

## UCS 2403 Design & Analysis of Algorithms

### Assignment 8

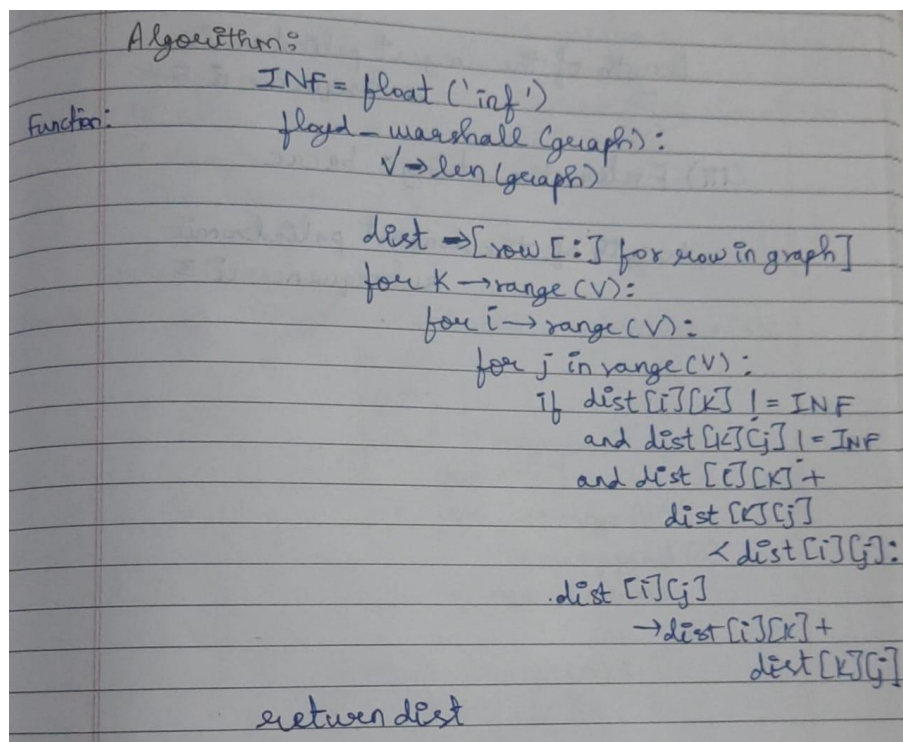
**Date of Exercise:** 02.05.2024

**Aim:** To gain understanding and proficiency on Dynamic Programming

#### **Question 1:**

Given the adjacency matrix representation of a simple weighted, directed graph  $G = (V, E)$ , write a Python program to implement the Floyd-Warshall algorithm.

#### **Algorithm:**



Handwritten algorithm for Floyd-Warshall:

```
Algorithm:  
Function:  
    INF = float('inf')  
    floyd_warshall(graph):  
        V = len(graph)  
  
        dist = [row[:] for row in graph]  
        for k in range(V):  
            for i in range(V):  
                for j in range(V):  
                    if dist[i][k] != INF  
                       and dist[k][j] != INF  
                       and dist[i][k] +  
                           dist[k][j]  
                           < dist[i][j]:  
                        dist[i][j]  
                        → dist[i][k] +  
                           dist[k][j]  
  
        return dist
```

#### **Code:**

```
INF = float('inf')  
  
def floyd_warshall(graph):  
    V = len(graph)  
  
    dist = [row[:] for row in graph]  
  
    # Floyd-Warshall algorithm  
    for k in range(V):  
        for i in range(V):
```

```
        for j in range(V):
            if dist[i][k] != INF and dist[k][j] != INF and dist[i][k] +
dist[k][j] < dist[i][j]:
                dist[i][j] = dist[i][k] + dist[k][j]

    return dist

V = int(input("Enter the number of vertices: "))
print("-----ADJACENCY MATRIX-----")
graph = []
for i in range(V):
    row = []
    for j in range(V):
        weight_input = input(f"Enter weight from vertex {i} to vertex {j} (enter
'INF' for infinity): ")
        if weight_input.upper() == 'INF':
            weight = INF
        else:
            weight = int(weight_input)
        row.append(weight)
    graph.append(row)

result = floyd_warshall(graph)

print("-----SHORTEST PATH MATRIX-----")
for i in range(V):
    for j in range(V):
        if result[i][j] == INF:
            print('INF', end='\t')
        else:
            print(result[i][j], end='\t')
    print()
```

### Output:

```
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/8.1.py"
Enter the number of vertices: 5
-----ADJACENCY MATRIX-----
Enter weight from vertex 0 to vertex 0 (enter 'INF' for infinity): 0
Enter weight from vertex 0 to vertex 1 (enter 'INF' for infinity): 4
Enter weight from vertex 0 to vertex 2 (enter 'INF' for infinity): INF
Enter weight from vertex 0 to vertex 3 (enter 'INF' for infinity): 5
Enter weight from vertex 0 to vertex 4 (enter 'INF' for infinity): INF
Enter weight from vertex 1 to vertex 0 (enter 'INF' for infinity): INF
Enter weight from vertex 1 to vertex 1 (enter 'INF' for infinity): 0
Enter weight from vertex 1 to vertex 2 (enter 'INF' for infinity): 1
Enter weight from vertex 1 to vertex 3 (enter 'INF' for infinity): INF
Enter weight from vertex 1 to vertex 4 (enter 'INF' for infinity): 6
Enter weight from vertex 2 to vertex 0 (enter 'INF' for infinity): 2
Enter weight from vertex 2 to vertex 1 (enter 'INF' for infinity): INF
Enter weight from vertex 2 to vertex 2 (enter 'INF' for infinity): 0
Enter weight from vertex 2 to vertex 3 (enter 'INF' for infinity): 3
Enter weight from vertex 2 to vertex 4 (enter 'INF' for infinity): INF
Enter weight from vertex 3 to vertex 0 (enter 'INF' for infinity): INF
Enter weight from vertex 3 to vertex 1 (enter 'INF' for infinity): INF
Enter weight from vertex 3 to vertex 2 (enter 'INF' for infinity): 1
Enter weight from vertex 3 to vertex 3 (enter 'INF' for infinity): 0
Enter weight from vertex 3 to vertex 4 (enter 'INF' for infinity): 2
Enter weight from vertex 4 to vertex 0 (enter 'INF' for infinity): 1
Enter weight from vertex 4 to vertex 1 (enter 'INF' for infinity): INF
Enter weight from vertex 4 to vertex 2 (enter 'INF' for infinity): INF
Enter weight from vertex 4 to vertex 3 (enter 'INF' for infinity): 4
Enter weight from vertex 4 to vertex 4 (enter 'INF' for infinity): 0
-----SHORTEST PATH MATRIX-----
0      4      5      5      7
3      0      1      4      6
2      6      0      3      5
3      7      1      0      2
1      5      5      4      0
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> █
```

### Time Complexity:

The Time Complexity of the Floyd Warshall algorithm is  $O(V^3)$ , where  $V$  is the number of vertices in the graph with  $E$  number of edges ( $G = (V, E)$ )

### Question 2:

You are given a string  $S$  consisting of lowercase letters. Find the length of the longest palindromic subsequence in the string. A palindromic sub-sequence is a subsequence of the string that is read the same forward and backward. Implement a dynamic programming algorithm to solve this problem efficiently.

Example: Input string: abacbcba

The solution is 5

**Algorithm:**

Algorithm:  
Func: longest\_palindromic\_subsequence(s):  
n = len(s)  
dp = []  
for i → range(n):  
row = []  
for j → range(n):  
row.append(0)  
dp.append(row)  
  
for i → range(n):  
dp[i][i] = 1  
for l → range(2, n+1):  
for i → range(n-l+1):  
j = i+l-1  
if s[i] == s[j]:  
if l == 2:  
dp[i][j] → 2  
  
else:  
dp[i][j] → dp[i+1][j-1] + 2  
else:  
dp[i][j] → max(dp[i+1][j], dp[i][j-1])  
return dp[0][n-1]

**Code:**

```
def longest_palindromic_subsequence(s):  
    n = len(s)  
    dp = []  
  
    for i in range(n):  
        row = []  
        for j in range(n):  
            row.append(0)  
        dp.append(row)  
  
    for i in range(n):  
        dp[i][i] = 1
```

```
for l in range(2, n + 1):
    for i in range(n - l + 1):
        j = i + l - 1
        if s[i] == s[j]:
            if l == 2:
                dp[i][j] = 2
            else:
                dp[i][j] = dp[i + 1][j - 1] + 2
        else:
            dp[i][j] = max(dp[i + 1][j], dp[i][j - 1])
return dp[0][n - 1]

string = str(input("Enter the string in lowercase letters: "))
print("Length of the longest palindromic subsequence:",
      longest_palindromic_subsequence(string))
```

### Output:

```
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/8.2.py"
Enter the string in lowercase letters: abacba
Length of the longest palindromic subsequence: 5
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/8.2.py"
Enter the string in lowercase letters: ababba
Length of the longest palindromic subsequence: 5
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/8.2.py"
Enter the string in lowercase letters: bacac
Length of the longest palindromic subsequence: 3
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\vscode> █
```

### Time Complexity:

The Time Complexity for the longest palindromic sequence problem is  $O(n^2)$

### Question 3:

*In computational linguistics and computer science, edit distance is a string metric, i.e. a way of quantifying how dissimilar two strings are to one another, that is measured by counting the minimum number of operations required to transform one string into the other (Source: Wikipedia).*

These operations include insert a character, remove a character or update a character. Develop and implement a bottom-up dynamic programming algorithm to compute the edit distance between two strings  $s_1$  and  $s_2$ .

Example:

Input:  $s_1$  = "intention",  $s_2$  = "execution"

Output: 5

**Explanation:**

intention -> inention (remove 't')

inention -> enention (replace 'i' with 'e')

enention -> exention (replace 'n' with 'x')

exention -> exection (replace 'n' with 'c')

exection -> execution (insert 'u')

**Algorithm:**

Algorithm:

Function: minimum(x, y, z):  
return min(x, min(y, z))

Function edit(X, Y, m, n):  
dp = [[0 for i in range(n+1)] for j in range(m+1)]  
for i in range(m+1):  
for j in range(n+1):  
if i == 0:  
dp[i][j] = j  
elif j == 0:  
dp[i][j] = i  
elif X[i-1] == Y[j-1]:  
dp[i][j] = dp[i-1][j-1]  
else:  
dp[i][j] = 1 + minimum(dp[i-1][j], dp[i][j-1], dp[i-1][j-1])  
return dp[m][n]

**Code:**

```
def minimum(x,y,z):  
    return min(x,min(y,z))  
  
def edit(X,Y,m,n):  
    dp=[[0 for i in range(n+1)] for j in range(m+1)]  
    for i in range(m+1):  
        for j in range(n+1):  
            if i == 0:  
                dp[i][j] = j  
            elif j == 0:  
                dp[i][j] = i  
            elif X[i-1] == Y[j-1]:  
                dp[i][j] = dp[i-1][j-1]  
            else:  
                dp[i][j] = 1 + minimum(dp[i-1][j], dp[i][j-1], dp[i-1][j-1])  
    return dp[m][n]
```



```
        elif j == 0:
            dp[i][j] = i
        elif X[i-1] == Y[j-1]:
            dp[i][j] = dp[i-1][j-1]
        else:
            dp[i][j] = 1 + minimum(dp[i-1][j], dp[i-1][j-1], dp[i][j-1])
    return dp[m][n]

X = input("Enter string 1: ")
Y = input("Enter string 2: ")
print("The number of operations to edit the strings is ",edit(X, Y, len(X),
len(Y)))
```

### Output:

```
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/8.3.py"
Enter string 1: intention
Enter string 2: exception
The number of operations to edit the strings is 4
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/8.3.py"
Enter string 1: intention
Enter string 2: execution
The number of operations to edit the strings is 5
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> & "C:/Users/Mugilkrishna D U/AppData/Local/Programs/Python/Python311/python.exe" "c:/Users/Mugilkrishna D U/OneDrive/Desktop/My Files/SSN/SEM4/DESIGN AND ANALYSIS OF ALGORITHMS/LAB/8.3.py"
Enter string 1: inception
Enter string 2: execution
The number of operations to edit the strings is 5
PS C:\Users\Mugilkrishna D U\OneDrive\Desktop\My Files\.vscode> |
```

### Time Complexity:

The Time Complexity for computing the edit distance between two strings is  $O(m*n)$ , where  $m$  and  $n$  are the lengths of the strings

### Learning Outcome:

Upon completing this exercise, I have understood the applications of Dynamic Programming and its various uses for solving problems in an effective manner. I have now learnt to implement the Floyd-Warshall algorithm. I have also understood how to find the longest palindromic subsequence length and how to compute the edit distance between two strings using Dynamic Programming