Amplitude Modulation

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Modulation

Modulation is a process of varying the characteristics of RF carrier wave in accordance with a modulating signal that typically contains information that is to be transmitted.

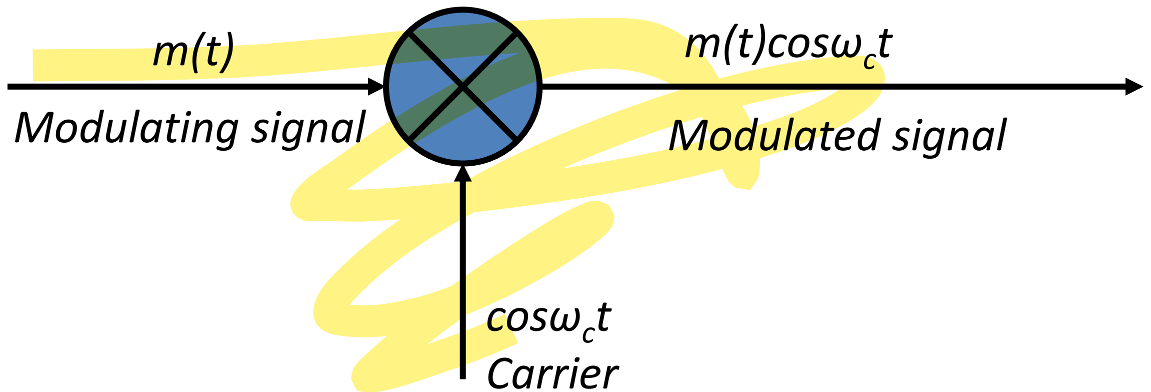
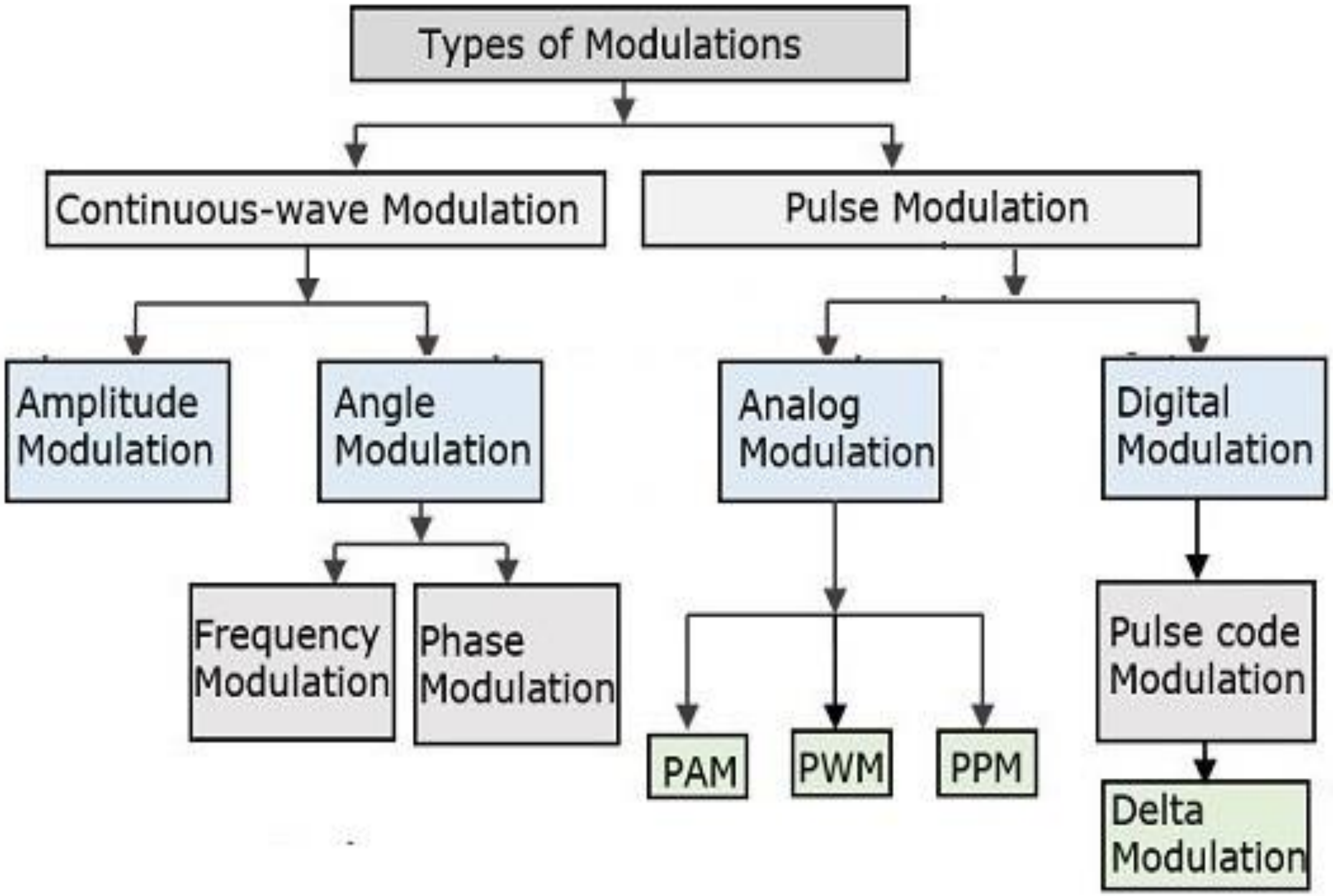


Fig. Modulator

- The original signal that generated from input transducer which actually we want to sent through transmitter is called Message Signal or modulating Signal or Baseband Signal.

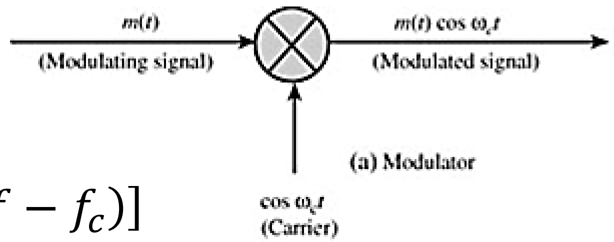
- The High Frequency Sinusoidal Signal Which is used to shift the frequency of message signal is called carrier signal.

Different types of modulation systems



Amplitude Modulation

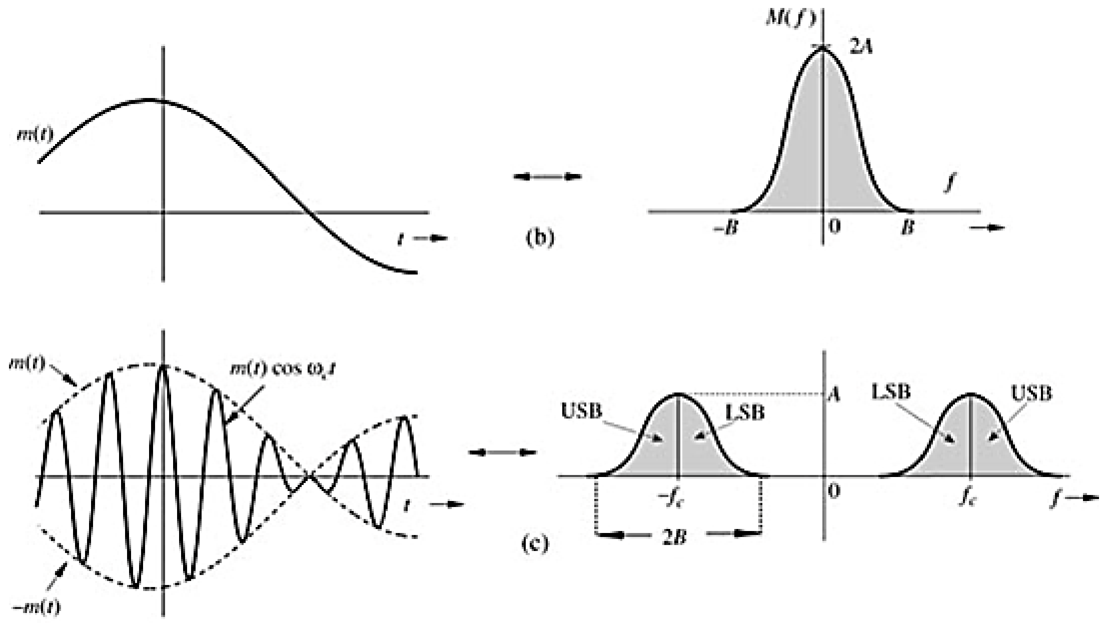
- **Amplitude Modulation** is a process of varying amplitude of high frequency carrier signal in accordance with the instantaneous amplitude of the information signal and also the frequency and phase are kept constant.



$$m(t) \Longleftrightarrow M(f)$$

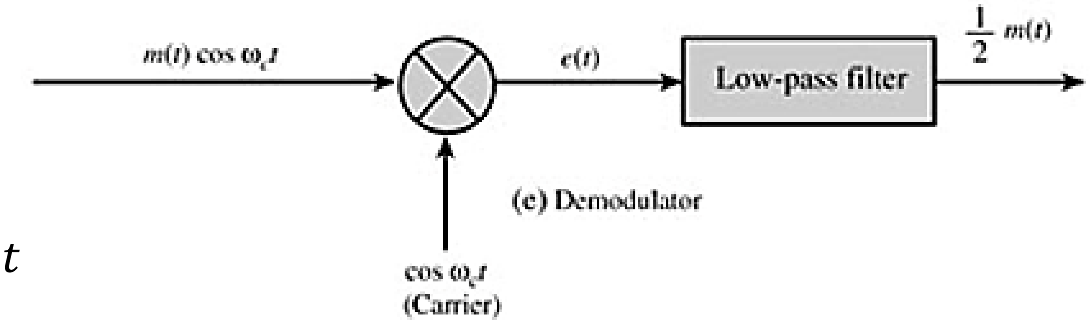
$$m(t)\cos 2\pi f_c t \Longleftrightarrow \frac{1}{2}[M(f+f_c)+M(f-f_c)]$$

If the bandwidth of modulating signal is B Hz, then bandwidth of modulated signal is 2B Hz. The band above f_c is USB, and below f_c is LSB. Since, modulated signal does not contain discrete component of carrier frequency, it is called double side band suppressed carrier (**DSB-SC).**



Demodulation

Demodulator: recovering the message signal at the receiver from the modulated signal.

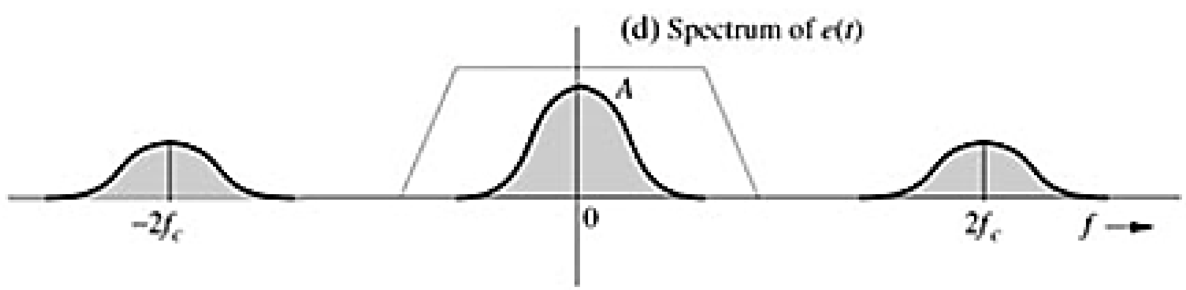


(c) Demodulator

$$e(t) = m(t)\cos^2\omega_c t$$

$$e(t) = \frac{1}{2}[m(t) + m(t)cos2\omega_c t$$

$$E(f) = \frac{1}{2}M(f) + \frac{1}{4}[M(f+2f_c) + M(f-2f_c)]$$



(d) Spectrum of $e(t)$

General Equation of AM and Spectrum

Let us consider, a sinusoidal carrier wave

$$c(t) = A\cos\omega_c t$$

Now, if $x(t)$ denotes the modulating or baseband signal, according to AM, So, amplitude modulated wave may be expressed as

$$s(t) = x(t)\cos\omega_c t + A\cos\omega_c t$$

$$s(t) = [x(t) + A]\cos\omega_c t$$

$$s(t) = E(t)\cos\omega_c t$$

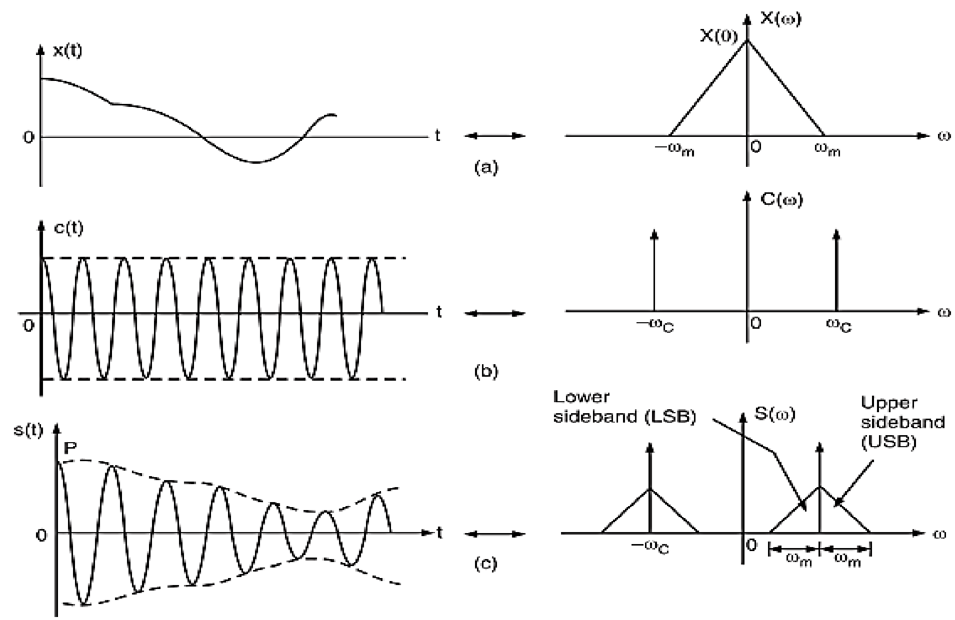
Where, $E(t) = [x(t) + A]$ is called the envelope of the AM wave.

If $\quad x(t) \leftrightarrow X(\omega)$

then $\quad e^{j\omega_c t}\, x(t) \leftrightarrow X(\omega - \omega_c)$

$$A\cos\omega_c t \leftrightarrow \pi A[\delta(\omega + \omega_c) + \delta(\omega - \omega_c)]$$

$$x(t)\cos\omega_c t \leftrightarrow \frac{1}{2}[X(\omega - \omega_c) + X(\omega + \omega_c)]$$



$$S(\omega) = \frac{1}{2}[X(\omega - \omega_c) + X(\omega + \omega_c)] + \pi A[\delta(\omega + \omega_c) + \delta(\omega - \omega_c)]$$

Modulation Index

- In AM wave, the modulation index (m) is defined as

$$m = \frac{E_m}{E_c}$$

Where, E_m and E_c is amplitude of the modulating and carrier waves respectively.

- When $E_m \leq E_c$, the modulation index has values between 0 and 1 and no distortion is introduced in the AM wave.
- $E_m \geq E_c$, the modulation index is greater than 1, this will distort the shape of AM signal.
- The distortion happens for over modulation.
- The modulation index is also called modulation factor.
- The modulation index as percentage can be written as:

$$Percent\ modulation, = \frac{E_m}{E_c} \times 100.$$

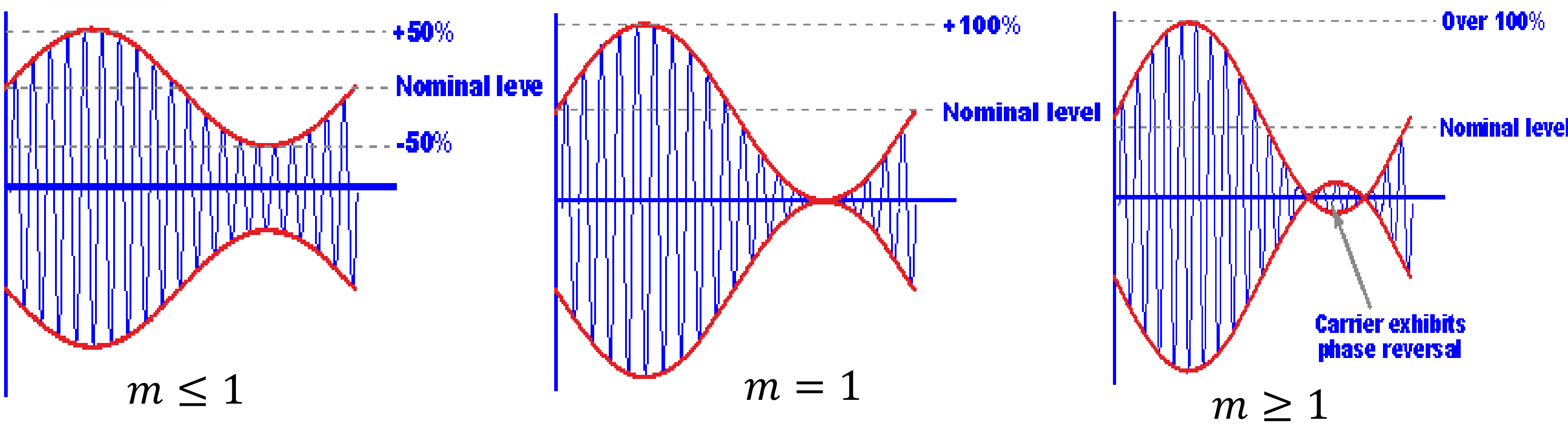
Linear and Over Modulation

- Based on the values of modulation index, it can be classified as:

  i. Linear modulation

  ii. Over modulation

Linear Modulation: If $m \leq 1$ or if the percentage modulation is less than 100, then the type of modulation is linear amplitude modulation.

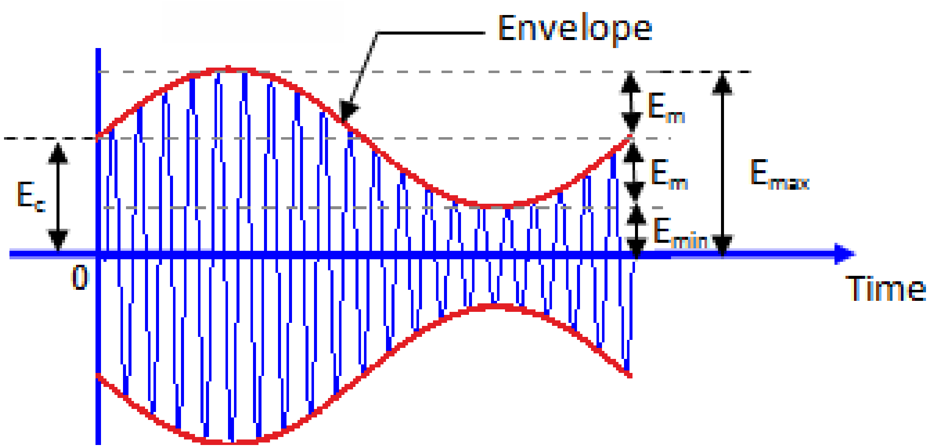


Over modulation: If$m \leq 1$, if the percentage modulation is greater than 100%, then the type of modulation is called as over modulation. Over modulation introduces envelope distortion . Therefore, it should be avoided .

Evaluation of Modulation Index

- The modulation index can be measured using the following two methods :
  I. Using the AM signal as it is
  II. Using the trapezoidal display of the AM wave

The AM Wave: If we see the AM wave in the cathode ray oscilloscope (CRO), it will appears to be the following figure:



$$m = \frac{E_m}{E_c}$$

$$E_m = \frac{E_{max} - E_{min}}{2}$$

$$and \qquad E_c = E_{max} - E_m$$

$$E_c = E_{max} - \left[\frac{E_{max} - E_{min}}{2}\right]$$

Or, $$E_c = \frac{E_{max} + E_{min}}{2}$$

But, $$m = \frac{(E_{max} - E_{min})/2}{(E_{max} + E_{min})/2}$$

Hence, $$m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

Using the trapezoidal display of the AM wave

The set up for evaluating the modulation index using trapezoidal display has been shown in the following figures:

$$A = 2(E_c + E_m)$$

$$and \quad B = 2(E_c - E_m)$$

$$A + B = 4E_c$$

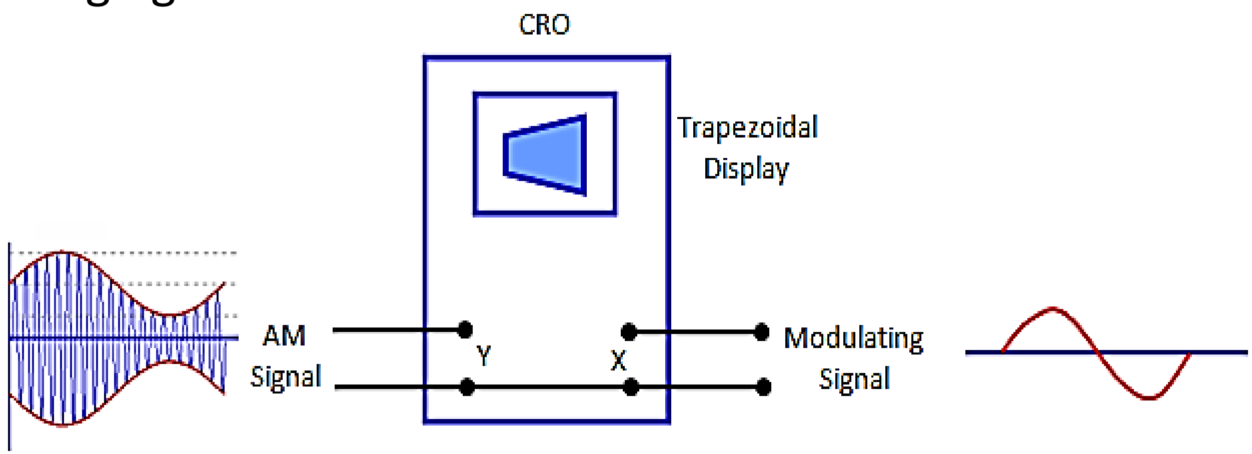
$$Or \qquad E_c = \frac{A+B}{4}$$

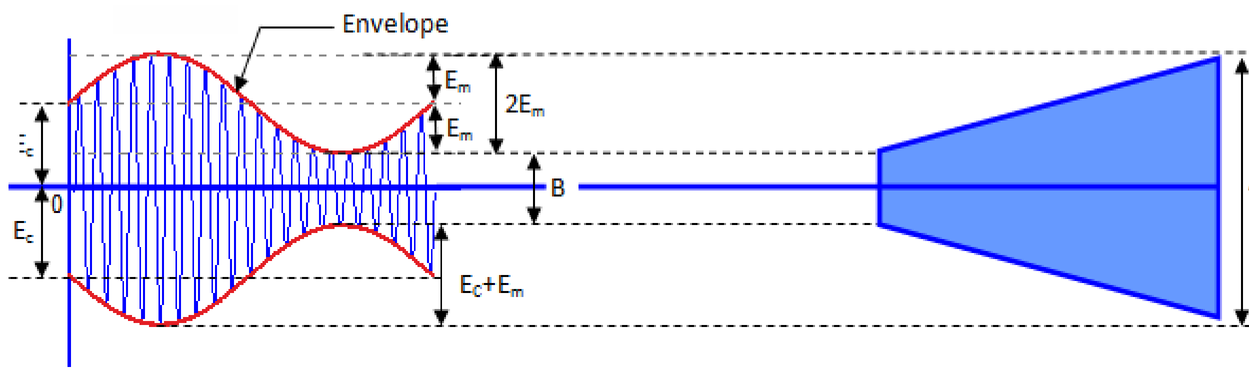
$$A - B = 4E_m$$

$$Or \qquad E_m = \frac{A-B}{4}$$

$$m = \frac{E_m}{E_c}$$

$$m = \frac{A-B}{A+B}$$





AM Wave

Oscilloscope Display

See example 3.2, 3.3, 3.4, 3.5 of SS books.

Transmission Efficiency

Transmission efficiency of an AM wave is the ratio of the transmitted power which contains the information (i.e. the total sideband power) to the total transmitted power.

$$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{[\frac{m^2}{4}P_c + \frac{m^2}{4}P_c]}{[1+\frac{m^2}{2}]P_c}$$

or,

$$\eta = \frac{\frac{m^2}{2}}{1+\frac{m^2}{2}} = \frac{m^2}{2+m^2} \qquad \text{..................(4)}$$

The percent transmission efficiency is given by ,

$$\eta = \frac{m^2}{2+m^2} \times 100\% \qquad \text{..................(5)}$$

The higher percentage of modulation is preferred for strong and more intelligible received signal because it increases the transmitted power P_t by the AM transmitter.

$$P_t = P_c\left(1 + \frac{m^2}{2}\right)$$

Single Tone AM

Till now we considered, the baseband signal as a random signal, now let us consider

$$x(t) = V_m cos\omega_m t$$

Let the carrier signal be : $c(t) = Acos\omega_c t$

We know that the general expression for AM signal is :

$$\mathrm{s(t) = x(t)cos\omega_c t + Acos\omega_c t}$$

$$\mathrm{s(t)} = V_m cos\omega_m t\mathrm{cos\omega_c t + Acos\omega_c t}$$

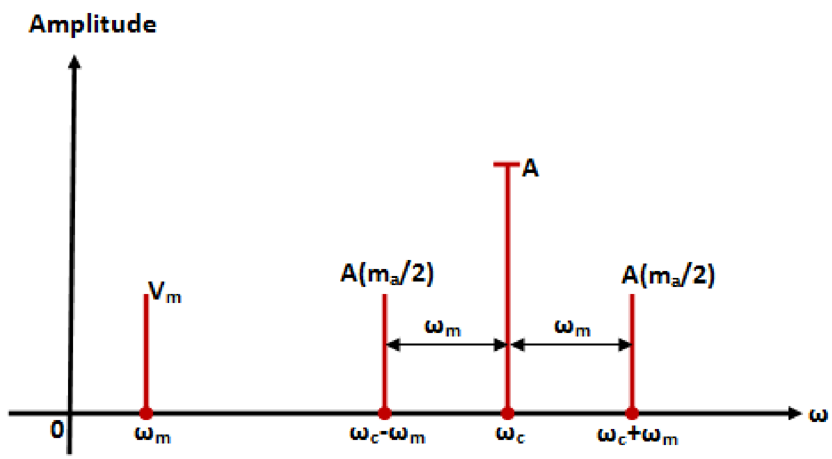
$$\mathrm{s(t) = Acos\omega_c t[1 +} \frac{V_m}{A} cos\omega_m t]$$

$$\mathrm{s(t) = Acos\omega_c t[1 +} m_a cos\omega_m t]$$

$$\mathrm{s(t) = Acos\omega_c t + A.} m_a \mathrm{cos\omega_c t} cos\omega_m t$$

$$\mathrm{s(t) = Acos\omega_c t + A.} m_a \mathrm{cos\omega_c t} cos\omega_m t$$

$$\mathrm{s(t) = Acos\omega_c t +} \frac{\mathrm{A.} m_a}{2} [\mathrm{cos(\omega_c} + \omega_m)\mathrm{t} + cos(\omega_c - \omega_m)t]$$



Single-sided frequency spectrum of single-tone AM wave

See example 3.7, 3.3, 3.4, 3.5 of SS books.

Power Content in AM Wave

We know that the general expression for AM signal is :

$$s(t) = \underbrace{x(t)\cos\omega_c t}_{\text{Sideband term}} + \underbrace{A\cos\omega_c t}_{\text{Carrier term only}}$$

The total power of P_{AM} is the sum of carrier power P_C and sideband power P_s.

Carrier power, $P_c = \frac{1}{2\pi}\int_0^{2\pi} A^2 cos^2\omega_c t dt = \frac{A^2}{2}$

Sideband Power, $P_s = \frac{1}{2\pi}\int_0^{2\pi} x^2(t) cos^2\omega_c t dt = \frac{1}{2}\overline{x^2(t)}$

Therefore, total power, $P_{AM} = \frac{1}{2}\left[A^2 + \overline{x^2(t)}\ \right]$

Transmission Efficiency of AM Wave

Total modulated power, $P_{AM} = \frac{1}{2}\left[A^2 + \overline{x^2(t)}\ \right]$

- The useful message power is carried by the sidebands, P_s
- The carrier power, P_c is a waste from transmission point of view because it does not carry any information.

Hence, efficiency of AM wave can be defined as

Transmission efficiency, $\eta = \frac{P_s}{P_{AM}} \times 100\%$

$$\eta = \frac{\frac{1}{2}\overline{x^2(t)}}{\frac{1}{2}\left[A^2 + \overline{x^2(t)}\ \right]} \times 100\%$$

$$\eta = \frac{\overline{x^2(t)}}{\left[A^2 + \overline{x^2(t)}\ \right]} \times 100\%$$

The maximum transmission efficiency of AM wave is 33.33%. It means that only one third of total power is carried by sides bands and rest two-third is wasted by carrier.

N.B.: Find the transmission efficiency of single-tone AM wave.

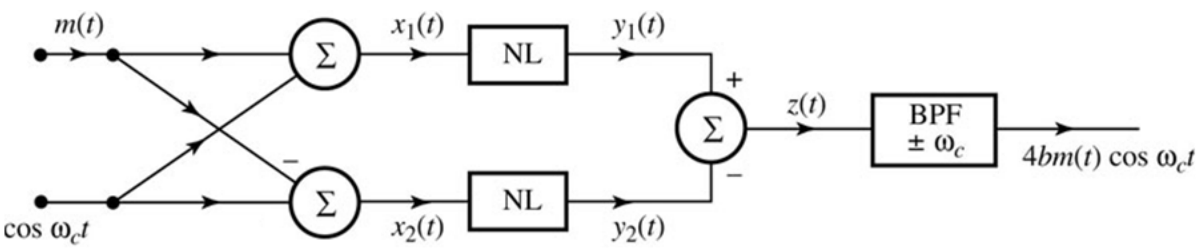
Type of Modulators

Multiplier Modulators: A variable gain amplifier in which the gain parameter (such as the β of transistor) is controlled by the message signal $m(t)$ and the input is the carrier signal.

- Difficult to maintain linearity in this kind of amplifier
- It is also expensive

Nonlinear Modulators: Nonlinear devices such as diode or transistors are used to output modulated signal.

$$y(t) = ax(t) + bx^2(t)$$



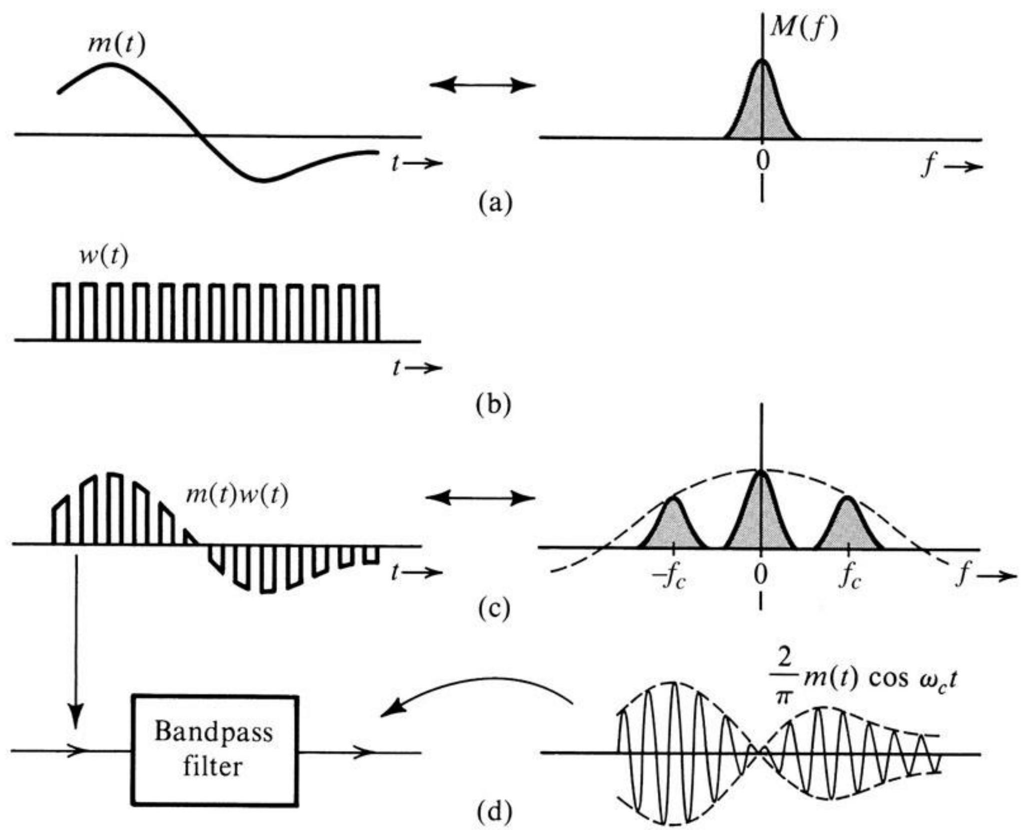
$$z(t) = y_1(t) - y_2(t) = [ax_1(t) + bx_1^2(t)]\text{-}[ax_2(t) + bx_2^2(t)]$$

$$z(t) = 2a.m(t) + 4b.m(t)cos\omega_c t$$

Single balanced modulator because one of the input does not appear at the output $z(t)$

Type of Modulators

Switching Modulators



$$w(t) = \sum_{n=0}^{\infty} C_n \cos(n\omega_c t + \theta_n)$$

$$w(t)m(t) = \sum_{n=0}^{\infty} C_n m(t) \cos(n\omega_c t + \theta_n)$$

$$w(t)m(t) = \frac{1}{2}m(t) + \frac{2}{\pi}\left[m(t)cos\omega_c t - \frac{1}{3}m(t)cos3\omega_c t + \frac{1}{5}m(t)cos5\omega_c t - \cdots\right]$$

Circuit of Switching Modulators

Diode-bridge electronic switch:

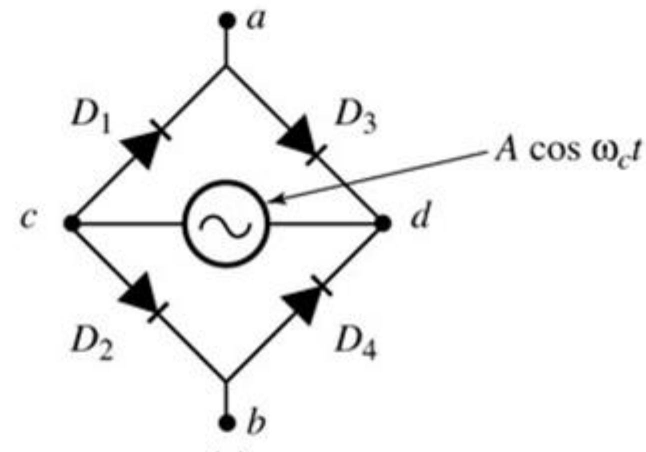
When $V_c > V_d$ all diodes are open and matched

$$V_{D_1} = V_{D_2}$$

$$V_{D_1} = V_c - V_a$$

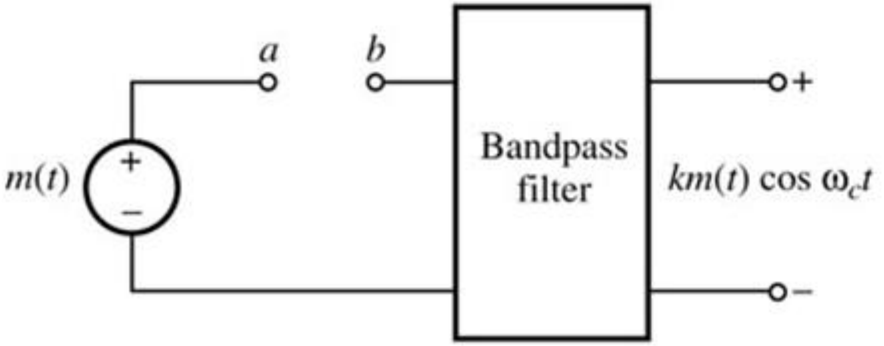
$$V_{D_2} = V_c - V_b$$

$$V_a = V_b$$



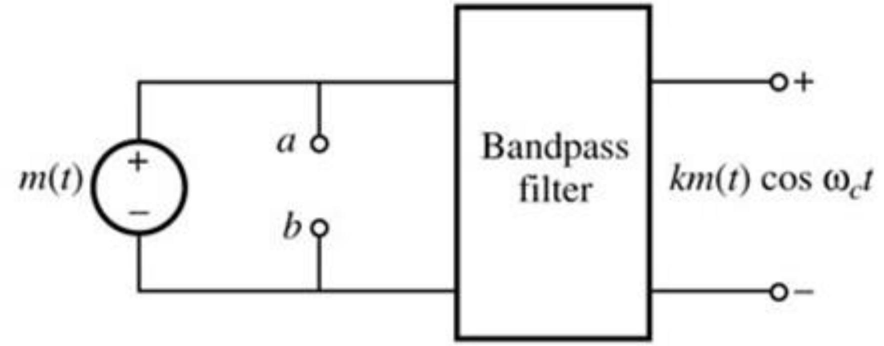
(a)

Diode-bridge electronic switch



(b)

Series-bridge diode modulator

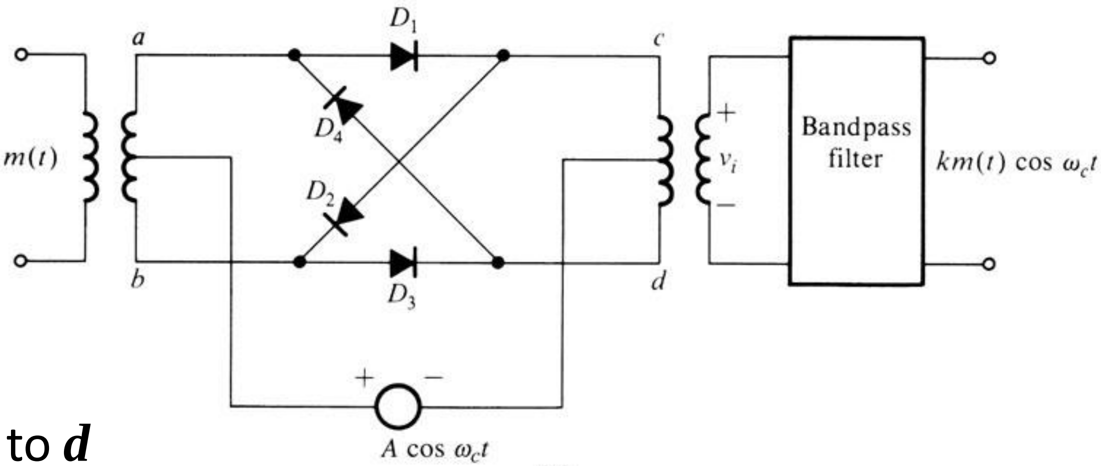


(c)

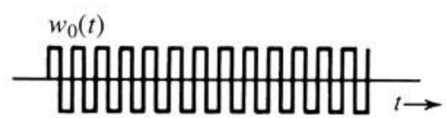
Shunt-bridge diode modulator

Circuit of Switching Modulators

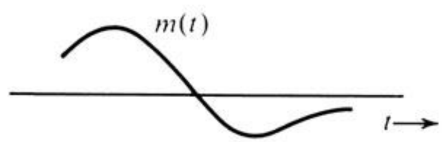
Ring Modulator



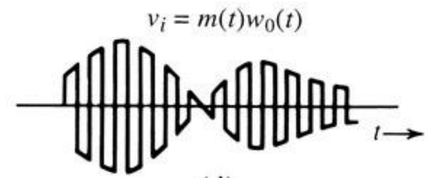
(a)



(b)



(c)



(d)

During Positive cycle of carrier:

- D_1 & D_3 Conducts
- $\boldsymbol{a}$ connected to $\boldsymbol{c}$ & $\boldsymbol{b}$ connected to $\boldsymbol{d}$
- output proportional to $m(t)$

During Negative cycle of carrier:

- D_2 & D_4 Conducts
- $\boldsymbol{a}$ connected to $\boldsymbol{d}$ & $\boldsymbol{b}$ connected to $\boldsymbol{c}$
- output proportional to $-m(t)$

$$\mathrm{w}(t)m(t) = \frac{4}{\pi}\left[m(t)cos\omega_c t - \frac{1}{3}m(t)cos3\omega_c t + \frac{1}{5}m(t)cos5\omega_c t - \cdots\right]$$

Double Balanced Modulator

Demodulation of DSB-SC Signals

The DSB-SC signal may be demodulated by the following methods:

- Synchronous or coherent detection method
- Using envelope detector after carrier reinsertion

Challenge of coherent demodulation for DSB-SC signals

- The received signal might suffer from some unknown frequency or phase shift.

$$r(t) = A_c m(t - t_0) cos[(\omega_c + \Delta\omega)(t - t_0)]$$

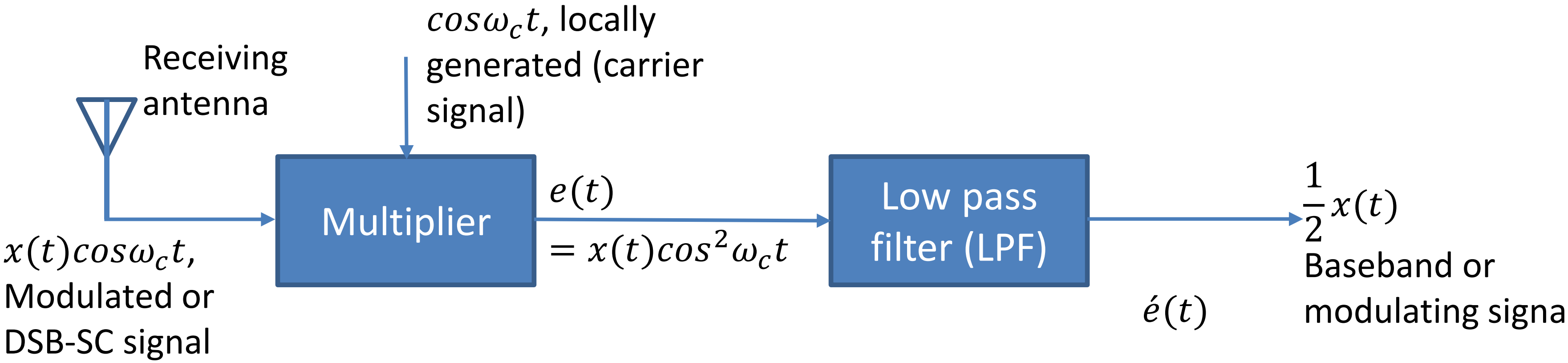
$$= A_c m(t - t_0) cos[(\omega_c + \Delta\omega)t - \theta_d]$$

- The receiver must be sophisticated to generate a local oscillator $cos[(\omega_c+\Delta\omega)t - \theta_d)]$ purely from the received signal $r(t)$.

- Amplitude modulation (AM) that transmit the carrier with the modulated signal will simplify the job of the receiver.

Synchronous Detection Method

It is a method of DSB-SC detection which is shown in the following figure:



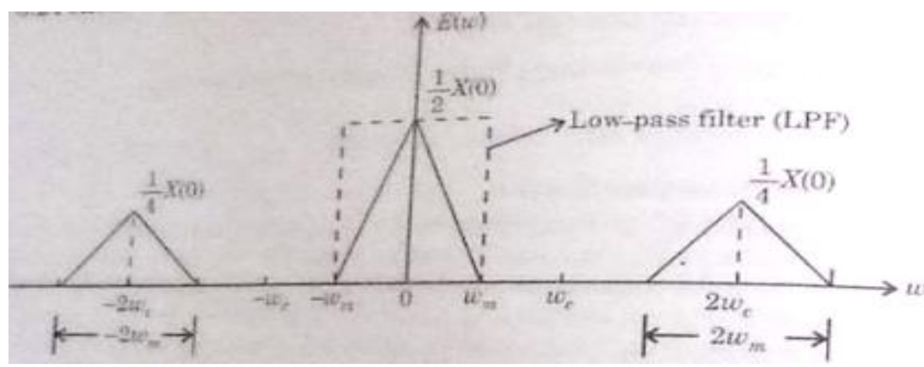
Mathematically, $e(t) = x(t)cos\omega_c t.\, cos\omega_c t$

$$= x(t)cos^2\omega_c t = \frac{1}{2}x(t).\, 2cos^2\omega_c t = \frac{1}{2}x(t) + \frac{1}{2}x(t)cos2\omega_c t$$

When *e(t)* is passed trhough a LPF, $\acute{e}(t) = \frac{1}{2}x(t)$

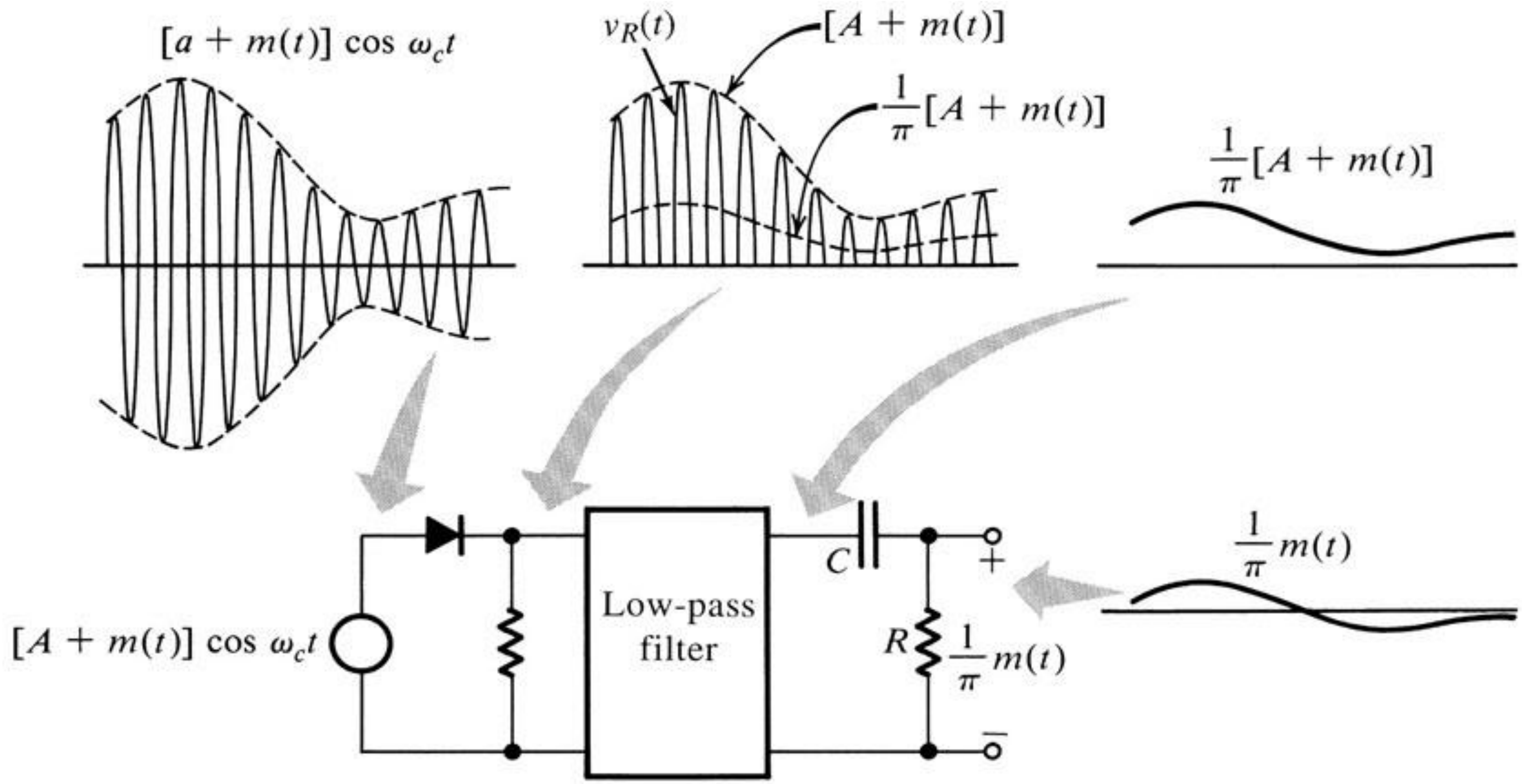
In frequency spectrum of *e(t)* using FT as*:*

$$x(t)cos^2\omega_c t \leftrightarrow \frac{1}{2}X(\omega) + \frac{1}{4}[X(\omega + 2\omega_c) + X(\omega - 2\omega_c)]$$



Envelope Detection

Rectifier

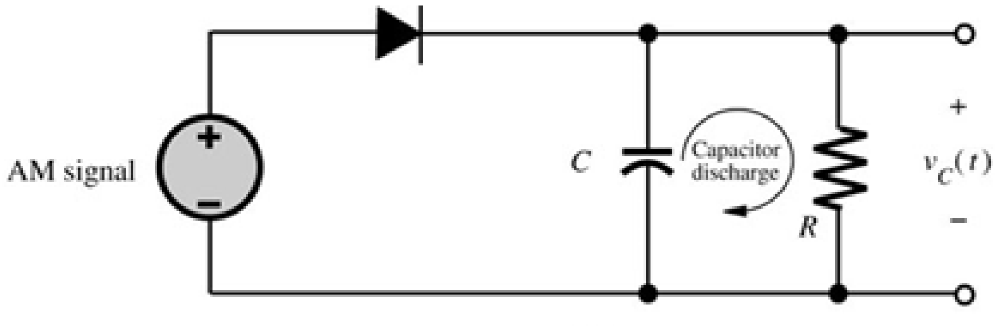


$$v_r(t) = \{[A + m(t)]cos\omega_c t\}w(t)$$

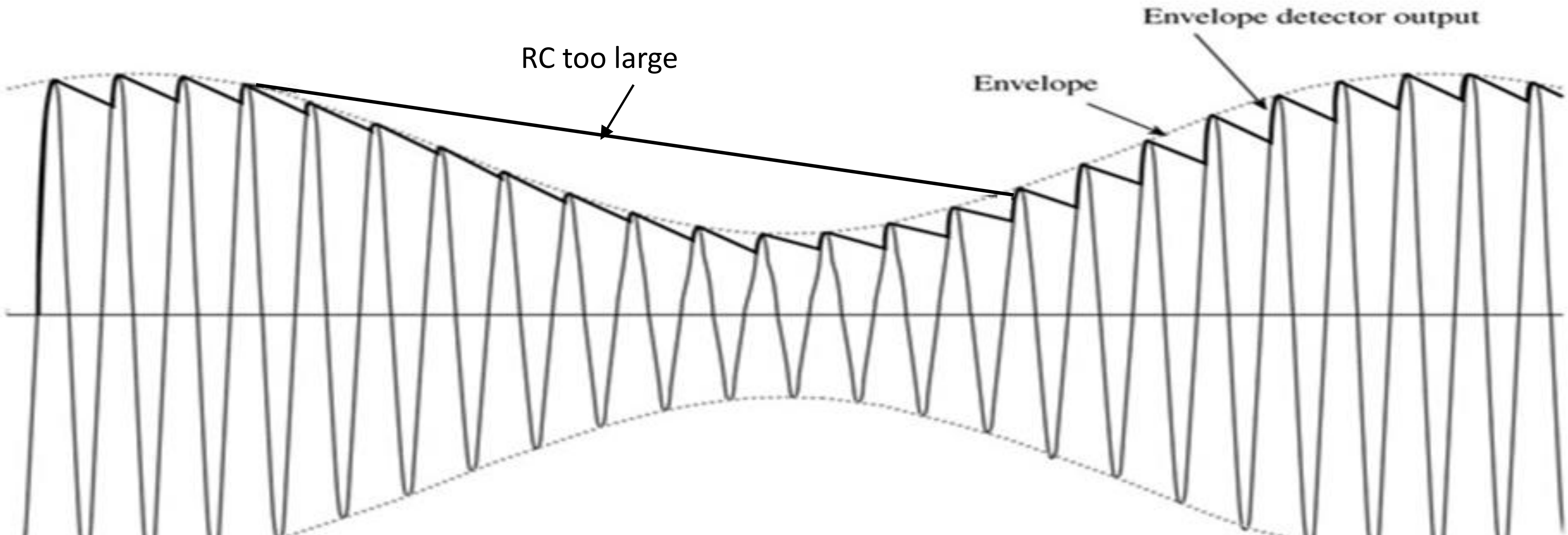
$$= [A + m(t)]cos\omega_c t\left[\frac{1}{2} + \frac{2}{\pi}\left(cos\omega_c t - \frac{1}{3}cos3\omega_c t + \frac{1}{5}cos5\omega_c t - \cdots\right)\right]$$

$$= \frac{1}{\pi}[A + m(t)] + \text{other terms of higher frequencies}$$

Envelope Detection



(a)



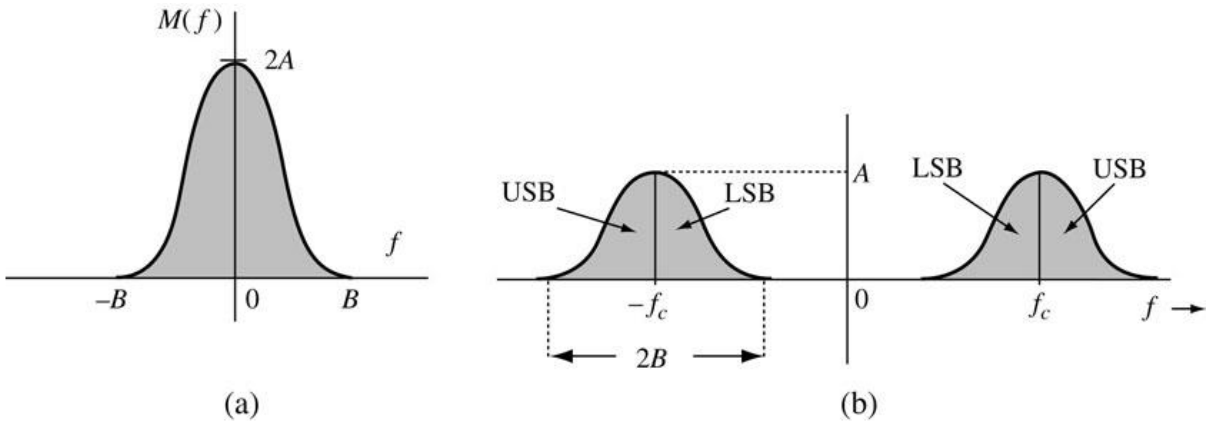
(b)

$1/f_c \ll RC < 1/(B)$

$B < 1/RC < f_c$

Bandwidth-Efficient Amplitude Modulations

The bandwidth of Amplitude Modulation is 2B Hz.



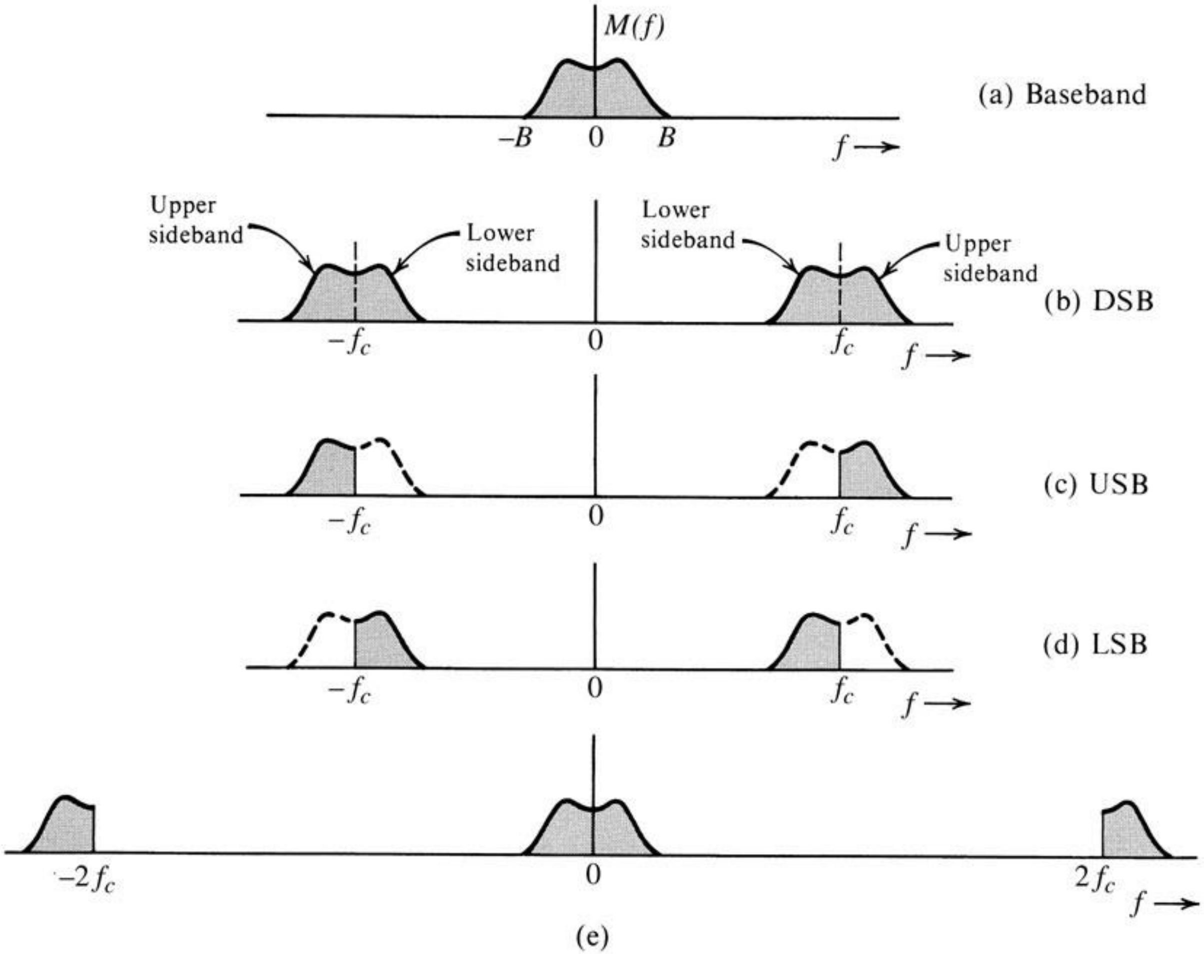
How to reduce bandwidth?

Single-Sideband (SSB) modulation, which remove either the LSB or the USB so that for one message signal $m(t)$, there is only a bandwidth of B Hz.

Quadrature Amplitude (QAM) modulation, which utilize spectral redundancy by sending two messages over the same bandwidth, 2B Hz .

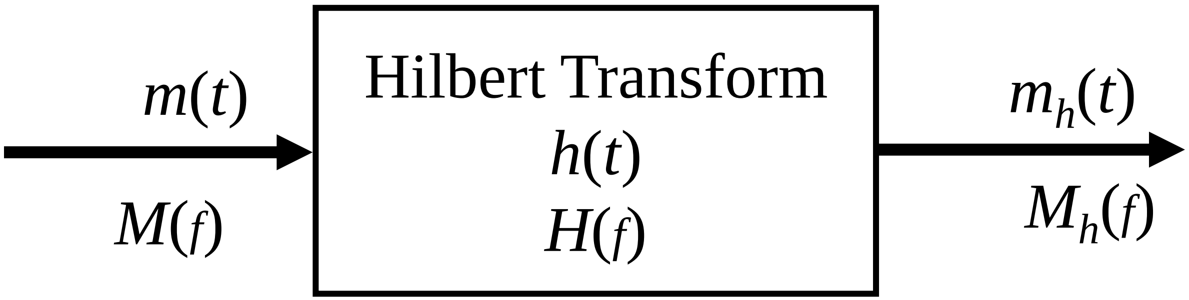
Amplitude Modulation: Single Sideband (SSB)

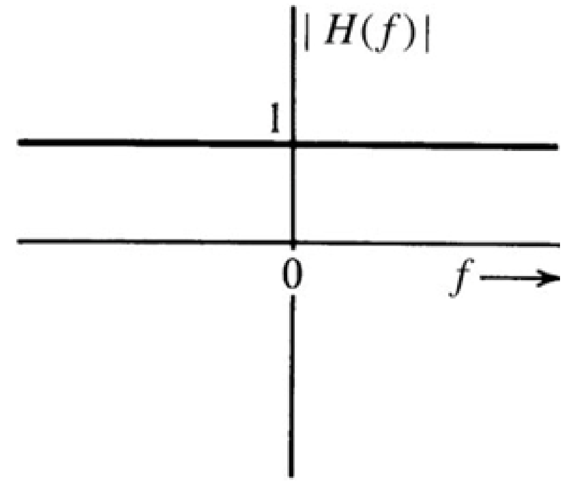
Single-Sideband (SSB) modulation, use Hilbert transform to remove the LSB or USB.



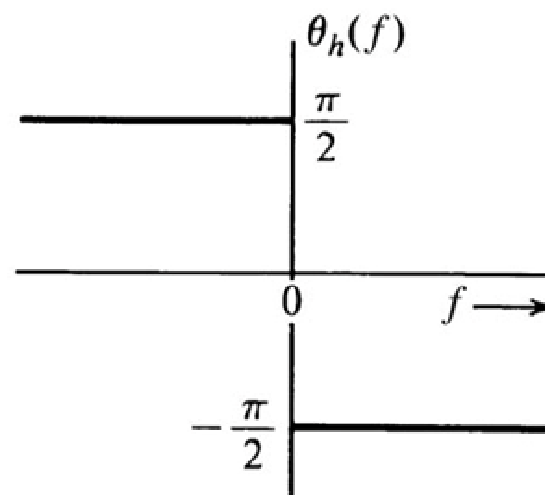
Hilbert Transform

Hilbert transform is an ideal phase shifter that shifts the phase of every positive spectral component by -π/2.





$$H(f) = \begin{cases} 1.\, e^{-\frac{j\pi}{2}} = -j & f > 0 \\ 1.\, e^{\frac{j\pi}{2}} = j & f < 0 \end{cases}$$



$$H(f) = -j\, sgn(f) \qquad\qquad h(t) = \frac{1}{\pi t}$$

$$m_h(t) = m(t) * h(t)$$

$$M_h(f) = -j\, sgn(f) M(f)$$

Time Domain Representation of SSB Signals

$$M_+(f) = M(f).u(f) = M(f)\frac{1}{2}[1 + sgn(f)]$$

$$M_+(f) = \frac{1}{2}[M(f) + jM_h(f)]$$

$$M_-(f) = M(f).u(-f) = M(f)\frac{1}{2}[1 - sgn(f)]$$

$$M_-(f) = \frac{1}{2}[M(f) - jM_h(f)]$$

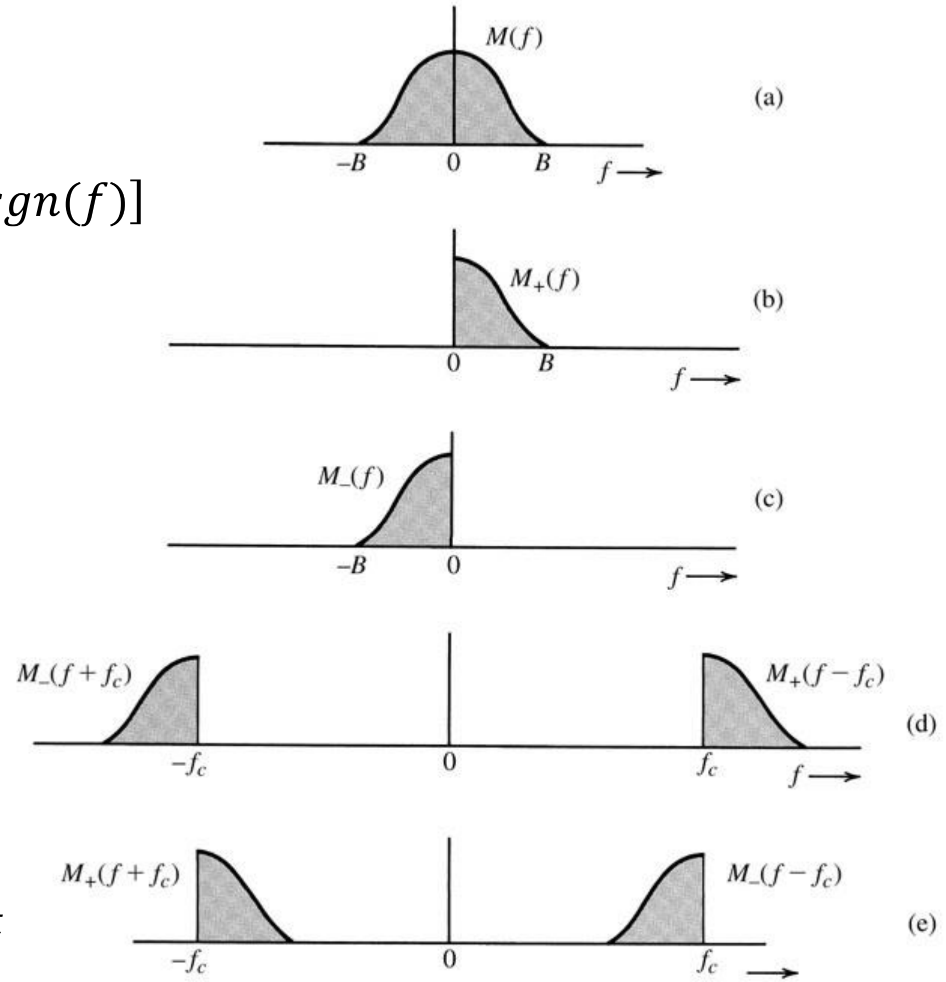
$$\Phi_{USB}(f) = M_+(f - f_c) + M_-(f + f_c)$$

$$\Phi_{USB}(f) = \frac{1}{2}[M(f - f_c) + M(f + f_c)]$$

$$-\frac{j}{2}[M_h(f + f_c) - M_h(f - f_c)]$$

$$\varphi_{USB}(t) = m(t)\, cos\omega_c t - m_h(t)\, sin\omega_c t$$

$$\varphi_{LSB}(t) = m(t)\, cos\omega_c t + m_h(t)\, sin\omega_c t$$



Coherent Demodulation of SSB-SC

$$\varphi_{SSB}(t)2\cos\omega_c t = [m(t)\ cos\omega_c t \mp m_h(t)\ sin\omega_c t]2\cos\omega_c t$$

$$= m(t)\ [1 + cos2\omega_c t] \mp m_h(t)\ sin2\omega_c t$$

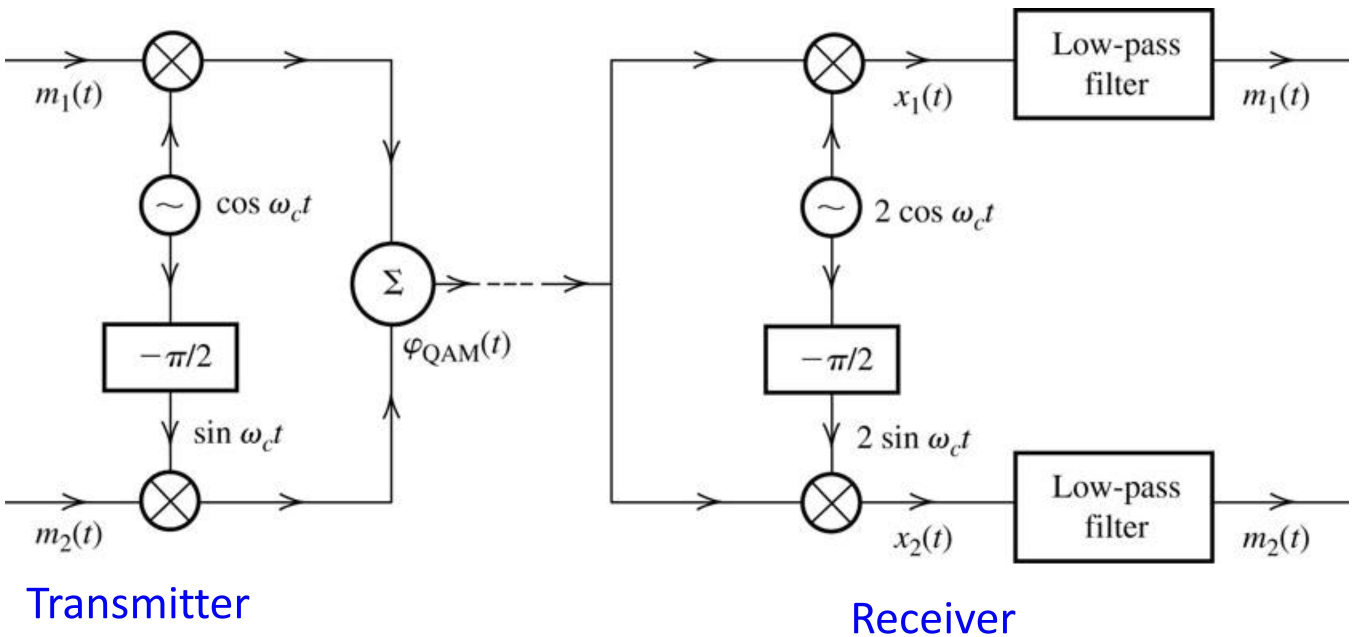
$$= m(t) + [m(t)cos2\omega_c t \mp m_h(t)\ sin2\omega_c t]$$

These terms can be filtered out by using low-pass filter

Quadrature Amplitude Modulation (QAM)

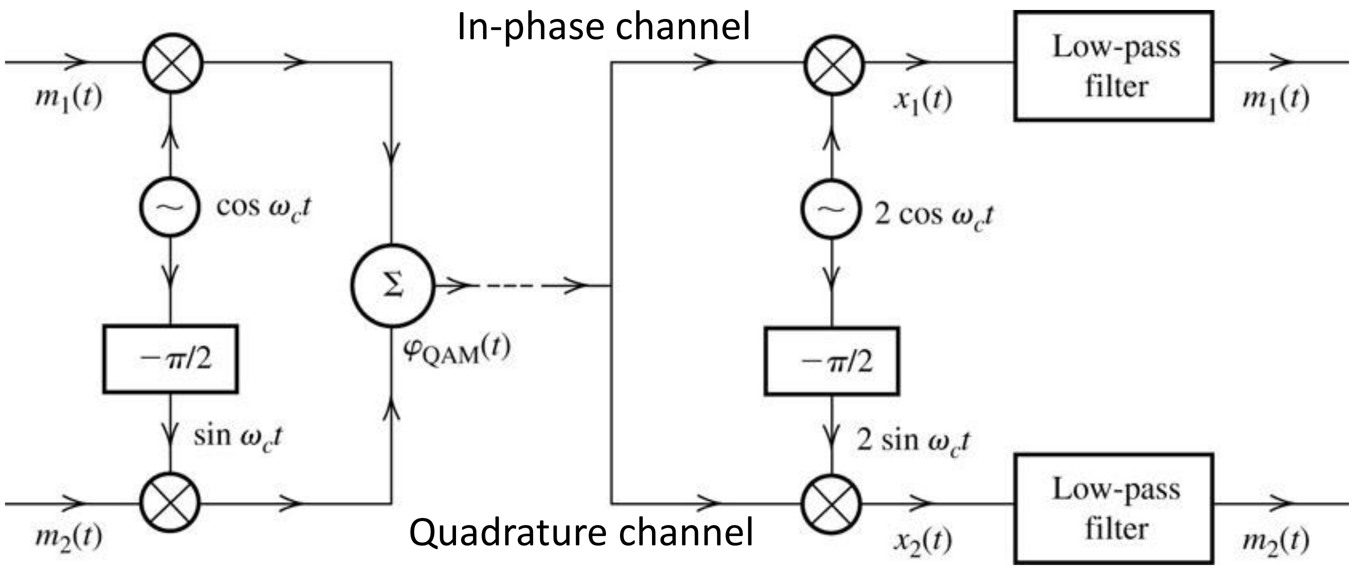
It is difficult to generate accurately SSB-SC and requires large power A >>|m(t)|, so QAM offers an attractive alternative.

QAM operates by transmitting two DSB signals via carrier of the same frequency but in phase quadrature.



$$\varphi_{QAM}(t) = m_1(t) cos\omega_c t + m_2(t)\ sin\omega_c t$$

Quadrature Amplitude Demodulation



$$\varphi_{QAM}(t) = m_1(t) cos\omega_c t + m_2(t)\ sin\omega_c t$$

$$x_1(t) = 2\varphi_{QAM}(t) cos\omega_c t$$

$$= [m_1(t) cos\omega_c t + m_2(t)\ sin\omega_c t\ ] 2cos\omega_c t$$

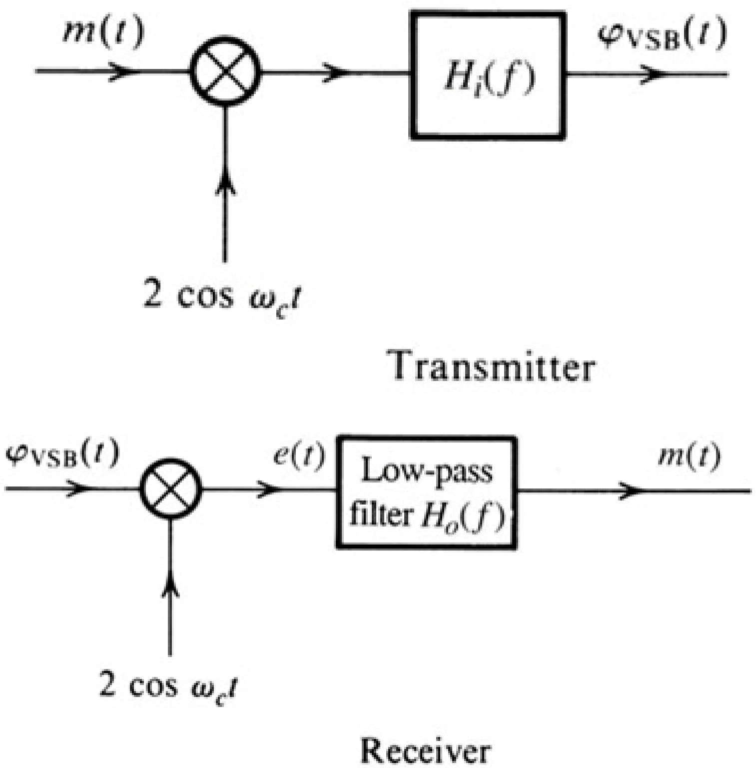
$$= m_1(t) + m_1(t) cos2\omega_c t + m_2(t)\ sin2\omega_c t$$

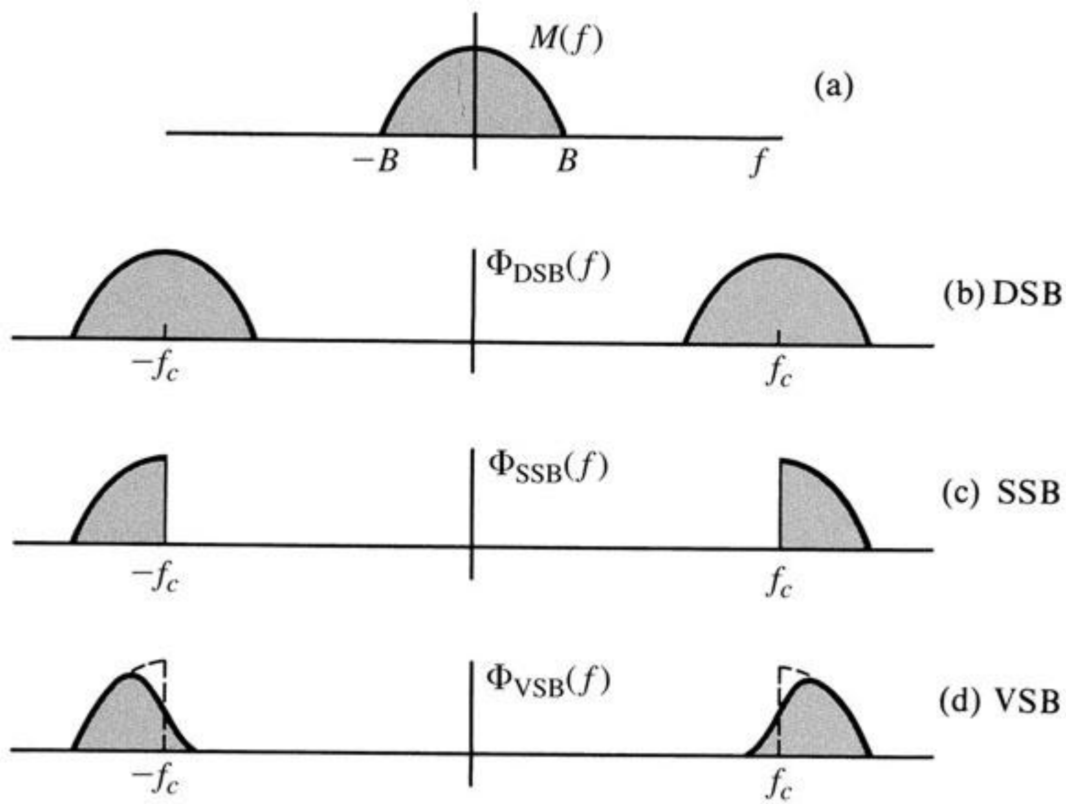
$$x_2(t) = m_2(t) - m_2(t) cos2\omega_c t + m_1(t)\ sin2\omega_c t$$

Amplitude Modulations: Vestigial Sideband (VSB)

VSB signals are relatively easy to generate, and their bandwidth is typically 25% greater than that of SSB signals.

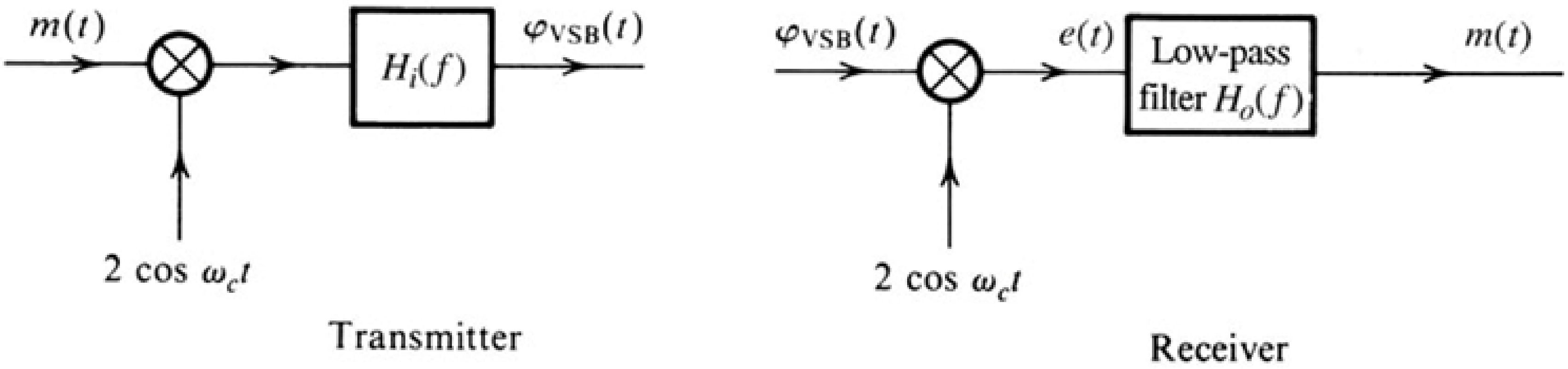
$$\Phi_{VSB}(f) = [M(f + f_c) + M(f - f_c)]H_i(f)$$





Demodulation of Vestigial Sideband (VSB)

$$\Phi_{VSB}(f) = [M(f+f_c) + M(f-f_c)]H_i(f) \qquad \text{---------->}\ 1$$



$$M(f) = [\Phi_{VSB}(f+f_c) + \Phi_{VSB}(f-f_c)]H_o(f)$$

Substitute equation 1 and filter out spectra at $\pm 2\, f_c$

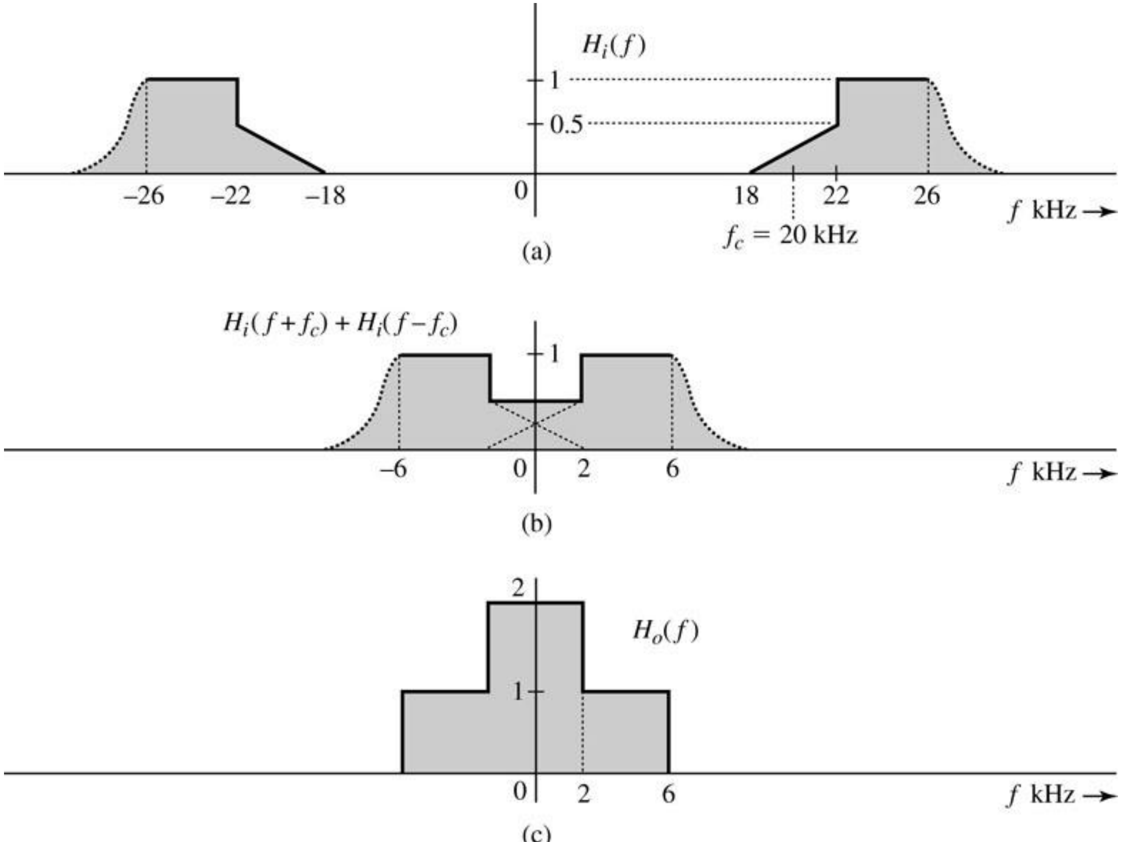
$$\text{M}(f) = M(f)[H_i(f+f_c) + H_i(f-f_c)]H_o(f)$$

$$H_o(f) = \frac{1}{H_i(f+f_c) + H_i(f-f_c)} \qquad |f| \leq B$$

Demodulation of Vestigial Sideband (VSB)

Example: The carrier frequency of a certain VSB signals is f_c = 20 kHz, and the baseband signal bandwidth is 6 kHz. The VSB shaping filter $H_i(f)$ at the transmitter is shown below, find the filter $H_0(f)$ at the receiver for distortionless reception.

$$H_o(f) = \frac{1}{H_i(f+f_c) + H_i(f-f_c)}$$



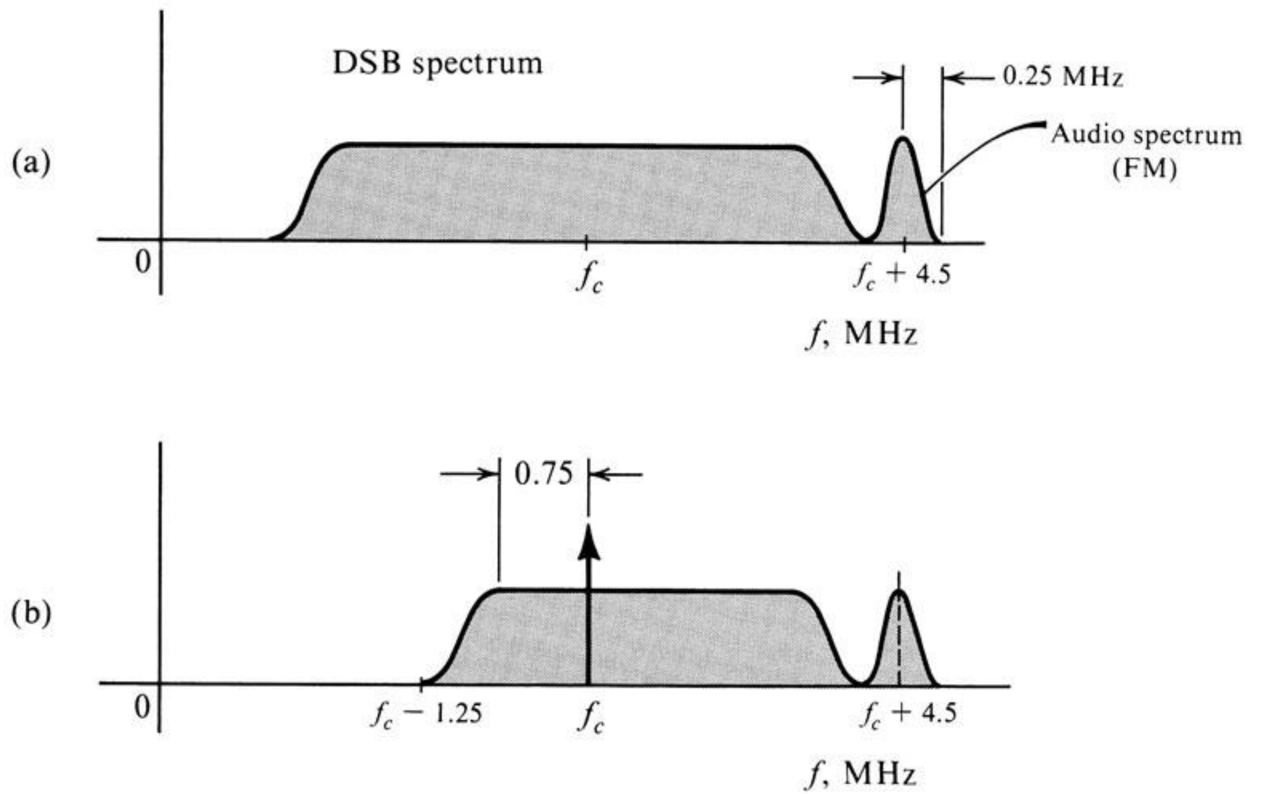
For envelope demodulation, VSB+C require larger carrier than DSB+C but less than SSB+C.

Use of VSB in Broadcast Television

TV Broadcasting

- Bandwidth 4.5 MHz
- Has sizable power in the low-frequency region
- Envelope detector is used instead of synchronous to reduce the cost of the receiver.

BW for SSB = 4.5 MHz
BW for DSB = 9 MHz
BW for VSB = 6 MHz



Comparison among DSB, SSB, and VSB

Parameter of Comparison	DSBFC	DSBSC	SSB	VSB
Carrier Suppression	NA	Fully	Fully	NA
Sideband Suppression	NA	NA	One SB completely	One SB suppressed partially
Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW > 2f_m$
Transmission efficiency	Minimum	Moderate	Maximum	Moderate
Number of modulating inputs	1	1	1	2
Applications	Radio broadcasting	Radio broadcasting	Point to point mobile communication	TV