

Coordinate Geometry

(Following)

" " " Two Dimensions ②+1

Three " ④ set

Two Dimensions ②+1

14
7
7

Two Dimensional Coordinate Geometry

Pair of straight lines

$$ax^2 + 2hxy + by^2 = b(y - m_1x)(y - m_2x)$$

$$= b[y^2 - (m_1 + m_2)xy + m_1m_2x^2]$$

Compare with $(a \neq 0)$

$$m_1 + m_2 = -\frac{2h}{b} \quad a = b$$

$$m_1m_2 = \frac{a^2 + 2h^2 + b^2}{b}$$

$$= \sqrt{a^2 + (b-h)^2}$$

$$= \frac{\sqrt{(m_1 - m_2)^2}}{1 + m_1m_2}$$

$$= \frac{\sqrt{4h^2 - 4ab}}{a+b}$$

$$\begin{cases} y = m_1x \\ y = m_2x \end{cases}$$

$$x^2 - y^2 = 0$$

~~$$ax^2 + 2hxy + by^2 = 0$$~~

general eqn.

of two straight lines

$x^2 - y^2 = 0 \rightarrow$ homogeneous eqn of 2nd degree

Homogeneous eqn हले तरीके से straight line पाएं जब origin

में पाएं । degree वाली straight line हो ।

$$(x+y)(x-y) = 0$$

$$x^2 - y^2 = 0$$

$$= \frac{\sqrt{4h^2 - 4ab}}{a+b}$$

$$\theta = \tan^{-1} \frac{2\sqrt{h^2 - ab}}{a+b}$$

১২-০১-৮০

$a+b=0$ হলে θ হবে 90° (Perpendicular)

$a+b \neq 0$ হলে θ " " 0° (Parallel)

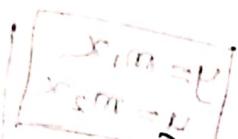
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 $a+b=0$

Parallel \rightarrow Distance থাকে—

Coincide \rightarrow আকলে মিল হয়।

একটির উপর আকলে
আকলে মিল হয়।
coincide (অস্পষ্ট)

$$0 = b - x$$



equal triplicate to নির্ণয়

Problem (Exm এ আভবে)

$$(x-m-y)(x-m+y)d = [pd + py]d^2 + \frac{xy}{d}$$

Show that the product of the perpendiculars
from the point (α, β) on the lines

$$ax^2 + 2hxy + by^2 = 0 \quad \text{is} \quad \frac{ds}{d} = \pm m + \frac{1}{m}$$

$$\frac{ax^2 + 2h\alpha\beta + b\beta^2}{\sqrt{(a-b) + 4h^2}} = \pm m, \frac{1}{m}$$

$$0 = (b-x)(b+x)$$

$$0 = b - x$$

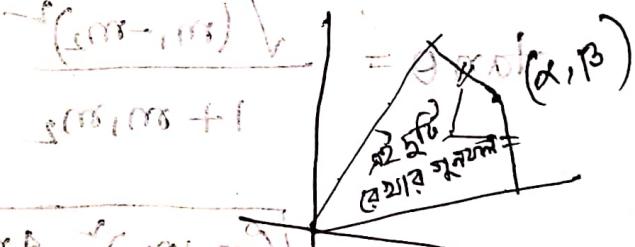
$$\frac{ds^2 - dp^2}{d^2} =$$

$$\frac{ds^2 - dp^2}{d^2} \text{ not } = 0$$

$$\frac{ds^2 - dp^2}{d^2} =$$

$$\frac{ds^2 - dp^2}{d^2} =$$

$$\frac{ds^2 - dp^2}{d^2} =$$



Soln
Given that, one straight line being $\alpha x^2 + 2hxy + by^2 = 0 \quad \text{.....(1)}$

$$\frac{\partial}{\partial x}(\alpha x^2 + 2hxy + by^2) = 2\alpha x + 2hy + 0 = 0 \quad \text{.....(2)}$$

Let the straight lines

Let $y = m_1 x$ and $y = m_2 x$ be the straight lines represented by eqn (1).

$$\begin{aligned}\therefore \alpha x^2 + 2hxy + by^2 &= b(y - m_1 x)(y - m_2 x) \\ &= b[y^2 - (m_1 + m_2)xy + m_1 m_2 x^2]\end{aligned}$$

Comparing the coefficients of like terms on both sides we get,

$$m_1 + m_2 = -\frac{2h}{b} \quad \text{and} \quad m_1 m_2 = \frac{a}{b}$$

$$m_1 m_2 = \frac{a}{b}$$

Now, the perpendicular distance from the point (α, β) on the line $y - m_1 x = 0$ is

$$\beta - m_1 \alpha = \frac{x\beta + \alpha m_1 \alpha + \beta d}{d}$$

$$\sqrt{1+m_1^2} \cdot \frac{\beta + d\alpha + d\beta + d}{d}$$

Also the perpendicular distance from the point.

$$(\alpha, \beta) \text{ on the line } y - m_2 x = \beta \text{ is } \frac{\beta - m_2 \alpha}{\sqrt{1+m_2^2}}$$

∴ The product of the perpendiculars is

$$\frac{(\beta - m_1 \alpha)}{\sqrt{1+m_1^2}} \cdot \frac{(\beta - m_2 \alpha)}{\sqrt{1+m_2^2}}$$

$$(x_m - \beta)(x_m - \beta) d = \beta d + \beta x d + x d$$

$$\left[\frac{\beta^2 - (m_1 + m_2)\beta + m_1 m_2 \alpha^2}{\sqrt{1+m_1^2} + \sqrt{1+m_2^2}} \right] d =$$

$$\sqrt{1+m_1^2 + m_2^2 + m_1^2 m_2^2}$$

$$= \frac{\beta^2 - (m_1 + m_2)\alpha\beta + m_1 m_2 \alpha^2}{\sqrt{1+(m_1 + m_2)^2 - 2m_1 m_2 + m_1^2 m_2^2}} = x_m + y_m$$

$$= \frac{\beta^2 + \frac{2h}{b}\alpha\beta + \frac{a}{b}\alpha^2}{\sqrt{1 + \left(\frac{-2h}{b}\right)^2 - 2\frac{a}{b} + \left(\frac{a}{b}\right)^2}} = \frac{m_1 m_2}{\frac{m_1 + m_2}{b}}$$

$$\sqrt{1 + \left(\frac{-2h}{b}\right)^2 - 2\frac{a}{b} + \left(\frac{a}{b}\right)^2} = \sqrt{b^2 + 4h^2 - 2ab + a^2} = \sqrt{b^2 + 1}$$

$$0 = x_m - \beta$$

$$= \frac{b\beta^2 + 2h\alpha\beta + a\alpha^2}{b(x_m - \beta)}$$

$$\sqrt{\frac{b^2 + 4h^2 - 2ab + a^2}{b^2}} = \sqrt{b^2 + 1}$$

$$\frac{ad^2 + 2hab + b\beta^2}{\sqrt{(a-b)^2 + 4h^2}} \quad \text{Page-54:}$$

$$= 11f + 53 \left(\begin{array}{l} ex-4^* \\ ex-8^* \\ ex-9^* \end{array} \right)$$

Examination marks (7 marks)
must be obtained from straight lines in book

Find the condition that the general eqn of

2nd degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
represent a pair of straight lines.

Soln:

$$\Delta = 0, ab = 0$$

$$\Delta = 0, h^2 = ab$$

Let the general eqn of 2nd degree be

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \dots \dots \dots \quad (1) \quad ax^2 + bxy + c = 0$$

Treating this as a quadratic eqn in x and

supposing $a \neq 0$, we have, $h^2 - ab < 0$

$$x = \frac{-2(hy+g) \pm \sqrt{4(hy+g)^2 - 4ab(y^2 + 2fy + c)}}{2a}$$

longer sit tortuous beginning at in this

$$to \Rightarrow ax^2 + hy^2 + g^2 = \sqrt{hy^2 + 2hgy + g^2} - aby^2 - 2fy - ac$$

, will triplicate

$$\Rightarrow ax + hy + g = \pm \sqrt{(h^2 - ab)y^2 + 2(hg - af)y + (g^2 - ac)}$$

Now an order that this may be capable of

being reduced to the form $ax + by + c = 0$,
 it is necessary and sufficient that the quantity
 under the radical sign should be a perfect
 square.

\therefore we get

$$4(hg - af)^2 = 4(h^2 - ab)(g^2 - ac)$$

$$\Rightarrow h^2g^2 - 2afhg + a^2f^2 = hg^2 - abg^2 - ach^2 + a^2bc$$

$$\Rightarrow 2f^2hg - af^2 = bg^2 + ch^2 - abc \quad [-a \text{ যুক্তি অঙ্গ}]$$

$$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\therefore \Delta = abc + 2fgh - af^2 - (bg^2 + ch^2) = 0$$

which is the required condition that the general eqn of 2nd degree represents a pair of straight lines.

Point of intersection: $x^2 + xy - 2x^2y - 2xy^2 + x^2y^2 = 0$

Let the point of intersection between the straight lines

represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

is (α, β)

$$\text{where } \alpha = \frac{hf - bg}{ab - h^2} \quad \frac{1}{\alpha} = \beta \quad \alpha \beta = d$$

$$\beta = \frac{gh - af}{ab - h^2} \quad \frac{1}{\beta} = \alpha \quad (\text{মুক্ত}) \quad \frac{1}{\alpha} \cdot \frac{1}{\beta} = d$$

$$(d - \beta d)^2 = \beta^2 - d\beta^2 + d^2 = 0 \quad \therefore$$

Exm ৭ আজবে:

Problem:

Prove that the eqn $12x^2 + 7xy - 12y^2 - x + 7y - 1 = 0$

represents a pair of straight lines.

Find also their equations, point of intersection and the angle between them for a work sheet

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The eqn of the straight lines represented by the given eqn are

for a work sheet

Solⁿ: Given that,

$$12x^2 + 7xy - 12y^2 - x + 7y - 1 = 0 \dots \text{Eqn. 1}$$

Comparing Eqn. 1 with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$,

$$a = 12$$

$$c = -1$$

$$b = -12$$

$$g = -\frac{1}{2}$$

$$h = \frac{7}{2}$$

$$f = \frac{7}{2}$$

$$\therefore \Delta = abc + 2fg - af^2 - bg^2 - ch^2$$

$$= (12) \cdot (-12) \cdot (-1) + 2 \left(\frac{7}{2} \right) \cdot \left(-\frac{1}{2} \right) \cdot \left(\frac{7}{2} \right) - 12 \left(\frac{7}{2} \right)^2$$

$$- (-12) \cdot \left(-\frac{1}{2} \right)^2 - (-1) \cdot \left(\frac{7}{2} \right)^2$$

which shows that the given eqn. represents a pair of straight lines.

Now,

$$12x^2 + (7y-1)x + (-12y^2 + 7y - 1) = 0$$

$$\Rightarrow 12x^2 + (7y-1)x - (12y^2 - 7y + 1) = 0$$

$$(\frac{1}{2} - \frac{1}{2}) \cdot (\frac{1}{2} - \frac{1}{2}) = \frac{1}{4} \cdot \frac{1}{4}$$

$$\therefore x = \frac{-(7y-1) \pm \sqrt{(7y-1)^2 + 4 \cdot 12 \cdot (12y^2 - 7y + 1)}}{2 \cdot 12} = \frac{-(7y-1) \pm \sqrt{49y^2 - 14y + 1 + 48y^2 - 336y + 48}}{24} = \frac{-(7y-1) \pm \sqrt{625y^2 - 350y + 49}}{24}$$

$$\Rightarrow 24x + 7y - 1 = \pm \sqrt{49y^2 - 14y + 1 + 576y^2 - 336y + 48}$$

$$= \pm \sqrt{625y^2 - 350y + 49}$$

$$\frac{24x + 7y - 1}{\sqrt{625y^2 - 350y + 49}} = \pm \sqrt{(25y - 7)^2}$$

$$\therefore 24x + 7y - 1 = \pm (25y - 7)$$

(\downarrow) \therefore the straight lines represented by eqn ① are

$$\therefore 24x + 7y - 1 = -25y + 7$$

$$4x - 3y + 1 = 0$$

$$\Rightarrow 24x + 32y - 8 = 0$$

$$\Rightarrow 3x + 4y - 1 = 0$$

\therefore The eqns of the straight lines represented by

the given eqn ① are

$$4x - 3y + 1 = 0$$

$$3x + 4y - 1 = 0$$

$$\frac{4}{3} \text{ and } =$$

$$\cdot \frac{4}{3} = 0$$

Let (α, β) be the point of intersection of these straight lines.

where

$$\alpha = \frac{hf - bg}{ab - h^2} = \frac{\frac{7}{2} \cdot \frac{7}{2} - (-12) \cdot (-\frac{1}{2})}{12 \cdot (-12) - (\frac{7}{2})^2} = 10$$

$$O = 1 + f\beta - xP = \frac{49}{4} + -\frac{144}{4} - \frac{49}{4}$$

$$(f - f\beta) \sqrt{1 + f^2} = \frac{25}{527}$$

$$\therefore \beta = \text{_____}$$

$$(f - f\beta) \pm = hf + xP \therefore$$

Let θ be the angle between the straight lines.

$$O = 1 + f\beta - xP$$

$$1 + f\beta - xP = hf + xP \therefore$$

$$\therefore \tan \theta = \frac{2 \sqrt{h^2 - ab}}{a+b}$$

$$O = 8 - f\beta \pm + xP \therefore$$

$$= \frac{2 \sqrt{(\frac{7}{2})^2 - (12)(-12)}}{12 - 12}$$

$$O = 1 - f\beta + xP \therefore$$

for best answer omit this part to appo edit.

$$= \alpha$$

then ① P is moving left

$$= \tan \frac{\pi}{2}$$

$$O = 1 + f\beta - xP$$

$$\therefore \theta = \frac{\pi}{2}$$

Q. Find the value of λ for which the eqn $12x^2 - 10xy + 2y^2 + 11x - 5y + \lambda = 0$ represents two straight lines. Also find the angle between them.

Soln: Comparing the given eqn with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ we get,

$$a = 12, b = 2, h = -\frac{1}{2}, g = \frac{11}{2}, f = -\frac{5}{2}, c = \lambda$$

As the given eqn represents a pair of straight lines, thus

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\Rightarrow 12 \cdot 2 \cdot \lambda + 2 \cdot \left(-\frac{5}{2}\right) \cdot \frac{1}{2} \cdot \left(-\frac{5}{2}\right) - 12 \cdot \left(-\frac{5}{2}\right)^2 - 2 \cdot \left(\frac{1}{2}\right)^2 = 0$$

$$\therefore \lambda = 2$$

From left) (θ -be the angle between the straight lines represented by eqn ①

$$= \sqrt{144 - 100} = \frac{2\sqrt{144 - 100}}{12 + 2} = \frac{2\sqrt{25 - 24}}{12 + 2}$$

$$\therefore \tan \theta = \frac{2\sqrt{25 - 24}}{12 + 2} = \frac{2\sqrt{1}}{14} = \frac{1}{7}$$

$$\therefore \theta = \tan^{-1} \left(\frac{1}{7} \right)$$

$$\Rightarrow \tan \theta = \frac{1}{7}$$

If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines, prove that the product of perpendiculars from the origin to the straight lines is

$$d = \sqrt{(a-b)^2 + 4h^2}$$

Soln:

Given that,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \text{... eqn 1}$$

Let the straight lines represented by eqn 1 be

$$y = m_1x + c_1 \quad \text{and} \quad y = m_2x + c_2$$

Then,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = b(y - m_1x - c_1)(y - m_2x - c_2)$$

$$\text{... eqn 1} = b[y^2 - (m_1 + m_2)xy +$$

$$\frac{1}{4}(m_1^2 + m_2^2)x^2 + (c_1m_2 + c_2m_1)x - (c_1 + c_2)y + m_1m_2x^2 + c_1c_2]$$

Comparing the coefficients of like terms on both sides we obtain

$$\frac{1}{4}(m_1^2 + m_2^2) = 0 \Rightarrow m_1^2 + m_2^2 = 0$$

$$m_1 + m_2 = -\frac{2h}{b}$$

$$c_1 m_2 + c_2 m_1 = \frac{2g}{b}$$

$$c_1 + c_2 = -\frac{2f}{b}$$

$$m_1 m_2 = \frac{a}{b}$$

$$c_1 c_2 = \frac{c}{b}$$

Now the perpendicular distance from the origin to the straight lines $y - m_1 x - c_1 = 0$ is $\frac{|c_1|}{\sqrt{1+m_1^2}}$

also the perpendicular distance from the origin to the straight lines $y - m_2 x - c_2 = 0$ is $\frac{|c_2|}{\sqrt{1+m_2^2}}$

According to the question, $(EP - d_{AP})k = EP \cdot AP$: Given
the product of the perpendiculars = $\frac{-c_1}{\sqrt{1+m_1^2}} \cdot \frac{-c_2}{\sqrt{1+m_2^2}}$

$$= \frac{c_1 c_2}{\sqrt{1+m_1^2 + m_2^2 + (m_1 m_2)^2}}$$

$$= \frac{\cancel{b} c_1 c_2}{\sqrt{1 + (m_1 + m_2)^2 - 2m_1 m_2 + (m_1 m_2)^2}}$$

$$= \frac{\frac{c}{b}}{\sqrt{1 + (-\frac{2b}{b})^2 - 2 \cdot \frac{a}{b} - (\frac{a}{b})^2}}$$

$$= \frac{\frac{c}{b}}{\sqrt{1 + \frac{4b^2}{b^2} - 2 \cdot \frac{a}{b} - \frac{a^2}{b^2}}}$$

of rimpino $\frac{c}{b}$ most ammabib evoluisibsqeqnog satt won
 $\frac{\sqrt{b^2 + 4b^2 - 2ab + a^2}}{\sqrt{a^2 + b^2}}$ si $O = \varrho - xim - y$, esemil tdpiorre satt

of rimpino $\frac{c}{\sqrt{(a-b)^2 + 4b^2}}$ (Proved) ibrisqasq satt oalo
 $\frac{\varrho -}{\sqrt{a^2 + b^2}}$, si $O = \varrho - xim - y$, esemil tdpiorre satt

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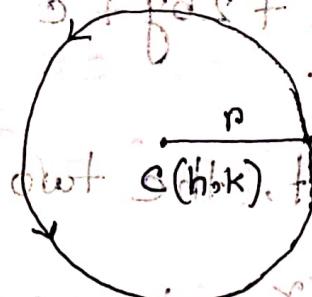
$\frac{\varrho -}{\sqrt{a^2 + b^2}} =$ evoluisibsqeqnog satt to fowbom satt

$$\frac{c}{\sin(\alpha) + \cos(\alpha) + i(\alpha + \beta)}$$

$$\frac{\varrho -}{1 + \sin(\alpha) - i(\alpha + \beta) + i\sqrt{1 - \sin^2(\alpha)}}$$

Circles and system of circles: (1 set of Q) ①

A circle in standard form $O = x^2 + y^2 + 2gx + 2fy + c = 0$



$O = x^2 + y^2 + 2gx + 2fy + c = 0$ is the general form of a circle about $C(h, k)$. To find the center

Let $O = x^2 + y^2 + 2gx + 2fy + c = 0$ be the general form of a circle.

Given that the center of the circle is (h, k) .

First, we will find the value of $x^2 + y^2 + 2gx + 2fy + c$ through

the points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , ..., (x_n, y_n) which lie on the circle.

Let (x_1, y_1) be one point on the circle. Then, the center of the circle is

the point (h, k) such that $(x_1 - h)^2 + (y_1 - k)^2 = r^2$.

Now, we will find the value of $x^2 + y^2 + 2gx + 2fy + c$ through the points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , ..., (x_n, y_n) which lie on the circle.

Let (x_1, y_1) be one point on the circle. Then, the center of the circle is

the point (h, k) such that $(x_1 - h)^2 + (y_1 - k)^2 = r^2$.

Thus, the center of the circle is (h, k) .

$$\sqrt{(x-h)^2 + (y-k)^2} = \sqrt{(x_1-h)^2 + (y_1-k)^2}$$

$$\sqrt{(x-h)^2 + (y-k)^2} = \sqrt{(x_1-h)^2 + (y_1-k)^2}$$

$$\sqrt{(x-h)^2 + (y-k)^2} = \sqrt{r^2}$$

① □ Prove that, the two circles $x^2 + y^2 + 2ax + c^2 = 0$
 and $x^2 + y^2 + 2by + c^2 = 0$ may touch if $\frac{1}{a^2} + \frac{1}{b^2} = 1$

② □ Prove that, the two circles $x^2 + y^2 + 2ax + c = 0$
 and $x^2 + y^2 + 2bx + c = 0$ may touch if $c = 0$ or $a = b$

① Solⁿ:

Given that,

$$x^2 + y^2 + 2ax + c^2 = 0$$

$$x^2 + y^2 + 2by + c^2 = 0$$

the centres of the circles are $(-a, 0)$ and $(0, -b)$
 and respectively and their radii are $\sqrt{a^2 - c^2}$ and
 $\sqrt{b^2 - c^2}$ respectively.

If the given circles touch each other, then the
 distance between their centres = sum or difference
 of their radii.

$$\Rightarrow \sqrt{(-a-0)^2 + (0+b)^2} = \sqrt{a^2 - c^2} \pm \sqrt{b^2 - c^2}$$

$$\Rightarrow a^2 + b^2 = a^2 - c^2 + b^2 - c^2 \pm 2\sqrt{a^2 - c^2} \cdot \sqrt{b^2 - c^2}$$

$$\Rightarrow c^2 = \pm \sqrt{a^2 - c^2} \cdot \sqrt{b^2 - c^2}$$

$$\Rightarrow c^4 = (a^2 - c^2)(b^2 - c^2)$$

if you subtract it from both sides

$$\Rightarrow c^4 = a^2b^2 - a^2c^2 - b^2c^2 + c^4$$

then factorise out c^2 to

$$\Rightarrow a^2b^2 - b^2c^2 - a^2c^2 = 0 \quad \text{both sides} \Rightarrow (a-b)^2 + (a+b)^2 = 0$$

$$\Rightarrow \frac{1}{c^2} - \frac{1}{a^2} - \frac{1}{b^2} = 0 \quad \text{or} \quad \frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

∴ $\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (Proved)

left to bonds remains left to original left hand

- Find the eqn of the circle which passes through the points $(1, 2)$, $(3, 4)$ and touches the line $3x+y-3=0$

: Mod ①

- * □ Find the eqn of the tangent to the circle $x^2 + y^2 - 4x + 6y - 3 = 0$, which are parallel to the straight line $3x - 4y + 9 = 0$

- * □ Find the eqn of the common chord of the circles $(x-a)^2 + y^2 = a^2$, $x^2 + (y-b)^2 = b^2$ and show that the circle described on the common chord as diameter is $(a^2 + b^2)(x^2 + y^2) = 2ab(bx + ay)$

$$O = (b-x)(d-y)$$

Solⁿ:

$$(x-a)(x-b) = P$$

□ Prove that the length of the common chord of the two circles whose equations are

$$(x-a)^2 + (y-b)^2 = c^2 \text{ and } (x-b)^2 + (y-a)^2 = c^2$$

is $\sqrt{4c^2 - 2(a-b)^2}$ where $a \neq b$

$$(\text{because}) \quad \frac{1}{r_d} + \frac{1}{r_o} = \frac{1}{r_c}$$

□ Find the length of the common chord of the two circles $x^2 + y^2 - 12x + 16y + 16 = 0$ and $x^2 + y^2 - 9x + 12y + 59 = 0$.

Given eqns of the circles are $x^2 + y^2 - 12x + 16y + 16 = 0$ and $x^2 + y^2 - 9x + 12y + 59 = 0$.

① Solⁿ:

Given eqns of the circles are $x^2 + y^2 - 12x + 16y + 16 = 0$ and $x^2 + y^2 - 9x + 12y + 59 = 0$.

$$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - c^2 = 0 \dots \text{①} \text{ and}$$

$$x^2 + y^2 - 2bx - 2ay + b^2 + a^2 - c^2 = 0 \dots \text{②}$$

∴ The eqn of the common chord is

$$-2ax - 2by + 2bx + 2ay = 0$$

$$\Rightarrow ax + by - bx - ay = 0$$

$$\Rightarrow (a+b)x - (a+b)y = 0$$

$$\Rightarrow (a+b)(x-y) = 0$$

$$\therefore x-y=0 \dots \dots \text{③}$$

Let the centre of the circle given by eqn ① is $C_1(a, b)$. Also let C_1D be the perpendicular distance from the centre to the common chord AB , where D is the middle point of AB .

$$\therefore C_1D = \frac{a-b}{\sqrt{1+1}} = \frac{a-b}{\sqrt{2}}$$

Now from the right angle triangle AC_1D , we get,

$$AC_1^2 = C_1D^2 + AD^2$$

$$\Rightarrow AD^2 = AC_1^2 - C_1D^2$$

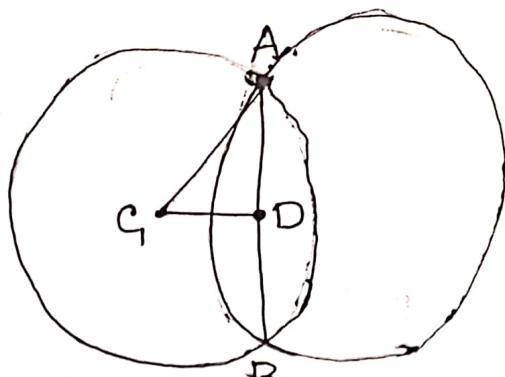
$$= c^2 - \left(\frac{a-b}{\sqrt{2}}\right)^2$$

$$= c^2 - \frac{(a-b)^2}{2}$$

$$\therefore AB = 2AD$$

$$= 2 \left[\sqrt{\frac{2c^2 - (a-b)^2}{2}} \right] = \sqrt{4c^2 - 2(a-b)^2}.$$

$$= 2 \sqrt{\frac{4c^2 - 2(a-b)^2}{2}}$$



Orthogonal circles: $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cut each other orthogonally.

Define Orthogonal circles. Find the condition that the two circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ cut each other orthogonally.

Find the eqn of the circle which cuts the circle

$x^2 + y^2 - 3x - 4y + 5 = 0$ and $4x^2 + 4y^2 - 7x + 8y + 11 = 0$ orthogonally and passes through the point $(2, -1)$

Solⁿ:

Given that,

$$x^2 + y^2 - 3x - 4y + 5 = 0 \dots \text{... } (1)$$

$$4x^2 + 4y^2 - 7x + 8y + 11 = 0 \dots \text{... } (2)$$

Let the eqn of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \dots \text{... } (3)$$

Since the circle (3) cuts the circles given by eqns (1) and (2) orthogonally.

Thus,

$$2g \cdot (-\frac{3}{2}) + 2f \cdot (-2) = c + 5$$

$$\Rightarrow 3g + 4f = -c - 5 \dots \text{... } (4)$$

১১-১-১০

and

$$2g \cdot (-\frac{7}{8}) + 2f \cdot 1 = c + \frac{11}{4} \text{ সেলরিস লম্বোপর্টন}$$

ফর্মুলা $\Rightarrow \frac{7}{4}g - 2f + c + \frac{11}{4} \text{ সেলরিস লম্বোপর্টন}$

$$\Rightarrow \frac{7}{4}g - 2f = c - \frac{11}{4} \text{ সেলরিস অফ ফর্মুলা}$$

ব্রেস $\Rightarrow 7g - 8f = -4c - 11 \dots \text{V}$

Also the circle (III) passes through the points

$$C(2, -1) \text{ Then } yf + xf \text{ bres } 0 = \bar{e} + yf - xf - y + x$$

$$(2, -1) \text{ ত্রিভুজ } + 1 + 8g - 2f + c = 0 \text{ bres ফর্মুলা}$$

$$\Rightarrow 8g - 2f = -c - 5 \dots \text{VI}$$

g, f, c এর value করে করে eqn (III) থেকে প্রাপ্ত হবে।

$$\text{II} \dots \dots \dots 0 = 11 + yf + xf - yf + xf$$

এখন সেলরিস অফ ফর্মুলা কি?

$$\text{III} \dots \dots \dots 0 = \bar{e} + yf + xf + y + x$$

ফর্মুলা নথিগ সেলরিস অফ কি? (III) সেলরিস অফ কি?
 ফর্মুলা (II) ব্রেস (I)

$$\bar{e} + \bar{g} = (\bar{x} -) \cdot f \cdot \bar{e} + (\bar{y} -) \cdot g \cdot \bar{e} \text{ একটি}$$

VI

* * * \square Prove that two circles that pass through the point $(0, a)$, $(0, -a)$ and touch the line $y = mx + c$ will cut orthogonally if $c^2 = a^2(m^2 + 2)^2$.

Soln:

$$O = (x_0 - m^2 a^2 + r^2) + pmr^2 + r^2 \beta$$

Let the eqns of the circles be

$$x^2 + y^2 + 2gx + 2fy + k = 0 \quad \dots \text{①}$$

Since the circle given by eqn ① passes through the point $(0, a)$ and $(0, -a)$

Then

$$a^2 + 2fa + k = 0 \quad \dots \text{②}$$

$$a^2 - 2fa + k = 0 \quad \dots \text{③}$$

Solving eqns ② and ③ we get,

$$k = -a^2, f = 0$$

$$O = x_0 - x_1 p^2 + r^2 \beta + r^2 x$$

$$\therefore \text{Eqn ①} \Rightarrow x^2 + y^2 + 2gx - a^2 = 0 \quad \dots \text{④}$$

The centre and radius of the circle given by eqn ④ is $(-g, 0)$ and $\sqrt{g^2 + a^2}$

According to the question, $b \cdot O \cdot S + r^2 \beta = 0$

$$\frac{mg - c}{\sqrt{1+m^2}} = \sqrt{g^2 + a^2} \quad x_0 - = r^2 \beta \leftarrow$$

P.T.O

friction coeff

$$\Rightarrow \frac{c(mg - c)}{B} = g + a^2$$

$\Rightarrow \frac{c + cm}{B} = 1 + m^2$ \Rightarrow coeff of friction c is same for both circles

$$\Rightarrow mg(-2cmg + c^2) = mg^2 + g^2 + a^2 + a^2m^2$$

$$\Rightarrow g^2 + 2cmg + (a^2 + a^2m^2 - c^2) = 0$$

which is a quadratic eqn in g , $0 = k + ft^2 + x^2 + f^2x$

Let the roots of this eqn be g_1 and g_2 $(\omega - \omega)$ b/w (ω, ω) fric.

$$g_1 + g_2 = -2cm \quad \text{... (4)}$$

$$g_1 g_2 = a^2 + a^2m^2 - c^2 \quad \text{... (5)}$$

\therefore Eqn (4) reduces to -

$$x^2 + y^2 + 2g_1 x - a^2 = 0$$

$$x^2 + y^2 + 2g_2 x - a^2 = 0 \quad \Leftrightarrow \text{Eqn (5)}$$

Since these two circles cut each other orthogonally,

Thus $x^2 + y^2 + 2g_1 x - a^2 = 0$ is Eqn (4)

$$2g_1 g_2 + 2 \cdot 0 \cdot 0 = -a^2 - a^2$$

$$\Rightarrow g_1 g_2 = -a^2 \quad \frac{x^2 + y^2}{B} = \frac{-a^2 - a^2}{m + 1}$$

O.T.Q

$$\Rightarrow a^2 + a^2 m^2 - c^2 = -a^2$$

$$\Rightarrow c^2 = 2a^2 + a^2 m^2$$

Divide by $a^2(m^2+2)$ to get radical axis

$\therefore O = f_1 - f_2 + f_1 f_2$ is the equation of radical axis

□ Radical axis:

$O = f_1 - f_2 + f_1 f_2$

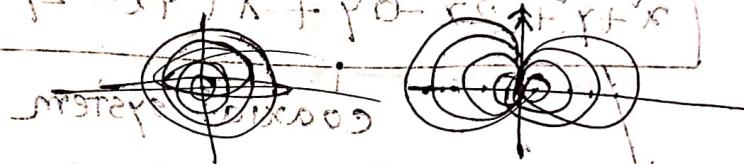
$$2(f_1 - f_2)x_1 + 2(f_1 f_2)y_1 + f_1 c_1 - f_2 c_2 = 0$$

→ radical axis इसे जीवन

common chord & radical axis same eqn

$$O = C - C' - x_1 P$$

Coaxial circles: $(x - f_1)^2 + (y - g_1)^2 = c_1^2$



$$x^2 + y^2 + 2gx + g^2 + c_1^2 - g^2 = 0 \rightarrow \text{radical axis}$$

$$\Rightarrow (x+g)^2 + y^2 = g^2 - c_1^2$$

$$\Rightarrow (x - (-g))^2 + (y - 0)^2 = g^2 - c_1^2 \rightarrow \text{coaxial axis}$$

$$g^2 = c_1^2 \quad \boxed{(1, c), (2, 0), (-g, 0)}$$

$$\Rightarrow g = \pm \sqrt{c} \quad \begin{array}{l} (-\sqrt{c}, 0) \\ (\sqrt{c}, 0) \end{array}$$

$$(\sqrt{c}, 0)$$

Problem:

Define limiting points of a coaxial system.

Find the limiting points of (the) coaxial systems of circles $x^2 + y^2 + 2x - 6y = 0$,

$$O = x^2 + 2x^2 + (2y^2 - 10y + 5) = 0$$

even looped

→ circle has two centers

(two points where $r_1 = r_2$)

→ if $r_1 = r_2$ (two lines)

$$4x - 2y - 5 = 0$$

radius = 0
point circle

$$\boxed{x^2 + y^2 + 2x - 6y + \lambda(4x - 2y - 5) = 0} \text{ no loops}$$

coaxial system

এখন center ও radius কে বর্ণনা করো। এবং

radius = 0 কিরণ করো।

$$\lambda = 0$$

$$O = x^2 + y^2 + 2x - 6y + 0 = 0$$

$$r^2 - p^2 = x^2 + y^2 - (x+2)^2 - (y-3)^2 = 0$$

$$(x^2 + y^2 + 2x - 6y + (2+4\lambda)x + (-6-2\lambda)y - 5\lambda = 0)$$

$$\boxed{(1, 2), (3, 1)}$$

$$\lambda = 0$$

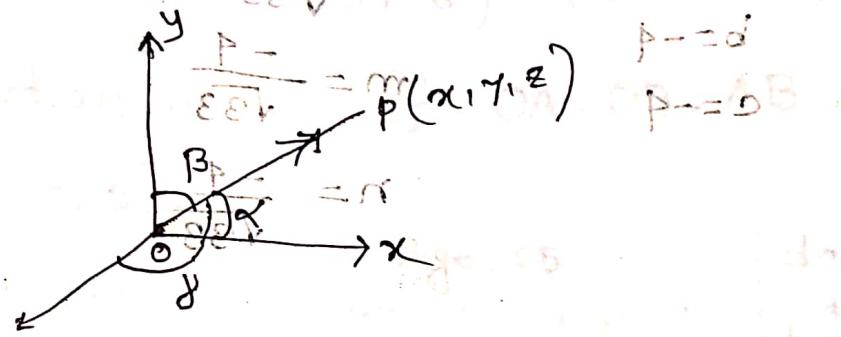
$$(0, 5)$$

$$(0, 1)$$

Three Dimensional Coordinate Geometry

বৰ্ষাৰ
10 AM CT

কোন একটি রেখা x অক্ষের, y , এবং (z) অক্ষের সমূহে ধনাত্মক দিক যে কোন ড্রুপন করে তাৰ cos এবং জ্ঞানকে বলা হয়. Direction Cosine.



$$\text{প্রতিটো } \pm \frac{z}{\sqrt{x^2+y^2+z^2}} = \cos \alpha \rightarrow 1, 0, 0$$

$$m = \cos \beta \rightarrow 0, 1, 0$$

$$n = \cos \gamma \rightarrow 0, 0, 1$$

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}$$

direction ratio (a, b, c)
direction cosine (l, m, n)

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

(cont'd.)

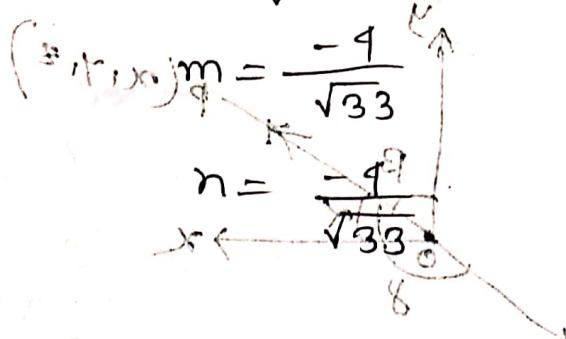
$P \equiv (2, \frac{1}{7}, 9)$ এর স্থানীয় ক্রমিক অনুপাত ক্ষেত্রের Direction ratio কোণ কোণ।

क्रमांक १०) $(1, 3, 5)$ ज्या \cosine ज्याएँ नाही

$$m_{12345} = \frac{-1}{\sqrt{33}} \quad \text{চল রংক ক্ষেত্র পরিসীমা মান}$$

$$b = -4$$

$$c = -4$$



$\therefore l^m + m^n + n^l \rightleftharpoons$ এর মীন অবস্থা ১ আয়ে।

$$= \frac{1}{33} \left(1 + \frac{16}{33} \right) + \frac{16}{33} \cdot 209 = \alpha$$

$$= 1(b \cdot d \cdot n) \text{ of the numbers} \quad \frac{m}{d} = \frac{m}{b} = \frac{1}{b}$$

$$(m, m, b) \text{ among the numbers} \quad \frac{m}{b} = \frac{m}{d} = \frac{1}{b}$$

$$\frac{\sqrt{a^2 + b^2 + c^2}}{\sqrt{d^2 + e^2 + f^2}} = \frac{r}{s} \equiv \frac{m}{n} = \frac{\lambda}{\delta}$$

$$\frac{m}{5} = \frac{m}{6} = \frac{l}{10} \quad \text{LHS}$$

$$\frac{d}{dx} \int_{\gamma_1}^x f(t) dt = f(x)$$

$$\frac{b^2}{4} + \frac{c^2}{4} + \frac{d^2}{4} = m$$

* Define Direction cosine and ratios of line.

Prove that $l^2 + m^2 + n^2 = 1$.

If A and B are $(2, 3, -6), (3, -4, 5)$,

Find the direction cosine of OA, OB, AB ,
where O is $(0, 0, 0)$

Page: 25

de's
↓
direction
cosines.

~~marks~~ If θ be the angle between the straight lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2)

Then show that ① $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ or if

$$\text{Also } \sin \theta = \pm \sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2}$$

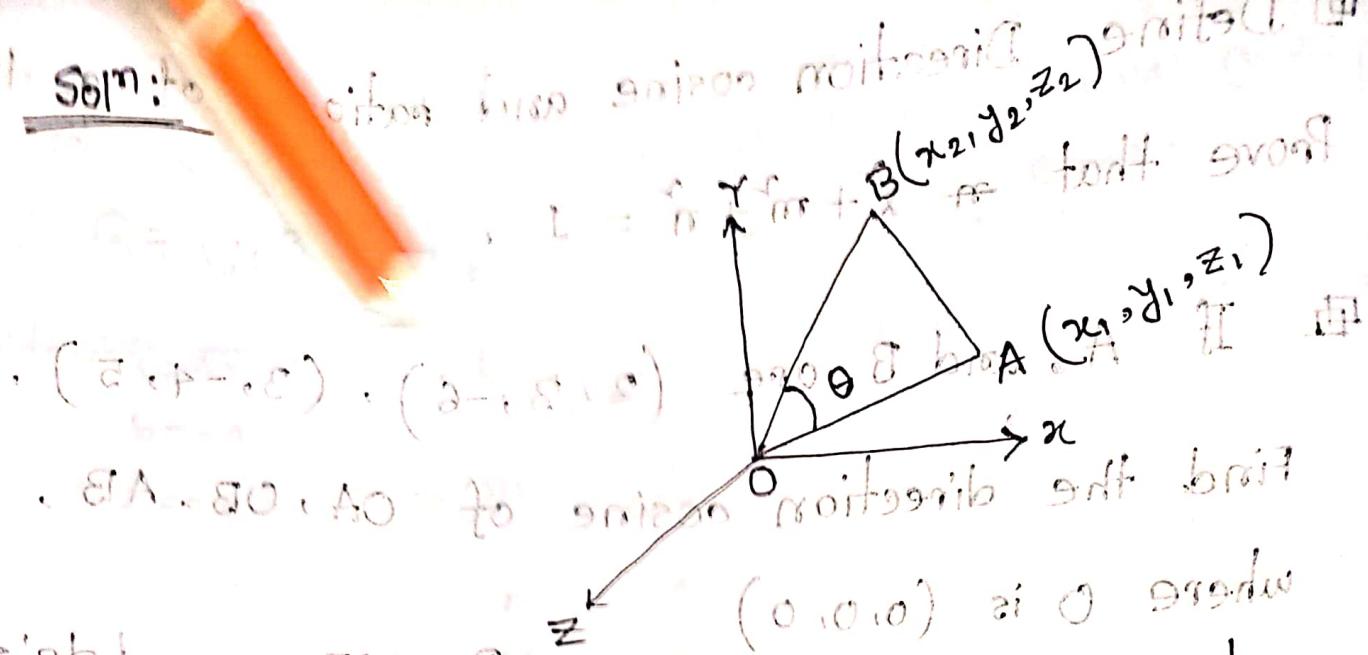
Also find the condition of Perpendicularity and

Parallelism.

$$\text{① } \dots \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$\text{② } \dots \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

Soln:-



Let OA and OB are two straight lines, whose

direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) respectively.

Also let the coordinates of A and B are

(x_1, y_1, z_1) and (x_2, y_2, z_2) respectively.

Then we get,

$$\frac{l_1}{x_1} = \frac{m_1}{y_1} = \frac{n_1}{z_1} \quad \dots \dots \textcircled{1}$$

$$\frac{l_2}{x_2} = \frac{m_2}{y_2} = \frac{n_2}{z_2} \quad \dots \dots \textcircled{11}$$

Eq ① \Rightarrow

$$\frac{l_1}{m_1} = \frac{m_1}{y_1} = \frac{n_1}{z_1} = \frac{\sqrt{l_1^2 + m_1^2 + n_1^2}}{\sqrt{x_1^2 + y_1^2 + z_1^2}} = \frac{1}{OA}$$

$$\theta^{200} - 1 = \theta^{200} \quad \boxed{\therefore l_1^2 + m_1^2 + n_1^2 = 1}$$

$$\Rightarrow x_1 = l_1 \cdot OA \quad (l_1, m_1, n_1) \cdot (A, B + Z_m + n_1) \cdot OA$$

Similarly, Eq ① \Rightarrow $x_2 = l_2 \cdot OB$

$$\frac{l_2}{x_2} = \frac{m_2}{y_2} = \frac{n_2}{z_2} = \frac{\sqrt{l_2^2 + m_2^2 + n_2^2}}{\sqrt{x_2^2 + y_2^2 + z_2^2}} = \frac{1}{OB}$$

$$\theta^{200} - 1 = \theta^{200} \quad \text{Eq ①} \Rightarrow x_1 = l_1 \cdot OA$$

$$x_2 = l_2 \cdot OB, \quad y_2 = m_2 \cdot OB, \quad z_2 = n_2 \cdot OB$$

Now, from the triangle $\triangle OAB$, we obtain,

$$\text{as for } OA^2 + OB^2 - AB^2 = 2 \cdot OA \cdot OB \cdot \cos \theta \quad \text{or } \theta = 90^\circ$$

$$\Rightarrow 2 \cdot OA \cdot OB \cdot \cos \theta = OA^2 + OB^2 - AB^2 \quad \text{or } \cos \theta = 0$$

$$\Rightarrow 2 \cdot OA \cdot OB \cdot \cos \theta = (x_1^2 + y_1^2 + z_1^2) + (x_2^2 + y_2^2 + z_2^2)$$

$$\text{as for } l_1^2 + l_2^2 - l^2 = (x_1^2 + y_1^2 + z_1^2) + (x_2^2 + y_2^2 + z_2^2)$$

$$\Rightarrow 2 \cdot OA \cdot OB \cdot \cos \theta = 2(x_1^2 + y_1^2 + z_1^2) + 2(x_2^2 + y_2^2 + z_2^2)$$

$$0 = (l_1^2 + l_2^2 - l^2) + (m_1^2 + m_2^2 - m^2) + (n_1^2 + n_2^2 - n^2)$$

$$\Rightarrow \cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2 \dots \dots \text{.....(11)} \Leftarrow \text{① } \text{प्र}$$

2nd Part Soln: $\frac{l}{AO} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{s^2 + b^2 + r^2}} = \frac{10}{\sqrt{18}} = \frac{10}{\sqrt{18}} = \frac{10}{\sqrt{18}}$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$A = \left(l_1^2 + m_1^2 + n_1^2 \right) \left(l_2^2 + m_2^2 + n_2^2 \right) - \frac{\left(l_1 l_2 + m_1 m_2 + n_1 n_2 \right)^2}{AO \cdot l_1 l_2} = 18 \text{ } \text{.....(2)}$$

$$= (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2$$

$$\frac{l}{AO} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{s^2 + b^2 + r^2}} = \frac{l}{\sqrt{18}} = \frac{l}{\sqrt{18}} \text{ [By Lagrange's identity]}$$

$$\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2 \dots \dots \text{.....(4)}$$

$$\therefore \sin \theta = \pm \sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2} \dots$$

मिति से यहां पर्याप्त अधिकारी का नाम जॉन वॉल्टर है।(5)

case - 1: If the two lines are perpendicular that is

$$\theta = 90^\circ \text{ then eqn (4) } \Rightarrow \theta 200 \cdot 80 \cdot AO \cdot s \Leftarrow$$

$$(s^2 + b^2 + r^2)_2 + (m_1^2 + m_2^2 + n_1^2 + n_2^2) = 0 \cdot 200 \cdot 80 \cdot AO \cdot s \Leftarrow$$

case - 2: If the two lines are parallel that is

$$\theta = 0^\circ \text{ then eqn (5) } \Rightarrow \theta 200 \cdot 80 \cdot AO \cdot s \Leftarrow$$

$$200 \cdot 80 \cdot AO \cdot s \sqrt{\frac{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2}{(s^2 + b^2 + r^2) \cdot AO \cdot s}} = 0$$

$$m_1 n_2 - m_2 n_1 = 0$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$n_1 d_2 - n_2 d_1 = 0$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{n_1}{n_2}$$

$$l_1 m_2 - m_1 l_2 = 0$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{m_1}{m_2}$$

similarly

$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}}$$

standard or perpendicular

angle between

$$0 = \pi - 180^\circ$$

$$0 = x - 90^\circ$$

$$0 = y - 90^\circ$$

$$0 = b + 90^\circ + 90^\circ + 90^\circ$$

$$0 = b + 90^\circ + 90^\circ + 90^\circ$$

Q Prove that the acute angle between the straight lines whose direction cosines are

given by the relations $l+m+n=0 \dots (1)$

ज्याति नहीं सिर्फ़ लाइन का दिशा कोण है

$l^2 + m^2 + n^2 = 0 \dots (2)$

$\frac{\pi}{3}$

32 page
example-6

~~Eliminating n between~~

$$\frac{a_1}{l} = \frac{a_2}{m}$$

The Plane

(অন্তল)

2 dimensional যেকোন straight line 3rd dimension

এ এটা plane.

$$\frac{a_1}{l} = \frac{a_2}{m} = \frac{a_3}{n}$$

$$xy \rightarrow z=0$$

$$yz \rightarrow x=0$$

$$zx \rightarrow y=0$$

0 = 3D line - plane

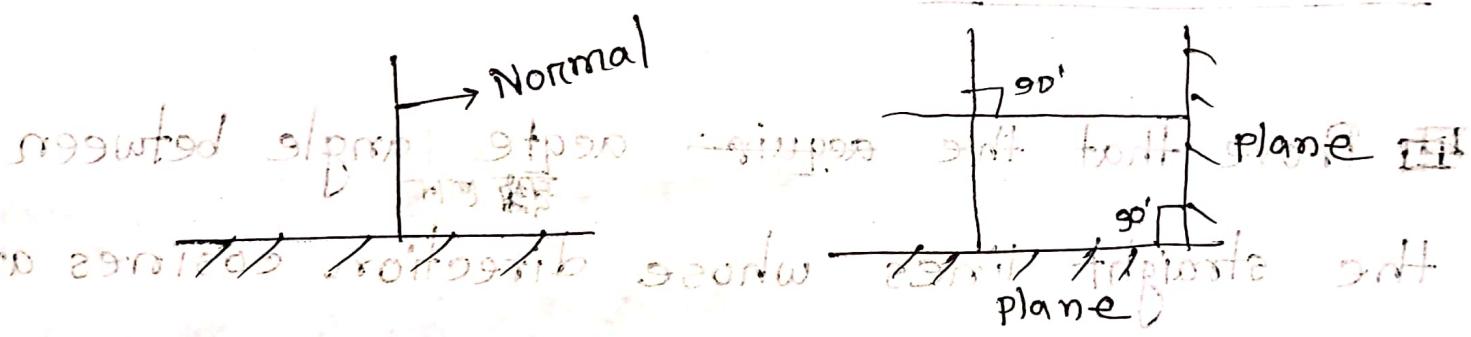
$$A_1x + B_1y + C_1z + d_1 = 0$$

$$A_2x + B_2y + C_2z + d_2 = 0$$

$$[A_1, B_1, C_1, A_2, B_2, C_2]$$

direction ratios

$$\frac{a_1}{l} = \frac{a_2}{m} = \frac{a_3}{n}$$

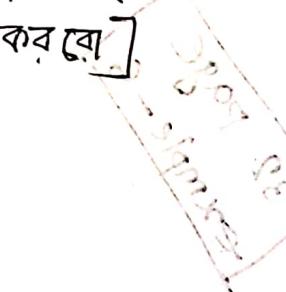


$$\cos \theta = \frac{l_1l_2 + m_1m_2 + n_1n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \sqrt{l_2^2 + m_2^2 + n_2^2}}$$

$$\text{এবং } \theta = \sqrt{l_1^2 + m_1^2 + n_1^2}$$

[cosine কোণ থাকলে]

এটা use করবো]



$$\cos \theta = \frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \sqrt{A_2^2 + B_2^2 + C_2^2}}$$

$$\Rightarrow A_1 A_2 + B_1 B_2 + C_1 C_2 = 0 \quad [1]$$

□ ~~stra~~ Page \rightarrow 13 (ex-1(e))

" \rightarrow 14 (3(a))

" \rightarrow 19 (19)

" \rightarrow 74 (6(c))

" \rightarrow 76 (8(b))

circle পুরাণী

general eqn of 2nd degree \rightarrow CT
বর্ম - circle পর্যন্ত

Assignment : Change of axis [Topic Name]

2টি Theorem

6টি problem

2টি exam আছে।

Submission date : 18. ~~Feb~~