Name: ....

Marks   / 50

Cheshire   Coveney   McRae   Loh   Staffe   Giblett

Time Allowed: 50 minutes

**Resources allowed:** Formula sheet provided, Classpad and Calculator allowed.

All necessary working and reasoning must be shown for full marks.

Marks may not be awarded for untidy, illegible or poorly arranged work.

Question 1   **[2, 3: 5 marks]**

The network below shows the flow of water through a system of underground pipes, where the flow is measured in thousands of litres per hour.

37

34
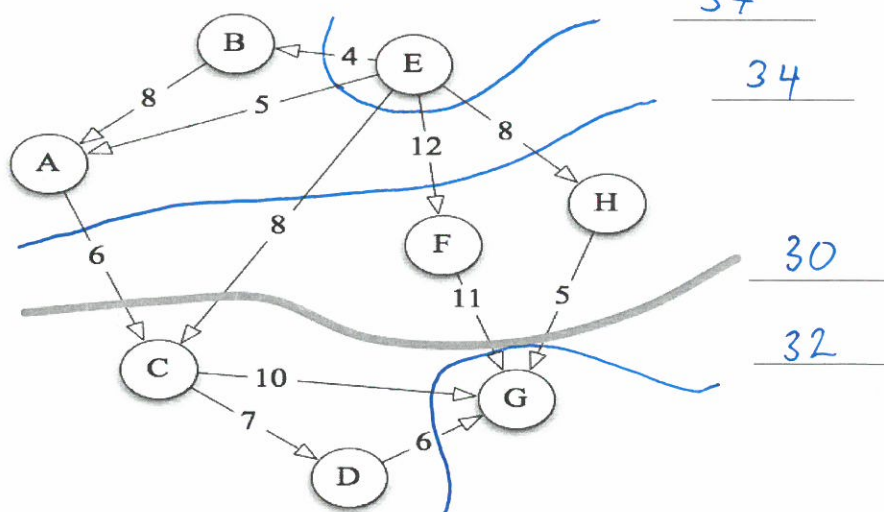
* ✓ 2 correct
or
* ✓ all 4 correct

30

32

a) Identify the source and sink for this network.   Source: __E__ ✓   Sink: __G__ ✓

b) Given that one of the cuts above achieves the maximum flow, determine the value of each cut shown and state the maximum flow through the underground pipe network.

30 000 L/hour   ✓

Question 2 [3, 3, 2: 8 marks]

Caleb retires with $350 000 in an annuity earning interest at 8% per annum, compounded annually. He withdraws $40 000 at the end of each year after the interest has been added to the account.

a) Write a recursive formula to model the amount, C, left in the account after n years.

$$C_{n+1} = 1.08\,C_n - 40000 \qquad C_0 = 350\,000$$

✓ uses C.   ✓ recursive rule   ✓ C_0

b) Determine:

i) The number of full withdrawals that Caleb will be able to make.

15 full withdrawals ✓

ii) The value of the final withdrawal.

final withdrawal = $40\,000 - 13891.40$ ✓ working

$= \$26\,108.60$ ✓ final value

(2 marks if only answer is correct)

c) If Caleb had decided to only take out $35 000 at the end of each year, determine how this would affect the length of the original annuity.

The annuity will last 21 years. ∴ this will increase the length of the annuity by 5 years. ✓ increase length   ✓ 5 years.

Question 3 [4 marks]

Samantha was fortunate enough to win $50 000 in the lottery one weekend. She decided to invest the money into an annuity and take out money each year to use towards a holiday. The annuity will pay interest at 4.8% p.a. compounded quarterly and Samantha will take out a sum of money at the end of each year (after the interest has been added). Her first withdrawal will be $5 000 at the end of the first year, however, to keep up with inflation she has decided to take out an extra 2.5% each year (for example at the end of the second year she will withdraw $5 125).

Write a recursive rule to represent the amount, A, left in this annuity after n years.

$$A_{n+1} = \left(1 + \frac{0.048}{4}\right)^4 A_n - \left(1.025^n \times 5000\right) \qquad A_0 = 50\,000$$

✓ uses A   ✓ compounded quarterly   ✓ increasing withdrawals   ✓ A_0

Question 4 [3, 2: 5 marks]

The network below shows the distance, in metre, between various cottages on a camp site. The manager needs to upgrade the water pipes connecting each chalet as the old system is becoming hazardous to holiday makers.

a) Show, on the network below, how the manager can connect all the cottages with new piping and state the length required to minimise potential costs.
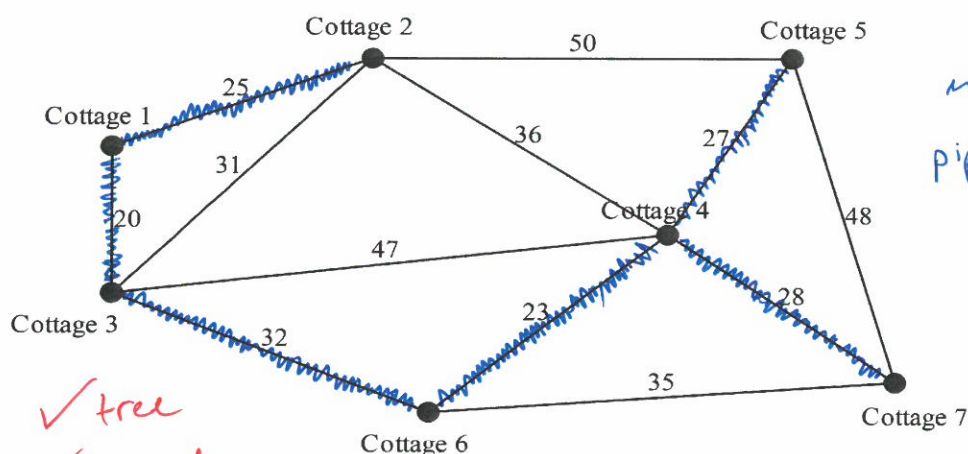
min length of piping is 155 m

✓ min length.

✓ tree

✓ correct edges used.

b) After doing some digging it was found that the piping between Cottage 1 and Cottage 2 can not be replaced. What affect will this have on the minimum amount of piping needed to connect all the cottages?

increases the minimum length of piping by 6 m.

✓ increase

✓ by 6m.

Question 5 [3, 2: 5 marks]

Lucy invested a sum of money into a perpetuity that paid $32 500 a year. The money was invested in an account that paid interest at a rate of 6.5% p.a., compounded monthly.

(a) How much did Lucy invest?

$$32\,500 + P = P\left(1 + \frac{0.065}{12}\right)^{12}$$

Using Classpad $P = \$485\,278.50$

✓ use of effective interest rate

✓ correct equation

✓ answer to 2.d.p.

(b) If instead, Lucy had put her sum of money in a shoebox under the bed and just spent $32 500 each year, how many years would the money have lasted?

$$time = \frac{485\,278.50}{32\,500}$$

$$= 14.9$$

$$\approx 15 \text{ years.}$$

✓ part a) answer divided by 32 500

✓ answer in years.

Question 6

The following grid details the distances (in metres) of internet cabling that connects the Administration office and several other offices throughout Shenton College.

|	Admin	Maths	HASS	English	PE	Arts
Admin	—	40	87	101	99	-
Maths	40	-	85	-	102	83
HASS	87	85	-	95	-	92
English	101	-	95	-	95	74
PE	99	102	-	95	-	-
Arts	-	83	92	74	-	-

(handwritten annotations:) ✓ all correct **or** ✓ at least 2 correct.

The cabling is to be replaced by new fibre optic cables to improve internet speeds throughout the campus.

a) Use Prim's algorithm on the grid above to indicate the shortest amount of cabling required to connect the offices and state this minimum amount below.

min length = 377m   ✓ min length

b) Using your calculations in part a), draw a network representing the cabling needed to connect all the offices.
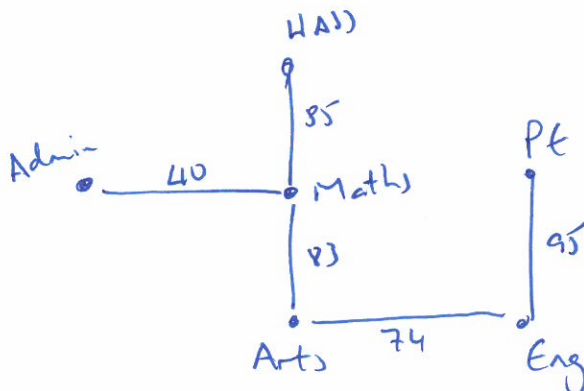
✓ A tree

✓ Labelled with correct connections. and values.

c) Determine the total cost of replacing the cabling if it costs $56.50 per metre plus an extra $35 for a new connection terminal at each office?

Total Cost = $(377 \times 56.50) + (6 \times 35)$

= $ 21 510.50

✓ cabling + connection prices

✓ correct cost to 2d.p.
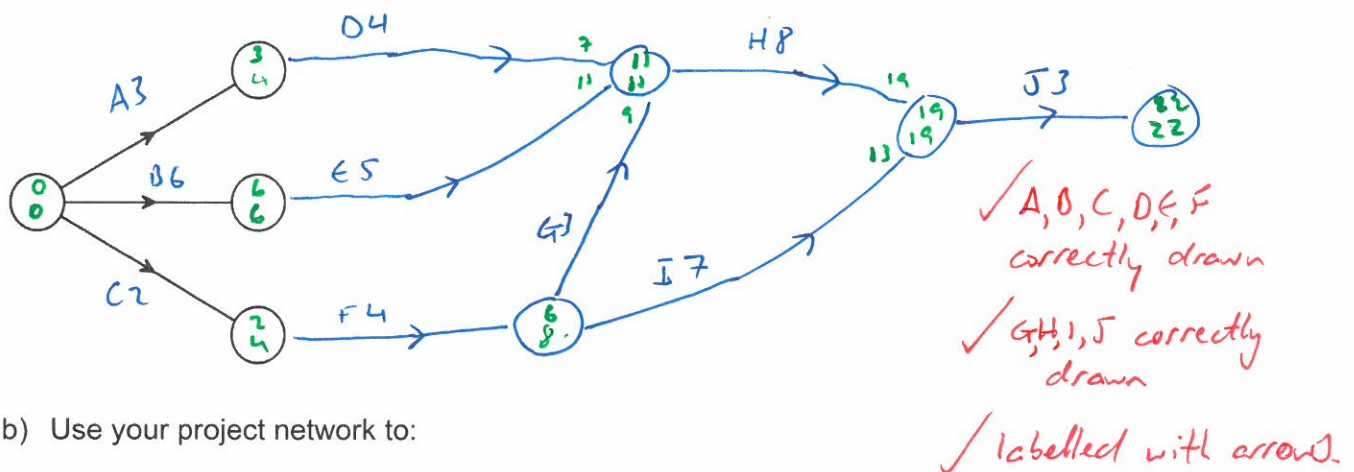
Question 7   [3, 2, 1, 2: 8 marks]

The table below shows an activity chart for the completion a renovation project.

Task	Completion Time (days)	Predecessors
A	3	-
B	6	-
C	2	-
D	4	A
E	5	B
F	4	C
G	3	F
H	8	D, E, G
I	7	F
J	3	H, I

a) Using the chart above, complete the project network for the renovation project.

✓ A, B, C, D, E, F correctly drawn

✓ G, H, I, J correctly drawn

✓ labelled with arrows.

b) Use your project network to:

i) identify the critical path for this renovation and state the minimum time needed to complete all scheduled tasks.   ✓ time   ✓ path.

Min time = 22 days.   Path = BEHJ

ii) State the latest starting time allowed for Task G without affecting the minimum completion time.
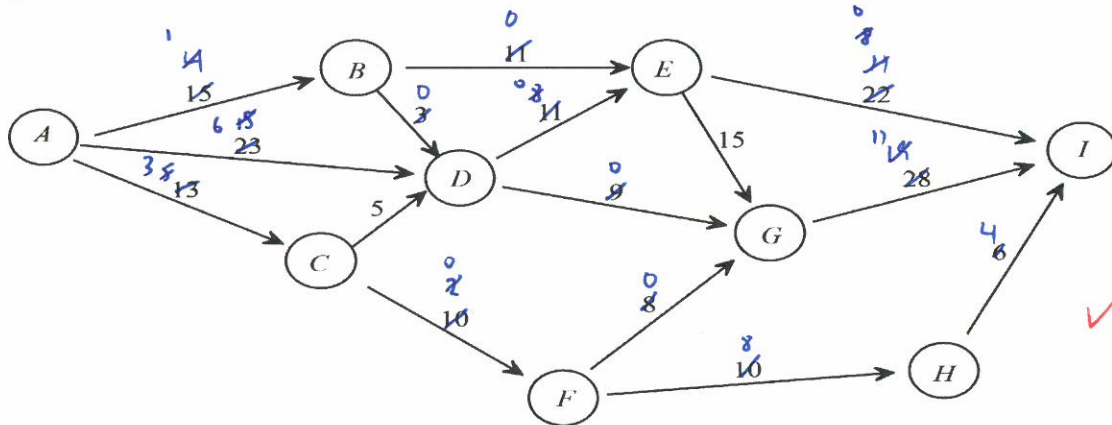
LST = 8 days.   ✓ states 8 days.

c) If Task B was reduced by two days, describe the affect this will have on the float time for Task F.

float time reduces to 0 days.

✓ reduces   ✓ 0 days.

Question 8

[4, 2, 2 : 8 marks]

The following network outlines a heavy freight rail system. The numbers show the maximum volume of freight (in hundreds of tonnes per day) that can be carried on each section of the system.

a) By listing your paths determine the maximum flow possible for each day. Clearly show your working on the diagram.

$ABEI = 11$

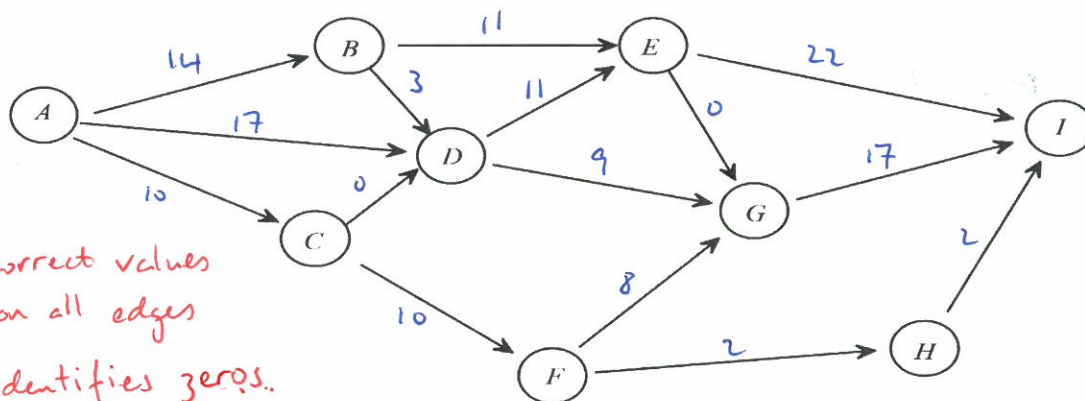$ABDEI = 3$

$ADEI = 8$

$ADGI = 9$

$ACFGI = 8$

$ACFHI = 2$

∴ maximum flow = 4100 tonnes/day

✓ lists paths

✓ states length of each path

✓ states maximum flow.

b) Show the volume of freight per day that achieves the maximum flow through system below.

✓ correct values on all edges

✓ identifies zeros.

Be careful with total flow thru.

c) If you could upgrade only one rail line, from the original network, to increase the maximum flow of freight, which one would you choose? State the effect this may have on the maximum volume of freight per day.

Path DG. Increases the flow by 900 tonnes/day

✓ correct path

✓ increases flow by 900

✓ sufficient working shown