Gunn Diode

A Gunn diode (also known as a transferred electron device) is a type of diode made from materials like Gallium Arsenide (GaAs) or Indium Phosphide (InP), which exhibit negative differential resistance. Unlike regular diodes, it does not have a p-n junction. Instead, it relies on the bulk properties of the semiconductor material. It is primarily used in microwave oscillators for generating high-frequency RF signals (typically in the GHz range).

Working Principle of a Gunn Diode

The operation of a Gunn diode is based on the Gunn Effect, discovered by J. B. Gunn in the 1960s.

Step-by-Step Operation:

1. **Material Structure**:
   - Made of n-type GaAs or InP.
   - Typically has three regions: heavily doped n^+ region (cathode), a lightly doped n region (active layer), and another n^+ region (anode).
2. **Electron Behavior in Conduction Band**:
   - In materials like GaAs, the conduction band has two valleys:
     - A lower-energy valley (high mobility of electrons),
     - And a higher-energy satellite valley (low mobility of electrons).
3. **Applying Voltage**:
   - When a voltage is applied and electric field increases beyond a threshold (~3.3 kV/cm for GaAs), electrons gain enough energy to jump from the lower valley to the higher-energy valley.
4. **Transferred Electron Effect**:
   - Electrons now have lower mobility → current decreases even though voltage increases → negative differential resistance is observed.
5. **Oscillations**:
   - The diode does not conduct smoothly but generates current oscillations (due to domain formation and movement).
   - These microwave oscillations can be extracted using a resonant cavity.

Negative Differential Resistance

- In a normal resistor, if voltage increases, current increases (Ohm's law: V = IR).
- In **negative resistance**, an increase in voltage causes a **drop in current**.

In Gunn Diode:

- **At low voltages**: diode behaves like a resistor → current increases with voltage.
- **Beyond threshold voltage**:
  - Current **starts to drop** even as voltage increases.
  - This region of the I–V curve shows **negative differential resistance**.

Graph: I–V Characteristics

```
I |
  |        /\
  |       /  \
  |      /    \___ Negative Resistance Region
  |_____/          \
  |                 \_____________
  +-------------------------------------> V
```

Example: A microwave transmission line with a characteristic impedance of Z0=50 Ω is terminated with a load impedance Z_L=100+j50 Ω. Calculate the reflection coefficient, Calculate the voltage standing wave ratio (VSWR) and Determine the magnitude of the reflected power if the incident power is 10 W

Given:

Characteristic impedance: $Z_0 = 50\,\Omega$

Load impedance: $Z_L = 100 + j50\,\Omega$

Incident power: $P_{\text{inc}} = 10\,\text{W}$

Reflection Coefficient Γ

The reflection coefficient Γ is given by:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Substitute the values:

$$\Gamma = \frac{(100 + j50) - 50}{(100 + j50) + 50} = \frac{50 + j50}{150 + j50}$$

Now compute this complex division.

Let's write:

$$\Gamma = \frac{50 + j50}{150 + j50}$$

Multiply numerator and denominator by the complex conjugate of the denominator:

$$\Gamma = \frac{(50 + j50)(150 - j50)}{(150 + j50)(150 - j50)}$$

Numerator:

$$(50 + j50)(150 - j50) = 50 \cdot 150 - 50 \cdot j50 + j50 \cdot 150 - j50 \cdot j50$$

$$= 7500 - j2500 + j7500 + 2500 = 10000 + j5000$$

Denominator:

$$(150 + j50)(150 - j50) = 150^2 - (j50)^2 = 22500 + 2500 = 25000$$

So:

$$\Gamma = \frac{10000 + j5000}{25000} = 0.4 + j0.2$$

Magnitude of Γ:

$$|\Gamma| = \sqrt{(0.4)^2 + (0.2)^2} = \sqrt{0.16 + 0.04} = \sqrt{0.20} \approx 0.447$$

$$\boxed{|\Gamma| \approx 0.447}$$

Voltage Standing Wave Ratio (VSWR)

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.447}{1 - 0.447} = \frac{1.447}{0.553} \approx 2.617$$

Reflected Power

The reflected power P_r is given by:

$$P_r = |\Gamma|^2 \cdot P_{\text{inc}} = (0.447)^2 \cdot 10 = 0.1998 \cdot 10 = 1.998\,\text{W}$$

$$\boxed{P_{\text{reflected}} \approx 2.0\,\text{W}}$$

Calculate the reflection coefficient, Calculate the voltage standing wave ratio (VSWR) and magnitude of the reflected power for following information

Characteristic impedance: $Z_0 = 75\,\Omega$

Load impedance: $Z_L = 25 - j25\,\Omega$

Incident power: $P_{\text{inc}} = 20\,\text{W}$

Example: A rectangular waveguide has inner dimensions of a=2.5 cm and b=1 cm. It is operating in the TE10 mode at a frequency of f=12 GHz.

i. Calculate the cutoff frequency for the TE_{10} mode.
ii. Determine the wavelength inside the waveguide λg.
iii. Find the phase velocity v_p and group velocity v_g in the waveguide.

Given:

Rectangular waveguide dimensions:

- $a = 2.5\,\text{cm} = 0.025\,\text{m}$
- $b = 1\,\text{cm} = 0.01\,\text{m}$

Mode: TE_{10}

Operating frequency: $f = 12\,\text{GHz} = 12 \times 10^9\,\text{Hz}$

Speed of light: $c = 3 \times 10^8\,\text{m/s}$

Cutoff Frequency for TE$_{10}$ Mode

The cutoff frequency for the TE$_{mn}$ mode in a rectangular waveguide is given by:

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For **TE**$_{10}$ mode: $m = 1, n = 0$

$$f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.025} = \frac{3 \times 10^8}{(\ )} = 6 \times 10^9\,\text{Hz} = \boxed{6\,\text{GHz}}$$

ii. Wavelength Inside the Waveguide (λ_g)

First, calculate:

- **Free-space wavelength:**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{12 \times 10^9} = 0.025\,\mathrm{m} = 2.5\,\mathrm{cm}$$

- **Cutoff wavelength λ_c:**

$$\lambda_c = \frac{c}{f_c} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05\,\mathrm{m} = 5\,\mathrm{cm}$$

- **Guide wavelength λ_g:**

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

$$\lambda_g = \frac{0.025}{\sqrt{1 - \left(\frac{0.025}{0.05}\right)^2}} = \frac{0.025}{\sqrt{1 - (0.5)^2}} = \frac{0.025}{\sqrt{0.75}}$$

$$\lambda_g = \frac{0.025}{0.866} \approx 0.0289\,\mathrm{m} = \boxed{2.89\,\mathrm{cm}}$$

iii. Phase Velocity v_p and Group Velocity v_g

Phase Velocity v_p:

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{6}{12}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - 0.25}} = \frac{3 \times 10^8}{\sqrt{0.75}}$$

$$v_p = \frac{3 \times 10^8}{0.866} \approx 3.464 \times 10^8 \, \mathrm{m/s}$$

Group Velocity v_g:

$$v_g = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 3 \times 10^8 \cdot \sqrt{0.75} = 3 \times 10^8 \cdot 0.866 = 2.598 \times 10^8 \, \mathrm{m/s}$$

Answer iii:

- **Phase velocity:** $\boxed{v_p \approx 3.46 \times 10^8 \, \mathrm{m/s}}$
- **Group velocity:** $\boxed{v_g \approx 2.60 \times 10^8 \, \mathrm{m/s}}$

Example: A microwave directional coupler has the following measured power levels:

- Power input at Port 1 (Input): P_1=100 mW
- Power output at Port 2 (Through): P_2=89 mW
- Power at Port 3 (Coupled): P_3=10 mW
- Power at Port 4 (Isolated): P_4=0.1 mW

Calculate the following:

i. Coupling factor (C)
ii. Directivity (D)
iii. Isolation (I)

Given:

- $P_1 = 100 \, \mathrm{mW}$ → Input power (Port 1)
- $P_2 = 89 \, \mathrm{mW}$ → Through port (Port 2)
- $P_3 = 10 \, \mathrm{mW}$ → Coupled port (Port 3)
- $P_4 = 0.1 \, \mathrm{mW}$ → Isolated port (Port 4)

i. Coupling Factor (C)

The **Coupling Factor** represents how much power is coupled from Port 1 to Port 3:

$$C = 10 \log_{10}\left(\frac{P_1}{P_3}\right)$$

$$C = 10 \log_{10}\left(\frac{100}{10}\right) = 10 \log_{10}(10) = \boxed{10 \, \mathrm{dB}}$$

ii. Directivity (D)

Directivity is the ratio of power at the **coupled port** to that at the **isolated port** (Ports 3 and 4):

$$D = 10\log_{10}\left(\frac{P_3}{P_4}\right) = 10\log_{10}\left(\frac{10}{0.1}\right) = 10\log_{10}(100) = \boxed{20\,\mathrm{dB}}$$

iii. Isolation (I)

Isolation is the ratio of power at the **input port** to the **isolated port** (Ports 1 and 4):

$$I = 10\log_{10}\left(\frac{P_1}{P_4}\right) = 10\log_{10}\left(\frac{100}{0.1}\right) = 10\log_{10}(1000) = \boxed{30\,\mathrm{dB}}$$

Example: A reflex klystron operates at a frequency of 3 GHz with a repeller voltage of 500 V. The transit time factor θ=ωt (where ω=2πf) for the electron bunching is 135°. Calculate:

i. The transit time t of the electrons in the drift space.
ii. The drift distance d between the cavity and the repeller, assuming electrons have zero initial velocity after the cavity.

Given:

- Frequency of operation: $f = 3\,\mathrm{GHz} = 3 \times 10^9\,\mathrm{Hz}$
- Repeller voltage: $V_r = 500\,\mathrm{V}$
- Transit angle: $\theta = 135^\circ = \frac{135\pi}{180} = \frac{3\pi}{4}\,\mathrm{rad}$
- Electrons are assumed to have **zero initial velocity** after the cavity.

i. Transit Time t

The transit time factor is:

$$\theta = \omega t$$

Where:

- $\omega = 2\pi f = 2\pi \times 3 \times 10^9 = 6\pi \times 10^9\,\mathrm{rad/s}$
- $\theta = \frac{3\pi}{4}$

So:

$$t = \frac{\theta}{\omega} = \frac{3\pi/4}{6\pi \times 10^9} = \frac{1}{8 \times 10^9}\,\mathrm{s}$$

$$\boxed{t = 1.25 \times 10^{-10}\,\mathrm{s}}$$

ii. Drift Distance d

Assuming electrons are **accelerated by the repeller voltage** V_r and their **initial velocity is zero**, we can use kinematic equations under constant acceleration.

Step 1: Electron acceleration

The repeller applies a **negative electric field**, and the **acceleration** a of an electron is:

$$a = \frac{eV_r}{m_e d}$$

But this equation contains d, which is what we are solving for. Instead, use the **equation of motion**:

$$d = \frac{1}{2}at^2 \Rightarrow d = \frac{1}{2} \cdot \left(\frac{eV_r}{m_e d}\right) t^2$$

Multiply both sides by d:

$$d^2 = \frac{1}{2} \cdot \frac{eV_r}{m_e} \cdot t^2$$

Solve for d:

$$d = \sqrt{\frac{1}{2} \cdot \frac{eV_r}{m_e} \cdot t^2}$$

Step 2: Plug in known constants:

- Electron charge: $e = 1.602 \times 10^{-19}\,\mathrm{C}$
- Electron mass: $m_e = 9.109 \times 10^{-31}\,\mathrm{kg}$
- $V_r = 500\,\mathrm{V}$
- $t = 1.25 \times 10^{-10}\,\mathrm{s}$

$$d = \sqrt{\frac{1}{2} \cdot \frac{(1.602 \times 10^{-19})(500)}{9.109 \times 10^{-31}} \cdot (1.25 \times 10^{-10})^2}$$

First, calculate the constants:

$$\frac{eV_r}{m_e} = \frac{(1.602 \times 10^{-19})(500)}{9.109 \times 10^{-31}} \approx \frac{8.01 \times 10^{-17}}{9.109 \times 10^{-31}} \approx 8.795 \times 10^{13}$$

Now:

$$d = \sqrt{\frac{1}{2} \cdot 8.795 \times 10^{13} \cdot (1.25 \times 10^{-10})^2} = \sqrt{0.5 \cdot 8.795 \times 10^{13} \cdot 1.5625 \times 10^{-20}}$$

$$= \sqrt{6.872 \times 10^{-7}} \approx 8.29 \times 10^{-4}\ \mathrm{m}$$

$$\boxed{d \approx 0.829\ \mathrm{mm}}$$

S-Parameters (Scattering Parameters) in Microwave Engineering

In microwave engineering, S-parameters (or scattering parameters) describe how RF/microwave signals behave in a network—particularly how they are reflected and transmitted when encountering ports in a system. Instead of using voltage and current like in Z (impedance) or Y (admittance) parameters, S-parameters work with incident and reflected power waves, which are more practical to measure at high frequencies.

Why S-parameters are preferred at microwave frequencies

At microwave frequencies (typically above 1 GHz):

1. Voltage and current are hard to measure directly due to distributed nature of circuits (e.g., waveguides, transmission lines).
2. Ports are often matched to a characteristic impedance (e.g., 50 ohms), so incident and reflected waves are more natural to describe system behavior.
3. Networks become non-lumped, so classical parameters (Z, Y) that assume lumped elements become less accurate.
4. Reflection and transmission (wave-based behavior) are fundamental at high frequencies.
5. Test equipment like Vector Network Analyzers (VNAs) are designed to measure S-parameters directly.

Definition of S-Parameters

For an **N-port network**, define:

Definition of S-Parameters

For an **N-port network**, define:

- a_n: incident wave at port n
- b_n: reflected wave at port n

The relationship is:

$$\mathbf{b} = \mathbf{S} \cdot \mathbf{a}$$

Where:

- $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$,
- $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}$,
- $\mathbf{S}$ is the **scattering matrix.**

Each element S_{ij} is defined as:

-

$$S_{ij} = \left.\frac{b_i}{a_j}\right|_{a_k=0,\ k\neq j}$$

This means: **"The signal reflected at port i due to an incident signal at port j, with all other ports terminated in matched loads."**

S-Matrix for a 2-Port Network

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Meaning of each element:

- S_{11}: Reflection coefficient at **port 1** (input).
  → How much of the input signal at port 1 is reflected back.
- S_{21}: Forward transmission coefficient from **port 1 to port 2**.
  → How much signal is transmitted from input to output.
- S_{12}: Reverse transmission coefficient from **port 2 to port 1.**
  → How much signal travels backward through the device.
- S_{22}: Reflection coefficient at **port 2** (output).
  → How much of the signal entering port 2 is refl d.

Example Interpretation:

If:

- $S_{21} = 0.8$: 80% of input power at port 1 is transmitted to port 2.
- $S_{11} = 0.1$: 10% of input power at port 1 is reflected back.

Summary

Parameter	Meaning
S_{11}	Input reflection coefficient
S_{21}	Forward transmission
S_{12}	Reverse transmission
S_{22}	Output reflection coefficient

S-parameters are critical tools in microwave design because they are physically meaningful, easy to measure, and directly relate to how energy flows through RF systems.

S-Matrix for a 2-Port Network

[b1b2]=[S11S12S21S22][a1a2]\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}[b1b2]=[S11S21S12S22][a1a2]

Meaning of each element:

- **S11S_{11}S11**: Reflection coefficient at **port 1** (input).
  → How much of the input signal at port 1 is reflected back.
- **S21S_{21}S21**: Forward transmission coefficient from **port 1 to port 2**.
  → How much signal is transmitted from input to output.
- **S12S_{12}S12**: Reverse transmission coefficient from **port 2 to port 1**.
  → How much signal travels backward through the device.
- **S22S_{22}S22**: Reflection coefficient at **port 2** (output).
  → How much of the signal entering port 2 is reflected.

Example Interpretation:

If:

- S21=0.8S_{21} = 0.8S21=0.8: 80% of input power at port 1 is transmitted to port 2.
- S11=0.1S_{11} = 0.1S11=0.1: 10% of input power at port 1 is reflected back.

📌 Summary

Parameter	Meaning
S11S_{11}S11	Input reflection coefficient
S21S_{21}S21	Forward transmission
S12S_{12}S12	Reverse transmission
S22S_{22}S22	Output reflection coefficient