PERTH MODERN SCHOOL

UNIT 3C/3D MAS – 2012

INVESTIGATION 3 – MATRICES

PART 1 – TAKE HOME SECTION

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NAME: *SOLUTIONS – 26 marks* **DATE:** ______________

[To achieve full marks, working and reasoning should be shown.]
[A maximum of 2 marks will be deducted for incorrect rounding, units, notation, etc.]

When investigating with matrix transformations in the plane it is usually very informative to look at what happens to the vertices of the unit square when applying a transformation? To do this we set the vertex points of the square out in a matrix M.

$$M = \begin{array}{c} \begin{array}{cccc} O & A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

To apply the transformation we pre-multiply M by the transformation matrix T. Hence we get:

$T_1 \times M = M'$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \times \begin{array}{c} \begin{array}{cccc} O & A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} O & A' & B' & C' \end{array} \\ \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Points A, B, C are called the object points and A', B', C' are the image points.

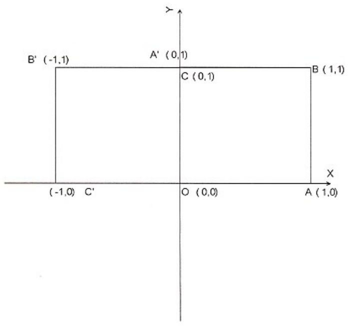
In the diagram below the object unit square is drawn. Plot the image points, label them and hence state what transformation has occurred.

Question 1 [1, 1, 1, 1 = 4 marks]

$T_1 \times M = M_1$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \times \begin{array}{c} \begin{array}{cccc} O & A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} O & A_1 & B_1 & C_1 \end{array} \\ \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a) Plot the image points.



✓[Draws the image figure correctly] *-1 overall if straight edge not used*

b) Name the transformation.

Rotation of 90⁰ anticlockwise. *✓[Recognises the transformation]*

c) Rewrite the matrix containing only the two image points A_1 and C_1

$$\begin{array}{c} \begin{array}{cc} A_1 & C_1 \end{array} \\ \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array} \Rightarrow \begin{array}{c} \begin{array}{cc} A_1 & C_1 \end{array} \\ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \end{array}$$

✓[Completes the matrix accurately]

d) What is the connection?

This image matrix is the same as the transformation matrix. *✓[Compares the two matrices]*

Repeat this investigation for the following transformation matrices.

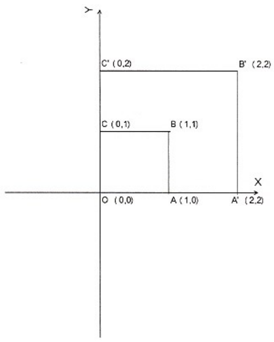
Question 2 [1, 1, 1, 1, 1 = 5 marks]

$T_2 \times M = M_2$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{array}{c} \begin{array}{cccc} O & A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} O & A_2 & B_2 & C_2 \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{bmatrix} \end{array}$$

✓[Completes the matrix accurately]

a) Plot the image points.



✓[Draws the image figure correctly]

b) Name the transformation.

Enlargement matrix factor 2. *✓[Recognises the transformation]*

c) Rewrite the matrix containing only the two image points A_2 and C_2

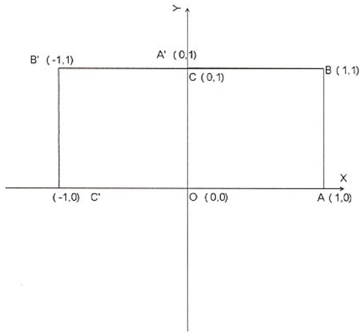
$$\begin{array}{c} \begin{array}{cc} A_2 & C_2 \end{array} \\ \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \end{array}$$

✓[Completes the matrix accurately]

d) What is the connection?

This image matrix is the same as the transformation matrix. *✓[Compares the two matrices]*

Question 3 [1, 1, 1, 1, 1 = 5 marks]



$T_3 \times M = M_3$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \left[\begin{array}{cccc} O & A & B & C \\ \hline 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{cccc} O & A_3 & B_3 & C_3 \\ \hline 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

✓[Completes the matrix accurately]

a) Plot the image points.

✓[Draws the image figure correctly]

b) Name the transformation.

Reflection in the Y-axis. *✓[Recognises the transformation]*

c) Rewrite the matrix containing only the two image points A_3 and C_3

$$\left[\begin{array}{cc} A_3 & C_3 \\ \hline & \\ & \end{array}\right] \Rightarrow \left[\begin{array}{cc} A_3 & C_3 \\ \hline -1 & 0 \\ 0 & 1 \end{array}\right]$$

✓[Completes the matrix accurately]

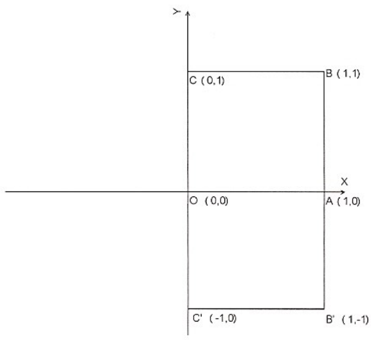
d) What is the connection?

This image matrix is the same as the transformation matrix. *✓[Compares the two matrices]*

We are now going to complete various transformations on the unit square, by considering the image points A′ and C′ we should be able to identify the transformation matrix T.

Question 4 [1, 1, 1 = 3 marks]

Suppose matrix T_4 performs a reflection in the x-axis on the unit square M.



a) Complete this transformation in the diagram opposite.

✓[Draws the image figure correctly]

b) Fill in the results to the matrix operation.

$T_4 \times M = M_4$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \left[\begin{array}{cccc} O & A & B & C \\ \hline 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{cccc} O & A_4 & B_4 & C_4 \\ \hline 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

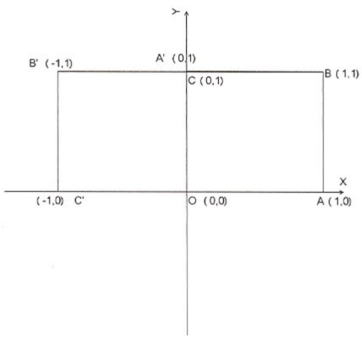
✓[Completes the image matrix from the diagram]

c) Hence complete the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

✓[Defines the transformation matrix]

Question 5 [1, 1, 1 = 3 marks]
Suppose matrix T_5 performs a reflection in the y-axis on the unit square M.
Complete this transformation in the diagram opposite and fill in the results to the matrix operation.



a) Complete this transformation in the diagram opposite.

✓[Draws the image figure correctly]

b) Fill in the results to the matrix operation.

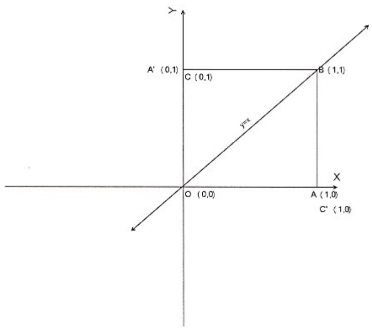
$T_5 \times M = M_5$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \left[\begin{array}{cccc} O & A & B & C \\ \hline 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{cccc} O & A_5 & B_5 & C_5 \\ \hline 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

✓[Completes the image matrix from the diagram]

c) Hence complete the following: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ *✓[Defines the transformation matrix]*

Question 6 [1, 1, 1 = 3 marks]
Suppose matrix T_6 performs a reflection in the line $y = x$ on the unit square M.
Complete this transformation in the diagram opposite and fill in the results to the matrix operation.



a) Complete this transformation in the diagram opposite.

✓[Draws the image figure correctly]

b) Fill in the results to the matrix operation.

$T_6 \times M = M_6$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \left[\begin{array}{cccc} O & A & B & C \\ \hline 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{cccc} O & A_6 & B_6 & C_6 \\ \hline 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

✓[Completes the image matrix from the diagram]

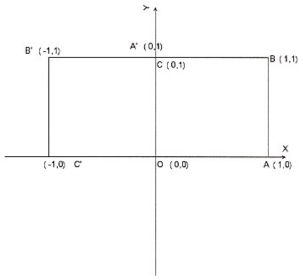
c) Hence complete the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

✓[Defines the transformation matrix]

Question 7 [1, 1, 1 = 3 marks]

Suppose matrix T_7 performs a rotation 90^0 anti-clockwise about the origin (0, 0) on the unit square M.
Complete this transformation in the diagram opposite and fill in the results to the matrix operation.



a) Complete this transformation in the diagram opposite.

✓[Draws the image figure correctly]

b) Fill in the results to the matrix operation.

$T_7 \times M = M_7$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{array}{c} \begin{array}{cccc} O & A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} O & A_7 & B_7 & C_7 \end{array} \\ \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

✓[Completes the image matrix from the diagram]

c) Hence complete the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

✓[Defines the transformation matrix]