



WESLEY COLLEGE

By daring & by doing

## YEAR 12 MATHEMATICS METHODS

Logarithmic functions and  
calculus involving logarithmic functions

### Test 4

Name: \_\_\_\_\_

Marks: /50

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#### Calculator Free (32 marks)

1. [5 marks]

Determine the exact values of  $a$ ,  $b$  and  $c$ .

a)  $3^a = 11$

[1]

b)  $\log_2(2b - 1) = 4$

[2]

c)  $\log_2 4^{2c} = \log_3 27^{c+1}$

[2]

2. [3 marks]

Express  $\log 9 - 3\log \sqrt{3} + \log 81$  in the form  $k\log 3$  where  $k$  is an exact fraction.

3. [7 marks]

Differentiate:

a)  $5\log_e x$

[1]

b)  $\log_e (2x^3 - 5x)$

[1]

c)  $\ln[(2x-1)(4x+3)]$  (no need to simplify your answer)

[2]

d)  $\frac{\ln x}{x^2}$  (simplify your answer)

[3]

4. [4 marks]

Show that  $\int_2^4 (x - 4x^{-1} + 4x^{-3}) dx = \frac{51}{8} - 4 \ln 2$

5. [5 marks]

Show the only solution to  $3^{2x+1} - 11(3^x) - 4 = 0$  is  $x = \log_3 4$ .

6. [8 marks]

a) Given that

$$p = \log_2 x \text{ and } q = \log_2 y$$

find expressions in terms of  $p$  and  $q$  for

i.  $\log_2(x^2 y)$

[2]

ii.  $\log_2\left(\frac{\sqrt{y}}{x^3}\right)$

[2]

b) Hence, or otherwise, solve the simultaneous equations

$$\log_2(x^2 y) = 2$$

$$\log_2\left(\frac{\sqrt{y}}{x^3}\right) = -11$$

**End of Part A**

[4]



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**Calculator Section**

NAME: \_\_\_\_\_

**(18 marks)**

7. [2 marks]

Biologists use the logarithmic model  $n = k \log(A)$  to estimate the number of species,  $n$ , that live in a region of area  $A \text{ km}^2$ . In the model,  $k$  represents a constant.

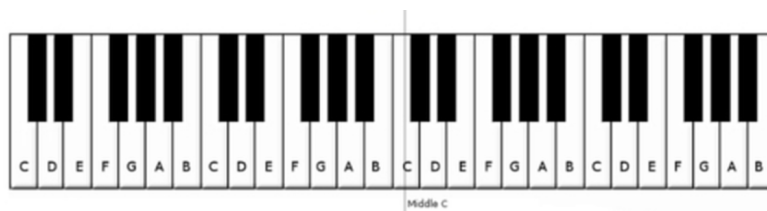
If 2800 species live in a rainforest of 500 square kilometres, then how many species will be left when half of this rainforest is destroyed by logging?

[2]

8. [2 marks]

In the modern scale of musical notes the note names repeat every octave, and each note is double the frequency of the note

of the same name in the octave below. The A note below middle C has a standard frequency of 440 Hertz.



There are actually 12 different notes, including flats and sharps, in an octave. This is called a chromatic scale and the ratio of the frequency of one note to the previous note in the chromatic scale is a constant.

a) What is the ratio of a musical note to the previous note in the chromatic scale?

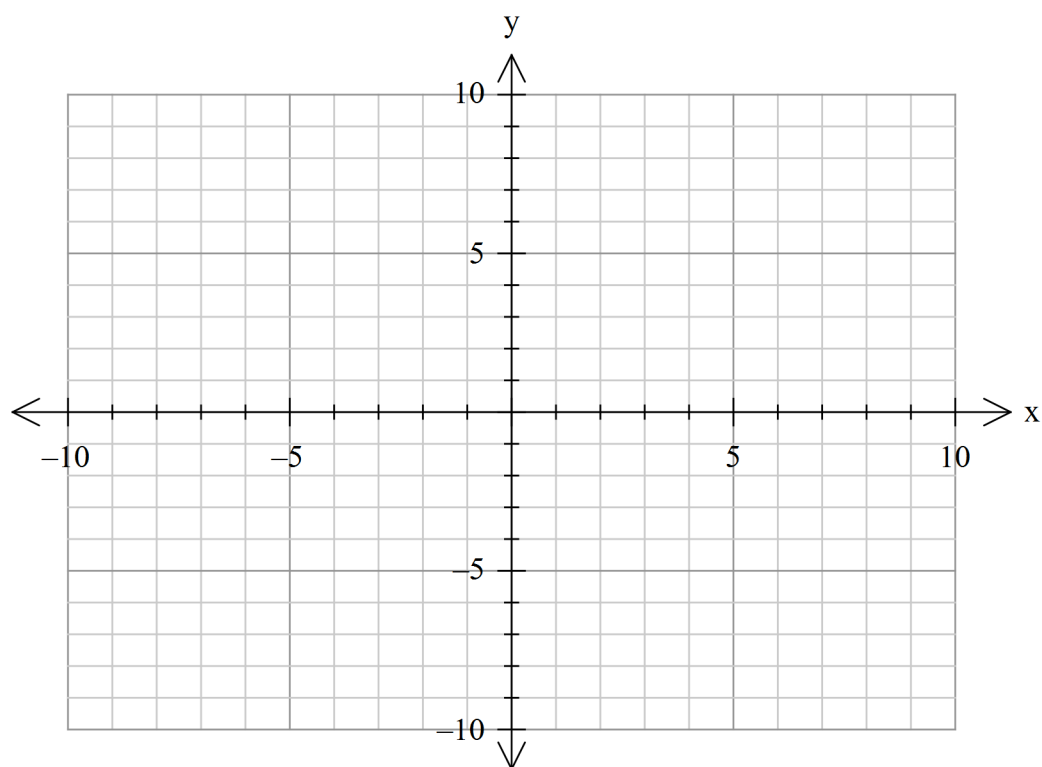
[1]

b) What is the frequency of middle C?

[1]

9. [5 marks]

- a) Sketch the graph of  $y = \log_2(x + 3) + 1$  on the grid below, labelling important features.



- b) Show how you use your graph to solve  $\log_2(x + 3) = 1.6$

[3]

[2]

10. [8 marks]

A small colony of black peppered moths lives on a small isolated island. In summer the population begins to increase. If  $t$  is the number of days after 12 midnight on 1 January, the equation that best models the number of moths in the colony at any given time is

$$N(t) = 500 \ln(21t + 3) \quad 0 \leq t \leq 40$$

a) What is the population of the species on 1 January?

[1]

b) What is the population of moths after 30 days?

[1]

c) On which day is the population first greater than 2000?

[1]

A related species, the white peppered moth, shares the same habitat. It reproduces in a similar pattern to the black peppered moth, with its population modelled by

$$P(t) = P \ln(Qt + 3) \quad 0 \leq t \leq 40$$

d) The initial population is 769 and the population when  $t = 15$  is 2750.  
Find the value of  $P$ , correct to the nearest whole number and  $Q$ , correct to 3 decimal places.

[2]

Please turn over ...

- e) The populations of  $P(t)$  and  $N(t)$  are briefly equal on the first day. Determine an approximate time when the populations of the black and white peppered moths will again be the same.

[2]

- f) What are the population growth rates at this time?

[2]

**End of Part B**