Median: Median is the middle most value of a set of observations when the values are arranged in order of magnitude. That means, it devides the whole ordered observations in to two equal parts, It is also called a position or location measure of central tendency.

Median from ungrouped data:

First order the observations in ascending or deseending order of magnitude.

Rule: 1 = if number of observations n is odd, then median will be $\left(\frac{n+1}{2}\right)^{th}$ observation.

Rule: 2 = if number of observations n is even, then median will be arithmatic mean of the $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ orderedered observation.

$$\text{median} = \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2} \text{ observation.}$$

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Example: The following data give the montly wages in taka of 7 workers of a factory.

wage (in Taka): 2700, 27500, 2680, 2790, 2760, 2720, 2740

Compute the median wage of workers.

~~Here n = odd~~

Solution: First arrange the data set in ascending order of magnitude.

2680, 2700, 2720, $\underline{2740}$, 2750, 2760, 2780

here n is odd.

$n = 7$

Median $= \left(\frac{n+1}{2}\right)^{th}$ ordered observation

$= \left(\frac{7+1}{2}\right)^{th}$ " "

$= 4^{th}$ ordered "

$= 2740$

Example:2

The following data ~~are~~ refer to the profits of a store in thousands taka for the last 12 months are

3, 6, 8, 9, 6, 10, 5, 12, 9, 8, 11, 7

Compute the median profit of the store.

Solⁿ:

First arrange the observations in ascending order of magnitude.

3, 5, 6, 6, 7, 8, 8, 9, 9, 10, 11, 12

Here $n=12$ is even

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)th \text{ observation}}{2}$$

$$= \frac{\left(\frac{12}{2}\right)th \text{ obs}^n + \left(\frac{12}{2}+1\right)th \text{ obs}^n}{2}$$

$$= \frac{6^{th} \text{ observation} + 7^{th} \text{ observation}}{2}$$

$$= \frac{8+8}{2} = 8$$

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Computation of median from a grouped data of continous variable:

The formula for median is

$$M_e = L + \frac{\frac{n}{2} - F}{f} \times c$$

Here, M_e = median,

L = lower limit of the median class,

n = Total of observations

F = Cumulative frequency of pre-median class.

f = frequency of the median class

c = width of the median class.

Example: The following frequency distribution refers to the number of hours worked per month of 50 workers of factory.

Compute ~~the~~ median of the frequency distribution

Number of hours worked per month	30-55	55-80	80-105	105-130	130-155	155-80	180-205
Number of workers	3	4	6	9	12	11	5

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<u>Interpretation</u>: 50% of the workers worked for 136.26 hours.

<u>Mode:</u> Mode is the value of a variable which occurs the maximum number of times.

<u>For ungrouped data;</u>

Example: Find Mode of the data sets.

a) 4, 5, 5, 5, 6, 6, 7, 8, 12

<u>Solun:</u> Mode is 5

<u>Example:</u>

class interval	Frequency f	Cumulative frequency F

<u>Mode from grouped data:</u> the formula for computing mode is

$$M_o = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$$

Here, M_o = mode, L = lower limit of the modal class,

Δ_1 = Frequency difference btween the modal class and pre modal-class.

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Solution:

class interval	Frequency f	Cumulative frequency F
30 – 55	3	3
55 – 80	4	7
80 – 105	6	13
105 – 130	9	22
130 – 155	12	34
155 – 180	11	45
180 – 205	9	50

$\sum f_i = n = 50$

Here, $n = 50$, then $\frac{50}{2} = 25^{th}$ observation

lies in the class 130 – 155

Median class is 130 – 155

Here $L = 130$, $\frac{n}{2} = 25$, $F = 22$ $f = 12$

$c = 25$

$$M_e = L + \frac{\frac{n}{2} - F}{f} \times c$$

$$= 130 + \frac{25 - 22}{12} \times 25$$

$$= 136.26$$

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Δ_2 = Frequency ~~between~~ difference between the modal class and most modal class.

c = width of the modal class.

Example:

class interval	frequency
30-55	3
55-80	4
80-105	6
105-130	9
130-155	12
155-180	11
180-205	5

Soln: From the table, we can see that class 130-155 contains highest frequency

$$Mo = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$$

$L = 130$, $\Delta_1 = 12 - 9 = 3$, $\Delta_2 = 12 - 11 = 1$, $c = 25$

$$Mo = 130 + \frac{3}{3+1} \times 25$$

$= 130 + 18.75 = 148.75$ hours per month.

Geometric mean:- it is denoted by G.M, of n positive and non-zero observations. $x_1, x_2, \dots, x_n$ is nth root of their product and is defined by

$$G.M = \sqrt[n]{x_1 \times x_2 \times \cdots \times x_n}$$

$$= (x_1 \times x_2 \times \cdots \times x_n)^{1/n}$$

$$\log G.M = \log \{x_1 \times x_2 \times \cdots \times x_n\}^{1/n}$$

$$= \frac{1}{n} \log \{x_1 \times x_2 \times \cdots \times x_n\}$$

$$= \frac{\log x_1 + \log x_2 + \cdots + \log x_n}{n}$$

$$\log G.M = \frac{\sum_{i=1}^{n} \log x_i}{n}$$

Harmonic mean: It is defined as the reciprocal of the arithmatic mean of the reciprocal of the individual observations. Suppose $x_1, x_2 \dots x_n$ are non-zero observations of a data set.

It is computed by the formula

$$H.M = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \left(\frac{1}{x}\right)}$$ for ungrouped data

$$H.M = \frac{n}{\sum \left(\frac{f_i}{x_i}\right)}$$ for grouped data.

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Theorem: For two-positive non-zero quantities

$$A.M \geq G.M \geq H.M$$

Here, A.M = Arithmatic mean

G.M = Geometric mean

H.M = Harmonic mean.

Proof: Suppose, x_1 and x_2 are two positive and non-zero quantities.

Then $A.M = \frac{x_1+x_2}{2}$

$G.M = \sqrt{x_1 \times x_2}$

$H.M = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}}$

Here $\left(\sqrt{x_1}-\sqrt{x_2}\right)^2 \geq 0$ Since x_1 and x_2 are positive.

$$(\sqrt{x_1})^2 - 2\sqrt{x_1}\sqrt{x_2} + (\sqrt{x_2})^2 \geq 0$$

$$\Rightarrow x_1 + x_2 - 2\sqrt{x_1 x_2} \geq 0$$

$$\Rightarrow x_1 + x_2 \geq 2\sqrt{x_1 x_2}$$

$$\frac{x_1+x_2}{2} \geq \sqrt{x_1 x_2}$$

$\therefore$ A.M $\geq$ G.M. —(i)

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Again $\left(\frac{1}{\sqrt{x_1}}-\frac{1}{\sqrt{x_2}}\right)^2 \geq 0$

$$\left(\frac{1}{\sqrt{x_1}}\right)^2 - 2\frac{1}{\sqrt{x_1}\sqrt{x_2}} + \left(\frac{1}{\sqrt{x_2}}\right)^2 \geq 0$$

$$\Rightarrow \frac{1}{x_1}+\frac{1}{x_2} - 2\frac{1}{\sqrt{x_1 x_2}} \geq 0$$

$$\Rightarrow \frac{1}{x_1}+\frac{1}{x_2} \geq \frac{2}{\sqrt{x_1 x_2}}$$

$$\Rightarrow \sqrt{x_1}\sqrt{x_2} \geq \frac{2}{\frac{1}{x_1}+\frac{1}{x_2}}$$

$\therefore$ G.M $\geq$ H.M —— (ii)

From (i) and (ii) we have

A.M $\geq$ G.M $\geq$ H.M

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