

# 6.3000: Signal Processing

## Convolution

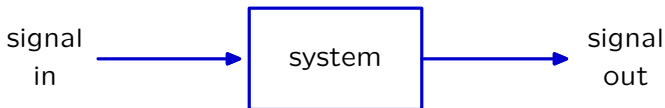
- Unit Sample Signal and Unit Sample Response
- Discrete-Time Convolution
- Impulse Function and Impulse Response
- Continuous-Time Convolution

*October 12, 2023*

## Last Time: The System Abstraction

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Represent a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



## Properties of Systems

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We will focus primarily on systems that have two important properties:

- **linearity**
- **time invariance**

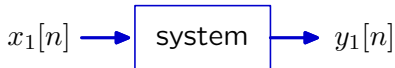
Such systems are both prevalent and mathematically tractable.

## Additivity

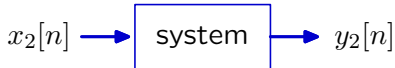
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A system is additive if its **response to a sum** of signals is equal to the **sum of the responses** to each signal taken one at a time.

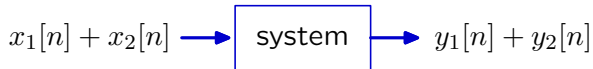
Given



and



the **system is additive** if



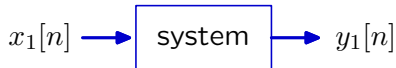
for all possible inputs and all times  $n$ .

## Homogeneity

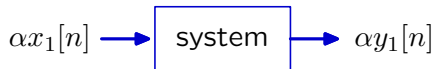
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A system is homogeneous if multiplying its input signal by a constant multiplies the output signal by the same constant.

Given



the **system is homogeneous** if



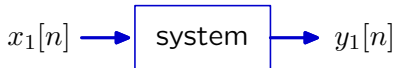
for all  $\alpha$  and all possible inputs and all times  $n$ .

## Linearity

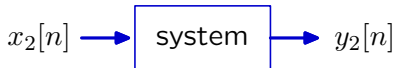
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A system is linear if its **response to a weighted sum** of input signals is equal to the **weighted sum of its responses** to each of the input signals.

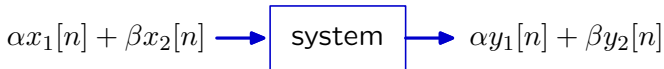
Given



and



the **system is linear** if



for all  $\alpha$  and  $\beta$  and all possible inputs and all times  $n$ .

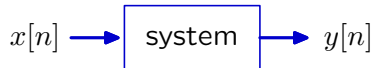
A system is linear if it is both additive and homogeneous.

## Time-Invariance

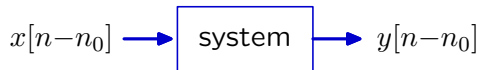
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A system is time-invariant if delaying the input signal simply delays the output signal by the same amount of time.

Given



the **system is time invariant** if



for all  $n_0$  and for all possible inputs and all times  $n$ .

# Linear Difference Equations with Constant Coefficients

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If a discrete-time system can be described by a linear difference equation with constant coefficients, then the system is linear and time-invariant.

**General form:**

$$\sum_l c_l y[n-l] = \sum_m d_m x[n-m]$$

Such systems are easily shown to be linear and time-invariant.

**Additivity:** output of sum is sum of outputs

$$\sum_l c_l (y_1[n-l] + y_2[n-l]) = \sum_m d_m (x_1[n-m] + x_2[n-m])$$

**Homogeneity:** scaling an input scales its output

$$\sum_l c_l (\alpha y[n-l]) = \sum_m d_m (\alpha x[n-m])$$

**Time invariance:** delaying an input delays its output

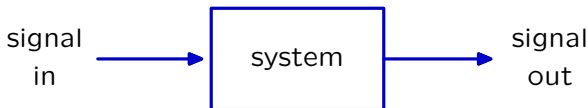
$$\sum_l c_l y[(n-n_0)-l] = \sum_m d_m x[(n-n_0)-m]$$



## Today: Representing a System by its Unit-Sample Response

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Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.



This abstraction is particularly powerful for **linear and time-invariant** systems, which are both **prevalent** and **mathematically tractable**.

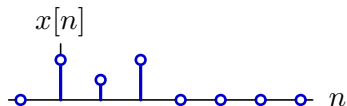
Three important representations for LTI systems:

- **Difference Equation:** algebraic **constraint** on samples ✓
- **Convolution:** represent a system by its **unit-sample response**
- **Filter:** represent a system by its **frequency response**

## Superposition

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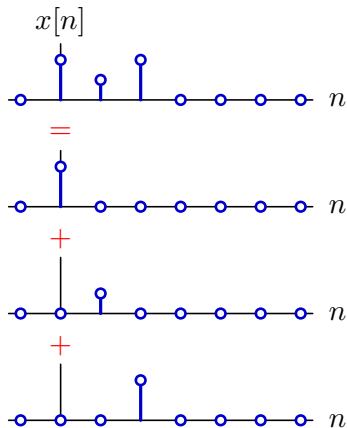
Break the input signal into additive parts and sum responses to the parts.



## Superposition

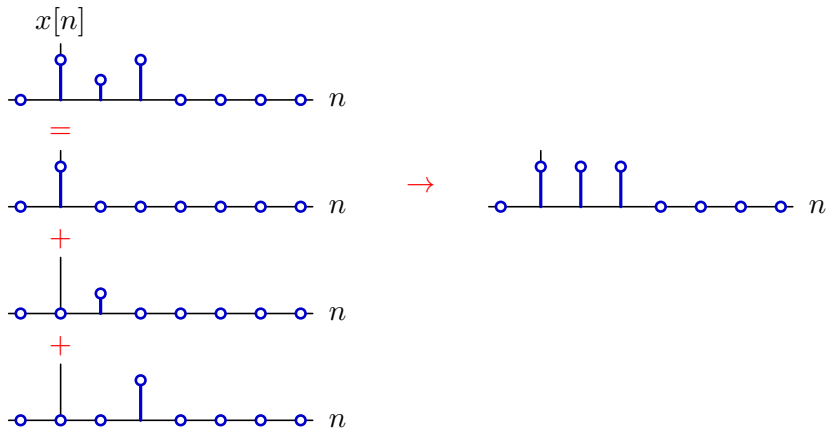
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Break the input signal into additive parts.



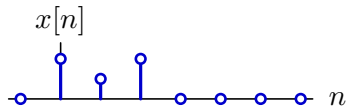
# Superposition

Find the response to each part.

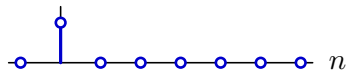


# Superposition

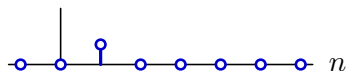
Find the response to each part.



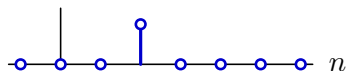
=



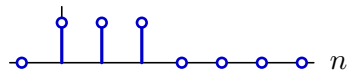
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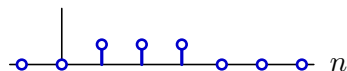
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→

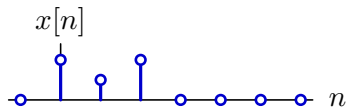


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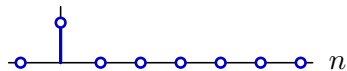


# Superposition

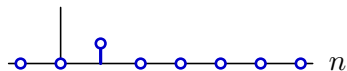
Find the response to each part.



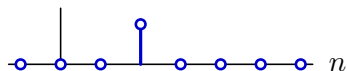
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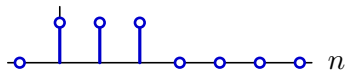
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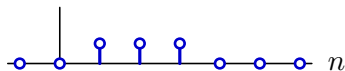
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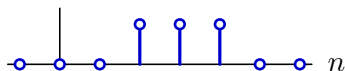
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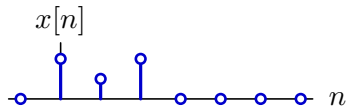


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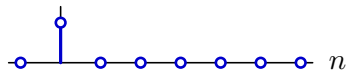


# Superposition

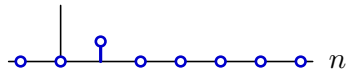
Add the responses to the parts.



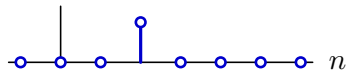
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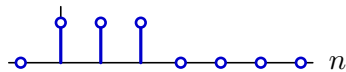
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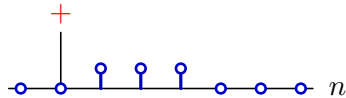
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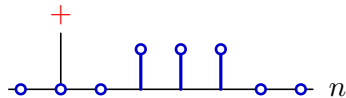
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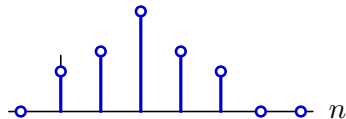
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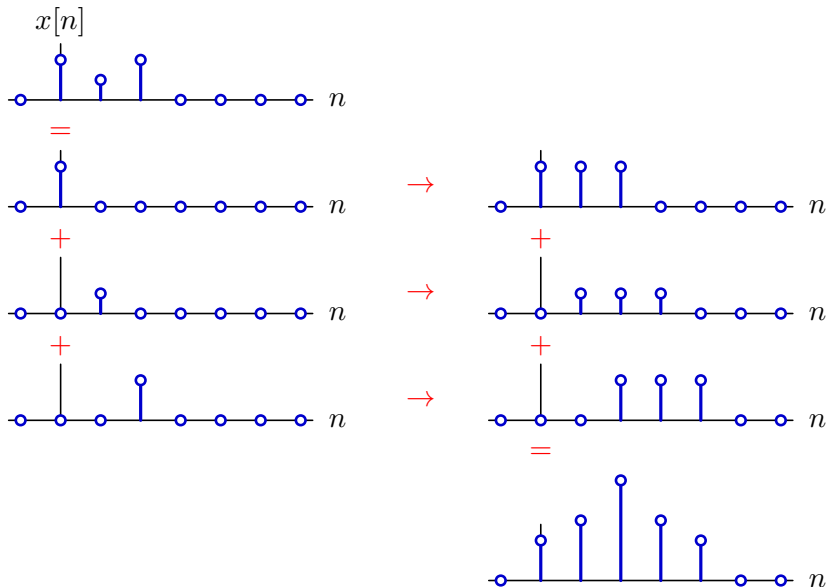


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# Superposition

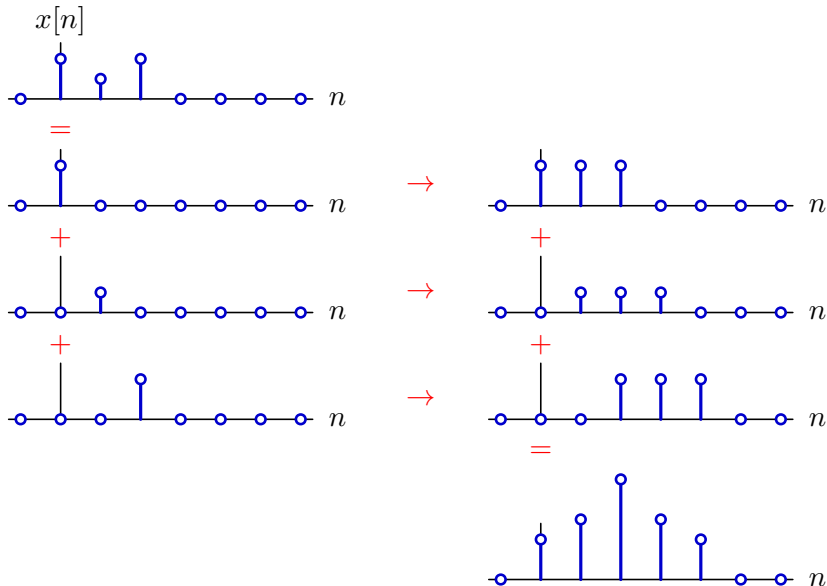
Superposition only works if the system is **additive**.





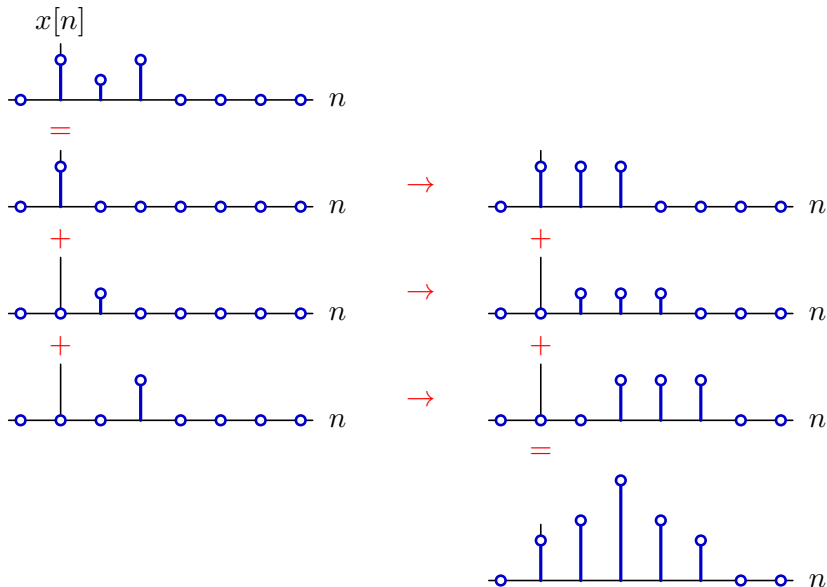
# Superposition

Superposition is easy if system is also **homogeneous** and **time-invariant**.



# Superposition

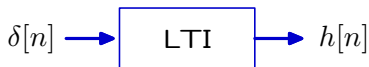
Response to **any** input is determined by the system's response to  $\delta[n]$ .



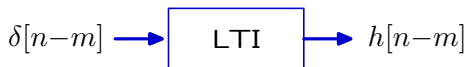
## Unit-Sample Response and Convolution

If a system is linear and time-invariant (LTI), its input-output relation is **completely specified** by the system's unit-sample response  $h[n]$ .

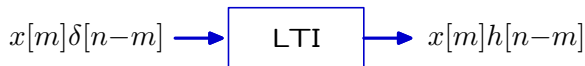
1. One can always find the unit-sample response of a system.



2. Time invariance implies that shifting the input simply shifts the output.



3. Homogeneity implies that scaling the input simply scales the output.



4. Additivity implies that the response to a sum is the sum of responses.

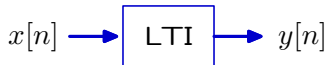
$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m] \rightarrow \text{LTI} \rightarrow y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

This rule for combining the input  $x[n]$  with the unit-sample response  $h[n]$  is called **convolution**.

## Convolution

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The response of an LTI system to an arbitrary input  $x[n]$  can be found by **convolving** that input with the unit-sample response  $h[n]$  of the system.



$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] \equiv (x * h)[n]$$

## Notation

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Convolution is represented with an asterisk.

$$\sum_{m=-\infty}^{\infty} x[m]h[n-m] \equiv (x * h)[n]$$

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Convolution is represented with an asterisk.

$$\sum_{m=-\infty}^{\infty} x[m]h[n-m] \equiv (x * h)[n]$$

It is customary (but confusing) to abbreviate this notation:

$$(x * h)[n] = x[n] * h[n]$$

$x[n] * h[n]$  looks like an operation of samples; but it is not!

$$x[1] * h[1] \neq (x * h)[1]$$

Convolution operates on signals not samples.

Unambiguous notation:

$$y = x * h$$

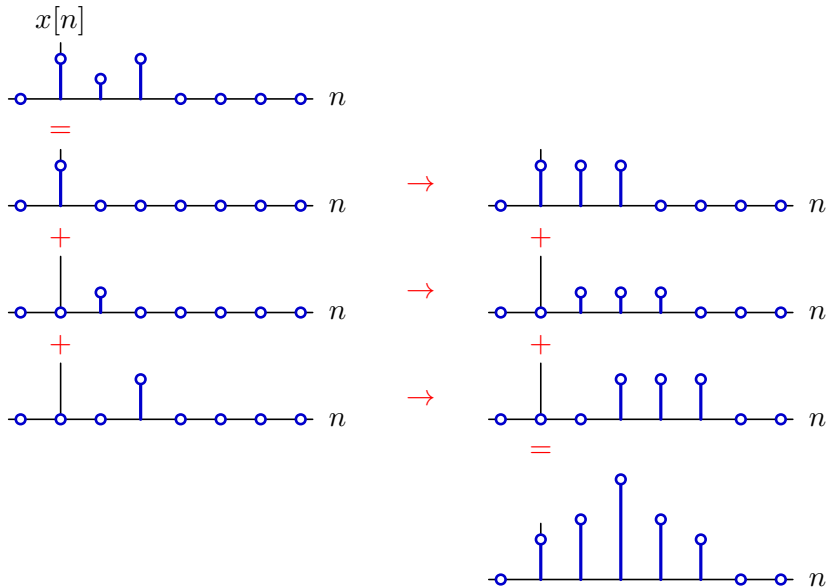
$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] \equiv (x * h)[n]$$

The symbols  $x$  and  $h$  represent DT signals.

Convolving  $x$  with  $h$  generates a new DT signal  $y = x * h$ .

# Structure of Convolution

Convolution as an application of superposition.

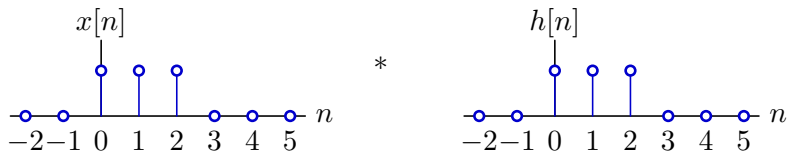


## Structure of Convolution

---

Focus on computing the  $n^{\text{th}}$  output sample.

$$y[\textcolor{red}{n}] = \sum_{m=-\infty}^{\infty} x[m]h[\textcolor{red}{n} - m]$$



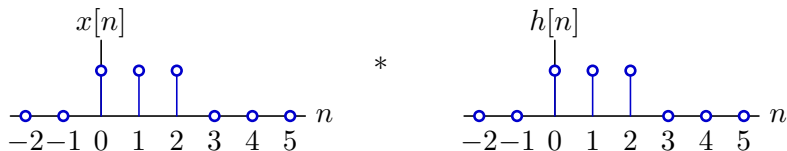


## Structure of Convolution

---

Focus on computing the  $n^{\text{th}}$  output sample: start with  $n = 0$ .

$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[0 - m]$$

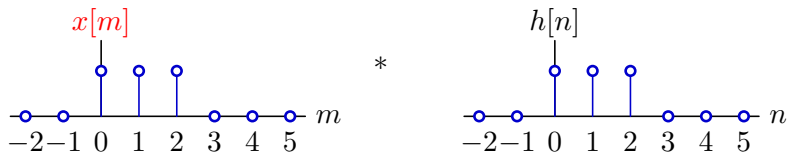


## Structure of Convolution

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The sum is over  $x[m]$  (not  $x[n]$ ).

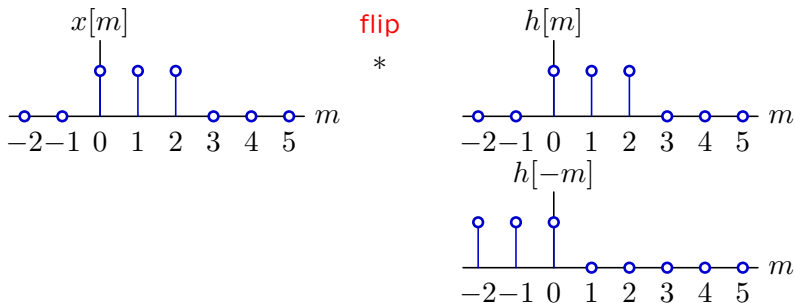
$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[0-m]$$



## Structure of Convolution

The sum is over  $x[m]$  and  $h[-m]$  ( $h[m]$  is **flipped**).

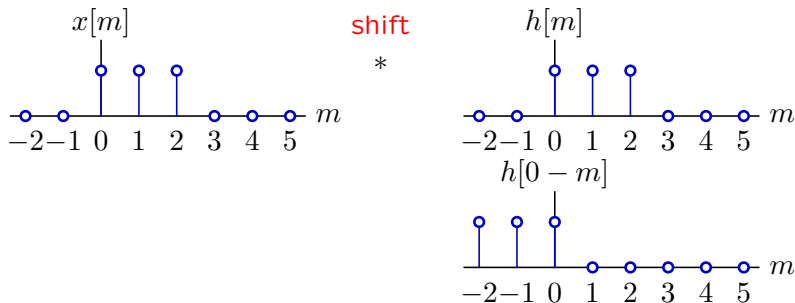
$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[0 - m]$$



## Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

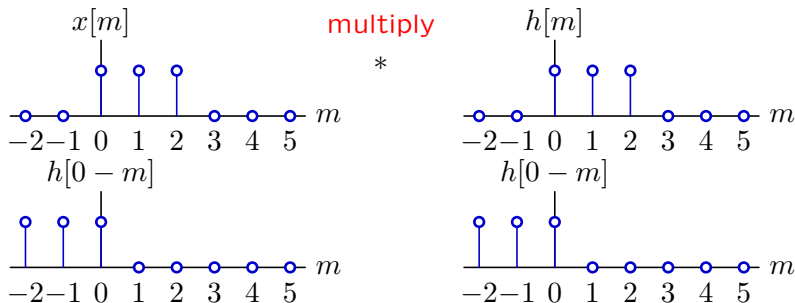
$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[0-m]$$



## Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

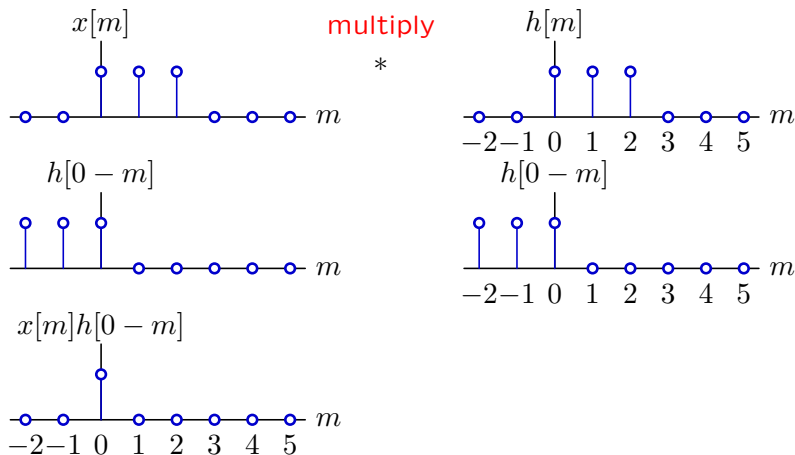
$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[0-m]$$



## Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

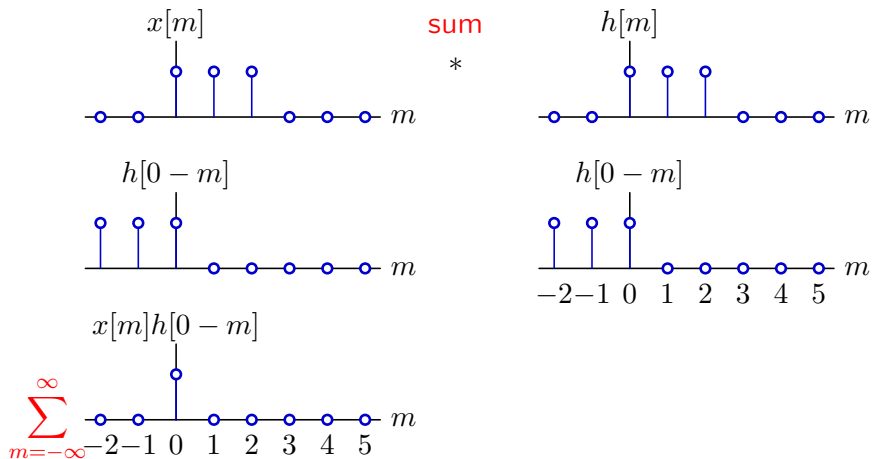
$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[0-m]$$



# Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

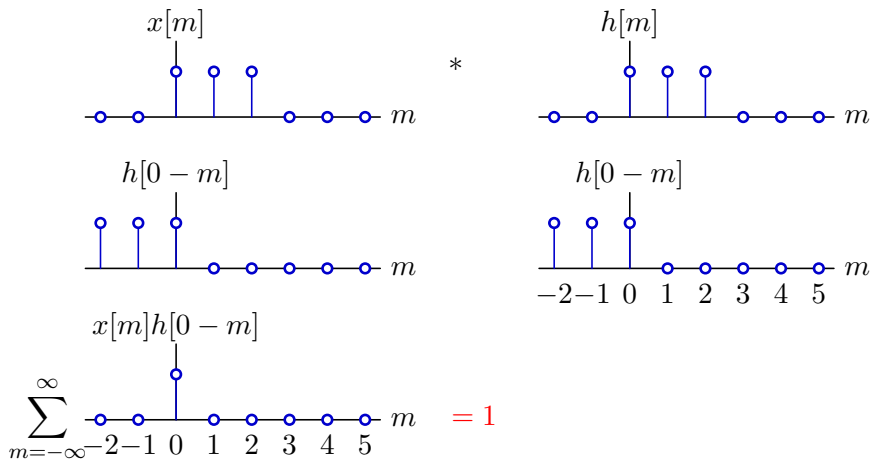
$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[0-m]$$



## Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

$$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[0-m]$$

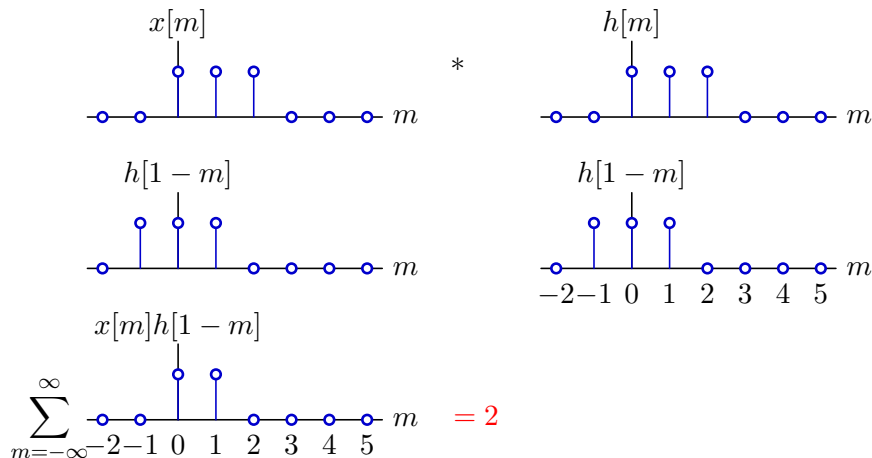




## Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

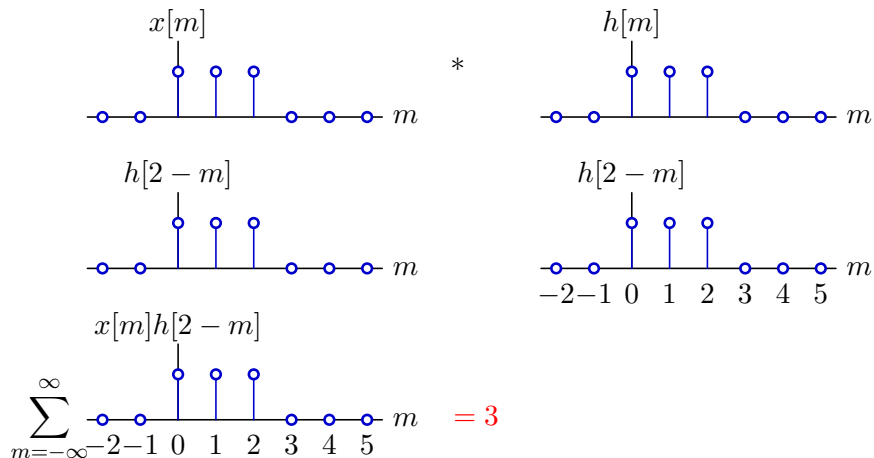
$$y[1] = \sum_{m=-\infty}^{\infty} x[m]h[1-m]$$



# Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

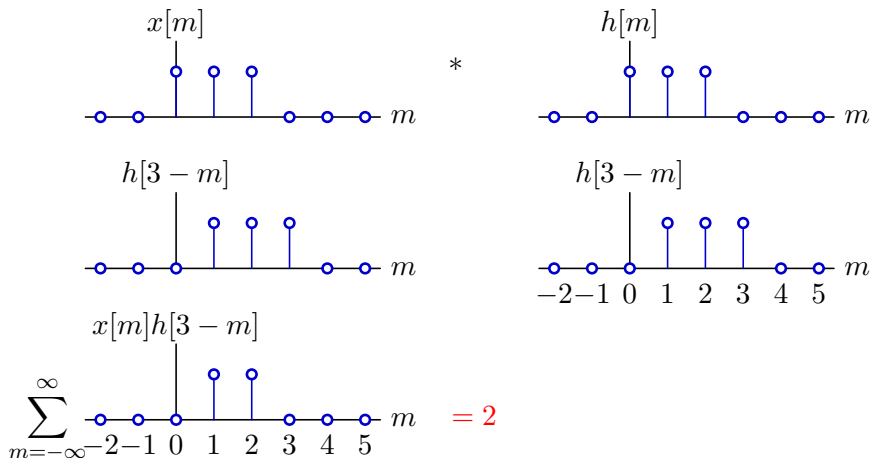
$$y[2] = \sum_{m=-\infty}^{\infty} x[m]h[2-m]$$



# Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

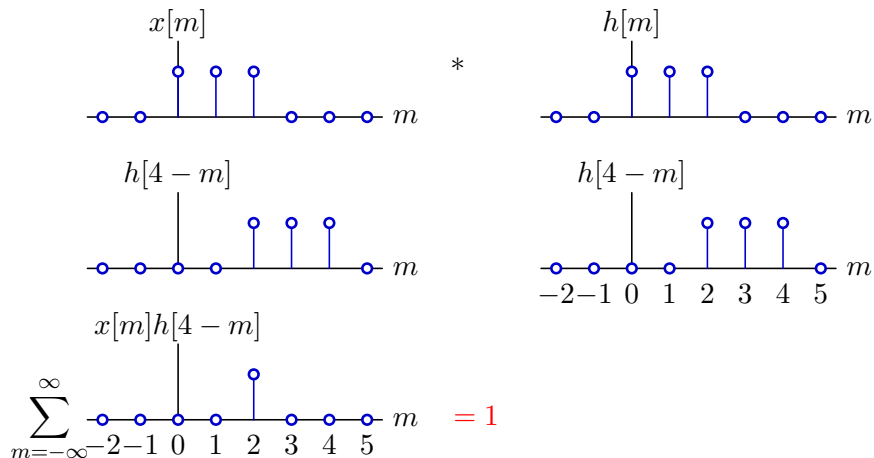
$$y[3] = \sum_{m=-\infty}^{\infty} x[m]h[3-m]$$



# Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

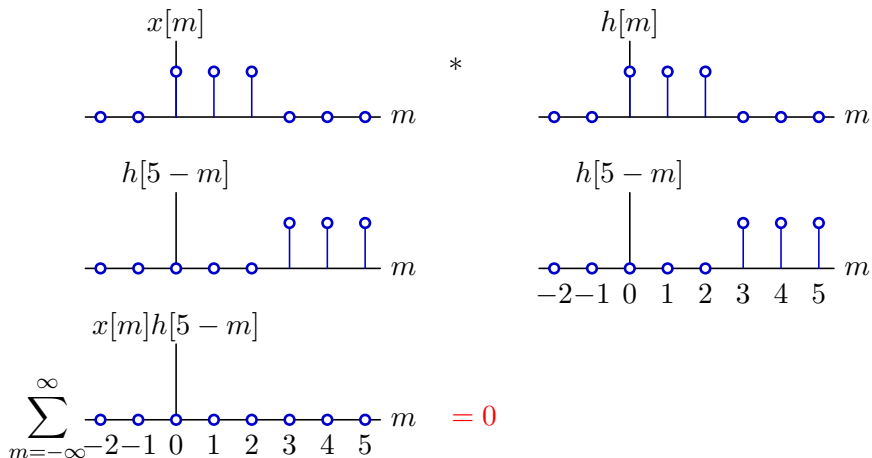
$$y[4] = \sum_{m=-\infty}^{\infty} x[m]h[4-m]$$



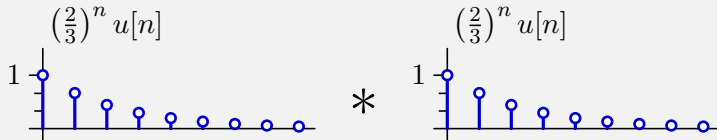
## Structure of Convolution

Focus on computing the  $n^{\text{th}}$  output sample.

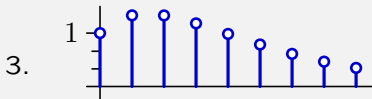
$$y[5] = \sum_{m=-\infty}^{\infty} x[m]h[5-m]$$



## Check Yourself



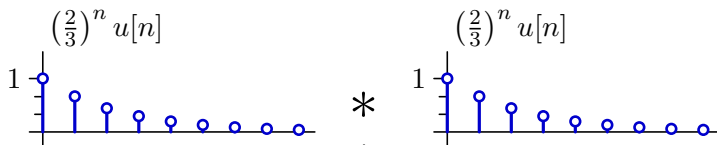
Which plot shows the result of the convolution above?



5. none of the above

## Check Yourself

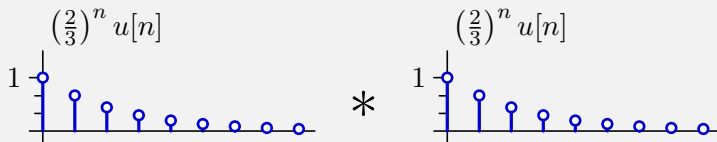
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Express mathematically:

$$\begin{aligned} \left( \left( \frac{2}{3} \right)^n u[n] \right) * \left( \left( \frac{2}{3} \right)^n u[n] \right) &= \sum_{m=-\infty}^{\infty} \left( \left( \frac{2}{3} \right)^m u[m] \right) \times \left( \left( \frac{2}{3} \right)^{n-m} u[n-m] \right) \\ &= \sum_{m=0}^n \left( \frac{2}{3} \right)^m \times \left( \frac{2}{3} \right)^{n-m} \\ &= \sum_{m=0}^n \left( \frac{2}{3} \right)^n = \left( \frac{2}{3} \right)^n \sum_{m=0}^n 1 \\ &= (n+1) \left( \frac{2}{3} \right)^n u[n] \\ &= 1, \frac{4}{3}, \frac{4}{3}, \frac{32}{27}, \frac{80}{81}, \dots \end{aligned}$$

## Check Yourself



Which plot shows the result of the convolution above? **3**

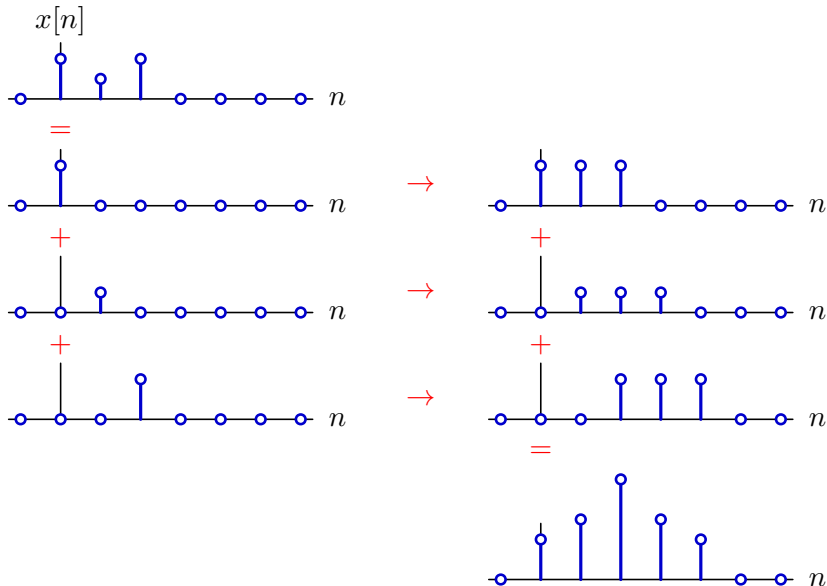


5. none of the above



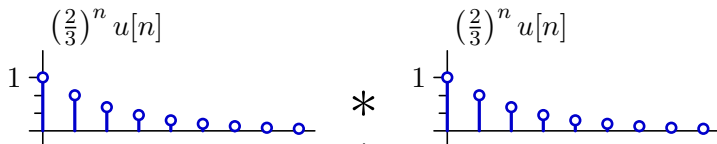
# Superposition

Superposition is often an easy way to implement convolution.



## Check Yourself

---

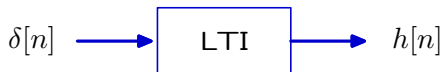


$n:$	0	1	2	3	4	5	
$x[0] \times h[n]:$	1	$\frac{2}{3}$	$\frac{4}{9}$	$\frac{8}{27}$	$\frac{16}{81}$	$\frac{32}{243}$	$\dots$
$x[1] \times h[n-1]:$		$\frac{2}{3}$	$\frac{4}{9}$	$\frac{8}{27}$	$\frac{16}{81}$	$\frac{32}{243}$	$\dots$
$x[2] \times h[n-2]:$			$\frac{4}{9}$	$\frac{8}{27}$	$\frac{16}{81}$	$\frac{32}{243}$	$\dots$
$x[3] \times h[n-3]:$				$\frac{8}{27}$	$\frac{16}{81}$	$\frac{32}{243}$	$\dots$
$x[4] \times h[n-4]:$					$\frac{16}{81}$	$\frac{32}{243}$	$\dots$
$x[5] \times h[n-5]:$						$\frac{32}{243}$	$\dots$
<hr/>							
$\sum x[m]h[n-m]:$	1	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{32}{27}$	$\frac{80}{81}$	$\frac{192}{243}$	$\dots$

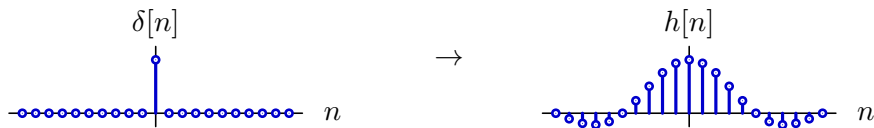
## Unit-Sample Response

---

The unit-sample response is a **complete** description of a system.



It can be used to determine the response to **any** other input.



Given  $h[n]$  one can compute the response to any arbitrary input signal.

$$y[n] = (x * h)[n] \equiv \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

## Continuous-Time Systems

---

Superposition and convolution are of equal importance for CT systems.

# Linear Differential Equations with Constant Coefficients

---

If a continuous-time system can be described by a linear differential equation with constant coefficients, then the system is linear and time-invariant.

**General form:**

$$\sum_l c_l \frac{d^l y(t)}{dt^l} = \sum_m d_m \frac{d^m x(t)}{dt^m}$$

Such systems are easily shown to be linear and time-invariant.

**Additivity:** output of sum is sum of outputs

$$\sum_l c_l \left( \frac{d^l y_1(t)}{dt^l} + \frac{d^l y_2(t)}{dt^l} \right) = \sum_m d_m \left( \frac{d^m x_1(t)}{dt^m} + \frac{d^m x_2(t)}{dt^m} \right)$$

**Homogeneity:** scaling an input scales its output

$$\sum_l c_l \left( \alpha \frac{d^l y(t)}{dt^l} \right) = \sum_m d_m \left( \alpha \frac{d^m x(t)}{dt^m} \right)$$

**Time invariance:** delaying an input delays its output

$$\sum_l c_l \frac{d^l y(t - \tau)}{dt^l} = \sum_m d_m \frac{d^m x(t - \tau)}{dt^m}$$

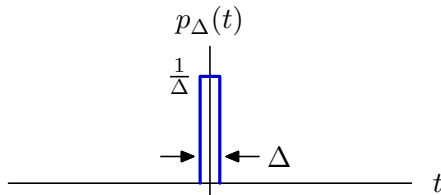
## Impulse Response

A CT system is completely characterized by its **impulse response**, much as a DT system is completely characterized by its unit-sample response.

We have worked with the impulse (Dirac delta) function  $\delta(t)$  previously.

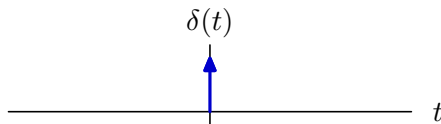
It's defined in a limit as follows.

Let  $p_{\Delta}(t)$  represent a pulse of width  $\Delta$  and height  $\frac{1}{\Delta}$  so that its area is 1.



Then

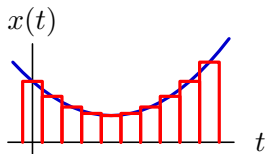
$$\delta(t) = \lim_{\Delta \rightarrow 0} p_{\Delta}(t)$$



## Impulse Response

An arbitrary CT signal can be represented by an infinite sum of infinitesimal impulses (which define an integral).

Approximate an arbitrary signal  $x(t)$  (blue) as a sum of pulses  $p_\Delta(t)$  (red).



$$x_\Delta(t) = \sum_{m=-\infty}^{\infty} x(m\Delta) p_\Delta(t - m\Delta) \Delta$$

and the limit of  $x_\Delta(t)$  as  $\Delta \rightarrow 0$  will approximate  $x(t)$ .

$$\lim_{\Delta \rightarrow 0} x_\Delta(t) = \lim_{\Delta \rightarrow 0} \sum_{m=-\infty}^{\infty} x(m\Delta) p_\Delta(t - m\Delta) \Delta \rightarrow \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$

The result in CT is much like the result for DT:

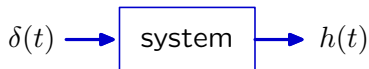
$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau \qquad x[n] = \sum_{m=-\infty}^{\infty} x[m] \delta(n - m)$$

## Impulse Response

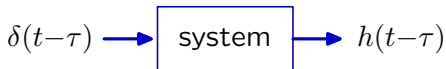
---

If a system is linear and time-invariant (LTI), its input-output relation is **completely specified** by the system's impulse response  $h(t)$ .

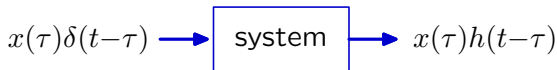
1. One can always find the impulse response of a system.



2. Time invariance implies that shifting the input simply shifts the output.



3. Homogeneity implies that scaling the input simply scales the output.



4. Additivity implies that the response to a sum is the sum of responses.

A block diagram representing a system. On the left, the input is the integral  $x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$ . A blue arrow points from this input to a rectangular box labeled "system". Another blue arrow points from the "system" box to the output  $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$  on the right.

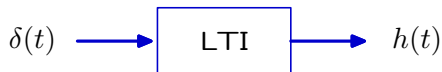
This rule for combining the input  $x(t)$  with the impulse response  $h(t)$  is called **convolution**.



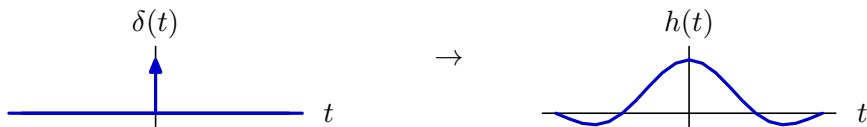
## Impulse Response

---

The impulse response is a **complete** description of a system.



It can be used to determine the response to **any** other input.



Given  $h(t)$  one can compute the response to any arbitrary input signal.

$$y(t) = (x * h)(t) \equiv \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$$

## Comparison of CT and DT Convolution

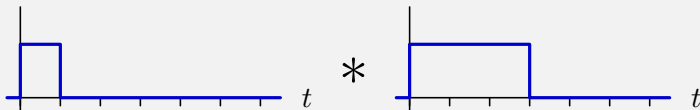
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Convolution of CT signals is analogous to convolution of DT signals.

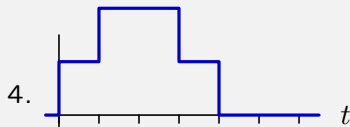
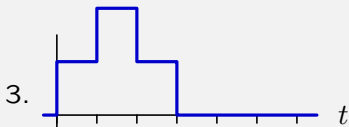
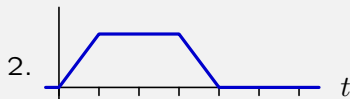
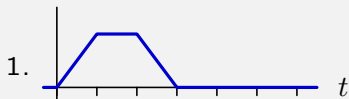
$$\text{DT: } y[n] = (x * h)[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

$$\text{CT: } y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

## Check Yourself



Which plot shows the result of the convolution above?

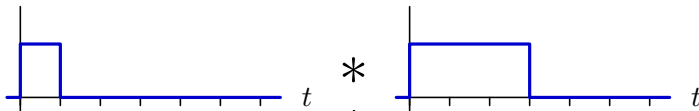


5. none of the above

## Check Yourself

---

Which plot shows the result of the following convolution?



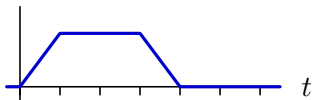
Start by flipping the first function about  $t = 0$ .

Multiply by the second function. No overlap  $\rightarrow 0$ .

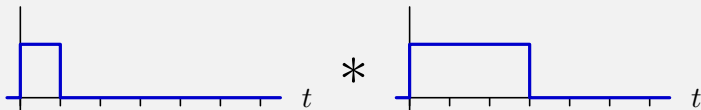
As the flipped function slides right, the overlap increases linearly for first unit.

Then there is constant overlap for 2 units.

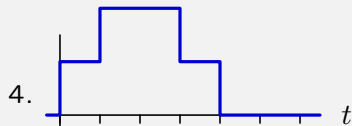
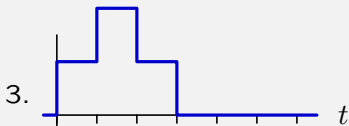
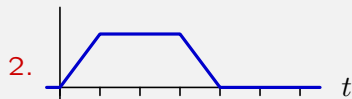
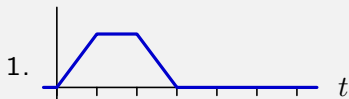
Then the overlap linearly decreases for the next unit till it is back to zero.



## Check Yourself

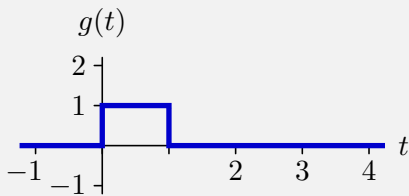
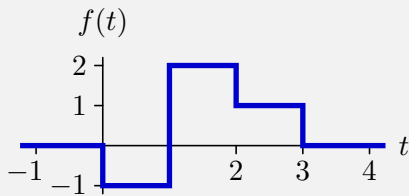


Which plot shows the result of the convolution above? **2**



5. none of the above

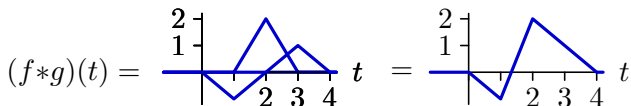
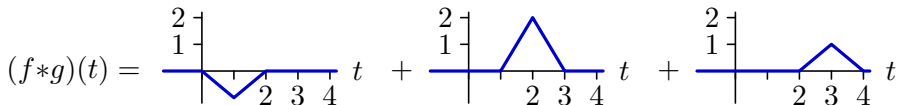
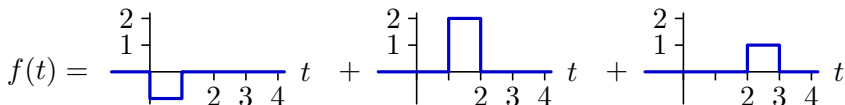
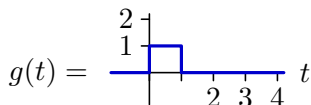
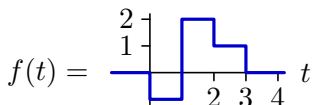
## Check Yourself



For what value of  $t$  is  $(f*g)(t)$  greatest?

What is the maximum value of  $(f*g)(t)$ ?

## Check Yourself



The peak value is 2 and it occurs at  $t = 2$ .

# Properties of Convolution

---

## Commutativity:

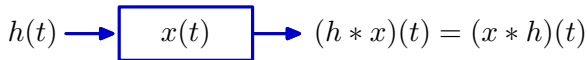
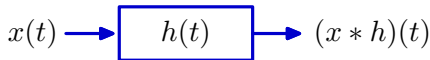
$$(x * y)(t) = (y * x)(t)$$

---

$$(x * y)(t) \equiv \int_{-\infty}^{\infty} x(\tau)y(t-\tau) d\tau$$

let  $\lambda = t - \tau$

$$\begin{aligned}(x * y)(t) &= \int_{\infty}^{-\infty} x(t-\lambda)y(\lambda)(-d\lambda) \\ &= \int_{-\infty}^{\infty} y(\lambda)x(t-\lambda) d\lambda \\ &= (y * x)(t)\end{aligned}$$





# Properties of Convolution

---

## Associativity.

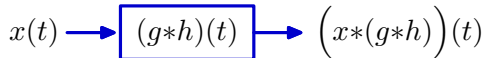
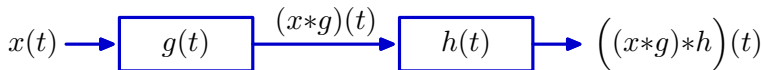
$$\left( (x * y) * z \right)(t) = \left( x * (y * z) \right)(t)$$

---

$$\left( (x * y) * z \right)(t) \equiv \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} x(\tau) y(\lambda - \tau) d\tau \right) z(t - \lambda) d\lambda$$

replace  $\lambda$  with  $\lambda + \tau$ :

$$\begin{aligned} \left( (x * y) * z \right)(t) &= \int_{-\infty}^{\infty} x(\tau) \left( \int_{-\infty}^{\infty} y(\lambda) z(t - \lambda - \tau) d\lambda \right) d\tau \\ &= \left( x * (y * z) \right)(t) \end{aligned}$$



# Properties of Convolution

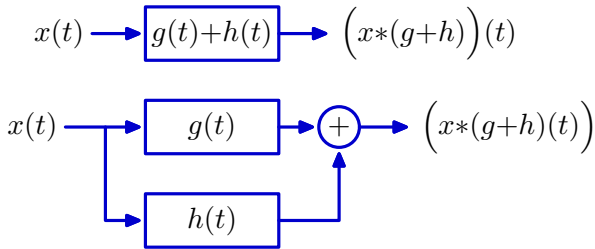
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## Distributivity over addition.

$$\left(x * (g+h)\right)(t) = (x*g)(t) + (x*h)(t)$$

---

$$\begin{aligned}\left(x * (g+h)\right) &= \int_{-\infty}^{\infty} x(\tau) \left(g(t-\tau) + h(t-\tau)\right) d\tau \\ &= \int_{-\infty}^{\infty} x(\tau) g(t-\tau) d\tau + \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau \\ &= (x*g)(t) + (x*h)(t)\end{aligned}$$



## Check Yourself

Match expressions on the left with functions on the right where

$$f(t) = e^{-t} u(t)$$

$$g(t) = e^t u(-t)$$

$$(f * f)(t)$$

☐

$$(g * g)(t)$$

☐

$$(f * g)(t)$$

☐

$$(g * f)(t)$$

☐

A



B



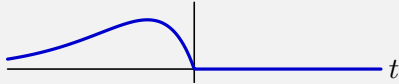
C



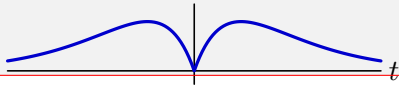
D



E



F



## CT Convolution

---

Let

$$f(t) = e^{-t} u(t)$$

$$g(t) = e^t u(-t)$$

Find  $(f * f)(t)$ ,  $(g * g)(t)$ ,  $(f * g)(t)$ , and  $(g * f)(t)$ .

$$(f * f)(t) = \int_{-\infty}^{\infty} f(\tau) f(t - \tau) d\tau = \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{-(t-\tau)} u(t - \tau) d\tau$$

The integrand is zero unless  $\tau > 0$  and  $t - \tau > 0$ .

Therefore  $0 < \tau < t$  and  $t > 0$ .

$$(f * f)(t) = e^{-t} \int_0^t d\tau u(t) = t e^{-t} u(t)$$

Answer: D

## CT Convolution

---

Let

$$f(t) = e^{-t} u(t)$$

$$g(t) = e^t u(-t)$$

Find  $(f * f)(t)$ ,  $(g * g)(t)$ ,  $(f * g)(t)$ , and  $(g * f)(t)$ .

Since  $g(t) = f(-t)$ ,

$$(g * g)(t) = \int_{-\infty}^{\infty} g(\tau)g(t - \tau) d\tau = \int_{-\infty}^{\infty} f(-\tau)f(-t + \tau) d\tau$$

Now let  $\lambda = -\tau$ :

$$\begin{aligned}(g * g)(t) &= \int_{\infty}^{-\infty} f(\lambda)f(-t - \lambda) (-d\lambda) \\ &= \int_{-\infty}^{\infty} f(\lambda)f(-t - \lambda) d\lambda \\ &= (f * f)(-t)\end{aligned}$$

Answer: E

## CT Convolution

---

Let

$$f(t) = e^{-t} u(t)$$

$$g(t) = e^t u(-t)$$

Find  $(f * f)(t)$ ,  $(g * g)(t)$ ,  $(f * g)(t)$ , and  $(g * f)(t)$ .

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau = \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{t-\tau} u(\tau - t) d\tau$$

The integrand is zero unless  $\tau > 0$  and  $\tau > t$ . Therefore  $\tau > \max(0, t)$ .

$$(f * g)(t) = \begin{cases} e^t \int_0^{\infty} e^{-2\tau} d\tau = \frac{1}{2} e^t & \text{if } t < 0 \\ e^t \int_t^{\infty} e^{-2\tau} d\tau = \frac{1}{2} e^{-t} & \text{if } t > 0 \end{cases}$$

$$= \frac{1}{2} e^{-|t|}$$

$$= (g * f)(t) \quad \text{since convolution is commutative}$$

Answer: C

## Check Yourself

Match expressions on the left with functions on the right where

$$f(t) = e^{-t} u(t)$$

$$g(t) = e^t u(-t)$$

$$(f * f)(t)$$

D

$$(g * g)(t)$$

E

$$(f * g)(t)$$

C

$$(g * f)(t)$$

C

A



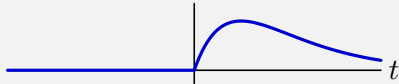
B



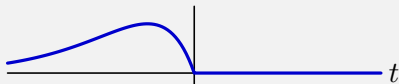
C



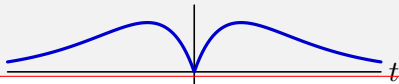
D



E



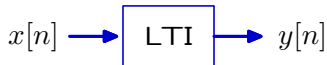
F



## Summary

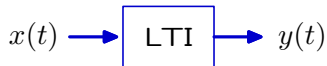
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The response of a discrete-time, LTI system to an input  $x[n]$  can be computed by convolving the input with the system's **unit sample response**.



$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] \equiv (x * h)[n]$$

The response of a continuous-time, LTI system to an input  $x(t)$  can be computed by convolving the input with the system's **impulse response**.



$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \equiv (x * h)(t)$$

Convolution allows us to represent a system by a **single signal!**