-The dataset that we are using is not good, we have to find another dataset

Eigen values

det = $\det(A - \lambda I) = 0$

our matrix :

$$A = \begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

So λ (identity matrix)

$$\cancel{A} \cdot I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So $\lambda \cdot I = \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$

then $\det(A - \lambda I) = 0$

$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{bmatrix} = \det(A - \lambda I)$$

Part 2

$$\det(A-\lambda I) = \begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

$$= 4-\lambda \begin{vmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{vmatrix} -$$

$$8 \begin{vmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{vmatrix} +$$

$$-1 \begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix} -$$

$$-2 \begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$$

1^{st} matrix : $A = \begin{bmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{bmatrix}$

$\det(A) = (aei + bfg + cdh - ceg - bdi - afh)$

Suppose : ~~$a = -9-\lambda$~~

$a = -9-\lambda$, $b = -2$, $c = -4$, $d = 10$

$e = 5-\lambda$, $f = -10$, $g = -13$, $h = -14$

$i = -13-\lambda$

So $aei = (-9-\lambda)(5-\lambda)(-13-\lambda)$

$\cancel{= (-9-\lambda)(5-\lambda)(-13-\lambda}$

$= (-9-\lambda)(-65+8\lambda+\lambda^2)$

$= (-9)(-65+8\lambda+\lambda^2) - \lambda(-65+8\lambda+\lambda^2)$

$= (585 - 72\lambda - 9\lambda^2) - (-65\lambda + 8\lambda^2 + \lambda^3)$

$= (585 - 72\lambda - 9\lambda^2 + 65\lambda - 8\lambda^2 - \lambda^3)$

$\boxed{= 585 - 7\lambda - 17\lambda^2 - \lambda^3}$

$bfg = (-2)(-10)(-13) = \boxed{-260}$

$cdh = (-4)(10)(-14) = (-4)(-140) = \boxed{560}$

$ceg = (-4)(5-\lambda)(-13)$

$= (5-\lambda)(-13) = -65 + 13\lambda$

$(-4)(-65 + 13\lambda) = \boxed{260 - 52\lambda}$

$bdi,\ (-2)(10)(-13-\lambda) = -20(-13-\lambda)$

$\boxed{= 260 + 20\lambda}$

$afh : = (-9-\lambda)(-10)(-14) = (-9-\lambda)(140)$

$\boxed{= -1260 - 140\lambda}$

So, $\det(A) = aei + bfg + cdh - ceg - bdi - afh$

$= (585 - 7\lambda - 17\lambda^2 - \lambda^3) + (-260) + 560 - (260 - 52\lambda)$

$- (260 + 20\lambda) - (-1260 - 140\lambda)$

$= 585 - 7\lambda - 17\lambda^2 - \lambda^3 - 260 + 560 - 260 +$

$52\lambda - 260 + 1260 - 20\lambda + 140\lambda$

$\boxed{= \det A = 1625 + 165\lambda - 17\lambda^2 - \lambda^3}$

first matrix

2nd matrix

$$A = \begin{bmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{bmatrix}$$

We use the same formula

$\det(A) = aei + bfg + cdh - ceg - bdi - af$

$aei = (-2)(5-\lambda)(-13-\lambda)$

$(5-\lambda)(-13-\lambda)$

$(5-\lambda)(-13-\lambda) = -65 - 5\lambda + 13\lambda + \lambda^2$

$= -65 + 8\lambda + \lambda^2$

$= (-2)(-65 + 8\lambda + \lambda^2)$

$\boxed{= 130 - 16\lambda - 2\lambda^2}$

$bfg: (-2)(-10)(-1) = \boxed{-20}$

$cdh = (-4)(0)(-14) = \boxed{0}$

$ceg = (-4)(5-\lambda)(-1)$

$(5-\lambda)(-1) = -5+\lambda$

$\boxed{(-4)(-5+\lambda) = 20 - 4\lambda}$

$bdi = (-2)(0)(-13-\lambda) = \boxed{0}$

$afh = (-2)(-10)(-14) = (-2)(140) = \boxed{-280}$

So,

$= (130 - 16\lambda - 2\lambda^2) + (-20) + (0) - (20 - 4\lambda)$

$- (0) - (-280)$

$= 130 - 16\lambda - 2\lambda^2 - 20 - (20 - 4\lambda) + 280$

$= (130 - 20 - 20 + 280) + (-16\lambda + 4\lambda) - 2\lambda^2$

$= (130 + 240) - 12\lambda - 2\lambda^2$

$\boxed{= 370 - 12\lambda - 2\lambda^2}$

$\boxed{\det(A) = 370 - 12\lambda - 2\lambda^2}$

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Third matrix

$$A = \begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \end{vmatrix}$$

Third matrix

$$A = \begin{vmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{vmatrix}$$

det A = $aei + bfg + cdh - ceg - bdi - afh$

$aei = (-2)(10)(-13-\lambda)$

$= -20(-13-\lambda)$

$= \underline{\underline{260 + 20\lambda}}$

$bfg : (-9-\lambda)\cdot(-10)(-1) = (-9-\lambda)(10)$

$= \underline{-90 - 10\lambda}$

$cdh = (-4)(0)(-13) = 0$

$ceg = (-4)(10)(-1) = 40$

$bdi = (-9-\lambda)(0)(-13-\lambda)$

$= 0$

~~ap~~ $afh = (-2)(-10)(-13) = -260$

So $= (260 + 20\lambda) + (-90 - 10\lambda) + 0 - 40 - 0 - (-260)$

$= (260 + 20\lambda) + (-90 - 10\lambda) - 40 + 260$

$\det A = 390 + 10\lambda$ (third matrix)

4^{th} matrix : $A = \begin{vmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{vmatrix}$

$\det(A) = aei + bfg + cdh - ceg - bdi - afh$

$aei = (-2)(10)(-14) = (-2)(-140) = 280$

$bfg = (-9-\lambda)(5-\lambda)(-1)$

So $(5-\lambda)(-1) = -5 + \lambda$

$(-9-\lambda)(-5+\lambda) = (-9)(-5) + (-9)\lambda +$

$(-\lambda)(-5) + (-\lambda)(\lambda)$

$= 45 - 9\lambda + 5\lambda - \lambda^2$

$bfg = 45 - 9\lambda + 5\lambda - \lambda^2$

$cdh = (-2)(0)(-13) = 0$

$ceg = (-2)(10)(-1) = 20$

$bdi = \cancel{(-9)}\ (-9-\lambda)(0)(-14) = 0$

$afh = (-2)(5-\lambda)(-13)$

$= (5-\lambda)(-13)$

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$= \underline{-65 + 13\lambda}$

$(-2)(-65 + 13\lambda) = 130 - 26\lambda$

$det(A) = 280 + (45 - 4\lambda - \lambda^2) + 0 - 20 - 130 + 26\lambda$

$det(A) = \underline{175 + 22\lambda - \lambda^2}$ (4^{th} matrix)

So, we are to find the solution

$= (4-\lambda)(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625) - 8(-2\lambda^2 - 12\lambda + 370) - 1(10\lambda + 390) + 2(\lambda^2 + 22\lambda + 175)$

$\Rightarrow \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$

$\Rightarrow \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$

So eigen Values are:

$\lambda_1 = 2.675$, $\lambda_2 = -5.604$, $\lambda_3 \approx 11.054$

$\lambda_4 = -21.125$

Eigen Values

We tried from many routes it did not work.
We will show it in last slides but those above worked!

for eigen vectors

for $\lambda = -21.125$

$$A - \lambda I = \begin{bmatrix} 4-(-21.125) & 8 & -1 & -2 \\ -2 & -9-(-21.125 & -2 & -4 \\ 0 & 10 & 5-(-21.125 & -10 \\ -1 & -13 & -4 & -13-(-21.125) \end{bmatrix}$$

$$= \begin{bmatrix} 25.125 & 8 & -1 & -2 \\ -2 & 12.125 & -2 & -4 \\ 0 & 10 & 26.125 & -10 \\ -1 & -13 & -14 & 8.125 \end{bmatrix}$$

Using homogeneous system

Let $x = [x_1, x_2, x_3, x_4]^T$;

$$\begin{cases} 25.125x_1 + 8x_2 - x_3 - 2x_4 = 0 \\ -2x_1 + 12.125x_2 - 2x_3 - 4x_4 = 0 \\ 10x_2 + 26.125x_3 - 10x_4 = 0 \\ -1x_1 - 13x_2 - 14x_3 + 8.125x_4 = 0 \end{cases}$$

Let $x_4 = 1$ from 3^{rd} equation

$$10x_2 + 26.125x_3 - 10 = 0$$

$$\Rightarrow 10x_2 + 26.125x_3 = 10$$

$$x_2 = \frac{10 - 26.125x_3}{10}$$

Put it in eq 1,

$$25.125x_1 + 8x_2 - x_3 - 2 = 0$$

$$25.125x_1 + 8\left(\frac{10 - 26.125x_3}{10}\right) - x_3 - 2 = 0$$

$$25.125x_1 + 8 - 20.9x_3 - x_3 - 2 = 0$$

$$25.125x_1 + 80/10 - (8 \times 26.125)x_3/10 - x_3 - 2 = 0$$

$$25.125x_1 + 8 - 20.9x_3 - x_3 - 2 = 0$$

$$25.125x_1 + 6 - 21.9x_3 = 0$$

$$25.125x_1 = 21.9x_3 - 6$$

$$\boxed{x_1 = \frac{21.9x_3 - 6}{25.125}}$$

from eq 2:

$$-2x_1 + 12.125x_2 - 2x_3 - 4 = 0$$

$$-2x_1 + 12.125x_2 - 2x_3 = 4$$

Remember, $x_2 = \dfrac{10 - 26.125x_3}{10}$, $x_3 = x_3$

$$x_1 = \frac{21.9x_3 - 6}{25.125}, \quad x_4 = 1$$

$$-2\left(\frac{21.9x_3 - 6}{25.125}\right) + 12.125\left(\frac{10 - 26.125x_3}{10}\right) - 2x_3 = 4$$

$$-2 \times \frac{21.9x_3 - 6}{25.125} = \frac{-43.8x_3 + 12}{25.125}$$

$$1\cancel{2x}\ 12.125 \cdot \frac{10 - 26.125x_3}{10} = 12.125 - 31.688x_3$$

$$-2x_3$$

$$\frac{-43.8x_3 + 12}{25.125} + 12.125 - 31.688x_3 - 2x_3 = 4$$

$$-43.8x_3 + 12 + 25.125\,(12.125 - 31.688x_3 - 2x_3)$$

$$= 4 \times 25.125$$

$$\Rightarrow -43.8x_3 + 12 + 304.781 - 795.618x_3 - 50.25x_3 = 100.$$

$$(12 + 304.781) + (-43.8 - 795.618 - 50.25)\,x_3 = 100.$$

$$316.781 - 889.668\,x_3 = 100.5$$

$$-889.668\,x_3 = 100.5 - 316.781$$

$$= -216.281$$

$$x_3 = \frac{-216.281}{-889.668}$$

$$x_3 \simeq 0.243$$

$$x_2 = \frac{10 - 26.125 \times 0.243}{10} = \frac{10 - 6.353}{10}$$

$$x_2 = \frac{3.647}{10} \simeq \boxed{0.365 = x_2}$$

$$x_1 = \frac{21.9 \times 0.243 - 6}{25.125} = \frac{5.322 - 6}{25.125}$$

$$\boxed{X_1 \simeq -0.027}$$

Remember $x_4 = 1$

$$X = \begin{bmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{bmatrix} \quad \text{for } \lambda_1 = -21.125$$

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for $\lambda_2 = -5.604$

$$A - \lambda I = \begin{bmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{bmatrix}$$

let $x_4 = 1$

$$\begin{cases} 9.604x_1 & 8x_2 & -x_3 & -2 = 0 \\ -2x_1 & -3.396x_2 & -2x_3 & -4 = 0 \\ 10x_2 & 10.604x_3 & -10 & = 0 \\ -1x_1 & -13x_2 & -14x_3 & -7.396 = 0 \end{cases}$$

$$\begin{cases} 9.604x_1 + 8x_2 - x_3 = 2 \\ -2x_1 - 3.396x_2 - 2x_3 = 4 \\ 10x_2 + 10.604x_3 = 10 \\ -1x_1 - 13x_2 - 14x_3 = 7.396 \end{cases}$$

$10x_2 + 10.604x_3 = 10$

$\Rightarrow x_2 = \dfrac{10 - 10.604x_3}{10}$

x_2 into eq 1: $9.604x + 8x_2 - x_3 = 2$

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$$9.604x_1 + 8\left(\frac{10-10.604x_3}{10}\right) - x_3 = 2$$

$$9.604x_1 + \frac{80-84.832x_3}{10} - x_3 = 2$$

$$9.604x_1 + 8 - 8.4832x_3 - x_3 = 2$$

$$9.604x_1 + 8 - 9.483x_2 = 2$$

$$x_1 = \frac{9.483x_3 - 6}{9.604}$$

$$-2x_1 - 3.396x_2 - 2x_3 = 4$$

$$x_1 = \frac{9.483x_3 - 6}{9.604}$$

$$x_2 = \frac{10-10.604x_3}{10}$$

$$-2\left(\frac{9.483x_3-6}{9.604}\right) - 3.396\left(\frac{10-10.604x_3}{10}\right)$$

$$-2x_3 = 4$$

$$\cancel{-2x} \quad -2 \times \frac{9.483x_3 - 6}{9.604} = \frac{-18.966x_3 + 12}{9.604}$$

$$-3.396 \times \frac{10-10.604x_3}{10} = -3.396 + 3.599x$$

So, $$\frac{-18.966x_3 + 12}{9.604} - 3.396 + 3.599x_3 - 2x_3$$

$$= 4$$

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$$-18.966 x_3 + 12 + 9.604(-3.396 + 3.599 x_3 - 2x_3) = 38.4$$

$$-18.966 x_3 + 12 - 32.626 + 34.562 x_3 - 19.208 x_3 = 38.416$$

$$-20.626 + (-18.966 + 34.362 - 19.208) x_3 = 38.416$$

$$-20.626 - 3.612 x_2 = 38.416$$

$$-3.612 x_3 = 38.416 + 20.626 = 59.042$$

$$X_3 = \frac{59.042}{-3.612}$$

$$X_3 \simeq -16.35$$

$$X_2 = \frac{10 - 10.604 x_3}{10} = \frac{10 - 10.604(-16.35)}{10}$$

$$= \frac{10 + 173.396}{10} = \frac{183.396}{10}$$

$$X_2 \simeq 18.34 \; ; \; X_1 = \frac{9.483 - 6}{9.604} = \frac{9.483(-16.35) - 6}{9.604}$$

$$= \frac{-154.96 - 6}{9.604} = \frac{-160.96}{9.604} \simeq -16.76$$

$$X_2 = \begin{bmatrix} -16.76 \\ 18.34 \\ -16.35 \end{bmatrix} \text{ for } \lambda_2 = -5.604$$

$$-20.626 + (-18.966 + 34.562 - 19.208)x_3 = 38.416$$

$$-20.626 - 3.612x_3 = 38.416$$

$$-3.612x_3 = 38.416 + 20.626 = 59.042$$

$$x_3 = \frac{59.042}{-3.612}$$

$$x_3 \simeq -16.35$$

$$x_2 = \frac{10 - 10.604x_3}{10} = \frac{10 - 10.604 \times (-16.35)}{10}$$

$$= \frac{10 + 173.396}{10} = \frac{183.396}{10}$$

$$x_2 \simeq 18.34$$

$$x_1 = \frac{9.483 - 6}{9.604} = \frac{9.483 \times (-16.35) - 6}{9.604}$$

$$= \frac{-154.96 - 6}{9.604} = \frac{-160.96}{9.604} \simeq -16.76$$

$$\boxed{x_2 = \begin{bmatrix} -16.76 \\ 18.34 \\ -16.35 \end{bmatrix} \text{ for } \lambda_2 = -5.604}$$

for $x_3 = 2,675$

$$A - \lambda I = \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{bmatrix}$$

let $x_4 = 1$

$$\begin{cases} 1.325x_1 + 8x_2 - x_3 - 2 = 0 \\ -2x_1 - 11.675x_2 - 2x_3 - 4 = 0 \\ 10x_2 + 2.325_3 - 10 = 0 \\ -1x_1 - 13x_2 - 14x_3 - 15.675 = 0 \end{cases}$$

$$\begin{cases} 1.325x + 8x_2 - x_3 = 2 \\ -2x_1 - 11.675x_2 - 2x_3 = 4 \\ 10x_2 + 2.325x_3 = 10 \\ -1x_1 - 13x_2 - 14x_3 = 15.675 \end{cases} \Longrightarrow \boxed{x_2 = \frac{10 - 2.325x_3}{10}}$$

x_2 into eq1: $1.325x_1 + 8x_2 - x_3 = 2$

$$1.325x_1 + 8\left(\frac{10 - 2.325x_3}{10}\right) - x_3 = 2$$

$$1.325x_1 + 8 - 1.86x_3 - x_3 = 2$$

$$1.325x_1 + 8 - 2.86x_3 = 2$$

$$\frac{1.325x_1}{1.325} = \frac{2.86x_3 - 6}{1.325}$$

$$\boxed{x_1 = \frac{2.86x_3 - 6}{1.325}}$$

into equ 2: $-2x_1 - 11.675x_2 - 2x_3 = 4$

$$-2\left(\frac{2.86x_3 - 6}{1.325}\right) - 11.675\left(\frac{10 - 2.325x_3}{10}\right) - 2x_3 = 4$$

$$-2x \cdot \frac{286x_3 - 6}{1.325} = \frac{-5.72x_3 + 12}{1.325}$$

$$-11.675 \cdot \frac{10 - 2.325x_3}{10} = -11.675 + 2.715x$$

$$\frac{-5.72x_3+12}{1.325} - 11.675 + 2.715x_3 - 2x_3 = 4$$

$$-5.72x_3 + 12 + 1.325(-11.675 + 2.715x_3 - 2x_3) = 5.3$$

$$-5.72x_3 + 12 - 15.465 + 3.598x_3 - 2.65x_3 = 5.3$$

$$(12 - 15.465) + (-5.72x_3 + 3.598x_3 - 2.65x_3) = 5.3$$

$$-3.465 + (-5.72 + 3.598 - 2.65)x_3 = 5.3$$

$$-3.465 - 4.772x_3 = 5.3$$

$$-4.772x_3 = 5.3 + 3.465 = 8.765$$

$$x_3 = \frac{8.765}{-4.772} \simeq -1.837$$

$$x_2 = \frac{10 - 2.325x_3}{10} = \frac{10 - 2.325 \times (-1.832)}{10} = \frac{14.774}{10} \simeq \boxed{1.427}$$

$$x_1 = \frac{2.86x_3 - 6}{1.325} = \frac{2.86 \times (-1.832) - 6}{1.325} = \frac{-5.25 - 6}{1.325}$$

$$x_1 = \frac{-11.25}{1.325} \simeq \boxed{-8.491}$$

$$\cancel{\Rightarrow x_3 = \begin{bmatrix} -8.491 \\ 1.427 \\ -1.83 \end{bmatrix}}$$

for $x_3 = \begin{bmatrix} -8.491 \\ 1.427 \\ -1.837 \\ 1 \end{bmatrix}$

for $\lambda_3 = 2.675$

for $\lambda_4 = 11.054$

$$A - \lambda I = \begin{bmatrix} -7.054 & 8 & -1 & -2 \\ -2 & -20.054 & -2 & -4 \\ 0 & 10 & -6.054 & -10 \\ -1 & -13 & -14 & -24.054 \end{bmatrix}$$

let $x_4 = 1$

$$\begin{cases} -7.054x_1 + 8x_2 - x_3 - 2 = 0 \\ -2x_1 - 20.054x_2 - 2x_3 - 4 = 0 \\ 10x_2 - 6.054x_3 - 10 = 0 \\ -1x_1 - 13x_2 - 14x_3 - 24.054 = 0 \end{cases}$$

$$\begin{cases} -7.054x_1 + 8x_2 - x_3 = 2 \\ -2x_1 - 20.054x_2 - 2x_3 = 4 \\ 10x_2 - 6.054x_3 = 10 \\ -1x_1 - 13x_2 - 14x_3 = 24.054 \end{cases}$$

eq 3 : $10x_2 - 6.054x_3 = 10 \Rightarrow x_2 = \dfrac{10 + 6.054x_3}{10}$

x_2 into eq 1 : $-7.054x_1 + 8x_2 - x_3 = 2$

$$-7.054x_1 + 8\left(\frac{10 + 6.054x_3}{10}\right) - x_3 = 2$$

$$-7.054x_1 + 8 + 4.843x_3 - x_3 = 2$$

$$-7.054x_1 + 8 + 3.843x_3 = 2$$

$$-7.054x_1 = 2 - 8 - 3.843x_3 =$$

$$= -6 - 3.843x_3$$

$$x_1 = \frac{6 + 3.843x_3}{7.054}$$

into eq 2 : $-2x_1 - 20.054x_2 - 2x_3 = 4$

$$-2\left(\frac{6 + 3.843x_3}{7.054}\right) - 20.054\left(\frac{10 + 6.054x_3}{10}\right) - 2x_3 = 4$$

$$-2 \times \frac{6 + 3.843x_3}{7.054} = \frac{-12 - 7686x_3}{7.054}$$

So, $\dfrac{-12 - 7.686x_3}{7.054} - 20.054 - 12.138x_3 - 2x_3 = 4$

$$-12 - 7.686x_3 + 7.054(-20.054 - 12.138x_3 - 2x_3) = 28.216$$

$$-12 - 7.686x_3 - 141.562 - 85.667x_3 - 14.108x_3 = 28.216$$

$$(-12 - 141.562) + (-7.686 - 85.669 - 14.108)x_3 = 28.216$$

$$-153.562 - 107.463x_3 = 28.216$$

$$-107.463x_3 = 28.216 + 153.562 = 181.778$$

$$x_3 = \frac{181.778}{-107.463}$$

$$\boxed{x_3 \simeq -1.692}$$

$$x_2 = \frac{10 + 6.054x_3}{10} = \frac{10 + 6.054 \times (-1.692)}{10}$$

$$= \frac{10 - 10.242}{10} = \frac{-0.242}{10} \simeq \boxed{-0.024}$$

$$x_1 = \frac{6 + 3.843 x_3}{7.054} = \frac{6 + 3.843 \times (-1.692)}{7.054}$$

$$= \frac{6 - 6.504}{7.054} = \frac{-0.504}{7.054}$$

$$x_1 \simeq -0.071$$

$$\boxed{X_4 = \begin{bmatrix} -0.071 \\ -0.024 \\ -1.692 \\ 1 \end{bmatrix} \text{ fo } \lambda_4 = 11.054}$$

So all eigen vectors

for $\lambda_1 \simeq -21.125$, $x = \begin{bmatrix} -0.027 \\ 0.365 \\ 0.243 \\ 1 \end{bmatrix}$

for $\lambda_2 \simeq -5.604$, $x = \begin{bmatrix} -16.864 \\ 18.439 \\ -16.446 \\ 1 \end{bmatrix}$

for $\lambda_3 \simeq 2.675$, $x = \begin{bmatrix} -8.494 \\ 1.428 \\ -1.838 \\ 1 \end{bmatrix}$

for $\lambda_4 \simeq 11.054$, $x = \begin{bmatrix} -0.671 \\ -0.024 \\ -1.691 \\ 1 \end{bmatrix}$