Chapter 5 – Variation

Solutions to Exercise 5A

1 a $y = kx^2$

$8 = k \times 2^2$

$8 = 4k$

$\therefore\ k = 2$

$x = 6: \quad y = 2 \times 6^2 = 72$

$y = 128: \quad 2x^2 = 128$

$x^2 = 64$

$x = 8$ (assuming $x > 0$)

x	2	4	6	8
y	8	32	72	128

b $y = kx$

$\frac{1}{6} = k \times \frac{1}{2}$

$1 = 3k$

$\therefore\ k = \frac{1}{3}$

$x = 1: \quad y = \frac{1}{3} \times 1 = \frac{1}{3}$

$y = \frac{2}{3}: \quad \frac{1}{3}x = \frac{2}{3}$

$x = 2$

x	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$

c $y = k\sqrt{x}$

$6 = k \times \sqrt{4}$

$6 = 2k$

$\therefore\ k = 3$

$x = 49: \quad y = 3 \times \sqrt{49} = 21$

$y = 90: \quad 3\sqrt{x} = 90$

$\sqrt{x} = 30$

$x = 900$

x	4	9	49	900
y	6	9	21	90

d $y = kx^{\frac{1}{5}}$

$\frac{1}{5} = k \times \left(\frac{1}{32}\right)^{\frac{1}{5}}$

$\frac{1}{5} = k \times \frac{1}{2}$

$\therefore\ k = \frac{2}{5}$

$x = 32: \quad y = \frac{2}{5} \times 32^{\frac{1}{5}} = \frac{4}{5}$

$y = \frac{8}{5}: \quad \frac{2}{5} \times x^{\frac{1}{5}} = \frac{8}{5}$

$x^{\frac{1}{5}} = 4$

$\left(x^{\frac{1}{5}}\right)^5 = 4^5$

$x = 1024$

x	$\frac{1}{32}$	1	32	1024
y	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{8}{5}$

2 $V = kr^3$

$125 = k \times 2.5^3$

$125 = 15.625k$

$\therefore \quad k = \dfrac{1.25}{15.625}$

$= 8$

a $V = 8 \times 3.2^3$

$= 262.144$

b $200 = 8r^3$

$r^3 = 25$

$r = \sqrt[3]{25}$

≈ 2.924

3 $a = b^{\frac{2}{3}}$

$\dfrac{2}{3} = k \times 1^{\frac{2}{3}}$

$\therefore \quad k = \dfrac{2}{3}$

a $a = \dfrac{2}{3} \times 2^{\frac{2}{3}}$

≈ 1.058

b $2 = \dfrac{2}{3} \times b^{\frac{2}{3}}$

$b^{\frac{2}{3}} = 2 \times \dfrac{3}{2} = 3$

$b^{\frac{2}{3}} = 3^{\frac{3}{2}}$ (assuming $b > 0$)

$b \approx 5.196$

4 $A = kh$

$60 = k \times 10$

$\therefore \quad k = 6$

a $A = 6 \times 12$

$= 72 \text{ cm}^2$

b $120 = 6h$

$h = 20 \text{ cm}$

5 $E = kw$

$3.2 = k \times 452$

$\therefore \quad k = \dfrac{3.2}{452}$

$= \dfrac{8}{1130}$

a $E = \dfrac{8}{1130} \times 810$

$= \dfrac{648}{113} \text{ cm}$

b $10 = \dfrac{8}{1130} \times w$

$w = 10 \times \dfrac{1130}{8}$

$= \dfrac{2825}{2}$

$= 1412.5 \text{ g}$

6 $W = kL^2$

$18 = k \times 20^2$

$18 = 400k$

$\therefore \quad k = \dfrac{18}{400}$

$= \dfrac{9}{200}$

$L = \sqrt{225} = 15$

$W = \dfrac{9}{200} \times 15^2$

$= 10.125 \text{ kg}$

7

$$V = kr^3$$
$$4188.8 = k \times 10^3$$
$$\therefore \quad k = 4.1888$$
$$1\text{ m}^3 = 1\,000\,000\text{ cm}^3$$
$$1\,000\,000 = 4.1888\,r^3$$
$$r^3 = \frac{1\,000\,000}{4.1888}$$
$$\approx 238\,731.85$$
$$r \approx 62.035\text{ cm}$$

8 $S \propto r^2$ and $S = kr^2$ where k is the constant of proportionality.
Initially set $r = 1$
Then $S = k$

a If r is doubled set $r = 2$
Then $S = k(2^2) = 4k$
The surface area is increased by a factor of 4.

b If r is tripled set $r = 3$
Then $S = k(3^2) = 9k$
The surface area is increased by a factor of 9.

c If r is increased by 10% set $r = 1.1$
Then $S = k(1.1^2) = 1.21k$
The surface area is increased by 21%.

9 $E \propto v^3$ and $E = kv^3$ where k is the constant of proportionality. Initially set $v = 1$
Then $E = k$.
If the wind increases by 15 %
Then $E = k(1.15)^3 = 1.52087k$
The energy is increased by 52%

10

$$T = k\sqrt{L}$$
$$1.55 = k \times \sqrt{60}$$
$$\therefore \quad k = \frac{1.55}{\sqrt{60}}$$
$$T = \frac{1.55}{\sqrt{60}} \times \sqrt{90}$$
$$= 1.55 \times \sqrt{1.5}$$
$$\approx 1.898\text{ seconds}$$

11 a

$$d = k\sqrt{h}$$
$$4.8 = k \times \sqrt{1.8}$$
$$\therefore \quad k = \frac{4.8}{\sqrt{1.8}}$$

Person's height above ground $= 4 + 1.8$
$= 5.8$ m

$$d = \frac{4.8}{\sqrt{1.8}} \times \sqrt{5.8}$$
$$\approx 8.616\text{ km}$$

b Height difference between person and yacht $= 5.8 + 10 = 15.8$ m

$$d = \frac{4.8}{\sqrt{1.8}} \times \sqrt{15.8}$$
$$\approx 14.221\text{ km}$$

12 In each case set the initial value of x to 1.
Initial value of $y = k$ (when $x = 1$).

a i $y = k \times 2^2$
$= 4k$ (300% increase)

ii $y = k \times \sqrt{2}$
$\approx 1.41k$ (41% increase)

iii $y = k \times 2^3$
$= 8k$ (700% increase)

b **i** $y = k \times 0.5^2$

$= 0.25k$ (75% decrease)

ii $y = k \times \sqrt{0.5}$

$\approx 0.71k$ (29% decrease)

iii $y = k \times 0.5^3$

$= 0.125k$ (87.5% decrease)

c **i** $y = k \times 0.8^2$

$= 0.64k$ (36% decrease)

ii $y = k \times \sqrt{0.8}$

$\approx 0.89k$ (11% decrease)

iii $y = k \times 0.8^3$

$= 0.512k$ (48.8% decrease)

d **i** $y = k \times 1.4^2$

$= 1.96k$ (96% increase)

ii $y = k \times \sqrt{1.4}$

$\approx 1.18k$ (18% increase)

iii $y = k \times 1.4^3$

$= 2.744k$ (174.4% increase)

Solutions to Exercise 5B

1 a $y = \dfrac{2}{x}$

$2 = \dfrac{k}{1}$

$\therefore \quad k = 2$

$x = 6: \qquad y = \dfrac{2}{6} = \dfrac{1}{3}$

$y = \dfrac{1}{16}: \quad \dfrac{2}{x} = \dfrac{1}{16}$

$x = 2 \times 16 = 32$

x	2	4	6	32
y	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{16}$

b $y = \dfrac{k}{\sqrt{x}}$

$\dfrac{1}{2} = \dfrac{k}{\sqrt{1}}$

$\therefore \quad k = \dfrac{1}{2}$

$y = \dfrac{1}{2\sqrt{x}}$

$y = \dfrac{1}{4}: \quad \dfrac{1}{2\sqrt{x}} = \dfrac{1}{4}$

$2\sqrt{x} = 4$

$x = 4$

$x = 9: \qquad y = \dfrac{1}{2\sqrt{9}}$

$= \dfrac{1}{6}$

x	$\frac{1}{4}$	1	4	9
y	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$

c $y = \dfrac{k}{x^2}$

$3 = \dfrac{k}{1^2}$

$\therefore \quad k = 3$

$x = 3: \qquad y = \dfrac{3}{3^2} = \dfrac{1}{3}$

$y = \dfrac{1}{12}: \qquad \dfrac{3}{x^2} = \dfrac{1}{12}$

$x^2 = 36$

$x = 6$ (assuming $x > 0$)

x	1	2	3	6
y	3	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{12}$

d $y = \dfrac{k}{x^{\frac{1}{3}}}$

$\dfrac{1}{3} = \dfrac{k}{1^{\frac{1}{3}}}$

$\therefore \quad k = \dfrac{1}{3}$

$y = \dfrac{1}{3x^{\frac{1}{3}}}$

$y = \dfrac{1}{9}: \qquad \dfrac{1}{3x^{\frac{1}{3}}} = \dfrac{1}{9}$

$x^{\frac{1}{3}} = 3$

$x = 3^3 = 27$

$x = 125: \qquad y = \dfrac{1}{3 \times 125^{\frac{1}{3}}}$

$= \dfrac{1}{15}$

x	$\frac{1}{8}$	1	27	125
y	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{15}$

2 $a = \dfrac{k}{b^3}$

$4 = \dfrac{k}{(\sqrt{2})^3}$

$\therefore \quad k = 4 \times (\sqrt{2})^2 \times \sqrt{2}$

$= 8\sqrt{2}$

a $a = \dfrac{8\sqrt{2}}{b^3}$

$= \dfrac{8\sqrt{2}}{(2\sqrt{2})^3}$

$= \dfrac{8\sqrt{2}}{8 \times \sqrt{8}}$

$= \dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$

b $a = \dfrac{8\sqrt{2}}{b^3}$

$\dfrac{1}{16} = \dfrac{8\sqrt{2}}{b^3}$

$b^3 = 8\sqrt{2} \times 16$

$= 128\sqrt{2}$

≈ 181.01

$b \approx 5.657$

3 $a = \dfrac{k}{b^4}$

$5 = \dfrac{k}{2^4}$

$\therefore \quad k = 5 \times 2^4 = 80$

a $a = \dfrac{80}{4^4}$

$= \dfrac{80}{256} = 0.3125$

b $20 = \dfrac{80}{b^4}$

$b^4 = \dfrac{80}{20}$

$= 4 = 2^2$

$b = (2^2)^{\frac{1}{4}}$ (assuming $b > 0$)

$= 2^{\frac{1}{2}} = \sqrt{2}$

4 $V = \dfrac{k}{P}$

$22.5 = \dfrac{k}{1.9}$

$\therefore \quad k = 1.9 \times 22.5$

$= 42.75$

$15 = \dfrac{42.75}{P}$

$P = \dfrac{42.75}{15}$

$= 2.85$ kg/cm^2

5 $I = \dfrac{k}{R}$

$3 = \dfrac{k}{80}$

$\therefore \quad k = 3 \times 80$

$= 240$

a $I = \dfrac{240}{100}$

$= 2.4$ amperes

b 80% of 3 = 2.4

$2.4 = \dfrac{240}{R}$

$R = \dfrac{240}{2.4}$

$= 100$ ohms

This is an increase of 20 ohms from the original 80 ohms, i.e. an increase of 25%

6 $I = \dfrac{k}{d^2}$

$100 = \dfrac{k}{20^2}$

$\therefore \quad k = 100 \times 400$

$= 40\,000$

$I = \dfrac{40\,000}{25^2}$

$= 64$ candela

7 $r = \dfrac{k}{\sqrt{h}}$

$5.64 = \dfrac{k}{\sqrt{10}}$

$\therefore \quad k = 5.64\sqrt{10}$

$r = \dfrac{5.64\sqrt{10}}{\sqrt{12}}$

$= 5.15$ cm

8 In each case set the initial value of x to 1.
Initial value of $y = k$ (when $x = 1$).

a **i** $y = \dfrac{k}{4}$

$= 0.25k$ (75% decrease)

ii $y = \dfrac{k}{\sqrt{2}}$

$\approx 0.71k$ (29% decrease)

iii $y = \dfrac{k}{2^3}$

$= 0.125k$ (87.5% decrease)

b **i** $y = \dfrac{k}{0.5^2}$

$= 4k$ (300% increase)

ii $y = \dfrac{k}{\sqrt{0.5}}$

$\approx 1.41k$ (41% increase)

iii $y = \dfrac{k}{0.5^3}$

$= 8k$ (700% increase)

c **i** $y = \dfrac{k}{0.8^2}$

$= 1.5625k$ (56.25% increase)

ii $y = \dfrac{k}{\sqrt{0.8}}$

$\approx 1.12k$ (12% increase)

iii $y = \dfrac{k}{0.8^3}$

$\approx 1.95k$ (95% increase)

d **i** $y = \dfrac{k}{1.4^2}$

$\approx 0.51k$ (49% decrease)

ii $y = \dfrac{k}{\sqrt{1.4}}$

$\approx 0.85k$ (15% decrease)

iii $y = \dfrac{k}{1.4^3}$

$\approx 0.36k$ (64% decrease)

Solutions to Exercise 5C

1 **a** From the table, $y = \frac{2}{3}x$, which is a direct relationship.

b From the table, $y = 4x^2$, which is a direct square relationship.

c From the table, $y = \frac{5}{x}$, which is an inverse relationship.

d From the table, $y = 2\sqrt{x}$, which is a direct square root relationship.

e From the table, $y = \frac{4}{x^2}$, which is an inverse square relationship.

2 If direct variation exists, then the graph of y vs x^n will be a straight line through the origin.
Graphs **b** and **e** fit these criteria. Graph **f** is a straight line but does not pass through the origin.

3 If inverse variation exists, then the graph of y vs $\frac{1}{x^n}$ will be a straight line that is undefined at the origin.
Graphs **a**, **b** and **e** fit these criteria.
Graph *a* is a curve when showing y vs x, but will straighten out when showing y vs $\frac{1}{x^n}$.

4 **a** Gradient $= \frac{3}{1} = 3$

$$y = 3x$$

b Gradient $= \frac{6}{2} = 3$

$$y = 3 \times \frac{1}{x} = \frac{3}{x}$$

c Gradient $= \frac{10}{3}$

$$y = \frac{10}{3}x^2$$

d Gradient $= \frac{2}{1} = 2$

$$y = 2\sqrt{x}$$

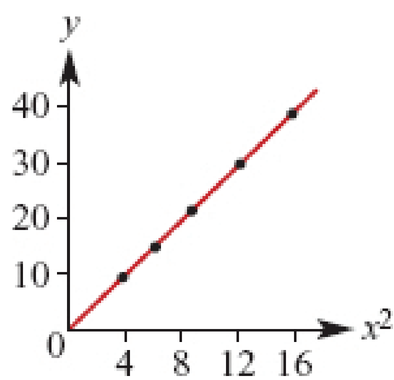
e Gradient $= \frac{3}{9} = \frac{1}{3}$

$$y = \frac{1}{3} \times \frac{1}{\sqrt{x}} = \frac{1}{3\sqrt{x}}$$

f Gradient $= \frac{6}{1} = 6$

$$y = 6x^3$$

5



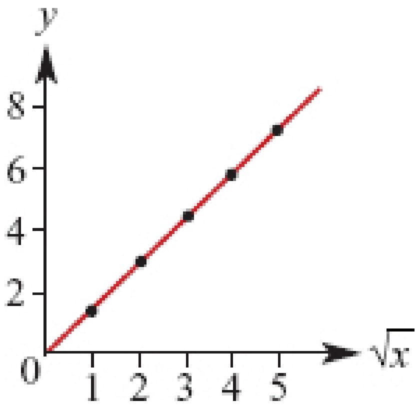
The graph is a straight line through the origin.

Gradient $= \frac{21.6}{9} = 2.4$

$$y = 2.4x^2$$

Note: Any point can be used to calculate the gradient.

6



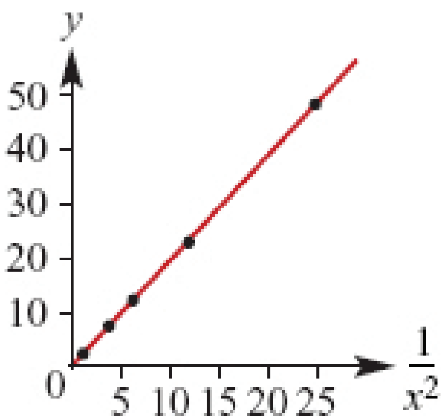
The graph is a straight line through the origin.

$$\text{Gradient} = \frac{7.5}{5} = 1.5$$

$$y = 1.5\sqrt{x}$$

Note: Any point can be used to calculate the gradient.

7



The graph is a straight line through the origin.

$$\text{Gradient} = \frac{2}{1} = 2$$

$$y = \frac{2}{x^2}$$

Note: Any point can be used to calculate the gradient.

8 CAS calculator's Power Regression function.

a $y = \frac{1}{4}\sqrt{x}$

b $y = 2x^{\frac{5}{4}}$

c $y = 3.5x^{0.4}$

d $y = 10x^{\frac{2}{3}}$

e $y = 2x^{-\frac{5}{2}}$

f $y = 3.2x^{-0.4}$

9 **a** Use your CAS calculator's power regression function to determine a and b.

$a = 100$

$b = 0.2$

b $C = at^b$

$= 100 \times 10^{0.2}$

≈ 158.49

10 **a** Use your CAS calculator's power regression function to determine a and b.

$a = 1500$

$b = -0.5$

b $I = at^b$

$= 1500 \times 10^{-0.5}$

≈ 474.34

Solutions to Exercise 5D

1 $y = \dfrac{kx}{z}$

$1 = \dfrac{2k}{10}$

$\therefore \quad k = 5$

When $y = \dfrac{1}{2}$ and $z = 60$:

$\dfrac{1}{2} = \dfrac{5x}{60}$

$x = 6$

When $x = 10$ and $y = 4$:

$4 = \dfrac{5 \times 10}{z}$

$z = \dfrac{50}{4} = 12.5$

x	2	4	6	10
z	10	2	60	12.5
y	1	10	0.5	4

2 $y = kxz$

$10 = k \times 2 \times 10$

$\therefore \quad k = \dfrac{1}{2}$

When $y = 25$ and $z = 50$:

$25 = \dfrac{50x}{2}$

$x = 1$

When $x = 10$ and $y = 15$:

$15 = \dfrac{10z}{2}$

$z = 3$

x	2	4	1	10
z	10	8	50	3
y	10	16	25	15

3 $y = \dfrac{kz}{x^2}$

$\dfrac{15}{2} = \dfrac{10k}{2^2}$

$\therefore \quad k = \dfrac{15 \times 4}{2 \times 10}$

$= 3$

When $y = 6$ and $z = 50$:

$6 = \dfrac{3 \times 50}{x^2}$

$x^2 = \dfrac{150}{6} = 25$

$x = 5$

When $x = 10$ and $y = 4$:

$4 = \dfrac{3z}{10^2}$

$3z = 400$

$z = \dfrac{400}{3}$

x	2	3	5	10
z	10	4	50	$\frac{400}{3}$
y	$\frac{15}{2}$	$\frac{4}{3}$	6	4

4 $a = \dfrac{kb^2}{c}$

$0.54 = \dfrac{k \times 1.2^2}{2}$

$k = \dfrac{0.54 \times 2}{1.2^2}$

$= 0.75$

$a = \dfrac{0.75 \times 2.6^2}{3.5}$

≈ 1.449

5
$$z = \frac{k\sqrt{x}}{y^3}$$
$$14.6 = \frac{k \times \sqrt{5}}{1.5^3}$$
$$k = \frac{1.46 \times 1.5^3}{\sqrt{5}}$$
$$= \frac{4.9275}{\sqrt{5}}$$
$$z = \frac{4.9275\sqrt{x}}{y^3\sqrt{5}}$$
$$= \frac{4.9275\sqrt{48}}{2.3^3\sqrt{5}}$$
$$\approx 0.397$$

6 **a** 9.8 J/kg.m

b 5880 J

7
$$I = krt$$
$$130 = k \times 6.5 \times 2$$
$$k = \frac{130}{13} = 10$$
$$I = 10 \times 5.8 \times 3$$
$$= \$174$$

8
$$E = kmv^2$$
$$281.25 = k \times 25 \times 15^2$$
$$k = \frac{281}{2.5 \times 225}$$
$$= 0.5$$
$$E = 0.5 \times 1.8 \times 20^2$$
$$= 360 \text{ joules}$$

9 In both cases, set the initial length and diameter to 1.

Initial value of $y = k$ (when $l = 1$, $d = 1$).

a
$$y = \frac{k \times 1.5}{0.5^2}$$
$$= 6k \text{ (500\% increase)}$$

b
$$y = \frac{k \times 0.5}{1.5^2}$$
$$\approx 0.22k \text{ (78\% decrease)}$$

10 **a** $W = \dfrac{kd^2}{L}$

Let the diameter be a and the length b for a supported weight of C.

$$C = \frac{ka^2}{L}$$

Let the new diameter be x.

If the length doubles and the weight remains the same, then

$$C = \frac{kd^2}{2L}$$
$$\frac{kx^2}{2L} = \frac{ka^2}{L}$$
$$\therefore \quad x^2 = \frac{ka^2}{L} \times \frac{2l}{k}$$
$$= 2a^2$$
$$x = \sqrt{2}a$$

The diameter has increased by a factor of $\sqrt{2} \approx 1.41$ or approximately 41%.

b
$$W = \frac{k \times (2a)^2}{3L}$$
$$= \frac{4ka^2}{3L}$$
$$= \frac{4C}{3}$$

The weight has increased by a factor of $\frac{4}{3} \approx 1.33$ or approximately 33%.

11 In both cases, set the initial values of p and q to 1.
Initial value of $y = k$ (when $p = 1$, $q = 1$).

a $y = \dfrac{k \times 2^2}{\sqrt{2}}$

$= 2.83k$ (183% increase)

b $y = \dfrac{k \times 2^2}{\sqrt{0.5}}$

$= 5.66k$ (466%increase)

12 **a** $T = \dfrac{kx}{l}$

For the first spring,

$T = \dfrac{k \times 1}{3} = \dfrac{k}{3}$

For the second spring,

$T = \dfrac{k \times 0.9}{2.7} = \dfrac{k}{3}$

The tensions will both be the same.

b $T = \dfrac{kx^2}{l}$

For the first spring,

$T = \dfrac{k \times 1^2}{3} = \dfrac{k}{3}$

For the second spring,

$T = \dfrac{k \times 0.9^2}{2.7} = \dfrac{3k}{10}$

$$\begin{aligned}\text{The ratio of tension} &= \frac{T(\text{second spring})}{T(\text{first spring})}\\ &= \frac{0.3k}{\frac{k}{3}}\\ &= \frac{3k}{10} \times \frac{3}{k}\\ &= \frac{9}{10} = 0.9\end{aligned}$$

This is a 10% decrease; the tension in the second spring is 90% that in the first.

Solutions to Exercise 5E

1
$$C = b + kd$$
$$42.4 = b + 22k \quad ①$$
$$47.8 = b + 25k \quad ②$$
② − ①:
$$5.4 = 3k$$
$$\therefore \quad k = 1.8$$
$$42.4 = b + 22 \times 1.8$$
$$42.4 = b + 39.6$$
$$b = 2.8$$
$$C = 2.8 + 1.8 \times 17$$
$$= \$33.40$$

2 **a**
$$C = b + kd$$
$$13\,125 = b + 50k \quad ①$$
$$17\,875 = b + 70k \quad ②$$
② − ①:
$$4750 = 20k$$
$$\therefore \quad k = 237.5$$
$$13\,125 = b + 50 \times 237.5$$
$$b = 1250$$
The fixed overhead charge is \$1250 and the cost per guest is \$237.50

b
$$C = b + kd$$
$$= 1250 + 237.5 \times 100$$
$$= \$25\,000$$

3
$$p = k_1 x + k_2 y^2$$
$$14 = 3k_1 + 16k_2 \quad ①$$
$$14.5 = 5k_1 + 9k_2 \quad ②$$
Multiply ① by 5 and ② by 3:
$$70 = 15k_1 + 80k_2 \quad ①$$
$$43.5 = 15k_1 + 27k_2 \quad ②$$
① − ②:
$$26.5 = 53k_2$$
$$k_2 = 0.5$$
$$14 = 3k_1 + 16 \times 0.5$$
$$14 = 3k_1 + 8$$
$$3k_1 = 6$$
$$k_1 = 2$$
$$p = 2 \times 4 + 0.5 \times 25$$
$$= 20.5$$

4
$$C = k_1 n + \frac{k_2}{n}$$
$$32 = 200k_1 + \frac{k_2}{200} \quad ①$$
$$61 = 400k_1 + \frac{k_2}{400} \quad ②$$
Multiply ① by 0.5:
$$16 = 100k_1 + \frac{k_2}{400} \quad ①$$
$$61 = 400k_1 + \frac{k_2}{400} \quad ②$$
② − ①:
$$45 = 300k_1$$
$$k_1 = \frac{45}{300} = 0.15$$
$$32 = 200 \times 0.15 + \frac{k_2}{200}$$
$$32 = 30 + \frac{k_2}{200}$$
$$\frac{k_2}{200} = 2$$
$$k_2 = 400$$
$$C = 0.15 \times 360 + \frac{400}{360}$$
$$\approx \$55.11$$

5 a $s = k_1 t + k_2 t^2$

$142.5 = 3k_1 + 9k_2$ ①

$262.5 = 5k_1 + 25k_2$ ②

Multiply ① by 5 and ② by 3:

$712.5 = 15k_1 + 45k_2$ ①

$787.5 = 15k_1 + 75k_2$ ②

② − ①:

$75 = 30k_2$

$k_2 = \frac{75}{30} = 2.5$

$142.5 = 3k_1 + 9 \times 2.5$

$3k_1 = 120$

$k_1 = 40$

$s = 40 \times 6 + 2.5 \times 36$

$= 330$ m

b The sixth second is the time from $t = 5$ to $t = 6$.

Distance travelled $= 330 - 262.5$

$= 67.5$ m

6 $t = k_1\, b + \frac{k_2}{m}$

$45 = 10k_1 + \frac{k_2}{1}$

$45 = 10k_1 + k_2$ ①

$30 = 8k_1 + \frac{k_2}{2}$ ②

Multiply ② by 2:

$45 = 10k_1 + k_2$ ①

$60 = 16k_1 + k_2$ ②

② − ①:

$15 = 6k_1$

$k_1 = \frac{15}{6} = 2.5$

$45 = 10 \times 2.5 + k_2$

$k_2 = 20$

$t = 2.5 \times 16 + \frac{20}{4}$

$= 45$ minutes

Solutions to Review: Short-answer questions

1 **a**

$$a = kb^2$$
$$\frac{3}{2} = k \times 2^2$$
$$\therefore k = \frac{3}{8}$$
$$b = 4: \qquad a = \frac{3}{8} \times 4^2 = 6$$
$$a = 8: \qquad \frac{3}{8} \times b^2 = 8$$
$$b^2 = \frac{64}{3}$$
$$b = \pm\frac{8}{\sqrt{3}}$$

b

$$y = kx^{\frac{1}{3}}$$
$$10 = k \times 2^{\frac{1}{3}}$$
$$k = \frac{10}{2^{\frac{1}{3}}}$$
$$x = 27: \qquad y = \frac{10}{2^{\frac{1}{3}}} \times 27^{\frac{1}{3}} = \frac{30}{2^{\frac{1}{3}}}$$
$$y = \frac{1}{8}: \qquad \frac{10}{2^{\frac{1}{3}}} \times x^{\frac{1}{3}} = \frac{1}{8}$$
$$x^{\frac{1}{3}} = \frac{2^{\frac{1}{3}}}{80}$$
$$x = \frac{2}{80^3}$$
$$x = \frac{1}{256\,000}$$

c

$$y = \frac{k}{x^2}$$
$$\frac{1}{3} = \frac{k}{2^2}$$
$$\therefore k = \frac{4}{3}$$
$$x = \frac{1}{2}: \qquad y = \frac{4}{3x^2} = \frac{4}{3 \times 0.5^2} = \frac{16}{3}$$
$$y = \frac{4}{27}: \qquad \frac{4}{3x^2} = \frac{4}{27}$$
$$3x^2 = 27$$
$$x^2 = 9$$
$$x = \pm 3$$

d

$$a = \frac{kb}{\sqrt{c}}$$
$$\frac{1}{4} = \frac{k \times 1}{\sqrt{4}}$$
$$\therefore k = \frac{2}{4} = \frac{1}{2}$$
$$a = \frac{b}{2\sqrt{c}} = \frac{\frac{4}{9}}{2\sqrt{\frac{16}{9}}} = \frac{4}{9} \times \frac{1}{2} \times \frac{3}{4} = \frac{1}{6}$$

2 a $d = kt^2$

$78.56 = k \times 4^2$

$k = \dfrac{78.56}{16}$

$= 4.91$

$d = 4.91t^2$

b $d = 4.91 \times 10^2$

$= 491$ m

c $19.64 = 4.91t^2$

$t^2 = \dfrac{19.64}{491} = 4$

$t = 2$ seconds

3 a $v = k\sqrt{s}$

$7 = k\sqrt{2.5}$

$\therefore k = \dfrac{7}{\sqrt{2.5}}$

$v = \dfrac{7\sqrt{s}}{\sqrt{2.5}}$

$= 7\sqrt{\dfrac{s}{2.5}}$

$= 7\sqrt{\dfrac{10}{2.5}}$

$= 14$ m/s

b $28 = 7\sqrt{\dfrac{s}{2.5}}$

$\sqrt{\dfrac{s}{2.5}} = 4$

$\dfrac{s}{2.5} = 16$

$s = 40$ m

c Plot v against $\sqrt{s}$

4 $t = \dfrac{k}{v}$

$4 = \dfrac{k}{30}$

$\therefore k = 4 \times 30 = 120$

$t = \dfrac{120}{50}$

$= 2.4$ hours

5 $y \propto \dfrac{1}{x}$

a $y \propto \dfrac{1}{2x}$

$\therefore y$ is halved.

b $2y \propto \dfrac{1}{x}$

$x \propto \dfrac{1}{2y}$

$\therefore x$ is halved.

c $y \propto \dfrac{1}{\frac{x}{2}}$

$2y \propto \dfrac{1}{x}$

$\therefore y$ is doubled.

d $\dfrac{y}{2} \propto \dfrac{1}{x}$

$x \propto \dfrac{2}{y}$

$\therefore x$ is doubled.

6 $C = ktRI^2$

$9 = k \times 2.5 \times 60 \times 16$

$\therefore k = 0.00375$

$C = 0.00375 \times 1.5 \times 80 \times 9$

$= 4.05$ cents

7
$$C = a + kn$$
$$20 = a + 100k \quad ①$$
$$30 = a + 500k \quad ②$$
② − ① :
$$10 = 400k$$
$$\therefore \quad k = \frac{10}{400} = \frac{1}{40}$$
$$20 = a + \frac{100}{40}$$
$$20 = a + 2.5$$
$$a = 17.5$$
$$C = 17.5 + \frac{700}{40}$$
$$= \$35$$

8
$$v = kI$$
$$24 = k \times 6$$
$$\therefore k = 4$$
$$72 = 4I$$
$$I = 18 \text{ amps}$$

9 $I = \dfrac{k}{d^2}$

Let the initial distance be d_1. The final distance will be d_2.
$$I_1 = \frac{k}{(d_1)^2}$$
$$I_2 = \frac{k}{(2d_1)^2}$$
$$= \frac{k}{4(d_1)^2}$$
$$= \frac{1}{4}I_1$$

10 Set the initial values of x and z to 1.

Initial value of $y = k$
$$y = \frac{k \times 1.1^2}{0.9}$$
$$\approx 1.34 \text{ (34\% increase)}$$

Solutions to Review: Multiple-choice questions

1 C $y = kx^2$

$3 = k \times 9$

$k = \frac{1}{3}$

2 A $y = \frac{k}{x}$

$\frac{1}{4} = \frac{k}{2}$

$k = \frac{2}{4} = \frac{1}{2}$

3 B $a = kb^3$

$32 = k \times 8$

$k = 4$

$a = 4 \times 64$

$= 256$

4 C $p = \frac{k}{q^2}$

$\frac{1}{3} = \frac{k}{9}$

$k = \frac{9}{3} = 3$

$1 = \frac{3}{q^2}$

$q^2 = 3$

$q = \sqrt{3}$ (assuming $q > 0$)

5 B Gradient $= \frac{6}{2} = 3$

$y = 3x^2$

6 D Gradient $= \frac{4}{1} = 4$

$y = 4\sqrt{x}$

7 E $y = \frac{kx}{z^2}$

$\frac{1}{3} = \frac{k \times 2}{2^2} = \frac{k}{2}$

$k = \frac{2}{3}$

8 D $a = \frac{kp^2}{q}$

$8 = \frac{k \times 4}{5}$

$k = \frac{40}{4} = 10$

$a = \frac{10 \times 9}{6}$

$= 15$

9 D Set the initial value of q to 1.
Initial value of $p = k$

$p = k \times 1.1^2$

$= 1.21k$

21% increase

10 B Set the initial value of q to 1.
Initial value of $p = k$

$p = \frac{k}{0.8}$

$= 1.25k$

25% increase

Solutions to Review: Extended-response questions

1
$$m \propto d^2$$
$$\therefore \quad m = kd^2, \quad \text{where } k \in \mathbb{R} \setminus \{0\}$$
$$\therefore \quad k = \frac{m}{d^2}$$

When $m = 0.10, d = 9$

$$\therefore \quad k = \frac{0.10}{9^2} = \frac{1}{810}$$
$$\therefore \quad m = \frac{d^2}{810}$$

a When $d = 14$, $m = \frac{14^2}{810}$

$= 0.241\,97\ldots$

The mass of the second sphere is 0.24 kg, correct to two decimal places.

b When $m = 0.15$, $d^2 = 810m$

$= 810 \times 0.15$

$= 121.5$

$\therefore \quad d = 11.022\,70\ldots$

The diameter of the third sphere is 11 cm, to the nearest centimetre.

2 a
$$h \propto n^2$$
$$\therefore \quad h = kn^2, \text{ where } k \in \mathbb{R} \setminus \{0\}$$
$$\therefore \quad k = \frac{h}{n^2}$$

When $h = 13.5, n = 200$

$$\therefore \quad k = \frac{13.5}{200^2} = 0.000\,3375$$
$$\therefore \quad h = 0.000\,3375n^2$$

b When $n = 225$, $h = 0.000\,3375 \times 225$

$= 17.085\,93\ldots$

The water can be raised to a height of 17.1 m, correct to one decimal place.

c Now $n = \sqrt{\dfrac{h}{0.000\,3375}}$

When $h = 16,\ n = \sqrt{\dfrac{16}{0.000\,3375}}$

$= 217.732\,42\ldots$

The required speed is 218 revs/min, to the nearest rev/min.

3 Let s be the maximum speed of the yacht (in knots) and l be the length of the yacht (in metres).

$$s \propto \sqrt{l}$$

$\therefore$ $s = k\sqrt{l}$, for $k \in R\setminus\{0\}$

$\therefore$ $k = \dfrac{s}{\sqrt{l}}$

When $l = 20$, $s = 15$

$$\therefore \quad k = \frac{15}{\sqrt{20}} = \frac{15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{10} = \frac{3\sqrt{5}}{2}$$

$\therefore$ $s = \dfrac{3\sqrt{5l}}{2}$

When $l = 15$, $s = \dfrac{3\sqrt{5 \times 15}}{2}$

$= 12.990\,38\ldots$

The maximum speed of the yacht is 13 knots, to the nearest knot.

4 a $V \propto \dfrac{1}{P}$

$\therefore$ $V = \dfrac{k}{P}$, where $k \in R\setminus\{0\}$

$\therefore$ $k = VP$

When $V = 43.5$, $P = 2.8$

$$\therefore \quad k = 43.5 \times 2.8 = 121.8$$

$\therefore$ $V = \dfrac{121.8}{P}$

b Now $P = \dfrac{121.8}{P}$

When $V = 12.7$, $P = \dfrac{121.8}{127}$

$= 9.590\,55\ldots$

The pressure is 9.6 kg/cm^2, correct to one decimal place.

5 a $w \propto \dfrac{1}{d}$

$\therefore$ $w = \dfrac{k}{d}$, where $k \in \mathbb{R}\setminus\{0\}$

$\therefore$ $k = dw$

When $d = 6, w = 500$

$\therefore$ $k = 6 \times 500$

$= 3000$

$\therefore$ $w = \dfrac{3000}{d}$

b When $d = 5$, $w = \dfrac{3000}{5}$

$= 600$

A weight of 600 kg could be carried.

c When $d = 9$, $w = \dfrac{3000}{9}$

$= 333.333\,33\ldots$

A weight of 333 kg could be carried, to the nearest kilogram.

6 a By inspection, it can be conjectured that some type of inverse variation exists. As p increases, v decreases.

Assume $v \propto \dfrac{1}{p^n}$ for some positive number n.

$\therefore$ $v = \dfrac{k}{p^n}$, for $k \in \mathbb{R}\setminus\{0\}$

$\therefore$ $k = vp^n$

Let $n = 1$, $\therefore k = vp$

When $p = 12, v = 12$

$\therefore$ $k = 12 \times 12$

$= 144$

When $p = 16$, $v = 9$

$$\therefore \quad k = 16 \times 9 = 144$$

When $p = 18$, $v = 8$

$$\therefore \quad k = 18 \times 8 = 144$$

$\therefore\ k = 144$ and $n = 1$

i.e. $$v = \frac{144}{p}$$

b **i** When $p = 72$,

$$v = \frac{144}{72} = 2$$

The volume is 2 units.

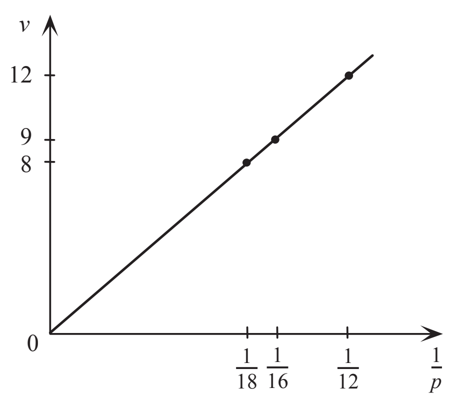
ii Now $$p = \frac{144}{v}$$

When $v = 3$, $$p = \frac{144}{3} = 48$$

The pressure is 48 units.

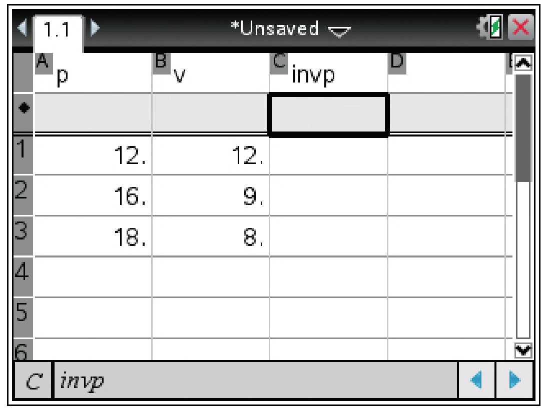
$\frac{1}{p}$	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{18}$
v	12	9	8

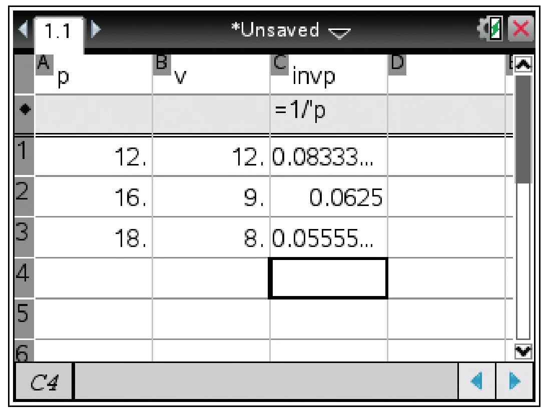
c



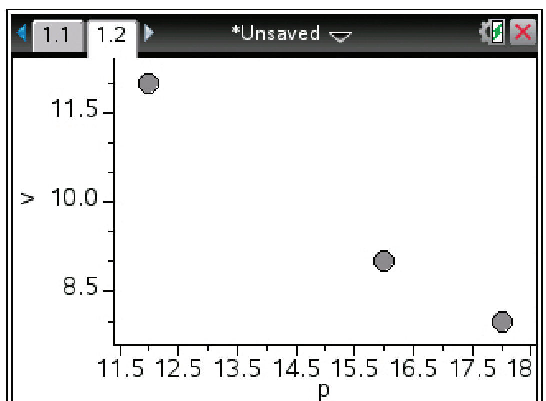
CAS calculator techniques for Question 6

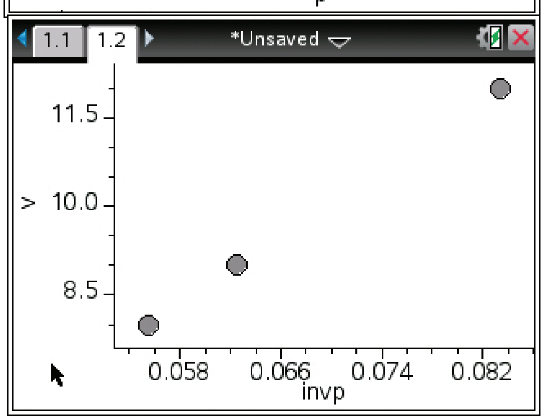
TI: Open a Lists & Spreadsheet application. Call column A, **p**, column B, **v** and column C, **invp**. Input the data for the first two columns.
In the Grey box below the name **invp** type **=1/p** then ENTER and choose Variable Reference.
Open a Data & Statistics application.
Add the variable **p** to the horizontal axis and variable **v** to the vertical axis.
Now add the variable **invp** to the horizontal leaving **v** on the vertical axis.
It can be seen that v is inversely proportional to p.





CP: Open the Statistics application. Type the data for **p** into **list1** and **v** into **list2**.
At the bottom of list3 tap inside the Cal box and type **1/list1** followed by EXE.
Tap **SetGraph** → **Settings** and set Draw: On, Type: Scatter, XList: list1, YList: list2 then tap SET
Tap y to see the graph
Repeat this process changing XList to list3





7 Let t be the manufacturing time (in minutes), d be the diameter of the item (in cm) and n be the number of parts.
There exist constants k_1 and k_2 such that $t = k_1 d + k_2 n$.
When $t = 30, d = 3$ and $n = 8$

$$30 = 3k_1 + 8k_2 \qquad \ldots\boxed{1}$$

When $t = 38, d = 5$ and $n = 10$

$$38 = 5k_1 + 10k_2 \qquad \ldots\boxed{2}$$

$\boxed{1} \times 5$ $$150 = 15k_1 + 40k_2 \qquad \ldots\boxed{1'}$$

$\boxed{2} \times 3$ $$114 = 15k_1 + 30k_2 \qquad \ldots\boxed{2'}$$

$\boxed{1'} - \boxed{2'}$ $$36 = 10k_2$$

$\therefore$ $$k_2 = 3.6$$

Substitute $k_2 = 3.6$ in $\boxed{1}$

$\therefore$ $$30 = 3k_1 + 8 \times 3.6$$

$\therefore$ $$k_1 = \frac{30 - 8 \times 3.6}{3} = 0.4$$

$\therefore$ $$t = 0.4d + 3.6n$$

When $d = 4$ and $n = 12$,

$$t = 0.4 \times 4 + 3.6 \times 12 = 44.8$$

It will take 44.8 minutes.

8 Let C be the cost of wrought iron (in \$) and l be the length of wrought iron (in metres).
There exist constants k_1 and k_2 such that $C = k_1 l + k_2 l^2$.
When $l = 2, C = 18.4$

$\therefore$ $$18.4 = 2k_1 + 4k_2 \qquad \ldots\boxed{1}$$

When $l = 3, C = 33.6$

$\therefore$ $$33.6 = 3k_1 + 9k_2 \qquad \ldots\boxed{2}$$

$\boxed{1} \times 3$ $$55.2 = 6k_1 + 12k_2 \qquad \ldots\boxed{1'}$$

$\boxed{2} \times 2$ $$67.2 = 6k_1 + 18k_2 \qquad \ldots\boxed{2'}$$

$\boxed{2'} - \boxed{1'}$ $$12 = 6k_2$$

$\therefore$ $$k_2 = 2$$

Substitute $k_2 = 2$ into $\boxed{1}$

$$18.4 = 2k_1 + 4 \times 2$$

$$\therefore \quad k_1 = \frac{18.4 - 4 \times 2}{2}$$

$$= 5.2$$

$$\therefore \quad C = 5.2l + 2l^2$$

When $l = 5$, $$C = 5.2 \times 5 + 2 \times 5^2$$

$$= 76$$

The cost is \$76.

9 Let S_n be the sum of the first n natural numbers. There exist constants k_1 and k_2 such that $S_n = k_1 n + k_2 n^2$.

Now $$S_3 = 1 + 2 + 3 = 6$$

$$S_4 = 1 + 2 + 3 + 4 = 10$$

When $n = 3, S_3 = 6$

$$\therefore \quad 6 = 3k_1 + 9k_2 \quad \ldots \boxed{1}$$

When $n = 4, S_4 = 10$

$$\therefore \quad 10 = 4k_1 + 16k_2 \quad \ldots \boxed{2}$$

$\boxed{1} \times 4$ $$24 = 12k_1 + 36k_2 \quad \ldots \boxed{1'}$$

$\boxed{2} \times 3$ $$30 = 12k_1 + 48k_2 2 \quad \ldots \boxed{2'}$$

$\boxed{2'} - \boxed{1'}$ $$6 = 12k_2$$

$$\therefore \quad k_2 = \frac{1}{2}$$

Substitute $k_2 = \frac{1}{2}$ into $\boxed{1}$

$$6 = 3k_1 + 9 \times \frac{1}{2}$$

$$\therefore \quad k_1 = \frac{6 - 9 \times \frac{1}{2}}{3}$$

$$= \frac{1}{2}$$

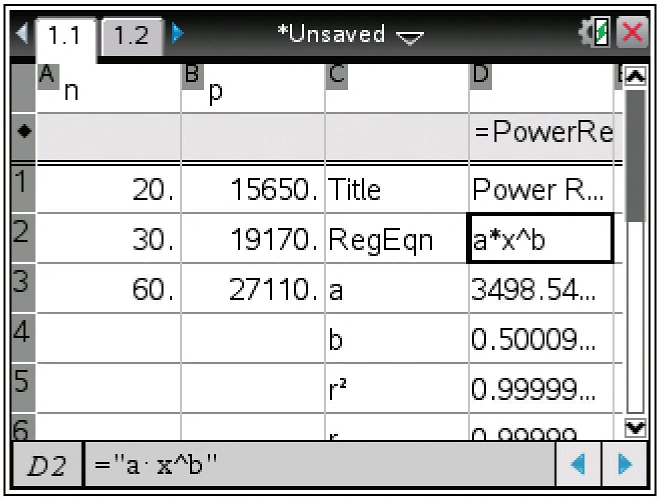
$$\therefore \quad S_n = \frac{1}{2}n + \frac{1}{2}n^2$$

$$= \frac{1}{2}n(n + 1)$$

10 a Using a CAS calculator,

TI: In a Lists & Spreadsheet application input 20, 30 and 60 into a column called **n** and input 15 650, 19 170 and 27 110 to a column called **p**. Press **Menu→4**:**Statistics→1**:**Stat Calculations→9**:**Power Regression**. Set *X* List to **n** and *Y* List to **p** and Save RegEqn to **f1**. The relationship between *N* and *P* is found to be

$P = 3498.544\,689N^{0.500\,099\,300\,8}$



CP: In the Statistics application input 20, 30 and 60 into **list1** and input 15 650, 19 170 and 27 110 **list2**. Tap **Calc** → **Power Reg** and set XList to **list1**, *Y*List to **list2** and Copy Formula to **yl**

b **TI**: In the Calculator application type **f1(55)**

CP: In the Main application type **y1(55)**. Use the letter y from the 0 tab (do not use the variable y) to find that the caterers would anticipate selling 25 956 pies on that day.

c Sketch ***f*2 = 25 000**.

TI: Scroll to $f1(x)$ = and press ENTER. Press **Menu** → **6:Analyze Graph** → **4:Intersection**

CP: Tap **Analysis** → **G - Solve→Intersect**

The point of intersection is (51.023 01, 25 000). The caterers would be hoping for a maximum crowd of 51 000, to the nearest thousand.

11 a Using a CAS calculator, and **Power Regression** as in **10 a**, the relationship is found to be $t = \dfrac{3600}{d^2}$

b Using a CAS calculator, the relationship is found to be $T = 0.14d^2$.

c On a CAS calculator, let $\boldsymbol{f1} = \mathbf{0.14x^2}$ and $\boldsymbol{f2} = \mathbf{80}$.
The point of intersection is found to be (23.904 572, 80).
The maximum dosage that should be given is 23.9 ml, correct to one decimal place.

d $t = 3600d^{-2}$
When $d = 23.904\,57\ldots$, $t = 6.300\,00\ldots$
It would take 6.3 minutes for the patient to lose consciousness.

e When $d = 20$,
$$t = 3600 \times 20^{-2}$$
$$= 9$$
and
$$T = 0.14 \times 20^2$$
$$= 56$$
The patient would lose consciousness after nine minutes and remain unconscious for 56 minutes.

12 a i Using a CAS calculator, and **Power Regression** as in **10 a** so the relationship is pasted directly into $\boldsymbol{f1}$, the relationship is found to be approximately $T = 0.000\,539R^{1.501}$.

ii On the CAS calculator,
TI: In the Calculator application type $\boldsymbol{f1}$({**228**, **779**, **1427**, **2872**, **4497**, **5900**}) followed by ENTER to find the corresponding T values.
CP: In the Main application type $\boldsymbol{y1}$({**228**, **779**, **1427**, **2872**, **4497**, **5900**}) followed by EXE to find the corresponding T values. (Use the letter y from the 0 tab)
The values (correct to two decimal places) are shown in the following table.

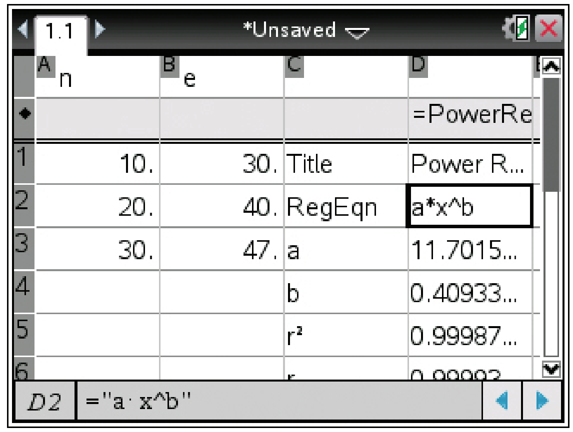
Planet	R	T
Mars	228	1.87
Jupiter	779	11.86
Saturn	1427	29.45
Uranus	2870	84.09
Neptune	4497	165.05
Pluto	5900	248.16

b Using $\boldsymbol{f1}$ as in **a ii**, let $\boldsymbol{f2} = \mathbf{70}$ and set the screen to show the point of intersection.
The point of intersection is found to be (2540.0837, 70).
The radius of the comet's orbit is 2.540×10^9 km, correct to four significant figures.

13

Number of advertisements (n)	10	20	30
Number of enquiries (E)	30	40	47

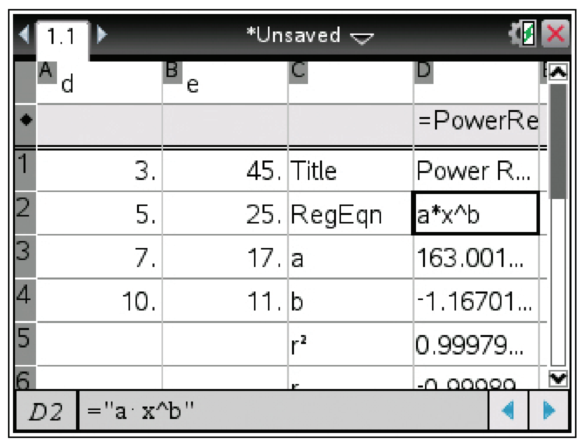
Consider $E = an^b$.
From a CAS calculator,
$a = 11.7016$ and $b = 0.4093$, correct to four decimal places.



When $n = 100$, $E = 77$, correct to the nearest whole number.
There will be approximately77 enquiries if 100 advertisements are placed.

Number of days (d)	3	5	7	10
Number of enquiries (E)	45	25	17	11

Consider $E = kd^p$.
From a CAS calculator,
$k = 163$ and $p = -1.167$, correct to three decimal places.



When $d = 14$, $E = 7.49$, correct to two decimal places.
After 14 days there will be about 7 enquiries.