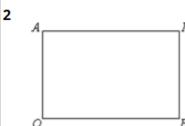
$$egin{aligned} \mathbf{1} & \stackrel{
ightarrow}{AB} = (3oldsymbol{i} - 5oldsymbol{j}) - (oldsymbol{i} + 2oldsymbol{j}) \ &= 3oldsymbol{i} - 5oldsymbol{i} - oldsymbol{i} - 2oldsymbol{j} \ &= 2oldsymbol{i} - 7oldsymbol{j} \end{aligned}$$



a 
$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$
  
=  $5i + 6j$ 

$$egin{aligned} \mathbf{b} & \stackrel{
ightarrow}{AB} = \stackrel{
ightarrow}{AO} + \stackrel{
ightarrow}{OB} \ &= -5oldsymbol{i} + 6oldsymbol{j} \end{aligned}$$

c 
$$\overrightarrow{BA} = -\overrightarrow{AB}$$
  
=  $5i - 6j$ 

$$\mathbf{a} \quad |5\boldsymbol{i}| = \sqrt{5^2} = 5$$

**b** 
$$|-2j| = \sqrt{(-2)^2} = 2$$

c 
$$|3m{i}+4m{j}|=\sqrt{3^2+4^2} \ =\sqrt{9+16}=5$$

$$\begin{aligned} \mathsf{d} & \quad |{-5}\boldsymbol{i} + 12\boldsymbol{j}| = \sqrt{{(-5)}^2 + 12^2} \\ & = \sqrt{25 + 144} = 13 \end{aligned}$$

4 a 
$$egin{aligned} m{u} - m{v} &= (7m{i} + 8m{j}) - (2m{i} - 4m{j}) \\ &= 7m{i} + 8m{j} - 2m{i} + 4m{j} \\ &= 5m{i} + 12m{j} \\ |m{u} - m{v}| &= |5m{i} + 12m{j}| \end{aligned}$$

$$|u - v| = |5i + 12j|$$
  
=  $\sqrt{25 + 144}$   
= 13

$$\begin{array}{ll} \mathbf{b} & x \boldsymbol{u} + y \boldsymbol{v} = x(7\boldsymbol{i} + 8\boldsymbol{j}) + y(2\boldsymbol{i} - 4\boldsymbol{j}) \\ &= 7x\boldsymbol{i} + 8x\boldsymbol{j} + 2y\boldsymbol{i} - 4y\boldsymbol{j} \\ &= 44\boldsymbol{j} \end{array}$$

$$7x + 2y = 0$$

$$14x + 4y = 0$$

$$8x - 4y = 44$$

1

$$(1) + (2)$$
:

$$22x = 44$$

$$x = 2$$

$$7\times 2 + 2y = 0$$
$$2y = -14$$

$$y = -7$$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$= -\overrightarrow{OA} + \overrightarrow{OB}$$

$$= -10\mathbf{i} + (4\mathbf{i} + 5\mathbf{j})$$

$$= -6\mathbf{i} + 6\mathbf{j}$$

$$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB}$$

$$= -3\mathbf{i} + \frac{5}{2}\mathbf{j}$$

$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

$$=10oldsymbol{i}+\left(-3oldsymbol{i}+rac{5}{2}oldsymbol{j}
ight)$$

$$=7oldsymbol{i}+rac{5}{2}oldsymbol{j}$$

a i 
$$\stackrel{
ightarrow}{OM}=rac{1}{5}\stackrel{
ightarrow}{OP}$$
  $=rac{2}{5}i$ 

$$egin{aligned} \mathbf{i}\mathbf{i} & \overrightarrow{MQ} = \overrightarrow{MO} + \overrightarrow{OQ} \ &= -\overrightarrow{OM} + \overrightarrow{OQ} \ &= -rac{2}{5} \mathbf{i} + \mathbf{j} \end{aligned}$$

iii 
$$\overrightarrow{MN}=rac{1}{6}\overrightarrow{MQ}$$
  $=rac{1}{6}igg(-rac{2}{5}m{i}+m{j}igg)$   $=-rac{1}{15}m{i}+rac{1}{6}m{j}$ 

$$egin{aligned} \mathbf{iv} \quad \overrightarrow{ON} &= \overrightarrow{OM} + \overrightarrow{MN} \ &= rac{2}{5} oldsymbol{i} + \left( -rac{1}{15 oldsymbol{i}} + rac{1}{6} oldsymbol{j} 
ight) \ &= rac{1}{3} oldsymbol{i} + rac{1}{6} oldsymbol{j} \end{aligned}$$

$$egin{aligned} \mathbf{v} & \stackrel{
ightarrow}{OA} = \stackrel{
ightarrow}{OP} + \stackrel{
ightarrow}{PA} \ &= 2oldsymbol{i} + oldsymbol{j} \end{aligned}$$

$$egin{aligned} \mathbf{b} \ \mathbf{i} & \stackrel{
ightarrow}{ON} = rac{1}{3} m{i} + rac{1}{6} m{j} \ &= rac{1}{6} (2 m{i} + m{j}) \ &= rac{1}{6} \stackrel{
ightarrow}{OA} \end{aligned}$$

Since ON is parallel to OA and they share a common point O, ON must be on the line OA. Hence N is on OA.

7 
$$\overrightarrow{OA} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \mathbf{i} + 3\mathbf{j}$$

$$\overrightarrow{OB} = \begin{bmatrix} 5 \\ -1 \end{bmatrix} = 5\mathbf{i} - \mathbf{j}$$

$$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$$

$$= -\mathbf{i} - 3\mathbf{j} + 5\mathbf{i} - \mathbf{j}$$

$$= 4\mathbf{i} - 4\mathbf{j}$$

$$|\overrightarrow{AB}| = \sqrt{4^2 + (-4)^2}$$

$$= \sqrt{16 + 16}$$

$$= \sqrt{32} = 4\sqrt{2} \text{ units}$$

8 a 
$$\boldsymbol{i} + 3\boldsymbol{j} = 2\ell\boldsymbol{i} + 2k\boldsymbol{j}$$

$$2\ell=1$$
  $\ell=rac{1}{2}$ 

$$2k = 3$$

$$k=rac{3}{2}$$

$$x - 1 = 5$$

$$x = 6$$

$$y = x - 4$$

$$= 2$$

$$x + y = 6$$

$$+y-0$$

$$x-y=0$$
 2

$$2x = 6$$

$$x = 3$$

$$3 + y = 6$$

$$y = 3$$

d 
$$k=3+2l$$

$$k = -2 - l$$

$$3+2l=-2-l$$

$$3l=-5 \ 5$$

$$l=-rac{5}{3}$$

$$k=-2--rac{5}{3}$$

$$=-2+rac{5}{3}$$

$$=-rac{1}{3}$$

9 
$$\overrightarrow{AB} = \begin{bmatrix} 5-2\\1-3 \end{bmatrix}$$

$$= \begin{bmatrix} 3\\-2 \end{bmatrix}$$

$$= 3\mathbf{i} - 2\mathbf{j}$$

$$|\overrightarrow{AB}| = \sqrt{3^2 + (-2)^2}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13}$$

$$egin{array}{ll} {\sf 10_a} & \stackrel{
ightarrow}{AB} = m{i} + 4m{j} - 3m{i} \ = -2m{i} + 4m{j} \end{array}$$

$$egin{array}{ll} \mathbf{b} & \stackrel{
ightarrow}{AC} = -3m{i} + m{j} - 3m{i} \ &= -6m{i} + m{j} \end{array}$$

$$\mathbf{c} \qquad \overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} \\ = -6\boldsymbol{i} + \boldsymbol{j} - (-2\boldsymbol{i} + 4\boldsymbol{j}) \\ = -4\boldsymbol{i} - 3\boldsymbol{j} \\ |\overrightarrow{BC}| = \sqrt{(-4)^2 + (-3)^2}$$

 $=\sqrt{16+9}$ 

11a

Let 
$$D=(a,b)$$
.  $\overrightarrow{AB}=-5oldsymbol{i}+3oldsymbol{j}$ 

$$\overset{
ightarrow}{CD} = (a+1)i + bj$$

$$a + 1 = -5$$

$$a = -6$$
  
 $b = 3$ 

$$D$$
 is  $(-6,3)$ .

**b** Let 
$$F = (c, d)$$
.

$$\overrightarrow{BC} = -\boldsymbol{i} - 4\boldsymbol{j}$$

$$\overset{
ightarrow}{AF}=(c-5)oldsymbol{i}+(d-1)oldsymbol{j}$$

$$c - 5 = -1$$

$$c=4$$

$$d-1=-4$$
$$d=-3$$

$$F$$
 is  $(4, -3)$ .

**c** Let 
$$G = (e, f)$$
.

$$\stackrel{
ightarrow}{AB} = -5m{i} + 3m{j}$$

$$2\overrightarrow{GC}=2(-1-e)oldsymbol{i}+2(-f)oldsymbol{j} \ 2(-1-e)=-5$$

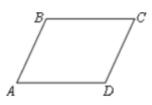
$$e=\frac{3}{2}$$

$$-2f=3$$

$$-2f = 3$$
$$f = -\frac{3}{2}$$

$$G$$
 is  $\left(\frac{3}{2}, -\frac{3}{2}\right)$ .

12 
$$\overrightarrow{OA} = -\overrightarrow{AO}$$
  
 $= -\mathbf{i} - 4\mathbf{j}$   
 $A \text{ is } (-1, -4).$   
 $B \text{ is } (-2, 2).$   
 $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$   
 $\overrightarrow{OC} = \overrightarrow{BC} + \overrightarrow{OB}$   
 $= 2\mathbf{i} + 8\mathbf{j} + (-2\mathbf{i} + 2\mathbf{j})$   
 $= 10\mathbf{j}$ 



C is (0, 10)

$$\mathsf{a}_{\mathsf{i}} \quad 2m{i}-m{j}$$

ii 
$$-5i + 4j$$

iii 
$$i + 7j$$

iv 
$$6i + 3j$$

$$egin{aligned} \mathbf{v} & \stackrel{
ightarrow}{AD} = \stackrel{
ightarrow}{BC} \ &= 6oldsymbol{i} + 3oldsymbol{j} \end{aligned}$$

$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$$

$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$$

$$\overrightarrow{OD} = \overrightarrow{AD} + \overrightarrow{OA}$$

$$= 6\mathbf{i} + 3\mathbf{j} + 2\mathbf{i} - \mathbf{j}$$

$$= 8\boldsymbol{i} + 3\boldsymbol{j}$$
  
 $= 8\boldsymbol{i} + 2\boldsymbol{j}$ 

$$= 8i + 2i$$
  
 $D ext{ is } (8,2).$ 

14a 
$$\overrightarrow{OP} = 12i + 5j$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$
  
=  $18\mathbf{i} + 13\mathbf{j} - 12\mathbf{i} - 5\mathbf{j}$ 

$$=6i+8j$$

$$\begin{array}{ll} \mathbf{b} & |\overrightarrow{RQ}| = |\overrightarrow{OP}| \\ & = \sqrt{12^2 + 5^2} \end{array}$$

$$=13$$
  $|\overrightarrow{OR}| = |\overrightarrow{PQ}|$ 

$$=\sqrt{6^2+8^2}$$

$$=10$$

15a j
$$|\overrightarrow{AB}|=|2m{i}-5m{j}| = \sqrt{2^2+5^2}=\sqrt{29}$$

$$\overrightarrow{BC}| = |10\mathbf{i} + 4\mathbf{j}|$$

$$= \sqrt{10^2 + 4^2}$$

$$= \sqrt{116} = 2\sqrt{29}$$

iii 
$$|\stackrel{
ightarrow}{CA}|=|12m{i}-m{j}| = \sqrt{12^2+1^2}=\sqrt{145}$$

$$\begin{array}{ll} \mathbf{b} & AB^2 + BC^2 = 29 + 116 \\ & = 145 = AC^2 \end{array}$$

 $\therefore$  ABC is a right-angled triangle.

16a j 
$$\stackrel{
ightarrow}{AB}=-m{i}-3m{j}$$

ii 
$$\stackrel{
ightarrow}{BC}=4m{i}+2m{j}$$

iii 
$$\overrightarrow{CA} = -3i + j$$

$$|\overrightarrow{AB}| = \sqrt{1^2 + 3^2} \ = \sqrt{10}$$

ii 
$$|\overrightarrow{BC}|=\sqrt{4^2+2^2} \ =\sqrt{20}=2\sqrt{5}$$

$$AB = CA$$
 $= \sqrt{10}$ 
 $AB^{2} + CA^{2} = 10 + 10$ 
 $= 20 = BC^{2}$ 

 $\therefore ABC$  is an isosceles right-angled triangle.

17a j
$$\stackrel{
ightarrow}{OA}=-3m{i}+2m{j}$$

ii 
$$\overrightarrow{OB} = 7j$$

iii 
$$\stackrel{
ightarrow}{BA} = -3i - 5j$$

iv 
$$\stackrel{
ightarrow}{BM}=rac{1}{2}\stackrel{
ightarrow}{BA} \ =rac{1}{2}(-3m{i}-5m{j})$$

$$egin{aligned} \mathbf{b} & \overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM} \ \overrightarrow{OD} = 7 oldsymbol{j} + -rac{3}{2} oldsymbol{i} - rac{5}{2} oldsymbol{j} \ &= -rac{3}{2} oldsymbol{i} + rac{9}{2} oldsymbol{j} \ M = \left(-rac{3}{2},rac{9}{2}
ight) \end{aligned}$$

18a 
$$a=3i+4j$$
 $|a|=\sqrt{3^2+4^2}$ 
 $=5$ 

$$\hat{a} = \frac{1}{5}(3\boldsymbol{i} + 4\boldsymbol{j})$$

$$egin{aligned} \mathbf{b} & b = 3oldsymbol{i} - oldsymbol{j} \ |b| = \sqrt{3^2 + (-1)^2} \end{aligned}$$

$$=\sqrt{10}$$

$$\hat{b}=\sqrt{10} \ \hat{b}=rac{1}{\sqrt{10}}(3m{i}-m{j})$$

$$egin{aligned} \mathbf{c} & c = -oldsymbol{i} + oldsymbol{j} \ |c| &= \sqrt{(-1)^2 + 1^2} \ &= \sqrt{2} \end{aligned}$$

$$\hat{c}=rac{1}{\sqrt{2}}(-m{i}+m{j})$$

$$\mathbf{d} \qquad d = \boldsymbol{i} - \boldsymbol{j}$$

$$\hat{d}=rac{1}{\sqrt{2}}(m{i}-m{j})$$

$$oldsymbol{e} = rac{1}{2}oldsymbol{i} + rac{1}{3}oldsymbol{j}$$

$$|e| = \sqrt{\left(rac{1}{2}
ight)^2 + \left(rac{1}{3}
ight)^2}$$

$$=\sqrt{\frac{1}{4}+\frac{1}{9}}$$

$$=\sqrt{\frac{13}{36}}$$

$$=\frac{\sqrt{13}}{6}$$

$$\hat{e}=rac{6}{\sqrt{13}}igg(rac{1}{2}m{i}+rac{1}{3}m{j}igg)$$

$$=rac{1}{\sqrt{13}}(3m{i}+2m{j})$$

f 
$$f-6i-4$$

$$f=6oldsymbol{i}-4oldsymbol{j}\ |f|=\sqrt{6^2+\left(-4
ight)^2}$$

$$= \sqrt{52}$$

$$= \sqrt{52} \\ = 2\sqrt{13}$$

$$\hat{f}=rac{1}{2\sqrt{13}}(6m{i}-4m{j})$$

$$=rac{1}{\sqrt{13}}(3m{i}-2m{j})$$