Modeling Pollinator Movement and Behavior

By: Alec Henriksen, Jade Schack, and Mugdha Danda

Advisor: Sergey Berg, Ph.D

1. Abstract

The study of pollinator behavior and movement is an especially important factor in understanding rapidly declining pollinator populations. The University of St. Thomas's Pollinator Path is a series of gardens that provides floral resources for bees while encouraging the study and conservation of pollinators. The goal of this investigation is to assess how different spatial organizations of pollinator path sites and plantings influence bee populations. Utilizing a network model to represent the Pollinator Path, an adjacency matrix, and the statistical software RStudio, this research offers theoretical insight to understanding the Pollinator Path's efficiency and risks. With a current efficiency rating of 66.437±29.42312, the elimination of Pollinator Path Site 9 would have the greatest positive impact on the efficiency of the Path. In the future, additional bee data collection, the introduction of new parameters, seasonality, and a bee flight time variable would help improve the accuracy of the model.

2. Introduction

2.1 Bees

Bees are vitally important pollinators of wild plants and agricultural crops worldwide. In 2010, in the United States, pollination by bees directly or indirectly (e.g., pollination required to produce seeds for the crop) contributed to over 19 billion crops, whereas pollination by other insect pollinators contributed to nearly 10 billion crops. Bees provide pollination services to over 180,000 different plant species and more than 1,200 crops. In terms of percentages, these hard-working animals help pollinate over 75% of our flowering plants and nearly 75% of our crops. That means that one out of every three bites of four bites of food we eat is because of pollinators, mainly bees. If we want to talk in dollars and cents, pollinators add $217 billion to the global economy, and bees alone are responsible for between $1.2 and 5.4 billion in agricultural productivity in the United States. In addition to the food that we eat, pollinators support healthy ecosystems that clean the air, stabilize soils, protect from severe weather, and support other wildlife.

However, a recent study about the status of pollinators in North America by the National Academy of Sciences (National Research Council, 2007) found that populations of bees and some wild pollinators are declining. They suggest that 30% of the total bee population in the U.S. has disappeared in the last five years. Specifically, honey bees, a managed species not native to North America, have experienced declines since 2006 due to multiple interacting stressors such as habitat loss and fragmentation, loss of floral resources, non-target impacts of pesticides, climate change, diseases, and parasites. These stressors also affect Minnesota's native insect pollinators, who are facing challenges on many fronts. Even though Minnesota is one of the few states with laws addressing pollinator health, several native species have experienced declines in population and distribution, with some once common species now disappeared from the state.

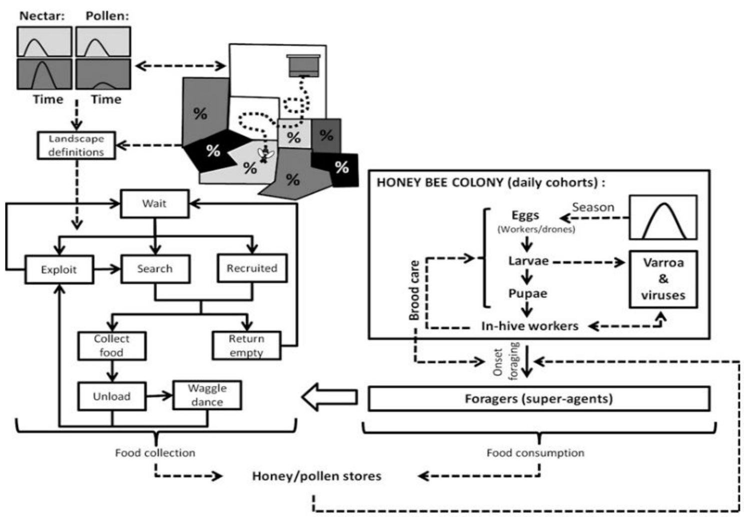
2.1.1 Past Bee Models

In the past few years, researchers started assessing how different factors are causing bee population decline by using quantitative and computer models. These models include BEEHAVE (Becher, Matthias A et al), compartment model of honey bee colony population dynamics (Khoury DS et al, 2011), and a demographic model of bee colony (Khoury DS et al, 2013).

2.1.1.1 BEEHAVE

BEEHAVE is a computer model to simulate the development of a honeybee colony and its nectar and pollen foraging behavior in different landscapes (Figure 1). The purpose of BEEHAVE is to allow multiple stressors of honeybee colonies within a hive and in the landscape to be represented, either alone or in combination, to understand their potential influence on colony development and survival.

Figure 1

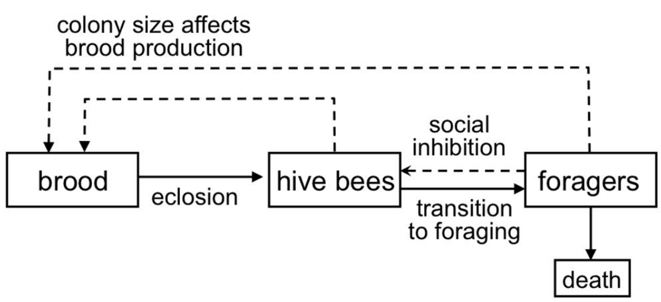


Overview of the BEEHAVE model structure

2.1.1.2 Compartmental Model

A compartment model of honey bee colony population dynamics (Figure 2) was developed to explore the impact of different death rates of forager bees on colony growth and development and suggests that colony failure can be understood in terms of observed principles of honey bee population dynamics. The model provides a theoretical framework for experimental investigation of the problem.

Figure 2

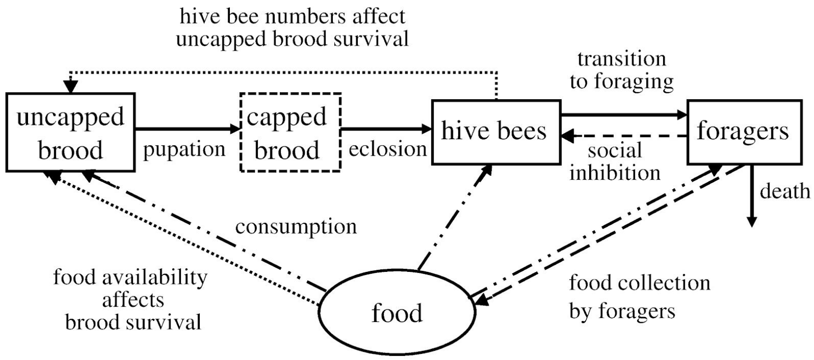


Elements of honey bee social dynamics considered by Compartmental model

2.1.1.3 Demographic Model

A demographic model of bee colony (Figure 3) presents a simple mathematical framework for exploring the interactions of food and forager mortality on colony fate and provides the mathematical basis for more involved simulation models of hive performance.

Figure 3

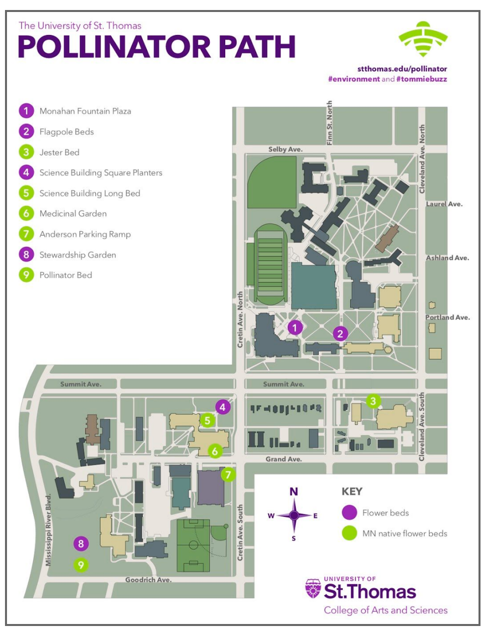


Honey bee social dynamics represented in the Demographic model

2.2 Study Site

The Pollinator Path, located on the University of St. Thomas's St. Paul Campus, was created in 2016 by the College of Arts and Sciences' Biology Department. Lead investigators of the project are Doreen Schroeder, M.A. and Catherine Grant, B.A. The Path is a series of gardens designed to attract and promote the study of pollinators. Allowing students, faculty, and the community to study pollinator behavior, the Path is also a resource for learning about the importance of supporting decreasing pollinator populations.

Figure 4



Pollinator Path Map

The Pollinator Path is comprised of nine unique sites, or beds, some designed for pollinators and others for their aesthetics contributions to campus. Figure 4 includes the name and number of each site, the spatial layout of the path, and distinguishes Minnesota native flower bed sites. From this point forward, name and corresponding site number will be used interchangeably in reference to each of the sites. Each site is unique in the type of pollinators it attracts as well as plant composition. Pollinators most commonly found at the Path include bees, wasps, syrphid flies, butterflies, moths, and beetles. For the purpose of this investigation, we chose to focus on bees. This includes honey bees, bumble bees, and other native bees such as carpenter, miner, sweat and carder bees. Additionally, plant composition, specifically diversity and the presence of Minnesota native flowers, is an important and varying factor at each of the sites. Diversity takes into account the sheer number of different plant species while top Minnesota native flowers that are of interest include Wild Bergamot, Milkweed, Golden Alexander, Culver's Root, and St. John's Wort. For a full list of plants and Minnesota native plants included in the Pollinator Path, see Appendix A and B. The table below shows a further break down of the sites including plant types, plant diversity, pollinator diversity, and the design purpose.

Site Number	Site Name	Plant Types	Plant Diversity	Pollinator Diversity	Designed for Pollinators?
1	Monahan Fountain Plaza	Perennials	Low	Low	Yes
2	Flagpole Beds	Annuals	Low	Low	No, aesthetic
3	Jester Bed	Annuals	High	High	Yes
4	Science Building Square Planters	Annuals	Medium	Low	No, aesthetic
5	Science Building Long Bed	Perennials/Annuals	High	High	Yes
6	Medicinal Garden	Perennials/Annuals	High	High	No, medicinal
7	Anderson Parking Ramp	Perennials	High	High	Yes
8	Stewardship Garden	Annuals	High	High	No, food
9	Pollinator Bed	Perennials	High	High	Yes

2.3 Main Idea

The main idea of our summer research was to assess how different spatial organizations of path sites and plantings influence bees and their interaction with the Pollinator Path. We wanted to look at current risks as well as efficiency metrics that could improve the movement of bees between sites.

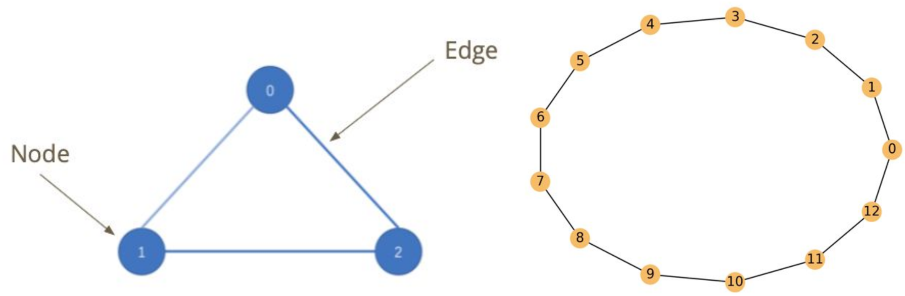
3. Methodology

3.1 Network Models

Network models are the most useful mathematical model to study dynamics in a heterogeneous population. Networks are a simple yet effective way to model a large class of technological, social, economic, and biological systems that can be described as a set of entities with interactions between them.

For instance, in real world application, three or more individuals can form a network (Figure 5(i)) by forming connections with each other. In network terminology, individuals are represented by the nodes, and the connection between them is referred to as an edge.

Figure 5



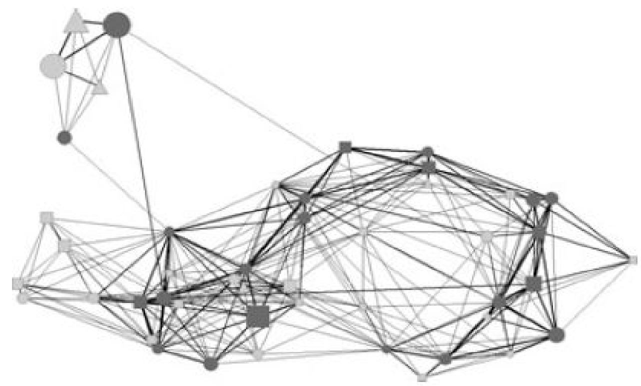
(i) **(ii)**

Graphical representation of Network Models

3.1.1 Examples of Network Model

The network model was originally developed for application in statistical physics. However, in recent years it is mainly used in epidemiology to study the transmission of disease or infection between individuals or groups of individuals. One of the first few frameworks using contact networks, in epidemiology, was to investigate the dynamics of indirectly transmitted diseases in the subpopulation of sleepy lizards (Figure 6). In this setting, each node represents a lizard in the study population and the connection between them is defined as the spatial proximity. The closer the lizards are, the higher their frequency of contact is, which in turn reflects the weightedness of that edge and connection strength of the two nodes connected by that edge.

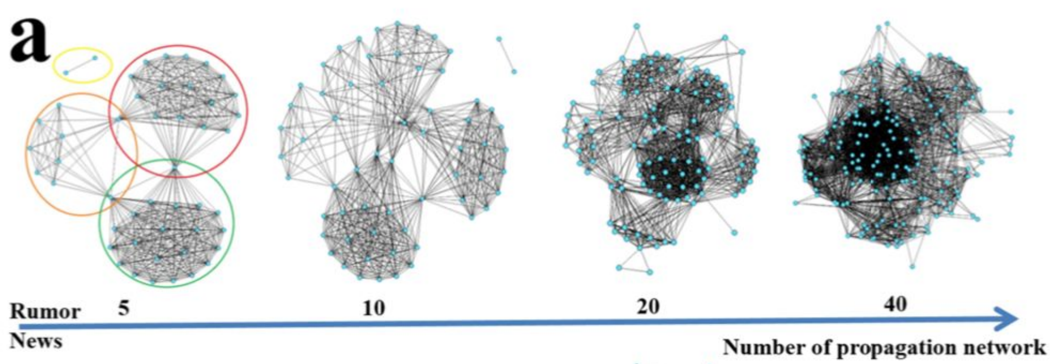
Figure 6



Network Model in Epidemiology

The application of network models is not limited to STEM fields, they have also been used in other disciplines, such as sociology (Figure 7), to measure actor prestige (Korfiatis and Sicilia 2007) and to investigate dynamics of rumor spreading (Moreno et al. 2004).

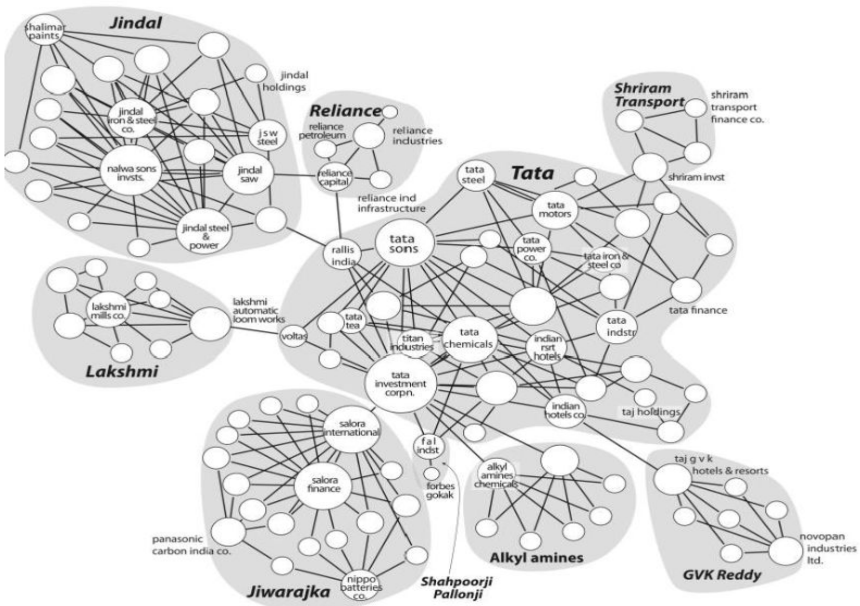
Figure 7



Rumor Network Model (Sociology)

In addition to this, they are used in economics to investigate the interconnectivity in modern economic systems. This allows researchers to better understand and explain some economic phenomena. In Figure 8, the authors (Mani et al., 2014) are trying to explain the interconnectivity of Tata business group and its ties with other Indian firms.

Figure 8



Tata business group and other Indian business group with ties with Tata

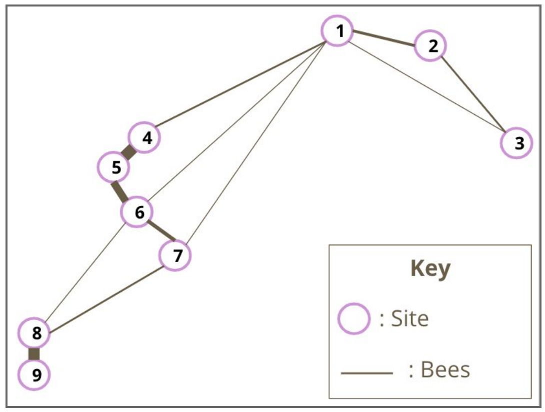
3.1.2 Example of Network Model for Bee Population

A past research, done by a group of researchers from Toulouse University (Pasquaretta et. al, 2017), applied network model to bee population to describe how spatial network statistics can be used to analyze the foraging patterns of bees moving between feeding sites at various spatial and temporal scales. Along with this, they also explain how spatial movement networks can be derived theoretically and experimentally to describe bee foraging decisions. They also suggest that bee site fidelity behavior can be manipulated by training the bees to visit all the sites with fewer movements. In this study, the pollinator sites are defined as the nodes, and the movement of bees between the two sites represents the connection or the edge that connects the two nodes. We are using this research as the foundation for our current study. This is because in our study we are trying to improve the efficiency of the Pollinator Path by predicting locations for pollinator sites, whereas they are trying to train the bees to solve the Travelling Salesman Problem, i.e., visit all the nodes using the shortest path possible.

3.2 Analysis of Model

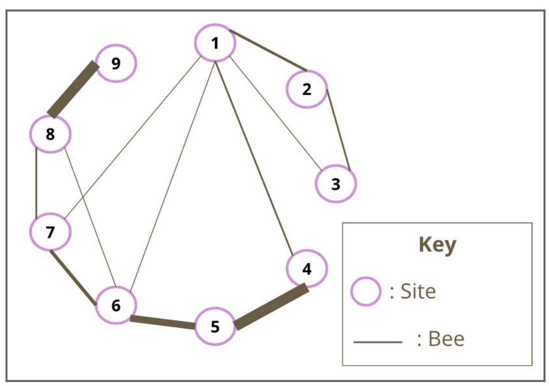
We are utilizing network models to represent the Pollinator Path. In our model, each node represents a different site while the edges represent bee activity and connectivity. As previously stated, edges with wider widths correspond to sites with higher connectivity and interaction. Figure 9 shows a possible Pollinator Path network model which takes spatial organization into consideration while Figure 10 is a visualization of the same network model but without concern for spatial layout. If we look at Site 8 and Site 9, Figure 9 shows they are relativity close, so the edge between the two sites is wider, portraying a higher degree of bee connectivity, than between the more distantly spaced Site 1 and Site 7.

Figure 9



Example Pollinator Path Network Model with Spatial Organization

Figure 10



Example Pollinator Path Network Model Without Spatial Organization

This theoretical model helps gain insight into the interaction of bees between the nine Pollinator Path sites. It is a useful visual for recognizing connections as well as which sites have stronger relationships, more edges, with other sites. However, without concrete data, it was necessary for us to create a probability table to construct the network model's edges.

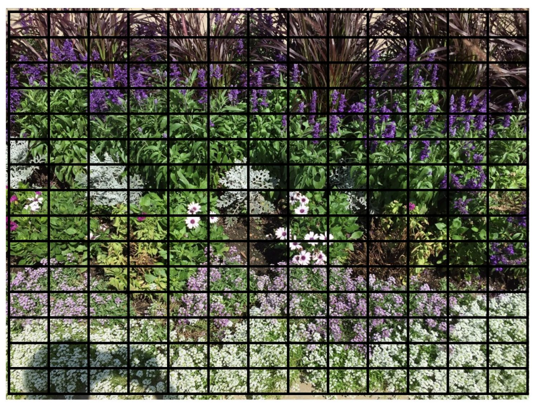
3.3 Parameterization

A network model's edges are determined by manually observed or collected data or probabilities. In order to construct the edges for our network model, we need to consider a number of different parameters that reflect bee behavior and movement. For our particular model, it was important to consider the factors that determine a bee's likelihood of remaining at a particular site or moving to visit another site within the Path. We separate this into two probabilities: the probability of staying at a particular site and the probability of leaving said site.

3.3.1 Probability of Staying

For the probability of staying, we looked at the percentage of purple, percentage of native plants, percentage of preferred plants, and diversity of plants at each site. Bees have a preference for colors between the range of violet to purple. This range is attractive to bees due to their ability to see visible and ultraviolet light (Reverté, Retana, Gómez & Bosch, 2016). For the percentage of purple, we measured how purple a site appears to bees. To do this, we took several photographs of each site and laid a grid over each photograph, see Figure 11. Next, we counted each of the squares that contained purple and divided this number by the total number of squares. After averaging the photographs from each site, this gave us the approximate percentage of purple of each path site.

Figure 11



Grid

For the percentage of native plants, we noted the number Minnesota native plant species found at each site and divided by the total number of plants, see Appendix B for a Minnesota native plant list. Minnesota native plants are an important factor in bee pollination and behavior as multiple studies showcase this preference. A study at Capitol Reef National Park, Utah, found native plants were visited by approximately two times as many bee species as exotic plants (Evans, Smart, Cariveau & Spivak, 2018) while a University of California Berkeley study found 77% of bee species at mature hedgerows were only found on native plant species (Morandin & Kremen, 2013). For the percentage of preferred plants, we looked at fifteen plant species known to be preferred by bees. These species include Coneflower, Goldenrod, Yarrow, Hyssop, Black-Eyed Susan, Joe Pye Weed, Bee Balm, Milkweed, Asters, Sedum, Mint, Bergamot, Foxglove, Zinnia, and Borage. These plant species were totaled at each site and divided by the total number of plant species, resulting in the percentage of preferred plants. Finally, for the diversity of plants, we used the number of different plant species found at each of the nine sites. Diversity of plants is important as sites with more plant species tend to attract a higher number of bees as well as increase pollinator diversity in general (Blaauw & Isaacs, 2014). These numbers went into four separate diagonal matrices, Figures 12, 13, 14, and 15.

Figure 12

	1	2	3	4	5	6	7	8	9
1	0.28	0	0	0	0	0	0	0	0
2	0	0.74	0	0	0	0	0	0	0
3	0	0	0.17	0	0	0	0	0	0
4	0	0	0	0.62	0	0	0	0	0
5	0	0	0	0	0.33	0	0	0	0
6	0	0	0	0	0	0.05	0	0	0
7	0	0	0	0	0	0	0.005	0	0
8	0	0	0	0	0	0	0	0.17	0
9	0	0	0	0	0	0	0	0	0.48

Percentage of Purple

Figure 13

	1	2	3	4	5	6	7	8	9
1	0.25	0	0	0	0	0	0	0	0
2	0	0.166666667	0	0	0	0	0	0	0
3	0	0	0.315789474	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0.466666667	0	0	0	0
6	0	0	0	0	0	0.22	0	0	0
7	0	0	0	0	0	0	0.692307692	0	0
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0.833333333

Percentage of Native Plants

Figure 14

	1	2	3	4	5	6	7	8	9
1	0.375	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0.210526316	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0.6	0	0	0	0
6	0	0	0	0	0	0.2	0	0	0
7	0	0	0	0	0	0	0.538461538	0	0
8	0	0	0	0	0	0	0	0.071428571	0
9	0	0	0	0	0	0	0	0	0.416666667

Percentage of Preferred Plants

Figure 15

	1	2	3	4	5	6	7	8	9
1	8	0	0	0	0	0	0	0	0
2	0	6	0	0	0	0	0	0	0
3	0	0	19	0	0	0	0	0	0
4	0	0	0	10	0	0	0	0	0
5	0	0	0	0	15	0	0	0	0
6	0	0	0	0	0	50	0	0	0
7	0	0	0	0	0	0	13	0	0
8	0	0	0	0	0	0	0	14	0
9	0	0	0	0	0	0	0	0	24

Plant Diversity

3.3.2 Probability of Leaving

For the probability of leaving, we looked at distance, roads, and buildings. For distance, we used global positioning system coordinates, found through the use of Google Maps, to calculate the shortest distances between each of the nine sites. In addition, we noted if these distances included roads or buildings that would hinder a bee's path. These numbers were also input into separate matrices with the presence of roads and buildings represented by the number one, Figures 16, 17, and 18 show these matrices. Distance is measured in miles. In the roads and buildings matrices, one represents the presence of a road or building between the starting site, row, and ending location, column, while a zero indicates the absence of a road or building.

Figure 16

Site	1	2	3	4	5	6	7	8	9
1	0	0.062	0.147	0.131	0.167	0.201	0.205	0.401	0.39
2	0.062	0	0.12	0.182	0.218	0.241	0.286	0.447	0.435
3	0.147	0.12	0	0.19	0.214	0.207	0.195	0.406	0.393
4	0.131	0.182	0.19	0	0.036	0.081	0.095	0.27	0.26
5	0.167	0.218	0.214	0.036	0	0.057	0.077	0.234	0.224
6	0.201	0.241	0.207	0.081	0.057	0	0.023	0.206	0.194
7	0.205	0.286	0.195	0.095	0.077	0.023	0	0.213	0.2
8	0.401	0.447	0.406	0.27	0.234	0.206	0.213	0	0.013
9	0.39	0.435	0.393	0.26	0.224	0.194	0.2	0.013	0

Distance

Figure 17

site	1	2	3	4	5	6	7	8	9
1	0	0	1	1	1	1	1	1	1
2	0	0	1	1	1	1	1	1	1
3	1	1	0	1	1	1	1	1	1
4	1	1	1	0	0	0	1	1	1
5	1	1	1	0	0	0	1	1	1
6	1	1	1	0	0	0	1	1	1
7	1	1	1	1	1	1	0	1	1
8	1	1	1	1	1	1	1	0	0
9	1	1	1	1	1	1	1	0	0

Roads

Figure 18

Site	1	2	3	4	5	6	7	8	9
1	0	0	1	1	1	1	1	1	1
2	0	0	1	1	1	1	1	1	1
3	1	1	0	0	1	1	1	1	1
4	1	1	0	0	0	1	1	1	1
5	1	1	1	0	0	1	1	1	1
6	1	1	1	1	1	0	0	1	1
7	1	1	1	1	1	0	0	1	1
8	1	1	1	1	1	1	1	0	0
9	1	1	1	1	1	1	1	0	0

Buildings

3.3.3 Weights

After selecting the parameters, we assigned a weight to each parameter based on general knowledge and previous research. Since the diversity matrix had a scale from 0 to 50, the following transformation was needed to convert the diagonal values to values on a scale from 0 to 1, like the other matrices, before a weight could be applied:

DiversityMatrix * 0.02 = **Transformed Diversity Matrix**

```
0.16 0.00 0.00  0.0  0.0    0 0.00 0.00 0.00
0.00 0.12 0.00  0.0  0.0    0 0.00 0.00 0.00
0.00 0.00 0.38  0.0  0.0    0 0.00 0.00 0.00
0.00 0.00 0.00  0.2  0.0    0 0.00 0.00 0.00
0.00 0.00 0.00  0.0  0.3    0 0.00 0.00 0.00
0.00 0.00 0.00  0.0  0.0    1 0.00 0.00 0.00
0.00 0.00 0.00  0.0  0.0    0 0.26 0.00 0.00
0.00 0.00 0.00  0.0  0.0    0 0.00 0.28 0.00
0.00 0.00 0.00  0.0  0.0    0 0.00 0.00 0.48
```

For the probability of staying, the weights are as follows: percentage of purple, 0.2; percentage of native plants, 0.3; percentage of preferred plants, 0.5; and diversity of plants, 0.4. This created the diagonal matrix in Figure 19. This matrix shows the probability of a bee starting at one site, rows, and remaining at the same site, columns. Consequently, only the diagonals of the matrix have probabilities. The rest of the matrix is populated with zeros, as the other elements indicate the bee has changed locations.

Figure 19

	X1	X2	X3	X4	X5	X6	X7	X8	X9
[1,]	0.3825	0.000	0.000	0.000	0.000	0.000	0.0000000	0.0000000	0.0000000
[2,]	0.0000	0.246	0.000	0.000	0.000	0.000	0.0000000	0.0000000	0.0000000
[3,]	0.0000	0.000	0.386	0.000	0.000	0.000	0.0000000	0.0000000	0.0000000
[4,]	0.0000	0.000	0.000	0.204	0.000	0.000	0.0000000	0.0000000	0.0000000
[5,]	0.0000	0.000	0.000	0.000	0.626	0.000	0.0000000	0.0000000	0.0000000
[6,]	0.0000	0.000	0.000	0.000	0.000	0.576	0.0000000	0.0000000	0.0000000
[7,]	0.0000	0.000	0.000	0.000	0.000	0.000	0.5819231	0.0000000	0.0000000
[8,]	0.0000	0.000	0.000	0.000	0.000	0.000	0.0000000	0.1817143	0.0000000
[9,]	0.0000	0.000	0.000	0.000	0.000	0.000	0.0000000	0.0000000	0.7463333

Probability of Staying Diagonal Matrix

For the probability of leaving, a few matrix transformations were needed before we could successfully apply weights. For the distance matrix, we used the following equation which subtracted each distance from 0.5:

MatrixA =

0.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.0	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.0	0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.0	0.5	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.0	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5	0.0	0.5	0.5
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.0	0.5
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.0

MatrixA - DistanceMatrix = **Transformed Distance Matrix**

0.000	0.438	0.353	0.369	0.333	0.299	0.295	0.099	0.110
0.438	0.000	0.380	0.318	0.282	0.259	0.214	0.053	0.065
0.353	0.380	0.000	0.310	0.286	0.293	0.305	0.094	0.107
0.369	0.318	0.310	0.000	0.464	0.419	0.405	0.230	0.240
0.333	0.282	0.286	0.464	0.000	0.443	0.423	0.266	0.276
0.299	0.259	0.293	0.419	0.443	0.000	0.477	0.294	0.306
0.295	0.214	0.305	0.405	0.423	0.477	0.000	0.287	0.300
0.099	0.053	0.094	0.230	0.266	0.294	0.287	0.000	0.487
0.110	0.065	0.107	0.240	0.276	0.306	0.300	0.487	0.000

This was done to invert our values, as shorter distances should correspond to a higher probability of leaving while further distances should correspond to a lower probability of leaving. For the roads and buildings matrices, we applied the following transformation:

$$\textbf{MatrixB} = \begin{matrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{matrix}$$

(RoadsMatrix - MatrixB) * -1 = **Transformed Roads Matrix**

$$\begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{matrix}$$

(BuildingsMatrix - MatrixB) * -1 = **Transformed Buildings Matrix**

$$\begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{matrix}$$

This was done to invert the ones and zeros as the presence of a road or building should correspond to a higher probability of leaving than the absence of a road or building. For the probability of leaving, the weights are as follows: distance, 0.85; roads, 0.1; and buildings, 0.15. Additionally, we zeroed out the diagonals as this matrix is meant to showcase only the probability of leaving to distinct path sites. The combined weighted probability of distance, roads, and buildings matrix is seen in Figure 20, creating our probability of leaving matrix. This matrix shows the probability of a bee start at one site, rows, and moving to each of the eight other sites, columns.

Figure 20

	X1	X2	X3	X4	X5	X6	X7	X8	X9
[1,]	0.00000	0.62230	0.30005	0.31365	0.28305	0.25415	0.25075	0.08415	0.09350
[2,]	0.62230	0.00000	0.32300	0.27030	0.23970	0.22015	0.18190	0.04505	0.05525
[3,]	0.30005	0.32300	0.00000	0.41350	0.24310	0.24905	0.25925	0.07990	0.09095
[4,]	0.31365	0.27030	0.41350	0.00000	0.64440	0.45615	0.34425	0.19550	0.20400
[5,]	0.28305	0.23970	0.24310	0.64440	0.00000	0.47655	0.35955	0.22610	0.23460
[6,]	0.25415	0.22015	0.24905	0.45615	0.47655	0.00000	0.55545	0.24990	0.26010
[7,]	0.25075	0.18190	0.25925	0.34425	0.35955	0.55545	0.00000	0.24395	0.25500
[8,]	0.08415	0.04505	0.07990	0.19550	0.22610	0.24990	0.24395	0.00000	0.66395
[9,]	0.09350	0.05525	0.09095	0.20400	0.23460	0.26010	0.25500	0.66395	0.00000

Probability of Leaving Matrix

The weights show the parameters we believe to have more significance. Significance is reflected in higher weights, as more significant parameters have greater weights. For example, the percentage of purple was given a lower weight than the percentage of native plants considering the grid measurement of purple is less accurate. To quantify how purple a hue appears and what percentage of a site is purple from a bee's perspective is much more difficult to precisely measure than calculating the sheer percentage of Minnesota native plants contained in a given site.

3.4 Adjacency Matrix

Once our probability of staying and probability of leaving matrices were constructed, we combined the two matrices into one adjacency matrix. An adjacency matrix is a square matrix that indicates the connections between each of the rows and columns. For our adjacency matrix, all rows must sum to one, or 100%. To do this, we kept the probability of staying matrix, or diagonal matrix, the same and redistributed the other entries using the following RStudio code:

````
```{r}
if(i != j){
  adjacencyMatrix[i,j] <- (leaveMatrix[i,j]/sum(leaveMatrix[i,]))*(1-stayMatrix[i,i])
}
```
````

This code scans a given matrix, and if the row does not equal the column, i.e., the diagonals or probability of staying, it redistributes the other entries of the row. For example, if the row and column are both one, it would leave the first entry, and redistribute the remaining entries in row one. To do this, the code takes a single entry, leaveMatrix[i, j], and divides the entry by the sum of the entire row, sum(leaveMatrix[i,]). This gives the percentage a single entry accounts for in the row. Once this percentage is obtained, the code multiplies the percentage by the remaining probability after the probability of staying is subtracted from the row, 1-stayMatrix[i,i]. The final adjacency matrix is shown in Figure 21.

Figure 21

	1	2	3	4	5	6	7	8	9
1	0.38250000	0.174541400	0.08415737	0.08797187	0.07938925	0.07128344	0.07032982	0.02360221	0.02622468
2	0.23968240	0.246000000	0.12440530	0.10410760	0.09232181	0.08479202	0.07005982	0.01735126	0.02127985
3	0.09405284	0.101246700	0.38600000	0.12961460	0.07620145	0.07806652	0.08126378	0.02504523	0.02850893
4	0.08785622	0.075713490	0.11582510	0.20400000	0.18050230	0.12777180	0.09642755	0.05476133	0.05714225
5	0.03910556	0.033116420	0.03358615	0.08902887	0.62600000	0.06583909	0.04967463	0.03123747	0.03241181
6	0.03959566	0.034298590	0.03880110	0.07106654	0.07424479	0.57600000	0.08653713	0.03893353	0.04052265
7	0.04278715	0.031038810	0.04423756	0.05874168	0.06135242	0.09478014	0.58192310	0.04162682	0.04351235
8	0.03850083	0.020611560	0.03655635	0.08944638	0.10344670	0.11433580	0.11161350	0.18171430	0.30377460
9	0.01276972	0.007545742	0.01242145	0.02786120	0.03204038	0.03552303	0.03482650	0.09067865	0.74633330

Adjacency Matrix

In this particular case, the adjacency is a nine by nine matrix, one row and one column for each of the nine Pollinator Path sites. The rows represent the starting site of a particular bee while the columns represent the ending location of said bee. The probability of moving between sites is the number found in the corresponding element. For example, the first entry, adjacencyMatrix[1,1], shows the probability of a bee starting at Site 1 and remaining at Site 1 while the second entry, adjacencyMatrix[1,2], shows the probability of a bee starting at Site 1 and leaving to end at Site 2.

3.5 Code

Our network model was coded using the statistical software RStudio version 1.2.1335. The full code can be found in Appendix C.

4. Bee Tracking Data

We attempted to watch bees at a few pollinator sites in close proximity to other pollinator sites. We recorded the current site of the bee every five minutes. Watching bees during this time helped us realize that bees move much more frequently between flowers than they do between sites. We were unable to see any bees move between sites but saw movement within sites frequently. For a full list of collected data, see Appendix D.

5. Results

The main calculation we made to determine how well the Pollinator Path is currently functioning is called ‘efficiency.’ Efficiency is a single number representing the average number of moves it takes a single bee to visit every site in the path and return to its starting site. This is the average of 1,000 simulations. The efficiency is currently 66.437±29.42312.

In order to find out the current risks of the Pollinator Path, we wanted to see which sites had the largest impact on the efficiency score. To do this, we had to remove each site from the adjacency matrix one at a time and run the simulations with only eight sites. This gave us nine new efficiency scores, one for each of the sites, see Figure 22. The lower the efficiency score means the faster the bee moved through the entire Pollinator Path. Because of this, we think the site with the lowest efficiency score is the safest to remove from the pollinator path if any site were to be removed or lost. In accordance, the simulations that do not include the Pollinator Bed (Site 9) have the lowest efficiency score meaning the Pollinator Bed is the least useful site for maximizing the efficiency of the Pollinator Path. Since removing Site 9 is the best for improving efficiency score, we compared every other site's efficiency score to Site 9 in order to calculate $\Delta\eta_9$.

Figure 22

Site	η	$\Delta\eta_9$
1	60.387	+11.217
2	58.053	+8.883
3	59.19	+10.02
4	60.264	+11.094
5	54.408	+5.238
6	55.859	+6.689
7	52.418	+3.248
8	55.568	+6.398
9	49.17	0

However, this method of analyzing efficiency fails to take into account why the efficiency number is changing. The Pollinator Bed is the largest site designed for pollinators in the entire Pollinator Path. Subsequently, the likelihood of a bee staying at the Pollinator Bed is very high compared to most other sites. A higher chance of staying means that bees can get stuck at Site 9 for several moves rather than moving on to new sites and returning to the starting point. In real life, bees staying where they are to pollinate more flowers is not a bad thing. In our simulations, this drastically alters the efficiency score and results in poor efficiency.

Another goal was to add a site to the pollinator path and see how that would change the efficiency score. Each member of our group added one site to the path, a specific location with specific qualities, Figure 23. We then ran our simulations with ten sites and found the efficiency scores of each new site. Adding a tenth site is almost certainly going to make the efficiency score higher because the bee has to stop at an additional site before it can leave the path. Site 12 was positioned on the corner of Cretin Avenue and Summit Avenue and had the best efficiency score of the new sites.

Figure 23

Site	η	$\Delta\eta_{12}$
10	80.976	+4.806
11	79.571	+3.401
12	76.17	0

6. Conclusion

6.1 Summary

Our research and simulations have led us to believe that the flower sites in the pollinator path work well individually and are an excellent way to help pollinators around campus. However, we believe that the sites are too far apart and the difference in flower types makes the interconnectivity of sites difficult to impossible.

6.2 Limitations

Finding evidence to support the hypothesis that bees move to different sites in the pollinator path is challenging without using technology. Our only available option is to follow bees on foot. This process is inaccurate and extremely time consuming. Past researchers have been able to use camera tracking or Global Positioning System (GPS) units in order to collect more accurate data. Without either of these two options at our disposal, the idea of tracking a bee between flower sites on campus is impossible.

6.3 Future

We have several goals for the future of this project and continuation of Pollinator Path research. One of these goals is to increase the number of parameters in our simulations. These parameters could include weather (temperature, rain, clouds, sun, etc.), seasonality (spring, early summer, late summer, etc.), and total bee flight time. Another improvement to our research will be using data from actual bees along with regression analysis to improve the accuracy of our model. We also realize that bees will likely never visit every site in the Pollinator Path, and we would like to add a chance that a bee leaves the path at any given point rather than a specified point.

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8. Works Cited

Becher, Matthias A et al. "BEEHAVE: a systems model of honeybee colony dynamics and foraging to explore multifactorial causes of colony failure."

Blaauw, B. & Isaacs, R. (2014). Flower plantings increase wild bee abundance and the pollination services provided to a pollination-dependent crop. *Journal of Applied Ecology, 51*, 890-898. doi:10.1111/1365-2664.12257

Evans, E., Smart, M., Cariveau, D., & Spivak, M. (2018). Wild, native bees and managed honey bees benefit from similar agricultural land uses. *Agriculture, Ecosystem & Environment, 268*, 162-170. dio:10.1016/j.agee.2018.09.014

Fenner, Aaron L., et al. "Using Social Networks to Deduce Whether Residents or Dispersers Spread Parasites in a Lizard Population

Khoury DS, Barron AB, Myerscough MR (2013) Modelling Food and Population Dynamics in Honey Bee Colonies.

Khoury DS, Myerscough MR, Barron AB (2011) A Quantitative Model of Honey Bee Colony Population Dynamics

Mani, D., & Moody, J. (2014). Moving beyond Stylized Economic Network Models: The Hybrid

World of the Indian Firm Ownership Network.

Morandin, LA. & Kremen, C. (2013). Bee Preference for Native versus Exotic Plants in Restored Agricultural Hedgerows. *Restoration Ecology, 21*(1), 1-7. doi:10.1111/j.1526-100X.2012.00876.x

Moreno Y, Nekovee M, Pacheco AF (2004) Dynamics of rumor spreading in complex networks.

National Research Council. 2007. *Status of Pollinators in North America*. Washington, DC: The National Academies Press.

Pasquaretta, C., Jeanson, R., Andalo, C., Chittka, L., & Lihoreau, M. (2017). Analysing plant-pollinator interactions with spatial movement networks. *Ecological Entomology, 42*(1), 4-7. doi:10.1111/een.12446

Reverté, S., Retana, J., Gómez, J., & Bosch, J. (2016). Pollinators show flower colour preferences but flowers with similar colours do not attract similar pollinators. *Annals of Botany, 118*(2), 249-257. doi:10.1093/aob/mcw103

University of St. Thomas. (2019). Pollinator Path. Retrieved from https://www.stthomas.edu/biology/research/greenhouses/pollinatorpath/

9. Appendix

Appendix A

2019 Pollinator Path Plants

SITE 1: MONAHAN FOUNTAIN PLAZA	
Agastache foeniculum 'Blue Fortune'	Blue Fortune Giant Hyssop
Ajuga reptans 'Chocolate Chip'	Chocolate Chip Ajuga
Allium tanguiticum 'Summer Beauty'	Summer Beauty Ornamental Onion
Calamagrostis acutiflora 'Karl Foerster'	Karl Foerster Korean Feather Grass
Echinacea pupurea	Purple Coneflower
Nepeta faassenii 'Walker's Low'	Walker's Low Catmint
Salvia nemorosa 'Caradonna'	Caradonna Salvia
Verbena bonariensis	Verbena bonariensis

SITE 2: FLAGPOLE BEDS	
Artemesia 'Dusty Miller'	Dusty Miller
Lobularia maritima	Alyssum (purple & white)
Osteospermum spp	White Osteospermum
Osteospermum spp	Purple Osteospermum
Pennisetum purpurescens	Purple Fountain Grass
Salvia 'Evolution Violet'	Evolution Violet Salvia

SITE 3: JESTER BED	
Agastache foeniculum	Lavender Hyssop
Allium aflatunense 'Purple Sensation'	Purple Sensation Allium
Allium sphaerocephalon	Drumstick Allium
Anemone sylvestris 'Snowdrop'	Grape Leaf Anemone
Aruncus dioicus	Goat's Beard
Cephalanthus occidentalis 'Balloptics'	Fiber Optics Button Bush
Chelone glabra	White Turtlehead
Chelone lyonii 'Hot Lips'	Hot Lips Turtlehead
Eupatorium maculatum 'Phantom'	Phantom Joe Pye Weed
Geum triflorum	Prairie Smoke
Nepeta faassenii 'Walker's Low'	Walker's Low Catmint
Pulmonaria 'Raspberry Splash'	Raspberry Splash Lungwort
Salvia nemorosa 'Caradonna'	Salvia Caradonna
Salvia nemorosa 'Mainacht'	Salvia May Night
Sedum spectabile 'Autumn Fire'	Autumn Fire Sedum
Tulipa biflora	Biflora Tulip
Tulipa humilis alba coerulea oculata	Oculata Tulip
Tulipa turkestanica	Turkestanica Tulip
Zizea aurea	Golden Alexander

SITE 4: SCIENCE BUILDING SQUARE PLANTERS	
Calibrachoa parviflora 'Cabaret Deep Blue'	Cabaret Deep Blue Calibrachoa
Euphorbia graminea 'Glitz'	Glitz Euphorbia
Heliotropium arborescens 'Fragrant Delight'	Fragrant Delight Heliotrope
Juncus inflexus 'Blue Mohawk'	Blue Mohawk Juncus
Lobularia maritima	Purple Alyssum
Origanum x 'Kirigami'	Kirigami Oregano
Scaevola aemula 'Surdiva Blue Violet'	Surdiva Blue Violet Scaevola
Talium paniculatum 'Limon'	Limon Talium
Tradescantia pallida 'Purpurea'	Purple Heart
Verbena rigida 'Santos Purple'	Santos Purple Verbena

SITE 5: SCIENCE BUILDING LONG BED	
Agastache foeniculum 'Blue Fortune'	Blue Fortune Giant Hyssop
Allium aflatunense 'Purple Sensation'	Purple Sensation Allium
Calamagrostis acutiflora 'Karl Foerster'	Karl Foerster Feather Reed Grass
Echinacea pallida	Pale Purple Coneflower
Echinacea purpurea	Purple Coneflower
Eupatorium dubium 'Baby Joe'	Baby Joe Joe Pye Weed
Liatris spicata 'Kobold'	Kobold Prairie Blazing Star
Monarda didyma 'Purple Rooster'	Purple Rooster Bee Balm
Nepeta faassenii 'Walker's Low'	Walker's Low Catmint
Nepeta faassenii 'Blue Wonder'	Blue Wonder Catmint
Ratibida pinnata	Yellow Coneflower
Salvia nemorosa 'Mainacht'	May Night Salvia
Sedum 'Purple Emperor'	Purple Emperor Sedum
Veronicastrum virginianum	Culver's Root
Verbena bonariensis 'Buenos Aires'	Buenos Aires Verbena

SITE 6: MEDICINAL GARDEN	
NATIVE AMERICAN MEDICINAL PLANTS	
Aralia racemosa	American Spikenard, Indian Root
Asclepias incarnata	Swamp Milkweed
Asclepias tuberosa	Butterflyweed
Chelone lyonii 'Hot Lips'	Hot Lips Turtlehead
Cimicifuga racemosa	Black Cohosh
Ceonothus americanus	New Jersey Tea
Echinacea purpurea, Echinacea pallida	Coneflower
Eupatorium perfoliatum	Boneset
Lobelia siphilitica	Great Blue Lobelia
Monarda fistulosa	Wild Bergamot
Podophyllum peltatum	American May Apple
Sanguinaria canadensis	Bloodroot
Scutellaria lateriflora	Blue Skullcap
MODERN PHARMACEUTICAL PLANTS	
Achillea spp.	Yarrow
Catharanthus roseus	Madagascar Periwinkle
Digitalis purpurea 'Dalmatian Purple'	Dalmation Purple Foxglove
Glycyrrhiza glabra	Licorice root
Melilotus officinalis	Yellow Sweet Clover
Mentha x piperita	Peppermint
Narcissus albus plenus odoratus	Daffodil
Narcissus poeticus 'Pheasant's Eye'	Pheasant's Eye Daffodil
Narcissus pseudo-narcissus	Daffodil
Papaver somniferum spp.	Opium poppy/breadseed poppy
Ricinus communis	Castor Bean
Tanacetum parthenium	Feverfew
Yucca filimentosa	Yucca

TRADITIONAL MEDICIAL (OFFICINALIS) PLANTS	
Althea officinalis	Marshmallow
Anchusa officinalis	Bugloss
Borago officinalis	Borage
Hyssopus officinalis	Hyssop
Lavandula officinalis (angustifolia)	English Lavender
Levisticum officinale	Lovage
Melissa officinalis	lemon balm
Rosmarinus officinalis	rosemary
Salvia officinalis	Sage
Sanguisorba officinalis	Great burnet
Saponaria officinalis	Soapwort
Stachys officinalis	Betony
Symphytum officinale	Comfrey, Knitbone
Valeriana officinalis	Valerian
Verbena officinalis	Vervain

MODERN HERBAL MEDICINE PLANTS	
Allium sativum var. sativum	Garlic
Althea officinalis	Marshmallow
Echinacea purpurea	Purple Coneflower
Filipendula ulmaria/ Spirea ulmaria	Meadowsweet
Hydrastis canadensis	Goldenseal
Matricaria recutita	Chamomile
Melilotus officinalis	Yellow Sweet Clover
Tanacetum parthenium	Ferverfew
Thymus vulgaris	Thyme

SITE 7: ANDERSON PARKING RAMP	
Achillea millefolium	Yarrow
Agastache foeniculum	Giant Hyssop
Asclepias tuberosa	Butterflyweed
Chelone lyonii 'Hot Lips'	Hot Lips Turtlehead
Hyssopus officinalis	Hyssop
Lobelia siphilitica	Giant Blue Lobelia
Nepeta faassenii 'Walker's Low'	Walker's Low Catmint
Physocarpus opulifolius	Ninebark (Diablo?)
Rudbeckia hirta	Black-Eyed Susan
Sedum 'Autumn Joy'	Autumn Joy Sedum
Silphium perfoliatum	Cup Plant
Spiraea betulifolia 'Tor'	Tor Spirea
Vernonia fasciculata	Ironweed
SITE 8: STEWARDSHIP GARDEN	
VEGETABLE GARDEN	
Beta vulgaris	Beets
Brassica oleracea 'Flash'	Flash Collards
Brassica oleracea var. botrytis	Cauliflower
Brassica oleracea var. italica	Broccoli
Capsicum annum	Pepper
Coriandrum sativum	Cilantro
Cucumis sativus	Cucumber
Ocimum basilicum	Basil
Phaseolus vulgaris	Bush Beans
Solanum lycopersicum	Tomato
ZINNIA BORDER	
Ipomea purpurea	Morning Glory
Osteospsermum spp.	White Osteospermum
Salvia 'Evolution Violet'	Evolution Violet Salvia
Zinniea elegans 'Benary's Giant'	Benary's Giant Zinnia

SITE 9: POLLINATOR GARDEN	
Agastache foeniculum	Anise Hyssop
Aquilegia canadensis	Wild Columbine
Asclepias incarnata	Swamp Milkweed
Asclepias syriaca	Common Milkweed
Asclepias tuberosa	Butterfly Weed
Aster novae-angliae	New England Aster
Borago officinalis	Borage
Calendula officinalis	Calendula
Eupatorium maculatum	Joe Pye Weed
Hypericum pyramidatum	Great St. John's Wort
Liatris aspera	Rough Blazingstar
Liatris ligulistylis	Meadow Blazingstar
Liatris pycnostachya	Prairie Blazingstar
Lobelia cardinalis	Cardinal Flower
Lobelia silphilitica	Blue Lobelia
Monarda fistulosa	Wild Bergamot
Penstemon calycosus	Calico Beardtongue
Penstemon digitalis	Smooth Beardstongue
Rudbeckia hirta	Black-eyed Susan
Solidago speciosa	Showy Goldenrod
Stachys byzantium	Lamb's Ear
Thalictrum spp.	Meadow Rue
Veronicastrum viginiana	Culver's Root
Zizia aurea	Golden Alexander

Appendix B

Minnesota Native Plant List

Scientific Name	Common Name	Scientific Name	Common Name
Agastache foeniculum	Hyssop	*Liatris ligulistylis*	Meadow Blazingstar
Aquilegia canadensis	Wild Columbine	*Liatris pycnostachya*	Prairie Blazingstar
Aralia racemosa	American Spikenard, Indian Root	*Lobelia cardinalis*	Cardinal Flower
Artemesia 'Dusty Miller'	Dusty Miller	*Lobelia silphilitica*	Blue Lobelia
Aruncus dioicus	Goat's Beard	*Monarda didyma 'Purple Rooster'*	Purple Rooster Bee Balm
Asclepias incarnata	Swamp Milkweed	*Monarda fistulosa*	Wild Bergamot
Asclepias syriaca	Common Milkweed	*Penstemon calycosus*	Calico Beardtongue
Asclepias tuberosa	Butterflyweed	*Penstemon digitalis*	Smooth Beardstongue
Aster novae-angliae	New England Aster	*Physocarpus opulifolius*	Ninebark
Chelone glabra	White Turtlehead	*Ratibida pinnata*	Yellow Coneflower
Echinacea pallida	Pale Purple Coneflower	*Rudbeckia hirta*	Black-Eyed Susan
Echinacea pupurea	Purple Coneflower	*Sanguinaria canadensis*	Bloodroot
Eupatorium dubium 'Baby Joe'	Baby Joe Joe Pye Weed	*Sanguisorba officinalis*	Great burnet
Eupatorium maculatum	Joe Pye Weed	*Silphium perfoliatum*	Cup Plant
Eupatorium maculatum 'Phantom'	Phantom Joe Pye Weed	*Solidago speciosa*	Showy Goldenrod
Geum triflorum	Prairie Smoke	*Spiraea betulifolia* 'Tor'	Tor Spirea
Hydrastis canadensis	Goldenseal	*Vernonia fasciculata*	Ironweed
Hypericum pyramidatum	Great St. John's Wort	*Veronicastrum viginiana*	Culver's Root
Liatris aspera	Rough Blazingstar	*Zizia aurea*	Golden Alexander

Appendix C

movementCalculator.Rmd (Rstudio version: 1.2.1335)

```
singleCellEffMatrix <- 0.5 * PreferredMatrix + 0.4* diversityMatrix + 0.2 *
purplePercentMatrix + 0.3 *NativePlantMatrix
##Finding the singleCell matrix by adding up the values of Preferred, diversity, percent of
purple and native plant matrices.

movementProbability <- Distance*.85 + Roads*.1+Buildings*.15
##Finding the total movement probability matrix by adding up the values of roads, buildings
and Distance matrices.

removeLocation <- 0
### The number of site we want to remove

resultMat <- produceMatrix(removeLocation)
## Matrix obtained after the removing the desired site

initialLocation <- getInitialLocation(removeLocation)
## Randomly generating starting location,by considering the removed location.

Efficiency <- replicate(1000, numOfMovements(resultMat,initialLocation,removeLocation))
## Running the function a 1000 times, and obtained values in Efficiency

mean(Efficiency)
## Finding the mean of total number of movements after running them 1000 times.
```

produceMatrix Function

```
produceMatrix <- function(removeLocation){
##This function produces matrix of movement probabilties calculated by combining the other two matrices.
for (i in (1:ncol(movementProbability)))
{
  movementProbability[i,i]=0
}
mat <- singleCellEffMatrix
move <- movementProbability

if(removeLocation >0 && removeLocation < (nrow(probabilty)+1))
{
  for(i in (1:nrow(mat)))
  {
    move[removeLocation,i]=0.0
    move[i,removeLocation]=0.0
  }
  mat[removeLocation,removeLocation]=0.0
}

for (i in (1:nrow(mat)))
{
  for(j in (1:ncol(mat)))
  {
    if(i != j && removeLocation != j && i != removeLocation)
    {
      mat[i,j]=(move[i,j]/sum(move[i,]))*(1-singleCellEffMatrix[i,i])
    }
  }
}
return (mat)
}
```

getInitialLocation Function

```
getInitialLocation <- function(removeLocation)
{ ## This function randomly generates the starting location of the bee

  locations <- matrix(0L, nrow=ncol(resultMat)-1)
  for(i in (1:ncol(resultMat)))
  {
    if (i != removeLocation)
    {
     locations[i]=i
    }
    else
    {
      locations[i]=i+1
    }
  }
  location <- sample(locations,1)
  return(location)
}
```

numOfMovements Function

```
numOfMovements <- function(mat,initialLocation, removeLocation){
  ##This functions returns the number of movements taken by the bee to reach its initial
position after visiting each node.
  nmovements <- 1
  probMat <- mat
  prevLocation <- 0
  visMat <- matrix(FALSE, nrow=ncol(probMat))
  hold <- FALSE
  holdValue <- 0
  beelocation <- initialLocation
  visMat[initialLocation]=TRUE
  if(removeLocation > 0 && removeLocation <=  ncol(probMat))
  {
    visMat[removeLocation]=TRUE
  }
  while(!hold)
  {
    colNum <- 1
    prev <- 0.0
    num=runif(1)
    rowNum=beelocation
    if(prevLocation == beelocation)
    {
      holdValue <- probMat[prevLocation,beelocation]
      rowSum <- 0
      for( colNum in (1:ncol(probMat)))
      {
        if(colNum != beelocation)
        {
          rowSum=rowSum+probMat[beelocation, colNum]
        }
      }
      probMat[prevLocation,beelocation] = (holdValue-holdValue*0.05)
      sum=1-probMat[prevLocation, beelocation]
       for( colNum in (1:ncol(probMat)))
      {
        if(colNum != beelocation)
        {
           res=(sum-rowSum)*(probMat[beelocation,colNum]/rowSum)
          probMat[beelocation,colNum]=probMat[beelocation,colNum]+res
        }
      }
    }
```

```
  for(colNum in (1:ncol(probMat)) )
    {
      sum=prev+probMat[rowNum,colNum]
      if(num >= prev && num <= sum)
      {
        prevLocation = beelocation
        beelocation <- colNum
        visMat[beelocation]=TRUE
         break
      }

       prev=sum
   }
  if(nmovements >= ncol(probMat) & initialLocation == beelocation)
     {
       for(i in (1:ncol(probMat)))
       {
         if(visMat[i] == FALSE)
         {
           hold=FALSE
           break
         }
         hold=TRUE
       }

     }
     nmovements <- nmovements+1
 }
 return(nmovements)
}
```

Appendix D

Bee Tracking Data Table

Bee	Starting Position	5 Minutes	10 Minutes	15 Minutes	Total Time (in minutes)
1	1	1			7
2	1	1	1		10
3	1	1	1	1	15
4	5	5			5
5	5	5			5
6	6	6	6	6	17

7	8				1
8	9				3
9	9				2
10	9	9			6