REVIEW - LINEAR (CHAPTER 4)

1. calculate the distance between:

a) (0,3) and (6,9)

$d=\sqrt{(x_2-x_1)^2+(y_2-x_1)^2}$

$d=\sqrt{(6-0)^2+(9-3)^2}$

$d=8.49$ units

b) (-4,4) and (2,1)

$d=\sqrt{(2--4)^2+(1-4)^2}$

$d=6.71$ units

2. state the gradient and the y-intercept of:

a) $y=2x-5$

$m=2$

y-int $=(0,-5)$

b) $y+3x-8=0$

$y=-3x+8$

$m=-3$

y-int $=(0,8)$

c) $3y+9=x$

$y=-3+\frac{x}{3}$

$m=\frac{1}{3}$

y-int $=(0,-3)$

d) $2x+10+4y=6$

$4y=-2x-4$

$y=-\frac{x}{2}-1$

$m=-\frac{1}{2}$

y-int $=(0,-1)$

3. Determine the equation between:

a) (2,-1) and (-3,5)

$m=\frac{5--1}{-3-2}=\frac{-6}{5}$

$y=\frac{-6}{5}x+c$

$-1=\frac{-6}{5}(2)+c$

$c=-1+2.4$

$c=1.4$

$\therefore y=\frac{-6}{5}x+1.4$

b) (1,6) and (0,2)

$m=\frac{2-6}{0-1}=\frac{-4}{1}=-4$

$y=-4x+c$

$y=4x+2$

4. Determine the equation of the line perpendicular to $6y+2x+12=0$ that goes through (2,-4)

$6y=-2x-12$

$y=\frac{-x}{3}-2$

$m=\frac{1}{3}$

$\perp m=3$

$y=3x+c$

$-4=3(2)+c$

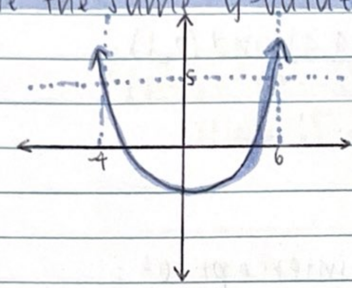
$c=-10$

$\therefore y=3x-10$

FUNCTIONS VS RELATIONS

A function (e.g quadratic, cubic, linear etc.) can have several $x$-values that give the same $y$-value

for example:

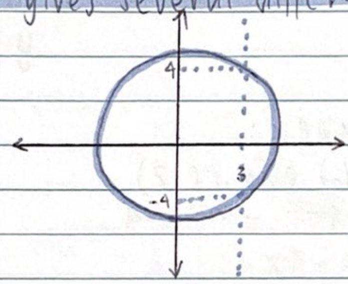



this is an example of a 'many-to-one' function

this shows that when $x=-4$ or when $x=6$, $y$ will equal 5

unlike a function, a relation (e.g circle, $y^2=x$) can have one $x$-value that gives several different $y$-values

fore example:




this is an example of a 'one-to-many' relation

the method (shown in examples) used to determine relations from functions is called the 'vertical line test'.

* note: if a function is 'one-to-one', it is also a function *

# ~~Ex~~FUNCTION NOTATION

we say '$y$ is a function of $x$' because as $x$ changes, $y$ changes in response

we can write this as $f(x)$ ('ef of ex')

and simply put, it replaces '$y$' in an equation

so $y=3x-2$ can be $f(x)=3x-2$

Then we could show $y$ equals when $x=5$ as:

$= f(5)=3(5)-2$

$= 13$

to determine $x$ when the function is equal to 19, we write:

$f(x)=19$ , $3x-2=19$ , $3x=21$ , $x=7$

*determine $a$ whe $f(a)=19$ *

FACTORISING CUBICS

if $y = ax^3 + bx^2 + cx$ (or any combination of these 3 terms)

① Factor out '$x$'

② Factorise quadratic using any method already learned

EXAMPLES:

① Factorise:

a) $y = x^3 + 8x^2 + 12x$

$= x(x^2 + 8x + 12)$

$= (x)(x+2)(x+6)$

b) $y = -x^3 + 4x^2 + 5x$

$= -x(x^2 - 4x - 5)$

$= (-x)(x-5)(x+1)$

② Determine all axis intercepts of:

a) $y = 4x^3 - 6x$

y axis int, $x = 0$:

$y = 4(0)^3 - 6(0) = 0$ $\therefore (0,0)$

x axis int, $y = 0$:

$0 = 4x^3 - 6x$

$0 = (2x)(2x^2 - 3)$ $\therefore 2x = 0,\ x = 0$

and

$2x^2 - 3 = 0,\ 2x^2 = 3$

$x^2 = \frac{3}{2},\ x^2 = \pm\sqrt{\frac{3}{2}}$

b) $y = 2x^3 - 12x^2 + 18x$

y-int: $(0,0)$

x axis int, $y = 0$:

$0 = 2x^3 - 12x^2 + 18x$

$= 2x(x^2 - 6x + 9)$

$= 2x(x-3)^2$

$\therefore 2x = 0,\ x = 0$

$x - 3 = 0,\ x = 3$

$(x = 0)$ and $(x = 3)$

$(0,0)$ and $(3,0)$

if $y = ax^3 + bx^2 + cx + d$

① Determine one root of function first (note: sometimes the question is scaffolded to help you)

Try $x = 1$... if $y = 0$ when $x = 1$ then $(x-1)$ is a factor

$\therefore y = (x-1)(ax^2 + bx + c)$

② 'By inspection' determine a and c of quadratic factor

③ Expand

④ Group coefficients of either '$x$' (or '$x^2$') and solve for c (or b). Use 'c' value from original expanded cubic

⑤ Now factorise quadratic factor of $(x-1)(ax^2 + bx + c)$ + give final answer

FACTORISING CUBICS cont.

**EXAMPLE:**

factorise $y = x^3 - 6x^2 - x + 6$

try $x = 1$

$y = (1)^3 - 6(1)^2 - (1) + 6$

$= 1 - 6 - 1 + 6$

$= 0$ $\therefore$ one root $= (1, 0)$

*opposite sign

$\therefore y = (x-1)(ax^2 + bx + c)$

$a = 1,\ c = -6$

$y = (x-1)(x^2 + bx - 6)$

$= x^3 + bx^2 - 6x - x^2 - bx + 6$

$-6x - bx = -x$ or another way: $b + (-1) = -6$ | $-6 - b = -1$

what does b have to be? $b = -6 + 1$ $b = -5$ | $-b = -1 + 6$ $\therefore b = -5$

$y = (x-1)(x^2 - 5x - 6)$

$= (x-1)(x-6)(x+1)$

**EXAMPLE 2:** (qu 6,

$f(x) = x^3 - 10x^2 + 31x - 30$

try $x = 1$ a.ka $f(1)$

$y = (1)^3 - 10(1)^2 + 31(1) - 30$

$= 1 - 10 + 31 - 30$

$= -8$ $\therefore x = 1$ is not an x-axis intercept

try $f(2)$

$y = (2)^3 - 10(2)^2 + 31(2) - 30$

$= 8 - 40 + 62 - 30$

$= 0$ $\therefore x = 2$ is an x-axis intercept

opposite sign

$\therefore f(x) = (x-2)(ax^2 + bx + c)$

inspection: a must $= 1$ so it $= x^3$ what does c have to be to make $-30$ (from the top bit)

$a = 1$

$c = 15$ $\therefore f(x) = (x-2)(x^2 + bx + 15)$ → now just b is left

expand: $x^3 + bx^3 + 15x - 2x^2 - 2bx - 30$

$b - 2 = -10$ ← use the coefficients + original coefficient

$b = -8$

* check using x coefficients: $15 - 2b = 31$, $-2b = 31 - 5$, $b = \frac{16}{-2}$, $b = -8$ $\therefore$ correct

$\therefore f(x) = (x-2)(x^2 - 8x + 15)$ x-ints: $(2,0)$ $(3,0)$ $(5,0)$

$= (x-2)(x-3)(x-5)$

REVIEW

① State the Domain + Range

a) $y = -x^2 + 4$

Domain: $\{x \in \mathbb{R}\}$ ✓ Range: $\{y \in \mathbb{R}, y \leq 4\}$ ✓ it is a max t.p, moved 4 units up

b) $y = \frac{1}{x} - 4$

Domain: $\{x \in \mathbb{R}, x \neq 0\}$ ✓ has an asymptote Range: $\{y \in \mathbb{R}, y \neq -4\}$ since $x$ will never be 0 (the fraction will never = 0)

c) $f(x) = 3 - \sqrt{x-2}$

Domain: $\{x \in \mathbb{R}, x \geq 2\}$ ✓ Range: $\{y \in \mathbb{R}, y \leq 3\}$ is negative so its opposite sign

② Consider the functions:

$f(x) = 3x - 5$ $g(x) = 3x^2 - 2x + 1$ $h(x) = 2x + 4$

a) determine $f(6)$

$f(6) = 3(6) - 5$, $f(6) = 18 - 5$ $f(6) = 13$ ✓

b) determine $k$ if $f(k) = -11$

$-11 = 3k - 5$, $-11 + 5 = 3k$, $-6 = 3k$, $\frac{-6}{3} = k$, $k = -2$ ✓

c) state the value of $m$ if $h(m) = g(m)$

$2m + 4 = 3m^2 - 2m + 1$, $2m + 2m + 3 = 3m^2$ $0 = 3m^2 - 4m - 3$ ← need quadratic formula

$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-3)}}{2(3)}$ $= \frac{4 \pm \sqrt{52}}{6}$

d) determine $h(2a+b)$ in terms of $a$ and $b$

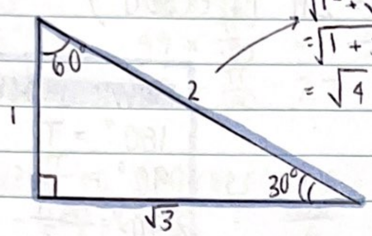
$= 2(2a + b) + 4$ , $= 4a + 2b + 4$

# EXACT VALUES - TRIGONOMETRY

classpad = on standard, not decimal

e.g. decimal of sin(60) $= 0.8660254038\ldots$

standard of sin(60) $= \frac{\sqrt{3}}{2}$




$\sqrt{1^2 + \sqrt{3}^2}$
$= \sqrt{1+3}$
$= \sqrt{4} = 2$

find $\sin(30) = \frac{O}{H} = \frac{1}{2}$

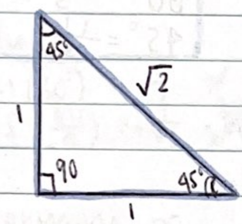
find $\tan(30) = \frac{O}{A} = \frac{1}{\sqrt{3}}$

find $\sin(60) = \frac{\sqrt{3}}{2}$

find $\tan(60) = \frac{\sqrt{3}}{1}$

find $\cos(30) = \frac{A}{H} = \frac{\sqrt{3}}{2}$

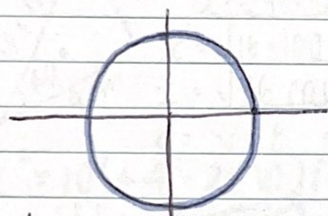
find $\cos(60) = \frac{\sqrt{3}}{2}$




find $\sin(45) = \frac{O}{H} = \frac{1}{\sqrt{2}}$

find $\cos(45) = \frac{A}{H} = \frac{1}{\sqrt{2}}$

find $\tan(45) = \frac{O}{A} = \frac{1}{1} = 1$



(x)
$\cos(0) = 1$
$\cos(90) = 0$
$\cos(180) = -1$

(y)
$\sin(0) = 0$
$\sin(90) = 1$
$\sin(180) = 0$

$\tan(0) = \frac{\sin(0)}{\cos(0)} = \frac{0}{1} = 0$

$\tan(90) = \frac{\sin(90)}{\cos(90)} = \frac{1}{0} =$ DNE*

*DNE = does not exist

## QUADRANTS

| | |
|---|---|
| S<br>sin | A<br>all |
| T<br>tan | C<br>cos |

$\sin(150) = \sin(30)$, $\cos(150) = -\cos(30)$,

$\tan(150) = -\tan(30) = \frac{-1}{\sqrt{3}}$

what values will be positive in the quadrants?

RADIANS

The number of radians in half a circle = $\pi$ (i.e $180^\circ$)

The number of radians in a full circle = $2\pi$ (i.e $360^\circ$)

Converting degrees $\rightarrow$ radians

$$\boxed{\text{radians} = \text{degrees} \times \frac{\pi}{180}}$$

Converting radians $\rightarrow$ degrees

$$\boxed{\text{degrees} = \text{radians} \times \frac{180}{\pi}}$$

Some standards

$180^\circ = \pi$

$90^\circ = \frac{\pi}{2}$

$270^\circ = \frac{3\pi}{2}$

$360^\circ = 2\pi$

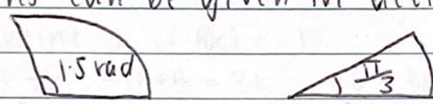
$30^\circ = \frac{\pi}{6}$

$60^\circ = \frac{\pi}{3}$

$45^\circ = \frac{\pi}{4}$

Radians can be given in decimal or exact form:

e.g



## SOLVING TRIG. FUNCTIONS

$\sin\theta = \frac{1}{2}$

$\theta = 30°, 150°, 390°, 510°, 750°, 870°$ etc. etc.

in general terms.
$\theta = 360° n + 30°$ or $\theta = 360° n + 150°$
where $n$ is any real number (i.e. $n \in \mathbb{R}$)

So, when solving functions, the domain **must** be set and it will always be provided (unless question is in a context)
Also, you have to give all answers that are within the given domain

### Further examples

① Solve $\sin\theta = -\frac{1}{2}$ given $0 \leq \theta \leq 360°$

$\theta = -30°$ and $\theta = -150°$
$\theta = 210°$ or $\theta = 330°$

② $\tan(x) = 1$ given $0 \leq x \leq 360°$

$\theta = 45°, 225°$

③ $\cos\theta = -\frac{1}{\sqrt{2}}$ given $0 \leq \theta \leq 2\pi$

$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

**Always check the domain!!**

④ $\cos(x) = \frac{1}{\sqrt{2}}$ given $-\pi \leq x \leq \pi$

$x = \frac{\pi}{4}, -\frac{\pi}{4}$ or $\pm\frac{\pi}{4}$

⑤ $\sin\theta = -\frac{\sqrt{3}}{2}$ given $-360° \leq \theta \leq 360°$

$\theta = (180+60), (360-60), -60°, -120°$
$\theta = 240°, 300°, -60°, -120°$ $\therefore \theta = -120°, -60°, 240°, 300°$

⑥ $\cos(2x) = 0.5$ given $0 \leq x \leq 2\pi$

$2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$
$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

⑦ $3\tan x + \sqrt{3} = 0$, $0 \leq x \leq 360°$

$3\tan x = -\sqrt{3}$
$\tan x = \frac{-\sqrt{3}}{3} = \frac{-\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$x = 150, 330$

# DIFFERENTIATING FURTHER POWERS OF $x$ - CALCULUS

① $y = \frac{1}{x}$ ⟵ reciprocal

$y = x^{-1}$

$\frac{dy}{dx} = -1x^{-1-1}$

$= -x^{-2}$

$= \frac{-1}{x^2}$

② $y = \sqrt{x}$ ⟵ root function

$y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$

$= \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2x^{1/2}}$ or $\frac{1}{2\sqrt{x}}$

③ $y = \frac{-8}{x^2}$

$= -8x^{-2}$

$\frac{dy}{dx} = 16x^{-3}$

$= \frac{16}{x^3}$

④ $y = \frac{1}{\sqrt[3]{x}}$

$= \frac{1}{x^{1/3}}$

$= x^{-\frac{1}{3}}$

$\frac{dy}{dx} = -\frac{1}{3}x^{-\frac{1}{3}-1}$

$= -\frac{1}{3}x^{-\frac{4}{3}}$

$= \frac{-1}{3x^{4/3}}$

⑤ $y = \sqrt[3]{x^2} + 10x$

$= (x^2)^{1/3} + 10x$

$= x^{2/3} + 10x$

$\frac{dy}{dx} = \frac{2}{3}x^{\frac{2}{3}-1} + 10$

$= \frac{2}{3}x^{-\frac{1}{3}} + 10$

$= \frac{2}{3x^{\frac{1}{3}}} + 10$ or

$\frac{2}{3\sqrt[3]{x}} + 10$

⑥ $y = 3x^2 - 2x + 7 - \frac{5}{x^2} + \frac{6}{x}$

$= 3x^2 - 2x + 7 - 5x^{-2} + 6x^{-1}$

$\frac{dy}{dx} = 6x - 2 + 10x^{-3} - 6x^{-2}$

$= 6x - 2 + \frac{10}{x^3} - \frac{6}{x^2}$

⑦ find the equation of the tangent to $y = 12\sqrt{x}$ at the point $(4, 24)$.

$y = 12x^{\frac{1}{2}}$

$= 12\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$

$= \frac{6}{\sqrt{x}}$

$x = 4,\ m = ?$

$\frac{dy}{dx} = \frac{6}{\sqrt{4}}$

$m = 3$

$y = 3x + c$

sub $(4, 24)$

$24 = 3(4) + c$

$12 = c$

$\therefore\ y = 3x + 12$