Instructions to candidates

1. **All** questions should be attempted.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare answer pages are included at the end of this booklet. If you need to use the space to continue an answer, indicate in the original answer space where the question is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Structure of this paper

	Questions	Marks available	Your score
Section 1	1	2	
	2	6	
	3	5	
	4	6	
	5	4	
	6	7	
	7	5	
	8	5	
	Total:	**40**	
Section 2	9	3	
	10	9	
	11	6	
	12	8	
	13	6	
	14	5	
	15	5	
	16	7	
	17	5	
	18	6	
	19	10	
	20	10	
	Total:	**80**	
	Total marks = 120		
			%

See next page

Section One: Calculator-free **(40 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the space provided.

Suggested working time for this section is 50 minutes.

Question 1 **(2 marks)**

Given the matrix $A = \begin{bmatrix} x & 12 \\ 3 & 9 \end{bmatrix}$ find all values of x such that the matrix is singular.

Solution
For $\boldsymbol{A}$ to be singular determinant is equal to 0 Hence $\lvert\boldsymbol{A}\rvert = 0$ $9x - 36 = 0$ $x = 4$
Specific behaviours
✓ determines determinant is equal to 0 ✓solves for x

DO NOT WRITE IN THIS AREA

Question 2 **(6 marks)**

If $z_1 = 2 + 2\sqrt{3}\,i$ and $z_2 = 6\text{cis}\left(-\frac{\pi}{3}\right)$, determine in simplest form:

(a) iz_1

[1]

Solution
$iz_1 = -2\sqrt{3} + 2i$ or $4\text{cis}\left(\frac{5\pi}{6}\right)$
Specific behaviours
✓correct value for iz_1

(b) $\frac{1}{z_2}$

[1]

Solution
$\frac{1}{z_2} = \frac{1}{6}cis\left(\frac{\pi}{3}\right)$ or $\frac{1}{12} + \frac{\sqrt{3}}{12}\,i$
Specific behaviours
✓ correct value for $\frac{1}{z_2}$

(c) z_1z_2

[2]

Solution
$z_1z_2 = 4cis\left(\frac{\pi}{3}\right).6cis\left(\frac{-\pi}{3}\right)$ $= 24$
Specific behaviours
✓ convert z_1 to polar form ✓correct value for z_1z_2

(d) $z_1 + z_2$

[2]

Solution
$z_2 = 3 - 3\sqrt{3}\,i$ $z_1 + z_2 = 2 + 2\sqrt{3}i + 3 - 3\sqrt{3}i$ $= 5 - \sqrt{3}i$
Specific behaviours
✓ convert z_2 to Cartesian form ✓ correct value of $z_1 + z_2$

See next page

Question 3 **(5 marks)**

(a) Prove the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

[3]

Solution
LHS = $\cos(\theta + 2\theta)$ = $\cos\theta\cos 2\theta - \sin\theta\sin 2\theta$ = $\cos\theta(2\cos^2\theta - 1) - 2\sin^2\theta\cos\theta$ = $2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ = $4\cos^3\theta - 3\cos\theta$ = RHS
Specific behaviours
✓uses compound angle identity ✓uses $2\cos^2\theta - 1 = \cos 2\theta$ identity ✓uses $2\sin\theta\cos\theta = \sin 2\theta$ identity

(b) Hence or otherwise determine the indefinite integral $\int 2\cos^3\theta\ d\theta$

[2]

Solution
$\int 2\cos^3\theta\, d\theta = \frac{1}{2}\int (\cos 3\theta + 3\cos\theta)\, d\theta$ $= \frac{1}{2}\left(\frac{\sin 3\theta}{3} + (3\sin\theta)\right) + c$ $= \frac{\sin 3\theta}{6} + \frac{3\sin\theta}{2} + c$
Specific behaviours
✓rewrite $\int 2\cos^3\theta\, d\theta$ to the form $\frac{1}{2}\int(\cos 3\theta + 3\cos\theta)\, d\theta$ ✓integrates expression correctly

DO NOT WRITE IN THIS AREA

Question 4 **(6 marks)**

Determine the following integrals, writing your answers in simplified form.

(a) $\int \cos 2t \sin^5 2t \, dt$

[2]

Solution
$\int \cos 2t \sin^5 2t \, dt$ $= \frac{\sin^6 2t}{12} + c$
Specific behaviours
✓ applies the chain rule ✓ correct solution including the constant

(b) $\int \frac{4+4\cos x}{x+\sin x} dx$

[2]

Solution
$\int \frac{4+4\cos x}{x+\sin x} dx$ $= 4\ln\lvert x+\sin x\rvert + c$
Specific behaviours
✓ recognizes $\frac{f'(x)}{f(x)}$ ✓ correct solution

(c) $\int_1^e \frac{(1+\ln x)^2}{x} dx$

[2]

Solution
$\int_1^e \frac{(1+\ln x)^2}{x} dx$ $= \left[\frac{(1+\ln x)^3}{3}\right]_1^e$ $= \frac{7}{3}$
Specific behaviours
✓ correct antiderivative ✓ correct answer

DO NOT WRITE IN THIS AREA

Question 5 **(4 marks)**

Prove $\left(1-i\sqrt{3}\right)^n+\left(1+i\sqrt{3}\right)^n=2^{n+1}\cos\left(\frac{\pi n}{3}\right)$ where $n=1,2,3,\ldots.$

Solution
$\text{LHS}=\left[2cis\left(-\frac{\pi}{3}\right)\right]^n+\left[2cis\left(\frac{\pi}{3}\right)\right]^n$ $=2^n cis\left(-\frac{\pi n}{3}\right)+2^n cis\left(\frac{\pi n}{3}\right)$ $=2^n\left[\cos\left(-\frac{\pi n}{3}\right)+i\sin\left(-\frac{\pi n}{3}\right)+\cos\left(\frac{\pi n}{3}\right)+i\sin\left(\frac{\pi n}{3}\right)\right]$ $=2^n\left[\cos\left(\frac{\pi n}{3}\right)-i\sin\left(\frac{\pi n}{3}\right)+\cos\left(\frac{\pi n}{3}\right)+i\sin\left(\frac{\pi n}{3}\right)\right]$ $=2^n\left[2\cos\left(\frac{\pi n}{3}\right)\right]$ $=2^{n+1}\cos\left(\frac{\pi n}{3}\right)$ $=\text{RHS}$
Specific behaviours
✓expresses complex numbers on LHS in polar form ✓applies de Moivre's theorem ✓uses sin θ = -sin θ and cos θ = cos (-θ) ✓establish correct conclusion

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See next page

Question 6 **(7 marks)**

Given the curve $\sin(xy) + y^2 - \left(\frac{4}{\pi}\right)x = \frac{4}{\pi}$ find:

(a) $\frac{dy}{dx}$

[3]

Solution
$\cos(xy)y + \cos(xy)x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{4}{\pi} = 0$ $\frac{dy}{dx}(\cos(xy)x + 2y) = \frac{4}{\pi} - \cos(xy)y$ $\frac{dy}{dx} = \frac{\frac{4}{\pi} - \cos(xy)y}{\cos(xy)x + 2y}$
Specific behaviours
✓ correctly differentiates implicitly ✓ correctly applies product rule ✓correctly rearranges equation for $\frac{dy}{dx}$

(b) the value of x when $y = 0$

[1]

Solution
$y = 0 \quad \sin(x(0)) + 0^2 - \left(\frac{4}{\pi}\right)x = \frac{4}{\pi}$ $-\left(\frac{4}{\pi}\right)x = \frac{4}{\pi}$ $x = -1$
Specific behaviours
✓ correct value of x

DO NOT WRITE IN THIS AREA

(c) using the incremental formula, find the approximate change in x when y changes from 0 to 0.1.

[3]

Solution
$\delta x \approx \frac{dx}{dy}\delta y$ $\approx \frac{\cos(xy)x + 2y}{\frac{4}{\pi} - \cos(xy)y}\delta y$ $\approx \frac{\cos(-1\times 0)(-1) + 2(0)}{\frac{4}{\pi} - \cos(-1\times 0)(0)}0.1$ $= -\frac{\pi}{40}$ i.e. x decreases by $\frac{\pi}{40}$
Specific behaviours
✓ chooses increment formula in correct form ✓ correct substitutions for x, y and δy ✓ correct value of δx

DO NOT WRITE IN THIS AREA

Question 7 **(5 marks)**

A particle in simple harmonic motion has acceleration such that $\frac{d^2x}{dt^2} + \pi^2 x = 0$ where is x the displacement from the origin O.
The particle is instantaneously at rest at time $t = 0$ seconds and at position $x = 4$.

(a) Find the displacement of x as a function of t.

[3]

Solution
Since $x = 4$ and $v = 0$ when $t = 0$, then $x = 4\cos(nt)$ Since $\frac{d^2x}{dt^2} = -\pi x$, then $n = \pi$. Hence, $x = 4\cos(\pi t)$ or $x = 4\sin\left(\pi t + \frac{\pi}{2}\right)$
Specific behaviours
✓correct period ✓correct amplitude ✓correctly states displacement of x as a function of t

(b) Find the maximum velocity of the particle.

[2]

Solution
$v = -4\pi\sin(\pi t)$ Since $-4\pi \leq v \leq 4\pi$ Maximum velocity is 4π ms^{-1}
Specific behaviours
✓correctly differentiates ✓correct maximum velocity

DO NOT WRITE IN THIS AREA

Question 8 **(5 marks)**

Use the substitution $u = e^{2x} - 1$ to evaluate $\int_0^{\ln 3} (4e^{2x}) \sqrt[3]{e^{2x} - 1}\, dx$.

Solution	
$\int_0^{\ln 3} (4e^{2x}) \sqrt[3]{e^{2x} - 1}\, dx$	$u = e^{2x} - 1$ $du = 2e^{2x} dx$
$= \int_0^8 2u^{\frac{1}{3}}\, du$	When $x = 0$, $u = 0$ When $x = \ln 3$, $u = 8$
$= \left[\frac{3u^{\frac{4}{3}}}{2} \right]_0^8$	
$= 24$	

Specific behaviours
✓ Substitutes for x
✓ Substitutes for dx
✓ Changes upper and lower limits
✓ Simplifies and finds antiderivative
✓ Evaluates

DO NOT WRITE IN THIS AREA

End of questions

Student Name: **SOLUTIONS**

Methodist Ladies' College Semester 2, 2011

3CD MATHEMATICS: SPECIALIST

Question/Answer Booklet – Section 2 – Calculator-assumed

Teacher's Name: _____

Time allowed for this paper

Section	Reading	Working
~~**Calculator-free**~~	~~5 minutes~~	~~50 minutes~~
Calculator-assumed	10 minutes	100 minutes

Materials required/recommended for this paper

Section Two (Calculator-assumed): 80 marks

To be provided by the supervisor
Section Two Question/Answer booklet Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important Note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

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Section Two: Calculator-assumed **(80 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the space provided.

Suggested working time for this section is 100 minutes.

Question 9 **(3 marks)**

Use proof by exhaustion to prove that 127 is a prime number.

Solution
127 ÷ 2 leaves a remainder of 1 127 ÷ 3 leaves a remainder of 1 127 ÷ 5 leaves a remainder of 2 127 ÷ 7 leaves a remainder of 1 127 ÷ 11 leaves a remainder of 6 As $13 > \sqrt{127}$ there can be no other prime factors of 127. $\therefore$ 127 has no other factors other than 1 and itself i.e. 127 is prime
Specific behaviours
✓ Follows process of exhaustion for dividing 127 by all possible prime numbers ✓ States why the process is exhausted ✓ establishes correct conclusion

DO NOT WRITE IN THIS AREA

Question 10 **(9 marks)**

The triangle ABC with vertices $A(0, 0)$, $B(3, 1)$ and $C(-1, 2)$ is transformed by the matrix $\mathbf{M} = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$ to produce triangle $A'B'C'$.

(a) Determine the coordinates of A' and C'.

[2]

Solution
$A'(0, 0)$. $C'(4, 1)$
Specific behaviours
✓ correctly determines A' ✓ correctly determines C'

(b) The area of triangle $A'B'C'$ is 7 square units. What is the area of triangle ABC?

[2]

Solution
$\lvert\mathbf{M}\rvert = 0 - (-2)$ $= 2$ Area of ABC = $7 \div \lvert 2 \rvert$ $= 3.5$ units^2
Specific behaviours
✓ correctly calculates the value of the determinant ✓ correct area of triangle

(c) Matrix $\boldsymbol{M}$ represents a combination of transformation $\boldsymbol{X}$ followed by transformation $\boldsymbol{Y}$. If the matrix for transformation $\boldsymbol{X} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, determine the matrix for transformation $\boldsymbol{Y}$ and describe the geometric transformation $\boldsymbol{Y}$ represents.

[3]

Solution
$\mathbf{YX} = \mathbf{M}$ $\mathbf{Y} = \mathbf{M}\,\mathbf{X}^{-1}$ $\boldsymbol{Y}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$ $\boldsymbol{Y} = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^{-1}$ $\boldsymbol{Y} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ $\boldsymbol{Y}$ represents a dilation scale factor 2 parallel to x-axis
Specific behaviours
✓ correctly acknowledges $\boldsymbol{YX} = \boldsymbol{M}$ ✓ correctly calculates matrix $\boldsymbol{Y}$ ✓ correctly describes matrix $\boldsymbol{Y}$ geometrically

See next page

(d) The triangle $A'B'C'$ then undergoes a shear of factor k parallel to the y-axis such that the image of the coordinate C' is (4, -3). Determine the value of k.

[2]

Solution
$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$ $4k + 1 = -3$ $k = -1$
Specific behaviours
✓ obtains an expression to solve involving k ✓ correctly calculates k

DO NOT WRITE IN THIS AREA

Question 11 **(6 marks)**

The complex number $z = x + iy$ satisfies the inequality $|(\bar{z})^2 - z^2| \leq 16$.

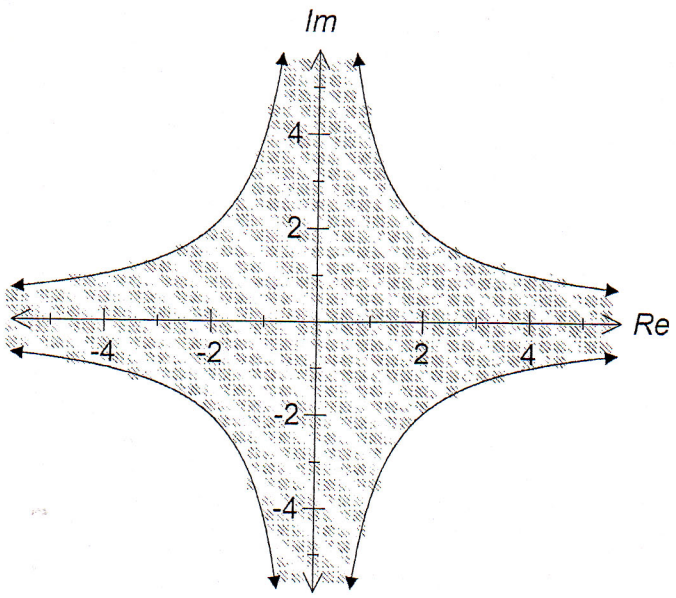
(a) Show that $|xy| \leq 4$.

[3]

Solution
$\left\|(x-yi)^2-(x+yi)^2\right\| \leq 16$ $\left\|x^2-2xyi-y^2-x^2-2xyi+y^2\right\| \leq 16$ $\left\|-4xyi\right\| \leq 16$ $\left\|xy\right\| \leq 4$
Specific behaviours
✓ substitutes $x - yi$ and $x + yi$ into the inequality ✓ expands and simplifies expression ✓ deduces the required result

(b) Hence sketch the set of all complex numbers z that satisfy the inequality $|(\bar{z})^2 - z^2| \leq 16$ on the axes below.

[3]

Solution

Specific behaviours
✓ correct plotting of $xy = 4$ ✓ correct plotting of $xy = -4$ ✓ solid line and correct shading

See next page

Question 12 **(8 marks)**

(a) According to research carried out by a company the proportion of households switching between oil, gas and electric heat in the United States after 1 year is shown in the table below.

		To		
		Oil	Gas	Electric
From	Oil	70%	30%	0
	Gas	10%	80%	10%
	Electric	20%	0	80%

If the pattern of switching types of heating continues

(i) determine the proportion of current households using gas who will be using gas in 5 years time.

[2]

Solution

$$\begin{bmatrix} 0.7 & 0.3 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0.2 & 0 & 0.8 \end{bmatrix}^5 = \begin{bmatrix} 0.3242 & 0.5366 & 0.1392 \\ 0.2717 & 0.5031 & 0.2253 \\ 0.3577 & 0.2783 & 0.3639 \end{bmatrix}$$

50.3%

Specific behaviours

✓Sets up transition matrix from table and raises to the power of 5
✓ correct proportion of households

(ii) determine in the long term what proportion, to the nearest percent, of households will be using each of the three forms of heating?

[2]

Solution

As $n \to \infty$

$$\begin{bmatrix} 0.7 & 0.3 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0.2 & 0 & 0.8 \end{bmatrix}^n = \begin{bmatrix} 0.3077 & 0.4615 & 0.2308 \\ 0.3077 & 0.4615 & 0.2308 \\ 0.3077 & 0.4615 & 0.2308 \end{bmatrix}$$

Oil 31%, gas 46% and electric 23%

Specific behaviours

✓determines steady state matrix
✓ correct proportions for each method of heating

See next page

(b) A farmer is breeding marron in one of his dams. He has collected the following data on their breeding and survival rates in 2003.

Age (years)	1	2	3	4
Population	750	1200	900	600
Birth Rate	0	0.7	1.4	0.5
Survival Rate	0.7	0.6	0.5	0

(i) Construct a Leslie matrix, $\boldsymbol{L}$, to represent this population.

[1]

Solution

$$\begin{bmatrix} 0 & 0.7 & 1.4 & 0.5 \\ 0.7 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}$$

Specific behaviours

✓correctly sets up matrix from information in the table

(ii) What is the total population in 2009?

[1]

Solution

$$[1 \quad 1 \quad 1 \quad 1]\boldsymbol{L}^6 \begin{bmatrix} 750 \\ 1200 \\ 900 \\ 600 \end{bmatrix} = [5123.11]$$

The total population in 2009 is approximately 5123 marron.

Specific behaviours

✓correct total population

(iii) Over a period of time the population growth reaches a steady state of 6.5%. If in the long term the farmer wishes to maintain a stable population level in the dam what culling rate of each age group will the farmer need to set?

[2]

Solution

Let h % be culled each year.

$$\left(1 - \frac{h}{100}\right)(1.065) = 1$$

$$1 - \frac{h}{100} = \frac{1}{1.065}$$

$\therefore$ The culling rate will need to be 6.1%

Specific behaviours

✓ correctly determines k
✓ correctly determines the culling rate

See next page

Question 13 **(6 marks)**

The velocity of a particle that travels in a straight line is given by $v = 1 - \sqrt{2}\sin t, 0 \le t \le 2\pi$, where v is in m/s and t is in seconds.

(a) Determine the times when the particle is at rest.

[2]

Solution
$0 = 1 - \sqrt{2}\sin t$ $t = \frac{\pi}{4}$ sec or $t = \frac{3\pi}{4}$ sec
Specific behaviours
✓ recognises that $v = 0$ when the particle is at rest ✓ correctly determines the two times the particle is at rest for $0 \le t \le 2\pi$

(b) If the particle was initially at the origin determine an expression for its displacement.

[2]

Solution
$x = t + \sqrt{2}\cos t + c$ At $t = 0$ $x = 0$ $\therefore c = -\sqrt{2}$ $x = t + \sqrt{2}\cos t - \sqrt{2}$
Specific behaviours
✓ integrates velocity to determine displacement ✓ correctly determines the value of the constant and hence states the expression

(c) Determine the distance the particle travelled in the third second.

[2]

Solution
$\int_2^3 \left\|1 - \sqrt{2}\sin t\right\| dt$ $= 0.30\,\text{m}$
Specific behaviours
✓ integrates the absolute value of the velocity over the appropriate lower and upper limits ✓ correctly determines the distance travelled in the third second

DO NOT WRITE IN THIS AREA

Question 14 **(5 marks)**

A certain type of electronic circuit will remain in a stable state if the values of two variable resistors, x and y, satisfy the equation $\frac{1}{x}+\frac{1}{y}=0.005$.

In a particular circuit, the value of y is increasing at a rate of 15 units per second. At what rate must x be changing when $y = 1000$ for the circuit to remain stable?

Solution

EITHER

$$\frac{1}{x}+\frac{1}{y}=0.005$$

$$-\frac{1}{x^2}-\frac{1}{y^2}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{y^2}{x^2}$$

When $y=1000$ $x=250$

$$\frac{dx}{dt}=\frac{dx}{dy}.\frac{dy}{dt}$$

$$=-\frac{x^2}{y^2}.15$$

$$=-\frac{250^2}{1000^2}(15)$$

$$=-\frac{15}{16}$$

OR

$$\frac{1}{x}+\frac{1}{y}=0.005$$

$$-\frac{1}{x^2}\frac{dx}{dt}-\frac{1}{y^2}\frac{dy}{dt}=0$$

$$\frac{dx}{dt}=-\frac{x^2}{y^2}\frac{dy}{dt}$$

When $y=1000$ $x=250$

$$\frac{dx}{dt}=-\frac{250^2}{1000^2}(15)$$

$$=-\frac{15}{16}$$

x is decreasing at a rate of 0.9375 units/sec

Specific behaviours

- ✓ correctly determines expression for $\frac{dy}{dx}$
- ✓ correctly determines x when $y = 1000$
- ✓ uses chain rule with $\frac{dy}{dt}$
- ✓ correctly calculates $\frac{dx}{dt}$
- ✓ statement correctly interpreting the rate is decreasing

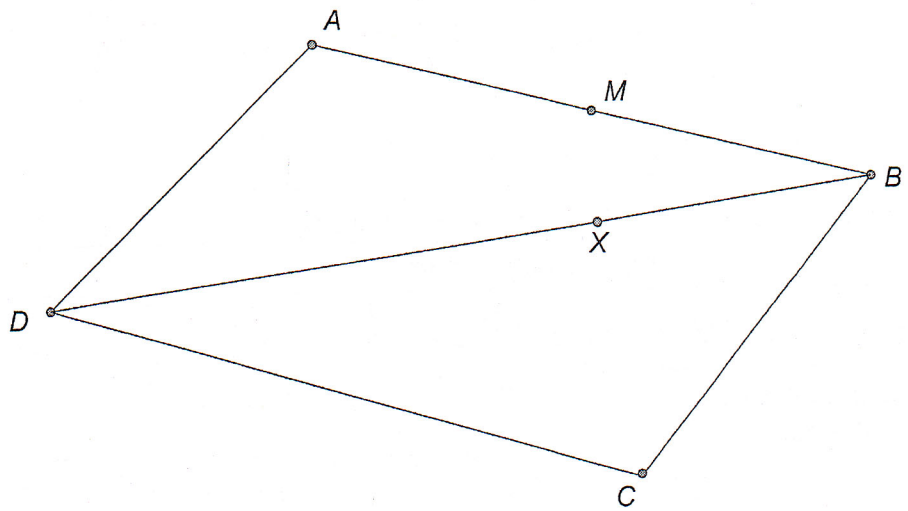
OR

- ✓ correctly differentiates with respect to t
- ✓ correctly determines expression for $\frac{dx}{dt}$
- ✓ correctly determines x when $y = 1000$
- ✓ correctly calculates $\frac{dx}{dt}$
- ✓ statement correctly interpreting the rate is decreasing

DO NOT WRITE IN THIS AREA

Question 15 **(5 marks)**

The diagram below shows parallelogram $ABCD$ where $\overrightarrow{AB} = \boldsymbol{a}$ and $\overrightarrow{BC} = \boldsymbol{b}$.
Point X divides DB internally in the ratio 2:1.
Point M is the midpoint of AB.



(a) Show that $\overrightarrow{DX} = \frac{2}{3}\boldsymbol{a} - \frac{2}{3}\boldsymbol{b}$

[1]

Solution
$\overrightarrow{DX} = \frac{2}{3}\overrightarrow{DB}$ $= \frac{2}{3}(\mathbf{a} - \mathbf{b})$
Specific behaviours
✓ correctly shows $\overrightarrow{DX}$

(b) Find $\overrightarrow{CX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

[1]

Solution
$\overrightarrow{CX} = \overrightarrow{CD} + \overrightarrow{DX}$ $= -\mathbf{a} + \frac{2}{3}(\mathbf{a} - \mathbf{b})$ $= -\frac{1}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}$
Specific behaviours
✓ determines correctly $\overrightarrow{CX}$

(c) Prove that points M, X and C are collinear.

[3]

Solution
$\overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM}$ $= -\mathbf{b} + \frac{1}{2}(-\mathbf{a})$ $= -\frac{1}{2}\mathbf{a} - \mathbf{b}$ $= \frac{3}{2}\overrightarrow{CX}$ Since $\overrightarrow{CM} = k\overrightarrow{CX}$ and point C is common, C, M and X are collinear.
Specific behaviours
✓ determines an expression for $\overrightarrow{CM}$ in terms of $\overrightarrow{CB}$ and $\overrightarrow{BM}$ ✓ shows is $\overrightarrow{CM}$ a scalar multiple of $\overrightarrow{CX}$ ✓ deduces the required result

DO NOT WRITE IN THIS AREA

Question 16 **(7 marks)**

A line and a plane are given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$ and $\boldsymbol{r} \bullet \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 29$.

(a) Given that the point (2, c, -2) lies on the plane determine c.

[2]

Solution
$\begin{pmatrix} 2 \\ c \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 29$ $c = 15.5$
Specific behaviours
✓ substitutes point into equation of plane ✓ correct value of c

(b) Find the position vector of the intersection between the line and the plane.

[3]

Solution
Intersect when $\begin{bmatrix} -1 \\ 1-2\lambda \\ 4+4\lambda \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = 29$ $\therefore \lambda = 5$ Hence point of intersection at $\begin{bmatrix} -1 \\ -9 \\ 24 \end{bmatrix}$
Specific behaviours
✓ substituting the in the equation of the plane ✓correctly determines value of λ ✓ correct point of intersection

DO NOT WRITE IN THIS AREA

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(c) Find the acute angle between the line and the plane.

[2]

Solution
Find angle, θ, between line and normal to plane: Using CAS, $\theta \approx 72.65°$ Angle between line and plane $= 90° - \theta \approx 17.35°$
Specific behaviours
✓ correctly determines θ ✓ correctly determines the acute angle between the line and plane

DO NOT WRITE IN THIS AREA

Question 17 **(5 marks)**

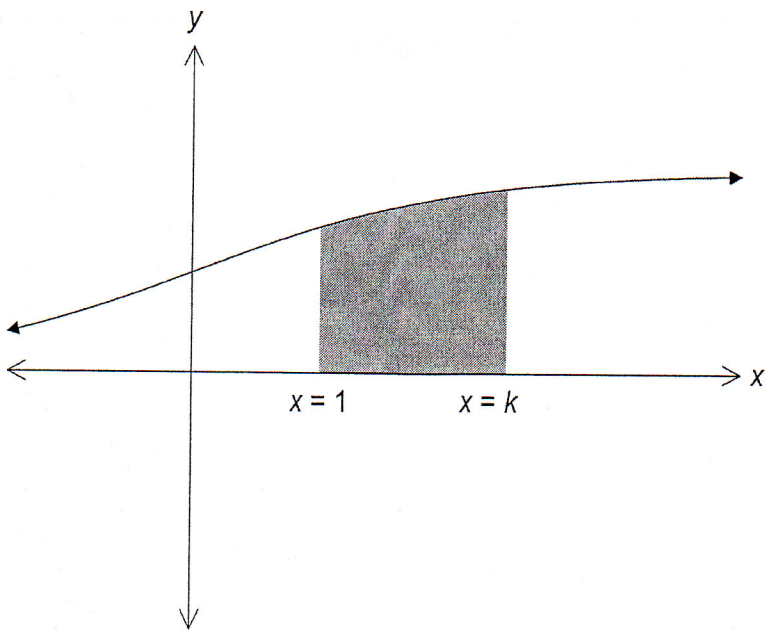
The solution to the differential equation $\frac{dy}{dx} = 3y - 6xy$ passes through the point where $x = 0$ and $y = 2$. Determine y when $x = 1$.

Solution
$\int \frac{dy}{y} = \int (3 - 6x)\, dx$ $\ln \lvert y \rvert = 3x - 3x^2 + c$ $\lvert y \rvert = e^c e^{3x-3x^2}$ $y = \pm e^c e^{3x-3x^2}$ $x = 0, y = 2 \quad 2 = \pm e^c$ $y = 2e^{3x-3x^2}$ When $x = 1,\ y = 2$
Specific behaviours
✓ separates variables ✓ correctly determines antiderivatives ✓ applies log definition ✓ substitutes (0, 2) to calculate constant ✓ deduce required result

DO NOT WRITE IN THIS AREA

Question 18 **(6 marks)**

The graph of $f(x) = \frac{e^x}{1+e^x}$ is shown below



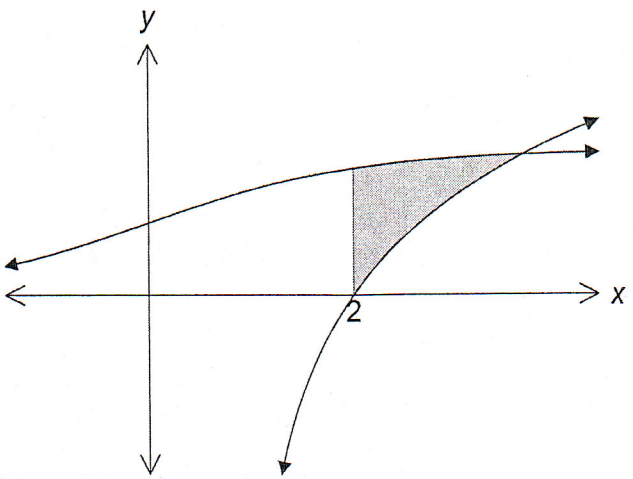
(a) Show that the area enclosed between the curve $y = f(x)$ and the x-axis between $x = 1$ and $x = k$, is $\ln\left(\frac{e^k+1}{e+1}\right)$.

[3]

Solution
$\int_1^k \frac{e^x}{1+e^x}\,dx$ $= \left[\ln\left\|1+e^x\right\|\right]_1^k$ $= \ln\left\|1+e^k\right\| - \ln\left\|1+e^1\right\|$ $= \ln\left(\frac{1+e^k}{1+e}\right)$
Specific behaviours
✓recognises $\frac{f'(x)}{f(x)}$ and integrates correctly ✓substitutes upper/lower limits ✓uses log laws to simplify expression and deduce required result

DO NOT WRITE IN THIS AREA

(b) The graphs of $f(x)$ and $g(x) = \ln(x-1)$ are shown below. Determine the area bound by the two curves and the line $x = 2$.



[3]

Solution
Value of x at point of intersection $x \approx 3.650$ $\int_2^{3.650}\left(\frac{e^x}{1+e^x} - \ln(x-1)\right)dx$ $= 0.62$ units^2
Specific behaviours
✓determines x value at point of intersection of functions ✓determines an expression for the area between the curves ✓ correctly determines the area of the required region

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Question 19 **(10 marks)**

An object, *P*, has an initial position of $\begin{pmatrix} 5 \\ 2 \\ -9 \end{pmatrix}$ metres and is moving with a constant velocity of $\begin{pmatrix} -5 \\ 3 \\ 1 \end{pmatrix}$ metres per second.

(a) A second object, *Q*, is moving with constant velocity of $\begin{pmatrix} -3 \\ 2 \\ -4 \end{pmatrix}$ metres per second and collides with object *P* after six seconds.

Determine the initial distance apart of object *P* and object *Q*.

[4]

Solution

After t seconds, $\boldsymbol{r}_P(t) = \begin{pmatrix} 5-5t \\ 2+3t \\ -9+t \end{pmatrix}$, $\boldsymbol{r}_Q(t) = \begin{pmatrix} a-3t \\ b+2t \\ c-4t \end{pmatrix}$

Since $\boldsymbol{r}_P(6) = \boldsymbol{r}_Q(6)$,

$-25 = a - 18, \quad a = -7$

$20 = b + 12, \quad b = 8$

$-3 = c - 24, \quad c = 21$

At $t = 0$, distance apart $= \left|\begin{pmatrix} 5 \\ 2 \\ -9 \end{pmatrix} - \begin{pmatrix} -7 \\ 8 \\ 21 \end{pmatrix}\right| = \left|\begin{pmatrix} 12 \\ -6 \\ -30 \end{pmatrix}\right|$

≈ 32.9 m

Specific behaviours

✓ determines point of collision
✓ determines $\mathbf{r}_Q$
✓ determines ${}_Q\mathbf{r}_P$
✓ determine correct distance between *P* and *Q*

DO NOT WRITE IN THIS AREA

(b) A third object, *R*, is initially located at $\begin{pmatrix} 4 \\ -11 \\ 2 \end{pmatrix}$ metres and is also moving with a constant velocity $\begin{pmatrix} -7 \\ x \\ 2 \end{pmatrix}$ metres per second. Determine the value of x such that after 5 seconds the distance between objects *P* and *R* is minimised. State the minimum distance at this time.

[6]

Solution

$\boldsymbol{r}_R(t) = \begin{pmatrix} 4-7t \\ -11+xt \\ 2+2t \end{pmatrix}$

At $t = 5$, require $\left|\begin{pmatrix} -31 \\ -11+5x \\ 12 \end{pmatrix} - \begin{pmatrix} -20 \\ 17 \\ -4 \end{pmatrix}\right|$ is a minimum

i.e. $\left|\begin{pmatrix} -11 \\ -28+5x \\ 16 \end{pmatrix}\right|$ is a min

Using CAS, $x = 5.6$ and the minimum distance is 19.4 m

Specific behaviours

✓ determines $\boldsymbol{r}_P$ when t = 5
✓ determines $\boldsymbol{r}_R$ when t = 5
✓ determines ${}_P\boldsymbol{r}_R$ when t = 5
✓ Recognise need to find minimum of $|{}_P\boldsymbol{r}_R|$
✓ correct value of *x*
✓ correct minimum distance

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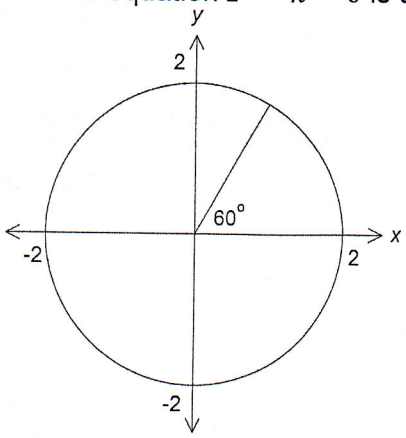
See next page

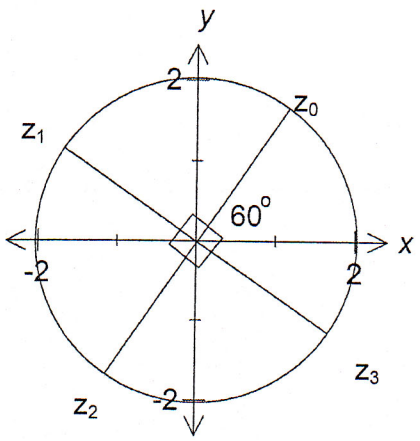
Question 20 **(10 marks)**

(a) One of the solutions to the equation $z^4 - k = 0$ is shown on the graph below.



(i) Make a sketch of the remaining roots on the axes above.

[1]

Solution

Specific behaviours
✓ correctly places remaining 3 roots on axes

(ii) Determine algebraically the value of k in Cartesian form.

[2]

Solution
$k = \left(2cis\frac{\pi}{3}\right)^4$ $= 16cis\frac{4\pi}{3}$ $= -8 - 8\sqrt{3}i$
Specific behaviours
✓ determines k in polar form ✓ determines k in Cartesian form

See next page

(b) (i) Solve $z^3 + 27 = 0$ algebraically, using de Moivre's theorem giving your answers in Cartesian form.

[5]

Solution
$z^3 = 27cis(\pi + 2k\pi)$ k an integer $z_k = 3cis\left(\frac{\pi + 2k\pi}{3}\right)$ $z_0 = 3cis\left(\frac{\pi}{3}\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}i$ $z_1 = 3cis(\pi) = -3$ $z_2 = 3cis\left(\frac{5\pi}{3}\right) = \frac{3}{2} - \frac{3\sqrt{3}}{2}i$
Specific behaviours
✓ Expresses z^3 in polar form ✓ Expresses z in polar form ✓✓ three roots in polar form (only ✓ for 2 roots in polar form and 0 for 1 root) ✓ each root in Cartesian form

(ii) Hence, solve $(z + 1)^3 + 27 = 0$ algebraically.

[2]

Solution
$z_k = 3cis\left(\frac{\pi + 2k\pi}{3}\right) - 1$ $z_0 = \frac{1}{2} + \frac{3\sqrt{3}}{2}i$ $z_1 = -4$ $z_2 = \frac{1}{2} - \frac{3\sqrt{3}}{2}i$
Specific behaviours
✓ recognises solutions are translation on $z^3 + 27 = 0$, real component decreases by1 ✓ correct three solutions in Cartesian form

DO NOT WRITE IN THIS AREA

End of Questions

Student Name: _____

Methodist Ladies' College Semester 2, 2011

3CD MATHEMATICS: SPECIALIST

Question/Answer Booklet – Section 1 – Calculators *NOT* allowed – Notes sheets *NOT* allowed

Teacher's Name: _____

Time allowed for this paper

Section	Reading	Working
Calculator-free	5 minutes	50 minutes
Calculator-assumed	10 minutes	100 minutes

Materials required/recommended for this paper

Section One (Calculator-free): 40 marks

To be provided by the supervisor
Section One Question/Answer booklet Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: nil

Section Two (Calculator-assumed): 80 marks

To be provided by the supervisor
Section Two Question/Answer booklet Formula sheet

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this course.

Important Note to candidates
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1234567-8

Student Name: _____

Methodist Ladies' College Semester 2, 2011

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Question/Answer Booklet – Section 1 – Calculators *NOT* allowed – Notes sheets *NOT* allowed

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1234567-8

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3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil** except in diagrams.

Structure of this paper

	Questions	Marks available	Your score
Section 1	1	2	
	2	6	
	3	5	
	4	6	
	5	4	
	6	7	
	7	5	
	8	5	
	Total:	**40**	
Section 2	9	3	
	10	9	
	11	6	
	12	8	
	13	6	
	14	5	
	15	5	
	16	7	
	17	5	
	18	6	
	19	10	
	20	10	
	Total:	**80**	
	Total marks = 120		
			%

See next page

Section One: Calculator-free **(40 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the space provided.

Suggested working time for this section is 50 minutes.

Question 1 **(2 marks)**

Given the matrix $A = \begin{bmatrix} x & 12 \\ 3 & 9 \end{bmatrix}$ find all values of x such that the matrix is singular.

Question 2 **(6 marks)**

If $z_1 = 2 + 2\sqrt{3}\,i$ and $z_2 = 6\text{cis}\left(-\frac{\pi}{3}\right)$, determine in simplest form:

(a) iz_1

[1]

(b) $\frac{1}{z_2}$

[1]

(c) $z_1 z_2$

[2]

(d) $z_1 + z_2$

[2]

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Question 3

(5 marks)

(a) Prove the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

[3]

(b) Hence or otherwise determine the indefinite integral $\int 2\cos^3\theta \ d\theta$

[2]

Question 4 **(6 marks)**

Determine the following integrals, writing your answers in simplified form.

(a) $\int \cos 2t \sin^5 2t \ dt$

[2]

(b) $\int \frac{4+4\cos x}{x+\sin x} dx$

[2]

(c) $\int_1^e \frac{(1+\ln x)^2}{x} dx$

[2]

DO NOT WRITE IN THIS AREA

Question 5

(4 marks)

Prove $\left(1-i\sqrt{3}\right)^n+\left(1+i\sqrt{3}\right)^n=2^{n+1}\cos\left(\frac{\pi n}{3}\right)$ where $n=1,2,3,\ldots$.

Question 6

(7 marks)

Given the curve $\sin(xy) + y^2 - \left(\frac{4}{\pi}\right)x = \frac{4}{\pi}$ find:

(a) $\frac{dy}{dx}$

[3]

(b) the value of x when $y = 0$

[1]

(c) using the incremental formula, find the approximate change in x when y changes from 0 to 0.1.

[3]

DO NOT WRITE IN THIS AREA

Question 7 **(5 marks)**

A particle in simple harmonic motion has acceleration such that $\frac{d^2x}{dt^2} + \pi^2 x = 0$ where is x the displacement from the origin O.
The particle is instantaneously at rest at time $t = 0$ seconds and at position $x = 4$.

(a) Find the displacement of x as a function of t.

[3]

(b) Find the maximum velocity of the particle.

[2]

Question 8 **(5 marks)**

Use the substitution $u = e^{2x} - 1$ to evaluate $\int_0^{\ln 3} (4e^{2x})\sqrt[3]{e^{2x} - 1}\, dx$.

End of Questions

Student Name: _____

Methodist Ladies' College Semester 2, 2011

3CD MATHEMATICS: SPECIALIST

Question/Answer Booklet – Section 2 – Calculator-assumed

Teacher's Name: _____

Time allowed for this paper

Section	Reading	Working
~~**Calculator-free**~~	~~5 minutes~~	~~50 minutes~~
Calculator-assumed	10 minutes	100 minutes

Materials required/recommended for this paper

Section Two (Calculator-assumed): 80 marks

To be provided by the supervisor
Section Two Question/Answer booklet Formula sheet

To be provided by the candidate
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1234567-8

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DO NOT WRITE IN THIS AREA

Section Two: Calculator-assumed **(80 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the space provided.

Suggested working time for this section is 100 minutes.

Question 9 **(3 marks)**

Use proof by exhaustion to prove that 127 is a prime number.

Question 10 **(9 marks)**

The triangle ABC with vertices $A(0, 0)$, $B(3, 1)$ and $C(-1, 2)$ is transformed by the matrix $\mathbf{M} = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$ to produce triangle $A'B'C'$.

(a) Determine the coordinates of A' and C'.

[2]

(b) The area of triangle $A'B'C'$ is 7 square units. What is the area of triangle ABC?

[2]

(c) Matrix $\boldsymbol{M}$ represents a combination of transformation $\boldsymbol{X}$ followed by transformation $\boldsymbol{Y}$. If the matrix for transformation $\boldsymbol{X} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, determine the matrix for transformation $\boldsymbol{Y}$ and describe the geometric transformation $\boldsymbol{Y}$ represents.

[3]

(d) The triangle $A'B'C'$ then undergoes a shear of factor k parallel to the y-axis such that the image of the coordinate C' is (4, -3). Determine the value of k.

[2]

DO NOT WRITE IN THIS AREA

Question 11 **(6 marks)**

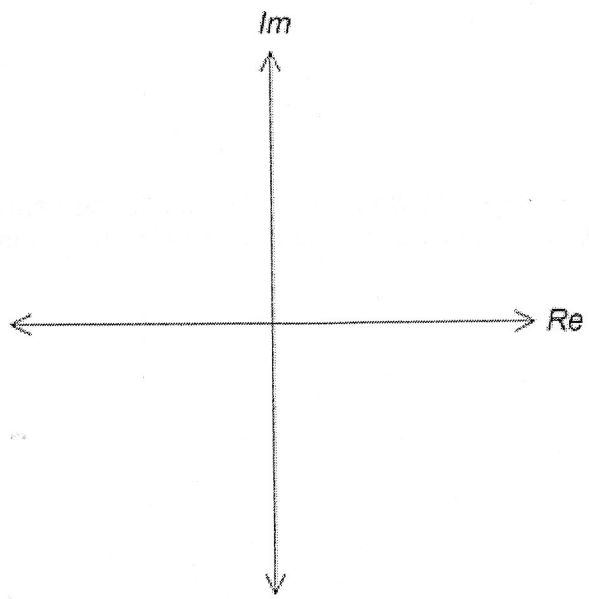
The complex number $z = x + iy$ satisfies the inequality $|(\bar{z})^2 - z^2| \leq 16$.

(a) Show that $|xy| \leq 4$.

[3]

(b) Hence sketch the set of all complex numbers z that satisfy the inequality $|(\bar{z})^2 - z^2| \leq 16$ on the axes below.

[3]



Question 12 **(8 marks)**

(a) According to research carried out by a company the proportion of households switching between oil, gas and electric heat in the United States after 1 year is shown in the table below.

		To		
		Oil	Gas	Electric
From	Oil	70%	30%	0
	Gas	10%	80%	10%
	Electric	20%	0	80%

If the pattern of switching types of heating continues

(i) determine the proportion of current households using gas who will be using gas in 5 years time.

[2]

(ii) determine in the long term what proportion, to the nearest percent, of households will be using each of the three forms of heating?

[2]

DO NOT WRITE IN THIS AREA

(b) A farmer is breeding marron in one of his dams. He has collected the following data on their breeding and survival rates in 2003.

Age (years)	1	2	3	4
Population	750	1200	900	600
Birth Rate	0	0.7	1.4	0.5
Survival Rate	0.7	0.6	0.5	0

(i) Construct a Leslie matrix, $\boldsymbol{L}$, to represent this population.

[1]

(ii) What is the total population in 2009?

[1]

(iii) Over a period of time the population growth reaches a steady state of 6.5%. If in the long term the farmer wishes to maintain a stable population level in the dam what culling rate of each age group will the farmer need to set?

[2]

Question 13 **(6 marks)**

The velocity of a particle that travels in a straight line is given by $v = 1 - \sqrt{2}\sin t, 0 \leq t \leq 2\pi$, where v is in m/s and t is in seconds.

(a) Determine the times when the particle is at rest.

[2]

(b) If the particle was initially at the origin determine an expression for its displacement.

[2]

(c) Determine the distance the particle travelled in the third second.

[2]

Question 14

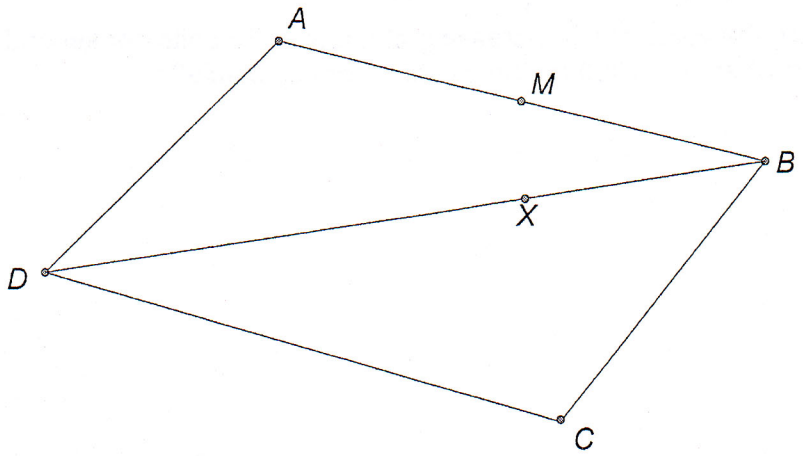
(5 marks)

A certain type of electronic circuit will remain in a stable state if the values of two variable resistors, x and y, satisfy the equation $\frac{1}{x} + \frac{1}{y} = 0.005$.

In a particular circuit, the value of y is increasing at a rate of 15 units per second. At what rate must x be changing when $y = 1000$ for the circuit to remain stable?

Question 15 **(5 marks)**

The diagram below shows parallelogram $ABCD$ where $\overrightarrow{AB} = \boldsymbol{a}$ and $\overrightarrow{BC} = \boldsymbol{b}$.
Point X divides DB internally in the ratio 2:1.
Point M is the midpoint of AB.



(a) Show that $\overrightarrow{DX} = \frac{2}{3}\boldsymbol{a} - \frac{2}{3}\boldsymbol{b}$

[1]

(b) Find $\overrightarrow{CX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

[1]

(c) Prove that points M, X and C are collinear. [3]

Question 16 **(7 marks)**

A line and a plane are given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$ and $\boldsymbol{r} \bullet \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 29$.

(a) Given that the point (2, c, -2) lies on the plane determine c.

[2]

(b) Find the position vector of the intersection between the line and the plane.

[3]

(c) Find the acute angle between the line and the plane.

[2]

DO NOT WRITE IN THIS AREA

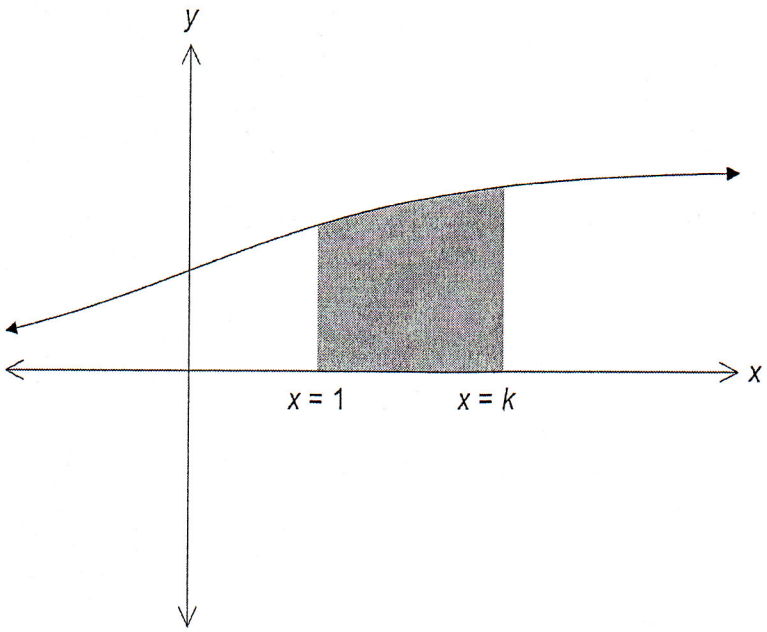
Question 17

(5 marks)

The solution to the differential equation $\frac{dy}{dx} = 3y - 6xy$ passes through the point where $x = 0$ and $y = 2$. Determine y when $x = 1$.

Question 18 **(6 marks)**

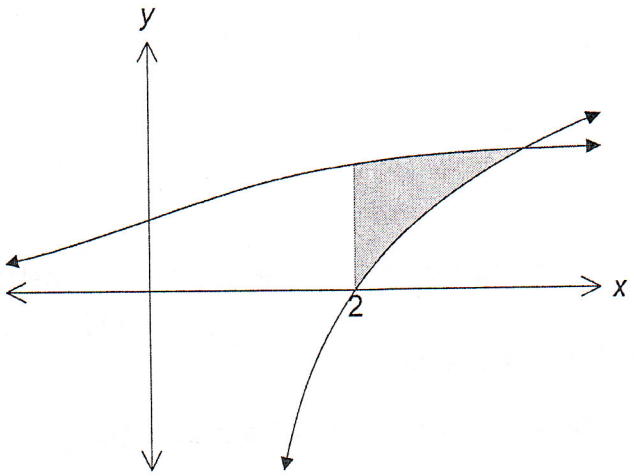
The graph of $f(x) = \frac{e^x}{1+e^x}$ is shown below



(a) Show that the area enclosed between the curve $y = f(x)$ and the x-axis between $x = 1$ and $x = k$, is $\ln\left(\frac{e^k+1}{e+1}\right)$.

[3]

(b) The graphs of $f(x)$ and $g(x) = \ln(x - 1)$ are shown below. Determine the area bound by the two curves and the line $x = 2$.



[3]

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(a) A second object, Q, is moving with constant velocity of $\begin{pmatrix} -3 \\ 2 \\ -4 \end{pmatrix}$ metres per second and collides with object P after six seconds.

Determine the initial distance apart of object P and object Q.

[4]

(b) A third object, R, is initially located at $\begin{pmatrix} 4 \\ -11 \\ 2 \end{pmatrix}$ metres and is also moving with a constant velocity $\begin{pmatrix} -7 \\ x \\ 2 \end{pmatrix}$ metres per second. Determine the value of x such that after 5 seconds the distance between objects P and R is minimised. State the minimum distance at this time.

[6]

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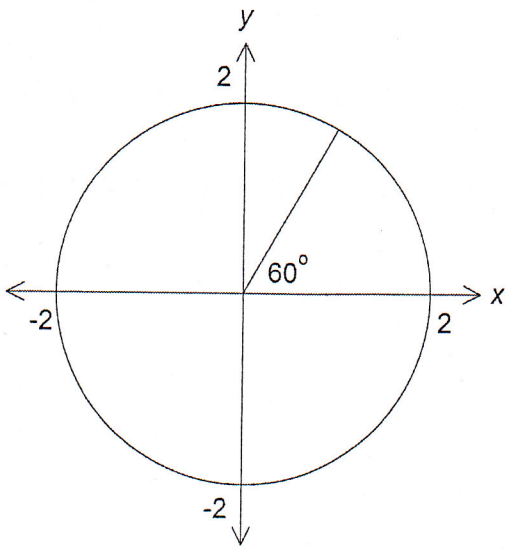

(i) Make a sketch of the remaining roots on the axes above.

[1]

(ii) Determine algebraically the value of k in Cartesian form.

[2]

(b) (i) Solve $z^3 + 27 = 0$ algebraically, using de Moivre's theorem giving your answers in Cartesian form.

[5]

(ii) Hence, solve $(z + 1)^3 + 27 = 0$ algebraically.

[2]