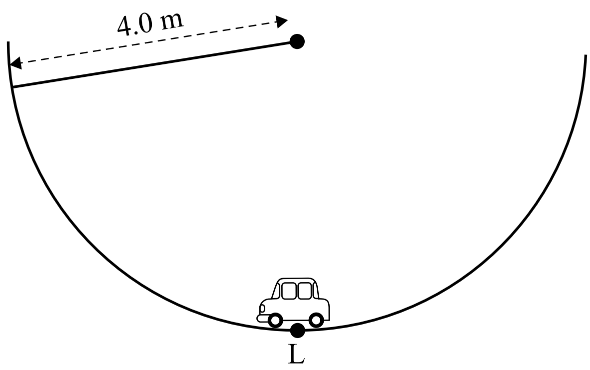
SECTION A (Short Response – 15 questions worth 80 marks)

Answer all questions in the spaces provided. Where appropriate, show all working.

Question 1

A toy car of mass 2.0 kg is on a track that is part of a vertical circle of radius 4.0 m, as shown in the diagram below.

At the lowest point, L, the car is moving at 6.0 ms^{-1}. Ignore friction.



a) At the lowest point (L), draw arrow to show the **direction of the resultant force** acting on the car. Label this arrow as F_R.

[2 marks]

Resultant force acts towards centre of circle (vertically upwards on diagram)
[1 mark for direction] [1 mark for label]

b) Calculate the magnitude of the reaction force exerted on the track by the car when it is at point L.

$R = mv^2 / r + mg = 2.0 \times 6.0^2 / 4.0 + (2.0 \times 9.8) = 37.6$ N
[1 mark] [1 mark] [1 mark]

[3 marks]

Question 2

A planet has a radius half the Earth's radius and a mass a quarter of the Earth's mass. What is the approximate gravitational field strength on the surface of this planet?

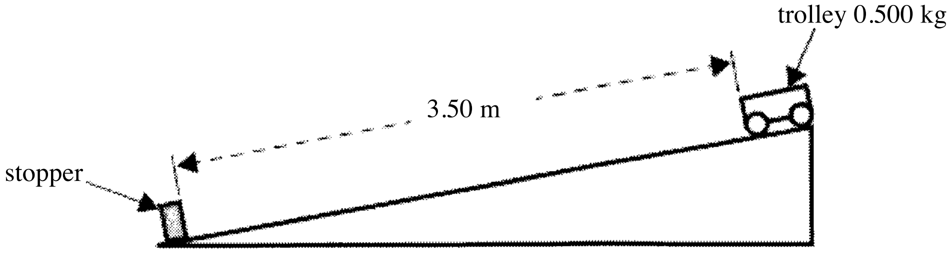
[3 marks]

$g = GM_P / r_P^2 = G \times 0.25\, M_E / (0.5\, R_E)^2 = 0.25 / 0.25\, (GM_E / R_E^2) = g_E = 9.80\ ms^{-2}$
[1 mark] [1 mark] [1 mark]

Question 3

A group of students set up an inclined plane as shown in the diagram below. They place a frictionless trolley of mass 0.500 kg at the top of the incline so that the distance from the front of the trolley to the stopper was 3.50 m.

They released the trolley from rest and found that it took 2.00 s to reach the stopper at the bottom.



a) Calculate the magnitude of the acceleration of the trolley.

u = 0 s = 3.50 m t = 2.00 sec

$3.50 = 0 + ½ a (2.00)^2$ [1 mark]

$a = 1.75\ ms^{-2}$ [1 mark]

[2 marks]

b) Hence, show that the angle of the slope is approximately 10°.

Acceleration down the slope = g sinθ

[1 mark]

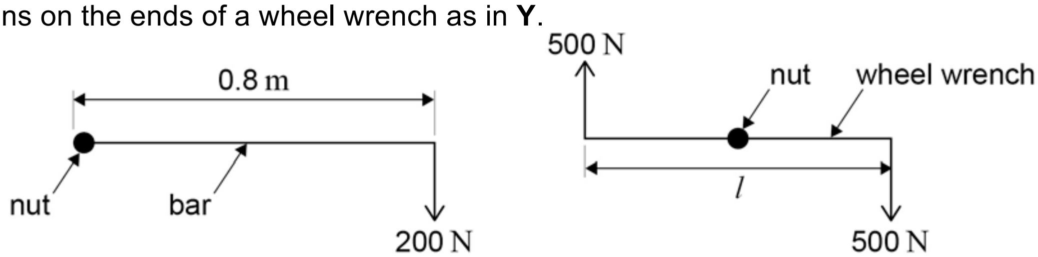
1.75 = 9.80 sinθ

sinθ = 1.75 / 9.8 so angle θ = 10.3° [1 mark]

[2 marks]

Question 4

A car wheel nut can be loosened by applying a force of 200 N on the end of a bar of length 0.8 m as in **X**. A car mechanic is capable of applying forces of 500 N simultaneously in opposite directions on the ends of a wheel wrench as in **Y**.



Calculate the length (l) of the wheel wrench required by the mechanic to loosen the nut.

In situation X: Torque required to loosen the nut: τ = F x r = 200 x 0.80 = 160 Nm

[2 marks]

In situation Y: same total torque is required so τ = F x r = 500 x *L* / 2 x 2 = 160 Nm

[1 mark]

L = 160 / 500 = 0.32 m [1 mark]

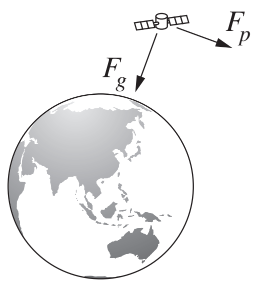
[4 marks]

Question 5

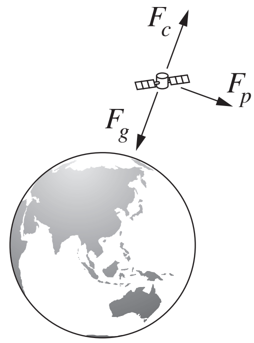
Which of the following diagrams correctly represents the force(s) acting on a satellite in a stable circular orbit around Earth?

F_g = gravitational force	F_p = propulsive force
F_c = centripetal force	F_r = reaction force

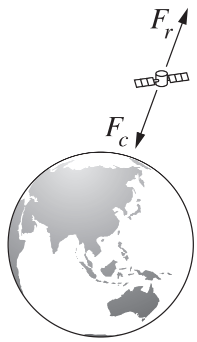
(A)



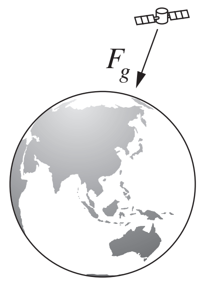
(B)



(C)



(D)



Answer = **D** **[1 mark]**

Briefly justify your answer.

For the satellite to remain in a stable orbit the required centripetal force is provided by the gravitational force of attraction between the Earth and the satellite.

[1 mark]

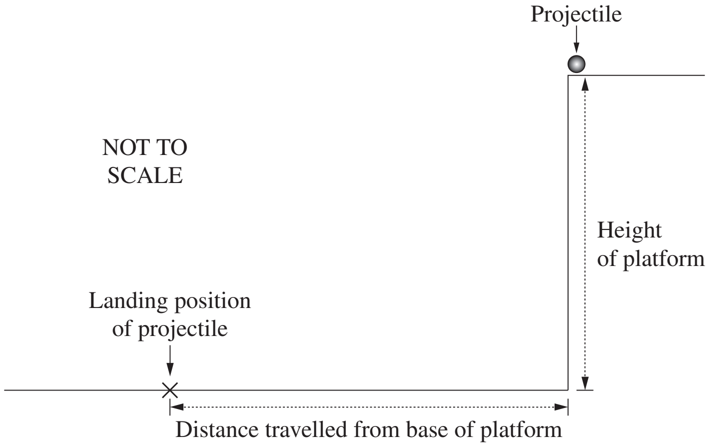
This is the only force acting on the satellite and is directed towards the centre of the Earth. The vector F_g shows this force.

[1 mark]

[3 marks]

Question 6

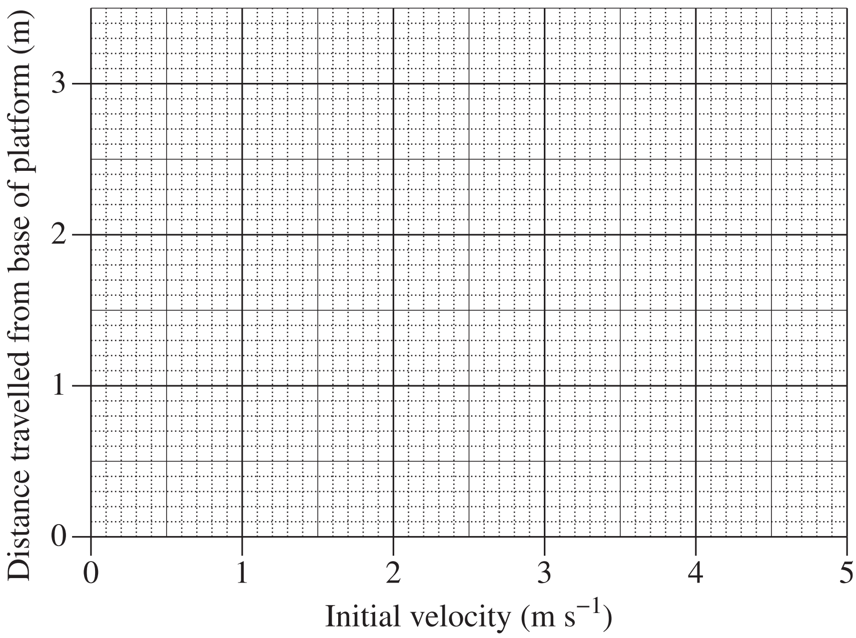
A projectile is fired **horizontally** from a platform.



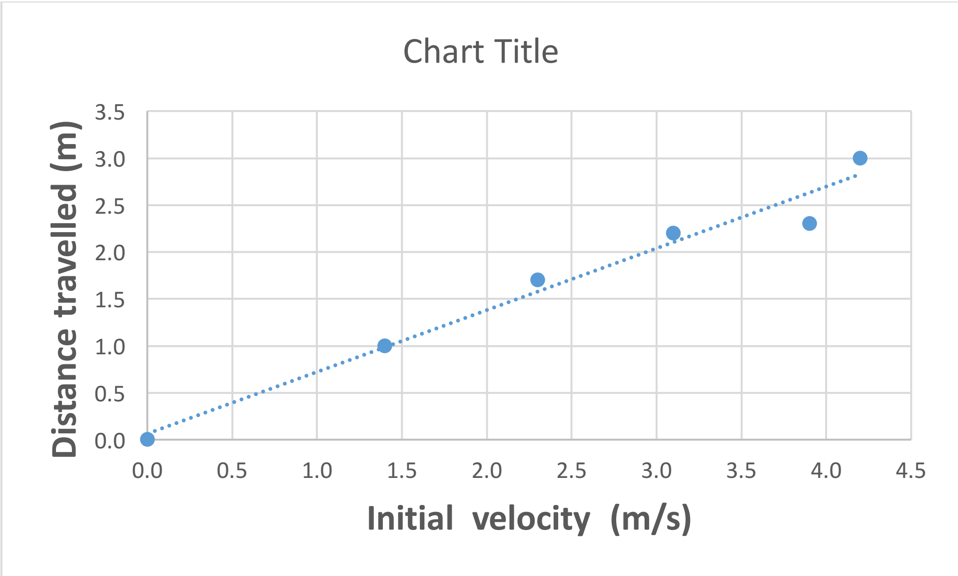
Measurements of the distance travelled by the projectile from the base of the platform are made for a range of initial velocities.

Initial velocity of projectile (m s^{-1})	*Distance travelled from base of platform* (m)
1.4	1.0
2.3	1.7
3.1	2.2
3.9	2.3
4.2	3.0

a) Graph the data on the grid provided and draw the line of best fit.



[2 marks]



b) Calculate the height of the platform.

The range of the projectile $R = v_H \times t$

$t = R / v_H$ = gradient of linear graph = 3.0 / 4.2 = 0.714 seconds **[1 mark]**

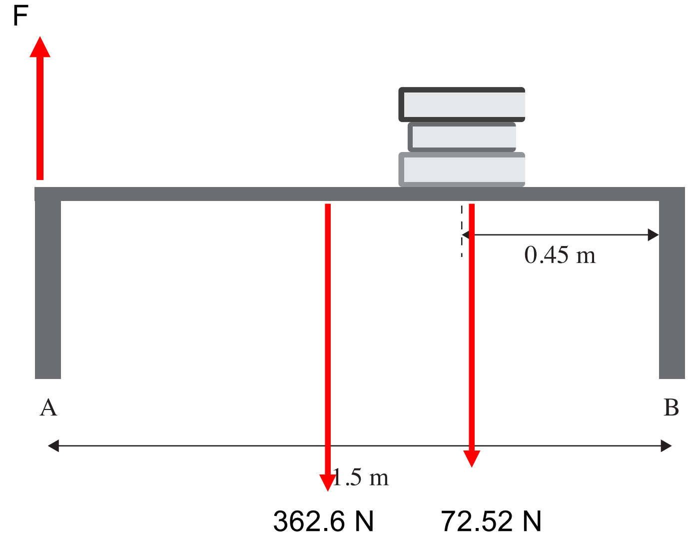
Vertical displacement $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 9.8 \times 0.714^2 = 2.50$ m **[2 marks]**

Hence, height of platform = 2.50 m **[1 mark]**

[4 marks]

Question 7

Alana's study table has two panels, one at each end. Alana has a pile of books on her table.



The table is uniform and has a mass of 37 kg. The mass of the books is 7.4 kg and the weight of the books acts at a distance of 0.45 m from end B of the table.

Calculate the magnitude of the support force provided by panel A of the study table.

Taking moments about B

Σ CW = Σ ACW **[1 mark]**

F x 1.5 = (72.52 x 0.45) + (326.6 x 0.75) **[1 mark]**

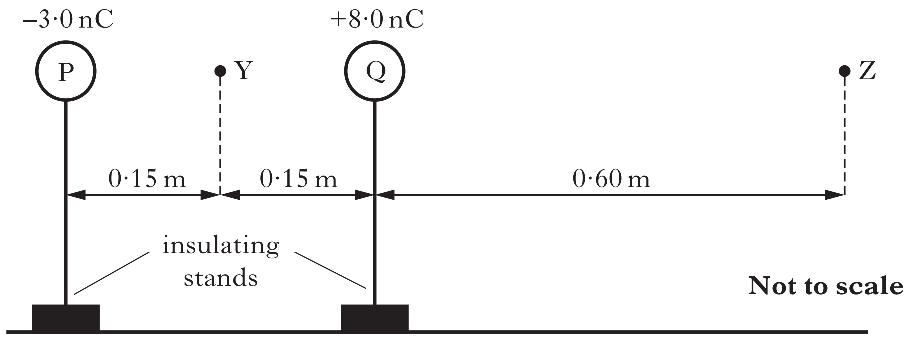
1.5 F = 304.58 **[1 mark]**

Hence: F = 304.58 / 1.5 = 203 N **[1 mark]**

[4 marks]

Question 8

Two identical conducting spheres P and Q are placed on insulating stands 0.30 m apart as shown below.



a) Calculate the magnitude and direction of the electrostatic force exerted by P on Q.

You may use the additional information:

$$\frac{1}{4\pi\epsilon} = 9.0\ x\ 10^9 \text{ units}$$

[3 marks]

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$ **[1 mark]**

F = 9.0 x 10^{9} x 3.0 x 10^{-9} x 8 x 10^{-9} / 0.30^{2} **[1 mark]**

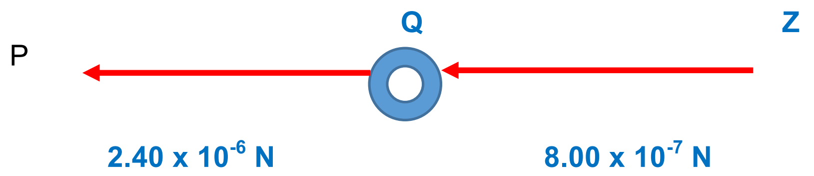
F = 2.40 x 10^{-6} N towards P **[1 mark]**

b) A third charge of + 4.0 nC is now placed at point Z. What is the **magnitude** and **direction** of the **resultant electrostatic force** now experienced by charge Q?

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

F = 9.0 x 10^{9} x 8 x 10^{-9} x 4 x 10^{-9} / 0.60^{2}

F = 8.00 x 10^{-7} N towards Z **[2 marks]**

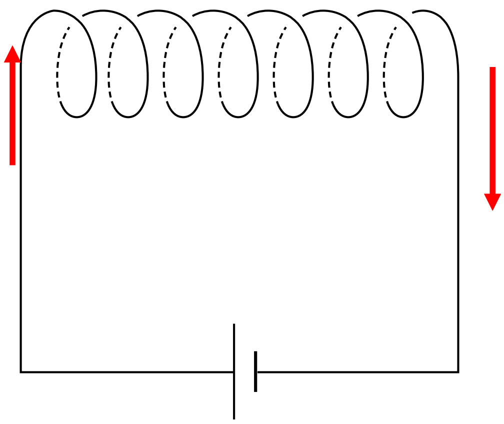


net force on Q = 2.40 x 10^{-6} + 8.00 x 10^{-7} = 3.20 x 10^{-6} N towards P
[1 mark] **[1 mark]**

[4 marks]

Question 9

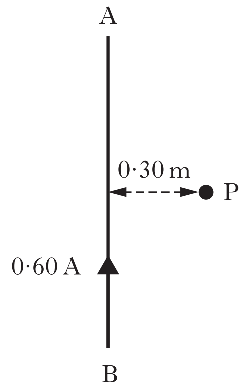
a) The diagram below shows a solenoid. Draw five flux lines (with arrows) to show the magnetic field of the solenoid. Clearly label the poles of the magnetic field of the solenoid.



1 for correct poles
1 for external field
1 for internal field

[3 marks]

a) A long straight conductor AB carries a current of 0.60 A in the direction B to A as shown.



Calculate the strength of the magnetic field at point P, a distance of 0.30 m from the conductor. State if the direction of the magnetic field is into the page or out of the page.

You may use the additional information:

$$\frac{\mu}{2\pi} = 2.0 \; x \; 10^{-7} \; units$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

[1 mark]

B = 2.0 x 10^{-7} x 0.60 /0.30 = 4.0 x 10^{-7} T (direction is into the page)

[1 mark] [1 mark] [1 mark]

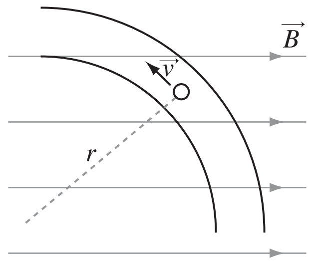
[4 marks]

Question 10

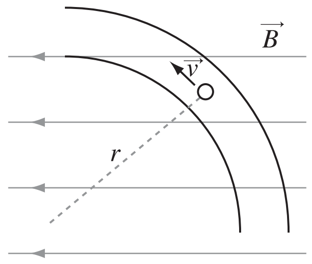
High energy particle accelerators can be used to accelerate protons to close to the speed of light. Magnetic fields are used to produce the circular paths that these protons will follow in the accelerator.

Which of the following diagrams shows the orientation that the magnetic field must have in order to deflect the path of the protons in the accelerator?

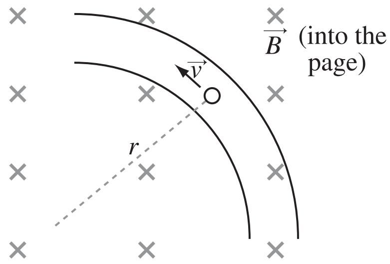
A.



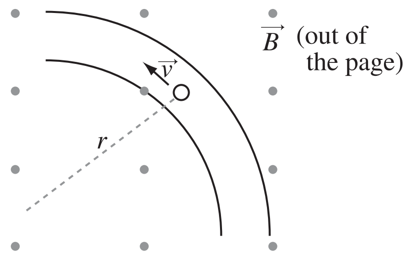
B.



C.



D.



Answer = **C** **[1 mark]**

Briefly justify your answer.

The magnetic force $F_M = qvB$ must provide the centripetal force needed to keep the charged particle moving in a circular path. [1 mark]

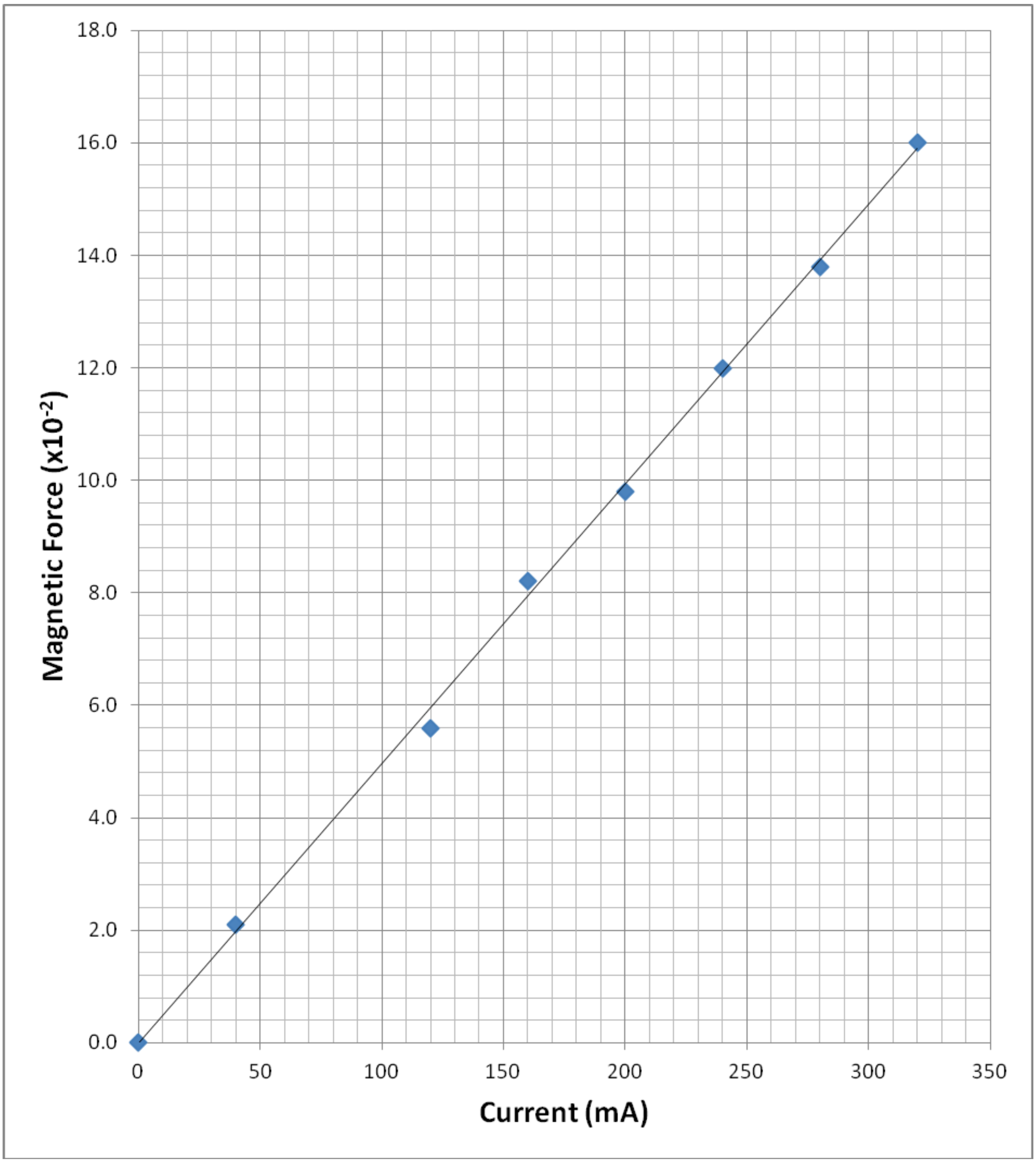
The centripetal force F_C acts towards the centre of the circular path. [1 mark]

Using the RH rule to determine the direction of the force will require the magnetic field (B) to be directed into the page. [1 mark]

[4 marks]

Question 11

An experiment is carried out to measure the magnitude of the force acting on a straight current-carrying wire when placed within a uniform magnetic field at an angle of 90° to the field. The electric current (**I**) is varied for each trial and the magnetic force **F** is measured. The results of the experiment are shown in the graph below.



a) Calculate the gradient of the line of best fit for the graph. State the units of the gradient of the graph.

Gradient = Rise / Run = 4.0×10^{-2} / 80×10^{-3} = 0.500 NA^{-1}
[1 mark] [1 mark] [1 mark]

[3 marks]

b) The magnetic field used has a magnitude of 0.262 T. Use your value for the gradient to find the length of the straight current-carrying wire placed in the magnetic field.

F = ILB

Since the graph of B against I is linear, the gradient must equal LB [1 mark]

0.500 = L x 0.262 [1 mark]

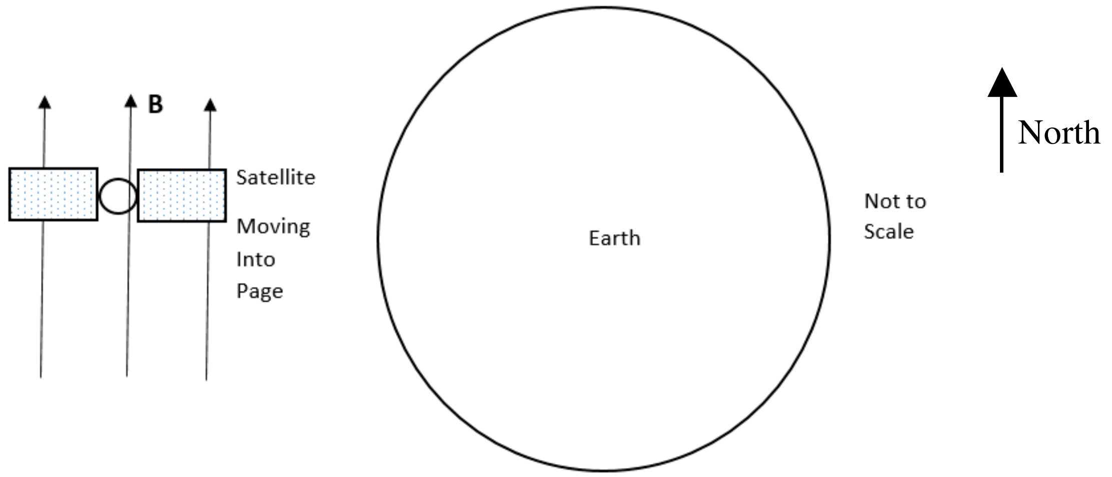
L = 0.500 / 0.262 = 1.91 m [1 mark]

[3 marks]

Question 12

A satellite is orbiting above the equator of the Earth. The satellite is powered by a large solar array of total width 150 m.

The satellite is moving at 2.34×10^4 kmh^{-1} **into** the page. At that point, the Earth's magnetic field has a magnitude of 250 μT northwards.



a) Given that the solar array is electrically conductive, calculate induced EMF between the tips of the solar array.

$v = 2.34 \times 10^4 / 3.6 = 6.50 \times 10^3$ ms^{-1} [1 mark]

EMF = v L B = $6.50 \times 10^3 \times 150 \times 250 \times 10^{-6}$ [2 marks]

= 244 V [1 mark]

[4 marks]

b) Which tip of the solar panel (left or right) will be positive? Justify your answer.

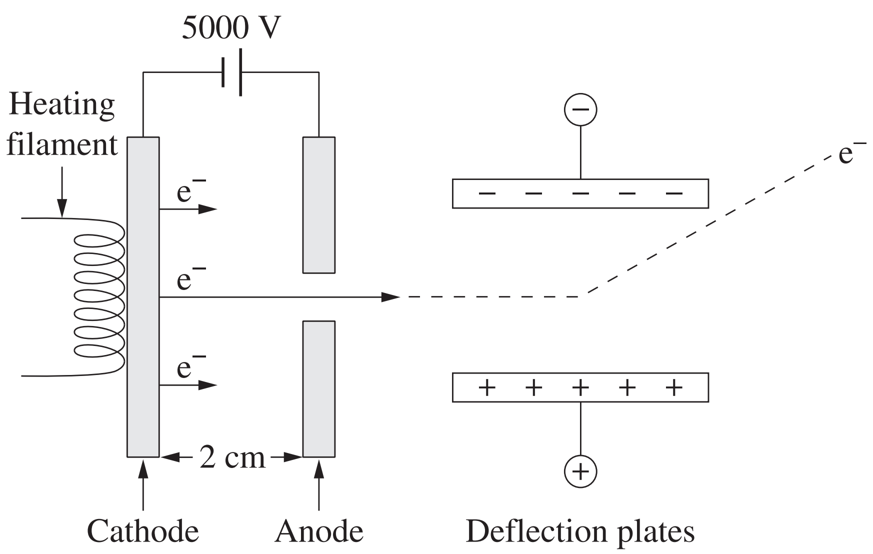
Using RH rule (fingers in field direction, thumb in current direction) gives the direction of the force as being towards the right. Hence, the right tip of the solar panel will be positive.

[1 mark for answer] **[1 mark for reason/justification]**

[2 marks]

Question 13

A part of a cathode ray oscilloscope was represented on a website as shown.



Electrons leave the cathode and are accelerated towards the anode.

a) Calculate the strength of the electric field between the anode and cathode.

$E = V / d = 5000 / 0.02 = 2.50 \times 10^5$ Vm^{-1}
[1 mark] **[1 mark]**

[2 marks]

b) Explain why the representation of the path of the electron between the **deflection plates** is inaccurate.

The electron is shown to move upwards towards the negative plate. [1 mark]

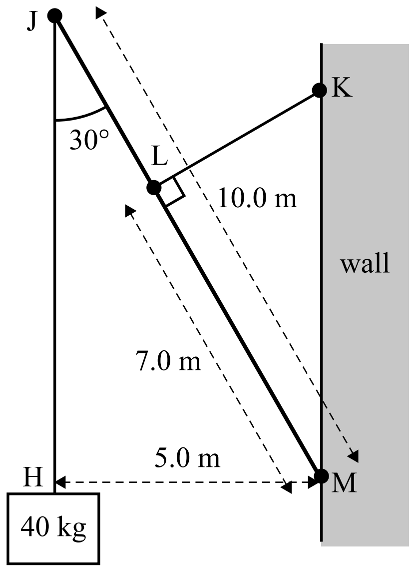
The force acting on the electron is downwards towards the positive plate. [1 mark]

The path should also be curved since the actual velocity of the electron will be the resultant of the original horizontal velocity and the (vertically) downwards velocity that comes from the force exerted by the electric field on the electron [1 mark]

[3 marks]

Question 14

A uniform beam, JM, of negligible mass and length 10 m is joined to a wall at point M by a frictionless hinge. A cable, KL, connects the beam to the wall, as shown in the diagram below. A mass of 40 kg hangs from point J with a cable JH.



a) What is the net force acting on the beam JM? Justify your answer.

[2 marks]

The net force = 0 since the system is in equilibrium (no rotation, no translation)
[1 mark] [1 mark]

b) Calculate the torque about the point **M** due to the 40 kg mass. Include the direction.

Torque $\tau = F \times r \times \sin\theta = (40 \times 9.8) \times 10.0 \times \sin 30 = 1.96 \times 10^3$ Nm.
[1 mark] [1 mark]

The direction of the torque is anti-clockwise (ACW) [1 mark]

[3 marks]

c) Hence, determine the tension force in the cable KL.

Σ ACW = Σ CW [1 mark]

$1.96 \times 10^3 = F \times 7.0$ where F = tension force in the cable [1 mark]

$F = 1.96 \times 10^3 / 7.0 = 2.80 \times 10^2$ N [1 mark]

[3 marks]

d) The maximum tension in cable KL can only be 500 N before it snaps. What is the largest mass that can be supported by cable JH?

mg x r x sin θ = F x 7.0 [1 mark]

m x 10.0 x 9.8 x sin 30 = 500 x 7.0 [1 mark]

m = 500 x 7.0 / (10.0 x 9.8 x sin 30) = 71.4 kg [1 mark]

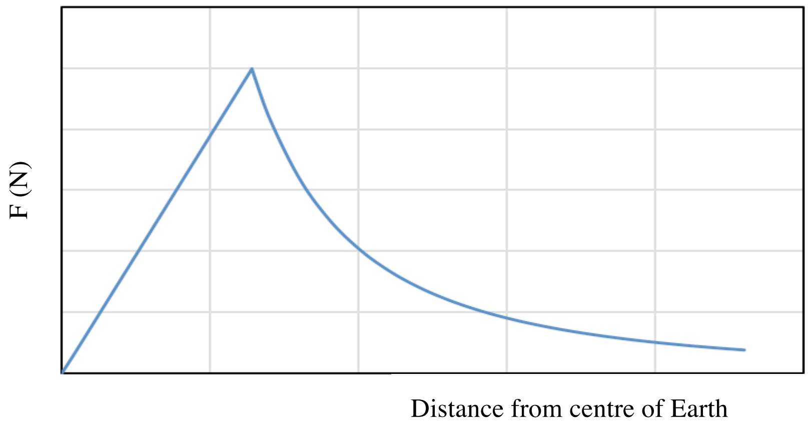
maximum mass possible = 71.4 kg

[3 marks]

Question 15

In Jules Verne's classic novel, *A Journey to the Centre of the Earth*, adventurers follow tunnels towards the Earth's centre. While this is improbable, we will consider a hypothetical vertical journey to the centre. The traveller finds that the **g**-field decreases to zero in approaching the centre of the Earth.

The graph shows the gravitational force on a 100 kg traveller while moving from a distance of approximately 2 earth radii above the surface to the centre of the Earth.



a) Explain why the g-field decreases inside the Earth.

Since $g = GM / r^2$ **[1 mark]**

As you move closer to the centre of the Earth, there is less mass between you and the centre of the Earth. Hence, the gravitational force of attraction will be smaller. This means that the gravitational field strength must decrease as you approach the centre of the Earth.

[2 marks]

[See detailed mathematical answer at end of the solutions to the exam]

[3 marks]

Assume the Earth is a uniform sphere of radius 6.37×10^6 m.

b) What are the co-ordinates of the highest point of the graph?

The highest point on the graph will be when you are on the surface of the Earth.

$F = W = mg = 100 \times 9.8 = 9.80 \times 10^2$ N **[1 mark]**

The distance from the centre of the Earth = $R_E = 6.37 \times 10^6$ m

Hence, the co-ordinates will be (6.37×10^6 , 9.80×10^2) **[1 mark]**

[2 marks]

END of SECTION A – GO ON TO SECTION B

SECTION B (Problem Solving – 7 questions worth 90 marks)

Answer all 7 questions in the spaces provided. Show all working.

Question 16

A spacecraft is placed in orbit around Saturn so that it is Saturn-stationary (the Saturn equivalent of geostationary – the spacecraft is always above the same point on Saturn's surface on the equator).

The following information may be needed to answer some/all of the following questions.

Mass of Saturn M = 5.68×10^{26} kg

Mass of spacecraft m = 2.0×10^{3} kg

Period of rotation of Saturn T = 10 hours 15 minutes.

a) Calculate the period, in seconds, of this spacecraft's orbit.

Period T = (10 x 60 x 60) + (15 x 60) = 3.69×10^{4} seconds
[1 mark] [1 mark]

[2 marks]

b) Derive the following mathematical relationship. The symbol *r* represents the orbital radius of the spacecraft as it orbits Saturn.

$$M = \frac{4\pi^2 r^3}{GT^2}$$

For a stable orbit around Saturn:

$$F_C = F_G$$

$$mv^2 / r = G M m / r^2 \quad \textbf{[1 mark]}$$

$$v^2 = GM / r \quad \textbf{[1 mark]}$$

Since $v = 2\pi r / T$ **it follows that** $v^2 = 4\pi^2 r^2 / T^2$

$$4\pi^2 r^2 / T^2 = GM / r$$

Re-arranging gives $M = 4\pi^2 r^3 / GT^2$ **[1 mark]**

[3 marks]

c) Calculate the orbital radius of the spacecraft when it is in the "Saturn-stationary" orbit.

$R^3 = GM \times T^2 / 4\pi^2$ **[1 mark]**

$= 6.67 \times 10^{-11} \times 5.68 \times 10^{26} \times (3.69 \times 10^4)^2 / 4\pi^2$ **[1 mark]**

[3 marks]

$= 1.3248 \times 10^{24}$

Hence, the orbital radius $R = 1.10 \times 10^8$ m **[1 mark]**

d) Calculate the centripetal acceleration of the spacecraft as it orbits Saturn.

$a_C = v^2 / r = g = GM_S / r^2$ **[1 mark]**

$= 6.67 \times 10^{-11} \times 5.68 \times 10^{26} / (1.10 \times 10^8)^2$ **[1 mark]**

$= 3.13$ ms^{-2} **[1 mark]**

[3 marks]

e) Will an astronaut inside this spacecraft feel *weightless*? Explain your answer.

Yes, both the satellite and the astronaut are in "free-fall" around Saturn.

[1 mark]

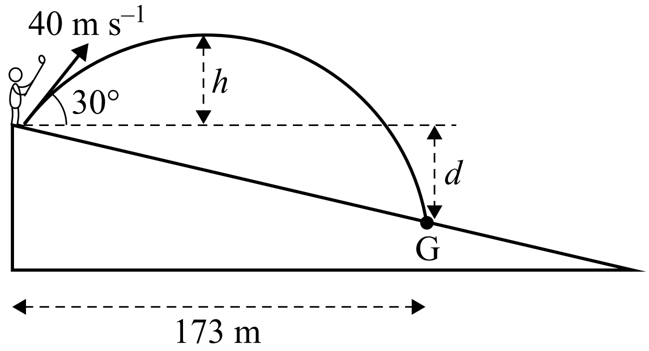
As a result, no reaction force will be experienced and "apparent weightlessness" will be experienced.

[1 mark]

[2 marks]

Question 17

A golfer hits a ball on a part of a golf course that is sloping downwards away from her, as shown in the diagram below.



The golfer hits the ball at a speed of 40 ms^{-1} at an angle of 30° to the horizontal.

Ignore the effects of air resistance.

a) Calculate the maximum height, *h*, that the ball rises above its initial position.

[3 marks]

Vertical component of velocity $v_H = v \sin\theta = 40 \sin 30 = 20.0$ ms^{-1} [1 mark]

At highest point: $v_v = 0$ $v^2 = u^2 + 2as$

$0^2 = 20^2 + (2 \times -9.8 \times s)$

[1 mark]

$s = 400 / 19.6 = 20.4$ m

Height h = 20.4 m [1 mark]

The ball lands at a point at a horizontal distance of 173 m from the hitting-off point, as shown above.

b) Calculate the vertical drop, *d*, from the hitting-off point to the landing point G.

$V_H = v \cos\theta = 40 \cos 30 = 34.64$ ms^{-1} [1 mark]

Range $R = 173 = v_H \times t$ [1 mark]

Flight time $t = 173 / 34.64 = 4.994$ seconds [1 mark]

Vertical displacement: $s = ut + \frac{1}{2}at^2 = (20.0 \times 4.994) - 4.9 \times 4.994^2 = -22.32$ [1 mark]

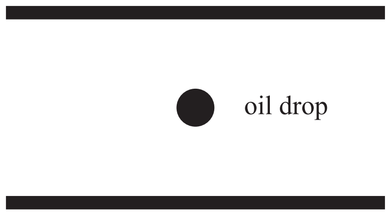
The vertical drop d = 22.32 m [1 mark]

[5 marks]

Question 18

In 1909 Robert Millikan performed an experiment to determine the size of the charge on an electron. He put a charge on a tiny drop of oil, and measured how strong an applied electric field had to be in order to stop the oil drop from falling.

Claire and Nikki are doing a similar experiment. An electrically charged oil drop is held stationary in an electric field, so that it floats.



The oil drop has a negative charge of 24×10^{-10} C, and is placed in a uniform electric field of strength 610 NC^{-1} directed vertically.

The oil drop "floats" between the plates.

a) Name the forces (including directions) acting on the oil drop while it "floats" between the plates.

Weight force W = mg (acting downwards) **[1 mark]**

Electrical force F_E = Eq (acting upwards) **[1 mark]**

[2 marks]

b) Calculate the mass of the oil drop.

If the oil drop "floats" then the resultant force acting on it must be zero **[1 mark]**

Hence: $Eq = mg$

$m = Eq / g = 610 \times 24 \times 10^{-10} / 9.8 = 1.49 \times 10^{-7}$ kg

[1 mark] **[1 mark]**

[3 marks]

c) If the distance between the plates is 85 mm, calculate the potential difference (V) between the plates.

Since $E = V / d$

$V = E \times d = 610 \times 85 \times 10^{-3} = 51.9$ V

[1 mark] **[1 mark]**

[2 marks]

d) Explain what the same charged oil drop would do if the plates were brought closer together. Assume the charge on the oil drop remains the same, and the voltage across the plates remains unchanged.

The electric field strength will increase in strength so the electrical force experienced by the charged particle will be greater. **[1 mark]**

Since the weight of the oil drop remains constant:

$F_E > W$ **[1 mark]**

The net force will now be directed upwards, so the oil drop will move upwards.

[1 mark]

[3 marks]

e) A free electron and a free proton are placed in identical electric fields of the same strength.

Compare:

- The strength of the **electric force** on each particle
- The **acceleration** of each particle (you may neglect gravity)

Give reasons to justify your comparisons.

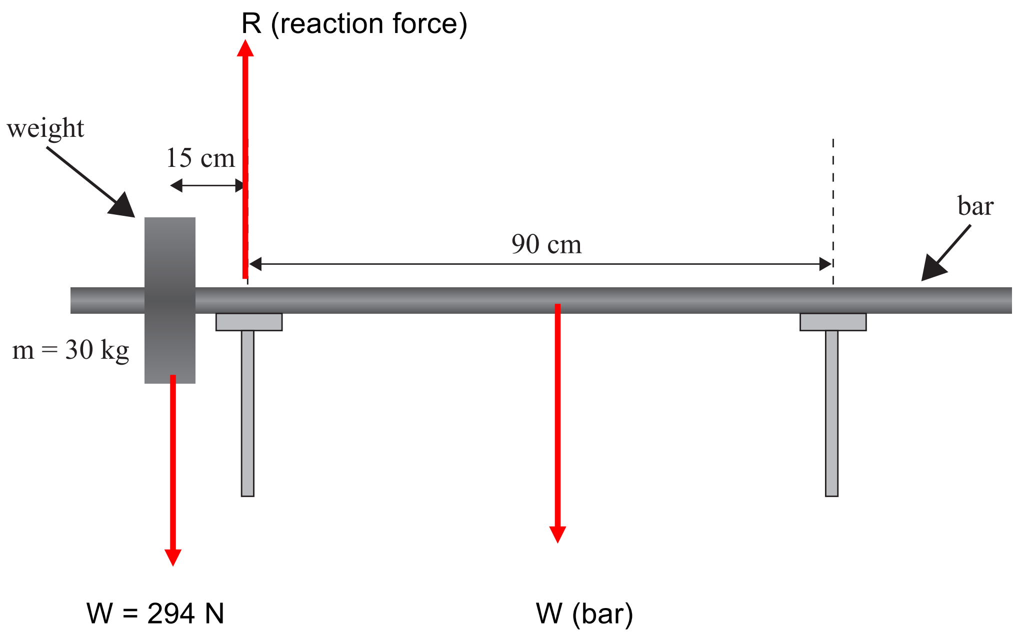
Electric force	**Acceleration**
The force is equal to Eq **Both particles have the same charge and so the electric force will be the same strength. [1 mark]** **The forces will, however, act in opposite directions.** **[1 mark]**	**Since F = ma it follows that a = F / m** **[1 mark]** **Electrons have a smaller mass than protons and therefore they will experience a greater acceleration,** **[1 mark]**

[4 marks]

Question 19

Grace often uses the gym available at her school. One of her favourite activities is to work-out using weights.

Grace puts the barbell on two supports and changes the weights on the bar.



With no weights on one end and a 30 kg weight on the other end, the support force provided by the **right hand** support is zero.

a) Draw **labelled arrows** on the diagram showing the forces acting on the bar.

[1 mark] + [1 mark] + [1 mark]

[3 marks]

b) Use the concept of torque to calculate the **mass** of the bar. Assume it is a uniform bar.

[4 marks]

Taking moments about the LH support

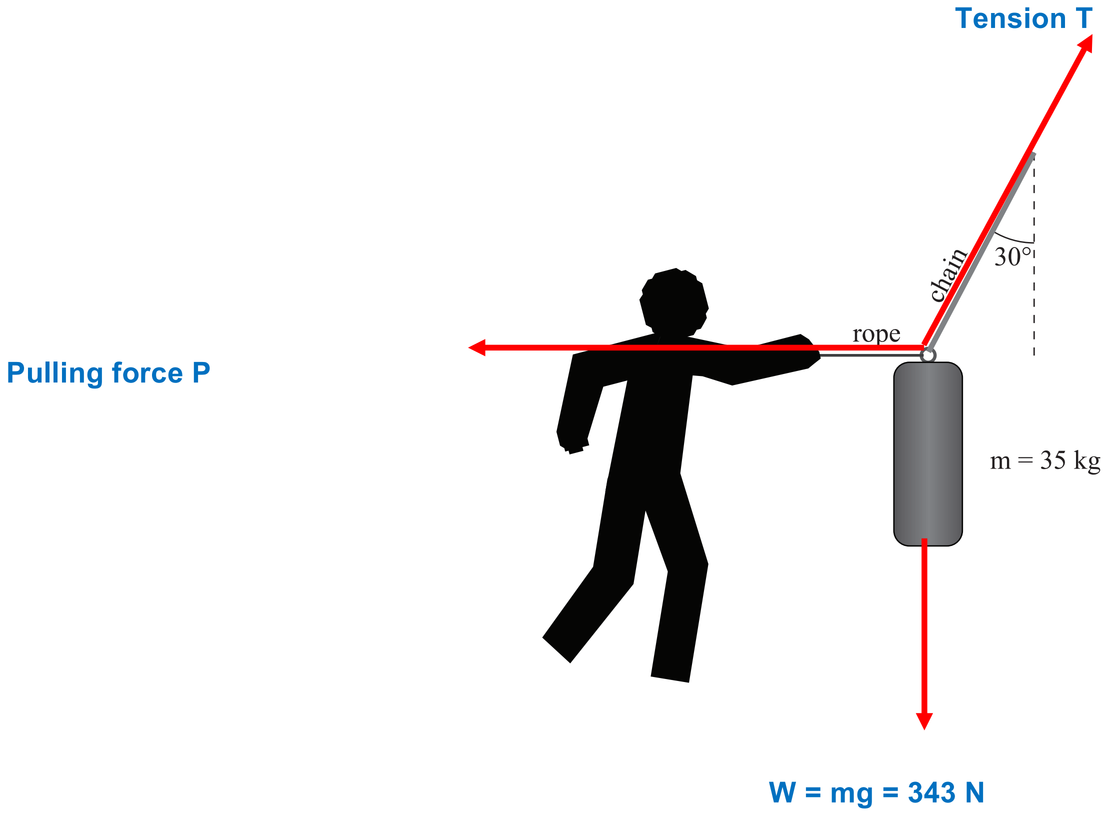
Σ ACW = Σ CW [1 mark]

294 x 0.15 = W x 0.45 [1 mark]

W = 294 x 0.15 / 0.45 = 98.0 N [1 mark]

Mass m = 98.0 / 9.8 = 10.0 kg [1 mark]

While Grace is doing her weights another student, Mia, comes in to use the punch-bag. The punch-bag has a mass of 35 kg and is supported by a chain attached to the ceiling of the gym.



Mia pulls the bag horizontally, using the rope tied to a ring at the top of the bag, until the chain is at an angle of 30° to the vertical, as shown in the diagram above.

c) Draw labelled arrows to represent the three forces acting on the ring at the top of the bag.

[1 mark] + [1 mark] + [1 mark]

[3 marks]

d) Determine the size of the tension force on the chain.

Net force = 0

$F_{UP} = F_{DOWN}$ **[1 mark]**

T sin 60 = W = 343 **[1 mark]**

T = 343 / sin 60 = 396 N **[1 mark]**

[3 marks]

e) Mia now pulls harder on the rope so that the angle between the chain and the vertical is increased. Does the tension force on the chain increase, decrease or stay the same? Justify your answer with appropriate reasoning.

T = W / sin θ

Since W is a constant, the tension T is inversely proportional to sinθ. **[1 mark]**

Smaller angle θ will produce a greater tension in the chain. **[1 mark]**

As the angle to the vertical increases, the value of the angle θ decreases. This means the tension in the chain will increase in magnitude. **[1 mark]**

[3 marks]

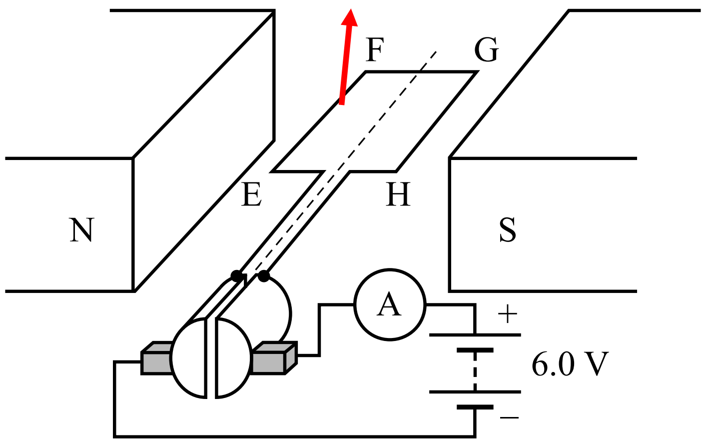
Question 20

A group of students have a model that can be used as either a DC motor or as a generator, depending on the connections used.

The magnets provide a uniform magnetic field of 2.25 mT.

EFGH is a square coil of each side length 4.0 cm with 30 turns.

A 6.0 V battery and an ammeter are connected to the shaft through a commutator.



The ammeter shows a reading of 4.15 A.

a) With the coil horizontal as shown in the diagram above, what is the total force on the side EF? Give the magnitude and direction. Show your working.

F = N(ILB) = 30 x 4.15 x 0.04 x 2.25 x 10^{-3} = 1.12 x 10^{-2} N
[1 mark] **[1 mark]**

Using RH rule on side EF will give the direction of the force as upwards
[1 mark]

[3 marks]

b) What is the maximum torque produced when the coil rotates? Include the appropriate direction.

Maximum torque: $\tau = 2\ (F \times r) = 2 \times 1.12 \times 10^{-2} \times 0.02$ **[1 mark]**

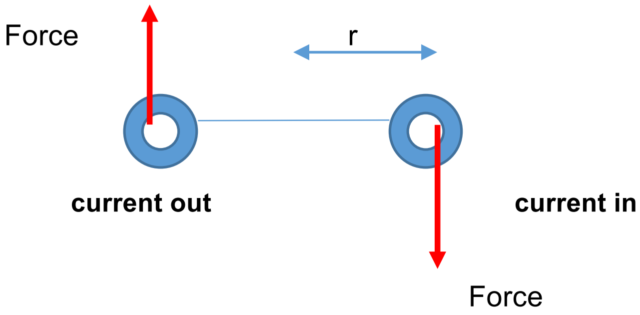
$= 4.48 \times 10^{-4}$ Nm in clockwise direction
[1 mark] **[1 mark]**

[3 marks]

c) The students notice that when the wire coil starts to rotate it does not undergo a "smooth" rotation. With the aid of clearly labelled diagrams, explain why this is so.

As the coil rotates, the size of the torque varies and so the rate of rotation is not constant.

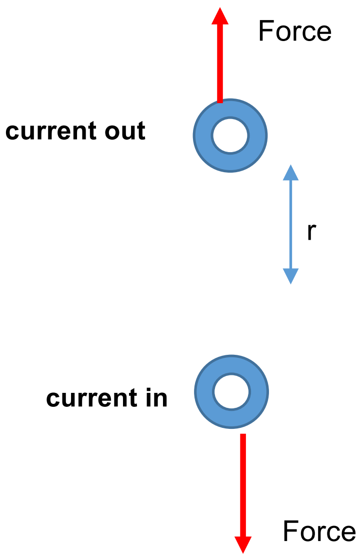
When viewed from E to F magnetic field is directed from left to right



Here the angle between the force and distance from the axis of rotation = 90°

After a 90° rotation of the coil

Here the angle between the force and distance from the axis of rotation = 0°



As the coil rotates the angle between the force acting on the wire and the distance from the axis of rotation varies from 90° to 0° and hence the magnitude of the torque produced varies from 4.48×10^{-4} Nm to zero

d) What is the role of the commutator in the operation of the motor?

The purpose of the split-ring commutator is to reverse the direction of current flow in the coil every half-rotation. **[1 mark]**

This ensures the torque acts in the same direction after every half-rotation **[1 mark]**

This means that the coil will rotate continuously in the same direction (full 360° rotation)
[1 mark]

[3 marks]

e) What are 3 different ways in which the torque output of the motor can be increased?

1	**Increase the size of the current flowing in the coil** **[1 mark]**
2	**Increase the strength of the magnetic field** **[1 mark]**
3	**Use more turns on the coil / mount the coil on a soft iron core** **[1 mark]**

[3 marks]

Question 21

In 2015, the *New Horizons* space probe passed between the dwarf planet Pluto and its largest moon, Charon. This provided the first detailed look at these two bodies.

The radius of Pluto's orbit around the Sun = 5.87×10^{12} m

The radius of Charon's orbit around Pluto = 1.75×10^{4} km

The period of Charon's orbit around Pluto = 153 hours

The mass of the Sun = 1.99×10^{30} kg

a) What is the orbital speed of Charon as it moves around Pluto?

[3 marks]

orbital period $T = 153 \times 60 \times 60 = 5.51 \times 10^{5}$ seconds **[1 mark]**

orbital speed $v = 2\pi r/T = 2 \times 3.142 \times 1.75 \times 10^{7} / 5.51 \times 10^{5} = 1.996 \times 10^{2} = 2.00 \times 10^{2}$ ms^{-1}
[1 mark] **[1 mark]**

b) Hence, calculate the mass of Pluto.

$v^2 = GM_P / r$ so, $M_P = v^2 r / G = 199.6^2 \times 1.75 \times 10^{7} / 6.67 \times 10^{-11} = 1.05 \times 10^{22}$ kg
[1 mark] **[1 mark]** **[1 mark]** **[1 mark]**

[4 marks]

c) What is the magnitude of the gravitational force between Pluto and the Sun?

[3 marks]

$F = GM_S M_P / r^2 = 6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.05 \times 10^{22} / (5.87 \times 10^{12})^2$
[1 mark] **[1 mark]**

Gravitational force = 4.04×10^{16} N
[1 mark]

d) To determine weight of a space probe on the surface of Pluto, what **additional** information would be required? Give reasons to support your answer.

Mass of the spacecraft **[1 mark]**

Radius of Pluto (to calculate "g" on the surface of Pluto) **[1 mark]**

[2 marks]

Question 22

The Physics of Black Holes

Anything on the surface of a large body (for example a planet) is affected by the gravitational field surrounding that body and to escape from the surface an object must move very quickly. The force of gravity will slow such an object but providing it is moving fast enough it can escape, that is, move sufficiently far away from the large body that the gravitational pull never manages to slow the escaping object's velocity to zero.

The minimum velocity needed to leave a surface is called the escape velocity and the equation used to calculate it is

$$v = \sqrt{\frac{2GM_E}{r_E}}$$

Imagine all the matter of the Sun, a ball of gas (mainly hydrogen and helium) with radius 6.96×10^8 m, squeezed together, so that it has a radius of only 2420 km. The Sun would now be a 'white dwarf' and its gases would have become a mixture of atomic nuclei and loose electrons.

Compressing the Sun even more would cause the electrons to fuse into the nuclei leaving nothing but neutrons. The Sun would be a 'neutron star' with a radius of 7250 m. The escape velocity of a neutron star is 1.93×10^8 m s^{-1}. It is hard to imagine anything being able to achieve such huge velocities but light would be able to escape since it travels at 3×10^8 m s^{-1}.

If the Sun continued to shrink past the neutron star stage the escape velocity would increase, eventually, to the speed of light. Then nothing, not even light, would be able to escape from the Sun's surface. Anything can fall into such an object, but nothing can escape. It is a 'black hole'.

The critical radius at which a neutron star becomes a black hole is given by the formula

$$r = \frac{2GM}{c^2}$$

In fact, our Sun is too small to become a black hole. Stars more massive than the Sun explode before they begin to collapse, losing some of their mass. If the amount of mass remaining after such an explosion is more than 3.2 times the mass of our Sun, the collapsing star will become a black hole.

a) Calculate the escape velocity from the Earth.

[3 marks]

$v^2 = 2GM_E / R_E = 2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24} / 6.38 \times 10^6$

[1 mark] [1 mark]

Hence, escape velocity = 1.12×10^4 ms^{-1}

[1 mark]

b) Describe how the strength of the gravitational field changes as you move further away from a planet.

[2 marks]

The strength of the gravitational field is inversely proportional to the square of the distance from the centre of the planet.

[1 mark]

The field strength significantly weakens as the distance from the centre of the planet increases

[1 mark]

c) Calculate the radius of a black hole having the same mass of the Sun.

$r = 2GM / c^2 = 2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30} / (3 \times 10^8)^2 = 2.95 \times 10^3$ m

[1 mark] [1 mark] [1 mark]

[3 marks]

d) A very large black hole has a mass equal to 4.2 times the mass of the Sun. What is the gravitational field strength (g) at a distance of 200 km from the centre of the black hole?

$g = GM / r^2 = 6.67 \times 10^{-11} \times 4.2 \times 1.99 \times 10^{30} / (200 \times 10^3)^2 = 1.39 \times 10^{10}$ ms^{-2}

[1 mark] [1 mark] [1 mark]

[3 marks]

END of SECTION B – GO ON TO SECTION C

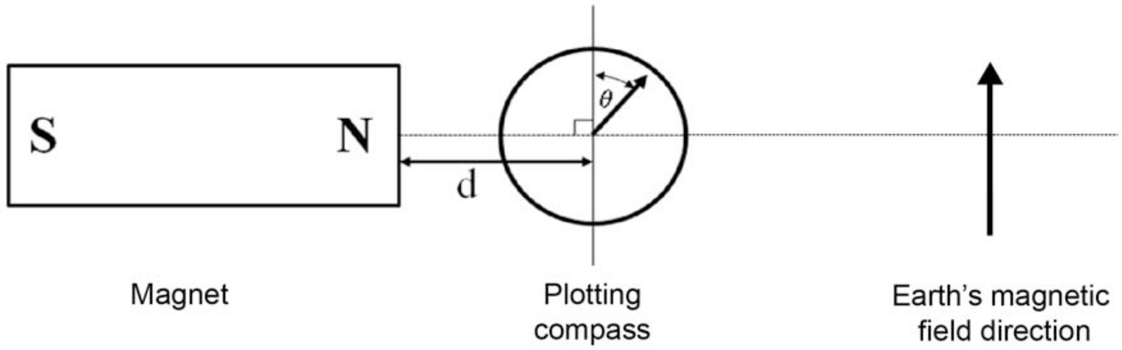
SECTION C (Comprehension and Interpretation – worth 30 marks)

Read the following information very carefully and answer the questions which follow.

The Earth's Magnetic Field

A student performed an experiment to investigate how the magnetic field strength of a bar magnet varied with distance from the magnet along a line through the long axis of the magnet. She measured the angle θ between the pointer of a plotting compass and geographic north, as she moved the plotting compass to various distances (d) away from the magnet. She measured the angle θ at intervals of 3.0 cm, as shown in diagram A.

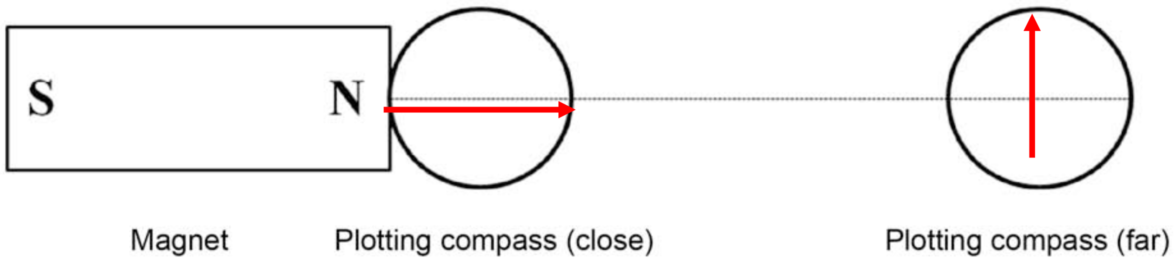
Diagram A



The arrow on the end of the compass needle represents the north end of the compass needle.

a) On diagram B, draw arrows on each of the plotting compasses to indicate the angle you would expect the needle to make when it is close to, and when it is far (more than 50 cm) from, the bar magnet,

Diagram B



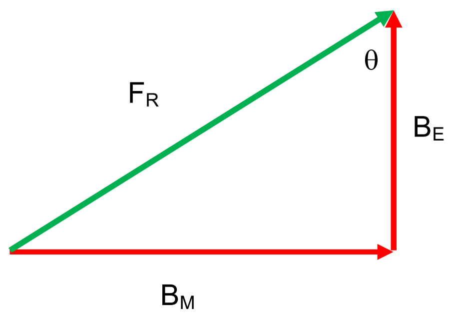
[1 mark] **[1 mark]**

[2 marks]

b) Both the Earth's magnetic field (B_E) and the bar magnet's magnetic field (B_M) affect the compass.

✷ Draw a vector diagram that shows these two magnetic fields and the resultant magnetic field experienced by the plotting compass shown in Diagram A.

[3 marks]



✷ Use your diagram to derive a relationship between the Earth's magnetic field, the magnet's magnetic field and the angle θ.

[2 marks]

$\tan\theta = B_M / B_E$ **[1 mark]**

$B_M = B_E \tan\theta$ **[1 mark]**

c) Calculate the strength of the magnetic field due to the bar magnet at a point on the axis, 10.0 cm from the end of the bar magnet, if the value of θ at this point is 82°, and the Earth's magnetic field strength is 2.0×10^{-5} T.

$B_M = B_E \tan\theta = 2.0 \times 10^{-5} \times \tan 82 = 1.42 \times 10^{-2}$ T

[1 mark] **[1 mark]** **[1 mark]**

[3 marks]

The student collected the following data in her experiment.

Distance from magnet (m)	**$1/d^2$ (m^{-2})**	**θ (degrees)**	**tan θ**
0.15 ± 0.01	**44.4**	58 ± 1	1.60 ± 0.03
0.18 ± 0.01	**30.9**	42 ± 1	0.90 ± 0.03
0.21 ± 0.01	**22.7**	40 ± 1	0.84 ± 0.03
0.24 ± 0.01	**17.4**	33 ± 1	0.65 ± 0.02
0.27 ± 0.01	**13.7**	27 ± 1	0.51 ± 0.02
0.30 ± 0.01	**11.1**	23 ± 1	0.42 ± 0.02

[4 marks]

d) Complete the table by filling in the values for $1/d^2$. Write the values to **three** significant figures.

[4 marks]

e) Using the data from the **third** row of the table, determine the **relative error** involved with the following measurements. Show all working.

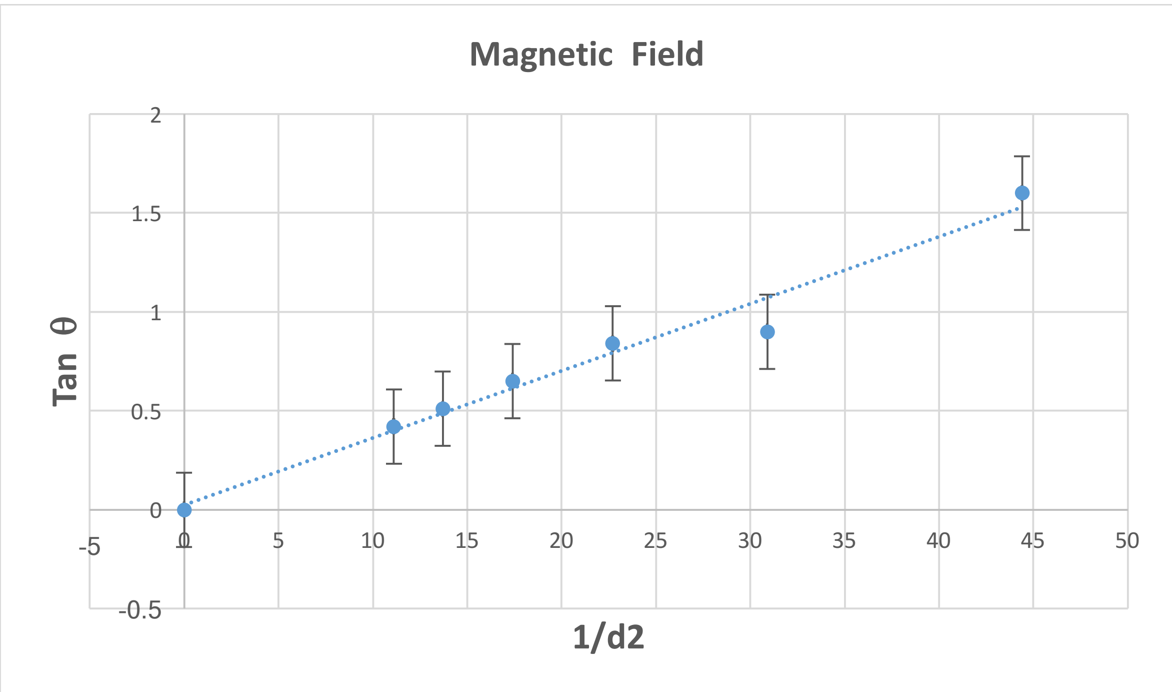
Distance d	Angle θ	tan θ
Relative error = 0.01 / 0.21 x 100 = 4.76 % [1 mark]	Relative error = 1 / 40 x 100 = 2.50 % [1 mark]	Relative error = 0.03 / 0.84 x 100 = 3.57 % [1 mark]

[3 marks]

f) Plot a graph of tan θ (vertical axis) versus $1/d^2$ (horizontal axis). Include error bars and draw a line of best fit. Clearly identify any outliers in the data collected.

Use the graph paper provided.

[5 marks]



g) Determine the gradient of the straight line graph. Show all working and express the value to three significant figures. Include the appropriate units.

Gradient = Rise / Run = 0.72 / 20 = 3.60 x 10^{-2} m^2
[1 mark] [1 mark] [1 mark] [1 mark] for units

[4 marks]

h) Mark and label the point on your graph where the strength of the Earth's magnetic field (B_E) is **equal** to the strength of the magnetic field (B_M) of the bar magnet.

Use this point to determine the distance from the magnet where the two fields are equal.

[4 marks]

$B_E = B_M$ where $\tan\theta = 1$ or when $\theta = 45^\circ$ [1 mark]

$B_M = B_E \tan\theta$ [1 mark]

From the graph, when $\tan\theta = 1$, the value of $1/d^2 = 27$ [1 mark]

$1/d^2 = 27$ so then $d^2 = 1/27$

d = 0.192 m = 19.2 cm [1 mark]

DETAILED ANSWER to SECTION A (Question 15)

The gravitational field strength is given by the expression $g = GM_E / R^2$

If you are on the surface of the Earth or moving away from the surface of the Earth, you will be attracted by **all** of the Earth's mass. Hence, "g" will be inversely proportional to the square of the distance from the centre of the Earth. Thus, "g" will decrease as you move further away from the centre of the Earth.

If you are **inside** the Earth and moving towards the centre of the Earth, then the following must be true:

a) The distance R from the centre of the Earth is decreasing
b) The amount of the Earth's mass between you and the centre of the Earth which is exerting the gravitational force on you is also decreasing

Assuming the Earth is spherical in shape and has a uniform density then

$\rho = M / V$ (ρ = density, M = mass, V = volume)

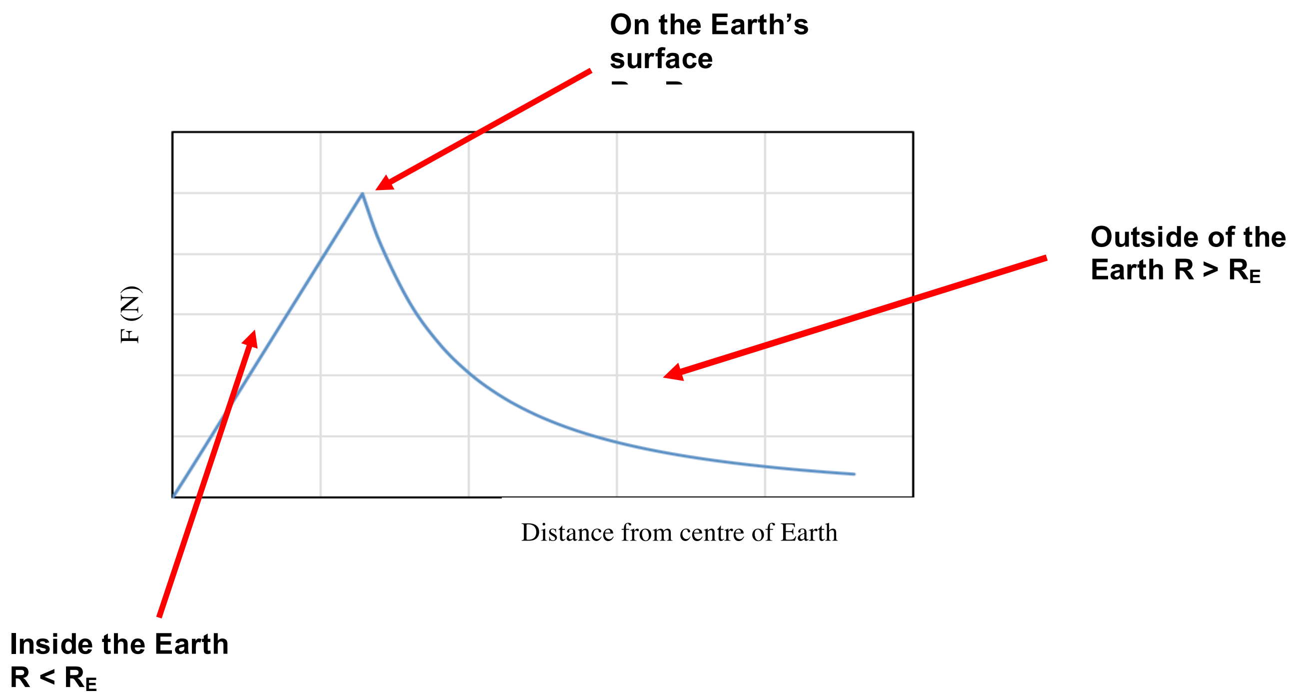
$M = \rho \times V = \rho \times 4/3\,\pi R^3$

Hence, the gravitational field strength can be calculated thus:

$$g = GM_E / R^2 = G \rho \times 4/3 \pi R^3 / R^2 \quad = 4/3 \pi \rho G R$$

g = **$4/3 \pi \rho G$** R where **$4/3 \pi \rho G$ equals a constant number**

Therefore "g" is directly proportional to R and will increase as R increases as you move from inside the Earth towards the surface. Similarly, when you are at the centre of the Earth R = 0 and so g = 0



END of SECTION C – THERE ARE NO MORE QUESTIONS