Sampling Theory and Pulse Code Modulation

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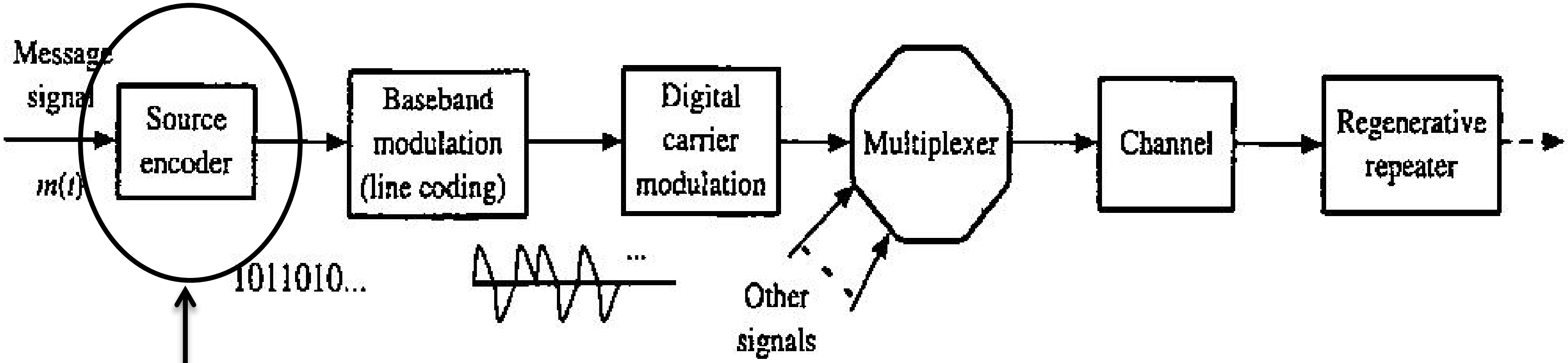
Pulse Modulation: Introduction

- Many Signals in Modern Communication Systems are digital . Also, analog signals are transmitted digitally.
- Reduced distortion and improvement in signal to noise ratios.
- PAM, PWM , PPM , PCM and DM.
- Data transmission, digital transmission, or digital communications is the physical transfer of data (a digital bit stream or a digitized analogue signal) over a point-to-point or point-to multipoint communication channel

Digital Communication System



Analog to Digital Conversion is carried out here

Digital Communication System

Advantages of digital communication systems:

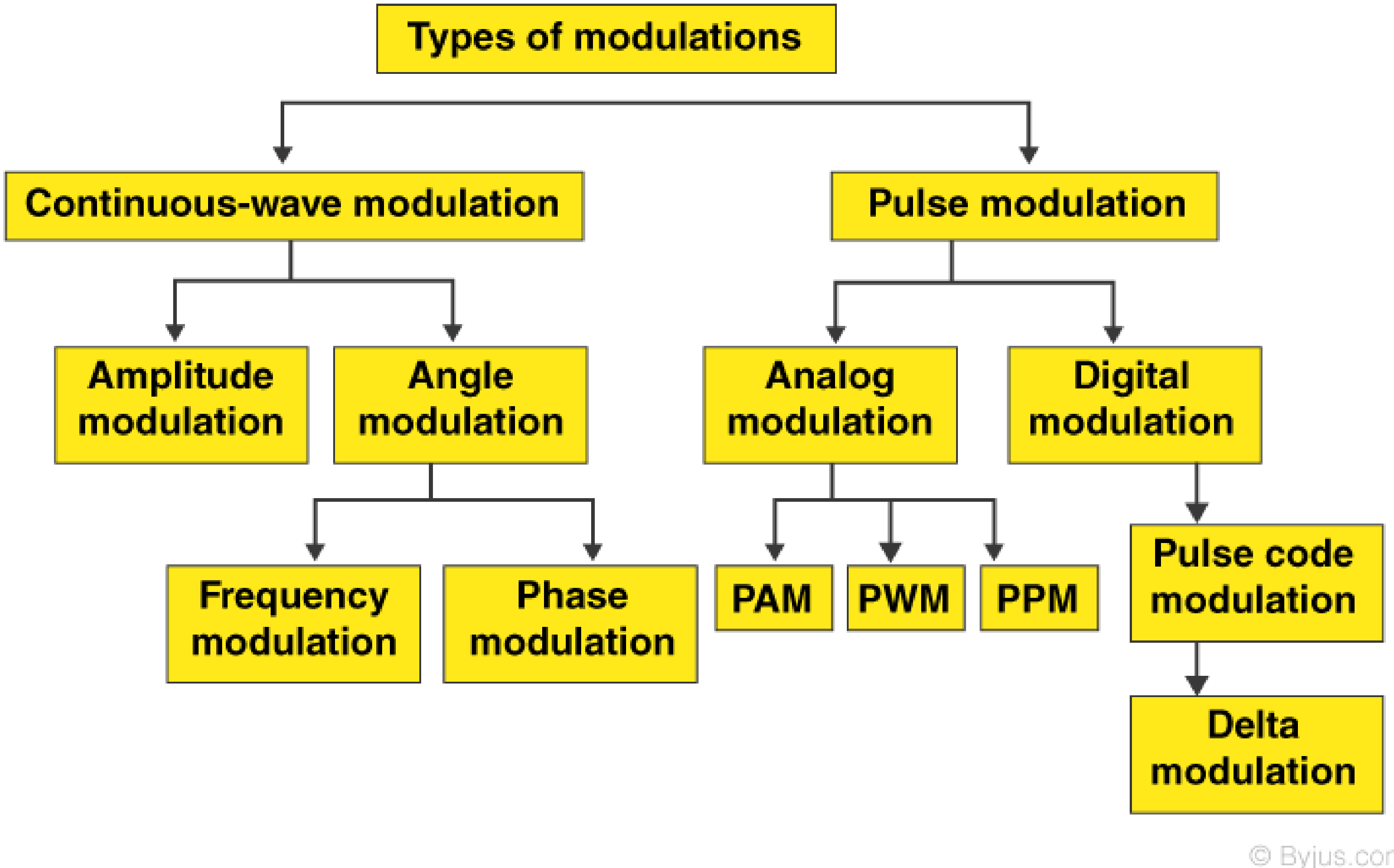
- Easy way of transmission of signals
- Connection of more calls through one channel i.e., Multiplexing is possible using Digital Communication.
- Source Encoding and Channel Encoding can be used to detect errors at the received signal.
- Using repeaters between source and destination, we can reproduce the original signal with less distortions.
- Security is the major advantage of digital communication compared to Analog Communication.
- Transmitting analogue signals digitally allows for greater signal processing capability.
- Digital communication can be done over large distances through internet and other things.
- The messages can be stored in the device for longer times, without being damaged.
- Advancement in communication is achieved through Digital Communication.

Digital Communication System

Disadvantages of digital communication systems:

- Sampling Error
- Digital communications require greater bandwidth than analogue to transmit the same information.
- The detection of digital signals requires the communications system to be synchronized, whereas generally speaking this is not the case with analogue systems.
- Digital signals are often the approximation of voice signals, ie, we don't get the exact analogue signal.

Pulse Modulation: Introduction



Analog Pulse Modulation

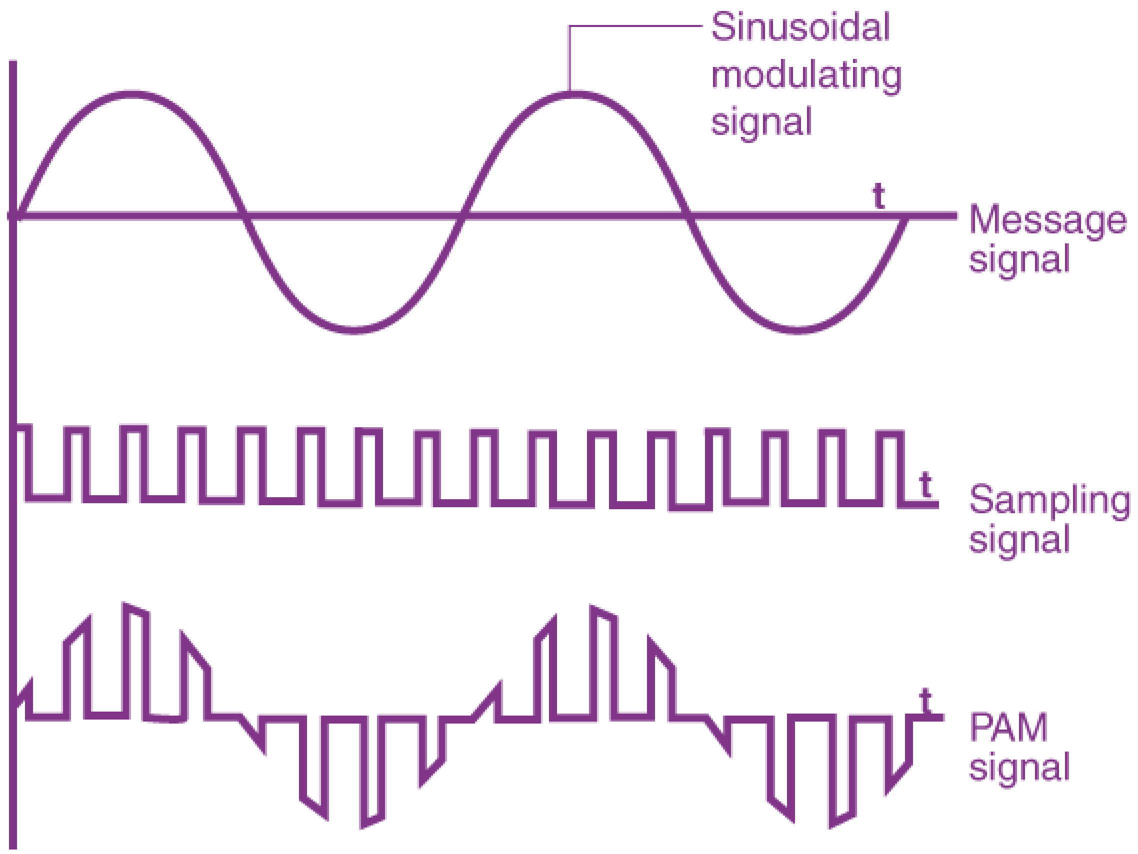
- Analog pulse modulation results when some attribute of a pulse varies continuously in one-to-one
- In analog pulse modulation systems, the amplitude, width, or position of a pulse can vary over a continuous range in accordance with the message amplitude at the sampling instant.

Three types of pulse modulation:

1. Pulse Amplitude Modulation (PAM)
2. Pulse Width Modulation (PWM)
3. Pulse Position Modulation (PPM)

Pulse Amplitude Modulation (PAM)

- In this scheme high frequency carrier (pulse) is varied in accordance with sampled value of message signal



Pulse Amplitude Modulation (PAM)

Advantages of PAM

- Both Modulation and demodulation are simple.
- Easy construction of transmitter and receiver circuits.

Disadvantages of PAM

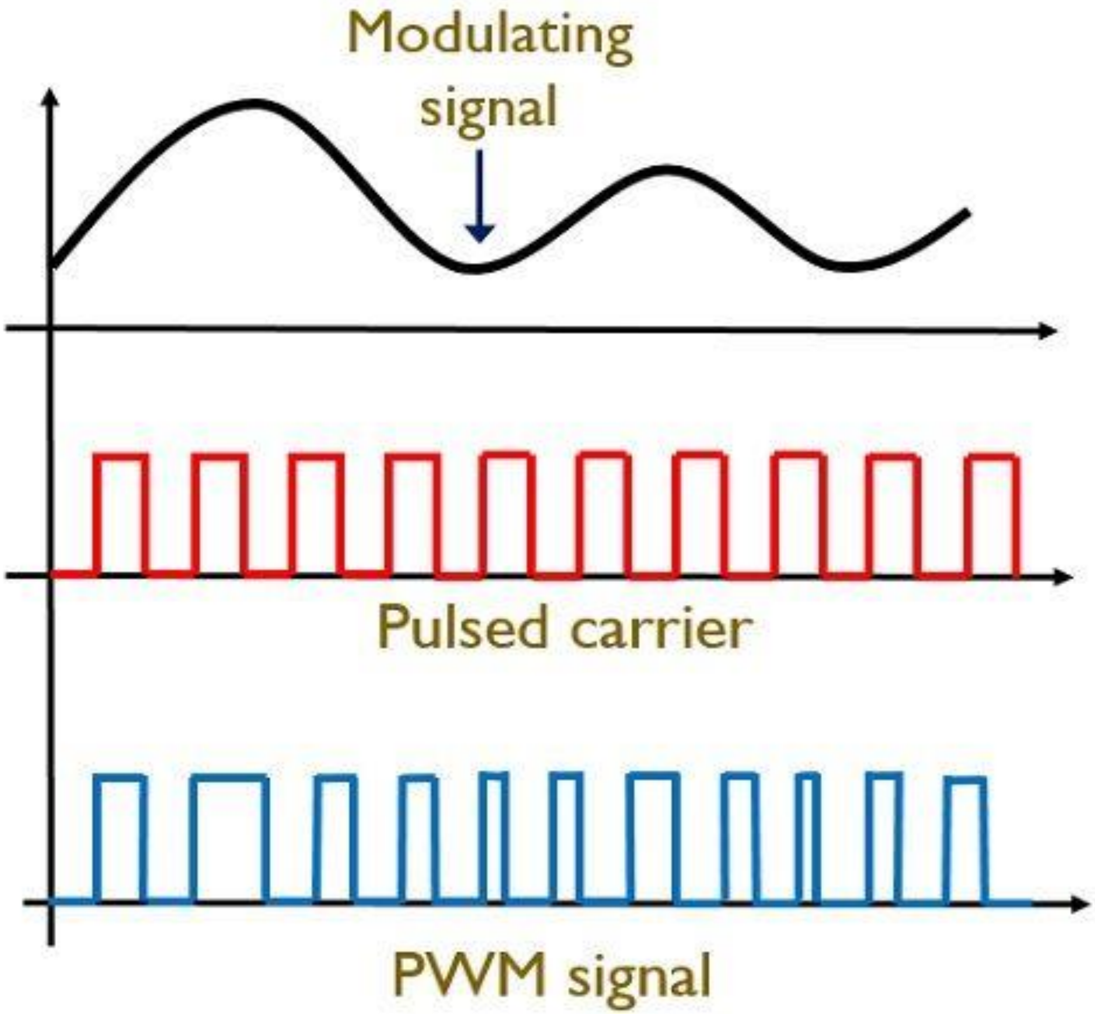
- Large bandwidth is required for transmission.
- More noise.
- Here the amplitude is varying. Therefore, the power required will be more.

Applications of PAM

- Mainly used in Ethernet communication.
- Many microcontrollers use this technique in order to generate control signals.
- It is used in Photo-biology.
- It acts as an electronic driver for LED circuits

Pulse Width Modulation (PWM)

- In this width of carrier pulses are varied in accordance with sampled values of message signal. Example: Speed control of DC Motors. It is also called pulse time modulation



Pulse Width Modulation (PWM)

Advantages of PTM

- Low power consumption.
- It has an efficiency of about 90 per cent.
- Noise interference is less.
- High power handling capacity.

Disadvantages of PTM

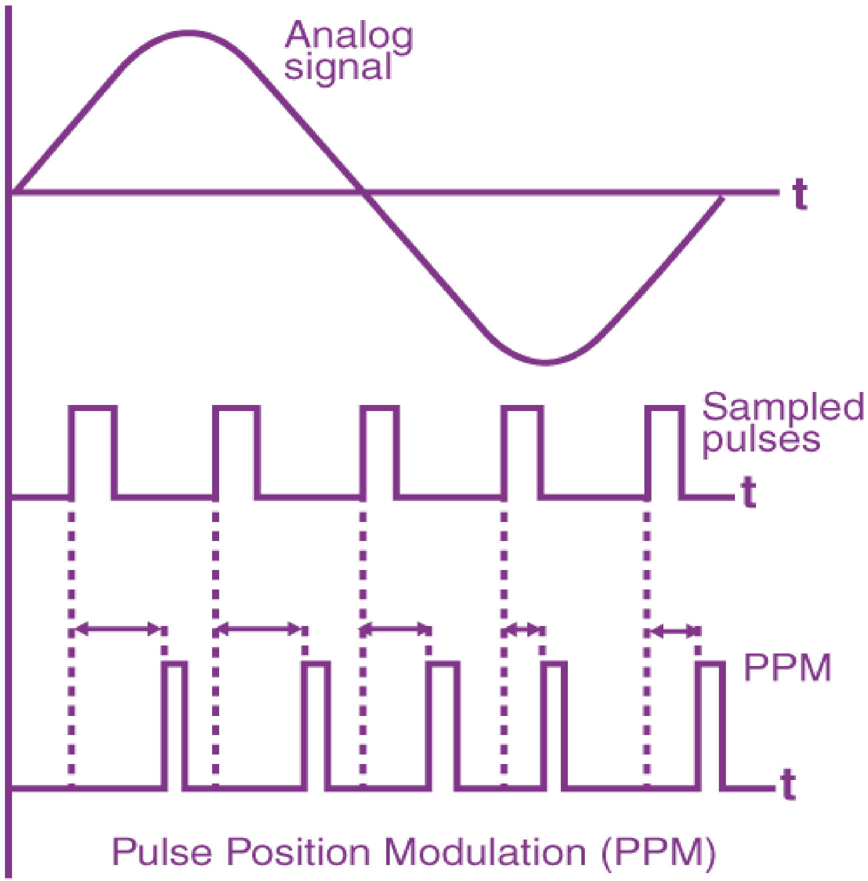
- The circuit is more complex.
- Voltage spikes can be seen.
- The system is expensive as it uses semiconductor devices.
- Switching losses will be more due to high PWM frequency.

Applications of PTM

- Used in encoding purposes in the telecommunication system.
- Used to control brightness in a smart lighting system.
- Helps to prevent overheating in LED’s while maintaining it’s brightness.
- Used in audio and video amplifiers.

Pulse Position Modulation (PPM)

- In this scheme position of high frequency carrier pulse is changed in accordance with the sampled values of message signal.



Pulse Position Modulation (PPM)

Advantages of PPM

•As it has constant amplitude noise interference is less.

•We can easily separate signal from a noisy signal.

•Among all three types, it has the most power efficiency.

•Requires less power when compared to pulse amplitude modulation.

Disadvantages of PPM

•The system is highly complex.

•The system requires more bandwidth.

Applications of PPM

•It is used in the air traffic control system and telecommunication systems.

•Remote controlled cars, planes, trains use pulse code modulations.

•It is used to compress data and hence it is used for storage.

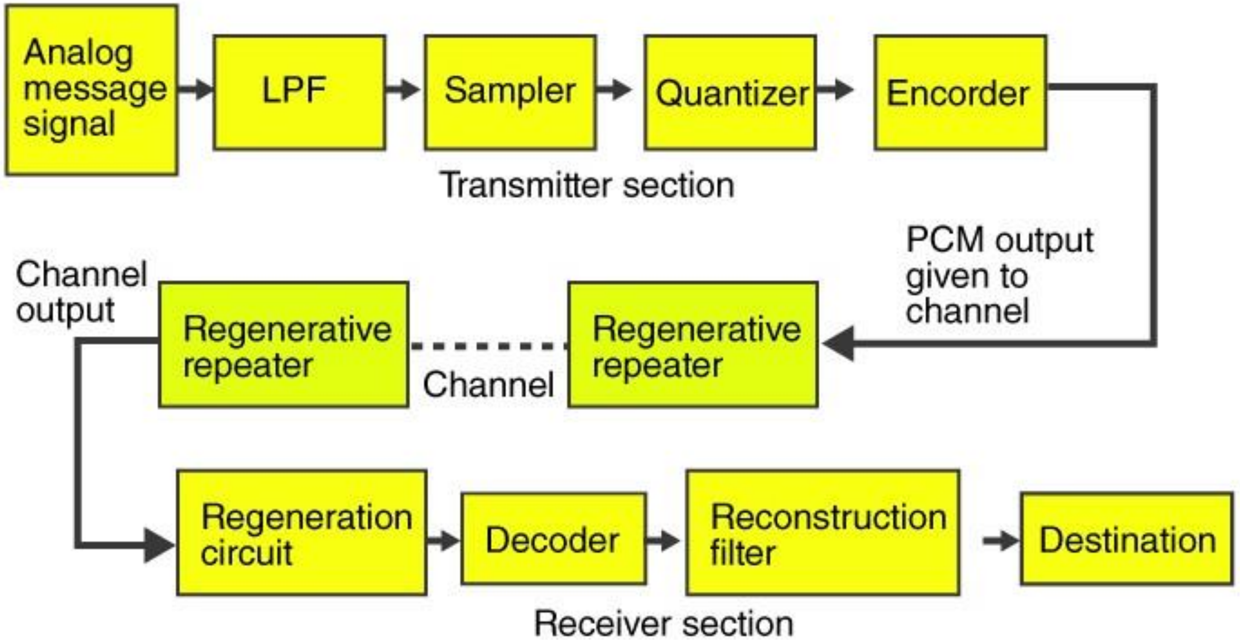
Digital Pulse Modulation

In systems utilizing digital pulse modulation, the transmitted samples take on only discrete values. Two important types of digital pulse modulation are:

1. Pulse Code Modulation (PCM)
2. Delta Modulation (DM)

Pulse Code Modulation (PCM)

- PCM is most useful and widely useful for all pulse modulation.
- PCM basically is a tool for converting an analog signal into a digital signal (A/D conversion).
- An analog signal can be converted into a digital signal by means of sampling and quantizing.
- Quantizing is a rounding off signal value to one of the closest permissible number (or quantizing levels).
- A binary digital signal is very desirable because of its simplicity, economy, and ease of engineering.



Pulse Code Modulation (PCM)

Changing analog signal to digital signal: Sampling → Quantizing → Coding

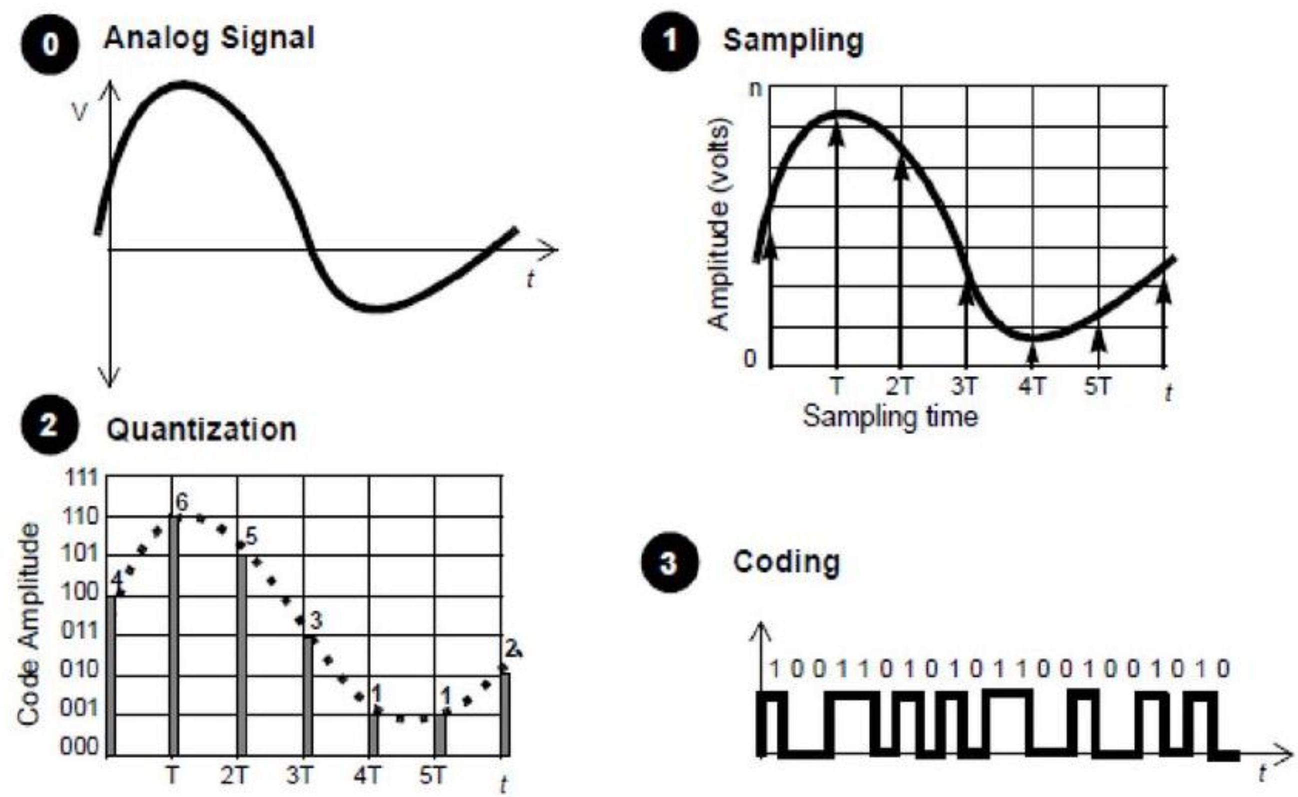


Figure 1.2 The three steps of digitalization of a signal: sampling of the signal, quantization of the amplitude, and binary encoding.

Pulse Code Modulation (PCM)

Advantages of PCM

- It is mainly used in long distant communication.
- Transmitter efficiency is more.
- It has higher noise immunity when compared to other methods.

Disadvantages of PCM

- More bandwidth is required when compared to analogue systems.
- In this method encoding, decoding and quantization of the circuit have to be done. This makes it more complex.

Applications of PCM

- It is used in the satellite transmission system.
- It is also used in space communication.
- Used in Telephony.
- One of the recent applications is the compact disc.

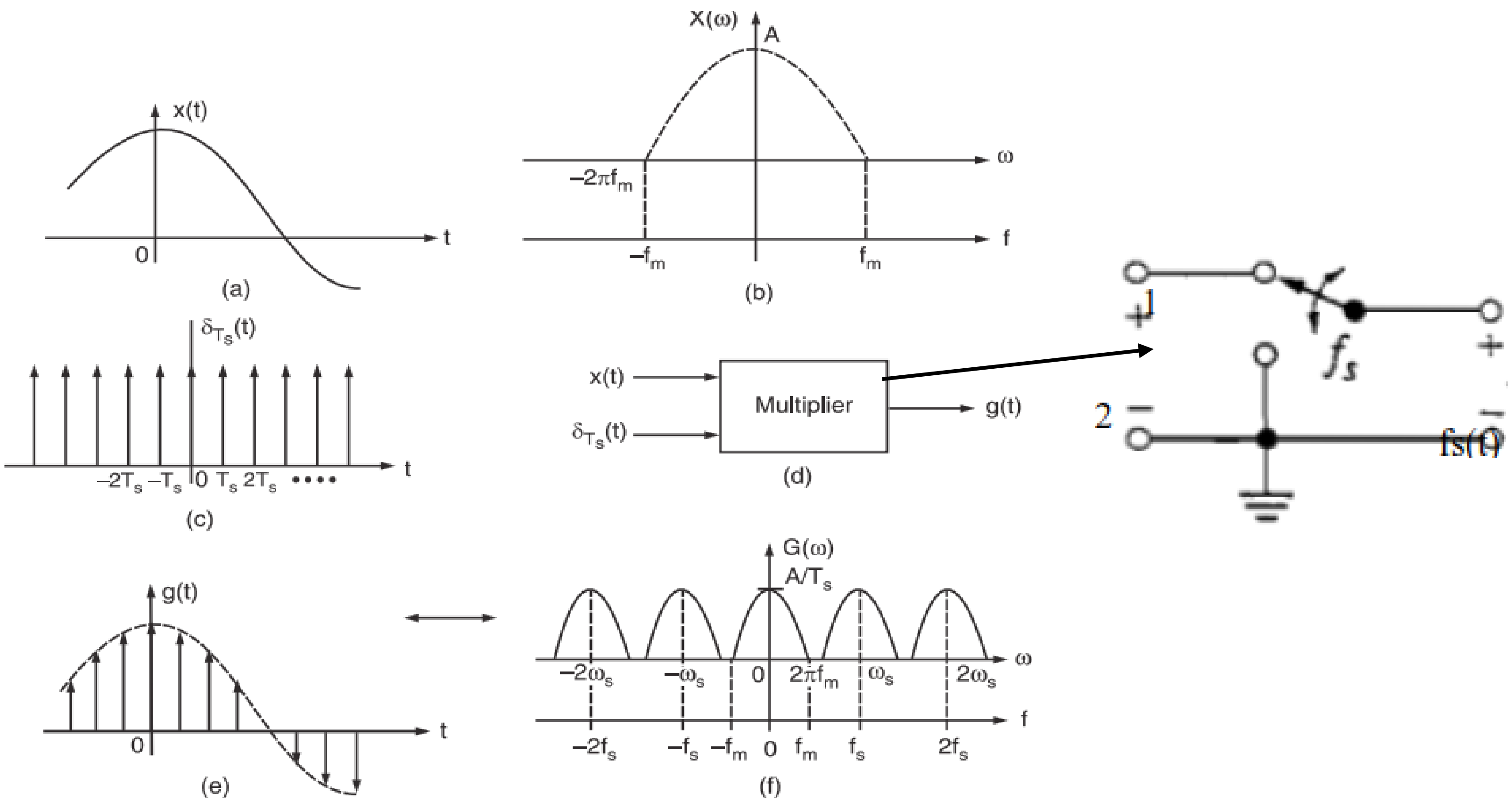
Sampling Theorem

The minimum sampling frequency required to reconstruct the original waveform from the sampled sequence is given by Nyquist criterion or theorem which can be stated as

$$f_s \geq 2f_m$$

Where, f_s = sampling frequency or the Nyquist rate

f_m = highest frequency component in the input analog waveform



Sampling Theorem

Consider a continuous time signal *x(t),* whose spectrum is band limited to f_m*. It has no frequency component beyond* f_m*. This signal is converted to digital, this is done by sampling x(t) at the rate of*

$$f_s = \frac{1}{T_s}.$$

The sample signal g(t), can be written as

$$g(t) = x(t)\delta_{T_s}(t)$$

The Fourier series expansion of pulse train $\delta_{T_s}(t)$ *is*

$$\delta_{T_s}(t) = \frac{1}{T_s}[1 + 2cos\omega_s t + 2cos2\omega_s t + 2cos3\omega_s t + \cdots]$$

$$g(t) = \frac{1}{T_s}[x(t) + 2x(t)cos\omega_s t + 2x(t)cos2\omega_s t + 2x(t)cos3\omega_s t + \cdots]$$

Now, to obtain $G(\omega)$*,*

The Fourier transform of x(t)is X(ω)

The Fourier transform of $2x(t)cos\omega_s t$ *is* $[X(\omega - \omega_s) + X(\omega + \omega_s)]$

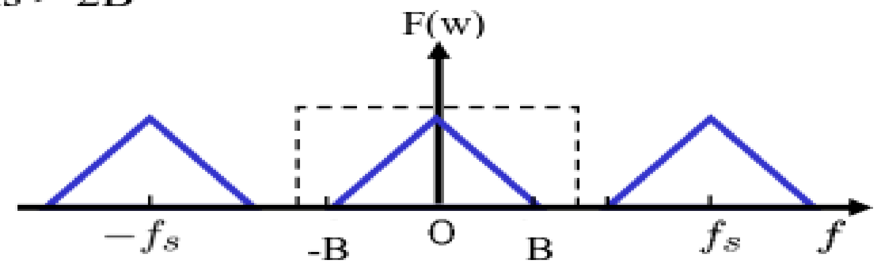
The Fourier transform of $2x(t)cos2\omega_s t$ *is* $[X(\omega - 2\omega_s) + X(\omega + 2\omega_s)]$ *and so on*

$$G(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

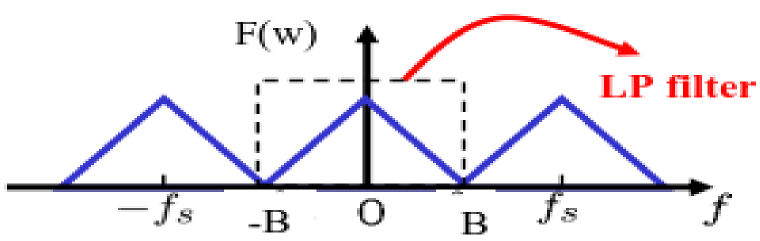
Now, to reconstruct the signal, the sampling rate should be $f_s \geq 2f_m$*.*

Sampling Theorem

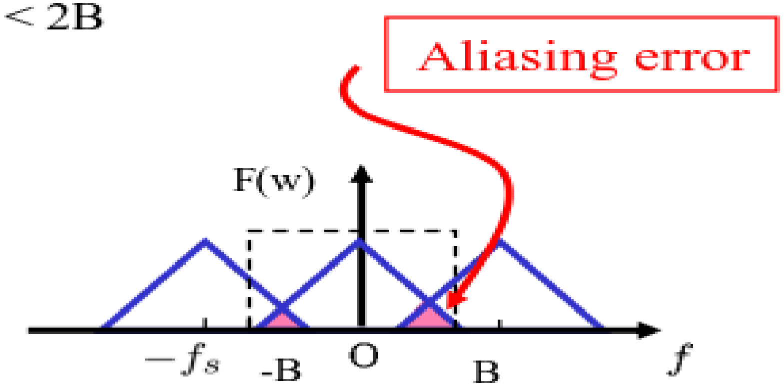
- if $fs > 2B$



- if $fs = 2B$



- if $fs < 2B$



How to Avoid Aliasing

- To prevent aliasing, two things can be done
  - Increase the sampling rate
  - Introduce an anti-aliasing filter
- Anti-aliasing filter - restricts the bandwidth of the signal to satisfy the sampling condition.
  - This is not satisfiable in reality since a signal will have *some* energy outside of the bandwidth.
  - The energy can be small enough that the aliasing effects are negligible (not eliminated completely).
- Anti-aliasing filter: low pass filters, band pass filters, non-linear filters
- Always remember to apply an anti-aliasing filter prior to signal down-sampling

Example

See example 9.1, 9.2, 9.3, 9.4 from Sanjay Sharma Books

Quantizing

- Quantizing is a process of reducing the excessive bits and confining the data. The sampled output when given to Quantizer, reduces the redundant bits and compresses the value.
- In order to process the sampled signal digitally, the sample values have to be quantized to a finite number of levels, and each value can then be represented by a string of bits.
- To quantize a sample value is to round it to the nearest point among a finite set of permissible values,
- Therefore, a distortion will inevitably occur. This is called quantization noise (or error).

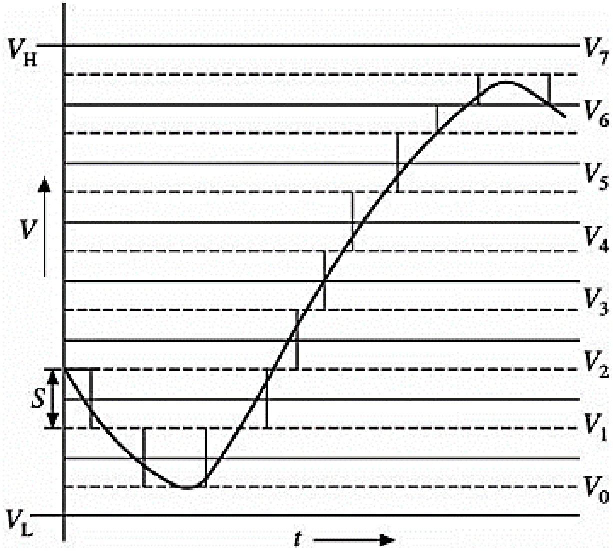
- Signal V is confined to a range from V_L to V_H
- This range is divided into M (M=8 in the figure)
- The step size S is given by

$$S = (V_H - V_L)/M$$

- The center of each steps, locate the quantization levels $V_0, V_1, \ldots V_{M-1}$
- The quantized signal V_q takes on any one of the quantised level values.
- A signal V is quantized to its nearest quantisation level.

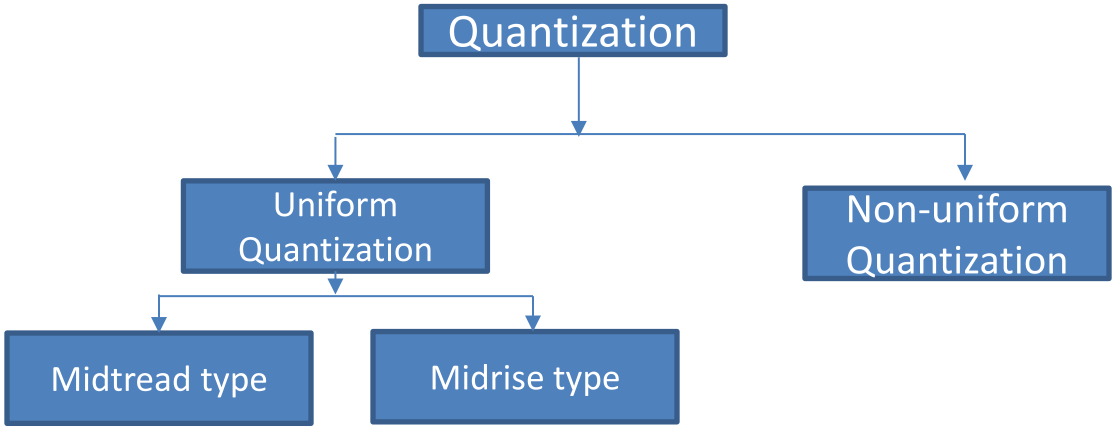
$$V_q = V_3 \text{ if } (V_3 - S/2) \leq V < (V_3 + S/2)$$

$$V_q = V_4 \text{ if } (V_4 - S/2) \leq V < (V_4 + S/2)$$



Quantizing

The quantization process can be classified into two types as under:



Uniform Quantizer: A uniform Quantizer has the same step size through the signal range

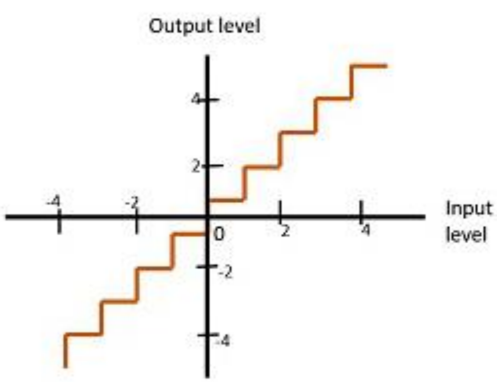


Fig 1 : Mid-Rise type Uniform Quantization

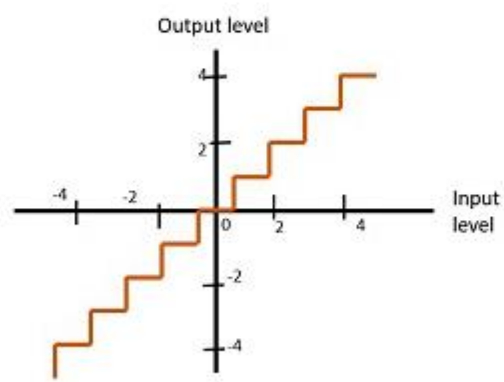
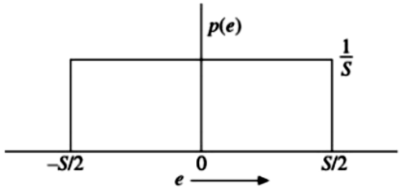


Fig 2 : Mid-Tread type Uniform Quantization

Quantization Noise or Error

- An approximation to the original signal is called the quantized signal and an error is introduced in the signal due to the approximation.
- So, the quantization error is $e = V - V_q$ that is randomly distributed within the range S/2.
- For linear quantization, the probability density function p(e) is constant within the range ±S/2.
- So, the average quantisation noise output power is given by its variance.

$$\sigma^2 = \int_{-\infty}^{\infty} (e-\mu)^2 p(e)de$$



Where, where μ = mean, which is zero for quantisation noise

- The range of quantisation error ±(S/2) determines the limits for integration. Therefore,

$$\sigma^2 = \int_{-S/2}^{S/2} (e-0)^2 \frac{1}{S} de = \frac{1}{S}\int_{-S/2}^{S/2} e^2 de$$

$$= \frac{1}{S}\left(\frac{e^3}{3}\right)_{-S/2}^{S/2} = \frac{S^2}{12}$$

Quantization Noise or Error

- Signal to quantisation noise ratio (SQR) is a good measure of performance of a PCM system transmitting speech.
- If V_r is the r.m.s. value of the input signal and if we assume (for convenience) a resistance level of 1 ohm, then SQR is given by

$$SQR = 10log\left[\frac{{V_r}^2}{({S^2}/{12})}\right]dB$$
$$= 10\log(12) + 20log\left[\frac{V_r}{S}\right]dB$$
$$= 10.8 + 20log\left[\frac{V_r}{S}\right]dB$$

If the input signal is a sinusoidal wave with V_m as the maximum amplitude, SQR may be calculated for the full range sine wave as

$$SQR = 10log\left[\frac{\left({V_m}/{\sqrt{2}}\right)^2}{({S^2}/{12})}\right]dB = 7.78 + 20log\left[\frac{V_m}{S}\right]dB$$

Expressing S in terms of Vm and the number of steps, M, we have

$$SQR = 10log\left[\frac{\left({V_m}/{\sqrt{2}}\right)^2}{({4{V_m}^2}/{12M^2})}\right]dB = 20\log(1.225M)\,dB$$

Now, $M = 2^n$, where n is the number of bits used to code a quantisation level.

$$SQR = 20\log(1.225M)\,dB = 20\log(1.225) + 20nlog(2) \quad dB$$
$$SQR = 1.76 + 6.02n \;\; dB$$

Quantization Noise or Error

The following table gives the values of SQR for different binary code word sizes for sinusoidal input systems. It may be seen that every additional code bit gives an improvement of 6 dB in SQR.

n (bits)	M(levels)	Voltage ratio (1.225M)	SQR (dB)
2	4	4.9	13.8
4	16	19.6	25.8
6	64	78.4	37.8
7	128	156.8	43.8
8	256	313.6	49.8
9	512	627.2	55.8
10	1024	1254.4	61.8

Binary PCM

The essential features of binary PCM are illustrated by means of an example shown in the figure below