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Discrete Convolution and Correlation

Convolution is a mathematical operation equivalent to finite impulse response (FIR) filtering. It is a method of finding the zero-state response of relaxed linear time-invariant systems.

Correlation is a measure of similarity between two signals and is found using a process similar to convolution. There are two types of correlation:

(i) Cross correlation.

(ii) autocorrelation.

Convolution sum represented as, → convolution operator

$$y(n) = x(n) * h(n) = h(n) * x(n)$$
$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

Properties of convolution:

1. Commutative property: $x(n) * h(n) = h(n) * x(n)$

2. Associative property: $[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$

3. Distributive property: $x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

4. Shifting property:

   If $x(n) * h(n) = y(n)$, then $x(n-k) * h(n-m) = y(n-k-m)$

5. Convolution with an impulse: $x(n) * \delta(n) = x(n)$

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Ex 2.2 Find $y(n)$ if $x(n) = n+3$ for $0 \le n \le 2$

$h(n) = a^n u(n)$ for all n

$\Rightarrow$ We have $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

Given $x(n) = n+3$ for $0 \le n \le 2$

$h(n) = a^n u(n)$ for all n

$h(n) = 0$ for $n < 0$, so the system is causal. $x(n)$ is a causal finite duration sequence whose value is zero for $n > 2$. Therefore,

$$y(n) = \sum_{k=0}^{2} x(k)\, h(n-k)$$

$$= \sum_{k=0}^{2} (k+3)\, a^{n-k} u(n-k)$$

$$= 3a^n u(n) + 4a^{n-1} u(n-1) + 5a^{n-2} u(n-2)$$

Ex-2.3

Given, $x(n) = 3^n u(n)$ and $h(n) = \left(\frac{1}{3}\right)^n u(n)$

A causal signal is applied to a causal system [Both are Causal]

$$\therefore\ y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} 3^k \left(\frac{1}{3}\right)^{n-k}$$

$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} 3^k \times 3^k$$

$\sum_{k=0}^{n} r^k = \frac{r^{n+1}-1}{r-1}$

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$$= \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} (3^2)^k$$

$$= \left(\frac{1}{3}\right)^n \left[\frac{1-(3^2)^{n+1}}{1-3^2}\right]$$

$$= \left(\frac{1}{3}\right)^n \left[\frac{9^{n+1}-1}{9-1}\right] = \left(\frac{1}{3}\right)^n \left[\frac{9^{n+1}-1}{8}\right]$$

Ex. 2.4

Given, $x(n) = 3^n u(n)$ and $h(n) = 2^n u(n)$

Since both $x(n)$ and $h(n)$ are causal, we have

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=0}^{n} x(k)h(n-k)$$

$$\cancel{= \sum_{k=0}^{n} 3^k . 2^{n-k} =}$$

$$= \sum_{k=0}^{n} 3^k . 2^{n-k} = 2^n \sum_{k=0}^{n} 3^k . 2^{-k} = 2^n \sum_{k=0}^{n} \left(\frac{3}{2}\right)^k$$

$$= 2^n \left[\frac{1-\left(\frac{3}{2}\right)^{n+1}}{1-\frac{3}{2}}\right]$$

$$= 2^n \left[\frac{\left(\frac{3}{2}\right)^{n+1}-1}{1/2}\right]$$

$$= 2^{n+1} \left[\left(\frac{3}{2}\right)^{n+1}-1\right]$$

EX-2.5

Given $x(n) = \cos n\pi \, u(n)$ and $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$= (-1)^n u(n)$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} (-1)^k u(k) \left(\frac{1}{2}\right)^{(n-k)} u(n-k)$$

$$= \sum_{k=0}^{n} (-1)^k \left(\frac{1}{2}\right)^{n-k}$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} (-1)^k (2)^k$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} (-2)^k$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1 - (-2)^{n+1}}{1-(-2)}\right]$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1 + 2(-2)^n}{3}\right]$$

$$= \frac{1}{3}\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n \cdot \frac{2(-2)^n}{3}$$

$$= \frac{1}{3}\left(\frac{1}{2}\right)^n + \frac{2(-1)^n}{3} \quad \text{for } n > 0$$

$$= \frac{1}{3}\left(\frac{1}{2}\right)^n u(n) + \frac{2}{3}(-1)^n u(n)$$

Ex 2.6

Given $x(n) = u(n)$, $h(n) = u(n-3)$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} u(k)\, u(n-3-k)$$

$$\therefore\ 0 \le k \le n-3$$

$$\therefore\ y(n) = \sum_{k=0}^{n-3} u(k)\, u(n-3-k)$$

$$= \sum_{k=0}^{n-3} (1)(1) = n-2$$

number of terms = (last value − first value) + 1

$= (n-3-0)+1$

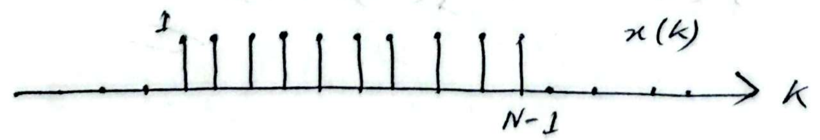
$= n-2$

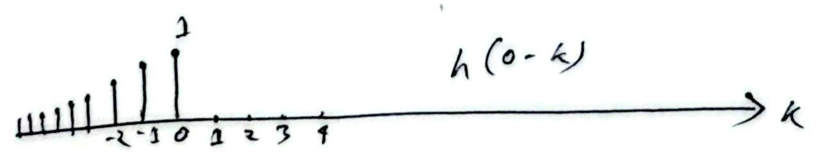
Ex 2.7

We know that the output $y(n)$ is given by

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

From this we can observe that to obtain the nth value of the output sequence we must form the product $x(k)h(n-k)$ and sum the values of the resulting sequence. The two component sequences are shown in figure as a function of k, with $h(n-k)$ shown for several values of n





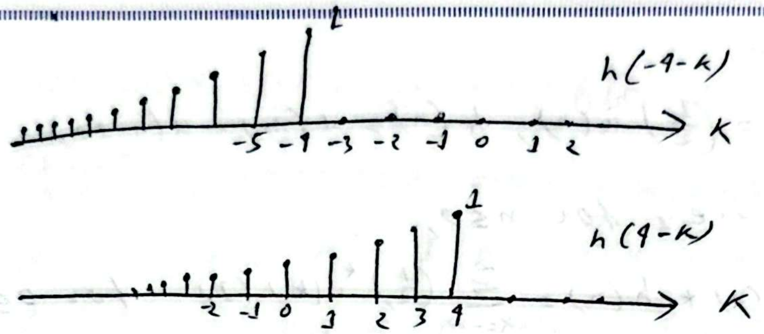


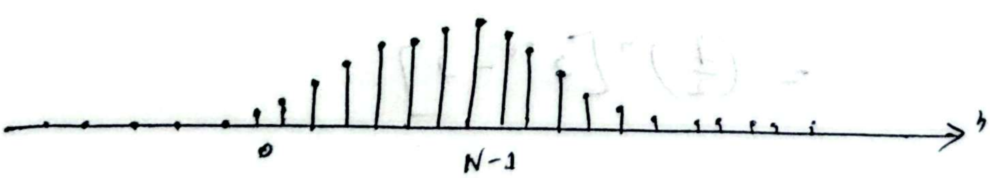
Fig: Component sequences in evaluating the convolution sum with $h(~~k~~\,n-k)$ show for several values of n.

As we see in figure, for $n<0$, $h(n-k)$ and $x(k)$ have no non-zero samples that overlap, and consequently $y(n)=0$, $n<0$. For n greater than or equal to zero but less than N, $h(n-k)$ and $x(k)$ have ~~zor~~ nonzero samples that overlap from $k=0$ to $k=n$; thus for $0 \le n < N$,

$$y(n) = \sum_{k=0}^{n} a^{n-k} = a^n \frac{1-a^{-(n+1)}}{1-a^{-1}} = \frac{1-a^{n+1}}{1-a}, \quad 0 \le n < N$$

For, $N-1 \le n$, the nonzero samples that overlap extend from $k=0$ to $k=N-1$ and thus:

$$y(n) = \sum_{k=0}^{N-1} a^{n-k} = a^n \frac{1-a^{-N}}{1-a^{-1}} = a^{n-(N-1)}\left[\frac{1-a^N}{1-a}\right], \quad N \le n$$



Example 2.8

Given $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = u(n) - u(n-10)$

For $n < 10$, i.e., for $n \le 9$,

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u(k)(1) \quad \text{for } 0 \le n \le 9$$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \quad 0 \le n \le 9$$

$$= \frac{1 - \left(\frac{1}{2}\right)^{n+1}}{1 - \frac{1}{2}} = \left(\frac{1}{2}\right)^n \left[\frac{1 - \left(\frac{1}{2}\right)^{-(n+1)}}{1 - \left(\frac{1}{2}\right)^{-1}}\right]$$

$$= \left(\frac{1}{2}\right)^n \left[2^{n+1} - 1\right] \quad 0 \le n \le 9$$

For $n \ge 10$, i.e., for $n > 9$,

$$y(n) = \sum_{k=n-9}^{n} \left(\frac{1}{2}\right)^k$$

$$= \left(\frac{1}{2}\right)^{n-9} + \left(\frac{1}{2}\right)^{n-9+1} + \left(\frac{1}{2}\right)^{n-9+2} + \cdots + \left(\frac{1}{2}\right)^{n-9+9}$$

$$= \left(\frac{1}{2}\right)^{n-9} \left[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \cdots + \left(\frac{1}{2}\right)^9\right]$$

$$= \left(\frac{1}{2}\right)^{n-9} \left[\frac{1 - \left(\frac{1}{2}\right)^{9+1}}{1 - \frac{1}{2}}\right] \quad \text{for } n \gtrsim 9$$

$$= 2\left(\frac{1}{2}\right)^{n-9} - 2\left(\frac{1}{2}\right)^{n+1}$$

$$= \left(\frac{1}{2}\right)^n \left[2^{10} - 1\right]$$

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Exa 2.9

Let $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

~~$x(k) = \left(\frac{1}{3}\right)^{-k} u(-k-1)$ and $h(k) = u(k-1)$ or~~

Here, $x(k) = 0$ for $k > -1$ and $h(n-k) = 0$ for $k > n-1$.

For $n-1 < -1$, i.e., for $n < 0$, the interval of summation is from $k = -\infty$ to ~~n~~ $n-1$.

$$\therefore\ y(n) = \sum_{k=-\infty}^{n-1} \left(\frac{1}{3}\right)^{-k} \text{ for } n-1 \le -1 \text{ or for } n \le 0$$

$$= \left(\frac{1}{3}\right)^{-(n-1)} + \left(\frac{1}{3}\right)^{-(n-2)} + \cdots$$

$$= \left(\frac{1}{3}\right)^{-(n-1)} \left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \cdots\right]$$

$$= 3^{n-1} \left[\frac{1}{1 - 1/3}\right] = 0.5(3)^n$$

For $n-1 > -1$, i.e., for $n > 0$, the interval of summation is from $k = -\infty$ to $k = -1$.

$$\therefore\ y(n) = \sum_{k=-\infty}^{-1} \left(\frac{1}{3}\right)^{-k}$$

$$= \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{k}$$

$$= \frac{1/3}{1 - 1/3}$$

$$= 0.5$$

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Example 2.10

a) Given, $h(n) = \delta(n) - \delta(n-2)$

The step response $s(n) = h(n) * u(n)$

$$= [\delta(n) - \delta(n-2)] * u(n)$$

$$= \delta(n) * u(n) - \delta(n-2) * u(n)$$

$$= u(n) - u(n-2)$$

b) Given, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

$$\therefore s(n) = \left(\frac{1}{4}\right)^n u(n) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} u(k) \left(\frac{1}{4}\right)^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} \left(\frac{1}{4}\right)^{n-k}$$

$$= \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^{-k}$$

$$= \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} 4^k$$

$$= \left(\frac{1}{4}\right)^n \left[\frac{1 - 4^{n+1}}{1 - 4}\right]$$

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c) Given, $h(n) = n\,u(n)$

$$\therefore y(n) = h(n) * u(n)$$

$$= n\,u(n) * u(n)$$

$$= \sum_{k=0}^{n} k\,u(k)\,u(n-k)$$

$$= \sum_{k=0}^{n} k$$

d) Given, $h(n) = u(n)$

$$\therefore y(n) = h(n) * u(n)$$

$$= u(n) * u(n)$$

$$= \sum_{k=-\infty}^{\infty} u(k)\,u(n-k)$$

$$= \sum_{k=0}^{\cancel{\infty}\, n} 1$$

$$= n+1$$

\# Show that an impulse response and convolution

Methods to compute the Convolution sum of two sequences x(n) and h(n).

→ Folding method

Method 1 : Linear Convolution Using Graphical method

Method 2 : Linear Convolution using Tabular Array

Method 3 : Linear Convolution using Tabular Method

Method 4 : Linear Convolution using Matrices.

Method 5 : Linear Convolution using the Sum-by Column Method

Method 6 : Linear Convolution using the Flip, Shift, multiply and Sum method.

Example 2.11 Determine the Convolution sum of two sequences:

$$x(n) = \{4, 2, 1, 3\}, \quad h(n)\{1, \underset{\uparrow}{2}, 2, 1\}$$

⇒ x(n) starts at $n_1 = 0$ and h(n) starts at $n_2 = -1$. Therefore, the starting sample at y(n) is at

$$n = n_1 + n_2 = 0 - 1 = -1$$

x(n) has 4 samples, h(n) has 4 samples. Therefore, y(n) will have $N = 4 + 4 - 1 = 7$ samples, i.e., from $n = -1$ to $n = 5$.

Method 1 Graphical method:

We know that $$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

For $n = -1$, $$y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$$

$$= 4 \cdot 1 = 4$$

For $n=0$, $y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k) = 4\cdot 2 + 2\cdot 1 = 10$

For $n=1$, $y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k) = 4\cdot 2 + 2\cdot 2 + 1\cdot 1 = 13$

For $n=2$, $y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k) = 4\cdot 1 + 2\cdot 2 + 1\cdot 2 + 3\cdot 1 = 13$

For $n=3$, $y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k) = 2\cdot 1 + 1\cdot 2 + 3\cdot 2 = 10$

For $n=4$, $y(4) = \sum_{k=-\infty}^{\infty} x(k)\, h(4-k) = 1\cdot 1 + 3\cdot 2 = 7$

For $n=5$, $y(5) = \sum_{k=-\infty}^{\infty} x(k)\, h(5-k) = 3\cdot 1 = 3$

$$\therefore y(n) = \{4, \underset{\uparrow}{10}, 13, 13, 10, 7, 3\}$$

Method 2 Tabular Array:

K		-4	-3	-2	-1	0	1	2	3	4	5	6	7
x(k)		–	–	–	–	4	2	1	3	–	–	–	–
h(-k)		–	–	1	2	2	1	–	–	–	–	–	–
n = -1	h(-1-k)	–	1	2	2	1	–	–	–	–	–	–	–
n = 0	h(-k)	–	–	1	2	2	1	–	–	–	–	–	–
n = 1	h(1-k)	–	–	–	1	2	2	1	–	–	–	–	–
n = 2	h(2-k)	–	–	–	–	1	2	2	1	–	–	–	–
n = 3	h(3-k)	–	–	–	–	–	1	2	2	1	–	–	–
n = 4	h(4-k)	–	–	–	–	–	–	1	2	2	1	–	–
n = 5	h(5-k)	–	–	–	–	–	–	–	1	2	2	1	–

The starting value of $n = -1$. From the table, we can see that

For $n=-1$, $y(-1) = \sum_{k=-\infty}^{\infty} x(k)\,h(-1-k) = 4\cdot 1 = 4$

For $n=0$, $y(0) = \sum_{k=-\infty}^{\infty} x(k)\,h(-k) = 4\cdot 2 + 2\cdot 1 = 10$

For $n=1$, $y(1) = \sum_{k=-\infty}^{\infty} x(k)\,h(1-k) = 4\cdot 2 + 2\cdot 2 + 1\cdot 1 = 13$

For $n=2$, $y(2) = \sum_{k=-\infty}^{\infty} x(k)\,h(2-k) = 4\cdot 1 + 2\cdot 2 + 1\cdot 2 + 3\cdot 1 = 13$

For $n=3$, $y(3) = \sum_{k=-\infty}^{\infty} x(k)\,h(3-k) = 2\cdot 1 + 1\cdot 2 + 3\cdot 2 = 10$

For $n=4$, $y(4) = \sum_{k=-\infty}^{\infty} x(k)\,h(4-k) = 1\cdot 1 + 3\cdot 2 = 7$

For $n=5$, $y(5) = \sum_{k=-\infty}^{\infty} x(k)\,h(5-k) = 3\cdot 1 = 3$

$\therefore\ y(n) = \{4, \underset{\uparrow}{10}, 13, 13, 10, 7, 3\}$

Method 3 Tabular Method:

Given, $x(n) = \{4, 2, 1, 3\}$, $h(n) = \{1, \underset{\uparrow}{2}, 2, 1\}$

h(n) \ x(n)	4	2	1	3
1	4	2	1	3
2	8	4	2	6
2	8	4	2	6
1	4	2	1	3

$\therefore\ y(n) = 4, 10, 13, 13, 10, 7, 3$

The starting value of n is equal to -1, mark the symbol $\uparrow$ at time origin ($n=0$).

$\therefore\ y(n) = \{4, \underset{\uparrow}{10}, 13, 13, 10, 7, 3\}$

~~Method~~ Matrices Method:

The given sequences are: $x(n) = \{x(0), x(1), x(2), x(3)\}$
$= \{4, 2, 1, 3\}$

and $h(n) = \{h(0), h(1), h(2), h(3)\} = \{1, \underset{\uparrow}{2}, 2, 1\}$

The sequence $x(n)$ is starting at $n=0$ and the sequence $h(n)$ is starting at $n=-1$. So, the sequence $y(n)$ corresponding to the linear convolution of $x(n)$ and $h(n)$ will start at $n = 0 + (-1) = -1$. $x(n)$ is of length 4 and $h(n)$ is also of lenght 4. So length of $y(n) = 4+4-1 = 7$.

Substituting the sequence values in matrix ~~form~~ form and multiplying as shown below, we get,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 1 & 2 & 2 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4+0+0+0 \\ 4\cdot 2 + 2\cdot 1 + 0 + 0 \\ 4\cdot 2 + 2\cdot 2 + 1\cdot 1 + 0 \\ 4\cdot 1 + 2\cdot 2 + 1\cdot 2 + 3\cdot 1 \\ 40 + 2\cdot 1 + 1\cdot 2 + 3\cdot 2 \\ 0 + 0 + 1\cdot 1 + 3\cdot 2 \\ 0 + 0 + 0 + 3\cdot 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \\ 13 \\ 13 \\ 10 \\ 7 \\ 3 \end{bmatrix}$$

$\therefore y(n) = x(n) * h(n) = \{4, \underset{\uparrow}{10}, 13, 13, 10, 7, 3\}$

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Note:

To check my convolution is correct or not:

sum of $x(n)$ X sum of $*h(n)$ = sum of $y(n)$

Example 2.12 Find the convolution of the signals

$$x(n) = \begin{cases} 2 & n = -2, 0, 1 \\ 3 & n = -1 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \delta(n) - 2\,\delta(n-1) + 3\,\delta(n-2) - \delta(n-3)$$

$\Rightarrow$ Given $x(n) = \{2, 3, \underset{\uparrow}{2}, 2\}$; $h(n)\{1, -2, 3, -1\}$

$x(n)$ starts at $n_1 = -2$ and $h(n)$ starts at $n_2 = 0$. The starting sample of $y(n)$ is at $n = n_1 + n_2 = -2 + 0 = -2$.

Since number of samples in $x(n)$ is 4, and the number of samples in $h(n)$ is 4, the number of samples in $y(n)$ will be $4+4-1 = 7$. So $y(n)$ exists from -2 to 4.

Method 1 Graphical method:

We know that $y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$

For $n=-2$, $y(-2) = \sum_{k=-\infty}^{\infty} x(k)\,h(-2-k) = 2\cdot 1 = 2$

For $n=-1$, $y(-1) = \sum_{k=-\infty}^{\infty} x(k)\,h(-1-k) = 2\cdot(-2) + 3\cdot 1 = -1$

For $n=0$, $y(0) = \sum_{k=-\infty}^{\infty} x(k)\,h(-k) = 2\cdot 3 + 3\cdot(-2) + 2\cdot 1 = 2$

For $n=1$, $y(1) = \sum_{k=-\infty}^{\infty} x(k)\,h(1-k) = 2\cdot(-1) + 3\cdot 3 + 2\cdot(-2) + 2\cdot 1 = 5$

For $n=2$, $y(2) = \sum_{k=-\infty}^{\infty} x(k)\,h(2-k) = 3\cdot(-1) + \cancel{3}2\cdot 3 + 2\cdot(-2) = -1$

For $n=3$, $y(3) = \sum_{k=-\infty}^{\infty} x(k)\,h(3-k) = 2(-1) + 2\cdot 3 = 4$

For $n=4$, $y(4) = \sum_{k=-\infty}^{\infty} x(k)\,h(4-k) = 2\cdot(-1) = -2$

$\therefore\ y(n) = \{2, -1, \underset{\uparrow}{2}, 5, -1, 4, -2\}$

Method 2 Tabular array:

		-5	-4	-3	-2	-1	0	1	2	3	4	5
k		-5	-4	-3	-2	-1	0	1	2	3	4	5
x(n)		–	–	–	2	3	2	2	–	–	–	–
h(-n)		–	–	-1	3	-2	1	–	–	–	–	–
n = -2	h(-2-k)	-1	3	-2	1	–	–	–	–	–	–	–
n = -1	h(-1-k)	–	-1	3	-2	1	–	–	–	–	–	–
n = 0	h(-k)	–	–	-1	3	-2	1	–	–	–	–	–
n = 1	h(1-k)	–	–	–	-1	3	-2	1	–	–	–	–
n = 2	h(2-k)	–	–	–	–	-1	3	-2	1	–	–	–
n = 3	h(3-k)	–	–	–	–	–	-1	3	-2	1	–	–
n = 4	h(4-k)	–	–	–	–	–	–	-1	3	-2	1	–

The starting sample of $y(n)$ is at $n=-2$. $y(n)$ is calculated as shown below.

From the table, we can see that

For $n=-2$, $y(-2) = \sum_{k=-\infty}^{\infty} x(k)\,h(-2-k) = 2\cdot 1 = 2$

For $n=-1$, $y(-1) = \sum_{k=-\infty}^{\infty} x(k)\,h(-1-k) = 2\cdot(-2) + 3\cdot 1 = -1$

For $n=0$, $y(0) = \sum_{k=-\infty}^{\infty} x(k)\,h(-k) = 2\cdot 3 + 3\cdot(-2) + 2\cdot 1 = 2$

For $n=1$, $y(1) = \sum_{k=-\infty}^{\infty} x(k)\,h(1-k) = 2\cdot(-1) + 3\cdot 3 + 2\cdot(-2) + 2\cdot 1 = 5$

For $n=2$, $y(2) = \sum_{k=-\infty}^{\infty} x(k)\,h(2-k) = 3\cdot(-1) + 2\cdot 3 + 2\cdot(-2) = -1$

For $n=3$, $y(3) = \sum_{k=-\infty}^{\infty} x(k)\,h(3-k) = 2\cdot(-1) + 2\cdot 3 = 4$

For $n=4$, $y(4) = \sum_{k=-\infty}^{\infty} x(k)\,h(4-k) = 2\cdot(-1) = -2$

$\therefore y(n) = \{2, -1, \underset{\uparrow}{2}, 5, -1, 4, -2\}$

Method 3 Tabular Method:

$h(n)$ \ $x(n)$	2	3	2	2
1	2	3	2	2
-2	-4	-6	-4	-4
3	6	9	6	6
-1	-2	-3	-2	-2

$\therefore y(n) = \{2, -1, \underset{\uparrow}{2}, 5, -1, 4, -2\}$

Method 4 Matrices Method:

The given sequences are: $x(n) = \{x(0), x(1), x(2), x(3)\}$

$= \{2, 3, \underset{\uparrow}{2}, 2\}$

and $h(n) = \{h(0), h(1), h(2), h(3)\} = \{1, -2, 3, -1\}$

The sequence $x(n)$ is starting at $n=-2$ and the sequence $h(n)$ is starting at $n=0$. So the sequence $y(n)$ corresponding to the linear convolution of $x(n)$ and $y(n)$ will start at

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$n = -2+0 = -2$. $x(n)$ is of length 4 and $h(n)$ is also of length 4. So length of $y(n) = 4+4-1 = 7$.

Substituting the sequence values in matrix from and multiplying, we get the convolution of $x(n)$ and $h(n)$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 \\ -1 & 3 & -2 & 1 \\ 0 & -1 & 3 & -2 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2\cdot1 + 0 + 0 + 0 \\ 2\cdot(-2) + 3\cdot1 + 0 + 0 \\ 2\cdot3 + 3\cdot(-2) + 2\cdot1 + \cancel{2}\,0 \\ 2\cdot(-1) + 3\cdot3 + 2\cdot(-2) + 2\cdot1 \\ 0 + 3\cdot(-1) + 2\cdot3 + 2\cdot(-2) \\ 0 + 0 + 2\cdot(-1) + 2\cdot3 \\ 0 + 0 + 0 + 2\cdot(-1) \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \\ 5 \\ -1 \\ 4 \\ -2 \end{bmatrix}$$

$$\therefore y(n) = x(n) * h(n) = \{2, -1, \underset{\uparrow}{2}, 5, -1, 4, -2\}$$

Deconvolution:

Deconvolution is the process of finding the input $x(n)$ too [or impulse response $h(n)$] applied to the system once the output $y(n)$ and the impulse response $h(n)$ [or the input $x(n)$] of the system are known.

The Z-transform also can be used for deconvolution operation.

$$\boxed{Y(z) = X(z)H(z) \quad \text{or} \quad X(z) = \frac{Y(z)}{H(z)}}$$

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Ex-2.22

Given, $h(n) = \{2, 1, 0, -1, 3\}$

$$\therefore H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = \sum_{n=0}^{4} h(n) z^{-n}$$

$$= 2 + z^{-1} + 0 - z^{-3} + 3z^{-4} = 2 + z^{-1} - z^{-3} + 3z^{-4}$$

and $y(n) = \{2, -5, 1, 1, 6, -11, 6\}$

$$\therefore Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} = \sum_{n=0}^{6} y(n) z^{-n}$$

$$= 2 - 5z^{-1} + z^{-2} + z^{-3} + 6z^{-4} - 11z^{-5} + 6z^{-6}$$

$$\therefore X(z) = \frac{Y(z)}{H(z)} = \frac{2 - 5z^{-1} + z^{-2} + z^{-3} + 6z^{-4} - 11z^{-5} + 6z^{-6}}{2 + z^{-1} - z^{-3} + 3z^{-4}}$$

$$\begin{array}{r|l}
 & 1 - 3z^{-1} + 2z^{-2} \\
\hline
2 + z^{-1} - z^{-3} + 3z^{-4} & 2 - 5z^{-1} + z^{-2} + z^{-3} + 6z^{-4} - 11z^{-5} + 6z^{-6} \\
 & 2 + z^{-1} - z^{-3} + 3z^{-4} \\
\hline
 & -6z^{-1} + z^{-2} + 2z^{-3} + 3z^{-4} - 11z^{-5} + 6z^{-6} \\
 & -6z^{-1} - 3z^{-2} + 3z^{-4} - 9z^{-5} \\
\hline
 & 4z^{-2} + 2z^{-3} - 2z^{-5} + 6z^{-6} \\
 & 4z^{-2} + 2z^{-3} - 2z^{-5} + 6z^{-6} \\
\hline
 & 0
\end{array}$$

$$\therefore X(z) = 1 - 3z^{-1} + 2z^{-2}$$

Taking inverse z-transform, we have input

$$x(n) = \delta(n) - 3\delta(n-1) + 2\delta(n-2)$$

$$\therefore x(n) = \{1, -3, 2\}$$

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Deconvolution by Recursion:

Assuming $y(n)$ and $h(n)$ are one sided sequences,

$$y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

$$\boxed{\therefore\ x(n) = \frac{y(n) - \sum_{k=0}^{n-1} x(k)\, h(n-k)}{h(0)}}$$

Example 2.23

Given, $y(n) = \{1, 1, 2, 0, 2, 1\}$ and $h(n) = \{1, -1, 1\}$

Let N_1 be the number of samples in $x(n)$, and N_2 be the number of samples in $h(n)$. The number of samples in $y(n) = N_1 + N_2 - 1 = 6$, therefore, $N_1 = 6 - N_2 + 1 = 6 - 3 + 1 = 4$.

Let, $x(n) = \{x(0), x(1), x(2), x(3)\}$

We have, $x(n) = \dfrac{y(n) - \sum_{k=0}^{n-1} x(k)\, h(n-k)}{h(0)}$

For, $n=0$, $x(0) = \dfrac{y(0)}{h(0)} = \dfrac{1}{1} = 1$

For $n=1$, $x(1) = \dfrac{y(1) - x(0)\,h(1)}{h(0)} = \dfrac{1 - 1\cdot(-1)}{1} = 2$

For $n=2$, $x(2) = \dfrac{y(2) - x(0)\,h(2) - x(1)\,h(1)}{h(0)} = \dfrac{2 - 1\cdot 1 - 2\cdot(-1)}{1}$

$= 3$

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যেহেতু $h(3)$ exist করে না, so, $h(3)=0$

For $n=3$, $x(3) = \dfrac{y(3) - x(0)\,h(3) - x(1)\,h(2) - x(2)\,h(1)}{h(0)}$

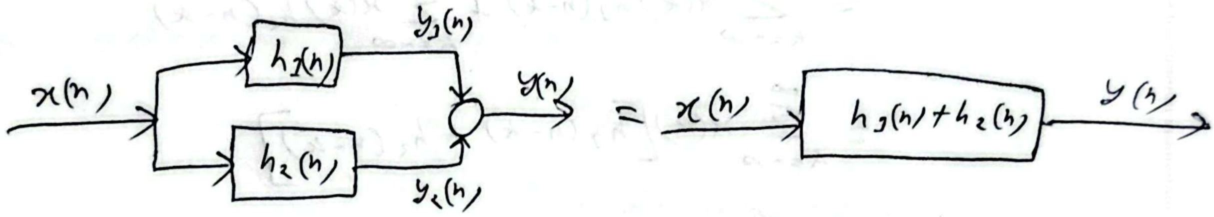
$$= \frac{0 - 1\cdot(0) - 2\cdot 1 - 3(-1)}{1} = \frac{-2+3}{1} = 1$$

$\therefore x(n) = \{1, 2, 3, 1\}$

Interconnection of LTI System:

LTI → Linear Time Invariant

Parallel Connection of Systems:



Prove that, If two systems are connected in parallel, the overall impulse response is equal to the sum of two impulse responses.

proof:

~~Let the output of system 1 is, $y_1(n) = x(n) * h_1(n)$~~

~~and the output of system 2 is, $y_2(n) = x(n) * h_2(n)$~~

~~$\therefore$ The~~

Consider two LTI systems connected in parallel having impulse responses $h_1(n)$ and $h_2(n)$. Let the input be $x(n)$.

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Output of system 1:

$$y_1(n) = x(n) * h_1(n)$$

output of system 2:

$$y_2(n) = x(n) * h_2(n)$$

The overall output of the system $y(n)$ is,

$$y(n) = y_1(n) + y_2(n)$$

$$= x(n) * h_1(n) + x(n) * h_2(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k) h_1(n-k) + \sum_{k=-\infty}^{\infty} x(k) h_2(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \left[h_1(n-k) + h_2(n-k) \right]$$

$$= \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

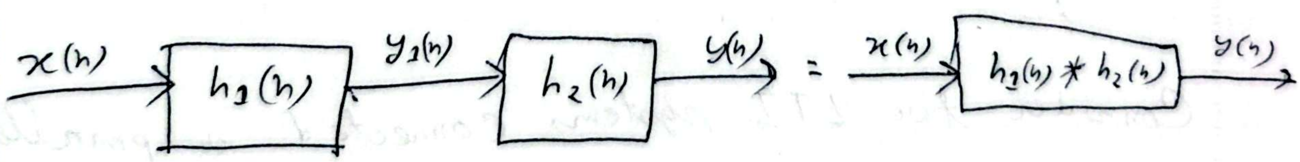
$$= x(n) * h(n)$$

where $h(n) = h_1(n) + h_2(n)$

[proved]

serial

Cascade Connection of Systems:



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Prove that, when two LTI systems are connected in cascade, the overall impulse response is the convolution of individual impulse responses.

proof:

Consider two LTI systems connected in cascade having impulse responses $h_1(n)$ and $h_2(n)$. Let the input be $x(n)$.

output of the system 1:

$$y_1(n) = x(n) * h_1(n)$$

The output becomes the input of the second system.

So, output of the system 2:

$$y_2(n) = y_1(n) * h_2(n)$$

$$= [x(n) * h_1(n)] * h_2(n)$$

$$= x(n) * [h_1(n) * h_2(n)]$$

$$= x(n) * h(n)$$

where $h(n) = h_1(n) * h_2(n)$

[proved]

Circular Shift: Normal shift-এ signal কে left/right-এ shift হয় এবং outside value 0 হয়। কিন্তু circular shift-এ, signal-এর last element আবার first-এ চলে আসে

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Periodic or Circular Convolution:

Methods of performing periodic or Circular Convolution:

Method 1: Graphical Method (Concentric Circle Method)

Method 2: Circular convolution using Tabular Array

Method 3: Circular convolution using Matrices.

Example 2.28

Let $x_3(n)$ be the circular convolution of $x_1(n)$ and $x_2(n)$. To find the circular convolution, both sequences must be of same length. Here $x_1(n)$ is of length 6 and $x_2(n)$ is of length 3. Therefore, we append three zeros to the sequence $x_2(n)$ and use concentric circle method to find circular convolution. So we have

$$x_1(n) = \{1, 2, -1, -2, 3, 1\}, \quad x_2(n) = \{3, 2, 1, 0, 0, 0\}$$

Graph all point of $x_1(n)$ on the outer circle in the anticlockwise direction. Starting at the same point as $x_1(n)$, graph all points of $x_2(n)$ on the inner circle in clockwise direction.

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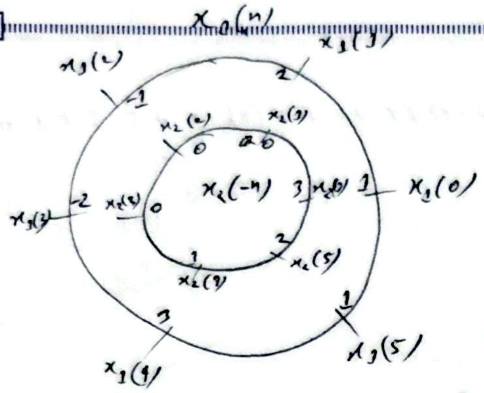
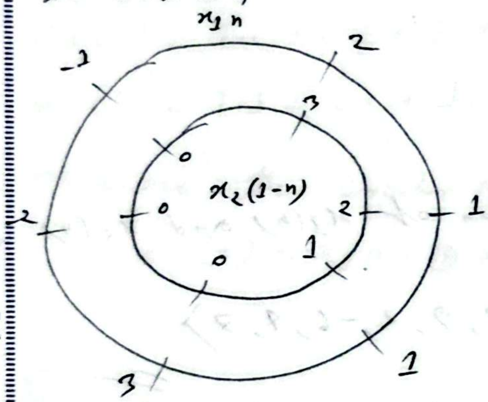


Multiply corresponding samples on the circles and add to obtain

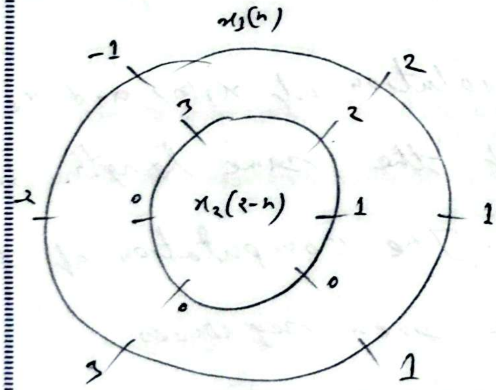
$$x_3(0) = 1\cdot 3 + 2\cdot 0 + (-1)\cdot 0 + (-2)\cdot 0 + 3\cdot 1 + 1\cdot 2$$
$$= 8$$

Rotate the inner circle in anticlockwise direction by one sample and multiply the corresponding samples and add the products to obtain

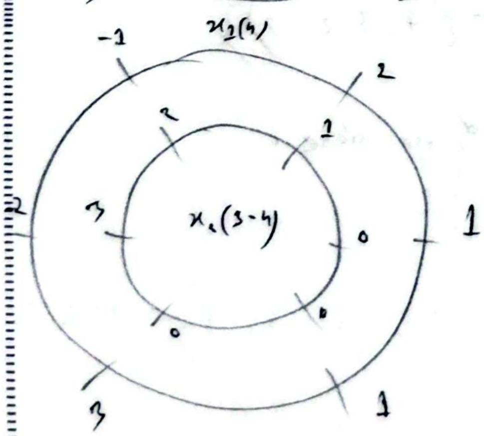


$$x_3(1) = 1\cdot 2 + 2\cdot 3 + (-1)\cdot 0 + (-2)\cdot 0 + 3\cdot 0 + 1\cdot 1$$
$$= 9$$

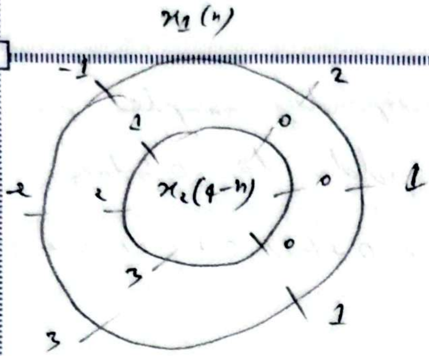
Continue it to obtain $x_3(2)$, $x_3(3)$, $x_3(4)$, and $x_3(5)$



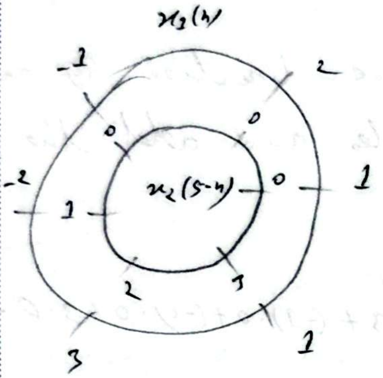
$$x_3(2) = 1\cdot 1 + 2\cdot 2 + (-1)\cdot 3 + (-2)\cdot 0 + 3\cdot 0 + 1\cdot 0$$
$$= 2$$



$$x_3(3) = 1\cdot 0 + 2\cdot 1 + (-1)\cdot 2 + (-2)\cdot 3 + 3\cdot 0 + 1\cdot 0$$
$$= -6$$



$$x_3(4) = 1\cdot 0 + 2\cdot 0 + (-1)\cdot 1 + (-2)\cdot 2 + 3\cdot 3 + 1\cdot 0$$
$$= 4$$



$$x_3(5) = 1\cdot 0 + 2\cdot 0 + (-1)\cdot 0 + (-2)\cdot 1 + 3\cdot 2 + 1\cdot 3$$
$$= 7$$

Therefore, the circular convolution of $x_1(n)$ and $x_2(n)$ is:

$$x_3(n) = x_1(n) \circledast x_2(n) = \{8, 9, 2, -6, 4, 7\}$$

Example 2.29

Let $x_3(n)$ be the circular convolution of $x_1(n)$ and $x_2(n)$. Here $x_1(n)$ and $x_2(n)$ are of the same length. So no padding of zeroes is required. The computation of the circular convolution $x_3(n)$ of the given sequences $x_1(n) = \{1, 2, 1, 2\}$ and $x_2(n) = \{4, 3, 2, 1\}$ by the concentric circles method is done.

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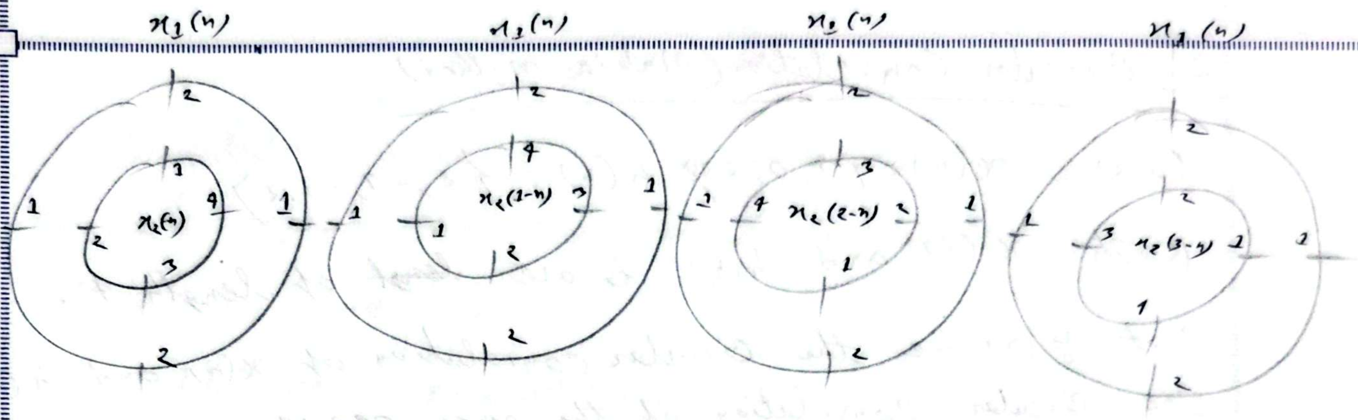

$x_3(0) = 1\cdot4 + 2\cdot1 + 1\cdot2 + 2\cdot3 = 14$

$x_3(1) = 1\cdot3 + 2\cdot4 + 1\cdot1 + 2\cdot2 = 16$

$x_3(2) = 1\cdot2 + 2\cdot3 + 1\cdot4 + 2\cdot1 = 14$

$x_3(3) = 1\cdot1 + 2\cdot2 + 1\cdot3 + 4\cdot2 = 16$

Therefore, the circular convolution of $x_1(n)$ and $x_2(n)$ is:

$$x_3(n) = x_1(n) \circledast x_2(n) = \{14, 16, 14, 16\}$$

CT 1 Given, $x(n) = \{2,1,0,5\}$ and $h(n) = \{2,0,1,1\}$

1. Compute the linear convolution of the following signals. 10
$x(n) = \{2,1,0,5\}$ $h(n) = \{2,0,1,1\}$
2. Compute the circular convolution of the following signals. 10
$x(n) = \{1,0,2,0\}$ $h(n) = \{2,-1,-1,2\}$

refore,

will

Tabular Method:

$h(n)$ \ $x(n)$	2	1	0	5
2	4	2	0	10
0	0	0	0	0
1	2	1	0	5
1	2	1	0	5

$\therefore y(n) = \{4, 2, 2, 13, 1, 5, 5\}$

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2) Circular Convolution (Matrix method)

Given, $x(n) = \{1, 0, 2, 0\}$ $h(n) = \{2, -1, -1, 2\}$

Both $x(n)$ and $h(n)$ ~~is~~ are ~~lengt~~ of length 4.

Let, $y(n)$ be the circular convolution of $x(n)$ and $h(n)$.
The circular convolution of the given sequences:

$$x(n) = \{x(0), x(1), x(2), x(3)\}$$
$$= \{1, 0, 2, 0\}$$

and $h(n) = \{h(0), h(1), h(2), h(3)\}$
$$= \{2, -1, -1, 2\}$$

Can be determined using matrices as follows:

$$\begin{bmatrix} h(0) & h(3) & h(2) & h(1) \\ h(1) & h(0) & h(3) & h(2) \\ h(2) & h(1) & h(0) & h(3) \\ h(3) & h(2) & h(1) & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -1 & -1 \\ -1 & 2 & 2 & -1 \\ -1 & -1 & 2 & 2 \\ 2 & -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2+0-2+0 \\ -1+0+4+0 \\ -1+0+4+0 \\ 2+0-2+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \\ 0 \end{bmatrix}$$

$\therefore y(n) = \{0, 3, 3, 0\}$

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Discrete Correlation

Correlation measures the similarity between two discrete signals as one signal is shifted over another. There are two types:

1) Cross - Correlation.

2) Auto - Correlation.

Applications:

1) used in radar and sonar system to find the location of a target by comparing the transmitted and reflected signals.

2) image processing.

3) Control engineering.

Cross Correlation:

Cross-Correlation is the measure of similarity between two discrete time signals as one signal is shifted with respect to the other.

The cross correlation between a pair of sequences $x(n)$ and $y(n)$,

$$R_{xy}(n) = \sum_{k=-\infty}^{\infty} x(k)\, y(k-n) \quad \text{------------} \quad (i)$$

$$= \sum_{k=-\infty}^{\infty} x(k+n)\, y(k) \quad ; n = 0, \pm 1, \pm 2, \ldots\ldots$$

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If we wish to fix the sequence $y(n)$ and to shift the sequence $x(n)$, then the correlation can be written as:

$$R_{yx}(n) = \sum_{k=-\infty}^{\infty} y(k)\, x(k-n) \quad \text{———— (ii)}$$

$$= \sum_{k=-\infty}^{\infty} y(k+n)\, x(k) \; ; \; n = 0, \pm 1, \pm 2, \ldots$$

$$\boxed{R_{xy}(n) \neq R_{yx}(n)}$$

But if the time shift $n=0$, then we get

$$R_{xy}(0) = R_{yx}(0) = \sum_{k=-\infty}^{\infty} x(k)\, y(k)$$

Comparing ~~the~~ (i) and (ii),

$$\boxed{R_{xy}(n) = R_{yx}(-n)}$$

P/ # Prove that, the cross-correlation is the convolution of one signal with a flipped version of the other, that is: $R_{xy}(n) = x(n) * y(-n)$

where, the symbols have their usual meanings.

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CamScanner

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~~Prove~~ Proof:

The cross-correlation between two discrete-time signals $x(n)$ and $y(n)$ is defined as,

$$R_{xy}(n) = \sum_{k=-\infty}^{\infty} x(k)\, y(k-n)$$

The convolution of two signals $x(n)$ and $h(n)$ is,

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Let $h(n) = y(-n)$, which is the time-reversed or flipped version of $y(n)$, then,

$$x(n) * y(-n) = \sum_{k=-\infty}^{\infty} x(k)\, y(-(n-k))$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, y(k-n) = R_{xy}(n)$$

$\therefore R_{xy}(n) = x(n) * y(-n)$ [proved]

Autocorrelation:

Auto-correlation is the correlation of a discrete-time signal with itself to measure similarity at different time shifts. The autocorrelation of a sequence $x(n)$ is defined as:

$$\boxed{R_{xx}(n) = \sum_{k=-\infty}^{\infty} x(k)\, x(k-n) = \sum_{k=-\infty}^{\infty} x(k+n)\, x(k)}$$

If the time shift $n=0$, then we have

$$R_{xx}(0) = \sum_{k=-\infty}^{\infty} x^2(k)$$

$R_{xx}(n)$ is given by,

$$\boxed{R_{xx}(n) = x(k) * x(-k)}$$

Example 2.51 Find the auto correlation of $x(n) = a^n u(n)$

→ To compute $R_{xx}(n)$ which is even symmetric, we need to compute the result only for $n \geq 0$ and create its even extension.

For $n \geq 0$, $R_{xx}(n) = \sum_{k=-\infty}^{\infty} x(k)\, x(k-n) = \sum_{k=n}^{\infty} a^k a^{k-n}$

$$= \sum_{m=0}^{\infty} a^{m+n} a^m = a^n \sum_{\substack{\cancel{k=n} \\ m=0}}^{\infty} a^{2m}$$

$$= \frac{a^n}{1-a^2}\, u(n)$$

$m = k - n$

$\therefore k = m + n$

$k = n,\ m = 0$

The even extension of this result gives

$$R_{xx}(n) = \frac{a^{|n|}}{1-a^2}$$