

Properties of Asymptotic Growth rates:

1. Big "oh" (O): The function $f(n) = O(g(n))$ (read as "f of n is big oh of g of n") iff (if and only if) there exist positive constants c and n_0 such that $f(n) \leq c * g(n)$ for all $n, n \geq n_0$. It also indicates the upper bound on the worst-case complexity.

2. Omega (Ω): The function $f(n) = \Omega(g(n))$ (read as "f of n is omega of g of n") iff there exist positive constants c and n_0 such that $f(n) \geq c * g(n)$ for all $n, n \geq n_0$.

3. Theta (Θ): The function $f(n) = \Theta(g(n))$ iff there exists positive constants c_1, c_2 and n_0 such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n, n \geq n_0$.

Comparing Sorting Algorithms

| Sorting Algorithm | Procedure | Time complexity (Best case, Average case, Worst case) | Memory Requirement | Best for large data? |
|-------------------|---------------------------------------------|----------------------------------------------------------|--------------------|-----------------------------------------|
| Insertion sort | place each elements in its correct position | $O(n)$ $O(n^2)$ $O(n^2)$ | $O(1)$ | No. Because of its time complexity |
| Merge sort | Divide and conquer | $O(n \log n)$ $O(n \log n)$ $O(n \log n)$ | $O(n)$ | Yes, Because of its time complexity |
| Heap sort | Builds a heap | $O(n \log n)$ $O(n \log n)$ $O(n \log n)$ | $O(1)$ | Yes, Because of its time complexity |
| Quick Sort | Partitioning based, recursive sort | $O(n \log n)$ $O(n \log n)$ $O(n^2)$ | $O(\log n)$ | Yes, Because of its time complexity |
| Bubble sort | Repeated swaps until sorted | $O(n)$ $O(n^2)$ $O(n^2)$ | $O(1)$ | No. Because of its high time complexity |

□ Cycle Detection in an Undirected graph

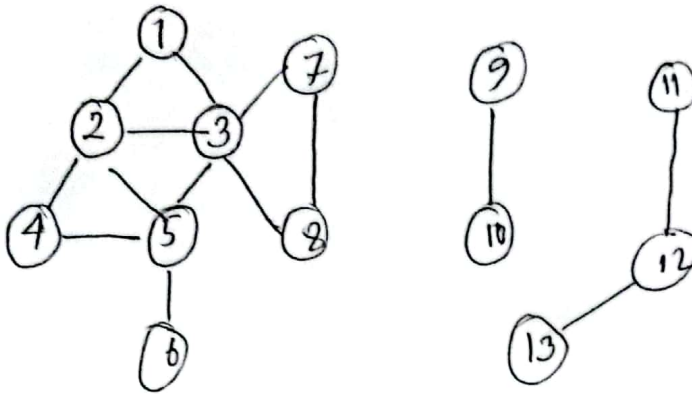
To detect cycles in an undirected graph, we can use DFS with parent tracking:

Algorithms:

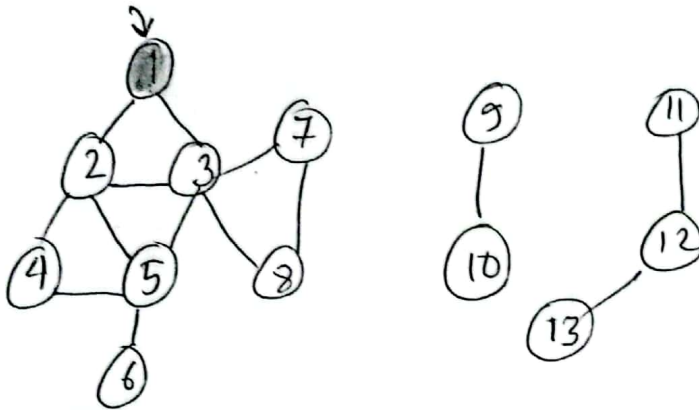
1. Start DFS traversal from any node.
2. Mark each visited node.
3. If a visited node is encountered again (excluding the immediate parent), a cycle exists.
4. Stop when the first cycle is found.

Time complexity: $O(m+n)$ (where $n = \text{nodes}$,
 $m = \text{edges}$)

i) Breadth first search

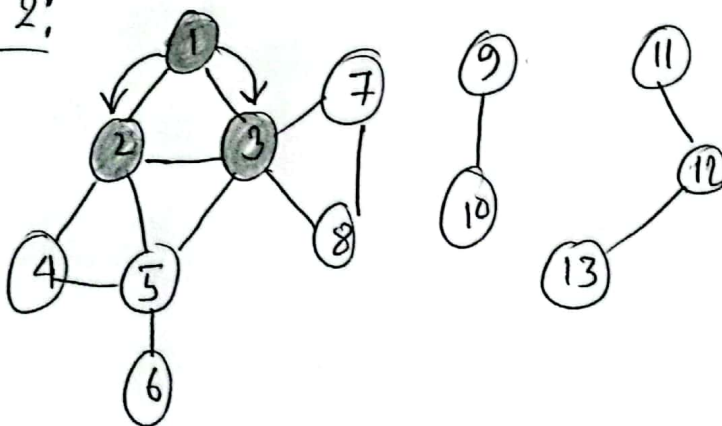


Step 1:



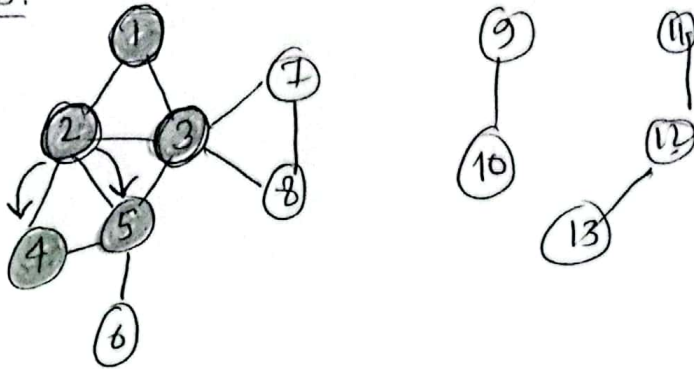
visited nodes: ① →

Step 2:



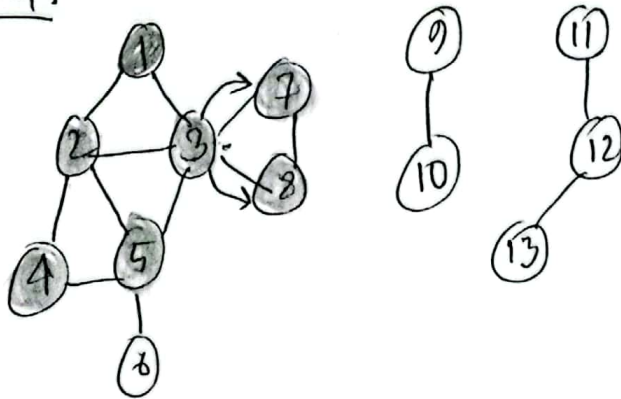
visited nodes: 1 → 2 → 3

Step 3:



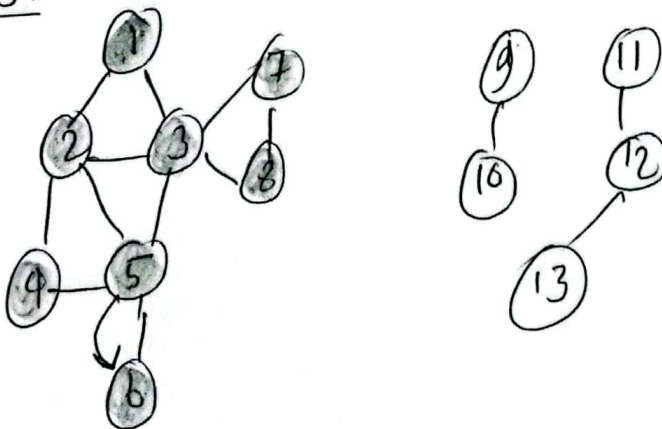
Visited nodes: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5$

Step 4:



Visited nodes: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 7 \rightarrow 8$

Step 5:

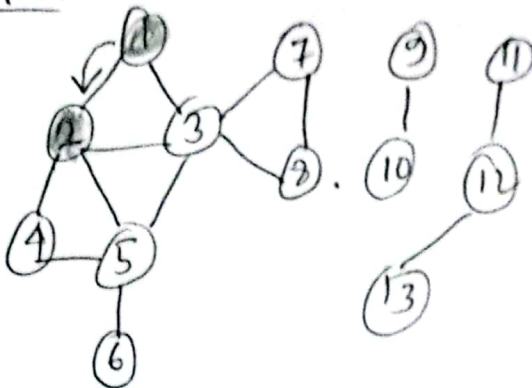


Visited nodes: $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 7 \rightarrow 8 \rightarrow 6$

nodes 9, 10, 11, 12, 13 are not possible to visit from node 1. Because there is no path from 1 to 9, 10, 11, 12, 13.

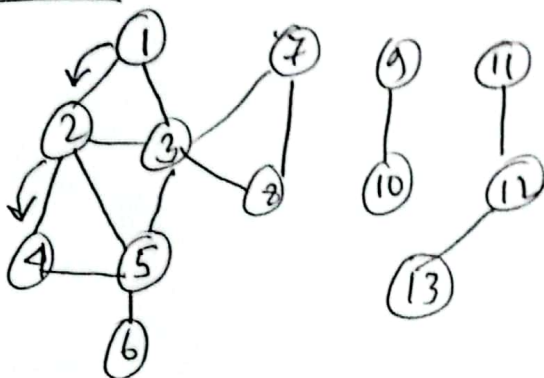
⑪ Depth first search

Step 1:



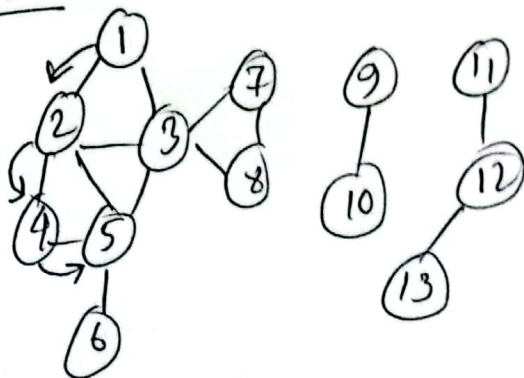
visited nodes: 1 → 2

Step 2:



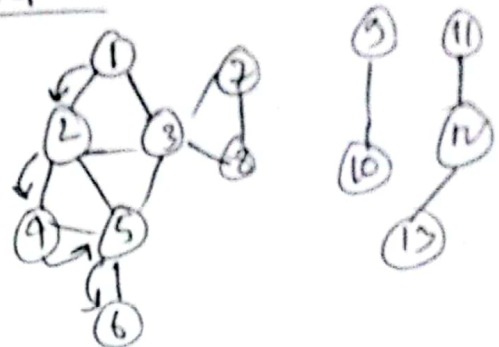
visited nodes: 1 → 2 → 4

Step 3:



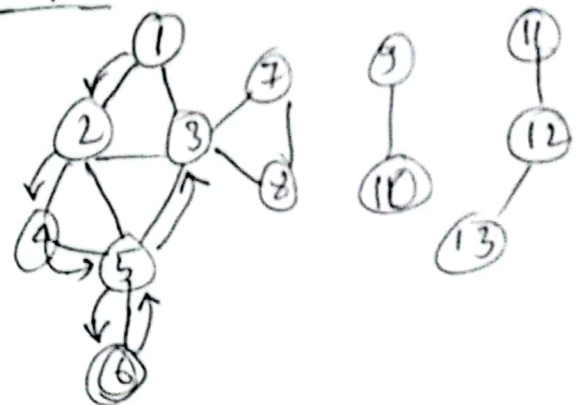
visited nodes: 1 → 2 → 4 → 5

Step 4:



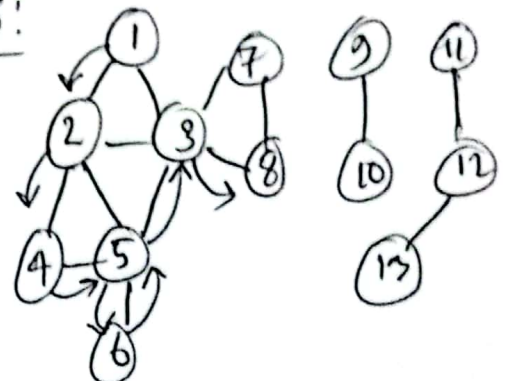
visited: 1 → 2 → 4 → 5 → 6

Step 5:



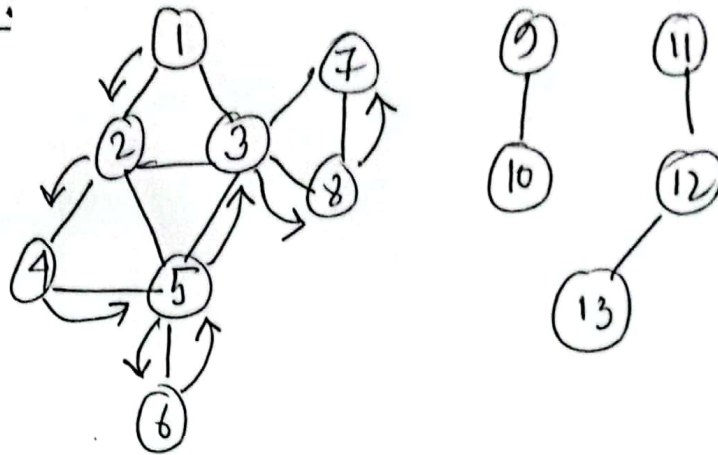
visited: 1 → 2 → 4 → 5 → 6 → 3

Step 6:



visited: 1 → 2 → 4 → 5 → 6 → 3 → 8

Step 7:



visited : $1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 3 \rightarrow 8 \rightarrow 7$

nodes 9, 10, 11, 12, 13 are not possible to visit from node 1. Because there is no path from 1 to 9, 10, 11, 12, 13.