UNIVERSITY EXAMINATIONS **UNIVERSITEITSEKSAMENS**

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NUMERICAL METHODS I
COS2633

Duration: 2 hours **100 Marks**

Examiners:

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INSTRUCTIONS:

This is a closed book examination.
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Follow the instructions on the handling of this online assessment when you upload your answer script on the myExams platform.
Late submissions beyond the allocated time will not be accepted.

SPECIFIC INSTRUCTIONS:

1. ANSWER ALL FOUR QUESTIONS.

2. A non-programmable calculator can be used for computations.

3. Start each question on a new page. Keep parts of a question together.

4. Show all essential work and formulas used in the computations.

5. Useful formulae are given on the last page

[TURN OVER]

QUESTION 1

Consider the nonlinear equation $f(x) = x^2 - 4\sin x = 0$ whose solution is to be approximated numerically. Apart from $x = 0$ the root equation has another root in $[1,\ 2.5]$. (*Hint: x in radians*)

(a) Sketch the graph to be used to approximate the root using the fixed point scheme, $x = \dfrac{4\sin x}{x}$ for $x \in [1,\ 2.5]$. Show on the graph a geometrical illustration of whether the fixed point scheme will converge to the root starting at $x_0 = 1.5$. (7)

(b) Perform three iterations of the fixed point scheme, $x = \dfrac{4\sin x}{x}$ to approximate the root using $x_0 = 1.5$ as the initial approximation. (8)

(c) Perform three iterations of Newton's method for the function in (a) above, using $x(0) = 1.5$ as the initial solution. (10)

[25]

QUESTION 2

Consider the experimental data below for a certain function:

x	0	1	2	3	4	5
$f(x)$	2.1	7.7	13.6	27.2	40.9	61.1

(a) Show (do not formulate) that the set of normal equations to be solved to construct a quadratic least squares polynomial $P(x) = a_0 + a_1x + a_2x^2$ is given by (7)

$$\begin{array}{rcrcrcr} 6a_0 & + & 15a_1 & + & 55a_2 & = & 152.60 \\ 15a_0 & + & 55a_1 & + & 225a_2 & = & 585.60 \\ 55a_0 & + & 225a_1 & + & 979a_2 & = & 2488.80 \end{array}$$

(b) Given that the coefficients of the quadratic least squares polynomial in (a) above are $a_0 = 2.4786$, $a_1 = 2.3593$, $a_2 = 1.8607$, write down the $P(x)$ and use it to approximate $f(2.5)$. (3)

(c) By selecting three suitable data points in the given set, construct a second degree Newton's divided difference polynomial that interpolates the three selected points, and use it to find an approximation of $f(2.5)$. (7)

(d) Approximate the value of $f(2.5)$ using the least squares exponential form $y = a_0e^{a_1x}$. Compute the error in the approximation. (8)

[25]

QUESTION 3

Consider the linear system

$$\begin{array}{rcrcrcr} 4.63x_1 & - & 1.21x_2 & + & 3.22x_3 & = & 2.22 \\ -3.07x_1 & + & 5.48x_2 & + & 2.11x_3 & = & -3.17 \\ 1.26x_1 & + & 3.11x_2 & + & 4.57x_3 & = & 5.11 \end{array}$$

(a) Determine whether the coefficient matrix is diagonally dominant and deduce whether iterative methods are likely to converge to the solution. (5)

(b) Compute an approximate solution of the system using three iterations of Gauss-Seidel method with $\mathbf{x}^{(0)} = (0,\ 0,\ 0)^T$ as the initial solution. (10)

(c) Use Gaussian elimination with scaled partial pivoting to solve the system. (10)

[25]

QUESTION 4

Consider the data below

x	$f(x)$	$f'(x)$
1	0	1
1.25	0.2789	1.2231
1.5	0.6082	1.4055
1.75	0.9793	1.5596
2	1.3863	1.6931

(a) Approximate $f'(1.5)$ using the three-point midpoint formula. (3)

(b) Compute an approximation of $\int_1^2 f(x)\,dx$ using the composite Simpson's rule. (7)

(c) If the data in given above is for $f(x) = x\ln x$, use the three-point Gauss quadrature method to approximate $\int_1^2 f(x)\,dx$. (9)

(Hint: Parameters, $c_0 = c_2 = 0.555555555,\ c_1 = 0.8888888888;\ r_0 = -r_2 = 0.774596669,\ r_1 = 0.0$)

(d) Approximate the actual error in the approximations obtained in (a) - (c) above. (6)

[25]

TOTAL MARKS: [100]