Test of Hypothesis

BBA's 2-2

Statistical Hypothesis: - A statistical hypothesis is an assumption or statement about a population parameter,) which we want to verify on the basis of information contained in a sample. A **statistical hypothesis** is an assumption about a population parameter. This assumption may or may not be true.

Examples:

(1) A physician may hypothesize that the recommended drug is effective in 90 percent cases.
(2) A sewing machine company claims that their new machine is superior to the one available in the market.
(3) A market representative of a company claims that average sales of his product per day is 400 kg.
(4)

Test of a statistical hypothesis: A test of a statistical hypothesis is a two-action decision problem after the experimental sample values have been obtained, the two-actions being the acceptance or rejection of the hypothesis under consideration.

Hypothesis testing refers to the formal procedures used by statisticians to accept or reject statistical hypotheses.

Null hypothesis:

- Null means the possible rejection of the hypothesis. Null hypothesis is a statement, which tells us that no difference exists between the parameter and the statistic being compared to it. A hypothesis which states that there is no difference between assumed and actual value of the parameter is the Null hypothesis. Null hypothesis is always denoted by H_0.
- The null hypothesis, H_0, is usually the hypothesis that corresponds to the status quo, the standard, the desired level/amount, or it represents the statement of "no difference."
- A theory about the values of one or more population parameters. The theory generally represents the status quo, which we adopt until it is proven false.

Example:

i) The average height of students of MBSTU is 5.2. That is $H_0: \mu = 50$
ii) H_0: There is no difference in the population between the rates of prevalence of malnutrition between the male and female children.
iii) The average production of machine A and machine B are equal. That is $H_0: \mu_A = \mu_B$

National Hypothesis Hypothesis Hypothesis Hypothesis Alternative hypothesis

Alternative hypothesis: The alternative hypothesis is the logical opposite of the null hypothesis. Alternative hypothesis is usually denoted by H_1 or H_a.

the complement of the null hypothesis

$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_2$

$H_0: \mu_A = \mu_B$
H_1 or H_a

Example:

i) Null hypothesis, $H_0: \mu = 50$ and Alternative hypothesis $H_1: \mu \neq 50$ or $H_1: \mu > 50$ or $H_1: \mu > 50$

ii) H_0: There is no difference in the population between the rates of prevalence of malnutrition between the male and female children.
H_1: There is a difference in the population between the rates of prevalence of malnutrition between the male and female children

iii) Null hypothesis, $H_0: \mu_A = \mu_B$ and Alternative hypothesis $H_1: \mu_A \neq \mu_B$ or $H_1: \mu_A > \mu_B$ or $H_1: \mu_A < \mu_B$

th this this this this !!

(i) New process is greater than standard
(ii) " " is inferior " "
(iii) No difference between two process

One-tailed test: A test of any statistical hypothesis where the alternative is one-sided such as

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$

or perhaps, $H_0 : \mu = \mu_0$

$$H_1 : \mu < \mu_0$$

is called a one-sided test.

Two-tailed test: A test of any statistical hypothesis where the alternative is two-sided such as

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

729

is called a two-tailed test.

Rejecting null hypothesis H_0 is ...

Type I error: The error of rejecting H_0 (accepting H_1) when H_0 is true is called type I error. The probability of type I error is denoted by α and it is called the level of significance.

$\alpha = P[\text{Rejecting } H_0 \mid H_0 \text{ true}]$

Type II error: The error of accepting H_0 when H_0 is false (H_1 is true) is called type II error. The probability of type II error is denoted by β.

Level of significance: The significance level of a test is a fixed probability of wrongly rejecting the null hypothesis when in fact it is true. The probability of type I error is called the level of significance and is denoted by α.

Critical value: the value of the sample statistic that separates the acceptance region and rejection region is called critical value

Power test: the probability of rejecting false null hypothesis is called power of a test.

Critical region or rejection region: A region of rejection is a set of possible values of the sample statistic, which provides evidence to contradict the null hypothesis and leads to a decision to reject the null hypothesis.

$1-\beta = P[\text{Reject } H_0 \mid H_1]$

Acceptance region: A region of acceptance is a set of possible values of the sample statistic, which provides evidence to support the null hypothesis and lead to a decision to accept it.

Test statistic: The statistic used to provide evidence about the null hypothesis is called the test statistic.

Five steps for testing a hypothesis:-

There is a five-step procedure that systematizes hypothesis testing: when we go to step 5, we are ready to reject or not reject the hypothesis.

1. Establish the null hypothesis (H_0) and the alternative hypothesis (H_1).
2. Select the level of significance, that is α. Generally we take α=5% or α=1%.
3. Select an appropriate test statistic.
4. Determine critical value from statistical table. Then formulate decision rule.
5. Compute the value of the test statistic from sample.
6. Compare the value of test statistic and critical value.
7. Make the decision. Reject the null hypothesis, if the calculated test statistic falls in the rejection region (calculated value > critical value). Otherwise accept the null hypothesis, if the calculated test statistic falls in the accepted region. (Calculated value < critical value).

5%=1.96(critical value)
1%=2.58
10%=1.64

728 X is normally distributed with SD 10 and mean 50. $H: \mu = 50$ is a simple hypothesis since a normal distribution is completely define by its mean and S.D.

simple hypothesis: is a hypothesis completely specifies the distribution of the popn it is called a simple hypothesis

$H_0: \mu \neq 50$, is a composite hypothesis bcoz we donot know the exact distribution of the popⁿ everify. we know that it is normal with a known SD.

Mean Test

a) A test of the mean of a normal distribution when σ is known:

For two tailed test

1. $H_0: \mu = \mu_0$, Vs $H_1: \mu \neq \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Critical value $Z_{\alpha/2}$ or $-Z_{\alpha/2}$. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_{\alpha/2}$, or $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > Z_{\alpha/2}$.
5. Make decision.

11.5

Example 01: A blub manufacturing company claims that the average longevity of their bulb is 3.65 years with standard deviation of 0.16 years. A random sample of 36 bulbs gave a mean longevity of 3.45 years. Does the sample mean justify the claim of the manufacturer? Use a 5% level of significance.

$\mu = 3.65$, $\sigma = .16$, $n = 36$

Solution:

$Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) = \dfrac{3.45 - 3.65}{.16/\sqrt{36}}$

1. $H_0: \mu = 3.65$, Vs $H_1: \mu \neq 3.65$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Critical value C=1.96 or -1.96. Reject H_0, if $Z = \dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} < -1.96$, or $Z = \dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} > 1.96$.

if $Z < -1.96$ or $Z > 1.96$

5. Now, $Z = \dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \dfrac{3.45 - 3.65}{0.16/\sqrt{36}} = -7.50$.

Comment: Since the calculate value -7.50 is less than critical value-1.96. Hence we reject the null hypothesis and conclude that the mean longevity of the bulb is not 3.65 years.

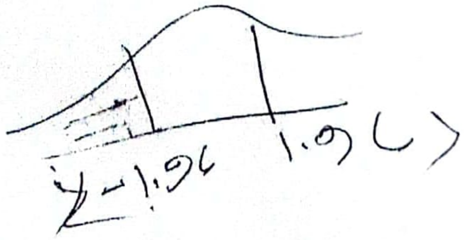
Exercise 01: The manufacturer of the MFR tire claims that the mean mileage the tire can be driven before the tread wears out is 60000 miles. The standard deviation of mileage is 5000 miles. The TATA company bought 48 tires and found that the mean mileage for their truck is 59500 miles. Is TATA company experience different from that claimed by the manufacturer at the 5% significance level?

$(-7.50) < -1.96$

$\mu = 600$, $n = 48$, $\sigma = 5$

For one tailed (Right tailed) test

1. $H_0: \mu = \mu_0$, Vs $H_1: \mu > \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Critical value Z_α. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > Z_\alpha$.
5. Make decision



Example: The production manager of Northern Windows Inc. has asked you to evaluate a proposed new procedure for producing its Regal Line of double-hung windows. The present process has a mean production of 80 units per hour with a population standard deviation of $\sigma = 8$. The manager indicates that she does not want to change to a new procedure unless there is strong evidence that the mean production level is higher with the new process. A random sample of 25 production hours was selected and the sample mean was 83 units per hour.

Solution:-

1. $H_0: \mu = 80$, Vs $H_1: \mu > 80$.
2. Let the level of significance $\alpha = 0.05$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Critical value C= Z_α =1.64. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} > Z_\alpha$.
5. Now, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{83 - 80}{8/\sqrt{25}} = 1.875$.

Comment: For a 5% level test $\alpha = 0.05$ and $Z_\alpha = Z_{0.05} = 1.645$. Thus since 1.875 is greater than 1.645 we would reject the null hypothesis and conclude that there was strong evidence to support the conclusion that the new process resulted in higher productivity.

Exercise 02: An internet server claims that its users spend on the average 20 hours per week with a standard deviation of 2.5 hours. To determine whether this is an underestimate, a competitor conducted a sample survey of 15 customers and found that the average time spent online was 21.8 hours per week. Do the data provide sufficient evidence to indicate that the average hours of use are greater than that claimed by the first internet server? Test at 1% level.

<u>For one tailed (Left tailed) test</u>

1. $H_0: \mu = \mu_0$, Vs $H_1: \mu < \mu_0$.
2. Level of significance $= \alpha$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$.
4. Critical value C=- Z_α. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_\alpha$.
5. Make decision

Example: An internet server claims that its users spend on the average 20 hours per week with a standard deviation of 2.5 hours on the information superhighway. To determine whether this is an overestimate, a competitor conducted a sample survey of 15 customers and found that the average time spent online was 21.8 hours per week. Do the data provide sufficient evidence to indicate that the average hours of use are less than that claimed by the first internet? Test at 5% level.

Solution:

1. $H_0: \mu = 20$, Vs $H_1: \mu < 20$
2. Level of significance, $\alpha = 0.05$

$\mu = 20$
$\sigma = 2.5$
$n = 15$
$\bar{x} = 21.8$

3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$

4. Critical value C=-Z_α=-1.645. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_\alpha$

$Z < Z_\alpha$

5. Now, $Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{21.8 - 20}{2.5/\sqrt{15}} = 2.79$

P-value

$P > \alpha$

Comment: For a 5% level test $\alpha = 0.05$ and $Z_\alpha = Z_{0.05} = 1.645$. Thus since computed value is greater than the critical value $-Z_\alpha = -1.645$, we cannot reject the null hypothesis and conclude that there is sufficient evidence to agree with the claim of the first internet server.

Exercise 03: The KFC claims that the waiting time of customers of service is normally distributed, [illegible] mean of 3 minutes and a standard deviation of 1 minute. The quality assurance department fo[illegible]in sample of 50 customers at the Baily road that the mean waiting time was 2.75 minutes. At the [illegible]vel of significance, can we conclude that the mean waiting time is less than 3 minutes?

$\mu = 3$

$\sigma = 1$

$n = 50$

(b) A [illegible]f the mean of a normal distribution: when σ is not known[illegible] population is large $n \geq$ [illegible]

For two tailed test

1. $H_0: \mu = \mu_0$, [illegible]$: \mu \neq \mu_0$
2. Level of signific[illegible] $= \alpha$
3. Test statistic, $Z =$ [illegible] $\sim N(0,1)$
4. Reject H_0, if $Z = \dfrac{\overline{X} - \text{[illegible]}}{s/\sqrt{n}}$ [illegible] $Z_{\alpha/2}(Critical\ \ value)$, or $Z =$ [illegible] $> Z_{\alpha/2}(Critical\ \ value)$
5. Make decision.

Example: Given the following hypot[illegible]

$H_0: \mu = 400$, Vs $H_1: \mu \neq 4$[illegible]

For a random sample of 32 observations, [illegible]nean was 407 and the standard deviation 6. Using the 0.05 significance level, what is you[illegible]n regarding the null hypothesis?

Solution:-

1. $H_0: \mu = 400$, Vs $H_1: \mu \neq 400$.
2. Level of significance, $\alpha = 0.05$.
3. Test statistic, $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} \sim N(0,$[illegible]
4. Reject H_0, if $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}}$ [illegible]$_{\alpha/2}$, or $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} > Z_{\alpha/2}$.

± 1.96, or 1.96

5. Now, $Z = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}} = \dfrac{407\text{[illegible]}0}{\text{[illegible]}2} = 6.60$.

P-value

Comment:- For a [illegible]evel test $\alpha = 0.05$ and $Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$[illegible]s since 6.60 is greater than 1.96, [illegible]ay reject the null hypothesis.