CANNING VALE COLLEGE

Year 12 Mathematics Methods Test 4
Logarithmic functions and Calculus of Log functions

Name:________________________________

Section 1: Calculator Free 38 marks ~~40~~ 35 minutes

1. [1, 1, 1, 2 marks]

Suppose that two variables x and y are related by $y = 6^x$.

a) Use the *definition of a logarithm* to express x in terms of y.

$\log_6 y = x$ ✓

b) Given that $\log_6 2 = q$, write the following in terms of q:

i) $\log_6 24 = \log_6(6 \times 2^2)$

$= \log_6 6 + 2\log_6 2$

$= 1 + 2q$ ✓

ii) $\log_6 0.5 = \log_6 2^{-1}$

$= -\log_6 2$

$= -q$ ✓

iii) $\log_6 3 = \log_6\left(\frac{6}{2}\right)$

$= \log_6 6 + \log_6 2^{-1}$ ✓

$= 1 - q$ ✓

✓ **2.** **[2, 2, 2 marks]**

Solve the following, giving your answers in exact form involving logarithms where necessary.

a) $3^{x-4} = 14$

$(x-4)\ln 3 = \ln 14$ ✓

$x - 4 = \dfrac{\ln 14}{\ln 3}$

$x = \dfrac{\ln 14}{\ln 3} + 4$ ✓

or

$x\log 3 - 4\log 3 = \log 14$

$x\log 3 = \log 14 + 4\log 3$

$x = \dfrac{\log 14 + \log 81}{\log 3}$

$x = \dfrac{\log 1134}{\log 3} = \dfrac{\log 14 + 4\log 3}{\log 3}$

$x = \log_3 14 + 4$

b) $\log(x+4) - \log(x-5) = 1$

$\log \dfrac{x+4}{x-5} = 1.$ ✓

$\dfrac{x+4}{x-5} = 10$

$x + 4 = 10x - 50$

$9x - 54 = 0$

$9x = 54$

$x = 6$ ✓

c) $11(3^x) = 5 + 3^{x+2}$

$11(3^x) = 5 + 3^x . 3^2$ put $3^x = y$

$11y = 5 + 9y$ ✓

$2y = 5$

$y = \dfrac{5}{2}$

$3^x = 2.5$

$x\log 3 = \log 5 - \log 2$

$x = \dfrac{\log 5 - \log 2}{\log 3}$ ✓ or $\dfrac{\log 2.5}{\log 3}$ or $\log_3\left(\dfrac{5}{2}\right)$

0.834

✓ **4. [3, 2 marks]**

Let $g(x) = \dfrac{\ln x}{x^2}$, for $x > 0$.

(a) Use the quotient rule to show that $g'(x) = \dfrac{1 - 2\ln x}{x^3}$.

$g(x) = \frac{u}{v}$ $\quad u = \ln x \quad u' = \frac{1}{x}$

$v = x^2 \quad v' = 2x$

$g'(x) = \dfrac{\frac{x^2}{x} - 2x\ln x}{(x^2)^2}$ ✓ ✓ $= \dfrac{\cancel{x}(1 - 2\ln x)}{x^{\cancel{4}\,3}}$ ✓

$= \dfrac{x - 2x\ln x}{x^4}$ $\quad = \dfrac{1 - 2\ln x}{x^3}$

(b) The graph of g has a maximum point at A. Find the x-coordinate of A.

$g'(x) = 0$

at $1 - 2\ln x = 0$ ✓

$2\ln x = 1$

$\ln x = \frac{1}{2}$

$x = e^{1/2}$ ✓

5. [2 marks]

Find the derivative with respect to x of $y = \ln(x^3 + x^2)$

$\dfrac{dy}{dx} = \dfrac{3x^2 + 2x}{x^3 + x^2}$ ✓

$= \dfrac{\cancel{x}(3x + 2)}{\cancel{x}(x^2 + x)}$

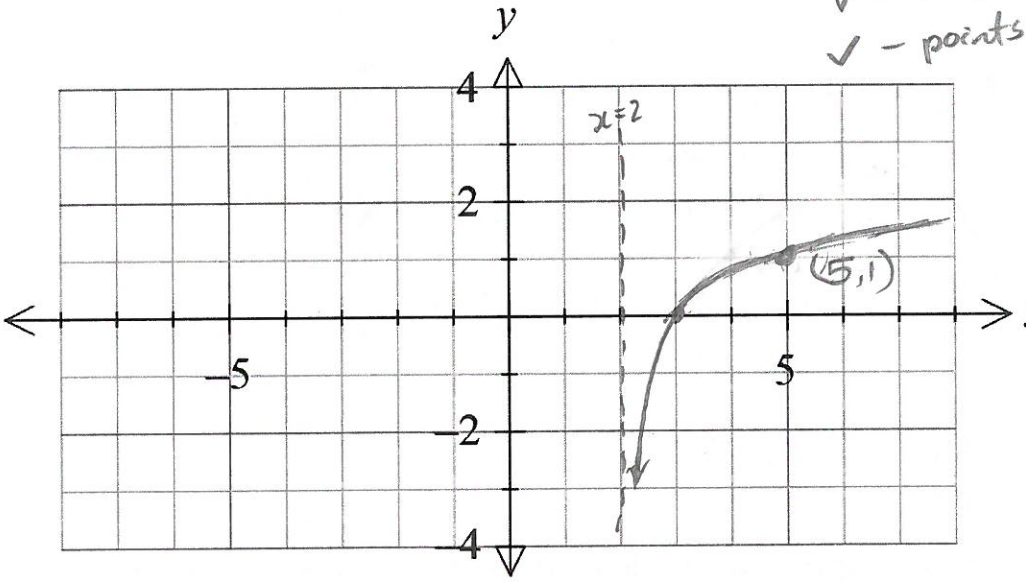
$= \dfrac{3x + 2}{x^2 + x}$ ✓

3. **[3, 3, 2 marks]**

On the sets of axes below, sketch the functions:

a) $y = \log_3(x-2)$

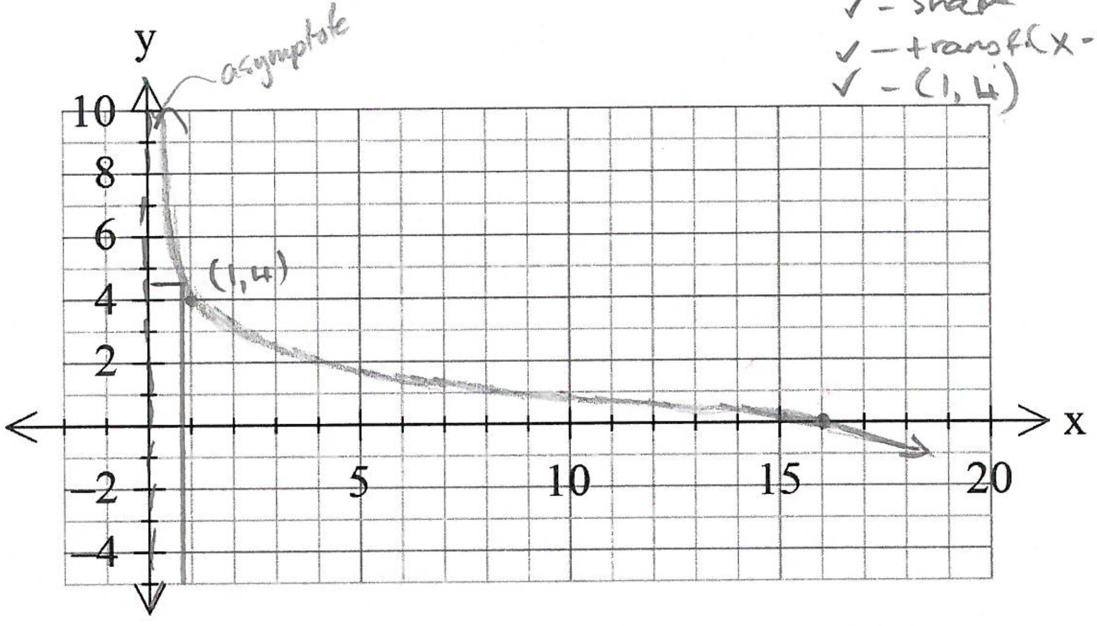
✓ - shape
✓ - transformation
✓ - points



Remind students to label significant points with coordinates.

b) $y = \log_{0.5} x + 4$

✓ - shape
✓ - transf (x-int)
✓ - (1, 4)



c) <u>Use the graph</u> to solve $\log_{0.5} x = 0.5$

$\log_{0.5} x + 4 = 0.5 + 4$

$\log_{0.5} x + 4 = 4.5$ ✓

$x \approx 0.8$ (actually 0.707) ✓

(according to graph)

v

6. [2, 1, 2, 2 marks]

a) Given the function $g(x) = x\ln x - x + 1$, determine $g'(x)$

$g'(x) = x \cdot \frac{1}{x} + \ln x - 1 + 0$ ✓

$= 1 + \ln x - 1$

$= \ln x$ ✓

b) Hence determine an expression for $\int \ln(x)\,dx$

$\int \ln x \, dx = x \ln x - x + c$ ✓

c) Evaluate $\int_1^2 \ln(x)dx$

$x \ln x - x \Big]_1^2$

$= (2\ln 2 - 2) - (\ln 1 - 1)$ ✓

$= 2\ln 2 - \ln 1 - 1$

$= \ln 4 - 1 \quad (\text{or } 2\ln 2 - 1)$ ✓

d) Evaluate $\int_1^2 \ln\sqrt{x}dx$

$= \frac{1}{2}\int_1^2 \ln x$ ✓

$= \frac{\ln 4 - 1}{2}$ or $\ln 2 - \frac{1}{2}$ ✓

e) Expression for $\int_a^b \ln(x)\,dx$; $b > a > 0$

$= x\ln x - x\Big]_a^b$

$= (b\ln b - b) - (a\ln a - a)$ ✓

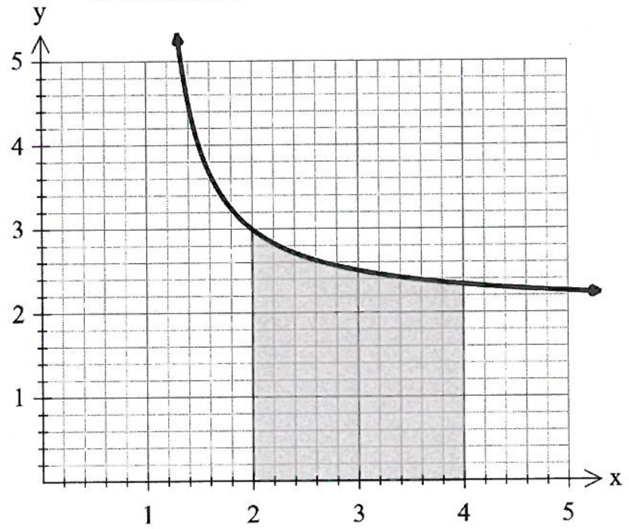
$= \ln b^b - b - \ln a^a + a$

$= \ln\left(\frac{b^b}{a^a}\right) - b + a$ ✓

✓7. **[2, 2 marks]**

Consider the function $f(x) = 2 + \dfrac{1}{x-1}$ $,\ x > 1$

The region enclosed by the graph of $f(x)$, the x-axis and the lines $x = 2$ and $x = 4$, is shaded below.



$\int 2\, dx + \int \frac{1}{x-1}\, dx$

(a) Find $\int f(x)\,\mathrm{d}x.$ $= 2x + \ln(x+1) + C$ ✓ ✓

(b) Find a simplified expression for the exact area of A.

$\left. 2x + \ln(x-1) \right]_2^4$

$= (8 + \ln 3) - (4 + \ln 1)$ ✓

$= 4 + \ln 3 - \ln 1$

$= 4 + \ln 3$ ✓

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Section 2: Calculator & Notes Allowed 13 marks 15 minutes

✓ 8. **[1, 1, 3 marks]**

The faintest sound that can be heard by the human ear has intensity

$$I_0 = 10^{-16} \text{ watts per square centimetre.}$$

Noise levels, β, are measured in decibels and are related to intensity:

$$\beta = 10 \log \frac{I}{I_0} \text{ decibels}$$

Where I is the intensity of sound in watts per square centimetre.

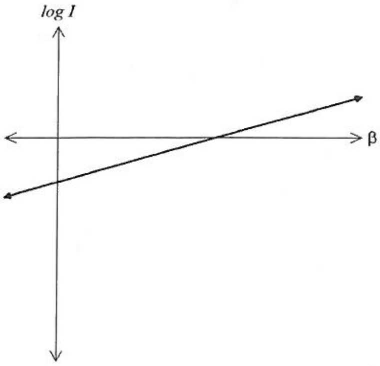
a) The maximum intensity which a human ear can tolerate is 10^{-4} watts per square centimetre. Determine the corresponding value of β.

$\beta = 10 \log_{10} \left(\frac{10^{-4}}{10^{-16}}\right)$
$= 10 \log_{10} 10^{12}$
$= 10 \times 12$
$= 120$ ✓

b) Busy motor traffic has a noise level of 70 decibels. Determine the corresponding intensity.

$7 = \log_{10} \left(\frac{I}{10^{-16}}\right)$
$10^7 = 10^x . x^{16}$
$I = 10^{-9}$ ✓

c) The graph (without scales) of $\log I$ against β is sketched below; it is linear.



By expressing $\log I$ in terms of β, determine the gradient and the intercept on the vertical axis.

$10(\log(I) - \log 10^{-16}) = \beta$
$\log I = \frac{\beta}{10} + \log 10^{-16}$
$\log I = \frac{\beta}{10} - 16$ ✓
$m = \frac{1}{10}$ ✓ $y\text{-int} = -16$ ✓

✓ **9.** **[3, ~~4~~ 2 marks]**

A particle P moves along a straight line. Its velocity, v ms^{-1} at time t seconds, is given by

$$v = 10ln(t+3) + 2 \text{ for } t \geq 0$$

(a) Find the initial velocity and acceleration

$V = 10 \ln(3) + 2$

$V = \underline{12.986 \text{ ms}^{-1}}$ ✓

$a = \frac{10}{x+3}$ ✓

at $t=0$ $a = \frac{10}{3} \text{ ms}^{-2}$ ✓

(b) Find the acceleration of P when its velocity is 20 ms^{-1}

$V = 20$ at $t = 3.05s$ ✓

at $t = 3.05$ $a = 1.65 \text{ ms}^{-2}$ ✓

10. **[~~2~~ 3 marks]**

✓ - from $x=0 \to x=1$

✓✓ - points $x \geq 1$

Draw ~~Sketch~~ the graph of $y = 2 \times 10^{2x}$,for $x \geq 0$; using semi-log grid.