09-10-23

Coordinate Geometry

Two Dimensions — ②+1

4 set

Three " — ②+①

Two-Dimensional Co-ordinate Geometry

14 — 7, 7

Pair of straight lines

$$\boxed{\begin{matrix} y=m_1x \\ y=m_2x \end{matrix}}$$

$$ax^2+2hxy+by^2 \equiv b(y-m_1x)(y-m_2x)$$

$$\equiv b\left[y^2-(m_1+m_2)xy+m_1m_2x^2\right]$$

Compare

$$m_1+m_2=-\frac{2h}{b}$$

$$m_1m_2=\frac{a}{b}$$

$$\tan\theta=\frac{\sqrt{(m_1-m_2)^2}}{1+m_1m_2}$$

$$=\frac{\sqrt{(m_1+m_2)^2-4m_1m_2}}{1+m_1m_2}$$

$$=\frac{\frac{\sqrt{4h^2-4ab}}{b}}{\frac{a+b}{b}}$$

$$=\frac{\sqrt{4h^2-4ab}}{a+b}$$

$$\boxed{\theta=\tan^{-1}\frac{2\sqrt{h^2-ab}}{a+b}}$$

$$x^2-y^2=0$$

$$ax^2+2hxy+by^2=0$$ → general eqn.

$x^2-y^2=0$ → Homogeneous eqn of 2nd degree

Homogeneous eqn হলে প্রত্যেক straight line পারে সব origin দিয়ে যাবে। degree যত straight line তত।

$$(x+y)(x-y)=0$$

$$x^2-y^2=0$$

⊞ $a+b=0$ হলে θ হবে 90° (Perpendicular)

⊞ $h^2-ab=0$ " θ " 0° (Parallel)

Parallel → Distance থাকে

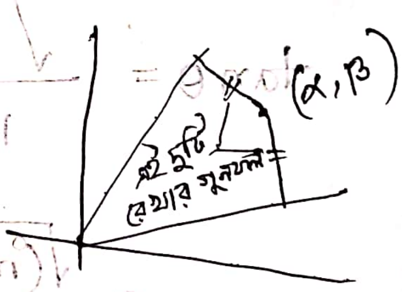
Coincide → থাকে না।

একটির উপর আরেকটি রেখা থাকলে সেটি হয় coincide (সমপাতিত)

⊞ Problem (Exam এ আসবে)

Show that the product (গুনফল) of the perpendiculars (লম্ব দূরত্ব) from the point (α, β) on the lines $ax^2+2hxy+by^2=0$ is

$$\frac{a\alpha^2+2h\alpha\beta+b\beta^2}{\sqrt{(a-b)+4h^2}}$$



Soln

~~Given that,~~

$$ax^2 + 2hxy + by^2 = 0 \quad \ldots \text{①}$$

~~Let the straight lines~~

Let $y = m_1 x$ and $y = m_2 x$ be the straight lines represented by eqn ①

$$\therefore \; ax^2 + 2hxy + by^2 \equiv b(y - m_1 x)(y - m_2 x)$$

$$= b\left[y^2 - (m_1 + m_2)xy + m_1 m_2 x^2\right]$$

Comparing the coefficients of like terms on both sides we get

$$m_1 + m_2 = -\frac{2h}{b}$$

$$m_1 m_2 = \frac{a}{b}$$

Now, the perpendicular distance from the point (α, β) on the line $y - m_1 x = 0$ is

$$\frac{\beta - m_1 \alpha}{\sqrt{1 + m_1^2}}$$

Also the perpendicular distance from the point (α, β) on the line $y - m_2 x = 0$ is $\dfrac{\beta - m_2\alpha}{\sqrt{1+m_2^2}}$

$\therefore$ The product of the perpendiculars is

$$\frac{(\beta - m_1\alpha)}{\sqrt{1+m_1^2}} \cdot \frac{(\beta - m_2\alpha)}{\sqrt{1+m_2^2}}$$

$$= \frac{\beta^2 - (m_1+m_2)\alpha\beta + m_1 m_2\alpha^2}{\sqrt{1+m_1^2+m_2^2+m_1^2 m_2^2}}$$

$$= \frac{\beta^2 - (m_1+m_2)\alpha\beta + m_1 m_2 \alpha^2}{\sqrt{1+(m_1+m_2)^2 - 2m_1m_2 + m_1^2 m_2^2}}$$

$$= \frac{\beta^2 + \frac{2h}{b}\alpha\beta + \frac{a}{b}\alpha^2}{\sqrt{1+\left(\frac{-2h}{b}\right)^2 - 2\frac{a}{b} + \left(\frac{a}{b}\right)^2}}$$

$\left[\because m_1 + m_2 = -\frac{2h}{b} \quad m_1 m_2 = \frac{a}{b}\right.$

$$= \frac{\dfrac{b\beta^2 + 2h\alpha\beta + a\alpha^2}{b}}{\sqrt{\dfrac{b^2 + 4h^2 - 2ab + a^2}{b^2}}}$$

$$= \frac{a\alpha^2 + 2h\alpha\beta + b\beta^2}{\sqrt{(a-b)^2 + 4h^2}}$$

Page-54
" - 53
" - 65 (ex-4*, ex-8*, ex-9*)

Exam এ আসবে (17 marks) must

15-10-23

☐ Find the condition that the general eqn of 2nd degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a pair of straight line.

$\Delta = 0, a+b=0$
$\Delta = 0, h^2 = ab$

Soln:

Let the general eqn of 2nd degree be

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \ldots\ldots (1) \quad ax^2+bx+c=0$$

$$\Rightarrow ax^2 + 2(hy+g)x + (by^2 + 2fg + c) = 0 \quad \ldots\ldots (1)$$

Treating this as a quadratic eqn in x and supposing $a \neq 0$ we have,

$$x = \frac{-2(hy+g) \pm \sqrt{4(hy+g)^2 - 4.a.(by^2+2fy+c)}}{2a}$$

$$\Rightarrow ax + hy + g = \pm\sqrt{h^2y^2 + 2hgy + g^2 - aby^2 - 2afy - ac}$$

$$\Rightarrow ax+hy+g = \pm\sqrt{(h^2-ab)y^2+2(hg-af)y+(g^2-ac)}$$

Now an order that this may be capable of being reduced to the form $ax+by+c=0$, it is necessary and sufficient that the quantity under the radical sign should be a perfect square.

$\therefore$ We get,

$$4(hg-af)^2 = 4(h^2-ab)(g^2-ac)$$

$$\Rightarrow h^2g^2 - 2afhg + a^2f^2 = h^2g^2 - abg^2 - ach^2 + a^2bc$$

$$\Rightarrow 2fhg - af^2 = bg^2 + ch^2 - abc \quad [-a \text{ দ্বারা ভাগ}]$$

$$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\therefore \Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

which is the required condition that the general eqn of 2nd degree represents a pair of straight line.

Point of intersection :

Let the Point of intersection between the straight lines represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is (α, β)

where $\alpha = \dfrac{hf - bg}{ab - h^2}$

$\beta = \dfrac{gh - af}{ab - h^2}$ (গুরুত্ব)

Exam এ আসবে : Problem :

Prove that the eqn $12x^2 + 7xy - 12y^2 - x + 7y - 1 = 0$ represents a pair of straight lines.

Find also there equations, point of intersection and the angle between them

Sol^n: Given that,

$$12x^2+7xy-12y^2-x+7y-1=0 \quad \ldots\ldots (1)$$

Comparing eq^n (1) with $ax^2+2hxy+by^2+2gx+2fy+c=0$ we get,

$a=12$ $\quad c=-1$

$b=-12$ $\quad g=-\frac{1}{2}$

$h=\frac{7}{2}$ $\quad f=\frac{7}{2}$

$$\therefore \Delta = abc+2fgh-af^2-bg^2-ch^2$$

$$=(12)\cdot(-12)(-1)+2\left(\frac{7}{2}\right)\cdot\left(-\frac{1}{2}\right)\cdot\left(\frac{7}{2}\right)-12\cdot\left(\frac{7}{2}\right)^2-(-12)\cdot\left(-\frac{1}{2}\right)^2-(-1)\cdot\left(\frac{7}{2}\right)^2$$

$=0$

which shows that the given eq^n represents a pair of straight lines.

Now,

$$12x^2 + (7y-1)x + (-12y^2 + 7y - 1) = 0$$

$$\Rightarrow 12x^2 + (7y-1)x - (12y^2 - 7y + 1) = 0$$

$$\therefore x = \frac{-(7y-1) \pm \sqrt{(7y-1)^2 + 4\cdot 12\cdot(12y^2 - 7y + 1)}}{2\cdot 12}$$

$$\Rightarrow 24x + 7y - 1 = \pm\sqrt{49y^2 - 14y + 1 + 576y^2 - 336y + 48}$$

$$= \pm\sqrt{625y^2 - 350y + 49}$$

$$= \pm\sqrt{(25y-7)^2}$$

$$\therefore 24x + 7y - 1 = \pm(25y - 7)$$

(−) (+)

$$\therefore 24x + 7y - 1 = -25y + 7 \qquad 4x - 3y + 1 = 0$$

$$\Rightarrow 24x + 32y - 8 = 0$$

$$\Rightarrow 3x + 4y - 1 = 0$$

$\therefore$ The eqns of the straigh lines represented by the given eqn (1) are

$$4x - 3y + 1 = 0$$

$$3x + 4y - 1 = 0$$

Let (α, β) be the point of intersection of these straight lines.

where

$$\alpha = \frac{hf - bg}{ab - h^2} = \frac{\frac{7}{2} \cdot \frac{7}{2} - (-12) \cdot (-\frac{1}{2})}{12 \cdot (-12) - (\frac{7}{2})^2}$$

$$= \frac{\frac{49}{4} - 6}{-144 - \frac{49}{4}}$$

$$= \frac{25}{527}$$

$\therefore \beta = (\quad)$

Let θ be the angle between the straight lines

$$\therefore \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$= \frac{2\sqrt{(\frac{7}{2})^2 - (12)(-12)}}{12 - 12}$$

$$= \infty$$

$$= \tan \frac{\pi}{2}$$

$\therefore \theta = \frac{\pi}{2}$.

30-10-23

Q. Find the value of λ for which the eqn $12x^2-10xy+2y^2+11x-5y+\lambda=0$ represents two straight lines. Also find the angle between them.

Soln: Comparing the given eqn with $ax^2+2hxy+by^2+2gx+2fy+c=0$ we get,

$a=12$, $b=2$, $h=-5$, $g=11/2$, $f=-5/2$, $c=\lambda$

As the given eqn represents a pair of straight lines, thus $abc+2fgh-af^2-bg^2-ch^2=0$

$\Rightarrow 12\cdot 2\cdot \lambda + 2\cdot\left(-\frac{5}{2}\right)\cdot 11/2\cdot(-5) - 12\cdot\left(-\frac{5}{2}\right)^2 - 2\cdot(11/2)^2 - \lambda\cdot(-5)^2 = 0$

$\therefore \lambda = 2$

let θ be the angle between the straight lines represented by eqn ①

$\therefore \tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$

$\Rightarrow \tan\theta = \frac{2\sqrt{25-24}}{14}$

$\Rightarrow \tan\theta = \frac{1}{7}$

$\therefore \theta = \tan^{-1}\left(\frac{1}{7}\right)$.

□ If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines, probe that the product of parpendiculars from the origin to the straight lines is $\dfrac{c}{\sqrt{(a-b)^2 + 4h^2}}$

Soln:

Given that,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad \ldots \text{①}$$

let the two straight lines represented by eqn ① be

$$y = m_1x + c_1 \quad \text{and} \quad y = m_2x + c_2$$

Then,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c \equiv b(y - m_1x - c_1)(y - m_2x - c_2)$$

$$= b\left[y^2 - (m_1+m_2)xy + (c_1m_2 + c_2m_1)x - (c_1+c_2)y + m_1m_2x^2 + c_1c_2\right]$$

Comparing the coofficients of like terms on both sides we obtain,

$$m_1+m_2 = -\frac{2h}{b}$$

$$c_1m_2+c_2m_1 = \frac{2g}{b}$$

$$c_1+c_2 = -\frac{2f}{b}$$

$$m_1m_2 = \frac{a}{b}$$

$$c_1c_2 = \frac{c}{b}$$

Now the parpendicular distence from the origin to the straight lines $y-m_1x-c_1=0$ is $\frac{-c_1}{\sqrt{1+m_1^2}}$.

also the parpendicular distance from the origin to the straight lines $y-m_2x-c_2=0$ is $\frac{-c_2}{\sqrt{1+m_2^2}}$

According to the question,

the product of the parpendiculars $= \frac{-c_1}{\sqrt{1+m_1^2}} \cdot \frac{-c_2}{\sqrt{1+m_2^2}}$

$$= \frac{c_1c_2}{\sqrt{1+m_1^2+m_2^2+(m_1m_2)^2}}$$

$$= \frac{\cancel{b}\, c_1c_2}{\sqrt{1+(m_1+m_2)^2-2m_1m_2+(m_1m_2)^2}}$$

$$= \frac{\frac{c}{b}}{\sqrt{1+\left(-\frac{2h}{b}\right)^2 - 2\cdot\frac{a}{b} - \left(\frac{a}{b}\right)^2}}$$

$$= \frac{\frac{c}{b}}{\sqrt{1+\frac{4h^2}{b^2} - 2\frac{a}{b} - \frac{a^2}{b^2}}}$$

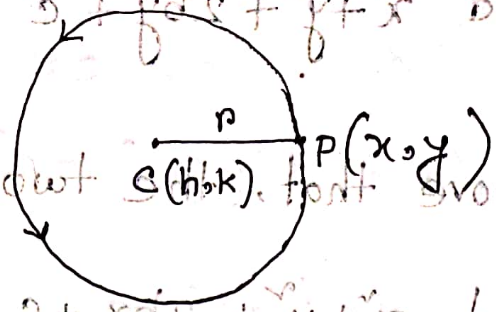
$$= \frac{c}{\sqrt{b^2+4h^2-2ab+a^2}}$$

$$\frac{c}{\sqrt{(a-b)^2+4h^2}} \text{ (Proved)}$$

Page: 48, 49 → (Prob- 4, 5)

□ Circles and system of circles : (1st set Q)

A



① Prove that the two circles $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$ may touch if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

⑪ Prove that, the two circles $x^2+y^2+2ax+c=0$ and $x^2+y^2+2bx+c=0$ may touch if $c=0$ or $a=b$

① Soln:

Given that,

$$x^2+y^2+2ax+c^2=0$$

$$x^2+y^2+2by+c^2=0$$

the centres of the circles are $(-a,0)$ and $(0,-b)$ ~~and~~ respectively and their radii are $\sqrt{a^2-c^2}$ and $\sqrt{b^2-c^2}$ respectively.

If the given circles touch each other, then the distance between their centres = sum or difference of their radii.

$$\Rightarrow \sqrt{(-a-0)^2+(0+b)^2} = \sqrt{a^2-c^2} \pm \sqrt{b^2-c^2}$$

$$\Rightarrow \cancel{a^2}+\cancel{b^2} = \cancel{a^2}-c^2+\cancel{b^2}-c^2 \pm 2\sqrt{a^2-c^2}\cdot\sqrt{b^2-c^2}$$

$$\Rightarrow c^2 = \pm\sqrt{a^2-c^2}\cdot\sqrt{b^2-c^2}$$

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$$\Rightarrow c^4 = (a^2-c^2)(b^2-c^2)$$

$$\Rightarrow c^4 = a^2b^2 - a^2c^2 - b^2c^2 + c^4$$

$$\Rightarrow a^2b^2 - b^2c^2 - a^2c^2 = 0$$

$$\Rightarrow \frac{1}{c^2} - \frac{1}{a^2} - \frac{1}{b^2} = 0$$

$$\therefore \frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2} \quad \text{(Proved)}$$

□ Find the eqn of the circle which passes through the points $(1,2)$, $(3,4)$ and touches the line $3x+y-3=0$

* □ Find the eqn of the tangent to the circle $x^2+y^2-4x+6y-3=0$, which are parallel to the straight line $3x-4y+9=0$

* □ Find the eqn of the common chord of the circles $(x-a)^2+y^2=a^2$, $x^2+(y-b)^2=b^2$ and show that the circle described on the common chord as diameter is $(a^2+b^2)(x^2+y^2) = 2ab(bx+ay)$

3. Sol^n:

□ Prove that the length of the common chord of the two circles whose equations are

$$(x-a)^2+(y-b)^2=c^2 \quad \text{and} \quad (x-b)^2+(y-a)^2=c^2$$

is $\sqrt{4c^2-2(a-b)^2}$ where $a \neq b$

(proved)

□ Find the length of the common chord of the circles $x^2+y^2-12x+16y-69=0$ and $x^2+y^2-9x+12y-59=0$

① Sol^n:

Given eq^{n}s of the circles are

$$x^2+y^2-2ax-2by+a^2+b^2-c^2=0 \quad \text{.....①}$$ and

$$x^2+y^2-2bx-2ay+b^2+a^2-c^2=0 \quad \text{.....⑪}$$

$\therefore$ The eq^n of the common chord is

$$-2ax-2by+2bx+2ay=0$$

$$\Rightarrow ax+by-bx-ay=0$$

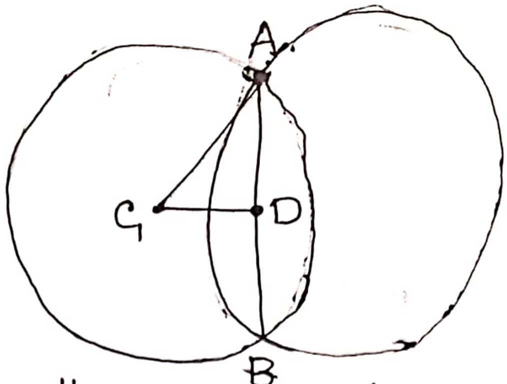
$$\Rightarrow (a+b)x-(a+b)y=0$$

$$\Rightarrow (a+b)(x-y)=0$$

CamScanner

$\therefore x - y = 0 \quad \ldots\ldots \quad (\text{III})$

~~Let~~ The centre of the circle given by eqn ① is $C_1(a, b)$. Also let C_1D be the parpendicular distence from the centre to the common chord AB, where D is the middle point of AB.



$$\therefore C_1D = \frac{a-b}{\sqrt{1+1}} = \frac{a-b}{\sqrt{2}}$$

Now from the right angle triangle AC_1D, we get,

$$AC_1^2 = C_1D^2 + AD^2$$

$$\Rightarrow AD^2 = AC_1^2 - C_1D^2$$

$$= c^2 - \left(\frac{a-b}{\sqrt{2}}\right)^2$$

$$= c^2 - \frac{(a-b)^2}{2}$$

$$\therefore AB = 2AD$$

$$= 2\left[\sqrt{\frac{2c^2 - (a-b)^2}{2}}\right]$$

$$= 2\sqrt{\frac{4c^2 - 2(a-b)^2}{2^2}}$$

$$= \sqrt{4c^2 - 2(a-b)^2}.$$

01-11-23

Orthogonal circles :

Define Orthogonal circles. Find the condition that the two circles $x^2+y^2+2g_1x+2f_1y+c_1=0$ and $x^2+y^2+2g_2x+2f_2y+c_2=0$ cut each other orthogonally.

□ Find the eqn of the circle which cuts the circle $x^2+y^2-3x-4y+5=0$ and $4x^2+4y^2-7x+8y+11=0$ orthogonally and passes through the point $(2,-1)$

Solⁿ:

Given that,

$$x^2+y^2-3x-4y+5=0 \quad \cdots\cdots (i)$$

$$4x^2+4y^2-7x+8y+11=0 \quad \cdots\cdots (ii)$$

Let the eqn of the circle be

$$x^2+y^2+2gx+2fy+c=0 \quad \cdots\cdots (iii)$$

Since the circle (iii) cuts the circles given by eqns (i) and (ii) orthogonally.

Thus,

$$2g\cdot(-3/2)+2\cdot f\cdot(-2)=c+5$$

$$\Rightarrow 3g+4f=-c-5 \quad \cdots\cdots (iv)$$

and $2g\cdot\left(-\frac{7}{8}\right) + 2f\cdot 1 = c + \frac{11}{4}$

$\Rightarrow$ ~~$\frac{7}{4}g - 2f = c + \frac{11}{4}$~~

$\Rightarrow \frac{7}{4}g - 2f = -c - \frac{11}{4}$

$\Rightarrow 7g - 8f = -4c - 11 \quad \ldots\ (v)$

Also the circle (iii) passes through the point $(2, -1)$. Then

$4 + 1 + 8g - 2f + c = 0$

$\Rightarrow 8g - 2f = -c - 5 \quad \ldots\ (vi)$

g, f, c এর value বের করে eqn (iii) তে বসাতে হবে।

** □ Prove that two circles that pass through the point $(0, a)$, $(0, -a)$ and touch the line $y = mx + c$ will cut orthogonally if $c^2 = a^2(m^2 + 2)$

Sol^n:

Let the eq^{n}s of the circles be

$$x^2 + y^2 + 2gx + 2fy + k = 0 \quad \ldots \ldots (1)$$

Since the circle given by eq^n (1) pass through the point $(0, a)$ and $(0, -a)$

Then $$a^2 + 2fa + k = 0 \quad \ldots \ldots (2)$$

$$a^2 - 2fa + k = 0 \quad \ldots \ldots (3)$$

Solving eq^{n}s (2) and (3) we get,

$$k = -a^2, \quad f = 0$$

$\therefore$ Eq^n (1) $\Rightarrow$ $x^2 + y^2 + 2gx - a^2 = 0 \quad \ldots \ldots (4)$

The centre and radious of the circle given by eq^n (4) is $(-g, 0)$ and $\sqrt{g^2 + a^2}$

According to the question,

$$\frac{mg - c}{\sqrt{1 + m^2}} = \sqrt{g^2 + a^2}$$

P.T.O

$$\Rightarrow \frac{(mg-c)^2}{1+m^2} = g^2+a^2$$

$$\Rightarrow m^2g^2 - 2cmg + c^2 = m^2g^2 + g^2 + a^2 + a^2m^2$$

$$\Rightarrow g^2 + 2cmg + (a^2+a^2m^2-c^2) = 0$$

which is a quadratic eq^n in g.

Let the roots of this eq^n be g_1 and g_2

$$g_1+g_2 = -2cm$$

$$g_1g_2 = a^2+a^2m^2-c^2$$

$\therefore$ Eq^n (4) reduces to –

$$x^2+y^2+2g_1x-a^2=0$$

$$x^2+y^2+2g_2x-a^2=0$$

Since these two circles cut each other orthogonally,

Thus

$$2g_1g_2 + 2\cdot0\cdot0 = -a^2-a^2$$

$$\Rightarrow g_1g_2 = -a^2$$

P.T.O

$\Rightarrow a^2 + a^2m^2 - c^2 = -a^2$

$\Rightarrow c^2 = 2a^2 + a^2m^2$

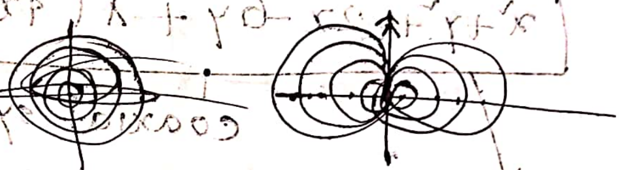
$= a^2(m^2+2)$

□ Radical axis: $2(g_1-g_2)x_1 + 2(f_1-f_2)y_1 + c_1 - c_2 = 0$

→ radical axis এর সমীকরণ

(common chord & radical axis same eqn)

Coaxial circles:



$x^2+y^2+2gx+g^2+c-g^2=0$ — Y axis → radical axis, $x=0$

$\Rightarrow (x+g)^2+y^2=g^2-c$ — $c_1-c_2=0$, $c_1=c_2$

$\Rightarrow (x-(-g))^2+(y-0)^2=g^2-c$ — x axis → coaxial axis, $y=0$

$g^2=c$ — $(-g,0)$

$\Rightarrow g=\pm\sqrt{c}$ — $(-\sqrt{c},0)$, $(\sqrt{c},0)$

Problem:

Define limiting points of a coaxial system.

Find the limiting points of the coaxial systems of circles $x^2+y^2+2x-6y=0$,

$2x^2+2y^2-10y+5=0$

$S_1-S_2=0$

radious $=0$ point circle

$4x-2y-5=0$

$$\boxed{x^2+y^2+2x-6y+\lambda(4x-2y-5)=0}$$

coaxial system

এর centre ও radious বের করতে হবে। এরপর radious $=0$ ধরতে হবে।

$x^2+y^2+(2+4\lambda)x+(-6-2\lambda)y-5\lambda=0$

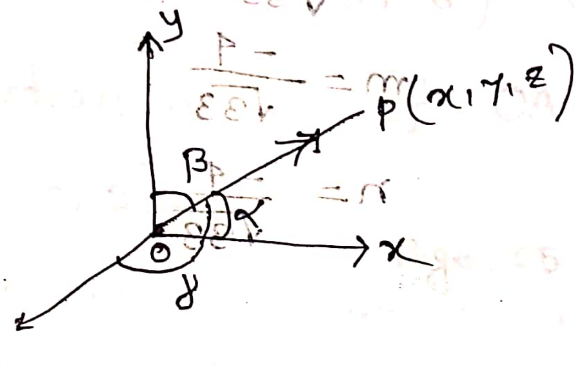
result হবে $\boxed{(1,2),(3,1)}$

22(

Three Dimensional coordinate Geometry

বুধবার CT
10 AM

কোন একটি রেখা x ~~অক্ষের~~, y, এবং z অক্ষের সাথে ধনাত্মক দিকে যে কোন উৎপন্ন করে তার $\cos$ এর মানকে বলা হয় Direction Cosine .



$l = \cos\alpha \longrightarrow 1, 0, 0$

$m = \cos\beta \longrightarrow 0, 1, 0$

$n = \cos\gamma \longrightarrow 0, 0, 1$

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$

→ direction ratio (a, b, c)

→ direction cosine (l, m, n)

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$P \equiv (2, 7, 9)$ PQ রেখার Direction ratio বের কর।

$Q \equiv (1, 3, 5)$ " " cosine " "

$a = -1$ $\qquad l = \frac{-1}{\sqrt{33}}$

$b = -4$

$c = -4$ $\qquad m = \frac{-4}{\sqrt{33}}$

$n = \frac{-4}{\sqrt{33}}$

$\therefore l^2 + m^2 + n^2 \longrightarrow$ এর মান সবসময় 1 আসবে।

$= \frac{1}{33} + \frac{16}{33} + \frac{16}{33}$

$= 1$

* Define: Direction cosine and ratioes of line.

Prove that $l^2 + m^2 + n^2 = 1$.

If A and B are $(2, 3, -6)$, $(3, -4, 5)$.

Find the direction cosine of OA, OB, AB,

where O is $(0, 0, 0)$

Page: 25

dc's = direction cosines.

*** If θ be the angle between the straight lines

আসবে

(7-8) marks

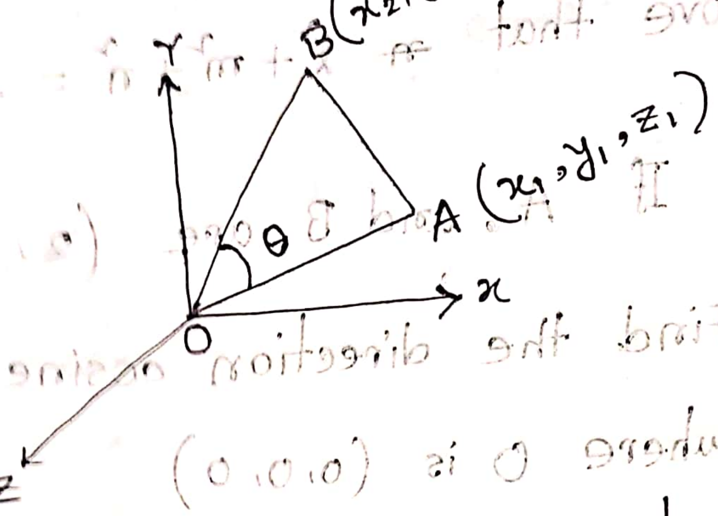
whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2)

Then show that (i) $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(ii) $\sin\theta = \pm\sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2}$

Also find the condition of Perpendicularity and Parallelism.

Solⁿ:



Let OA and OB are two straight lines, whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) respectively.

Also let the coordinates of A and B are (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively.

Then we get,

$$\frac{l_1}{x_1} = \frac{m_1}{y_1} = \frac{n_1}{z_1} \quad \dots \text{(i)}$$

~~OA রেখার ratio x_1 =~~

$$\frac{l_2}{x_2} = \frac{m_2}{y_2} = \frac{n_2}{z_2} \quad \dots \text{(ii)}$$

Eq^n ① $\Rightarrow$

$$\frac{l_1}{x_1} = \frac{m_1}{y_1} = \frac{n_1}{z_1} = \frac{\sqrt{l_1^2+m_1^2+n_1^2}}{\sqrt{x_1^2+y_1^2+z_1^2}} = \frac{1}{OA}$$

$$\Rightarrow x_1 = l_1 \cdot OA, \; y_1 = m_1 \cdot OA, \; z_1 = n_1 \cdot OA \quad \left[\because l_1^2+m_1^2+n_1^2 = 1\right]$$

Similarly, Eq^n ② $\Rightarrow$

$$\frac{l_2}{x_2} = \frac{m_2}{y_2} = \frac{n_2}{z_2} = \frac{\sqrt{l_2^2+m_2^2+n_2^2}}{\sqrt{x_2^2+y_2^2+z_2^2}} = \frac{1}{OB}$$

$$\Rightarrow x_2 = l_2 \cdot OB, \; y_2 = m_2 \cdot OB, \; z_2 = n_2 \cdot OB$$

Now, from the triangle $\triangle OAB$, we obtain,

$$OA^2 + OB^2 - AB^2 = 2 \cdot OA \cdot OB \cdot \cos\theta$$

$$\Rightarrow 2 \cdot OA \cdot OB \cdot \cos\theta = OA^2 + OB^2 - AB^2$$

$$\Rightarrow 2 \cdot OA \cdot OB \cdot \cos\theta = (x_1^2+y_1^2+z_1^2) + (x_2^2+y_2^2+z_2^2) - (x_1-x_2)^2 - (y_1-y_2)^2 - (z_1-z_2)^2$$

$$\Rightarrow 2 \cdot OA \cdot OB \cdot \cos\theta = 2(x_1x_2 + y_1y_2 + z_1z_2)$$

$$\Rightarrow 2 \cdot OA \cdot OB \cdot \cos\theta = 2 \cdot l_1 \cdot OA \cdot l_2 \cdot OB + 2 m_1 \cdot OA \cdot m_2 \cdot OB + 2 \cdot n_1 \cdot OA \cdot n_2 \cdot OB$$

$\Rightarrow \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 \quad \ldots\ldots\ldots (111)$

2nd Part soln:

$\sin^2\theta = 1 - \cos^2\theta$

$= (l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - (l_1 l_2 + m_1 m_2 + n_1 n_2)^2$

$= (m_1 n_2 - n_1 m_2)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2$

[By Lagrange's identity]

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 \quad \ldots\ldots\ldots (4)$

$\therefore \sin\theta = \pm\sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2} \quad \ldots\ldots (5)$

case-1: If the two lines are parpendicular that is

$\theta = 90^\circ$ then eqn (4) $\Rightarrow$

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

case-2: If the two lines are parallel that is

$\theta = 0^\circ$ then eqn (5) $\Rightarrow$

$\sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - m_1 l_2)^2} = 0$

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$$m_1 n_2 - m_2 n_1 = 0$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$n_1 l_2 - n_2 l_1 = 0$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{n_1}{n_2}$$

$$l_1 m_2 - m_1 l_2 = 0$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{m_1}{m_2}$$

$$\boxed{\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}}$$

Ex. Prove that the ~~acquire~~ acute (সূক্ষ্ম কোণ) angle between the straight lines whose direction cosines are given by the relations $l+m+n=0 \quad \ldots (1)$

$l^2+m^2+n^2=0 \quad \ldots (2)$ is $\pi/3$

32 page example-6

~~Eliminating n betwee~~

The Plane (সমতল)

2 dimension এ যেটা straight line 3 dimension এ সেটা plane.

$xy \longrightarrow z=0$

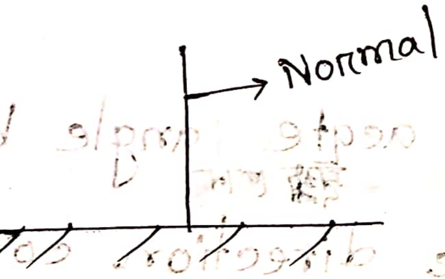
$yz \longrightarrow x=0$

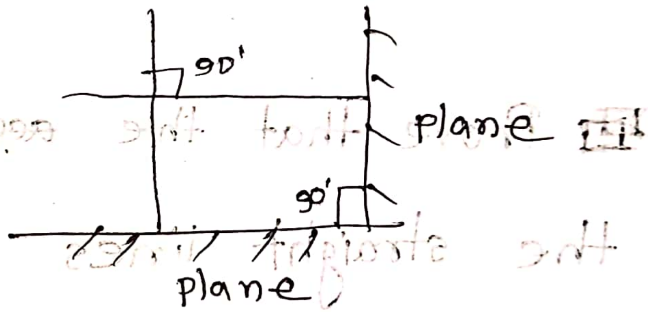
$zx \longrightarrow y=0$

$A_1x + B_1y + C_1z + d_1 = 0$

$A_2x + B_2y + C_2z + d_2 = 0$

[$A_1, B_1, C_1, A_2, B_2, C_2$ direction ratioes





$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$ [cosine জানা থাকলে এটা use করবো]

$$\cos\theta = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\,\sqrt{A_2^2 + B_2^2 + C_2^2}}$$

$$\Rightarrow A_1A_2 + B_1B_2 + C_1C_2 = 0$$

['

□ ~~stra~~ Page → 13 (ex → 1 (e))

" → 14 (3(d))

" → 19 (14)

" → 74 (6(c))

" → 76 (8(b))

circle পুরাটা

general eqn of 2nd degree → CT.

১ম - circle পর্যন্ত → CT.

Assignment : Change of axis [Topic Name]

2 টা Theorem

6 টা problem → 2 টা exm এ আসবে।

Submission date : 18. ~~[illegible]~~