SHENTON COLLEGE

Year 12 Mathematics Applications
2021 Test 3 – Time Series Analysis, Compound interest Loans, investments and reducing balance loans.
Calculator Assumed

Name: Solutions

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Time Allowed: 50 minutes

Marks: ___/55

Special Materials Allowed: Formula sheet, 1 A4 page notes, Classpad, calculator.
Marks may not be awarded for untidy or poorly arranged work.

Question 1. **[6 marks: 1, 1, 2, 2]**

James invests $145 000 into a savings account that earns compound interest of 2.33% per annum.

(a) Determine the total amount of the investment over six years, if the interest is compounded:

(i) annually.

$166 489.12 ✓ correct value

N 6
I 2.33
PV -145 000
PMT 0
FV 166 489.12
P/Y 1
C/Y 1

(ii) weekly.

$166 751.13 ✓ correct value

N (52×6)
I 2.33
PV -145 000
PMT 0
FV 166 751.13
P/Y 52
C/Y 52

(b) Determine the total amount of interest this account will accrue in the first year when the interest is compounded daily.

$148 418.06 - $145 000 ✓ calculates total amount after 365 days

= $3 418.06 ✓ subtracts total amount by principal amount

N 365
I 2.33
PV -145 000
PMT 0
FV 148 418.06
P/Y 365
C/Y 365

(c) Determine the minimum time, to the nearest month, for this investment of $145 000 to become $169 000 when the interest is compounded monthly.

N = 78.96 ✓ calculates N of 78.96

∴ 79 months ✓ gives answer to the next subsequent month

N 78.96
I 2.33
PV -145 000
PMT 0
FV 169 000
P/Y 12
C/Y 12

Question 2. **[7 marks: 3, 2, 2]**

The manager of a small curtain business recorded the company earnings (in $000's) each quarter from 2018 to the third quarter of 2021. The data is shown in the table below, together with moving average calculations.

Time (t)	Year	Quarter	Company Earnings ($000's)	4-point centered moving averages (m)	5 point moving average (g)
1	2018	1	38		
2		2	45		
3		3	20	45.125	43.6
4		4	**A**	44.75	44.6
5	2019	1	37	44.625	39.8
6		2	43	44.5	51
7		3	21	44	42.4
8		4	76	**B**	43.8
9	2020	1	35	43.875	39.2
10		2	44	43.5	49.8
11		3	20	43	41.2
12		4	74	42.375	42.4
13	2021	1	33	41.75	**C**
14		2	41		
15		3	18		

(a) Calculate the missing entries **A, B** and **C**.

A = 78 ✓A

B = 43.875 ✓B

C = 37.2 ✓C

(b) From the two sets of moving averages given in the table above, which is the most appropriate moving average for the manager to consider? Justify your choice.

✓states 4 CMA with reasoning

The four-point centred moving average is the most appropriate as:

→ The data is collected quarterly with peaks occurring every 4th quarter, or,

→ the data has a smooth decreasing trend, whereas the five point moving averages has fluctuations.

✓gives appropriate reasoning in context

The regression equation for the time, t, against the four-point centred moving averages, m, is $m = -0.3034t + 46.189$.

(c) Interpret the feature of the regression equation above which highlights the trend of this time series.

The negative gradient of the regression equation (-0.3043) as this indicates that the time series data has a decreasing trend.

✓states negative gradient of -0.3043

✓States decreasing trend or negative secular trend.

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Question 3. **[8 marks: 3, 1, 1, 3]**

Marcia places \$14 000 into an investment account with GNI bank for 5 years. Interest in this investment account is given at 2.5% p.a. compounding monthly. At the end of each month, after the interest is calculated, Marcia places an additional \$95 into the investment account.

(a) Write a recurrence relation, to give the value of the investment in the account, B_n, at the end of each month, *n*.

$B_{n+1} = \left(1 + \frac{0.025}{12}\right) B_n + 95$, $B_0 = 14\,000$

✓ uses correct B_n notation

✓ gives recursive rule with correct rate, compound period and payment

✓ states initial amount of \$14 000

(b) Calculate how much money will be in the investment account at the end of 6 months.

$B_6 = \$14\,748.89$ ✓ calculates B_6

(c) Calculate the final value of this investment account at the end of the 5 years.

$B_{60} = \$21\,926.87$ ✓ calculates B_{60}

(d) Calculate the total interest accrued by this investment account at the end of the 5 years.

Interest = \$21 926.87 − \$14 000 − (\$95 × 60)

= \$2 226.87 interest.

✓ Value from (c) subtracts principal

✓ value from (c) subtracts 60 lots of \$95

✓ states interest

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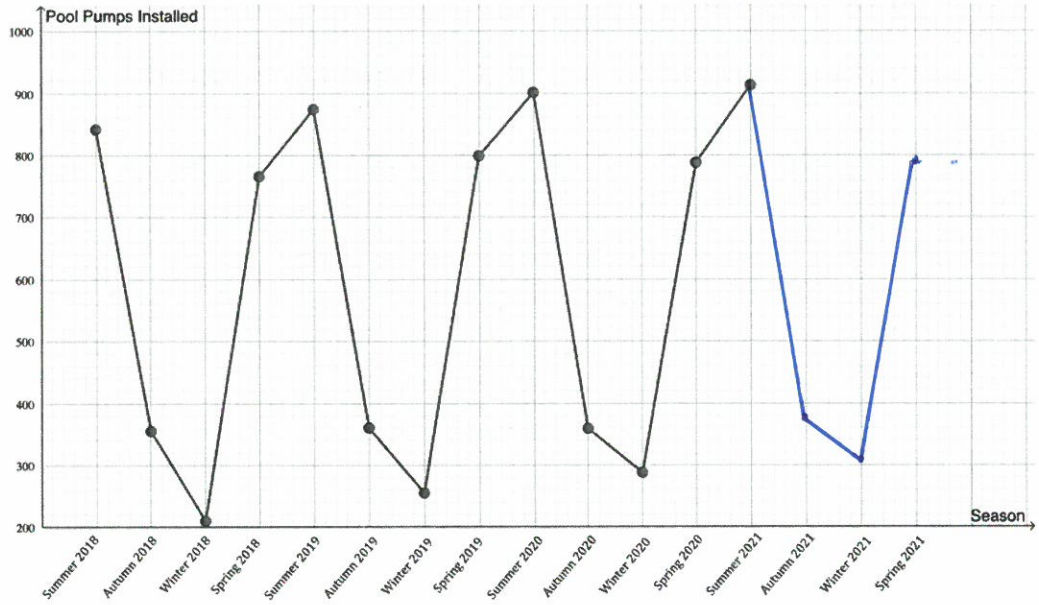
Question 4. **[6 marks: 1, 2, 1, 2]**

The graph below shows the number of pool pumps installed by a local pool business each season from 2018 onwards.



The data for the next three seasons are shown in the table below.

Season	Autumn 2021	Winter 2021	Spring 2021
Pool Pumps Installed	380	310	790

(a) Complete the time series plot above by including this additional information.

✓plots 3 points and connects with a straight line

(b) Determine the most appropriate moving average to smooth the data in order to predict future seasons. Justify your answer.

✓4 CMA is the most appropriate

A four-point centred moving average is the most appropriate as the :

- period of the time series data is 4 with peaks occuring each summer season, or,
- data is collected every season and there are clearly four seasons within one cycle.

✓gives appropriate reasoning in context

The seasonal indices (correct to two decimal places) are shown in the table below.

Season	Summer	Autumn	Winter	Spring
Seasonal Index	153.67%	63.32%	46.09%	136.92%

(c) Complete the table above by calculating the seasonal index for Spring.

✓calculates spring Seasonal Index

(d) The deseasonalised number of pool pumps sold in Autumn 2019 is 562. Determine the actual number of pool pumps sold in Autumn 2019.

$562 = \frac{\text{actual}}{0.6332}$

✓shows deseasonalised calculation

Actual sales = 356 pool pumps.

✓states solution to the nearest pool pump

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Question 5. **[5 marks: 1, 2, 2]**

Jacqueline recently won $40 000 from a competition in her local newspaper and wishes to invest all of her winnings into a savings account. Jacqueline has done some research and found that ZNA bank have a savings account that is 3.05% per annum compounding quarterly.

(a) If Jacqueline leaves this money in the savings account with ZNA bank for 8 years, calculate the total interest accrued from this account.

$$A = 40\,000\left(1 + \frac{0.0305}{4}\right)^{(4\times 8)}$$

✓ substitutes values correctly

$$A = 51\,006.54 - 40\,000$$

$$\text{Interest} = \$11\,006.54$$

✓ total amount subtract principal and states interest

OR

N 32
I 3.05
PV −40 000
PMT 0
FV 51 006.54
P/Y 4
C/Y 4

✓ calculates final value correctly

Interest = 51006.54 − 40000

Interest = $11 006.54

✓ final amount subtract Principal and calculates interest

(b) Calculate the effective annual interest rate for this account.

$$i_{effective} = \left(\left(\left(1 + \frac{0.0305}{4}\right)^{4} - 1\right)\right) \times 100$$

$$i_{effective} = 3.0851\ \%$$

✓ calculates correct effective interest rate to ≥ 2 d.p.

(c) Determine the annual interest rate, as a percentage, that a different savings account, compounding monthly, would need to offer to achieve the same effective annual interest rate as in part (b).

$$0.030851\ldots = \left(1 + \frac{x}{12}\right)^{12} - 1$$

✓ uses result from (b) in an equation compounding monthly

$$\therefore x = 0.030423$$

$$\therefore \text{Annual rate of } 3.0423\%$$

✓ calculates rate as a percentage ≥ 2 d.p.

OR

N (8×12)
I 3.0423
PV −40 000
PMT 0
FV 51 006.54
P/Y 12
C/Y 12

✓ substitutes values correctly into classpad

✓ gives rate as a percentage ≥ 2 d.p.

∴ Annual rate of 3.0423%

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Question 6. **[12 marks: 3, 2, 3, 2, 2]**

A local café has recently opened in town and is only open Monday – Friday. The total number of coffees sold each day for the first four weeks is shown in the table below.

n	Week	Day	Number of coffees sold	Weekly Mean	Number of coffees as a percentage of the weekly mean (%)
1	Week 1	Monday	458	419.6	109.15
2		Tuesday	383		**Y**
3		Wednesday	376		89.61
4		Thursday	405		96.52
5		Friday	476		113.44
6	Week 2	Monday	470	**Z**	109.56
7		Tuesday	384		89.51
8		Wednesday	379		88.34
9		Thursday	414		96.50
10		Friday	498		116.08
11	Week 3	Monday	471	435.2	108.23
12		Tuesday	388		89.15
13		Wednesday	381		87.55
14		Thursday	**X**		97.20
15		Friday	513		117.88
16	Week 4	Monday	480	443.6	108.21
17		Tuesday	399		89.95
18		Wednesday	391		88.14
19		Thursday	425		95.81
20		Friday	523		117.90

(a) Calculate the value of X, Y and Z.

X = 423 ✓ Y = 91.28 ✓ Z = 429 ✓

* Appropriate number of decimal places

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(b) Describe the trend and seasonality of this data.

✓describes trend as increasing

As the weeks increase, the trend of the data is increasing.

The period of the data is 5 with the peak number of coffees being sold on Fridays and the low number of coffees being sold on wednesdays.

✓describes seasonality with peaks/troughs in the context of days/weeks

(c) Show how to calculate the seasonal index for Wednesday and interpret this value in the context of this situation.

$$S.I.(\text{wednesday}) = \frac{89.61 + 88.34 + 87.55 + 88.14}{4}$$

✓shows seasonal index calculation

$$= 88.41\%$$

✓calculates seasonal index for wednesday

On wednesdays, the coffee shop sells 11.59% less coffees than the average weekday at the cafe

✓interprets this value in the context of days at the cafe

The equation of the least-squares regression line for deseasonalised data is $D = 1.1966n + 420.285$ and the seasonal index for Friday is 1.1633.

(d) Use this regression line to predict the number of coffees sold on Friday of Week 6, assuming the above seasonality and trend continues.

$D = 1.1966(30) + 420.285$

$D = 456.183$

✓substitutes $n = 30$ into regression equation

Prediction $= 456.183 \times 1.1633$

Prediction ≈ 531 coffees.

✓multiplies by seasonal index for Friday, giving answer to the nearest coffee.

(e) Comment on the reliability of your prediction made in part (d).

The prediction in part (d) is unreliable as Friday of week 6 is beyond one cycle of the data and is extrapolation.

✓states prediction is unreliable

✓states this prediction is beyond one cycle (extrapolation)

Question 7. **[11 marks: 1, 3, 2, 2, 2, 1]**

A small business took out an equipment loan of \$12 000 and made monthly repayments of \$750. The table below shows the progress of the loan for the first few months, with repayments and interest applied at the end of each month.

Month (n)	Balance at start of month (\$)	Interest (\$)	Repayment (\$)	Balance at the end of the month (\$)
1	12 000.00	144.00	750.00	11 394.00
2	11 394.00	136.73	750.00	10 780.73
3	10 780.73	129.37	750.00	10 160.10
4	10 160.10	**D**	750.00	**E**

(a) Show how to calculate the monthly interest of 1.2%.

$\frac{144}{12\,000} \times 100\% = 1.2\%$

✓ shows use of the table to generate 1.2%..

(b) The recurrence relation to model the balance at the end of the month, T_n, is $T_{n+1} = aT_n - b, T_0 = c$. State the values of a, b and c.

$T_{n+1} = \left(1 + \frac{0.144}{12}\right) T_n - 750, \quad T_0 = 12\,000$

✓a ∴ $a = \left(1 + \frac{0.144}{12}\right)$ or 1.012

✓b $b = 750$

✓c $c = 12\,000$

(c) Determine the values of **D** and **E** in the table above.

D = 121.92 ✓D E = 9532.02 ✓E

(d) Determine the:

(i) number of repayments to fully pay off the loan.

17.87 payments ✓ determines 17.87... payments

= 18 repayments. ✓ gives payments to the next subsequent integer

(ii) amount of the final repayment.

750 − 95.14 ✓ repayment + (negative value)

= \$654.86 ✓ states final repayment

(e) If the business opted to decrease the amount of each repayment, comment briefly on how this would change the total interest owed over the life of the loan.

The total interest owed on the loan would increase. ✓ states that the total interest would increase

END OF CLASS TEST