Find $L\{4e^{5t} + 6t^3 - 3\sin 4t + 2\cos 2t\}$

<u>Soln:</u> $L\{4e^{5t} + 6t^3 - 3\sin 4t + 2\cos 2t\}$

$$= 4\cdot L\{e^{5t}\} + 6\,L\{t^3\} - 3\,L\{\sin 4t\} + 2\,L\{\cos 2t\}$$

$$= 4\cdot\frac{1}{s-5} + 6\cdot\frac{3!}{s^4} - 3\cdot\frac{4}{s^2+4^2} + 2\cdot\frac{s}{s^2+2^2}$$

$$= \frac{4}{s-5} + \frac{36}{s^4} - \frac{12}{s^2+16} + \frac{2s}{s^2+4}$$ Ans:

Find $L\{t^2 e^{3t}\}$

<u>Soln:</u> $L\{t^2 e^{3t}\}$

Now, $L\{e^{3t}\} = \frac{1}{s-3}$

$$\therefore L\{t^2 e^{3t}\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{1}{s-3}\right)$$

$$= (-1)^2(-1)\cdot\frac{d}{ds}\left(\frac{1}{(s-3)^2}\right)$$

$$= (-1)(-2)\cdot\frac{1}{(s-3)^3}$$

$$= \frac{2}{(s-3)^3}$$ (Ans:)

☐ Find $L\{e^{-2t} \cdot \sin 4t\}$

Soln: $L\{\sin 4t\} = \dfrac{4}{s^2+16}$

$\therefore L\{e^{-2t} \sin 4t\} = \dfrac{4}{(s+2)^2+16}$

$= \dfrac{4}{s^2+4s+20}$ (Ans:)

☐ find $L\{e^{4t} \cosh 5t\}$

Soln: $L\{\cosh 5t\} = \dfrac{s}{s^2-25}$

$\therefore L\{e^{4t} \cosh 5t\} = \dfrac{s-4}{(s-4)^2-25}$

$= \dfrac{s-4}{s^2-8s+16-25}$

$= \dfrac{s-4}{s^2-8s-9}$ (Ans:)

$= a\left[\frac{1}{a}\tan^{-1}\frac{s}{a}\right]_s^{\infty}$

$= a\left[\tan^{-1}\infty - \tan^{-1}\frac{s}{a}\right]$

$= a\left[\frac{\pi}{2} - \tan^{-1}\frac{s}{a}\right]$

$= a\cot^{-1}\frac{s}{a}$

$L\left\{\frac{\sin at}{at}\right\} = a\tan^{-1}\frac{a}{s}$ Ans:

Evaluate $\int_0^{\infty} te^{-3t}\sin t\,dt$

Soln: $\int_0^{\infty} te^{-3t}\sin t\,dt = L\{t\sin t\}$ where $s = 3$

Now, $L\{\sin t\} = \frac{1}{s^2+1}$

$\therefore L\{t\sin t\} = (-1)\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$

$= (-1)\cdot\frac{2s}{(s^2+1)^2}$

$\therefore L\{t\sin t\} = \frac{2s}{(s^2+1)^2}$ ... ①

putting the value of s into the equation (i) and we get,

$$L\{t \sin t\} = \frac{2\times 3}{(3^2+1)^2} = \frac{6}{100} = \frac{3}{50} \text{ (Ans:)}$$

Find $L^{-1}\left\{\frac{5s^2-15s-11}{(s+1)(s-2)^3}\right\}$

soln:

let $$\frac{5s^2-15s-11}{(s+1)(s-2)^3} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{(s-2)^2} + \frac{D}{(s-2)^3} \quad \text{.... (i)}$$

$$\Rightarrow 5s^2-15s-11 = A(s-2)^3 + B(s+1)(s-2)^2 + C(s+1)(s-2) + D(s+1) \quad \text{.... (ii)}$$

$s=-1 \Rightarrow$

$$5+15-11 = -27A \quad \therefore A = -\frac{1}{3}$$

$$s=2 \Rightarrow 20-30-11 = 3D \quad \therefore D = -7$$

$$s=0 \Rightarrow -11 = -8A + 4B - 2C + D$$

$$\therefore \frac{-20}{3} = 4B - 2C \quad \text{.... (iii)}$$

coefficient of $s^3 \Rightarrow$

$$0 = A + B \quad \therefore B \Rightarrow \frac{1}{3}$$

Now from (iii) $\Rightarrow$

$$-\frac{20}{3} = \frac{4}{3} - 2c$$

$$\therefore c = 4$$

Now from equation (1) we get,

$$\frac{5s^2 - 15s - 11}{(s+1)(s-2)^3} = -\frac{1}{3(s+1)} + \frac{1}{3(s-2)} + \frac{4}{(s-2)^2} - \frac{7}{(s-2)^3}$$

$$\therefore L^{-1}\left\{\frac{5s^2-15s-11}{(s+1)(s-2)^3}\right\} = -\frac{1}{3}L^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{3}L^{-1}\left\{\frac{1}{s-2}\right\} + 4L^{-1}\left\{\frac{1}{(s-2)^2}\right\} - 7L^{-1}\left\{\frac{1}{(s-2)^3}\right\}$$

$$= -\frac{1}{3}e^{-t} + \frac{1}{3}e^{2t} + 4te^{2t} - \frac{7t^2e^{2t}}{2!}$$

$$= \frac{1}{3}(e^{2t} - e^{-t}) + 4te^{2t} - \frac{7}{2}t^2e^{2t}$$

$$= \frac{1}{3}(e^{2t} - e^{-t}) + te^{2t}\left(4 - \frac{7}{2}t\right)$$

Ans:

y

$L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$

Solⁿ: let $\frac{3s+1}{(s-1)(s^2+1)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+1}$ ... (i)

$\Rightarrow 3s+1 = A(s^2+1) + (Bs+C)(s-1)$ ... (ii)

putting $s=1$ and we get from (ii)

$4 = 2A \quad \therefore A = 2$

putting $s=0$ and we get from (ii)

$1 = A - C \quad \therefore C = 1$

putting $s=2$ and we get from (ii)

$7 = 5A + (2B+C)$

$\therefore B = -2$

From equation (i) we get

$$\frac{3s+1}{(s-1)(s^2+1)} = \frac{2}{s-1} + \frac{-2s+1}{s^2+1}$$

$$= \frac{2}{s-1} + -\frac{2s}{s^2+1} + \frac{1}{s^2+1}$$

$$\therefore L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\} = L^{-1}\left\{\frac{2}{s-1} - \frac{2s}{s^2+1} + \frac{1}{s^2+1}\right\}$$

$$= 2e^t - 2\cos t + \sin t$$

Ans:

Find $L^{-1}\left\{\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$

Solⁿ:

$$\frac{2s^2-4}{(s+1)(s-2)(s-3)} = \frac{A}{(s+1)} + \frac{B}{(s-2)} + \frac{C}{(s-3)} \quad \cdots (1)$$

$$2s^2-4 = A(s-2)(s-3) + B(s+1)(s-3) + C(s+1)(s-2) \quad \cdots (ii)$$

$s=2$, $8-4 = B(3)(-1) \therefore B = -\frac{4}{3}$

$s=-1$, $2-4 = A(-3)(-4) \Rightarrow A = -\frac{1}{6}$

$s=3$ $18-4 = C(4)(1) \Rightarrow C = \frac{7}{2}$

From (1) we get,

$$\frac{2s^2-4}{(s+1)(s-2)(s-3)} = -\frac{1}{6(s+1)} - \frac{4}{3(s-2)} + \frac{7}{2(s-3)}$$

$$L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\} = L^{-1}\left\{-\frac{1}{6(s+1)} - \frac{4}{3(s-2)} + \frac{7}{2(s-3)}\right\}$$

$$= -\frac{1}{6}e^{-t} - \frac{4}{3}e^{2t} + \frac{7}{2}e^{3t}$$

(Ans:)

Find $L^{-1}\left\{\dfrac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\}$

let, $\dfrac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \dfrac{As+B}{(s^2+2s+2)} + \dfrac{Cs+D}{(s^2+2s+5)}$ .... (I)

$\Rightarrow s^2+2s+3 = (As+B)(s^2+2s+5)+(Cs+D)(s^2+2s+2)$ ..... (II)

Coefficient of $s^3$ and $s^2$

$0 = A + C \quad \therefore A = -C$ ... (III)

$1 = 2A + B + 2C + D \quad \therefore 1 = B + D$ .. (IV) $\because A = -C$

Putting the value of $s = 0$ and 1 and we get,

$3 = 5B + 2D$

$\Rightarrow 3 = 5B + 2(1-B) \quad \because$ from (IV)

$\therefore B = \dfrac{1}{3}$

From (IV) we get

$D = \dfrac{2}{3}$

Now $s = 1$ and we get,

~~1 = 2A + 2C~~

$1+2+C = 8(A+B)+5(C+D)$

$\Rightarrow 6 = 8\left(A + \frac{1}{3}\right) + 5\left(C + \frac{2}{3}\right)$ $\quad \because B = \frac{1}{3}$ and $D = \frac{2}{3}$

$\Rightarrow 6 = 8\left(-C + \frac{1}{3}\right) + 5\left(C + \frac{2}{3}\right)$ $\quad \therefore C = -A$

$\therefore C = 0$

$\therefore A = 0$

From equation ① we get,

$$\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \frac{1}{3(s^2+2s+2)} + \frac{2}{3(s^2+2s+5)}$$

$$= \frac{1}{3\{(s+1)^2+1\}} + \frac{2}{3\{(s+1)^2+4\}}$$

$$\therefore L^{-1}\left\{\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\} = \frac{1}{3}L^{-1}\left\{\frac{1}{(s+1)^2+1}\right\} + \frac{1}{3}L^{-1}\left\{\frac{2}{(s+1)^2+4}\right\}$$

$$= \frac{1}{3}e^{-t}\sin t + \frac{1}{3}e^{-t}\sin 2t$$

$$= \frac{1}{3}e^{-t}(\sin t + \sin 2t)$$

Ans;

Ⓑ Solve the equation $y''+2y'+5y=0$ where $y=2$, $\frac{dy}{dx}=-4$ at $x=0$

<u>Soln:</u> $y''+2y'+5y=0$

$\Rightarrow L\{y''+2y'+5y\}=L\{0\}$

$\Rightarrow s^2Y(s)-sy'(0)-y'(0)+2(sY(s)-y(0))+5Y(s)=0$

$\Rightarrow s^2Y(s)-2s+4+2sY(s)-4+5Y(s)=0$

$\Rightarrow Y(s)(s^2+2s+5)=2s-4+4$

$$\Rightarrow Y(s)=\frac{2s}{s^2+2s+5}$$

$$\Rightarrow Y(s)=\frac{2s}{(s+1)^2+2^2}$$

$$\Rightarrow L^{-1}\{Y(s)\}=L^{-1}\left\{\frac{2s}{(s+1)^2+2^2}\right\}$$

$$=\cancel{2\cos 2x}\ L^{-1}\left\{\frac{2(s+1)-2}{(s+1)^2+2^2}\right\}$$

$$=L^{-1}\left\{\frac{2(s+1)}{(s+1)^2+2^2}-\frac{2}{(s+1)^2+2^2}\right\}$$

$$\Rightarrow y(x) = e^{-x}\, 2\cos 2x - e^{-x}\sin 2x$$

$$\therefore y(x) = e^{-x}\left(2\cos 2x - \sin 2x\right)$$

Ans!

☐ Solve the equation $y'' + 2y' + y = te^{-t}$ if $y(0) = 1$ and, $y'(0) = -2$.

Soln: $y'' + 2y' + y = te^{-t}$

$$\Rightarrow \mathcal{L}\{y'' + 2y' + y\} = \mathcal{L}\{te^{-t}\}$$

$$\Rightarrow s^2 Y(s) - s\,y(0) - y'(0) + 2\left(sY(s) - y(0)\right) + Y(s) = \frac{1}{(s+1)^2}$$

$$\Rightarrow \cancel{Y(s)(s^2}\ s^2 Y(s) - s + 2 + 2sY(s) - 2 + Y(s) = \frac{1}{(s+1)^2}$$

$$\Rightarrow Y(s)\left(s^2 + 2s + 1\right) = \frac{1}{(s+1)^2} + s$$

$$\Rightarrow Y(s) = \frac{1}{(s+1)^4} + \frac{s}{(s+1)^2}$$

$$= \frac{1}{(s+1)^4} + \frac{s+1-1}{(s+1)^2}$$

$$\Rightarrow Y(s) = \frac{1}{(s+1)^4} + \frac{s+1}{(s+1)^2} - \frac{1}{(s+1)^2}$$

$$\Rightarrow Y(s) = \frac{1}{(s+1)^4} + \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$\Rightarrow \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^4} + \frac{1}{s+1} - \frac{1}{(s+1)^2}\right\}$$

$$\Rightarrow y(t) = \frac{1}{6}e^{-t}t^3 + e^{-t} - e^{-t}\cdot t$$

$$= e^{-t}\left(\frac{t^3}{6} + 1 - t\right)$$

$$\therefore y(t) = e^{-t}\left(\frac{t^3}{6} + 1 - t\right)$$

<u>Ans:</u>

□ Solve the equation $y'' + y = x\cos 2x$ where $y(0) = y'(0) = 0$

<u>Soln:</u> $y'' + y = x\cos 2x$

$$\Rightarrow \mathcal{L}\{y'' + y\} = \mathcal{L}\{x\cos 2x\}$$

$$\Rightarrow s^2 Y(s) - s y(0) - y'(0) + Y(s) = \frac{s^2-4}{(s^2+4)^2}$$

$$\Rightarrow s^2 Y(s) - 0 - 0 + Y(s) = \frac{s^2-4}{(s^2+4)^2}$$

$$\Rightarrow Y(s)(s^2+1) = \frac{s^2-4}{(s^2+4)^2}$$

$$\Rightarrow Y(s) = \frac{s^2-4}{(s^2+4)^2(s^2+1)} \quad \cdots \cdots (\text{i})$$

Let, $$\frac{s^2-4}{(s^2+4)^2(s^2+1)} = \frac{As+B}{(s^2+4)} + \frac{Cs+D}{(s^2+4)^2} + \frac{Es+F}{(s^2+1)} \quad \cdots (\text{ii})$$

$$\Rightarrow s^2 - 4 = (As+B)(s^2+4)(s^2+1) + (Cs+D)(s^2+1) + (Es+F)(s^2+4)^2$$

$s=0 \Rightarrow$

$$-4 = 4B + D + 16F \quad \cdots (\text{iii})$$

coefficient of $s^5, s^4, s^3, s^2, s$ and we get,

$0 = A + E \quad \therefore A = -E$

$0 = B + F \quad \therefore B = -F$

$0 = C$

$1 = D$

Now from (iii) $\Rightarrow -4 = 4B + 1 - 16B$

$$\Rightarrow -5 = -12B \quad \therefore B = \frac{5}{12}$$

$\therefore F = -\frac{5}{12}$

$s = 1 \Rightarrow$

$-3 = (A+B)10 + (C+D)2 + (E+F)25$

$\Rightarrow -3 = 10A + 10B + 2C + 2D + 25(-A) + 25F$

$\Rightarrow -3 = -15A + 10\frac{5}{12} + 0 + 2 - 25\frac{5}{12}$

$\Rightarrow 15A = 3 + \frac{50}{12} + 2 - \frac{125}{12}$

$\Rightarrow 15A = \frac{5}{4} \quad \therefore A = \frac{1}{12}$

$\therefore E = -\frac{1}{12}$

putting the value of A, B, C, D, E, F into equation (11) and we get,

$$\frac{s^2-4}{(s^2+4)^2(s^2+1)} = \frac{\frac{s}{12}+\frac{5}{12}}{(s^2+9)} + \frac{1}{(s^2+9)^2} + \frac{-\frac{s}{12}-\frac{5}{12}}{(s^2+1)}$$

$$= \frac{s+5}{12(s^2+9)} + \frac{1}{(s^2+9)^2} - \frac{s+5}{12(s^2+1)}$$

$$= \frac{s}{12(s^2+9)} + \frac{5}{12(s^2+9)} + \frac{1}{(s^2+9)^2} - \frac{s}{12(s^2+1)} - \frac{5}{12(s^2+1)}$$

$$\therefore y(t) = \frac{1}{12}\cos 2t + \frac{5}{12}\sin 2t + t\sin 2t - \frac{1}{12}\cos t - \frac{5}{12}\sin t$$