3))((x-2)+(x+3))$

$= (x-2-x-3)(x-2+x+3)$

$= -5(2x+1)$

e

$(2x-3)^2 - (2x+3)^2$

$= ((2x-3)-(2x+3))((2x-3)+(2x+3))$

$= (-6)(4x)$

$= -24x$

f

$(2x-1)^2 - (3x+6)^2$

$= ((2x-1)-(3x+6))((2x-1)+(3x+6))$

$= (-x-7)(5x+5)$

$= -5(x+7)(x+1)$

7 **a** Check signs: must be + and −
$x^2 - 7x - 18 = (x-9)(x+2)$

b Check signs: must be − and −
$y^2 - 19y + 48 = (y-16)(y-3)$

c $a^2 - 14a + 24 = (a-12)(a-2)$

d $a^2 + 18a + 81 = (a+9)(a+9) =$
$(a+9)^2$

e $x^2 - 5x - 24 = (x-8)(x+3)$

f $x^2 - 2x - 120 = (x-12)(x+10)$

8 **a** Check signs: must be − and −
$3x^2 - 7x + 2 = (3x-a)(x-b)$
$a + 3b = 7;\ ab = 2$
$b = 2, a = 1$:
$3x^2 - 7x + 2 = (3x-1)(x-2)$

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b Check signs: must be + and +
$6x^2 + 7x + 2 = (6x + a)(x + b)$
$a + 6b = 7,\ ab = 2$; no solution.

Try:
$6x^2 + 7x + 2 = (3x + a)(2x + b)$
$2a + 3b = 7,\ ab = 2$
$a = 2,\ b = 1$
$6x^2 + 7x + 2 = (3x + 2)(2x + 1)$

c $5x^2 + 23x + 12 = (5x + a)(x + b)$
$a + 5b = 23; ab = 12$
$\therefore b = 4,\ a = 3$
$5x^2 + 23x + 12 = (5x + 3)(x + 4)$

d $2x^2 + 9x + 4$
$= 2x^2 + x + 8x + 4$
$= x(2x + 1) + 4(2x + 1)$
$= (2x + 1)(x + 4)$

e $6x^2 - 19x + 10$
$= 6x^2 - 15x - 4x + 10$
$= 3x(2x - 5) - 2(2x - 5)$
$= (2x - 5)(3x - 2)$

f $6x^2 - 7x - 3$
$= 6x^2 - 9x + (2x - 3)$
$= 3x(2x - 3) + (2x - 3)$
$= (2x - 3)(3x + 1)$

g $12x^2 - 17x + 6$
$= 12x^2 - 9x - 8x + 6)$
$= 3x(4x - 3) - 2(4x - 3)$
$= (4x - 3)(3x - 2)$

h $5x^2 - 4x - 12$
$= 5x^2 - 10x + 6x - 12)$
$= 5x(x - 2) + 6(x - 2)$
$= (x - 2)(5x + 6)$

i $5x^3 - 16x^2 + 12x$
$= x(5x^2 - 16x + 12)$
$= x(5x^2 - 10x - 6x + 12)$
$= x(5x(x - 2) - 6(x - 2))$
$= x(x - 2)(5x - 6)$

9 **a** Check signs: must be + and −
$3y^2 - 12y - 36 = 3(y^2 - 4y - 12)$
$= 3(y^2 - 4y - 12)$
$= 3(y + a)(y - b)$
$a - b = -4; ab = 12$
$\therefore a = 2,\ b = 6$
$3y^2 - 12y - 36 = 3(y + 2)(y - 6)$

b $2x^2 - 18x + 28 = 2(x^2 - 9x + 14)$
$= 2(x - 2)(x - 7)$

c $4x^2 - 36x + 72 = 4(x^2 - 9x + 18)$
$= 4(x - 6)(x - 3)$

d $3x^2 + 15x + 18 = 3(x^2 + 5x + 6)$
$= 3(x + 3)(x + 2)$

e $ax^2 + 7ax + 12a = a(x^2 + 7x + 12)$
$= a(x + 3)(x + 4)$

f
$48x - 24x^2 + 3x^3 = 3x(16 - 8x + x^2)$
$= 3x(4 - x)^2$ or $3x(x - 4)^2$

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10 **a** $(x-1)^2+4(x-1)+3$

Put $y = x-1$:

$= y^2+4y+3$

$= (y+3)(y+1)$

$= (x-1+3)(x-1+1)$

$= x(x+2)$

b $2(x-1)^2+5(x-1)-3$

Put $a = x-1$:

$= 2a^2+5a-3$

$= (2a-1)(a+3)$

$= (2(x-1)-1)(x-1+3)$

$= (2x-3)(x+2)$

c $(2x+1)^2+7(2x+1)+12$

Put $a = 2x+1$:

$= a^2+7a+12$

$= (a+3)(a+4)$

$= (2x+1+3)(2x+1+4)$

$= (2x+4)(2x+5)$

$= 2(x+2)(2x+5)$

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Solutions to Exercise 3C

1 a $(x-2)(x-3)=0, \therefore\ x=2,3$

b $x(2x-4)=0, \therefore\ 2x(x-2)=0$

$\therefore\ x=0,2$

c $(x-4)(2x-6)=0$

$\therefore\ 2(x-4)(x-3)=0$

$\therefore\ x=3,4$

d $(3-x)(x-4)=0$

$\therefore\ x=3,4$

e $(2x-6)(x+4)=0$

$\therefore\ 2(x-3)(x+4)=0$

$\therefore\ x=3,-4$

f $2x(x-1)=0, \therefore\ x=0,1$

g $(5-2x)(6-x)=0$

$\therefore\ 2\left(\frac{5}{2}-x\right)(6-x)=0$

$\therefore\ x=\frac{5}{2},6$

h $x^2=16, \therefore\ x^2-16=0$

$\therefore\ (x-4)(x+4)=0$

$\therefore\ x=4,-4$

2 a $x^2-4x-3=0$

$\therefore\ x=-0.65,\ 4.65$

b $2x^2-4x-3=0$

$\therefore\ x=-0.58,2.58$

c $-2x^2-4x+3=0$

$\therefore\ x=-2.58,0.58$

3 a $x^2-x-72=0$

$\therefore\ (x-9)(x+8)=0$

$\therefore\ x=9,-8$

b $x^2-6x+8=0$

$\therefore\ (x-2)(x-4)=0$

$\therefore\ x=2,4$

c Check signs: must be + and −

$x^2-8x-33=0$

$\therefore\ (x-a)(x+b)=0$

$a-b=8; ab=33$

$a=11; b=3$

$(x-11)(x+3)=0$

$\therefore\ x=11,-3$

d $x(x+12)=64$

$x^2+12x-64=0$

Check signs: must be + and −

$\therefore\ (x-a)(x+b)=0$

$b-a=12; ab=64;$

$b=16; a=4$

$(x-4)(x+16)=0$

$\therefore\ x=4,-16$

e Check signs: must be + and −

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$x^2 + 5x - 14 = 0$

$(x - a)(x + b) = 0$

$b - a = 5; ab = 14;$

$b = 7; a = 2$

$(x - 2)(x + 7) = 0$

$\therefore\ x = 2, -7$

f $x^2 = 5x + 24,\ \therefore\ x^2 - 5x - 24 = 0$

Check signs: must be + and −

$\therefore\ (x - a)(x + b) = 0$

$a - b = 5; ab = 24$

$a = 8; b = 3$

$(x - 8)(x + 3) = 0$

$\therefore x = 8, -3$

4 a $2x^2 + 5x + 3 = 0$

$\therefore\ (2x + a)(x + b) = 0$

$a + 2b = 5; ab = 3$

$a = 3; b = 2$

$(2x + 3)(x + 1) = 0$

$\therefore\ x = -\frac{3}{2}, -1$

b $4x^2 - 8x + 3 = 0$

$\therefore\ (2x - a)(2x - b) = 0$

$2a + 2b = 8; ab = 3$

$a = 3; b = 1$

$(2x - 3)(2x - 1) = 0$

$\therefore x = \frac{3}{2}, \frac{1}{2}$

c $6x^2 + 13x + 6 = 0$

$\therefore\ (3x + a)(2x + b) = 0$

$2a + 3b = 13; ab = 6$

$a = 2; b = 3$

$(3x + 2)(2x + 3) = 0$

$\therefore\ x = -\frac{2}{3}, -\frac{3}{2}$

d $2x^2 - x = 6$

$\therefore\ 2x^2 - x - 6 = 0$

$\therefore\ x = -\frac{3}{2}, 2$

e $6x^2 + 15 = 23x$

$\therefore\ 6x^2 - 23x + 15 = 0$

$\therefore\ (6x - a)(x - b) = 0$

$a + 6b = 23; ab = 15$

$b = 3; a = 5$

$(6x - 5)(x - 3) = 0$

$\therefore\ x = \frac{5}{6}, 3$

f Check signs: must be + and −

$2x^2 - 3x - 9 = 0$

$\therefore\ (2x - a)(x + b) = 0$

$2b - a = -3; ab = 9$

$b = -3; a = -3$

$(2x + 3)(x - 3) = 0$

$\therefore\ x = -\frac{3}{2}, 3$

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g $10x^2 - 11x + 3 = 0$

$\therefore\ (5x - a)(2x - b) = 0$

$2a + 5b = 11; ab = 3$

$a = 3; b = 1$

$(5x - 3)(2x - 1) = 0$

$\therefore\ x = \frac{3}{5}, \frac{1}{2}$

h $12x^2 + x = 6$

$\therefore\ 12x^2 + x - 6 = 0$

Check signs: must be + and −

$\therefore\ (6x - a)(2x + b) = 0$

$6b - 2a = 1; ab = 6;$ no solution

$\therefore\ (4x - a)(3x + b) = 0$

$4b - 3a = 1; ab = 6$

$a = -3; b = -2$

$(4x + 3)(3x - 2) = 0$

$\therefore\ x = -\frac{3}{4}, \frac{2}{3}$

i $4x^2 + 1 = 4x$

$\therefore\ 4x^2 - 4x + 1 = 0$

$\therefore\ (2x - 1)^2 = 0, \therefore\ x = \frac{1}{2}$

j $x(x + 4) = 5$

$x^2 + 4x - 5 = 0$

Check signs: must be + and −

$\therefore\ (x - a)(x + b) = 0$

$b - a = 4; ab = 5$

$b = 5; a = 1$

$(x - 1)(x + 5) = 0$

$\therefore x = 1, -5$

k $\frac{1}{7}x^2 = \frac{3}{7}x$

$\therefore\ x^2 = 3x, \therefore\ x^2 - 3x = 0$

$\therefore\ x(x - 3) = 0, \therefore\ x = 0, 3$

l $x^2 + 8x = -15$

$x + 8x + 15 = 0$

$(x + 5)(x + 3) = 0$

$\therefore\ x = -5, -3$

m $5x^2 = 11x - 2$

$\therefore\ 5x^2 - 11x + 2 = 0$

$\therefore\ (5x - a)(x - b) = 0$

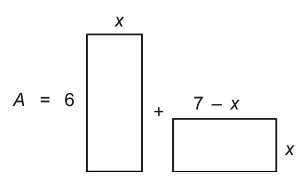
$a + 5b = 11; ab = 2$

$a = 1; b = 2$

$(5x - 1)(x - 2) = 0$

$\therefore\ x = \frac{1}{5}, 2$

5 Cut vertically down middle:



$\therefore\ A = 6x + x(7 - x) = 30$

$\therefore\ 6x + 7x - x^2 = 30$

$\therefore\ x^2 - 13x - 30 = 0$

$\therefore\ (x - 3)(x - 10) = 0$

$\therefore\ x = 3, 10$

However, $0 < x < 7$ so $x = 3$

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6 $M = \frac{wl}{2}x - \frac{w}{2}x^2$

$\therefore 104x - 8x^2 = 288$

$\therefore\ x^2 - 13x + 36 = 0$

$\therefore\ (x-4)(x-9) = 0$

$\therefore\ x = 4, 9$

7 $h = 70t - 16t^2 = 76$

$\therefore 16t^2 - 70t + 76 = 0$

$\therefore 8t^2 - 35t + 38 = 0$

$\therefore\ (8t - 19)(t-2) = 0$

$\therefore\ t = 2,\ \frac{19}{8}$ seconds

8 $D = \frac{n}{2}(n-3) = 65$

$\therefore\ n^2 - 3n - 130 = 0$

$\therefore\ (n-a)(n+b) = 0$

$b - a = -3; ab = 130$

$b = 10; a = 13$

$(n-10)(n+13) = 0$

$\therefore\ n = -10, 13$

Since $n > 0$, the polygon has 13 sides.

9 $R = 1.6 + 0.03v + 0.003v^2 = 10.6$

$\therefore 3v^2 + 1600 + 30v = 10600$

$\therefore 3v^2 + 30v - 9000 = 0$

$\therefore v^2 + 10v - 3000 = 0$

$\therefore (v-a)(v+b) = 0$

$b - a = 10;\ ab = 3000$

$b = 60, a = 50$

$(v-50)(v+60) = 0$

$\therefore\ v = 50, -60$

$v \geq 0, \therefore\ v = 50$ km/h

10 $P = 2L + 2W = 16$

$\therefore\ L = 8 - W$

$A = LW = W(8-W) = 12$

$\therefore 8W - W^2 = 12$

$\therefore\ W^2 - 8W + 12 = 0$

$\therefore\ (W-2)(W-6) = 0$

$\therefore\ w = 2, 6$

Length = 6 cm, width = 2 cm

11 $A = \frac{bh}{2} = 15$

$h = b - 1, \therefore\ A = \frac{b}{2}(b-1)$

$\frac{b}{2}(b-1) = 15$

$b^2 - b = 30, \therefore\ b^2 - b - 30 = 0$

$\therefore\ (b+5)(b-6) = 0$

$b = 6, -5$

$b \geq 0, \therefore\ b = 6$ cm

Therefore height (altitude) = 5 cm

12 $e = c + 30 \ldots (1)$

$\frac{1800}{e} + 10 = \frac{1800}{c} \ldots (2)$

Substitute (1) into (2):

$\frac{1800}{c+30} + 10 = \frac{1800}{c}$

$\therefore 1800c + 10c(c+30) = 1800(c+30)$

$\therefore 1800c + 10c^2 + 300c = 1800c + 54000$

$\therefore 10c^2 + 300c = 54000$

$\therefore c^2 + 30c - 5400 = 0$

$\therefore (c-a)\,(c+b) = 0$

$b - a = 30;$

$ab = 5400$

$b = 90, a = 60$

$(c-60)(c+90) = 0$

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$\therefore\ c = \$60$

Cheap seats are \$60, expensive \$90

13 Original cost per person = x

Original members = N where

$Nx = 2100$

$\therefore x = \dfrac{2100}{N}$

Later: $(N - 7)(x + 10) = 2100$

$\therefore\ (N - 7)(\dfrac{2100}{N} + 10) = 2100$

$\therefore\ (N - 7)(2100 + 10N) = 2100N$

$\therefore 2100N - 14700 + 10N^2 - 70N$

$= 2100N$

$\therefore -14700 + 10N^2 - 70N = 0$

$\therefore\ N^2 - 7N - 1470 = 0$

$\therefore\ (N - a)(N + b) = 0$

$a - b = 7; ab = 1470$

$a = 42; b = 35$

$\therefore\ (N - 42)(N + 35) = 0$

Since $N > 7,\ N = 42$

So 42 members originally agreed to go on the bus.

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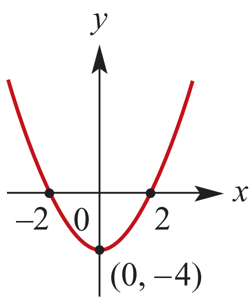
Solutions to Exercise 3D

1 a $y = x^2 - 4$

i turning point at $(0, -4)$

ii the axis of symmetry $x = 0$

iii the x-axis intercepts $(-2, 0)$ and $(2,0)$

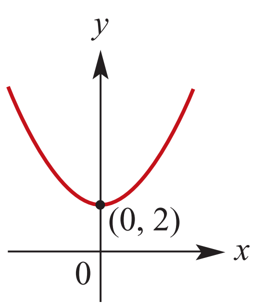


b $y = x^2 + 2$

i turning point at $(0,2)$

ii the axis of symmetry $x = 0$

iii No x-axis intercepts: $y(\text{min}) = 2$

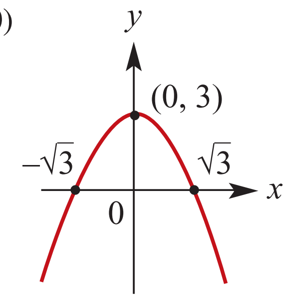


c $y = -x^2 + 3$

i turning point at $(0, 3)$

ii the axis of symmetry $x = 0$

iii the x-axis intercepts $(-\sqrt{3}, 0)$ and $(\sqrt{3}, 0)$

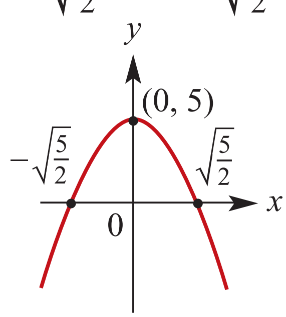


d $y = -2x^2 + 5$

i turning point at $(0, 5)$

ii the axis of symmetry $x = 0$

iii the x-axis intercepts $(-\sqrt{\frac{5}{2}}, 0)$ and $(\sqrt{\frac{5}{2}}, 0)$



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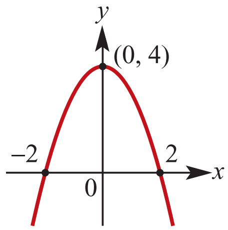
e $y = -x^2 + 4$

i turning point at $(0, 4)$

ii the axis of symmetry $x = 0$

iii the x-axis intercepts $(-2, 0)$ and $(2, 0)$

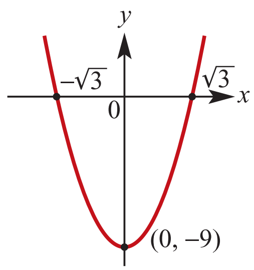


f $y = 3x^2 - 9$

i turning point at $(0, -9)$

ii the axis of symmetry $x = 0$

iii the x-axis intercepts $(-\sqrt{3}, 0)$ and $(\sqrt{3}, 0)$

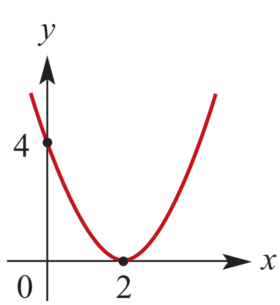


2 **a** $y = (x - 2)^2$

i turning point at $(2, 0)$

ii the axis of symmetry $x = 2$

iii the x-axis intercept $(2, 0)$ (turning pt)

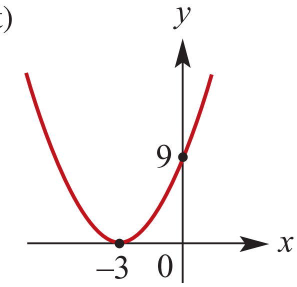


b $y = (x + 3)^2$

i turning point at $(-3, 0)$

ii the axis of symmetry $x = -3$

iii the x-axis intercept $(-3, 0)$ (= turning pt)

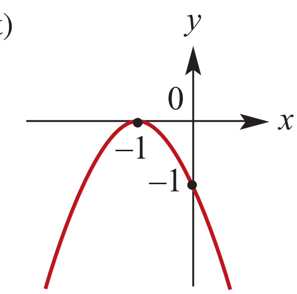


c $y = -(x + 1)^2$

i turning point at $(-1, 0)$

ii the axis of symmetry $x = -1$

iii the x-axis intercept $(-1, 0)$ (= turning pt)



d $y = -\dfrac{1}{2}(x - 4)^2$

i turning point at $(4, 0)$

ii the axis of symmetry $x = 4$

iii the x-axis intercept $(4, 0)$ (= turning pt)

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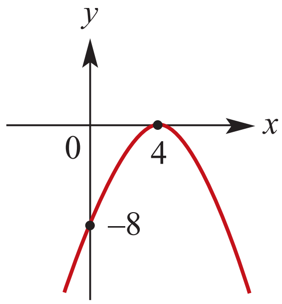
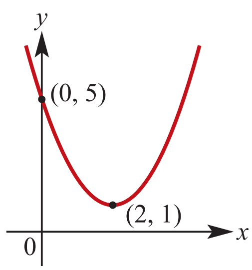


3 **a** $y = (x-2)^2 + 1$

i turning point at $(2, 1)$

ii the axis of symmetry $x = 2$

iii no x-axis intercepts.

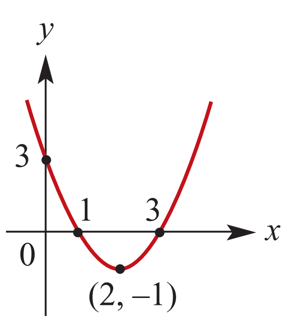


b $y = (x-2)^2 - 1$

i turning point at $(2, -1)$

ii the axis of symmetry $x = 2$

iii the x-axis intercepts $(1, 0)$ and $(3, 0)$

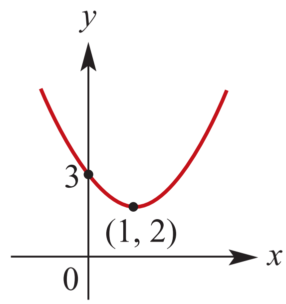


c $y = (x-1)^2 + 2$

i turning point at $(1, 2)$

ii the axis of symmetry $x = 1$

iii no x-axis intercepts

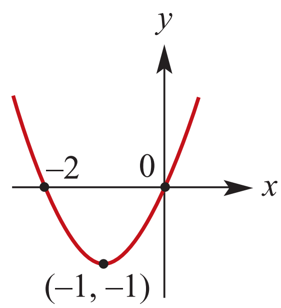


d $y = (x+1)^2 - 1$

i turning point at $(-1, -1)$

ii the axis of symmetry $x = -1$

iii the x-axis intercepts $(0, 0)$ and $(-2, 0)$



e $y = -(x-3)^2 + 1$

i turning point at $(3, 1)$

ii the axis of symmetry $x = 3$

iii the x-axis intercepts:

$$y = -(x-3)^2 + 1 = 0$$
$$\therefore (x-3)^2 = 1$$
$$\therefore x - 3 = \pm 1$$
$$\therefore x = 3 \pm 1$$

$(2, 0)$ and $(4,0)$

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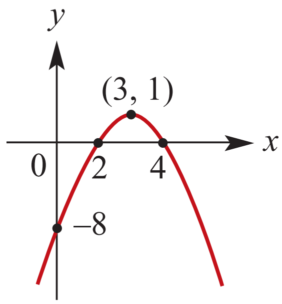
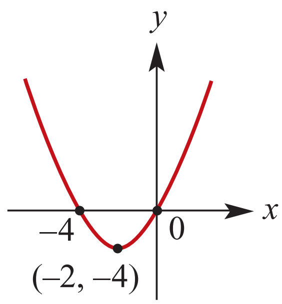


f $y = (x + 2)^2 - 4$

i turning point at $(-2, -4)$

ii the axis of symmetry $x = -2$

iii the x-axis intercepts $(0, 0)$ and $(-4, 0)$



g $y = 2(x + 2)^2 - 18$

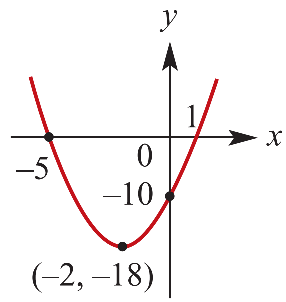
i turning point at $(-2, -18)$

ii the axis of symmetry $x = -2$

iii the x-axis intercepts:

$$\begin{aligned} y = y = 2(x+2)^2 - 18 &= 0 \\ \therefore 2(x+2)^2 &= 18 \\ \therefore (x+2)^2 &= 9 \\ \therefore x + 2 &= \pm 3 \\ \therefore x &= -2 \pm 3 \end{aligned}$$

$(-5, 0)$ and $(1, 0)$



h $y = -3(x - 4)^2 + 3$

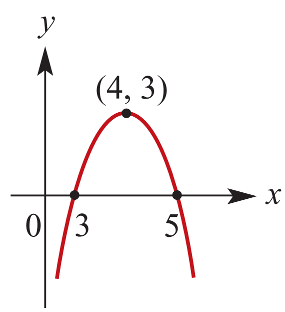
i turning point at $(4,3)$

ii the axis of symmetry $x = 4$

iii the x-axis intercepts:

$$\begin{aligned} y = -3(x-4)^2 + 3 &= 0 \\ \therefore 3(x-4)^2 &= 3 \\ \therefore (x-4)^2 &= 1 \\ \therefore x - 4 &= \pm 1 \\ \therefore x &= 4 \pm 1 \end{aligned}$$

$(5,0)$ and $(3,0)$



i $y = -\frac{1}{2}(x + 5)^2 - 2$

i turning point at $(-5, -2)$

ii the axis of symmetry $x = -5$

iii the x-axis intercepts:

$$\begin{aligned} y = -\frac{1}{2}(x+5)^2 - 2 &= 0 \\ \therefore -\frac{1}{2}(x+5)^2 &= 2 \\ \therefore (x+5)^2 &= -4 \end{aligned}$$

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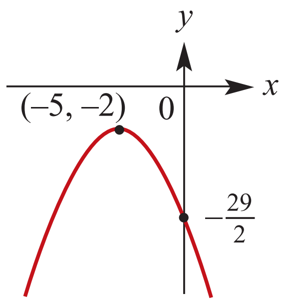
No x-axis intercepts because no real roots.



j $y = 3(x + 2)^2 - 12$

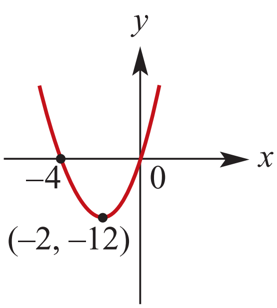
i turning point at $(-2, -12)$

ii the axis of symmetry $x = -2$

iii the x-axis intercepts:

$$y = 3(x + 2)^2 - 12 = 0$$
$$\therefore 3(x + 2)^2 = 12$$
$$\therefore (x + 2)^2 = 4$$
$$\therefore x + 2 = \pm 2$$
$$\therefore x = -2 \pm 2$$

(0,0) and $(-4, 0)$



k $y = -4(x - 2)^2 + 8$

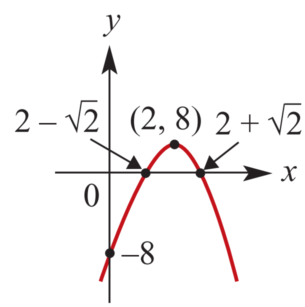
i turning point at $(2, 8)$

ii the axis of symmetry $x = 2$

iii the x-axis intercepts:

$$y = -4(x - 2)^2 + 8 = 0$$
$$\therefore 4(x - 2)^2 = 8$$
$$\therefore (x - 2)^2 = 2$$
$$\therefore x - 2 = \pm \sqrt{2}$$
$$\therefore x = 2 \pm \sqrt{2}$$

$(2 - \sqrt{2}, 0)$ and $(2 + \sqrt{2}, 0)$

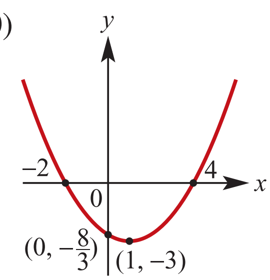


l $y = \frac{1}{3}(x - 1)^2 - 3$

i turning point at $(1, -3)$

ii the axis of symmetry $x = 1$

iii the x-axis intercepts $(-2, 0)$ and $(4, 0)$



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Solutions to Exercise 3E

1 **a** $(x-1)^2 = x^2 - 2x + 1$

b $(x+2)^2 = x^2 + 4x + 4$

c $(x-3)^2 = x^2 - 6x + 9$

d $(-x+3)^2 = x^2 - 6x + 9$

e $$\begin{aligned}(-x-2)^2 &= (-1)^2(x+2)^2\\ &= x^2 + 4x + 4\end{aligned}$$

f $(x-5)^2 = x^2 - 10x + 25$

g $\left(x - \frac{1}{2}\right)^2 = x^2 - x + \frac{1}{4}$

h $\left(x - \frac{3}{2}\right)^2 = x^2 - 3x + \frac{9}{4}$

2 **a** $x^2 - 4x + 4 = (x-2)^2$

b $x^2 - 12x + 36 = (x-6)^2$

c $$\begin{aligned}-x^2 + 4x - 4 &= -(x^2 - 4x + 4)\\ &= -(x-2)^2\end{aligned}$$

d $$\begin{aligned}2x^2 - 8x + 8 &= 2(x^2 - 4x + 4)\\ &= 2(x-2)^2\end{aligned}$$

e $$\begin{aligned}&-2x^2 + 12x - 18\\ &= -2(x^2 - 6x + 9)\\ &= -2(x-3)^2\end{aligned}$$

f $x^2 - x + \frac{1}{4} = \left(x - \frac{1}{2}\right)^2$

g $x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

h $x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$

3 **a** $x^2 - 2x - 1 = 0$

$$\begin{aligned}\therefore\ x^2 - 2x + 1 - 2 &= 0\\ \therefore\ (x-1)^2 - 2 &= 0\\ \therefore\ (x-1)^2 &= 2\\ \therefore\ x - 1 &= \pm\sqrt{2}\\ \therefore\ x &= 1 \pm \sqrt{2}\end{aligned}$$

b $x - 4x - 2 = 0$

$$\begin{aligned}\therefore\ x^2 - 4x + 4 - 6 &= 0\\ \therefore\ (x-2)^2 - 6 &= 0\\ \therefore\ (x-2)^2 &= 6\\ \therefore\ x - 2 &= \pm\sqrt{6}\\ \therefore\ x &= 2 \pm \sqrt{6}\end{aligned}$$

c $x^2 - 6x + 2 = 0$

$$\begin{aligned}\therefore\ x^2 - 6x + 9 - 7 &= 0\\ \therefore\ (x-3)^2 - 7 &= 0\\ \therefore\ (x-3)^2 &= 7\\ \therefore\ x - 3 &= \pm\sqrt{7}\\ \therefore\ x &= 3 \pm \sqrt{7}\end{aligned}$$

d $x^2 - 5x + 2 = 0$

$$\begin{aligned}\therefore\ x^2 - 5x + \frac{25}{4} + 2 - \frac{25}{4} &= 0\\ \therefore\ \left(x - \frac{5}{2}\right)^2 - \frac{17}{4} &= 0\\ \therefore\ \left(x - \frac{5}{2}\right)^2 &= \frac{17}{4}\\ \therefore\ x - \frac{5}{2} &= \pm\frac{1}{2}\sqrt{17}\\ \therefore\ x &= \frac{5 \pm \sqrt{17}}{2}\end{aligned}$$

e $2x^2 - 4x + 1 = 0$

$$\therefore \quad 2\left(x^2 - 2x + \frac{1}{2}\right) = 0$$

$$\therefore \quad x^2 - 2x + 1 - \frac{1}{2} = 0$$

$$\therefore \quad (x-1)^2 = \frac{1}{2}$$

$$\therefore \quad x - 1 = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \quad x = \frac{2 \pm \sqrt{2}}{2}$$

f $3x^2 - 5x - 2 = 0$

$$\therefore \quad 3\left(x^2 - \frac{5x}{3} - \frac{2}{3}\right) = 0$$

$$\therefore \quad x^2 - \frac{5x}{3} - \frac{2}{3} = 0$$

$$\therefore \quad x^2 - \frac{5x}{3} + \frac{25}{36} - \frac{2}{3} - \frac{25}{36} = 0$$

$$\therefore \quad \left(x - \frac{5}{6}\right)^2 - \frac{49}{36} = 0$$

$$\therefore \quad \left(x - \frac{5}{6}\right)^2 = \frac{49}{36}$$

$$\therefore \quad x - \frac{5}{6} = \pm\frac{7}{6}$$

$$\therefore \quad x = \frac{5}{6} \pm \frac{7}{6}$$

$$= 2, -\frac{1}{3}$$

g $x^2 + 2x + k = 0$

$$\therefore \quad x^2 + 2x + 1 - (1-k) = 0$$

$$\therefore \quad (x+1)^2 - (1-k) = 0$$

$$\therefore \quad x + 1 = \pm\sqrt{1-k}$$

h $kx^2 + 2x + k = 0$

$$\therefore \quad x^2 + \frac{2x}{k} + 1 = 0$$

$$\therefore \quad x^2 + \frac{2x}{k} + \frac{1}{k^2} - \frac{1}{k^2+1} = 0$$

$$\therefore \quad \left(x + \frac{1}{k}\right)^2 - \left(\frac{1}{k^2} - 1\right) = 0$$

$$\therefore \quad \left(x + \frac{1}{k}\right)^2 = \frac{1-k^2}{k^2}$$

$$\therefore \quad x + \frac{1}{k} = \pm\frac{1}{k}\sqrt{1-k^2}$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1-k^2}}{k}$$

i $x^2 - 3kx + 1 = 0$

$$\therefore \quad x^2 - 3kx + \frac{9}{4}k^2 - \left(\frac{9}{4}k^2 - 1\right) = 0$$

$$\therefore \quad \left(x - \frac{3k}{2}\right)^2 - \left(\frac{9}{4}k^2 - 1\right) = 0$$

$$\therefore \quad \left(x - \frac{3k}{2}\right)^2 = \left(\frac{9}{4}k^2 - 1\right)$$

$$\therefore \quad x - \frac{3k}{2} = \pm\sqrt{\frac{9}{4}k^2 - 1}$$

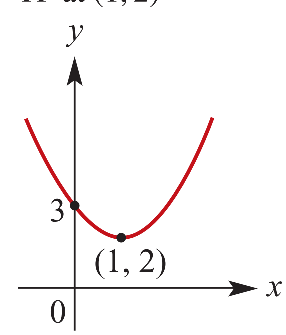
$$\therefore \quad x = \frac{3k \pm \sqrt{9k^2 - 4}}{2}$$

4 a $x^2 - 2x + 3$

$= x^2 - 2x + 1 + 2$

$= (x-1)^2 + 2$

TP at $(1, 2)$



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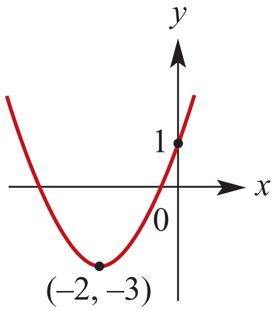
b $x^2 + 4x + 1$

$= x^2 + 4x + 4 - 3$

$= (x + 2)^2 - 3$

TP at $(-2, -3)$

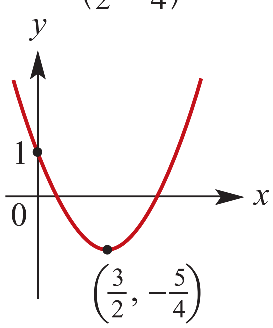


c $x^2 - 3x + 1$

$= x^2 - 3x + \dfrac{9}{4} - \dfrac{5}{4}$

$= \left(x - \dfrac{3}{2}\right)^2 - \dfrac{5}{4}$

TP at $\left(\dfrac{3}{2}, -\dfrac{5}{4}\right)$



5 **a** $y = 2x^2 - 2x - 5$

$= 2\left(x^2 - x - \dfrac{5}{2}\right)$

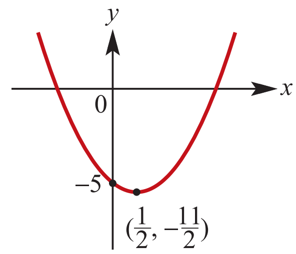
$= 2\left(x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - \frac{5}{2}\right)$

$= 2\left((x - \frac{1}{2})^2 - \left(\frac{1}{2}\right)^2 - \frac{5}{2}\right)$

$= 2\left((x - \frac{1}{2})^2 - \frac{11}{4}\right)$

$= 2\left((x - \frac{1}{2})^2\right) - \frac{11}{2}$

TP at $\left(\dfrac{1}{2}, -\dfrac{11}{2}\right)$



b $y = 4x^2 + 8x + 8$

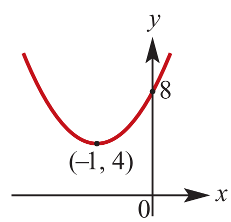
$= 4\left(x^2 + 2x + 2\right)$

$= 4\left(x^2 + 2x + 1 - 1 + 2\right)$

$= 4\left((x + 1)^2 + 1\right)$

$= 4(x + 1)^2 + 4$

TP at $(-1, 4)$



c $y = 3x^2 - 6x - 4$

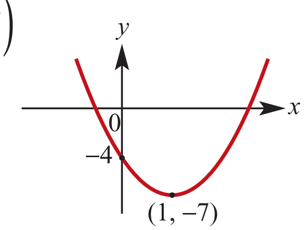
$= 3\left(x^2 - 2x - \dfrac{4}{3}\right)$

$= 3\left(x^2 - 2x + 1 - 1 - \dfrac{4}{3}\right)$

$= 3\left((x - 1)^2 - \left(\sqrt{\frac{7}{3}}\right)^2\right)$

$= 3(x - 1)^2 - 7$

TP at $\left(1, -7\right)$



6 **a** $x^2 - 8x + 12$

$= x^2 - 8x + 16 - 4$

$= (x - 4)^2 - 4$

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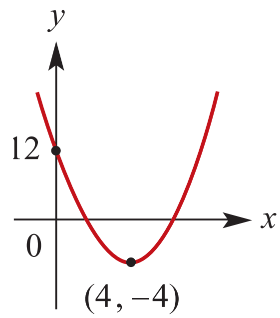
TP at $(4, -4)$

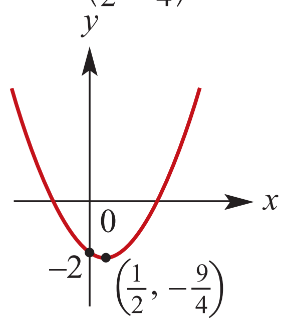


b $x^2 - x - 2$

$= x^2 - x + \frac{1}{4} - \frac{9}{4}$

$= \left(x - \frac{1}{2}\right)^2 - \frac{9}{4}$

TP at $\left(\frac{1}{2}, -\frac{9}{4}\right)$



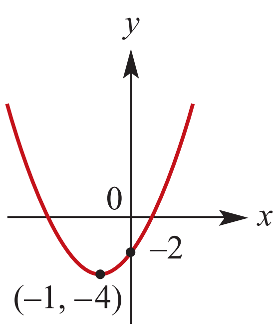
c $2x^2 + 4x - 2$

$= 2(x^2 + 2x - 1)$

$= 2(x^2 + 2x + 1 - 2)$

$= 2(x + 1)^2 - 4$

TP at $(-1, -4)$



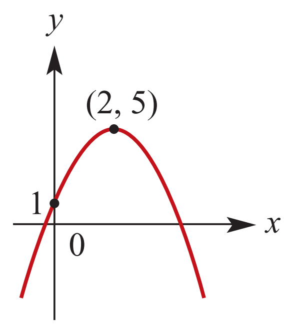
d $-x^2 + 4x + 1$

$= -(x^2 - 4x - 1)$

$= -(x^2 - 4x + 4 - 5)$

$= -(x - 2)^2 + 5$

TP at $(2, 5)$



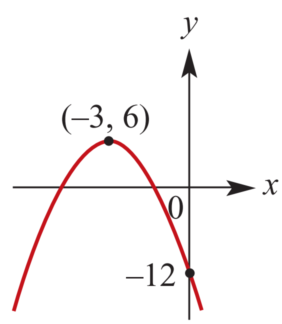
e $-2x^2 - 12x - 12$

$= -2(x^2 + 6x + 6)$

$= -2(x^2 + 6x + 9 - 3)$

$= -2(x + 3)^2 + 6$

TP at $(-3, 6)$



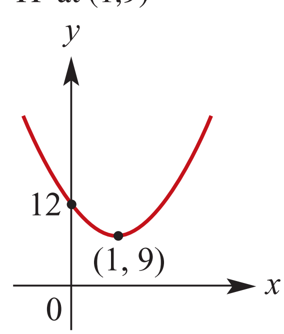
f $3x^2 - 6x + 12$

$= 3(x^2 - 2x + 4)$

$= 3(x^2 - 2x + 1 + 3)$

$= 3(x - 1)^2 + 9$

TP at $(1,9)$



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Solutions to Exercise 3F

1 **a** x-axis intercepts 4 and 10;
x-coordinate of vertex
$= \frac{1}{2}(4 + 10) = 7$

b x-axis intercepts 6 and 8;
x-coordinate of vertex $= \frac{1}{2}(6 + 8) = 7$

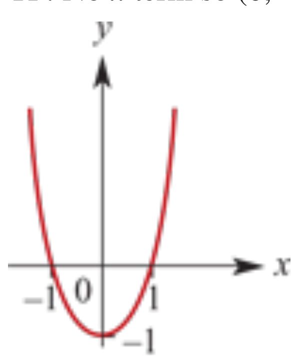
c x-axis intercepts -6 and 8;
x-coordinate of vertex
$= \frac{1}{2}(-6 + 8) = 1$

2 **a** x-axis intercepts a and 6;
x-coordinate of vertex $= \frac{1}{2}(a + 6) = 2$
$\therefore\ a + 6 = 4, \therefore\ a = -2$

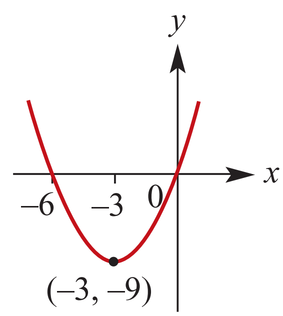
b x-axis intercepts a and -4;
x-coordinate of vertex $= \frac{1}{2}(a - 4) = 2$
$\therefore\ a - 4 = 4, \therefore\ a = 8$

c x-axis intercepts a and 0;
x-coordinate of vertex $\frac{1}{2}(a + 0) = 2$
$\therefore\ a = 4$

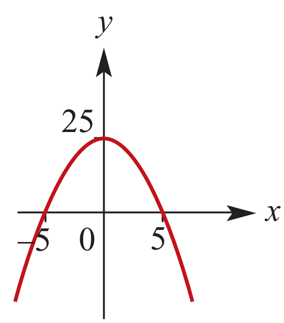
3 **a** $y = x^2 - 1$
x-intercepts: $y = x^2 - 1 = 0$
$\therefore\ (x - 1)(x + 1) = 0$
$\therefore x = 1, -1$
x-int: $(-1, 0)$ and $(1, 0)$
TP: No x term so $(0, -1)$



b $y = x^2 + 6x$
x-intercepts: $y = x^2 + 6x = 0$
$\therefore x(x + 6) = 0$
$\therefore x = 0, -6$
x-int: $(-6, 0)$ and $(0, 0)$
TP: $y = x^2 + 6x$
$= x^2 + 6x + 9 - 9$
$= (x + 3)^2 - 9$
TP at $(-3, -9)$



c $y = 25 - x^2$
x-intercepts: $y = 25 - x^2 = 0$
$\therefore\ (5 - x)(5 + x) = 0$
$\therefore x = 5, -5$
TP: No x term so (0,25)



d $y = x^2 - 4$
x-intercepts: $y = x^2 - 4 = 0$
$\therefore\ (x - 2)(x + 2) = 0$
$\therefore\ x = 2, -2$
x-int: $(-2, 0)$ and $(2, 0)$
TP: No x term so $(0, -4)$

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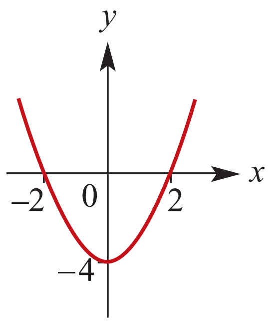


e $y = 2x^2 + 3x$

x-intercepts: $y = 2x^2 + 3x = 0$

$\therefore\ x(2x + 3) = 0$

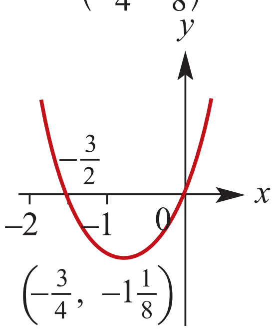
x-int: $\left(-\frac{3}{2}, 0\right)$ and $(0, 0)$

TP: $y = 2x^2 + 3x$

$$= 2\left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right)$$

$$= 2\left(x + \frac{3}{4}\right)^2 - \frac{9}{8}$$

TP at $\left(-\frac{3}{4}, -\frac{9}{8}\right)$



f $y = 2x^2 - 4x$

x-intercepts: $y = 2x^2 - 4x = 0$

$\therefore\ 2x(x - 2) = 0$

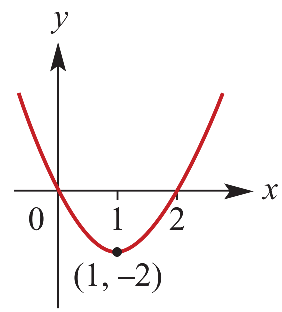
x-int: $(2, 0)$ and $(0, 0)$

TP: $y = 2x^2 - 4x$

$$= 2(x^2 - 2x + 1 - 1)$$

$$= 2(x - 1)^2 - 2$$

TP at $(1, -2)$



g $y = -2x^2 - 3x$

x-intercepts: $y = -2x^2 - 3x = 0$

$\therefore\ -x(2x + 3) = 0$

x-int: $\left(-\frac{3}{2}, 0\right)$ and $(0, 0)$

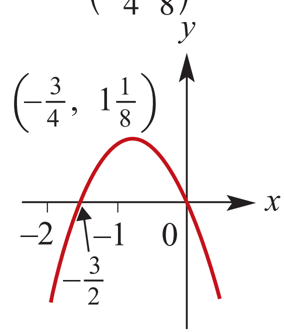
TP: $y = -2x^2 - 3x$

$$= -2\left(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right)$$

$$= -2\left((x + \frac{3}{4}\right)^2 + \frac{9}{8}$$

$$= -2\left((x + \frac{3}{4}\right)^2 + \frac{9}{8}$$

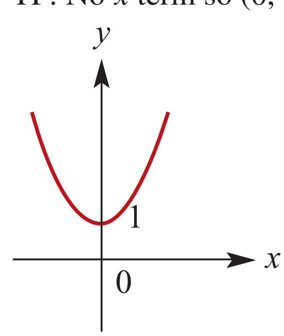
TP at $\left(-\frac{3}{4}, \frac{9}{8}\right)$



h $y = x^2 + 1$

No x-intercepts since $y > 0$ for all x

TP: No x term so $(0, 1)$



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4 a $y = x^2 + 3x - 10$

x-intercepts: $y = x^2 + 3x - 10 = 0$

$\therefore (x + 5)(x - 2) = 0$

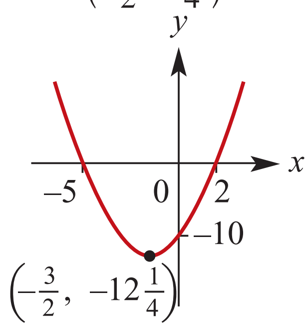
x-int: $(-5, 0)$ and $(2, 0)$

TP: $y = x^2 + 3x - 10$

$$y = x^2 + 3x + \frac{9}{4} - \frac{9}{4} - 10$$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{49}{4}$$

TP at $\left(-\frac{3}{2}, -\frac{49}{4}\right)$



b $y = x^2 - 5x + 4$

x-intercepts: $y = x^2 - 5x + 4 = 0$

$\therefore (x - 1)(x - 4) = 0$

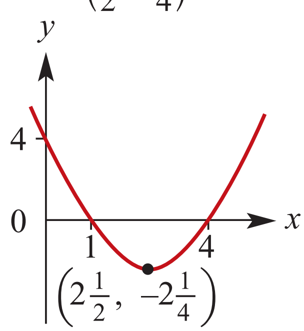
x-int: $(1, 0)$ and $(4, 0)$

TP: $y = x^2 - 5x + 4$

$$y = x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 4$$

$$y = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$$

TP at $\left(\frac{5}{2}, -\frac{9}{4}\right)$



c $y = x^2 + 2x - 3$

x-intercepts: $y = x^2 + 2x - 3$

$\therefore (x - 1)(x + 3) = 0$

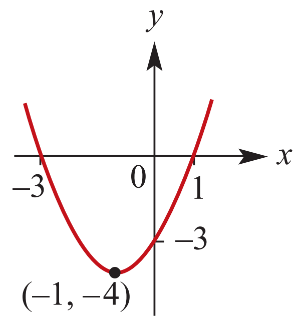
x-int: $(1, 0)$ and $(-3, 0)$

TP: $y = x^2 + 2x - 3$

$$y = x^2 + 2x + 1 - 4$$

$$y = (x + 1)^2 - 4$$

TP at $(-1, -4)$



d $y = x^2 + 4x + 3$

x-intercepts: $y = x^2 + 4x + 3 = 0$

$\therefore (x + 1)(x + 3) = 0$

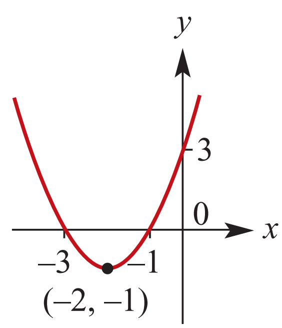
x-int: $(-1, 0)$ and $(-3, 0)$

TP: $y = x + 4x + 3$

$$y = x^2 + 4x + 4 - 1$$

$$y = (x + 2)^2 - 1$$

TP at $(-2, -1)$



e $y = 2x^2 - x - 1$

x-intercepts: $y = 2x^2 - x - 1 = 0$

$\therefore (2x + 1)(x - 1) = 0$

x-int: $\left(-\frac{1}{2}, 0\right)$ and $(1, 0)$

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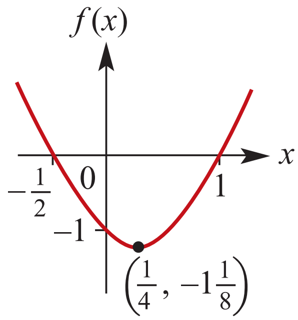
TP: $y = 2x^2 - x - 1$

$$y = 2\left(x^2 - \frac{x}{2} + \frac{1}{2}\right)$$

$$y = 2\left(x^2 - \frac{x}{2} + \frac{1}{16} - \frac{9}{16}\right)$$

$$y = 2\left(x - \frac{1}{4}\right)^2 - \frac{9}{8}$$

TP at $\left(\frac{1}{4}, -\frac{9}{8}\right)$



f $y = 6 - x - x^2$

x-intercepts: $y = -(x^2 + x - 6) = 0$

$\therefore -(x + 3)(x - 2) = 0$

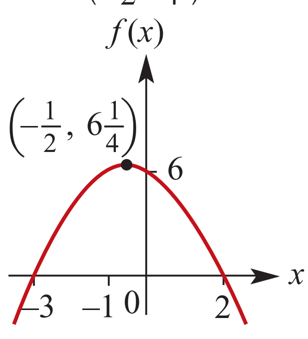
x-int: $(-3, 0)$ and $(2,0)$

TP: $y = -(x^2 + x - 6)$

$$y = -\left(x^2 + x + \frac{1}{4} - 6 - \frac{1}{4}\right)$$

$$y = -\left(x + \frac{1}{2}\right)^2 + \frac{25}{4}$$

TP at $\left(-\frac{1}{2}, \frac{25}{4}\right)$



g $y = -x^2 - 5x - 6$

x-intercepts: $y = -(x^2 + 5x + 6) = 0$

$\therefore -(x + 3)(x + 2) = 0$

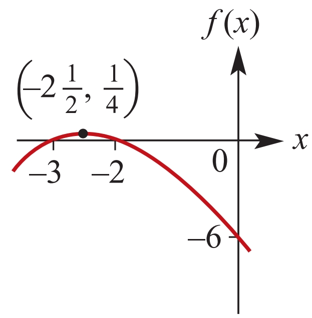
x-int: $(-3, 0)$ and $(-2, 0)$

TP: $y = -(x^2 + 5x + 6)$

$$y = -\left(x^2 + 5x + \frac{25}{4} + 6 - \frac{25}{4}\right)$$

$$y = -\left(x + \frac{5}{2}\right)^2 + \frac{1}{4}$$

TP at $\left(-\frac{5}{2}, \frac{1}{4}\right)$



h $y = x^2 - 5x - 24$

x-intercepts: $y = x^2 - 5x - 24 = 0$

$\therefore (x + 3)(x - 8) = 0$

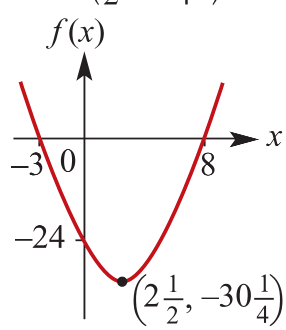
x-int: $(-3, 0)$ and $(8,0)$

TP: $y = x^2 - 5x - 24$

$$y = x^2 - 5x + \frac{25}{4} - 24 - \frac{25}{4}$$

$$y = \left(x - \frac{5}{2}\right)^2 - \frac{121}{4}$$

TP at $\left(\frac{5}{2} - \frac{121}{4}\right)$



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Solutions to Exercise 3G

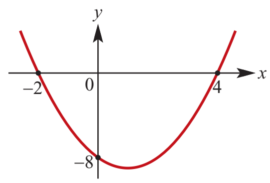
1 a $x^2 + 2x - 8 = 0$

$\therefore (x + 2)(x - 4) = 0$

$\therefore x = -2, 4$

'Positive coefficient of x^2':

b



c $x^2 - 2x - 8 \le 0 \Leftrightarrow -2 \le x \le 4$

d $x^2 - 2x - 8 > 0 \Leftrightarrow x > 4$ or $x < -2$

2 a Positive coefficient of x^2

$x \le -2$ or $x \ge 3$

b Positive coefficient of x^2

$-4 < x < -3$

c Positive coefficient of x^2

$-4 \le x \le \frac{1}{2}$

d Positive coefficient of x^2

$x < 2$ or $x > 6$

e Positive coefficient of x^2

$2 < x < 3$

f Negative coefficient of x^2

$\frac{3}{2} \le x \le \frac{7}{2}$

g Positive coefficient of x^2

$-\frac{7}{2} < x < 2$

h Positive coefficient of x^2

$-2 \le x \le \frac{5}{2}$

i Negative coefficient of x^2

$x < -5$ or $x > \frac{5}{2}$

j Negative coefficient of x^2

$-2 \le x \le \frac{7}{2}$

k Negative coefficient of x^2

$x < \frac{2}{5}$ or $x > \frac{7}{2}$

l Positive coefficient of x^2

$x \le \frac{5}{2}$ or $x \ge \frac{11}{2}$

3 a Negative coefficient of x^2

$x < -5$ or $x > 5$

b Negative coefficient of y^2

$-\frac{2}{3} \le y \le \frac{2}{3}$

c Negative coefficient of y^2

$y > 4$ or $y < -4$

d Negative coefficient of x^2

$-\frac{6}{5} \le x \le \frac{6}{5}$

e Negative coefficient of y^2

$y \le -\frac{1}{4}$ or $y \ge \frac{1}{4}$

f Negative coefficient of y^2

$y < -\frac{5}{6}$ or $y > \frac{5}{6}$

4 a $x^2 + 2x - 8 = 0$

$\therefore (x + 4)(x - 2) = 0$

$\therefore x = 2, -4$

'Positive coefficient of x^2':

$x \ge 2$ or $x \le -4$

b $x^2 - 5x - 24 = 0$

$\therefore (x+3)(x-8)=0$
$\therefore x=-3, 8$
'Positive coefficient of x^2':
$\{x: \ -3 < x < 8\}$

c $x-4x-12=0$
$\therefore (x+2)(x-6)=0$
$\therefore x=-2, 6$
'Positive coefficient of x^2':
$\{x: \ -2 \le x \le 6\}$

d $2x^2-3x-9=0$
$\therefore (2x+3)(x-3)=0$
$\therefore x=-\frac{3}{2}, 3$
'Positive coefficient of x^2':
$\{x: x < -\frac{3}{2}\} \cup \{x: x > 3\}$

e $6x^2+13x<-6$
$\therefore 6x^2+13x+6<0$
$6x^2+13x+6=0$
$\therefore (3x+2)(2x+3)=0$
$x=-\frac{2}{3}, -\frac{3}{2}$
'Positive coefficient of x^2':
$\left\{x: -\frac{3}{2} < x < -\frac{2}{3}\right\}$

f $-x^2-5x-6=0$
$\therefore -(x+2)(x+3)=0$
$\therefore x=-2, -3$
'Negative coefficient of x^2':
$\{x: \ -3 \le x \le -2\}$

g $12x^2+x>6$
$\therefore 12x^2+x-6>0$
$12x^2+x-6=0$
$\therefore (4x+3)(3x-2)=0$
$\therefore x=-\frac{3}{4}, \frac{3}{2}$
'Positive coefficient of x^2':
$\left\{x: x < -\frac{3}{4}\right\} \cup \left\{x: x > \frac{3}{2}\right\}$

h $10x^2-11x \le -3$
$\therefore 10x^2-11x+3 \le 0$
$10x^2-11x+3=0$
$\therefore (5x-3)(2x-1)=0$
$\therefore x=\frac{1}{2}, \frac{3}{5}$
'Positive coefficient of x^2':
$\left\{x: \frac{1}{2} \le x \le \frac{3}{5}\right\}$

i $x(x-1) \le 20$
$\therefore x^2-x-20 \le 0$
$x^2-x-20=0$
$\therefore (x-5)(x+4)=0$
$x=-4, 5$
'Positive coefficient of x^2':
$\{x: \ -4 \le x \le 5\}$

j $4+5p-p^2=0$
$\therefore p=\dfrac{-5 \pm \sqrt{41}}{-2}$
'Negative coefficient of x^2':
$\left\{p: \dfrac{5-\sqrt{41}}{2} \le p \le \dfrac{5+\sqrt{41}}{2}\right\}$

k $3+2y-y^2=0$
$\therefore (1+y)(3-y)=0$
$\therefore y=-1, 3$
'Negative coefficient of x^2':
$\{y: y < -1\} \cup \{y: y > 3\}$

l $x^2+3x \ge -2$
$\therefore x^2+3x+2 \ge 0$
$x^2+3x+2=0$
$\therefore (x+2)(x+1)=0$
$\therefore x=-2; -1$
'Positive coefficient of x^2':
$\{x: x \le -2\} \cup \{x: x \ge -1\}$

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5 a

$$x^2 + 3x - 5 \geq 0$$

$$\Leftrightarrow \left(x + \frac{3}{2}\right)^2 - \frac{29}{4} \geq 0$$

$$\Leftrightarrow \left(x + \frac{3}{2}\right)^2 \geq \frac{29}{4}$$

$$\Leftrightarrow x \leq -\frac{3}{2} - \frac{\sqrt{29}}{2} \text{ or } x \geq -\frac{3}{2} + \frac{\sqrt{29}}{2}$$

b $x^2 - 5x + 2 < 0$

$$\Leftrightarrow 2\left(x - \frac{5}{2}\right)^2 - \frac{17}{4} < 0$$

$$\Leftrightarrow \left(x - \frac{5}{2}\right)^2 < \frac{17}{4}$$

$$\Leftrightarrow \frac{5}{2} - \frac{\sqrt{17}}{2} < x < \frac{5}{2} + \frac{\sqrt{17}}{2}$$

c $2x^2 - 3x - 1 \leq 0$

$$\Leftrightarrow 2\left(x - \frac{3}{4}\right)^2 - \frac{17}{8} \leq 0$$

$$\Leftrightarrow 2\left(x - \frac{3}{4}\right)^2 \leq \frac{17}{8}$$

$$\Leftrightarrow \left(x - \frac{3}{4}\right)^2 \leq \frac{17}{16}$$

$$\Leftrightarrow \frac{3}{4} - \frac{\sqrt{17}}{4} < x < \frac{3}{4} + \frac{\sqrt{17}}{4}$$

d $2x^2 - 3x - 1 \leq 0$

$$\Leftrightarrow -\left(x + \frac{3}{2}\right)^2 + \frac{41}{4} > 0$$

$$\Leftrightarrow \left(x - \frac{3}{4}\right)^2 < \frac{41}{4}$$

$$\Leftrightarrow \left(x - \frac{3}{4}\right)^2 < \frac{41}{16}$$

$$\Leftrightarrow \frac{3}{4} - \frac{\sqrt{41}}{4} < x < \frac{3}{4} + \frac{\sqrt{41}}{4}$$

e $2x^2 + 7x + 1 \leq 0$

$$\Leftrightarrow 2\left(x^2 + \frac{7}{2} + \frac{49}{16} - \frac{49}{16} + \frac{1}{2}\right) < 0$$

$$\Leftrightarrow 2\left(x + \frac{7}{4}\right)^2 - \frac{41}{16} < 0$$

$$\Leftrightarrow \left(x + \frac{7}{4}\right)^2 < \frac{41}{16}$$

$$\Leftrightarrow \frac{-7 - \sqrt{41}}{4} < x < \frac{-7 + \sqrt{41}}{4}$$

f $2x^2 - 8x + 5 \geq 0$

$$\Leftrightarrow 2\left(x^2 - 4 + 4 - 4 + \frac{5}{2}\right) \geq 0$$

$$\Leftrightarrow 2(x - 2)^2 - \frac{3}{2} \geq 0$$

$$\Leftrightarrow (x - 2)^2 \geq \frac{3}{2}$$

$$\Leftrightarrow x \leq \frac{4 - \sqrt{6}}{2} \text{ or } x \geq \frac{4 + \sqrt{6}}{2}$$

6 The square of any real number is zero or positive.

7 The negative of the square of any real number is zero or negative.

8 $x^2 + 2x + 7$

$= x^2 + 2x + 1 - 1 + 7$

$= (x + 1)^2 + 6$

Since $(x + 1)^2 \geq 0$ for all x

$(x + 1)^2 + 6 \geq 6$ for all x

9 $-x^2 - 2x - 7$

$= -(x^2 + 2x + 1 - 1 + 7$

$= -((x + 1)^2 - 6$

Since $-(x + 1)^2 \leq 0$ for all x

$-(x + 1)^2 - 6 \leq -6$ for all x

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Solutions to Exercise 3H

1 **a** $a = 2, b = 4$ and $c = -3$

i $b^2 - 4ac = 4^2 - 4(-3)2 = 40$

ii $\sqrt{b^2 - 4ac} = \sqrt{40} = 2\sqrt{10}$

b $a = 1, b = 10$ and $c = 18$

i $b^2 - 4ac = 10^2 - 4(18)1 = 28$

ii $\sqrt{b^2 - 4ac} = \sqrt{28} = 2\sqrt{7}$

c $a = 1, b = 10$ and $c = -18$

i $b^2 - 4ac = 10^2 - 4(-18)1 = 172$

ii $\sqrt{b^2 - 4ac} = \sqrt{172} = 2\sqrt{43}$

d $a = -1, b = 6$ and $c = 15$

i $b^2 - 4ac = 6^2 - 4(15)(-1) = 96$

ii $\sqrt{b^2 - 4ac} = \sqrt{96} = 4\sqrt{6}$

e $a = 1, b = 9$ and $c = -27$

i $b^2 - 4ac = 9^2 - 4(-27)1 = 189$

ii $\sqrt{b^2 - 4ac} = \sqrt{189} = 3\sqrt{21}$

2 **a** $\dfrac{2 + 2\sqrt{5}}{2} = 1 + \sqrt{5}$

b $\dfrac{9 - 3\sqrt{5}}{6} = \dfrac{3 - \sqrt{5}}{2}$

c $\dfrac{5 + 5\sqrt{5}}{10} = \dfrac{1 + \sqrt{5}}{2}$

d $\dfrac{6 + 12\sqrt{2}}{6} = 1 + 2\sqrt{2}$

3 **a** $x^2 + 6x = 4$

$\therefore x^2 + 6x - 4 = 0$

$\therefore x = \dfrac{-6 \pm \sqrt{6^2 - 4(-4)1}}{2}$

$\therefore x = \dfrac{-6 \pm \sqrt{52}}{2}$

$\therefore x = -3 \pm \sqrt{13}$

b $x^2 - 7x - 3 = 0$

$\therefore x = \dfrac{7 \pm \sqrt{7^2 - 4(-3)1}}{2}$

$\therefore x = \dfrac{7 \pm \sqrt{61}}{2}$

c $2x^2 - 5x + 2 = 0$

$\therefore x = \dfrac{5 \pm \sqrt{5^2 - 4(2)2}}{4}$

$\therefore x = \dfrac{5 \pm \sqrt{9}}{4}$

$\therefore x = \dfrac{5 \pm 3}{4} = \dfrac{1}{2}, 2$

d $2x^2 + 4x - 7 = 0$

$\therefore x = \dfrac{-4 \pm \sqrt{4^2 - 4(-7)2}}{4}$

$\therefore x = \dfrac{-4 \pm \sqrt{72}}{4}$

$\therefore x = -1 \pm \dfrac{6}{4}\sqrt{2}$

$\therefore x = -1 \pm \dfrac{3}{2}\sqrt{2}$

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e $2x^2 + 8x = 1$

$\therefore 2x^2 + 8x - 1 = 0$

$\therefore x = \dfrac{-8 \pm \sqrt{8^2 - 4(-1)2}}{4}$

$\therefore x = -2 \pm \dfrac{\sqrt{72}}{4}$

$\therefore x = -2 \pm \dfrac{3}{2}\sqrt{2}$

f $5x^2 - 10x = 1$

$\therefore 5x^2 - 10x - 1 = 0$

$\therefore x = \dfrac{10 \pm \sqrt{10^2 - 4(-1)5}}{10}$

$\therefore x = 1 \pm \dfrac{\sqrt{120}}{10}$

$\therefore x = 1 \pm \dfrac{\sqrt{30}}{5}$

g $-2x^2 + 4x - 1 = 0$

$\therefore x = \dfrac{-4 \pm \sqrt{4^2 - 4(-1)(-2)}}{-4}$

$\therefore x = 1 \pm \dfrac{\sqrt{8}}{4}$

$\therefore x = 1 \pm \dfrac{\sqrt{2}}{2}$

h $2x^2 + x = 3$

$\therefore 2x^2 + x - 3 = 0$

$\therefore x = \dfrac{-1 \pm \sqrt{1^2 - 4(-3)2}}{4}$

$\therefore x = \dfrac{-1 \pm \sqrt{25}}{4}$

$\therefore x = \dfrac{-1 \pm 5}{4} \quad \therefore x = 1, -\dfrac{3}{2}$

i $2.5x^2 + 3x + 0.3 = 0$

$\therefore x = \dfrac{-3 \pm \sqrt{3^2 - 4(0.3)2.5}}{5}$

$\therefore x = \dfrac{-3 \pm \sqrt{6}}{5}$

j $-0.6x^2 - 1.3x = 0.1$

$\therefore -6x^2 - 13x - 1 = 0$

$\therefore 6x^2 + 13x + 1 = 0$

$\therefore x = \dfrac{-13 \pm \sqrt{13^2 - 4(1)6}}{12}$

$\therefore x = \dfrac{-13 \pm \sqrt{145}}{12}$

k $2kx^2 - 4x + k = 0$

$\therefore x = \dfrac{4 \pm \sqrt{4^2 - 4(2k)k}}{4k}$

$\therefore x = 1 \pm \dfrac{\sqrt{16 - 8k^2}}{4k}$

$\therefore x = \dfrac{2 \pm \sqrt{4 - 2k^2}}{2k}$

l $2(1 - k)x^2 - 4kx + k = 0$

$\therefore x = \dfrac{4k \pm \sqrt{16k^2 - 8k(1 - k)}}{4(1 - k)}$

$\therefore x = \dfrac{4k \pm \sqrt{24k^2 - 8k}}{4(1 - k)}$

$\therefore x = \dfrac{2k \pm \sqrt{6k^2 - 2k}}{2(1 - k)}$

4 a $y = x^2 + 5x - 1$

x-axis intercepts:

$x = \dfrac{-5 \pm \sqrt{29}}{2}$

$x = -\dfrac{5}{2};$

$y = \dfrac{25}{4} - \dfrac{25}{2} - 1 = -\dfrac{29}{4}$

TP at $(-2.5, -7.25)$

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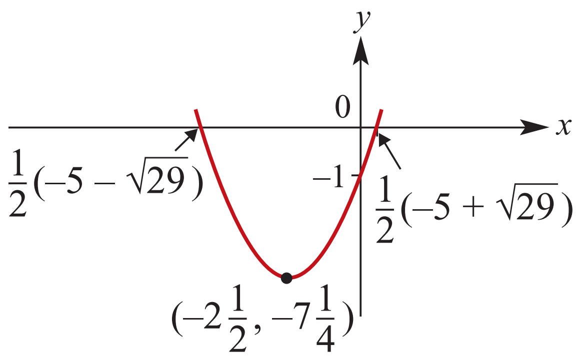


b $y = 2x^2 - 3x - 1$

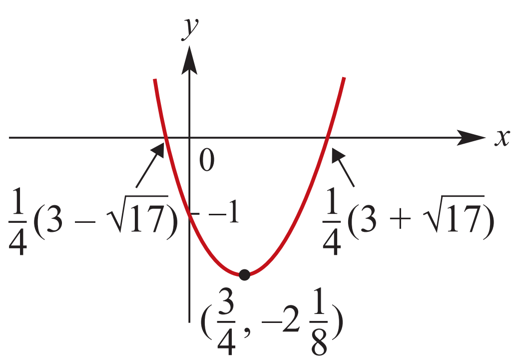
x-axis intercepts:

$\therefore x = \dfrac{3 \pm \sqrt{17}}{4}$

$x = \dfrac{3}{4}$;

$y = \dfrac{9}{8} - \dfrac{9}{4} - 1 = -\dfrac{17}{8}$

TP at $(0.75, -2.125)$



c $y = -x^2 - 3x + 1$

x-axis intercepts:

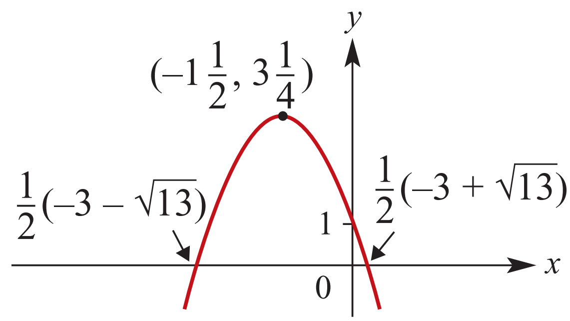
$\therefore x = \dfrac{3 \pm \sqrt{13}}{-2}$

$\therefore x = \dfrac{-3 \pm \sqrt{13}}{2}$

$x = -\dfrac{3}{2}$;

$y = -\dfrac{9}{4} + \dfrac{9}{2} + 1 = \dfrac{13}{4}$

TP at $(-1.5, 3.25)$



d $y + 4 = x^2 + 2x$

$\therefore y = x^2 + 2x - 4$

x-axis intercepts:

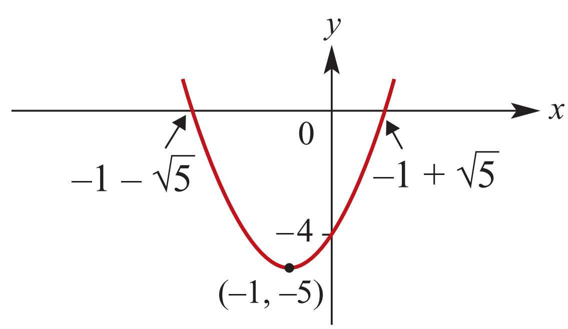
$\therefore x = \dfrac{-2 \pm \sqrt{20}}{2}$

$\therefore x = -1 \pm \sqrt{5}$

$x = -1$;

$y = 1 - 2 - 4 = -5$

TP at $(-1, -5)$



e $y = 4x^2 + 5x + 1$

x-axis intercepts:

$\therefore x = \dfrac{-5 \pm \sqrt{25-16}}{8}$

$\therefore x = \dfrac{-5 \pm 3}{8}$

$\therefore x = -1, -\dfrac{1}{4}$

$x = -\dfrac{5}{8}$;

$y = \dfrac{100}{64} - \dfrac{25}{8} + 1 = -\dfrac{9}{16}$

TP at $(-0.625, -0.5625)$

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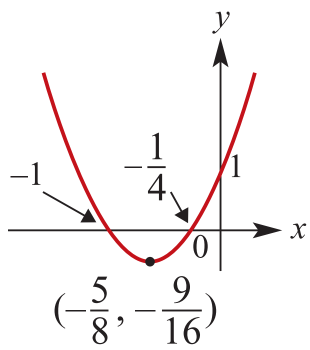


f $y = -3x^2 + 4x - 2$

x-axis intercepts:

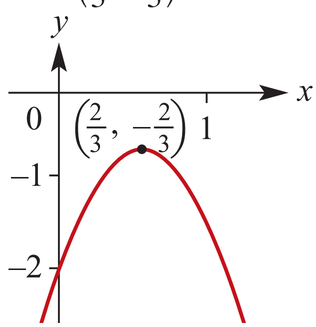
$\therefore x = \dfrac{-4 \pm \sqrt{16 - 24}}{-6}$

This is not defined, so no x-intercepts.

$x = \dfrac{2}{3}$;

$y = -\dfrac{4}{3} + \dfrac{8}{3} - 2 = -\dfrac{2}{3}$

TP at $\left(\dfrac{2}{3}, -\dfrac{2}{3}\right)$



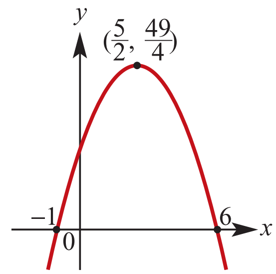
g $y = -x^2 + 5x + 6$ When $y = 0$

$$-x^2 + 5x + 6 = 0$$
$$x^2 - 5x - 6 = 0$$
$$(x - 6)(x + 1) = 0$$
$$x = 6 \text{ or } x = -1$$

When $x = 0, y = 6$

Axis of symmetry: $x = \dfrac{5}{2}$

Coordinates of turning point $\left(\dfrac{5}{2}, \dfrac{49}{4}\right)$

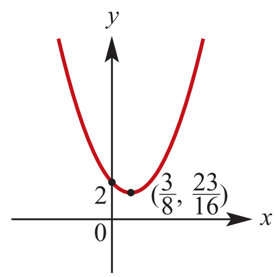


h $y = 4x^2 - 3x + 2$

$\Delta = 9 - 4 \times 4 \times 2 < 0$

Therefore no x-axis intercepts. Axis of symmetry: $x = \dfrac{3}{8}$

Coordinates of turning point $\left(\dfrac{3}{8}, \dfrac{23}{16}\right)$



i $y = 3x^2 - x - 4$

When $y = 0$,

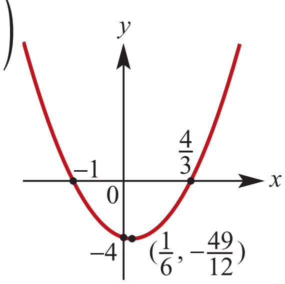
$x = \dfrac{1 \pm \sqrt{1 - 4 \times 3 \times (-4)}}{6}$

That is $x = \dfrac{1 \pm 7}{6}$

$x = -1$ or $x = \dfrac{4}{3}$

Axis of symmetry: $x = \dfrac{1}{6}$

Coordinates of turning point $\left(\dfrac{1}{6}, -\dfrac{49}{12}\right)$



Solutions to Exercise 3I

1 a $x^2 + 2x - 4;$
$\Delta = 2^2 - 4(-4) = 20$

b $x^2 + 2x + 4;$
$\Delta = 2^2 - 4(4) = -12$

c $x^2 + 3x - 4;$
$\Delta = 3^2 - 4(-4) = 25$

d $2x^2 + 3x - 4;$
$\Delta = 3^2 - 8(-4) = 41$

e $-2x^2 + 3x + 4;$
$\Delta = 3^2 - 4(-2)(4) = 41$

2 a $x^2 - 5x + 2;$
$\Delta = 5^2 - 4(2) = 17$
$\Delta > 0$ so graph crosses the x-axis

b $-4x^2 + 2x - 1;$
$\Delta = 2^2 - 4(-4)(-1) = -12$
$\Delta < 0$ so graph does not cross the x-axis

c $x^2 - 6x + 9;$
$\Delta = 6^2 - 4(9) = 10$
$\Delta = 0$ so graph touches the x-axis

d $-2x^2 - 3x + 8;$
$\Delta = 3^2 - 4(-2)8 = 73$
$\Delta > 0$ so graph crosses the x-axis

e $3x^2 + 2x + 5;$
$\Delta = 2^2 - 4(5)(3) = -56$
$\Delta < 0$ so graph does not cross the x-axis

f $-x^2 - x - 1;$
$\Delta = 1^2 - 4(-1)(-1) = -3$
$\Delta < 0$ so graph does not cross the x axis

3 a $x^2 + 8x + 7;$
$\Delta = 8^2 - 4(7) = 36$
$\Delta > 0$ so the equation has 2 real roots

b $3x^2 + 8x + 7;$
$\Delta = 8^2 - 4(7)(3) = -20$
$\Delta < 0$ so no real roots

c $10x^2 - x - 3;$
$\Delta = 1^2 - 4(-3)(10) = 121$
$\Delta > 0$ so the equation has 2 real roots

d $2x^2 + 8x - 7;$
$\Delta = 8^2 - 4(-7)2 = 120$
$\Delta > 0$ so the equation has 2 real roots

e $3x^2 - 8x - 7;$
$\Delta = 8^2 - 4(-7)3 = 148$
$\Delta > 0$ so the equation has 2 real roots

f $10x^2 - x + 3;$
$\Delta = 1^2 - 4(10)(3) = -119$
$\Delta < 0$ so the equation has no real roots

4 a $9x^2 - 24x + 16;$
$\Delta = 24^2 - 4(9)(16) = 0$
$\Delta = 0$ so the equation has 1 rational root

b $-x^2 - 5x - 6;$
$\Delta = 5^2 - 4(-6)(-1) = 1$
$\Delta > 0$ so the equation has 2 rational roots.

c $x^2 - x - 4;$

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$\Delta = 1^2 - 4(-4) = 17$
$\Delta > 0$ so the equation has 2 irrational roots, and is not a perfect square

d $25x^2 - 20x + 4$;
$\Delta = 20^2 - 4(25)(4) = 0$
$\Delta = 0$ so the equation has 1 rational root and is a perfect square.

e $6x^2 - 3x - 2$;
$\Delta = 3^2 - 4(6)(-2) = 57$
$\Delta > 0$ so the equation has 2 irrational roots and is not a perfect square

f $x^2 + 3x + 2$;
$\Delta = 3^2 - 4(2) = 1$
$\Delta > 0$ so the equation has 2 rational roots and is not a perfect square

5 a $x^2 - 4mx + 20 = 0$
$\Delta = 16m^2 - 80 = 16(m^2 - 5)$

i If $(m^2 - 5) < 0$, no real solutions:
$\{m: -\sqrt{5} < m < \sqrt{5}\}$

ii If $(m^2 - 5) = 0$, one real solution:
$\{m: m = \pm\sqrt{5}\}$

iii If $(m^2 - 5) > 0$, 2 distinct solutions:
$\{m: m < -\sqrt{5}\} \cup \{m: m > \sqrt{5}\}$

b $mx^2 - 3mx + 3 = 0$
$\Delta = 9m^2 - 12m = 3m(3m - 4)$

i If $\Delta < 0$, no real solutions:
$\Delta = 0$ at $m = 0, \frac{4}{3}$
Upright parabola, so
$\{m: 0 < m < \frac{4}{3}\}$

ii If $\Delta = 0$, one real solution;
$m = 0, \frac{4}{3}$ satisfies this, but there is no solution to the equation if $m = 0$, so $\{m: m = \frac{4}{3}\}$

iii If $(3m^2 - 4) > 0$, 2 distinct solutions:
$\{m: m < 0\} \cup \{m: m > \frac{4}{3}\}$

c $5x^2 - 5mx - m = 0$
$\Delta = 25m^2 + 20m = 5m(5m + 4)$

i If $5m(5m + 4) < 0$, no real solutions
$\Delta = 0$ at $m = 0, -\frac{4}{5}$
Quadratic in m is upright:
$\{m: -\frac{4}{5} < m < 0\}$

ii If $5m(5m + 4) = 0$, one real solution:
$\{m: m = 0, -\frac{4}{5}\}$

iii If $5m(5m + 4) > 0$, 2 distinct solutions:
$\{m: m < -\frac{4}{5}\} \cup \{m: m > 0\}$

d $x^2 + 4mx - 4(m - 2) = 0$
$\Delta = 16m + 16(m - 2)$
$= 16(m^2 + m - 2)$

i If $m^2 + m - 2 < 0$, no real solutions:
$m^2 + m - 2 = (m + 2)(m - 1)$
Quadratic in m is upright, so
$\{m: -2 < m < 1\}$

ii If $m^2 + m - 2 = 0$, one real solution:
$\{m: m = -2, 1\}$

iii If $m^2 + m - 2 > 0$, 2 distinct

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solutions:
$\{m: m < -2\} \cup \{m: m > 1\}$

6 $mx^2 + (2m + n)x + 2n = 0$

$$\begin{aligned}\Delta &= (2m + n)^2 - 8mn\\ &= 4m^2 + 4mn + n^2 - 8mn\\ &= 4m^2 - 4mn + n^2\\ &= (2m - n)^2\end{aligned}$$

This is a perfect square for all rational m and n, so the solution is rational also.

7 $px^2 + 2(p + 2)x + p + 7 = 0$

$$\begin{aligned}\Delta &= 4(p + 2)^2 - 4p(p + 7)\\ &= 4p^2 + 16p + 16 - 4p - 28p\\ &= 16 - 12p = 4(4 - 3p)\end{aligned}$$

This equation has no real solution if $\Delta < 0$, i.e. if $p > \frac{4}{3}$

8 $(1 - 2p)x^2 + 8px - (2 + 8p) = 0$

$$\begin{aligned}\Delta &= 64p^2 + 4(1 - 2p)(2 + 8p)\\ &= 64p^2 - 8(2p - 1)(4p + 1)\\ &= 64p^2 - 8(8p^2 - 2p - 1)\\ &= 8(2p + 1)\end{aligned}$$

This equation has one real solution if $\Delta = 0$;

$2p + 1 = 0$ or $p = -\frac{1}{2}$

9 **a** $px^2 - 6x + 9 = 0$

$\Delta = 36 - 4p^2$

One solution $\Leftrightarrow \Delta = 0$

$$\begin{aligned}36 - 4p^2 &= 0\\ 36 &= 4p^2\\ 9 &= p^2\\ p &= \pm 3\end{aligned}$$

b $2x^2 - 4x + 3 - p = 0$

$\Delta = 16 - 4 \times 2(3 - p)$

Two solution $\Leftrightarrow \Delta > 0$

$$\begin{aligned}16 - 4 \times 2(3 - p) &> 0\\ 8p - 8 &> 0\\ p &> 1\end{aligned}$$

c $3x^2 - 2x - p + 1 = 0$

$\Delta = 4 - 4 \times 3(1 - p)$

Two solution $\Leftrightarrow \Delta > 0$

$12p - 8 > 0$

$p > \frac{2}{3}$

d $x^2 - 2x + 2 - p = 0$

$\Delta = 4 - 4 \times (2 - p)$

Two solution $\Leftrightarrow \Delta > 0$

$4p - 4 > 0$

$p > 1$

10 $y = px^2 + 8x + p - 6$

$$\begin{aligned}\Delta &= 64 - 4p(p - 6)\\ &= 4(-p + 6p + 16)\end{aligned}$$

If the graph crosses the x-axis, $\Delta > 0$:

$\Delta = 0$ when $p = \dfrac{-6 \pm \sqrt{100}}{-2}$

$\therefore p = 3 \pm 5 = -2, 8$

Inverted quadratic, so $\Delta > 0$ when:

$\{p: -2 < p < 8\}$

11 $(p^2 + 1)x^2 + 2pqx + q^2 = 0$

$$\begin{aligned}\Delta &= 4p^2q^2 - 4q^2(p^2 + 1)\\ &= 4q^2(p^2 - p^2 - 1)\\ &= -4q^2\end{aligned}$$

This is negative for all values of p, and for all non-zero q, so there are no real solutions.

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12 **a** For $x^2 + 4mx + 24m - 44$

$\Delta = (4m)^2 - 4(24m - 44)$

$= 16m^2 - 96m + 176$

b $4mx^2 + 4(m - 1)x + m - 2 = 0$ has a solution for all values of m if and only if $\Delta \geq 0$ for all m.

$$\begin{aligned} &16m^2 - 96m + 176 \\ &= 16(m^2 - 6m + 11) \\ &= 16(m^2 - 6m + 9 + 2) \\ &= 16(m - 3)^2 + 32 \geq 0 \quad \text{for all } m \end{aligned}$$

13 $4mx^2 + 4(m - 1)x + m - 2$

a $$\begin{aligned} \Delta &= 16(m - 1)^2 - 4(4m)(m - 2) \\ &= 16m^2 - 32m + 16 - 16m^2 + 32m \\ &= 16 \end{aligned}$$

b Δ is a perfect square and thus the solutions are rational for all m.

14 $4x^2 + (m - 4)x - m = 0$

$$\begin{aligned} \Delta &= (m - 4)^2 - 4(4)(-m) \\ &= m^2 - 8m + 16 + 16m \\ &= m^2 + 8m + 16 \\ &= (m + 4)^2 \end{aligned}$$

$\therefore \Delta$ is a perfect square for all m

15 $x^2 - (m + 2n)x + 2mn = 0$

$$\begin{aligned} \Delta &= (m + 2n)^2 - 4 \times 2mn \\ &= m^2 + 4mn + 4n^2 - 8mn \\ &= m^2 - 4mn + 4n^2 \\ &= (m - 2n)^2 \end{aligned}$$

Therefore Δ is a perfect square. The roots of the equation are rational.

16 $\Delta = b^2 - 4(a)(-c) = b^2 + 4ac > 0 \therefore$ the graph of $y = x^2 + bx - c$ where a and c are positive always intersects with the x-axis.

17 $\Delta = b^2 - 4(a)(c) = b^2 - 4ac > 0$ if .
$\therefore$ the graph of $y = x^2 + bx + c$ where a is negative and c is positive always intersects with the x-axis.

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Solutions to Exercise 3J

1 a
$$y = x - 2 \ldots (1)$$
$$y = x^2 - x - 6 \ldots (2)$$
$$\therefore x^2 - x - 6 = x - 2$$
$$x^2 - 2x - 4 = 0$$
$$\therefore x = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$$
Therefore points of intersection are $(1 - \sqrt{5}, -1 - \sqrt{5})$ and $(1 + \sqrt{5}, -1 + \sqrt{5})$

b
$$x + y = 6 \ldots (1)$$
$$y = x^2 \ldots (2)$$
From (1), $y = 6 - x$
$$\therefore x^2 = 6 - x$$
$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$\therefore x = 2 \text{ or } x = -3$$
Therefore points of intersection are $(2, 4)$ and $(-3, 9)$

c
$$5x + 4y = 21 \ldots (1)$$
$$y = x^2 \ldots (2)$$
Substitute from (2) in (1),
$$5x + 4x^2 = 21$$
$$4x^2 + 5x - 21 = 0$$
$$(4x - 7)(x + 3) = 0$$
$$\therefore x = \frac{7}{4} \text{ or } x = -3$$
Therefore points of intersection are $(-3, 9)$ and $\left(\frac{7}{4}, \frac{49}{16}\right)$

d
$$y = 2x + 1 \ldots (1)$$
$$y = x^2 - x + 3 \ldots (2)$$
Substitute from (1) in (2),
$$x^2 - x + 3 = 2x + 1$$
$$x^2 - 3x + 2 = 0$$
$$(x - 2)(x - 1) = 0$$
$$\therefore x = 2 \text{ or } x = 1$$
Therefore points of intersection are $(2, 5)$ and $(1, 3)$

2 a $y = x^2 + 2x - 8$ and $y = 2 - x$ meet where $x^2 + 2x - 8 = 2 - x$
$\therefore x^2 + 3x - 10 = 0$
$\therefore (x + 5)(x - 2) = 0$
$\therefore x = -5, 2$
When $x = -5, y = 2 - (-5) = 7$
When $x = 2, y = 2 - 2 = 0$
Curves meet at $(-5, 7)$ and $(2,0)$.

b $y = x^2 - x - 3$ and $y = 4x - 7$ meet where $x^2 - x - 3 = 4x - 7$
$\therefore x^2 - 5x + 4 = 0$
$\therefore (x - 4)(x - 1) = 0$
$\therefore x = 4, 1$
When $x = 1, y = 4 - 7 = -3$
When $x = 4, y = 16 - 7 = 9$
Curves meet at $(1, -3)$ and $(4,9)$.

c $y = x^2 + x - 5$ and $y = -x - 2$ meet where $x^2 + x - 5 = -x - 2$
$\therefore x^2 + 2x - 3 = 0$
$\therefore (x + 3)(x - 1) = 0$
$\therefore x = -3, 1$
When $x = -3, y = 3 - 2 = 1$
When $x = 1, y = -1 - 2 = -3$
Curves meet at $(-3, 1)$ and $(1, -3)$.

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d $y = x^2 + 6x + 6$ and $y = 2x + 3$ meet
where $x^2 + 6x + 6 = 2x + 3$
$\therefore x^2 + 4x + 3 = 0$
$\therefore (x + 3)(x + 1) = 0$
$x = -3, -1$
When $x = -3, y = -6 + 3 = -3$
When $x = -1, y = -2 + 3 = 1$
Curves meet at $(-3, -3)$ and $(-1, 1)$.

e $y = -x^2 - x + 6$ and $y = -2x - 2$
meet where $-x^2 - x + 6 = -2x - 2$
$\therefore -x^2 + x + 8 = 0$
$\therefore x^2 - x - 8 = 0$
$\therefore x = \dfrac{1 \pm \sqrt{1 - 4(8)}}{2}$
$\therefore x = \dfrac{1 \pm \sqrt{33}}{2}$
When $x = \dfrac{1 - \sqrt{33}}{2}$, $y = -3 + \sqrt{33}$
When $x = \dfrac{1 + \sqrt{33}}{2}$, $y = -3 - \sqrt{33}$
Curves meet at $\left(\dfrac{1 - \sqrt{33}}{2}, -3 + \sqrt{33}\right)$
and $\left(\dfrac{1 + \sqrt{33}}{2}, -3 - \sqrt{33}\right)$.

f $y = x^2 + x + 6$ and $y = 6x + 8$ meet
where $x^2 + x + 6 = 6x + 8$
$\therefore x^2 - 5x - 2 = 0$
$\therefore x = \dfrac{5 \pm \sqrt{25 - 4(-2)}}{2}$
$\therefore x = \dfrac{5 \pm \sqrt{33}}{2}$
When $x = \dfrac{5 - \sqrt{33}}{2}$, $y = 23 - 3\sqrt{33}$
When $x = \dfrac{5 + \sqrt{33}}{2}$, $y = 23 + \sqrt{33}$
Curves meet at
$\left(\dfrac{5 - \sqrt{33}}{2}, 23 - 3\sqrt{33}\right)$ and
$\left(\dfrac{5 + \sqrt{33}}{2}, 23 + 3\sqrt{33}\right)$.

3 If the straight line meets the parabola only once, then the $y_1 = y_2$ quadratic will produce a perfect square.

a $x - 6x + 8 = -2x + 4$
$\therefore x^2 - 4x + 4 = 0$
$\therefore (x - 2)^2 = 0, \therefore x = 2$
Touches at (2,0).

b $x^2 - 2x + 6 = 4x - 3$
$\therefore x^2 - 6x + 9 = 0$
$\therefore (x - 3)^2 = 0, \therefore x = 3$
Touches at (3,9).

c $2x^2 + 11x + 10 = 3x + 2$
$\therefore 2x^2 + 8x + 8 = 0$
$\therefore 2(x + 2)^2 = 0, \therefore x = -2$
Touches at $(-2, -4)$.

d $x^2 + 7x + 4 = -x - 12$
$\therefore x^2 + 8x + 16 = 0$
$\therefore (x + 4)^2 = 0, \therefore x = -4$
Touches at $(-4, -8)$.

4 **a** $y = x^2 - 6x; y = 8 + x$

$$\begin{aligned} \therefore \quad x^2 - 6x &= 8 + x \\ x^2 - 7x - 9 &= 0 \\ (x - 8)(x + 1) &= 0 \\ \therefore \quad x &= 8, -1 \end{aligned}$$

$x = 8; y = 8 + 8 = 16$
$x = -1; y = 8 + 1 = 7$

b $y = 3x^2 + 9x; y = 32 - x$

$$\begin{aligned} \therefore \quad 3x^2 + 9x &= 32 - x \\ 3x^2 + 10x - 32 &= 0 \\ (3x + 16)(x - 2) &= 0 \\ \therefore \quad x &= -\frac{16}{3}, 2 \end{aligned}$$

$x = -\dfrac{16}{3}; y = 32 + \dfrac{16}{3} = \dfrac{112}{3}$
$x = 2; y = 32 - 2 = 30$

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c $y = 5x^2 + 9x; y = 12 - 2x$

$$\therefore \quad 5x^2 + 9x = 12 - 2x$$
$$5x^2 + 11x - 12 = 0$$
$$(5x - 4)(x + 3) = 0$$
$$\therefore \quad x = \frac{4}{5}, -3$$

$x = \frac{4}{5}; y = 12 - \frac{8}{5} = \frac{52}{5}$
$x = -3; y = 12 - (-6) = 18$

d $y = -3x^2 + 32x; y = 32 - 3x$

$$\therefore \quad -3x^2 + 32x = 32 - 3x$$
$$-3x^2 + 35x - 32 = 0$$
$$3x^2 - 35x + 32 = 0$$
$$(x - 1)(3x - 32) = 0$$
$$x = 1, \frac{32}{3}$$

$x = 1; y = 32 - 3 = 29$
$x = \frac{32}{3}; y = 32 - 32 = 0$

e $y = 2x^2 - 12; y = 3(x - 4)$

$$\therefore 2x^2 - 12 = 3x - 12$$
$$2x^2 - 3x = 0$$
$$x(2x - 3) = 0$$
$$x = 0, \frac{3}{2}$$

$x = 0; y = 3(-4) = -12$
$x = \frac{3}{2}; y = 3\left(\frac{3}{2} - 4\right) = -\frac{15}{2}$

f $y = 11x^2; y = 21 - 6x$

$\therefore 11x^2 + 6x - 21 = 0$

$$\therefore x = \frac{-6 \pm \sqrt{6^2 - 4(-21)(11)}}{22}$$
$$= \frac{-3 \pm \sqrt{240}}{11} = -3 \pm \frac{4}{11}\sqrt{15}$$

$x = \frac{-3 - 4\sqrt{15}}{11}$;
$y = 21 + \frac{6}{11}(3 + 4\sqrt{15}) = \frac{249 + 24\sqrt{15}}{11}$

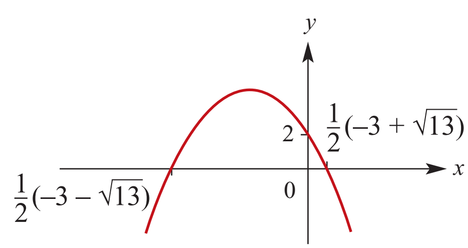
$x = \frac{-3 + 4\sqrt{15}}{11}$;
$y = 21 + \frac{6}{11}(3 - 4\sqrt{15}) = \frac{249 - 24\sqrt{15}}{11}$

Using a calculator: $x = 1.14, y = 14.19;$
$x = -1.68, y = 31.09$

5 a If $y = x + c$ is a tangent to the parabola $y = x^2 - x - 12$, then $x^2 - x - 12 = x + c$ must reduce to a quadratic with zero discriminant.

$x^2 - x - 12 = x + c$
$\therefore x^2 - 2x - (12 + c) = 0$
$\therefore \Delta = 4 + 4(12 + c)$
$= 4c + 52 = 0, \therefore c = -13$

b i $y = -2x^2 - 6x + 2$



ii If $y = mx + 6$ is a tangent to the parabola,

$-2x^2 - 6x + 2 = mx + 6$
$\therefore -2x^2 - (6 + m)x - 4 = 0$
$\therefore 2x^2 + (6 + m)x + 4 = 0$
For a tangent, $\Delta = 0$:
$\therefore \Delta = (6 + m)^2 - 4(4)(2) = 0$
$\therefore (6 + m)^2 = 32$
$\therefore 6 + m = \pm\sqrt{32} = \pm 4\sqrt{2}$
$m = -6 \pm 4\sqrt{2}$

6 a $y = x^2 + 3x$ has as a tangent

$y = 2x + c$
$\Delta = 0$ for $x^2 + 3x = 2x + c$
$\therefore x^2 + x - c = 0$
$\therefore \Delta = 1 + 4c = 0, \therefore c = -\frac{1}{4}$

b For two intersections, $\Delta > 0$ so
$c > -\frac{1}{4}$

7 $y = x$ is a tangent to the parabola
$y = x^2 + ax + 1$
$\therefore x^2 + ax + 1 = x$
$\therefore x^2 + (a - 1)x + 1 = 0$
$\Delta = (a - 1)^2 - 4 = 0$
$\therefore a - 1 = \pm 2$
$\therefore a = 1 \pm 2 = -1, 3$

8 $y = -x$ is a tangent to the parabola
$y = x^2 + x + b$
$\therefore x^2 + x + b = -x$
$\therefore x^2 + 2x + b = 0$
$\Delta = 4 - 4b = 0$
$\therefore b = 1$

9 A straight line passing through the point $(1, -2)$ has the form $y - (-2) = m(x - 1)$
$\therefore y = m(x - 1) - 2$
If this line is a tangent to $y = x^2$ then
$x^2 = m(x - 1) - 2$
$\therefore x^2 - m(x - 1) + 2 = 0$
$\therefore x^2 - mx + m + 2 = 0$
$\Delta = 0$ for a tangent here:
$\Delta = m^2 - 4(m + 2)$
$= m^2 - 4m - 8 = 0$
$m^2 - 4m - 8 = 0$
$\therefore m = \frac{4 \pm \sqrt{16 + 32}}{2}$
$\therefore m = 2 \pm \sqrt{12} = 2 \pm 2\sqrt{3}$
$\therefore y = (2 \pm 2\sqrt{3})(x - 1) - 2$
$y = 2(1 + \sqrt{3})x - 4 - 2\sqrt{3}$ and
$y = 2(1 - \sqrt{3})x - 4 + 2\sqrt{3}$

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Solutions to Exercise 3K

1 $y = ax^2 + c$ passes through $(0, 6)$ and $(-1, 2)$.
$\therefore a(0)^2 + c = 6, \therefore c = 6$
$a(-1)^2 + 6 = 2, \therefore a = -4$

2 $y = ax^2 + bx + 4$

a $\Delta = b^2 - 16a$

b If the turning point lies on the x axis, $\Delta = 0$.
$\therefore b^2 - 16a = 0$
This implies, $a = \dfrac{b^2}{16}$.

c Turning point when $x = -\dfrac{b}{2a}$
Therefore,

$$-4 = -\frac{b}{2a} \ldots (1)$$

$$a = \frac{b^2}{16} \ldots (2)$$

Rearranging(1)

$$a = \frac{b}{8}$$

$$\therefore \frac{b}{8} = \frac{b^2}{16}$$

$$\therefore b = 2 \qquad \text{(If } b = 0 \text{ then } a = 0\text{)}$$

$$\therefore a = \frac{1}{4}$$

3 a $y = k(x + 2)(x - 6)$
When $x = 1, y = -30$
$-30 = k(3)(-5)$

$k = 2$

$\therefore y = 2(x + 2)(x - 6)$

b $y = a(x - h)^2 + k$
Turning point $(-2, 4)$

$$\therefore y = a(x + 2)^2 + 4$$

Passes through $(3, -46)$

$$\therefore -46 = a(25) + 4$$

$$\therefore -50 = a(25)$$

$$\therefore a = -2$$

$$\therefore y = -2(x + 2)^2 + 4$$

c Passes through the points $(1, -2), (0, -3), (-1, -6)$
Use $y = ax^2 + bx + c$
Passes through $(0, -3)$,
$\therefore c = -3$

$$y = ax^2 + bx - 3$$

When$x = 1, y = -2$

$$\therefore -2 = a + b - 3$$

$$\therefore a + b = 1 \ldots (1)$$

When$x = -1, y = -6$

$$\therefore -6 = a + b - 3$$

$$\therefore a - b = -3 \ldots (2)$$

Add (1) and (2)

$$2a = -2$$

$$a = -1$$

$$\therefore b = 2$$

$$\therefore y = -x^2 + 2x - 3$$

4 $y = ax^2$ passes through (2,8).
$\therefore 8 = a(2)^2, \therefore a = 2$

5 $y = ax^2 + bx$ passes through (6,0) and $(-1, 4)$.
$\therefore a(6)^2 + 6b = 0$
$\therefore 36a + 6b = 0, \therefore b = -6a$
$a(-1)^2 - 6a(-1) = 4$

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$\therefore 7a = 4$
$\therefore a = \frac{4}{7}; b = -\frac{24}{7}$

6 $y = a(x - b)^2 + c$
The vertex is at (1,6) so $y = a(x - 1)^2 + 6$
$y = a(x - 1)^2 + 6$ passes through (2,4)
$\therefore a(2 - 1)^2 + 6 = 4$
$\therefore a = -2; b = 1; c = 6$

7 **a** $y = a(x - b)^2 + c$
The vertex is at (0,5) so
$y = (x - 0)^2 + 5$
$y = ax^2 + 5$
$y = ax^2 + 5$ passes through (0,4)
$\therefore a(4)^2 + 5 = 0$
$\therefore a = -\frac{5}{16}$
$y = -\frac{5x^2}{16} + 5$

b $y = a(x - b)^2 + c$
The vertex is at (0,0) so $y = ax^2$
$y = ax^2$ passes through $(-3, 9)$
$\therefore a(-3)^2 = 9$
$\therefore a = 1$
$y = x^2$

c $y = ax^2 + bx + c$
This is of the form $y = ax(x + 7)$
For (4,4)
$4 = a(4)(4 + 7)$
$4 = 44a$
Therefore $a = \frac{1}{11}$
And the rule is $y = \frac{x^2}{11} + \frac{7x}{11}$

d $y = a(x + b)(x + c)$
From x-intercepts, a and b are -1 and -3:
$y = a(x - 1)(x - 3)$
From y-intercept,
$a(-1)(-3) = 3, \therefore a = 1$
$\therefore y = (x - 1)(x - 3)$

e $y = a(x - b)^2 + c$
The vertex is at $(-1, 5)$ so
$y = a(x + 1)^2 + 5$
$y = a(x + 1)^2 + 5$ passes through $(1, 0)$
$\therefore a(2)^2 + 5 = 0$
$\therefore a = -\frac{5}{4}$
$y = -\frac{5}{4}(x + 1)^2 + 5$
OR $y = -\frac{5}{4}x^2 - \frac{5}{2}x + \frac{15}{4}$
Check with 3rd pt: $y = 0$ at $x = -3$

f $y = a(x - b)^2 + c$
The vertex is at (2, 2) so
$y = a(x + 2)^2 + 2$
$y = a(x - 2)^2 + 2$ passes through (0,6)
$\therefore a(-2)^2 + 2 = 6$
$\therefore a = 1$
$y = (x - 2)^2 + 2$
OR $y = x^2 - 4x + 6$
Check with 3rd pt: $y = 6$ at $x = 4$

8 $y = a(x - b)^2 + c$
The vertex is at $(-1, 3)$ so
$y = a(x + 1)^2 + 3$
$y = a(x + 1)^2 + 3$ passes through (3,8)
$\therefore a(4)^2 + 3 = 8$
$\therefore 16a = 5, \therefore a = \frac{5}{16}$
$y = \frac{5}{16}(x + 1)^2 + 3$

9 $y = a(x + b)(x + c)$
From x intercepts, a and b are 6 and -3:
$y = a(x - 6)(x + 3)$
Using (1, 10):
$a(1 - 6)(1 + 3) = 10$
$\therefore -20a = 10, \therefore a = -\frac{1}{2}$

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$\therefore y = -\frac{1}{2}(x - 6)(x + 3)$

OR $y = -\frac{1}{2}(x^2 - 3x - 18)$

10 $y = a(x - b)^2 + c$

The vertex is at $(-1, 3)$ so

$y = a(x + 1)^2 + 3$

$y = a(x + 1)^2 + 3$ passes through (0,4)

$\therefore a + 3 = 4, \therefore a = 1$

$y = (x + 1)^2 + 3$

OR $y = x^2 + 2x + 4$

11 The suspension cable forms a parabola:

$y = a(x - b)^2 + c$

The vertex is at (90, 30) so

$y = a(x - 90)^2 + 30$

When $x = 0, y = 75$, so:

$y = a(-90)^2 + 30 = 75$

$\therefore 8100a = 45, \therefore a = \frac{1}{180}$

$y = \frac{1}{180}(x - 90)^2 + 30$

$\therefore y = \frac{1}{180}x^2 - x + 75$

12 $y = 2(x - b)^2 + c$

$(1, -2) = \text{TP (vertex)} = (b, c)$

$\therefore y = 2(x - 1)^2 - 2$

OR $y = 2x^2 - 4x$

13 $y = a(x - b)^2 + c$

$(1, -2) = \text{TP (vertex)} = (b, c)$

$\therefore y = a(x - 1)^2 - 2$

Using the point (3,2),

$a(3 - 1)^2 - 2 = 2$

$\therefore 4a - 2 = 2, \therefore a = 1$

$\therefore y = (x - 1)^2 - 2$

OR $y = x^2 - 2x - 1$

14 **a** $y = \frac{1}{3}(x + 4)(8 - x)$

Squared term is negative, so inverted parabola; must be **A** or **C**.

The x-intercepts must be at 8 and -4, so **C**.

b $y = x^2 - x + 2$

Positive squared term gives an upright parabola; must be **B** or **D**.

The y-intercept is at (0,2) so only **B** is possible

c $y = -10 + 2(x - 1)^2$

Positive squared term gives an upright parabola; must be **B** or **D**.

Vertex is at $(1, -10)$ so **D**.

d $y = \frac{1}{2}(9 - x^2)$

Squared term is negative so inverted parabola; must be **A** or **C**.

Vertex at $\left(0, \frac{9}{2}\right)$ so **A**..

15 **a** $ax^2 + 2x + a$

$$= a\left(x^2 + \frac{2}{a}x + 1\right)$$

$$= a\left(x^2 + \frac{2}{a}x + \frac{1}{a^2} - \frac{1}{a^2} + 1\right)$$

$$= a\left(\left(x + \frac{1}{a}\right)^2 - \frac{1}{a^2} + 1\right)$$

$$= a\left(x + \frac{1}{a}\right)^2 - \frac{1}{a} + a$$

b Turning point: $\left(-\frac{1}{a}, a - \frac{1}{a}\right)$

c Perfect square when $a - \frac{1}{a} = 0$

That is, when $a^2 = 1$

$\therefore a = \pm 1$

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d Two solutions when $1 - a^2 > 0$, That is, $-1 < a < 1$

16 $y = a(x - b)^2 + c$
$(2, 2) = \text{TP (vertex)} = (b, c)$
$\therefore y = a(x - 2)^2 + 2$
Using the point $(4, -6)$,
$a(4 - 2)^2 + 2 = -6$
$\therefore 4a + 2 = -6 \therefore, a = -2$
$\therefore y = -2(x - 2)^2 + 2$
OR $y = -2x^2 + 8x - 6$

17 **(a)** has x-intercepts at 0 and 10, so
$y = a(x - b)(x - c)$
$b = 0, c = 10$
$\therefore y = ax(x - 10)$
$a > 0$ because upright parabola

(b) has x-intercepts at -4 and 10, so
$y = a(x - b)(x - c)$
$b = -4, c = 10$
$\therefore y = a(x + 4)(x - 10)$
$a < 0$ because upright parabola

(c) has no x intercepts, so
$y = a(x - b)^2 + c$
Vertex is at (6,6) so $b = c = 6$
$y = a(x - 6)^2 + 6$
y-intercept is at (0,8):
$a(0 - 6)^2 + 6 = 8$
$\therefore 36a = 2, \therefore a = \dfrac{1}{18}$
$y = \dfrac{1}{18}(x - 6)^2 + 6$

(d) $y = a(x - b)^2 + c$
Vertex is at (8,0) so $b = 8; c = 0$
$y = a(x - 8)^2$
$a < 0$ because inverted parabola

18 **(a)** $y = ax^2 + x + b$
Using $D = (0, 2), b = 2$
Using $A = (2, 3)$,

$$4a + 2 + 2' = 3$$
$$\therefore \quad a = -\frac{1}{4}$$
$$y = -\frac{1}{4}x^2 + x + 2$$

(b) $y = ax^2 + x + b$
Using $C = (0, -5), b = -5$
Using $B = (2, 1)$,

$$4a + 2 - 5 = 1$$
$$\therefore \quad a = 1$$
$$y = x^2 + x - 5$$

19 $r = at^2 + bt + c$
(1) $t = 5, r = 3 : 25a + 5b + c = 3$
(2) $t = 9, r = 6 : 81a + 9b + c = 6$
(3) $t = 13, r = 5 : 169a + 13b + c = 5$
(2) − **(1)** gives $56a + 4b = 3$
(3) − **(2)** gives $88a + 4b = -1$
From these 2 equations, $32a = -4$
so $a = -\dfrac{1}{8}$
Substitute into $56a + 4b = 3$:
$-\dfrac{56}{8} + 4b = 3$
$\therefore 4b = 10, \therefore b = \dfrac{5}{2}$
Substitute into **(1)**:
$-\dfrac{25}{8} + \dfrac{25}{2} + c = 3$
$\therefore c = -\dfrac{51}{8}$
$r = -\dfrac{1}{8}(t^2 - 20t + 51)$
$\therefore r = -\dfrac{1}{8}t^2 + \dfrac{5}{2}t - \dfrac{51}{8}$

20 **a** $y = (x - 4)^2 - 3$
Upright parabola, vertex $(4, -3)$ so **B**

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b $y = 3 - (x-4)^2$
Inverted parabola, vertex (4,3) so **D**

21 **a** $y = ax^2 + bx + c$
$(-2,-1): 4a - 2b + c = -1 \ldots$ **(1)**
$(1,2) : a + b + c = 2 \ldots$ **(2)**
$(3,-16)\ 9a + 3b + c = -16 \ldots$ **(3)**
(2) − **(1)** gives $3b - 3a = 3$ or
$b = a + 1$
(3) − **(2)** gives $8a + 2b = -18$ or
$b = -9 - 4a$
$b = a + 1 = -9 - 4a$
$\therefore 5a = -10, \therefore a = -2; b = -1$
Substitute into **(1)**:
$-8 + 2 + c = -1 \therefore c = 5$
$y = -2x^2 - x + 5$

b $y = ax^2 + bx + c$
$(-1,-2) : a - b + c = -2 \ldots$ **(1)**
$(1,-4) : a + b + c = -4 \ldots$ **(2)**
$(3, 10)$: $9a + 3b + c = 10 \ldots$ **(3)**
(2) − **(1)** gives $2b = -2$ or $b = -1$
(3) − **(2)** gives $8a + 2b = 14$
$\therefore 8a = 16 \therefore a = 2$
Substitute into **(2)**:
$2 - 1 + c = -4, \therefore c = -5$
$y = 2x^2 - x - 5$

c $y = ax^2 + bx + c$
$(-3,5) : 9a - 3b + c = 5 \ldots$ **(1)**
$(3,20): 9a + 3b + c = 20 \ldots$ **(2)**
$(5,57): 25a + 5b + c = 57 \ldots$ **(3)**
(2) − **(1)** gives $6b = 15$ or $b = \frac{5}{2}$
(3) − **(2)** gives $16a + 2b = 37$
$\therefore 16a + 5 = 37, \therefore a = 2$
Substitute into **(2)**:
$18 + \frac{15}{2} + c = 20, \therefore c = -\frac{11}{2}$
$y = 2x^2 + \frac{5}{2}x - \frac{11}{2}$

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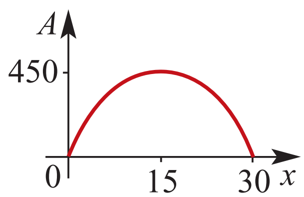
Solutions to Exercise 3L

1 a Width of paddock = x;
length = $60 - 2x$
$\therefore A = x(60 - 2x) = 60x - 2x^2$

b

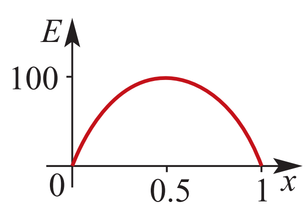


c Maximum area is at the vertex, i.e. when $x = 15$ (halfway between the two x-intercepts).
When $x = 15$,
$A = 15(60 - 30) = 450\text{ m}^2$

2 $A = x(10 - x)$; Maximum area = 25 m^2

3 $E = 400(x - x^2)$

a



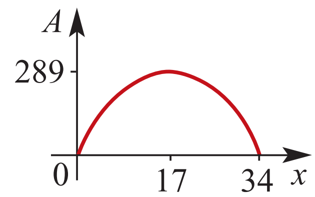
b Zero efficiency rating when $x = 0$ and 1

c Maximum efficiency rating is at the vertex where $x = 0.5$

d $E \geq 70$ when $400x - 400x^2 - 70 \geq 0$
i.e. $\{x\colon\ 0.23 < x < 0.77\}$

4 a If x cm = length of the rectangle, then
$2x + 2w = 68$, $\therefore w = 34 - x$
$A = lw = x(34 - x) = 34x - x^2$

b



c Maximum area formed is at the vertex where $x = 17$:
$A = 17(34 - 17) = 172 = 289\text{ cm}^2$

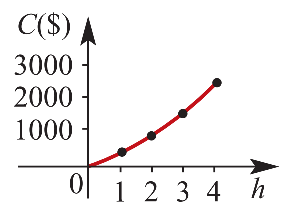
5 a $4x + 10y = 80$

b i $A = 1.64x^2 - 25.6x + 256$

ii 31.22 and 48.78

6 $C = 240h + 100h^2$

a



h is most unlikely to be less than zero in an alpine area, and will be less than 10, since the highest mountain on Earth is less high than 10 km above sea level.

b C's maximum value is at the top of the highest peak in the mountains (8.848 km for Mt Everest).

c For $h = 2.5$ km,
$C = 240(2.5) + 100(2.5)^2$
$= 600 + 625 = \$1225$

7 $T = 290(8t - 0.5t^2 - 1.4)$

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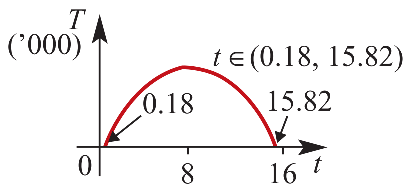
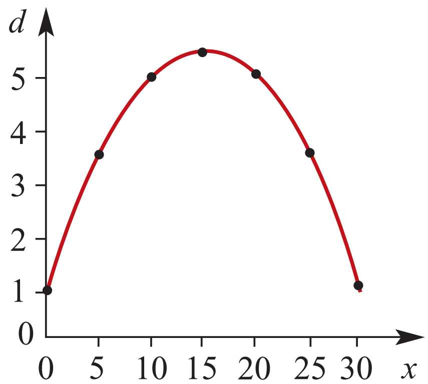
a 

Solving $8t - 0.5t^2 - 1.4 = 0$ with a CAS gives $t = 0.18, 15.82$. So $t \in (0.18, 15.82)$

b At the vertex $t = 8$, $T = 8874$ units

8 **a** $d = 1 + \frac{3}{5}x - \frac{1}{50}x^2, x \geq 0$



b **i** Maximum height = 5.5 m

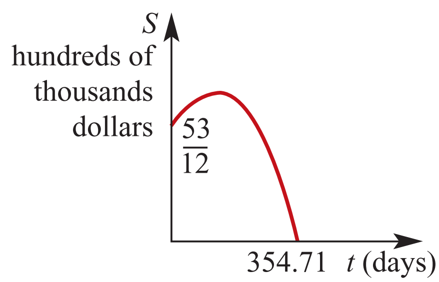
ii When $y = 2$, $x = 15 \pm 5\sqrt{7}$
($x = 1.9$ m or 28.1 m)

iii y-intercept = 1, so it was struck 1 metre above the ground.

9 The x-intercepts are 0 and 1.5

So $y = ax(x - 1.5)$
A is the point (0.75, 0.6) so:
$0.6 = a(0.75)(0.75 - 1.5)$
$\frac{3}{5} = -\frac{9}{16}a$
So $a = -16$
$y = -\frac{16}{15}x^2 + \frac{8}{5}x$
$a = -\frac{16}{15},\ b = \frac{8}{5}, c = 0$

10 **a** $s = at^2 + bt + c$
$900a + 30b + c = 7.2 \ldots$ **(1)**
$22\,500a + 150b + c = 12.5 \ldots$ **(2)**
$90\,000a + 300b + c = 6 \ldots$ **(3)**
(2) − **(1)** gives $21600a + 120b = 5.3$
(3) − **(2)** gives $67500a + 150b = -6.5$
Using a CAS, the solution is:
$a = -\frac{7}{21\,600}; b = \frac{41}{400}; c = \frac{53}{12}$

b 

c **i** $t = 180$, $s = 12.36$, so spending is estimated at $1 236 666.

ii $t = 350$, $s = 0.59259$, so spending is estimated at $59259

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Solutions to Review: Short-answer questions

1 a $x^2 + 9x + \frac{81}{4} = \left(x + \frac{9}{2}\right)^2$

b $x^2 + 18x + 81 = (x + 9)^2$

c $x^2 - \frac{4}{5}x + \frac{4}{25} = \left(x - \frac{2}{5}\right)^2$

d $x^2 + 2bx + b^2 = (x + b)^2$

e $9x^2 - 6x + 1 = (3x - 1)^2$

f $25x^2 + 20x + 4 = (5x + 2)^2$

2 a $-3(x - 2) = -3x + 6$

b $-a(x - a) = -ax + a^2$

c $(7a - b)(7a + b) = 49a^2 - b^2$

d $(x + 3)(x - 4) = x^2 + 3x - 4x - 12$
$= x^2 - x - 12$

e $(2x + 3)(x - 4) = 2x^2 + 3x - 8x - 12$
$= 2x^2 - 5x - 12$

f $(x + y)(x - y) = x^2 - y^2$

g $(a - b)(a^2 + ab + b^2)$
$= a^3 - a^2b + a^2b - ab^2 + ab^2 - b^3$
$= a^3 - b^3$

h
$(2x + 2y)(3x + y) = 6x^2 + 6xy + 2xy + 2y^2$
$= 6x^2 + 8xy + 2y^2$

i $(3a + 1)(a - 2) = 3a^2 + a - 6a - 2$
$= 3a^2 - 5a - 2$

j
$(x + y)^2 - (x - y)^2$
$= ((x + y) - (x - y))((x + y) + (x - y))$
$= (2y)(2x) = 4xy$

k $u(v + 2) + 2v(1 - u)$
$= uv + 2u + 2v - 2uv$
$= 2u + 2v - uv$

l $(3x + 2)(x - 4) + (4 - x)(6x - 1)$
$= (3x + 2)(x - 4) + (x - 4)(1 - 6x)$
$= (x - 4)(3x + 2 + 1 - 6x)$
$= (x - 4)(3 - 3x)$
$= -3x^2 + 15x - 12$

3 a $4x - 8 = 4(x - 2)$

b $3x^2 + 8x = x(3x + 8)$

c $24ax - 3x = 3x(8a - 1)$

d $4 - x^2 = (2 - x)(2 + x)$

e $au + 2av + 3aw = a(u + 2v + 3w)$

f $4a^2b^2 - 9a^4 = a^2(4b^2 - 9a^2)$
$= a^2(2b - 3a)(2b + 3a)$

g $1 - 36x^2a^2 = (1 - 6ax)(1 + 6ax)$

h $x^2 + x - 12 = (x + 4)(x - 3)$

i $x^2 + x - 2 = (x + 2)(x - 1)$

j $2x^2 + 3x - 2 = (2x - 1)(x + 2)$

k $6x^2 + 7x + 2 = (3x + 2)(2x + 1)$

l $3x^2 - 8x - 3 = (3x + 1)(x - 3)$

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m $3x^2 + x - 2 = (3x - 2)(x + 1)$

n $6a^2 - a - 2 = (3a - 2)(2a + 1)$

o $6x^2 - 7x + 2 = (3x - 2)(2x - 1)$

4 a
$$\begin{aligned} x^2 - 2x - 15 &= 0 \\ (x - 5)(x + 3) &= 0 \\ x = 5 \text{ or } x &= -3 \end{aligned}$$

b
$$\begin{aligned} x^2 - 9x &= 0 \\ x(x - 9) &= 0 \\ x = 0 \text{ or } x &= 9 \end{aligned}$$

c
$$\begin{aligned} 2x^2 - 10x + 12 &= 0 \\ 2(x^2 - 5x + 6) &= 0 \\ (x - 3)(x - 2) &= 0 \\ x = 3 \text{ or } x &= 2 \end{aligned}$$

d
$$\begin{aligned} x^2 - 24x - 25 &= 0 \\ (x - 25)(x + 1) &= 0 \\ x = 25 \text{ or } x &= -1 \end{aligned}$$

e
$$\begin{aligned} 3x^2 + 15x + 18 &= 0 \\ 3(x^2 + 5x + 6) &= 0 \\ (x + 3)(x + 2) &= 0 \\ x = -3 \text{ or } x &= -2 \end{aligned}$$

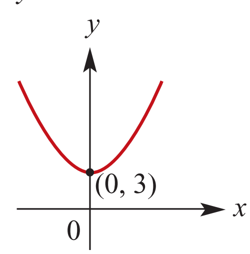
f
$$\begin{aligned} x^2 - 12x + 36 &= 0 \\ (x - 6)(x - 6) &= 0 \\ x &= 6 \end{aligned}$$

g
$$\begin{aligned} 2x^2 - 5x - 3 &= 0 \\ 2x^2 - 6x + x - 3 &= 0 \\ 2x(x - 3) + (x - 3) &= 0 \\ (x - 3)(2x + 1) &= 0 \\ x = 3 \text{ or } x &= -\frac{1}{2} \end{aligned}$$

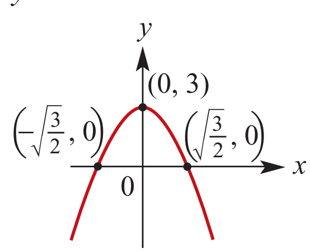
h
$$\begin{aligned} 12x^2 - 8x - 15 &= 0 \\ 12x^2 - 18x + 10x - 15 &= 0 \\ 6x(2x - 3) + 5(2x - 3) &= 0 \\ (6x + 5)(2x - 3) &= 0 \\ x = -\frac{5}{6} \text{ or } x &= \frac{3}{2} \end{aligned}$$

i
$$\begin{aligned} 5x^2 + 7x - 12 &= 0 \\ 5x^2 + 12x - 5x - 12 &= 0 \\ x(5x + 12) - (5x + 12) &= 0 \\ (5x + 12)(x - 1) &= 0 \\ x = 1 \text{ or } x &= -\frac{12}{5} \end{aligned}$$

5 a $y = 2x^2 + 3$



b $y = -2x^2 + 3$



c $y = 2(x - 2)^2 + 3$

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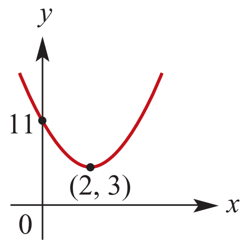
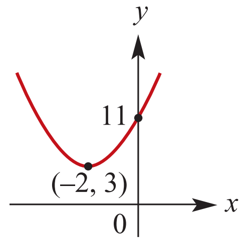
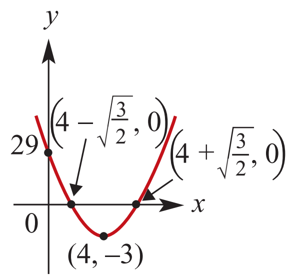


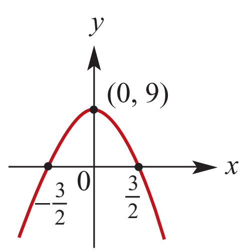
d $y = 2(x + 2)^2 + 3$



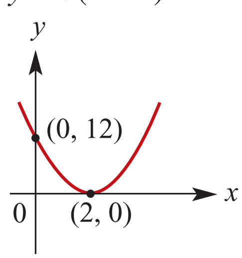
e $y = 2(x - 4)^2 - 3$



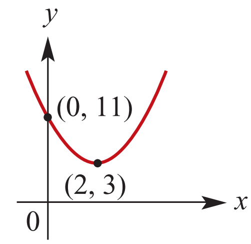
f $y = 9 - 4x^2$



g $y = 3(x - 2)^2$



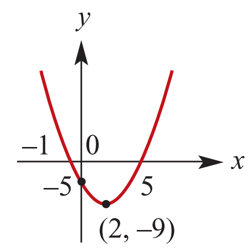
h $y = 2(2 - x)^2 + 3$



6 a $y = x^2 - 4x - 5$

$= x^2 - 4x + 4 - 9$

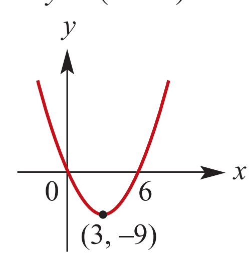
$\therefore y = (x - 2)^2 - 9$



b $y = x^2 - 6x$

$= x^2 - 6x + 9 - 9$

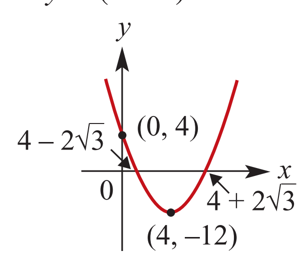
$\therefore y = (x - 3)^2 - 9$



c $y = x^2 - 8x + 4$

$= x^2 - 8x + 16 - 12$

$\therefore y = (x - 4)^2 - 12$



d $y = 2x^2 + 8x - 4$

$= 2(x^2 + 4x - 2)$

$\therefore y = 2(x^2 + 4x + 4 - 6)$

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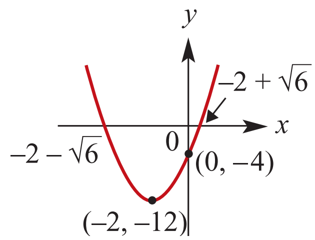
$\therefore y = 2(x+2)^2 - 12$

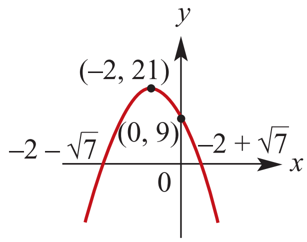


e $y = -3x^2 - 12x + 9$

$= -3(x^2 + 4x - 3)$

$= -3(x^2 + 4x + 4 - 7)$

$\therefore y = -3(x+2)^2 + 21$

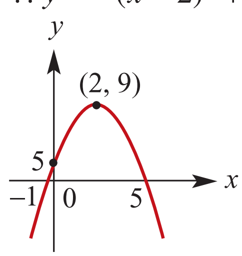


f $y = -x^2 + 4x + 5$

$\therefore y = -(x^2 - 4x - 5)$

$\therefore y = -(x^2 - 4x + 4 - 9)$

$\therefore y = -(x-2)^2 + 9$



7 **i** y-intercepts are at $(0, c)$ in each case; x-intercepts are where the factors equal zero.

ii The axis of the symmetry is at $x = -\dfrac{b}{2a}$

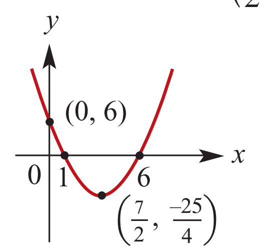
iii The turning point is on the axis of symmetry with the y-value for that point.

a $y = x^2 - 7x + 6 = (x-6)(x-1)$

i (0, 6), (6,0) and (1,0)

ii $x = -\dfrac{b}{2a} = \dfrac{7}{2}$

iii Turning point at $\left(\dfrac{7}{2}, -\dfrac{25}{4}\right)$



b $y = -x^2 - x + 12$

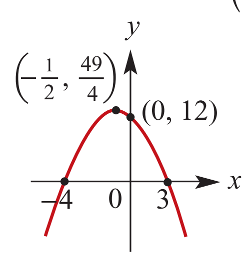
$= -(x^2 + x - 12)$

$= -(x+4)(x-3)$

i (0, 12), (–4, 0) and (3, 0)

ii $x = -\dfrac{b}{2a} = -\dfrac{1}{2}$

iii Turning point at $\left(-\dfrac{1}{2}, \dfrac{49}{4}\right)$



c $y = -x^2 + 5x + 14$

$= -(x^2 - 5x - 14)$

$= -(x-7)(x+2)$

i (0, 14), (–2, 0) and (7, 0)

ii $x = -\dfrac{b}{2a} = \dfrac{5}{2}$

iii turning point at $\left(\dfrac{5}{2}, \dfrac{81}{4}\right)$

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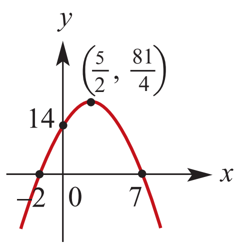
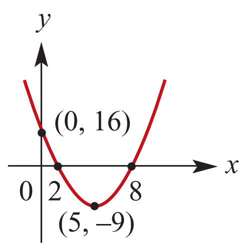


d $y = x^2 - 10x + 16 = (x - 8)(x - 2)$

i (0,16), (2,0) and (8,0)

ii $x = -\dfrac{b}{2a} = 5$

iii turning point at (5, –9)

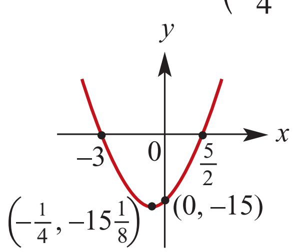


e $y = 2x^2 + x - 15 = (2x - 5)(x + 3)$

i $(0, -15), \left(\dfrac{5}{2}, 0\right)$ and $(-3, 0)$

ii $x = -\dfrac{b}{2a} = -\dfrac{1}{4}$

iii Turning point at $\left(-\dfrac{1}{4}, -\dfrac{121}{8}\right)$

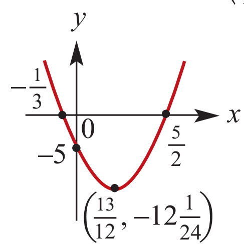


f $y = 6x^2 - 13x - 5 = (3x + 1)(2x - 5)$

i $(0, -5), \left(\dfrac{5}{2}, 0\right)$ and $\left(-\dfrac{1}{3}, 0\right)$

ii $x = -\dfrac{b}{2a} = \dfrac{13}{12}$

iii Turning point at $\left(\dfrac{13}{12}, -\dfrac{289}{24}\right)$

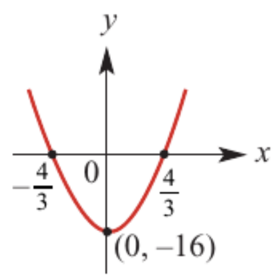


g $y = 9x^2 - 16 = (3x - 4)(3x + 4)$

i $(0, -16), \left(\dfrac{4}{3}, 0\right)$ and $\left(-\dfrac{4}{3}, 0\right)$

ii $x = -\dfrac{b}{2a} = 0$

iii Turning point at (0, –16)

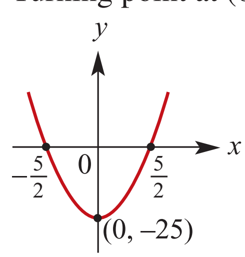


h $y = 4x^2 - 25 = (2x - 5)(2x + 5)$

i $(0, -25), \left(\dfrac{5}{2}, 0\right)$ and $\left(-\dfrac{5}{2}, 0\right)$

ii $x = -\dfrac{b}{2a} = 0$

iii Turning point at (0, –25)



8 $(5p - 1)x^2 - 4x + 2p - 1$

$$\begin{aligned}\Delta &= 16 - 4(2p-1)(5p-1)\\ &= 16 - 4(10p^2 - 7p + 1)\\ &= 16 - 40p^2 + 28p - 4\\ &= 12 - 40p^2 + 28p\\ &= -4(10p^2 - 7p - 3)\\ &= -4(10p-3)(p-1)\end{aligned}$$

$\Delta = 0 \Rightarrow p = -\dfrac{3}{10}$ or $p = 1$

9 **a** $x^2 > x$

$\Leftrightarrow x^2 - x > 0$

$\Leftrightarrow x(x-1) > 0$

$\Leftrightarrow x < 0$ or $x > 1$

b $(x+2)^2 \le 34$

$\Leftrightarrow (x+2)^2 - 34 \le 0$

$\Leftrightarrow (x + 2 - \sqrt{34})(x + 2 + \sqrt{34}) \le 0$

$\Leftrightarrow -2 - \sqrt{34} \le x \le -2 + \sqrt{34}$

c $3x^2 + 5x - 2 \le 0$

$\Leftrightarrow 3x^2 + 6x - x - 2 \le 0$

$\Leftrightarrow 3x(x+2) - (x+2) \le 0$

$\Leftrightarrow (x+2)(3x-1) \le 0$

$\Leftrightarrow -2 \le x \le \dfrac{1}{3}$

d $-2x^2 + 13x \ge 15$

$\Leftrightarrow -2x^2 + 13x - 15 \ge 0$

$\Leftrightarrow -(2x^2 - 13x + 15) \ge 0$

$\Leftrightarrow 2x^2 - 13x + 15 \le 0$

$\Leftrightarrow (2x-3)(x-5) \le 0$

$\Leftrightarrow \dfrac{3}{2} \le x \le 5$

10 $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a $x^2 + 6x + 3 = 0$

$\therefore x = \dfrac{-6 \pm \sqrt{36 - 12}}{2}$

$= \dfrac{-6 \pm 2\sqrt{6}}{2} = -3 \pm \sqrt{6}$

$x = -0.55, -5.45$ from calculator

b $x^2 + 9x + 12 = 0$

$\therefore x = \dfrac{-9 \pm \sqrt{81 - 48}}{2}$

$= \dfrac{-9 \pm \sqrt{33}}{2}$

$x = -1.63, -7.37$ from calculator

c $x^2 - 4x + 2 = 0$

$\therefore x = \dfrac{4 \pm \sqrt{16 - 8}}{2}$

$= 2 \pm \sqrt{2}$

$x = 3.414, 0.586$ from calculator

d $2x^2 + 7x + 2 = 0$

$\therefore x = \dfrac{-7 \pm \sqrt{49 - 16}}{4}$

$= \dfrac{-7 \pm \sqrt{33}}{4}$

$x = -0.314, -3.186$ from calculator

e $2x^2 + 7x + 4 = 0$

$\therefore x = -7 \pm \sqrt{49 - 32}$

$= \dfrac{-7 \pm \sqrt{17}}{4}$

$x = -0.719, -2.7816$ from calculator

f $3x^2 + 9x - 1 = 0$

$\therefore x = \dfrac{-9 \pm \sqrt{81 + 12}}{6}$

$= \dfrac{-9 \pm \sqrt{93}}{6}$

$x = -0.107, 3.107$ from calculator

11 $y = a(x-b)(x-c)$

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Assume the graph cuts the axis at (0,0) and (5,0), $b = 0$ and $c = 5$
Using (6, 10): $y = ax(x-5) = 10$

$$\therefore ax^2 - 5ax - 10 = 0$$
$$36a - 30a - 10 = 0$$
$$6a - 10 = 0$$
$$\therefore \quad a = \frac{5}{3}$$

$$y = \frac{5}{3}x(x-5)$$

12 A parabola has the same shape as $y = 3x^2$, but its vertex is at (5,2).
$y = 3(x-5)^2 + 2$

13 $(2m-3)x^2 + (5m-1)x + (3m-2) = 0$

$$\begin{aligned}\Delta &= (5m-1)^2 - 4\times(2m-3)(3m-2)\\ &= 25m^2 - 10m + 1 - 4(6m^2 - 13m + 6)\\ &= m^2 + 42m - 23\\ &= m^2 + 42m + 441 - 441 - 23\\ &= (m+21)^2 - 464\end{aligned}$$

$$\begin{aligned}\Delta > 0 &\Leftrightarrow (m-21)^2 - 464 > 0\\ &\Leftrightarrow (m + 21 - 4\sqrt{29})(m + 21 + 4\sqrt{29} > 0\\ &\Leftrightarrow x < -21 - 4\sqrt{29} \text{ or } m > -21 + 4\sqrt{29}\end{aligned}$$

14 Let a and b be the numbers.
$a + b = 30 \therefore b = 30 - a$
$P = ab = a(30 - a)$
Maximum occurs when $a = 15$.
Maximum product is 225

15 The vertex is at (1,5).
$\therefore y = a(x-1)^2 + 5$
Using (2, 10):
$y = a(2-1)^2 + 5 = 10$
$\therefore a = 5$
$\therefore y = 5(x-1)^2 + 5$
OR $y = 5x^2 - 10x + 10$

16 a $y = 2x + 3$ and $y = x^2$ meet where:
$x^2 = 2x + 3, \therefore x^2 - 2x - 3 = 0$
$\therefore (x-3)(x+1) = 0$
Where $x = 3, y = 9$; where $x = -1, y = 1$
Curves meet at (3,9) and (−1, 1).

b $y = 8x + 11$ and $y = 2x^2$ meet where:
$2x^2 = 8x + 11$
$$\therefore x = 2x^2 - 8x - 11 = 0$$
$$\therefore x = \frac{8 \pm \sqrt{64 + 88}}{4}$$
$$\therefore x = 2 \pm \frac{\sqrt{38}}{2}$$
Where $x = 2 - \frac{\sqrt{38}}{2}$, $y = 27 - 4\sqrt{38}$
Where $x = 2 + \frac{\sqrt{38}}{2}$, $y = 27 + 4\sqrt{38}$
From calculator: curves meet at (−1.08, 2.34) and (5.08,51.66).

c $y = 3x^2 + 7x$ and $y = 2$ meet where:
$3x^2 + 7x = 2$
$\therefore 3x^2 + 7x - 2 = 0$
$$\therefore x = \frac{-7 \pm \sqrt{49 - 24}}{6}$$
$$\therefore x = \frac{-7 \pm \sqrt{73}}{6}$$
Curves meet at $\left(\frac{-7 \pm \sqrt{73}}{6}, 2\right)$.
From calculator: (0.26, 2) and (−2.62, 2)

d $y = 2x^2$ and $y = 2 - 3x$ meet where
$2x^2 = 2 - 3x$
$\therefore 2x^2 + 3x - 2 = 0$
$\therefore (2x-1)(x+2) = 0, \therefore x = \frac{1}{2}, -2$

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Where $x = \frac{1}{2}, y = \frac{1}{2}$; where $x = -2, y = 8$

Curves meet at $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $(-2, 8)$.

17 **a** Equation is of the form

$y = k(x + 4)(x - 1)$

When $x = -1, y = -12$

Hence, $-12 = k(3)(-2)$

$\therefore k = 2$

$\therefore y = 2(x + 4)(x - 1)$

b Equation is of the form

$y = a(x + 1)^2 + 3$

When $x = 1, y = -5$

Hence, $-5 = a(4) + 3$

$\therefore a = -2$

$\therefore y = -2(x + 1)^2 + 3$

c Equation is of the form

$y = ax^2 + bx - 3$

When $x = 1, y = -3$

$\therefore -3 = a + b - 3 \ldots (1)$

When $x = -1, y = 1$

$\therefore 1 = a - b - 3 \ldots (2)$

Simplifying the equations

$a + b = 0 \ldots (1')$

$a - b = 4 \ldots (2')$

Add$(1')$ and $(2')$

$2a = 4$

$a = 2, b = -2$

$\therefore y = 2x^2 - 2x - 3$

18

$S = 9.42r^2 + 6(6.28)r = 125.6$

$\therefore 9.42r^2 + 37.68r - 125.6 = 0$

$$\therefore r = \frac{-37.68 \pm \sqrt{37.68^2 + 4(125.6)9.42}}{2(9.42)}$$

$$\therefore r = -2 \pm \frac{\sqrt{6152.4}}{18.84}$$

Since r > 0,

$$r = -2 + \frac{\sqrt{6152.4}}{18.84} = 2.16 \text{ m}$$

19 **a** $2x^2 + mx + 1 = 0$ has exactly one solution where $\Delta = 0$:

$\Delta = m^2 - 8 = 0, \therefore m^2 = 8$

$\therefore m = \pm 2\sqrt{2}$

b $x^2 - 4mx + 20 = 0$ has real solutions where $\Delta \geq 0$:

$\Delta = 16m^2 - 80 \geq 0$

$\therefore m^2 \geq 5$

Solution set:

$\{m : m \leq -\sqrt{5}\} \cap \{m : m \geq \sqrt{5}\}$

20 $y = x^2 + bx$

a When $y = 0, x(x + b) = 0$

$x = 0$ or $x = -b$

b Completing the square

$y = x^2 + bx + \frac{b^2}{4} - \frac{b^2}{4}$

$\therefore y = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4}$

The vertex is at $\left(-\frac{b}{2}, -\frac{b^2}{4}\right)$

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c $x^2 + bx = x$

i $x^2 + (b - 1)x = 0$
$\therefore x(x + (b - 1)) = 0$
$\therefore x = 0$ or $x = 1 - b$
The coordinates of the points of intersection are
$(0, 0)$ and $(1 - b, 1 - b)$

ii There is one point of intersection when $b = 1$.

iii There are two points of intersection when $b \neq 1$.

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Solutions to Review: Multiple-choice questions

1 A $12x^2 + 7x - 12 = (3x + 4)(4x - 3)$

2 C $x^2 - 5x - 14 = 0$
$\therefore (x - 7)(x + 2) = 0$
$\therefore x = -2, 7$

3 C $y = 8 + 2x - x^2$
$= 9 - (x^2 - 2x + 1)$
$= 9 - (x - 1)^2$
Maximum value of y is 9 when $x = 1$

4 E $y = 2x^2 - kx + 3$
If the graph of touches the x-axis then $\Delta = 0$:
$\Delta = (-k)^2 - 24 = 0$
$\therefore k^2 = 24$
$\therefore k = \pm\sqrt{24} = \pm 2\sqrt{6}$

5 B $x^2 - 56 = x$
$\therefore x^2 - x - 56 = 0$
$\therefore (x - 8)(x + 7) = 0$
$\therefore x = -7, 8$

6 C $x + 3x - 10$
$\Delta = 3^2 + 40 = 49$

7 E $y = 3x^2 + 6x - 1$
$= 3x + 6x + 3 - 4$
$= 3(x + 1)^2 - 4$
TP is at $(-1, -4)$.

8 E $5x^2 - 10x - 2$
$= 5(x^2 - 2x + 1) - 7$
$= 5(x - 1)^2 - 7$

9 D If two real roots of $mx^2 + 6x - 3 = 0$ exist, then $\Delta > 0$:
$\Delta = 6^2 + 12m = 12(m + 3)$
$m > -3$

10 A $6x^2 - 8xy - 8y^2$
$= (3x + 2y)(2x - 4y)$

11 B $y = x^2 - ax + \dfrac{a^2}{4} - \dfrac{a^2}{4}$
$y = \left(x - \dfrac{a}{2}\right) - \dfrac{a^2}{4}$
Therefore vertex $\left(\dfrac{a}{2}, -\dfrac{a^2}{4}\right)$

12 E $x^2 > b^2$
$(x - b)(x + b) > 0$
But $b < 0$ and therefore $-b > 0$
$(x - b)(x + b) > 0 \Leftrightarrow x > -b$ or $x < b$

13 D $\Delta = 4a^2 - 4b$
One solution when $\Delta = 0$
$\therefore a^2 = b$
$\therefore a = \pm\sqrt{b}$

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Solutions to Review: Extended-response questions

1 a The turning point (h, k) is $\left(25, \frac{9}{2}\right)$

$$\therefore \quad y = a(x-25)^2 + \frac{9}{2}$$

When $x = 0$, $\quad y = 0$

$$\therefore \quad 0 = a(0-25)^2 + \frac{9}{2}$$

$$\therefore \quad 0 = 625a + \frac{9}{2}$$

$$\therefore \quad 625a = \frac{-9}{2}$$

$$\therefore \quad a = \frac{-9}{1250}$$

Hence the equation for the parabola is $y = \frac{-9}{1250}(x-25)^2 + \frac{9}{2}$, for $0 \le x \le 50$.
This can also be written as $y = -0.0072x(x-50)$ [the intercept form].

b

x	0	5	10	15	20	25	30	35	40	45	50
y	0	1.62	2.88	3.78	4.32	4.5	4.32	3.78	2.88	1.62	0

You can find these values using a CAS calculator, or:

When $x = 10$,
$$\begin{aligned} y &= \frac{-9}{1250}(10-25)^2 + \frac{9}{2} \\ &= \frac{-9}{1250} \times 225 + \frac{9}{2} \\ &= \frac{-81}{50} + \frac{225}{50} \\ &= \frac{144}{50} \\ &= \frac{72}{25} \\ &= 2.88 \end{aligned}$$

When $x = 20$,
$$\begin{aligned} y &= \frac{-9}{1250}(20-25)^2 + \frac{9}{2} \\ &= \frac{-9}{1250} \times 25 + \frac{9}{2} \\ &= 4.32 \end{aligned}$$

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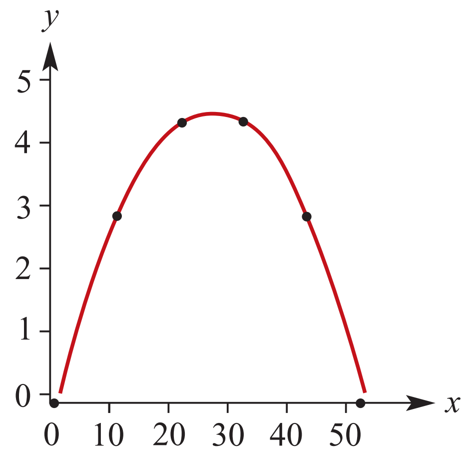


When $x = 30$, $y = \dfrac{-9}{1250}(30 - 25)^2 + \dfrac{9}{2}$

$= \dfrac{-9}{1250} \times 25 + \dfrac{9}{2}$

$= 4.32$

When $x = 40$, $y = \dfrac{-9}{1250}(40 - 25)^2 + \dfrac{9}{2}$

$= \dfrac{-9}{1250} \times 225 + \dfrac{9}{2}$

$= 2.88$

c

When $y = 3$, $\dfrac{-9}{1250}(x - 25)^2 + \dfrac{9}{2} = 3$

$\therefore \quad \dfrac{-9}{1250}(x - 25)^2 = \dfrac{-3}{2}$

$\therefore \quad (x - 25)^2 = \dfrac{-3}{2} \times \dfrac{-1250}{9} = \dfrac{625}{3}$

$\therefore \quad x - 25 = \pm\sqrt{\dfrac{625}{3}}$

$\therefore \quad x = 25 \pm \dfrac{25\sqrt{3}}{3}$

$\therefore \quad x \approx 10.57$ or $x \approx 39.43$

Hence the height of the arch is 3 m above water level approximately 10.57 m and 39.43 m horizontally from A. This can also be solved using a CAS calculator.

d When $x = 12$, $y = \dfrac{-9}{1250}(12 - 25)^2 + \dfrac{9}{2}$

$= \dfrac{-9}{1250} \times 169 + \dfrac{9}{2} = 3.2832$

The height of the arch is 3.2832 m at a horizontal distance of 12 m from A.

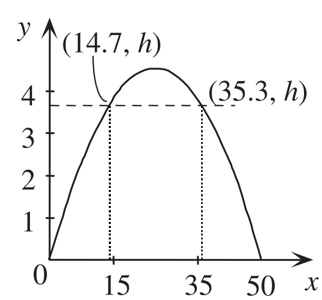
e The greatest height of the deck above water level, h m, is when

$x + 0.3 = 15$ and $x - 0.3 = 35$

i.e. when $x = 14.7$ and $x = 35.3$

$\therefore h = \dfrac{-9}{1250}(14.7 - 25)^2 + \dfrac{9}{2}$

$= 3.736152$



Hence the greatest height of the deck above water level is approximately 3.736 m.

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2 a If x cm is the side length of the square then $4x$ cm has been used to form the square, so the perimeter of the rectangle is $P = 12 - 4x$.

Let a cm be the width of the rectangle and $2a$ cm be the length of the rectangle,

So $\quad P = a + a + 2a + 2a = 6a$

$\therefore \quad 6a = 12 - 4x$

$\therefore \quad a = 2 - \frac{2}{3}x \qquad \text{and} \qquad 2a = 4 - \frac{4}{3}x$

Hence the dimensions of the rectangle are $\left(2 - \frac{2}{3}x\right)$ cm $\times \left(4 - \frac{4}{3}x\right)$ cm.

b Let A_1 be the area of the square and A_2 be the area of the rectangle.

$$\begin{aligned}\therefore A = A_1 + A_2 &= x^2 + \left(2 - \frac{2}{3}x\right)\left(4 - \frac{4}{3}x\right)\\ &= x^2 + 8 - \frac{8}{3}x - \frac{8}{3}x + \frac{8}{9}x^2\\ &= \frac{17}{9}x^2 - \frac{16}{3}x + 8\end{aligned}$$

Hence the combined area of the square and the rectangle in cm^2 is defined by the rule $A = \frac{17}{9}x^2 - \frac{16}{3}x + 8$.

c TP occurs when
$$\begin{aligned}x &= -\frac{b}{2a}\\ &= \frac{16}{3} \div \frac{34}{9}\\ &= \frac{24}{17}\end{aligned}$$

Minimum occurs when $x = \frac{24}{17}$.

When $x = \frac{24}{17}, \quad 4x = \frac{96}{17} \approx 5.65$

and $\qquad 12 - 4x = \frac{108}{17} \approx 6.35$

Hence, the wire needs to be cut into lengths of 5.65 cm and 6.35 cm (correct to 2 decimal places) for the sum of the areas to be a minimum.

3 a

$V = \text{rate} \times \text{time}$

When $x = 5, \quad V = 0.2 \times 60 = 12$

When $x = 10, \quad V = 0.2 \times 60 \times 5 = 60$

When $x = 0, \quad V = 0$

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$\therefore c = 0$ (y-axis intercept is 0)

$\therefore \quad V = ax^2 + bx$

When $x = 5$, $V = 12$, $\quad 12 = 25a + 5b \quad (1)$

When $x = 10$, $V = 60$, $\quad 60 = 100a + 10b \quad (2)$

$2 \times (1) \quad 24 = 50a + 10b \quad (3)$

$(2) - (3) \quad 36 = 50a$

$\therefore \quad a = \frac{36}{50} = \frac{18}{25}$

Substitute $a = \frac{18}{25}$ in (1) $\quad 12 = 25 \times \frac{18}{25} + 5b$

$\therefore \quad 12 = 18 + 5b$

$\therefore \quad 5b = -6$

$\therefore \quad b = -\frac{6}{5}$

Hence, the rule for V in terms of x is $V = \frac{18}{25}x^2 - \frac{6}{5}x$, $x \geq 0$, or $v = 0.72x^2 - 1.2x$

b When $x = 20$ (i.e. a depth of 20 cm),

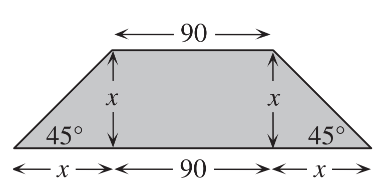
$$\begin{aligned} V &= \frac{18}{25}(20)^2 - \frac{6}{5}(20) \\ &= \frac{18 \times 400}{25} - 24 \\ &= 18 \times 16 - 24 = 264 \end{aligned}$$

Now $\quad V = \text{rate} \times \text{time}$

$$\begin{aligned} \therefore \quad \text{time} &= \frac{V}{\text{rate}} \\ &= \frac{264}{0.2} = 1320 \text{ minutes} \\ &= 22 \text{ hours} \end{aligned}$$

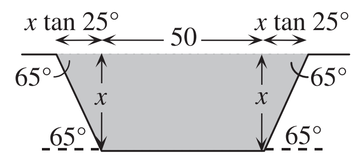
Water can be pumped into the tank for 22 hours before overflowing.

4 a Let V_E m^3 be the volume of the embankment.



$$\begin{aligned} V_E &= 120 \times \text{ shaded area} \\ &= 120\left(90x + \frac{1}{2}x^2 + \frac{1}{2}x^2\right) \\ &= 120x^2 + 10\,800x, x > 0 \end{aligned}$$

b Let V_C m^3 be the volume of the cutting.



$$\begin{aligned} V_C &= 100 \times \text{shaded area} \\ &= 100(50x + x^2 \tan 25°) \\ &\approx 100(50x + 0.466\,308x^2) \\ &\approx 46.63x^2 + 5000x,\ x > 0 \end{aligned}$$

c When $x = 4$, $\quad V_C \approx 46.63 \times 4^2 + 5000 \times 4$

$$\approx 20746.08$$

Now $V_E = L \times (x^2 + 90x)$, where L m is the length of the embankment.

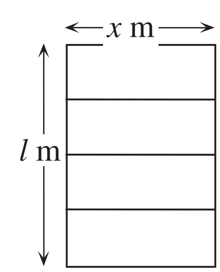
$\therefore \quad L = \dfrac{V_E}{x^2 + 90x}$

If using soil from the cutting, $V_C = V_E$

$$\therefore \quad L = \frac{V_C}{x^2 + 90x} = \frac{20746.08}{4^2 + 90 \times 4} \approx 55.18$$

Hence, when $x = 4$ m, an embankment 55.18 m long could be constructed from the soil taken from the cutting.

5 a $5x + 2l = 100$



$\therefore \quad 2l = 100 - 5x$

$\therefore \quad l = 50 - \dfrac{5}{2}x$

b $A = x \times l$

$$= 50x - \frac{5}{2}x^2$$

c When $A = 0$, $\quad -\dfrac{5}{2}x^2 + 50x = 0$

$\therefore \quad x\left(-\dfrac{5}{2}x + 50\right) = 0$

$\therefore$ either $x = 0$ or $\quad -\dfrac{5}{2}x + 50 = 0$

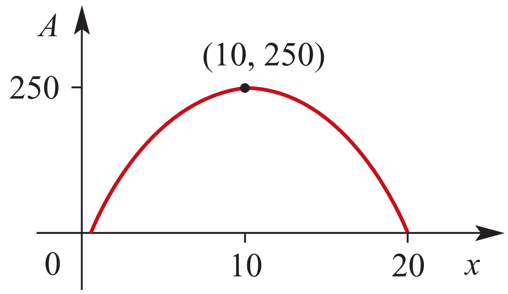
If $-\dfrac{5}{2}x + 50 = 0$, $\quad \dfrac{5}{2}x = 50$

$$\therefore \quad x = \frac{2 \times 50}{5} = 20$$

Turning point is halfway between the x-intercepts, i.e. at $x = 10$.

When $x = 10$,

$$A = -\frac{5}{2} \times 10^2 + 50 \times 10$$
$$= -250 + 500 = 250$$



Completing the square may also be used to find the vertex.

d The maximum area is 250 m^2 when x is 10 metres.

6 Given $AP = 1$, $AB = 1 - x$, $AD = x$ and $\frac{AP}{AD} = \frac{AD}{AB}$

then $\quad \frac{1}{x} = \frac{x}{1-x}$

$\therefore \quad 1 - x = x^2$

$\therefore\ x^2 + x - 1 = 0$

Using the general quadratic formula:

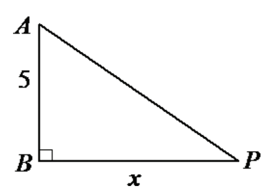
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{where } a = 1, b = 1, c = -1$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$= \frac{-1 - \sqrt{5}}{2} \text{ or } \frac{-1 + \sqrt{5}}{2}$$

but $x > 0$, so $\quad x = \frac{-1 + \sqrt{5}}{2}$

7 **a** Using Pythagoras' theorem

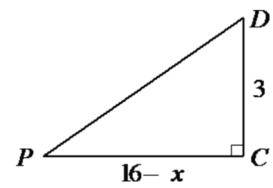


$$PA^2 = 5^2 + x^2$$
$$= x^2 + 25$$
$$\therefore \quad PA = \sqrt{x^2 + 25}$$

b **i** $PC = BC - BP$

$$= 16 - x$$

ii Using Pythagoras' theorem



$$PD^2 = (16 - x)^2 + 3^2$$
$$= x^2 - 32x + 256 + 9$$
$$\therefore \quad PD = \sqrt{x^2 - 32x + 265}$$

c If $PA = PD$, $\sqrt{x^2 + 25} = \sqrt{x^2 - 32x + 265}$

$\therefore \quad x^2 + 25 = x^2 - 32x + 265$

$\therefore \quad 25 = -32x + 265$

$\therefore \quad 32x = 240$

$\therefore \quad x = 7.5$

d If $PA = 2PD$, $\sqrt{x^2 + 25} = 2\sqrt{x^2 - 32x + 265}$

$\therefore \quad x^2 + 25 = 4(x^2 - 32x + 265)$

$\therefore \quad = 4x^2 - 128x + 1060$

$\therefore \quad 3x^2 - 128x + 1035 = 0$

Using the general quadratic formula,

$$x = \frac{128 \pm \sqrt{(-128)^2 - 4(3)(1035)}}{2(3)}$$
$$= \frac{128 \pm \sqrt{3964}}{6}$$
$$= \frac{128 \pm 2\sqrt{991}}{6}$$
$$= \frac{64 \pm \sqrt{991}}{3}$$
$$= 31.826\,71\ldots \text{ or } 10.839\,94\ldots$$
$$\approx 10.840 \text{ (as } 0 \le x \le 16)$$

e If $PA = 3PD$, $\sqrt{x^2 + 25} = 3\sqrt{x^2 - 32x + 265}$

$\therefore \quad x^2 + 25 = 9(x^2 - 32x + 265)$

$= 9x^2 - 288x + 2385$

$\therefore \quad 8x^2 - 288x + 2360 = 0$

$\therefore \quad 8(x^2 - 36x + 295) = 0$

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Using the general quadratic formula,

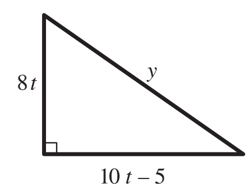
$$\begin{aligned} x &= \frac{36 \pm \sqrt{(-36)^2 - 4(1)(295)}}{2(1)} \\ &= \frac{36 \pm \sqrt{116}}{2} \\ &= \frac{36 \pm 2\sqrt{29}}{2} = 18 \pm \sqrt{29} \\ &= 23.385\,16\ldots \text{ or } 12.615\,83\ldots \end{aligned}$$

≈ 12.615 (as $0 \le x \le 16$)

Note: Parts **c**, **d** and **e** can be solved using the CAS calculator. Plot the graphs of ***f1* = √(*x*^2 + 25)**, ***f2* = √(*x*^2 − 32*x* + 265)**, ***f3* = 2 × *f*2(*x*)** and ***f4* = 3 × *f*2(*x*)** for $x \in [0, 16]$. The points of intersection of ***f1*** with each of the other graphs provide the solutions for x.

8 a i Consider AB and CD to be a pair of Cartesian axes with O at the point $(0, 0)$. The first jogger is at the point $(8t, 0)$ at time t. The second jogger is at the point $(0, 10t - 5)$ at time t.

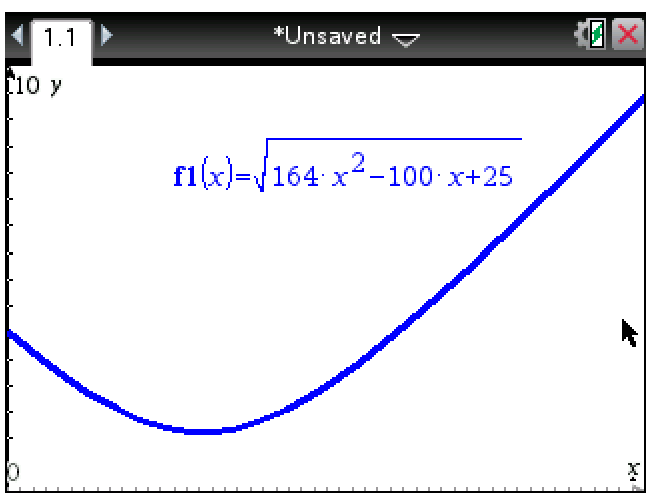
Using Pythagoras' theorem



$$\begin{aligned} y^2 &= (8t)^2 + (10t - 5)^2 \\ &= 64t^2 + 100t^2 - 100t + 25 \end{aligned}$$

$$\therefore \quad y = \sqrt{164t^2 - 100t + 25}$$

ii



iii On a CAS calculator, enter **solve(√(164*x*^2 − 100*x* + 25) = 4, *x*)**.

The points of intersection are $\left(\frac{9}{82}, 4\right)$ and $\left(\frac{1}{2}, 4\right)$.

Therefore joggers are 4 km apart after 0.11 hours (1.07 pm), correct to 2 decimal places, and after 0.5 hours (1.30 pm).

Or consider $\sqrt{164t^2 - 100t + 25} = 4$

$\therefore \quad 164t^2 - 100t + 9 = 0$

$$\therefore \quad t = \frac{100 \pm \sqrt{(-100)^2 - 4(9)(164)}}{2(164)}$$
$$= \frac{100 \pm \sqrt{4096}}{328}$$
$$= \frac{100 \pm 64}{328}$$
$$= \frac{1}{2} \text{ or } \frac{9}{82}$$

iv With the graph from part **ii** on screen
TI: Press **Menu→6:Analyze Graph→2:Minimum**
CP: Tap **Analysis→ G-Solve→Min**
to yield (0.30487837, 3.12 4752).
Therefore joggers are closest when they are 3.12 km apart after 0.30 hours, correct to 2 decimal places.
Alternatively, the minimum of $\sqrt{164t^2 - 100t + 25}$ occurs when $164t^2 - 100t + 25$ is a minimum.

This occurs when
$$t = \frac{100}{2 \times 164}$$
$$= \frac{25}{82} \text{(1.18 pm)}$$

$$\therefore \text{ minimum distance apart} = \frac{20}{\sqrt{41}}$$
$$= \frac{20\sqrt{41}}{41}$$
$$\approx 3.123 \text{ km}$$

b **i** When $y = 5$, $\quad 5 = \sqrt{164t^2 - 100t + 25}$

$\therefore \quad 25 = 164t^2 - 100t + 25$

$\therefore \quad 164t^2 - 100t = 0$

$\therefore \quad 4t(41t^2 - 25t) = 0$

$\therefore \quad t = 0 \text{ or } t = \dfrac{25}{41}$

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ii When $y = 6$, $\quad 6 = \sqrt{164t^2 - 100t + 25}$

$\therefore \quad 36 = 164t^2 - 100t + 25$

$\therefore \quad 164t^2 - 100t - 11 = 0$

Using the general quadratic formula,

$$t = \frac{100 \pm \sqrt{(-100)^2 - 4(164)(-11)}}{2(164)}$$

$$= \frac{100 \pm \sqrt{17216}}{328} = \frac{25 \pm 2\sqrt{269}}{82}$$

9 **a** $BC = x, CD = y, BD =$ diameter of circle $= 2a$

Using Pythagoras' theorem,

$$BC^2 + CD^2 = BD^2$$

$\therefore \quad x^2 + y^2 = 4a^2$, as required.

b Perimeter $= b$, $\quad$ but perimeter $= 2(x + y)$

$\therefore \quad 2(x + y) = b$

c $2(x + y) = b$ $\quad \therefore 2x + 2y = b$

$\therefore \quad 2y = b - 2x$

$\therefore \quad y = \frac{1}{2}b - x \qquad (1)$

Substituting (1) into $\quad x^2 + y^2 = 4a^2$ gives

$$x^2 + \left(\frac{1}{2}b - x\right)^2 = 4a^2$$

$\therefore \quad 2x^2 - bx + \frac{1}{4}b^2 - 4a^2 = 0 \qquad (2)$

$\therefore \quad 8x^2 - 4bx + b^2 - 16a^2 = 0$

d Now $x + y > 2a$ $\therefore$ using (1), $\quad x + \left(\frac{1}{2}b - x\right) > 2a$

$\therefore \quad \frac{1}{2}b > 2a$

$\therefore \quad b > 4a$

Considering the discriminant, Δ, of (2)

$$\Delta = (-b)^2 - 4(2)\left(\frac{1}{4}b^2 - 4a^2\right)$$

$$= b^2 - 8\left(\frac{1}{4}b^2 - 4a^2\right)$$

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$$= b^2 - 2b^2 + 32a^2$$
$$= 32a^2 - b^2$$

For the inscribed rectangle to exist, $\Delta \geq 0$

$\therefore\ 32a^2 - b^2 \geq 0$

$\therefore\quad b^2 \leq 32a^2$

$\therefore\quad b \leq 4\sqrt{2}a$

$\therefore\ 4a < b \leq 4\sqrt{2}a$, as required.

e **i** Substituting $a = 5$ and $b = 24$ into (2) gives

$$2x^2 - 24x + \left(\frac{1}{4}(24)^2 - 4(5)^2\right) = 0$$
$$\therefore\quad 2x^2 - 24x + 44 = 0$$
$$\therefore\quad x^2 - 12x + 22 = 0$$

Using the general quadratic formula,

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(22)}}{2(1)}$$
$$= \frac{12 \pm \sqrt{56}}{2}$$
$$= 6 \pm \sqrt{14}$$

Now $\quad y = \frac{1}{2}b - x$

$$= \frac{1}{2}(24) - x = 12 - x$$

When $x = 6 \pm \sqrt{14}$, $\quad y = 12 - (6 \pm \sqrt{14})$

When $x = 6 + \sqrt{14}$, $\quad y = 6 - \sqrt{14}$

When $x = 6 - \sqrt{14}$, $\quad y = 6 + \sqrt{14}$

ii If $b = 4\sqrt{2}a$, then (2) gives

$$2x^2 - 4\sqrt{2}ax + \left(\frac{1}{4}(4\sqrt{2}a\right)^2 - 4a^2) = 0$$
$$\therefore\quad 2x^2 - 4\sqrt{2}ax + 8a^2 - 4a^2 = 0$$
$$\therefore\quad 2x^2 - 4\sqrt{2}ax + 4a^2 = 0$$
$$\therefore\quad x^2 - 2\sqrt{2}ax + 2a^2 = 0$$
$$\therefore\quad (x - \sqrt{2}a)^2 = 0$$

$$\therefore \qquad x = \sqrt{2}a$$

$$\therefore \qquad y = \frac{1}{2}b - x$$
$$= 2\sqrt{2}a - \sqrt{2}a$$
$$= \sqrt{2}a$$

f If $\frac{b}{a} = 5$, then $b = 5a$ and, from (2):

$$2x^2 - (5a)x + \left(\frac{1}{4}(5a)^2 - 4a^2\right) = 0$$

$$\therefore \qquad 2x^2 - 5ax + \left(\frac{25}{4}a^2 - 4a^2\right) = 0$$

$$\therefore \qquad 2x^2 - 5ax + \frac{9}{4}a^2 = 0$$

Using the general quadratic formula,

$$x = \frac{5a \pm \sqrt{(-5a)^2 - 4 \times 2 \times \frac{9}{4}a^2}}{2(2)}$$
$$= \frac{5a \pm \sqrt{25a^2 = 18a^2}}{4}$$
$$= \frac{5a \pm \sqrt{7}a}{4}$$

Now $\qquad y = \frac{1}{2}b - x$
$$= \frac{1}{2}(5a) - x$$
$$= \frac{5}{2}a - x$$

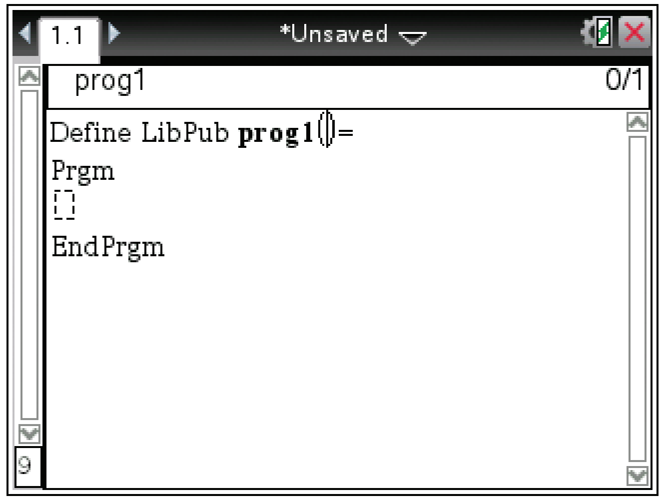
When $x = \frac{5a \pm \sqrt{7}a}{4}$, $y = \frac{5}{2}a - \frac{5a \pm \sqrt{7}a}{4}$

When $x = \frac{5a + \sqrt{7}a}{4}$, $y = \frac{5a - \sqrt{7}a}{4}$

When $x = \frac{5a - \sqrt{7}a}{4}$, $y = \frac{5a + \sqrt{7}a}{4}$

g The following program can be input into a CAS calculator to solve equation (2) in part **c** for x and y, given a and $b(a, b \in \mathbb{R})$, correct to 2 decimal places.

TI: In the calculator application press **menu→9:Functions & Programs→ 1**:**Program Editor1**:**New.** Name the program progl. The following information is shown automatically. Complete the screen as follows: Complete the screen as follows:




```
Define LibPub progl () =
Prgm
setMode(5,2)
setMode(1,16)
Local a,b,w,x,y,z
Request "a = ",a
Request "b = ",b
(b + √(32a^2 − b^2))/4 → x
b/2 − x → y
(b − √(32a^2 − b^2))/4 → w
b/2 − w → z
Disp "x= ",x
Disp "and y= ",y
Disp "OR"
Disp "x=",w
Disp "and y =", z
EndPrgm
```

10 a Equation of curve A is

$y = (x - h)^2 + 3$

$(0, 4)$: $4 = (0 - h)^2 + 3$

$h^2 = 1$

$h = 1$ (since $h > 0$)

So $y = (x - 1)^2 + 3$

$= x^2 - 2x + 4$

Giving $b = -2, c = 4$ and $h = 1$

b **i** The coordinates of P' are $(x, -6 + 4x - x^2)$

ii Let (m, n) be the coordinates of M.

$\therefore \quad m = x$

and $\quad n = \dfrac{(x^2 - 2x + 4) + (-6 + 4x - x^2)}{2}$

$= \dfrac{2x - 2}{2} = x - 1$

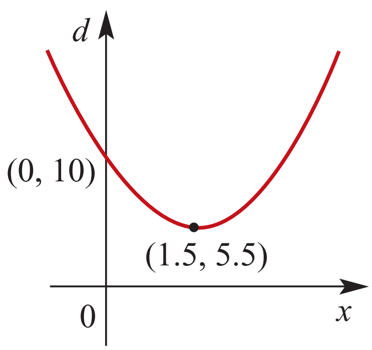
$\therefore$ the coordinates of M are $(x, x - 1)$.

iii The coordinates of M for $x = 0, 1, 2, 3, 4$ are $(0, -1), (1, 0), (2, 1), (3, 2)$ and $(4, 3)$ respectively.

iv $y = x - 1$ is the equation of the straight line on which the points $(0, -1)$, $(1, 0)$, $(2, 1)$, $(3, 2)$ and $(4, 3)$ all lie.

c **i** $d = (x^2 - 2x + 4) - (-6 + 4x - x^2) = 2x^2 - 6x + 10$

ii



iii

Consider $\quad 2(x^2 - 3x + 5) = 2\left(x^2 - 3x + \dfrac{9}{4} + 5 - \dfrac{9}{4}\right)$

$= 2\left(\left(x - \dfrac{3}{2}\right)^2 + \dfrac{11}{4}\right) = 2\left(x - \dfrac{3}{2}\right)^2 + \dfrac{11}{2}$

$\therefore$ minimum value of d is $\dfrac{11}{2}$ and occurs for $x = \dfrac{3}{2}$.

11 **a** Length of path $= \sqrt{(60 + 30)^2 + (30 + 15)^2}$

$= \sqrt{10\,125}$

$= 45\sqrt{5}$

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b **i** $y = ax^2 + bx + c$

$$\begin{aligned}
&\text{At } (-20, 45), && 45 = 400a - 20b + c && (1)\\
&\text{At } (40, 40), && 40 = 1600a + 40b + c && (2)\\
&\text{At } (30, 35), && 35 = 900a + 30b + c && (3)\\
&(2) - (1) \text{ gives} && -5 = 1200a + 60b && (4)\\
&(2) - (3) \text{ gives} && 5 = 700a + 10b && (5)\\
&6 \times (5) - (4) \text{ gives} && 35 = 3000a &&\\
&\therefore && a = \frac{35}{3000} = \frac{7}{600} &&
\end{aligned}$$

Substituting $a = \dfrac{7}{600}$ into (5) gives:

$$\begin{aligned}
5 &= 700\left(\frac{7}{600}\right) + 10b\\
&= \frac{49}{6} + 10b
\end{aligned}$$

$$\therefore \quad 10b = \frac{-19}{6}$$

$$\therefore \quad b = \frac{-19}{60}$$

Substituting $a = \dfrac{7}{600}$ and $b = \dfrac{-19}{60}$ into (1) gives:

$$\begin{aligned}
45 &= 400\left(\frac{7}{600}\right) - 20\left(\frac{-19}{60}\right) + C\\
&= \frac{14}{3} + \frac{19}{3} + C
\end{aligned}$$

$$\therefore \quad c = 34$$

$$\therefore \quad y = \frac{7}{600}x^2 - \frac{19}{60}x + 34$$

ii

Consider
$$\begin{aligned}
\frac{7}{600}x^2 - \frac{19}{60}x + 34 &= \frac{7}{600}\left(x^2 - \frac{190}{7}x + \frac{20400}{7}\right)\\
&= \frac{7}{600}\left(\left(x - \frac{95}{7}\right)^2 + \frac{133775}{49}\right)\\
&= \frac{7}{600}\left(x - \frac{95}{7}\right)^2 + \frac{5351}{168}
\end{aligned}$$

$\therefore$ minimum value is $\dfrac{5351}{168}$ and this occurs when $x = \dfrac{95}{7}$.

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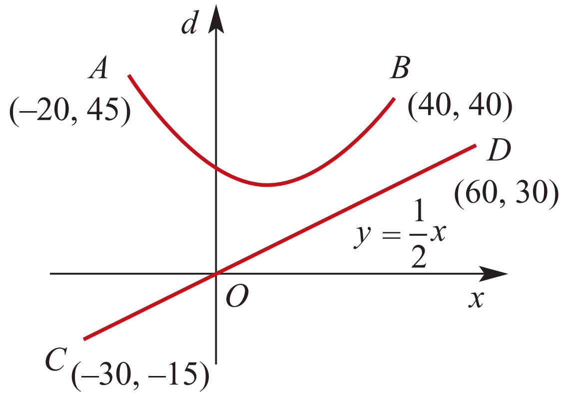
c



d **i** The expression $y = (ax^2 + bx + c) - \frac{1}{2}x$ determines the distance, perpendicular to the x-axis, between $y = ax^2 + bx + c$ and $y = \frac{1}{2}x$ at the point x. In this question, it is the distance between the path and the pond.

ii

Consider $\dfrac{7x^2}{600} - \dfrac{19x}{60} + 34 - \dfrac{x}{2} = \dfrac{7x^2}{600} - \dfrac{49x}{60} + 34$

$$= \frac{7}{600}\left(x^2 - 70x + \frac{20400}{7}\right)$$

$$= \frac{7}{600}\left(x^2 - 70x + 1225 + \frac{11825}{7}\right)$$

$$= \frac{7}{600}\left((x-35)^2 + \frac{11825}{7}\right)$$

$$= \frac{7}{600}(x-35)^2 + \frac{473}{24}$$

$\therefore$ minimum value is $\dfrac{473}{24}$ which occurs when $x = 35$.

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