SHENTON
COLLEGE

# SHENTON COLLEGE

**Examination Semester One 2017**
**Question/Answer Booklet**

# MATHEMATICS SPECIALIST UNIT 3

## Section Two
## (Calculator-assumed)

Your name _____

## Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

## Material required/recommended for this section

**To be provided by the supervisor**
Question/answer booklet for Section Two.
Formula Sheet (retained from Section One)

**To be provided by the candidate**
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this examination

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 53 | 35 |
| Section Two: Calculator-assumed | 11 | 11 | 100 | 99 | 65 |
| | | | **Total** | 152 | 100 |

## Instructions to candidates

The rules for the conduct of Western Australian external examinations are detailed in the Year 12 *Information Handbook 2017.* Sitting this examination implies that you agree to abide by these rules

Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: if you need to use the space to continue an answer, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

**Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil, except in diagrams**.

## STRUCTURE OF THIS PAPER

| QUESTION | MARKS AVAILABLE | MARKS AWARDED |
|---|---|---|
| 9 | 6 | |
| 10 | 8 | |
| 11 | 9 | |
| 12 | 9 | |
| 13 | 11 | |
| 14 | 8 | |
| 15 | 8 | |
| 16 | 13 | |
| 17 | 9 | |
| 18 | 12 | |
| 19 | 6 | |
| **TOTAL** | **99** | |

**Section Two: Calculator-assumed** **65% (99 Marks)**

This section has **eleven (11)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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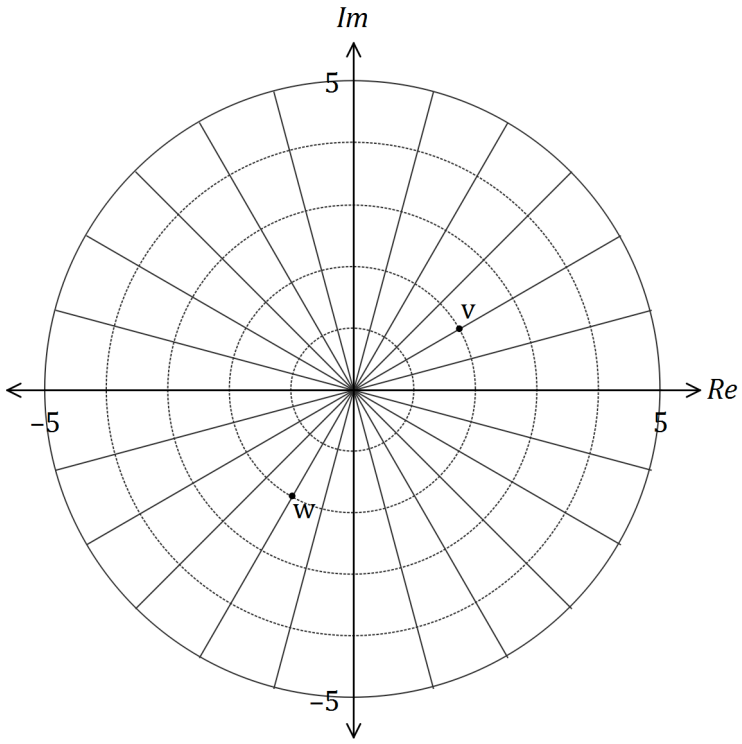
**Question 9** **(6 marks)**

The complex numbers $v$ and $w$ are shown on the Argand diagram below.




On the diagram, clearly mark the complex numbers

(a) $z_1 = vw$. (2 marks)

(b) $z_2 = \frac{v}{w}$. (2 marks)

(c) $z_3 = v - iw$. (2 marks)



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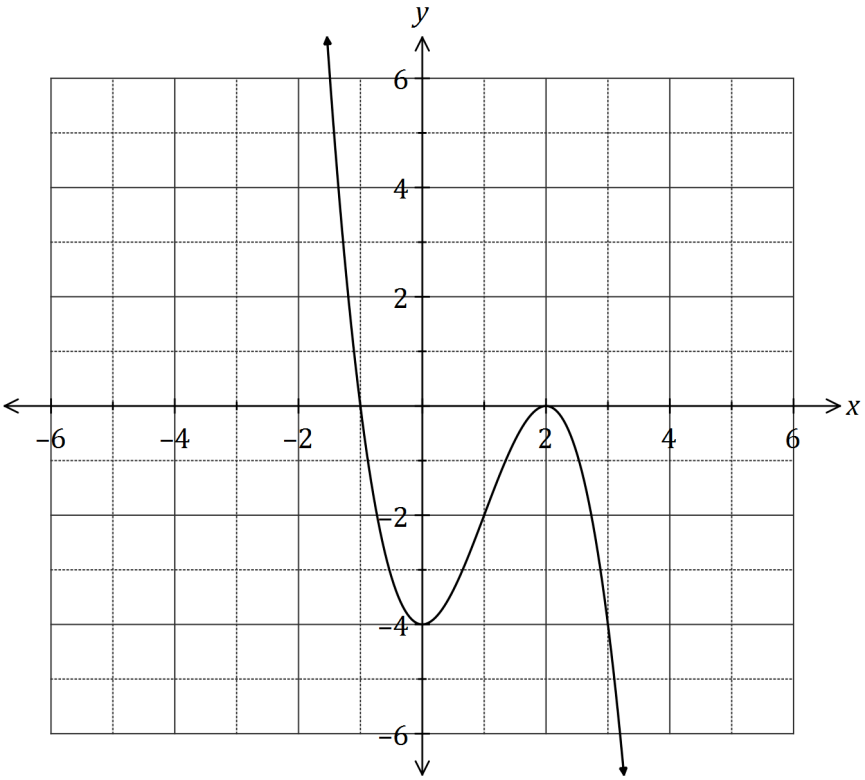
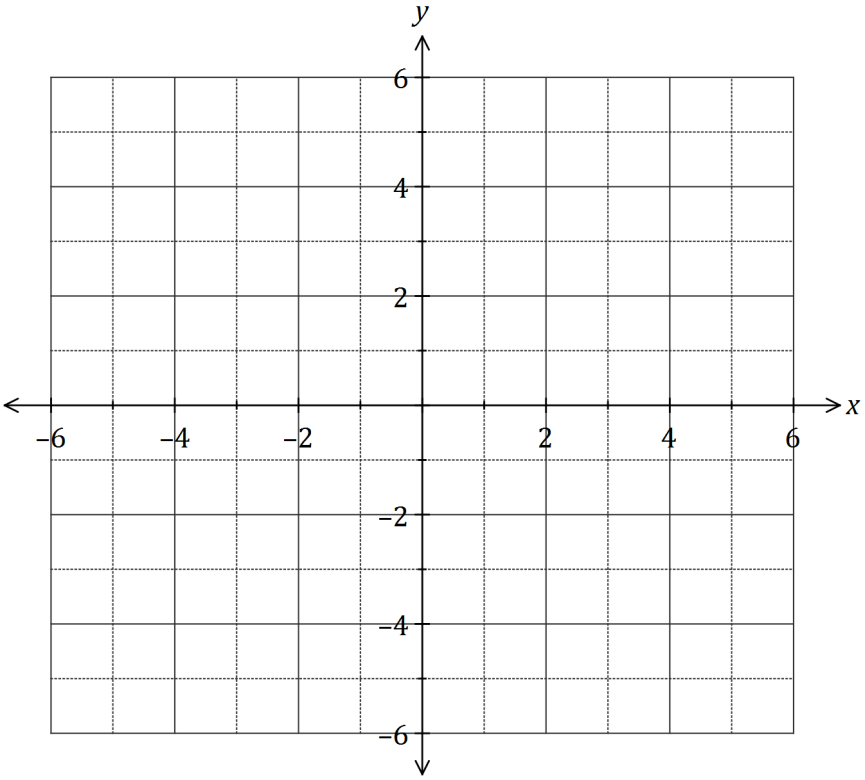
**Question 10** **(8 marks)**

The graph of $y = f(x)$ is drawn below.

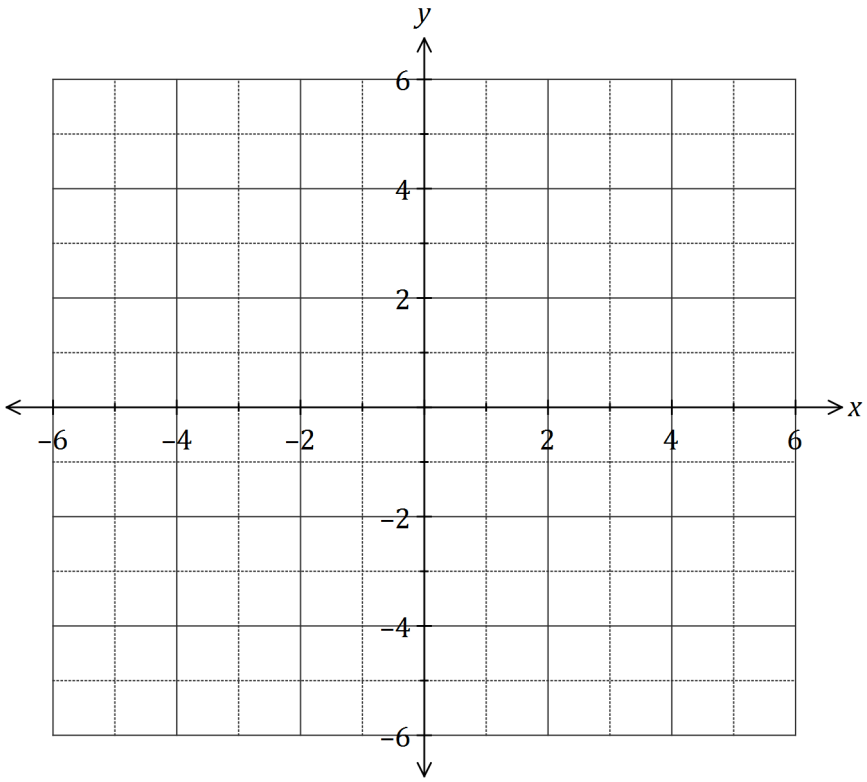



On the axes provided, sketch the graphs of

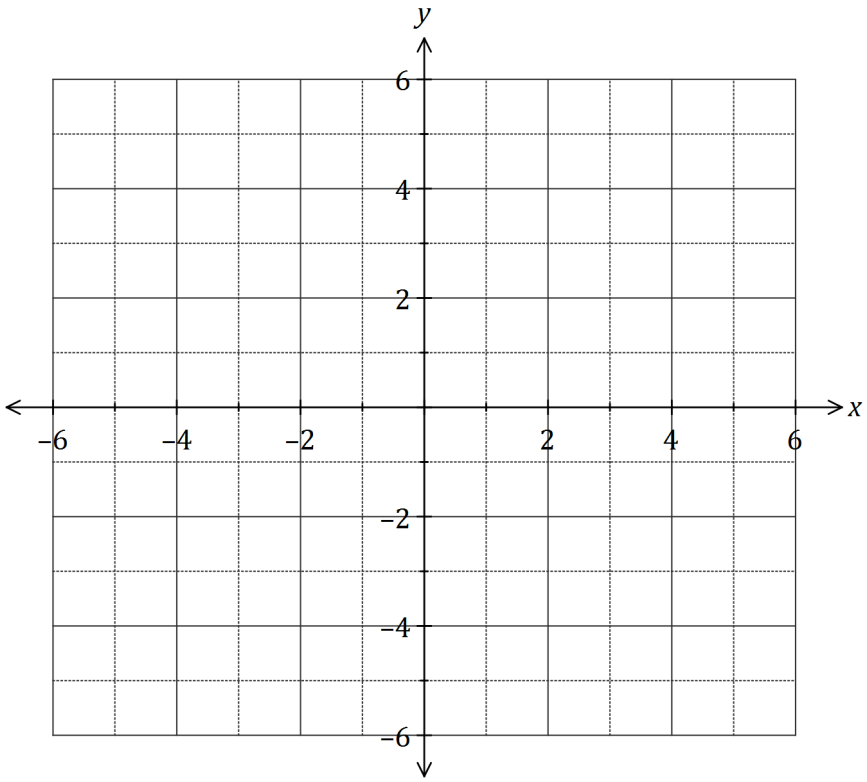
(a) $y = f(|x|)$. (2 marks)



(b) $y = f^{-1}(x)$. (2 marks)




(c) $y = \frac{1}{f(x)}$. (4 marks)



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**Question 11** **(9 marks)**

The position vectors of particles $A$ and $B$ are $\mathbf{r_A} = \begin{pmatrix} 15 - t \\ 4 - 3t \end{pmatrix}$ and $\mathbf{r_B} = \begin{pmatrix} 3t - 8 \\ 5 - t^2 \end{pmatrix}$, where $t$ is the time in seconds, $t \geq 0$, and distances are measured in metres.

(a) Determine the speed of particle $B$ when $t = 2$. (2 marks)

(b) Determine the Cartesian equation for the path of particle $A$, including any appropriate restrictions on the domain. (3 marks)

(c) Determine where the paths of the particles cross and whether the particles meet. Justify your answer. (4 marks)

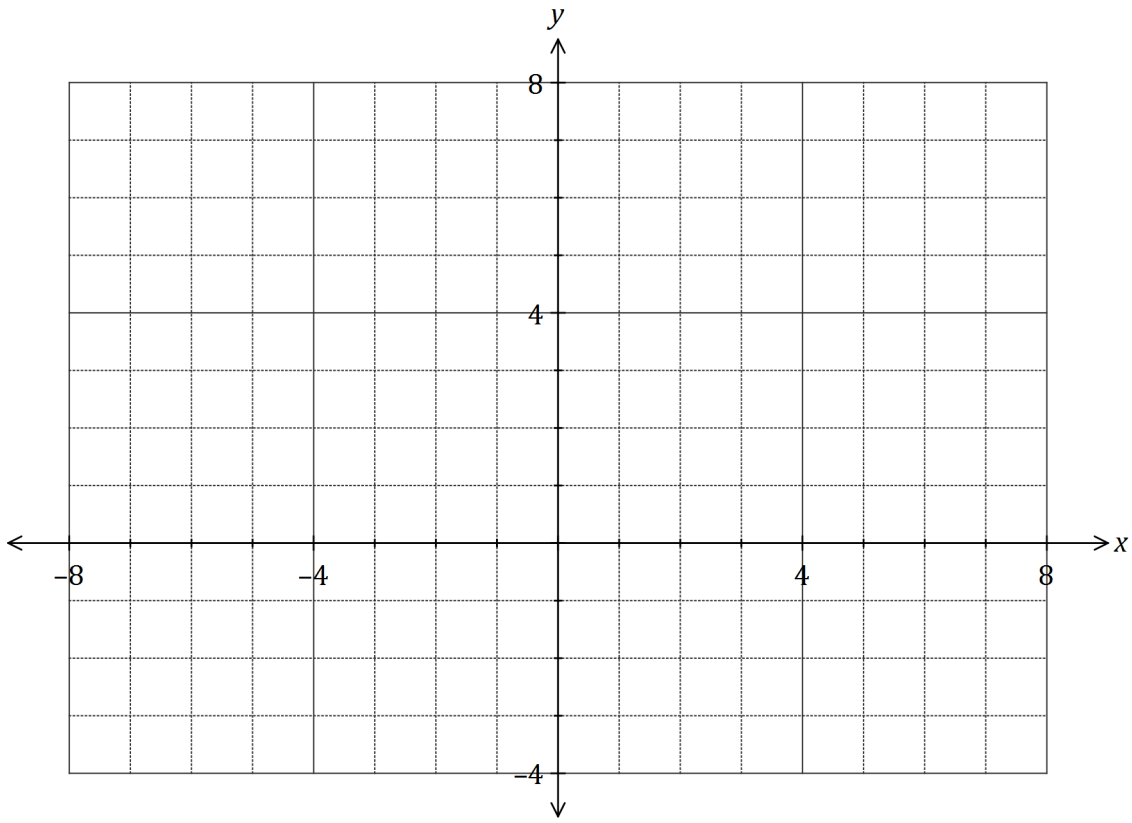
**Question 12** **(9 marks)**

A function is defined by $f(x) = \frac{x^2+4x-12}{3x-7}$, $x \neq 0$.

(a) Determine the exact coordinates of all stationary points of the graph of $y = f(x)$. (2 marks)

(b) Determine the equation(s) of the asymptote(s) of the graph $y = f(x)$. (3 marks)

(c) Sketch the graph $y = f(x)$ on the axes below. (4 marks)



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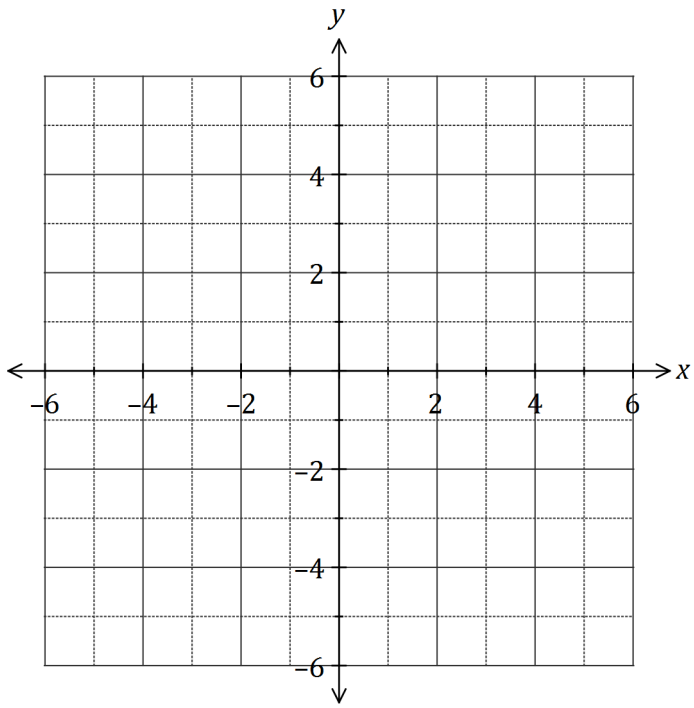
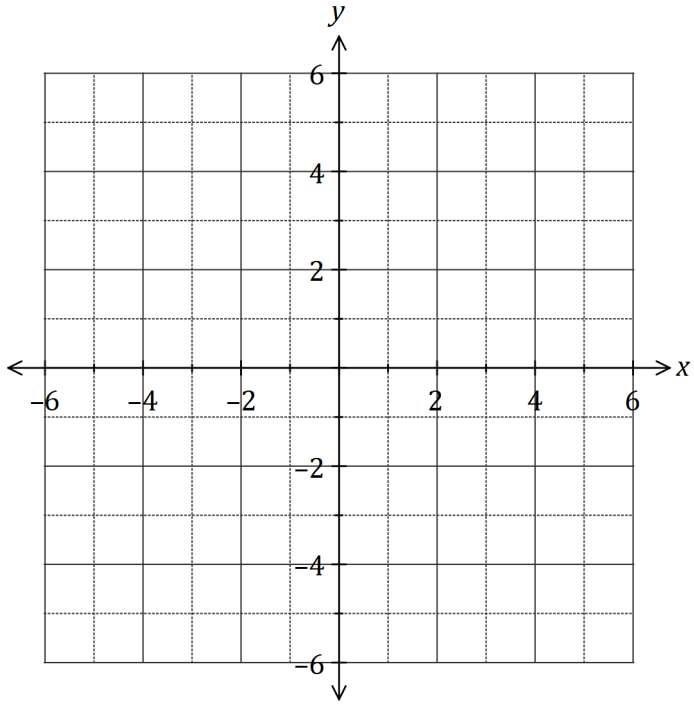
**Question 13** **(11 marks)**

(a) On the Argand planes below, sketch the subsets of the complex plane determined by

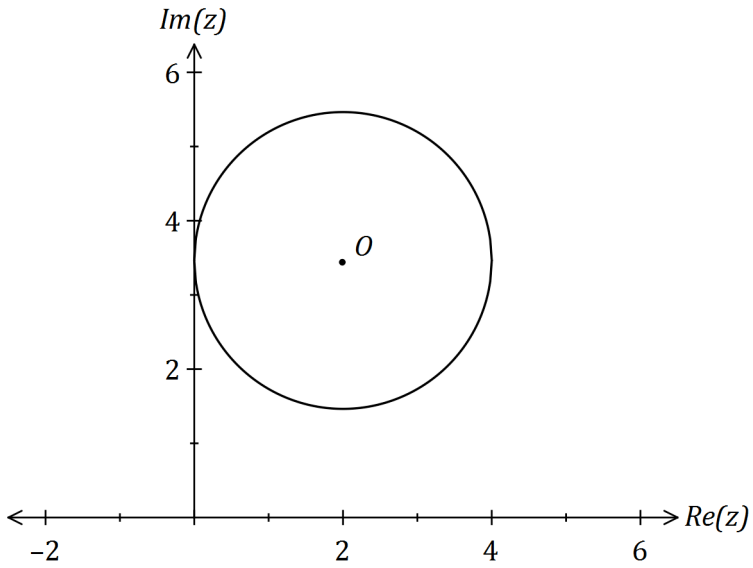
(i) $|z + 3i| = |z + 2 - i|$. (3 marks)




(ii) $|z + 3 + i| \leq 3$. (3 marks)



(b) A subset of the complex plane, a circle with centre $O$, is shown below.




(i) Mark the position on the subset where $|z|$ is maximised. Label this point (A).
(1 mark)

(ii) Mark the position on the subset where $|z - 2|$ is minimised. Label this point (B).
(1 mark)

(iii) If the subset shown is $\left|z - 2 - 2\sqrt{3}i\right| = 2$, determine the maximum and minimum values of $\arg z$. (3 marks)



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**Question 14** **(8 marks)**

The plane $P$ has equation $\mathbf{r} \cdot \mathbf{n} = 11$, where $\mathbf{n} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and the point $A$ has position vector $2\boldsymbol{i} + 5\mathbf{j} - 2\mathbf{k}$.

(a) Determine the Cartesian equation of plane $Q$ that is parallel to $P$ and passes through $A$. (2 marks)

(b) Determine the equation of the line $L$ that passes through $A$ and is perpendicular to $P$. (1 mark)

(c) Determine the position vector of $B$, the point of intersection of line $L$ with plane $P$. (3 marks)

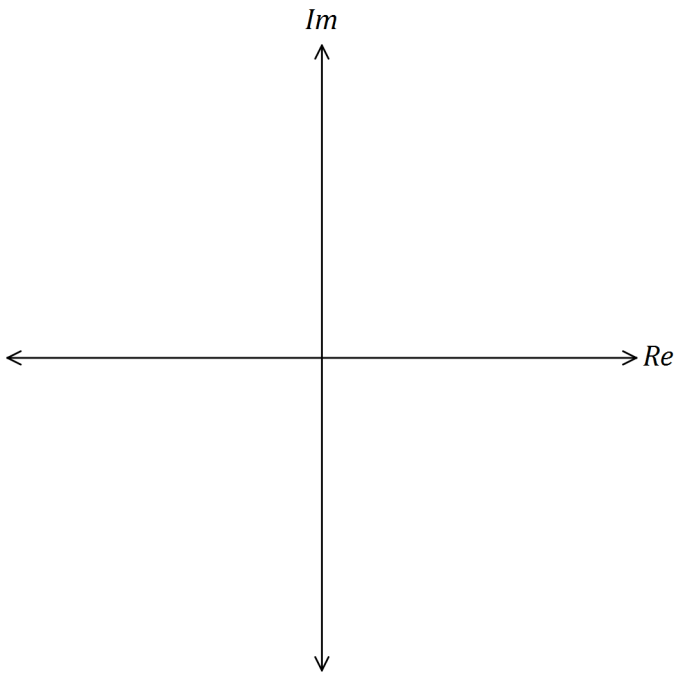
(d) Determine the exact distance between planes $P$ and $Q$. (2 marks)

**Question 15** **(8 marks)**

Consider the complex equation $z^5 = -16 + 16\sqrt{3}i$.

(a) Show use of De Moivre's theorem to solve the equation, giving all solutions in the form $r\,\text{cis}\,\theta\,,\ r > 0,\ -\pi \leq \theta \leq \pi$. (4 marks)

(b) Plot the solutions found in part (a) on the Argand diagram below, indicating all key features of the plot. (4 marks)



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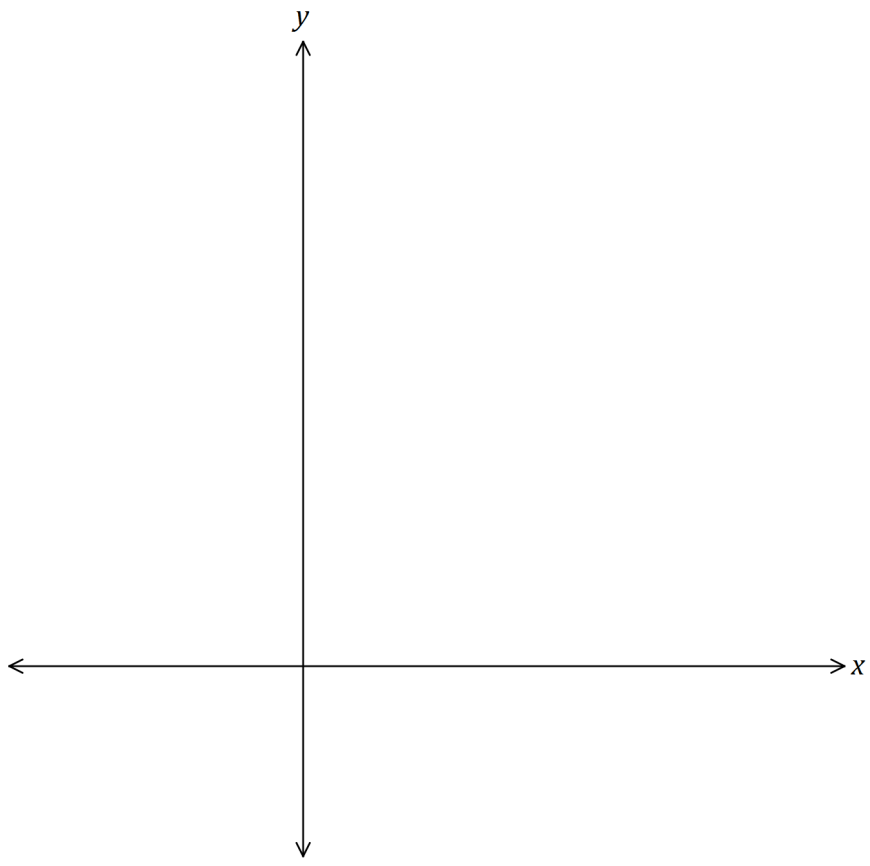
**Question 16** **(13 marks)**

The position vector, in centimetres, of a particle at time $t$ seconds is given below.

$$\mathbf{r}(t) = 2e^{t-1}\mathbf{i} + \frac{e^{2t}}{3}\mathbf{j}$$

(a) Show that the path of the particle can be expressed as a Cartesian equation in the form $y = ax^2$, and determine the value of $a$. (4 marks)

(b) Sketch the path of the particle for $0 \leq t \leq 2$. (3 marks)



(c) Determine the speed of the particle when $t = 1$. (3 marks)

(d) Write an expression in terms of $t$ for the speed of the particle. (1 marks)

(d) Hence write an expression in terms of $t$ for the total distance travelled by the particle along its path between $t = 0$ and $t = 2$. Do **not** evaluate this expression. (2 marks)



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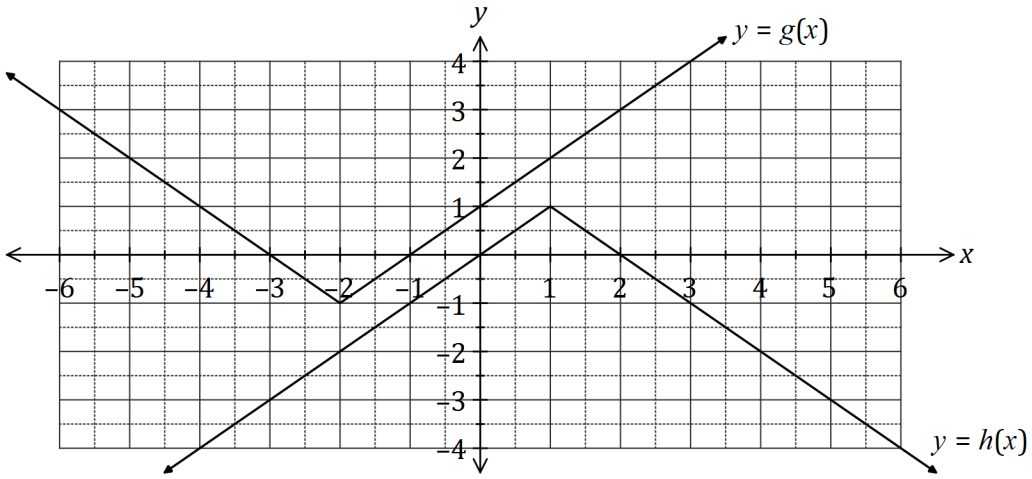
**Question 17** **(9 marks)**

(a) The graphs of the functions $g$ and $h$ are shown below.

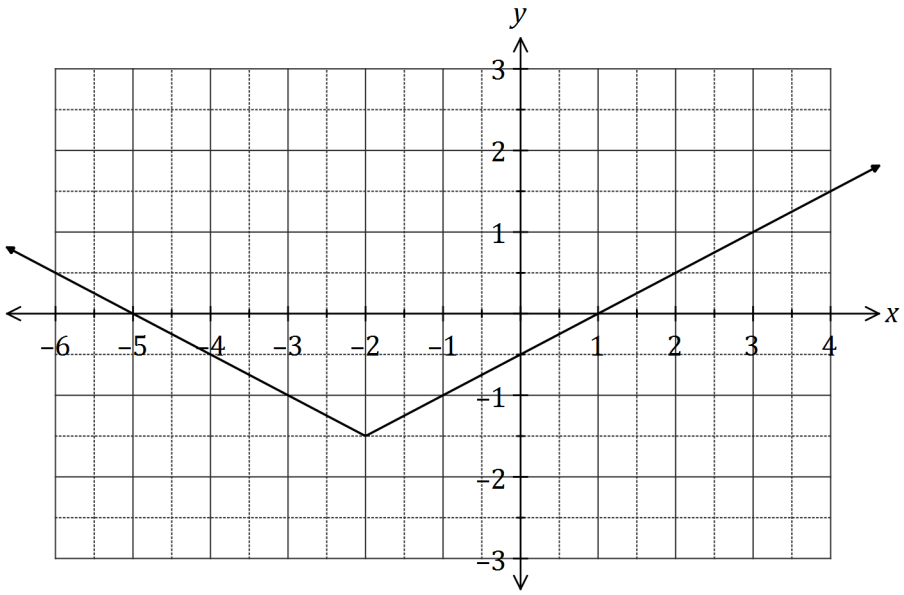



Determine the value(s) of $k$ if

(i) $k = h \circ g(2)$. (1 mark)

(ii) $g\big(h(k)\big) = 1$. (2 marks)

(b) The graph of $f(x) = a|x - p| + q$ is shown below.




(i) Determine the value of the constants $a$, $p$ and $q$. (3 marks)

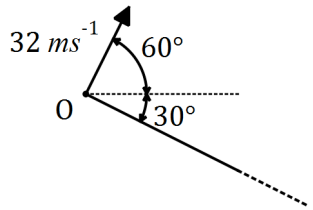
(ii) $g(x) = mx + c$ is a line with positive constants $m$ and $c$.
If the equation $|f(x)| = g(x)$ has an infinite number of solutions, determine the values of $m$ and $c$. (3 marks)



**See next page**

**Question 18** **(12 marks)**

A small body is projected upwards from the top of a hill with an initial velocity of 32 $ms^{-1}$ at an angle of 60° to the horizontal. The hill slopes downwards at a constant angle of 30° to the horizontal. Let the origin $O$ of a cartesian coordinate system be the top of the hill, with $\mathbf{i}$ a unit vector in the positive $x$ direction and $\mathbf{j}$ a unit vector in the positive $y$ direction. Displacement is measured in metres and time in seconds.




(a) Show that the initial velocity of the body is $16\mathbf{i} + 16\sqrt{3}\mathbf{j}$. (1 mark)

The acceleration of the body, $t$ seconds after projection, is given by $\mathbf{a} = -0.2t\mathbf{i} + (0.2t - 10)\mathbf{j}$.

(b) Determine an expression for the position vector of the body after $t$ seconds. (3 marks)

(c) Determine the time at which the body lands on the hillside. (3 marks)

(d) Calculate the distance of the body from $O$ at the instant it lands. (2 marks)

(e) Determine the maximum vertical height attained by the particle above the hillside. (3 marks)



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**Question 19** **(6 marks)**

Determine, where possible, a unique solution for the following systems of equations. In each case, interpret the system of equations geometrically.

(a) $8x + y + z = 15$, $2x + y - z = 3$, and $x - y + 2z = 3$. (2 marks)

(b) $x + y - z = 0$, $x - y + 2z = 10$ and $3x - y + z = 16$. (2 marks)

(c) $x + y = z + 2$, $x - y + z = 1$ and $x + z = y + 3$. (2 marks)

**End of questions**

Additional working space

Question number: _____