UNIVERSITY EXAMINATIONS UNIVERSITEITSEKSAMENS

Sep/Nov 2021

APM 2616
Computer Algebra

Examiners:
First: Mr M Kgarose
Second: Prof JMW Munganga

100 Marks
2 Hours

Closed book and online examination, which you have to write within 2 hours and submit online through the link: https://cset.myexams.unisa.ac.za/portal

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This examination allows PDF attachment documents only as part of your submission.

Declaration: I have neither given nor received aid on this examination.

Answer All Questions and Submit within the stipulated timeframe.

Late submission will not be accepted.

This paper consists of 6 pages.

SHOW ALL YOUR WORKING.

[TURN OVER]

QUESTION 1

(a) Using MuPad syntax, type statements to declare the following variables

(i) $MyRoot = \sqrt{\sqrt{x}+1}$. (2)

(ii) $MyPoly = 3x^3 + 2x^2 + x + 1$. (2)

(iii) $MyRad = a + \sqrt{a + \sqrt{a + \sqrt{x}}}$. (3)

(iv) $MyFraction = \frac{2}{\sqrt{1+x-\frac{1}{x}}}$. (3)

(b) Use MuPad functions to compute the general solution of the system of linear equations (15)

$$\begin{aligned} a + b + c + d + e &= 1, \\ a + 2b + 3c + 4d + 5e &= 2, \\ a - 2b - 3c - 4d - 5e &= 2, \\ a - b - c - d - e &= 3. \end{aligned}$$

[25]

QUESTION 2

(a) Use MuPad to show that (15)

$$\lim_{x\to\infty} x^a = \begin{cases} \infty & \text{for} \quad a > 0, \\ 1 & \text{for} \quad a = 0, \\ 0 & \text{for} \quad a < 0. \end{cases}$$

Hint: Use `assume` to distinguish the cases.

(b) Find the derivatives f'', $f^{(5)}$ and $f^{(20)}$ for the following functions

(i) f(x) = ln(ln(x)). (5)

(ii) f(x) = u(x) · v(x). (5)

Hint: Use the function `diff` and the $ operator.

[25]

QUESTION 3

We define the Collatz conjecture as the function $f : \mathbb{N} \to \mathbb{N}$ where

$$f(n) = \begin{cases} n/2 & \text{for} \quad n \quad \text{even}, \\ 3n+1 & \text{for} \quad n \quad \text{odd}. \end{cases}$$

Write a MuPad program that on input x_0, returns the smallest index i with $x_i = 1$.

[25]

QUESTION 4

Write a complete LaTeX code to re–create the paper on pages 4–6.
Please take note that manual referencing and manual labelling of equations, pages and citations will not be allowed.

Article about APM2616 September-November 2021 Exam

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Abstract

A new model for the transmission dynamics of *echinococcus multilocularis* in the population of red foxes and voles with environment as a source of infection is formulated and rigorously analyzed.Numerical experiments indicates that administering disinfection of environment only induce more positive impact than applying treatment only on red foxes in controlling the infection. Generally, interventions towards treating red foxes and environmental disinfection could be sufficient in tackling transmission of disease in the populations.

Keywords: Echinococcus m.; Treatment; Elasticity indices; Global sensitivity analysis; Global Stability.

1 Introduction

Echinococcus multilocularis (EM) is a parasitic taenid tapeworm and one of the six species of the Echinococcus genus. The disease is sylvatic and has indirect life cycle between two major hosts in a predator–prey interaction [4]. After oral ingestion of eggs by rodents (such as voles), regarded as intermediate hosts, a larval stage (metacestode) develops in any of the internal organs (liver, kidney, heart etc). The mature metacestode are capable of producing numerous protoscoleces, each having the potential to develop into an adult EM when a definitive host preyed on intermediate host and the cycle continue [1, 3]. The model formulation, equations and flow diagram are presented in Section 2. Basic properties of the model on existence, uniqueness, positivity and boundedness of solutions are discussed in Section 3. Furthermore, existence and global stabilities with systematic calculation of control reproduction number are presented in Section 4.

2 The Model

The total population of red foxes, which is assumed constant (birth and death rates, μ_f, are assumed equal) in the environment at time t, denoted by $N_f^*(t)$ is divided into susceptible ($S_f(t)$), exposed ($E_f(t)$), infected ($I_f(t)$), and recovered ($R_f(t)$) subpopulations, so that

$$N_f^*(t) \quad = S_f(t) + E_f(t) + I_f(t) + R_f(t). \tag{1}$$

3 Basic properties of the model

The dynamics of red foxes and voles populations, all its associated parameters are assumed nonnegative. Hence, we now present the basic results for the properties of the model.

Theorem 1. *The following region is positively invariant for the model* (1)*:* $\Omega = \Omega_f \times \Omega_v \times \Omega_b \subset \mathbb{R}_+^4 \times \mathbb{R}_+^3 \times \mathbb{R}_+$, *where* $\Omega_f = \{(S_f, E_f, I_f, R_f) \subset \mathbb{R}_+^4 : S_f + E_f + I_f + R_f = N_f^*\}$, $\Omega_v = \{(S_v, E_v, I_v) \subset \mathbb{R}_+^3 : S_v + E_v + I_v = N_v^*\}$ *and* $\Omega_b = \{B \subset \mathbb{R}_+ : B \leq \frac{\eta_f N_f^*}{\mu_b}\}$.

Proof. The detailed proof of Theorem 1 is presented in Appendix in [1]. □

4 Calculation of control reproduction number

The basic reproduction number in epidemic models, is an important threshold value that quantify the infection risk in order to effectively control the disease. Furthermore, it plays a vital role in stability analysis of equilibria of the models. It can be derived using the next generation matrix approach [5]. However, when there is intervention, it is referred as the control reproduction number. For detailed computation of the control reproduction number. Therefore the basic control reproduction number, denoted by $\mathcal{R}_c$ is given by

$$\mathcal{R}_c = \rho(FV^{-1}) = \left[\left(\frac{N_f^* sp\alpha_f}{\bar{\alpha_f}\bar{\xi_f}} \right) \left(\frac{N_v^* \beta_v \alpha_v}{\mu_v \bar{\alpha_v}} \right) \left(\frac{\eta_f}{K\mu_b} \right) \right]^{\frac{1}{3}}. \tag{2}$$

It is worth stating that $\mathcal{R}_c$ is the basic control reproduction number, which represent the number of secondary infection cases generated by introducing at least one infective into the population that is assumed wholly susceptible. This number is obtained from the contribution of average number of secondary infections through fox-to-environment-to vole transmission

$\left(\mathcal{R}_c^f = \frac{N_f^* sp\alpha_f}{\bar{\alpha_f}\bar{\xi_f}}\right)$, voles-to-fox transmission $\left(\mathcal{R}_c^v = \frac{N_v^* \beta_v \alpha_v}{\mu_v \bar{\alpha_v}}\right)$ and environment-to-voles transmission $\left(\mathcal{R}_c^b = \frac{\eta_f}{K\mu_b}\right)$ as a result of one infectious subject during its infectious period.

References

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