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"Differential Equation"

Define differential equation.

⇒ An equation involving a derivative is called a differential equation.

There are 2 Types of differential equation.

1| Ordinary differential equation.

2| Partial differential equation.

1| Ordinary differential equation:

An equation involving a derivative / A differential equation involving one or more dependent variable with respect to a single independent variable is called an ordinary differential equation.

Example: $\frac{dy}{dx} + 5y = 0$ → single independent

2| Partial differential equation:

A differential equation involving one or more dependent variable with respect to more than One independent variable is called partial differential equation.

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Example: $$\frac{dy}{dx} + \frac{dy}{dz} = 2y$$

Order: The order of the highest ordered derivative involved in a differential equation is called the order of the differential equation.

Example:
$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 6y = 0$$

This is a 3rd order ordinary differential equation.

Degree: The degree of a differential equation is the power of the highest order derivative present in it.

Example:
$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^7 + 5y = 0$$

This is a 2nd order 3rd degree ordinary differential equation.

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transcendental functions → $\log x$, $\sin x$, $\cos x$, e^x etc.

C.T. **Linear differential equation:**

A differential equation involving no product of the dependent variable and/or it's derivative and transcendental function of the dependent variable is called a linear differential equation.

Example: $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 5y = \sin x$

↳ No product

no product
no transcendental

C.T. **Non-linear differential equation:**

A differential equation involving the product of the dependent variable and/or it's derivative and transcendental function of the dependent variable is called a non-linear differential equation.

Example:

i) $y'' - 7y^2 = \sin x$

↳ Product of dependent variable

ii) $y'' - 7(y')^2 + 5y = \sin x$

↳ product of the derivative

iii) $y'' - 7yy' = 0$

↳ Product of the dependent variable and its derivative

iv) $y''' + 7y'' + 5y = e^y$

↳ transcendental function of the dependent variable.

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Solutions:

1) General Solution:

If the solution of nth ordered differential equation involving nth arbitrary ~~zero~~ constants is called a general solution.

$y''' + y'' + y = 0$, has 3 general solution.

2) Particular Solution:

The solution obtained from the general solution by choosing particular values of the arbitrary ~~cont~~ constants is called a particular solution.

It has two conditions:

⇒ i) Initial Condition: The condition involving the value of the dependent variable for the same value of the independent variable.

$y(0) = 3$

$y(0) = 5$

same

ii) Boundary Condition: The condition involving the value of the dependent variable for the different value of the independent variable.

$y(0) = 2$

$y(3) = 5$

different

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Formation of differential equation:

CT. # $y = e^x(A\cos x + B\sin x)$ ..... (i)

Differentiation with respect to x, we get from (i),

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

$$= y + e^x(-A\sin x + B\cos x)$$

$$\Rightarrow \frac{dy}{dx} - y = e^x(-A\sin x + B\cos x) \quad \text{.....(ii)}$$

Differentiation with respect to x, we get from (ii),

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x(-A\sin x + B\cos x) + e^x(-A\cos x - B\sin x)$$

$$= \frac{dy}{dx} - y - e^x(A\cos x + B\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} - \frac{dy}{dx} + y + y = 0 \quad [\text{From equation (i), (ii)}]$$

$$\therefore \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Identification:

This is a 2nd order 1st degree linear ordinary differential equation.

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$y = ax + bx^2$ ..... (i)

Differentiation with respect to x, we get,

$$\frac{dy}{dx} = a + 2bx \quad \text{..... (ii)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 0 + 2b$$

$$\Rightarrow b = \frac{1}{2}\frac{d^2y}{dx^2} \quad \text{..... (iii)},$$

From equation (ii),

$$\frac{dy}{dx} = a + 2 \cdot \frac{1}{2}\frac{d^2y}{dx^2} \cdot x$$

$$\therefore a = \frac{dy}{dx} - \frac{d^2y}{dx^2}x$$

Now, putting the value of a and b in equation (i),

$$y = ax + bx^2$$

$$\Rightarrow y = x\frac{dy}{dx} - x^2\frac{d^2y}{dx^2} + \frac{1}{2} \cdot x^2 \cdot \frac{d^2y}{dx^2}$$

$$\Rightarrow 2y = 2x\frac{dy}{dx} - 2x^2\frac{d^2y}{dx^2} + x^2\frac{d^2y}{dx^2}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

Identification:

This is a 2nd order 1st degree non-linear ordinary differential equation.

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$x^2+y^2 = (2a+x)^2$ ..... (i)

Differentiation with respect to x, we get from equation (i),

$$2x + 2yy' = 2(2a+x)$$

$$\Rightarrow yy' = 2a+x-x$$

$$\Rightarrow yy' = 2a$$

$$\therefore a = \frac{1}{2}yy'$$

Now, putting the value of a in equation (i),

$$x^2+y^2 = \left(2\cdot\frac{1}{2}yy'+x\right)^2$$

$$= (yy'+x)^2$$

$$= y^2y'^2 + 2xyy' + x^2$$

$$\Rightarrow y^2 = y^2y'^2 + 2xyy'$$

$$\Rightarrow yy'^2 + 2xy' - y = 0$$

$$\therefore y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$$

Identification:

This is a 1st order 2nd degree non-linear ordinary differential equation.

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Math Note (2 - 1) Mrinal Sir

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1) ~~Qualitative Variables~~

Separable Differential Equation:

An equation of the form $F(x)\,G(y)\,dx + f(x)\,g(x)\,dy = 0$

or, $\dfrac{F(x)}{f(x)}\,dx + \dfrac{g(y)}{G(y)}\,dy = 0$ is called a separable equation.

Example: $(x^2+2)y\,dx + x(y^2+3)\,dy = 0$

After separating variables we get,

$$\frac{x^2+2}{x}\,dx + \frac{y^2+3}{y}\,dy = 0$$

$x \sin y\,dx + (x^2+1)\cos y\,dy = 0$; $y(1) = \frac{\pi}{2}$

This is a separable equation. solution!

$\Rightarrow$ Given that,

$$x \sin y\,dx + (x^2+1)\cos y\,dy = 0$$

After separating variables we get,

$$\frac{x}{x^2+1}\,dx + \frac{\cos y}{\sin y}\,dy = 0$$

$$\Rightarrow \frac{x}{x^2+1}\,dx + \cot y\,dy = 0$$

Integrating both sides we get,

$$\int \frac{x}{x^2+1}\,dx + \int \cot y\,dy = 0$$

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$$= \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \int \cot y\,dy = 0$$

$$\Rightarrow \frac{1}{2}\log(x^2+1) + \log(\sin y) = \frac{1}{2}\log c$$

$$\Rightarrow \log(x^2+1) + 2\log(\sin y) = \log c$$

$$\Rightarrow \log(x^2+1) + \log(\sin y)^2 = \log c$$

$$\Rightarrow \log(x^2+1)(\sin y)^2 = \log c$$

$$\therefore (x^2+1)(\sin y)^2 = c \quad \text{———} \quad (i)$$

which is the required general solution.

Using $y = \pi/2$ and $x = 1$ in equation (i) we get,

$$(1+1)(\sin \pi/2)^2 = c$$

$$\Rightarrow 2 \cdot 1 = c$$

$$\therefore c = 2$$

$$\therefore (x^2+1)(\sin y)^2 = 2$$

which is the required particular solution.

$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$. Identify and solve the differential equation.

$\Rightarrow$ Given that,

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

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After separation variables we get,

$$\frac{dx}{1+x^2} = \frac{dy}{1+y^2}$$

Integrating both sides we get,

$$\int \frac{dx}{1+x^2} = \int \frac{dy}{1+y^2}$$

$$\Rightarrow \tan^{-1}(x) = \tan^{-1}(y) + c$$

~~c =~~

Solve for y:

$$y = \tan(\tan^{-1}(x) + c)$$

$$= \frac{\tan(\tan^{-1}(x)) + \tan c}{1 - \tan(\tan^{-1}(x)) \tan c}$$

$$= \frac{x + \tan c}{1 - x \tan c}$$

Solve for x:

$$x = \frac{y + \tan c}{1 - y \tan c}$$

This is the general solution to the given differential equation.

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$(x^2+1)(1-y)\,dx = xy(1+y)\,dy$; $y(1)=0$

=> Given that,

$$(x^2+1)(1-y)\,dx = xy(1+y)\,dy$$

After separating variables we get,

$$\frac{x^2+1}{x}\,dx = \frac{y(1+y)}{(1-y)}\,dy$$

Integrating both sides we get,

$$\int \frac{x^2+1}{x}\,dx = \int \frac{y(1+y)}{(1-y)}\,dy$$

$$\Rightarrow \int x\,dx + \int \frac{1}{x}\,dx = \int y\,dy + \int \frac{1+y}{1-y}\,dy$$

$$\Rightarrow \frac{x^2}{2} + \ln x = \frac{y^2}{2} - y - 2\ln(|y-1|) + c$$

$$C = \frac{x^2}{2} + \ln x - \frac{y^2}{2} + y + 2\ln(|y-1|) \quad \text{------ (i)}$$

which is the required general solution.

Using $y=0$ and $x=1$, in equation (i) we get,

$$C = \frac{1}{2} + \ln 1 - 0 + 0 + 2\ln(-1)$$

$$= \frac{1}{2}$$

$$\therefore\ x^2 + 2\ln x - y^2 + 2y + 4\ln(|y-1|) = 1$$

which is the required particular solution.

$$\Rightarrow -\int \frac{y+1}{y-1}\,dy$$

$$\Rightarrow -\int \frac{u+2}{u}\,du \quad \left| \begin{array}{l} u = y-1 \\ du = dy \end{array} \right.$$

$$\Rightarrow \int 1\,du - 2\int \frac{1}{u}\,du$$

$$\Rightarrow -u - 2\ln u$$

$$\Rightarrow -y+1 - 2\ln(y-1)$$

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Homogenious ~~Equation~~ Function:

A function $f(x,y)$ is said to be homogenious of degree n if $f(\lambda x, \lambda y) = \lambda^n f(x,y)$ or $g(x,y) = x^n h(y/x)$.

Homogenious Equation:

An equation of the form $Mdx + Ndy = 0$ is called a homogenious equation of M and N are homogenious function of the same degree.

Example: $(x^2+y^2)dx + 2xy\,dy = 0$

$M = x^2+y^2 \qquad N = 2xy$

Theorem:

C.I. If $Mdx + Ndy = 0$ is a homogenious equation then the transformation $y = vx$ ~~red~~ reduces to a separable equation.

Proof:

Since $Mdx + Ndy = 0$ is a homogenious equation is the form

$$\frac{dy}{dx} = g\left(\frac{y}{x}\right) \quad \text{——— (i)}$$

Let $y = vx$ ——— (ii)

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differentiating with respect to x we ~~gt~~ get from equation (ii),

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{———} \quad (iii)$$

putting (iii) and (ii), in (i), we get,

$$v + x\frac{dv}{dx} = g(v)$$

$$\Rightarrow x\frac{dv}{dx} = g(v) - v$$

$$\Rightarrow \frac{dv}{dx} = \frac{g(v) - v}{x}$$

$$\Rightarrow \frac{dv}{g(v) - v} = \frac{dx}{x}$$

$$\Rightarrow -\frac{dv}{v - g(v)} - \frac{dx}{x} = 0$$

$$\Rightarrow \frac{dx}{x} + \frac{dv}{v - g(v)} = 0$$

which is a separable equation.

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$(x^3+y^3)dx - 3xy^2dy = 0$ —— (i) is a homogeneous equation

$y(2) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{x^3+y^3}{3xy^2} \quad \text{—— (ii)}$$

Let $y = vx$ —— (iii)

differentiating with respect to x we get from equation (iii),

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (iv)}$$

putting equation (iii) and (iv) in equation (ii),

$$v + x\frac{dv}{dx} = \frac{x^3 + v^3x^3}{3x \cdot v^2x^2} = \frac{x^3(1+v^3)}{3v^2x^3}$$

$$= \frac{1+v^3}{3v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^3}{3v^2} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^3-3v^3}{3v^2} = \frac{1-2v^3}{3v^2}$$

$$\Rightarrow \frac{3v^2}{1-2v^3}dv = \frac{dx}{x}$$ This is the separable equation.

Integrating we get,

$$\int \frac{3v^2}{1-2v^3}dv = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\int \frac{-6v^2}{1-2v^3}dv = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\log(1-2v^3) = \log x + \log c$$

$$\Rightarrow -\frac{1}{2}\log(1-2v^3) = \log cx$$

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$$\Rightarrow \frac{1}{2}\log(1-2v^3) = \log(cx)^{-1}$$

$$\Rightarrow \frac{1}{2}\log(1-2v^3) = \log\frac{1}{cx}$$

$$\Rightarrow \log(1-2v^3) = 2\log\frac{1}{cx}$$

$$\Rightarrow \log(1-2v^3) = \log\frac{1}{c^2x^2}$$

$$\Rightarrow 1-2v^3 = \frac{1}{c^2x^2}$$

$$\Rightarrow 1-2\left(\frac{y}{x}\right)^3 = \frac{1}{c^2x^2} \quad [\text{From (iii)}]$$

$$\Rightarrow 1-\frac{2y^3}{x^3} = \frac{1}{c^2x^2}$$

$$\Rightarrow \frac{x^3-2y^3}{x^3} = \frac{1}{c^2x^2}$$

$$\Rightarrow x^3-2y^3 = \frac{x}{c^2} = C_1x$$

which is the general solution

$y(2)=1$ when $y=1$, $x=2$

$$\therefore 2^3-2\cdot1^3 = C_1\cdot 2$$

$$\Rightarrow 8-2 = 2C_1$$

$$\therefore C_1 = 3$$

$$\therefore x^3-2y^3 = 3x$$

which is the required particular solution.

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$(x^2+y^2)dx + 2xy\,dy = 0$ ..... (i) This is a homogeneous equation.

$$\Rightarrow \frac{dy}{dx} = \frac{-(x^2+y^2)}{2xy} \quad \text{..... (ii)}$$

Let, $y = vx$ ..... (iii)

differentiating with respect to x we get from equation (iii),

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{..... (iv)}$$

putting equation (iii) and (iv) in equation (ii),

$$v + x\frac{dv}{dx} = \frac{-x^2 - v^2x^2}{2vx^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-x^2 - v^2x^2 - 2v^2x^2}{2vx^2} = \frac{x^2(-1-3v^2)}{2vx^2}$$

$$= \frac{-1-3v^2}{2v}$$

$$\Rightarrow \frac{2v}{-(1+3v^2)}dv = \frac{dx}{x}$$

Integrating both sides we get,

$$\int \frac{2v}{-(1+3v^2)}dv = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{3}\int \frac{-6v}{-(1+3v^2)}dv = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{3}\log(1+3v^2) = \log x + \log c = \log cx$$

$$\Rightarrow \log(1+3v^2) = -3\log cx = \log\frac{1}{c^3x^3}$$

$$\Rightarrow 1+3v^2 = \frac{1}{c^3x^3}$$

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$\Rightarrow \log\left(x+\sqrt{x^2+a^2}\right)$

$$\Rightarrow 1+3\cdot\frac{y^2}{x^2} = C_1\cdot\frac{1}{x^3}$$

$$\Rightarrow \frac{x^2+3y^2}{x^2} = C_1\cdot\frac{1}{x^2}$$

$$\Rightarrow x^2+3y^2 = C_1\cdot\frac{1}{x}$$

Which is the required general solution.

$\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$ ..... (i) This is a homogeneous equation.

Let, $y = vx$ ..... (ii)

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ ..... (iii)

putting equation (ii) and (iii) in equation (i),

$$v+x\frac{dv}{dx} = \frac{vx+\sqrt{x^2+v^2x^2}}{x} = v+\sqrt{1+v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \sqrt{1+v^2}$$

$$\Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$ This is a separable equation

Integrating both sides we get,

$$\int\frac{dv}{\sqrt{1+v^2}} = \int\frac{dx}{x}$$

$$\Rightarrow \log\left(v+\sqrt{1+v^2}\right) = \log x + \log c = \log cx$$

$$\Rightarrow v+\sqrt{1+v^2} = cx$$

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$$\Rightarrow \frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = cx$$

$$\Rightarrow \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x} = cx$$

$$\Rightarrow y + \sqrt{x^2+y^2} = cx^2$$ This is the general solution

putting $x = 1$, $y = 0$ we have, [দেওয়া আছে]

$$0 + \sqrt{1^2+0} = c \cdot 1^2$$

$$\therefore c = 1$$

$$\therefore y + \sqrt{x^2+y^2} = x^2$$

which is the required particular solution.

Exact differential Equation:

An expression of the form $Mdx + Ndy = 0$ is called an exact differential equation, if there exist a function $F(x, y)$ such that $Mdx + Ndy = dF$.

- If the expression $Mdx + Ndy = 0$ is an exact differential then $Mdx + Ndy = 0$ is called an exact differential equation.

- A necessary and sufficient condition of the exactness of the differential equation $Mdx + Ndy = 0$ is,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

→ এটা হলে Exact

→ না হলে Non-Exact

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$(2x^2+y)\,dx + (x-y)\,dy = 0$

$\Rightarrow$ Here,

$$M = 2x^2+y \quad , \quad N = x-y$$

$$\frac{\partial M}{\partial y} = 1 \quad \left[\text{~~Differen~~ differentiating M with respect to } y\right]$$

$$\frac{\partial N}{\partial x} = 1 \quad \left[\text{" " N " " } x\right]$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ The given expression represents an exact differential equation.

$(x-y)\,dx - (x+y)\,dy = 0$ ------ (i)

$\Rightarrow$ Here,

$$M = x-y \quad , \quad N = -(x+y)$$

$$\frac{\partial M}{\partial y} = -1 \quad , \quad \frac{\partial N}{\partial x} = -1$$

$\therefore$ Differential equation (i) is exact.

Regrouping we get,

$$(x-y)\,dx - (x+y)\,dy = 0$$

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$\Rightarrow x\,dx - y\,dx - x\,dy - y\,dy = 0$

$\Rightarrow x\,dx - (y\,dx + x\,dy) - y\,dy = 0$

$\Rightarrow x\,dx - d(xy) - y\,dy = 0$ $\quad [d(xy) = x\,dy + y\,dx]$

Integrating we get,

$$\int x\,dx - \int d(xy) - \int y\,dy = 0$$

$$\Rightarrow \frac{x^2}{2} - xy - \frac{y^2}{2} = \frac{c}{2}$$

$$\therefore x^2 - 2xy - y^2 = c$$

which is the required general solution.

$(2x\cos y + 3x^2y)\,dx + (x^3 - x^2\sin y - y)\,dy = 0$ ....(i)

when $y(0) = 2$

Here,

$M = 2x\cos y + 3x^2y$ $\qquad N = x^3 - x^2\sin y - y$

$\Rightarrow \frac{\partial M}{\partial y} = -2x\sin y + 3x^2$ $\qquad \Rightarrow \frac{\partial N}{\partial x} = 3x^2 - 2x\sin y$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

so the differential equation is exact.

Regrouping we get,

$$(2x\cos y + 3x^2y)\,dx + (x^3 - x^2\sin y - y)\,dy = 0$$

$$\Rightarrow 2x\cos y\,dx + 3x^2y\,dx + x^3\,dy - x^2\sin y\,dy - y\,dy = 0$$

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$\Rightarrow (2x\cos y\,dx - x^2\sin y\,dy) + (3x^2y\,dx + x^3dy) - y\,dy = 0$

$\Rightarrow d(x^2\cos y) + d(x^3y) - y\,dy = 0$

Integrating we have,

$\int d(x^2\cos y) + \int d(x^3y) - \int y\,dy = 0$

$\Rightarrow x^2\cos y + x^3y - \frac{y^2}{2} = \frac{c}{2}$

$\therefore 2x^2\cos y + 2x^3y - y^2 = c$

This is the required general ~~equation~~ solution.

putting $y=2$ and $x=0$ we have,

$-4 = c$

$\therefore 2x^2\cos y + 2x^3y - y^2 = -4$

This is the required particular solution.

$(xy^2-1)\,dx + (x^2y-1)\,dy = 0$ and $y(1) = 2$

Here, $M = xy^2 - 1$ $\quad N = x^2y - 1$

$\frac{\partial M}{\partial y} = 2xy$ $\quad \frac{\partial N}{\partial x} = 2xy$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ so, the differential equation is exact.

Regrouping we get,

$(xy^2-1)\,dx + (x^2y-1)\,dy = 0$

$\Rightarrow xy^2\,dx - dx + x^2y\,dy - dy = 0$

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$\Rightarrow 2xy^2 dx - 2dx + 2x^2y\,dy - 2dy = 0$

$\Rightarrow 2xy^2 dx + 2x^2y\,dy - 2dx - 2dy = 0$

$\Rightarrow d(x^2y^2) - 2dx - 2dy = 0$

Integrating we have,

$$\int d(x^2y^2) - 2\int dx - 2\int dy = 0$$

$\Rightarrow x^2y^2 - 2x - 2y = c$

This is required general solution.

putting, $x = 1$ and $y = 2$ we have,

$$1^2 \cdot 2^2 - 2 \cdot 1 - 2 \cdot 2 = c$$

$\Rightarrow 4 - 2 - 4 = c$

$\therefore c = -2$

$\therefore x^2y^2 - 2x - 2y = -2$

This is the required particular solution.

C.T

Integrating Factor:

If a non exact equation multiplying by a factor becomes exact, then the factor is called integrating factor.

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Case: 1

if $\frac{M_y - N_x}{N} = f(x)$ then the integrating factor,

$$IF = e^{\int f(x)\,dx}$$

Case: 2

if $\frac{N_x - M_y}{M} = g(y)$ then the integrating factor,

$$IF = e^{\int g(y)\,dy}$$

$y\,dx + (3 + 3x - y)\,dy = 0$ ..... (i)

Here, $M = y$ $\therefore M_y = 1$

$N = 3 + 3x - y$ $\therefore N_x = 3$

$\therefore M_y \neq N_x$

$\therefore$ The differential equation (i) is not exact.

Now, $\frac{N_x - M_y}{M} = \frac{3-1}{y} = \frac{2}{y} = g(y)$

$$\therefore IF = e^{\int 2/y\,dy}$$

$$= e^{2\log y}$$

$$= e^{\log y^2}$$

$$= y^2$$

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Multiplying equation (i) by y^2 we get,

$$y^3dx + 3y^2dy + 3xy^2dy - y^3dy = 0 \quad \cdots \quad (ii)$$

Now the differential equation (ii) is exact.

Regrouping we get,

$$y^3dx + 3y^2dy + 3xy^2dy - y^3dy = 0$$

$$\Rightarrow (y^3dx + 3xy^2dy) + 3y^2dy - y^3dy = 0$$

$$\Rightarrow d(xy^3) + 3y^2dy - y^3dy = 0$$

Integrating we have,

$$\int d(xy^3) + 3\int y^2dy - \int y^3dy = 0$$

$$\Rightarrow xy^3 + \frac{3y^3}{3} - \frac{y^4}{4} = \frac{c}{4}$$

$$\Rightarrow \cancel{4x^3}\ 4xy^3 + 4y^3 - y^4 = c$$

which is the required general solution.

$(2x^2+y)dx + (x^2y - x)dy = 0 \quad \cdots\cdots \quad (i)$

C.T.

Here,

$M = 2x^2 + y \quad \therefore \quad My = 1$

$N = x^2y - x \quad \therefore \quad Nx = 2xy - 1$

$$\left[\begin{array}{l} My = \frac{\partial M}{\partial y} \\ Nx = \frac{\partial N}{\partial x} \end{array}\right]$$

$\therefore$ $My \neq Nx$. So, the differential equation (i) is not exact.

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Now, $\dfrac{My - Nx}{N} = \dfrac{1 - 2xy + 1}{x^2y - x}$

$= \dfrac{-2xy + 2}{x^2y - x}$

$= \dfrac{2(1 - xy)}{-x(1 - xy)}$

$= \dfrac{2}{-x} = f(x)$

$\therefore IF = e^{-\int \frac{2}{x} dx}$

$= e^{-2\log x} = e^{\log x^{-2}} = \dfrac{1}{x^2}$

multiplying equation (i) by $\frac{1}{x^2}$ we get,

$\left(2 + \frac{y}{x^2}\right) dx + \left(\frac{y}{x} - \frac{1}{x}\right) dy = 0$ ---- (ii)

Now the differential equation (ii) is exact.

Regrouping we get,

$2dx + \frac{y}{x^2} dx + y\,dy - \frac{dy}{x} = 0$

$\Rightarrow 2dx + d\left(-\frac{y}{x}\right) + y\,dy = 0$

Integrating we have,

$2\int dx + \int d\left(-\frac{y}{x}\right) + \int y\,dy = 0$

$\Rightarrow 2x - \frac{y}{x} + \frac{y^2}{2} = c$

which is the required general solution.

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1st order linear type equation:

An equation of the ~~factor~~ from ~~$\frac{dy}{dx} + P(x)y$~~ $\frac{dy}{dx} + P(x)y = Q(x)$ is called a 1st order linear differential equation.

Integrating factor, $\boxed{I.F. = e^{\int P(x)\,dx}}$

$xy' + 3y = x^2$ ..... (i)

$\Rightarrow \frac{dy}{dx} + \frac{3y}{x} = x$

This is a 1st order linear differential equation.

$I.F. = e^{\int \frac{3}{x} dx} = e^{3\log x} = e^{\log x^3} = x^3$

Multiplying equation (i) by x^3 and integrating,

$x^4 \frac{dy}{dx} + 3x^3 y = x^5$

$\Rightarrow x^3 \frac{dy}{dx} + 3x^2 y = x^4$

$\Rightarrow x^3 dy + 3x^2 y\, dx = x^4\, dx$

$\Rightarrow d(x^3 y) = x^4\, dx$

$\Rightarrow \int d(x^3 y) = \int x^4\, dx$

$\Rightarrow x^3 y = \frac{x^5}{5} + c$

which is the required general solution.

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$(x^2+1)y' + 4xy = x$ ..... (i)

$\Rightarrow y' + \frac{4x}{x^2+1} y = \frac{x}{x^2+1}$ ..... (ii)

This is a 1st order linear differential equation.

$$I.F. = e^{\int \frac{4x}{1+x^2} dx}$$

$$= e^{2\log(1+x^2)}$$

$$= (1+x^2)^2$$

Multiplying equation (ii) by $(1+x^2)^2$ and integrating,

$$(1+x^2)^2 y' + 4xy(x^2+1) = x(x^2+1)$$

$$\Rightarrow (1+x^2)^2 dy + 4xy(x^2+1)dx = x(x^2+1)dx$$

$$\Rightarrow d\left((1+x^2)^2 y\right) = x(x^2+1)dx$$

$$\Rightarrow \int d\left((1+x^2)^2 y\right) = \int x(x^2+1)dx$$

$$\Rightarrow (1+x^2)^2 y = \int x^3 dx + \int x\, dx$$

$$= \frac{x^4}{4} + \frac{x^2}{2} + c$$

$$\therefore (1+x^2)^2 y = \frac{x^4}{4} + \frac{x^2}{2} + c$$

which is the required general solution.

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Bernoulli Differential Equation:

An equation of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$ is called ~~a~~ Bernoulli's equation.

For 1st order non linear equation

How to transform Bernoulli's equation to 1st order linear differential equation.

Here,

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \quad (\text{when } n \neq 0, 1) \quad \ldots\ldots (i)$$

Dividing both sides by y^n we get,

$$y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x) \quad \ldots\ldots (ii)$$

Let, $y^{1-n} = v \quad \ldots\ldots (iii)$

$$\Rightarrow (1-n)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow y^{-n}\frac{dy}{dx} = \frac{dv}{dx}\cdot\frac{1}{1-n} \quad \ldots\ldots (iv)$$

putting (iii) and (iv) in equation (ii),

$$\frac{dv}{dx}\cdot\frac{1}{1-n} + vP(x) = Q(x)$$

$$\Rightarrow \frac{dv}{dx} + vP(x)(1-n) = Q(x)(1-n)$$

$\frac{dy}{dx} - y = xy^5 \quad \ldots\ldots (i)$

$\Rightarrow$ This is a Bernoulli's differential equation.

Now ~~dif~~ dividing both sides by y^5,

$$y^{-5}\frac{dy}{dx} - y^{-4} = x \quad \ldots\ldots (ii)$$

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Let, $v = y^{-4}$ ..... (iii)

$\Rightarrow \frac{dv}{dx} = -4y^{-5}\frac{dy}{dx}$

$\Rightarrow y^{-5}\frac{dy}{dx} = -\frac{1}{4}\cdot\frac{dv}{dx}$ ..... (iv)

putting (iii) and (iv) in equation (ii),

$-\frac{1}{4}\cdot\frac{dv}{dx} - v = x$

$\Rightarrow \frac{dv}{dx} + 4v = -4x$ ..... (v)

This is a 1st order linear differential equation.

$\therefore I.F. = e^{\int 4dx}$

$= e^{4x}$

Multiplying equation (v) by e^{4x} and integrating,

$e^{4x}\frac{dv}{dx} + 4ve^{4x} = -4xe^{4x}$

$\Rightarrow e^{4x}dv + 4ve^{4x}dx = -4xe^{4x}dx$

$\Rightarrow d(e^{4x}v) = -4xe^{4x}dx$

$\Rightarrow \int d(e^{4x}v) = -4\int xe^{4x}dx$

$\Rightarrow e^{4x}v = -4\left[x\int e^{4x}dx - \int\left(\frac{d}{dx}(x)\int e^{4x}dx\right)dx\right]$

$\cancel{= -4\left[\frac{xe^{4x}}{4} + \frac{xe^{4x}}{4}\right] + c}$

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SUBJECT: $\int uv\,dx \to u\int v\,dx - \int\left(\frac{d}{dx}(u)\int v\,dx\right)dx$

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$$\Rightarrow e^{4x}v = -4\left[x\cdot\frac{e^{4x}}{4} - \int \frac{1}{4}e^{4x}\,dx\right]$$

$$= -4\left[\frac{xe^{4x}}{4} - \frac{1}{16}e^{4x}\right] + c$$

$$= -xe^{4x} + \frac{1}{4}e^{4x} + c$$

$$\therefore v = -x + \frac{1}{4} + c\cdot\frac{1}{e^{4x}}.$$

$$\therefore y^{-4} = -x + \frac{1}{4} + ce^{-4x}$$

which is the required general solution.

$y' + xy = xy^3$ ..... (i)

This is a Bernoulli's differential equation.

Now dividing both sides by y^3,

$$y^{-3}\frac{dy}{dx} + xy^{-2} = x \quad \text{.....(ii)}$$

Let, $v = y^{-2}$ ..... (iii)

$$\Rightarrow \frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$$

$$\therefore y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx} \quad \text{.....(iv)}$$

Putting (iii) and (iv) in equation (ii),

$$-\frac{1}{2}\frac{dv}{dx} + xv = x$$

$$\Rightarrow \frac{dv}{dx} - 2xv = -2x \quad \text{.....(v)}$$

This is a 1st order linear differential equation.

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$\int x e^{-x^2} dx$

$\therefore I.F. = e^{\int -2x\, dx}$

$= e^{-2\frac{x^2}{2}}$

$= e^{-x^2}$

Multiplying equation (V) by e^{-x^2} and integrating

$$e^{-x^2}\frac{dV}{dx} - 2xVe^{-x^2} = -2xe^{-x^2}$$

$$\Rightarrow e^{-x^2}dV - 2xVe^{-x^2}dx = -2xe^{-x^2}dx$$

$$\Rightarrow d(Ve^{-x^2}) = -2xe^{-x^2}dx$$

$$\Rightarrow \int d(Ve^{-x^2}) = -2\int xe^{-x^2}dx$$

$$\Rightarrow Ve^{-x^2} = -2\left(-\frac{e^{-x^2}}{2} + c\right)$$

$$= e^{-x^2} + c$$

$$\Rightarrow V = 1 + ce^{x^2}$$

$$\therefore y^{-2} = 1 + ce^{x^2}$$

which is the required general solution.

$\frac{dy}{dx} = y(xy^2 - 1)$

$$\Rightarrow \frac{dy}{dx} + y = xy^3 \quad \text{------ (i)}$$

This is a Bernoulli's differential equation.

Now dividing both sides by y^3,

$$y^{-3}\frac{dy}{dx} + y^{-2} = x \quad \text{---- (ii)}$$

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$\int x e^{-2x} dx \rightarrow -\frac{(2x+1)e^{-2x}}{4} + c$

Let, $v = y^{-2}$ ..... (iii)

$\Rightarrow \frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$

$\Rightarrow y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}$ ..... (iv)

putting (iii) and (iv) in equation (ii),

$-\frac{1}{2}\frac{dv}{dx} + v = x$

$\Rightarrow \frac{dv}{dx} - 2v = -2x$ ..... (v)

This is a 1st order linear differential equation.

$\therefore I.F. = e^{\int -2\,dx}$

$= e^{-2x}$

multiplying equation (v) by e^{-2x} and integrating,

$e^{-2x}\frac{dv}{dx} - 2ve^{-2x} = -2xe^{-2x}$

$\Rightarrow e^{-2x}dv - 2ve^{-2x}dx = -2xe^{-2x}dx$

$\Rightarrow d(ve^{-2x}) = -2xe^{-2x}dx$

$\Rightarrow \int d(ve^{-2x}) = -2\int xe^{-2x}dx$

$\Rightarrow ve^{-2x} = -2\left[-\frac{(2x+1)e^{-2x}}{4}\right] + c$

$= \frac{(2x+1)e^{-2x}}{2} + c$

$\Rightarrow v = \frac{(2x+1)}{2} + ce^{2x}$

$= \left(x + \frac{1}{2}\right) + ce^{2x}$

$\therefore y^{-2} = \left(x + \frac{1}{2}\right) + ce^{2x}$

which is the required general solution.

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$\frac{dy}{dx} - \frac{y}{x} = xy^2$ ..... (i)

$\Rightarrow$ This is a Bernoulli's differential equation.

Now dividing both sides by y^2 we have,

$$y^{-2}\frac{dy}{dx} - \frac{1}{x}\cdot y^{-1} = x \quad \text{.....(ii)}$$

Let, $y^{-1} = v$ ..... (iii)

$$\Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow y^{-2}\frac{dy}{dx} = -\frac{dv}{dx} \quad \text{.....(iv)}$$

putting equation (iii) and (iv) in equation (ii),

$$-\frac{dv}{dx} - \frac{v}{x} = x$$

$$\Rightarrow \frac{dv}{dx} + \frac{v}{x} = -x \quad \text{.....(v)}$$

This is a 1st order linear differential equation.

$$\therefore I.F. = e^{\int 1/x\, dx}$$

$$= e^{\log x}$$

$$= x$$

Multiplying equation (v) by x and integrating,

$$x\frac{dv}{dx} + v = -x^2$$

$$\Rightarrow x\,dv + v\,dx = -x^2\,dx$$

$$\Rightarrow d(xv) = -x^2\,dx$$

$$\Rightarrow \int dr(xv) = -\int x^2\,dx$$

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$\Rightarrow xv = -\frac{x^3}{3} + c$

$\Rightarrow y^{-1} = -\frac{x^2}{3} + \frac{c}{x}$

$\therefore \frac{1}{y} = -\frac{x^2}{3} + \frac{c}{x}$

Which is the required general solution.

$\frac{dy}{dx} = x^3y^3 - xy$

$\Rightarrow \frac{dy}{dx} + xy = x^3y^3$ ..... (i)

This is a Bernoulli's differential equation.

Now dividing both sides by y^3 we have,

$y^{-3}\frac{dy}{dx} + xy^{-2} = x^3$ ..... (ii)

Let, $y^{-2} = v$ ..... (iii)

$\Rightarrow -2y^{-3}\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}$ ..... (iv)

putting (iii) and (iv) in equation (ii),

$-\frac{1}{2}\frac{dv}{dx} + xv = x^3$

$\therefore \frac{dv}{dx} - 2xv = -2x^3$ ..... (v)

This is a 1st order linear differential equation.

$\therefore I.F. = e^{\int -2x\,dx}$

$= e^{-2\cdot\frac{x^2}{2}}$

$= e^{-x^2}$

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SUBJECT: $\int x^3 e^{-x^2} dx \to -\frac{(x^2+1)e^{-x^2}}{2}$ NO +e DATE / / SAT SUN MON TUE WED THU FRI

Multiplying equation (v) by e^{-x^2} and integrating,

$$e^{-x^2}\frac{dv}{dx} - 2vxe^{-x^2} = -2x^3e^{-x^2}$$

$$\Rightarrow e^{-x^2}dv - 2vxe^{-x^2}dx = -2x^3e^{-x^2}dx$$

$$\Rightarrow d(ve^{-x^2}) = -2x^3e^{-x^2}dx$$

$$\Rightarrow \int d(ve^{-x^2}) = -2\int x^3e^{-x^2}dx$$

$$\Rightarrow ve^{-x^2} = -2\left[\frac{-(x^2+1)e^{-x^2}}{2}\right] + c$$

$$= (x^2+1)e^{-x^2} + c$$

$$\Rightarrow v = x^2+1+ce^{x^2}$$

$$\therefore \frac{1}{y^2} = x^2+1+ce^{x^2}$$

which is the required general solution.

Higher order Homogeneous equation:

- If an equation $=0$ $\to$ then it is homogeneous.
- If an equation $\neq 0$ or equation $= 12\sin x$ or equation $= 10x - 11x^2$ etc $\to$ then it is non-homogeneous.

$y'' + 7y' + 12y = 0$ ..... (i)

This is a higher order homogeneous differential equation.

Let, $y = e^{mx}$ be the trial solution of equation (i),

$$\Rightarrow y' = me^{mx}$$

$$\Rightarrow y'' = m^2e^{mx}$$

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putting the value of y'', y'', y' in equation (i),

$$m^2 e^{mx} + 7m e^{mx} + 12 e^{mx} = 0$$

$$\Rightarrow m^2 + 7m + 12 = 0$$

$$\Rightarrow m^2 + 4m + 3m + 12 = 0$$

$$\Rightarrow m(m+4) + 3(m+4) = 0$$

$$\Rightarrow (m+3)(m+4) = 0$$

$\therefore m = -3, -4$. [roots are not equal]

NOTE:

when roots are:

unequal: $C_1 e^{a_1 x} + C_2 e^{a_2 x} + C_3 e^{a_3 x} + \dots = y$

↳ according to order.

equal: $(C_1 + x C_2 + x_2 C_3) e^{ax} = y$

a = root.

imaginary:

Real x = Real parts co-efficient of root $\times x$

imaginary x = imaginary parts co-efficient of root $\times x$

$\therefore y = e^{\text{Real } x} \{ C_1 \cos(\text{imaginary } x) \times C_2 \sin(\text{imaginary } x) \}$

Hence the general solution is,

$$y = C_1 e^{-3x} + C_2 e^{-4x}$$ [as there are two roots]

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$y'' - 4y' + 4y = 0$ ..... (i)

This is a higher order homogeneous differential equation.

Let, $y = e^{mx}$ be the trial solution of equation (i),

$\Rightarrow y' = m e^{mx}$

$\Rightarrow y'' = m^2 e^{mx}$

Then the ~~aux~~ auxiliary equation is,

$m^2 e^{mx} - 4m e^{mx} + 4e^{mx} = 0$

$\Rightarrow m^2 - 4m + 4 = 0$

$\Rightarrow m^2 - 2m - 2m + 4 = 0$

$\Rightarrow m(m-2) - 2(m-2) = 0$

$\Rightarrow (m-2)(m-2) = 0$

$\therefore m = 2, 2$ [roots are equal]

Hence the general solution is,

$y = (c_1 + x c_2) e^{2x}$.

$y'' + 2y' + 2y = 0$ ..... (i)

This is a higher order homogeneous differential equation.

Let, $y = e^{mx}$ be the trail solution of equation (i),

$\Rightarrow y' = m e^{mx}$

$\Rightarrow y'' = m^2 e^{mx}$

Then the auxiliary equation is,

$m^2 e^{mx} + 2m e^{mx} + 2e^{mx} = 0$

$\Rightarrow m^2 + 2m + 2 = 0$

$\Rightarrow m = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$

$\therefore m = -1 \pm i$

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Hence the general solution is:

$$y = e^{-x}\{C_1 \cos x + C_2 \sin x\}$$

High order (2nd order) non-homogeneous equations:

$$\underbrace{y'' - 7y' + 12y}_{} = \underbrace{e^{3x}}_{}$$

→ solution of this part is particular integral (P.I.) or y_p

→ solution of this part is complemantary function (C.F) or y_c

General solution (G.S) = CF + PI

$$\therefore y = y_c + y_p$$

Method of undetermined co-efficient:

x	y_p
Algabric: x^2	$\longrightarrow A_0 + A_1 x + A_2 x^2$
exponential: e^{mx}	$\longrightarrow A e^{mx}$
trigonometric: $\sin x$, $\cos x$	$\longrightarrow A \sin x + B \cos x$
Algabric + exponential: $x^2 e^{mx}$	$\longrightarrow (A_0 + A_1 x + A_2 x^2) e^{mx}$

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$y'' + 4y' + 9y = x^2 + 3x$, solve the differential equation by the method of undetermined co-efficient.

$\Rightarrow y'' + 4y' + 9y = x^2 + 3x$ ..... (i)

Let, $y = e^{mx}$ be the trial solution of $y'' + 4y' + 9y = 0$

Then the auxiliary equation is,

$m^2 e^{mx} + 4m e^{mx} + 9e^{mx} = 0$

$\Rightarrow m^2 + 4m + 9 = 0$

$\Rightarrow m = \dfrac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 9}}{2}$

$= \dfrac{-4 \pm \sqrt{16 - 36}}{2}$

$= \dfrac{-4 \pm \sqrt{-20}}{2}$

$= \dfrac{-4 \pm 2\sqrt{5}\, i}{2}$

$= -2 \pm \sqrt{5}\, i$

Then the complementary solution is,

$y_c = e^{-2x} \{ C_1 \cos\sqrt{5}x + C_2 \cancel{\cos\sqrt{}} \sin\sqrt{5}x \}$

by the method of undetermined co-efficient:

$y_p = A + Bx + Cx^2$ ..... (ii)

$\left.\begin{aligned} \Rightarrow y'_p &= B + 2Cx \\ \Rightarrow y''_p &= 2C \end{aligned}\right\}$ ..... (iii)

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putting the values of equation (ii) and (iii) in equation (i),

$$2C + 4(B + 2Cx) + 9(A + Bx + Cx^2) = x^2 + 3x$$

$$\Rightarrow 2C + 4B + 8Cx + 9A + 9Bx + 9Cx^2 = x^2 + 3x$$

$$\Rightarrow 9Cx^2 + (8C + 9B)x + 9A + 4B + 2C = x^2 + 3x$$

equating co-efficients on both sides we have,

$9C = 1$ $\quad$ $9B + 8C = 3$ $\quad$ $9A + 4B + 2C = 0$

$\therefore C = 1/9$ $\quad$ $\Rightarrow 9B + \frac{8}{9} = 3$ $\quad$ $\Rightarrow 9A + 4 \times \frac{19}{81} + 2 \times \frac{1}{9} = 0$

$\Rightarrow 81B + 8 = 27$

$\therefore B = \frac{19}{81}$ $\quad$ $\therefore A = -\frac{94}{729}$

Therefore the particular integral is:

$$y_p = -\frac{94}{729} + \frac{19}{81}x + \frac{1}{9}x^2$$

Hence the general solution is,

$$y = y_c + y_p$$

$$= e^{-2x}(c_1 \cos\sqrt{5}x + c_2 \sin\sqrt{5}x) - \frac{94}{729} + \frac{19}{81}x + \frac{1}{9}x^2$$

where c_1 and c_2 are arbitrary constant. //

$y'' + 4y = xe^{2x}$, solve it by the method of undetermined Co-efficient.

$\Rightarrow y'' + 4y = xe^{2x}$ ..... (i)

Let, $y = e^{mx}$ be the trial solution of $y'' + 4y = 0$

$\Rightarrow y' = me^{mx}$

$\Rightarrow y'' = m^2 e^{mx}$

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Then the auxiliary equation is,

$$m^2 e^{mx} + 4e^{mx} = 0$$

$$\Rightarrow m^2 + 4 = 0$$

$$\therefore m = \pm 2i$$

Then the complementary function is,

$$y_c = C_1 \cos 2x + C_2 \sin 2x$$

By the method of undetermined co-efficient,

$$y_p = (A + Bx)e^{2x} \quad \ldots\ldots (ii)$$

$$\Rightarrow y'_p = Be^{2x} + 2(A+Bx)e^{2x}$$

$$\Rightarrow y''_p = 2Be^{2x} + 2Be^{2x} + 4(A+Bx)e^{2x}$$

$$= 4Be^{2x} + 4(A+Bx)e^{2x} \quad \ldots\ldots (iii)$$

putting (ii), (iii) in equation (i),

$$4Be^{2x} + 4(A+Bx)e^{2x} + 4(A+Bx)e^{2x} = xe^{2x}$$

$$\Rightarrow 8Ae^{2x} + 4Be^{2x} + 8Bxe^{2x} = xe^{2x}$$

equating co-efficient on both sides we have,

$$8B = 1 \qquad 8A + 4B = 0$$

$$\therefore B = \frac{1}{8} \qquad \Rightarrow 8A + \frac{4}{8} = 0$$

$$\Rightarrow 16A + 1 = 0$$

$$\therefore A = -\frac{1}{16}$$

$$\therefore y_p = \left(-\frac{1}{16} + \frac{1}{8}x\right)e^{2x}$$

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Hence the general solution is,

$$y = y_c + y_p$$

$$= C_1 \cos 2x + C_2 \sin 2x + \left(-\frac{1}{16} + \frac{1}{8}x\right)e^{2x}$$

where C_1 and C_2 are arbitrary constant.

2 set [illegible] .....

Method of Variation of Parameter

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = f(x) \quad \text{.....} \quad (i)$$

$$y_c = C_2 y_2(x) + C_1 y_1(x)$$

$$y_p = V_1(x)\, y_1(x) + V_2(x)\, y_2(x) \quad \text{.....} \quad (ii)$$

$$y_p' = V_1 y_1' + V_1' y_1 + V_2 y_2' + V_2' y_2$$

Let,

$$V_1' y_1 + V_2' y_2 = 0 \quad \text{.....} \quad (iii)$$

$$\therefore \; y_p' = V_1 y_1' + V_2 y_2'$$

$$\therefore \; y_p'' = V_1 y_1'' + V_1' y_1' + V_2 y_2'' + V_2' y_2' \quad \text{....} \quad (iv)$$

putting (ii) and (iv) in equation (i),

$$V_1' y_1' + V_2' y_2' = \frac{f(x)}{a_0(x)} \quad \text{.....} \quad (v)$$

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From (iii) and (V),

$$V_1' = \frac{\begin{vmatrix} 0 & y_2 \\ \frac{f(x)}{a_0(x)} & y_2' \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}} \qquad V_2' = \frac{\begin{vmatrix} y_1 & 0 \\ x' & \frac{f(x)}{a_0(x)} \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}}$$

Solve $y'' + y = \tan x$ by the method of variation of parameter.

$\Rightarrow$ Given that,

$$y'' + y = \tan x \quad \text{------} \quad (i)$$

Let, $y = e^{mx}$ be the ~~trail~~ trial solution of $y'' + y = 0$

Then the auxiliary equation is $m^2 + 1 = 0$

$$\Rightarrow m = \pm i$$

Then the complementary function is,

$$y_c = c_1 \cos x + c_2 \sin x.$$

By the method of variation of parameter we let,

$$y_p = V_1 \cos x + V_2 \sin x \quad \cdots\cdots (ii)$$

where V_1 and V_2 are such that

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$$\cancel{\left.\begin{aligned} V_1' \cos x + V_2' \sin x &= 0 \\ -V_1' \sin x + V_2' \cos x &= 0 \end{aligned}\right\} \cdots\cdots (iii)}$$

$$\cancel{\rightarrow V_1' \sin x + V_2' \cos x =}$$

$$\left.\begin{aligned} V_1' \cos x + V_2' \sin x &= 0 \\ -V_1' \sin x + V_2' \cos x &= \frac{\tan x}{1} \end{aligned}\right\} \cdots\cdots (iii)$$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

solving ~~equ~~ (iii) with cramer's rule we get,

$$V_1' = \frac{\begin{vmatrix} 0 & \sin x \\ \tan x & \cos x \end{vmatrix}}{\Delta} = \frac{-\sin x \tan x}{1}$$

$$= -\frac{\sin^2 x}{\cos x} = \frac{-(1-\cos^2 x)}{\cos x}$$

$$= -\left(\frac{1}{\cos x} - \cos x\right)$$

$$= \cos x - \sec x$$

Integrating, we have

$$V_1 = \sin x - \ln(\tan x + \sec x)$$

$$V_2' = \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix}}{\Delta} = \frac{\cos x \tan x}{1} = \sin x$$

Integrating, we have

$$V_2 = -\cos x$$

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$\therefore y_p = \sin x \cos x - \ln(\tan x + \sec x)\cos x - \sin x \cos x$

$= -\cos x \ln(\tan x + \sec x)$

Hence the general solution is,

$y = y_c + y_p$

$= C_1 \cos x + c_2 \sin x - \cos x \ln(\tan x + \sec x)$

where c_1 and c_2 are arbitrary constant.

solve $y'' + y = \sec x$ by the method of variation of parameter.

$\Rightarrow$ Given that,

$y'' + y = \sec x$ ..... (i)

Let,

$y = e^{mx}$ be the trial solution of $y'' + y = 0$

Then the auxiliary equation is $m^2 + 1 = 0$

$\therefore m = \pm i$

Then the complementary function is,

$y_c = C_1 \cos x + c_2 \sin x$

by the method of variation of parameter we get,

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$$y_p = V_1 \cos x + V_2 \sin x \quad \ldots\ldots (ii)$$

$$\left.\begin{aligned} V_1' \cos x + V_2' \sin x &= 0 \\ -V_1' \sin x + V_2' \cos x &= \frac{\sec x}{1} \end{aligned}\right\} \ldots\ldots (iii)$$

$$\Delta = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$V_1' = \frac{\begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix}}{\Delta} = -\sec x \sin x = -\tan x$$

$$\therefore V_1 = \ln(\cos x)$$

$$V_2' = \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix}}{\Delta} = \cos x \sec x = 1$$

$$\therefore V_2 = x$$

$$\therefore y_p = \ln(\cos x) \cos x + x \sin x$$

Hence the general solution is,

$$y = y_c + y_p$$
$$= c_1 \cos x + c_2 \sin x + \ln(\cos x) \cos x + x \sin x$$

where c_1 and c_2 are arbitrary constant.

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Operator Method / Short Method:

$D = \frac{d}{dx}$ $D^2 = \frac{d^2}{dx^2}$ $\frac{1}{D} = \int$ Integral operator ①

$\frac{1}{D^2} = \int$ Integral operator ②

$(D^2 - 3D + 2)y = x^2$ solve it by short method.

$\Rightarrow$ Given that,

$$(D^2 - 3D + 2)y = x^2 \text{ ..... (i)}$$

Let, $y = e^{mx}$ be the trail solution of

$$(D^2 - 3D + 2)y = 0$$

Then the auxiliary equation is, $m^2 - 3m + 2 = 0$

$\Rightarrow m^2 - 2m - m + 2 = 0$

$\Rightarrow m(m-2) - 1(m-2) = 0$

$\therefore m = 1, 2.$

The complementary equation is,

$$y_c = C_1 e^x + C_2 e^{2x}$$

By the operator method, we have,

$$y_p = \frac{x^2}{(D^2 - 3D + 2)}$$

$$= \frac{x^2}{2\left(\frac{D^2}{2} - \frac{3D}{2} + 1\right)}$$

$$= \frac{x^2}{2\left\{1 - \left(\frac{3D}{2} - \frac{D^2}{2}\right)\right\}}$$

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$$= \frac{1}{2}\left\{1-\left(\frac{3D}{2}-\frac{D^2}{2}\right)\right\}^{-1} x^2$$

$$= \frac{1}{2}\left\{1+\left(\frac{3D}{2}-\frac{D^2}{2}\right)+\left(\frac{3}{2}D-\frac{D^2}{2}\right)^2+\cdots\right\} x^2$$

$$= \frac{1}{2}\left\{1+\frac{3D}{2}-\frac{D^2}{2}+\frac{9D^2}{4}+\cdots\right\} x^2$$

$$= \frac{1}{2}\left\{x^2+\frac{3}{2}\cdot 2x-\frac{1}{2}\cdot 2+\frac{9}{4}\cdot 2\right\}$$

$$= \frac{1}{2}\left\{x^2+3x-1+\frac{9}{2}\right.$$

$$= \frac{1}{2}\left(x^2+3x+\frac{7}{2}\right)$$

Hence the general solution,

$$y = y_c + y_p$$

$$= c_1 e^{x} + c_2 e^{2x} + \frac{1}{2}\left(x^2+3x+\frac{7}{2}\right)$$

where c_1 and c_2 are arbitrary constant.

Formulas for exponential function : (operator method)

i) $\frac{1}{f(D)} e^{mx} = \frac{1}{f(m)} e^{mx}$ if $f(m) \neq 0$

ii) $\frac{1}{f(D)} e^{mx} = e^{mx} \frac{1}{f(D+m)} \cdot 1$ if $f(m) = 0$

iii) $\frac{1}{f(D)}\left(e^{mx} t(x)\right) = e^{mx} \frac{1}{f(D+m)} t(x)$

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$(D^2+9)y = e^{3x}$ solve it by operator method.

$\Rightarrow$ Given that,

$$(D^2+9)y = e^{3x} \quad \ldots\ldots (i)$$

Let,

$y = e^{mx}$ be the trial solution of $(D^2+9)y = 0$

Then the auxiliary equation is $m^2+9 = 0$

$$\therefore m = \pm 3i$$

$\therefore$ The complementary equation,

$$y_c = C_1 \cos 3x + C_2 \sin 3x$$

By the operator method, we have

$$y_p = \frac{e^{3x}}{D^2+9}$$

$$= \frac{e^{3x}}{3^2+9}$$

$$= \frac{1}{18} e^{3x}$$

Hence the general solution,

$$y = y_c + y_p$$

$$= C_1 \cos 3x + C_2 \sin 3x + \frac{1}{18} e^{3x}$$

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$(D^2-4)y = e^{-2x}$ solve it by operator method.

$\Rightarrow$ Given that,

$$(D^2-4)y = e^{-2x} \quad \text{.....} (i)$$

Let, $y = e^{mx}$ be the trial solution of $(D^2-4)y = 0$

$$\therefore m^2 - 4 = 0$$

$$\therefore m = 2$$

$\therefore$ The complementary equation,

$$y_c = c_1 \cos 2x + c_2 \sin 2x$$

By the operator method, we have

$$y_p = \frac{e^{-2x}}{D^2-4}$$

$$= e^{-2x} \cdot \frac{1}{(D-2)^2-4} \cdot 1$$

$$= e^{-2x} \cdot \frac{1}{D^2-4D+4-4} \cdot 1$$

$$= e^{-2x} \frac{1}{D^2-4D} \cdot 1$$

$$= e^{-2x} \frac{1}{-4D(1-D/4)} \cdot 1$$

$$= e^{-2x} \cdot \frac{1}{-4D} (1-D/4)^{-1} \cdot 1$$

$$= \frac{e^{-2x}}{-4D} \left(1 + \frac{D}{4} + \frac{D^2}{16} + \cdots\right) \cdot 1$$

$$= \frac{e^{-2x}}{-4D} (1)$$

$$= -\frac{e^{-2x}}{4} \frac{1}{D} (1) = \frac{-e^{-2x}}{4} x$$

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$\therefore y = y_c + y_p$

$\therefore y = c_1 \cos 2x + c_2 \sin 2x - \frac{x}{4} e^{-2x}$.

$(D^2+4D+4)y = x^3 e^{-2x}$ solve the equation by operator method.

$\Rightarrow$ Given that,

$$(D^2+4D+4)y = x^3 e^{-2x} \quad \text{.....(i)}$$

Let, $y = e^{mx}$ be the trial solution of $(D^2+4D+4)y = 0$

Then the auxiliary equation is $m^2+4m+4 = 0$

$$\Rightarrow m^2 + 2m + 2m + 4 = 0$$

$$\Rightarrow m(m+2) + 2(m+2) = 0$$

$$\therefore m = -2, -2$$

$\therefore$ The complementary equation,

$$y_c = (c_1 + xc_2)e^{-2x}$$

By the operator method,

$$y_p = \frac{1}{D^2+4D+4} x^3 e^{-2x}$$

$$= \frac{1}{(D+2)^2} x^3 e^{-2x}$$

$$= \frac{e^{-2x}}{(D-2+2)^2} x^3$$

$$= e^{-2x} \frac{1}{D^2} x^3$$

$$= e^{-2x} \frac{1}{D} \cdot \frac{1}{4} x^4$$

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$$= e^{-2x} \frac{1}{20} x^5$$

Hence the general solution,

$$y = y_c + y_p$$
$$= (c_1 + x c_2) e^{-2x} + e^{-2x} \frac{1}{20} x^5$$

where c_1 and c_2 are arbitrary constant.

Partial Differential Equation:

Notation: If $z = f(x, y)$

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}, \quad r = \frac{\partial^2 z}{\partial x^2}$$

$$t = \frac{\partial^2 z}{\partial y^2}, \quad s = \frac{\partial^2 z}{\partial y \partial x}$$

Linear equation: A first order equation $f(x, y, z, p, q) = 0$ is known as linear if it is linear in p, q and z, that is, if given equation is of the form $P(x,y)p + Q(x,y)q = R(x,y)z + S(x,y)$.

For example: $yx^2p + xy^2q = xyz + x^2y^3$ and $p + q = z + xy$ are both first order linear partial differential equations.

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Lect 2

Semi-linear equation: A first order partial differential equation $f(x, y, z, p, q) = 0$ is known as a semi-linear equation, if it is linear in p and q and the coefficients of p and q are functions of ~~x~~ x and y only i.e. if the given equation is of the form $P(x,y)p + Q(x,y)q = R(x, y, z)$.

For example, $xyp + x^2yq = x^2y^2z^2$ and $yp + xq = (x^2z^2/y^2)$ are both first order semi-linear partial differential equations.

Lect 3

Quasi-linear equation: A first order partial differential equation $f(x, y, z, p, q) = 0$ is known as ~~a semi-linear equation if~~ quasi-linear equation, if it is linear in p and q, i.e., if the given equation is of the form $P(x, y, z)p + Q(x, y, z)q = R(x, y, z)$.

For example, $x^2zp + y^2zp = xy$ and $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ are first order quasi-linear partial differential equations.

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Lect

Non-linear equation: A first order partial differential equation $f(x, y, z, p, q) = 0$ which does not come under the above three types, is known as a non-linear equation. For examples, $p^2+q^2=1$, $pq = z$ and $x^2p^2 + y^2q^2 = z^2$ are all non-linear partial differential equations.

$z = ax + by + cxy$, eliminate arbitrary constant from a, b, c.

$\Rightarrow$ Given that,

$$z = ax + by + cxy \quad \text{.....(i)}$$

Differentiating (i) with respect to x and y we get,

$$\therefore p = \frac{\partial z}{\partial x} = a + cy \quad \text{-----(ii)}$$

$$\therefore q = \frac{\partial z}{\partial y} = b + cx \quad \text{.....(iii)}$$

Again differentiating (ii) and (iii) partially with respect to x and y we get,

$$\therefore \frac{\partial p}{\partial x} = 0$$

$$\Rightarrow \frac{\partial^2 z}{\partial x^2} = 0$$

$$\therefore r = 0$$

$$\frac{\partial q}{\partial y} = 0$$

$$\Rightarrow \frac{\partial^2 z}{\partial y^2} = 0$$

$$\therefore t = 0$$

Again differentiating (ii) and (iii) partially with respect to y and x we get,

$$\therefore \frac{\partial p}{\partial y} = c$$

$$\Rightarrow \frac{\partial z^2}{\partial x \partial y} = c$$

$$\therefore s = c$$

$$\frac{\partial q}{\partial x} = c$$

$$\therefore s = c$$

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From (ii) and (iii) we have,

$$p = a + cy$$

$$\Rightarrow a = p - cy = p - sy$$

and, $q = b + cx$

$$\Rightarrow b = q - cx = q - sx$$

putting the value of equation a, b, c in equation (i)

$$z = (p - sy)x + (q - sx)y + sxy$$

$$= px - sxy + qy - sxy + sxy$$

$$= px + qy - sxy$$

This is the 2nd order 1st degree partial differential equation.

Show that the general solution of the linear partial differential equation $Pp + Qq = R$ is $\rho(u, v) = 0$ where ρ is an arbitrary ~~constant~~ function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ are solution of equation

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$ where P, Q, R are the function of x, y, z.

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→ **Proof:** Let, $u = U(x, y, z)$ and $V = v(x, y, z)$ be two functions of x, y, z connected by the relation $\rho(u, v) = 0$ ..... (i)

Regarding z as a dependent variable and differentiating with respect to x and y.

$$\therefore \frac{\partial \rho}{\partial u}\left(\frac{\partial u}{\partial x} + \frac{\partial z}{\partial x} \cdot \frac{\partial u}{\partial z}\right) + \frac{\partial \rho}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial x}\right) = 0$$

$$\therefore \frac{\partial \rho}{\partial u}\left(\frac{\partial u}{\partial x} + p \cdot \frac{\partial u}{\partial z}\right) + \frac{\partial \rho}{\partial v}\left(\frac{\partial v}{\partial x} + p \cdot \frac{\partial v}{\partial z}\right) = 0 \quad \text{.....(ii)}$$

$$\therefore \frac{\partial \rho}{\partial u}\left(\frac{\partial u}{\partial y} + q \cdot \frac{\partial u}{\partial z}\right) + \frac{\partial \rho}{\partial v}\left(\frac{\partial v}{\partial y} + q \cdot \frac{\partial v}{\partial z}\right) = 0 \quad \text{---(iii)}$$

Now eleminating $\frac{\partial \rho}{\partial u}$ and $\frac{\partial \rho}{\partial v}$ from (ii) and (iii) we have,

$$\begin{vmatrix} \frac{\partial u}{\partial x} + p\frac{\partial x}{\partial z} & \frac{\partial v}{\partial x} + p\frac{\partial v}{\partial z} \\ \frac{\partial x}{\partial y} + q\frac{\partial x}{\partial z} & \frac{\partial v}{\partial y} + q\frac{\partial v}{\partial z} \end{vmatrix} = 0$$

$$= \frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial x} + p\left(\frac{\partial u}{\partial z} \cdot \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial z}\right)$$

$$+ q\left(\frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial z} - \frac{\partial v}{\partial x} \cdot \frac{\partial v}{\partial z}\right) = 0$$

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Writing $\lambda P = \left(\frac{\partial u}{\partial z} \cdot \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial z}\right)$

$$\lambda Q = \left(\frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial z} - \frac{\partial v}{\partial x} \cdot \frac{\partial v}{\partial z}\right)$$

and $-\lambda R = \left(\frac{\partial u}{\partial x} \cdot \frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial x}\right)$

Then the above equation (iv) takes the form

$$-\lambda R + \lambda P p + \lambda Q q = 0$$

$$\therefore P p + Q q = R$$

This is a 1st order partial differential equation linear in p and q.

And we have,

$$\lambda P, \ \lambda Q, \ -\lambda R.$$

where $u = a$ and $v = b$ are two integrals of (i)

Differentiating these integrals we have,

$$\frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz = 0$$

$$\frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz = 0$$

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solving these equations for dx, dy and dz we have,

$$\frac{dx}{\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial z}-\frac{\partial v}{\partial y}\cdot\frac{\partial u}{\partial z}} = \frac{dy}{\frac{\partial u}{\partial z}\cdot\frac{\partial v}{\partial x}-\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial z}}$$

$$= \frac{dz}{\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y}-\frac{\partial v}{\partial x}\cdot\frac{\partial u}{\partial y}}$$

$$\Rightarrow \frac{dx}{\lambda P} = \frac{dy}{\lambda Q} = \frac{dz}{\lambda R}$$

$$\therefore \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \quad \text{------ (V)}$$

This are called Lagrange's auxiliary equation.

if the solution of (V),

$u = a$ and $v = b$ then the solution of the given equation $\phi(u,v) = 0$

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Find the partial differential equation from

$$\rho(x+y+z,\ x^2+y^2+z^2) = 0$$

$\Rightarrow$ Let, $u = x+y+z$, $v = x^2+y^2+z^2$

Then the above equation takes the form $\rho(u,v)=0$

Differentiating with respect to x and y partially, we have,

$$\therefore \frac{\partial \rho}{\partial u}\left(\frac{\partial u}{\partial x} + p\frac{\partial u}{\partial z}\right) + \frac{\partial \rho}{\partial v}\left(\frac{\partial v}{\partial x} + p\frac{\partial v}{\partial z}\right) = 0$$

$$\Rightarrow \frac{\partial \phi}{\partial u}(1+p) + \frac{\partial \phi}{\partial v}(2x - 2pz) = 0 \quad \cdots\cdots (i)$$

and similarly,

$$\frac{\partial \phi}{\partial u}(1+q) + \frac{\partial \phi}{\partial v}(2y - 2qz) = 0 \quad \cdots\cdots (ii)$$

Now eleminating $\frac{\partial \phi}{\partial u}$ and $\frac{\partial \phi}{\partial v}$ from equation (i) and (ii)

$$\begin{vmatrix} 1+p & 2x-2pz \\ 1+q & 2y-2qz \end{vmatrix} = 0$$

$$\Rightarrow (1+p)(2y-2qz) - (1+q)(2x-2pz) = 0$$

$$\Rightarrow (1+p)(y-qz) - (1+q)(x-pz) = 0$$

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$\Rightarrow y + py - qz - pqz - (x + qx - pz - pqz) = 0$

$\Rightarrow p(x+z) - q(x+z) = x - y$

This is a partial differential equation with 1st degree 1st order.

Find the ~~integral~~ integral surface of $x^2p + y^2q + z^2 = 0$, which passes through the hyperbola $xy = x+y$, $z = 1$.

$\Rightarrow$ Given that,

$$x^2p + y^2q + z^2 = 0 \quad \text{.....(i)}$$

$$xy = x + y \quad \text{....(ii)}$$

The Lagrange's auxiliary equation is,

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{-R}$$

$$\Rightarrow \frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{-z^2}$$

from 1st and 3rd term, we have,

$$\frac{dx}{x^2} = \frac{dz}{-z^2}$$

$$\Rightarrow \int \frac{dx}{x^2} = \int \frac{dz}{-z^2}$$

$$= -\frac{1}{x} = \frac{1}{z}$$

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$$\Rightarrow \frac{1}{x} + \frac{1}{z} = c_1$$

$$\Rightarrow \frac{x+z}{xz} = c_1 \quad \text{------ (iii)}$$

From 2nd and 3rd term, we have,

$$\frac{y+z}{yz} = c_2 \quad \text{------ (iv)}$$

Using $z = 1$ in (iii) and (iv) we have,

$$\frac{x+1}{x} = c_1$$

$$\Rightarrow x + 1 = c_1 x$$

$$\Rightarrow 1 = x(c_1 - 1)$$

$$\therefore x = 1/(c_1 - 1) \quad \text{---- (v)}$$

$$\frac{y+1}{y} = c_2$$

$$\therefore y = \frac{1}{(c_2 - 1)} \quad \text{---- (vi)}$$

Using (v) and (vi) in (ii) we get,

$$\frac{1}{(c_1 - 1)} \cdot \frac{1}{(c_2 - 1)} = \frac{1}{(c_1 - 1)} + \frac{1}{c_2 - 1}$$

$$\Rightarrow \frac{1}{c_1 c_2 - c_1 - c_2 + 1} = \frac{c_1 - 1 + c_2 - 1}{c_1 c_2 - c_1 - c_2 + 1}$$

$$\therefore c_1 + c_2 = 3$$

Now adding (iii) and (iv) we have,

$$\frac{x+z}{xz} + \frac{y+z}{yz} = c_1 + c_2 = 3$$

$$\therefore 2xy + (x+y)z = 3xyz$$

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Find the general solution of $yp - xq + x^2 - y^2 = 0$.

$\Rightarrow$ Given that,

$$yp - xq + x^2 - y^2 = 0 \quad \text{---} (i)$$

The Lang Lagranges auxiliary equation is,

$$\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{-(x^2-y^2)}$$

$$\therefore \frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{-(x^2+y^2)}$$

$$\Rightarrow = \frac{y\,dx + x\,dy - dz}{y^2 - x^2 - y^2 + x^2} = \frac{y\,dx + x\,dy - dz}{0}$$

from the last ratio, we have,

$$y\,dx + x\,dy - dz = 0$$

$$\Rightarrow d(xy) - dz = 0$$

integrating,

$$xy - z = c_1$$

again, taking 1st and 2nd ratio, we have,

$$\frac{dx}{y} = \frac{dy}{-x}$$

$$\Rightarrow -x\,dx = y\,dy$$

$$\Rightarrow x\,dx + y\,dy = 0$$

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Integrating,

$$\frac{x^2}{2} + \frac{y^2}{2} = \frac{c_2}{2}$$

$\therefore x^2 + y^2 = c_2$

Hence the general solution of given equation is,

$\phi(xy - z, x^2 + y^2) = 0$ where ϕ is an arbitrary function.

Thus the complete solution is,

$$xyz = a(x^2 + y^2) + b$$

where a and b are arbitrary constant.

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