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Probability

Sample space:

The collection or totality of all possible outcomes of a random experiment is called sample space.

Sample space denoted by Ω or S.

Example: If we toss a coin, the sample space is $\Omega = [H, T]$ where H and T denote the head and tail of the coin.

Events:

An event is subset of the sample space and is usually denoted by capital letters A, B, C, D, E etc.

There are two types of events:—

i) Simple event: An event is called if it contains only one sample point.

ii) Compound event: An event is called compound event if it contains more than one sample points.

Union of two events: Union of two events A and B also an event which contains all the ~~union~~ elements of a ~~event~~ A or B both. It is denoted by $A \cup B$.

Intersection of events: Intersection of two events is also an event which contains all common elements of both A and B. It is denoted by $A \cap B$.

[Note: $N(A \cup B) = N(A) + N(B) - N(A \cap B)$

Example: Suppose two brands refrigerator, say A and B are available in the market. A survey was conducted on 1000 people. 500 liked A, 400 liked B, 200 liked both A and B.

A person is selected at random from these 1000 people. How many persons liked

i) A or B

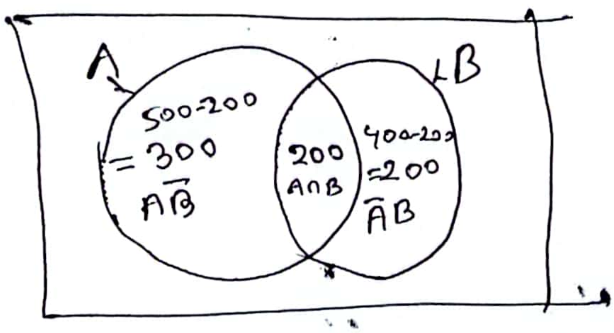
ii) only A

iii) only one.

Soln: Let A be the event that a person liked brand A and B be the event that person liked brand B.

$$N(A) = 500, \quad N(B) = 400 \quad N(A \cap B) = 200$$

The Ven diagram of the problem is



i) $N(A \cup B) = N(A) + N(B) - N(A \cap B)$

$= 500 + 400 - 200 = 700$

ii) only A, $N(A\bar{B}) = N(A) - N(A \cap B)$

$= 500 - 200$

$= 300$

(iii) only one, $N(A\bar{B} \cup \bar{A}B) = N(A\bar{B}) + N(\bar{A}B)$

$= 300 + 200 = 500$

Experiment: Experiment is an act that can be repeated under given condition.

Random experiment: An experimeriment is called experiment whose outcomes cannot be predicted with certainty. For example

i) In tossing a coin one is not sure if a head or tail will be obtained.

outcomes: The results of an experiment are known as outcomes.

Probability of an events / classical or mathmatical or a priori probability:

If there are n mutually exclusive, equally likely and exhaustive outcomes of a random experiment and if m of these outcomes are favourable to an event A, then the the probability of event A which is denoted by P[A] is defined by

$$P[A] = \frac{\text{Favourable outcome of an event } A}{\text{Total number of outcomes of the experiment}} = \frac{m}{n}.$$

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Conditional probability:

Definition: If A and B are two events in a probability space $(\Omega, \mathcal{A}, P[\cdot])$, then the conditional probability of A given B, denoted by $P[A|B]$ is defined by

$$P[A|B] = \frac{P[A\cap B]}{P[B]} \qquad [P[A\cap B] = P(AB)]$$

Example: In a class of 120 Students, 60 are studying English, 50 are studying France and 20 are studying both English and France. If a student is selected at random from this class, what is the probability that he is studying English if it is given that he is studying France.

complement $[\bar{A} = U - A]$

Soln: let E and F be the event that a student is studying English and France.

$N(E) = 60 \qquad N(S) = 120$

$N(F) = 50$

$N(E\cap F) = 20$

$$\therefore P[E|F] = \frac{P[E\cap F]}{P[F]} = \frac{\frac{N(E\cap F)}{N(S)}}{\frac{N(F)}{N(S)}} = \frac{N(E\cap F)}{N(F)}$$

$$= \frac{20}{50}$$

$$= \frac{2}{5}$$

Statement of Bayes Theorem:

Suppose $B_1, B_2 \cdots B_n$ are n mutually exclusive and exhaustive events and A be the another events such that $P(A) > 0$, then Bayes theorem states that

$$P[B_j|A] = \frac{P[B_j]\, P[A|B_j]}{\sum_{i=1}^{n} P(B_i)\, P[A|B_i]}$$

Prior Probability: The probabilities which we know before the experiments are called prior probability.

Hence, $P[B_1]$ $P[B_2] \cdots P[B_n]$ are called prior probability.

Posterior probability: The probabilities which are calculated after the experiment by using prior probability are called posterior probability.

Hence, $P[B_1|A]$, $P[B_2|A] \cdots P[B_n|A]$ are called posterior probability.

Example: In a course 65% students are female. The probability that a female student passes the course is .8 and the probability that a male student passes the courses is .75. A student number 2575 is selected at random from this class and is found to be passed. What is the probability that the student number 2575 is a female student?

Soln:

B_1 = a male student

B_2 = a female "

A = a student passes the course.

We are given,

$$P[B_1] = .35 \quad P[B_2] = .65$$

$$P[A|B_1] = .75 \quad P[A|B_2] = .8$$

$$\therefore P[B_2|A] = ?$$

$$\therefore P[B_2|A] = \frac{P[B_2] \cdot P[A|B_2]}{P[B_1]\, P[A|B_1] + P[B_2]\, P[A|B_2]}$$

$$= \frac{.65 \times .80}{.35 \times .75 + .65 \times .80} = .6645$$

Example: 2 In a bolt factory machine, A produces 45% of the output and machine B produces the rest. One the average, 9 items in 1000 produced by machine A are defective and 2 items in 500 produced by B are defective. An item is drawn at random from a days output and is found to be defective. Calculate the probability that it was produced by machine B_1?

Soln:

B_1 = item produced by machine A

B_2 = " " " " B

A = A defective item.

∴ we have, $P[B_1] = .45$, $P[B_2] = .65$

$P[A|B_1] = \frac{9}{1000} = .009$, ~~$P[A|B_2] = \frac{2}{1000} = .002$~~

$P[A|B_2] = \frac{2}{500} = .004$

find $P[B_1|A] = ?$

$$\therefore P[B_1|A] = \frac{P[B_1]\,P[A|B_1]}{P[B_1]\,P[A|B_1] + P[B_2].P[A|B_2]}$$

$$= \frac{.45 \times .009}{.45 \times .009 + .55 \times .004}$$

$$= \frac{81}{125}$$

Statement of Chebyshev's Inequality:

if X is a random variable with mean μ and Standard deviation σ, then for any positive number k, we have

$$P[|X-\mu| \geq k\sigma] \leq \frac{1}{k^2} \text{ or}$$

$$P[|X-\mu| \leq k\sigma] \geq 1-\frac{1}{k^2}$$