CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3, 4 TEST ONE 2016
NON-Calculator Section
Chapters 1, 2,

Name______________________________

Time: 50 minutes
Total: 46 marks

1. [3, 2, 6 marks]

(a) If $z_1 = 2cis\left(\frac{\pi}{12}\right)$ and $z_2 = 5cis\left(\frac{\pi}{6}\right)$, prove that: $z_1z_2 = 5\sqrt{2}(1 + i)$

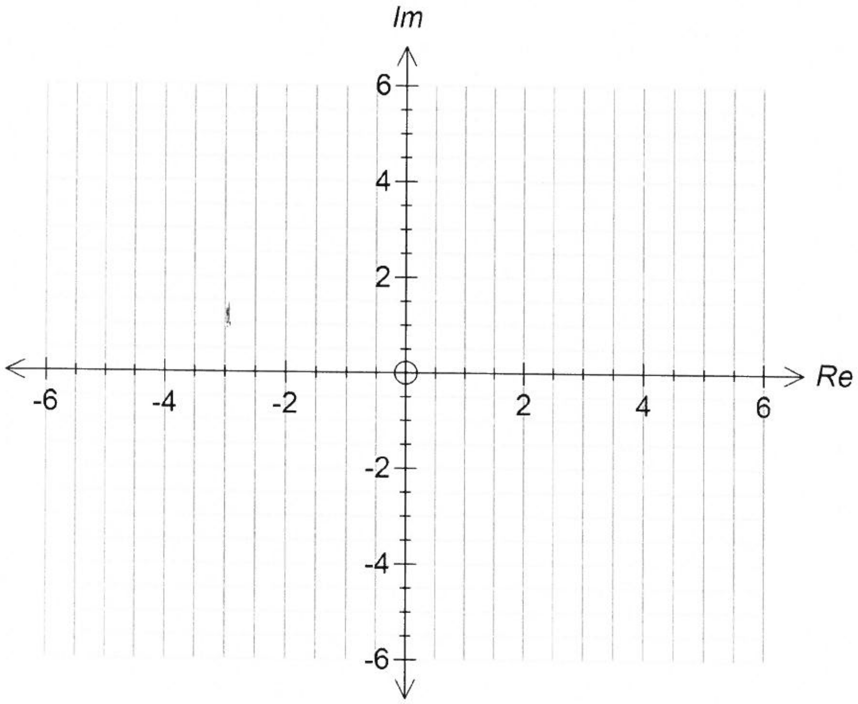
(b) Simplify $\dfrac{3cis\left(-\frac{\pi}{2}\right) \times 4cis\left(\frac{2\pi}{3}\right)}{2cis\left(\frac{5\pi}{4}\right) \times cis\left(-\frac{7\pi}{12}\right)}$

(c) Determine z if: $z\overline{z} + 2z = \frac{1+4i}{4}$

2. [3, 1, 1, 2, 3 marks]

(a) Represent the following set on the Argand diagram below.

$$\{Z : |Z + 4 - 2i| \leq 2\}$$



(b) Find

i) the minimum possible value of $Im(z)$

ii) the maximum possible value of $|Re(z)|$

iii) the minimum value of $|z|$

iv) the maximum possible value of $\arg(z)$, leave your answer in trig form.

3 [2, 1, 3 marks]

(a) Find the remainder when $2x^3 - x^2 + 2$ is divided by $x - 3$

(b) If $(x - 2)$ is a factor of $ax^2 - 12x + 4$ find a.

(c) The function $f(x) = x^4 - 7x^3 + px^2 + qx - 30$ has $(x - 3)$ as a factor but a remainder of -48 is left when $f(x)$ is divided by $(x + 1)$. Find p and q.

4 [6 marks]

Find two complex numbers, w and z, in Cartesian form, such that

$$iz + 2w = 3 \text{ and } z - (1 + i) = -2 \text{ where } i = \sqrt{-1}.$$

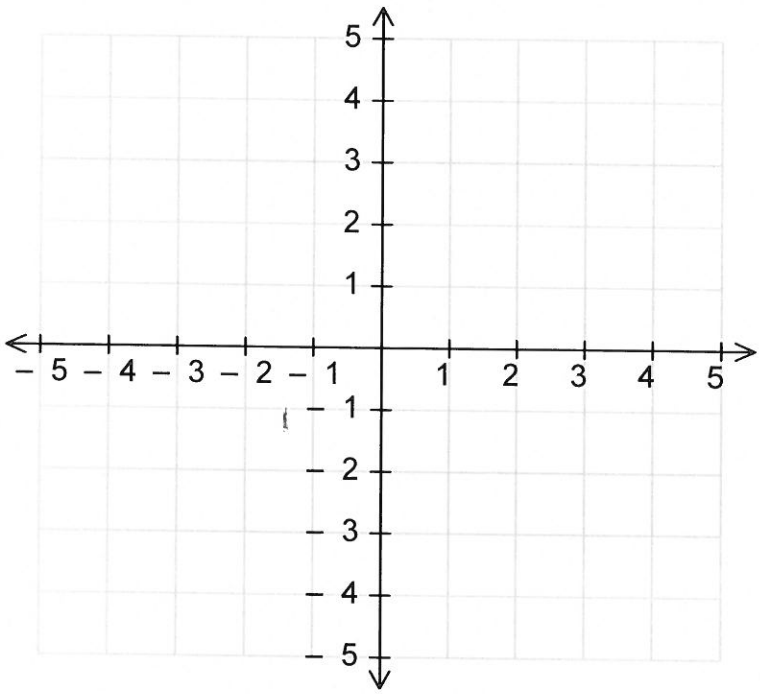
5 [5 Marks]

Use de Moivre's Theorem to prove that $cos4\theta = 8cos^4\theta - 8cos^2\theta + 1$

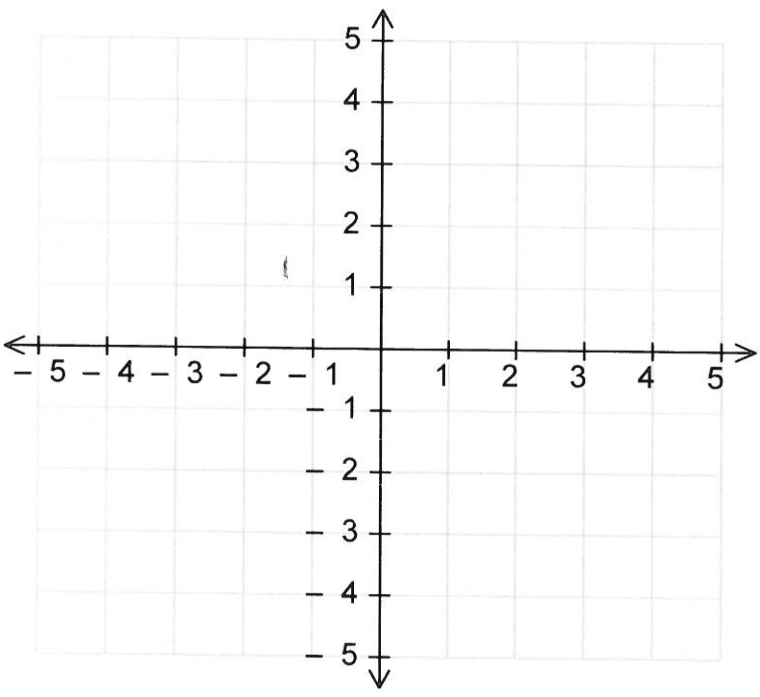
6 [2+4+2= 8 marks]

Draw separate sketches of the following sets of points in the complex plane.

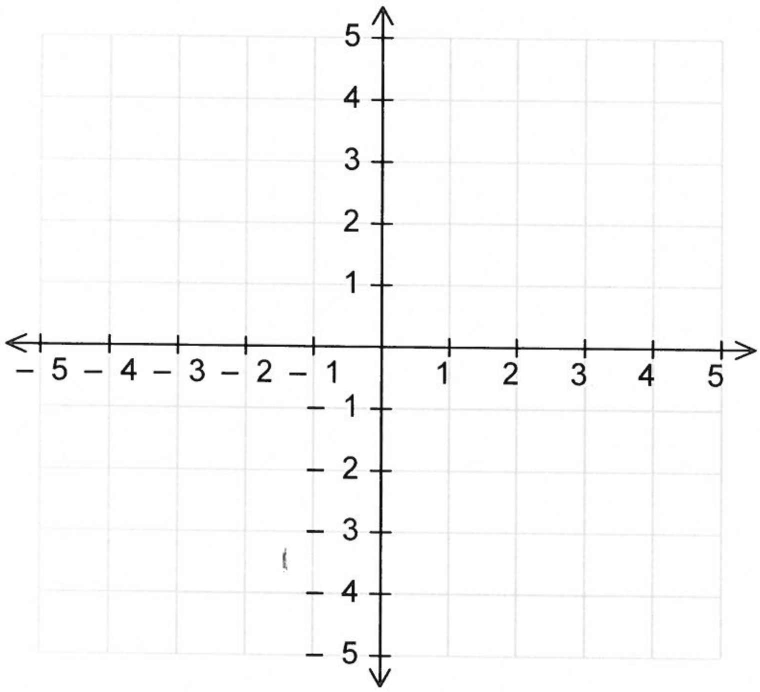
(a) $\{z : |z - 2 + 3i| = 1\}$



(b) $\{z : |z + 2 - i| < |z - 2 + 3i|\}$



(c) $\{z : \bar{z} = iz\}$



Monday

CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3, 4 TEST ONE 2016
NON-Calculator Section
Chapters 1, 2,

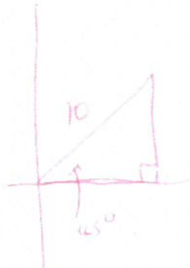
Name________________________________

Time: 50 minutes
Total: ~~48~~ 45 marks

1. [3, 2, 6 marks]

(a) If $z_1 = 2cis\left(\frac{\pi}{12}\right)$ and $z_2 = 5cis\left(\frac{\pi}{6}\right)$, prove that: $z_1z_2 = 5\sqrt{2}(1+i)$

$$z_1z_2 = 2\,cis(\pi/12)\,5\,cis(\pi/6)$$
$$= 10\,cis(\pi/12 + \pi/6)$$
$$= 10\,cis(\pi/4) \quad ✓$$
$$= 10(\cos\pi/4 + i\sin\pi/4)$$
$$= 10\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\frac{\sqrt{2}}{\sqrt{2}} \quad ✓$$
$$= 5\sqrt{2}(1+i) \quad ✓$$



$a^2 + a^2 = 100$
$a^2 = 50$
$a = 5\sqrt{2}$
$5\sqrt{2}(1+i)$

(b) Simplify $\dfrac{3cis\left(-\frac{\pi}{2}\right)\times 4cis\left(\frac{2\pi}{3}\right)}{2cis\left(\frac{5\pi}{4}\right)\times cis\left(-\frac{7\pi}{12}\right)}$

$$= \frac{12\,cis\,\pi/6}{2\,cis\,8\pi/12} \quad ✓$$
$$= 6\,cis(-6\pi/12) \quad \{\pi/6 - 8\pi/12\}$$
$$= 6\,cis(-\pi/2) \quad ✓$$

(c) Determine z if: $z\bar{z} + 2z = \frac{1+4i}{4}$

let $z = x + iy$ so $\bar{z} = x - iy$

LHS: $\Rightarrow$ $(x+iy)(x-iy) + 2(x+iy) = x^2 + y^2 + 2x + 2iy$ ✓

$\Rightarrow$ $Re(LHS) = x^2 + y^2 + 2x$ ✓

$Im(LHS) = 2y$

$Re(RHS) = \frac{1}{4}$ ✓

$Im(RHS) = 1$

$\therefore$ $2y = 1$

$\Rightarrow y = \frac{1}{2}$ ✓

And $x^2 + \left(\frac{1}{2}\right)^2 + 2(x) = \frac{1}{4}$

If not 2nd soln? -1

$x^2 + 2x = 0$

$\Rightarrow x(x+2) = 0$

$\Rightarrow x = -2$ or $x = 0$ ✓

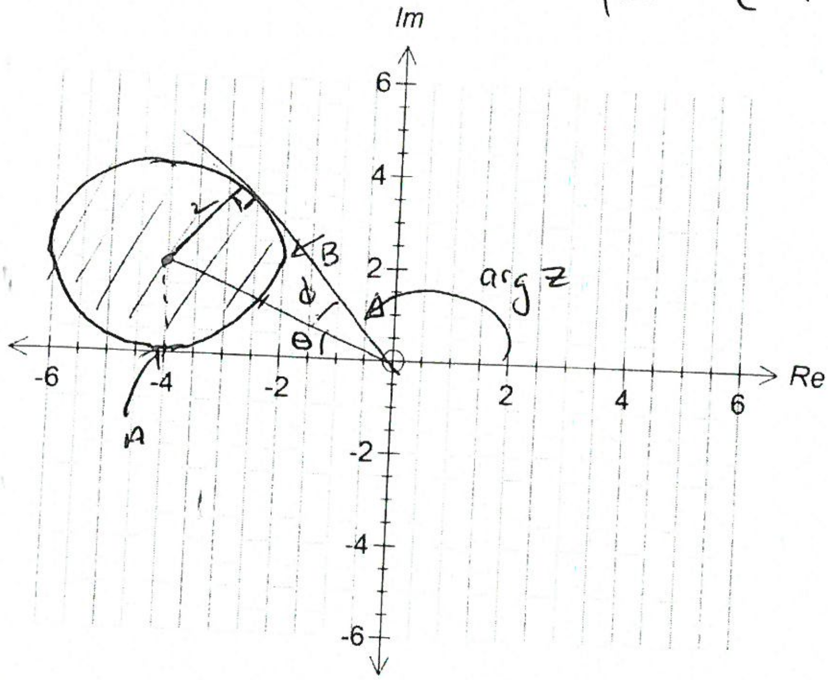
so $z = -2 + \frac{1}{2}i$ or $z = 0.5i$ ✓

2. [3, 1, 1, 2, 3 marks]

(a) Represent the following set on the Argand diagram below.

$\{Z : |Z + 4 - 2i| \leq 2\}$

$|Z - (-4+2i)| \leq 2$



(b) Find

i) the minimum possible value of $Im(z)$

0 at pt A

ii) the maximum possible value of $|Re(z)|$

~~-2~~ 6

iii) the minimum value of $|z|$

(4, 2) 2 2 4

$\sqrt{4^2+2^2} - 2$

$= \sqrt{20} - 2$

$2\sqrt{5} - 2 = 2(\sqrt{5}-1)$

iv) the maximum possible value of arg(z), leave your answer in trig form.

now $\tan\theta = \frac{2}{4}$

$\tan\theta = \frac{1}{2}$

$\Rightarrow \theta = \tan^{-1}\frac{1}{2}$

by symmetry $\phi = \theta$

Arg z is the angle from +ve Re axis

$\arg(z) = \pi - 2\tan^{-1} 1/2$

c) $\{z : \bar{z} = iz\}$

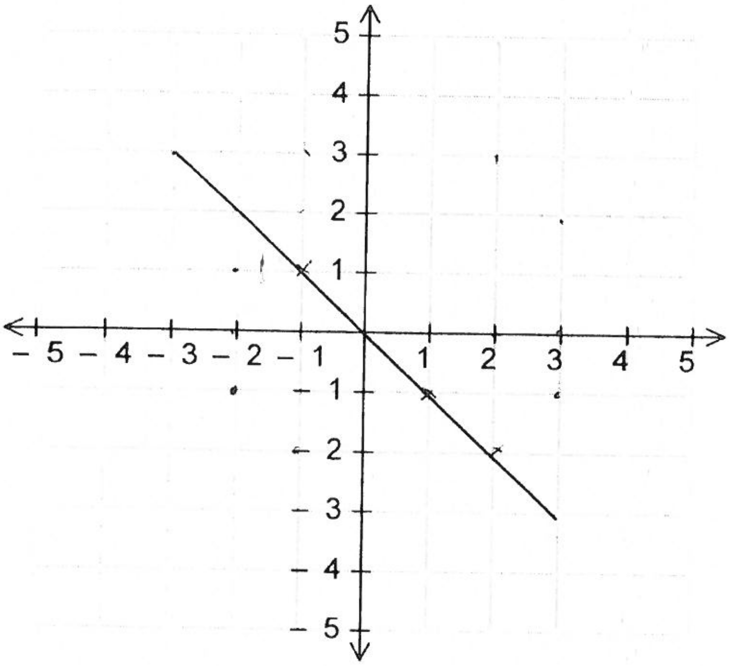
$a - bi = i(a + bi)$

$a - bi = ai - b$

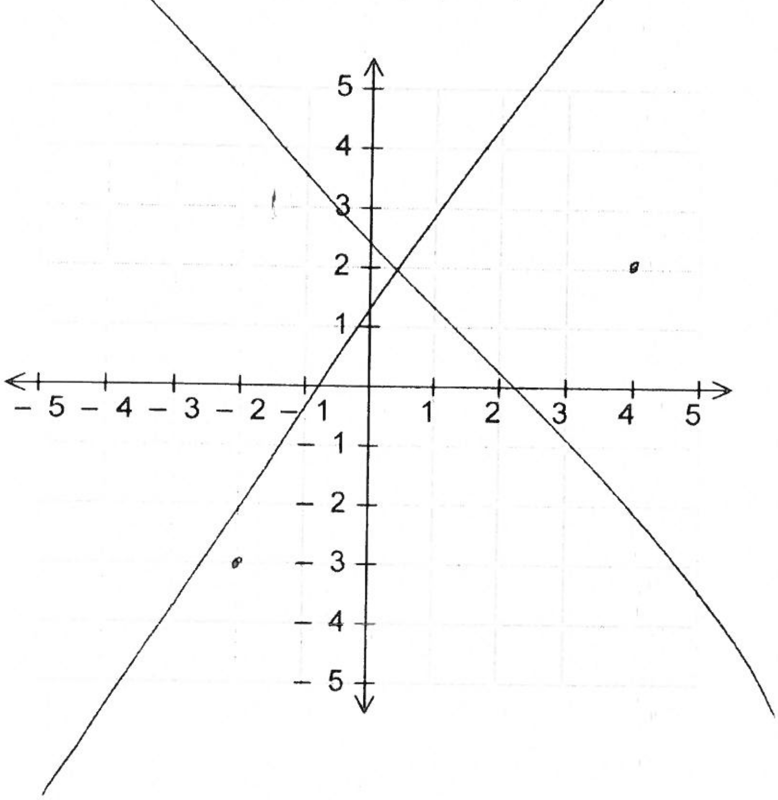
$\underline{a = -b}$

ie $a = 1 \quad b = -1$

$a = 2 \quad b = -2$



d) $\{z : |z - 2 + 3i| + |z - 4 - 2i| = |2 + 5i|$



3 [2, 1, 3 marks]

(a) Find the remainder when $2x^3 - x^2 + 2$ is divided by $x - 3$

$$\begin{array}{r} 2x^2 + 5x + 15 \\ x-3 \overline{) 2x^3 - x^2 + 2} \\ 2x^3 - 6x^2 \\ \hline 5x^2 + 2 \\ 5x^2 - 15x \\ \hline 2 + 15x \\ -45 \quad 15x \end{array}$$

or $x = 3$

$f($

$2(3)^3 - 3^2 + 2$

$= 54 - 9 + 2$

$= 47$

Rem $= 47$

(b) If $(x - 2)$ is a factor of $ax^2 - 12x + 4$ find a.

Let $f(x) = ax^2 - 12x + 4$

$f(2) = 0$ so $4a - 24 + 4 = 0$

$4a - 20 = 0$

$\Rightarrow a = 5$

(c) The function $f(x) = x^4 - 7x^3 + px^2 + qx - 30$ has $(x - 3)$ as a factor but a remainder of -48 is left when $f(x)$ is divided by $(x + 1)$. Find p and q.

$f(3) = 0 \quad \therefore \quad 3^4 - 7.3^3 + 9p + 3q - 30 = 0$

$\Rightarrow 81 - 189 + 9p + 3q - 30 = 0$

$9p + 3q = 138$

$3p + q = 46$ --- (1)

$$\begin{array}{r} 27 \\ \times 7 \\ \hline 189 \end{array}$$

also $f(-1) = -47$

$\therefore \quad 1 + 7 + p - q - 30 = -48$

$p - q = -26$ -- (2)

$\therefore \quad 3p + q = 46$

$p - q = -26$

$4p = 20$

$p = 5$ $\quad \therefore q = 31$

4 [6 marks]

Find two complex numbers, w and z, in Cartesian form, such that

$$iz + 2w = 3 \text{ and } z - (1+i) = -2 \text{ where } i = \sqrt{-1}.$$

Let $z = a + bi$

$w = c + di$

$i(a+bi) + 2(c+di) = 3$

$ai - b + 2c + 2di = 3$

$-b + 2c + (a+2d)i = 3$ -- ①

also $a + bi - (1+i) = -2$

$a - 1 + (b-1)i = -2$ -- ②

From ②

Eq Reals : $a - 1 = -2$

$a = -1$

$z = -1 + i$

Eq Im: $b - 1 = 0$

$\Rightarrow b = 1$

from ① $-b + 2c = 3$

$-1 + 2c = 3$

$2c = 4$

$c = 2$

$w = 2 + \frac{1}{2}i$

$a + 2d = 0$

$2d = 1$

$d = 1/2$

5 [5 Marks]

Use de Moivre's Theorem to prove that $cos4\theta = 8cos^4\theta - 8cos^2\theta + 1$

Using $(\cos\theta)^4 = (\cos 4\theta + i\sin 4\theta)$

now $(\cos\cancel{4}\theta + i\sin\theta)^4 = \underline{\cos^4\theta} + 4\cos^3\theta\, i\sin\theta + 6\cos^2\theta\, i^2\sin^2\theta$

$+ 4\cos\theta\, i^3\sin^3\theta + i^4\sin^4\theta$

Equating Re: $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$= \cos^4\theta - 6\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$

$= \cos^4\theta - 6\cos^2\theta + 6\cos^4\theta + 1 - 2\cos^2\theta + \cos^4\theta$

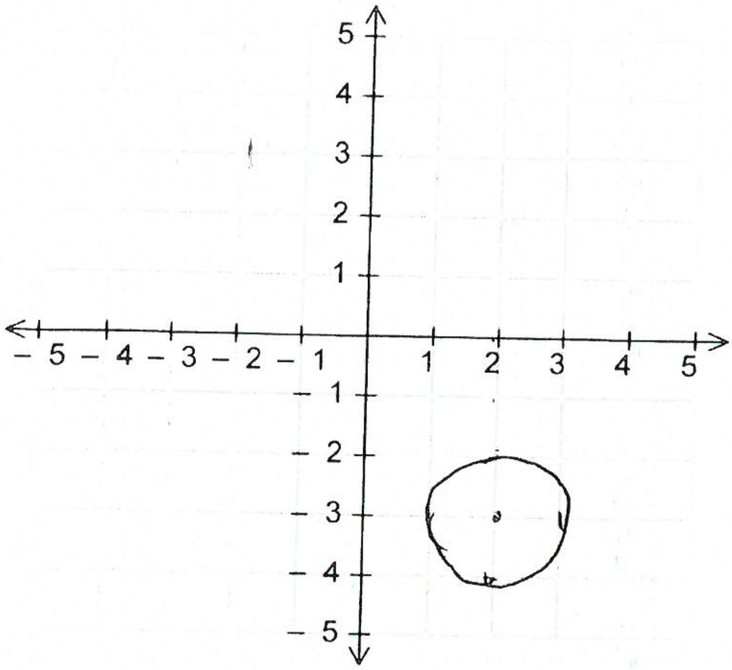
$= \underline{8\cos^4\theta - 8\cos^2\theta + 1}$

5

6 [2+4+2+2=10 marks]

Draw separate sketches of the following sets of points in the complex plane.

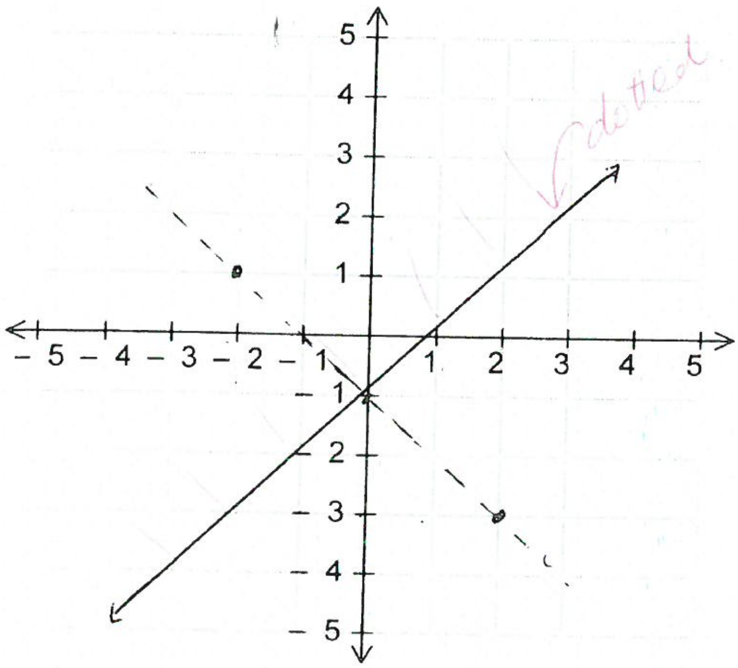
a) $\{z : |z-2+3i| = 1\}$ $\Rightarrow$ $|z-(2-3i)| = 1$



circle centre (2, -3)
radius 1

b) $\{z : |z+2-i| < |z-2+3i|\}$ $|z-(-2+i)| < z-(2-3i)|$

set of pts where the Dist from (-2, 1) is the same as dist from (2, -3)



✓ dotted

✓ Identify pts above

✓✓ " ⊥ Bisector

/ shading