Integration

formula:

(i) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

(ii) $\int \frac{1}{x} \, dx = \ln|x|$

(iii) $\int e^x \, dx = e^x$

(iv) $\int e^{mx} \, dx = \frac{e^{mx}}{m}$

(v) $\int \sin x \, dx = -\cos x$

(vi) $\int \cos x \, dx = \sin x$

(vii) $\int \sec^2 x \, dx = \tan x$

(viii) $\int \operatorname{cosec}^2 x \, dx = -\cot x$

(ix) $\int \sec x \tan x \, dx = \sec x$

(x) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x$

(xi) $\int \sinh x \, dx = \cosh x$

(xii) $\int \cosh x \, dx = \sinh x$.

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$\boxed{1}$ $\int \cos x \cos 2x \cos 3x \, dx$

let,

$$I = \int \cos x \cos 2x \cos 3x \, dx$$

$$= \frac{1}{2}\int \cos x \left(2\cos 3x \cdot \cos 2x\right) dx$$

$$= \frac{1}{2}\int \cos x \left\{\cos(3x+2x) + \cos(3x-2x)\right\} dx$$

$$= \frac{1}{2}\int \cos x \left(\cos 5x + \cos x\right) dx$$

$$= \frac{1}{2}\int \left(\cos x \cos 5x + \cos^2 x\right) dx$$

$$= \frac{1}{2} \cdot \frac{1}{2}\int \left(2\cos 5x \cdot \cos x + 2\cos^2 x\right) dx$$

$$= \frac{1}{4}\int \left(\cos 6x + \cos 4x + \cos 2x - 1\right) dx$$

$$= \frac{1}{4}\left[\int \cos 6x \, dx + \int \cos 4x \, dx + \int \cos 2x \, dx - \int 1 \, dx\right]$$

$$= \frac{1}{4}\left[\frac{\sin 6x}{6} + \frac{\sin 4x}{4} + \frac{\sin 2x}{2} - x\right] + c.$$

(Ans)

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$\boxed{2}$ $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$

$= \int \frac{e^{\log x^5} - e^{\log x^4}}{e^{\log x^3} - e^{\log x^2}} dx$

$= \int \frac{x^5 - x^4}{x^3 - x^2} dx$

$= \int \frac{x^2(x^3 - x^2)}{(x^3 - x^2)} dx$

$= \int x^2 dx = \frac{x^3}{3} + c$. (Ans)

$\boxed{3}$ $I = \int \frac{\operatorname{cosec} x + \tan^2 x + \sin^2 x}{\sin x} dx$

$= \int \operatorname{cosec} x \cdot \frac{1}{\sin x} dx + \int \frac{\tan^2 x}{\sin x} dx + \int \frac{\sin^2 x}{\sin x} dx$

$= \int \operatorname{cosec}^2 x \, dx + \int \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin x} dx + \int \sin x \, dx$

$= -\cot x + \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} . dx - \cos x$

$= -\cot x + \int \sec x . \tan x \, dx - \cos x$

$= -\cot x + \sec x - \cos x + c.$

(Ans)

$\boxed{4}$ $\int \sqrt{(1+\sin x)}\, dx$

$= \int \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + \sin 2\cdot\frac{x}{2}}\; dx$

$= \int \sqrt{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\; dx$

$= \int \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}\; dx$

$= \int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) dx$

$= \dfrac{-\cos\frac{x}{2}}{1/2} + \dfrac{\sin\frac{x}{2}}{1/2} + c.$

$= -2\cos\frac{x}{2} + 2\sin\frac{x}{2} + c.$ (Ans)

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$\boxed{5}$ $\int \sqrt{1+\cos x}\, dx$

$= \int \sqrt{2\cos^2 \frac{x}{2}}\, dx$

$= \sqrt{2} \int \cos \frac{x}{2}\, dx$

$= \sqrt{2}\ \dfrac{\sin \frac{x}{2}}{\frac{1}{2}} + c$. (Ans)

$\boxed{6}$ $\int \sqrt{1-\cos x}\, dx$

$= \int \sqrt{2\sin^2 \frac{x}{2}}\, dx$

$= \sqrt{2} \int \sin \frac{x}{2}\, dx$

$= \sqrt{2}\ \dfrac{\cos \frac{x}{2}}{\frac{1}{2}} + c$.
(Ans)

$\boxed{7}$ $\int \dfrac{dx}{1+\sin x}$

$= \int \dfrac{(1-\sin x)}{(1+\sin x)(1-\sin x)}\, dx$

$= \int \dfrac{(1-\sin x)}{1-\sin^2 x}\, dx$

$$= \int \frac{1-\sin x}{\cos^2 x}\, dx$$

$$= \int \frac{1}{\cos^2 x}\, dx - \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\, dx$$

$$= \int \sec^2 x\, dx - \int \tan x \cdot \sec x\, dx$$

$$= \tan x - \sec x + c. \text{ (Ans)}$$

$\boxed{8}$ $\int \frac{dx}{1+\cos x}$

$$= \int \frac{1-\cos x}{1+\cos x}\, dx$$

$$= \int \frac{1-\cos x}{\sin^2 x}\, dx$$

$$= \int \frac{1}{\sin^2 x} - \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\, dx$$

$$= \int \operatorname{cosec}^2 x\, dx - \int \cot x \cdot \operatorname{cosec} x\, dx$$

$$= -\cot x + \operatorname{cosec} x + c. \text{ (Ans)}$$

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$\boxed{9}$ $\int \sin mx \cdot \sin nx \cdot dx$

$= \frac{1}{2}\int 2\sin mx \cdot \sin nx \, dx$

$= \frac{1}{2}\int [\cos(mx - nx) - \cos(mx + nx)] \, dx$

$= \frac{1}{2}\int [\cos x(m-n) - \cos x(m+n)] \, dx$

$= \frac{1}{2} \cdot \frac{\sin x(m-n)}{m-n} - \frac{1}{2} \frac{\sin x(m+n)}{m+n} + c$. (Ans)

Method of substitution

$\boxed{1}$ $I = \int \frac{2\sin x}{5+3\cos x} dx \quad \ldots \ldots \ldots \text{(i)}$

Let, $5 + 3\cos x = z$

$\Rightarrow -3\sin x = \frac{dz}{dx}$

$\Rightarrow -3\sin x \, dx = dz$

$\Rightarrow \sin x \, dx = -\frac{1}{3} dz$.

from eq^n (i)

$$I = \int \frac{2}{z}\left(-\frac{1}{3}\right) dz$$

$$= -\frac{2}{3}\int \frac{1}{z}\, dz$$

$$= -\frac{2}{3} \ln|z| + c$$

$$= -\frac{2}{3} \ln|5 + 3\cos x| + c. \text{ (Ans)}$$

$\boxed{2}$ $\displaystyle\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\, dx$

let, $\sin^{-1}x = z$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}\, dx = dz$$

$$\therefore \int \frac{\sin^{-1}x}{\sqrt{1-x^2}} = \int z \cdot dz = \frac{z^2}{2} + c = \frac{(\sin^{-1}x)^2}{2} + c. \text{ (Ans).}$$

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$\boxed{3}$ $\dfrac{dx}{x^3(a+bx)^2}$

$I = \dfrac{dx}{x^3(a+bx)^2}$ ....(i)

Let, $a+bx = zx$

$\Rightarrow \dfrac{a}{x} + b = z$; [dividing both sides by x] .....(ii)

$\Rightarrow -\dfrac{a}{x^2} dx = dz$

$\Rightarrow dx = -\dfrac{x^2}{a} dz$

from eq 2 we get

$\Rightarrow \dfrac{a}{x} = z-b$

$\Rightarrow x = \dfrac{a}{z-b}$

from eq^n (i), we have,

$I = \int \dfrac{-\frac{x^2}{a} dz}{x^3 z^2 x^2}$

$= -\dfrac{1}{a}\int \dfrac{dz}{z^2 x^3}$

$= -\dfrac{1}{a}\int \dfrac{1}{z^2} \cdot \dfrac{1}{\left(\frac{a}{z-b}\right)^3} dz$

$$= -\frac{1}{a}\int \frac{1}{z^2}\left(\frac{z-b}{a}\right)^3 dz$$

$$= -\frac{1}{a^4}\int \frac{(z-b)^3}{z^2}\, dz$$

$$= -\frac{1}{a^4}\int \frac{z^3-3z^2b+3zb^2-b^3}{z^2}\, dz$$

$$= -\frac{1}{a^4}\int\left(\frac{z^3}{z^2}-\frac{3z^2b}{z^2}+\frac{3zb^2}{z^2}-\frac{b^3}{z^2}\right)dz$$

$$= -\frac{1}{a^4}\int\left(z-3b+\frac{3b^2}{z}-b^3z^{-2}\right)dz$$

$$= -\frac{1}{a^4}\left[\frac{z^2}{2}-3zb+3b^2\ln|z|-\frac{b^3z^{-1}}{-1}\right]+c$$

$$= -\frac{1}{a^4}\left[\frac{1}{2}\left(\frac{a+bx}{x}\right)^2-3\left(\frac{a+bx}{x}\right)b+3b^2\ln\left|\frac{a+bx}{x}\right| + b^3\cdot\frac{x}{a+bx}\right.+c.$$

(Ans)

$\boxed{4}$ $\int \frac{1+\cos x}{\sqrt[3]{x+\sin x}} dx$

let, $I = \int \frac{1+\cos x}{\sqrt[3]{x+\sin x}} dx$ .... (i)

let, $x+\sin x = z$

$\Rightarrow (1+\cos x) dx = dz$

from eq^n (i) we get,

$I = \int \frac{dz}{\sqrt[3]{z}} = \int \frac{1}{z^{1/3}} dz = \int z^{-1/3} dz = \frac{z^{-1/3+1}}{-1/3+1} + c$

$= \frac{z^{2/3}}{2/3} + c$

$= \frac{3}{2} (x+\sin x)^{2/3} + c.$

(Ans)

$\boxed{5}$ $\int \frac{e^x - 1}{e^x + 1} dx$

let, $I = \int \frac{e^x - 1}{e^x + 1} dx$

$= \int \frac{e^{x/2}(e^{x/2} - e^{-x/2})}{e^{x/2}(e^{x/2} + e^{-x/2})} dx$

let, $e^{x/2}+e^{-x/2}=z$

$\Rightarrow \left(\frac{1}{2}e^{x/2}-\frac{1}{2}e^{-x/2}\right)dx=dz$

$\Rightarrow \left(e^{x/2}-e^{-x/2}\right)dx=\cancel{2z}$ 2dz

Now, from eqn (i), we get,

$I=\int \frac{2\,dz}{z}=2\ln|z|+c=2\ln\left|e^{x/2}+e^{-x/2}\right|+c$

(Ans)

$\boxed{6}$ $\int \frac{dx}{\sqrt{x+1}-\sqrt{x-1}}$

$=\int \frac{\left(\sqrt{x+1}+\sqrt{x-1}\right)}{\left(\sqrt{x+1}-\sqrt{x-1}\right)\left(\sqrt{x+1}+\sqrt{x-1}\right)}dx$

$=\int \frac{\left(\sqrt{x+1}+\sqrt{x-1}\right)}{x+1-x+1}dx$

$=\int \frac{\left(\sqrt{x+1}+\sqrt{x-1}\right)}{2}dx$

$=\frac{1}{2}\int\left[(x+1)^{1/2}+(x-1)^{1/2}\right]dx$

$$= \frac{1}{2}\left[\frac{(x+1)^{3/2}}{3/2} + \frac{(x-1)^{3/2}}{3/2}\right] + c$$

$$= \frac{1}{2}\cdot\frac{2}{3}\left[(x+1)^{3/2} + (x-1)^{3/2}\right] + c$$

$$= \frac{1}{3}\left[(x+1)^{3/2} + (x-1)^{3/2}\right] + c. \text{ (Ans)}$$

VVI $\boxed{7}$ $I = \int \sqrt{\frac{a+x}{a-x}}\, dx$

let, $x = a \cos 2\theta \quad \ldots \ldots \ldots (i)$

$\Rightarrow dx = -2a \sin 2\theta \, d\theta.$

from eq^n (i),

$\frac{x}{a} = \cos 2\theta$

$\Rightarrow \cos^{-1}\frac{x}{a} = 2\theta$

$\Rightarrow \theta = \frac{1}{2}\cos^{-1}\left(\frac{x}{a}\right).$

$$\therefore I = \int \sqrt{\frac{a + a\cos 2\theta}{a - a\cos 2\theta}} \cdot (-2a \sin 2\theta \, d\theta)$$

$$= \int \sqrt{\frac{a(1+\cos 2\theta)}{a(1-\cos 2\theta)}} \cdot (-2a \sin 2\theta)\, d\theta.$$

$$= \int \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}} \left(-2a \sin 2\theta \, d\theta\right)$$

$$= -2a \int \frac{\cos\theta}{\sin\theta} \cdot 2\sin\theta \cdot \cos\theta \cdot d\theta$$

$$= -2a \int 2\cos^2\theta \, d\theta$$

$$= -2a \int (1+\cos 2\theta) \, d\theta$$

$$= -2a \left[\theta + \frac{\sin 2\theta}{2}\right] + c.$$

$$= -2a \left[\frac{1}{2}\cos^{-1}\frac{x}{a} + \frac{\sin\left(\cos^{-1}\frac{x}{a}\right)}{2}\right] + c. \text{ (Ans)}$$

VVI $\boxed{8}$ $I = \int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}} \, dx$ — (i) $\because a^3 - x^3 = \sin\theta$

Let, $x^{3/2} = a^{3/2} \sin\theta$ .... (ii)

$$\Rightarrow \frac{3}{2} x^{3/2 - 1} \, dx = a^{3/2} \cos\theta \, d\theta$$

$$\Rightarrow x^{1/2} \, dx = \frac{2}{3} a^{3/2} \cos\theta \, d\theta$$

Here, $x^3 = a^3 \sin^2\theta$

$$\Rightarrow \left(\frac{x}{a}\right)^3 = \sin^2\theta$$

$$\Rightarrow \sqrt{\left(\frac{x}{a}\right)^3} = \sin\theta$$

$$\Rightarrow \left(\frac{x}{a}\right)^{3/2} = \sin\theta$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)^{3/2}$$

From eq^n (i),

$$I = \int \frac{\frac{2}{3} a^{3/2} \cos\theta \, d\theta}{\sqrt{a^3 - a^3 \sin^2\theta}}$$

$$= \frac{2a^{3/2}}{3} \int \frac{\cos\theta \quad \text{dtheta}}{\sqrt{a^3(1 - \sin^2\theta)}}$$

$$= \frac{2a^{3/2}}{3} \int \frac{\cos\theta \, d\theta}{a^{3/2}\sqrt{\cos^2\theta}}$$

$$= \frac{2a^{3/2 - 3/2}}{3} \int \frac{\cos\theta}{\cos\theta} \, d\theta$$

$$= \frac{2}{3}\int d\theta = \frac{2}{3}\theta + c$$

$$= \frac{2}{3} \sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c \,.$$ (Ans)

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$\boxed{9}$ $\int \frac{dx}{x\sqrt{x^4-1}}$

$= \int \frac{x\,dx}{x^2\sqrt{x^4-1}}$

Let, $x^2 = \sec\theta$

$\Rightarrow 2x\,dx = \sec\theta \cdot \tan\theta\, d\theta$

$\therefore I = \int \frac{\frac{1}{2}\sec\theta \tan\theta\, d\theta}{\sec\theta\sqrt{(\sec\theta)^2-1}}$

$= \frac{1}{2}\int \frac{\tan\theta}{\tan\theta}\, d\theta$

$= \frac{1}{2}\int d\theta = \frac{1}{2}\theta + c = \frac{1}{2}\{\sec^{-1}(x^2) + c$.

(Ans)

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