Jodall

Bisection method

Algorithm

(i) start

(ii) Define function $f(x)$

(iii) choose initial guess x_0 and x_1

(iv) Read tolerable error ϵ

(v) Calculate new approximated root as, $x_2 = \frac{x_0 + x_1}{2}$

(vi) If $f(x_2) = 0$ Then goto step 8.

if $f(x_2) > 0$ Then $x_1 = x_2$

if $f(x_2) < 0$ Then $x_0 = x_2$

$$\text{Error} = \frac{\text{new value} - \text{old value}}{\text{new value}}$$

(7) If error > ϵ Then goto 5 otherwise goto 8.

(8) Display x_2 as root

(9) stop

* Example : I

$f(x) = x^2 - d$ where $d = 25$ and $x_0 = 2$ and $x_1 = 7$, solve this using bysection method.

Soln :

Given that

$$f(x) = x^2 - d = 0$$

$$d = 25, x_1 = 7, x_0 = 2$$

let's find the root of the function using this value for(x_2) ↑

iteration	x_0	x_1	x_2	$f(x_0)$	$f(x_1)$	$f(x_2)$	Error
0	2.0000	7.0000	4.5000	-21	24	-4.7500	1
1	4.5000	7.0000	5.7500	-4.7500	24	8.0625	0.2173
2	4.5000	5.7500	5.1250	-4.7500	8.0625	1.2656	0.1219
3	4.5000	5.1250	4.8125	-4.7500	1.2656	-1.8398	0.0649
4	4.8125	5.1250	4.9687	-1.8398	1.2656	-0.3120	0.0314
5	4.9687	5.1250	5.0468	-0.3120	1.2656	0.4706	0.0154

Ans:

Exm 2: $f(x) = \sin 5x + \cos 2x$ where $x_0 = -0.6$

$x_1 = -0.5$ Error $= 0.0005$ solve this using bysection method.

$f(x) = \sin 5x + \cos 2x$; $\epsilon_s = 0.0005$

$x_0 = -0.6$; $x_1 = -0.5$

solved by using bisection method,

x_0	x_1	x_2	$f(x_0)$	$f(x_1)$	$f(x_2)$	Error
−0.6	−0.5	−0.55	0.2212	−0.0581	0.0719	~~0.0434~~
−0.6	−0.55	−0.575	0.2212	0.0719	0.14504	0.0434
−0.6	−0.575	−0.5875	0.2212	0.14504	0.1828	0.0212
−0.6	−0.5875	−0.59375	0.2212	0.1828	0.20199	0.01052
−0.6	−0.59375	−0.5968	0.2212	0.20199	0.2115	0.00523
−0.6	−0.5968	−0.59843	0.2212	0.2115	0.21639	0.00261
−0.6	−0.59843	−0.59921	0.2212	0.21639	0.21880	0.0013
−0.6	−0.59921	−0.5996	0.2212	0.21880	0.2200	!
−0.6	−0.5996	−0.5998				0.0003

∴ root is −0.5998

False Position method

Algorithm :

① Start

② Define function $f(x)$

③ choose initial guess a and b.

④ choose tolerable error e .

⑤ calculate new approximat root

$$x_i = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

⑥ calculate error $= \dfrac{x_{i+1} - x_i}{x_{i+1}}$

⑦ if $f(a) * f(b) < 0 \longrightarrow$ goto 9.

if $f(a) * f(x_i) < 0 \longrightarrow b = x_i$

if $f(b) * f(x_i) < 0 \longrightarrow a = x_i$

⑧ if error > e then goto 5 otherwise goto 9.

⑨ Display x_i as root.

⑩ stop.

* Example: 1

Determine the roots of the equation: $f(x) = \sin 5x + \cos 2x$ where $a = -0.6$, $b = -0.5$ where $\epsilon = 0.0005$

do not need to calculate x_i by calcul

root

iteration	a	b	x_i	$f(a)$	$f(b)$	$f(x_i)$	error
0	−0.6	−0.5	−0.520819	0.221	−0.05817	−0.00718	1
1	−0.6	−0.520819	~~0.522~~ ~~−0.522308~~ −0.523308	0.221	−0.00718	−0.000754	0.00475
2	−0.6	−0.523308	−0.523569	0.221	−0.000754	−0.000077	0.000498

Ans: −0.523569 .

* Example: Determine the roots of the equation $f(x) = x^{10} - 1$ where $a = 0$, $b = 1.3$ and $\epsilon_s = 0.0005$.

$$f(x) = x^{10} - 1$$

$$x_i = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

a	b	x_i	$f(a)$	$f(b)$	$f(x_i)$	error
0	1.3	0.0943	−1	12.78	−0.9999	1
0.0943	1.3	0.18178	−0.9999	12.78	−0.9999	0.481
0.18178	1.3	0.26292	−0.9999	12.78	−0.9999	0.308
0.26292	1.3	0.33817	−0.9999	12.78	−0.9999	0.222
0.33817	1.3	0.40796	−0.9999	12.78	~~0.40796~~ −0.9998	0.[illegible]

Newton Rapshon method

Algorithm :

1. Start
2. Define function $f(x)$
3. Define first derivative of $f(x)$ as $f'(x)$
4. Read initial guess x_0
5. Read tolerable error e
6. if $f'(x)=0$ Then Print 'mathmatical error' and goto .

(7) Calculate $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

(8) if error > e Than set $x_0 = x_1$

otherwise goto 9.

(9) Print root as x_{n+1}.

(10) stop

Example 1:

Prove newton-Rapshon method from taylor series expansion.

Ans:

we know,

$$f(x+a) = f(a) + f'(a)(x-a)$$

$$\Rightarrow 0 = f(a) + f'(a)(x-a)$$

$$\Rightarrow -\frac{f(a)}{f'(a)} = x - a$$

$$\Rightarrow x = a - \frac{f(a)}{f'(a)}$$

* Find by newton Rapshon method a root of equation, $x^3-3x-5=0$ where $x_0=2$

Sol^n:

let,

$f(x)=x^3-3x-5$

$f'(x)=3x^2-3$

we know,

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{0+1}=x_0-\frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow x_1=2-\frac{-3}{9}$$

$$=2.333$$

again,

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

$$=2.333-\frac{0.699}{13.328}=2.2805$$

$x_3 = 2.2790$

$x_4 = 2.2790$

Example: 2

$$f(x) = \sin 5x + \cos 2x \quad \text{where } x_0 = 0.4$$

$\epsilon = 0.00005$

Soln: we know that,

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x) = \sin 5x + \cos 2x$$

$$f'(x) = 5\cos 5x - 2\sin 2x$$

iteration	x_i	error
1	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.85684$	1
2	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.59742$	0.4342
3	$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.67403$	0.11365
4	$x_4 = x_3 - \frac{f(x_3)}{f(x_3)} = 0.67319$	0.00124
5	$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.67319$	0.

$f(x): x^2-25=0$ where $x_0=7$

Soln:

Given,

$$f(x) = x^2-25$$

$$f'(x) = 2x$$

we know for newton Rapshon method

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)} = 7 - \frac{24}{14} = 5.285714$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 5.285714 - \frac{2.9387}{10.5714}$$

$$= 5.0077$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 5.0077 - \frac{0.07766}{10.0154}$$

$$= 5.0000$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 5.0000 - \frac{0}{10.0000}$$

$$x_4 = 5.0000$$

Ans : 5.0000

Secant method

① Star

Algorithm :

① start

② Define function as $f(x)$

③ input initial guess x_0 and x_1 and tolerable error e.

④ if $f(x_0) = f(x_1)$ Then Print "mathmatical error" and goto otherwise 5.

⑤ calculate $x_2 = x_1 - \frac{\cancel{f(x_1)} (x_1 - x_0)}{f(x_1) - f(x_0)} \times f(x_1)$

⑥ incriment iteration $i = i+1$

⑦ If error > e then set $x_0 = x_1$

$x_1 = x$ and goto 5 otherwise

goto 9.

⑧ Print root as x_2.

⑨ Stop

* Determine the root of $f(x) = \sin 5x + \cos 2x$

where $x_0 = 0.4$ and $x_1 = 1$ by secant

method.

Soln:

Given that,

$f(x) = \sin 5x + \cos 2x$ $x_0 = 0.4$ and $x_1 = 1$

we know,

$$x_2 = x_1 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \times f(x_1)$$

$$= 1 - \frac{(1 - 0.4)}{-1.37507 - 1.6060} * -1.37507$$

$$= 0.72323$$

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$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} \times f(x_2)$$

$$= 0.72323 - \frac{(0.72323 - 1)}{-0.33292 - (-1.37507)} * -0.33292$$

$$= 0.63481$$

$$x_4 = 0.63481 - \frac{(0.63481 - 0.72323) * 0.26419}{0.26419 - (-0.33292)}$$

$$= 0.67393$$

$$x_5 = 0.67393 - \frac{(0.67393 - 0.63481) * -0.00499}{-0.00499 - 0.26419}$$

$$= 0.67320$$

$\therefore$ root is 0.67320

Q: use graphical method to solve

$$3x_1 + 2x_2 = 18 \quad \text{------} \quad (i)$$

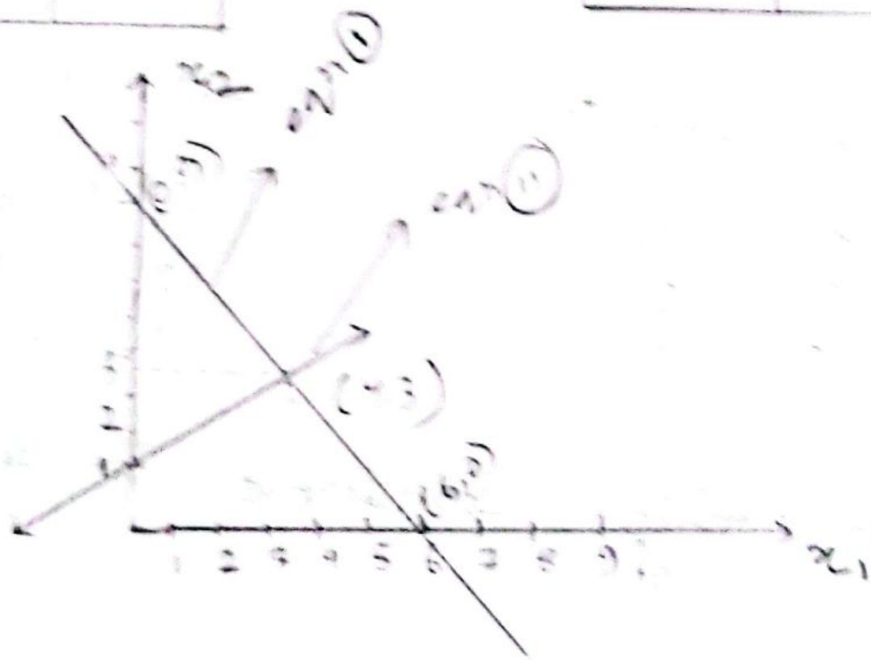
$$-x_1 + 2x_2 = 2 \quad \text{------} \quad (ii)$$

for eqn (i)

x_1	x_2
6	0
0	9

for eqn (ii)

x_1	x_2
-2	0
0	1



__Ans:__ $x_1 = 4$, $x_2 = 3$

* solve this equation using graphical method

$$4x_1 + 3x_2 = 12$$
$$x_1 - x_2 = 10$$

Soln:

Given,

$$4x_1 + 3x_2 = 12 \quad \text{---- (I)}$$
$$x_1 - x_2 = 10 \quad \text{---- (II)}$$

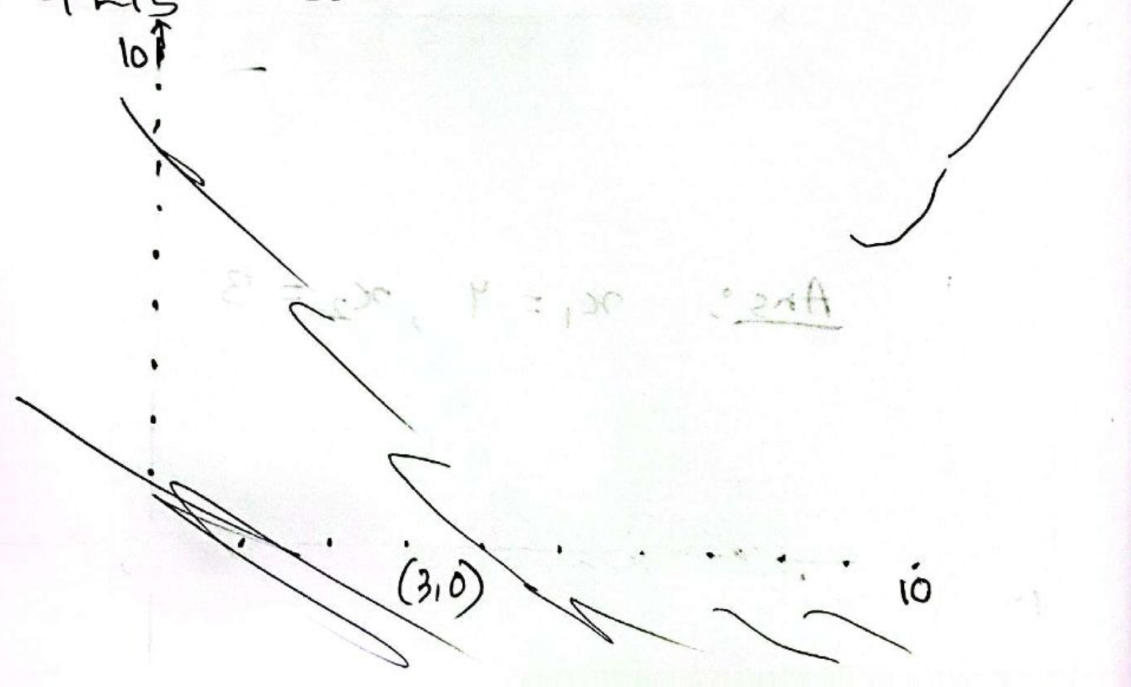
for eqn (I)

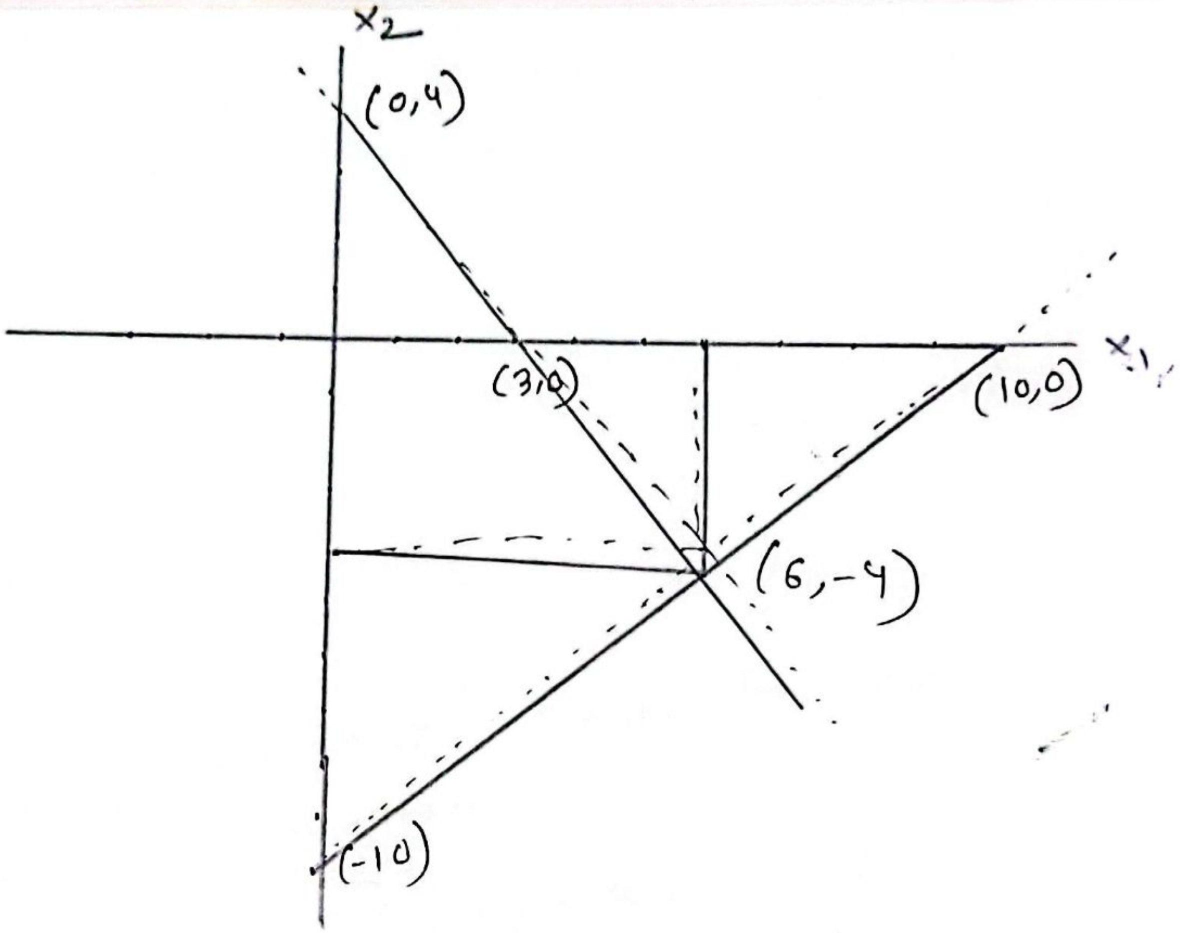
x_1	x_2
3	0
0	4

for eqn (II)

x_1	x_2
10	0
0	-10

Plot this coordinate in graph.





x_2
(0,4)
(3,0)
(10,0)
x_1
(6,-4)
(-10)

Crammers method

Ex. use cramer rule to solve the following equations:

$$0.3x_1 + 0.52x_2 + x_3 = -0.01$$
$$0.5x_1 + x_2 + 1.9x_3 = 0.67$$
$$0.1x_1 + 0.3x_2 + 0.5x_3 = -0.44$$

$$D = \begin{vmatrix} 0.3 & 0.52 & 1 \\ 0.5 & 1 & 1.9 \\ 0.1 & 0.3 & 0.5 \end{vmatrix} = -0.0022$$

$$\Delta x_1 = \begin{vmatrix} -0.01 & 0.52 & 1 \\ 0.67 & 1 & 1.9 \\ -0.44 & 0.3 & 0.5 \end{vmatrix} = 0.03278$$

~~Δx_2~~

$$\Delta x_2 = \begin{vmatrix} 0.3 & -0.01 & 1 \\ 0.5 & 0.67 & 1.9 \\ 0.1 & -0.44 & 0.5 \end{vmatrix} = 0.0649$$

$$\Delta x_3 = \begin{vmatrix} 0.3 & 0.52 & -0.01 \\ 0.5 & 1 & 0.67 \\ 0.1 & 0.3 & -0.44 \end{vmatrix} = -0.04356$$

$$\therefore x_1 = \frac{\Delta x_1}{D} = \frac{0.03278}{-0.0022} = -14.9$$

$$\therefore x_2 = \frac{\Delta x_2}{D} = \frac{0.0649}{-0.0022} = -29.5$$

$$\therefore x_3 = \frac{\Delta x_3}{D} = \frac{-0.04356}{-0.0022}$$

$$= 19.8$$

Gauss elimination method

Algorithm:

1. Start
2. Read number of unknowns: n
3. Read Augmented matrix (A) of n by n+1 size.
4. Transform Augmented Matrix (A) to upper triangular Matrix by row operations

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(5) obtain solution by back substitution

(6) Display result.

(7) stop

Ex.1

use Gauss elimination to solve

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$

$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$

$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

Soln:

Given that,

$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$ ---- (i)

$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$ ---- (ii)

$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$ ---- (iii)

Operation counting.

(i) $\times \frac{0.1}{3} \Rightarrow$

$0.1x_1 - 0.0033x_2 - 0.0066x_3 = 0.2616$ — (iv)

(ii) - (iv)

$(0.1-0.1)x_1 + (7+0.0033)x_2 - (0.3+0.0066)x_3$
$= -19.3 - 0.2616$

$\Rightarrow 7.0033x_2 - 0.2934x_3 = -19.56397$ — (v)

(i) $\times \frac{0.3}{3} \Rightarrow$

$0.3x_1 - 0.01x_2 - 0.02x_3 = 0.785$ — (vi)

(iii) - (vi)

$-0.19x_2 + 10.02x_3 = 70.615$ ---- (vii)

$(v) \times \frac{-0.19}{7.0033}$

$-0.19x_2 + 0.00796x_3 = 0.53677$ ... (vii)

(vii) - (viii)

$10.01204x_3 = 70.08423$

$x_3 = 6.9999$

Putting (x_3) value in (vii)

$$x_2 = \frac{70.615 - (10.02 \times 6.9999)}{-.19}$$

$= -2.5052$

Putting x_2 and x_3 value in

(1)

$\Rightarrow$

$$x_1 = \frac{7.85 + (0.1 \times -2.5052) + 0.2 \times 6.9999}{3}$$

$= 2.99982$

Ans:

$x_1 = 2.99982$

$x_2 = -2.5052$

$x_3 = 6.9999$

Gauss Seidel method

Algorithm :

① start

② Arrange given system of linear equations in diagonally dominant form

③ Read tolerable error.

④ convert the first equation in terms of firs variable, second equation in terms of second variable and so on.

⑤ set initial guesses x_0, x_1, x_2 and so on.

⑥ substitute value of x_1 and x_2 from step ⑤, in first equation obtained from step 4 to calculate new value of x_1,

(7) if error satisfy the given condition then goto step (9)

(8) goto step 6.

(9) Print value of x_0, x_1, x_2.

(10) stop.

Exmples

use the gauss-seidel method to obtain the solution of the following system,

$$3x_1 - 0.1x_2 - 0.2x_3 = 7.85$$

$$0.1x_1 + 7x_2 - 0.3x_3 = -19.3$$

$$0.3x_1 - 0.2x_2 + 10x_3 = 71.4$$

where $\epsilon_s = 0.001$

$$\therefore x_1 = \frac{7.85 + 0.1x_2 + 0.2x_3}{3}$$

$$x_2 = \frac{-19.3 - 0.1x_1 + 0.3x_3}{7}$$

$$x_3 = \frac{71.4 - 0.3x_1 + 0.2x_2}{10}$$

let initially $x_1 = 0$, $x_2 = 0$, $x_3 = 0$

iteration	x_1	x_2	x_3	ϵx_1	ϵx_2	ϵx_3
1	2.6166	-2.7945	7.0056	1	1	1
2	2.9905	-2.4996	7.000293	0.125	-0.1179	-0.0003
3	3.0000	-2.4999	7.000293	.003	.0001	0.000

Ans: $x_1 = 3.0000$

$x_2 = -2.4999$

$x_3 = 7.000293$

Fixed Point iteration (finding roots

1. Start

2. Define $f(x)$

3. Convert $f(x) = 0$ into the form of $x = g(x)$

   log/sin/cos/e^x

4. Let the initial guess 0.

5. DO $x_{i+1} = g(x_i)$

   while (none of the convergence iteration is matched)

Example:

Fixed Point iteration of non-linear equations

1. start
2. Read function $f(x_1)$ and $f(x_2)$
3. Define function $g(x)$ and $g'(x)$ which is obtained from $f(x_1)$ and $f(x_2)$ such that,

$$x_1 = g(x) \quad \text{and} \quad x_2 = g'(x)$$

4. Choose initial guess $x_1 = 0$ and $x_2 = 0$
5. Read tolerable error e.
6. set an initial iteration, step $= 1$
7. calculate value of x_1 and x_2
8. increment iteration step $=$ step $+ 1$
9. set $x_1 =$ new x_1 and $x_2 =$ new x_2

(10) if $\epsilon x_1 > e$ and $\epsilon x_2 > e$ goto 7 otherwise goto 11.

(11) Display x_1 and x_2

(12) Stop

Example I;

$$f(x) = x_1^2 + x_1 x_2 = 10 \; ; \qquad \epsilon_s = 0.0001$$

$$f(x) = x_2 + 3x_1 x_2^2 = 57 \; ;$$

Given that,

$$x_1^2 + x_1 x_2 = 10$$

$$\therefore x_1 = \sqrt{10 - x_1 x_2} \quad \text{———} \; (1)$$

$$x_2 + 3x_1 x_2^2 = 57$$

$$\therefore x_2 = \sqrt{\frac{57 - x_2}{3x_1}}$$

Let, initial Guess $x_1 = 0$, $x_2 = 0$

Then Put this value into equation and find new value of x_1 and x_2

iteration	x_1	x_2	ϵx_1	ϵx_2
1	3.16227	2.45119		
2	1.49955	3.48218		
3				
4				
5				
6				
7				
8				
9				

Successive over Relaxion method

Reloxation represents a slight modification of the Gauss-sedel method and is designed to enhance Convergence.

$$x_{i+1} = (1-\lambda)x_i + \lambda x_{i+1}$$

λ ↳ relaxtion factor

$0 < \lambda < 1$

when $\lambda = 1$ Than No change (successive = Gauss Seidel)

$\lambda < 1 \longrightarrow$ under relaxation method

$\lambda > 1 \longrightarrow$ over relaXation method

* what is SOR?

The successive over-Relaxation (SOR) method is an iterative algorithm used to solve a system of linear equation of the form $Ax = b$. It is an enhancement of the Gauss-Seidel method, designed to improve the rate of convergence by introducing a relaxation factor λ.

Q) why do we use (SOR) method?

The SOR method is used to solve system of linear equation. Particularly those that arise from discretizing partial differential equations.

* all are same as Gauss-Seidel,

Just, $x_1 = (1-\lambda) x_1 + \dfrac{\lambda (b_1 - a_{12} x_2 - a_{13} x_3)}{a_{11}}$

interpolation

(i) linear interpolation

(ii) Non-linear interpolation

(iii) Lagrange interpolation

(iv) Newton interpolation (forward / backward)

Newton forward interpolation

Algorithm :

(1) start

(2) Read n (Number of arguments)

(3) For i=1 to n

Read x_i and y_i

(4) construct forward difference table and print the table Like

(Here n = 5)

x	$f(x)$	Δdf	$\Delta^2 df$	$\Delta^3 df$	$\Delta^4 df$
x_1	$f(x_1)$				
		Δdf_1			
x_2	$f(x_2)$		$\Delta^2 df_1$		
				$\Delta^3 df_1$	
		Δdf_2			$\Delta^4 df_1$
x_3	$f(x_3)$		$\Delta^2 df_2$		
				$\Delta^3 df_2$	
		Δdf_3			
x_4	$f(x_4)$		$\Delta^2 df_3$		
		Δdf_4			
x_5	$f(x_5)$				

Calculate the value of forward difference,

$$d(x) = f(x_1) + \Delta df_1 (x - x_1) + \Delta^2 df_1 (x - x_1)(x - x_2)$$
$$+ \Delta^3 df_1 (x - x_1)(x - x_2)(x - x_3) + \Delta^4 df_1 (x - x_1)$$
$$(x - x_2)(x - B)(x - 4)$$

(Ne

Algorithm:

① start

② Read

③ For

④ constr

and dis

$d(x) =$

(Newton backward interpolation)

Algorithm:

1. Start
2. Read n (number of argument)
3. For i = 1 to n
   Read x_i and $f(x_i)$
4. Construct backward difference table and display the table.

$$\boxed{\text{table same as forward}}$$

$$d(x) = f(x_5) + \Delta d f_4 (x - x_5) + \Delta^2 d f_3 (x - x_5)(x - x_4) + \Delta^3 d f_2 (x - x_5)(x - x_4)(x - x_3) + \Delta^4 d f_1 (x - x_5)(x - x_4)(x - x_3)(x - x_2)$$

Ex A table of a Polynomial function is given below. Fit a polynomial and find the value of $f(x)$ at $x = 2.5$

x	-3	-1	0	3	5
$f(x)$	-30	-22	-12	330	3458

x	$f(x)$	Δdf	$\Delta^2 df$	$\Delta^3 df$	$\Delta^4 df$
-3	-30				
		4			
-1	-22		2		
		10		4	
0	-12		26		5
		114		44	
3	330		290		
		1564			
5	3458				

Newton forward :

$$f(x) = -30 + 4(2.5+3) + 2(2.5+3)(2.5+1) + 4(2.5+3)(2.5+1)(2.5)$$
$$+ 5(2.5+3)(2.5+1)(2.5)(2.5-3)$$

$= 102.6875$

Newton backword:

$$f(x) = 3458 + 1564(2.5-5) + 290(2.5-5)(2.5-3) + 44(2.5-5)(2.5-3)(2.5-0) + 5(2.5-5)(2.5-3)(2.5-0)(2.5+1)$$

$= 102.6875$

lagrange interpolation

Algorithm:

① Start

② Read number of data

③ Read data x_i and y_i for $i=1$ to n.

④ Read value of independent variables say x_p whose corresponding value of dependent say y_p is to be determined

⑤ initialize $i=1$ to n

set $p=1$

for $j=1$ to n

if $j \neq i$ then calculate

$$p = p * \frac{(x_p - x_j)}{(x_i - x_j)}$$

$$\therefore f(x) = \sum_{i=1}^{n} f(x_i) \prod_{\substack{j=1 \\ j \neq i}}^{j=n} \frac{x - x_j}{x_i - x_j}$$

main formula

n=3

$$= f(x_1) \cdot \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + f(x_2) \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}$$

$$+ f(x_3) \cdot \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$= -0.25 \cdot \frac{(4-3)(4-6)}{(1.5-3)(1.5-6)} + 2 \cdot \frac{(4-1.5)(4-6)}{(3-1.5)(3-6)}$$

$$+ \frac{20(4-1.5)(4-3)}{(6-1.5)(6-3)}$$

$f(4) = 6$

linear interpolation

Ex. 1. $\cosh x$, where $x = 0.16$

x	0.1	0.2	0.3	0.4
$\cosh x$	1.005	1.020	1.045	1.087

$$\cosh(0.16) = 1.0128$$

$$f(x) = a_1 + a_2 x$$

$$f(x_1) = \cosh x_1 = \cosh(0.1) = a_1 + a_2 x_1 = 1.005$$

$$f(x_2) = \cosh x_2 = \cosh(0.2) = a_1 + a_2 x_2 = 1.020$$

$$f(0.16) = a_1 + a_2 (0.16)$$

$$a_1 + a_2 x_1 = f(x_1)$$

$$a_1 = f(x_1) - a_2 x_1$$

$$a_1 + a_2 x_2 = f(x_2)$$

$$\Rightarrow a_2 x_2 = f(x_2) - a_1$$

$$= f(x_2) - f(x_1) + a_2 x_1$$

$$\Rightarrow a_2 x_2 - a_2 x_1 = f(x_2) - f(x_1)$$

$$\therefore a_2 = \frac{f(x_2) - f(x_1)}{(x_2 - x_1)}$$

$$\Rightarrow a_2 = \frac{1.020 - 1.005}{0.2 - 0.1} = 0.15$$

$$\therefore a_1 = f(x_1) - a_2 x_1 = 1.005 - 0.15 \times 0.1$$
$$= 0.99$$

$$\therefore f(x) = a_1 + a_2 x = 0.99 + 0.15(0.16)$$
$$= 1.014$$

$$\text{error} = \frac{1.0128 - 1.014}{1.0128} = 0.001184$$

Ans: thats why we don't use linear interpolation intigration

Trapezoidal Rule

Simpson's Rule

Trapezoidal Rule:

Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using trapezoidal Rule,

Soln: Here $f(x) = \frac{1}{1+x^2}$; width $h = \frac{b-a}{n}$

$= \frac{6-0}{6}$

$= 1$

Divide the interval $[0, 6]$ into 6 Parts.

x	0	1	2	3	4		
$y = f(x)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

formula of trapezoidal Rule:

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027)+2(0.5+0.2+0.1+0.0588+0.0385)\right]$$

$$= 1.4108$$

Ex. 2 $\int_1^2 \frac{1}{x}\,dx$ and $n=10$

Soln:

width $h = \frac{2-1}{10} = 0.1$

Divide the interval [1,2] into 10 Parts

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8 ...
$y=f(x)$	1								
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

y_9 y_{10}

formula of trapezoidal Rule.

$$\int_1^2 \frac{1}{x}\,dx = \frac{h}{2}\left\{(y_0+y_{10})+2(y_1+y_2+y_3+y_4 \cdots\cdots +y_9)\right\}$$

$$= 0.6935$$

Ans

Simpson's Rule

Ex: $\int_1^2 \frac{1}{x}dx$ and $n=10$

Soln: width $h = \frac{b-a}{n} = \frac{1}{10} = 0.1$

Divide the interval $[1,2]$ into 10 Parts

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
$y=f(x)$											
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

$\therefore$ Simpson's $\frac{1}{3}$ rule,

$$\int_1^2 \frac{1}{x}dx = \frac{h}{3}\{(y_0+y_{10}) + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\}$$

$\therefore$ Simpson's 3/8 rule:

$$\int_1^2 \frac{1}{x}dx = \frac{3h}{8}[(y_0+y_{10}) + 3(y_1+y_2+y_4+y_5) + 2(y_3)]$$