Exercise 11C: Solutions

1 a $\text{Maximum} = \sqrt{4^2+3^2} = 5$
$\text{Minimum} = -5$

b $\text{Maximum} = \sqrt{3+1} = 2$
$\text{Minimum} = -2$

c $\text{Maximum} = \sqrt{1+1} = \sqrt{2}$
$\text{Minimum} = -\sqrt{2}$

d $\text{Maximum} = \sqrt{1+1} = \sqrt{2}$
$\text{Minimum} = -\sqrt{2}$

e $\text{Maximum} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$
$\text{Minimum} = -2\sqrt{3}$

f $\text{Maximum} = \sqrt{1+3} = 2$
$\text{Minimum} = -2$

g $\text{Maximum} = \sqrt{1+3}+2 = 4$
$\text{Minimum} = -\sqrt{1+3}+2 = 0$

h
$$\begin{aligned}\text{Maximum} &= 5+\sqrt{3^2+2^2}\\ &= 5+\sqrt{13}\\ \text{Minimum} &= 5-\sqrt{3^2+2^2}\\ &= 5-\sqrt{13}\end{aligned}$$

2 a
$$\begin{aligned}r &= \sqrt{1+1} = \sqrt{2}\\ \cos\alpha &= \frac{1}{\sqrt{2}};\ \sin\alpha = -\frac{1}{\sqrt{2}}\\ \alpha &= -\frac{\pi}{4}\end{aligned}$$
$$\begin{aligned}\sqrt{2}\sin\left(x-\frac{\pi}{4}\right) &= 1\\ \sin\left(x-\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}}\\ x-\frac{\pi}{4} &= \frac{\pi}{4},\ \frac{3\pi}{4}\\ x &= \frac{\pi}{2},\ \pi\end{aligned}$$

b
$$\begin{aligned}r &= \sqrt{3+1} = 2\\ \cos\alpha &= \frac{\sqrt{3}}{2};\ \sin\alpha = \frac{1}{2}\\ \alpha &= \frac{\pi}{6}\end{aligned}$$
$$\begin{aligned}2\sin\left(x+\frac{\pi}{6}\right) &= 1\\ \sin\left(x+\frac{\pi}{6}\right) &= \frac{1}{2}\\ x+\frac{\pi}{6} &= \frac{\pi}{6},\ \frac{5\pi}{5},\ \frac{7\pi}{6}\\ x &= 0,\ \frac{2\pi}{3},\ 2\pi\end{aligned}$$

c

$$r = \sqrt{3+1} = 2$$
$$\cos\alpha = \frac{1}{2};\ \sin\alpha = -\frac{\sqrt{3}}{2}$$
$$\alpha = -\frac{\pi}{3}$$
$$2\sin\left(x - \frac{\pi}{3}\right) = -1$$
$$\sin\left(c - \frac{\pi}{3}\right) = -\frac{1}{2}$$
$$x - \frac{\pi}{3} = -\frac{\pi}{6}, \frac{7\pi}{6}$$
$$x = \frac{\pi}{6}, \frac{3\pi}{2}$$

d

$$r = \sqrt{9+3} = \sqrt{12}$$
$$= 2\sqrt{3}$$
$$\cos\alpha = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$
$$\sin\alpha = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$
$$\alpha = \frac{\pi}{6}$$
$$2\sqrt{3}\cos\left(x + \frac{\pi}{6}\right) = 3$$
$$\cos\left(x + \frac{\pi}{6}\right) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$
$$x + \frac{\pi}{6} = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}$$
$$x = 0, \frac{5\pi}{3}, 2\pi$$

e

$$r = \sqrt{4^2 + 3^2}$$
$$= \sqrt{25} = 5$$
$$\cos\alpha = \frac{4}{5};\ \sin\alpha = \frac{3}{5}$$
$$\alpha \approx 36.87^\circ$$
$$5\sin(\theta + 36.87) \approx 5$$
$$\sin(\theta + 36.87) \approx 1$$
$$\theta + 36.87 \approx 90^\circ$$
$$\theta \approx 53.13^\circ$$

f

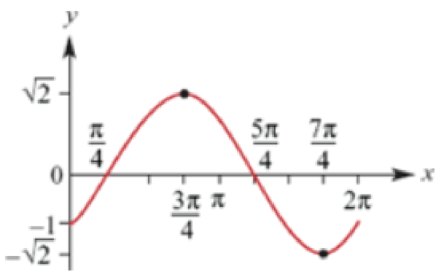
$$r = \sqrt{8+4} = \sqrt{12} = 2\sqrt{3}$$
$$\cos\alpha = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$
$$\sin\alpha = -\frac{2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}}$$
$$\alpha \approx -35.26^\circ$$
$$2\sqrt{3}\sin(\theta - 35.26) \approx 3$$
$$\sin(\theta - 35.26) \approx \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$
$$\theta - 35.26 \approx 60^\circ, 120^\circ$$
$$\theta \approx 95.26^\circ, 155.26^\circ$$

3
$$r = \sqrt{3+1} = 2$$
$$\cos\alpha = \frac{\sqrt{3}}{2};\ \sin\alpha = \frac{1}{2}$$
$$\alpha = \frac{\pi}{6}$$
$$2\cos\left(2x + \frac{\pi}{6}\right)$$

4
$$r = \sqrt{1+1} = \sqrt{2}$$
$$\cos\alpha = -\frac{1}{\sqrt{2}};\ \sin\alpha = -\frac{1}{\sqrt{2}}$$
$$\alpha = \frac{5\pi}{4}$$
$$\sqrt{2}\sin\left(3x - \frac{5\pi}{4}\right)$$

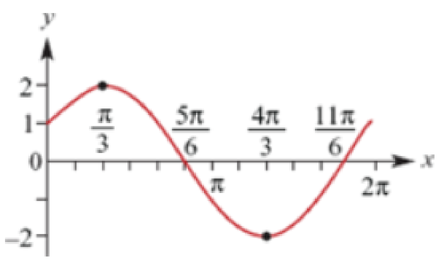
5 a
$$r = \sqrt{1+1} = \sqrt{2}$$
$$\cos\alpha = \frac{1}{\sqrt{2}};\ \sin\alpha = -\frac{1}{\sqrt{2}}$$
$$\alpha = -\frac{\pi}{4}$$
$$f(x) = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$$

The graph will have amplitude $\sqrt{2}$, period 2π, and be translated $\frac{\pi}{4}$ units right.



b
$$r = \sqrt{3+1} = 2$$
$$\cos\alpha = \frac{\sqrt{3}}{2},\ \sin\alpha = \frac{1}{2}$$
$$\alpha = \frac{\pi}{6}$$
$$f(x) = 2\sin\left(x + \frac{\pi}{6}\right)$$

The graph will have amplitude 2, period 2π, and be translated $\frac{\pi}{6}$ units left.



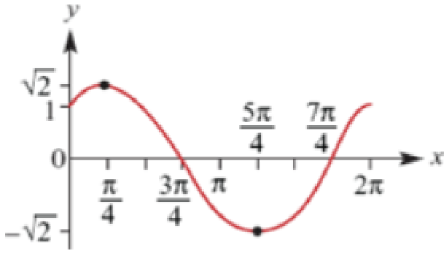
c $$r = \sqrt{1+1} = \sqrt{2}$$

$$\cos\alpha = \frac{1}{\sqrt{2}};\ \sin\alpha = \frac{1}{\sqrt{2}}$$

$$\alpha = \frac{\pi}{4}$$

$$f(x) = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

The graph will have amplitude $\sqrt{2}$, period 2π, and be translated $\frac{\pi}{4}$ units left.



d $$r = \sqrt{1+3} = 2$$

$$\cos\alpha = \frac{1}{2};\ \sin\alpha = -\frac{\sqrt{3}}{2}$$

$$\alpha = -\frac{\pi}{3}$$

$$f(x) = 2\sin\left(x - \frac{\pi}{3}\right)$$

The graph will have amplitude 2, period 2π, and be translated $\frac{\pi}{3}$ units right.