I a Add indices:
$$x^3 \times x^4 = x^{3+4} = x^7$$

b Add indices:
$$a^5 \times a^{-3} = a^{5+-3} = a^2$$

c Add indices:
$$x^2 \times x^{-1} \times x^2 = x^{2+-1+2} = x^3$$

d Subtract indices:
$$\frac{y^3}{y^7} = y^{3-7} = y^{-4}$$

e Subtract indices:
$$rac{x^8}{x^{-4}}=x^{8-(-4)}=x^{12}$$

f Subtract indices:
$$\frac{p^{-5}}{n^2} = p^{-5-2} = p^{-7}$$

g Subtract indices:
$$a^{\frac{1}{2}} \div a^{\frac{2}{3}} = a^{\frac{3}{6} - \frac{4}{6}} = a^{-\frac{1}{6}}$$

$$\begin{array}{ll} \mathbf{h} & \text{Multiply indices:} \\ & (a^{-2})^4 = a^{-2 \times 4} = a^{-8} \end{array}$$

i Multiply indices:
$$(y^{-2})^{-7} = y^{-2 \times (-7)} = y^{14}$$

j Multiply indices:
$$(x^5)^3 = x^{5 imes 3} = x^{15}$$

k Multiply indices:
$$(a^{-20})^{\frac{3}{5}}=a^{-20\times\frac{3}{5}}=a^{-12}$$

Multiply indices:
$$\left(x^{-rac{1}{2}}
ight)^{-4}=x^{-rac{1}{2} imes-4}=x^2$$

m Multiply indices:
$$(n^{10})^{rac{1}{5}}=n^{10 imesrac{1}{5}}=n^2$$

n Multiply the coefficients and add the indices:
$$2x^{rac{1}{2}} imes 4x^3 = (2 imes 4)x^{rac{1}{2}+3} = 8x^{rac{7}{2}}$$

$$(a^2)^{rac{5}{2}} imes a^{-4} = a^{2 imes rac{5}{2}} imes a^{-4} = a^{5+(-4)} = a^1 = a$$

$$\mathsf{p} \quad rac{1}{x^{-4}} = x^{1 \div rac{1}{4}} = x^4$$

$$\mathsf{q} \quad \left(2n^{-\frac{2}{5}}\right)^5 \div (4^3n^4) = 2^5n^{-\frac{2}{5}\times 5} \div ((2^2)^3n^4)$$

$$= 2^5n^{-2} \div (2^6n^4)$$

$$= 2^{5-6}n^{-2-4}$$

$$=2^{-1}n^{-6}=\frac{1}{2n^6}$$

r Multiply the coefficients and add the indices.

$$x^3 imes 2x^{ frac{1}{2}} imes -4x^{- frac{3}{2}} = (1 imes 2 imes -4)x^{3+ frac{1}{2}+\left(- frac{3}{2}
ight)} \ = -8x^2$$

$$\begin{array}{ll} \mathbf{s} & (ab^3)^2 \times a^{-2}b^{-4} \times \frac{1}{a^2b^{-3}} = a^2b^6 \times a^{-2}b^{-4} \times a^{-2}b^3 \\ & = a^{2+-2+-2}b^{6+-4+3} \\ & = a^{-2}b^5 \end{array}$$

t
$$(2^2p^{-3} \times 4^3p^5 \div ((6p^{-3}))^0 = 1$$

Anything to the power zero is 1.

2 a
$$25^{\frac{1}{2}} = \sqrt{25} = 5$$

b
$$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$$

c
$$\left(\frac{16}{9}\right)^{\frac{1}{2}} = \frac{16^{\frac{1}{2}}}{9^{\frac{1}{2}}}$$

$$= \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$$

d
$$16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}}$$

= $\frac{1}{\sqrt{16}} = \frac{1}{4}$

$$e \quad \left(\frac{49}{36}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{49}{36}\right)^{\frac{1}{2}}}$$

$$= \frac{1}{\frac{\sqrt{49}}{\sqrt{36}}}$$

$$= \frac{\sqrt{36}}{\sqrt{49}} = \frac{6}{7}$$

f
$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$$

$$\mathbf{g} \quad 144^{\frac{1}{2}} = \sqrt{144} = 12$$

h
$$64^{\frac{2}{3}} = \left(64^{\frac{1}{3}}\right)^2 = 4^2 = 16$$

i
$$9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3$$

= $3^3 = 27$

$$\mathbf{j} \qquad \left(\frac{81}{16}\right)^{\frac{1}{4}} = \frac{81^{\frac{1}{4}}}{16^{\frac{1}{4}}} \\
= \frac{3}{2}$$

$$\mathbf{k} \quad \left(\frac{23}{5}\right)^0 = 1$$

3 a
$$4.35^2 = 18.9225 \approx 18.92$$

$$\textbf{b} \quad 2.4^5 = 79.62624 \approx 79.63$$

$$\sqrt{34.6921} = 5.89$$

d
$$0.02^{-3} = 125\ 000$$

e
$$\sqrt[3]{0.729} = 0.9$$

f
$$\sqrt[4]{2.3045} = 1.23209... \approx 1.23$$

$$g (345.64)^{-\frac{1}{3}} = 0.14249... \approx 0.14$$

$$\text{h} \quad (4.558)^{\frac{2}{5}} = 1.83607\ldots \approx 1.84$$

$$\mathbf{i} \quad \frac{1}{\left(0.064\right)^{-\frac{1}{3}}} = \left(0.064\right)^{\frac{1}{3}} = 0.4$$

a
$$\frac{a^2b^3}{a^{-2}b^{-4}}=a^{2--2}b^{3--4}$$

$$\begin{array}{ll} \mathbf{b} & \frac{2a^2(2b)^3}{(2a)^{-2}b^{-4}} = \frac{2a^2 \times 2^3b^3}{2^{-2}a^{-2}b^{-4}} \\ & = \frac{2^4a^2b^3}{2^{-2}a^{-2}b^{-4}} \\ & = 2^{4--2} \ a^{2--2}b^{3--4} \\ & = 2^6a^4b^7 = 64a^4b^7 \end{array}$$

$$\mathbf{c} \qquad rac{a^{-2}b^{-3}}{a^{-2}b^{-4}} = a^{-2--2}b^{-3--4} \ -a^0b^1 - b$$

$$\begin{array}{ll} \mathbf{d} & \frac{a^2b^3}{a^{-2}b^{-4}} \times \frac{ab}{a^{-1}b^{-1}} = \frac{a^{2+1}b^{3+1}}{a^{-2+-1}b^{-4+-1}} \\ & = \frac{a^3b^4}{a^{-3}b^{-5}} \\ & = a^{3--3}b^{4--5} = a^6b^9 \end{array}$$

$$\begin{array}{l} \mathbf{e} & \frac{(2a)^2 \times 8b^3}{16a^{-2}b^{-4}} = \frac{4a^2 \times 8b^3}{16a^{-2}b^{-4}} \\ & = \frac{32a^2b^3}{16a^{-2}b^{-4}} \\ & = \frac{32}{16}a^{2--2}b^{3--4} \\ & = 2a^4b^7 \end{array}$$

$$\begin{split} \mathbf{f} & \quad \frac{2a^2b^3}{8a^{-2}b^{-4}} \div \frac{16ab}{(2a)^{-1}b^{-1}} = \frac{2a^2b^3}{8a^{-2}b^{-4}} \times \frac{(2a)^{-1}b^{-1}}{16ab} \\ & = \frac{2a^2b^3}{8a^{-2}b^{-4}} \times \frac{2^{-1}a^{-1}b^{-1}}{16ab} \\ & = \frac{2^{1+-1}a^{2+-1}b^{3+-1}}{16ab} \\ & = \frac{2^{1+-1}a^{2+-1}b^{3+-1}}{8 \times 16 \times a^{-2+1}b^{-4+1}} \\ & = \frac{2^0a^1b^2}{128a^{-1}b^{-3}} \\ & = \frac{1}{128}a^{1--1}b^{2--3} = \frac{a^2b^5}{128} \end{split}$$

5
$$\frac{2^{n} \times 8^{n}}{2^{2n} \times 16} = \frac{2^{n} \times (2^{3})^{n}}{2^{2n} \times 2^{4}}$$
$$= \frac{2^{n} \times 2^{3n}}{2^{2n} \times 2^{4}}$$
$$= \frac{2^{n+3n-2n}}{2^{4}}$$
$$= 2^{2n} \times 2^{-4}$$
$$= 2^{2n-4}$$

$$2^{-x} imes 3^{-x} imes 6^{2x} imes 3^{2x} imes 2^{2x} = (2 imes 3)^{-x} imes 6^{2x} imes (2 imes 3)^{2x}
onumber = 6^{-x} imes 6^{2x} imes 6^{2x}
onumber = 6^{-x+2x+2x}
onumber = 6^{3x}$$

7 In each case, add the fractional indices.

$$\begin{array}{ll} \textbf{a} & 2^{\frac{1}{3}} \times 2^{\frac{1}{6}} \times 2^{-\frac{2}{3}} = 2^{\frac{2}{6} + \frac{1}{6} + -\frac{4}{6}} \\ & = 2^{-\frac{1}{6}} = \left(\frac{1}{2}\right)^{\frac{1}{6}} \end{array}$$

$$\begin{array}{ll} \mathbf{b} & a^{\frac{1}{4}} \times a^{\frac{2}{5}} \times a^{-\frac{1}{10}} = a^{\frac{5}{20} + \frac{8}{20} + -\frac{2}{20}} \\ & = a^{\frac{11}{20}} \end{array}$$

$$\begin{array}{ll} \mathbf{c} & 2^{\tfrac{2}{3}} \times 2^{\tfrac{5}{6}} \times 2^{-\tfrac{2}{3}} = 2^{\tfrac{4}{6} + \tfrac{5}{6} + -\tfrac{4}{6}} \\ & = 2^{\tfrac{5}{6}} \end{array}$$

$$\begin{array}{ll} \mathbf{d} & \left(2^{\frac{1}{3}}\right)^2 \times \left(2^{\frac{1}{2}}\right)^5 = 2^{\frac{2}{3}} \times 2^{\frac{5}{2}} \\ & = 2^{\frac{4}{6} + \frac{15}{6}} = 2^{\frac{19}{6}} \end{array}$$

e
$$\left(2^{\frac{1}{3}}\right)^2 \times 2^{\frac{1}{3}} \times 2^{-\frac{2}{5}} = 2^{\frac{2}{3}} \times 2^{\frac{1}{3}} \times 2^{-\frac{2}{5}}$$

$$= 2^{\frac{2}{3} + \frac{1}{3} + -\frac{2}{5}} = 2^{\frac{3}{5}}$$

8 a
$$\sqrt[3]{a^3b^2}\div\sqrt[3]{a^2b^{-1}}=(a^3b^2)^{\frac{1}{3}}\div(a^2b^{-1})^{\frac{1}{3}}$$

$$=a^1b^{\frac{2}{3}}\div a^{\frac{2}{3}}b^{-\frac{1}{3}}$$

$$=a^{1-\frac{2}{3}}b^{\frac{2}{3}-\frac{1}{3}}=a^{\frac{1}{3}}b$$

$$\begin{array}{ll} \mathbf{b} & \sqrt{a^3b^2} \times \sqrt{a^2b^{-1}} = (a^3b^2)^{\frac{1}{2}} \times (a^2b^{-1})^{\frac{1}{2}} \\ & = a^{\frac{3}{2}}b^1 \times a^1b^{-\frac{1}{2}} \\ & = a^{\frac{3}{2}+1}b^{1+-\frac{1}{2}} = a^{\frac{5}{2}}b^{\frac{1}{2}} \end{array}$$

c
$$\sqrt[5]{a^3b^2} \times \sqrt[5]{a^2b^{-1}} = (a^3b^2)^{\frac{1}{5}} \times (a^2b^{-1})^{\frac{1}{5}}$$

 $= a^{\frac{3}{5}}b^{\frac{2}{5}} \times a^{\frac{2}{5}}b^{-\frac{1}{5}}$
 $= a^{\frac{3}{5}+\frac{2}{5}}b^{\frac{2}{5}+-\frac{1}{5}} = ab^{\frac{1}{5}}$

$$\begin{array}{ll} \mathbf{d} & \sqrt{a^{-4}b^2} \times \sqrt{a^3b^{-1}} = (a^{-4}b^2)^{\frac{1}{2}} \times (a^3b^{-1})^{\frac{1}{2}} \\ & = a^{-2}b^1 \times a^{\frac{3}{2}}b^{-\frac{1}{2}} \\ & = a^{-2+\frac{3}{2}}b^{1+-\frac{1}{2}} \\ & = a^{-\frac{1}{2}}b^{\frac{1}{2}} \\ & = \frac{b^{\frac{1}{2}}}{a^{\frac{1}{2}}} = \left(\frac{b}{a}\right)^{\frac{1}{2}} \end{array}$$

$$\begin{array}{ll} \mathbf{e} & \sqrt{a^3b^2c^{-3}} \times \sqrt{a^2b^{-1}c^{-5}} = (a^3b^2c^{-3})^{\frac{1}{2}} \times (a^2b^{-1}c^{-5})^{\frac{1}{2}} \\ & = a^{\frac{3}{2}}b^1c^{-\frac{3}{2}} \times a^1b^{-\frac{1}{2}}c^{-\frac{5}{2}} \\ & = a^{\frac{3}{2}+1}b^{1+-\frac{1}{2}}c^{-\frac{3}{2}+-\frac{5}{2}} \\ & = a^{\frac{5}{2}}b^{\frac{1}{2}}c^{-4} \end{array}$$

$$\mathbf{g} \quad \frac{\sqrt{a^3b^2}}{a^2b^{-1}c^{-5}} \times \frac{\sqrt{a^{-4}b^2}}{a^3b^{-1}} \times \sqrt{a^3b^{-1}} = \frac{(a^3b^2)^{\frac{1}{2}}}{a^2b^{-1}c^{-5}} \times \frac{(a^{-4}b^2)^{\frac{1}{2}}}{a^3b^{-1}} \times (a^3b^{-1})^{\frac{1}{2}}$$

$$= \frac{a^{\frac{3}{2}}b^1}{a^2b^{-1}c^{-5}} \times \frac{a^{-2}b^1}{a^3b^{-1}} \times a^{\frac{3}{2}}b^{-\frac{1}{2}}$$

$$= a^{\frac{3}{2}-2}b^{1--1}c^{0--5} \times a^{-2-3}b^{1--1} \times a^{\frac{3}{2}}b^{-\frac{1}{2}}$$

$$= a^{-\frac{1}{2}}b^2c^5 \times a^{-5}b^2 \times a^{\frac{3}{2}}b^{-\frac{1}{2}}$$

$$= a^{-\frac{1}{2}+-5+\frac{3}{2}}b^{2+2+-\frac{1}{2}}c^5$$

$$= a^{-4}b^{\frac{7}{2}}c^5$$