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Chapter - 1

Discrete - Time Signals and Systems

Q # What is Signal?

=> A signal is defined as a single-valued function of one or more independent variables which contain some information. A signal is also defined as a physical quantity that varies with time, space or any other independent variable.

Q # What is system?

=> A system is defined as an entity that acts on an input signal and transforms it into an output signal. A system is also defined as a set of elements or fundamental blocks which are connected together and produces an output in response to an input signal.

Q # Advantages of digital signals over analog signals:

① Noise resistance: Digital signals are less affected by noise and interference.

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② **Better long-distance transmission:** Signal quality remains high over ~~lag~~ long distances.

③ **Easy storage and processing:** Can be stored and processed without loss of quality.

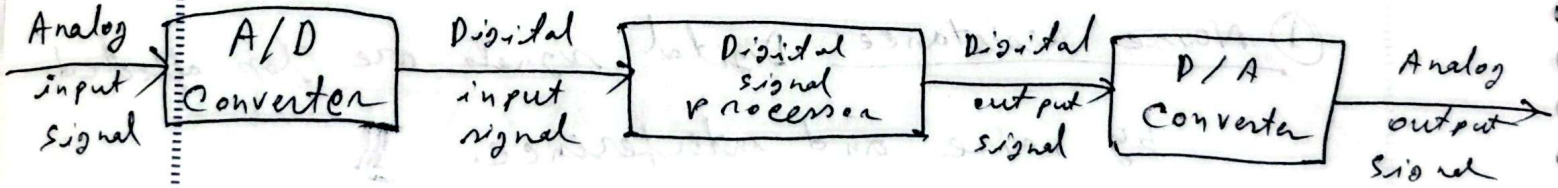
④ **Reproducibility:** Can be copied exactly without degradation.

⑤ **security:** Can be encrypted for secure communication.

⑥ **Compression and multiplexing:** Efficient use of ~~bandwith~~ bandwidth and easier to combine multiple signals.

⑦ **Compatibility with modern devices:** Works easily with computers, smartphones and other digital technology.

Block diagram of digital signal processing system.



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Representation of Discrete-Time Signals:

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequence representation

\# $x(n) = \begin{cases} n+1 & 0 \le n \le 2 \\ n & 2 \le n \le 3 \\ n+3 & 3 < n \le 5 \end{cases}$ Functional

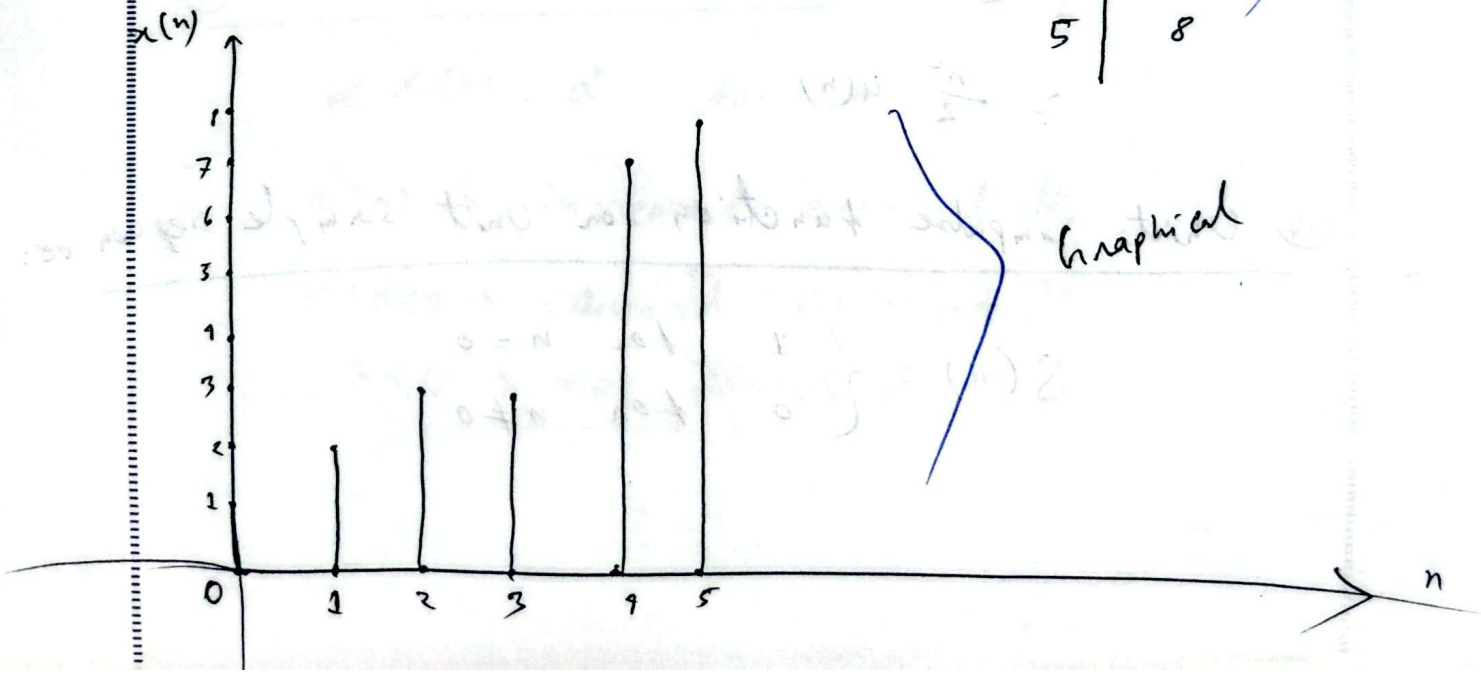
Sequence

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 3, 7, 8\}$

origin

n	$x(n)$
0	1
1	2
2	3
3	3
4	7
5	8

Tabular



Graphical

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Elementary Discrete-Time Signals:

(i) Unit step sequence:

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

(ii) Unit Ramp sequence:

$$R(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$= n\,u(n)$$

(iii) Unit Parabolic sequence:

$$P(n) = \begin{cases} \frac{n^2}{2} & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

$$= \frac{n^2}{2}\,u(n)$$

(iv) Unit impulse function or Unit sample sequence:

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

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Properties of discrete-time unit sample sequence:

1) $\delta(n) = u(n) - u(n-1)$, $u(n) = \sum_{m=0}^{n} \delta(m)$ [Relation between the unit sample sequence and the unit step sequence]

2) $\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$

3) $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

4) $\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

(V) Sinusoidal sequence:

$$x(n) = A \sin(\omega n + \phi)$$

$A > 1 \rightarrow$ Amplification

$A < 1 \rightarrow$ Attuenation

$A = 1 \rightarrow$ Unchanged

The period of the discrete-time sinusoidal sequence is:

$$N = \frac{2\pi}{\omega} m$$

(VI) Real Exponential Sequence:

$x(n) = a^n$ for all n

$a > 1 \rightarrow$ increased exponentially

$0 < a < 1 \rightarrow$ decreased exponentially

$a < 0 \rightarrow$ sign alternative

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(vii) Complex Exponential Sequence:

$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

$$= a^n \cos(\omega_0 n + \phi) + j a^n \sin(\omega_0 n + \phi)$$

$|a| = 1 \longrightarrow$ sinusoidal

$|a| > 1 \longrightarrow$ rise

$|a| < 1 \longrightarrow$ decay

Example 1.1 Find the following summations:

a) $\sum_{n=-\infty}^{\infty} e^{3n} \delta(n-3)$

Given, $\sum_{n=-\infty}^{\infty} e^{3n} \delta(n-3)$

we know that $\delta(n-3) = \begin{cases} 1 & \text{for } n=3 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \sum_{n=-\infty}^{\infty} e^{3n} \delta(n-3) = \left[e^{3n}\right]_{n=3} = e^9$$

b) $\sum_{n=-\infty}^{\infty} \delta(n-2) \cos 3n$

Given, $\sum_{n=-\infty}^{\infty} \delta(n-2) \cos 3n$

we know that $\delta(n-2) = \begin{cases} 1 & \text{for } n=2 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \sum_{n=-\infty}^{\infty} \delta(n-2) \cos 3n = \left[\cos 3n\right]_{n=2} = \cos 6$$

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c) $\sum_{n=-\infty}^{\infty} n^2 \delta(n+4)$

Given, $\sum_{n=-\infty}^{\infty} n^2 \delta(n+4)$

we know that $\delta(n+4) = \begin{cases} 1 & \text{for } n=-4 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \sum_{n=-\infty}^{\infty} n^2 \delta(n+4) = [n^2]_{n=-4} = 16$

d) $\sum_{n=-\infty}^{\infty} \delta(n-2) e^{n^2}$

Given, $\sum_{n=-\infty}^{\infty} \delta(n-2) e^{n^2}$

we know that $\delta(n-2) = \begin{cases} 1 & \text{for } n=2 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \sum_{n=-\infty}^{\infty} \delta(n-2) e^{n^2} = [e^{n^2}]_{n=2} = e^4$

e) $\sum_{n=0}^{\infty} \delta(n+1) 4^n$

Given, $\sum_{n=0}^{\infty} \delta(n+1) 4^n$

we know that $\delta(n+1) 4 = \begin{cases} 1 & \text{for } n=-1 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \sum_{n=0}^{\infty} \delta(n+1) 4^n = 0$ [as -1 is not in the limit]

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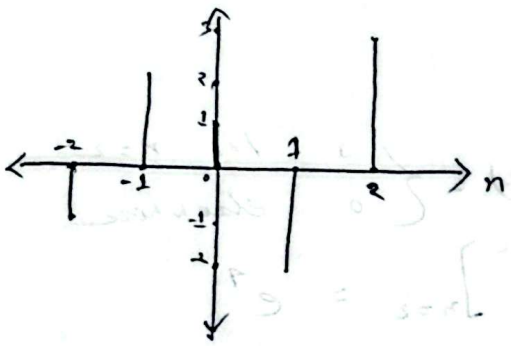
Basic Operations on Sequence

1| Time Shifting:

$y(n) = x(n-k)$

- If k is positive = delay and right shift
- If k is negative = advance and left shift

#



$x(n) = \{-1, 2, \underset{\uparrow}{1}, -2, 3\}$

for $k = 2$,

$y(-2) = x(-2-2) = x(-4) = 0$

$y(-1) = x(-1-2) = x(-3) = 0$

$y(0) = x(0-2) = x(-2) = -1$

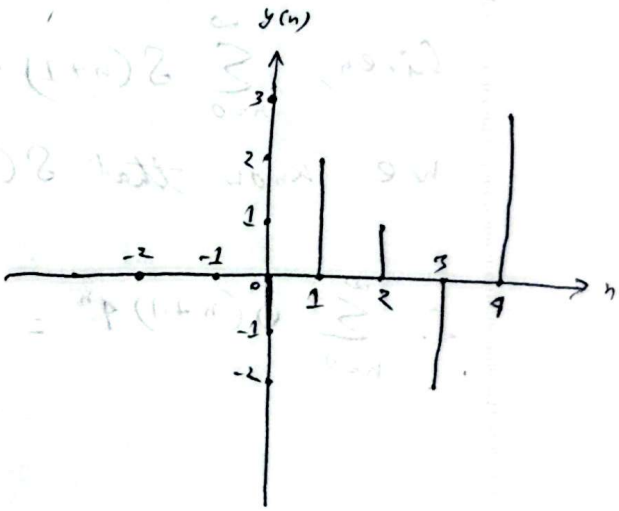
$y(1) = x(1-2) = x(-1) = 2$

$y(2) = x(2-2) = x(0) = 1$

$y(3) = x(3-2) = x(1) = -2$

$y(4) = x(4-2) = x(2) = 3$

$\therefore y(n) = \{\underset{\uparrow}{-1}, 2, 1, -2, 3\}$



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2) Time Reversal:

$$y(n) = x(-n)$$

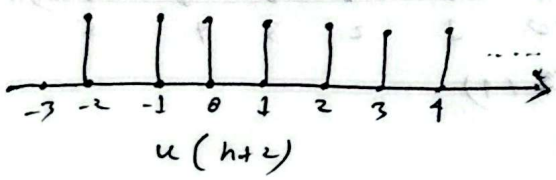
Example 3.2 Sketch the following signals:

VVI

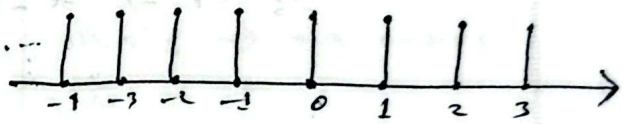
a) $u(n+2)\,u(-n+3)$

Given, $u(n+2)\,u(-n+3)$

The signal $u(n+2)\,u(-n+3)$ can be obtained by first drawing the signal $u(n+2)$, then drawing $u(-n+3)$



$u(n+2)$

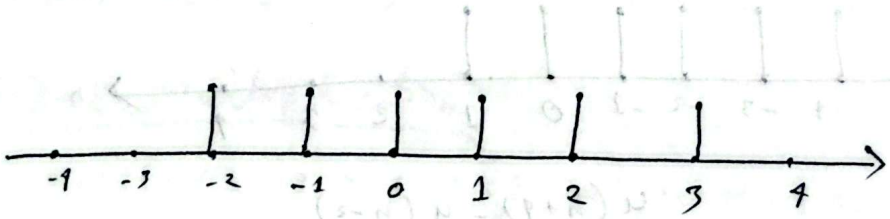


$u(-n+3)$

and then multiplying these sequences element to obtain $u(n+2)\,u(-n+3)$.

$x(n) = 0$ for $n < -2$ and $n > 3$

$x(n) = 1$ for $-2 \le n \le 3$



$u(n+2)\,u(-n+3)$

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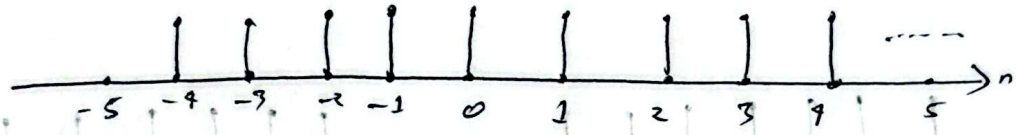
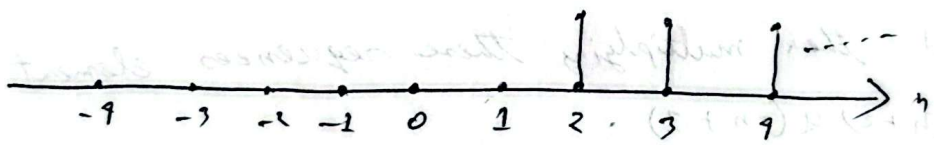
b) $x(n) = u(n+4) - u(n-2)$

Given, $x(n) = u(n+4) - u(n-2)$

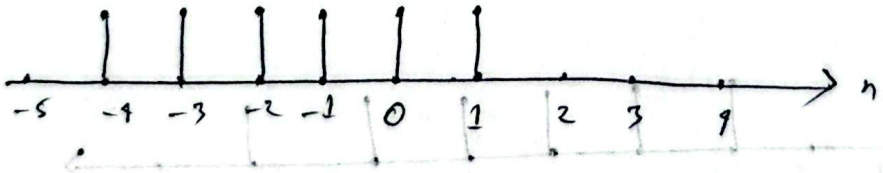
The signal $u(n+4) - u(n-2)$ can be obtained by first plotting $u(n+4)$, then plotting $u(n-2)$, and then subtracting each element of $u(n-2)$ from the corresponding element of $u(n+4)$ to obtain the result.



$u(n+4)$



$u(n-2)$



$u(n+4) - u(n-2)$

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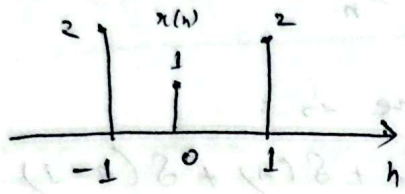
3) Amplitude Scaling:

$$y(n) = a\,x(n)$$

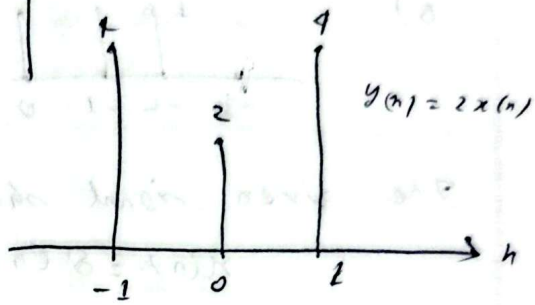
$a > 1 \rightarrow$ amplification

$a < 1 \rightarrow$ attenuation.

If $y(n) = 2x(n)$



$\rightarrow$



4) Time Scaling:

$$y(n) = x(an)$$

$a > 1 \rightarrow$ Compression

$a < 1 \rightarrow$ expansion

5) Signal Addition:

If $x_1(n) = \{1, 2, 3, 1, 5\}$ and $x_2(n) = \{2, 3, 4, 1, -2\}$

$\therefore x_1(n) + x_2(n) = \{1+2, 2+3, 3+4, 1+1, 5-2\} = \{3, 5, 7, 2, 3\}$

$\therefore x_1(n) - x_2(n) = \{1-2, 2-3, 3-4, 1-1, 5+2\} = \{-1, -1, -1, 0, 7\}$

6) Signal Multiplication:

If $x_1(n) = \{1, -3, 2, 4, 1.5\}$ and $x_2(n) = \{2, -1, 3, 1.5, 2\}$

$\therefore x_1(n)\,x_2(n) = \{1\times 2, -3\times -1, 2\times 3, 4\times 1.5, 1.5\times 2\}$

$= \{2, 3, 6, 6, 3\}$

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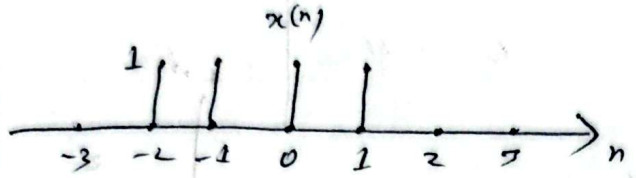
Example 1.3 Express the signals shown in figure as the sum of singular functions.

a)



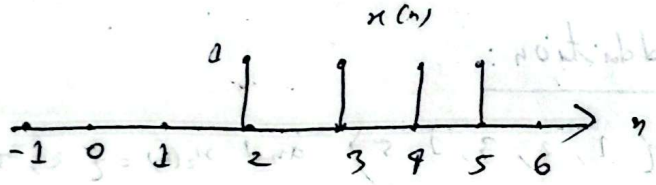
The given signal shown in figure is:

$$x(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1)$$

$$x(n) = \begin{cases} 0 & \text{for } n \le -3 \\ 1 & \text{for } -2 \le n \le 1 \\ 0 & \text{for } n \ge 2 \end{cases}$$

$$\therefore \cancel{n(x)}\ x(n) = u(n+2) - u(n-2)$$

b)



The given signal shown in figure is:

$$x(n) = \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5)$$

$$x = \begin{cases} 0 & \text{for } n \le 1 \\ 1 & \text{for } 2 \le n \le 5 \\ 0 & \text{for } n \ge 6 \end{cases}$$

$$\therefore x(n) = u(n-2) - u(n-6)$$

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May be a question:

given $x(n)$ or Figure of $x(n)$

i) $x(n-2)$ + Figure ii) $x(n+2)$ + Figure

iii) $y(n) = x(n-2) + x(n+2)$ + Figure

iv) $z(n) = x(n-2) \times x(n+2)$ + Figure

(2) Classification of Discrete-Time Signals:

Both continuous-time and discrete-time signals are further classified as follow:

1. Deterministic and random signals.
2. Periodic and non-periodic signals
3. Energy and power signals.
4. Causal and non-causal signals.
5. Even and odd signals.

Classification of Discrete-Time ~~Signals~~ Systems:

$y(n) = \text{Operation on } x(n)$

$y(n) = T[x(n)]$

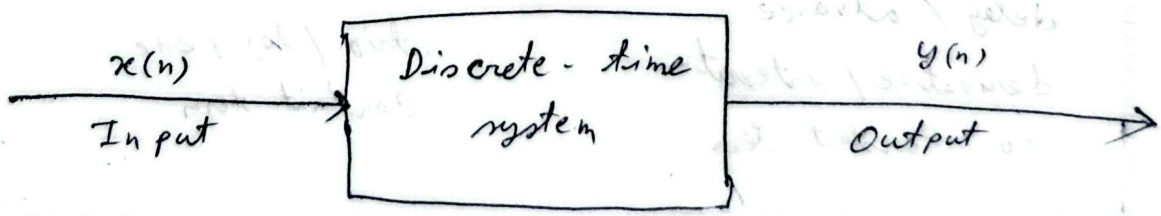

Fig: Block diagram of discrete-time system

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Both continuous-time and discrete time systems are further classified as follows:

1. Static (memoryless) and dynamic (memory) system
2. Causal and non-causal systems.
3. Linear and non-linear systems.
4. Time-invariant and time varying system.
5. Stable and unstable systems.
6. Invertible and non-invertible systems.
7. FIR and IIR systems.

$x(n \pm k)$ থাকলে → Dynamic
শুধু $x(n)$ থাকলে → Static
Delay / advance / summation → Dynamic

$x(n)$ or $x(n-k)$ → Causal
$x(n+k)$ → non-causal

Linear system	non linear system
শুধু addition (+)	Power ≠ 1
শুধু scaling (constant × x)	Production of signals
delay / advance	absolute value
derivative / integration	trig / log / exp
no constant term	constant term

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4| Output এ শুধু delay / advance হলে → Time-Invariant
output-এ n, t, sin(n), e^n direct multiply করলে → Time Varying

5| Output-এ n, t, e^n multiply করলে, summation / integration (∞ এ grow)
→ Unstable
শুধু Constant × x → stable

bounded input → bounded output → stable
bounded input → unbounded output → unstable

6| square / | | / sin / cos → NON-invertible
delay / scale / + Constant → Invertable
even operation (x(n) + x(−n)) → NON-invertible

7| RHS এ y(n−k) করলে → IIR
RHS এ x(n−k করলে → FIR

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1. How to represent an arbitrary sequence from delayed and weighted impulse sequence? Explain Briefly. 7
2. State whether the following system is linear, causal, time-invariant and stable: 8

$$y(n) + y(n-1) = x(n) + x(n-2)$$

d and

e

impulse sequences.

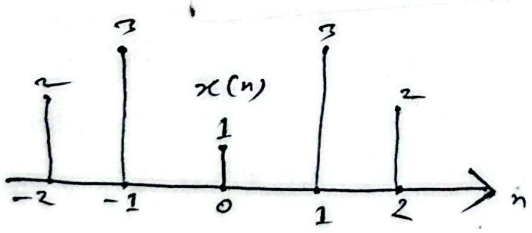
This representation is based on the ~~of~~ shifting property of the unit impulse sequence.

The unit impulse sequence $\delta(n)$ is defined as:

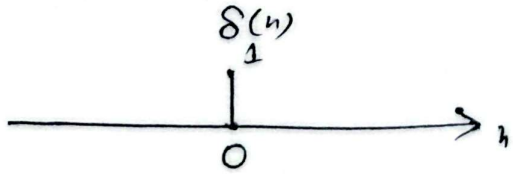
$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Any arbitrary sequence $x(n)$ can be represented in terms of delayed and weighted impulse sequence $\delta(n)$.

Consider a finite 5 sample sequence.



A unit impulse.



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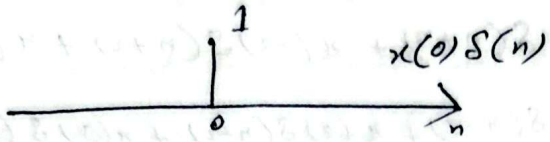
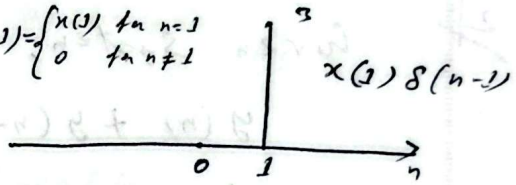
The sample $x(0)$ can be obtained by multiplying $x(0)$, the magnitude with unit impulse $\delta(n)$.

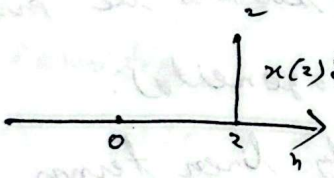


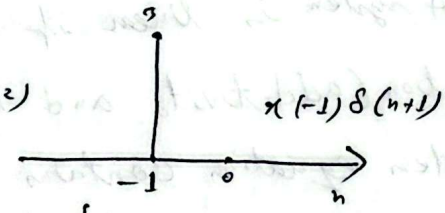
$$x(0)\,\delta(n) = \begin{cases} x(0) & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

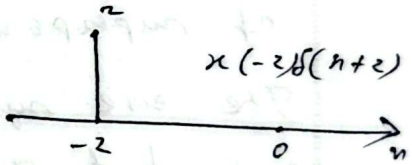
Similarly the sample $x(1)$ can be obtained by multiplying $x(1)$, the magnitude with $\delta(n-1)$ which is one sample advanced unit impulse. $x(1)\delta(n-1) = \begin{cases} x(1) & \text{for } n=1 \\ 0 & \text{for } n \neq 1 \end{cases}$



$x(2)$, $x(-1)$, $x(-2)$ can be obtained,







$x(2)\,\delta(n-2) = \begin{cases} x(2) & \text{for } n=2 \\ 0 & \text{for } n \neq 2 \end{cases}$ | $x(-1)\,\delta(n+1) = \begin{cases} x(-1) & \text{for } n=-1 \\ 0 & \text{for } n \neq -1 \end{cases}$ | $x(-2)\,\delta(n+2) = \begin{cases} x(-2) & \text{for } n=-2 \\ 0 & \text{for } n \neq -2 \end{cases}$

In this case,

$x(0)\delta(n) = \delta(n)$, $x(1)\delta(n-1) = 3\delta(n-1)$, $x(2)\delta(n-2) = 2\delta(n-2)$,

$x(-1)\delta(n+1) = 3\delta(n+1)$, $x(-2)\delta(n+2) = 2\delta(n+2)$

$$\therefore x(n) = 2\delta(n+2) + 3\delta(n+1) + \delta(n) + 3\delta(n-1) + 2\delta(n-2)$$

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In general, we can write an infinite sequence $x(n)$ for $-\infty \leq n \leq \infty$ as:

$$x(n) = \cdots + x(-3)\delta(n+3) + x(-2)\delta(n+2) + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2) + x(3)\delta(n-3) + \cdots$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

where $\delta(n-k)$ is unity for $n=k$ and zero for all other terms.

(P) 2) Given system,

$$y(n) + y(n-1) = x(n) + x(n-2)$$

(i) Linearity: A system is linear if it satisfies the principles of superposition (additivity and homogeneity).

The give system equation contains only linear terms of input and output. There is no multiplication, power, or non linear operation on $x(n)$ or $y(n)$.

Hence, the system satisfies superposition.

$\therefore$ The system is linear.

(ii) Causality: A system is causal if the output at time n depends only on present and past inputs.

Rearranging the equation,

$$y(n) = x(n) + x(n-2) - y(n-1)$$

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Here, $x(n) \rightarrow$ Present input

$x(n-2) \rightarrow$ past input

$y(n-1) \rightarrow$ Past output

No future input or output is involved.

$\therefore$ The system is causal.

(iii) <u>Time-Invariance:</u> A system is time-invariant if a time shift in input produces the same time shift in output.

Replacing n by $n-n_0$,

$$y(n-n_0) + y(n-n_0-1) = x(n-n_0) + x(n-n_0-2)$$

The form of the equation remains unchanged.

$\therefore$ The system is time-invariant.

(iv) <u>Stability:</u> A system is BIBO stable if bounded input produces bounded output.

Rewriting,

$$y(n) = x(n) + x(n-2) - y(n-1)$$

Let the input be bounded, $x(n) = 1$

Then, $y(n) = 2 - y(n-1)$

Assuming $y(-1) = 0$,

$$y(0) = 2,\ y(1) = 0,\ y(2) = 2,\ y(3) = 0, \ldots\ldots$$

The output oscillates and does not decay with time.

$\therefore$ The system is not BIBO stable.

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