PRESBYTERIAN LADIES' COLLEGE
A COLLEGE OF THE UNITING CHURCH IN AUSTRALIA

MATHEMATICS DEPARTMENT
MATHEMATICAL METHODS YEAR 12 – TEST 1

DATE: 2nd December 2015 Name: MARK ALLWRIGHT.

Reading Time: 3 minutes

SECTION ONE: CALCULATOR FREE
WORKING TIME: Maximum 25 minutes
TOTAL: 24 marks
EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, formula sheet (provided)

SECTION TWO: CALCULATOR ASSUMED
WORKING TIME: Minimum 25 minutes
TOTAL: 26 marks
EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, templates, up to 3 calculators, formula sheet (provided) one A4 page of notes (one side only)

Question	Marks available	Marks awarded	Question	Marks available	Marks awarded
1	6		3	4	
2	4		4	7	
3	7		5	8	
4	7		6	7	
Sect 1 Total	**24**		**Sect 2 Total**	**26**	
			TOTAL	**50**	

Question 1 **(6 marks)**

Find the antiderivative of each of the following

a) $\dfrac{4}{(2x-5)^3}$ $= 4(2x-5)^{-3}$ (3 marks)

$$\int 4(2x-5)^{-3} = \frac{4(2x-5)^{-2}}{-2\times 2}$$ ✓

$$= -(2x-5)^{-2}$$ ✓

$$= \frac{-1}{(2x-5)^2}$$ ✓

(b) $(10x+5)(x^2+x-3)^4$ (3 marks)

$$\int (10x+5)(x^2+x-3)^4$$

$$= \frac{(10x+5)(x^2+x-3)^5}{5(2x+1)}$$ ✓ ✓

$$= (x^2+x-3)^5$$ ✓

Question 2 **(4 marks)**

Let $f(x)=(x+3)(1-x^2)^5$.

The derivative of $f(x)$ can be written in the form $f'(x)=(1-x^2)^4(ax^2+bx+c)$.

Determine the value of a, b and c.

$$f'(x) = 1(1-x^2)^5 + (x+3)\times 5(1-x^2)^4\times(-2x)$$

$$= (1-x^2)^5 + (-10x)(x+3)(1-x^2)^4$$

$$= (1-x^2)^5 + (-10x^2-30x)(1-x^2)^4$$

$$= (1-x^2)^4\left[(1-x^2)+(-10x^2-30x)\right]$$

$$= (1-x^2)^4(-11x^2-30x+1)$$

$\therefore$ $a=-11$

$b=-30$

$c=1$

-1 per error in working.

Question 3 **(7 marks)**

Let A, B, C, D, E, F and G be points on the graph of a continuous function $f(x)$.
The table below shows the information about the sign of $f(x)$, $f'(x)$ and $f''(x)$ at these points.

Point	A	B	C	D	E	F	G
x	-4	-3	-1	0	1	2	4
$f(x)$	+	0	-	0	+	+	+
$f'(x)$	-	-	0	+	+	0	+
$f''(x)$	+	+	+	0	-	0	+

There are no other points at which $f(x)$, $f'(x)$ or $f''(x)$ are equal to zero.

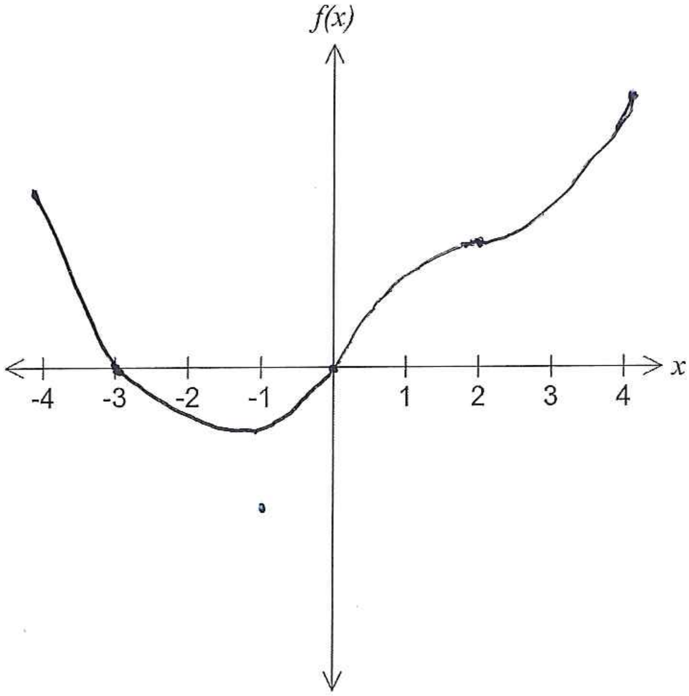
(a) Which point is a local minimum? (1 mark)

C ✓

(b) Describe the nature of the graph at point F. (2 marks)

Horizontal ✓ Point of Inflexion ✓.

(c) Sketch the function on the axes below. (4 marks)



✓✓✓✓

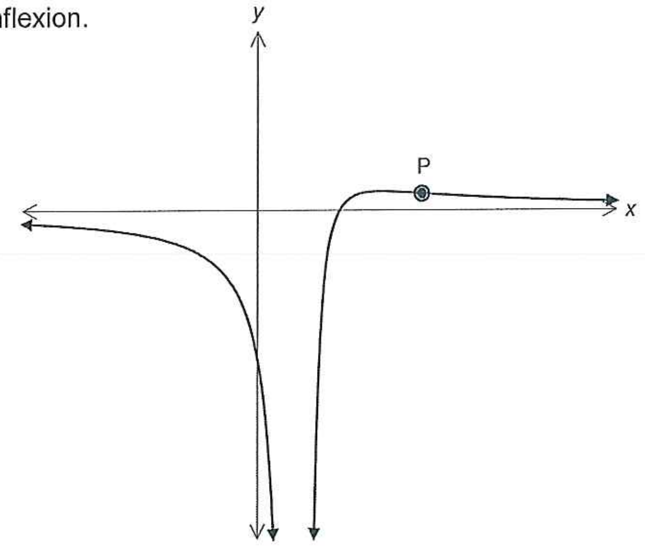
−1 per error.

Question 4 **(7 marks)**

Consider the function $y=\dfrac{x-2}{(x-1)^2}, x\neq 1$.

A sketch of part of the graph of h is given below.

The point P is a point of inflexion.



(a) Find $h'(x)$, writing your answer in the form $\dfrac{a-x}{(x-1)^n}$

where a and n are constants to be determined. (4 marks)

$$y' = \frac{(x-1)^2 \times 1 - (x-2)\times 2(x-1)^1}{(x-1)^4}$$ ✓ ✓

$$= \frac{(x-1)\left[(x-1)-(2x-4)\right]}{(x-1)^4}$$

$$= \frac{(x-1)(3-x)}{(x-1)^4}$$

$$= \frac{3-x}{(x-1)^3}$$

$\therefore a = 3$ ✓

$n = 3$ ✓

−1 per error in working.

Question 4 continued...

(b) Given that $h''(x) = \frac{2x-8}{(x-1)^4}$, calculate the coordinates of P. (3 marks)

$$0 = \frac{2x-8}{(x-1)^4}$$

$$0 = 2x - 8$$

$$8 = 2x$$

$$x = 4$$

$$y = \frac{4-2}{(4-1)^2}$$

$$= \frac{2}{9}$$

$$P = \left(4, \frac{2}{9}\right)$$

End of Section 1

Section 2 Calculator Assumed. Name: ______________________

Question 5 **(4 marks)**

Given that $y = x^{\frac{1}{3}}$, use $x = 1000$ and the increments formula $\delta y = \frac{dy}{dx}\delta x$ to determine an appropriate value for $\sqrt[3]{1006}$.

$$y = x^{1/3}$$

$$\frac{dy}{dx} = \frac{1}{3}x^{-2/3} \quad \checkmark \qquad \delta x = 6.$$

$$\delta y = \frac{dy}{dx} \times \delta x$$

$$= \frac{1}{3}(1000)^{-2/3} \times 6. \checkmark$$

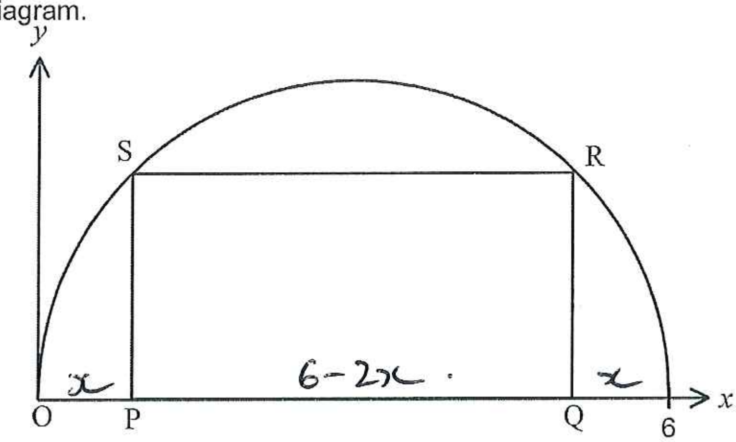
$$= 0 \cdot 02 \quad \checkmark$$

$$\therefore \sqrt[3]{1006} \approx 10 \cdot 02 \quad \checkmark$$

Question 6 **(7 marks)**

Consider the graph of the semicircle given by $f(x)=\sqrt{6x-x^2}$, for $0\le x\le 6$.

A rectangle PQRS is drawn with upper vertices R and S on the graph of $f(x)$, and PQ on the x-axis, as shown in the following diagram.



Let $\mathrm{OP}=x$.

(a) Explain why an equation for the area of the rectangle can be written as

$$Area=(6-2x)\sqrt{6x-x^2}$$ (3 marks)

length of rectangle (PQ) $= 6-2x$ ✓

height of rectangle (PS) $=\sqrt{6x-x^2}$ ✓

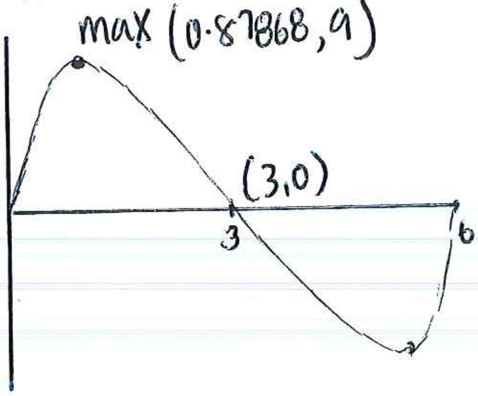
$\therefore$ Area of Rectangle = length × height $=(6-2x)\sqrt{6x-x^2}$ ✓

(b) (i) Find the rate of change of area when x = 2. (2 marks)

$$A' = \frac{4x^2-24x+18}{\sqrt{-x^2+6x}}$$ ✓

$A'(2) = -4.95$ ✓

(ii) The area is decreasing for $a<x<b$. Find the value of a and of b. (2 marks)

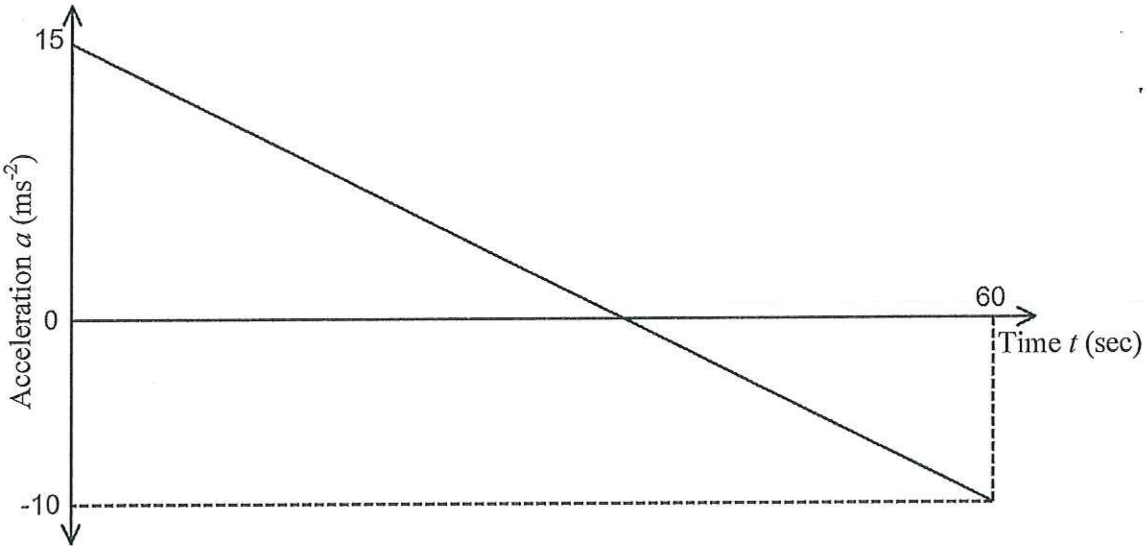


$a = 0.87868$ ✓

$b = 3.$ ✓

Question 7 **(8 marks)**

A jet plane travels horizontally along a straight path for one minute, starting at time $t = 0$, where t is measured in seconds. The acceleration, a, measured in ms^{-2}, of the jet plane is given by the straight line graph below.



(a) Find an expression for the acceleration of the jet plane during this time, in terms of t. (2 marks)

$m = \frac{-25}{60}$

$= -\frac{5}{12}$

$a = -\frac{5}{12}t + 15$

(b) Given that when $t = 0$ the jet plane is travelling at 125 ms^{-1}, find its maximum velocity in ms^{-1} during the minute that follows. (3 marks)

$-\frac{5}{12}t + 15 = 0$

$t = 36$

or

$V = \frac{-5t^2}{24} + 15t + c$

$V = -\frac{5t^2}{24} + 15t + 125$

max at (36, 395)

∴ max velocity of 395 m/s

(c) Given that the jet plane breaks the sound barrier at 295 ms^{-1}, find out for how long the jet plane is travelling greater than this speed. (3 marks)

$295 = -\frac{5x^2}{24} + 15x + 125$

$x_1 = 14.091098 \quad x_2 = 57.908902$

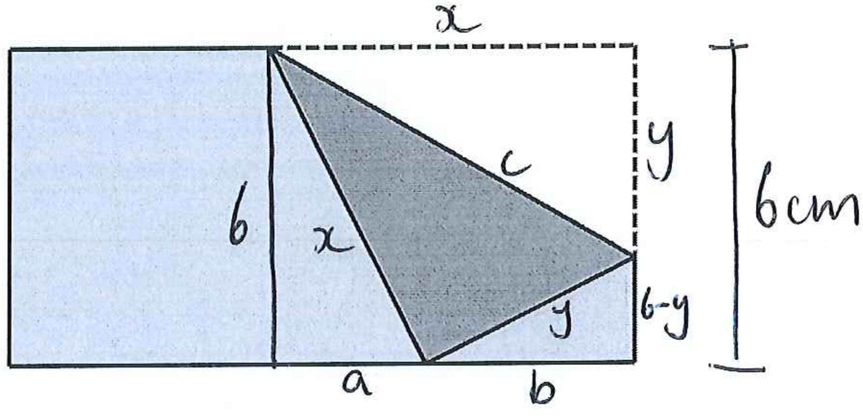
$x_2 - x_1 = 43.817804$

∴ The plane is breaking the sound barrier for 43.82 seconds.

Question 8 **(7 marks)**

A rectangular piece of paper measures 12 cm by 6 cm. One corner of the sheet of paper is folded up to just reach the opposite side as shown below.

What is the minimum length of the resulting crease in the paper?



$x^2+y^2=c^2$ ✓

$a=\sqrt{x^2-6^2}$ ✓ $\quad b=\sqrt{y^2-(6-y)^2}$ ✓

$x=a+b$

$x=\sqrt{x^2-36}+\sqrt{y^2+(6-y)^2}$ ✓

$y=\dfrac{x\left(x-\sqrt{x^2-36}\right)}{6}$ ✓ From C.A.S.

$c=\sqrt{x^2+\left(\dfrac{x\left(x-\sqrt{x^2-36}\right)}{6}\right)^2}$ ✓

From C.A.S. min at $(6.36, 7.79)$ ✓

$\therefore$ minimum length of crease is 7.79cm

End of Test