Example:

A microwave directional coupler has the following measured power levels:

- Power input at Port 1 (Input): P_1=100 mW
- Power output at Port 2 (Through): P_2=89 mW
- Power at Port 3 (Coupled): P_3=10 mW
- Power at Port 4 (Isolated): P_4=0.1 mW

Calculate the following:

i. Coupling factor (C)
ii. Directivity (D)
iii. Isolation (I)

Given:

- Input power $P_1 = 100\,\mathrm{mW}$
- Through power (Port 2) $P_2 = 89\,\mathrm{mW}$
- Coupled power (Port 3) $P_3 = 10\,\mathrm{mW}$
- Isolated port power (Port 4) $P_4 = 0.1\,\mathrm{mW}$

---

i. Coupling Factor (C)

The **coupling factor** represents how much power is coupled from the input port (Port 1) to the coupled port (Port 3):

$$C = 10\log_{10}\left(\frac{P_1}{P_3}\right)$$

$$C = 10\log_{10}\left(\frac{100}{10}\right) = 10\log_{10}(10) = 10\,\mathrm{dB}$$

ii. Directivity (D)

Directivity is a measure of how well the coupler isolates the forward and reverse signals. It's defined as the ratio of power at the coupled port to the isolated port:

$$D = 10\log_{10}\left(\frac{P_3}{P_4}\right)$$

$$D = 10\log_{10}\left(\frac{10}{0.1}\right) = 10\log_{10}(100) = 20\,\mathrm{dB}$$

iii. Isolation (I)

Isolation is the ratio of input power to the power at the isolated port:

$$I = 10\log_{10}\left(\frac{P_1}{P_4}\right)$$

$$I = 10\log_{10}\left(\frac{100}{0.1}\right) = 10\log_{10}(1000) = 30\,\mathrm{dB}$$

iii. Isolation (I)

Isolation is the ratio of input power to the power at the isolated port:

$$I = 10\log_{10}\left(\frac{P_1}{P_4}\right)$$

$$I = 10\log_{10}\left(\frac{100}{0.1}\right) = 10\log_{10}(1000) = 30\,\mathrm{dB}$$

Example:

A directional coupler used in a microwave system has the following measured power levels:

- Power input at Port 1 (Input): $P_1 = 200\,\mathrm{mW}$
- Power measured at Port 2 (Through): $P_2 = 178\,\mathrm{mW}$
- Power measured at Port 3 (Coupled): $P_3 = 20\,\mathrm{mW}$
- Power measured at Port 4 (Isolated): $P_4 = 0.2\,\mathrm{mW}$

Calculate:

1. Coupling Factor (C)
2. Directivity (D)
3. Isolation (I)
4. Insertion Loss (IL)

Coupling Factor (C)

$$C = 10\log_{10}\left(\frac{P_1}{P_3}\right) = 10\log_{10}\left(\frac{200}{20}\right) = 10\log_{10}(10) = 10\,\text{dB}$$

Coupling Factor = 10 dB

Directivity (D)

$$D = 10\log_{10}\left(\frac{P_3}{P_4}\right) = 10\log_{10}\left(\frac{20}{0.2}\right) = 10\log_{10}(100) = 20\,\text{dB}$$

Directivity = 20 dB

Isolation (I)

$$I = 10\log_{10}\left(\frac{P_1}{P_4}\right) = 10\log_{10}\left(\frac{200}{0.2}\right) = 10\log_{10}(1000) = 30\,\text{dB}$$

Isolation = 30 dB

Insertion Loss (IL)

Insertion Loss quantifies the loss of power between the input (Port 1) and the through port (Port 2):

$$IL = 10\log_{10}\left(\frac{P_1}{P_2}\right) = 10\log_{10}\left(\frac{200}{178}\right) \approx 10\log_{10}(1.1236) \approx 10 \times 0.050 = 0.5\,\text{dB}$$

Insertion Loss ≈ 0.5 dB

Example:

A rectangular waveguide has the following dimensions:

- **Width** $a = 2.286\,\text{cm}$
- **Height** $b = 1.016\,\text{cm}$

An **electromagnetic wave** with a frequency of **10 GHz** is propagating through the waveguide in the **dominant mode.**

Calculate the following:

1. **Cutoff frequency** f_c for the dominant mode (TE_{10})
2. **Guide wavelength** λ_g
3. **Phase velocity** v_p
4. **Group velocity** v_g

Given:

- Width $a = 2.286\,\text{cm} = 0.02286\,\text{m}$
- Height $b = 1.016\,\text{cm} = 0.01016\,\text{m}$
- Frequency $f = 10\,\text{GHz} = 10 \times 10^9\,\text{Hz}$
- Speed of light $c = 3 \times 10^8\,\text{m/s}$

1. Cutoff frequency f_c for TE_{10} mode:

For TE_{10} (dominant mode), the cutoff frequency is:

$$f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.02286} \approx \frac{3 \times 10^8}{0.04572} \approx 6.56\,\text{GHz}$$

Cutoff frequency $f_c \approx 6.56\,\text{GHz}$

2. Guide wavelength λ_g

First, calculate the **free-space wavelength** λ:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03\,\text{m} = 3\,\text{cm}$$

Now calculate the **guide wavelength:**

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{0.03}{\sqrt{1 - \left(\frac{6.56}{10}\right)^2}} = \frac{0.03}{\sqrt{1 - 0.4303}} = \frac{0.03}{\sqrt{0.5697}} \approx \frac{0.03}{0.7548} \approx 0.03975\,\text{m} = 3.975\,\text{cm}$$

Guide wavelength $\lambda_g \approx 3.975\,\text{cm}$

3. Phase velocity v_p

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - 0.4303}} = \frac{3 \times 10^8}{0.7548} \approx 3.975 \times 10^8\,\text{m/s}$$

Phase velocity $v_p \approx 3.975 \times 10^8\,\text{m/s}$

4. Group velocity v_g

$$v_g = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 3 \times 10^8 \times \sqrt{0.5697} = 3 \times 10^8 \times 0.7548 \approx 2.264 \times 10^8\,\text{m/s}$$

Group velocity $v_g \approx 2.264 \times 10^8\,\text{m/s}$

Example:
Determine the group velocity and phase velocity for a dominant mode propagating through a waveguide of breadth 10cms at frequency of 2.5GHz.

Given:

- Waveguide **breadth** $a = 10\,\text{cm} = 0.1\,\text{m}$
- **Frequency** $f = 2.5\,\text{GHz} = 2.5 \times 10^9\,\text{Hz}$
- Mode: **Dominant mode → TE$_{10}$**
- Speed of light $c = 3 \times 10^8\,\text{m/s}$

Step 1: Cutoff Frequency for TE$_{10}$ Mode

$$f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.1} = 1.5\,\text{GHz}$$

Step 2: Check if Frequency > Cutoff Frequency

$$f = 2.5\,\text{GHz} > f_c = 1.5\,\text{GHz} \Rightarrow \text{Wave is propagating}$$

Step 3: Phase Velocity v_p

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$v_p = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{1.5}{2.5}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - 0.36}} = \frac{3 \times 10^8}{\sqrt{0.64}} = \frac{3 \times 10^8}{0.8} = \boxed{3.75 \times 10^8\,\text{m/s}}$$

Step 4: Group Velocity v_g

$$v_g = c \cdot \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 3 \times 10^8 \cdot \sqrt{0.64} = 3 \times 10^8 \cdot 0.8 = \boxed{2.4 \times 10^8\,\text{m/s}}$$

Example:
A waveguide having dimensions a = 5 cm, b = 2 cm. The signal applied to waveguide is10GHz. Determine the cutoff frequency of in the rectangular waveguide.

Given:

- Waveguide dimensions:
  - a=5 cm=0.05 m (broad dimension)
  - b=2 cm=0.02 m (narrow dimension)
- Signal frequency: f=10 GHz

1. Cutoff Frequency Formula (Rectangular Waveguide)

For **TE**$_{mn}$ modes in a rectangular waveguide:

$$f_{c_{mn}} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Example:

A microwave transmission line has the following parameters:

- **Characteristic impedance** $Z_0 = 50\,\Omega$
- **Load impedance** $Z_L = 100\,\Omega$
- **Input power** $P_{\text{in}} = 10\,\text{mW}$

The operating frequency is **5 GHz**, and the line is **lossless.**

Calculate:

1. **Reflection coefficient** Γ
2. **Voltage Standing Wave Ratio (VSWR)**
3. **Power delivered to the load**
4. **Return loss in dB**

1. Reflection Coefficient Γ

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{50}{150} = \frac{1}{3} \approx 0.333$$

Reflection Coefficient $\Gamma = 0.333$

---

2. Voltage Standing Wave Ratio (VSWR)

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.333}{1 - 0.333} = \frac{1.333}{0.667} \approx 2$$

VSWR = 2

3. Power Delivered to the Load

Power delivered is the incident power minus the reflected power:

$$P_{\text{reflected}} = |\Gamma|^2 \cdot P_{\text{in}} = (0.333)^2 \cdot 10\,\text{mW} = 0.111 \cdot 10 = 1.11\,\text{mW}$$

$$P_{\text{delivered}} = P_{\text{in}} - P_{\text{reflected}} = 10 - 1.11 = 8.89\,\text{mW}$$

Power Delivered to Load = 8.89 mW

4. **Return Loss (RL)**

Return Loss (in dB) is given by:

$$\text{Return Loss} = -20\log_{10}(|\Gamma|) = -20\log_{10}(0.333) \approx -20 \times (-0.4771) = 9.54\,\text{dB}$$

Return Loss ≈ 9.54 dB

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Return Loss ≈ 9.54 dB

Example:

A **lossless microwave transmission line** has:

- **Characteristic impedance** $Z_0 = 75\,\Omega$
- It is terminated with a **load impedance** $Z_L = 30 + j40\,\Omega$
- The operating frequency is $f = 3\,\text{GHz}$
- The **wavelength in the line** is $\lambda = 0.1\,\text{m}$

Find:

1. **Reflection coefficient** Γ (magnitude and phase)
2. **Voltage Standing Wave Ratio (VSWR)**
3. **Position of the first voltage minimum from the load**
4. **Impedance at a distance of** $d = \lambda/8$ **from the load toward the generator**

1. Reflection Coefficient Γ

Given:

- $Z_0 = 75\,\Omega$
- $Z_L = 30 + j40\,\Omega$

Use:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 + j40) - 75}{(30 + j40) + 75} = \frac{-45 + j40}{105 + j40}$$

Now compute the complex division:

Let:

- Numerator: $N = -45 + j40$
- Denominator: $D = 105 + j40$

Use:

$$\Gamma = \frac{N}{D} = \frac{-45 + j40}{105 + j40} \times \frac{105 - j40}{105 - j40}$$

Multiply numerator and denominator:

Numerator:

$$(-45 + j40)(105 - j40) = -4725 + 1800 + j(-1800 - 4200) = -2925 - j6000$$

Denominator:

$$(105)^2 + (40)^2 = 11025 + 1600 = 12625$$

So:

$$\Gamma = \frac{-2925 - j6000}{12625} \approx -0.2316 - j0.475$$

Magnitude:

$$|\Gamma| = \sqrt{(-0.2316)^2 + (-0.475)^2} = \sqrt{0.0537 + 0.2256} = \sqrt{0.2793} \approx 0.5285$$

Phase:

$$\angle\Gamma = \tan^{-1}\left(\frac{-0.475}{-0.2316}\right) = \tan^{-1}(2.05) \approx -64.7^\circ$$

(Note: Since both parts are negative, angle is in 3rd quadrant → Add 180°)

$$\angle\Gamma \approx 180^\circ - 64.7^\circ = 115.3^\circ$$

Reflection coefficient:

- Magnitude: $|\Gamma| \approx 0.5285$
- Phase: $\angle\Gamma \approx 115.3^\circ$

2. VSWR

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.5285}{1 - 0.5285} = \frac{1.5285}{0.4715} \approx 3.24$$

VSWR ≈ 3.24

3. Position of First Voltage Minimum from the Load

For a lossy or lossless line:

$$d_{\min} = \frac{\lambda}{4\pi} \cdot (\pi - \angle\Gamma) = \frac{\lambda}{4\pi}(180^\circ - \angle\Gamma)$$

Convert angle to radians:

$$\angle\Gamma = 115.3^\circ = 2.012\,\text{rad}$$

$$d_{\min} = \frac{0.1}{4\pi}(\pi - 2.012) = \frac{0.1}{4\pi}(1.13) \approx \frac{0.1 \times 1.13}{12.566} \approx \frac{0.113}{12.566} \approx 0.009\,\text{m} = 0.9\,\text{cm}$$

First voltage minimum is at approx. 0.9 cm from the load

4. Impedance at $d = \lambda/8$ toward the generator

The impedance at distance d is given by:

$$Z(d) = Z_0 \cdot \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$

Where:

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.1} = 20\pi\,\text{rad/m}$$

$$d = \frac{\lambda}{8} = 0.0125\,\text{m} \Rightarrow \beta d = 20\pi \cdot 0.0125 = 0.25\pi\,\text{rad}$$

Microwave power measurement:

Errors in microwave power measurement can arise from several sources and can significantly affect the accuracy of the readings. Here's a brief explanation of the common types of errors:

Calibration Errors

- Result from using improperly calibrated instruments or standards.
- Can lead to systematic deviation from the true power value.

Mismatch Errors

- Occur due to impedance mismatches between components (e.g., source, cable, sensor).
- Cause reflected power, leading to standing waves and inaccurate readings.

Insertion Losses

- Losses in cables, connectors, or adapters between the source and the measurement device.
- These losses reduce the actual power reaching the sensor.

Temperature Effects

- Microwave components (especially sensors) can be temperature-sensitive.
- Changes in ambient temperature can alter measurement accuracy.

Frequency Response Errors

- Sensors and instruments may not have flat response across all frequencies.
- If not corrected, this causes errors when measuring signals at different frequencies.

Detector Non-Linearity

- Power sensors (like diode detectors) can behave non-linearly, especially at very low or very high-power levels.
- This leads to inaccurate readings if not compensated.

Noise and Drift

- Electronic noise and long-term drift in components can cause fluctuations in readings, especially at low power levels.

❖ VSWR Measurement Methods for Low and High VSWR:

VSWR (Voltage Standing Wave Ratio) indicates the impedance matching quality in a microwave system. The method of measurement depends on whether the VSWR is low (close to 1) or high (much greater than 1).

. Low VSWR Measurement (VSWR ≈ 1 to ~3)

Method: Using Directional Coupler with Power Meter or VSWR Meter

Setup:

- Microwave source → Directional coupler → Device Under Test (DUT)
- Coupled ports feed into a power meter or detector for forward and reflected power

Procedure:

1. Measure forward power P_f
2. Measure reflected power P_r
3. Compute reflection coefficient:

$$|\Gamma| = \sqrt{\frac{P_r}{P_f}}$$

4. Compute **VSWR**:

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Advantages:

- High accuracy for small reflections
- Ideal for well-matched systems (low return loss)

Example:

If $P_f = 10\,\text{mW}, P_r = 0.1\,\text{mW}$, then

$$|\Gamma| = \sqrt{0.1/10} = 0.1 \Rightarrow \text{VSWR} = \frac{1 + 0.1}{1 - 0.1} = 1.22$$

2. High VSWR Measurement (VSWR > 3 or when mismatch is large)

Method: Using Slotted Line or Network Analyzer

A. Slotted Line Method (Classical)

Setup:

- Microwave source → Slotted waveguide → DUT (like antenna)
- A probe moves along the slot to detect standing wave pattern

Procedure:

1. Measure **maximum voltage (Vmax)** and **minimum voltage (Vmin)** in the standing wave pattern
2. Calculate **VSWR**:

$$\text{VSWR} = \frac{V_{max}}{V_{min}}$$

Advantages:

- Suitable for high VSWR values
- Visualizes standing wave pattern directly

Disadvantages:

- Limited to waveguide systems
- Manual and time-consuming

Example:

A 20 dB directional coupler gives 3dbm in output power through coupled port. If the isolation specified as 55 dB, find the power available at the isolated port.

Given:

- Coupling factor = **20 dB**
- Coupled port output power = **3 dBm**
- Isolation = **55 dB**

We are to find the **power at the isolated port**.

Step 1: Find Input Power

A **20 dB directional coupler** means that the coupled port receives **20 dB less power** than the input.

So, if the **coupled port outputs 3 dBm**, then the **input power** is:

$$P_{\text{in}} = P_{\text{coupled}} + 20 = 3\,\text{dBm} + 20\,\text{dB} = \boxed{23\,\text{dBm}}$$

Step 2: Apply Isolation

Isolation is defined as how much lower the power is at the **isolated port** compared to the **input port**. So:

$$P_{\text{isolated}} = P_{\text{in}} - \text{Isolation} = 23\,\text{dBm} - 55\,\text{dB} = \boxed{-32\,\text{dBm}}$$

Helix in Travelling Wave Tube (TWT)

The helix is a critical component in a Travelling Wave Tube (TWT) — a specialized vacuum tube used to amplify high-frequency (microwave) signals.

Purpose of the Helix

The helix acts as a slow-wave structure, which slows down the phase velocit**y** of the RF signal so it can interact continuously with the electron beam traveling along the tube.

Why Slowing Down is Needed

- The electron beam travels at a speed close to the speed of light.
- The RF signal in a normal waveguide travels too fast to interact effectively.
- The helix structure delays the RF wave by making it spiral, effectively reducing its axial (along-the-tube) velocity.

- This synchronizes the wave and electron beam, enabling continuous interaction and energy transfer.

Structure of the Helix

- It's a spiral-shaped wire, typically made of metal (like tungsten or molybdenum).
- It runs along the axis of the TWT inside a vacuum envelope.
- The RF signal propagates along the helix.

How the Helix Works in TWT

- Electron gun emits a focused electron beam down the axis.
- RF input signal is fed into one end of the helix.
- As the RF signal travels along the helix, its axial electric field interacts with the electron beam.
- Electrons bunch due to this interaction, transferring energy to the RF wave.
- The RF wave is amplified as it travels down the helix.
- At the end, the amplified signal is extracted through an output coupler.

Key Parameters of the Helix

Parameter	Description
Pitch	Distance between turns — controls wave speed
Diameter	Affects impedance and interaction strength
Material	Must withstand high temperatures & voltages
Support rods	Hold the helix in place and provide insulation

Advantages of Helix TWTs

- Wide bandwidth (GHz range)
- Linear gain
- Suitable for satellite communications, radar, and electronic warfare