

## UCS 2403 Design & Analysis of Algorithms

### Assignment 5

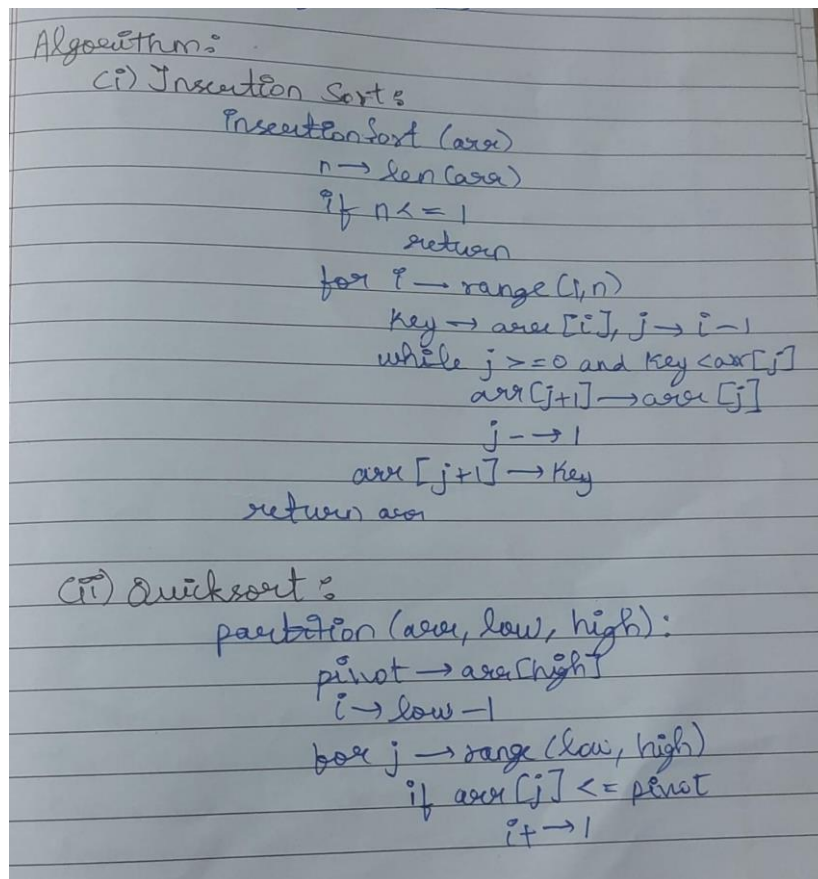
**Date of Exercise:** 21.03.2024

**Aim:** To gain understanding and proficiency on the divide and conquer algorithm

#### **Question 1:**

**Find the kth smallest element:** First, find the kth smallest element in an unsorted list using insertion sort. Next, find the same by modifying the divide-and-conquer algorithm of Quicksort. Compare the time complexities of both the algorithms.

#### **Algorithm:**



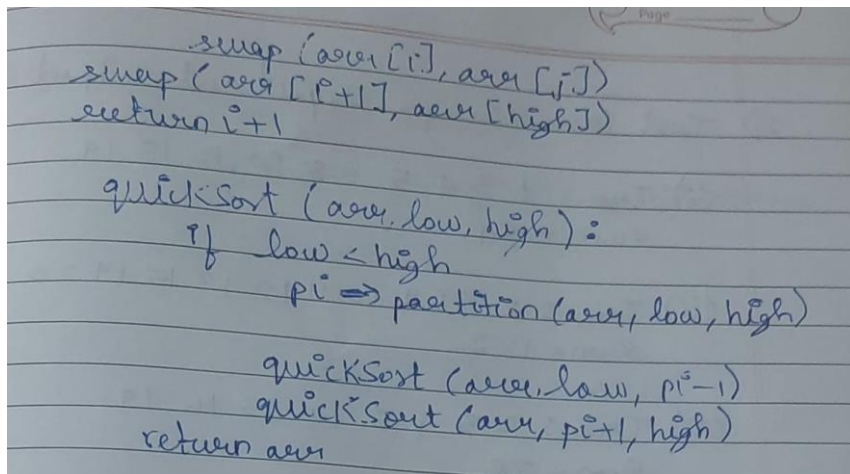
Algorithm:

(i) Insertion Sort:

```
InsertionSort(arr)
n → len(arr)
if n ≤ 1
    return
for i → range(1, n)
    key → arr[i], j → i - 1
    while j ≥ 0 and key < arr[j]
        arr[j+1] → arr[j]
        j → j - 1
    arr[j+1] → key
return arr
```

(ii) Quicksort:

```
partition(arr, low, high):
    pivot → arr[high]
    i → low - 1
    for j → range(low, high)
        if arr[j] ≤ pivot
            i → i + 1
```



Handwritten code for Quick Sort algorithm:

```
swap(arr[i], arr[j])  
swap(arr[p+1], arr[high])  
return i+1  
  
quickSort(arr, low, high):  
    if low < high:  
        pi = partition(arr, low, high)  
        quickSort(arr, low, pi-1)  
        quickSort(arr, pi+1, high)  
    return arr
```

**Code:**

```
#Input list  
size = int(input("Enter the size of the List: "))  
nums = []  
  
for k in range(size):  
    nums.append(int(input("Enter element " + str(k+1) + ": ")))  
print("\nList:", nums)  
  
# Insertion Sort  
def insertionSort(arr):  
    n = len(arr)  
    if n <= 1:  
        return  
    for i in range(1, n):  
        key = arr[i]  
        j = i - 1  
        while j >= 0 and key < arr[j]:  
            arr[j + 1] = arr[j]  
            j -= 1  
        arr[j + 1] = key  
    return arr  
  
print("Sorted List using insertion sort: ",insertionSort(nums))  
  
kthele1 = int(input("\nEnter the value of k: "))  
print("The " + str(kthele1) + " smallest element using Insertion Sort algorithm is:  
", nums[kthele1-1])  
  
#Quick Sort  
def partition(arr, low, high):  
    pivot = arr[high]  
    i = low - 1
```

```
    for j in range(low, high):
        if arr[j] <= pivot:
            i += 1
            arr[i], arr[j] = arr[j], arr[i]

    arr[i + 1], arr[high] = arr[high], arr[i + 1]
    return i + 1

def quickSort(arr, low, high):
    if low < high:
        pi = partition(arr, low, high)

        quickSort(arr, low, pi - 1)
        quickSort(arr, pi + 1, high)

    return arr

sorted_nums_quickSort = quickSort(nums, 0, len(nums) - 1)
print("Sorted List using Quick sort:", sorted_nums_quickSort)

kthele2 = int(input("Enter the value of k: "))
print("The " + str(kthele2) + " smallest element using Quick Sort algorithm is:",
sorted_nums_quickSort[kthele2 - 1])
```

### Output:

```
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local
/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIG
N AND ANALYSIS OF ALGORITHMS/LAB/5.1.py"
Enter the size of the List: 6
Enter element 1: 23
Enter element 2: 30
Enter element 3: 12
Enter element 4: 11
Enter element 5: 50
Enter element 6: 45

List: [23, 30, 12, 11, 50, 45]
Sorted List using insertion sort: [11, 12, 23, 30, 45, 50]

Enter the value of k: 4
The 4 smallest element using Insertion Sort algorithm is: 30
Sorted List using Quick sort: [11, 12, 23, 30, 45, 50]
Enter the value of k: 2
The 2 smallest element using Quick Sort algorithm is: 12
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local
/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIG
N AND ANALYSIS OF ALGORITHMS/LAB/5.1.py"
Enter the size of the List: 4
Enter element 1: 123
```

```
Enter element 2: 43
Enter element 3: 254
Enter element 4: 11

List: [123, 43, 254, 11]
Sorted List using insertion sort: [11, 43, 123, 254]

Enter the value of k: 2
The 2 smallest element using Insertion Sort algorithm is: 43
Sorted List using Quick sort: [11, 43, 123, 254]
Enter the value of k: 3
The 3 smallest element using Quick Sort algorithm is: 123
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> █
```

### Time Complexity:

- **Time Complexity for Insertion Sort:**  $O(n^2)$
- **Time Complexity for Quick Sort:**  $O(n \log n)$

### Question 2:

Find the sum of the values in the nodes of a binary tree: Consider the code given below that has to find the sum of the values in the nodes of a binary tree.

# Code to populate a tree starts here

```
import random
```

```
class TreeNode:
```

```
    def __init__(self):
        self.data = 0
        self.left = None
        self.right = None
```

```
    def insert(self, data):
        if data < self.data:
            if self.left == None:
                tempNode = TreeNode()
                self.left = tempNode
                self.left.data = data
            else:
                self.left.insert(data)
        elif data > self.data:
            if self.right == None:
                tempNode = TreeNode()
                self.right = tempNode
                self.right.data = data
            else:
                self.right.insert(data)
```

```
    def traverseInOrder(self):
        if self.left != None:
            self.left.traverseInOrder()
        print(self.data, end=' ')
```

```
        if self.right != None:
            self.right.traverseInOrder()

def createRoot():
    i = random.randint(0, 10)
    rootNode = TreeNode()
    rootNode.data = i
    return rootNode

def createTree():
    rootNode = createRoot()
    numNodes = random.randint(1, 10)
    currentNode = rootNode
    j = 0
    L = []
    while (j <= numNodes):
        newVal = random.randint(1,20)
        if newVal not in L:
            currentNode.insert(newVal)
            L.append(newVal)
        j+=1
    rootNode.traverseInOrder()
    return rootNode
# Code to populate the tree ends here

def getSum(node):
    if node == None:
        return 0
    else:
        leftSum = getSum(node.left)
        rightSum = getSum(node.right)
        return leftSum + rightSum
rootNode = createTree()
print("Sum = ",getSum(rootNode))
```

This code is known to have some bugs. Modify the given program to correctly find the sum. Ensure that the number of nodes in the tree and the value in each node are generated randomly.

**Algorithm:**

```
Algorithm:
class Treenode:
    data → 0, left → None, right → None

Treenode():
    self.data → 0
    self.left → None
    self.right → None

insert(data):
    if data < self.data:
        if self.left == None:
            tempNode → Treenode()
            tempNode.data → data
            self.left → tempNode
        else:
            self.left.insert(data)
    elif data > self.data:
        if self.right == None:
            tempNode → Treenode()
            tempNode.data → data
            self.right → tempNode
        else:
            self.right.insert(data)

traverseInOrder():
    if self is not None:
        if self.left is not None:
            self.left.traverseInOrder()
        print(self.data, end = ' ')
```

```
if self.right is not None:
    self.right.traverseInOrder()

createRoot():
    i → random.randint(0,10)
    rootNode → Tree Node()
    rootNode.data → i
    return rootNode

createTree():
    rootNode → createRoot()
    numNodes → random.randint(1,10)
    currentNode → rootNode
    j → 0, L → []

    while j < numNodes:
        newVal → random.randint(1,20)
        if newVal not in L:
            currentNode.insert(newVal)
            L.append(newVal)
            j += 1

    rootNode.traverseInOrder()
    return rootNode

getSum(node):
    if node is None:
        return 0
    else:
        leftSum → getSum(node.left)
        rightSum → getSum(node.right)
        return node.data → leftSum
                                + rightSum

rootNode → createTree()
print(getSum(rootNode))
```

**Code:**

```
import random
class TreeNode:
    def __init__(self):
        self.data = 0
        self.left = None
        self.right = None
    def insert(self, data):
        if data < self.data:
            if self.left == None:
                tempNode = TreeNode()
                tempNode.data = data
                self.left = tempNode
```

```
        else:
            self.left.insert(data)
    elif data > self.data:
        if self.right == None:
            tempNode = TreeNode()
            tempNode.data = data
            self.right = tempNode
        else:
            self.right.insert(data)

    def traverseInOrder(self):
        if self is not None:
            if self.left is not None:
                self.left.traverseInOrder()
            print(self.data, end=' ')
            if self.right is not None:
                self.right.traverseInOrder()

    @staticmethod
    def createRoot():
        i = random.randint(0, 10)
        rootNode = TreeNode()
        rootNode.data = i
        return rootNode

    @classmethod
    def createTree(cls):
        rootNode = cls.createRoot()
        numNodes = random.randint(1, 10)
        currentNode = rootNode
        j = 0
        L = []

        while j < numNodes:
            newVal = random.randint(1, 20)
            if newVal not in L:
                currentNode.insert(newVal)
                L.append(newVal)
                j += 1
        rootNode.traverseInOrder()
        return rootNode

    @staticmethod
    def getSum(node):
        if node is None:
            return 0
```



```
        else:
            leftSum = TreeNode.getSum(node.left)
            rightSum = TreeNode.getSum(node.right)
            return node.data + leftSum + rightSum

rootNode = TreeNode.createTree()
print("\nSum =", TreeNode.getSum(rootNode))
```

### Output:

```
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/5.2.py"
1 3 4 5 6 8 10 13 15 19
Sum = 84
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/5.2.py"
2 5 6 9 10 11 12 13 15 19 20
Sum = 122
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/5.2.py"
2 3 4 5 14 15 16 19
Sum = 78
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/5.2.py"
1 2 3 5 7 10 12 17 19 20
Sum = 96
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> □
```

### Time Complexity:

- Time Complexity in Average Case:  $O(\log n)$
- Time Complexity in Worst Case:  $O(n)$

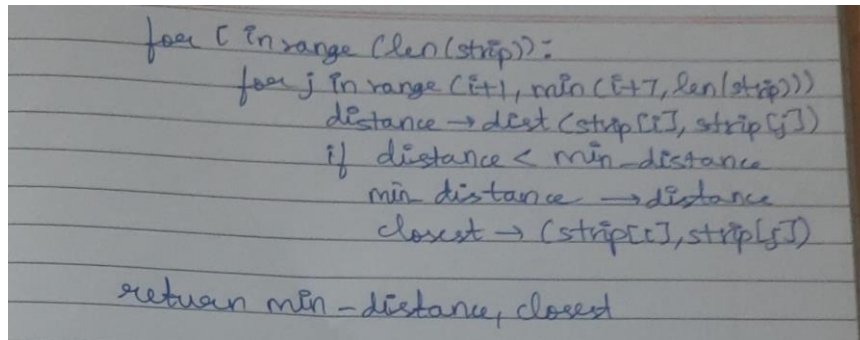
### Question 3:

Given a set of points in a 2D plane, find the pair of points with the smallest Euclidean distance between them, using divide-and-conquer strategy. For example. if the given set of points is  $\{(1, 2), (3, 5), (6, 9), (8, 12), (10, 15)\}$ , then the closest pair of points is (3, 5) and (6, 9).

**Algorithm:**

Algorithm:  
brute-force-closest(points):  
    min-dist  $\rightarrow$  99999  
    closest  $\rightarrow$  None  
    for i in range  $\rightarrow$  len(points)  
        for j in range (i+1, len(points))  
            distance  $\rightarrow$  dist (points[i],  
                                    points[j])  
            if distance < min-dist  
                min-dist  $\rightarrow$  distance  
                closest  $\rightarrow$  (points[i],  
                                points[j])  
    return min-dist, closest

closest-pair-dc(points):  
    if len(points)  $\leq$  3  
        return brute-force-closest(points)  
    sorted-points  $\rightarrow$  sorted(points, key=  
                                    lambda x: x[0])  
    mid  $\rightarrow$  len(points) // 2  
    left-min-dist, left-closest  
         $\rightarrow$  closest-pair-dc (points[:mid])  
    right-min-dist, right-closest  
         $\rightarrow$  closest-pair-dc  
            (sorted-points  
            [mid:])  
    min-dist = min(left-min-dist, right-min-dist)  
    closest  $\rightarrow$  left-closest if left-min-dist  
                == min-dist  
    else right-closest  
  
    mid-x  $\rightarrow$  sorted-points[mid][0]  
    strip  $\rightarrow$  [point for point in sorted-points  
                if abs(point[0] - mid-x)  
                        < min-dist]  
    strip-min-dist, strip-closest  
         $\rightarrow$  strip-closest-in-strip  
            (strip, min-dist)  
    if strip-min-dist < min-dist:  
        min-dist  $\rightarrow$  strip-min-dist  
        closest = strip-closest  
    return min-dist, closest  
  
strip-closest-in-strip(strip, min-dist):  
    min-distance  $\rightarrow$  min-dist  
    closest  $\rightarrow$  None



```
for i in range(len(strip)):  
    for j in range(i+1, min(i+7, len(strip))):  
        distance → dist(strip[i], strip[j])  
        if distance < min_distance:  
            min_distance → distance  
            closest → (strip[i], strip[j])  
  
return min_distance, closest
```

**Code:**

```
import math  
  
def dist(p1, p2):  
    return math.sqrt((p1[0] - p2[0])**2 + (p1[1] - p2[1])**2)  
  
def brute_force_closest(points):  
    min_dist = 99999  
    closest = None  
    for i in range(len(points)):  
        for j in range(i+1, len(points)):  
            distance = dist(points[i], points[j])  
            if distance < min_dist:  
                min_dist = distance  
                closest = (points[i], points[j])  
    return min_dist, closest  
  
def closest_pair_dc(points):  
    if len(points) <= 3:  
        return brute_force_closest(points)  
  
    sorted_points = sorted(points, key=lambda x: x[0])  
    mid = len(points) // 2  
    left_min_dist, left_closest = closest_pair_dc(sorted_points[:mid])  
    right_min_dist, right_closest = closest_pair_dc(sorted_points[mid:])  
  
    min_dist = min(left_min_dist, right_min_dist)  
    closest = left_closest if left_min_dist == min_dist else right_closest  
  
    mid_x = sorted_points[mid][0]  
    strip = [point for point in sorted_points if abs(point[0] - mid_x) < min_dist]  
  
    strip_min_dist, strip_closest = strip_closest_in_strip(strip, min_dist)  
    if strip_min_dist < min_dist:  
        min_dist = strip_min_dist  
        closest = strip_closest
```

```
    return min_dist, closest

def strip_closest_in_strip(strip, min_dist):
    min_distance = min_dist
    closest = None
    for i in range(len(strip)):
        for j in range(i+1, min(i+7, len(strip))):
            distance = dist(strip[i], strip[j])
            if distance < min_distance:
                min_distance = distance
                closest = (strip[i], strip[j])
    return min_distance, closest

def generate_random_points(n):
    points = []
    for i in range(n):
        x = random.randint(0, 100)
        y = random.randint(0, 100)
        points.append((x, y))
    return points

# Example usage:
num_points = 10
points = generate_random_points(num_points)
print("Randomly generated points:", points)

min_dist, closest_pair = closest_pair_dc(points)
print("Closest pair:", closest_pair, "with distance:", min_dist)
```

### Output:

```
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/5.3.py"
Randomly generated points: [(75, 14), (65, 54), (82, 68), (50, 86), (92, 89), (43, 63), (21, 72), (0, 76), (70, 56), (24, 76)]
Closest pair: ((21, 72), (24, 76)) with distance: 5.0
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/5.3.py"
Randomly generated points: [(84, 80), (52, 40), (28, 54), (55, 39), (2, 35), (96, 65), (65, 68), (90, 1), (33, 16), (70, 95)]
Closest pair: ((52, 40), (55, 39)) with distance: 3.1622776601683795
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/5.3.py"
Randomly generated points: [(25, 7), (92, 7), (88, 40), (77, 82), (80, 31), (62, 52), (97, 25), (50, 38), (16, 38), (66, 49)]
Closest pair: ((62, 52), (66, 49)) with distance: 5.0
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/5.3.py"
Randomly generated points: [(79, 0), (41, 22), (44, 89), (81, 52), (34, 11), (73, 63), (99, 73), (32, 67), (38, 64), (53, 66)]
Closest pair: ((32, 67), (38, 64)) with distance: 6.708203932499369
```

**Time Complexity:**

- Time Complexity of closed\_pair\_dc function:  $O(n \cdot \log n)$
- Time Complexity of brute\_force\_closest:  $O(n^2)$

**Learning Outcome:**

Upon completing this exercise, I have understood the applications of insertion sort and quick sort algorithms in finding the kth smallest element and also the principles of divide and conquer strategy in finding the Euclidean distance and the sum of a binary tree