## Circle Geometry Revision
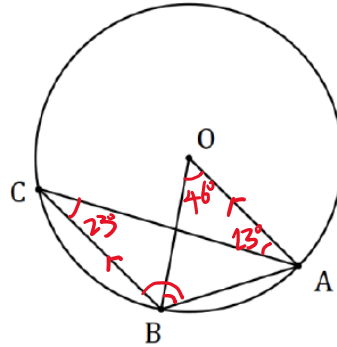
**Question 1**                                                          **(4 marks)**

In the diagram below (not drawn to scale) $A, B$ and $C$ lie on the circle with centre $O$ and $OA$ is parallel to $CB$.



Determine, with reasons, the size of $\angle OBA$ and the size of $\angle ABC$ when $\angle OAC = 23°$.

$\because OA /\!/ CB$

$\therefore \angle ACB = \angle OAC = 23°$ (alternate angles)

$\therefore \angle AOB = 2\angle ACB = 46°$ (angle at centre)

$\because OA = OB$ (radii)

$\therefore \angle OBA = (180 - 46) \div 2 = 67°$ (base angle in an isosceles $\triangle$)

$\because \angle CBO = \angle AOB = 46°$ (alternate angles)

$\therefore \angle ABC = 67 + 46 = 113°$

✓ $\angle ACB$ with reason

✓ $\angle AOB$ with reason

✓ $\angle OBA$ with reason

✓ $\angle ABC$

**Question 2** **(8 marks)**

(a)   $A, B, C$ and $D$ lie on a circle with diameter $AC$ (diagram not to scale). Determine, with reasons, the size of $\angle BDC$ when $\angle BCA = 40°$.   (4 marks)
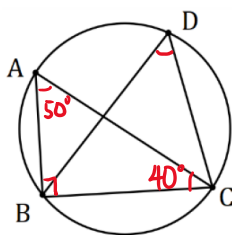
$\angle ABC = 90°$ (angle in semi-circle)

$\therefore \angle BAC = 90 - 40 = 50°$

$\therefore \angle BDC = 50°$ (Angles in the same segment are equal.)

✓ $\angle BAC = 90°$

✓ Gives reason (angle in semi-circle)

✓ Correct value of $\angle BDC$

✓ Gives reason

(b)   $K, L$ and $M$ lie on a circle (diagram not to scale). Secant $KN$ cuts the circle at $L$ and $JN$ is a tangent to the circle at $M$. Given that $\angle LNM = 33°$ and $\angle LMN = 43°$, determine the size of $\angle MKL$ and the size of $\angle KMJ$. Justify your answer.   (4 marks)
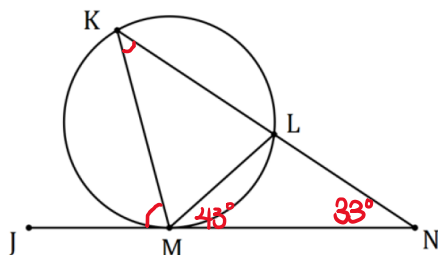
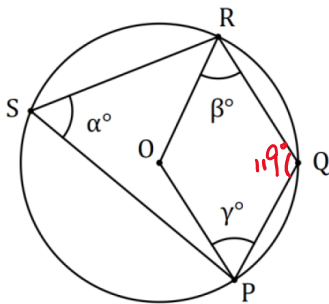$\angle MKL = \angle LMN = 43°$ (alternate segments)

$\angle KMJ = \angle LNM + \angle MKL$

$= 33° + 43°$

$= 76°$

(exterior angle)

✓ Correct value of $\angle MKL$

✓ Gives reason (alternate segments)

✓ Correct value of $\angle KMJ$

✓ Gives reason (exterior angle)

**Question 3**                                                                                    **(5 marks)**

In the diagram below (not drawn to scale) $P, Q, R$ and $S$ lie on the circle with centre $O$. Determine, with reasons, the size of angles $\alpha$, $\beta$ and $\gamma$ given that $\angle PQR = 119°$ and $3\beta = 4\gamma$.

$\alpha = 180 - 119 = 61°$

(Opposite angles in <mark>cyclic quadrilateral</mark> are supplementary.)

$\angle ROP = 2\alpha = 2 \times 61 = 122°$

( <mark>Angle at the centre</mark> is twice angle at the circumference.)

$\beta + \gamma = 360 - 119 - 122 = 119°$

( <mark>Angle sum of a quadrilateral</mark> equals to $360°$.)

$\therefore \beta = \frac{4}{3}\gamma$

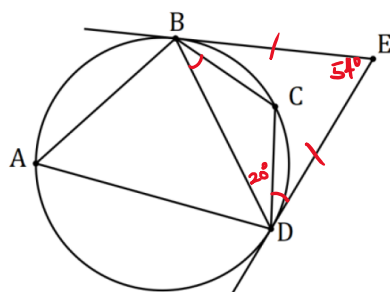$\therefore \frac{4}{3}\gamma + \gamma = 119°$

$\Rightarrow \gamma = 51°$

$\Rightarrow \beta = 68°$

✓ correct $\alpha$ with reason

✓ correct value of $\angle ROP$ with reason

✓ Forms equation for $\beta + \gamma$

✓ correct $\beta$

✓ correct $\gamma$

**Question 4**                                                   **(9 marks)**

(a)     In the diagram below (not drawn to scale) $A, B, C$ and $D$ lie on a circle and $EB$ and $ED$ are tangents to the circle. If $\angle BED = 54°$ and $\angle CDB = 20°$, determine the size of $\angle CBD$. Justify your answer.

                                                                          (4 marks)



$BE = DE$

( Two tangents to a circle are equal.)

$\therefore \angle EDB = \angle EBD = \frac{1}{2}(180-54) = 63°$

( Base angles in an isosceles triangle are equal.)

$\therefore \angle EDC = 63-20 = 43°$

$\therefore \angle CBD = \angle EDC = 43°$

( Alternate segments )
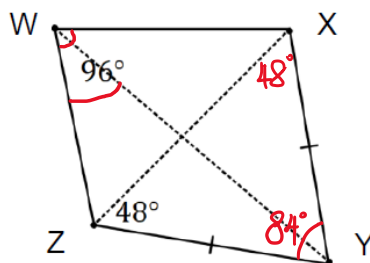
✓ $BE = DE$ with reason
✓ correct value of $\angle EDB$ with reason
✓ Correct value of $\angle EDC$
✓ correct value of $\angle CBD$ with reason

(b)     Quadrilateral $WXYZ$ is such that $YX = YZ$, $\angle XWZ = 96°$ and $\angle XZY = 48°$.

    (i)      Sketch a diagram to show this information.                      (1 mark)



✓ Correct diagram

    (ii)     Show that $WXYZ$ is cyclic and hence determine, with reasons, the size of $\angle YWZ$.                                                         (4 marks)

$\therefore YX = YZ$

$\therefore \angle XZY = \angle ZXY = 48°$ ( base angles in an isosceles $\triangle$)

$\therefore \angle XYZ = 180 - 2 \times 48 = 84°$

$\therefore \angle XZY + \angle XYZ = 96 + 84 = 180°$

$\therefore WXYZ$ is cyclic. ( opposite angles are supplementary.)

$\therefore \angle YWZ = \angle YXZ = 48°$

( angles in the same segment are equal.)

✓ Correct value of $\angle XYZ$ with reason
✓ Uses supplementary angles to prove cyclic
✓ Correct value of $\angle YWZ$
✓ Gives reason ( angles in the same segment)