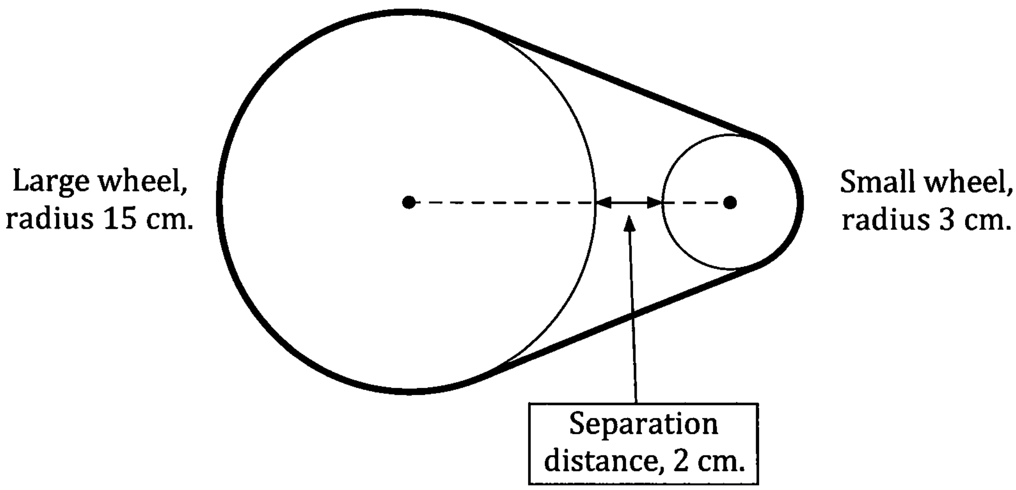
Chapter 2.

Radian measure.

Situation

A new machine is being designed and the design company is building a prototype. Part of the machine involves a continuous belt passing over two wheels, as shown in the diagram below (not drawn to scale).



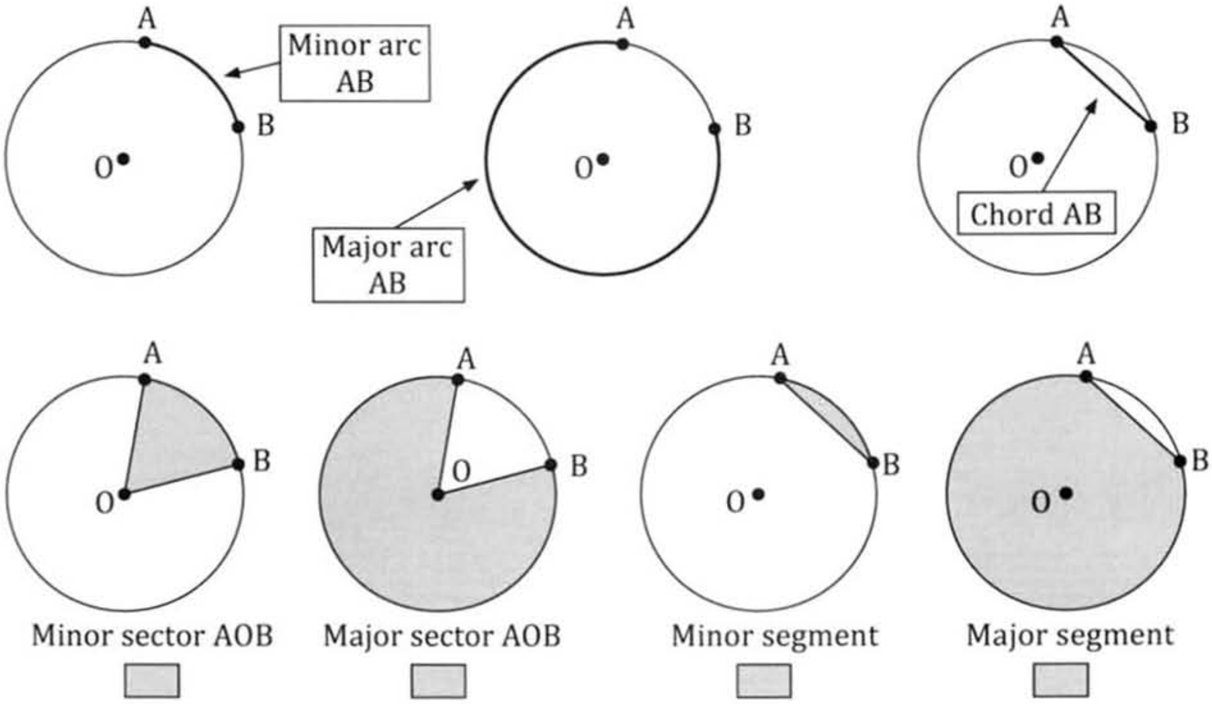
The belt is to be made from a "negligible stretch, high friction compound" and is made circular and then fitted exactly over the wheels. The belt is made by a computer controlled machine that only requires the operator to input the length of the radius of the circle and a circular belt of that radius will be produced.

Your task is to determine what the radius of the circular belt should be, giving your answer in centimetres correct to two decimal places.

Arcs, sectors and segments.

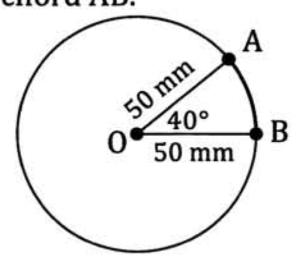
The situation on the previous page required you, amongst other things, to find the length of circular arcs. To determine the arc length you needed to determine the angle the arc subtended at the centre of the circle. Arc lengths, chord lengths, segment and sector areas can all be determined if we know the angle subtended at the centre of the circle and the radius of the circle.

If you have forgotten what terms like arc, segment and sector mean then the following diagrams should refresh your memory.



Example 1

Points A and B lie on the circumference of a circle centre O, radius 50mm and are such that $\angle AOB = 40°$. Find the length of (a) minor arc AB, (b) chord AB.



(a) Circumference of circle $= 2 \times \pi \times 50$ mm

$= 100\pi$ mm

Length of minor arc AB $= \dfrac{40}{360} \times 100\pi$ mm

$\approx 34{\cdot}9$ mm

Minor arc AB is of length 35 mm (to nearest mm.)

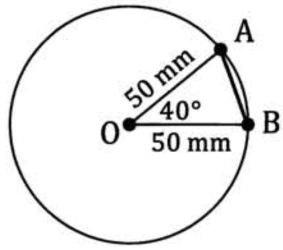
Alternatively we could give the answer in exact form as $\dfrac{100\pi}{9}$ mm.

$\dfrac{40}{360} \cdot 100\pi$

$\dfrac{100\pi}{9}$

$\dfrac{100\pi}{9}$ ▶ Decimal

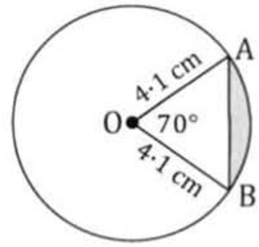
34.90658504



(b) By the cosine rule $AB^2 = 50^2 + 50^2 - 2(50)(50)\cos 40°$

$\therefore \quad AB \approx 34{\cdot}2$ cm

The chord AB is of length 34 mm (to nearest mm.).



Example 2
Find the area of the segment shown shaded in the diagram on the right.

Area of circle $= \pi \times 4{\cdot}1^2 \text{ cm}^2$

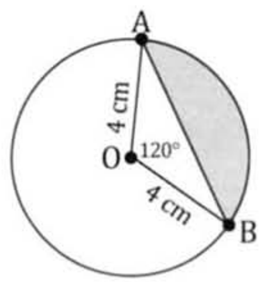
$\therefore$ Area of minor sector AOB $= \dfrac{70}{360} \times \pi \times 4{\cdot}1^2 \text{ cm}^2$

Shaded area $=$ Area of sector AOB $-$ Area of ΔAOB

$$= \frac{70}{360} \times \pi \times 4{\cdot}1^2 - \frac{1}{2} \times 4{\cdot}1 \times 4{\cdot}1 \sin 70°$$

$$\approx 2{\cdot}37 \text{ cm}^2$$

The shaded segment has an area of $2{\cdot}4 \text{ cm}^2$ (correct to one d.p.).



Example 3
Find the area of the segment shown shaded in the diagram on the right giving your answer in exact form.

Area of circle $= \pi \times 4^2$

$= 16\pi \text{ cm}^2$

$\therefore$ Area of minor sector AOB $= \dfrac{120}{360} \times 16\pi \text{ cm}^2$

Shaded area $=$ Area of sector AOB $-$ Area of ΔAOB

$$= \frac{120}{360} \times 16\pi - \frac{1}{2} \times 4 \times 4 \sin 120°$$

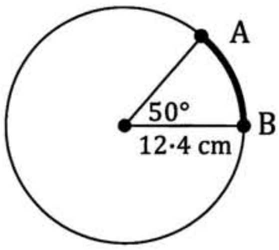
$$= \left(\frac{16\pi}{3} - 4\sqrt{3}\right) \text{ cm}^2$$

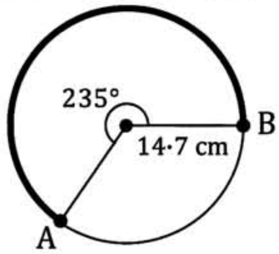
$$\frac{120}{360} \cdot 16 \cdot \pi - \frac{1}{2} \cdot 4 \cdot 4 \sin(120)$$

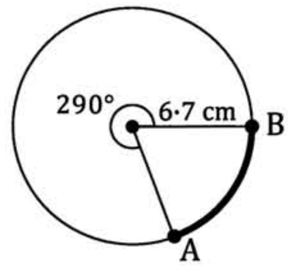
$$\frac{16 \cdot \pi}{3} - 4 \cdot \sqrt{3}$$

Exercise 2A

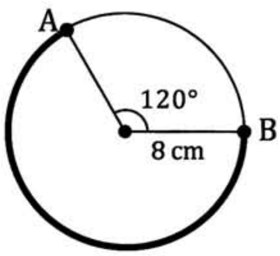
For numbers 1, 2 and 3 calculate the length of the arc AB shown in heavy type, giving each answer in centimetres and correct to one decimal place.

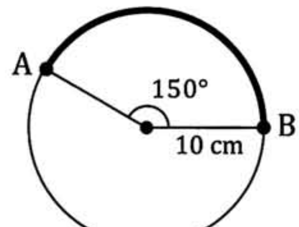
1. 

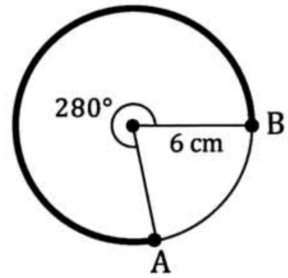
2. 

3. 

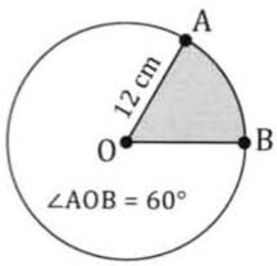
For numbers 4, 5 and 6 calculate the length of the arc AB shown in heavy type, giving each answer as an exact value.

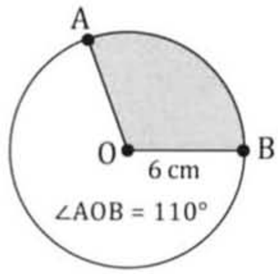
4. 

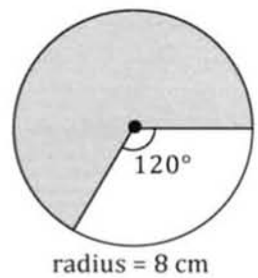
5. 

6. 

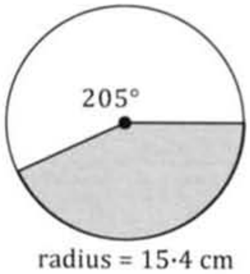
For numbers 7, 8 and 9 calculate the area of the shaded sector, giving each answer as an exact value.

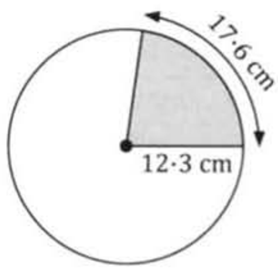
7. 

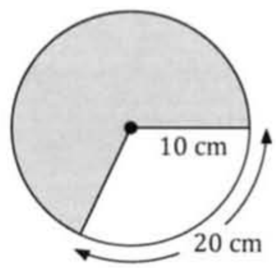
8. 

9. 

For numbers 10, 11 and 12 calculate the area of the shaded sector, giving each answer to the nearest square centimetre.

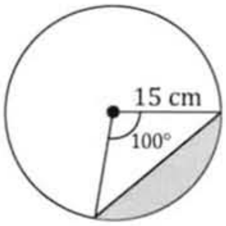
10. 

11. 

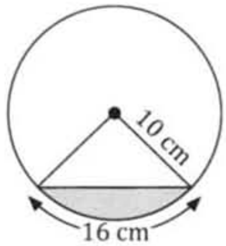
12. 

For numbers 13, 14 and 15 calculate the area of the shaded segment, giving each answer to the nearest square centimetre.

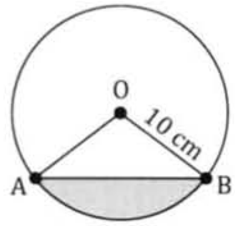
13.



14.



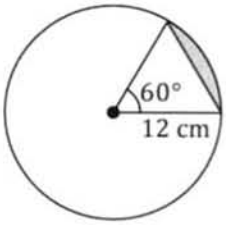
15.



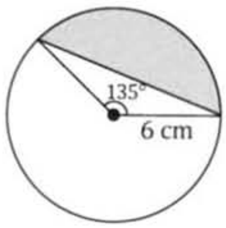
Minor sector OAB has area 90 cm^2.

For numbers 16, 17 and 18 calculate the area of the shaded segment, giving each answer as an exact value.

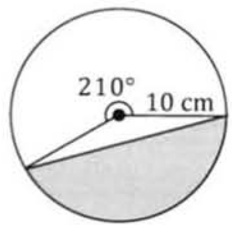
16.



17.



18.



19. A and B are two points on the circumference of a circle centre O and radius 15·2 cm. If $\angle AOB = 112°$ find the length of (a) the minor arc AB,
(b) the major arc AB.

20. A and B are two points on the circumference of a circle centre O. Angle AOB is of size 75° and the minor arc AB is of length 24 cm. Calculate the radius of the circle in centimetres, correct to one decimal place.

21. Points A and B lie on the circumference of a circle centre O, radius 15 cm. Find the area of the minor sector AOB given that $\angle AOB = 50°$.

22. A and B are two points on the circumference of a circle centre O and radius 18 cm. Find the area of the minor segment cut off by the chord AB given that $\angle OAB = 20°$.

23. Find the size of the acute angle AOB, correct to the nearest degree, given that A and B are two points on the circumference of a circle centre O, radius 12 cm, and the major sector AOB has an area of 378 cm^2.

24. Find the area of the minor segment cut off by a chord of length 10 cm drawn in a circle of radius 12 cm, giving your answer in square centimetres correct to one decimal place.

25. A clock has an hour hand of length 8 cm and a minute hand of length 12 cm.
Calculate the distance travelled by the tip of each hand in half an hour.
(Give your answers in exact form.)

26. One nautical mile is defined to be the distance on the surface of the earth that subtends an angle of one minute at the centre of the earth (1 degree = 60 minutes). How many nautical miles are travelled by a ship travelling due North and changing its latitude from 5°N to 8°N?
Assuming the earth to be a sphere of radius 6350 km express one nautical mile in kilometres, correct to two decimal places.

27. A minor sector is removed from a circular piece of card (see figure 1). By joining OA to OB the remaining major sector forms a conical hat (see figure 2). Find h and r, the height and base radius of the hat.

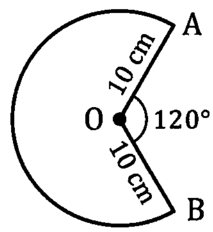


Figure 1

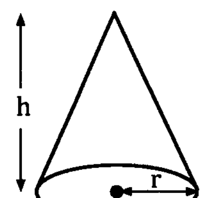
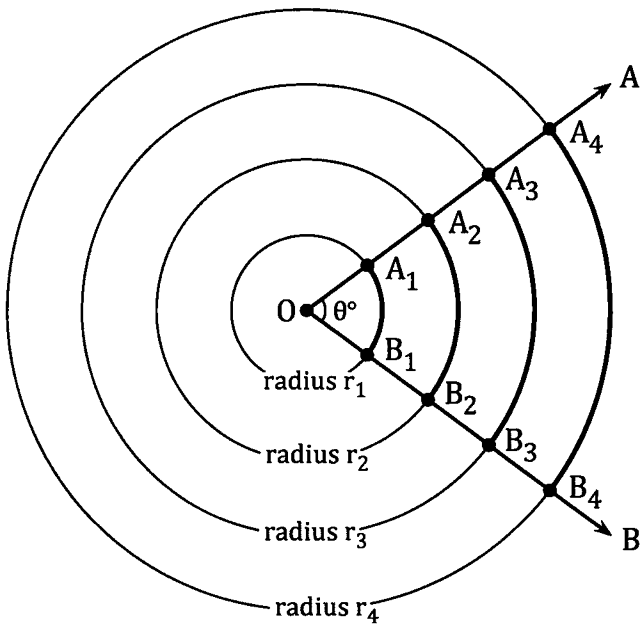


Figure 2

Radians.

Consider the concentric circles shown on the right. Lines OA and OB are drawn from the common centre O (see diagram). The minor arcs A_1B_1, A_2B_2, A_3B_3 and A_4B_4 each subtend an angle θ° at O and will be of increasing length.



$$A_1B_1 = \frac{\theta}{360} \times 2\pi r_1$$

$$A_2B_2 = \frac{\theta}{360} \times 2\pi r_2$$

$$A_3B_3 = \frac{\theta}{360} \times 2\pi r_3$$

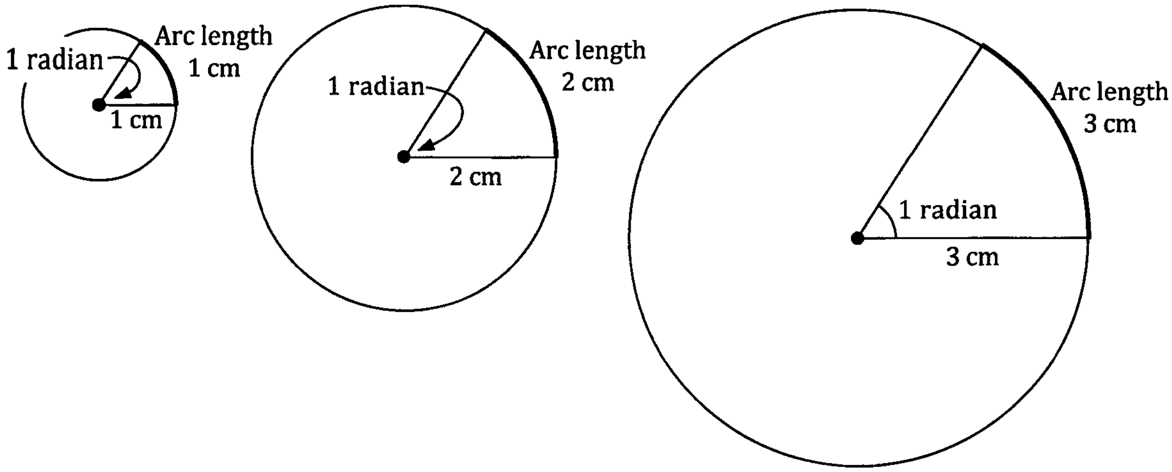
$$A_4B_4 = \frac{\theta}{360} \times 2\pi r_4$$

Note that the ratio of arc length, A_nB_n, to radius, r_n, is constant:

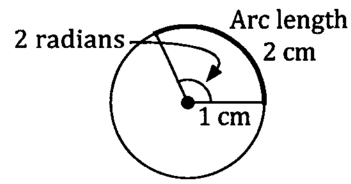
$$\frac{A_1B_1}{r_1} = \frac{A_2B_2}{r_2} = \frac{A_3B_3}{r_3} = \frac{A_4B_4}{r_4} = \frac{\theta}{360} \times 2\pi$$

This ratio of arc length to radius can be used as a measure of angle and gives us an alternative unit for measuring angles.

This unit is called a **radian** and proves to be a very useful measure of angle for advanced mathematics. If the ratio of arc length to radius is equal to 1 the angle subtended at the centre is 1 radian.



An arc of length 1 unit, in a circle of unit radius, subtends an angle of 1 radian at the centre of the circle.



An arc of length 2 units, in a circle of unit radius, subtends an angle of 2 radians at the centre of the circle, and so on.

Radians ⟷ degrees.

An arc of length 1 unit, in a circle of unit radius, subtends an angle of 1 radian at the centre of the circle. Thus an arc of length 2π units, in a circle of unit radius, will subtend an angle of 2π radians at the centre of the circle. However, if the radius is 1 unit an arc of $2\pi(1)$ is the full circumference of the circle and will subtend an angle of 360° at the centre.

Thus $\quad 2\pi$ radians $= 360°$

i.e. $\quad \boxed{\pi \text{ radians} = 180°}$

Thus, correct to one decimal place, 1 radian is equivalent to 57·3°.

Example 4

Convert (a) 60° (b) 90° and (c) 125° to radians, leaving your answers in terms of π.

(a) $180° = \pi$ radians

$\therefore\ 1° = \dfrac{\pi}{180}$ radians

$\therefore\ 60° = \dfrac{\pi}{180} \times 60$

$= \dfrac{\pi}{3}$ radians

(b) $180° = \pi$ radians

$\therefore\ 1° = \dfrac{\pi}{180}$ radians

$\therefore\ 90° = \dfrac{\pi}{180} \times 90$

$= \dfrac{\pi}{2}$ radians

(c) $180° = \pi$ radians

$\therefore\ 1° = \dfrac{\pi}{180}$ radians

$\therefore\ 125° = \dfrac{\pi}{180} \times 125$

$= \dfrac{25\pi}{36}$ radians

Note
- To convert degrees ⟷ radians we use the exact result π radians = 180° rather than the approximation 1 radian ≈ 57·3°.
- When an angle is given in radians the word radian is optional. The answer to example 4(a) could be given as $\frac{\pi}{3}$ radians or simply as $\frac{\pi}{3}$.

  When an angle is given with no units stated then the angle should be assumed to be in radians.
- Knowing conversions such as $\frac{\pi}{3}$ radians = 60° it follows that $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

Example 5

Convert (a) $\frac{\pi}{8}$ radians (b) 2·3 radians to degrees, correct to the nearest degree if rounding is necessary.

(a)
$$\begin{aligned} \pi \text{ radians} &= 180° \\ \therefore \quad \frac{\pi}{8} \text{ radians} &= \frac{180}{8} \text{ degrees} \\ &= 22{\cdot}5° \end{aligned}$$

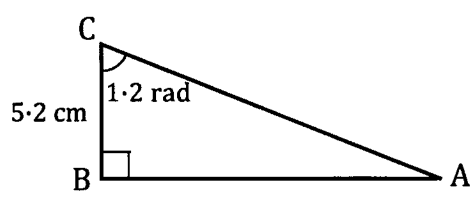
(b)
$$\begin{aligned} \pi \text{ radians} &= 180° \\ \therefore \quad 1 \text{ radian} &= \frac{180}{\pi} \text{ degrees} \\ \therefore \quad 2{\cdot}3 \text{ radians} &= \frac{180}{\pi} \times 2{\cdot}3 \\ &= 132° \text{ to nearest degree} \end{aligned}$$

☞ Explore the capability of your calculator to change between the various units for measuring angle.

The next example shows that the trigonometrical ratios can still be applied with angles given in radians. We do not need to change the angles to degrees but instead set our calculator to read angles as radians.

Example 6

Find the length of side AB as shown in the diagram on the right.



Let AB be of length x cm.

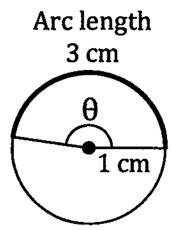
$$\begin{aligned} \tan 1{\cdot}2 &= \frac{x}{5{\cdot}2} \\ \therefore \quad x &= 5{\cdot}2 \tan 1{\cdot}2 \\ &\approx 13{\cdot}38 \end{aligned}$$

The side AB is of length 13·4 cm, correct to one decimal place.

Exercise 2B

For numbers 1 to 6 state the size of angle θ in radians.

1.



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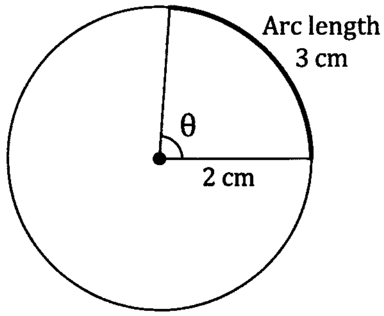

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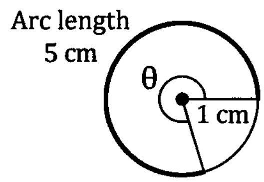

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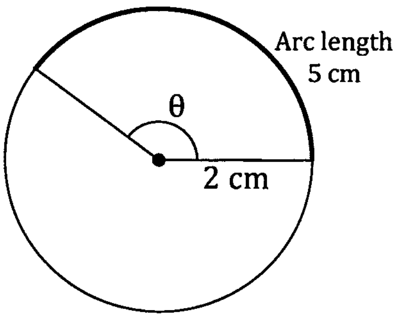

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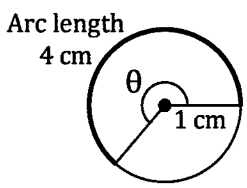

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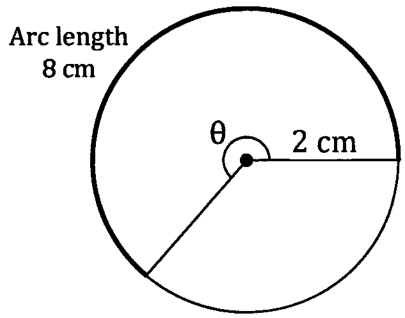


Express the following in radians as exact values in terms of π.

7. 90°	8. 30°	9. 150°	10. 135°
11. 5°	12. 18°	13. 80°	14. 130°

Express the following as degrees.

15. $\frac{\pi}{4}$ rads	16. $\frac{\pi}{3}$ rads	17. $\frac{2\pi}{3}$ rads	18. π rads
19. $\frac{\pi}{12}$ rads	20. $\frac{\pi}{5}$ rads	21. $\frac{7\pi}{36}$ rads	22. $\frac{7\pi}{18}$ rads

Express the following as radians, correct to two decimal places.

23. 32°	24. 63°	25. 115°	26. 170°
27. 16°	28. 84°	29. 104°	30. 26°

Change the following to degrees giving answers correct to the nearest degree.

31. 1·5 rads 32. 2·3 rads 33. 1·4 rads 34. 0·6 rads

Without using a calculator state the exact values of the following.

35. $\sin\frac{\pi}{4}$ 36. $\sin\frac{5\pi}{6}$ 37. $\cos\frac{3\pi}{4}$ 38. $\sin\frac{\pi}{2}$

39. $\sin\frac{2\pi}{3}$ 40. $\sin\frac{3\pi}{4}$ 41. $\cos\frac{\pi}{4}$ 42. $\tan\frac{2\pi}{3}$

43. $\cos\frac{\pi}{2}$ 44. $\tan\frac{\pi}{2}$ 45. $\cos\frac{2\pi}{3}$ 46. $\tan\frac{5\pi}{6}$

47. $\cos\frac{5\pi}{6}$ 48. $\tan\pi$ 49. $\cos\frac{\pi}{3}$ 50. $\sin\pi$

Use your calculator to determine the following correct to two decimal places.

51. sin 1 52. cos 2 53. tan 2·5 54. sin 3

55. cos 0·6 56. cos 0·15 57. tan 1·3 58. sin 2·3

Find the acute angle θ in each of the following giving your answers in radians correct to two decimal places.

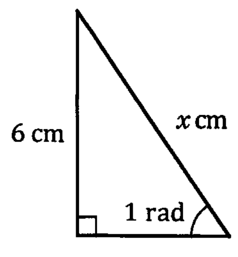
59. $\sin\theta = 0{\cdot}2$ 60. $\cos\theta = 0{\cdot}2$ 61. $\tan\theta = 0{\cdot}35$ 62. $\tan\theta = 1{\cdot}7$

63. Convert the following angular speeds to radians/second.
(a) 3 revolutions/second, (b) 15 revolutions/minute, (c) 90 degrees/second.

64. Convert the following angular speeds to revolutions/minute.
(a) 2π radians/minute, (b) $\frac{3\pi}{4}$ radians/second, (c) $\frac{\pi}{3}$ radians/second.

Find the value of x in each of the following, giving your answers correct to one decimal place.

65.



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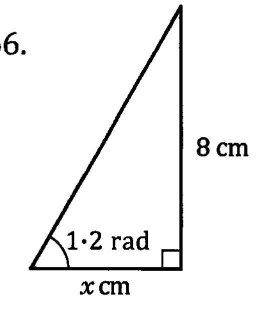

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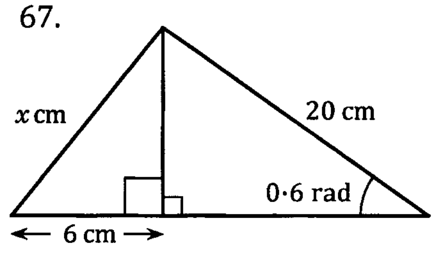

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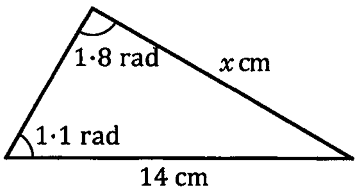

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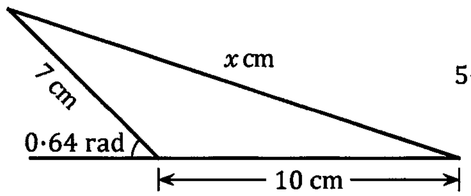
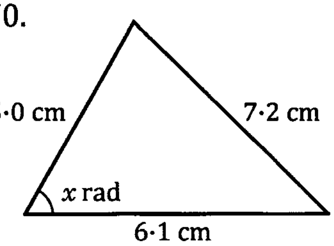


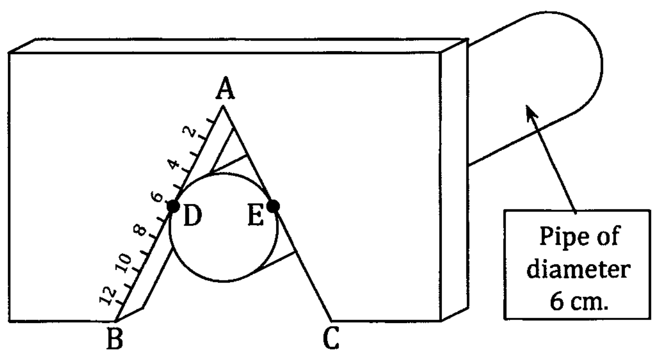
70.



71. Through what angle, in radians, does the minute hand of a clock rotate in
(a) 15 minutes, (b) 40 minutes, (c) 50 minutes, (d) 55 minutes?

72. A grad is another unit that can be used to measure angles. One right angle = 100 grads. Convert the following to radians.
(a) 50 grads, (b) 75 grads, (c) 10 grads, (d) 130 grads?

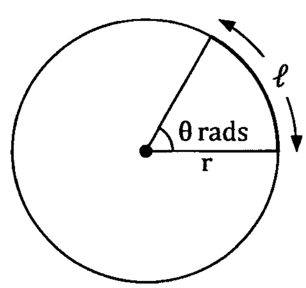
73. A simple gauge is to be made for measuring the diameter of pipes. The gauge will be in the form of a rectangular piece of wood from which a V-shape has been cut. The V is then placed on the pipe and the point of contact, D, (see diagram) allows the diameter to be read directly from the graduations on AB.



The V shape is cut such that $\angle BAC = 1$ radian and $AB = AC = 12$ cm.
(a) Draw a line 12 cm long to represent BA. Calibrate it so that diameters from 1 cm to 12 cm could be read **directly** from the point of contact.
(b) Would calibration have been simpler if $\angle BAC = \frac{\pi}{2}$ radians? Explain your answer.

Arcs, sectors and segments revisited.

Let us now consider again arc lengths, sector areas and segment areas but this time let the angle subtended at the centre of the circle be in radians.



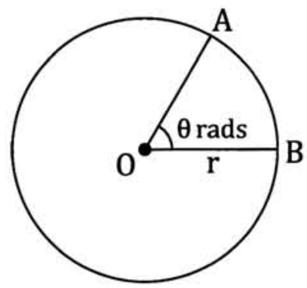
The central angle, in radians, is given by $\dfrac{\text{arc length}}{\text{radius}}$.

Thus $\theta = \dfrac{\ell}{r}$ with θ, and r as defined in the diagram on the right.

Thus $\ell = r\theta$.

$$\boxed{\text{Arc length} = r\theta}$$

Remembering that 1 revolution is 2π radians it follows that, with θ as shown in the diagram on the right,



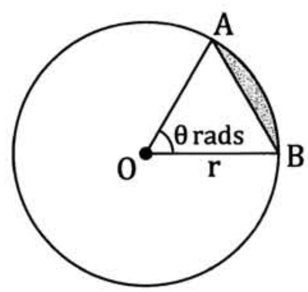
$$\frac{\theta}{2\pi} = \frac{\text{Area of sector AOB}}{\pi r^2}$$

$$\therefore \quad \text{Sector area} = \frac{\theta}{2\pi} \times \pi r^2$$

$$= \frac{1}{2} r^2\theta$$

Thus $\boxed{\text{Sector area} = \frac{1}{2}r^2\theta}$

From the diagram on the right we see that



$$\text{Shaded area} = \text{Area of sector AOB} - \text{Area of } \Delta\text{AOB}$$

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

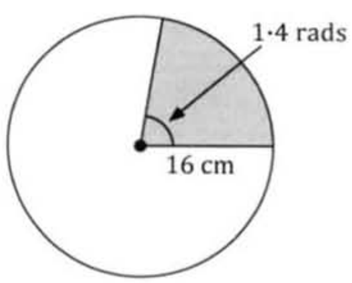
$$= \frac{1}{2}r^2(\theta - \sin\theta)$$

Thus $\boxed{\text{Segment area} = \frac{1}{2}r^2(\theta - \sin\theta)}$

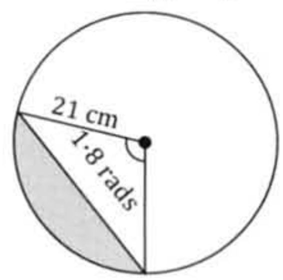
Example 7

Calculate the area of the shaded region in each of the following diagrams.

(a)



(b)



(a) Sector area $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 16^2 \times 1{\cdot}4$

$= 179{\cdot}2\text{ cm}^2$

The shaded region has area 179 cm^2. (To the nearest cm^2.)

(b) Area $= \frac{1}{2}r^2(\theta - \sin\theta)$

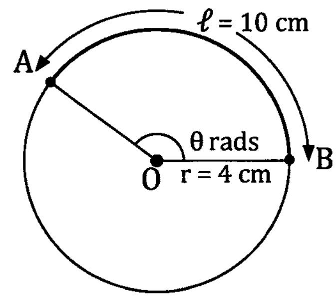
$= \frac{1}{2} \times 21^2 \times (1{\cdot}8 - \sin 1{\cdot}8)$

$\approx 182{\cdot}2\text{ cm}^2$

The shaded region has area 182 cm^2. (To the nearest cm^2.)

Example 8

Points A and B are points on the circumference of a circle, centre O and radius 4 cm. If the minor arc AB is of length 10 cm find the area of the minor sector AOB.



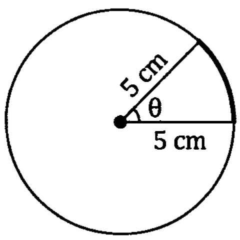
First draw a diagram:

$$\begin{aligned}
\text{Arc length, } \ell, &= r\theta \\
\text{Thus} \quad 10 &= 4\,\theta \\
\text{and so} \quad \theta &= 2{\cdot}5 \\
\therefore \quad \text{Sector area} &= \frac{1}{2}(4)^2\,(2{\cdot}5) \\
&= 20\ \text{cm}^2
\end{aligned}$$

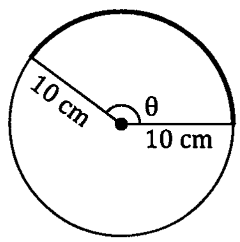
The minor sector AOB has an area of 20 cm^2.

Exercise 2C

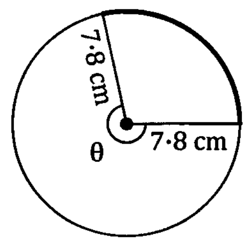
Find the lengths of the arcs shown by heavy type in the following diagrams.

1.


θ = 0·8 radians

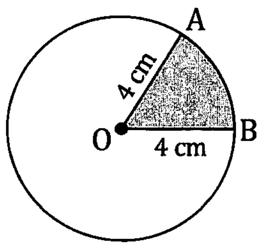
2.


θ = 2·5 radians

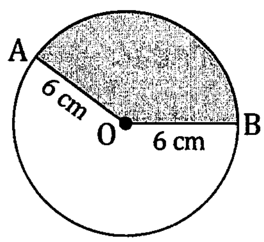
3.


θ = 4·5 radians

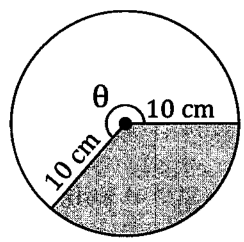
Find the areas of the sectors shown shaded in the following diagrams.

4.


∠AOB = 1 radian

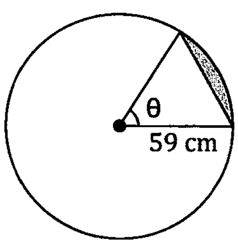
5.


∠AOB = 2·5 radians

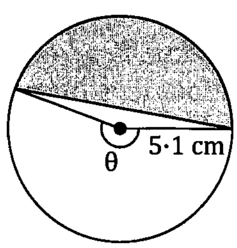
6.


θ = 4 radians

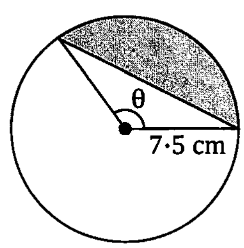
Find the areas of the segments shown shaded in the following diagrams.

7.


θ = 1 radian

8.


θ = 3·5 radians

9.


θ = 2·2 radians

10. Find the length of the arc AB given that it subtends an angle of $1{\cdot}2$ radians at the centre of the circle of which it forms a part and the radius of the circle is 15 cm.

11. Points A and B lie on the circumference of a circle, centre O, radius 15 cm. If the minor arc AB subtends an angle of $0{\cdot}8$ radians at O find the area of
    (a) the minor sector OAB,
    (b) the major sector OAB (to the nearest square centimetre).

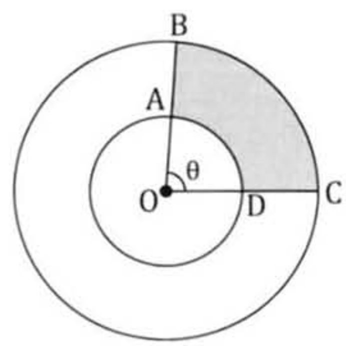
12. A and B are two points on a circle centre O and radius 8 cm. If arc AB subtends an angle of 1 radian at O find
    (a) the length of the minor arc AB,
    (b) the area of that part of the minor sector OAB not lying in triangle OAB. (Give your answer in square centimetres correct to one decimal place.)

13. A and B lie on the circumference of a circle centre O and radius 5 cm. The minor sector OAB has an area of 15 cm^2.
    (a) Calculate the length of the minor arc AB,
    (b) Calculate the area of the minor segment cut off by the chord AB. (Give your answer in square centimetres correct to two decimal places).

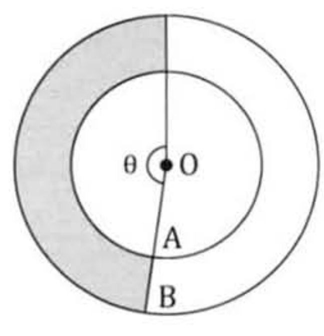
14. Points A and B lie on the circumference of a circle, centre at point O and with radius 8 cm.
    If the minor arc AB is of length 20 cm find the area of the minor sector OAB.

15. Points A and B lie on the circumference of a circle, centre O and of radius 6 cm. If the minor sector OAB has an area of 9 cm^2 find the area of the minor segment cut off by the chord AB. (Giving your answer in square centimetres correct to two decimal places.)

16. Find the area of the shaded region shown sketched on the right given that O is the centre of both circles, OD = DC = 6 cm and $\theta = 1{\cdot}5$ radians.



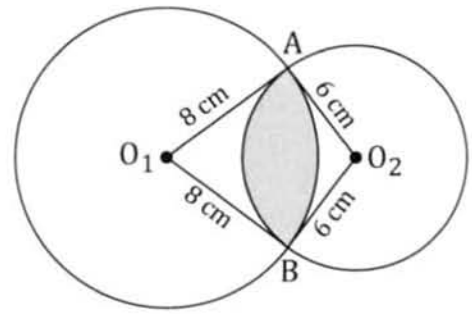
17. Find the area of the shaded region shown sketched on the right given that O is the centre of both circles, OA = 5 cm, AB = 4 cm and $\theta = 3$ radians.



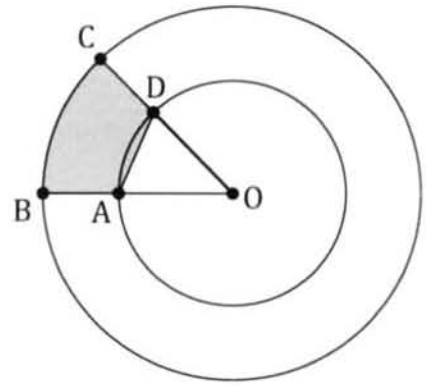
18. The diagram on the right shows two overlapping circles with the region common to both circles shown shaded.

    Find the area of this shaded region given that $\angle O_1AO_2$ and $\angle O_1BO_2$ are right angles.

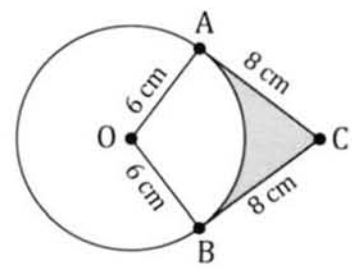
    (Give your answer in square centimetres, and correct to one decimal place.)



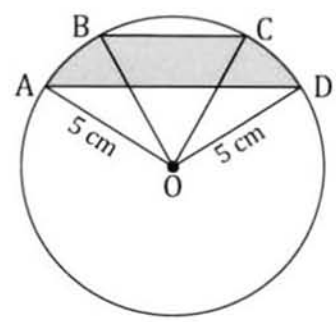
19. The diagram on the right shows two circles with common centre O. The region shown shaded is bounded by the minor arc BC, the chord AD and the lines CD and BA. Calculate the area of this region given that OA = 5 cm, AB = 3 cm and $\angle AOD = 0{\cdot}8$ radians.
    (Give your answer in square centimetres, and correct to one decimal place.)



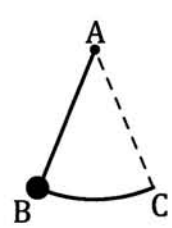
20. The diagram on the right shows the two tangents drawn from the point C to a circle centre O and radius 6 cm, touching the circle at the points A and B. Find the area of the region shown shaded.
    (The angle between a tangent and the radius drawn at the point of contact is a right angle.)
    (Give your answer in square centimetres and correct to one decimal place.)



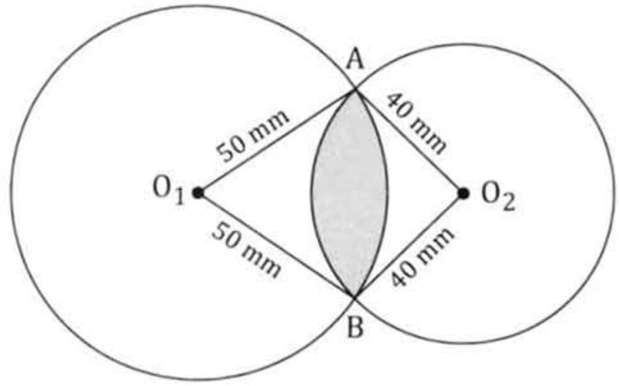
21. Calculate the area of the region shown shaded in the diagram on the right given that $\angle AOB = 0{\cdot}5$ radians, $\angle BOC = 1$ radian, $\angle COD = 0{\cdot}5$ radians and the circle is of radius 5 cm. (Give your answer in square centimetres and correct to two decimal places.)



22. A pendulum AB has end A fixed and a weight attached at B. In one swing the weight travels from B to C and back again (see diagram).
    The pendulum is of length 75 cm and $\angle BAC = 0{\cdot}8$ radians.
    (a) How far does the weight travel in one swing?
    (b) By how much does the length of the arc BC exceed that of the chord BC? (Answer to the nearest millimetre)

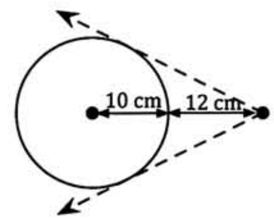


23. Find, to the nearest 10 mm^2, the area of the shaded region shown sketched on the right given that $O_1O_2 = 70$ mm.



24. Two circles of radius 10 cm and 7 cm have their centres 15 cm apart. Find the perimeter of the region common to both circles, giving your answer in centimetres and correct to one decimal place.

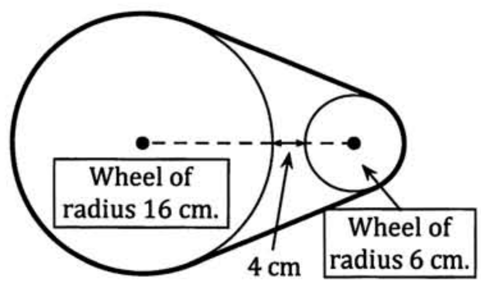
25. What percentage of the circumference of a circular disc of radius 10 cm can be illuminated from a point source of light in the plane of the disc and 12 cm from it (see diagram)? Give your answer to the nearest percent.



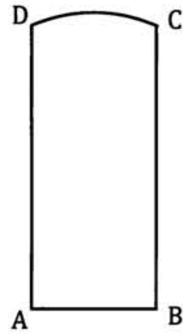
26. A goat is tethered to a post by a rope that is ten metres long. The goat is able to graze over any area that the rope allows it to reach other than that excluded by a straight fence. The perpendicular distance from the post to the fence is 6 m. Over what area can the goat graze (to the nearest square metre)?

27. A goat is tethered to a post by a rope that is twelve metres long. If the post is situated half way between two parallel fences that are ten metres apart. Over what area can the goat graze (to the nearest square metre)?

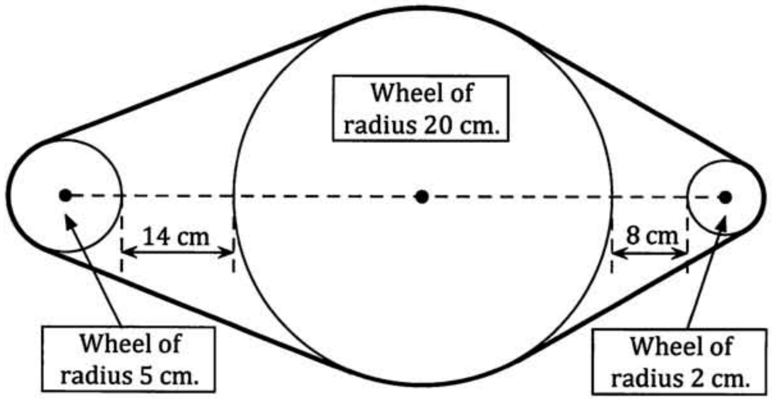
28. Find, to the nearest centimetre, the length of the continuous belt passing around two wheels as shown in the diagram on the right.



29. A door is to be made to the specifications shown in the diagram i.e. a circular segment on top of a rectangle. The top segment is part of a circle having its centre at the intersection of the diagonals of the rectangle ABCD.
If AB = 80 cm and AD = 200 cm find the area of the door correct to the nearest ten square centimetres.



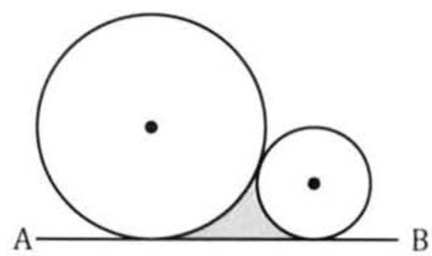
30. Find, to the nearest centimetre, the length of the continuous belt passing around the three wheels as shown in the diagram on the right (not drawn to scale).



31. (a) A minor sector of a circle has perimeter 14 cm and area 10 cm^2.
Find the radius of the circle.

(b) A major sector of a circle has perimeter 14 cm and area 10 cm^2.
Find the radius of the circle.

32. Circles of radius 10 cm and 5 cm touch each other tangentially and both touch the line AB (see diagram). Find the area of the region shown shaded in the diagram.
(Answer to nearest 0·1 cm^2.)



33. Triangle ABC has AB = 7 cm, AC = 6 cm and BC = 5 cm. Three circles are drawn, one with centre A and radius 4 cm, one with centre B and radius 3 cm and one with centre C and radius 2 cm. What percentage of the area of the triangle fails to lie in any of the circles? (Answer to nearest 0·1%.)

Miscellaneous Exercise Two.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary work section at the beginning of the book.

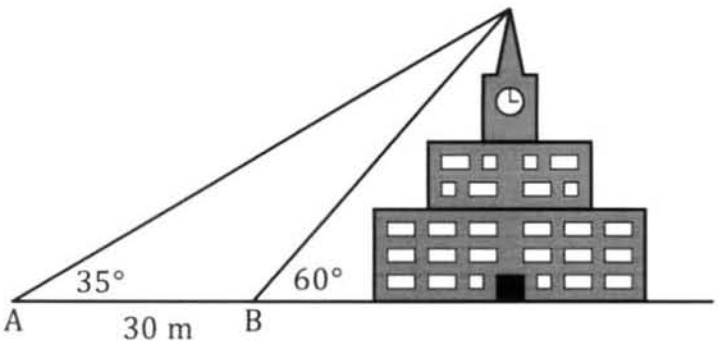
1. Without the assistance of a calculator, expand each of the following.
   (a) $(x+3)(2x-1)$ (b) $(x+7)(3x-4)$
   (c) $(x+5)(x-1)(x+3)$ (d) $(2x+1)(x-3)(x-2)$

2. Simplify each of the following by expressing them as equivalent statements without any square roots in the denominators (i.e. rationalize the denominators).
   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{5}{\sqrt{2}}$ (d) $\frac{6}{\sqrt{3}}$

   Hint for (e) to (h): To simplify $\frac{a}{b+\sqrt{c}}$ multiply by 1, written in the form $\frac{b-\sqrt{c}}{b-\sqrt{c}}$.

   (e) $\frac{1}{3+\sqrt{5}}$ (f) $\frac{1}{3-\sqrt{2}}$ (g) $\frac{2}{1+\sqrt{5}}$ (h) $\frac{3}{\sqrt{5}+\sqrt{2}}$

3. From a point A, level with the base of the town hall, the angle of elevation of the topmost point of the building is 35°.
   From point B, also at ground level but 30 metres closer to the hall, the same point has an angle of elevation of 60°.
   Find how high the topmost point is above ground level. (Give your answer correct to the nearest metre.)



4. A playground roundabout of radius 1·8 m makes one revolution every five seconds. Find, to the nearest centimetre, the distance travelled by a point on the roundabout in one second if the point is
   (a) 1·8 m from the centre of rotation,
   (b) 1 m from the centre of rotation.

5. From a lighthouse, ship A is 17·2 km away on a bearing S60°E and ship B is 14·1 km away on a bearing N80°W.
   How far, and on what bearing, is B from A?

6. The diagram on the right shows the sketch made by a surveyor after taking measurements for a block of land ABCD.
   Find the area and the perimeter of the block.