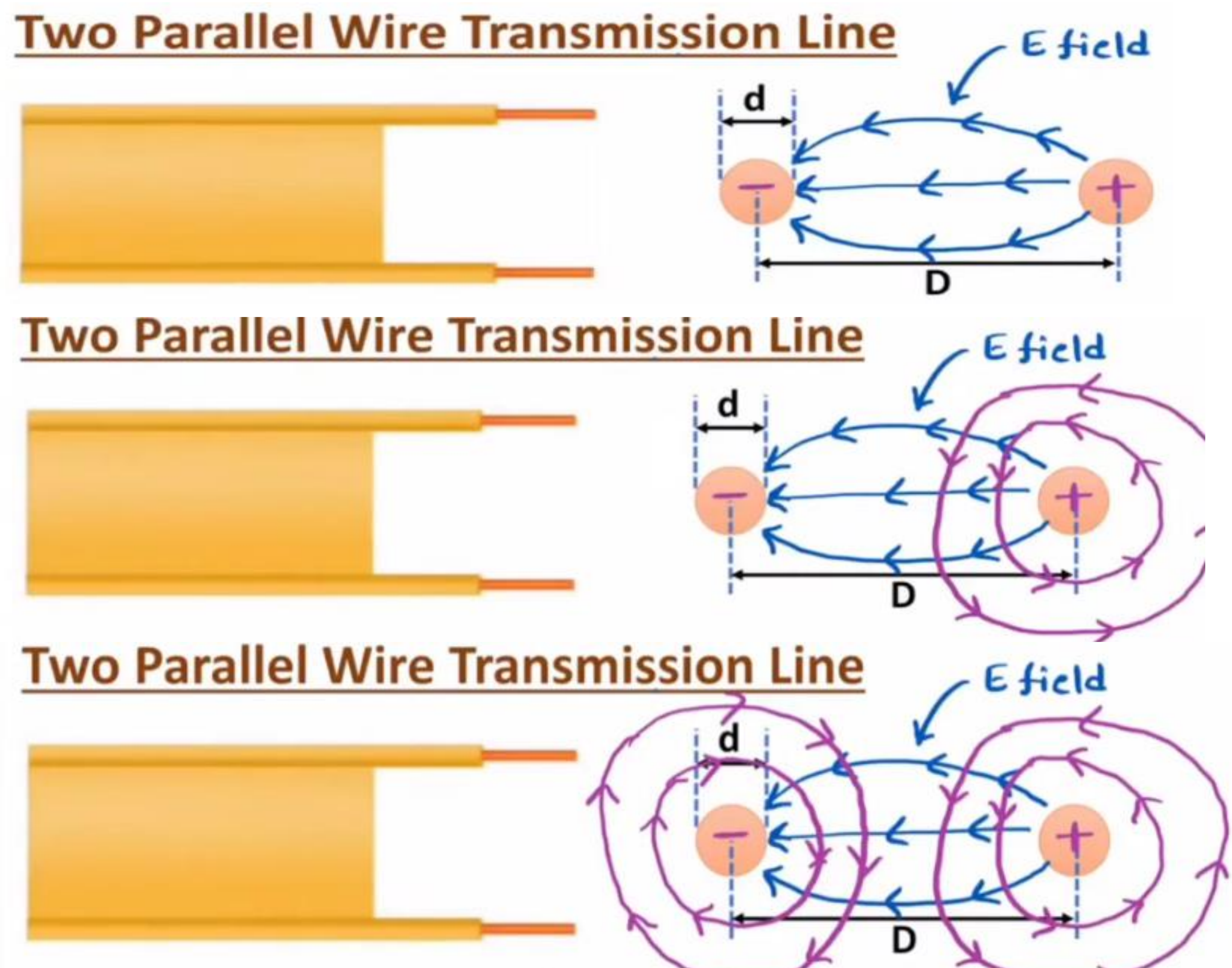
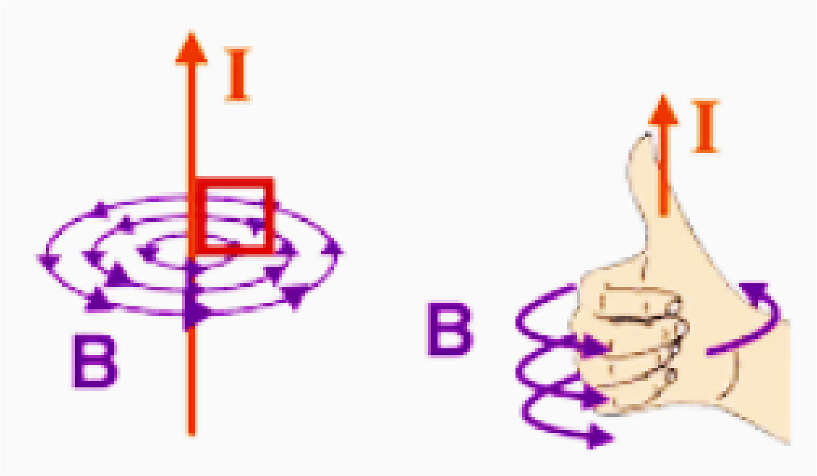
ICT 3101
Microwave Engineering

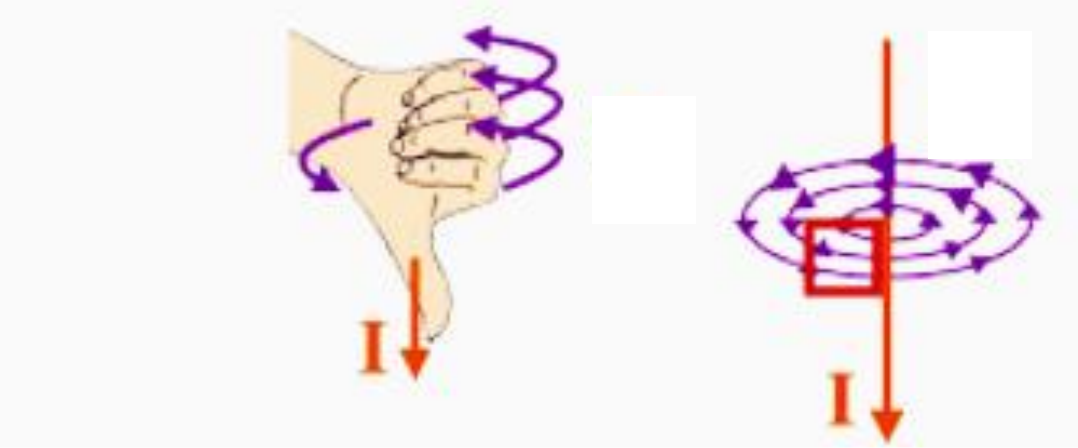
Transmission Line Equations

Transmission Lines of Microwave



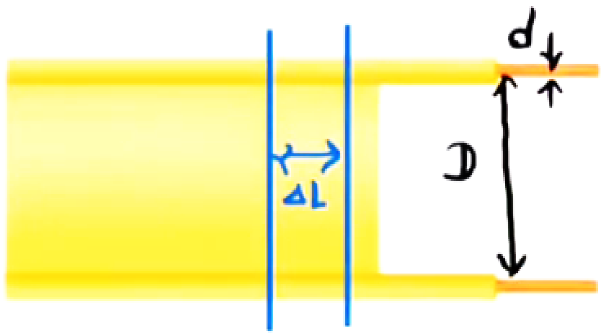
Right Hand Rule

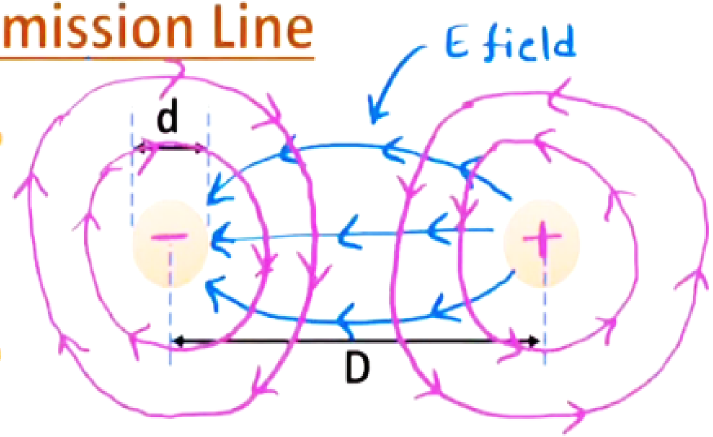




Transmission Line Equations

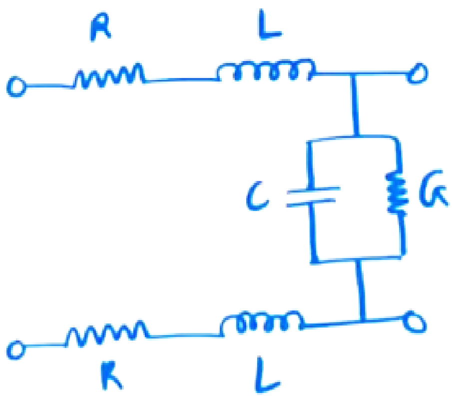
Two Parallel Wire Transmission Line





$$L = \frac{\mu}{\pi} \ln\left[\frac{2D}{d}\right]$$

$$C = \frac{\pi \epsilon}{\ln\left[\frac{2D}{d}\right]}$$



→ Characteristics Impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

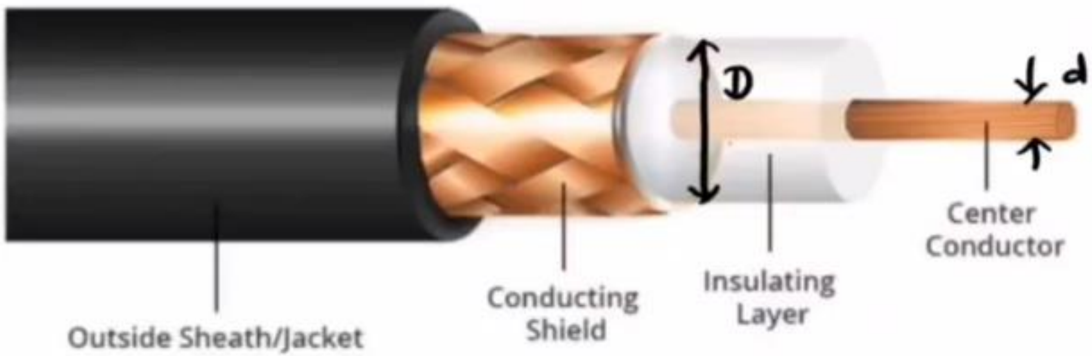
→ For Lossless Tx line

$$R = G = 0$$

$$Z_0 = \sqrt{L/C}$$

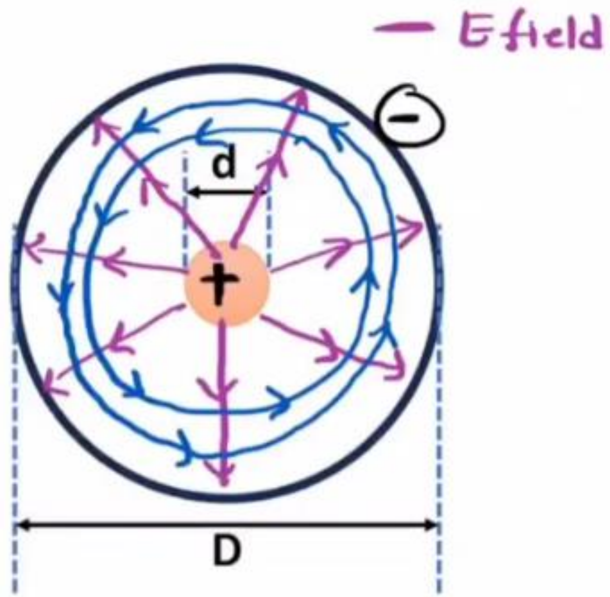
Transmission Line Equations

Coaxial Transmission Line



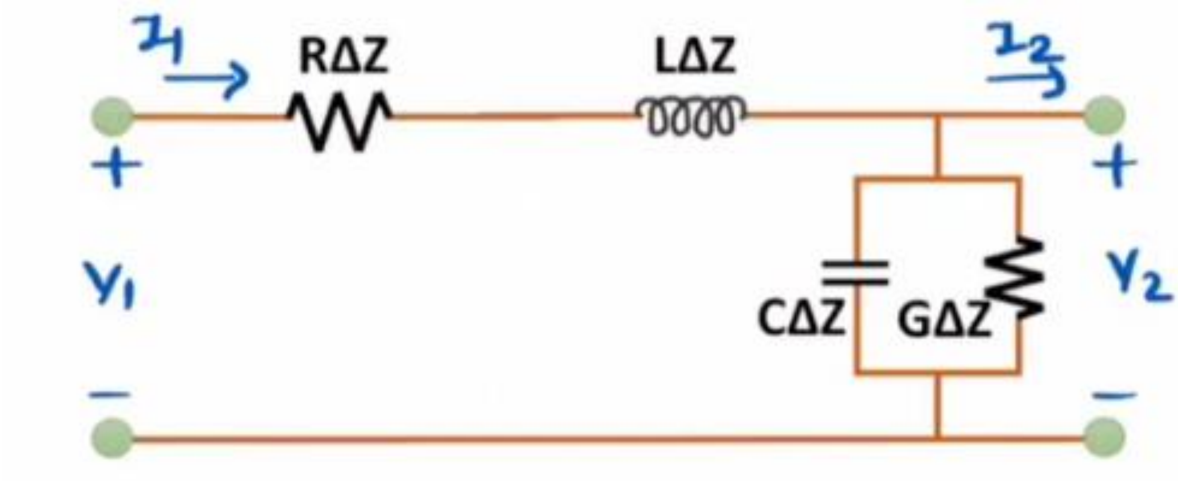
$$L = \frac{\mu}{2\pi} \ln\left[\frac{D}{d}\right]$$

$$C = \frac{2\pi\varepsilon}{\ln\left[\frac{D}{d}\right]}$$



$$\rightarrow Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

Transmission Line Equations



→ As per KVL, Voltage drop Across Tx Line

$$\Rightarrow \Delta V = V_2 - V_1 = -\left[R\Delta z I_1 + L\Delta z \frac{dI_1}{dt} \right]$$

$$\Rightarrow \frac{\Delta V}{\Delta z} = -\left[R + L\frac{d}{dt} \right] I_1 \quad \text{— (1)}$$

→ As per KCL, Current drop Across Tx Line

$$\Rightarrow \Delta I = I_2 - I_1 = -\left[G\Delta z V_2 + C\Delta z \frac{dV_2}{dt} \right]$$

$$\Rightarrow \frac{\Delta I}{\Delta z} = -\left[G + C\frac{d}{dt} \right] V_2 \quad \text{— (2)}$$

Transmission Line Equations

$$\Rightarrow \frac{\Delta V}{\Delta z} = -\left[R + L\frac{d}{dt}\right] I_1 \quad -① $$

$$\Rightarrow \frac{\Delta I}{\Delta z} = -\left[G + C\frac{d}{dt}\right] V_2 \quad -② $$

→ Write eqⁿ ① & ② in form of V & I.

$$\Rightarrow \frac{dV}{dz} = -\left[R + L\frac{d}{dt}\right] I \quad -③ $$

$$\Rightarrow \frac{dI}{dz} = -\left[G + C\frac{d}{dt}\right] V \quad -④ $$

→ Here, $\frac{d}{dt} = j\omega$

$$\Rightarrow \frac{dV}{dz} = -\left[R + j\omega L\right] I \quad -⑤ $$

$$\Rightarrow \frac{dI}{dz} = -\left[G + j\omega C\right] V \quad -⑥ $$

Transmission Line Equations

⤳ Here, $\frac{d}{dt} = j\omega$

$$\Rightarrow \frac{dV}{dz} = -[R + j\omega L]\, I \quad — (5)$$

$$\Rightarrow \frac{dI}{dz} = -[G + j\omega C]\, V \quad — (6)$$

⤳ To get Tx line eq.^{n} differentiate eq.^{n} (5) w.r.t. z

$$\Rightarrow \frac{d^2V}{dz^2} = -[R + j\omega L]\, \frac{dI}{dz}$$

$$\Rightarrow \frac{d^2V}{dz^2} = [R + j\omega L][G + j\omega C]\, V \quad — (A)$$

Transmission Line Equations

→ To get Tx line eq.^{n} differentiate eq.^{n} 5 w.r.t. z

$$\Rightarrow \frac{d^2V}{dz^2} = -[R+j\omega L]\frac{dI}{dz}$$

$$\Rightarrow \frac{d^2V}{dz^2} = [R+j\omega L][G+j\omega C]V \quad \text{— (A)}$$

→ eq.^{n} (A) Tx line eq.^{n} and it is similar to wave eq.^{n}

$$\Rightarrow \frac{d^2V}{dz^2} = \gamma^2 V \quad \text{— (7)}$$

→ γ = propagation constant

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

→ Sol.^{n} of eq.^{n} (7) is

$$\Rightarrow V = V_1 e^{-\gamma z} + V_2 e^{+\gamma z}$$

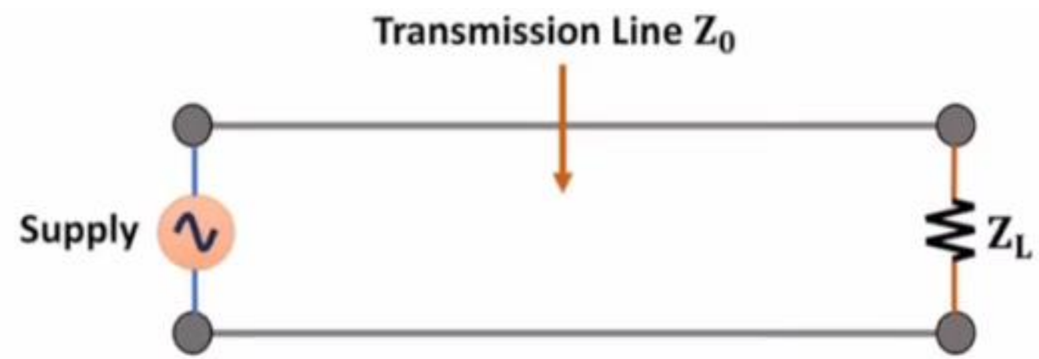
Transmission Line Equations

→ Sol.^{n} of eq.^{n} (7) is

$$\Rightarrow V = \underbrace{V_1 e^{-\gamma z}}_{\text{Incident signal}} + \underbrace{V_2 e^{+\gamma z}}_{\text{Reflected signal.}}$$

→ Simillarly,

$$\Rightarrow I = \underbrace{I_1 e^{-\gamma z}}_{\text{Incident signal}} + \underbrace{I_2 e^{+\gamma z}}_{\text{Reflected sign}}$$



Incident signal $[V_1 e^{-\gamma z}]$ Reflected signal $[V_2 e^{+\gamma z}]$

Propagation Constant, Attenuation Constant and Phase Constant of Transmission Line

→ From Tx line wave eqⁿ

$$\Rightarrow \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \underbrace{\alpha}_{\text{Attenuation Constant}} + j\underbrace{\beta}_{\text{phase constant}}$$

→ For lossless Tx line, [$R = G = 0$]

$$\Rightarrow \gamma = \sqrt{(0+j\omega L)(0+j\omega C)}$$

$$\Rightarrow \gamma = j\omega\sqrt{LC}$$

→ $\alpha = 0$, $\beta = \omega\sqrt{LC}$

→ α and β

$$\Rightarrow \alpha = \frac{1}{2}\left[G\sqrt{\frac{L}{C}} + R\sqrt{\frac{C}{L}}\right]$$

$$\Rightarrow \beta = \omega\sqrt{LC}\left[1+\frac{1}{8}\left(\frac{R}{\omega L}-\frac{G}{\omega C}\right)^2\right]$$

Propagation Constant, Attenuation Constant and Phase Constant of Transmission Line

~> Sol.^{n} of wave eq.^{n}

$\Rightarrow V = V_1 e^{-\gamma z} + V_2 e^{+\gamma z}$

~> T_x line eq.^{n}

$\Rightarrow \frac{dV}{dz} = -(R + j\omega L) I$

$\Rightarrow I = \frac{-1}{R + j\omega L} \left(\frac{dV}{dz} \right)$

$\Rightarrow I = \frac{-1}{R + j\omega L} \frac{d(V_1 e^{-\gamma z} + V_2 e^{\gamma z})}{dz}$

$\Rightarrow I = \frac{-1}{R + j\omega L} (V_1(-\gamma) e^{-\gamma z} + V_2(\gamma) e^{\gamma z})$

Characteristics Impedance of Transmission Line

⇝ [Here, $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\Rightarrow I = \frac{\sqrt{(R+j\omega L)(G+j\omega C)}}{R+j\omega L}\left(V_1 e^{-\gamma z} - V_2 e^{\gamma z}\right)$$

$$\Rightarrow I = \sqrt{\frac{G+j\omega C}{R+j\omega L}}\left(V_1 e^{-\gamma z} - V_2 e^{\gamma z}\right)$$

$$\Rightarrow I = \frac{1}{Z_0}\left(V_1 e^{-\gamma z} - V_2 e^{\gamma z}\right)$$

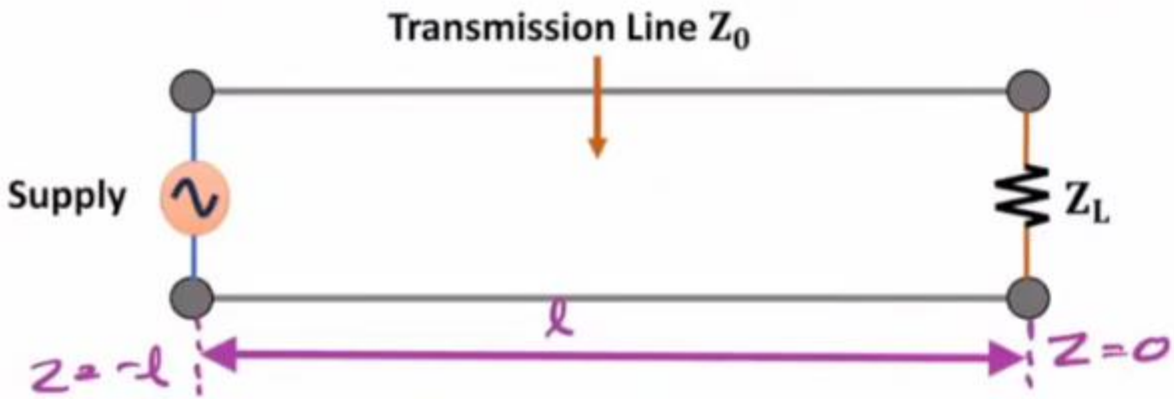
⇝ Hence, characteritics Impedance Z_0 is

$$\Rightarrow Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

⇝ For lossless T_x line, $[\,R = G = 0\,]$

$$\Rightarrow Z_0 = \sqrt{L/C}$$

Reflection Coefficient of Transmission Line



~> Based on Voltage & current eqⁿ of Tx line

$$\Rightarrow V = \underbrace{V_1 e^{-\gamma z}}_{\text{Incident Signal}} + \underbrace{V_2 e^{\gamma z}}_{\text{Reflected Signal}}$$

$$\Rightarrow I = \underbrace{I_1 e^{-\gamma z}}_{\text{Incident Signal}} + \underbrace{I_2 e^{\gamma z}}_{\text{Reflected Signal}}$$

~> So, above eqⁿ are :

$$\Rightarrow V = V_i e^{-\gamma z} + V_r e^{\gamma z}$$

$$\Rightarrow I = I_i e^{-\gamma z} + I_r e^{\gamma z}$$

Reflection Coefficient of Transmission Line

- At load $z = 0$

$$\Rightarrow V_L = V_i + V_r \qquad \Rightarrow I_L = I_i + I_r$$

- At Source $z = -l$

$$\Rightarrow V_S = V_i e^{\gamma l} + V_r e^{-\gamma l} \qquad \Rightarrow I_S = I_i e^{\gamma l} + I_r e^{-\gamma l}$$

- Characteristics Impedance Z_0

$$\Rightarrow Z_0 = \frac{V_i}{I_i} = -\frac{V_r}{I_r}$$

- Reflection Coefficient γ

$$\Rightarrow \gamma = \frac{V_r}{V_i} = -\frac{I_r}{I_i}$$

Reflection Coefficient of Transmission Line

What is VSWR?

Voltage
Standing
Wave
Ratio

?

What is VSWR?

Voltage
Standing
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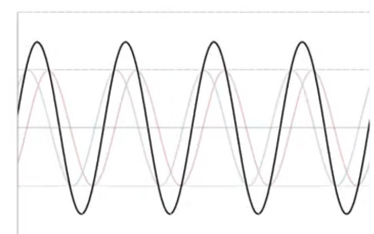
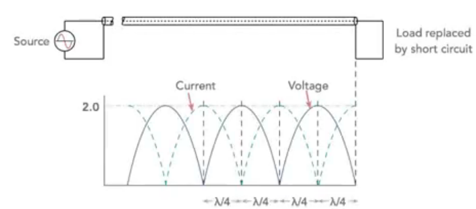
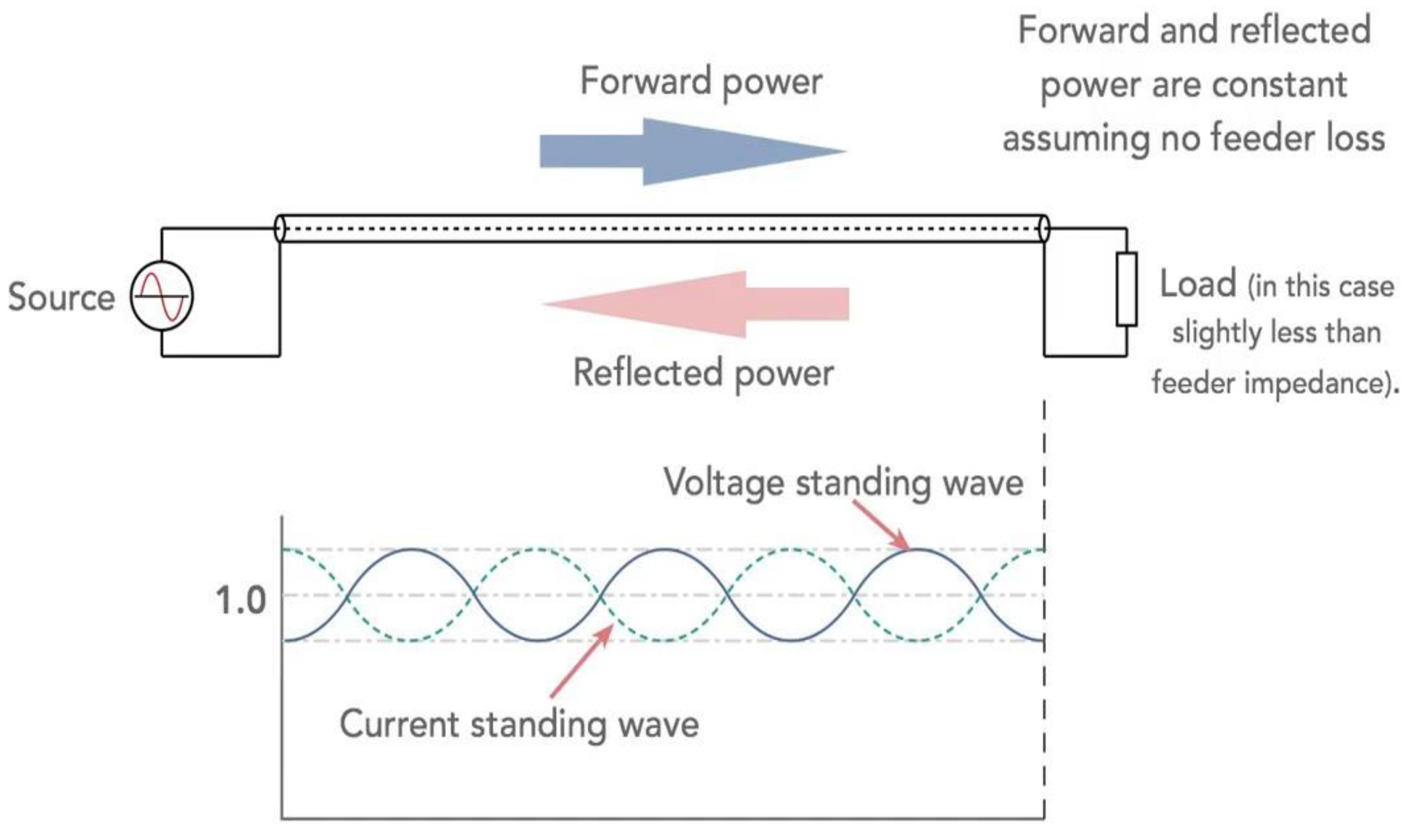


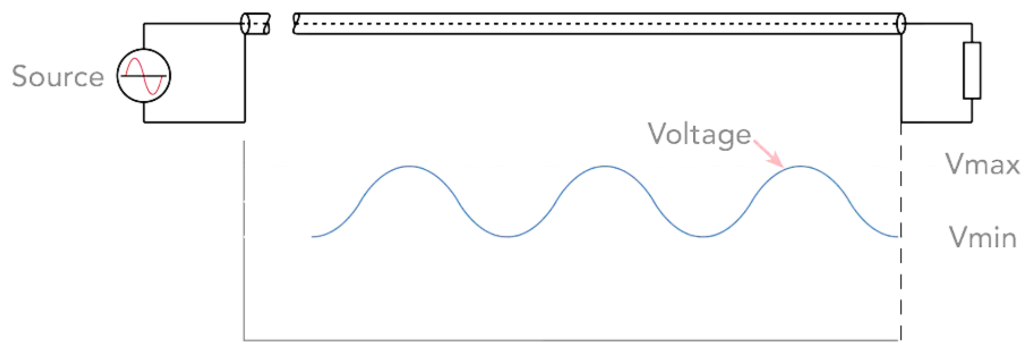


Reflection Coefficient of Transmission Line



Reflection Coefficient of Transmission Line

VSWR Definition



$$VSWR = \frac{V_{max}}{V_{min}}$$

Typical example values

3 : 1 & 2 : 1

Open short ciruit = ∞ : 1

Perfect match = 1 : 1

Reflection Coefficient of Transmission Line

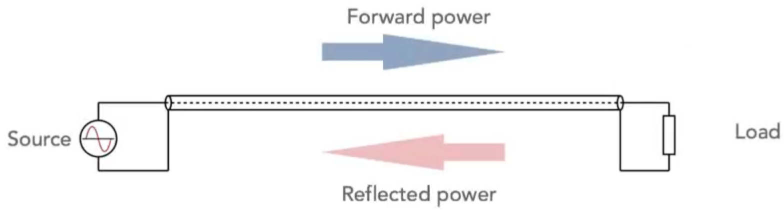
What is VSWR?

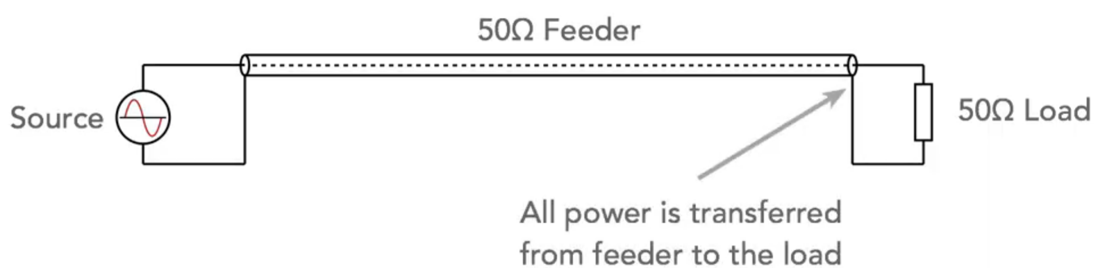
Voltage

Standing

Wave

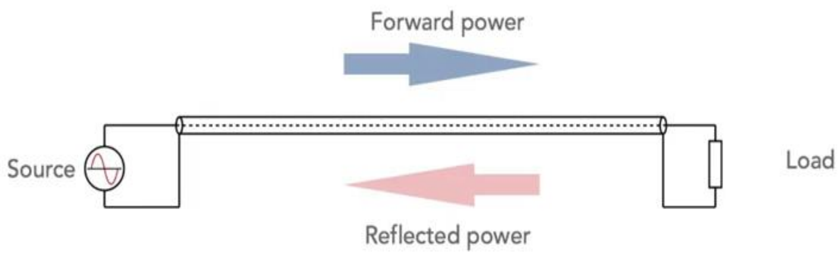
Ratio





Reflection Coefficient of Transmission Line

Forward & Reverse Power Levels



$$VSWR = \frac{1 + \sqrt{P_{ref} / P_{fwd}}}{1 - \sqrt{P_{ref} / P_{fwd}}}$$

Reflection Coefficient of Transmission Line

Summary

1) Maximum power is absorbed by a load when load impedance matches the feeder impedance
2) When there is a mismatch between feeder and load impedances, power is reflected
3) The voltages and currents from the forward and reflected power sum and form standing waves
4) It is easy to calculate and also measure VSWR