

## Problem A.

Given a relation  $R = \{(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)\}$  consisting of  $n$  tuples, check if  $R$  is reflexive, symmetric and transitive

### Input

The first line consists of a single integer  $n$  the number of tuples in the relation  $R$ .

The next  $n$  lines consist of two integers  $(a, b)$  representing a tuple in  $R$ .

### Output

Print 3 strings each being *YES/NO*, the first one indicating if it is reflexive, second one for symmetric and third one if  $R$  is transitive or not.

### Example

test	answer
6 1 1 2 2 3 3 1 2 1 3 2 3	YES NO YES
6 1 1 2 2 3 3 1 2 2 1 2 3 3 2	YES YES NO

### Note

The first example is reflexive and transitive but not symmetric.

The second example is reflexive and symmetric but not transitive.

## Problem B.

Given a relation  $R = \{(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)\}$  consisting of  $n$  tuples, find the transitive closure of  $R$ .

### Input

The first line consists of a single integer  $n$  - the number of tuples in the relation  $R$ .

The next  $n$  lines consist of two integers  $(a, b)$  representing a tuple in  $R$ .

### Output

First print the number of tuples added in the transitive closure, followed by the tuples added to the closure.

### Examples

test	answer
6 1 1 2 2 3 3 1 2 2 1 2 3 3 2	1 1 3
6 1 1 2 2 3 3 1 2 1 3 2 3	0

## Problem C.

Let  $G$  be a graph containing  $n$  vertices labelled from 0 to  $n - 1$ . You are given an array  $a$  of length  $n$  satisfying the following conditions  $\forall i$  from 0 to  $n - 1$  :

- $a_i \in \{0, 1\}$
- If  $a_i = 0$ , the node labelled  $i$  is colored red.
- If  $a_i = 1$ , the node labelled  $i$  is colored blue.

Check if the array  $a$  represents a valid 2-coloring of  $G$ .

### Input

The first line of the input contains 2 integers -  $n$  and  $m$ , representing the number of nodes and the number of edges of  $G$  respectively. The second line contains  $n$  integers which represent the elements of array  $a$ . Each of the next  $m$  lines contains 2 integers which represent an edge of the given graph.

### Output

The output should consist of just one line - print **YES** if the array  $a$  represents a valid 2-coloring of  $G$ , and **NO** otherwise. (Print it in the exact same format, the output will be considered **case-sensitive**).

### Examples

test	answer
4 4 0 1 1 0 0 1 0 2 1 3 2 3	YES
4 4 0 1 1 1 0 1 0 2 1 3 2 3	NO

## Problem D.

You are given a **simple** graph containing  $n$  vertices and  $m$  edges. The vertices are numbered from 1 to  $n$ . Find the minimum number of edges that need to be added to make this graph connected.

### Input

The first line contains 2 integers - the value of  $n$  and  $m$  respectively. Each of the next  $m$  lines contains 2 integers which represent an **undirected** edge of the given graph.

### Output

The output should consist of a single line - the minimum number of edges to be added to make the graph connected.

### Examples

test	answer
5 3 1 2 1 3 4 5	1
5 4 1 2 1 3 1 4 1 5	0

## Problem E.

You are given a **tree** containing  $n$  vertices labelled from 1 to  $n$ . The tree is rooted at the vertex 1. Find the height of the tree.

### Note

The height of a rooted tree is the length of the longest path from the root to any vertex.

### Input

The first line contains the value of  $n$ . Each of the next  $n - 1$  lines contains 2 integers which represent an **undirected** edge of the tree.

### Output

The output should consist of a single line - print the height of the tree rooted at vertex 1.

### Examples

test	answer
4 1 2 1 3 2 4	2
4 1 2 1 3 1 4	1

## Problem F.

You are given a tree consisting of  $n$  vertices numbered from 1 to  $n$ . Find the diameter of the tree. The diameter of the tree is defined as the length of the longest path between any two vertices in the tree.

### Input

The input consists of  $n$  lines. The first line contains the value of  $n$ , the number of vertices in the tree. Each of the next  $n - 1$  lines contains 2 integers which represent an **undirected** edge of the tree.

### Output

Print a single integer - the diameter of the tree.

### Examples

test	answer
5 1 2 1 3 3 4 3 5	3
10 6 4 1 3 10 8 9 3 2 7 5 4 2 4 8 5 9 5	6

### Note

In the first example the diameter corresponds to the path 2–1–3–5

In the second example the diameter corresponds to the path 1–2–9–5–4–2–7

## Problem G.

Given a directed graph consisting of  $n$  vertices labeled 1 to  $n$  and  $m$  edges, report the vertices of a cycle in  $G$  in the cyclic order (if a cycle exist). Otherwise print -1. If there are multiple cycles print the vertices of any of them

Note that the graph is represented using an edge list.

### Input

The first line of the input consists of two integers -  $n$  and  $m$  where  $n$  and  $m$  denote the number of nodes and edges in the graph.

Then  $m$  lines follow, each line consists of two integers  $u$  and  $v$  denoting an directed edge from  $u$  to  $v$ .

### Output

If there exists a cycle, print the vertices of the cycle in the cyclic order, if there are no cycles print -1. If there are multiple cycles, print any.

### Examples

test	answer
5 6 1 2 1 3 2 4 3 4 4 5 5 2	2 4 5
5 5 1 2 1 3 2 4 3 4 4 5	-1

### Note

For the first example  $4 - 5 - 2$ ,  $5 - 2 - 4$  are also valid cyclic ordering, however  $5 - 4 - 2$  is not valid as it is a directed graph and there are no edges from 5 to 4, 4 to 2, 2 to 5 to form a cycle

## Problem H.

Given 2 simple graphs  $G_1$  and  $G_2$ , check if they are **isomorphic** or not. Note that simple graphs  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  are isomorphic iff  $\exists$  a bijection  $f : V_1 \rightarrow V_2$  such that  $\forall a, b \in V_1$ ,  $a$  and  $b$  are adjacent in  $G_1$  iff  $f(a)$  and  $f(b)$  are adjacent in  $G_2$ .

**Hint:** Check out `next_permutation` function in C++ to generate permutations of an array

### Note

Consider the nodes to be labelled with 1-indexing.

### Input

The first line of the input consists of two integers -  $n_1$  and  $m_1$  - the number of nodes and edges respectively in the graph  $G_1$ . Then  $m_1$  lines follow, each line consisting of two integers  $u$  and  $v$  denoting an undirected edge of graph  $G_1$ . The next line of the input consists of two integers -  $n_2$  and  $m_2$  - the number of nodes and edges respectively in the graph  $G_2$ . Then  $m_2$  lines follow, each line consisting of two integers  $u$  and  $v$  denoting an undirected edge of graph  $G_2$ .

### Output

The output should consist of just one line - print **YES** if the two graphs are isomorphic, and **NO** otherwise. (Print it in the exact same format, the output will be considered **case-sensitive**).

### Example

test	answer
4 3 1 2 2 4 1 3 4 3 1 4 2 4 2 3	YES
4 3 1 2 2 4 1 3 4 3 2 1 2 4 2 3	NO



## Problem I.

You are given a directed graph consisting of  $n$  nodes numbered 1 to  $n$  and  $m$  edges. A node is said to be safe if either there is no outgoing edge or all the outgoing edges incident with a safe node.

More formally a node  $u$  is said to be safe if one of the following conditions is satisfied -

- There exists no edge  $(u, v)$
- For all edges of the form  $(u, v)$ ,  $v$  is a safe node.

Find all the safe nodes in the graph and print them in ascending order.

### Input

The first line contains 2 integers - the value of  $n$  and  $m$  respectively. Each of the next  $m$  lines contains 2 integers -  $u, v$ .  $(u, v)$  represents an directed edge of the graph from  $u$  to  $v$ .

### Output

The output should consist of a two lines - the first line should be the number of safe nodes.

The second line should consist of all the safe nodes in sorted order.

### Examples

test	answer
5 6 2 1 3 1 4 2 3 4 4 5 5 3	2 1 2
7 8 1 2 2 3 3 1 3 4 5 4 5 6 6 7 7 8	1 4

## Problem J.

Given a **directed** graph  $G$  with  $n$  vertices and  $m$  edges such that any vertex in  $G$  has **atmost one** outgoing edge. Find the length of the longest cycle in  $G$ .

### Note

The length of a cycle is the number of edges present in the cycle.

### Input

The first line contains 2 integers - the value of  $n$  and  $m$  respectively. Each of the next  $m$  lines contains 2 integers which represent a **directed** edge of the given graph. It is guaranteed that any vertex has atmost one outgoing edge.

### Output

The output contains a single integer - the length of the longest cycle in  $G$ .

### Examples

test	answer
7 7 1 2 2 3 3 4 4 5 5 2 6 7 7 6	4
2 1 2 2 1	2