KINGSWAY CHRISTIAN COLLEGE

MATHS DEPARTMENT5

Course: Mathematics Methods Year 12

Assessment Task: Test 5 – Discreet Random Variables and The Binomial Distribution

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Date: 10th August 2017

Assessment Score: ____________ / 45

Year Score: ____________

Comments: ____________

Teacher signature: ____________

Parent/ Guardian signature: ____________

Comments: ____________

METHODS YEAR 12 **Test 5 2017**
Discreet Random variables and Distributions

Calculator Allowed **Time: 45 mins** **Marks: / 45**

Calculators are allowed for this test, but no notes. Please show work out where needed.

Question 1 **(3,4,3 = 10 marks)**

The discrete random variable X can only take the values 0, 1, 2, 3, 4, 5. The probability distribution of X is given by the following

$$P(X=0)=P(X=1)=P(X=2)=a$$
$$P(X=3)=P(X=4)=P(X=5)=b \qquad \text{where } a \text{ and } b \text{ are constants.}$$
$$P(X\geq 2)=3P(X<2)$$

(a) Determine the values of a and b.

$a+3b = 3(2a)$ ✓ and $3a+3b=1$. ✓

3

Solve $\left\{\begin{array}{l} a+3b=6a \\ 3a+3b=1 \end{array} ; a,b\right\} \rightarrow a=\frac{1}{8}$ ✓

$b=\frac{5}{24}$.

(b) Show that the expectation of X is $\frac{23}{8}$ and determine the exact variance of X.

$E(X) = (0\times\frac{1}{8}) + (1\times\frac{1}{8}) + (2\times\frac{1}{8}) + (3\times\frac{5}{24}) + (4\times\frac{5}{24}) + (5\times\frac{5}{24})$

$= \frac{3}{8} + \frac{60}{24}$ ✓ any working.

$= \frac{23}{8}$ ✓

4

$V(X) = (0^2\times\frac{1}{8}) + (1^2\times\frac{1}{8}) + (2^2\times\frac{1}{8}) + (3^2\times\frac{5}{24}) + (4^2\times\frac{5}{24})$

$- \left(\frac{23}{8}\right)^2$ ✓ any working.

$= \frac{533}{192}$. ✓

(c) Determine the exact probability that the sum of two independent observations from this distribution exceeds 7.

all combinations:

5, 5 $\rightarrow \left(\frac{5}{24}\right)^2$

5, 4 or (4, 5) $\rightarrow 2 \times \left(\frac{5}{24}\right)^2$

5, 3 or (3, 5) $\rightarrow 2 \times \left(\frac{5}{24}\right)^2$

4, 4 $\rightarrow \left(\frac{5}{24}\right)^2$

✓ ③

$\therefore P(\text{sum} > 7) = 6 \times \left(\frac{5}{24}\right)^2$ ✓

$= \frac{150}{576} = \frac{25}{96}$ ✓

Question 2 **(3,2,2,3= 10 marks)**

On a long train journey, a statistician is invited by a gambler to play a dice game. The game uses two ordinary dice which the statistician is to throw.

If the total score is 12, the statistician is paid \$6 by the gambler. If the total score is 8, the statistician is paid \$3 by the gambler. However, if both or either dice show a 1, the statistician pays the gambler \$2. Otherwise, no money changes hands.

Let \$ X be the amount paid to the statistician by the gambler.

(a) Complete the table below. ✓✓✓ (−1) per mistake

x	-2	0	3	6
$P(X=x)$	$\frac{11}{36}$	$\frac{19}{36}$	$\frac{5}{36}$	$\frac{1}{36}$.

(b) Explain why the table in part (a) describes a probability distribution for the discrete random variable X.

$\sum p(x) = 1$ and $0 \le p \le 1$. ✓ ✓ 2

(c) Show that, if the statistician played the game 100 times, his expected loss would be \$2.78, to the nearest cent.

$E(x) = (-2 \times \frac{11}{36}) + (0 \times \frac{15}{36}) + (3 \times \frac{5}{36}) + (6 \times \frac{1}{36})$

$= -0{,}02\dot{7}$. ✓ 2

$\therefore$ In 100 games he would lose $100 \times 0{,}02\dot{7}$

$= -2{,}\dot{7}$ which is a loss of

\$2,78. (2 dp). ✓

(d) Find the amount, $\$a$, that the \$6 would have to be changed to in order to make the game unbiased.

For the game to be unbiased: $E(x) = 0$. ✓

$\therefore (-2 \times \frac{11}{36}) + (0 \times \frac{15}{36}) + (3 \times \frac{5}{36}) + (a \times \frac{1}{36}) = 0$. ✓ (working)

$\therefore$ solve$\left(-\frac{22}{36} + \frac{15}{36} + \frac{a}{36} = 0, a\right)$ 3

$\therefore a = 7$ ✓

Question 3 **(3 marks)**

Given that $X \sim B(15, p)$ find the value of p such that $P(X > 13) = 0.4$

Show your working

• realize that it is ≥ 14

$\therefore P(X>13) = P(X \geq 14)$ ✓

3

$= P(X=14) + P(X=15)$

$\therefore 0{,}4 = \binom{15}{14} p^{14}(1-p)^1 + \binom{15}{15} p^{15}(1-p)^0$ • any working ✓

$\therefore$ solve $(0{,}4 = 15p^{14}(1-p) + p^{15}, p) \big|_{0 \leq p \leq 1}$

$\therefore p = 0{,}869698$

$p \approx 0{,}87$. ✓ • correct answer

Question 4 **(2,4 = 6 marks)**

In a school of 480 students, 25% said they barracked for the Dockers.

(a) State why " Supported the Dockers" is a Binomial random variable in this context.

Independent trials. ✓

Success | Failure. ✓ ($\therefore$ Bernuli trials)

25%   75%.

2

(b) Determine μ and σ.

4

$n = 480$

$p = 0{,}25$

$\therefore \mu = np$ ✓

$= 480 \times 0{,}25$

$\mu = 120$ ✓

$\sigma = \sqrt{npq}$ ✓

$= \sqrt{480 \times 0{,}25 \times 0{,}75}$

$= \sqrt{90}$

$\sigma \approx 9{,}49$. ✓

Question 5 **(1,3,1,2 = 7 marks)**

A Study found that 80 per cent of people exhibiting common influenza symptoms recovered without taking any medication. A random sample of 30 people who had developed influenza symptoms was taken.

Let X denote the number of people in this sample who recovered without taking any medication.

(a) State why X is classified as discrete and not continuous?

Discrete as X is an integer ∴ something we "count" and do not "measure"

1

(b) State the probability distribution of X and the mean and standard deviation of this distribution.

$X \sim Bi(n; p)$

$\therefore X \sim Bi(30; 0{,}8)$

$\mu = 30 \times 0{,}8 = 24$

$\sigma = \sqrt{npq}$

$= \sqrt{30 \times 0{,}8 \times 0{,}2}$

$= 2{,}191$

3

(c) What is the probability, correct to three decimal places that

(i) Exactly 25 people recovered without any medication?

$\therefore P(X = 25) = 0{,}172$

1

(ii) At least 24 but no more than 28 recovered without any medication?

$P(24 \le X \le 28) = 0{,}596$

(also accept 0,597).

2

Question 6 **(3,2,2,2 = 9 marks)**

A manufacturer of chocolate produces 3 times as many soft centred chocolates as hard centred ones. The chocolates are randomly packed in boxes of 20.

Let the Discreet Random Variable X = the number of hard centred chocolates per box.

(a) Find the probability that in a box there are

(i) an equal number of soft centred and hard centred chocolates

$X \sim Bi(20; 0{,}25)$

$P(X=10) = 0{,}00992$

(ii) at least one hard centred chocolate.

$P(x \geq 1) = 1 - P(X=0)$

$1 - 0{,}003171.$

$= 0{,}997$

(iii) fewer than 5 hard centred chocolates.

$P(X < 5) = P(X \leq 4)$

$= 0{,}41484$

$\approx 0{,}4148$

(b) A random sample of 5 boxes is taken from the production line. Use your answer from question (iii), to find the probability that exactly 3 of the boxes contain fewer than 5 hard centred chocolates.

Let the Discreet Random Variable Y= the number of boxes that contain fewer than 5 hard centred chocolates.

$Y \sim Bi(5; 0{,}41484)$

$\therefore P(Y=3) = 0{,}24445$

$\approx \underline{0{,}2445}$