SHENTON COLLEGE

Examination Semester One 2019
Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 3
Section One
(Calculator-free)

Your name _____

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this section
To be provided by the supervisor
Question/answer booklet for Section One.
Formula sheet.

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Student Score
Section One: Calculator-free	8	8	50	52	
Section Two: Calculator-assumed	13	13	100	98	
			Total	150	

Instructions to candidates

The rules for the conduct of Western Australian external examinations are detailed in the Year 12 *Information Handbook 2019.* Sitting this examination implies that you agree to abide by these rules

Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
  Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil, except in diagrams**.

STRUCTURE OF THIS PAPER

QUESTION	MARKS AVAILABLE	MARKS AWARDED
1	6	
2	4	
3	6	
4	6	
5	7	
6	7	
7	8	
8	8	
TOTAL	**52**	

Section One: Calculator-free 35% (52 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

(a) Determine the modulus and argument of $\dfrac{3}{1-i}$. (3 marks)

(b) Determine z^2 in the form $a + bi$, where $a, b \in \mathbb{R}$, when $z = 4\cos\left(\dfrac{\pi}{6}\right) + 4i\sin\left(\dfrac{\pi}{6}\right)$.

(3 marks)

Question 2 **(4 marks)**

The equations of three planes are shown below.

$$x - y + 3z = 11$$
$$x + 2y - 2z = 0$$
$$x - y + z = 9$$

(a)   Determine the coordinates of the point of intersection of the planes.   (3 marks)

(b)   Determine the distance of the point of intersection of the planes from the origin.   (1 mark)

Question 3 **(6 marks)**

(a) State whether the planes with equations $2x - y + z = 2$ and $x + 3y + 2z = 1$ are perpendicular. Justify your answer. (2 marks)

(b) Determine the Cartesian equation of the plane that passes through the three points with position vectors shown below. (4 marks)

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \qquad \mathbf{c} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

Question 4 **(6 marks)**

Functions f and g are defined over their natural domains by $f(x) = \sqrt{8-x}$ and $g(x) = 3 + \dfrac{4}{\sqrt{x}}$.

(a)   State the domain of

  (i) $g(x)$. (1 mark)

  (ii) $g^{-1}(x)$. (2 marks)

(b)   Determine $f \circ g(x)$ and the natural domain of this composite function. (3 marks)

Question 5 (7 marks)

Four functions are defined as

$$f(x) = x^2 + 4x - 5, \qquad g(x) = 3x^2 + 2x - 1, \qquad h(x) = x + 5, \qquad k(x) = x - 1$$

Determine the equations of all asymptotes of the following graphs.

(a) $y = \dfrac{h(x)}{f(x)}$. (2 marks)
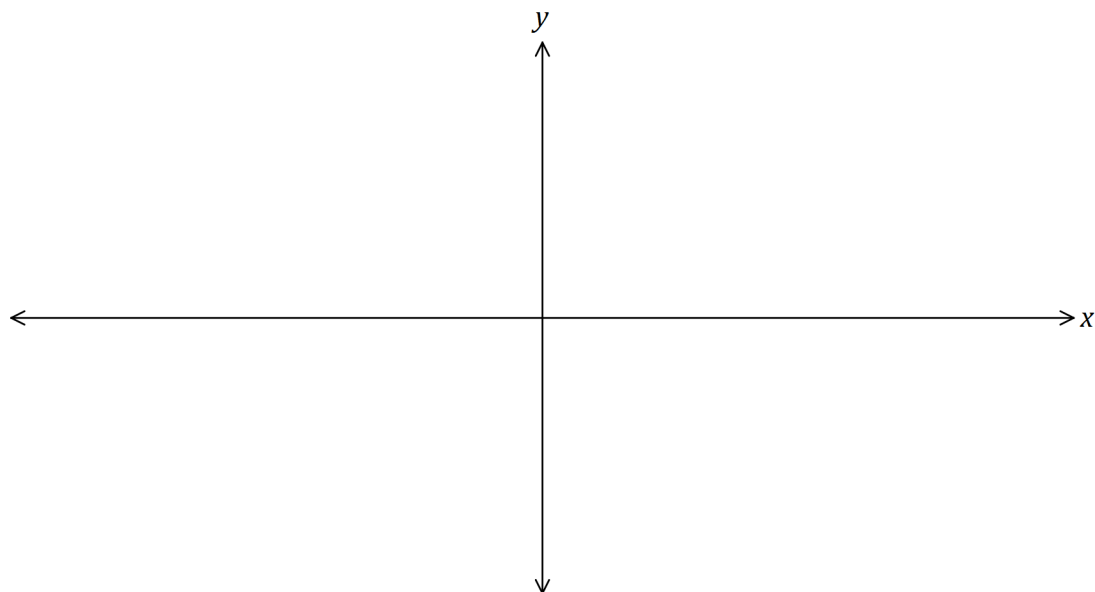
(b) $y = \dfrac{f(x)}{g(x)}$. (2 marks)

(c) $y = \dfrac{g(x)}{k(x)}$. (3 marks)

Question 6   **(7 marks)**
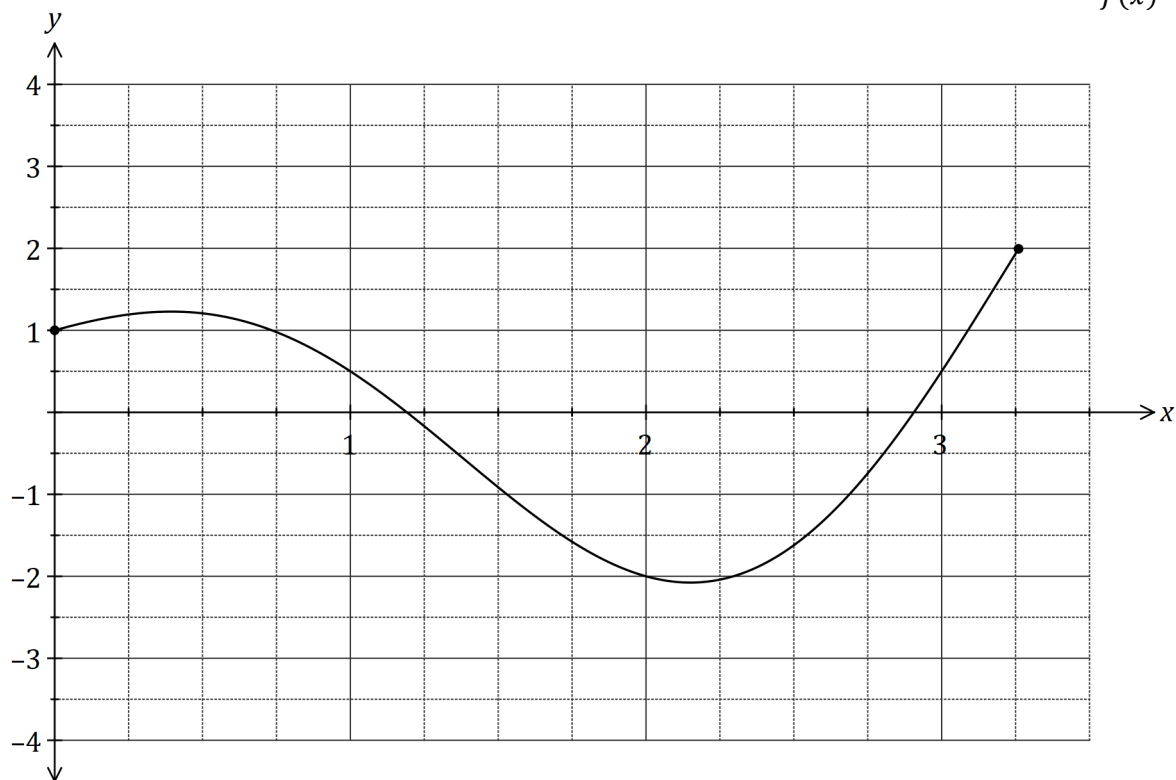
(a)   On the axes below, sketch the graph of $y = \dfrac{2x}{|x|}$.   (3 marks)

(b)   The graph of $y = f(x)$ is shown below. On the same axes draw the graph of $y = \dfrac{1}{f(x)}$.   (4 ma
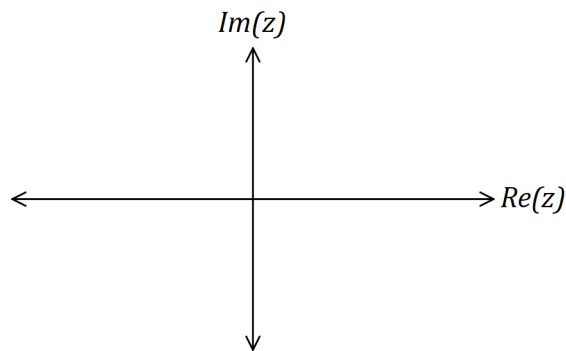

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Question 7   **(8 marks)**

(a)   Sketch the locus of points z in the complex plane determined by $\arg(z + 3i) = \frac{3\pi}{4}$.
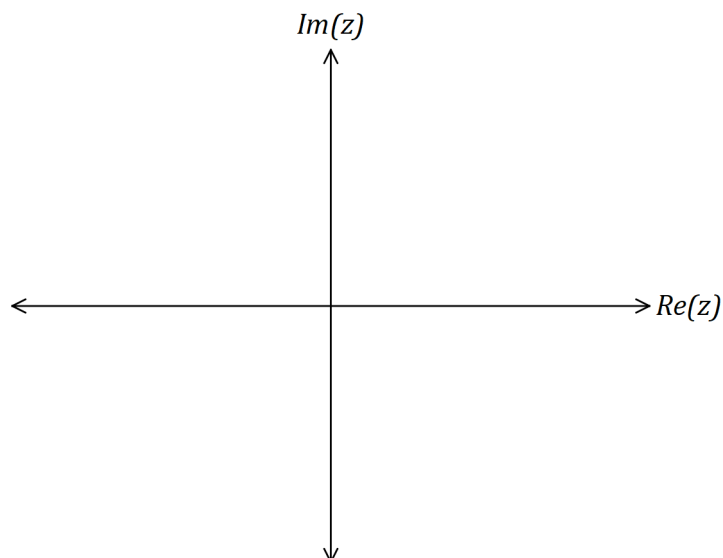
(3 marks)

(b)   Another locus of points z in the complex plane is determined by $z\bar{z} + z + \bar{z} = 8$.

  (i)   Show that this locus can also be defined in the form $|z - w| = k$, clearly showing
          the value of constant w and the value of constant k.   (3 marks)

  (ii)   Sketch the locus on the axes below.   (2 marks)

Question 8   **(8 marks)**

Let $z = x + yi$ and $z^2 = a + bi$ where $a, b, x, y \in \mathbb{R}$.

(a)   Show that $\sqrt{a^2 + b^2} + a = 2x^2$.   (4 marks)

(b)   By solving the equation $z^4 - 16z^2 + 100 = 0$ for z^2 or otherwise, determine the roots of the equation in Cartesian form.   (4 marks)

Supplementary page

Question number: _____