



SHENTON  
COLLEGE

# ATMAS Mathematics Specialist

## 2018 Test 2

Calculator Free

Name: .....

Time Allowed : 50 minutes

Marks	/52
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**Materials allowed:** No special materials.

***All necessary working and reasoning must be shown for full marks.***

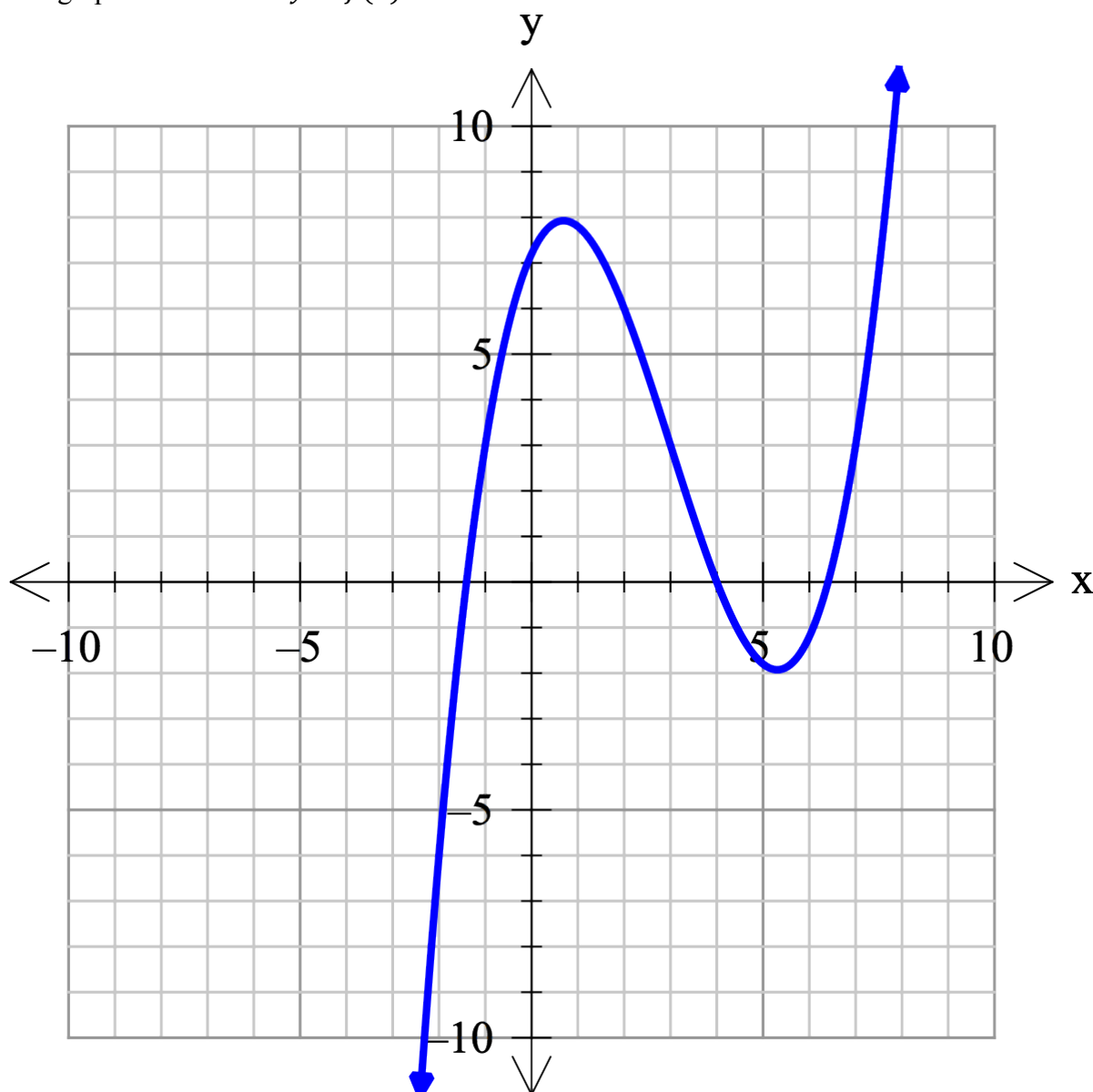
*Where appropriate, answers should be given in exact values.*

*Marks may not be awarded for untidy or poorly arranged work.*

**1** If  $f(x) = \frac{1}{x-1}$  and  $g(x) = x^2 - 3$ ,

Determine the domain and range of the composition  $f(g(x))$ . (5)

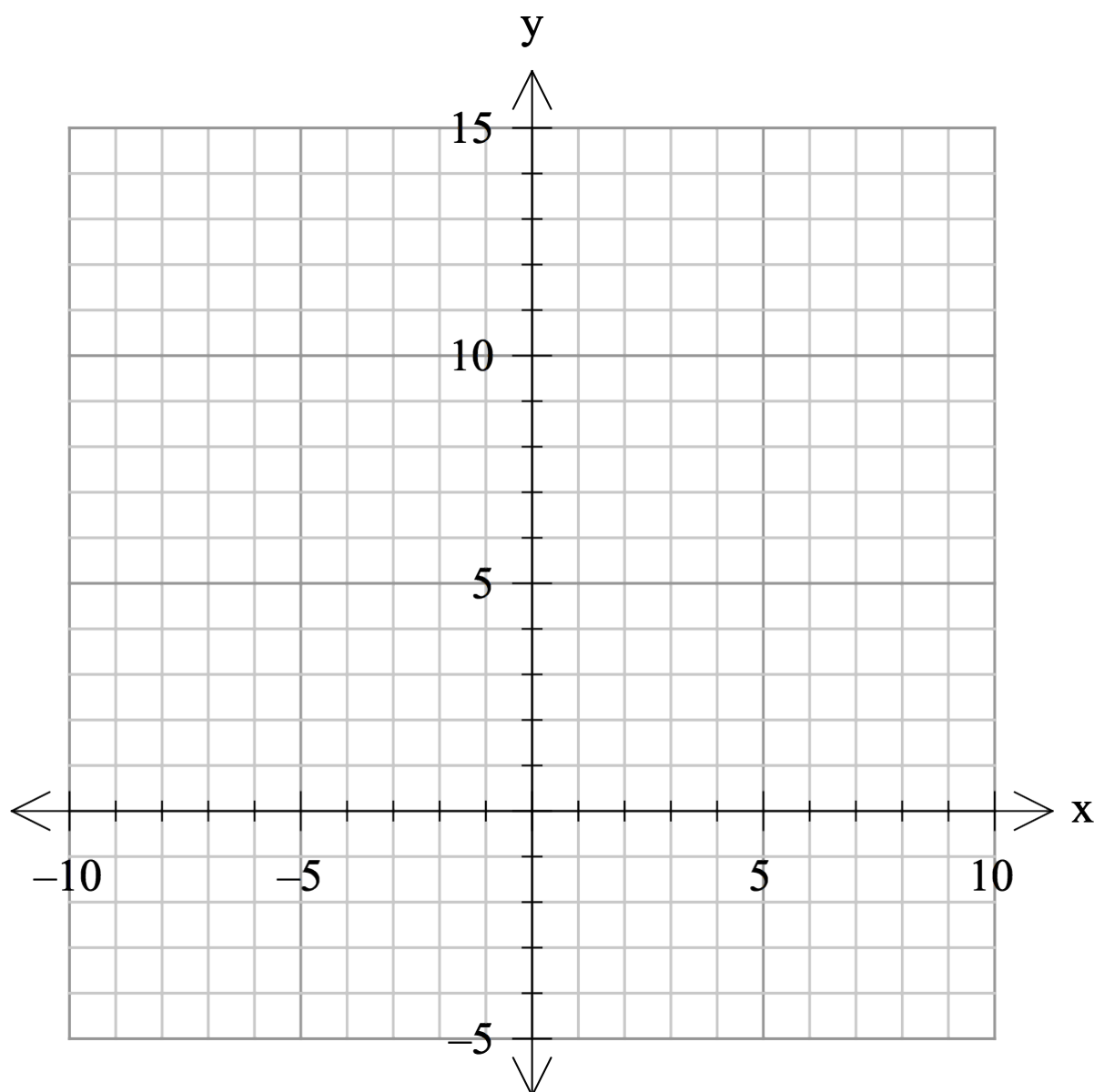
- 2 The graph below shows  $y = f(x)$ .



- a) Add a sketch of  $f^{-1}(x)$  to the axes above, indicating at least 3 key points. (4)
- b) Explain why  $f^{-1}(x)$  is not a function. (1)
- c) Mark on your sketch of  $f^{-1}(x)$  the points where it would intersect with  $\frac{1}{f^{-1}(x)}$ . (2)  
(Do not graph  $\frac{1}{f^{-1}(x)}$ .)

**3** a) Sketch the graph of  $y = |x - 3| + |x + 5|$

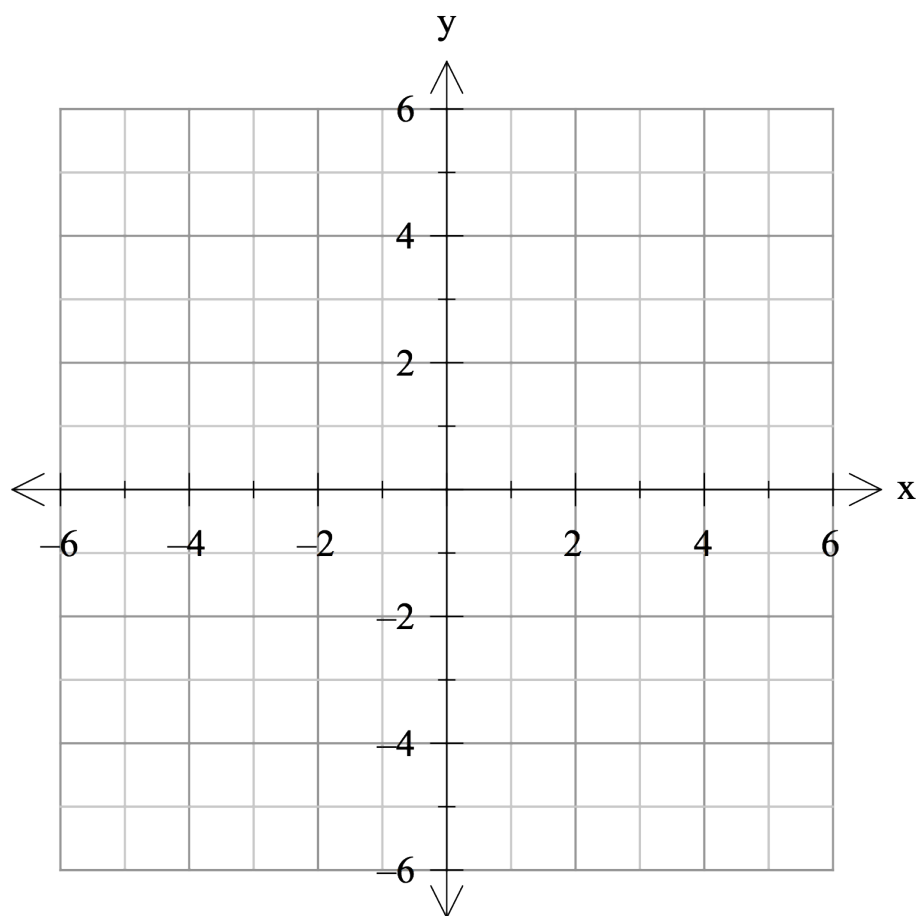
(4)



b) Hence or otherwise solve  $|x - 3| + |x + 5| = 12$

(2)

- 4** If  $f(x) = (x - 2)^2 - 1$ , sketch  $|f(|x|)|$  on the axes below. (3)



- 5** a) Determine a vector equation for the line parallel to  $5\mathbf{i} - 4\mathbf{k}$  and passing through the point  $-3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ . (2)

- b) Show whether the line from part a) intersects with the line  $\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}$  (3)

**6** Two spheres are defined by the equations  $S_1: \left| \mathbf{r} - \begin{pmatrix} -3 \\ 5 \\ -4 \end{pmatrix} \right| = 4$  and  $S_2: \left| \mathbf{r} - \begin{pmatrix} -1 \\ -1 \\ -7 \end{pmatrix} \right| = 3$  (3)

Determine whether or not the spheres touch, and if they do, describe the nature of their contact.

**7** A plane contains the points given by the position vectors  $\begin{pmatrix} 1 \\ 6 \\ -1 \end{pmatrix}$ ,  $\begin{pmatrix} 5 \\ 8 \\ -1 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ .  
a) Write a vector equation for the plane. (3)

b) Write the Cartesian equation for the plane.

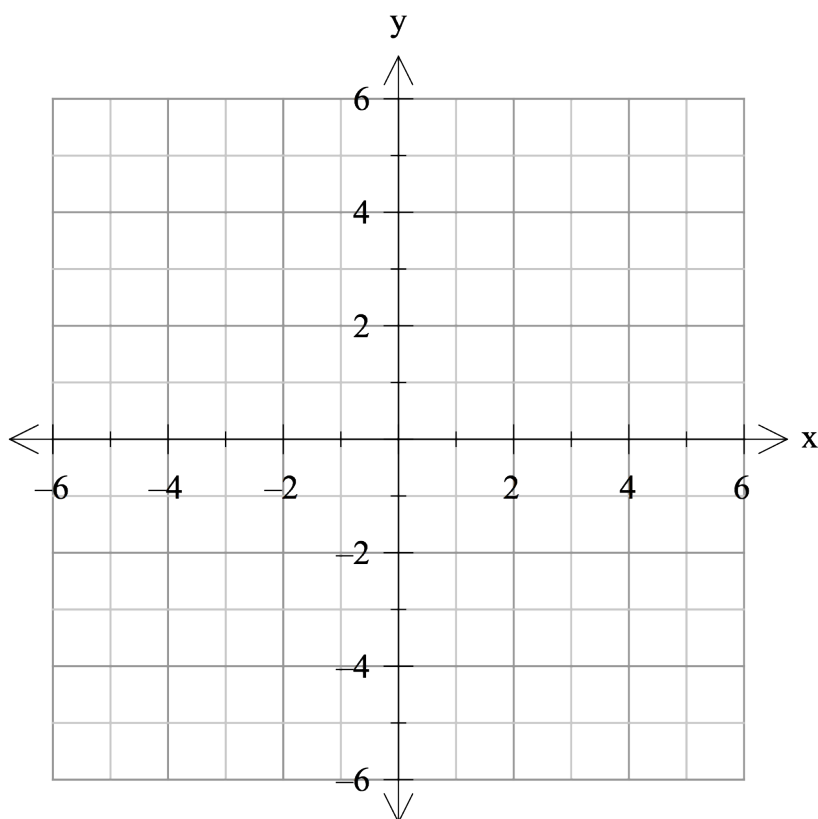
(4)

c) Give the equation of a line parallel to the plane and passing through the point  $\begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}$ . (1)

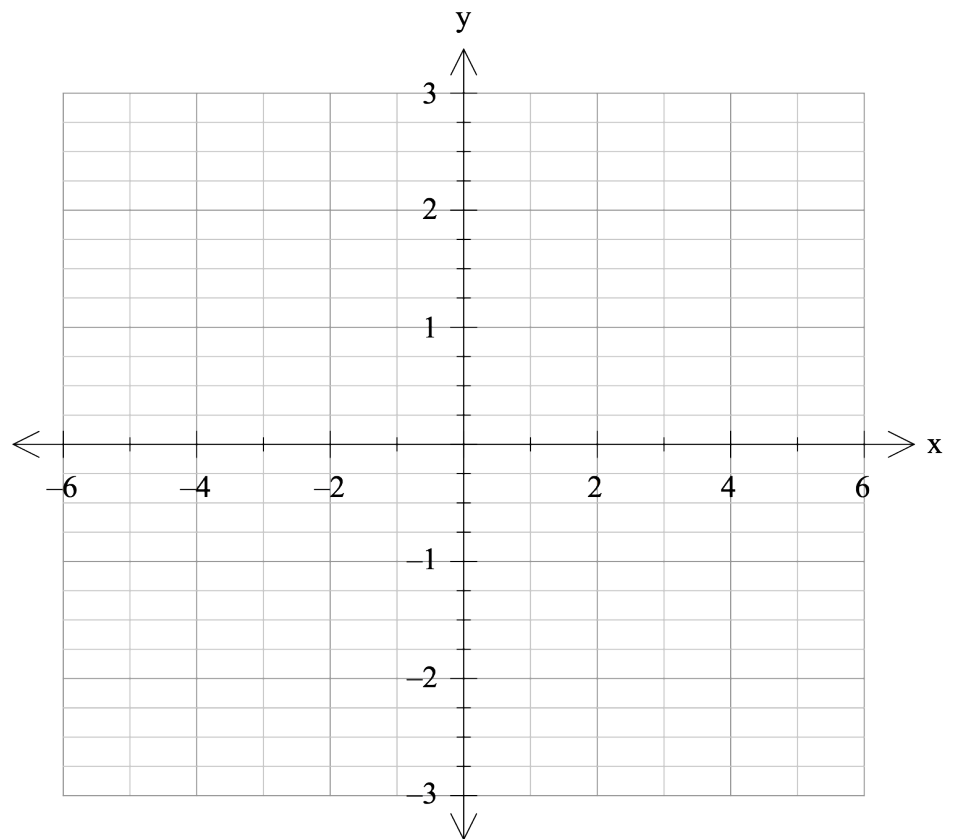
**8** Sketch the following rational functions.

a)  $y = \frac{2x^2 - 7x + 4}{2x - 1}$

(4)



b)  $y = \frac{9}{x^2 - 2x - 8}$  , given that  $f''(x) = -\frac{54(x^2 - 2x + 4)}{(x^2 - 2x - 8)^3}$  and  $f''(1) = -\frac{2}{9}$  (5)





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(6)

Two particles are moving through free space. Particle A starts at position  $\begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix}$  and is moving with constant velocity  $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ . Particle B is initially at  $\begin{pmatrix} 5 \\ -6 \\ -8 \end{pmatrix}$  and moving with velocity  $\begin{pmatrix} -3 \\ 1 \\ 3 \end{pmatrix}$ . All distances are in kilometres and time is in seconds. Determine the time at which the two particles are closest to each other, and the size of that minimum separation.